Programmation spatiale

- l'analyse des contes de fées
- et l'analyse musicale

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http://mgs.spatial-computing.org

Plusieurs formalismes informatiques tentent de capturer la notion d'interaction dans un système. MGS, un langage de programmation expérimental, est fondé sur la constatation que l'ensemble des interactions possibles s'organise suivant une structure topologique qui permet de spécifier la description du système et son évolution. Le style de programmation qui en résulte, la programmation spatiale, s'appuie sur des relations topologiques (connexité, bord, obstruction...) pour renouveler la notion de structure de données et a trouvé des applications effectives dans la modélisation et la simulation en biologie des systèmes mais aussi en intelligence artificielle (représentation des connaissances et analogie) ainsi qu'en analyse musicale. Nous illustrerons l'approche MGS dans ces deux derniers domaines.

Un « manifeste » topologique

- L'approche logique en informatique
  calculer = déduire
  (isomorphisme de Curry-Howard)

- D'autres paradigmes peuvent être fertiles
  calculer = se déplacer dans un espace

- Essayer de voir l'espace (et le temps) dans un programme (plutôt que des opérations logiques)

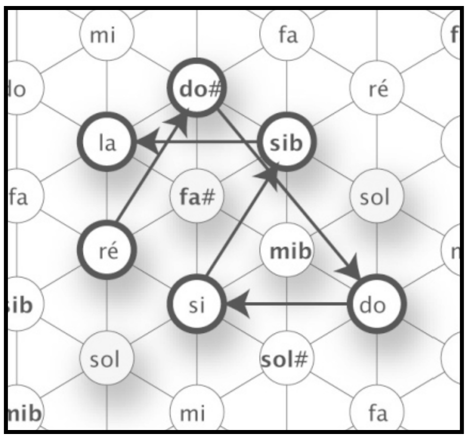
visées : *heuristiques*, *techniques*, *pédagogiques*

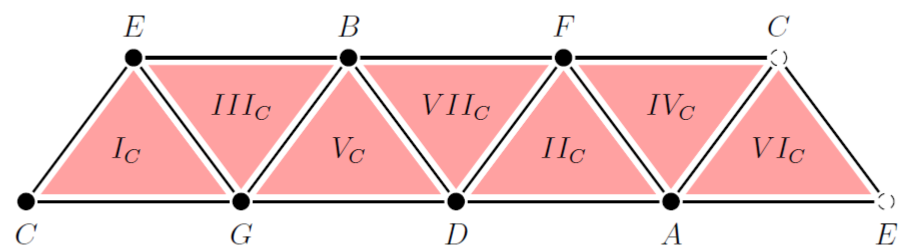
1ère partie

- L'analyse musicale du point de vue spatial
- L'analyse des contes de fées
- Résoudre des analogies
- Quizz

2ème partie

- Les systèmes dynamiques à structure dynamique
- MGS
- L'espace d'un accord
  et le prélude N°4 op. 28 de Chopin



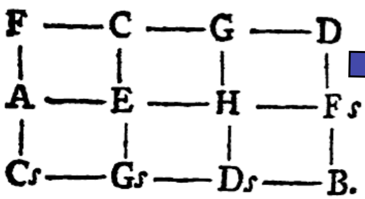


Music in a space

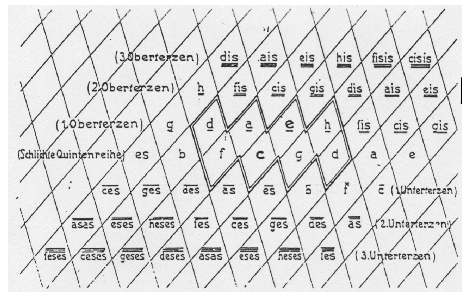
Computing the space of music

Music and spatial computing

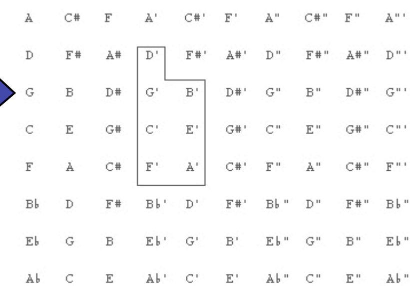
Représentations topologiques de structures musicales



