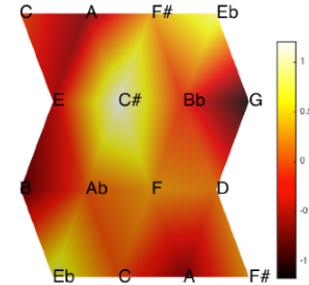


Theme : Human and Artificial Creativity (HAC)

# Mathemusal Morphologies

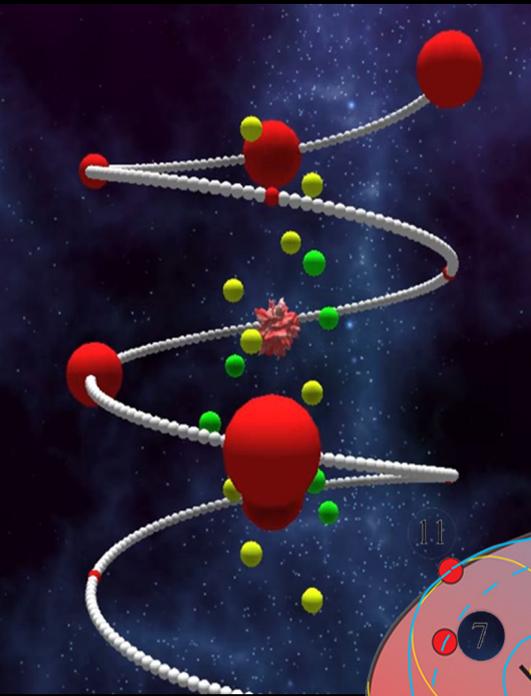


Moreno ANDREATTA<sup>1</sup>, Jean-Louis GIAVITTO<sup>2</sup> *et al.*<sup>3</sup>

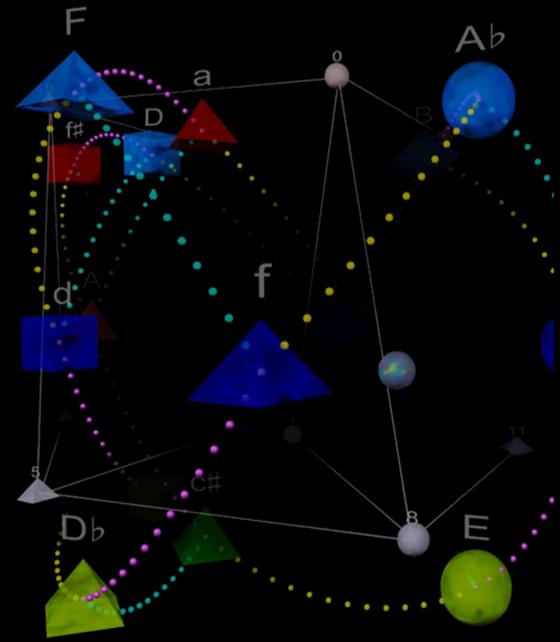
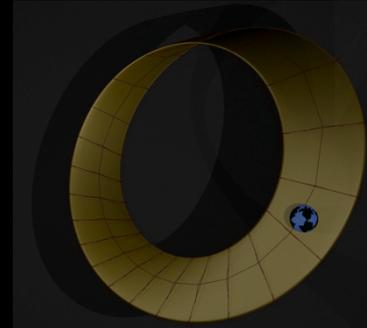
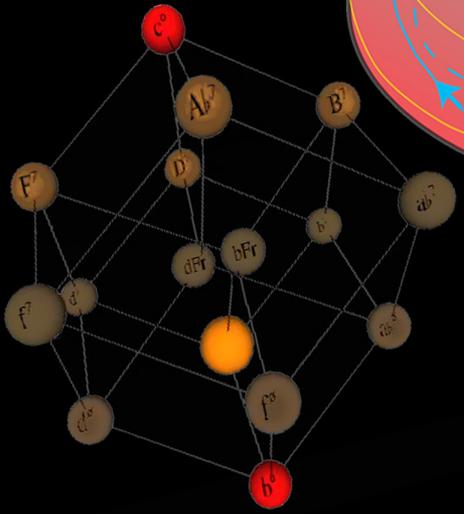
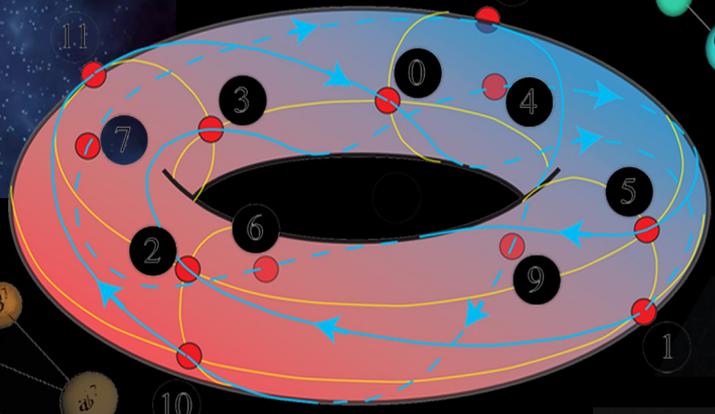
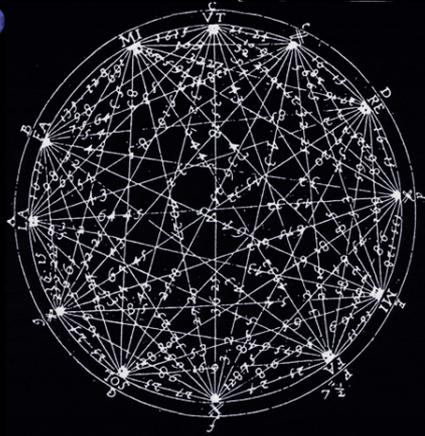
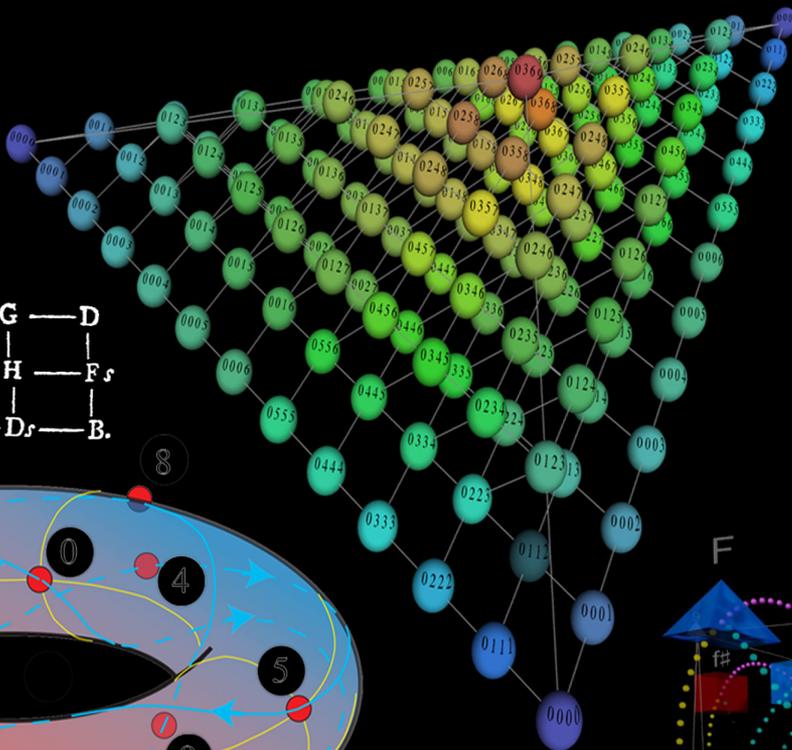
<sup>1</sup>CNRS / UPMC / Equipe Représentations Musicales

<sup>2</sup>CNRS / INRIA / Equipe MuTanT

<sup>3</sup>RepMus, Institut of Algebra (TU-Dresden), IReMus (Sorbonne Universités), GDR ESARS, SMCM, ...



F	—	C	—	G	—	D
A	—	E	—	H	—	F <sub>s</sub>
C <sub>s</sub>	—	G <sub>s</sub>	—	D <sub>s</sub>	—	B.



