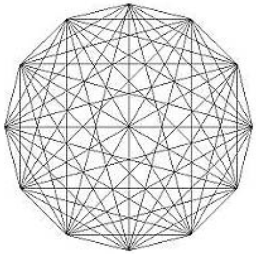
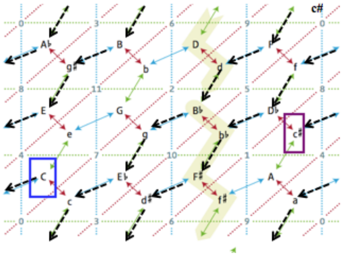


Journées LAC – GeoCal



Laboratoire d'Informatique de l'Ecole Polytechnique

24-25 novembre 2011

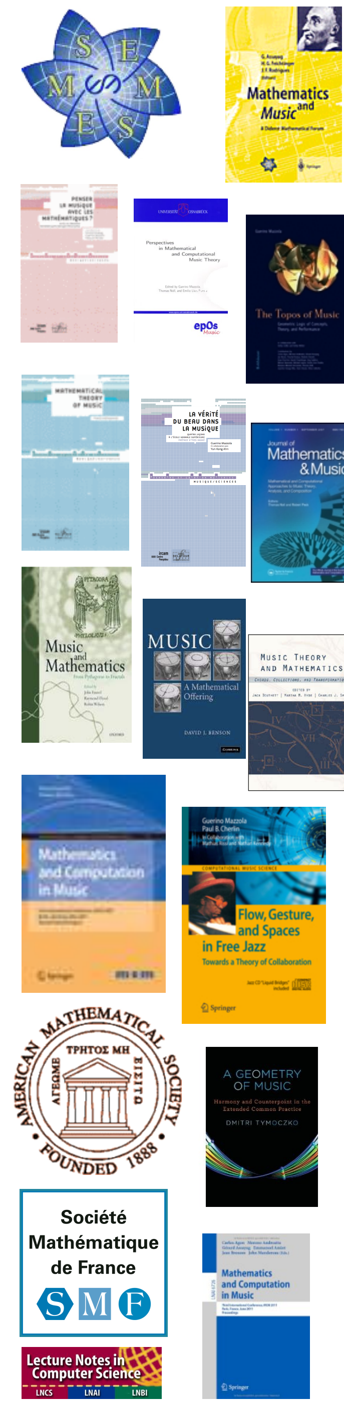
Outils algébriques et représentations géométriques en musicologie computationnelle :

le paradigme de la programmation spatiale et ses applications en informatique musicale

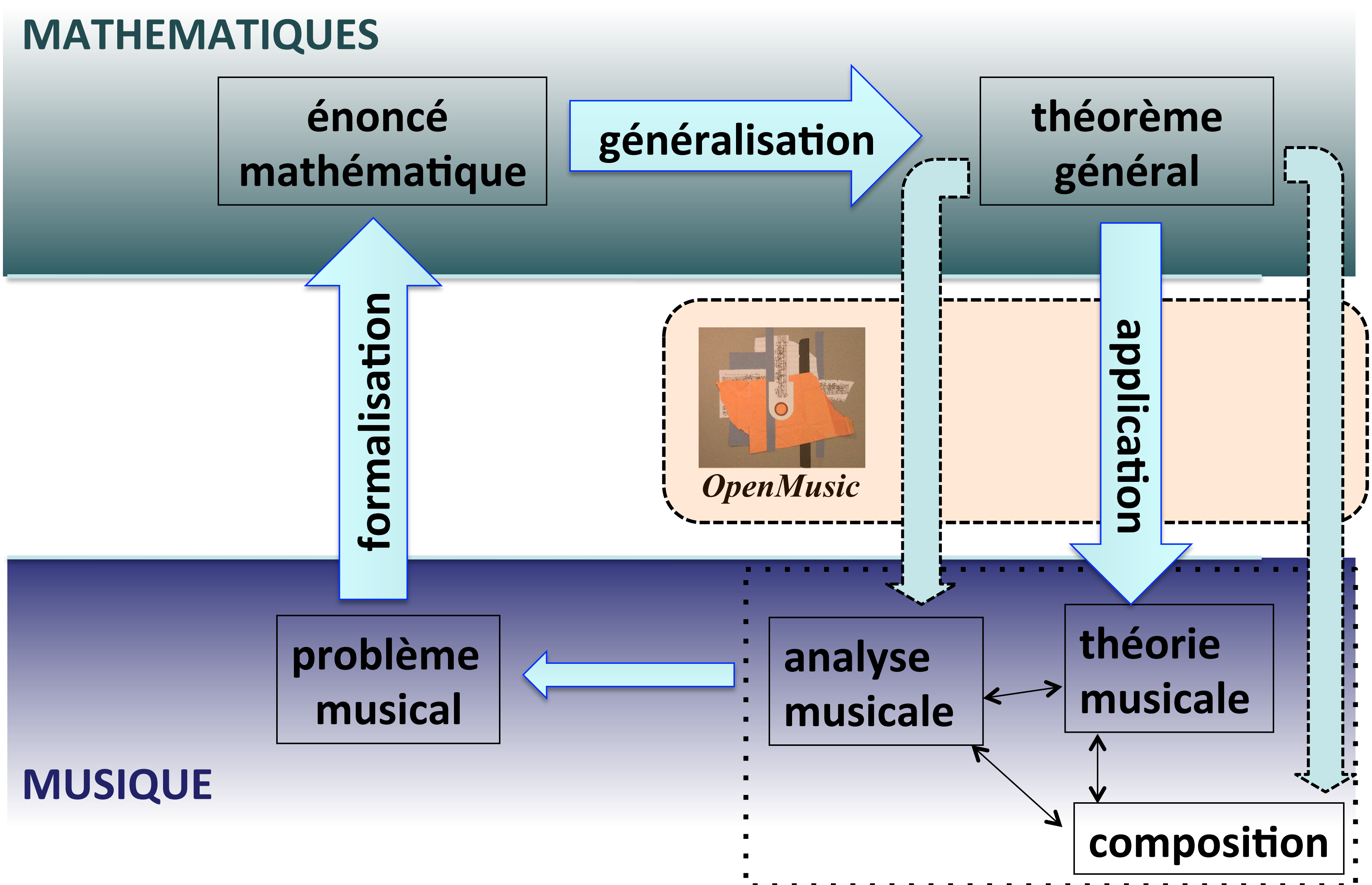
C. Agon, M. Andreatta, L. Bigo, J.-L. Giavitto, O. Michel, A. Spicher
Equipe Représentations Musicales (IRCAM/CNRS/UPMC)
Laboratoire d'Algorithmique, Complexité et Logique (lacl, Université Paris-Est, Creteil)

Mathématiques/Musique…une histoire récente!

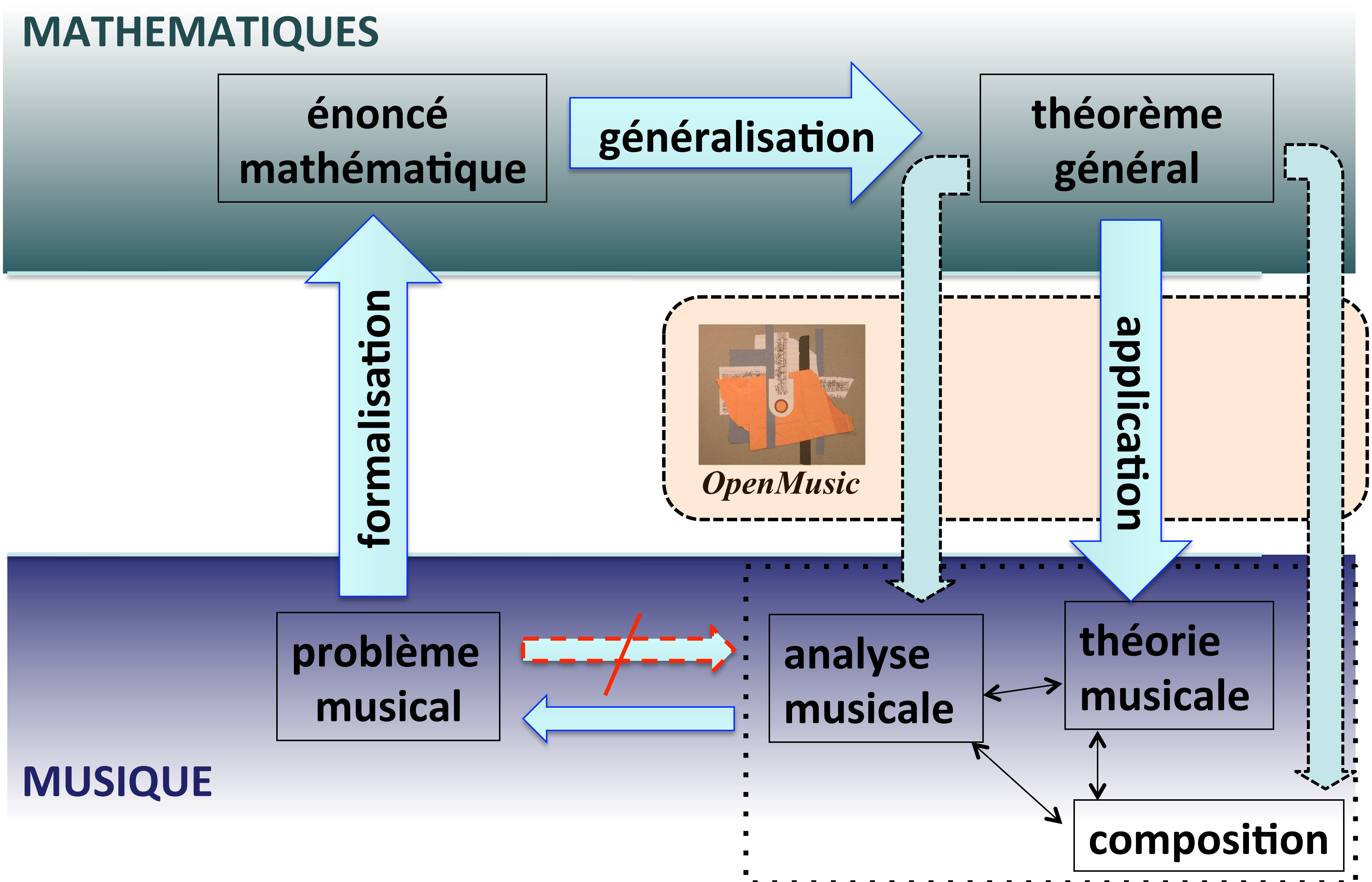
- 1999 : 4e Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (G. Assayag, H.G. Feichtinger, J.F. Rodrigues, Springer, 2001)
- 2000-2001 : Séminaire *MaMuPhi, Penser la musique avec les mathématiques ?* (Assayag, Mazzola, Nicolas éds., Coll. « M/S », Ircam/Delatour, 2006)
- 2000-2003 : International Seminar on *MaMuTh* (*Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Luis-Puebla eds, epOs, 2004)
- 2003 : *The Topos of Music* (G. Mazzola et al.)
- 2003 : *Music and Mathematics. From Pythagoras to Fractals* (J. Fauvel et al.)
- 2001 - 2011 : Séminaire *MaMuX* de l'Ircam
- 2004 - 2011 : Séminaire *mamuphi* (Ens/Ircam)
- 2006 : *Mathematical Theory of Music* (F. Jedrzejewski), Coll. « M/S »
- 2007 : *La vérité du beau dans la musique* (G Mazzola), Coll. « Musique/Sciences »
- 2007 : Journal of Mathematics and Music (Taylor & Francis) et MCM 2007
- 2007 : *Music. A Mathematical Offering* (Dave Benson), CUP
- 2008 : *Music Theory and Mathematics* (Jack Douthett et al.), URP
- 2009 : *Computational Music Science Series* (Springer)
- 2009 : MCM 2009 (Yale) et Proceedings chez Springer
- 2010 : Mathematics Subject Classification : 00A65 Mathematics and music
- 2011 : Conférence de la SMCM (Ircam, 15-17 juin 2011)



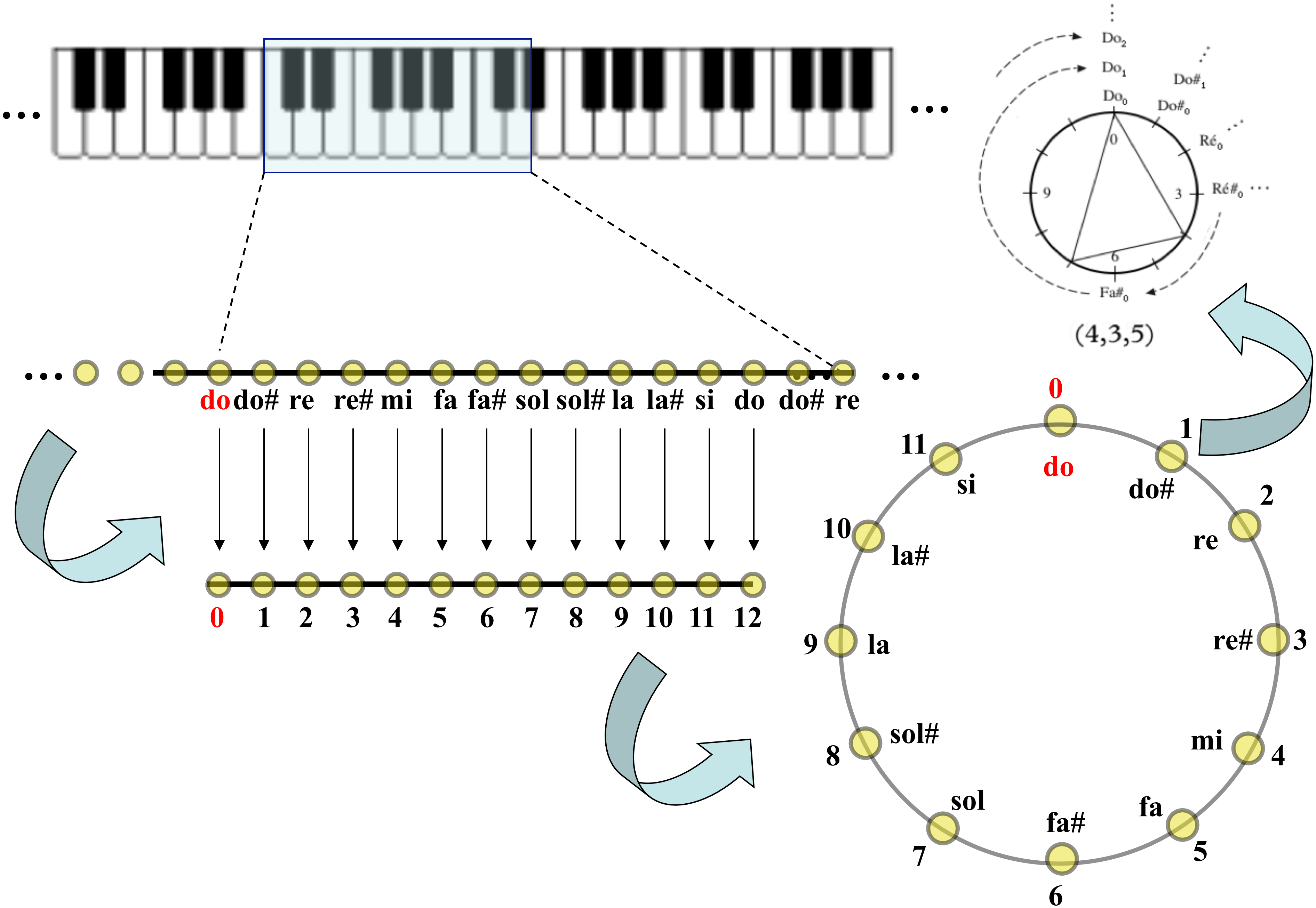
Double mouvement d'une dynamique mathémusicale



Double mouvement d'une dynamique mathémusicale



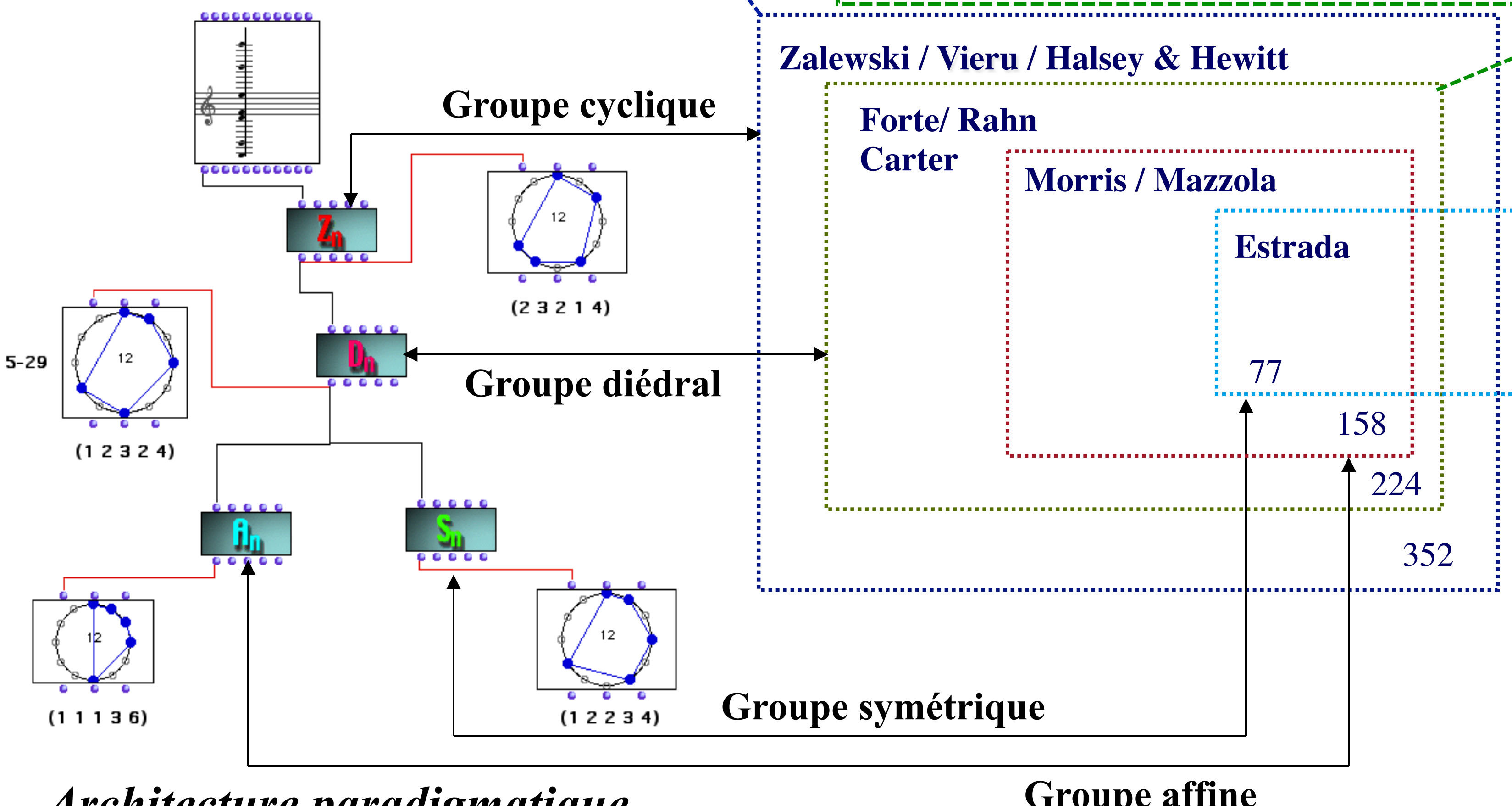
Réduction à l'octave et congruence modulo 12



Classification paradigmatique des structures musicales

$$\#\text{ of } k\text{-chords} = \frac{1}{n}\sum_{j|(n,k)} \phi(j)\binom{n/j}{k/j} = \frac{1}{n}\Phi_n(k)$$

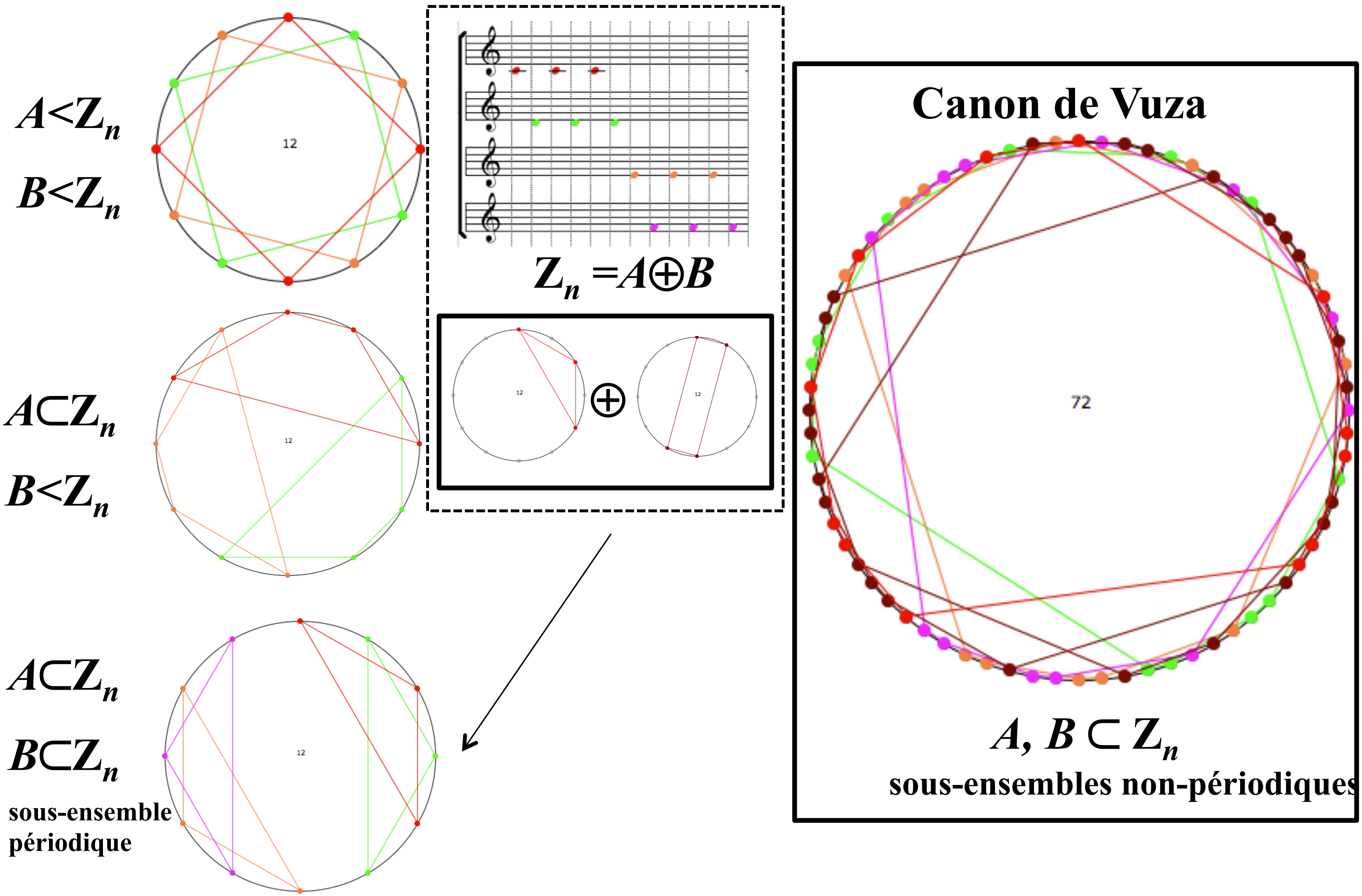
$$\#\text{ of } k\text{-chords} = \begin{cases} \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n-1)/2}{[k/2]}\right], & \text{if } n \text{ is odd,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{n/2}{k/2}\right], & \text{if } n \text{ is even and } k \text{ is even,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n/2)-1}{[k/2]}\right], & \text{if } n \text{ is even and } k \text{ is odd.} \end{cases}$$



Architecture paradigmatique

- D. Halsey & E. Hewitt: « Eine gruppentheoretische Methode in der Musik-theorie », *Jahresber. Der Dt. Math.-Vereinigung*, 80, 1978.
- D. Reiner: «Enumeration in Music Theory», *Amer. Math. Month*. 92:51-54, 1985
- R.C. Read: « Combinatorial problems in the theory of music », *Discrete Math.*, 1997
- H. Fripertinger: « Enumeration of mosaics », *Discrete Math.*, 1999

Factorisations de groupes et canons rythmiques mosaïques



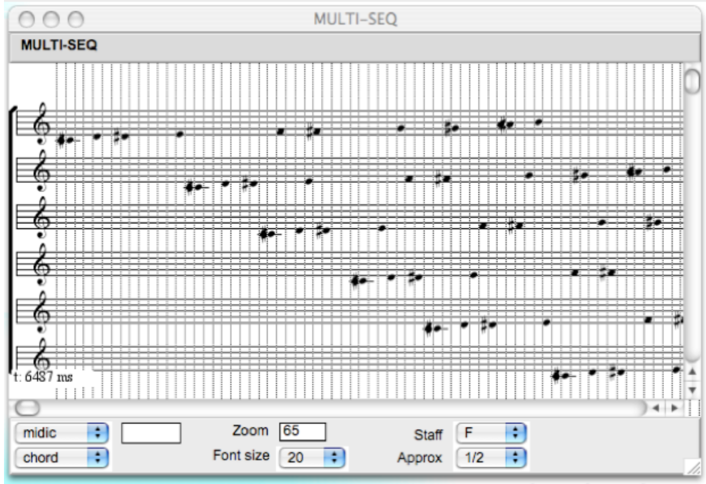
Canons mosaïques et conjecture spectrale

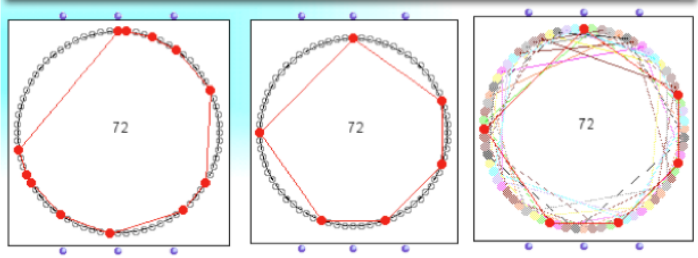
La conjecture de Fuglede (1974)

Un sous-ensemble de R^n pave par translation *ssi* il est spectral (i.e. il admet une décomposition hilbertienne d'exponentiels complexes)

***J. Func. Anal.* 16, 1974.**

Fausse en dim. $n \geq 3$
Ouverte en dim. 1 et 2





Canons de Vuza de période *n*

72
108 120 144 168 180
200 216 240 252 264 270 280 288
300 312 324 336 360 378 392 396
400 408 432 440 450 456 468 480
500 504 520 528 540 552 560 576 588 594
600 612 616 624 648 672 675 680 684 696
700 702 720 728 744 750 756 760 784 792
800 810 816 828 864 880 882 888…

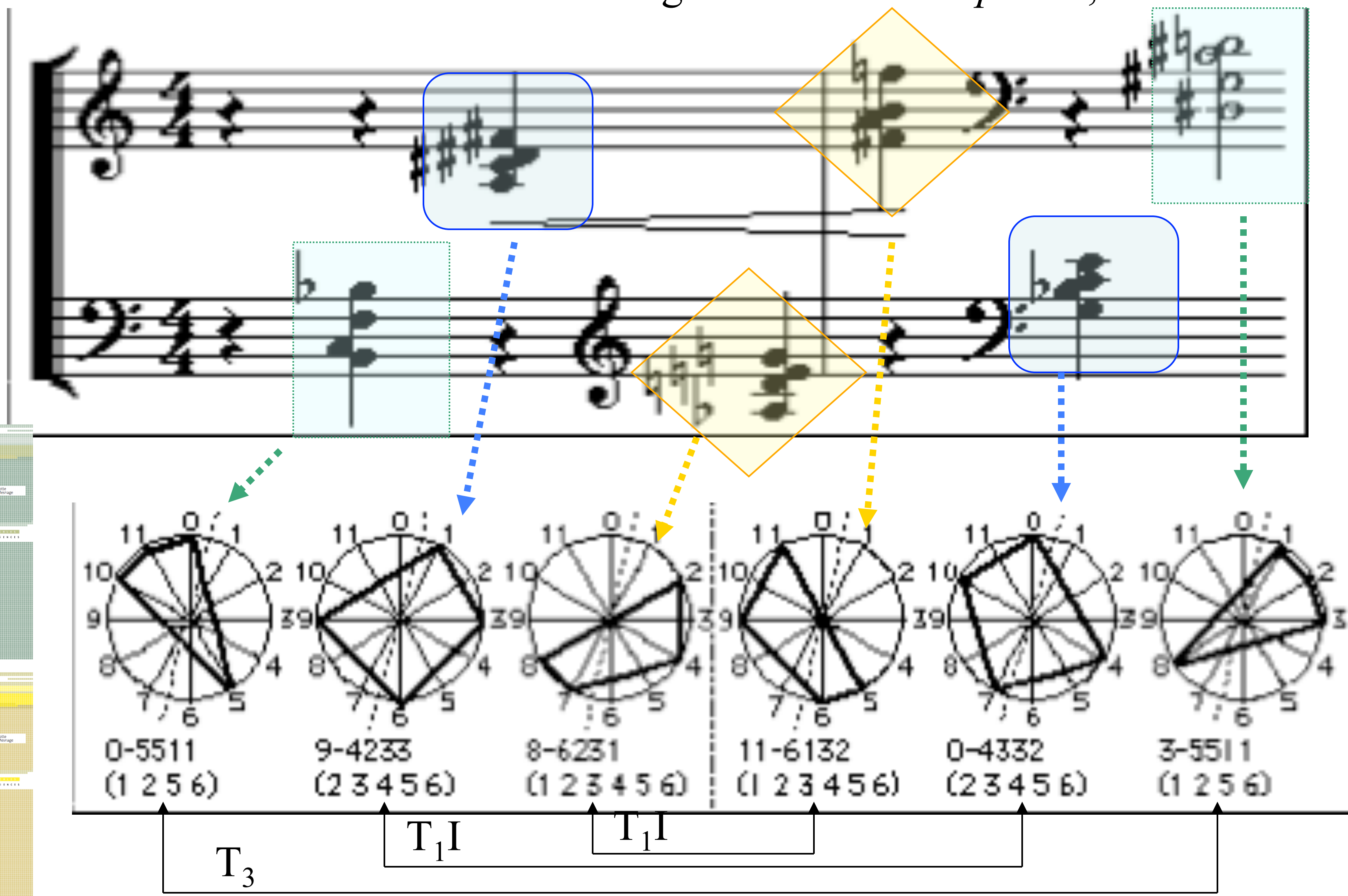
Résultat (Amiot 2009) :
Si *A* pave mais il n'est pas spéctral
=> *A* est le rythme d'un canon de Vuza

