MGS: a declarative spatial computing programming language

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We sketch the rationals of the MGS programming language. MGS is an experimental programming language developed to study the application of several spatial computing concepts to the specification and simulation of dynamical systems with a dynamical structure. MGS extends the notion of rewriting by considering more general structure than terms. The basic computation step in MGS replaces in a topological collection $A$, some subcollection $B$, by another topological collection $C$. A topological collection is a set of element structured by a neighborhood relationships describing an underlying space representing a data structure or constraints that must be fulfilled by the computation. This process proposes a unified view on several computational mechanisms initially inspired by biological or chemical processes (Gamma and the CHAM, Lindenmayer systems, Paun systems and cellular automata).
1. $(DS)^2$
2. Gamma, P systems, L systems, cellular automata...
3. Spatial generalization
4. MGS
5. Algorithmic examples
6. Biological modeling
Dynamical systems and Dynamical Structures
Specifying a dynamical system (for simulation)

Specification of
• structure of state
• structure of time
• evolution function
Formalism for Dynamical System

- State: often structured by space (e.g. fields)
- Time
- Evolution function

<table>
<thead>
<tr>
<th>C: continuous, D: discrete</th>
<th>PDE</th>
<th>Coupled ODE</th>
<th>Iteration of functions</th>
<th>Cellular automata</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>state</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>D</td>
<td>…</td>
</tr>
<tr>
<td>time</td>
<td>C</td>
<td>C</td>
<td>D</td>
<td>D</td>
<td>…</td>
</tr>
<tr>
<td>space</td>
<td>C</td>
<td>D</td>
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The medium/process problem

THE CHEMICAL BASIS OF MORPHOGENESIS

By A. M. TURING, F.R.S. University of Manchester

(Received 9 November 1951—Revised 15 March 1952)

a falling ball

\[ (p_x, p_y, v_x, v_y) \]

at any time a state is a position and a speed

A dynamical system (DS)
The medium/process problem

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A dynamical system (DS)

a falling ball

a developing embryo

at any time a state is a position and a speed

the structure of the state is changing in time
(chemical and mechanical state of each cell)

A dynamical system
with a dynamical structure

(DS)^2
Bio-inspired models of $(DS)^2$
Cellular Automata

• Von Neumann

• Avoids the dynamic structure problems
  – Predefined underlying (unbounded) space

• Replace a cell $X$
  in an NEWS grid
  by another one (with a new state)
Lindenmayer systems

It might be possible, however, to treat a few particular cases in detail with the aid of a digital computer. This method has the advantage that it is not so necessary to make simplifying assumptions as it is when doing a more theoretical type of analysis. It might even be possible to take the mechanical aspects of the problem into account as well as the chemical, when applying this type of method. The essential disadvantage of the method is that one only gets results for particular cases. But this disadvantage is probably of comparatively little importance.

Lindenmayer systems

• The structure of a tree can be coded by a string of parenthetised symbols

• A symbol is an elementary part of the plant

• The symbol between [ and ] represents a sub-tree

• Additional conventions are used to represent main axis, orientation, depth, etc.

• A rule

\[ s_0 \rightarrow s_1 s_2 s_3 \ldots \]

represents the evolution of \( s_0 \)
Lindemayer Systems

• Replace a substring $X$ in a string by another one

$$e / (C(e) \& (e.x \geq lm) \& (e.p == L))$$

$$\Rightarrow \{\text{type}="C", a=e.a, h=e.h, x=e.x*\text{shorter}, p=L\}, \{\text{type}="C", a=e.a, h=e.h, x=e.x*\text{longer}\}$$

morphogene concentration

Linear cell structure

time
Replace a sub-multiset $X$ in a multiset by another one.
A general device
A general device?

A general rewriting mechanism

1. In a *collection* of elements
2. *Replace* a subcollection $X$
3. With a collection $Y$ computed from $X$ and its *neighbors*

<table>
<thead>
<tr>
<th>Collection</th>
<th>Neighborhood</th>
<th>data structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>father/son</td>
<td>free</td>
</tr>
<tr>
<td>Set</td>
<td>all</td>
<td>$A + C + \text{Idempotent}$</td>
</tr>
<tr>
<td>Multiset</td>
<td>all</td>
<td>$A + \text{Commutative}$</td>
</tr>
<tr>
<td>Sequence</td>
<td>left and right (with jump for trees)</td>
<td>Associative</td>
</tr>
<tr>
<td>Grid</td>
<td>NEWS</td>
<td></td>
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monoidal
Rewriting systems (and abstract transition systems)