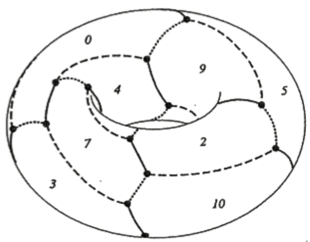
Euler : *Speculum musicum*, 1773



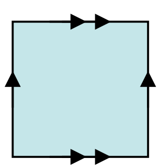
Hugo Riemann : « Ideen zu einer Lehre von den Tonvorstellung », 1914

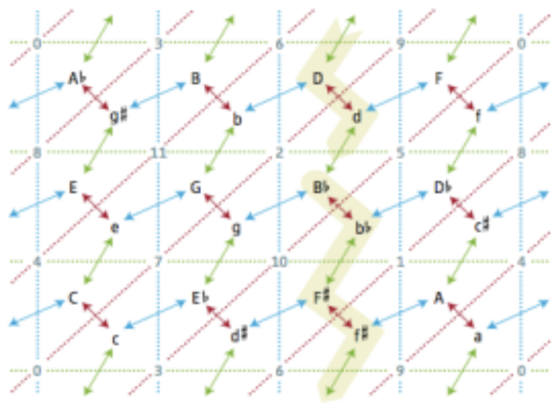


Longuet-Higgins (1962)

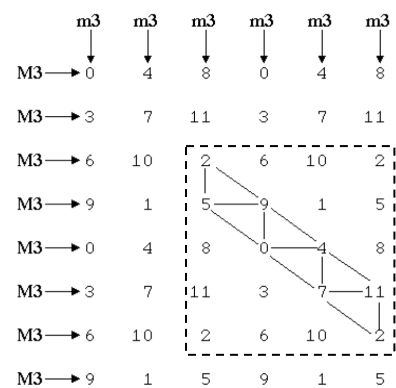


Douthett & Steinbach, *JMT*, 1998





J. Hook, « Exploring Musical Space », *Science, 2006*

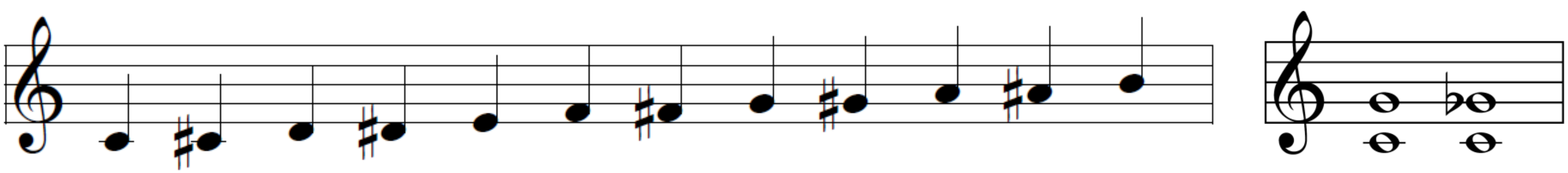


Balzano (1980)

Neo-Riemannian Problematic

- Traditional western music representation
  - Based on the use of a *staff*
  - Main drawbacks for visualization of harmony
    - *Contrapuntal* proximity

      The distance between notes on the score is not relevant for harmony
    - Spatially close patterns can sound very different



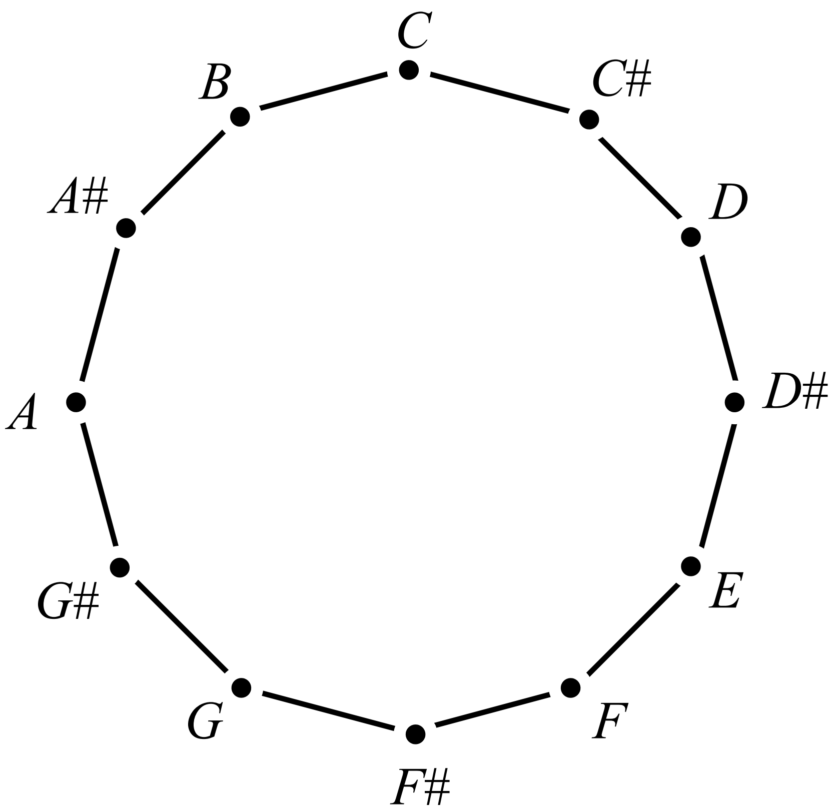
- Neo-Riemannian representation of music
  - Graphical representation of harmony rules
  - Consonance used as a graphical criterion