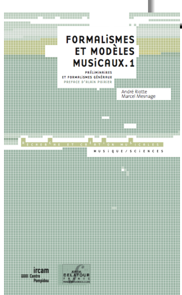
PROBLEME OUVERT : Trouver un algorithme qui permet d'obtenir toutes le factorisations d'un groupe cyclique non-Hajos en somme directe de deux sous-ensembles non périodiques (i.e. classifier tous les Canons de Vuza)

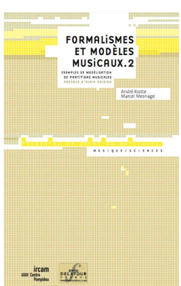
L'analyse formalisée ou les entités formelles en musique

André Riotte & Marcel Mesnage

A. Schoenberg : *Klavierstück Op. 33a*, 1929



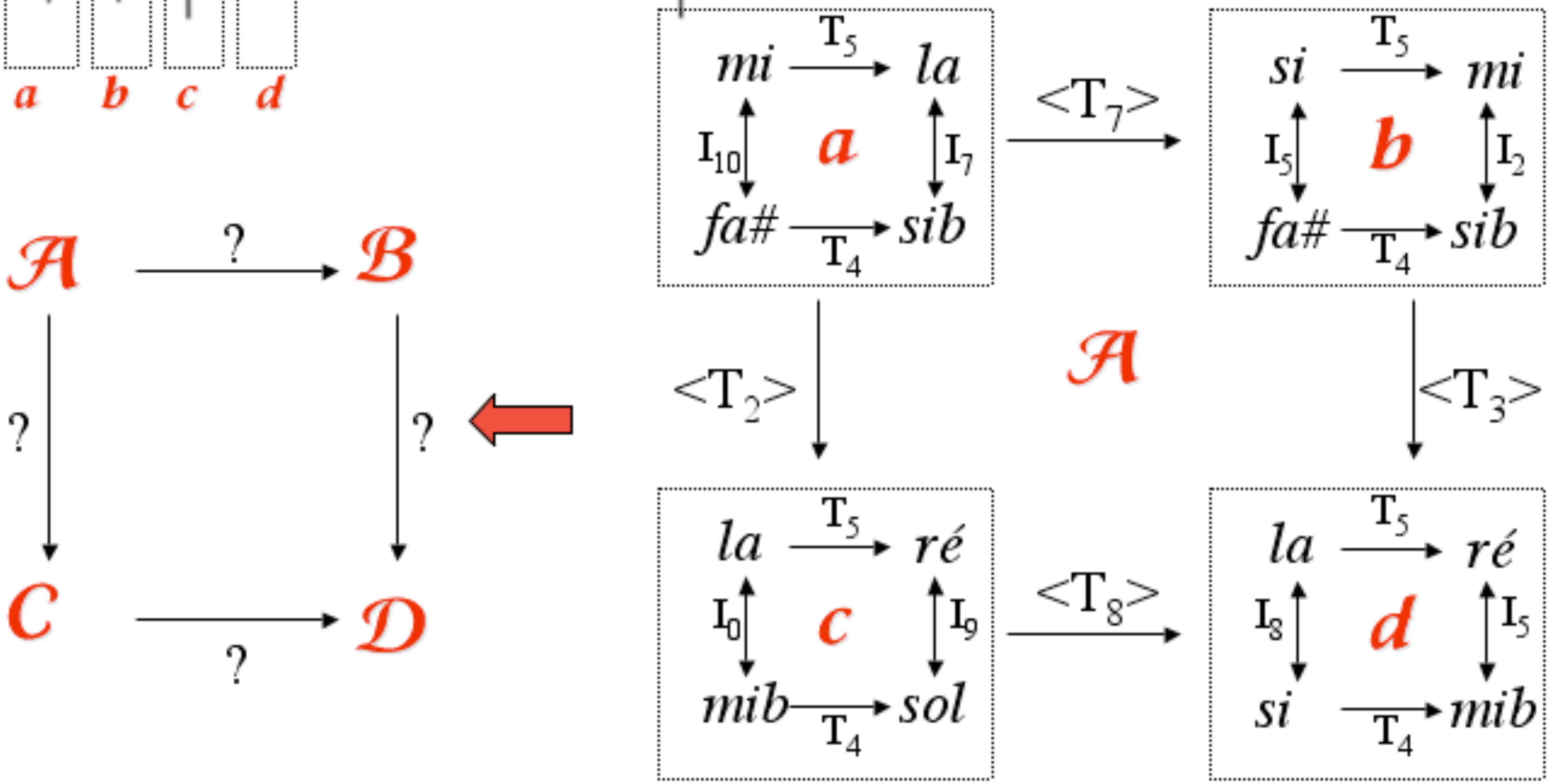




Klumpenhouwer Networks (K-réseaux) : isographies positives et récursivité

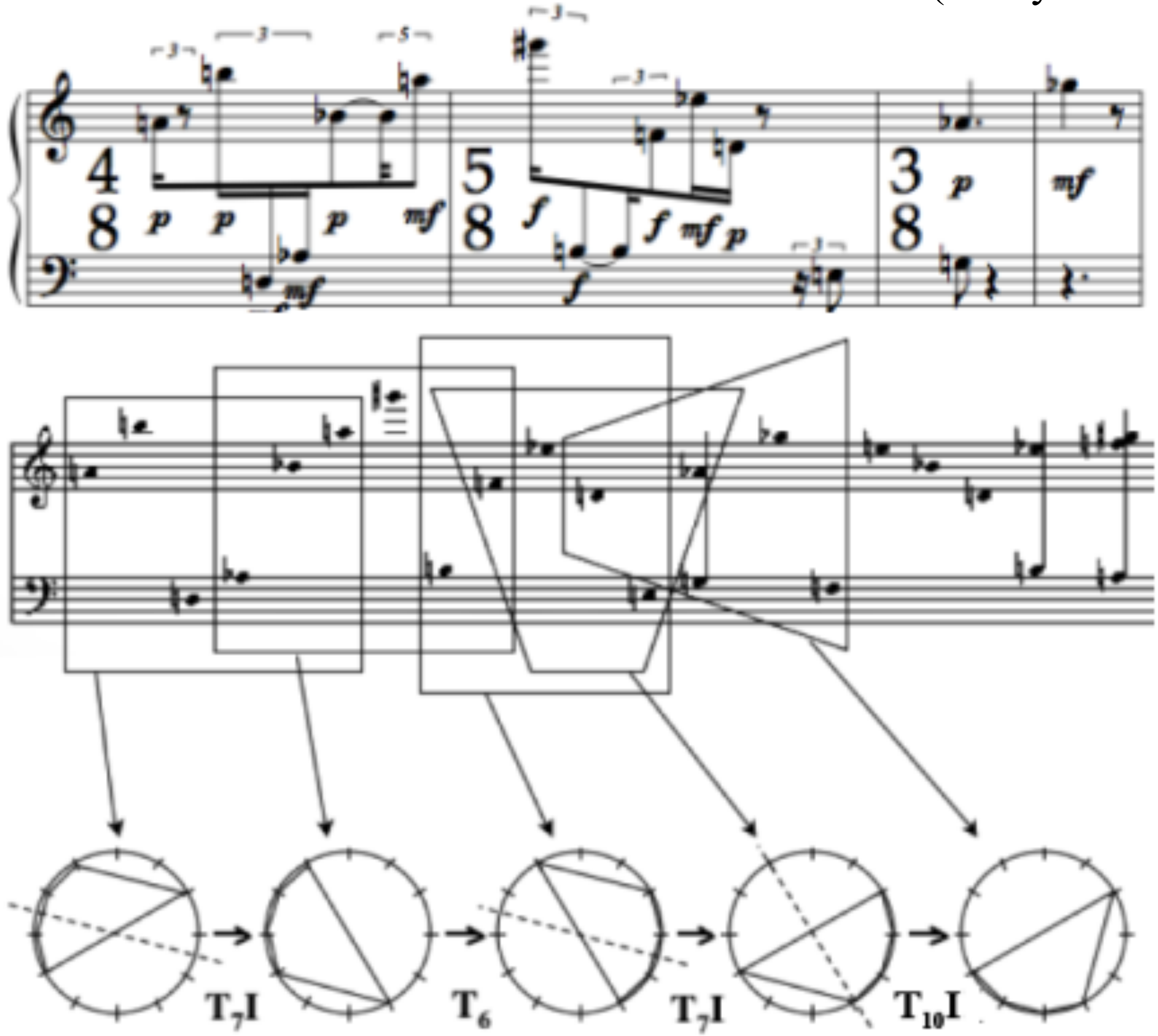
David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

$$\langle T_k \rangle : T_m \rightarrow T_m$$
$$I_m \rightarrow I_{k+m}$$



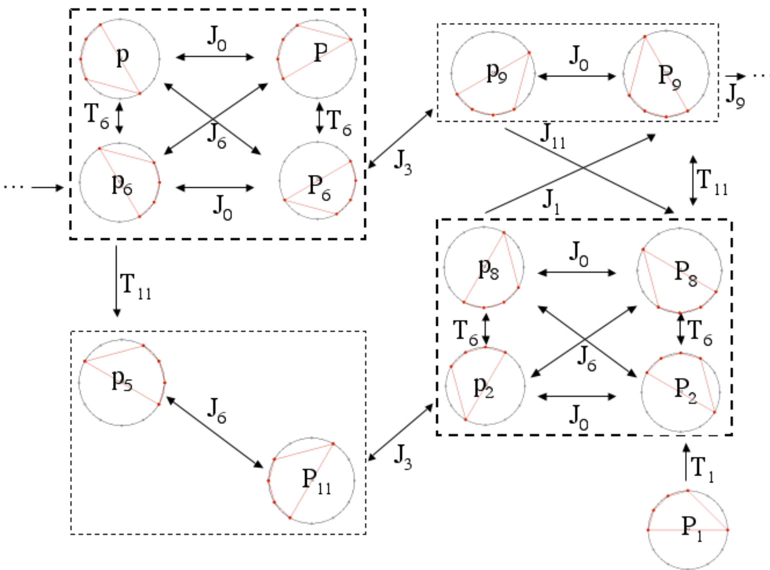
Segmentation par « imbriction »: progression transformationnelle

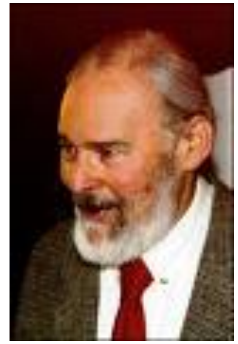
Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

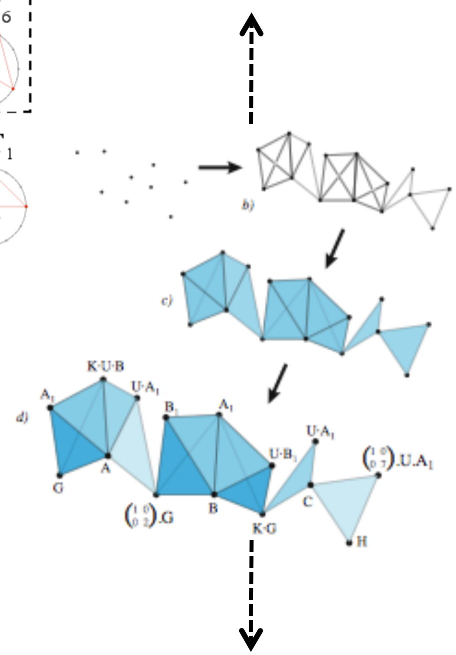


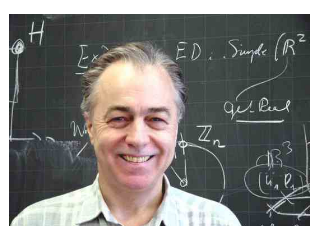
Vers une modélisation informatique de l’analyse transformationnelle

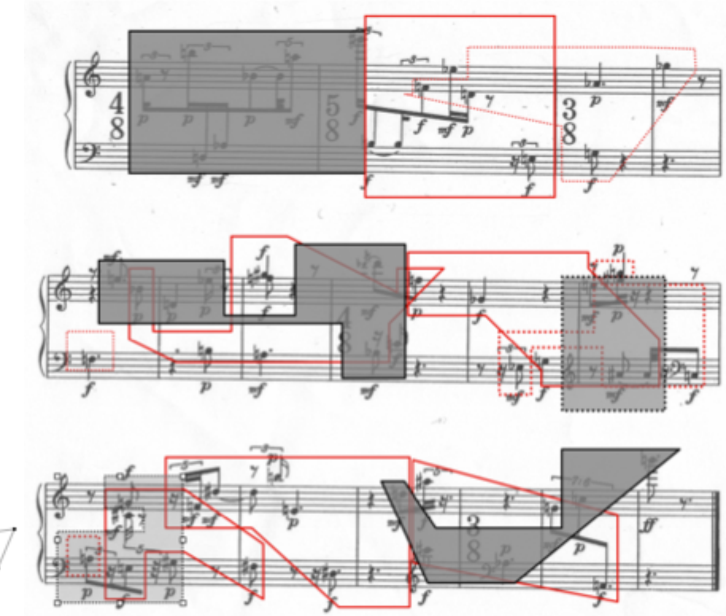
YunKang Ahn, L’analyse musicale computationnelle, thèse, Université de Paris VI / Ircam, déc 2009

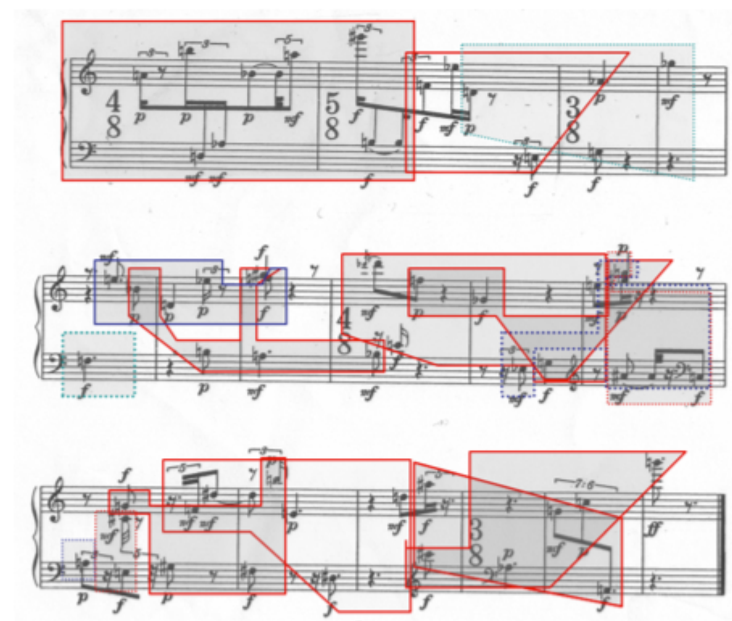




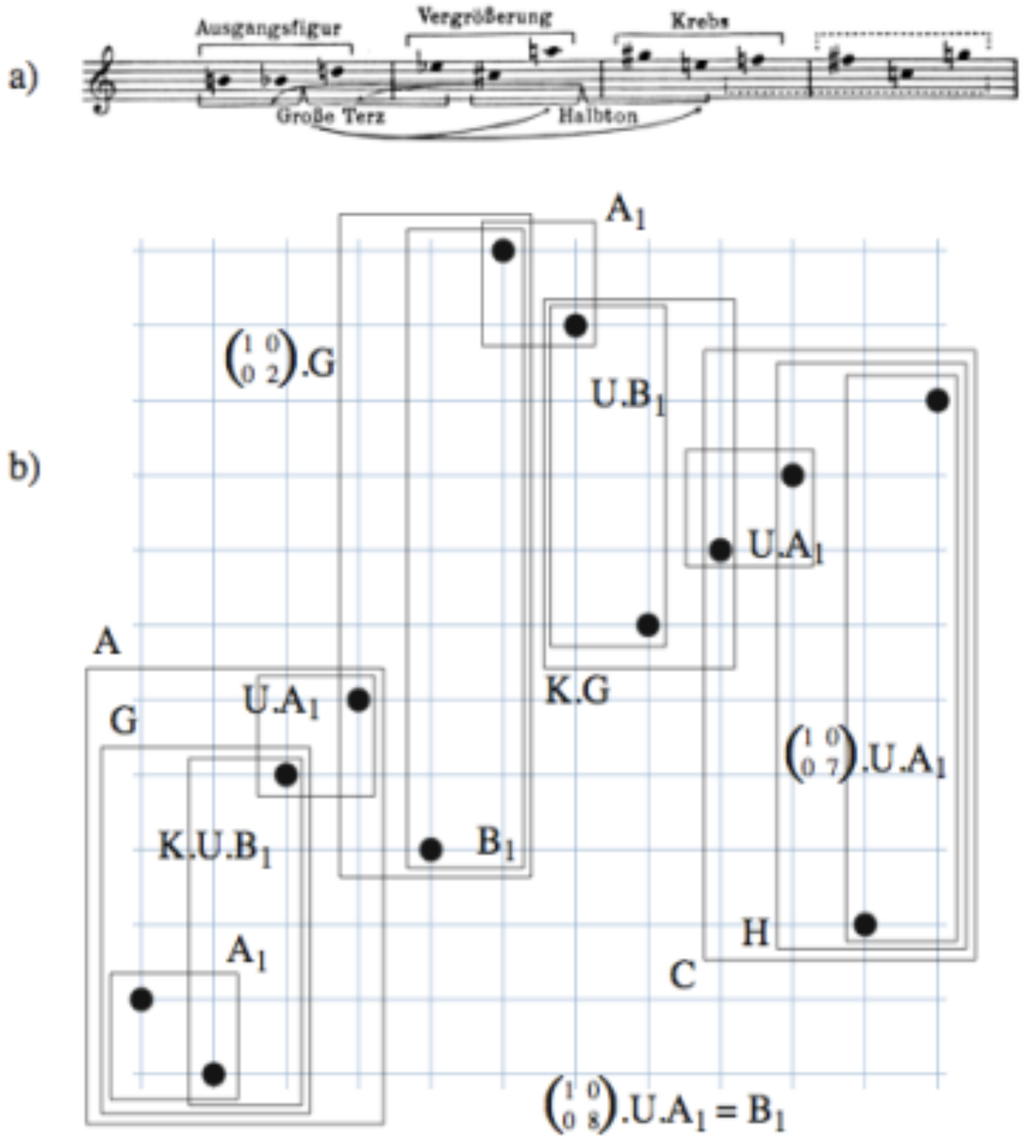


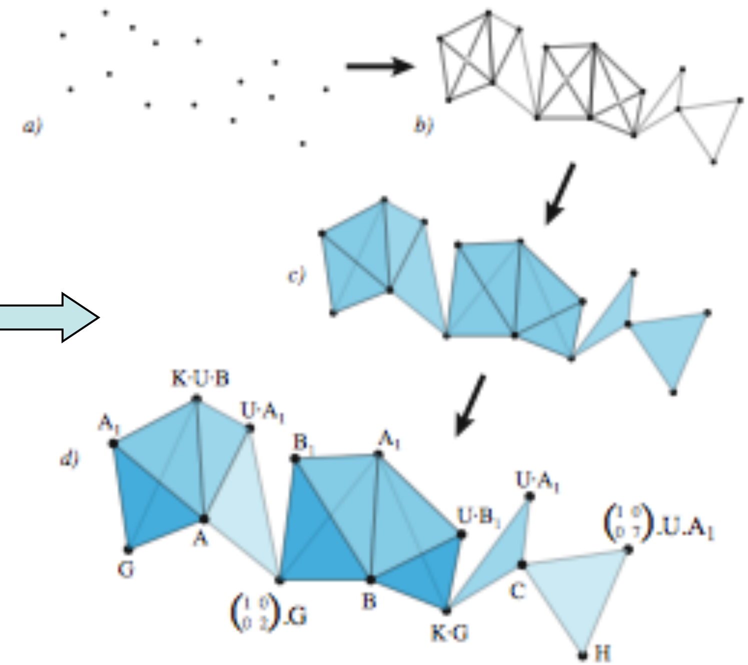




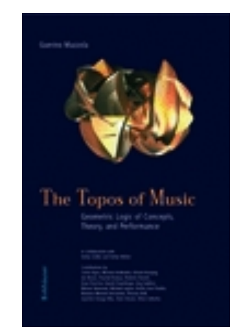


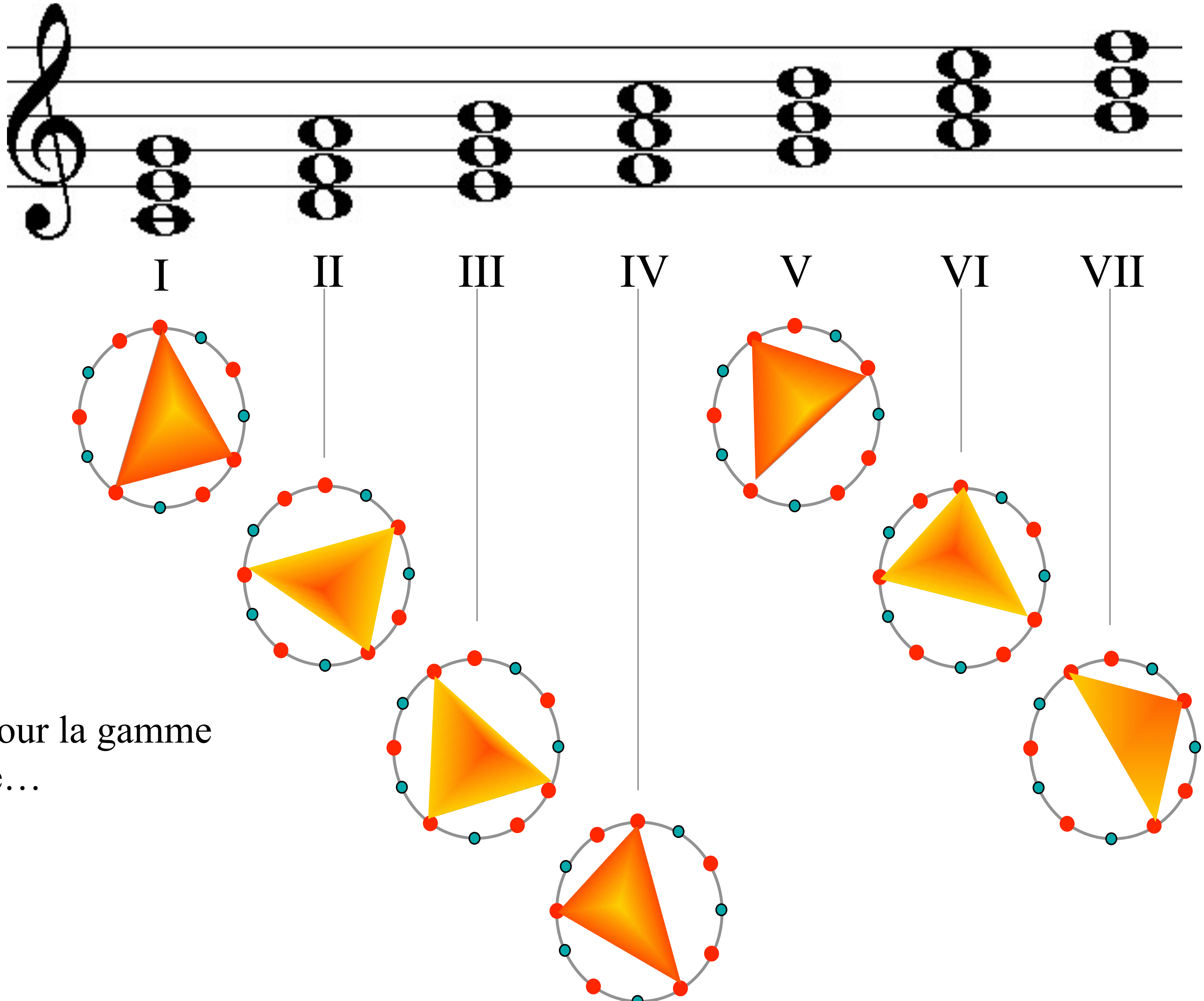
Nerf topologique et analyse musicale



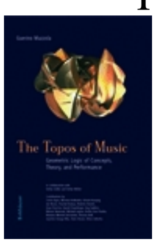


G. Mazzola : *The Topos of Music*, ch. 13 - “What are global compositions ?”



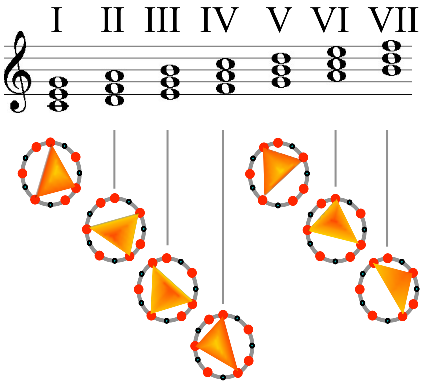


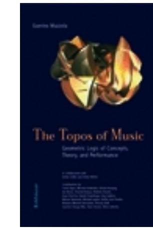
Un atlas pour la gamme diatonique…

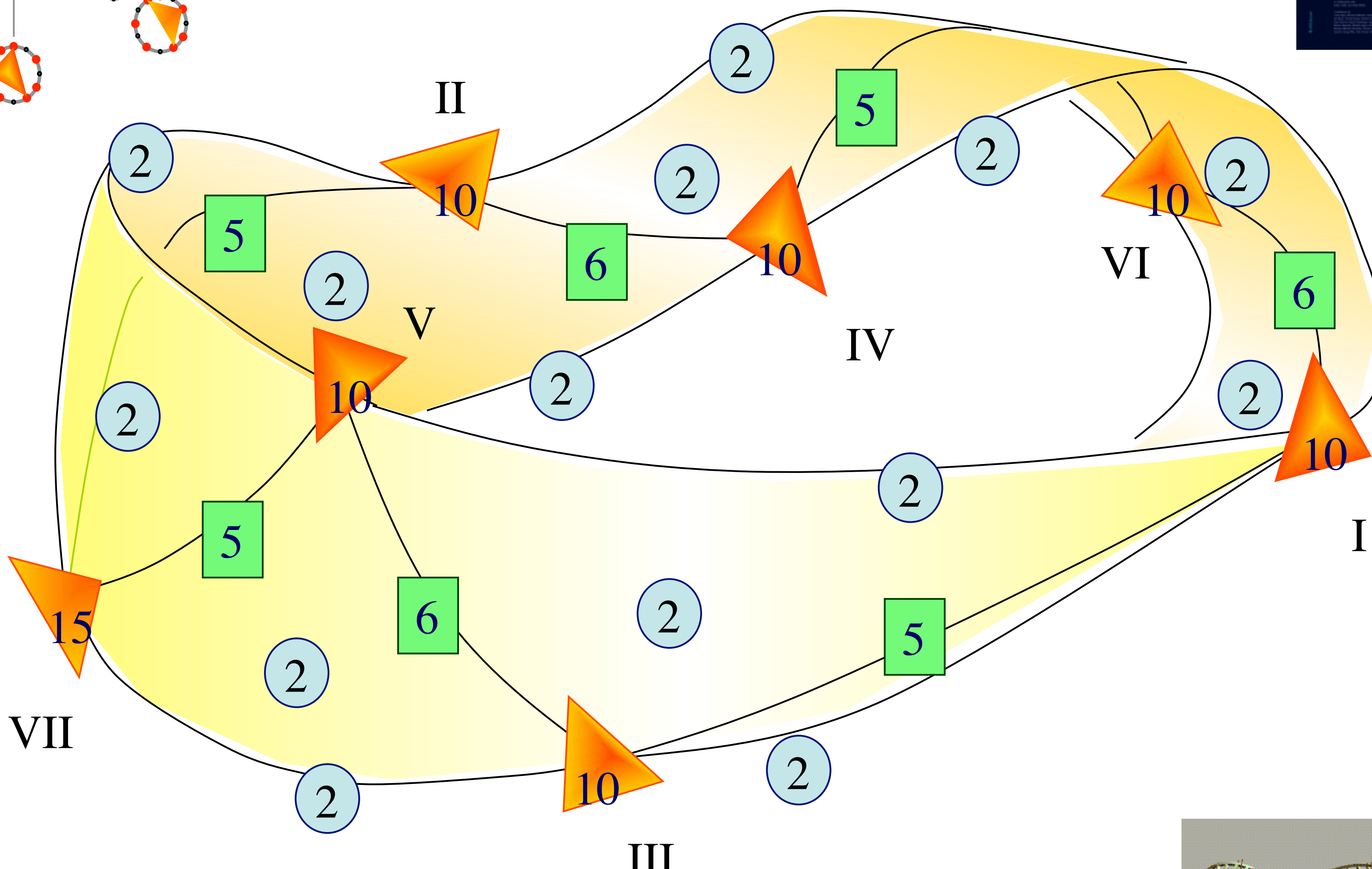


G. Mazzola, *The Topos of Music*

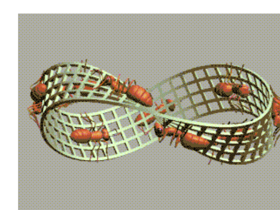
G. Mazzola, *The Topos of Music*



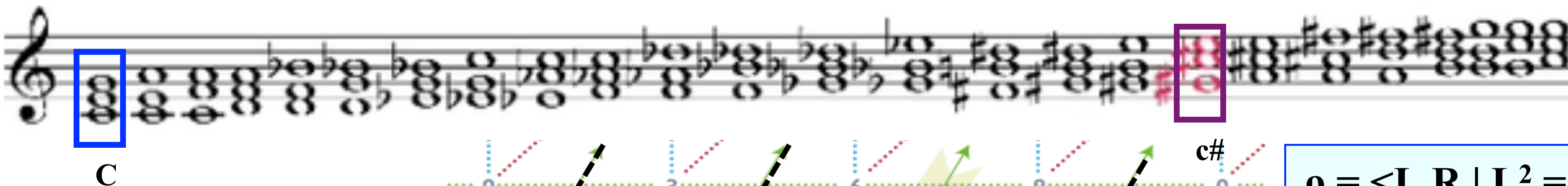




…et le nerf topologique associé



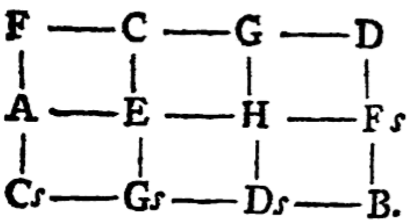
Du réseau d'Euler à la programmation spatiale