• Rewriting system
  – Used to formalize equationnal reasoning
  – A generative device (grammar)
  – Replace a sub-part of an entity by an other
  – Set of rewriting rules $\alpha \rightarrow \beta$
    • $\alpha$: pattern specifying a sub-part
    • $\beta$: expression evaluating a new sub-part

• Example: arithmetic expressions simplification
Rewriting on terms+property

1 + 2 → ...  \hspace{1cm} \text{(arithmetic) term rewriting}

a \cdot b \rightarrow ... \hspace{1cm} \text{string rewriting (~ L systems)}

2H + O → H₂O \hspace{1cm} \text{multiset rewriting (~ chemistry)}

- arithmetic operation
- string concatenation: « . » is a formal associative operation
- multiset concatenation (= the chemical soup): « . » is AC
A general device?

A general rewriting mechanism

1. **In a collection of elements**
2. **Replace a subcollection X**
3. **With a collection Y computed from X and its neighbors**

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The Spatial Approach

- Use space (topology) to unify the various collection structures
  - space as a resource
  - space as a constraint
  - space as an input/output

- **Neighborhood relationships:**
  - the structure of the collection
  - the structure of the subcollection
  - the computation dependencies

- Substitution (replacement) topological surgery
Properties

- **local evolution rules**
  mandatory when you cannot express a global function/relation because the domain of the function/relation is changing in time

- **interaction based approach**
  the l.h.s. of a rule specifies a set of elements in interaction, the r.h.s. the result of the interaction

- **the phase space is well defined but not well known**
  a generative process enumerates the elements but membership-test can be very hard

- **various kind of time evolution** (for the same set of rules)

- **demonstration by induction**
  on the rules or on the derivation (e.g. growth function in L system)
MGS Proposition

- Topological collections
  - Structure
    - A collection of topological cells
    - An *incidence relationship*

vertex

edge

surface

volume
MGS Proposition

• Topological collections
  – Structure
    • A collection of topological cells
    • An incidence relationship
  – Data: association of a value with each cell

0-cell

1-cell

2-cell

3-cell
Abstract (Simplicial) Complex and (Simplicial) Chains

*Incidence relationship and lattice of incidence:*  
- \( \text{boundary}(f) = \{ v_1, v_2, v_3, e_1, e_2, e_3 \} \)  
- \( \text{faces}(f) = \{ e_1, e_2, e_3 \} \)  
- \( \text{cofaces}(v_1) = \{ e_1, e_3 \} \)

*Topological chain*  
- coordinates with vertices  
- lengths with edges  
- area with \( f \)

\[
\begin{pmatrix} 0 \\ 4 \end{pmatrix} v_1 + \begin{pmatrix} 3 \\ 0 \end{pmatrix} v_2 + \begin{pmatrix} -3 \\ 0 \end{pmatrix} v_3 + 5 e_1 + 6 e_2 + 5 e_3 + 12 f
\]
Topology $\leftrightarrow$ simplicial complex $\leftrightarrow$ lattice

Topology (open and closed sets)

- simplices $=$ Closed sets

Simplicial Complex (set of sets closed by inclusion/intersection)

- simplices $=$ Lattice cones

Lattice (order relation: $\land$, $\lor$)

- Concise reformulation of classical approaches
- Extension
MGS Proposition

• Transformations
  – Functions defined by case on collections
    Each case (pattern) matches a sub-collection
  – Defining a rewriting relationship: topological rewriting

\[
\text{trans } T = \{
\begin{align*}
\text{pattern}_1 & \Rightarrow \text{expression}_1 \\
\text{...} & \\
\text{pattern}_n & \Rightarrow \text{expression}_n
\end{align*}
\}
\]
MGS Proposition

• Transformations

\[ T = \{ \text{pattern}_1 \Rightarrow \text{expression}_1, \ldots, \text{pattern}_n \Rightarrow \text{expression}_n \} \]
Topological rewriting = transformation

1 + 2 → ... (arithmetic) term rewriting

a . b → ... string rewriting (~ L systems)

2H + O → H₂O multiset rewriting (~ chemistry)

v₁ . σ₁ + v₂ . σ₂ → ... topological rewriting (MGS)

gluing cell in a cell complex: ... (AC and algebraic machinery)
Pattern matching: specifying a sub-collection of elements in interaction

- *Path transformation* (path = sequence of neighbor elements)
  - Concise but limited expressiveness
- *Patch transformation* (arbitrary shape)
  - Longer but higher expressiveness
Example: Diffusion Limited Aggregation (DLA)