    Two notes that sound well must be spatially close

Formalization of Notes Neighborhoods

- Strong algebraic structure of music
  - Set N of notes (i.e. pitch classes)

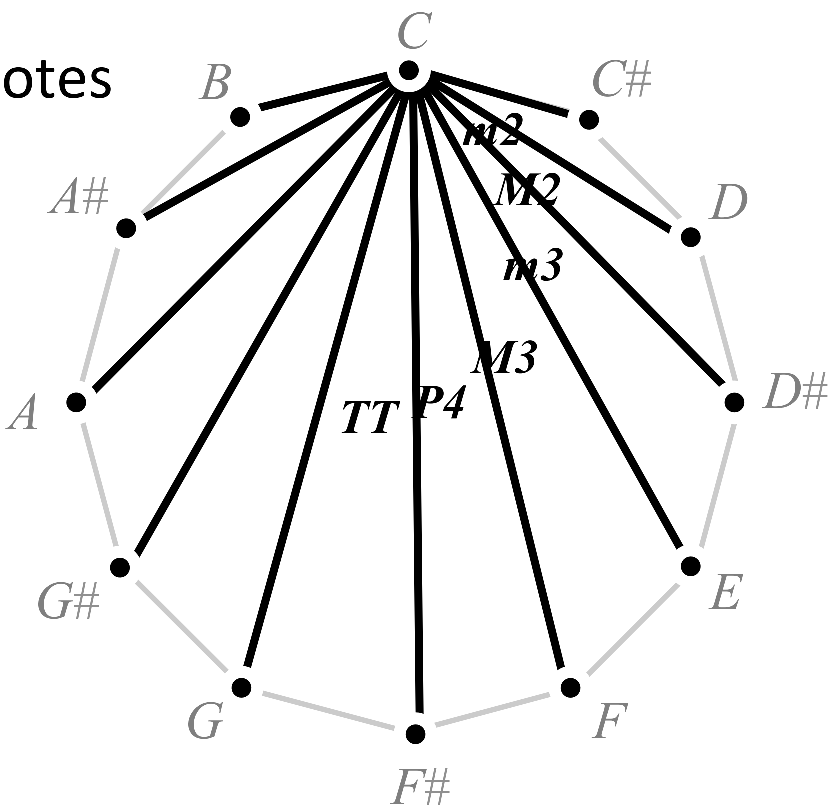
We do not consider octaves



$$\mathrm{N} = \{\ C,\ C\#,\ D,\ D\#,\ E,\ F,\ F\#,\ G,\ G\#,\ A,\ A\#,\ B\ \}$$

Formalization of Notes Neighborhoods

- Strong algebraic structure of music
  - Set N = { *C*, *C#*, …, *A#*, *B* }
  - Group (I,+) of intervals
    - Relative difference between notes