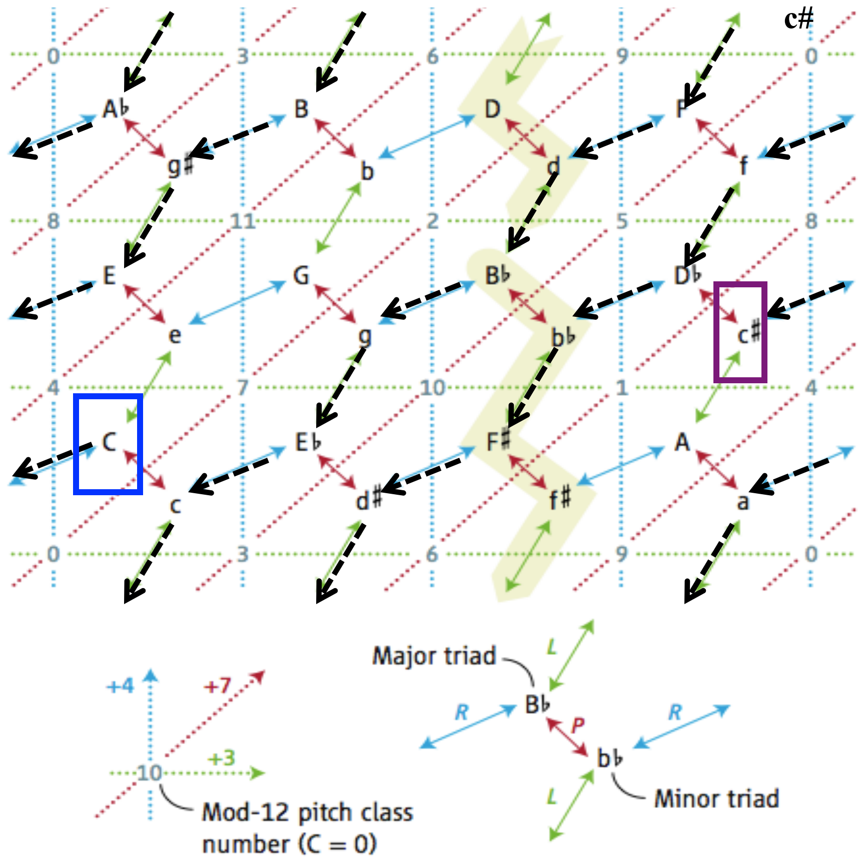
$\rho = \langle L,R \mid L^2 = (LR)^{12} = 1$
$LRL = L(LR)^{-1} \rangle$

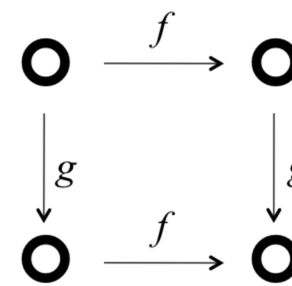
↕ **dualité**

$D_{12} = \langle I, T \mid I^2 = T^{12} = 1$
$ITI = I(IT)^{-1} \rangle$



Euler : *Speculum musicum*, 1773

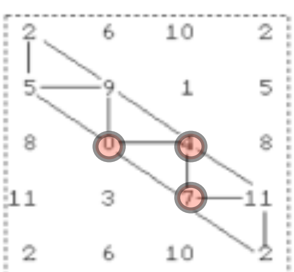


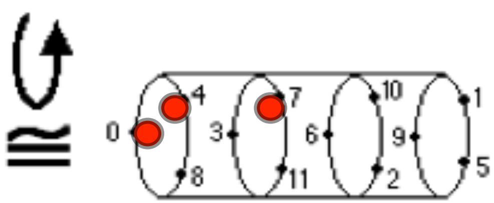


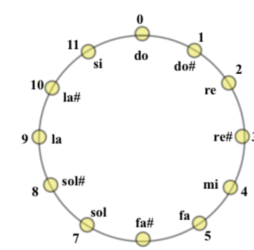
Tout diagramme commute

$\forall f \in D_{12}$

$\forall g \in \rho$





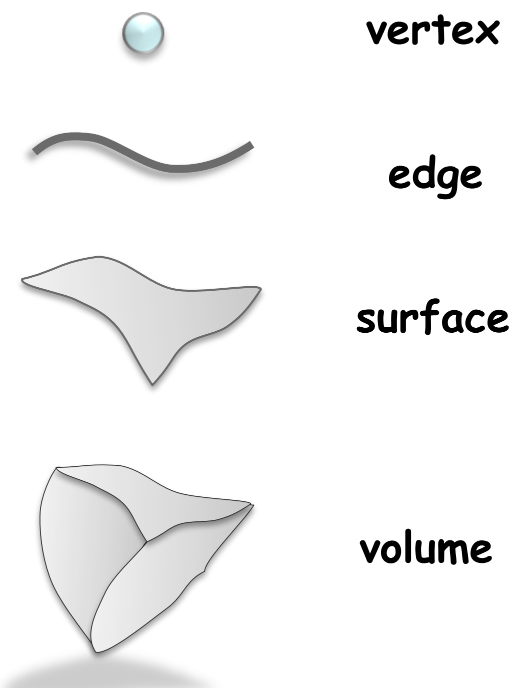


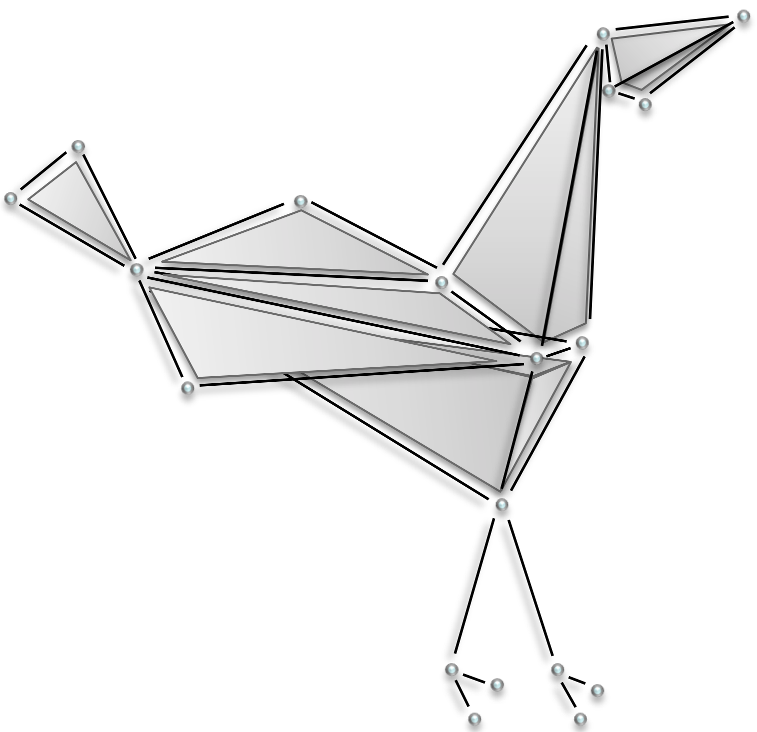
The MGS project

- Abstract rewriting of complex spatial structures
  - Data structure = topological collections

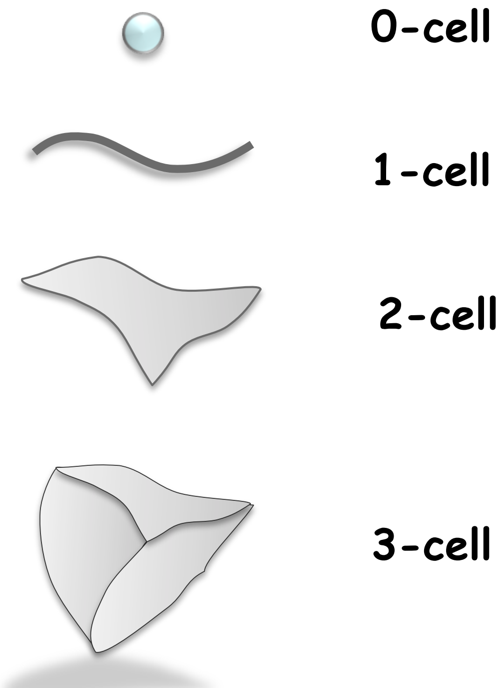
    sequence, generalized array, (multi-)set, arbitrary graph, Delaunay triangulation, g-map, …, <u>cell complexes</u>
  - Control structure = transformation
    - two powerful languages to specify sub-collections (elements in interaction)
    - Various rule application strategies: maximal parallel, asynchronous, stochastic, Gillespie-like, …
- Applications
  - modeling and simulation of $(DS)^2$: morphogenesis, systems biology, knowledge representation, complex systems
  - topological representation of musical processes

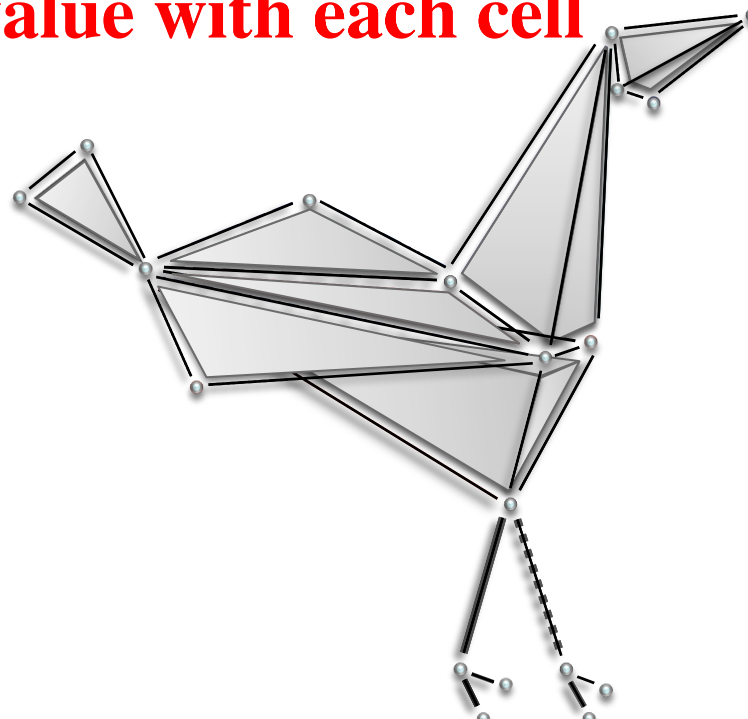
- Topological collections
  - Structure
    - A collection of topological cells
    - An ***incidence relationship***





- Topological collections
  - Structure
    - A collection of topological cells
    - An incidence relationship
  - Data: **association of a value with each cell**



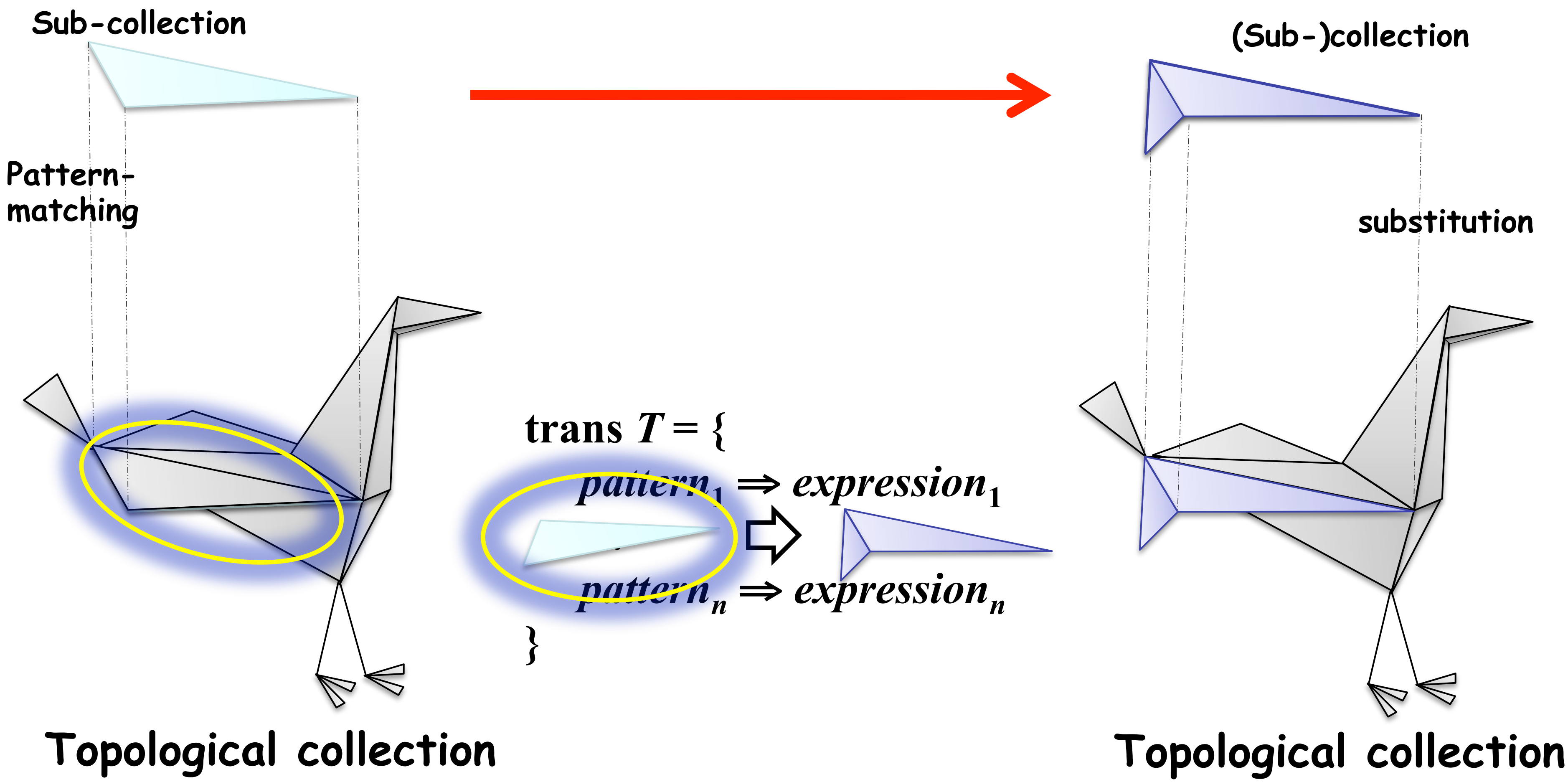


- Transformations
  - Functions defined by case on collections

    Each case (pattern) matches a sub-collection
  - Defining a rewriting relationship: ***topological rewriting***

trans T = {
  $pattern_1 \Rightarrow expression_1$
  …
  $pattern_n \Rightarrow expression_n$
}

- Transformations



Topological rewriting = transformation

1 + 2 → … (arithmetic) term rewriting

arithmetic operation

a . b → … string rewriting (~ L systems)

string concatenation: « . » is a formal associative operation

2H + O → H_2O multiset rewriting (~ chemistry)

multiset concatenation (= the chemical soup): « . » is AC

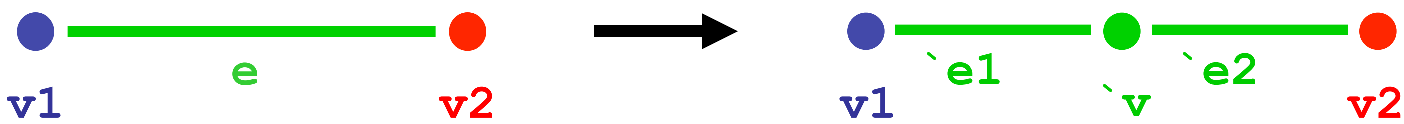
$v_1.\sigma_1 + v_2.\sigma_2 \rightarrow \ldots$ topological rewriting (MGS)

gluing cell in a cell complex: … (AC and algebraic machinery)

Topological rewriting ≠ graph rewriting

$v_1.\sigma_1 + v_2.\sigma_2 \rightarrow \ldots$ **topological rewriting (MGS)**

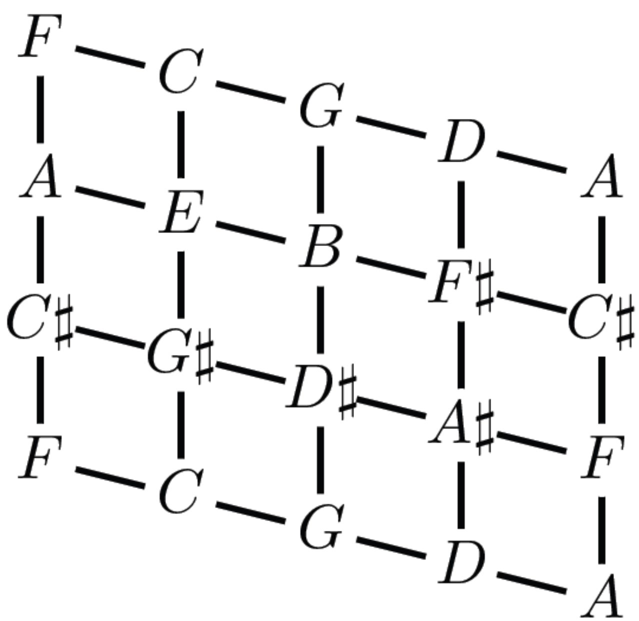
the structure is in the cells σ not in +



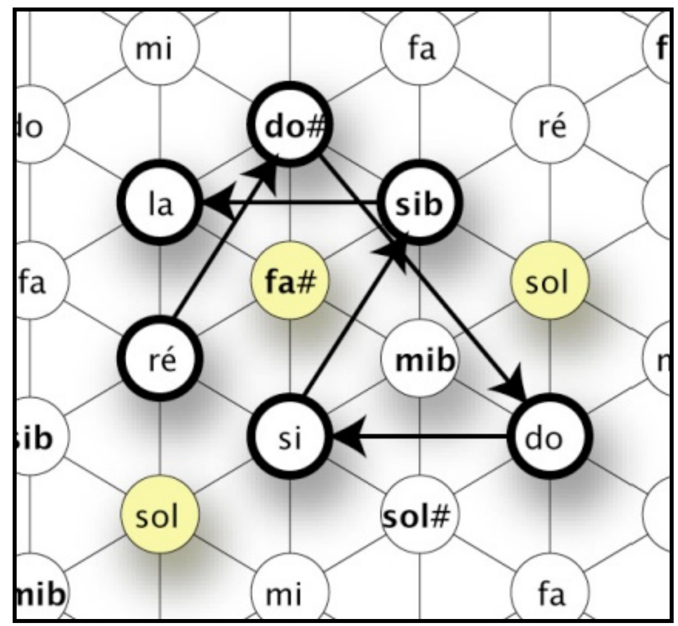
$$
\begin{aligned}
& x_1.v_1 + (x.e + x_2.v_2) \\
&= (x_1.v_1 + x.e) + x_2.v_2 \\
&= (x_1.v_1 + x_2.v_2) + x.e \\
&= x.e + (x_1.v_1 + x_2.v_2) \\
&= \ldots
\end{aligned}
$$

```
v1 < e:[ dim = 1 ] > v2 =>
   v1
   `e1:[ dim = 1,    faces = (^v1,`v),   val = … ]
   `v  :[ dim = 0, cofaces = (`e1,`e2), val = (v1+v2)/2 ]
   `e2:[ dim = 1,    faces = (^v2,`v),   val = … ]
   v2
```

Neo-Riemannian Problematic



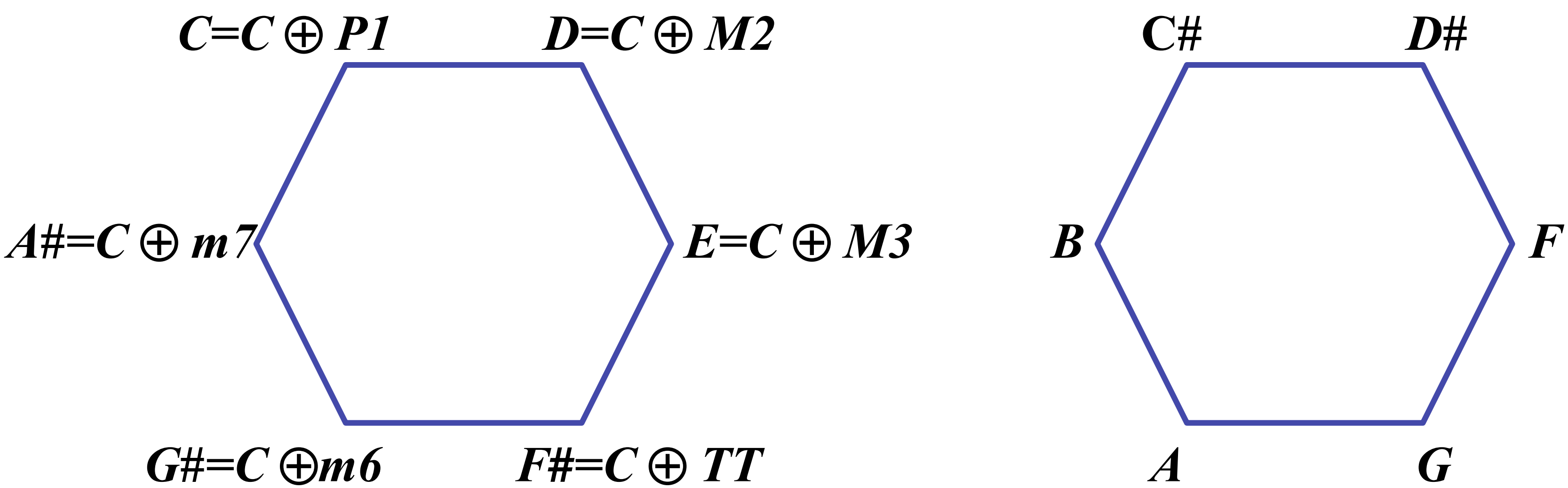
Euler's tonnetz



Hexagonal network of notes (J.-M. Chouvel)

Formalization of Notes Neighborhoods

- Spatial representation of *S*
  - *I* as a set of group generators
  - Graph representation of < *I* >
  - Representation of *S* based on Cayley graph
    - Action of < *I* > on N
    - Example with *I* = { *M2* }



Examples

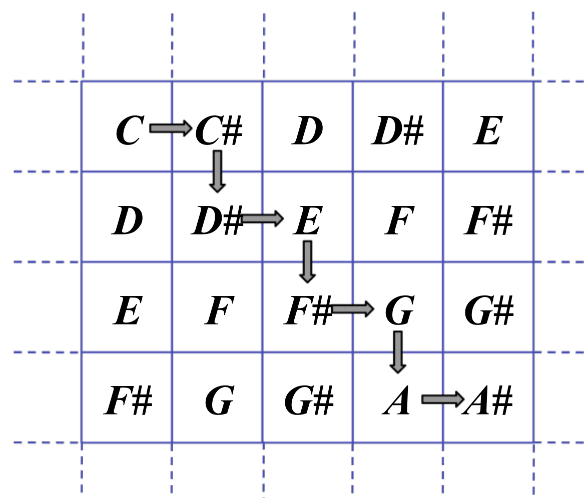
- Scale representations
  - Chromatic scale *I* = { *m2* }

C	*C#*	*D*	*D#*	*E*	*F*	*F#*	*G*	*G#*	*A*	*A#*	*B*

  - Whole-tone scale *I = { M2 }*

C	*D*	*E*	*F#*	*G#*	*A#*
C#	*D#*	*F*	*G*	*A*	*B*

  - Diminished scale *I* = { *m2, M2* }

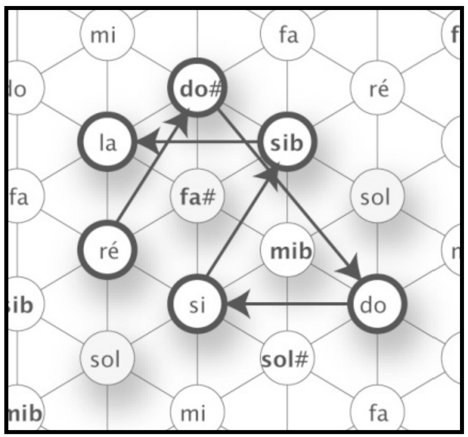


Applications

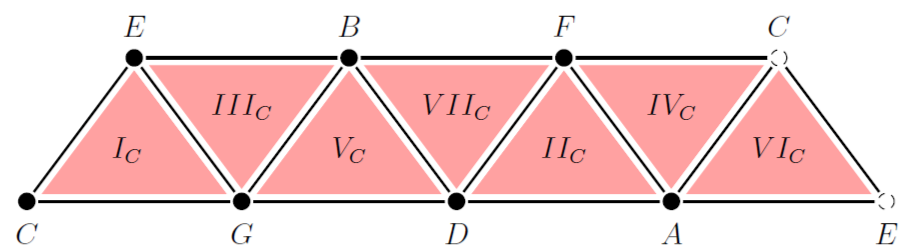
- Analysis example
  - Signature of a piece
  - Example : F. Chopin Prelude

Extract of the 2nd movement of the Symphony No. 9 L. van Beethoven

Extract of the Prelude N.4 Op28 of F. Chopin



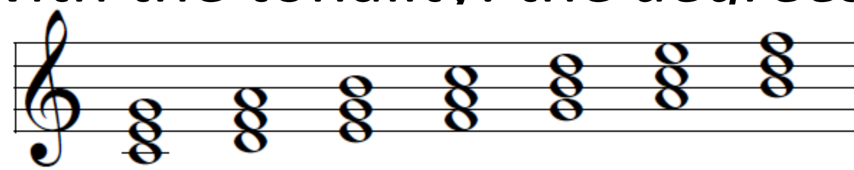
Music in a space



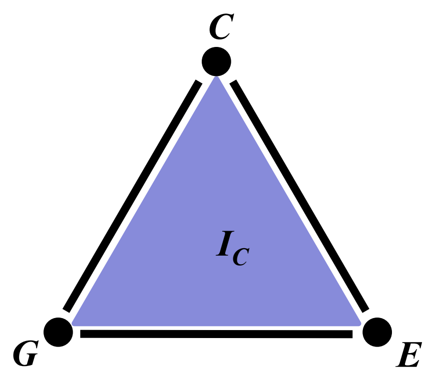
Computing the space of music

Tonality and Möbius Strip

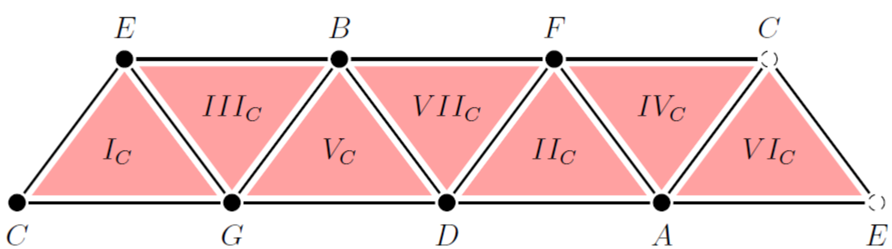
- Motivation: spatial visualization of tonality
- Association of a chord set with the tonality: the *degrees*
  - Example: C-major tonality