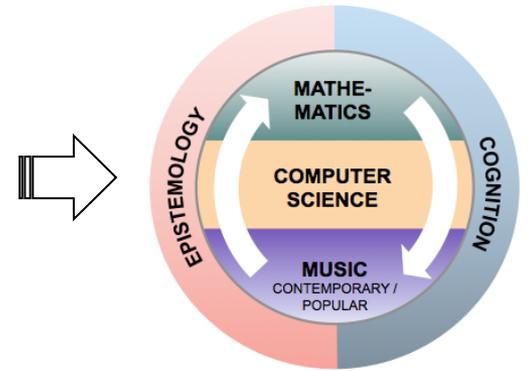
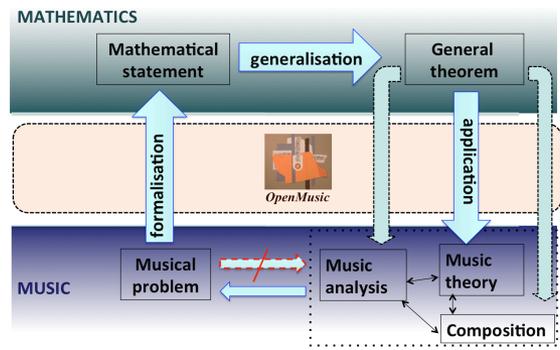
# Mathemusical research within the STMS Lab

**Rhythmic Tiling Canons**

**Set/Transformation Theory and Categorical Music Analysis**

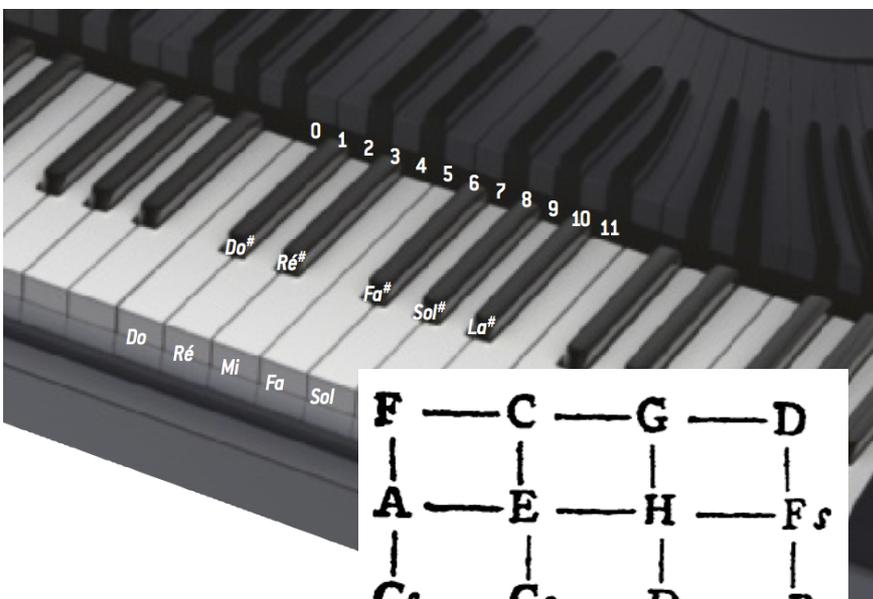
**Z-Relation, Homometric Sets and Dynamical Systems**

**Neo-Riemannian Theory, Spatial Computing and FCA**



<b>Programming paradigms for music</b>	<b>Computational Musicology</b>	<b>Symbolic Interaction Systems</b>	<b>Sign-signal articulations</b>
Specialized Languages (OpenMusic)	<b>MISA</b> <b>Modélisation Informatique des Structures Algébriques en Musique</b>	Improvisation modeling	Signal/Symbolic Unified Representations
Computer-Human Interfaces		Tempo and Rhythm extraction	Synthesis Control
<b>Music Algorithmics (PhD H. Cauré)</b>	Musicologie cognitive	<b>Interactive Scores (PhD D. Ghisi)</b>	Computer-assisted Orchestration
<b>Representation of temporal processes (PhD G. Genuys, K. Haddad)</b>	Computer-aided music analysis and complex systems (PhD Ch. De Paiva)		
<b>Spatial Computing, Homology Persistency and Formal Concept Music Analysis (Master A. Freund, PhD L. Bigo, M. Bergomi)</b>			

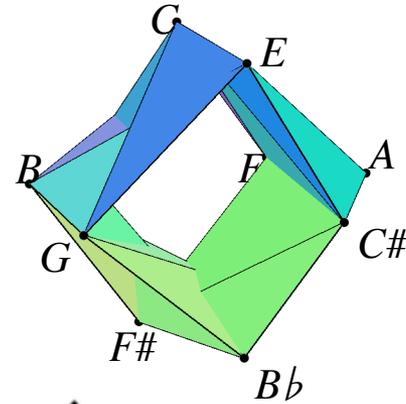
# Generalizing the topological Structure of the Tonnetz



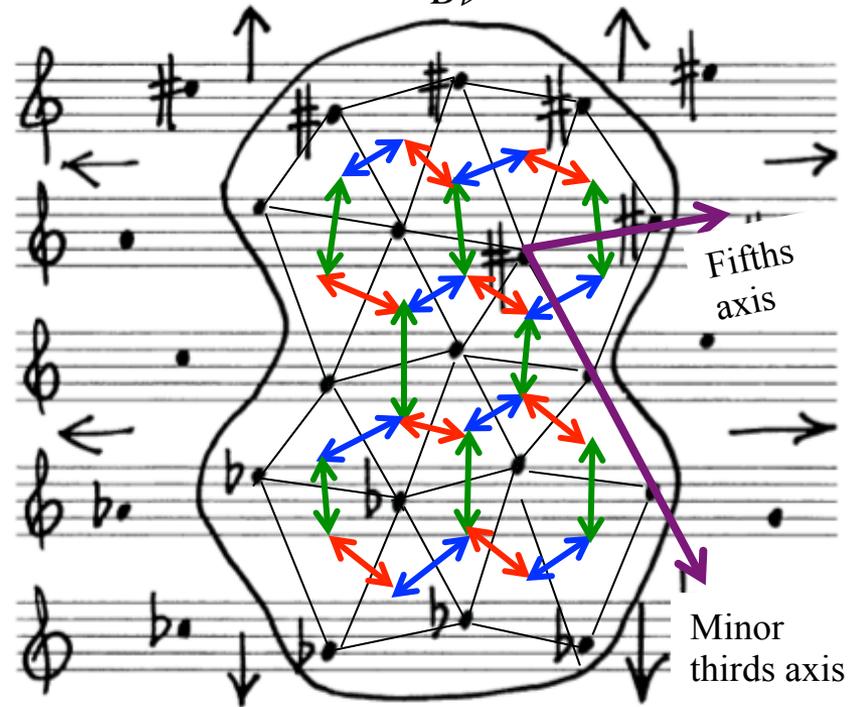
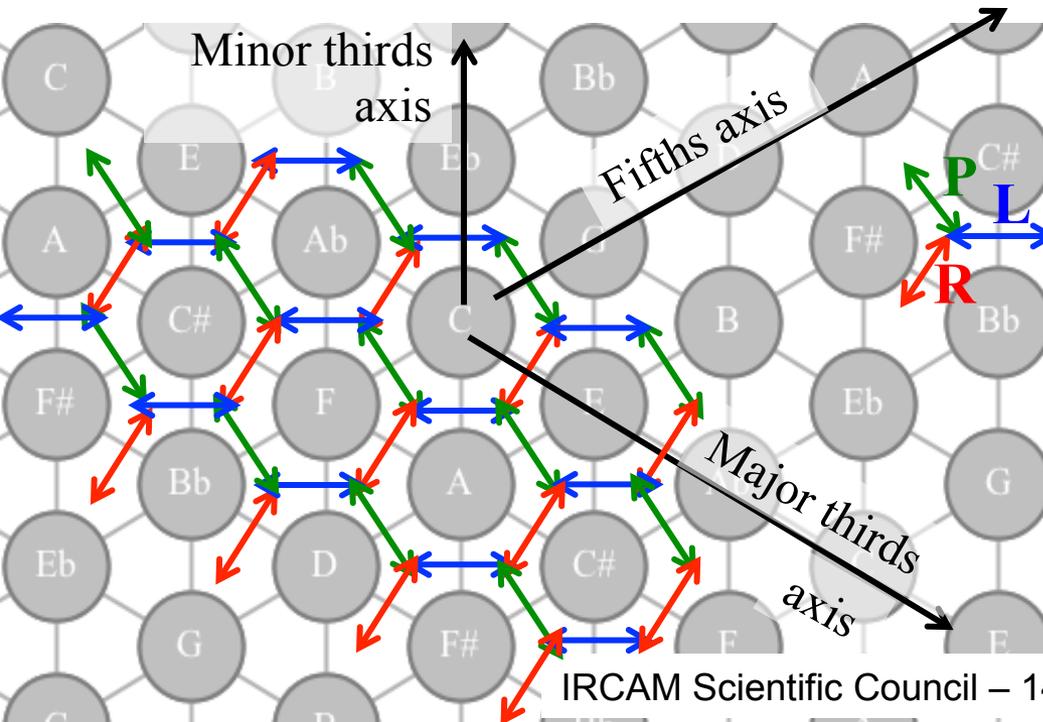
*Speculum Musicum* (Euler, 1773)



(L. Euler)



(J.-Ph. Rameau)