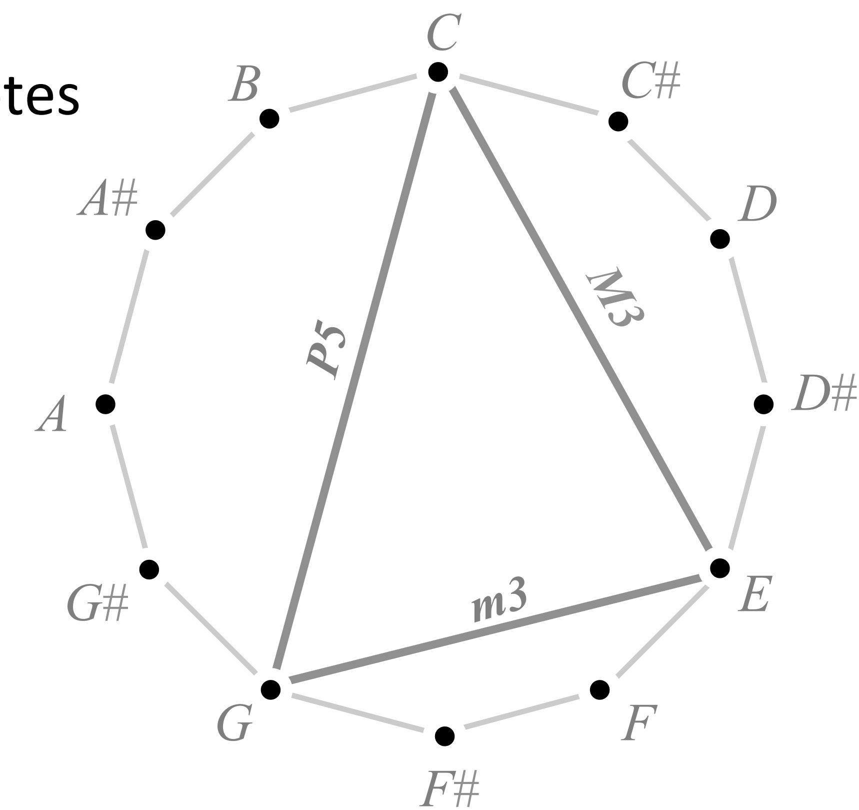


I = { *P1*, *m2*, *M2*, *m3*, *M3*, *P4*, *TT*, *P5*, *m6*, *M6*, *m7*, *M7* }

Formalization of Notes Neighborhoods

- Strong algebraic structure of music
  - Set N = $\{ C, C\#, \ldots, A\#, B \}$
  - Group (I,+) of intervals
    - Relative difference of two notes
    - $(I,+) \cong (\mathbb{Z}_{12},+)$ (isomorphism)
    - Example