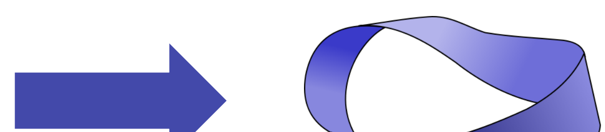


- Spatial representations
  - Note = vertex
  - Chord = surface



- Fusion of the common notes for the 7 degrees

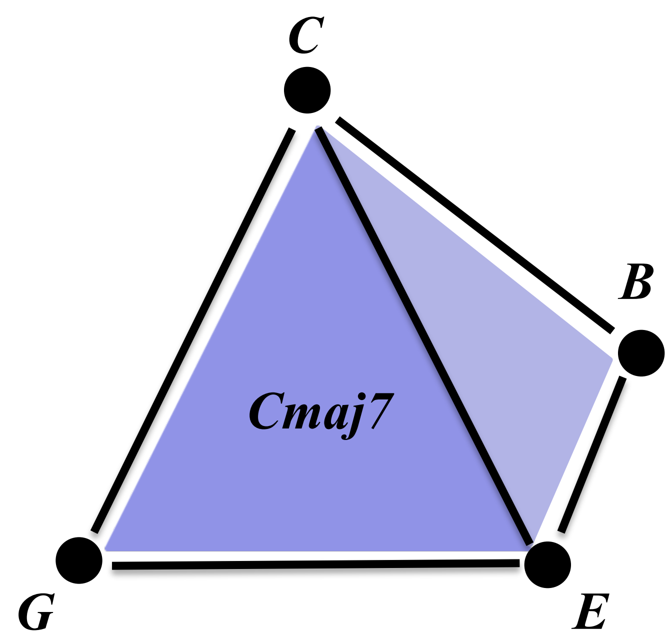




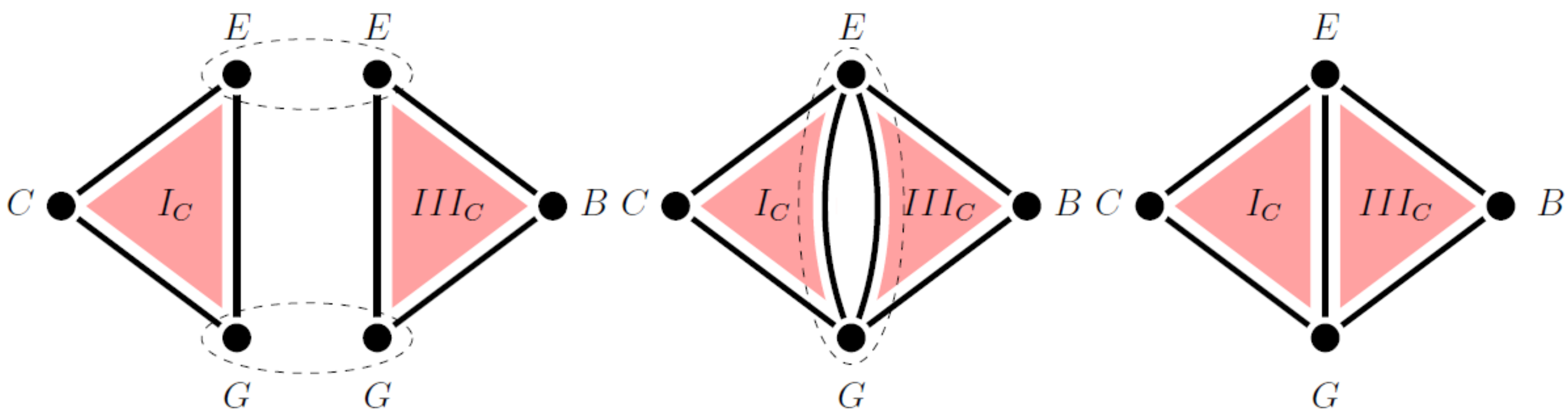
[Mazzola02]

Self-Assembly of Chords

- Automation of the process for the analysis of other chords sequences
- *Reaction* of the chords between themselves
- Simplicial representation of musical objects
  - Note: 0-simplex
  - 2-note chord: 1-simplex
  - 3-note chord: 2-simplex
  - 4-note chord: 3-simplex



Self-Assembly of Chords

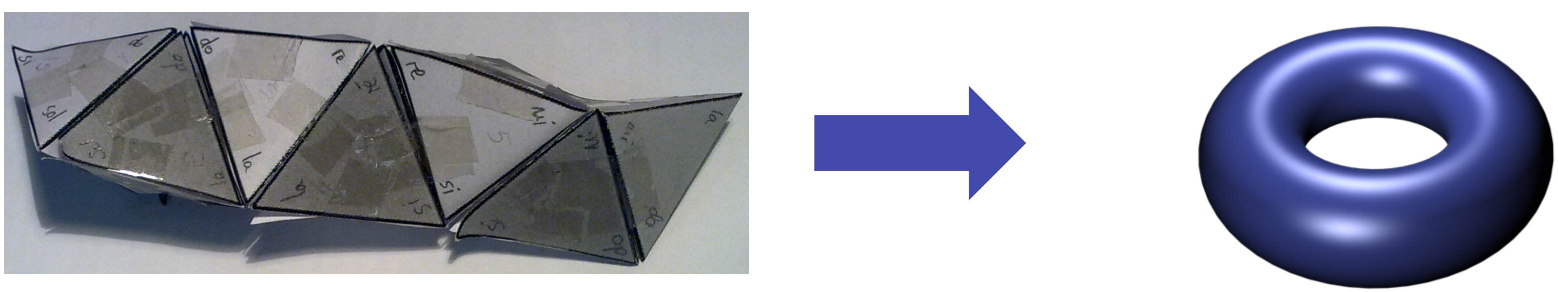



```
Trans identification = {
  s1 s2 / (s1 == s2 & (faces s1) == (faces s2))
    =>
      let c = new_cell (dim s1)
                       (faces s1)
                       (union (cofaces s1)
                              (cofaces s2))
      in s1 * c
}
```

Applications

- Four-note degrees of C-major tonality

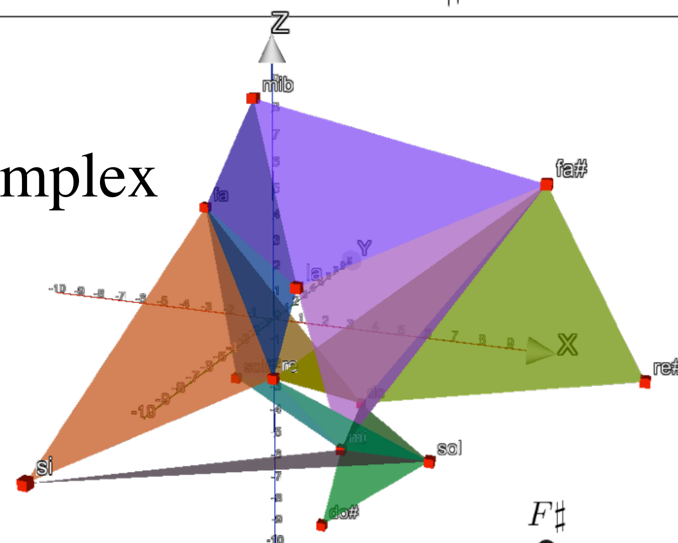
- Chord = 3-simplex (tetrahedrons)
- Self-assembly



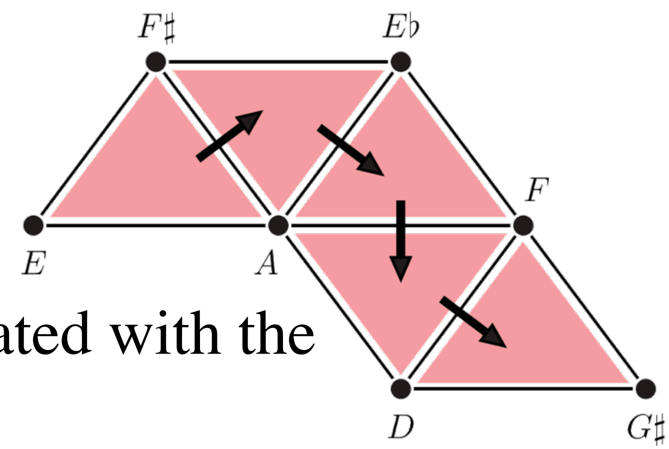
Applications

- Extract of the Prelude No. 4 Op. 28 of F. Chopin

- Associated simplicial complex



- Analysis of the path under the chords



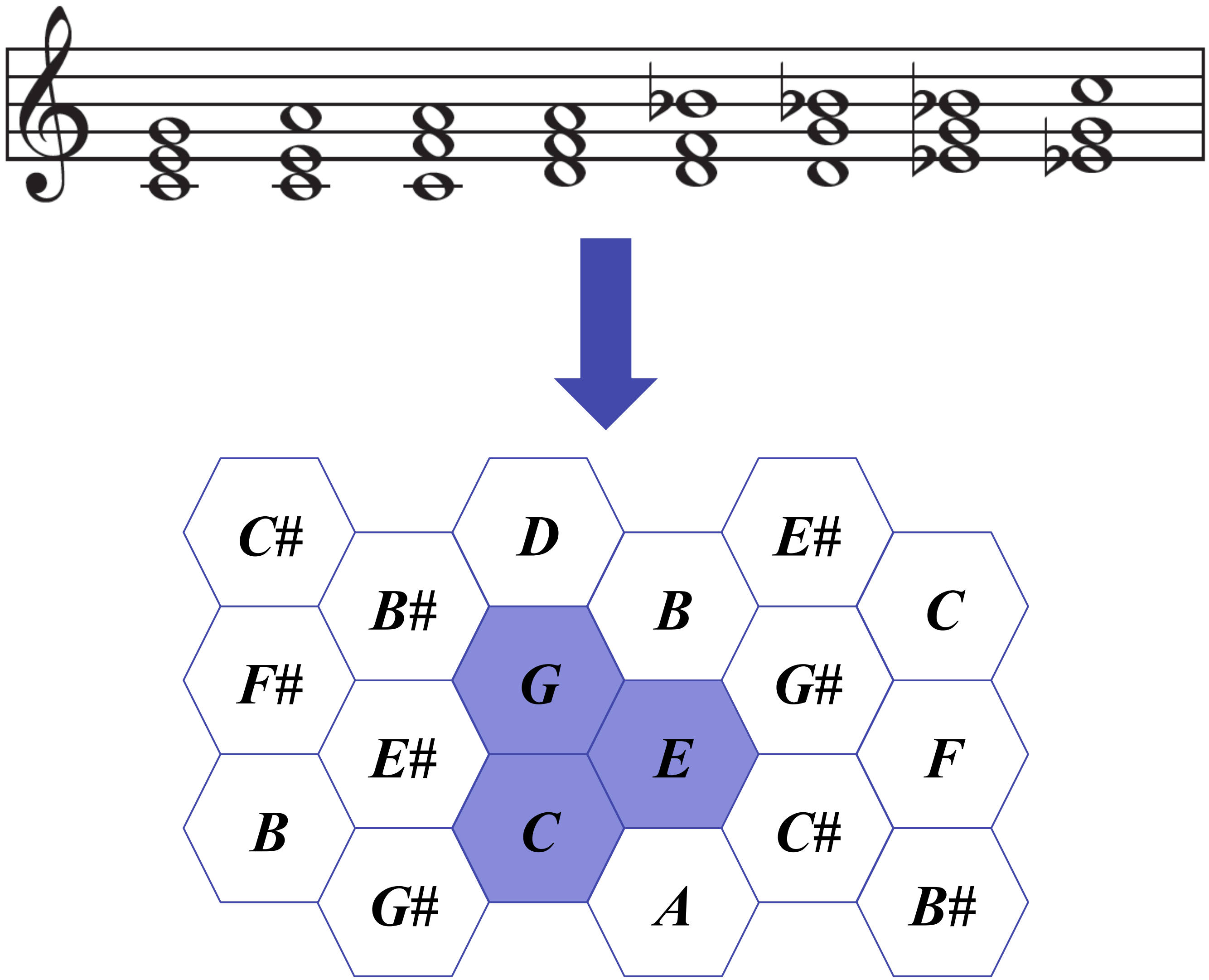
- The path chosen by F. Chopin is associated with the smallest movements on the chords

Music & spatial computing ?

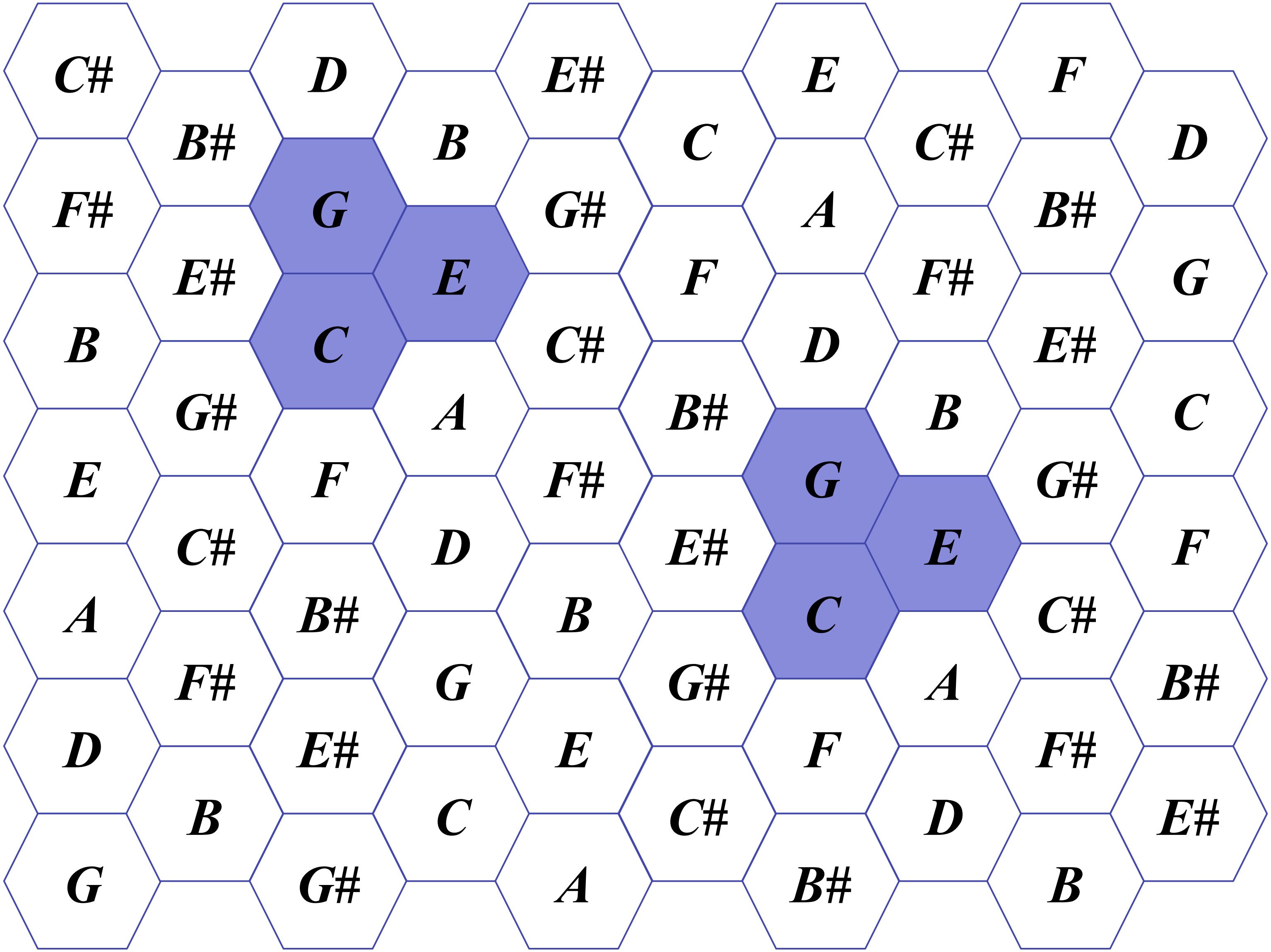
- Preliminary work
- Strong collaborations with composers / musicologists
- Extend the validation on more musical problems
- Extension to study musical styles
- Spatial properties ⇔ musical properties
  - "cinematic" of musical processes
  - rule as evolution function (derivative)

http://mgs.spatial-computing.org

Exemple :Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



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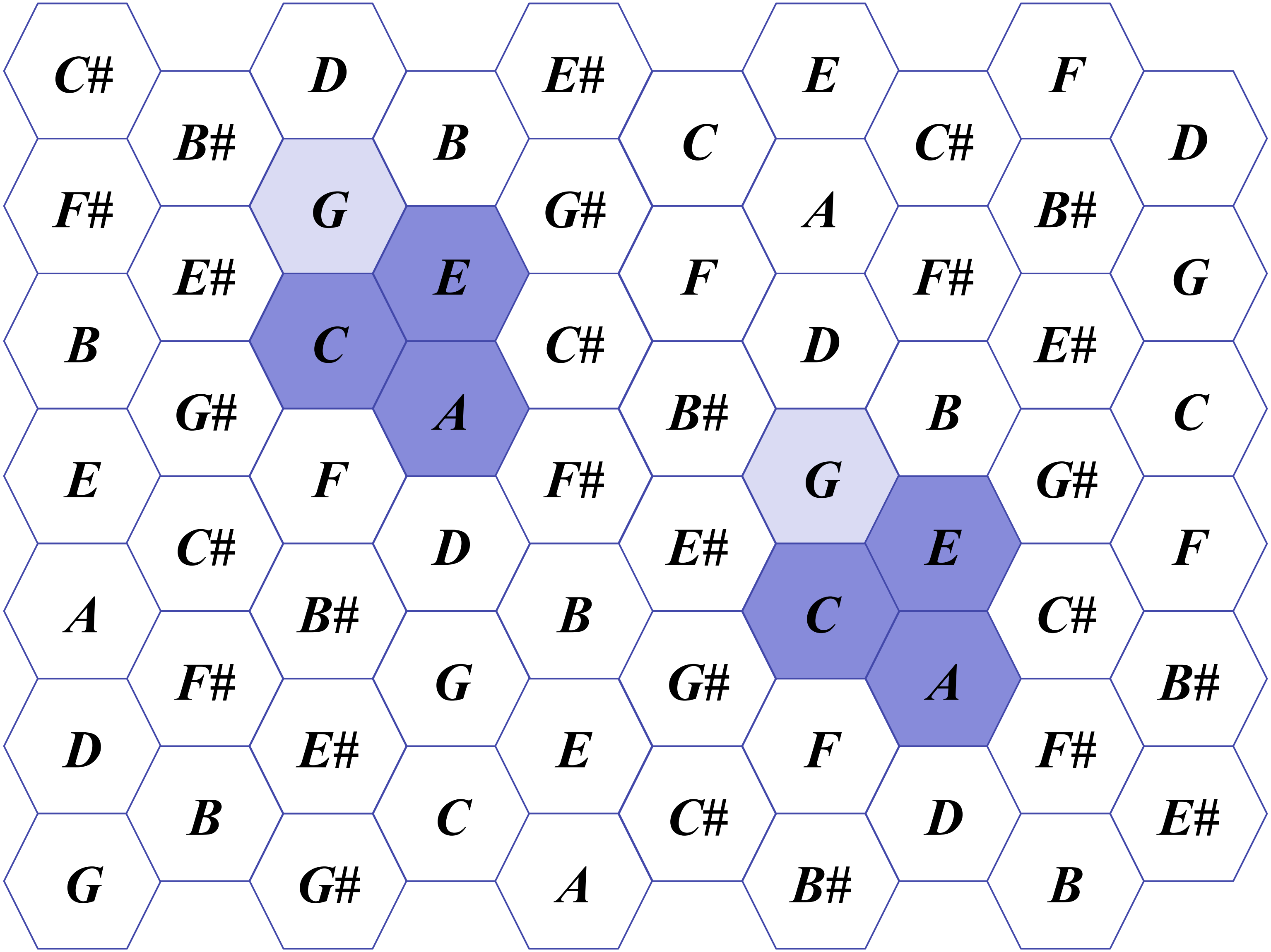
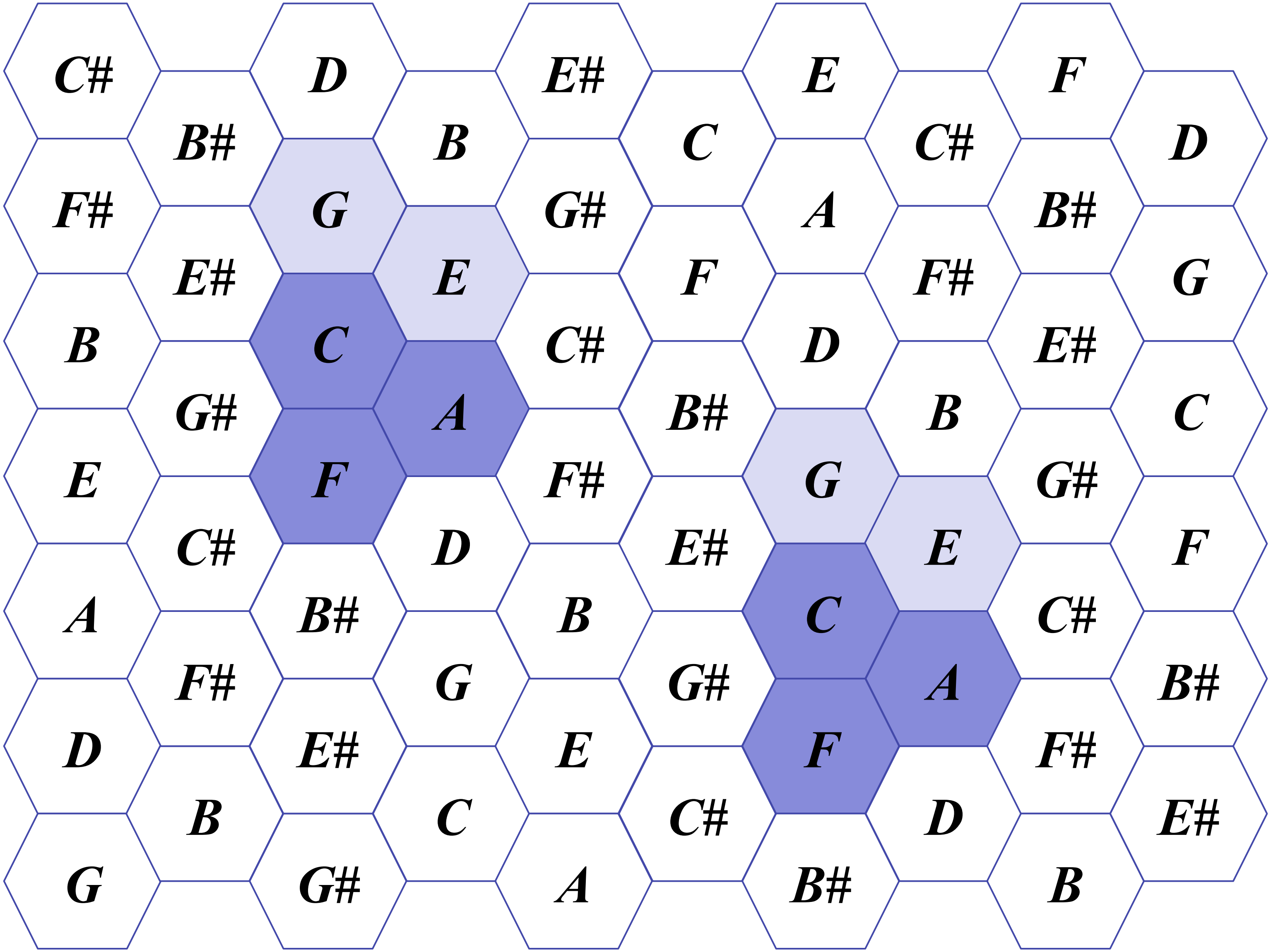


Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



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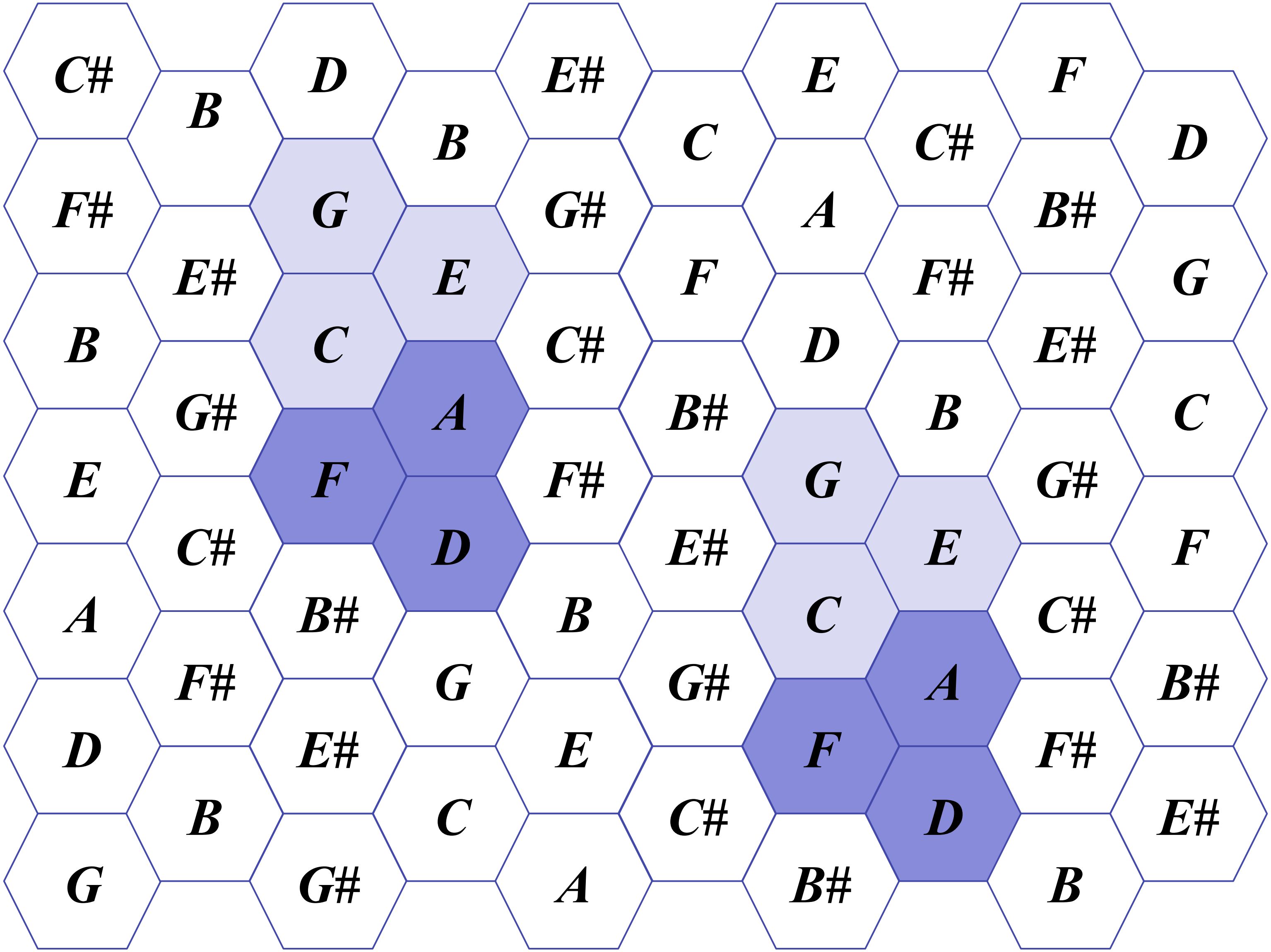

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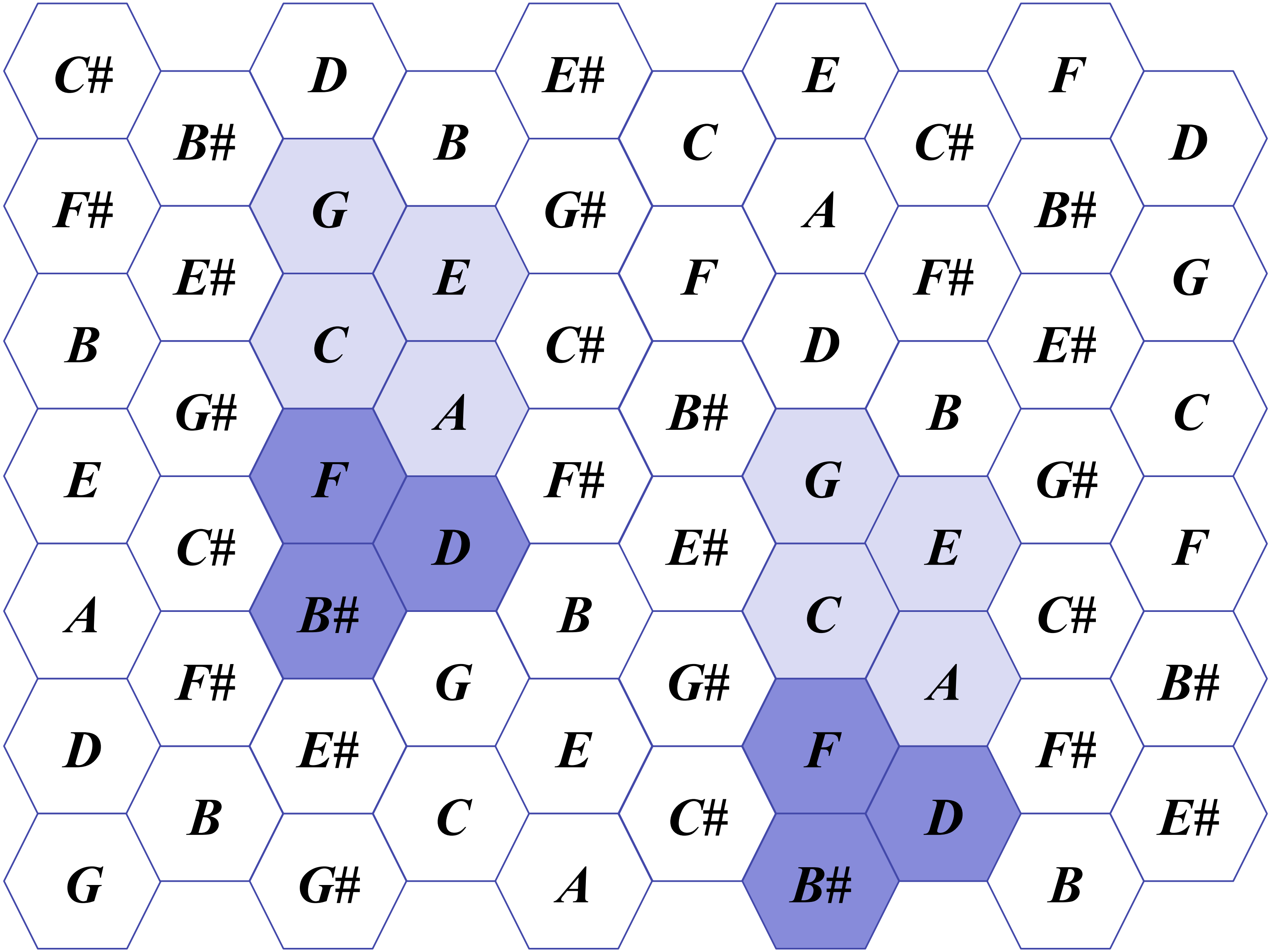