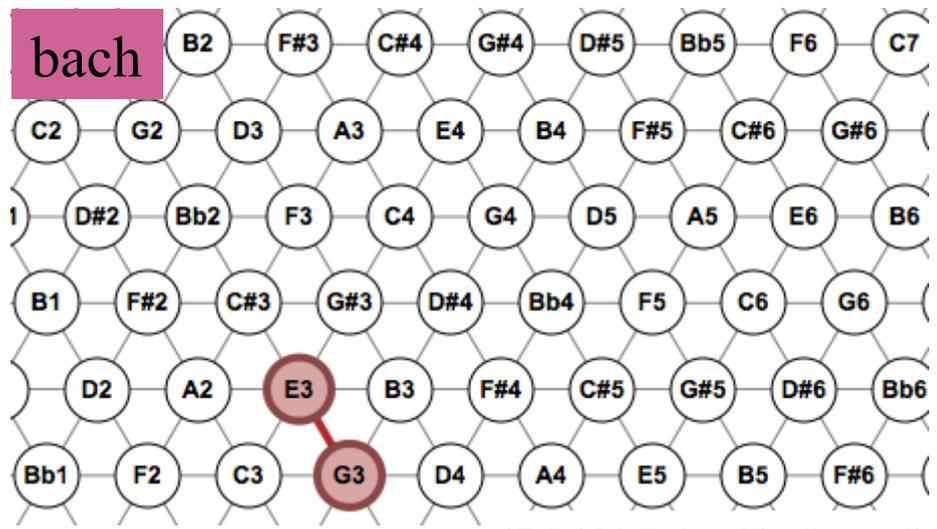
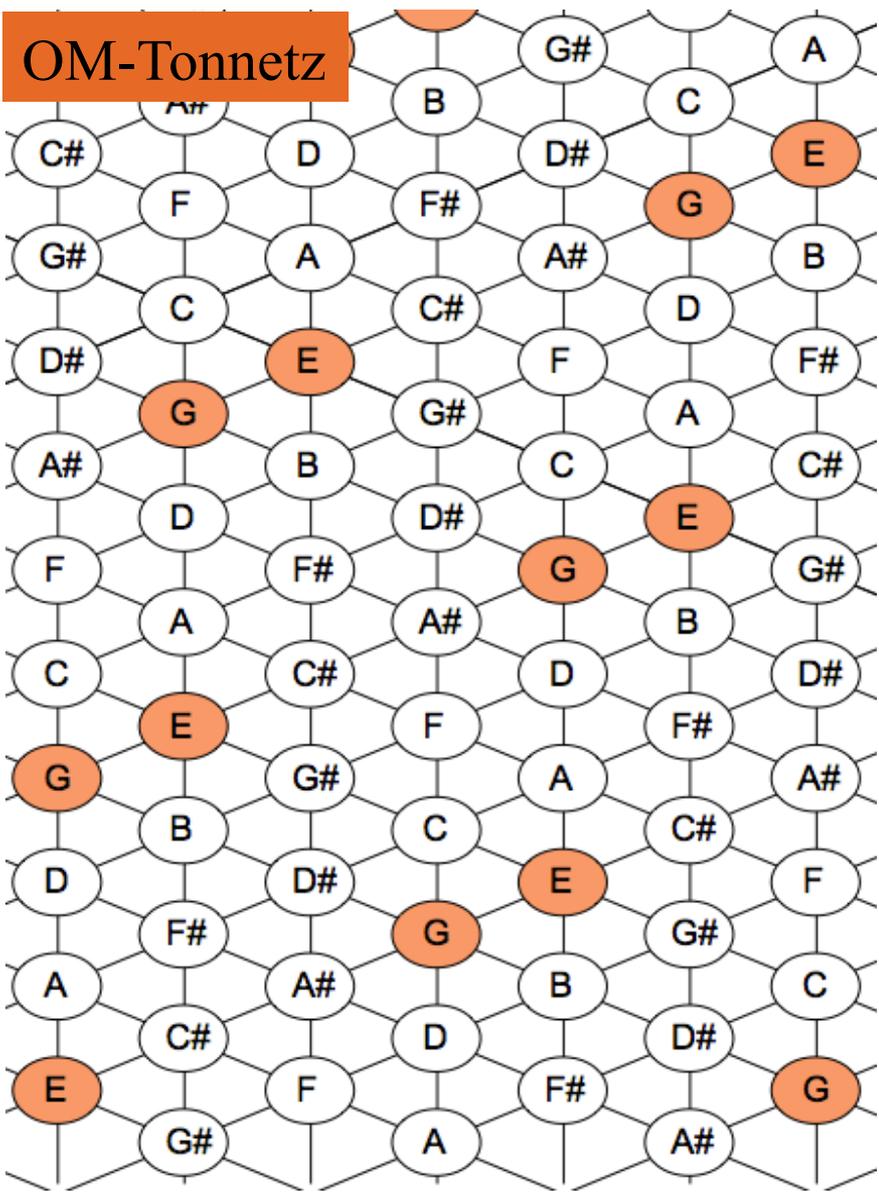


# Three environments for a Tonnetz-based Analysis

The screenshot shows the Plex Viewer interface with several components:

- Plex Viewer:** A 3D visualization of a Tonnetz network as a green and blue polyhedron.
- Tonnetz - K[3,4,5]:** A 2D grid of notes with a path highlighted in yellow.
- InfoBox:** A control panel with a tempo slider (0-20), play/stop buttons, and options for chromatic complexes (K[2,3,7]), heptatonic complexes (CM), and path transformation (Origin complex: K[3,4,5], Destination complex: K[3,4,5]).
- Tonnetzs network:** A graph showing nodes like K[1,1,9], K[1,2,9], K[1,3,8], K[2,2,8], K[2,3,7], K[2,4,6], K[1,4,7], K[1,5,6], and K[4,4,4].
- Chart:** A bar chart titled "bwv0281" showing 2-compactness values for various complexes. A prominent red bar is visible for K[4,4,4].

Hexachord



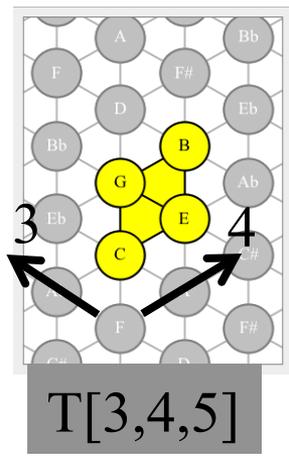
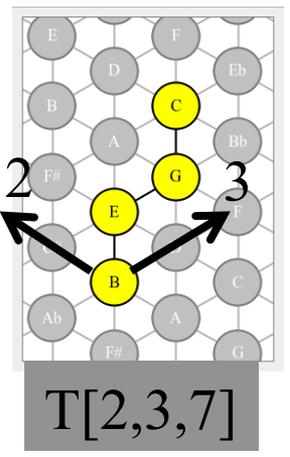
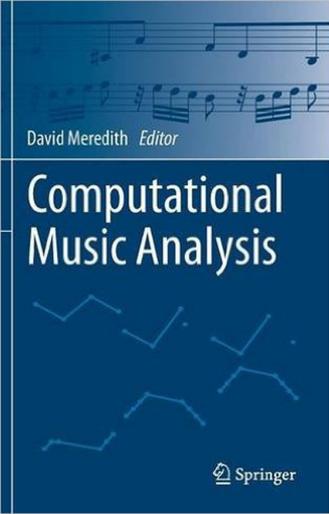
# Spatial music analysis via *Hexachord*

The screenshot displays the Hexachord software interface, which is used for spatial music analysis. It consists of several windows:

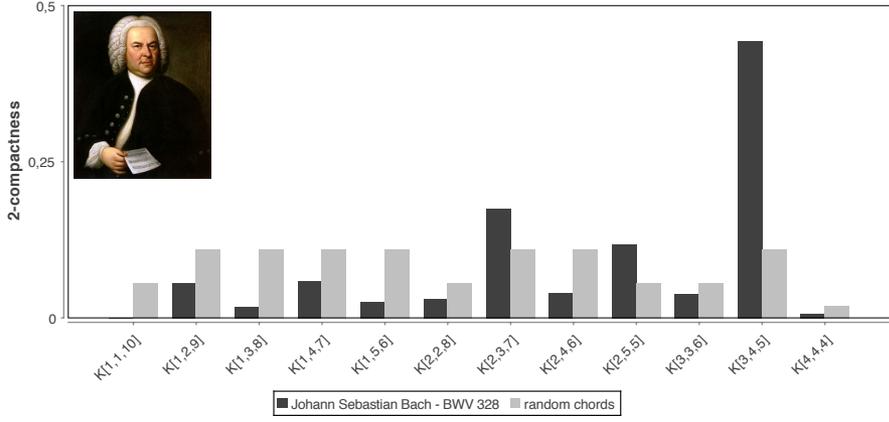
- Plex Viewer:** Shows a 3D visualization of a plex, a geometric structure representing the space of chords.
- Tonnetz : K[3,4,5]:** A 2D grid of notes (chromatic complex) where a specific path of notes is highlighted in yellow, representing the chord progression of the piece.
- InfoBox:** Contains playback controls (Tempo slider, Play/Stop buttons), file selection, and analysis options for chromatic and heptatonic complexes. It also shows settings for vertical compactness and path transformation.
- Tonnetzs network:** A window showing the cover of the journal *Computer Music Journal*, Volume 39, Number 3, Fall 2015, which features a similar visualization.
- Chart:** A bar chart titled "bww0281" showing the 2-compactness of various complexes. The y-axis ranges from 0 to 0.8. The x-axis lists complexes from K[1,1,10] to K[4,4,4]. The bar for K[3,4,5] is significantly higher than the others, reaching approximately 0.8.
- Chart:** A bar chart titled "2-compactness : bww0281" showing the complex compliance over time. The y-axis is "Complex compliance" (0 to 1) and the x-axis is "time" (0 to 35000). The chart shows a series of vertical bars, with a legend indicating different complexes: K[1,1,10], K[1,2,9], K[1,3,8], K[1,4,7], K[1,5,6], K[2,2,8], K[2,3,7], K[2,4,6], K[2,5,5], K[3,3,6], K[3,4,5], and K[4,4,4].