$$M3 + m3 = P5$$

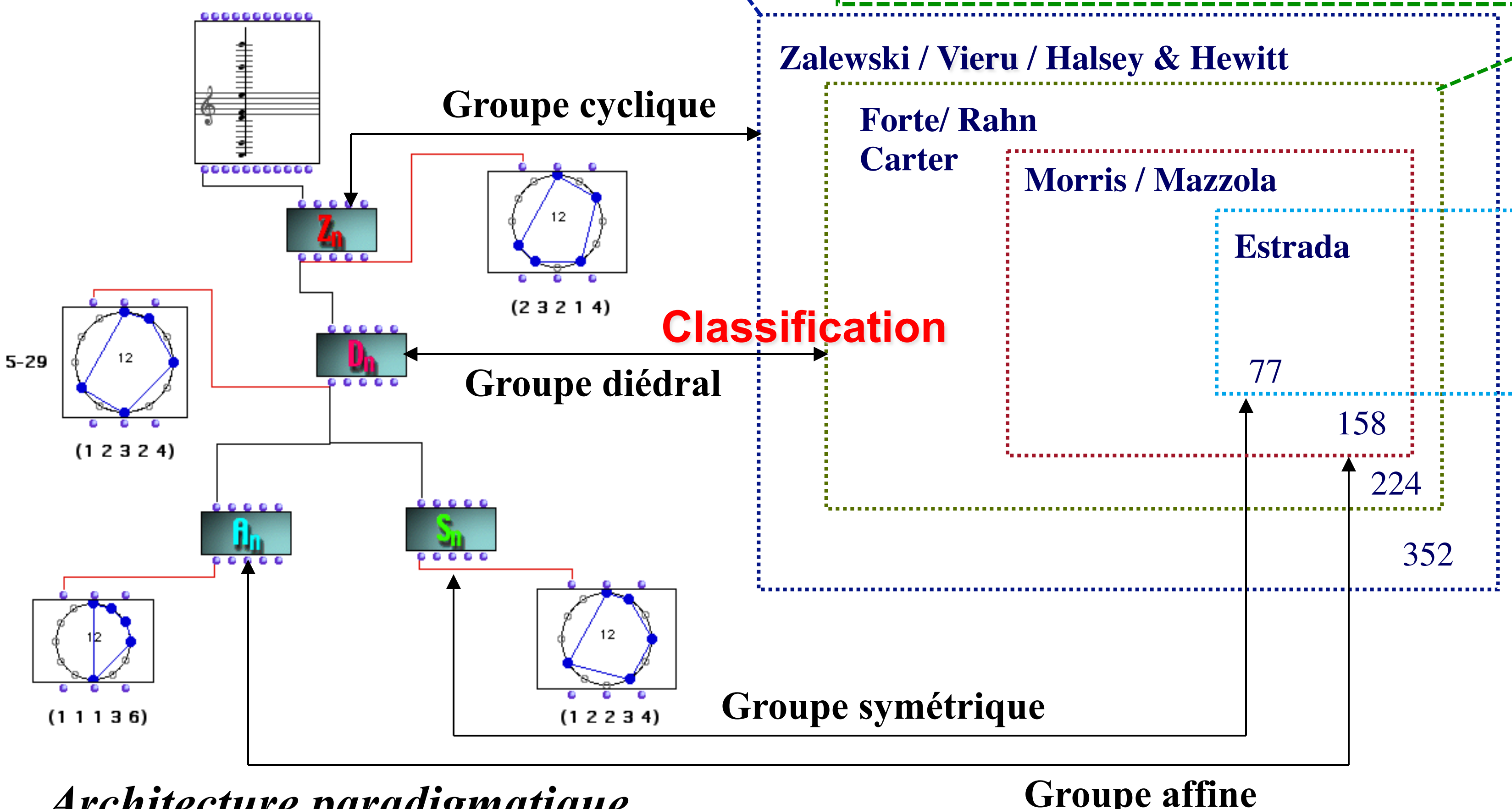


$$I = \{ P1, m2, M2, m3, M3, P4, TT, P5, m6, M6, m7, M7 \}$$

Classification paradigmatique des structures musicales

$$\#\text{ of }k\text{-chords} = \frac{1}{n}\sum_{j|(n,k)} \phi(j)\binom{n/j}{k/j} = \frac{1}{n}\Phi_n(k)$$

$$\#\text{ of }k\text{-chords} = \begin{cases} \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n-1)/2}{[k/2]}\right], & \text{if } n \text{ is odd,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{n/2}{k/2}\right], & \text{if } n \text{ is even and } k \text{ is even,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n/2)-1}{[k/2]}\right], & \text{if } n \text{ is even and } k \text{ is odd.} \end{cases}$$



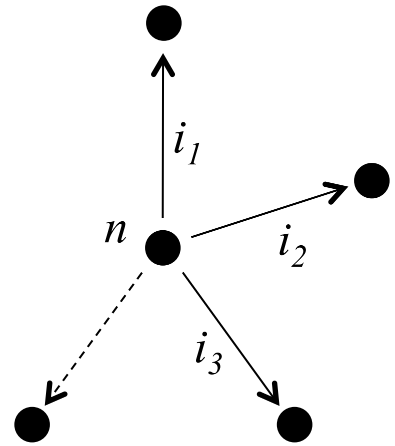
Architecture paradigmatique

- D. Halsey & E. Hewitt: « Eine gruppentheoretische Methode in der Musik-theorie », *Jahresber. Der Dt. Math.-Vereinigung*, 80, 1978.
- D. Reiner: «Enumeration in Music Theory», *Amer. Math. Month*. 92:51-54, 1985
- R.C. Read: « Combinatorial problems in the theory of music », *Discrete Math.*, 1997
- H. Fripertinger: « Enumeration of mosaics », *Discrete Math.*, 1999

Cf. Moreno Andreatta

Formalization of Notes Neighborhoods

- Consonance as a neighborhood relationship
  - $S \subset \mathrm{N} \times \mathrm{N}$
  - $(n_1, n_2) \in S$ if n_1 "sounds well" with n_2



- Assumptions on S
  - S is symmetric
    - $(n_1, n_2) \in S \Rightarrow (n_2, n_1) \in S$
  - S is defined up to a transposition
    - $\forall i \in \mathrm{I}, \quad (n_1, n_2) \in S \Rightarrow (n_1 \oplus i, n_2 \oplus i) \in \mathrm{S}$
    - (C, G) sounds well $\Rightarrow$ $(E = C \oplus M3, B = G \oplus M3)$ sounds well
- S characterized by a subset J of I
    - $J = \{ i_1, i_2, \ldots, i_n \}$
    - $\forall n \in \mathrm{N}, \forall \mathrm{i} \in J, \quad (n, n \oplus i) \in S$