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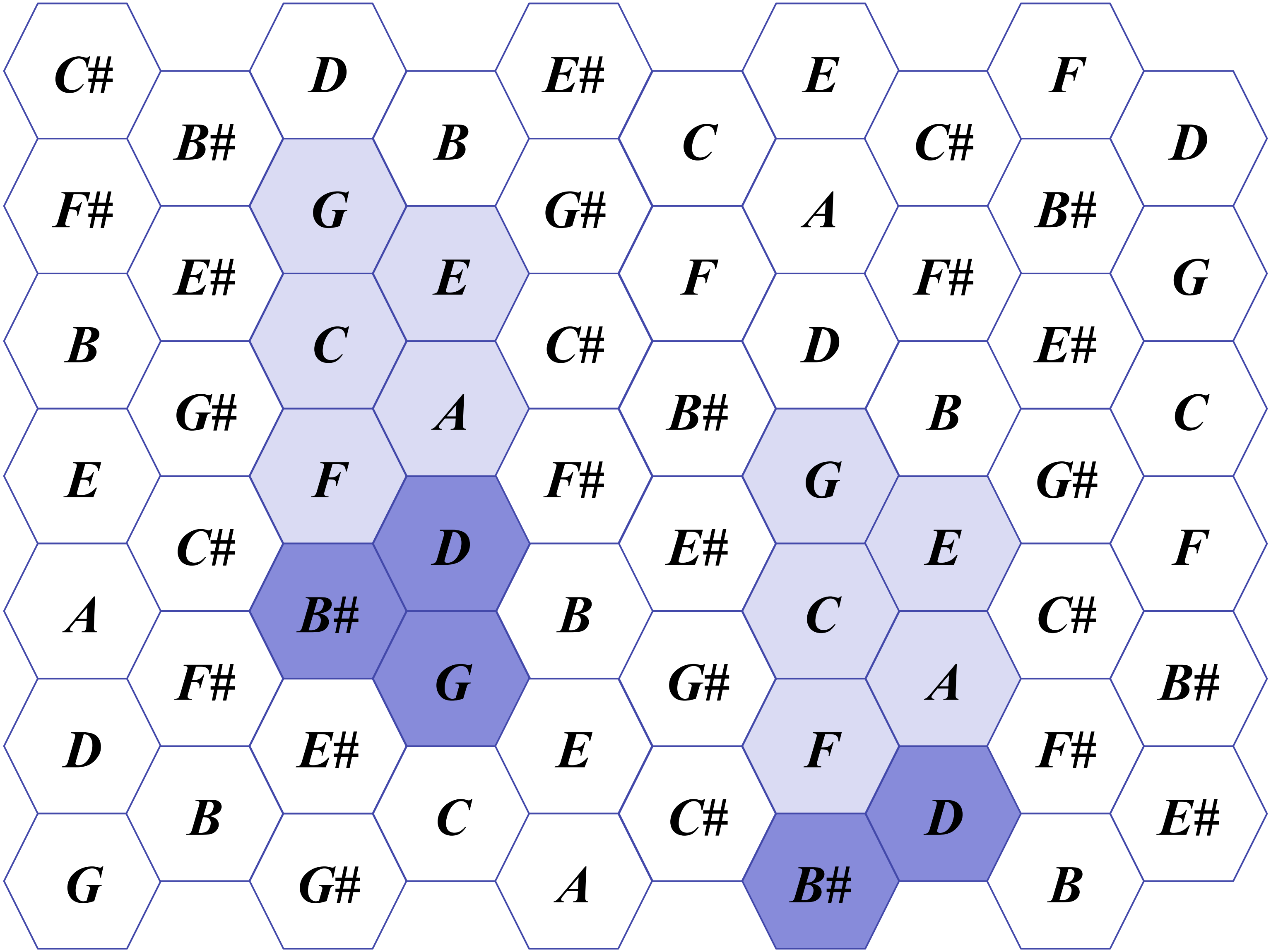

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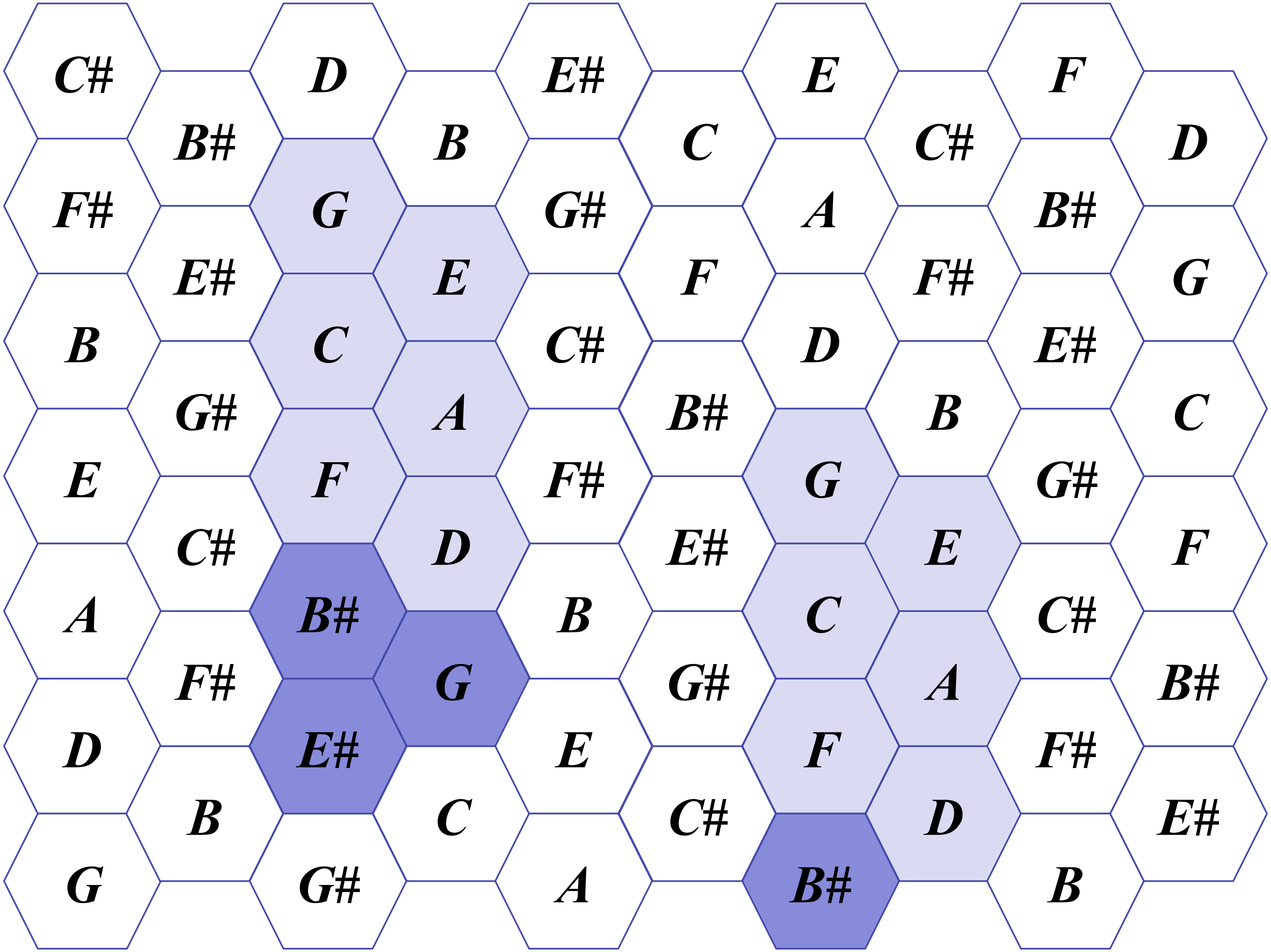

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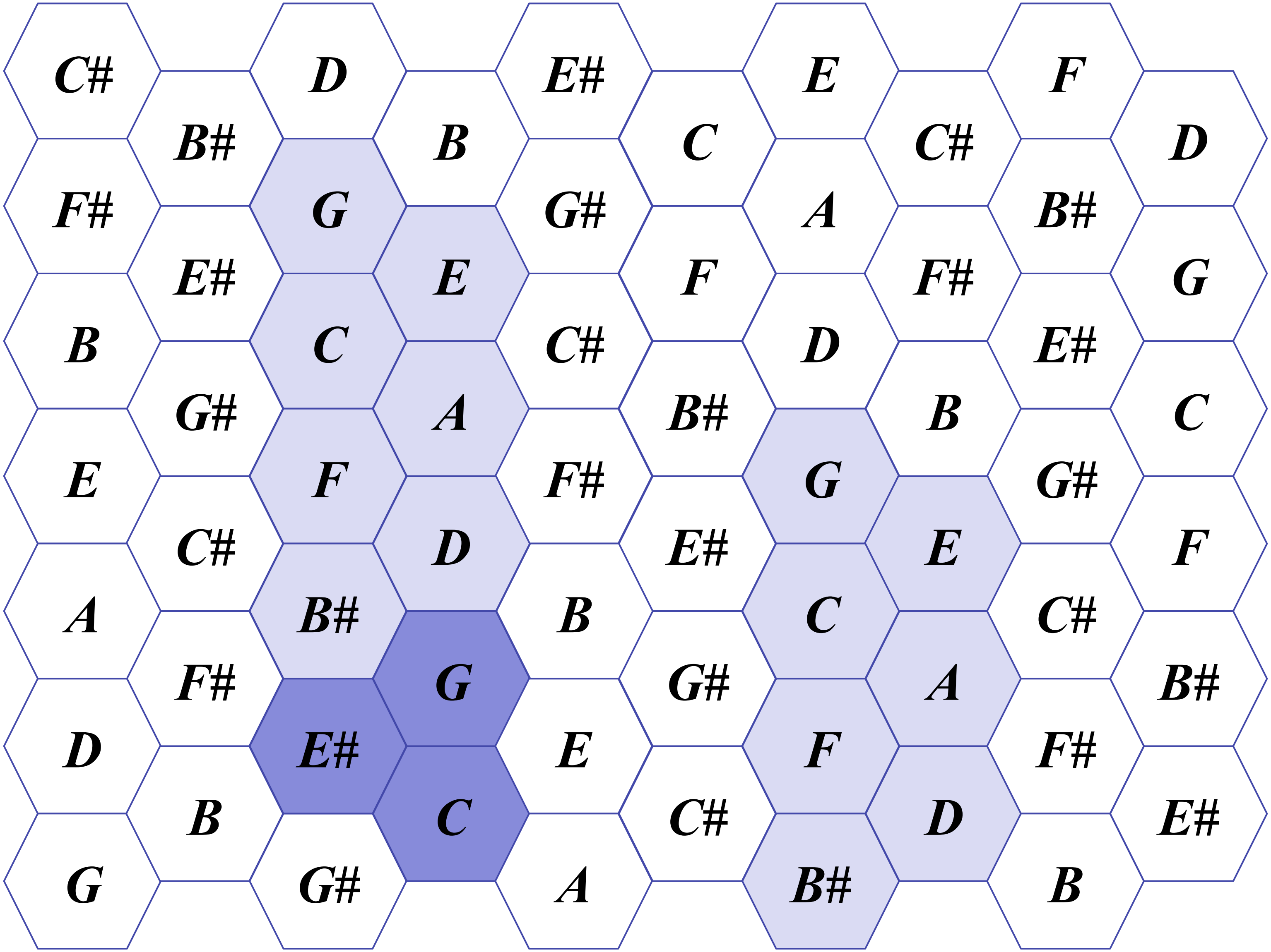

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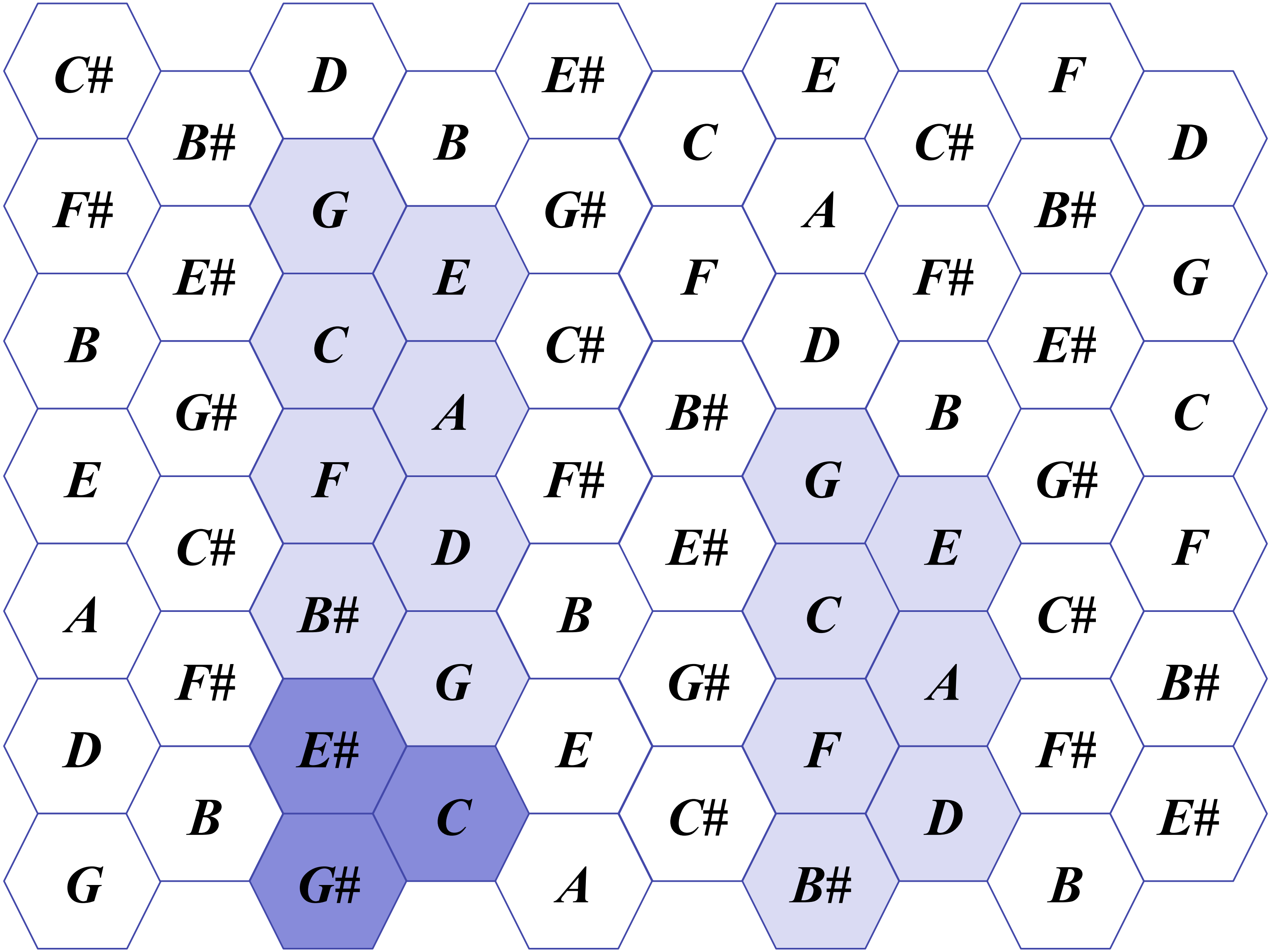
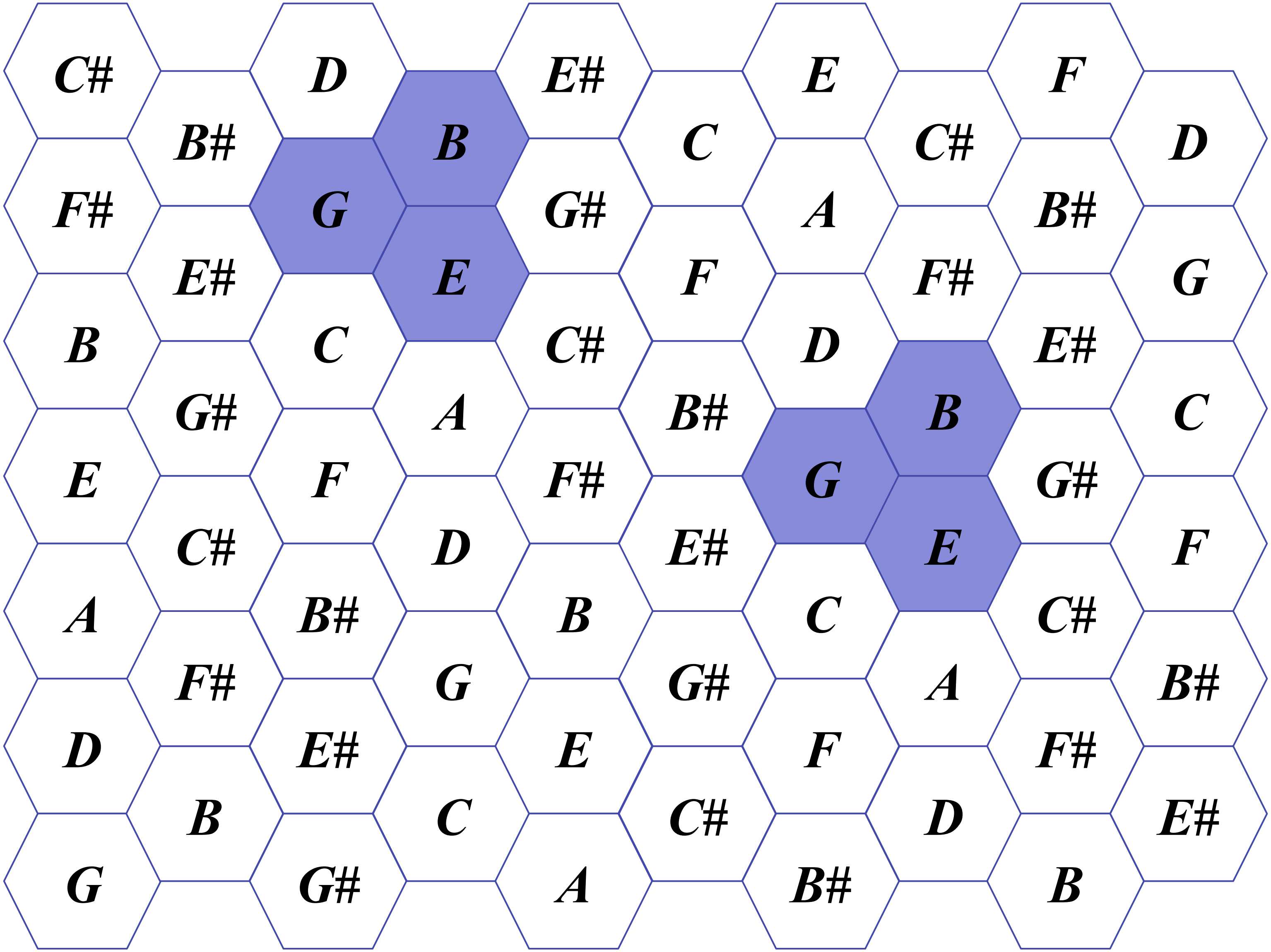


Extract of the Prelude Op.28 N.4
(F. Chopin)



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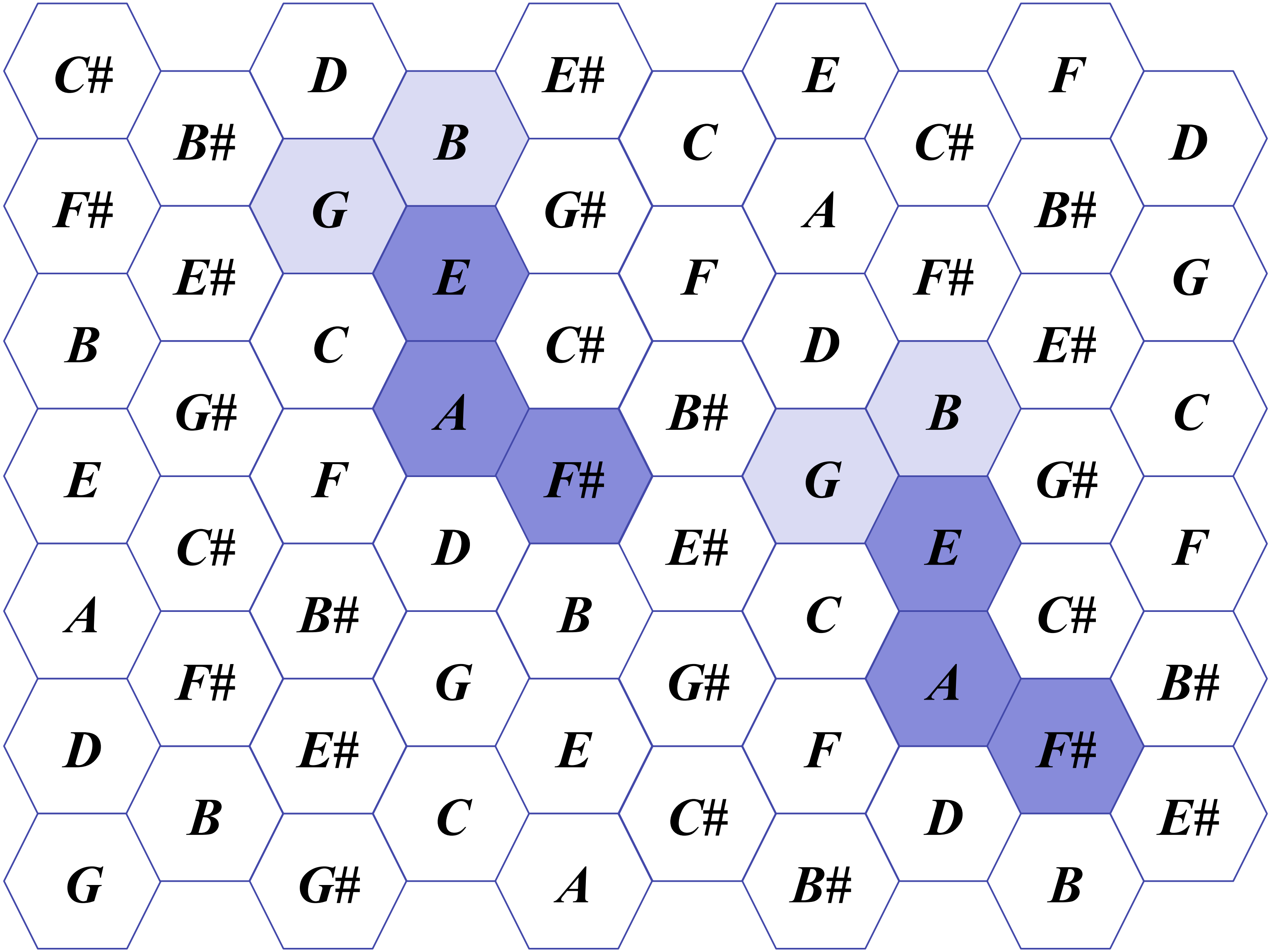

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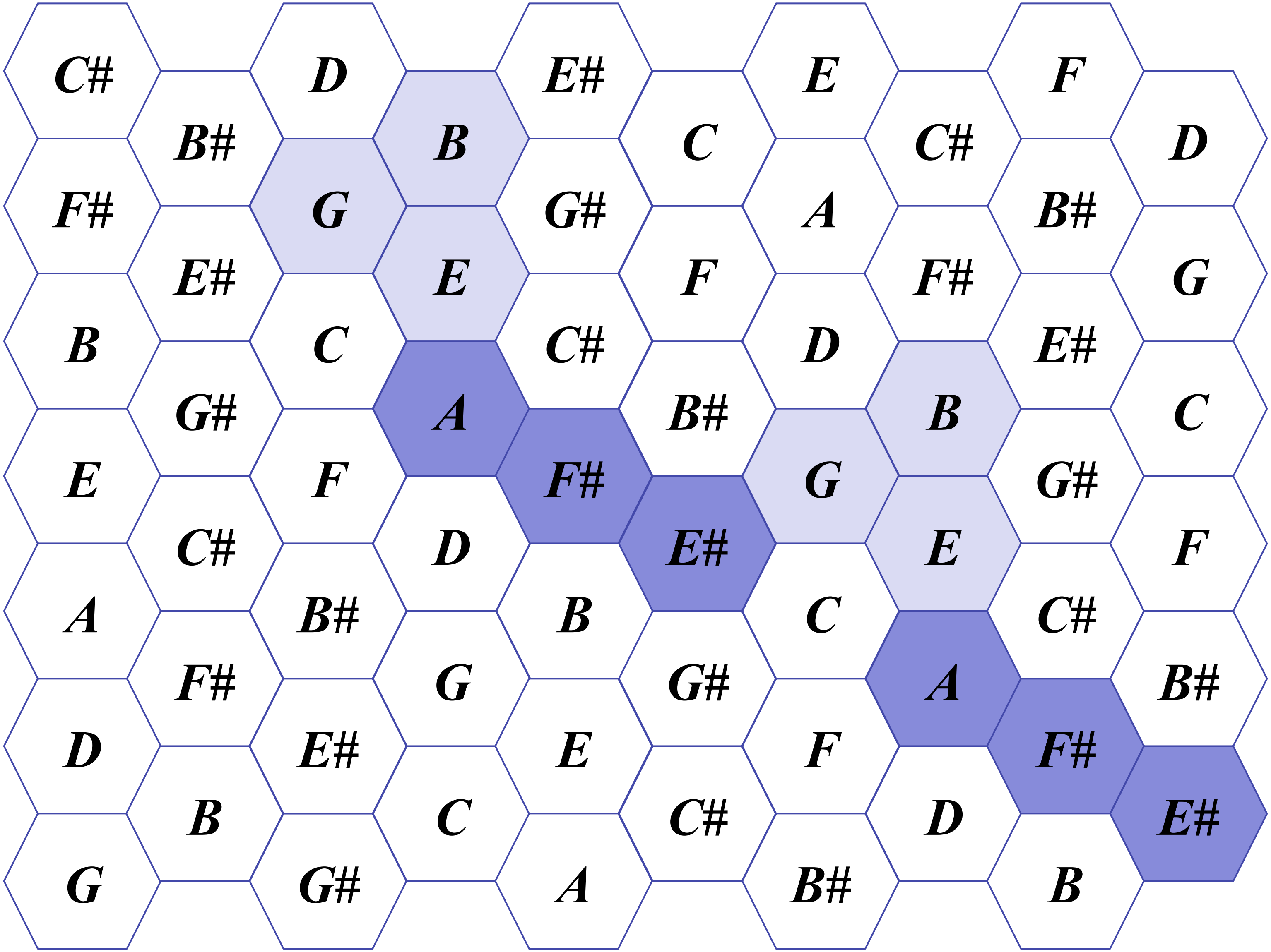

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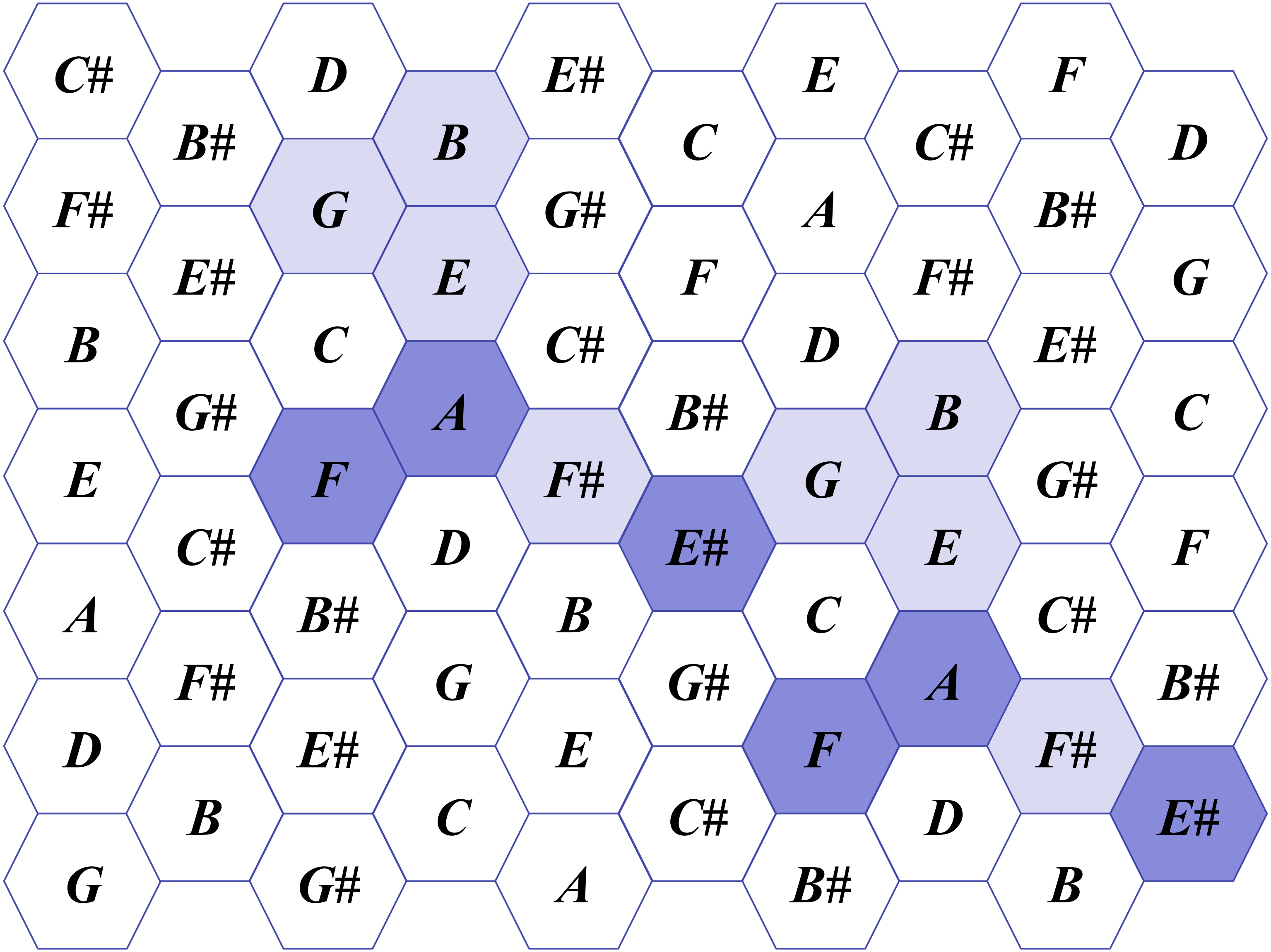