→ <http://www.lacl.fr/~lbigio/hexachord>

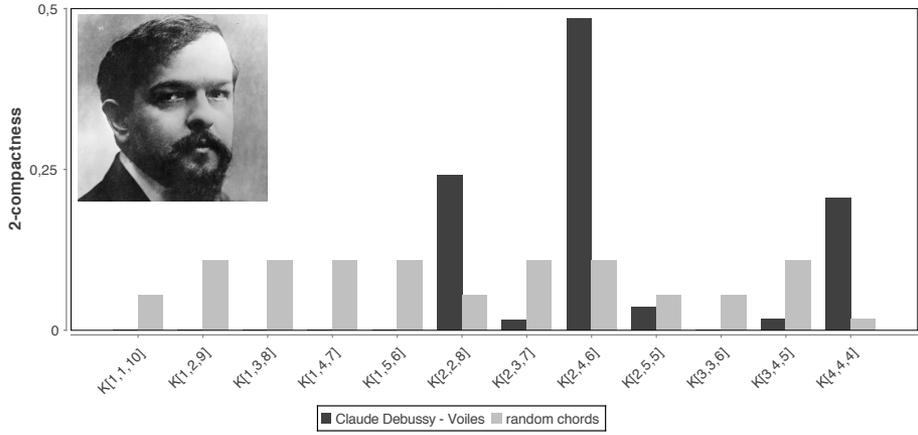
# The geometric character of musical logic



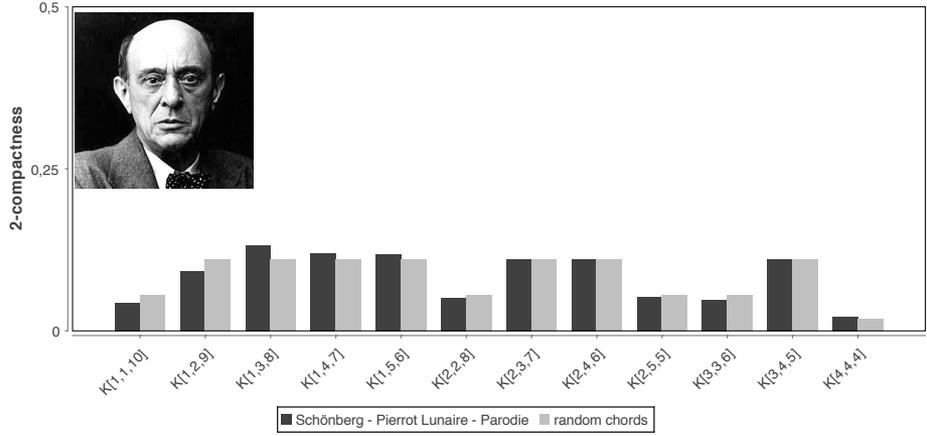
Johann Sebastian Bach - BWV 328



Claude Debussy - Voiles



Schönberg - Pierrot Lunaire - Parodie



# Pedagogical applications in Popular Music

## Hamiltonian cycles / Hamiltonian Songs

**CABARET HAMILTONIEN**

FABRICE GUEDY  
FORMALISÉS DANS LA MUSIQUE SAVANTE

MORENO ANDREATTA  
MATHÉMATIQUES APPLIQUÉES À LA MUSIQUE

POLO PIERRE LAMY  
ÉCRITURE APPLIQUÉE AUX FORMES MUSICALES

ALEXIA - VOIX, GUITARE, ÉCRITURE  
AUGUSTE - VOIX, CLAVIER, ÉCRITURE  
CLARA - VOIX, GUITARE, ÉCRITURE  
ERNEST - VOIX, PERCUSSIONS, ÉCRITURE  
SOPHIE - VOIX, FLOTE, ÉCRITURE  
THOMAS - VOIX, CLAVIERS, ÉCRITURE

ÉCRIRE SOUS LA CONTRAINTE...  
TACHER DU PAPIER SOUS  
L'OBLIGATION DE FAIRE  
QUELQUE CHOSE...  
SALIR DU BOIS CONDITIONNÉ  
AVEC LE DEVOIR MORAL DE  
TRANSFORMER UN OBJET  
INDÉFINI...

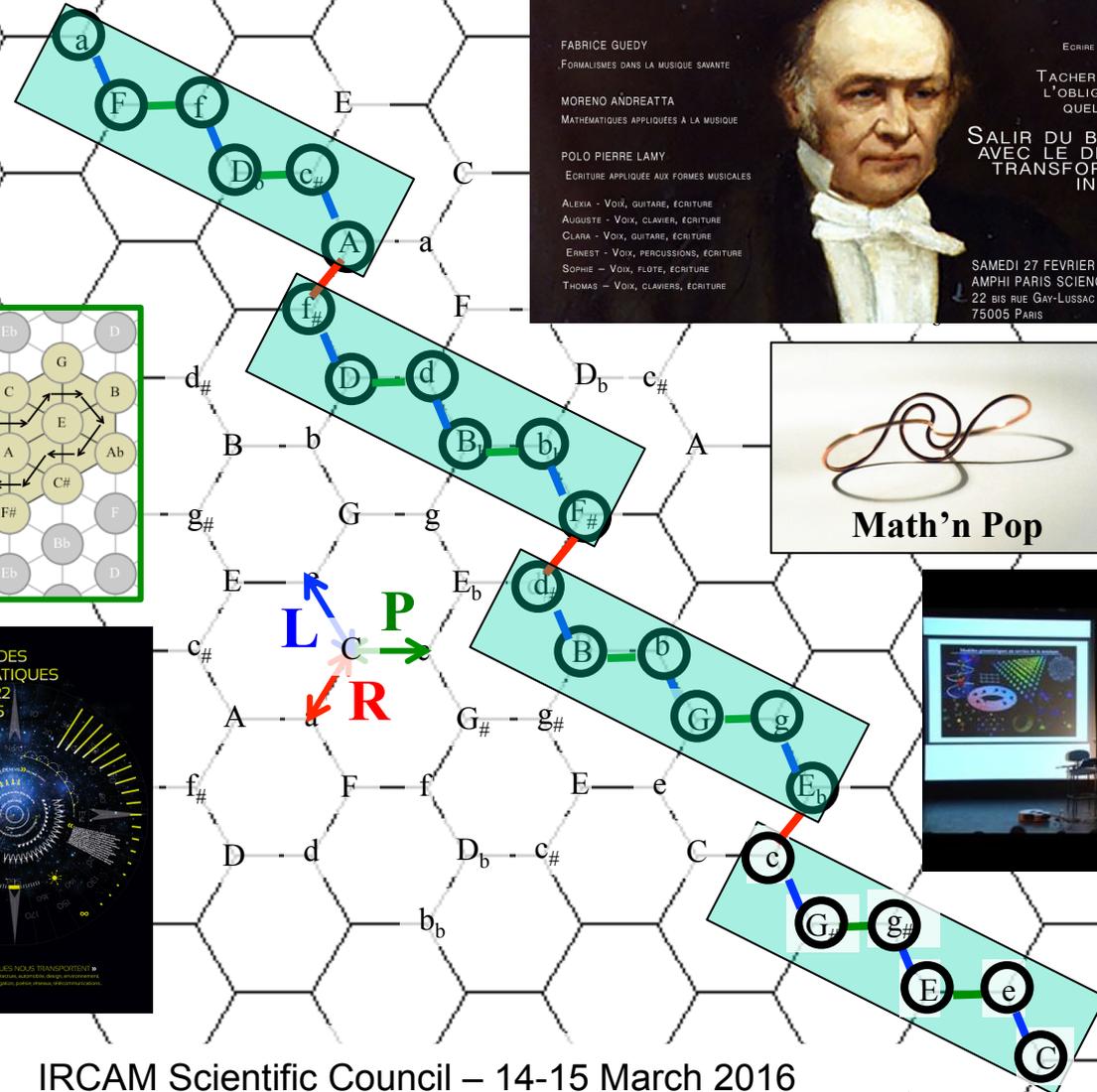
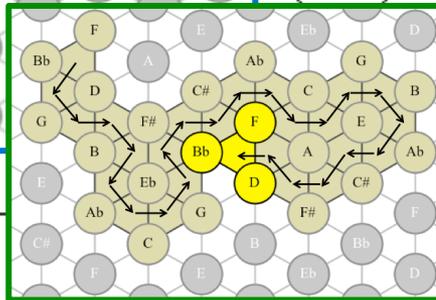
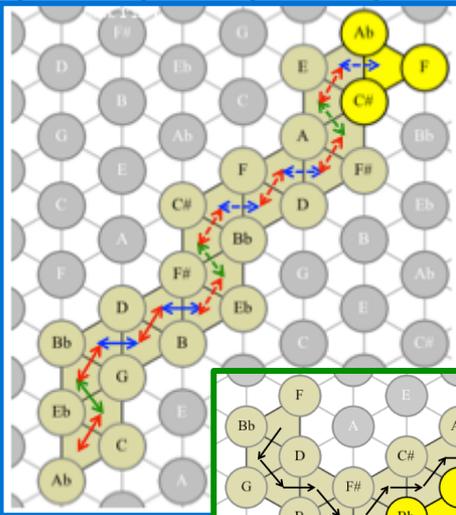
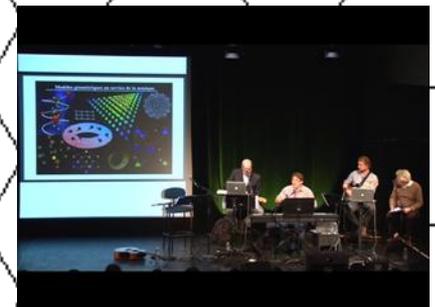
SAMEDI 27 FÉVRIER 2016 À 11H  
AMPHI PARIS SCIENCE ET LETTRES  
22 BIS RUE GAY-LUSSAC  
75005 PARIS

