Description of Chord Progressions by Minimal Transport Graphs Using the System & Contrast Model

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ICMC 2016
Outline of the work

1. Introduction to the S&C model
2. Computational approach
   - Chords relation description
   - S&C Description estimation
3. Experiments
   - Algorithm design
   - Evaluation using perplexity measure
4. Conclusion
Understand musical structure

Various points of view:

- Structure music as a **succession of events**? (for instance Tymoczko 2008)
- Structure music using **formal language theory**? (for instance De Haas 2011)
- Structure music considering the simplest explanation using **minimum description length** principle? (for instance Meredith 2013)

Our view? ⇒ Music structure also depends on an **expectation process**. (Narmour 2000)
Let's guess

1 2 3 ?

Relations ⇒ Expectation
System as an extension of progressions

The expectation is induced by the system formed by the first three elements.
System as an extension of progressions

The expectation is induced by the system formed by the first three elements.

Progressions also form square systems.
The contrast

Modeling the difference between the expectation and the observation.

Denial $\Rightarrow$ Contrast (creates some closure to the system)
Recent formalization and exploration of the model

- The S&C model has been formalized to analyse structures from 3 to 6 motifs. (Bimbot ISMIR 2012)
- It generalizes the Narmour’s Implication Realization model. (Bimbot Music Perception 2016)
- It is found in music from different types. (Bimbot ISMIR 2012, Deruty Master Thesis 2013)
A very “popular” music example

Have you ever analyzed the Macarena?

Introduction to the S&C model

Computational approach

Experiments

Conclusion
Multi-scale principle

The first element of the chorus of the Macarena can also be modelized as an S&C.

The S&C model works at multiple scales.
Multi-scale description

Macarena Chorus

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Multi-scale description

Just another way of organizing information.
From theory to application

Two more steps to go

1. A need to explicitly describe the relations between elements.
2. A need to design and evaluate an algorithmic implementation of the model.
Introduction to the S&C model

Computational approach

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Formalisation of the S&C model

Notations

The System and Contrast model in its square form defines the organisation of a sequence of four elements as:

\[ X = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix} \]

with

- \( x_0 \) the primer of the system
- \( x_1 = f(x_0) \) and \( x_2 = g(x_0) \)
- \( x_3 = \gamma(\hat{x}_3) \)

where \( \hat{x}_3 = f(g(x_0)) \) is the virtual element

Objective: modeling \( f, g, \) and \( \gamma \).
Describing relations between chords

The relations between chords are described in terms of minimal transport.

**Definition**

A *transport* between two chords $P = (p_i)_{0 \leq i \leq m_p}$ and $Q = (q_j)_{0 \leq j \leq m_q}$ is a set:

$$T = \{(p_k, q_k) \mid p, q \in [0; 11], k \in [0; n]\}$$

The cost of a transport is determined by the taxicab norm ($L_1$).
Minimal transport is used to describe relations in the S&C representation.

Example: $Cm - Fm - G - Cm$ chord progression

\[
\begin{align*}
Cm &= \begin{array}{c}
\text{\ding{73}}
\end{array} \\
Fm &= \begin{array}{c}
\text{\ding{73}}
\end{array} \\
G &= \begin{array}{c}
\text{\ding{73}}
\end{array} \\
\text{?} &= \text{?} \\
Cm &= \begin{array}{c}
\text{\ding{73}}
\end{array}
\end{align*}
\]
Minimal transport is used to describe relations in the S&C representation.

Example: $C_m \rightarrow F_m \rightarrow G \rightarrow C_m$ chord progression
Minimal transport is used to describe relations in the S&C representation.

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Example: $Cm \rightarrow Fm \rightarrow G \rightarrow Cm$ chord progression
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.

Cm Cm Cm Bb Ab Ab Ab G F F F Cm Cm Bb Bb
Biscale description for chord sequences

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\[ C_m \xrightarrow{f} C_m \]
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Optimization of an S&C: finding the set of transport \( \{f, g, \gamma\} \) that minimizes the global transport cost S&C.

Optimization in two steps:

1. optimization of the S&C U
2. optimization of the four sub-S&C.
Multiple tensorial configurations

30 possible choices of combination of squares corresponding to the four sub-S&Cs in the tensor.

A choice of four sub-S&C is a permutation of the initial sequence.
Using permutations, the four sub-S&C and the upper-scale S&C are different.

equivalent to
Experiments

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Evaluating with perplexity

What is Perplexity?

Model prediction ability

\[ PP_x = 2^{-\log(P_{model}(data))} = 2^{\text{Negative Log Likelihood (i.e. NLL)}} \]

\[ model_1 > model_2 \iff PP_{x_1} < PP_{x_2} \]

Calculating \( P(Y|X) \)

\[
\log P(Y|X) = \frac{1}{K} \sum_{i=1}^{K} \log p(y_{\tau(i)}|x_i)
\]
Perplexity calculation process

Sequence: $X_0 \ X_1 \ldots \ X_n$

**Bigram model**

$$NLL_{bigram} = -\frac{1}{n} \left[ \log p(X_0) + \log p(X_1|X_0) + \cdots + \log p(X_n|X_{n-1}) \right]$$

**Tensorial model**

1. Estimate configuration $\phi$ that minimizes global minimal transport.
2. Compute perplexity given $\phi$

$$NLL_\phi = -\frac{1}{n} \left[ \log p(X_0) + \log p(X_1|X_{\phi(1)}) + \cdots + \log p(X_n|X_{\phi(n)}) \right]$$

Both are first order models.
Experiment protocol

Data

- A corpus of 45 sections of recent pop songs represented as sequences of 16 chords.
- The probabilities are estimated using leave-one-out cross-validation strategy.

Results

<table>
<thead>
<tr>
<th></th>
<th>Perplexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bigram</td>
<td>3.84</td>
</tr>
<tr>
<td>Best piece-specific permutation</td>
<td>2.73</td>
</tr>
<tr>
<td>Best global permutation</td>
<td>3.17</td>
</tr>
</tbody>
</table>

Additional experiments show the benefit from the virtual element. (see paper)
These are only preliminary results which indicate the potential following trends:

**Outcomes**

- In our experiments the S&C model outperforms the sequential model on chord sequence prediction.
- Using minimal transport with S&C model appears as an effective way to model chord progression.
Future work

- Improve optimization complexity using crossing free property (Tymokzco 2006)
- Constraint transports to comply with musicological rules
- Investigate the effectiveness of S&C model to other musical dimensions (melody, rhythm)
Questions ?