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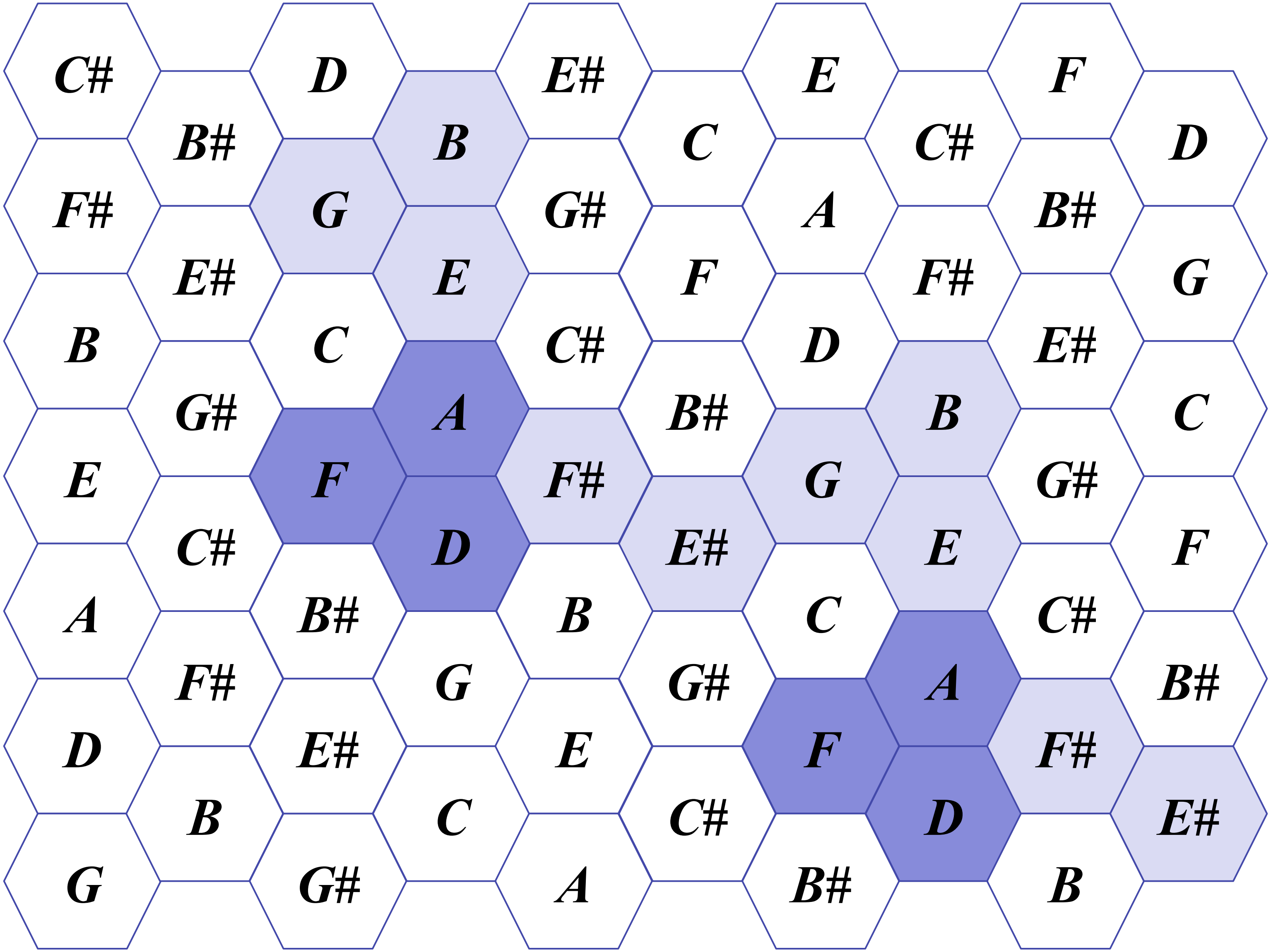

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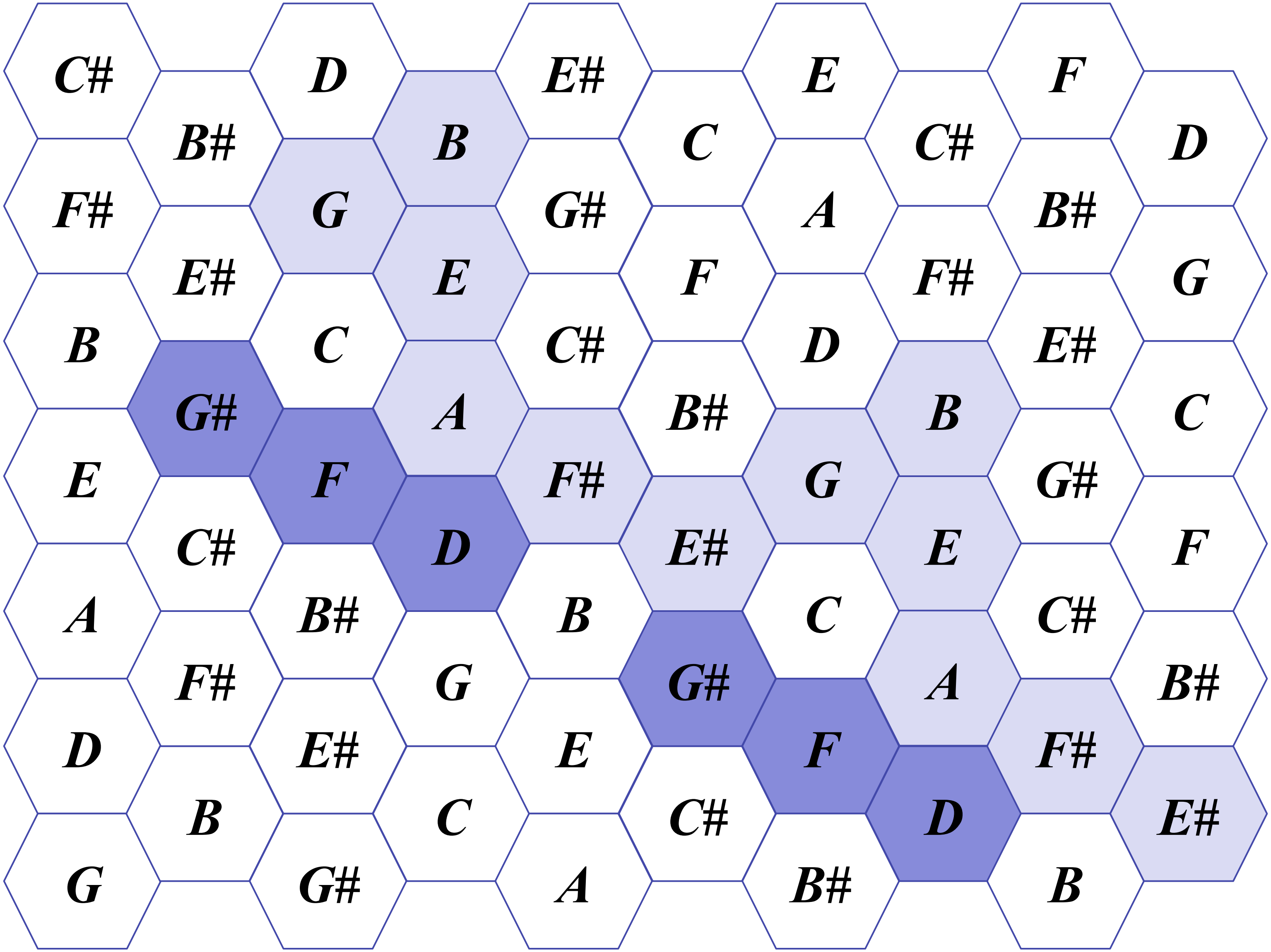

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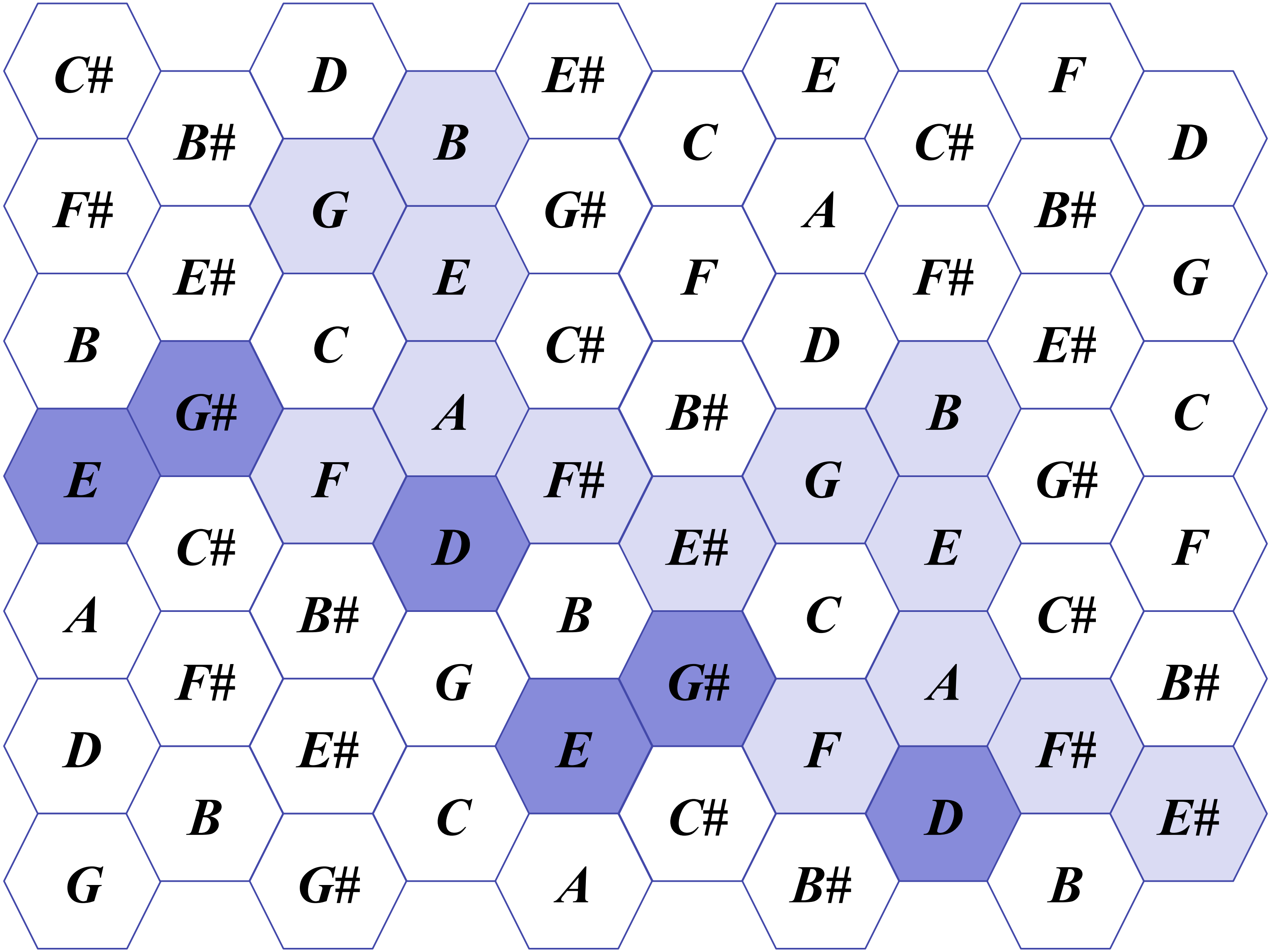

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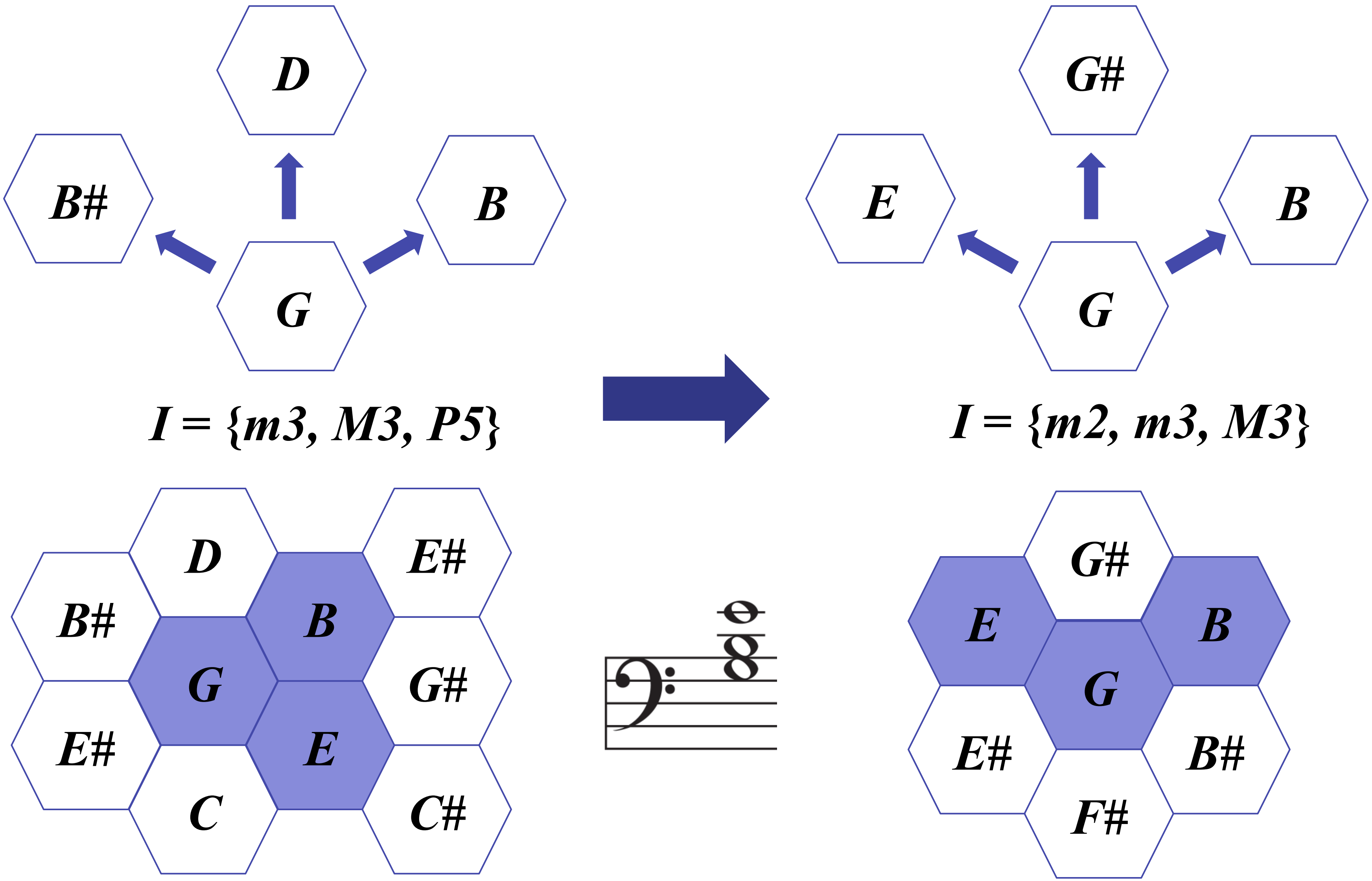

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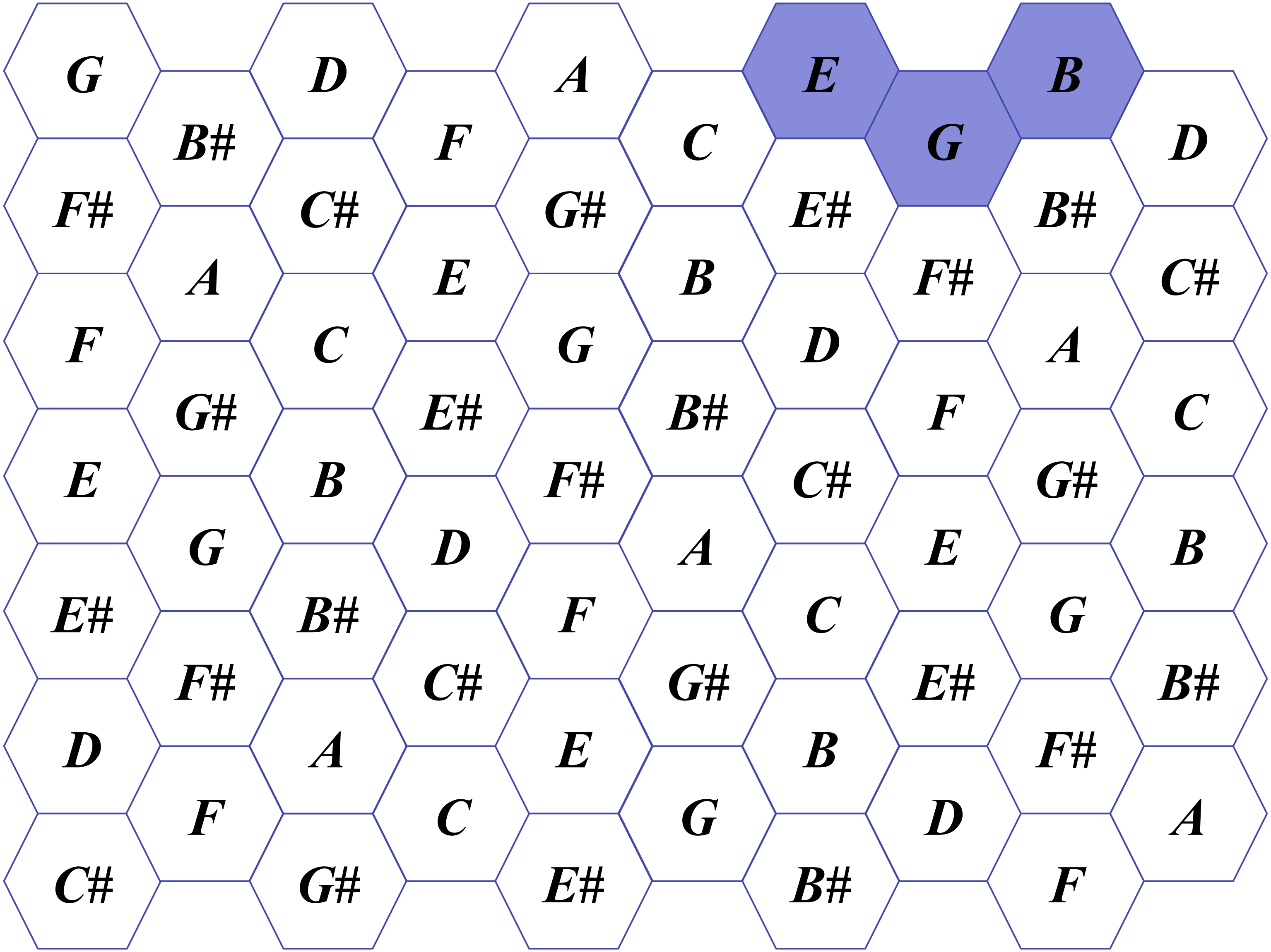

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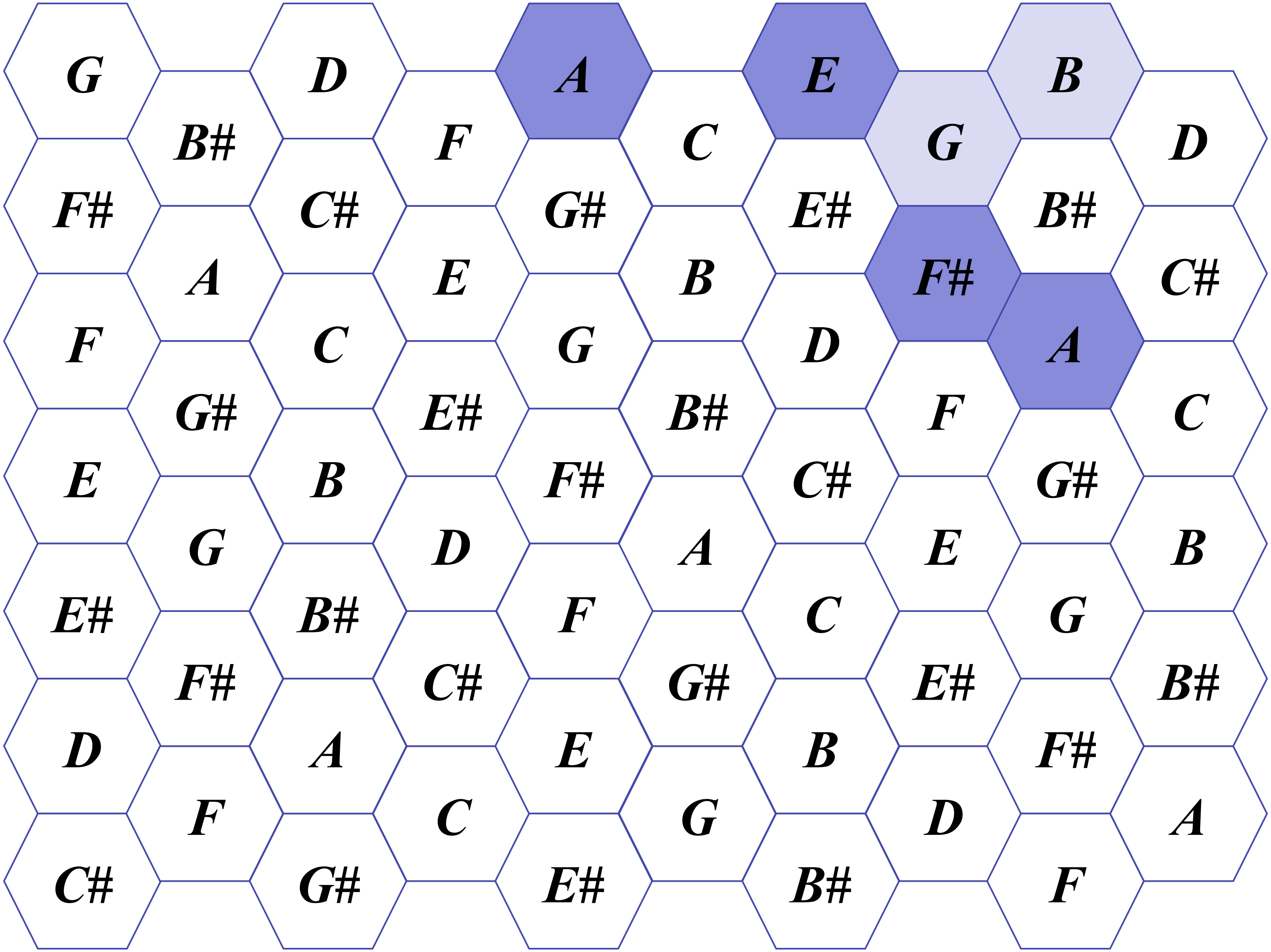

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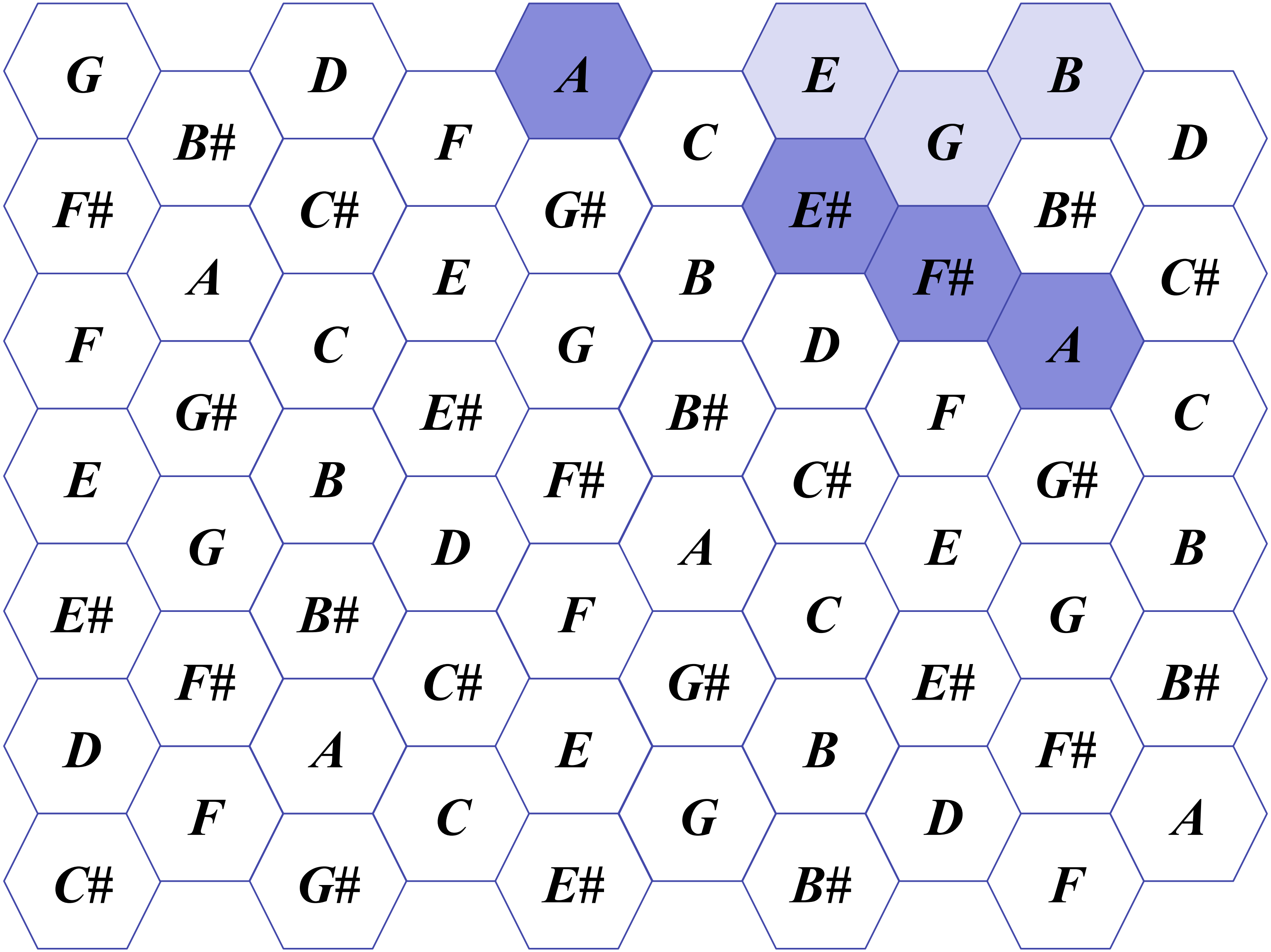

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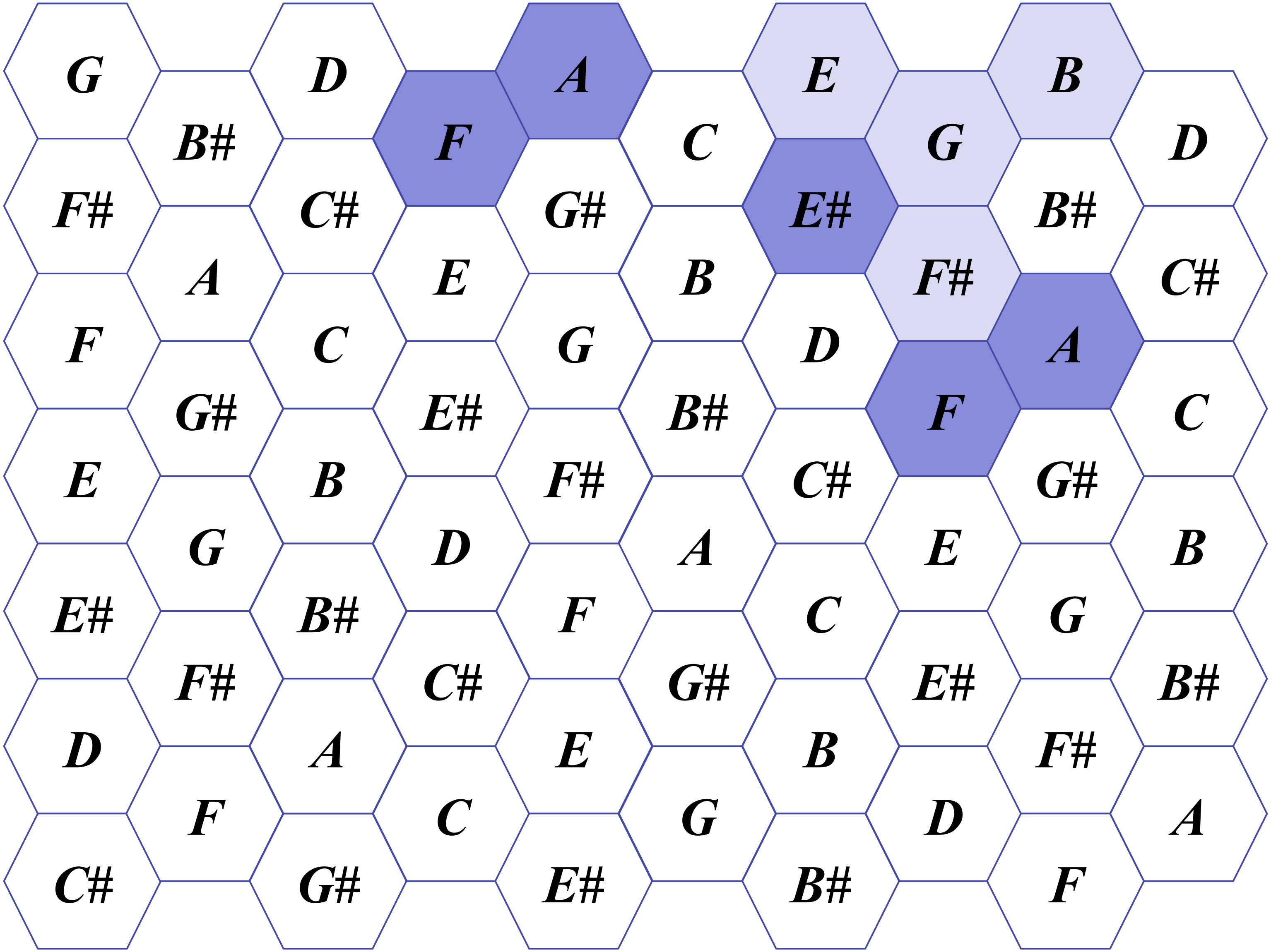