ENTRÉE LIBRE

PSL  
RESEARCH  
UNIVERSITY

**Math'n Pop**

**Bpi**



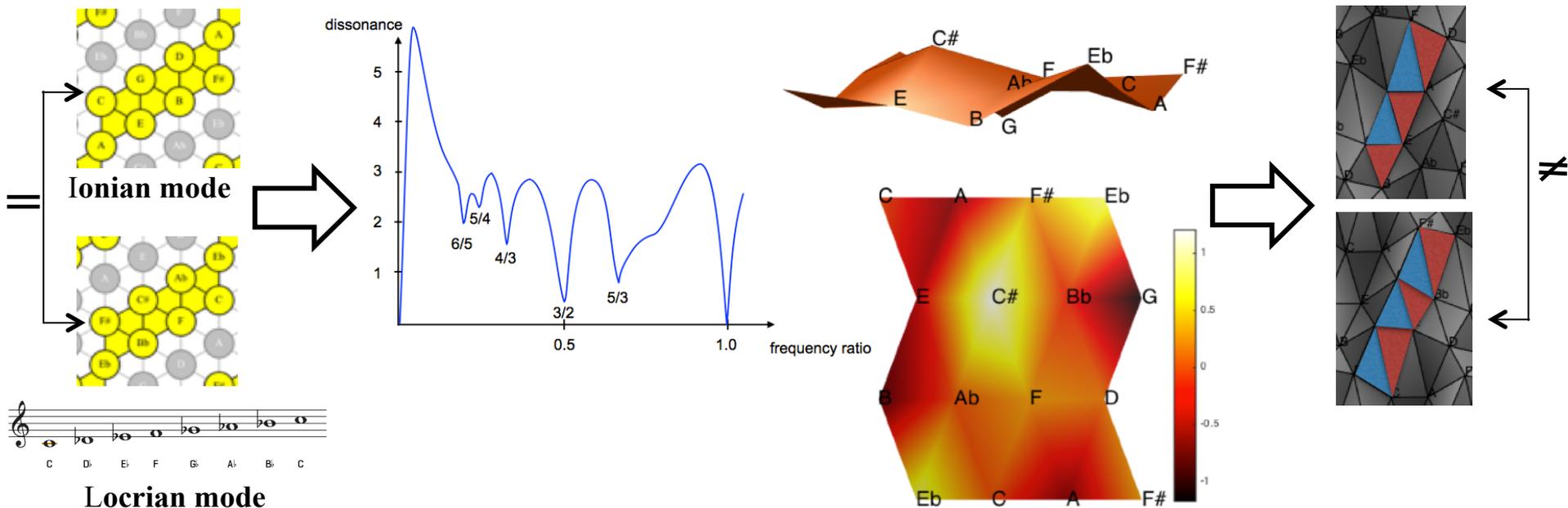
# Towards a Tonnetz-computer-aided composition

The image shows a Pure Data patch window titled "test\_video". The patch is designed for a Tonnetz-based composition. It features several key components:

- Audio Input:** A "play" button is connected to a "bach.ezmidisplay" object, which displays a spectrogram of the audio input.
- Feature Extraction:** Two "getattr" objects ("getattr zoom @listen 1" and "getattr center @listen 1") are connected to "p" objects, which then feed into a "metro 700" object.
- Grid Generation:** The "metro 700" object is connected to a "lattice" object (set to "1: Triangular") and a "shape" object (set to "1: Hexagon"). These objects generate a grid of hexagons, which is displayed as a colorful Tonnetz grid. The grid is defined by the parameters "cents '6000 + 700\*x + 500 \* y'" and "stayalive 3, born 2".
- Transcription:** A "bach.transcribe" object is connected to the grid, which outputs a list of musical notes. The notes are displayed in a text box, showing a sequence of notes such as "B0 F#1 C#2 Ab2 Eb3 Bb3 F4 C5 G5 D6 A6 E7 B7 F#8 C#9", "0 C#1 Ab1 Eb2 Bb2 F3 C4 G4 D5 A5 E6 B6 F#7 C#8 Ab8 E", "Ab0 Eb1 Bb1 F2 C3 G3 D4 A4 E5 B5 F#6 C#7 Ab7 Eb8 Bb", "0 Bb0 F1 C2 G2 D3 A3 E4 B4 F#5 C#6 Ab6 Eb7 Bb7 F8 C", "F0 C1 G1 D2 A2 E3 B3 F#4 C#5 Ab5 Eb6 Bb6 F7 C8 G8", "0 G0 D1 A1 E2 B2 F#3 C#4 Ab4 Eb5 Bb5 F6 C7 G7 D8", "D0 A0 E1 B1 F#2 C#3 Ab3 Eb4 Bb4 F5 C6 G6 D7 A7 E8", "1 E0 B0 F#1 C#2 Ab2 Eb3 Bb3 F4 C5 G5 D6 A6 E7 B7 F", "F#-1 C#0 Ab0 Eb1 Bb1 F2 C3 G3 D4 A4 E5 B5 F#6 C#7 Ab7", "1 Ab-1 Eb0 Bb0 F1 C2 G2 D3 A3 E4 B4 F#5 C#6 Ab6 Eb7 B", "Eb-1 Bb-1 F0 C1 G1 D2 A2 E3 B3 F#4 C#5 Ab5 Eb6 Bb6 F7", "2 F-1 C0 G0 D1 A1 E2 B2 F#3 C#4 Ab4 Eb5 Bb5 F6 C7 C", "C-1 G-1 D0 A0 E1 B1 F#2 C#3 Ab3 Eb4 Bb4 F5 C6 G6 D7", "2 D-1 A-1 E0 B0 F#1 C#2 Ab2 Eb3 Bb3 F4 C5 G5 D6 A6", "A-2 E-1 B-1 F#0 C#1 Ab1 Eb2 Bb2 F3 C4 G4 D5 A5 E6 B6", "2 B-2 F#-1 C#0 Ab0 Eb1 Bb1 F2 C3 G3 D4 A4 E5 B5 F#6 C", "F#-2 C#-1 Ab-1 Eb0 Bb0 F1 C2 G2 D3 A3 E4 B4 F#5 C#6 Ab6".
- Output:** The transcribed notes are connected to a "bach.ezmidisplay" object, which outputs a musical score on a staff.

At the bottom left, there is a video player showing a waveform for "CULP\_3.aif" and a video frame of a person writing on a board, with the name "D. Ghisi" and the "france musique" logo. The video player includes a "Come un lasciapassare" button and a volume control icon.

# Signal/Symbolic articulation in MIR



M. Bergomi, *Dynamical and Topology Tools for Modern Music Analysis*, PhD, LIM-Milan/UPMC/Ircam, Dec. 2015

➔ Towards a geometric dynamic modeling of a musical piece ?