Formalization of Notes Neighborhoods

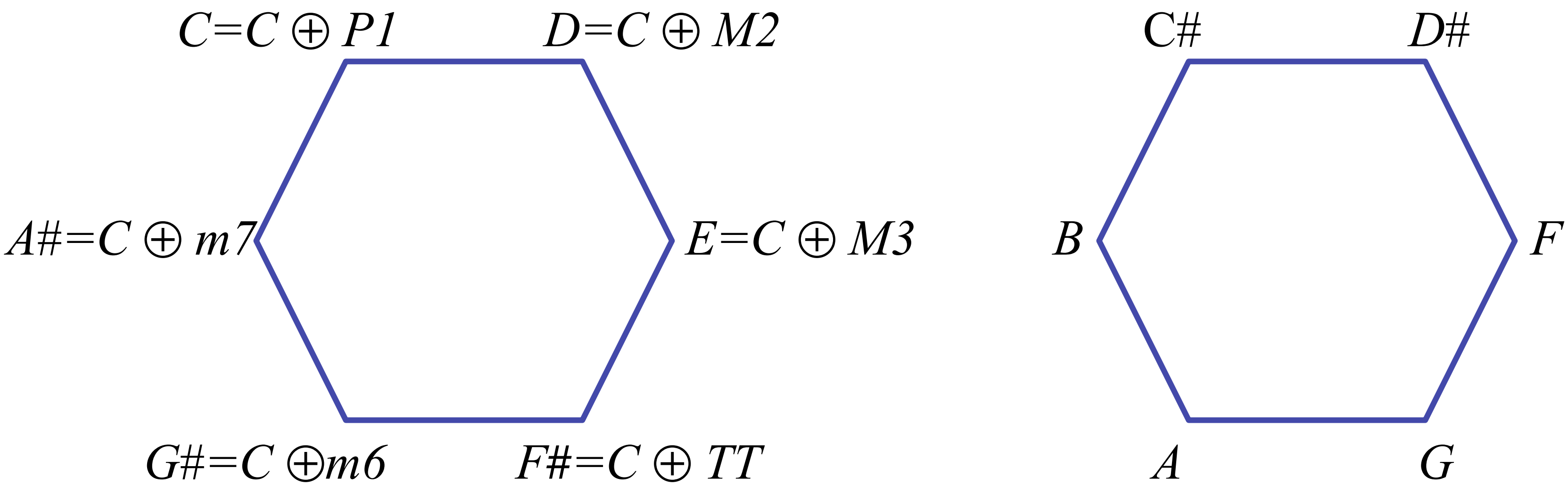
- Spatial representation of S
  - J as a set of group generators
    - $< J >$ subgroup of I generated by the elements of J
    - Example with $J = \{ M2 \}$

$$\begin{aligned} < J > \;&= \{M2,\ 2.M2\ (= M2 + M2),\ \ldots,\ 12.M2\ (= P1 = 0)\ \} \\ &= \{\ P1,\ M2,\ M3,\ TT,\ m6,\ m7\ \} \end{aligned}$$

  - On peut faire *agir* le groupe $< J >$ sur les notes (ou sur I)

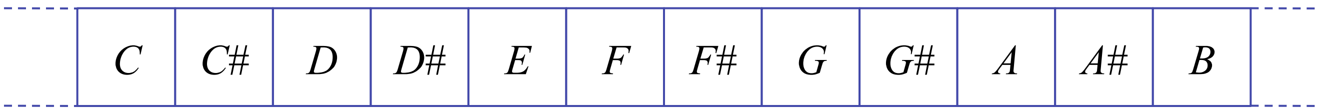
Formalization of Notes Neighborhoods

- Spatial representation of S
  - I as a set of group generators
  - Action of $< J >$ on $\mathbb{N}$ (or $\mathbb{I}$ to stay in a group)
    - Example with $J = \{ M2 \}$