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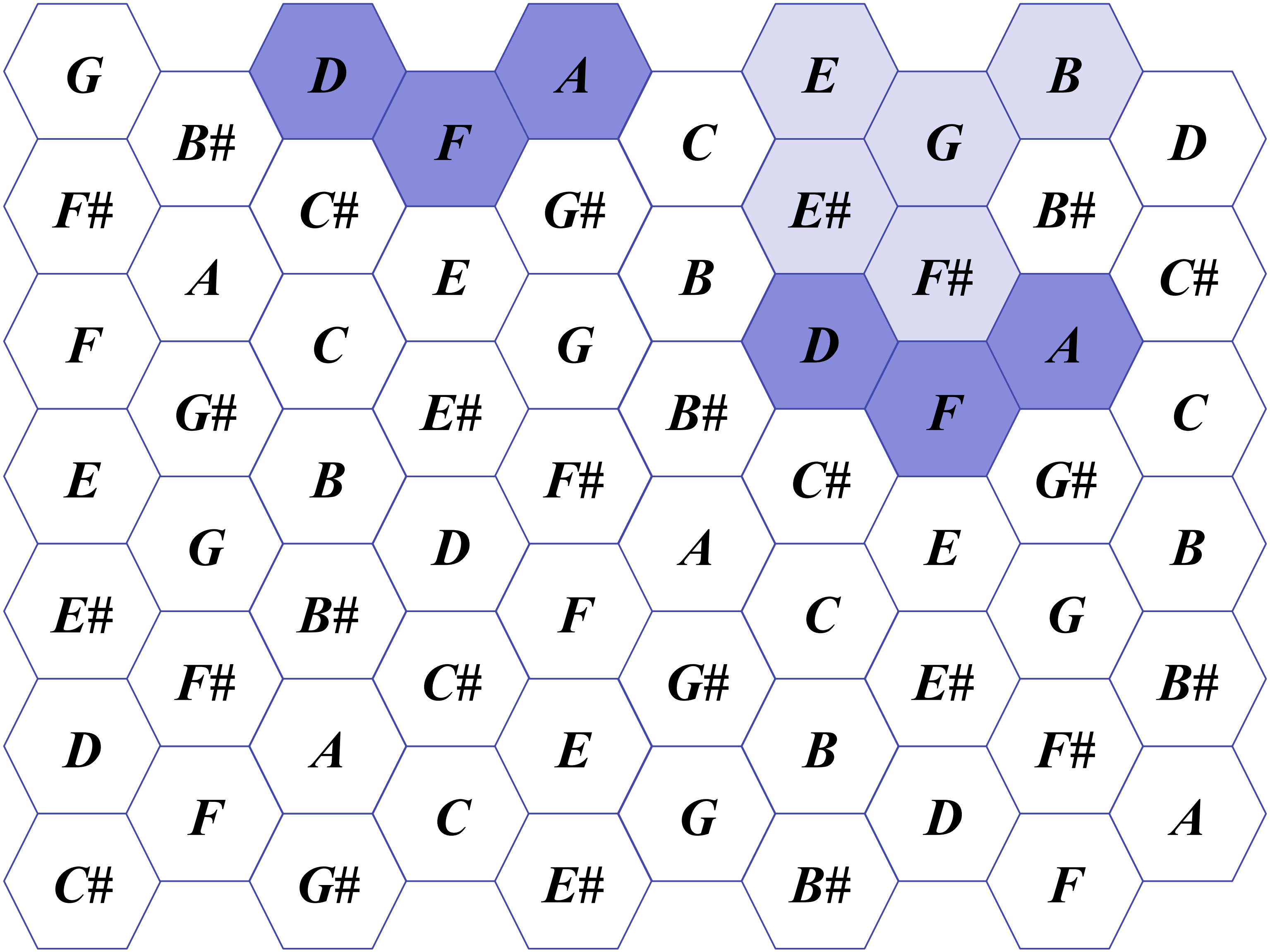

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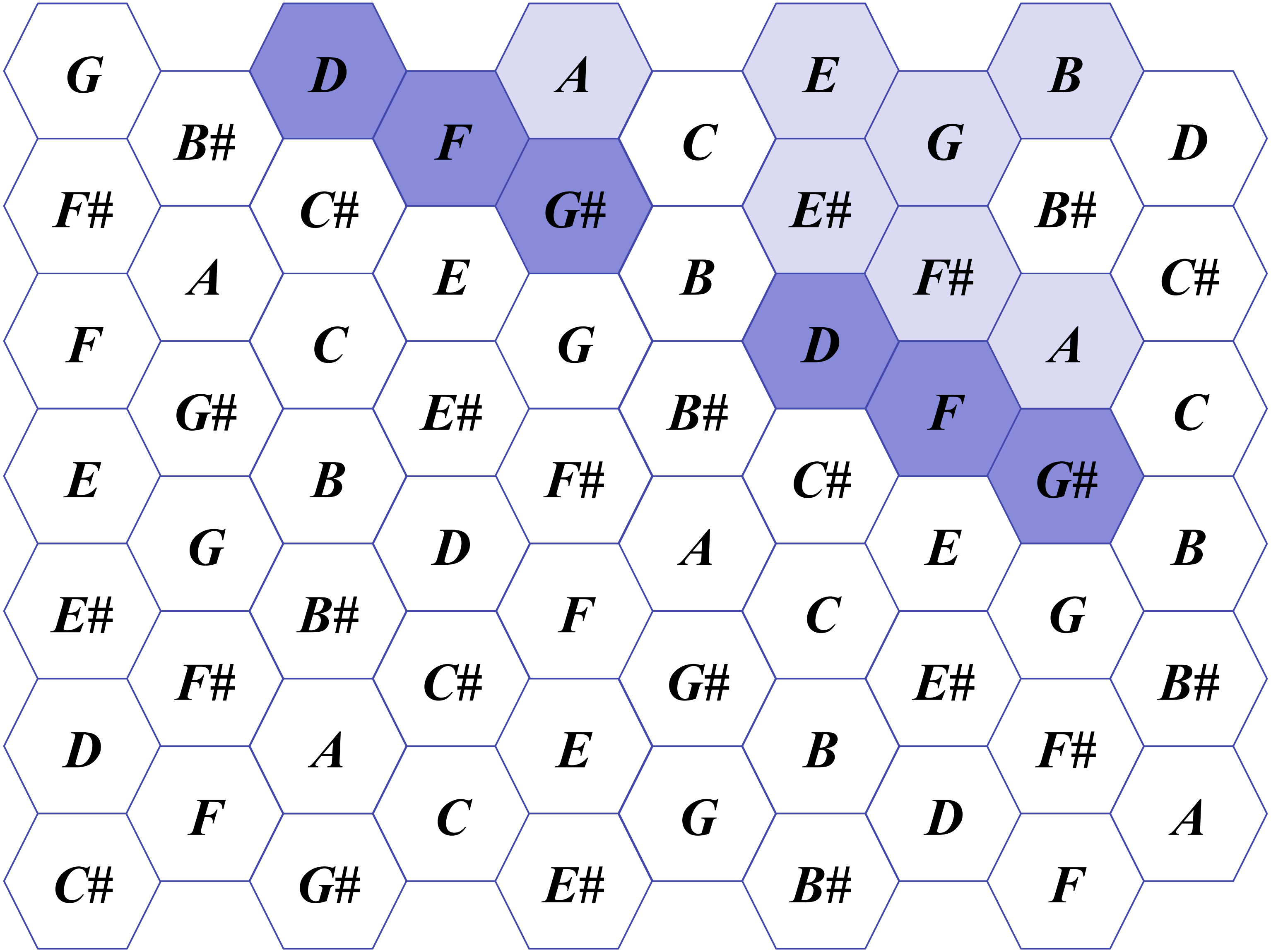

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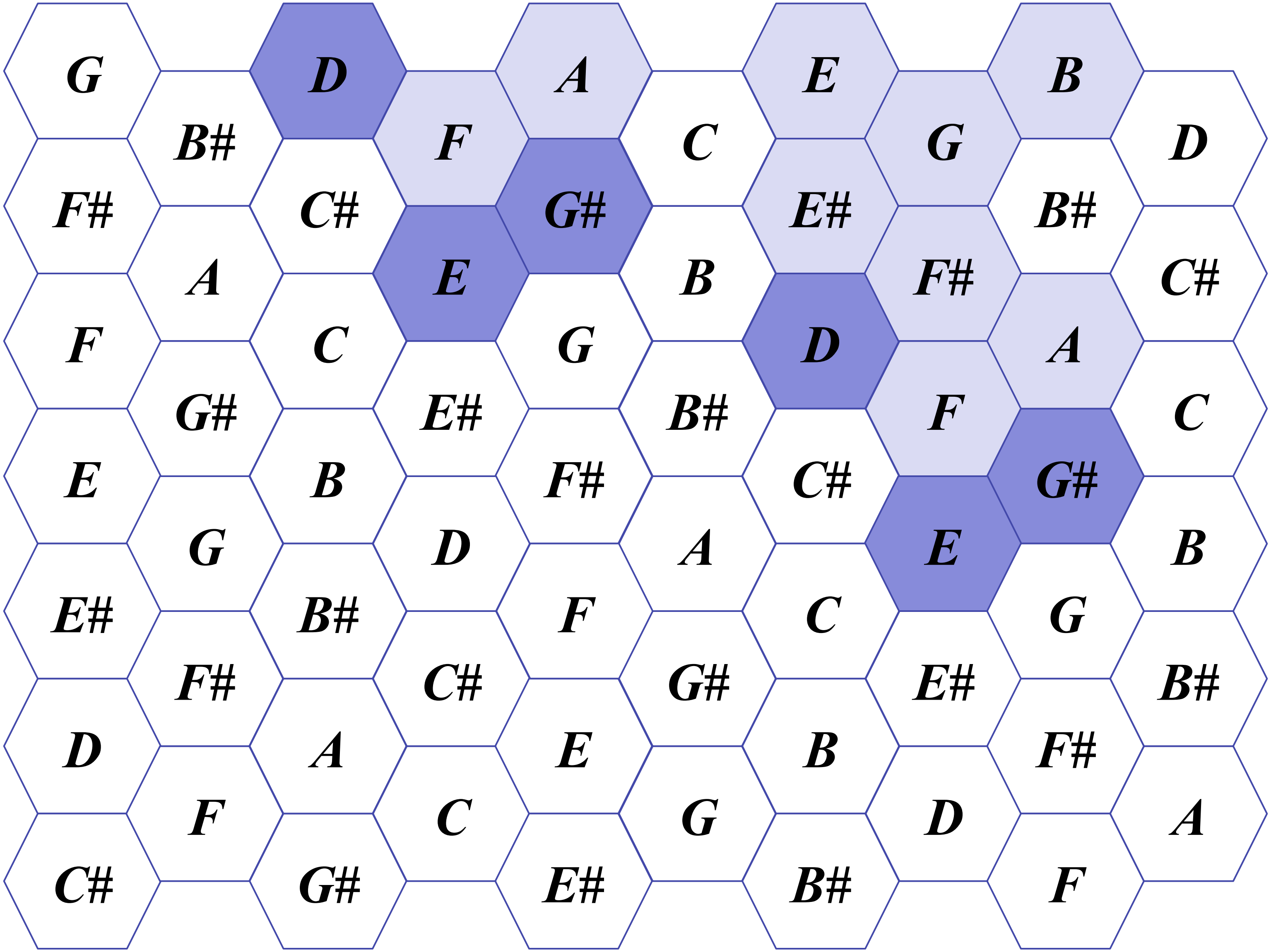

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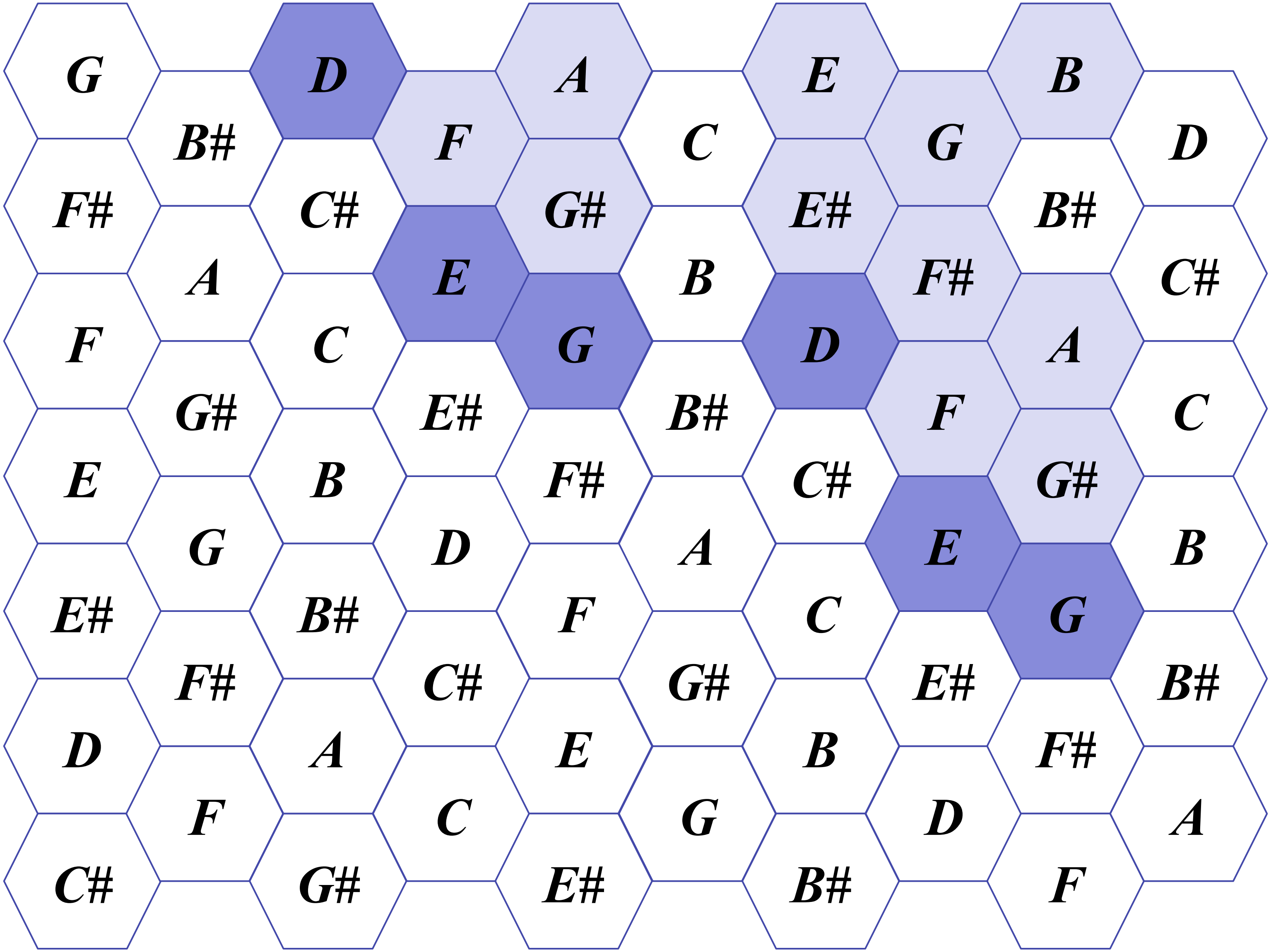

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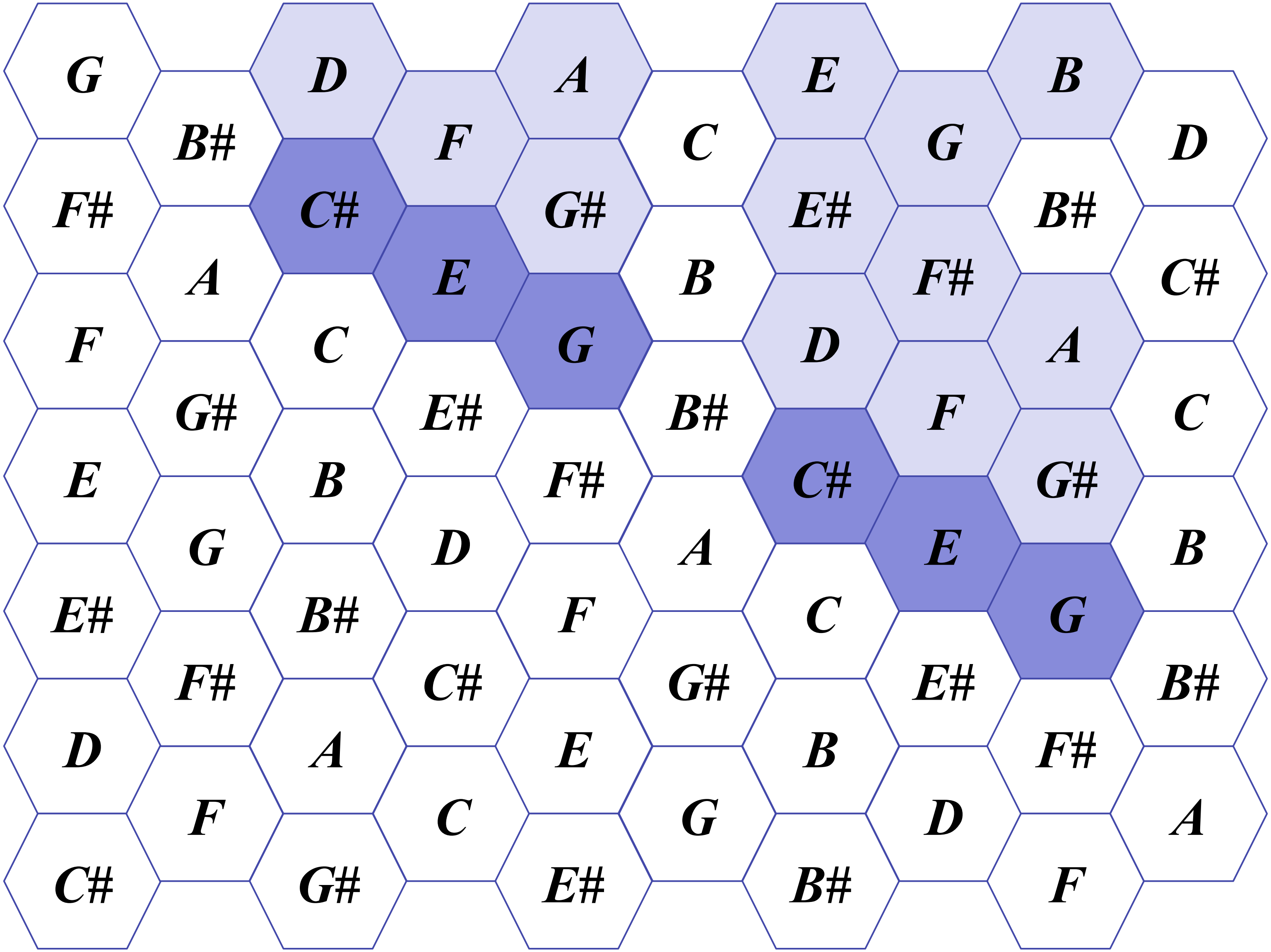

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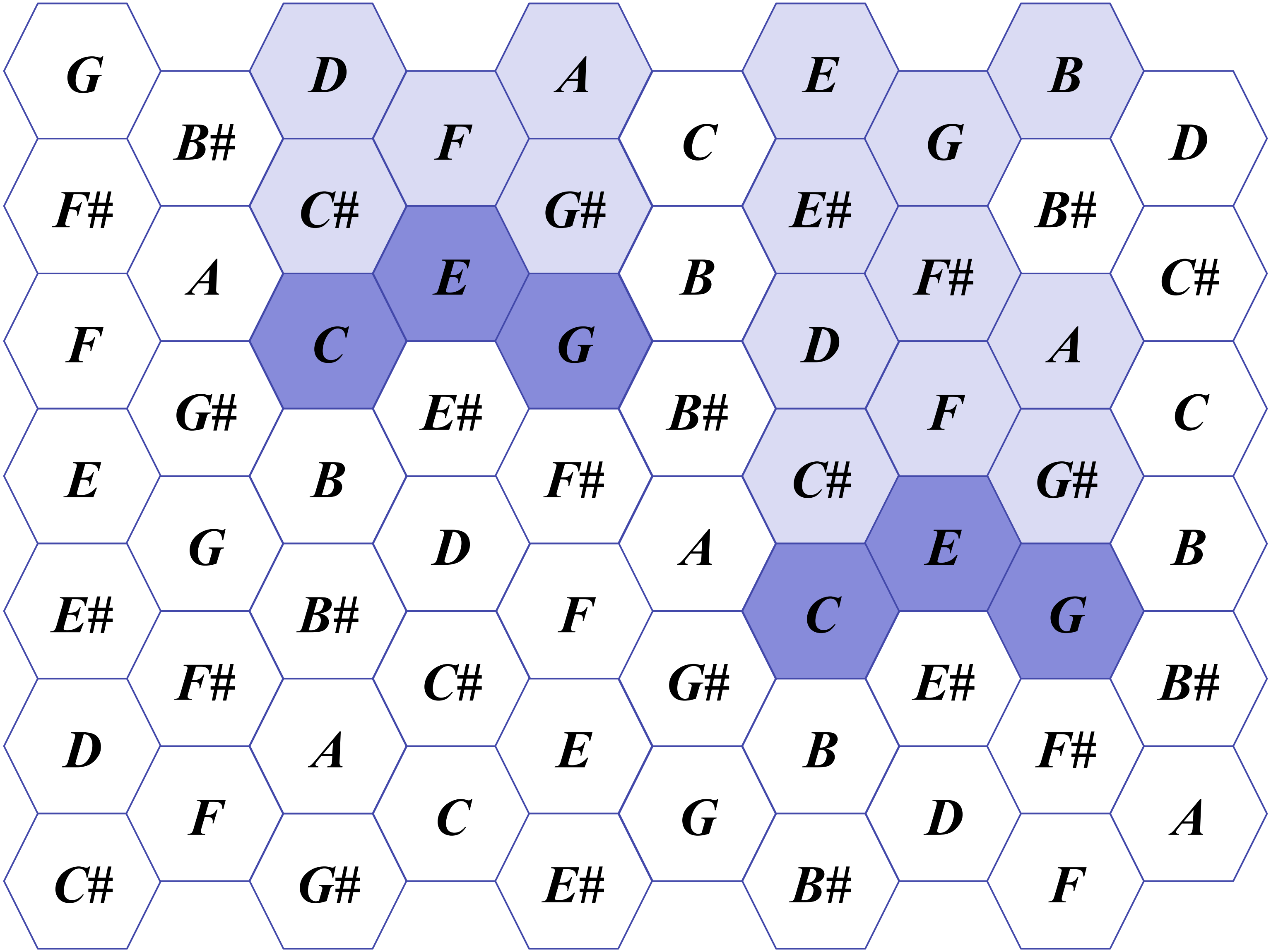

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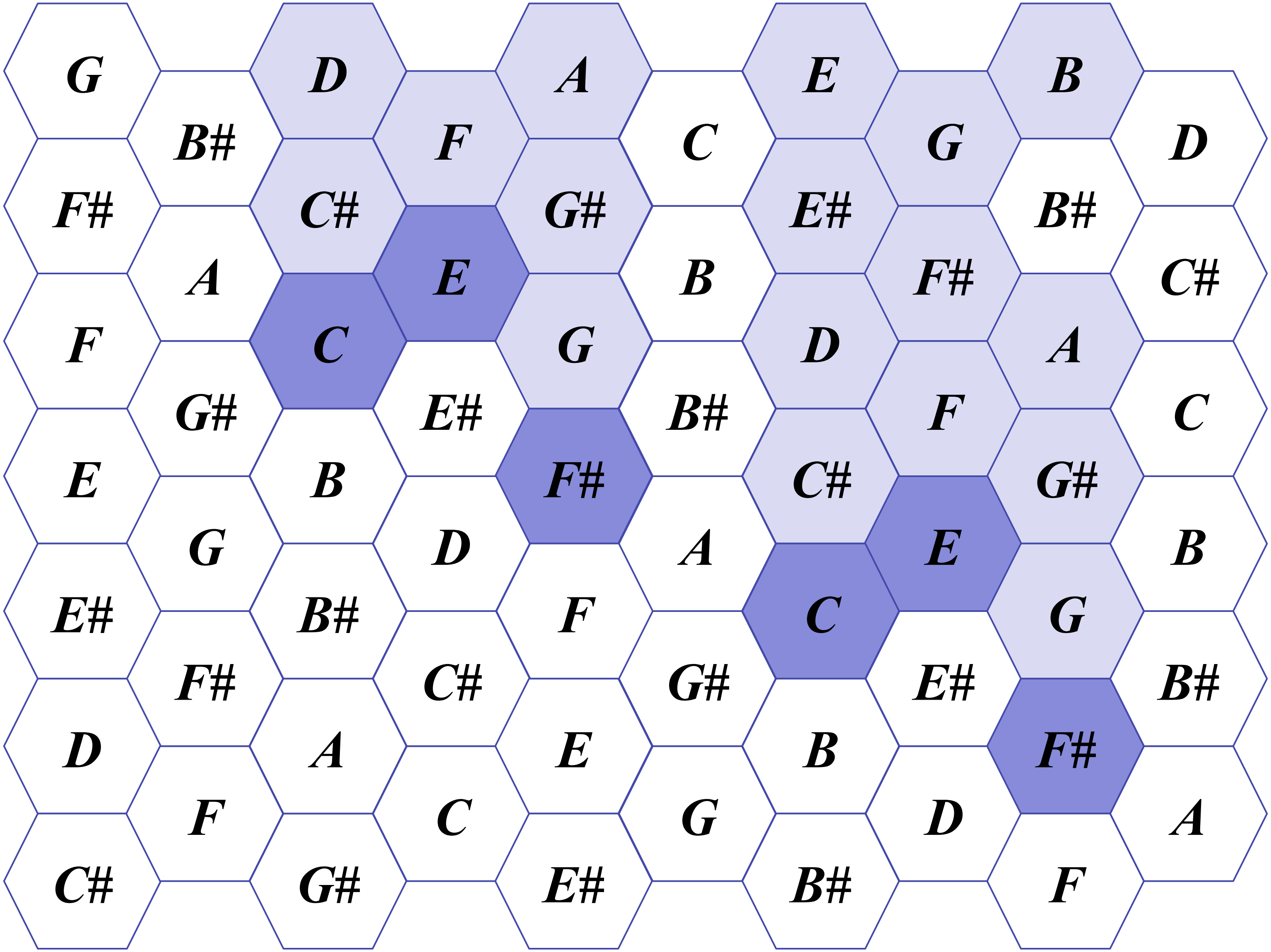

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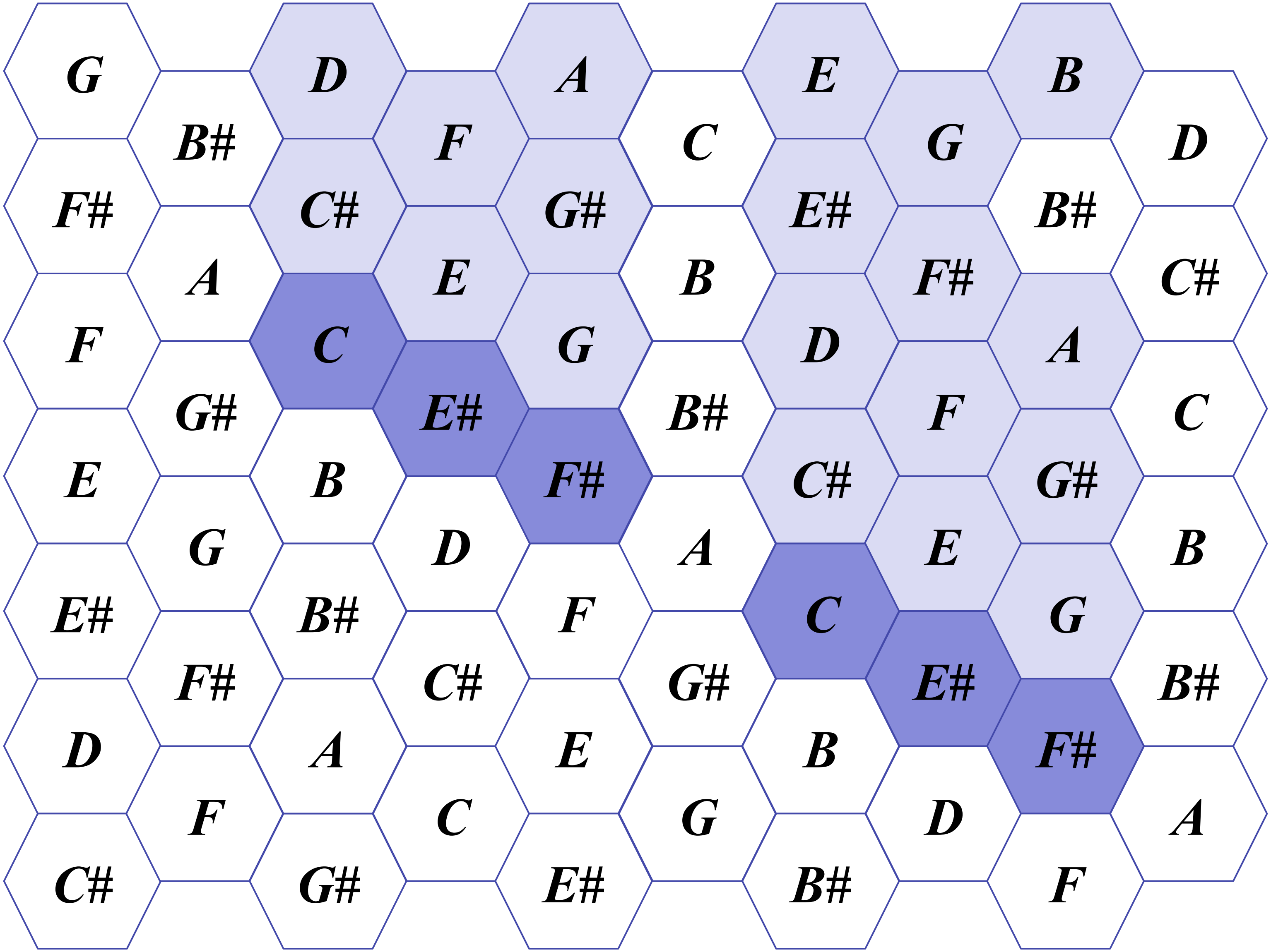