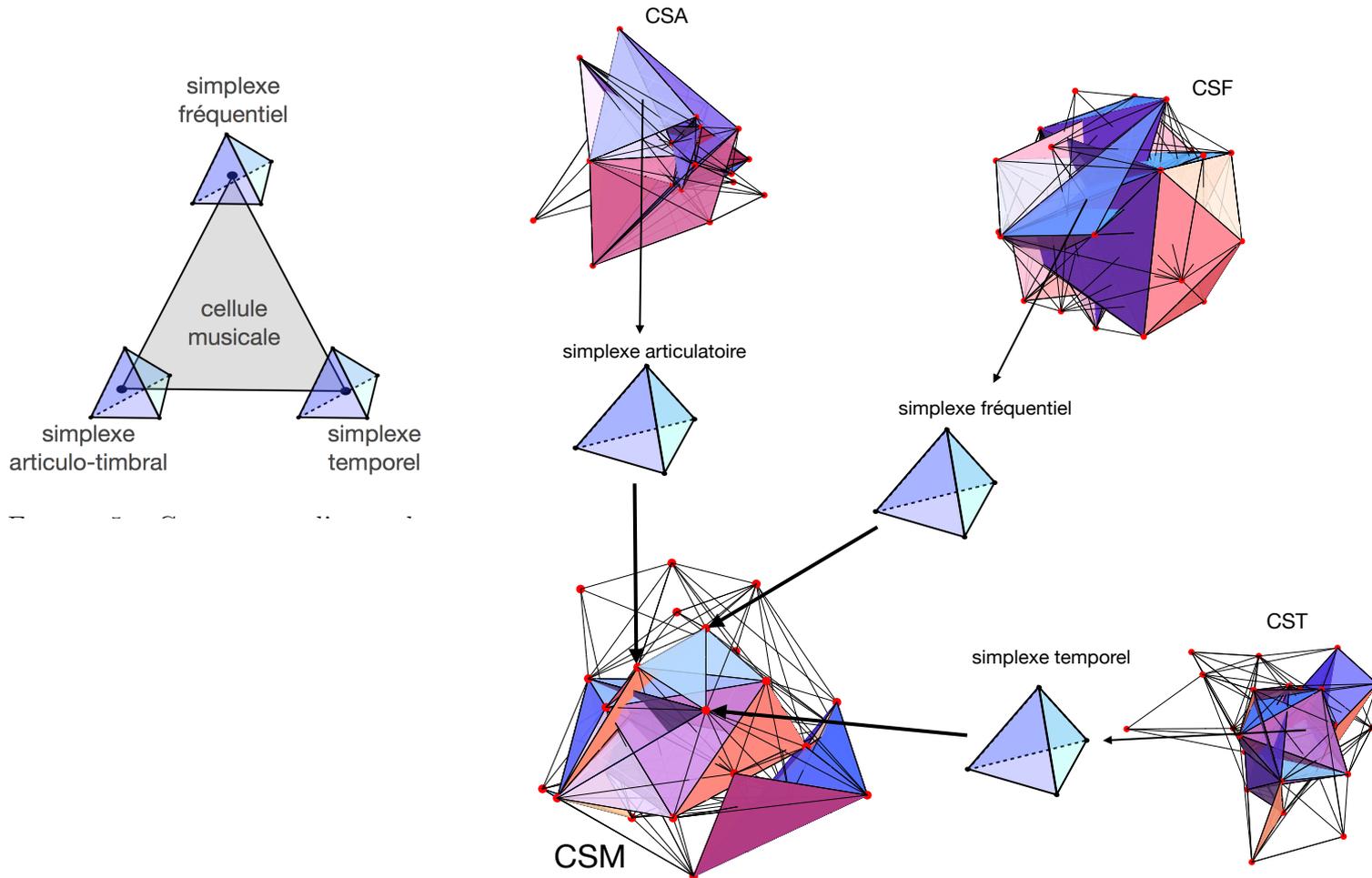
SPACE



MUSIC

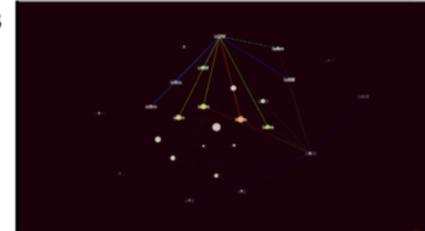
➔ Towards new perspectives on topological compositional spaces ?

# Julia Blondeau: Compositional spaces



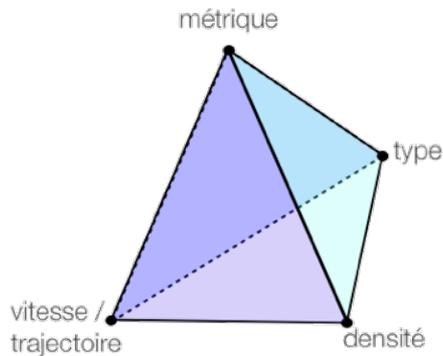
# Julia Blondeau: Compositional spaces

Réseau d'intervalles

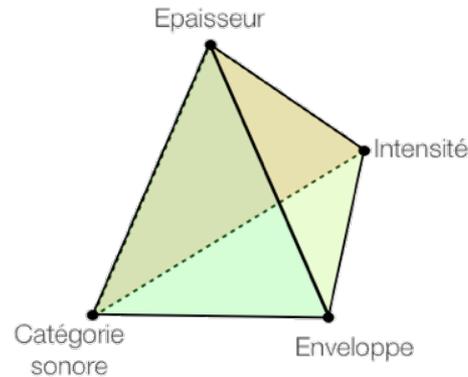


## Espaces compositionnels et réseaux intervalliques

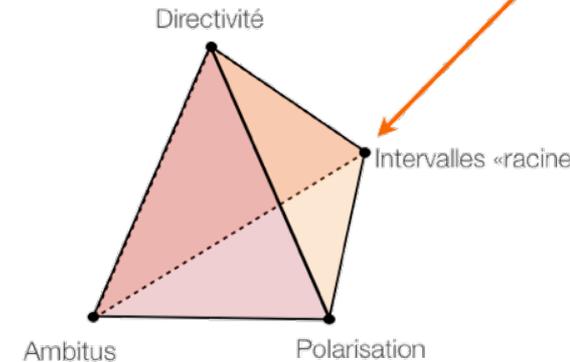
3-simplexe temporel



3-simplexe articulatoire



3-simplexe fréquentiel



métrique	division d'une durée en segments «relatifs» courts ou longs
vitesse	très lent → très rapide, accel, ral, rit ...etc
catégorie	régulier / irrégulier / lisse / strié
densité	faible, moyenne, forte, très forte

enveloppe	Enveloppes => // enveloppes en synthèse <i>Q, trajectoire, fonction d'évolution dyn...</i>
épaisseur	épaisseur «polyphonique» <i>Largeur de bande, champ de propagation....</i>
«catégorie»	pur, mat, bruité, enrichi => liens vers modes de jeu <i>Forme d'onde, nb de partiels, type de filtrage....</i>
intensité	pppp → mp mf → ffff .... <i>Amplitude moyenne</i>

Directivité	V, Λ, /, \, mixte ...etc.
Intervalles	intervalle(s) racine dans le réseau d'intervalles
Ambitus	restreint, moyen, large, très large
Polarisation	faible, moyenne, forte..

# Julia Blondeau: TESLA ou l'effet d'étrangeté

Julia Blondeau

Atlas I

♩ = 48

Alto solo

Très posé

♩ = 76

souple et libre

Electronique

Source\_1 et  
a181a\_curtain  
bLa1 03 930  
bCafor 033 H

Source source direct  
id\_tromp 0.90

carre tromp  
(R17 F1) 2 [ 500 ] 2 [ 999 ]

SRAPUP gmk2  
(R1C2\_Spnt2) (R10,0,10,070,1,1,3,35)

Shenker (R1) 2 (1,1) 2 (1,1) 4 (1,1)

Tromp 1 2200  
SRAPUP control 0.1,0  
non\_lev 7:translocat  
bLach 038 1050  
GRAPUPCNC\_03\_show7 (10,30)

id\_tromp 0.90

♩ = 66

A. 8

e

V. 1

V. 2

A.

Vc.

Cb.

Source de b

Source de pl

CLT cv

non régulier

The image displays a complex musical score for 'Atlas I' by Julia Blondeau. The score is written for Alto solo, Electronique, and a string quartet (Violins 1 & 2, Violas, and Cellos/Double Basses). The tempo is marked as ♩ = 48, and the time signature is 4/4. The score is annotated with various performance instructions such as 'Très posé', 'sfz', 'mf sostenuto', 'f', 'pp', 'mp', 'non régulier', and 'souple et libre'. A large, colorful geometric diagram, consisting of interconnected triangles in shades of purple, pink, and blue, is overlaid on the score. This diagram features numerous numerical labels (e.g., a30, f25, a24, f23, a22, f22, a21, f19, a16, f15, a9, f27, f25, a18, f11, a11, f46, f45, a13, f44, f43, f41, f46) and letters (A, B, S1, S2, S3) that likely correspond to specific musical elements or structural points. Handwritten notes and diagrams are also present, including a section labeled 'A' with rhythmic patterns like '1-2-1' and '1-2-1' and dynamics like 'pp', 'f', 'mp', 'ppp'. The score includes various musical notations such as notes, rests, and dynamic markings.

Many musical applications:

classification using topological characteristics: can we catch the notion of style ?

generative power of topological transformation ?

representation of other musical relationships...