- Diffusion: some particles are randomly diffusing; others are fixed
- Aggregation: if a mobile particle meets a fixed one, it stays fixed

```
trans dla = {
    'mobile', 'fixed' => 'fixed', 'fixed';
    'mobile', <undef> => <undef>, 'mobile
}
```

`NEIGHBOR OF`
Example: Diffusion Limited Aggregation (DLA)

- Diffusion: some particles are randomly diffusing; others are fixed
- Aggregation: if a mobile particle meets a fixed one, it stays fixed

\[
\text{trans dla} = \{
  \text{`mobile, `fixed => `fixed, `fixed;}
  \text{`mobile, <undef> => <undef>, `mobile}
\}
\]

This transformation is an abstract process that can be applied to any kind of space.
Polytypisme
Algorithmic examples
Bead Sort
Bead Sort

2
4
1
3

1
2
3
4
Bead Sort

Gbf NEWS = < North, South, East, West;
          North+South=0, East+West=0>

trans dla = {
            `bead | south> `empty  => `empty, `bead ;
        }
Eratosthenes’s Sieve

\[
\text{trans } \text{Generate} \quad = \quad \{x, \text{true}\} \quad \Rightarrow \quad x, \{x + 1, \text{true}\};
\]

\[
\text{trans } \text{Succed} \quad = \quad \{x, \text{true}\} \quad \Rightarrow \quad x;
\]

\[
\text{trans } \text{Eliminate} \quad = \quad (x, y \mid y \mod x = 0) \quad \Rightarrow \quad x;
\]

\[
\text{Eliminate[fixrule]} \left( \text{Succed} \left( \text{Generate[N]} \left( \{2, \text{true}\}, \text{set} : () \right) \right) \right)
\]
Eratosthenes’s Sieve
trans Eratos = {
  Genere1 = n : integer / ~right n => n, \{prime = n\};
  Genere2 = n : integer, \{prime as x, ~candidate, ~ok\}
            => n + 1, \{prime = x, candidate = n\};
  Test1  = \{prime as x, candidate as y, ~ok\} / y \mod x = 0 => \{prime = x\};
  Test2  = \{prime as x, candidate as y, ~ok\} / y \mod x <> 0
            => \{prime = x, ok = y\};
  Next   = \{prime as x1, ok as y\}, \{prime as x2, ~ok, ~candidate\}
            => \{prime = x1\}, \{prime = x2, candidate = y\};
  NextCreate = \{prime as x, ok as y\} as s / ~right s
               => \{prime = x\}, \{prime = y\};
}
Hamiltonian path

trans h_path = { `start , x* as p , `stop /
size(p) = n-2 => return p }

trans maze = { `input, c* as p , `output => return p }
Fractal construction by carving

Sierpinsky sponge (4 steps)

Menger sponge (2 steps)
Modeling of (bio)physical systems

- ...  
- Various models of Phage $\lambda$  
- Sperm crawling  
- Neurulation  
- Prototyping a « synthetic multicellular bacteria »  
- ...
Higher dimensional objects for complex simulations

Example of electrostatic Gauss law [Tonti 74]

- Electric charge content \( \rho \): dimension 3
- Electric flux \( \Phi \): dimension 2
- Law available on an arbitrary complex domain

\[
\phi = \iiint w \cdot dS = \frac{Q^c}{\varepsilon_0} = \iiint_{(V)} \frac{\rho}{\varepsilon_0} d\tau
\]

**electric field in space:**
- \( V \): electric potential (dim 0)
- \( E \): voltage (dim 1)
- \( w \): electric flux (dim 2)
- \( Q^c \): electric charge (dim 3)

*A Direct Discrete Formulation of Field Laws: The Cell Method*
A Synthetic « Multicellular Bacterium »
The Paris iGEM project: a « multicellular bacteria »
to decouple growth and transgene expression

- germinal cell
- somatic cell

+ : positive/negative influence
- : process
Implementation using BioBricks

DAP feeding

Differentialion

Germinal cell

Differentiation control

cre

lox71  gfp  T  ftsK  T  lox66  dapA

DAP starvation → RECOMBINAISON → Differentiation

Somatic cell

cre

No replication origin

loxSc  dapA

gfp  T  ftsK

loxScar  T

cre  différentiation control

irreversible recombination

loxsca  X  T

loxsca  Y

ftsK needed for cellular division
Proof of Concept: Simulation to answer 4 questions

• How does differentiation induces feeding? (proof of concept)
  cellular automaton (in MGS)

