Description of Chord Progressions by Minimal Transport Graphs Using the System & Contrast Model

LOUBOUTIN Corentin, BIMBOT Frédéric

ICMC 2016
Outline of the work

1. Introduction to the S&C model
2. Computational approach
   - Chords relation description
   - S&C Description estimation
3. Experiments
   - Algorithm design
   - Evaluation using perplexity measure
4. Conclusion
Understand musical structure

Various points of view:

- Structure music as a **succession of events**?  
  (for instance Tymoczko 2008)
- Structure music using **formal language theory**?  
  (for instance De Haas 2011)
- Structure music considering the simplest explanation using **minimum description length** principle?  
  (for instance Meredith 2013)

Our view? ⇒ Music structure also depends on an **expectation process**. (Narmour 2000)
Let’s guess

1  2  3  ?

Relations ⇒ Expectation
The expectation is induced by the system formed by the first three elements.
System as an extension of progressions

The expectation is induced by the *system* formed by the first three elements.

Progressions also form square systems.
The contrast

Modeling the difference between the expectation and the observation.

Denial $\Rightarrow$ Contrast (creates some closure to the system)
S&C in music

Recent formalization and exploration of the model

- The S&C model has been formalized to analyse structures from 3 to 6 motifs. (Bimbot ISMIR 2012)
- It generalizes the Narmour’s Implication Realization model. (Bimbot Music Perception 2016)
- It is found in music from different types. (Bimbot ISMIR 2012, Deruty Master Thesis 2013)
A very “popular” music example

Have you ever analyzed the Macarena?

System

Contrast

Hey, Ma ca re na!

... Macarena

... cosa buena

... Macarena

... Macarena
Multi-scale principle

The first element of the chorus of the Macarena can also be modelized as an S&C.

The S&C model works at multiple scales.
Multi-scale description

Macarena Chorus

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Multi-scale description

Just another way of organizing information.
From theory to application

Two more steps to go

1. A need to explicitly describe the relations between elements.
2. A need to design and evaluate an algorithmic implementation of the model.
Computational approach

1. Introduction to the S&C model

2. Computational approach
   - Chords relation description
   - S&C Description estimation

3. Experiments
   - Algorithm design
   - Evaluation using perplexity measure

4. Conclusion
Formalisation of the S&C model

Notations

The System and Contrast model in its square form defines the organisation of a sequence of four elements as:

\[ X = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix} \]

with

- \( x_0 \) the primer of the system
- \( x_1 = f(x_0) \) and \( x_2 = g(x_0) \)
- \( x_3 = \gamma(\hat{x}_3) \)

where \( \hat{x}_3 = f(g(x_0)) \) is the virtual element

Objective: modeling \( f \), \( g \), and \( \gamma \).
Describing relations between chords

The relations between chords are described in terms of minimal transport.

**Definition**

A *transport* between two chords \( P = (p_i)_{0 \leq i \leq m_p} \) and \( Q = (q_j)_{0 \leq j \leq m_q} \) is a set:

\[
T = \{(p_k, q_k) | p, q \in [0; 11], k \in [0; n]\}
\]

The cost of a transport is determined by the **taxicab norm** \((L_1)\).
Minimal transport is used to describe relations in the S&C representation.

Example: $Cm - Fm - G - Cm$ chord progression
Minimal transport is used to describe relations in the S&C representation.

Example: $Cm – Fm – G – Cm$ chord progression

![Diagram of chord progression](image)
Minimal transport is used to describe relations in the S&C representation.

Example: $Cm - Fm - G - Cm$ chord progression
Minimal transport is used to describe relations in the S&C representation.

Example: $Cm \rightarrow Fm \rightarrow G \rightarrow Cm$ chord progression
Minimal transport is used to describe relations in the S&C representation.

Example: $Cm \rightarrow Fm \rightarrow G \rightarrow Cm$ chord progression
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.

Cm Cm Cm Bb Ab Ab Ab G F F F Cm Cm Bb Bb
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.
Biscale description for chord sequences

A sequence of 16 chords can be described as 4 square S&Cs linked by an upper-scale S&C, resulting in a tensor structure.
Nested S&C description estimation

Optimization of an S&C: finding the set of transport \( \{f, g, \gamma\} \) that minimizes the global transport cost S&C.

Optimization in two steps:

1. optimization of the S&C U
2. optimization of the four sub-S&C.
Multiple tensorial configurations

30 possible choices of combination of squares corresponding to the four sub-S&Cs in the tensor.

A choice of four sub-S&C is a permutation of the initial sequence.
Tensorial permutations

Using permutations, the four sub-S&C and the upper-scale S&C are different.
Experiments

1. Introduction to the S&C model

2. Computational approach
   - Chords relation description
   - S&C Description estimation

3. Experiments
   - Algorithm design
   - Evaluation using perplexity measure

4. Conclusion
Experiments

1. Introduction to the S&C model

2. Computational approach
   - Chords relation description
   - S&C Description estimation

3. Experiments
   - Algorithm design
   - Evaluation using perplexity measure

4. Conclusion
Evaluating with perplexity

What is Perplexity?

Model prediction ability

\[ PP_x = 2^{-\log(P_{\text{model}}(\text{data}))} = 2^{-\text{Negative Log Likelihood (i.e. NLL)}} \]

\[ \text{model}_1 > \text{model}_2 \iff PP_{x1} < PP_{x2} \]

Calculating \( P(Y|X) \)

\[ \log P(Y|X) = \frac{1}{K} \sum_{i=1}^{K} \log p(y_{\tau(i)}|x_i) \]
Perplexity calculation process

Sequence: \( X_0 \ X_1 \ \ldots \ \ X_n \)

**Bigram model**

\[
NLL_{\text{bigram}} = -\frac{1}{n} [\log p(X_0) + \log p(X_1 | X_0) + \cdots + \log p(X_n | X_{n-1})]
\]

**Tensorial model**

1. Estimate configuration \( \phi \) that minimizes global minimal transport.
2. Compute perplexity given \( \phi \)

\[
NLL_{\phi} = -\frac{1}{n} [\log p(X_0) + \log p(X_1 | X_{\phi(1)}) + \cdots + \log p(X_n | X_{\phi(n)})]
\]

Both are first order models.
Experiment protocol

Data

- A corpus of 45 sections of recent pop songs represented as sequences of 16 chords.
- The probabilities are estimated using leave-one-out cross-validation strategy.

Results

<table>
<thead>
<tr>
<th></th>
<th>Perplexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bigram</td>
<td>3.84</td>
</tr>
<tr>
<td>Best piece-specific permutation</td>
<td>2.73</td>
</tr>
<tr>
<td>Best global permutation</td>
<td>3.17</td>
</tr>
</tbody>
</table>

Additional experiments show the benefit from the virtual element. (see paper)
Conclusions

These are only preliminary results which indicate the potential following trends:

**Outcomes**

- In our experiments the S&C model outperforms the sequential model on chord sequence prediction.
- Using minimal transport with S&C model appears as an effective way to model chord progression.
Perspectives

Future work

- Improve optimization complexity using crossing free property (Tymokzco 2006)
- Constraint transports to comply with musicological rules
- Investigate the effectiveness of S&C model to other musical dimensions (melody, rhythm)
Thank you

Questions ?