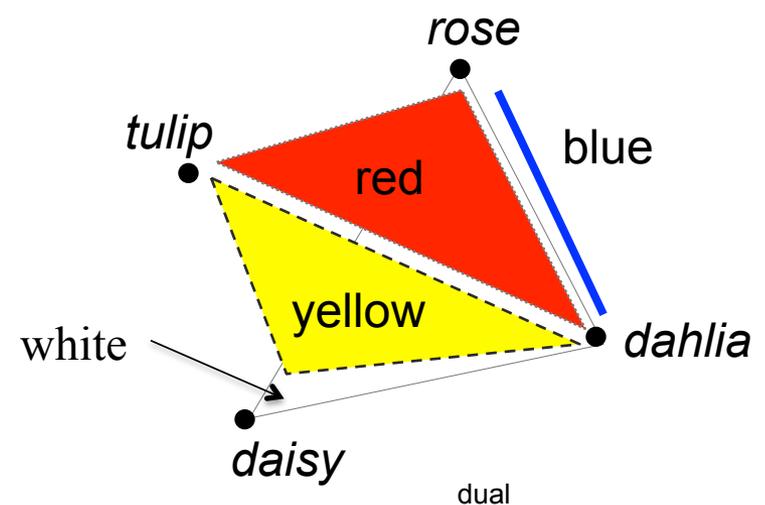
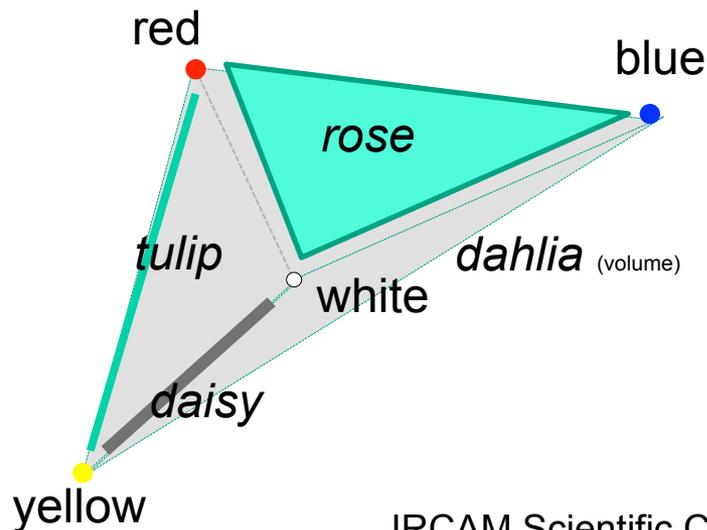
... What kind of relationships ?

# **TOPOLOGICAL vs ALGEBRAIC ANALYSIS OF (MUSICAL) RELATIONS**

# Q-analysis: a topological representation of a binary relationship

$\lambda \subset \{\text{tulipe, rose, marguerite, dahlia}\} \times \{\text{rouge, bleu, blanc, jaune}\}$

$\lambda$	red	blue	white	yellow
<i>tulip</i>	1	0	0	1
<i>rose</i>	1	1	1	0
<i>daisy</i>	0	0	1	1
<i>dahlia</i>	1	1	1	1

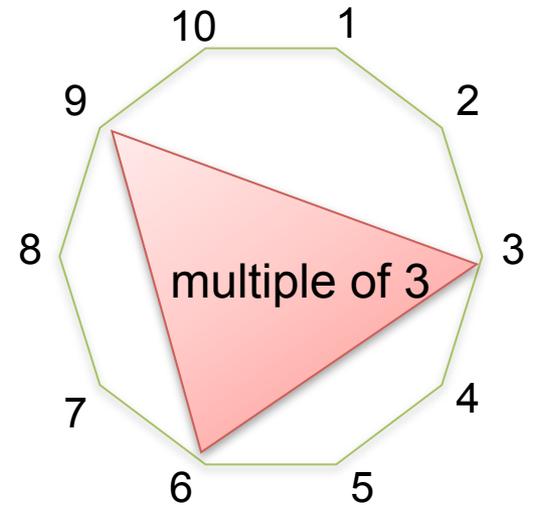
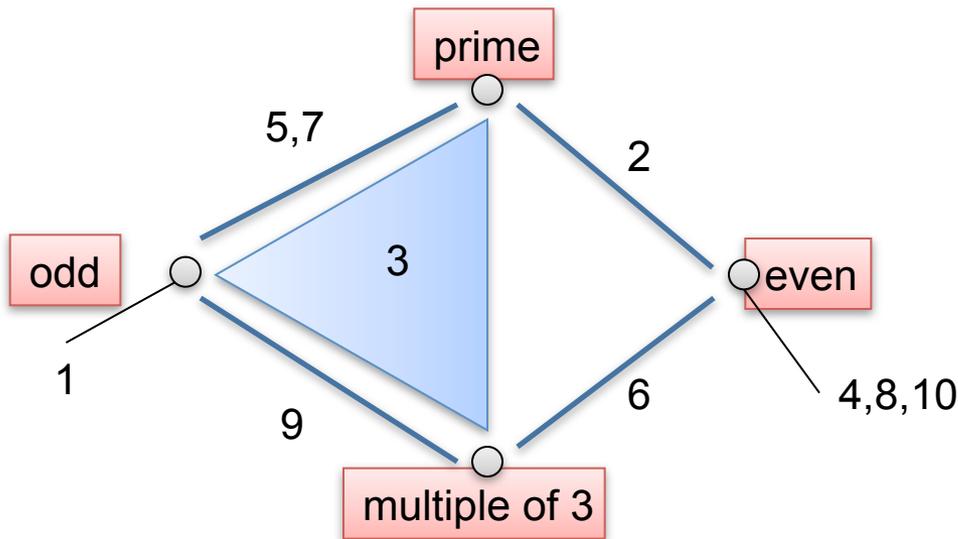


# Representation of a set of predicates

$$\lambda \subset \text{Objects} \times \text{Predicates} : (o,p) \in \lambda \Leftrightarrow p(o)$$

Objects = {1, 2, 3, ..., 10}

Predicates = {prime, even, odd, multiple-of-3}



# Formal Concept Analysis (FCA)

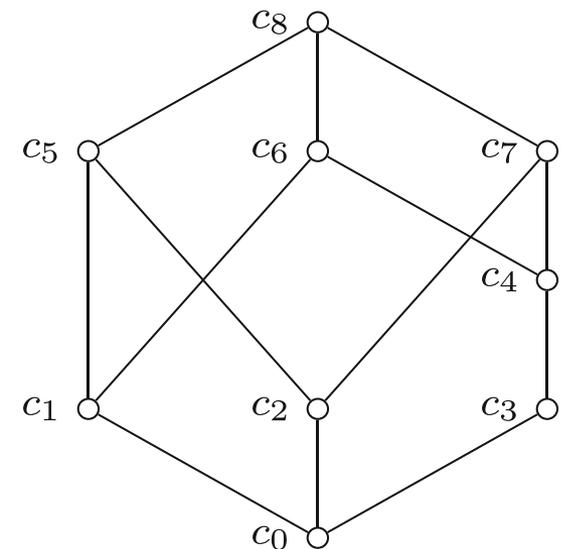
## it can also be represented as a lattice

$\lambda$ : interval  $g_i$  appear in a musical motive  $m_j$

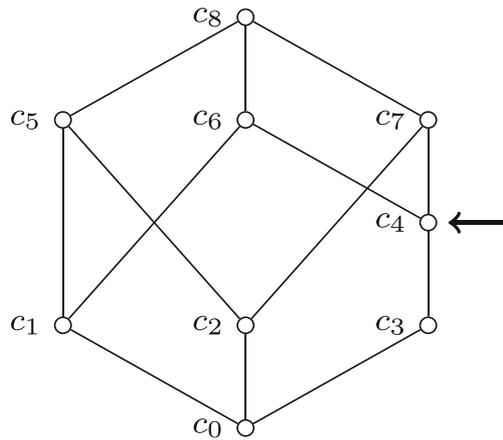
	$m_1$	$m_2$	$m_3$	$m_4$
$g_1$	×	×		×
$g_2$	×	×		
$g_3$	×		×	
$g_4$		×	×	
$g_5$			×	

let  $H \in G$  and  $N \in M$

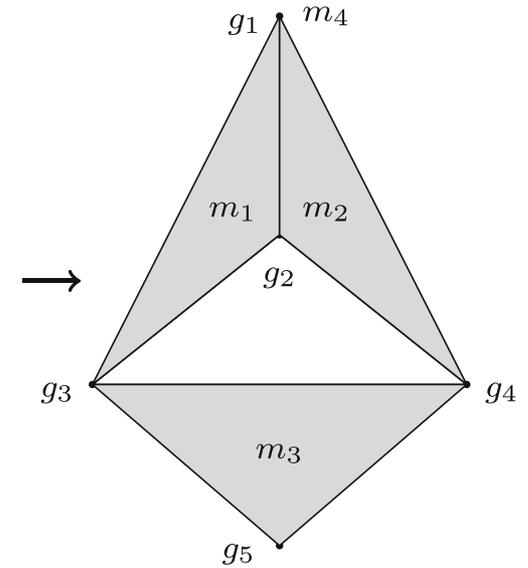
- $H' = \{ m \text{ in } M \mid (h, m) \in \lambda \text{ for all } h \in H \}$
- $N' = \{ g \text{ in } G \mid (g, n) \in \lambda \text{ for all } n \in N \}$
- $(H, N)$  is a *formal concept* iff  $H' = N$  and  $H = N'$
- formal concept are ordered:  $(H, N) < (I, P)$  iff  $H \subset I$