- diffusion of DAP
- somatic and germ cell
Proof of Concept: Simulation to answer 4 questions

• How does differentiation induces feeding? (proof of concept) cellular automaton (in MGS)

• How do spatial organization and distribution evolve? agents based system (in MGS)
Proof of Concept: Simulation to answer 4 questions

• How does differentiation induces feeding? (proof of concept) cellular automaton (in MGS)
• How do spatial organization and distribution evolve? agents based system (in MGS)
• How robust and tunable is the model? ODE kinetics (matlab)
Proof of Concept: Simulation to answer 4 questions

- How does differentiation induces feeding? (proof of concept) cellular automaton (in MGS)
- How do spatial organization and distribution evolve? agents based system (in MGS)
- How robust and tunable is the model? ODE kinetics
- How sensitive is the system to noise? Gillespie based simulation (in MGS)
Conclusions and Perspectives
MGS drawbacks and successes

Success

• Polytypisme is good
• Rule application strategies are good
• Patterns/rules are expressive and usually concise
• Clean semantics

Shortcomings

• Rules may be heavy (e.g. 100 variables for the fractal sponge)
  graphical drawing of rules
  look for better notations (e.g. path pattern)
• Efficiency
  well...
• Implicit methods (solvers) are hairy
  use explicit ones
Perspectives

• An intrinsic complexity theory
e.g., w.r.t. interactions

• A logic of spatial interactions

• Relationships to physics
  a discrete differential calculus (cf. PhysicaD 08)

• Internalizing time

• Implementation
  pattern-matching compilation and optimization, specific
abstract combinatorial complex, parallelism

• Non standard applications
  e.g., in knowledge representation
  or in music analysis (Louis Bigo talk)
A topological manifesto

Spatial computing proposes to celebrate corporeality of data rather than trying to deny it.

*Simon Greenworld* (MIT medialab)

- The logical approach in computer science
  
  \[ \text{computation} = \text{deduction} \]
  (the Curry-Howard isomorphism)

- Other paradigms can be fruitfull: topology
  
  \[ \text{computation} = \text{moving in a space} \]

- Try to perceive space (and time) in programs (rather than logical operations)
  
  purposes: *technical, heuristic, didactical*
Call for Papers: Spatial Computing Workshop 2011

at 5th IEEE International Conference on Self-Adaptive and Self-Organizing Systems
Ann Arbor, Michigan, USA, October 3, 2011

Organizers: Jacob Beal (BBN Technologies, USA) Stefan Dulman (Delft University, the Netherlands) Olivier Michel (University Paris-Est Créteil, France) Antoine Spicher (University Paris-Est Créteil, France)

Submission Deadline: July 4th, 2011

Many self-organizing or self-adaptive systems are “spatial computers” – collections of local computational devices distributed through physical space, in which:

- the difficulty of moving information between any two devices is strongly dependent on the distance between them, and
- the “functional goals” of the system are generally defined in terms of the system’s spatial structure.

Systems that can be viewed as spatial computers are abundant, both natural and man-made. For example, in wireless sensor networks and animal or robot swarms, inter-agent communication network topologies are determined by the distance between devices, while the agent objectives as a whole solve spatially-defined problems like “analyze and react to spatial temperature variance” or “surround and destroy an enemy.”

Similarly, in reconfigurable microchip platforms, moving data between adjacent logic blocks is much faster than moving it across the chip, which in turn favors problems with spatial structure like stream processing. In biological embryos, each developing cell’s behavior is controlled only by its local chemical and physical environment, but the eventual structure of the organism is a global property of the
Thanks

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- Olivier Michel

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    E. Delsinne, V. Larue, F. Letierce, B. Calvez,
    F. Thonereux, D. Boussié and the others...

- Past and presents Collaborations
  - A. Lesne (IHES, stochastic simulation)
  - P. Prusinkiewicz (UoC, declarative modeling)
  - P. Barbier de Reuille (meristeme model)
  - C. Godin (CIRAD, biological modeling)
  - H. Berry (INRIA, stochastic simulation)
  - G. Malcolm (Liverpool, rewriting)
  - J.-P. Banâtre (IRISA, programming)
  - P. Fradet (Inria Alpes, programming)
  - F. Delaplace (IBISC, synthetic biology)
  - P. Dittrich (Jena, chemical organization)
  - F. Gruau (LRI, language and hardware)
  - P. Liehnard (Poitier, CAD, Gmap and quasi-manifold)

http://mgs.spatial-computing.org