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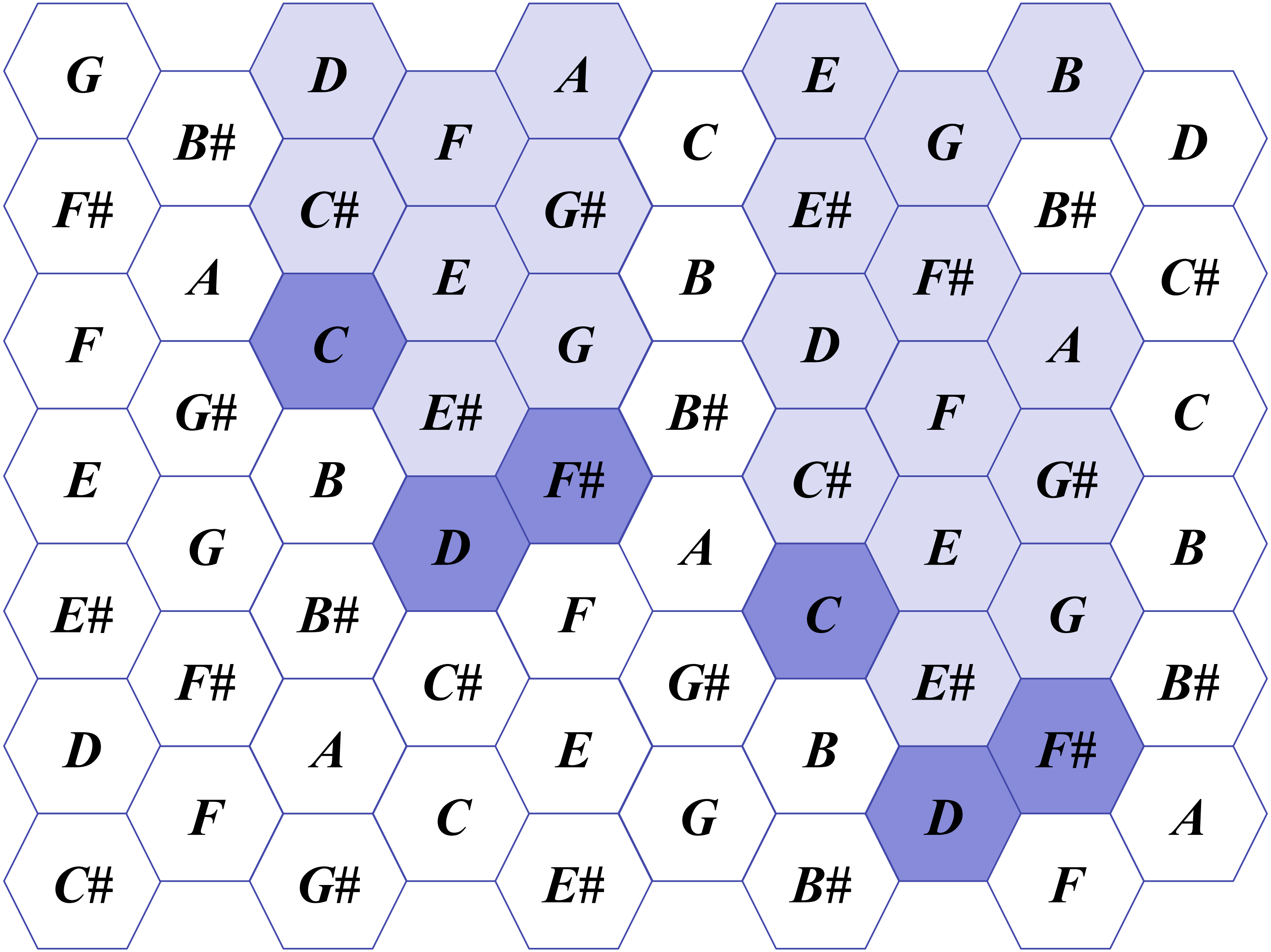

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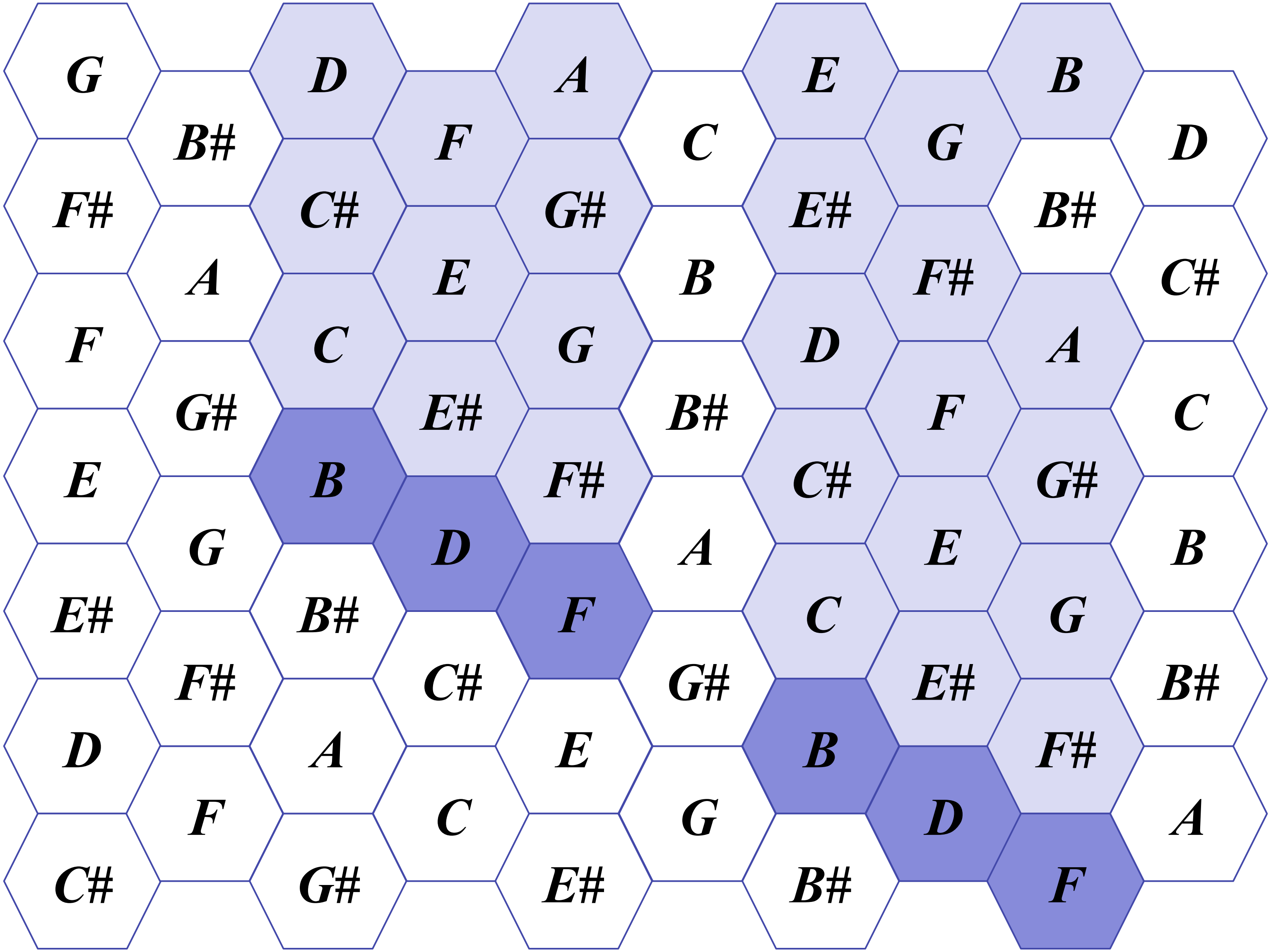
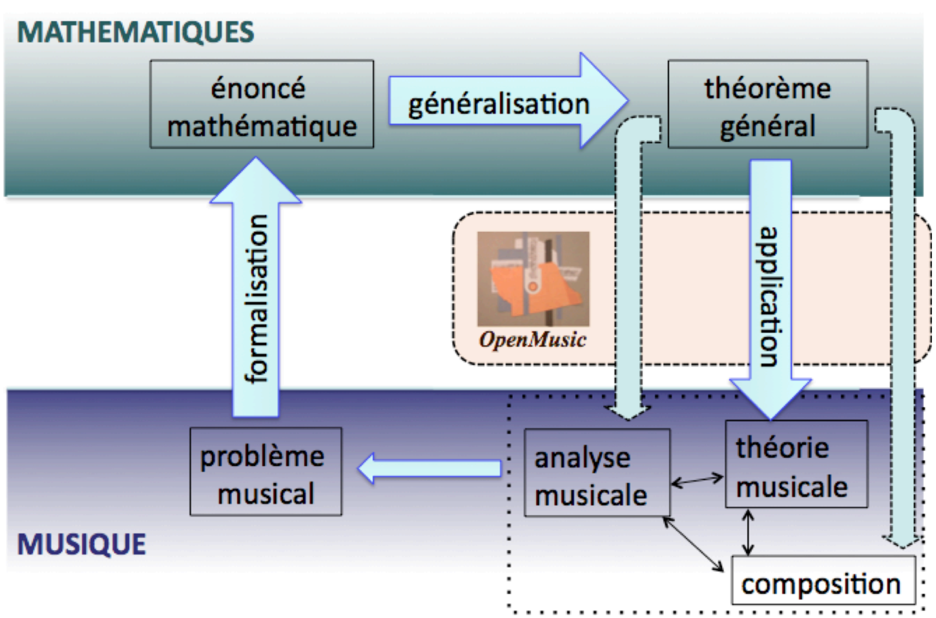
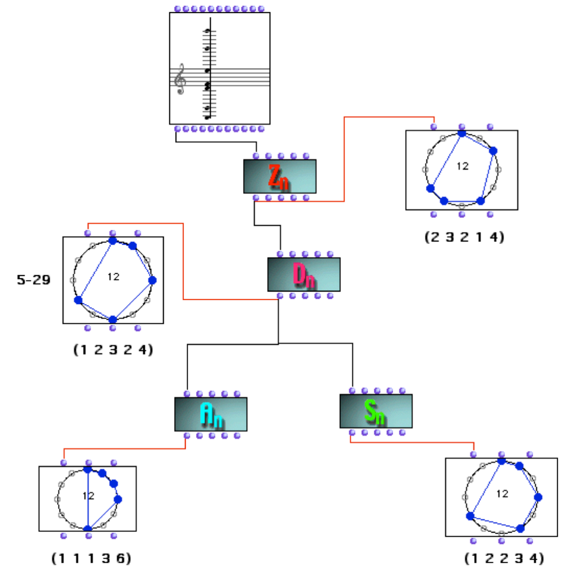
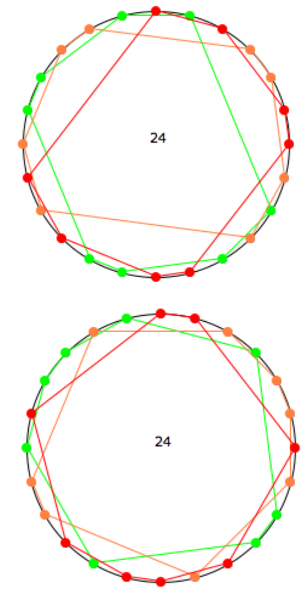


Quelques exemples de problèmes « mathémusicaux »





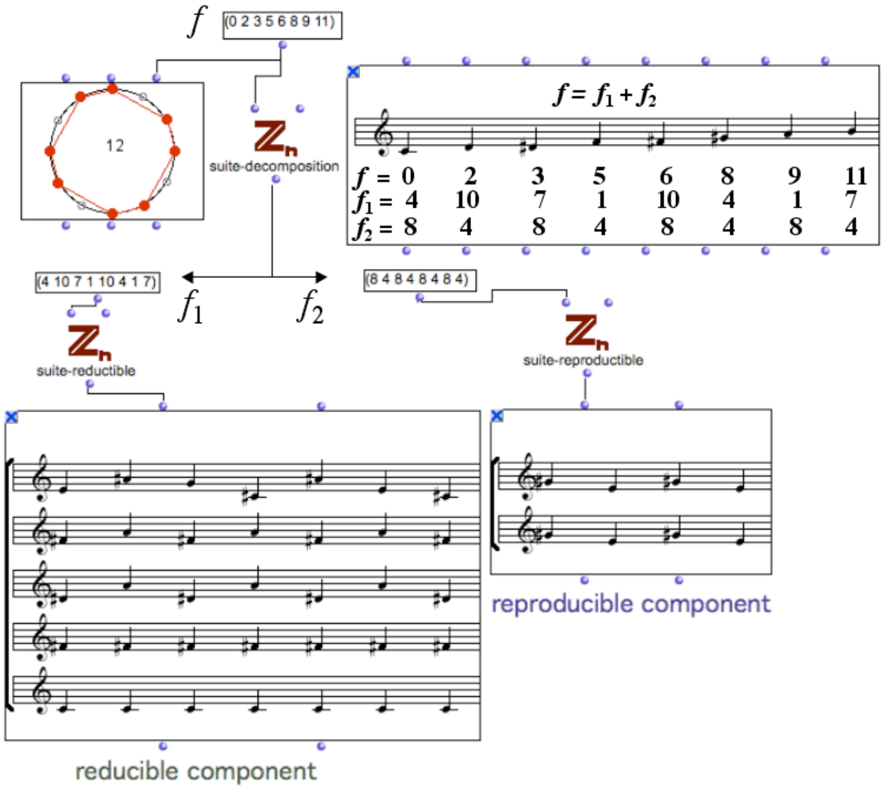
Actions de groupe et classification/ énumération des structures musicales



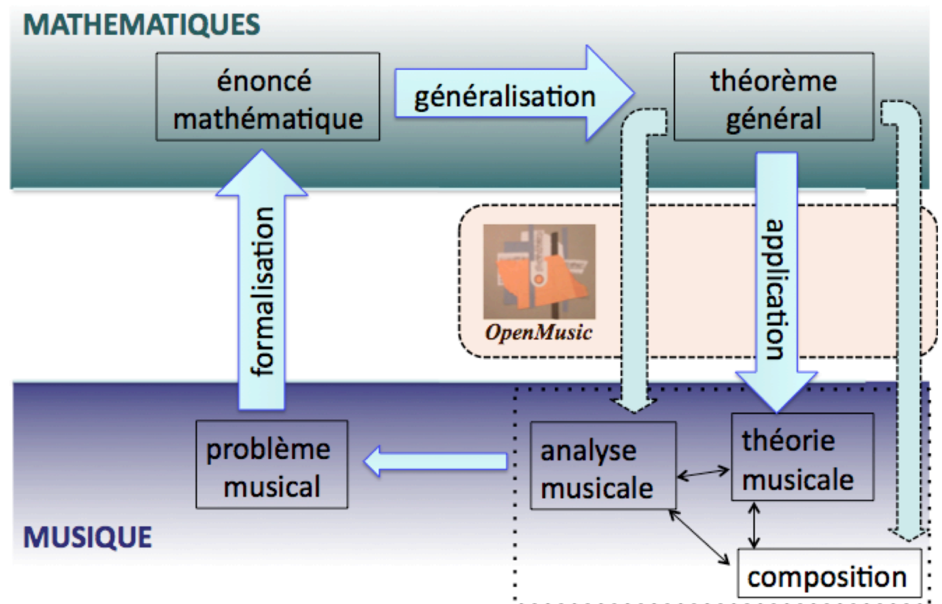
Pavages et homométrie

- *Set Theory* et théorie transformationnelle ;
- La construction des mosaïques et des pavages en musique ;
- La relation Z en musique, la DFT et l'homométrie ;
- La théorie des suites périodiques et le calcul des différences finies ;
- Les théories diatoniques et les ensembles maximalement repartis ;
- La théorie des *block-designs* en composition algorithmique ;
- …

Suites périodiques et calcul de différences finies



- **Théorème de décomposition**
- **Lemme de Fitting**



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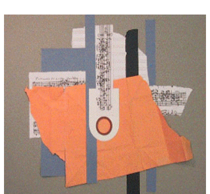


$Df(x)=f(x)-f(x-1)$.

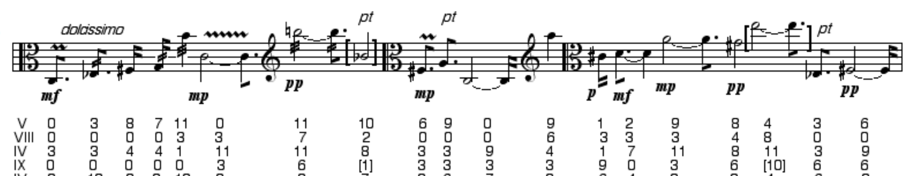
7 11 10 11 7 2 7 11 10 11 7 2 7 11…

4 11 1 8 7 5 4 11 1 8 7 5 4 11…

7 2 7 11 10 11 7 2 7 11 10 11…

7 5 4 11 1 8 7 5 4 11 1 8…

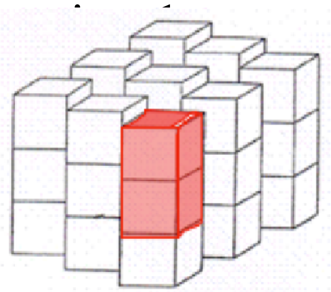
……



Les canons mosaïques comme problème « mathémusical »

Le problème de Minkowski/Hajos

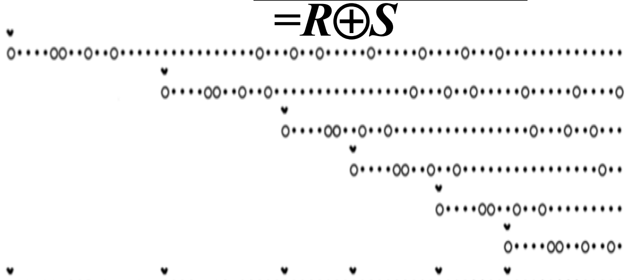
Dans un pavage simple [*simple lattice tiling*] d'un espace à *n* dimensions par des cubes unités, il y a au moins un couple de cubes qui ont en commun une face entière de



Les canons mosaïques de Vieru/Vuza

Un canon de Vuza est une factorisation d'un groupe cyclique en somme directe de deux sous-ensembles non-périodiques

$\mathbb{Z}/n\mathbb{Z} = R \oplus S$



Lien entre Minkowski et Vuza (Andreatta, 1996)

Groupes de Hajós (*good groups*)

$\mathbb{Z}/n\mathbb{Z}$ avec $n \in \{p^{\alpha}, p^{\alpha}q, pqr, p^2q^2, p^2qr, pqrs\}$ où p, q, r, s, sont des nombres premiers distincts

Groupes non-Hajós (*bad groups*)

72
108 120 144 168 180
200 216 240 252 264 270 280 288
300 312 324 336 360 378 392 396
400 408 432 440 450 456 468 480
500 504 520 528 540 552 560 576 588 594
600 612 616 624 648 672 675 680 684 696
700 702 720 728 744 750 756 760 784 792
800 810 816 828 864 880 882 888…

1907-1942 — **1991** — **1996**

M. Andreatta, « Constructing and Formalizing Tiling Rhythmic Canons: a historical Survey of a "Mathemusical" Problem », invited paper, Special Issue « Tiling Problems in Music », *Perspectives of New Music*, J. Rahn (ed.), University of Washington, Seattle (2011).

Classification « paradigmatique » des canons mosaïques

Résultat : uniquement deux « types » de canons différents (à une *transformation affine* près, i.e.
$f : Z_{72} \rightarrow Z_{72}$ t.q.
$f(x)=ax+b$ avec $a \in (Z_{72})^*$ et $b \in Z_{72}$

⇕

- **R. Tijdeman: "Decomposition of the Integers as a direct sum of two subsets", *Number Theory*, Cambridge University Press, 1995.**

The fundamental Lemma: *R* pave Z_n => *aR* pave Z_n $<a,n>=1$

R
(1 3 3 6 11 4 9 6 5 1 3 20)
(20 3 1 5 6 9 4 11 6 3 3 1)
(1 4 1 19 4 1 6 6 7 4 13 6)
(6 13 4 7 6 6 1 4 19 1 4 1)
(1 5 15 4 5 6 6 3 4 17 3 3)
(3 3 17 4 3 6 6 5 4 15 5 1)

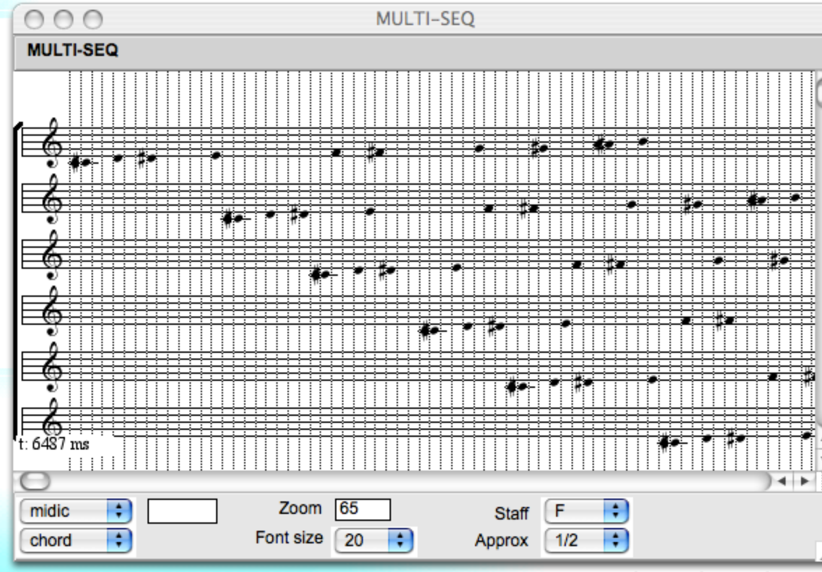
S
(8 8 2 8 8 38)
(16 2 14 2 16 22)
(14 8 10 8 14 18)

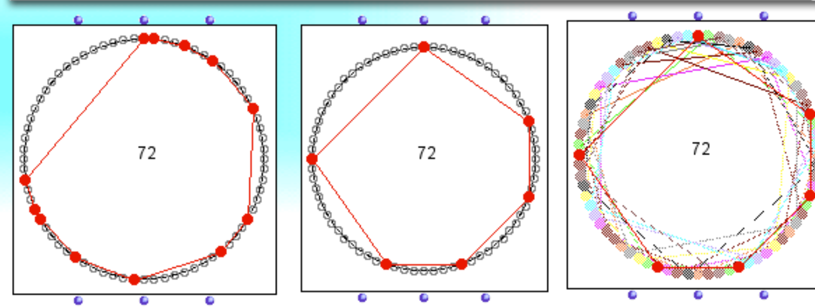
R
(1 3 3 6 11 4 9 6 5 1 3 20)
(1 4 1 19 4 1 6 6 7 4 13 6)
(1 5 15 4 5 6 6 3 4 17 3 3)

S
(8 8 2 8 8 38)
(16 2 14 2 16 22)
(14 8 10 8 14 18)

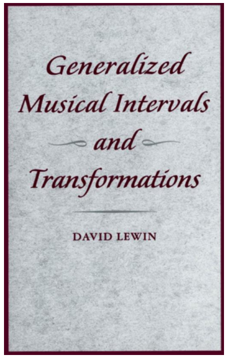
R
(1 3 3 6 11 4 9 6 5 1 3 20)
(1 4 1 19 4 1 6 6 7 4 13 6)

S
(14 8 10 8 14 18)





Z/72Z =R⊕S



Equivalence entre GIS et action de groupe

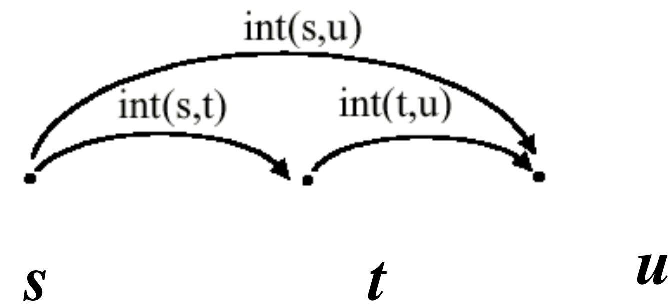
GIS = (S, G, int)

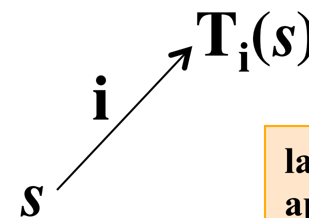
S = ensemble

(G,•) = groupe d'intervalles

int = fonction intervallique

$$S \times S \xrightarrow{\text{int}} G$$





Action simplément transitive

1. Pour tout objets s, t, u dans S : int(s,t)•int(t,u) = int(s,u)

2. Pour tout objet s dans S et tout intervalle i dans G il y a un seul objet t dans S tel que int(s,t) = i

la transposition T_i est une application $T_i : S \to S$ telle que int (s, T_i (s)) = i pour tout élément s dans S

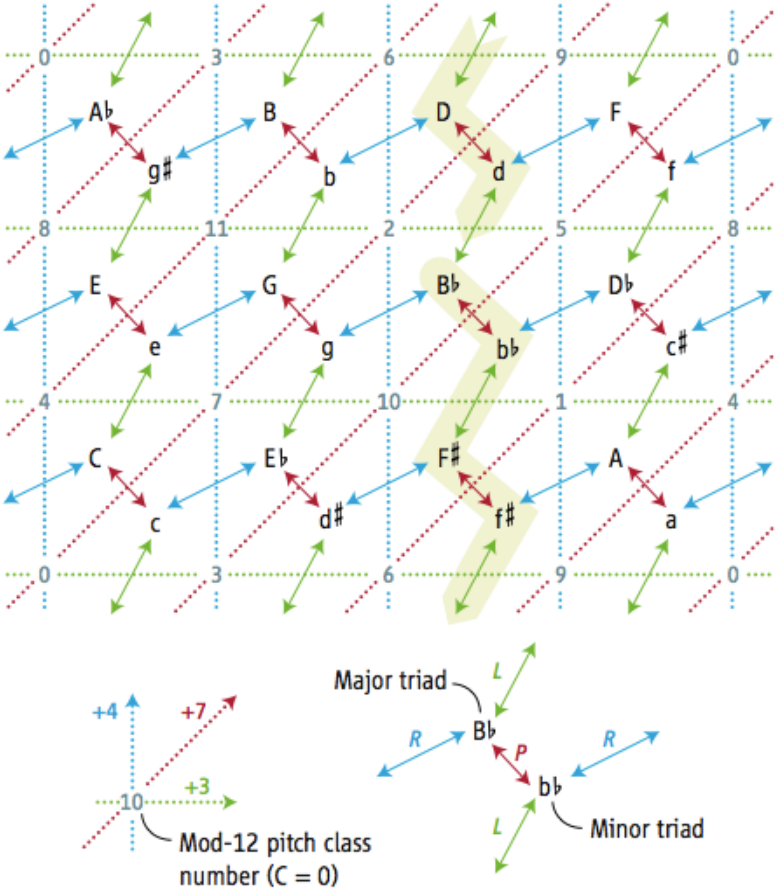
Soit $\tau = \{T_i \,;\, i \in G\}$ le groupe des transpositions

GIS = (S, G, int) $\Leftrightarrow$ $\tau \times S \to S$ telle que $(T_i, s) \to T_i(s)$

Terminologies équivalentes :

- Un GIS est un G-torseur à gauche
- S est un ensemble principal homogène [Bourbaki]

Le *Tonnetz* en tant que GIS



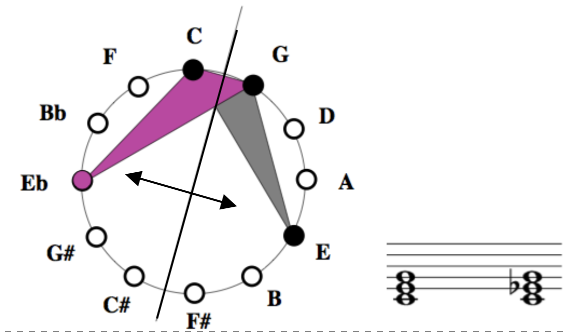
$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \; ; \; LRL = L(LR)^{-1} \rangle$

• ρ opère de façon simplement transitive sur l'ensemble *S* des 24 triades consonantes

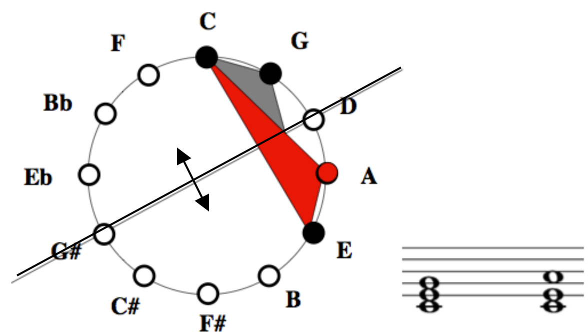
⇒ (*S*, ρ, int) est un GIS

(Neo-)Riemannian Operation **P** = „**Parallel**“

[Noll04]



(Neo-)Riemannian Operation **R** = „**Relative**“



(Neo-)Riemannian Operation **L** = „**Leading-Tone**“