# Relation between FCA and Q-analysis

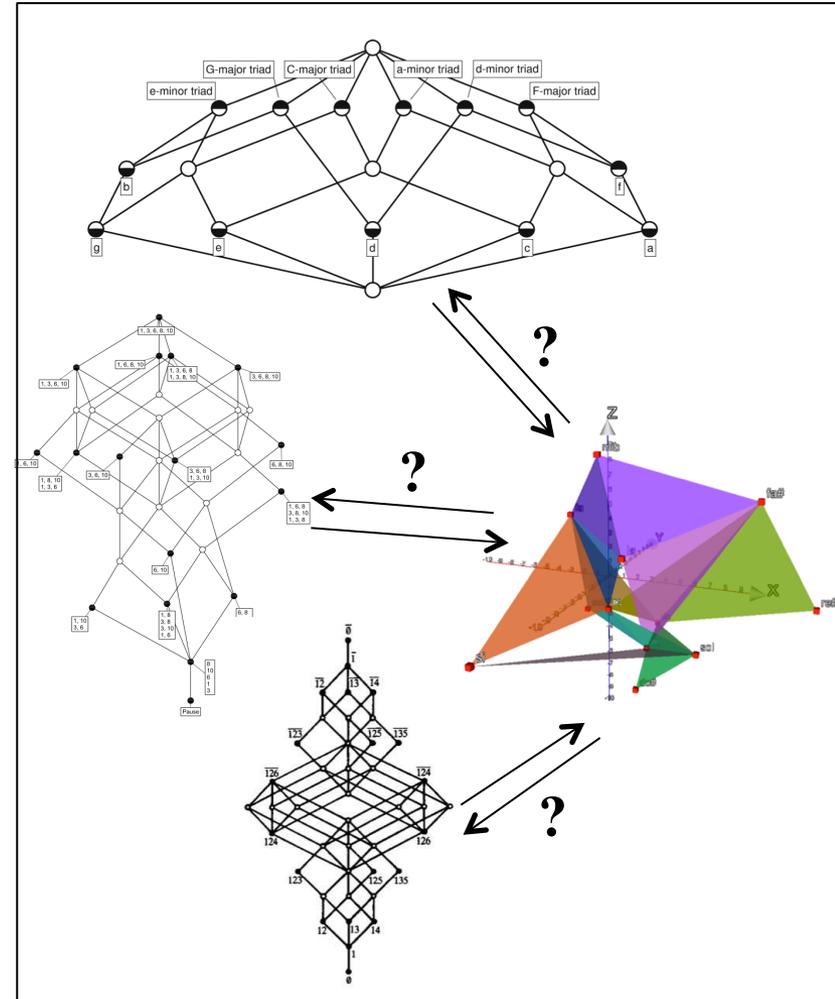
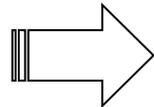
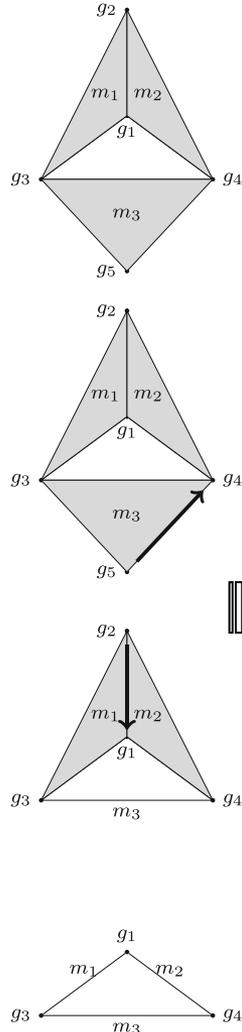
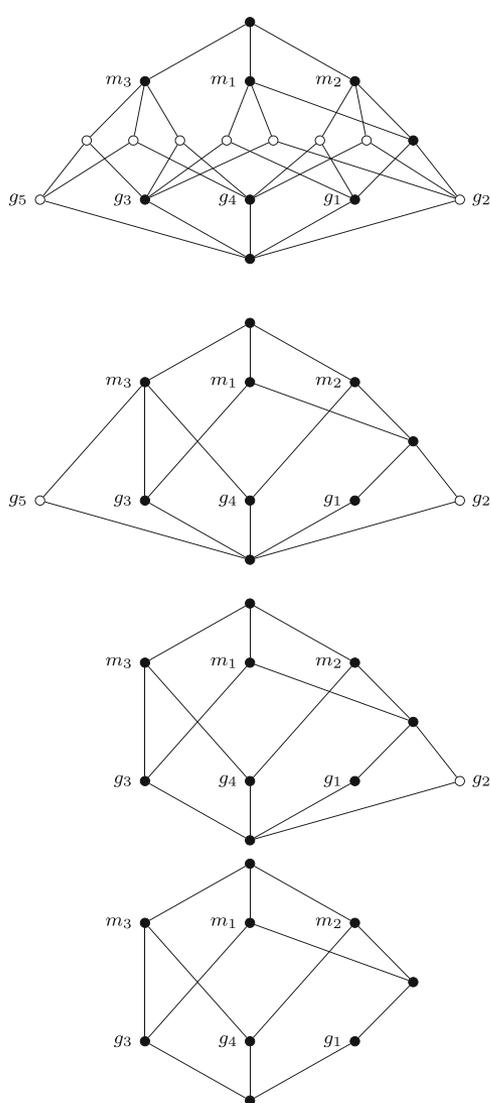


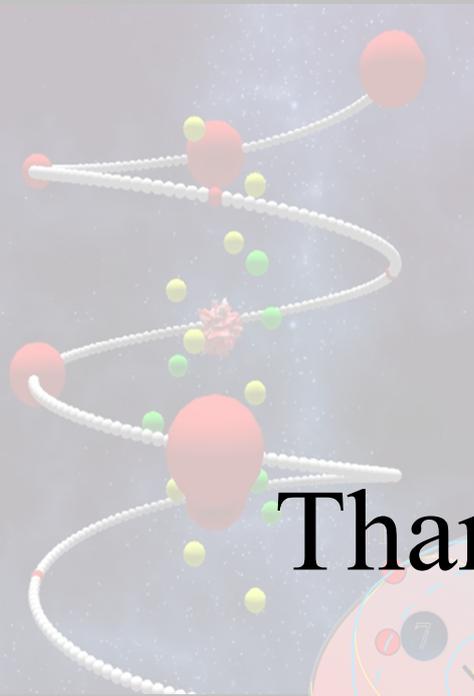
	$m_1$	$m_2$	$m_3$	$m_4$
$g_1$	×	×		×
$g_2$	×	×		
$g_3$	×		×	
$g_4$		×	×	
$g_5$			×	



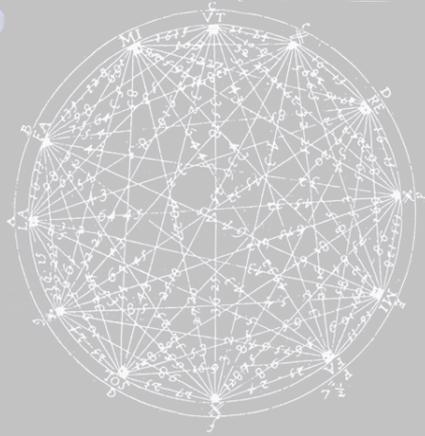
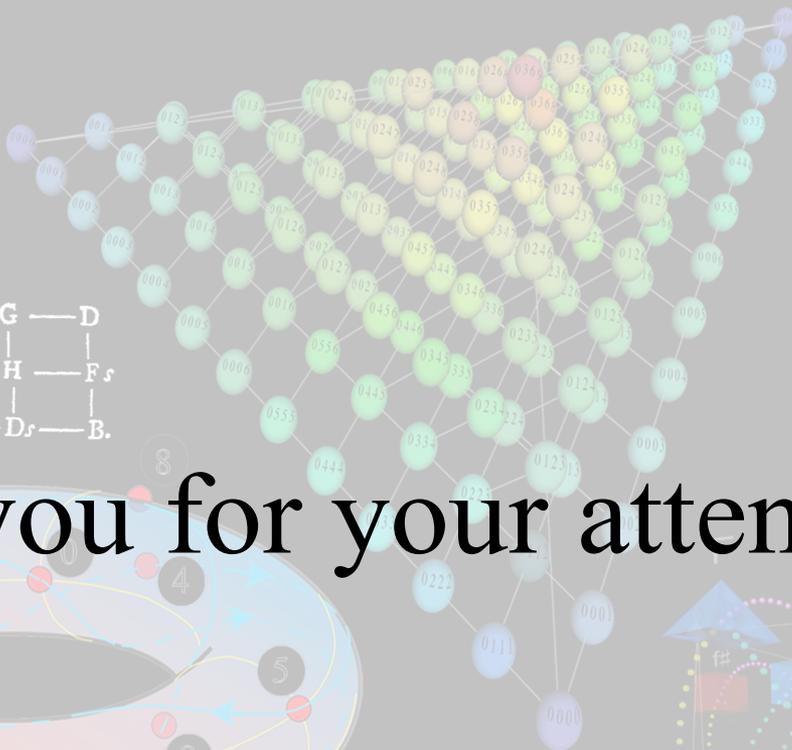
# Relation between FCA and Q-analysis

➔ Freund A., M. Andreatta, J.-L. Giavitto (2015), « Lattice-based and Topological Representations of Binary Relations with an Application to Music », *Annals of Mathematics and Artificial Intelligence*





F — C — G — D  
 A — E — H — F<sub>s</sub>  
 C<sub>s</sub> — G<sub>s</sub> — D<sub>s</sub> — B.



Thank you for your attention