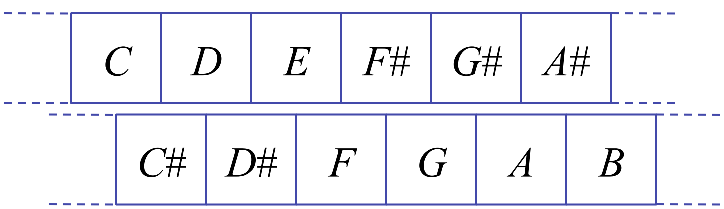


Applications

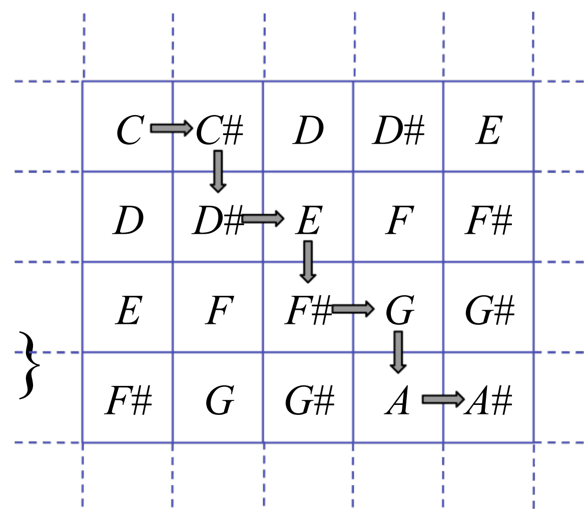
- Scale representations
  - Chromatic scale $J = \{ m2 \}$



  - Whole-tone scale $J = \{ M2 \}$

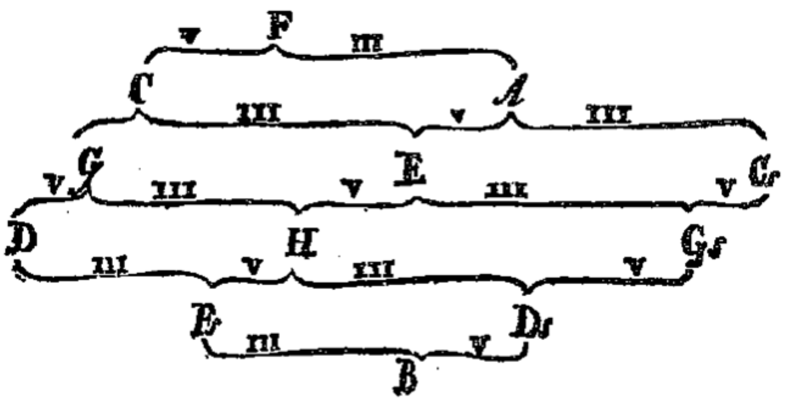


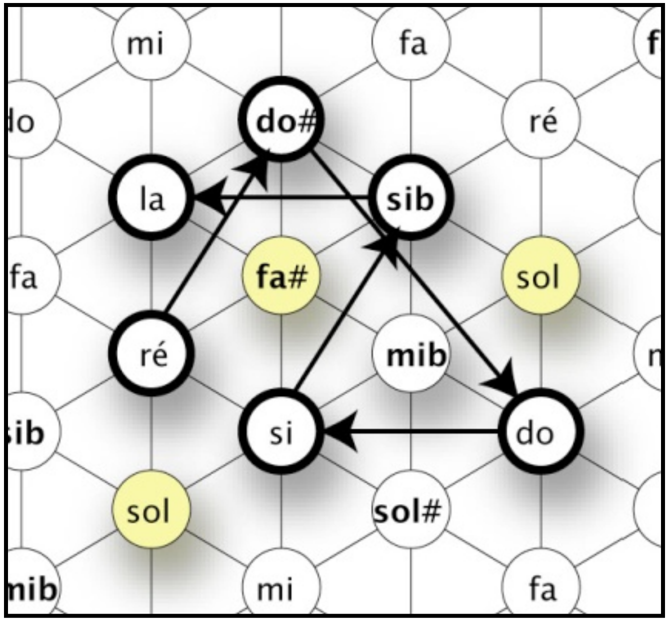
  - Diminished scale $J = \{ m2, M2 \}$



Applications

- Traditional harmony representation
  - $J = \{ M3, P5 \}$ (Euler's Tonnetz)
  - $J = \{ m3, M3, P5 \}$ (Harmonic table)





(J.-M. Chouvel)

Applications

- Instruments conception

$J = \{ m2, P4 \}$ (Guitar)

$J = \{ m2, P5 \}$ (Violin)

$J = \{ m2, M2, m3 \}$ (Accordion)

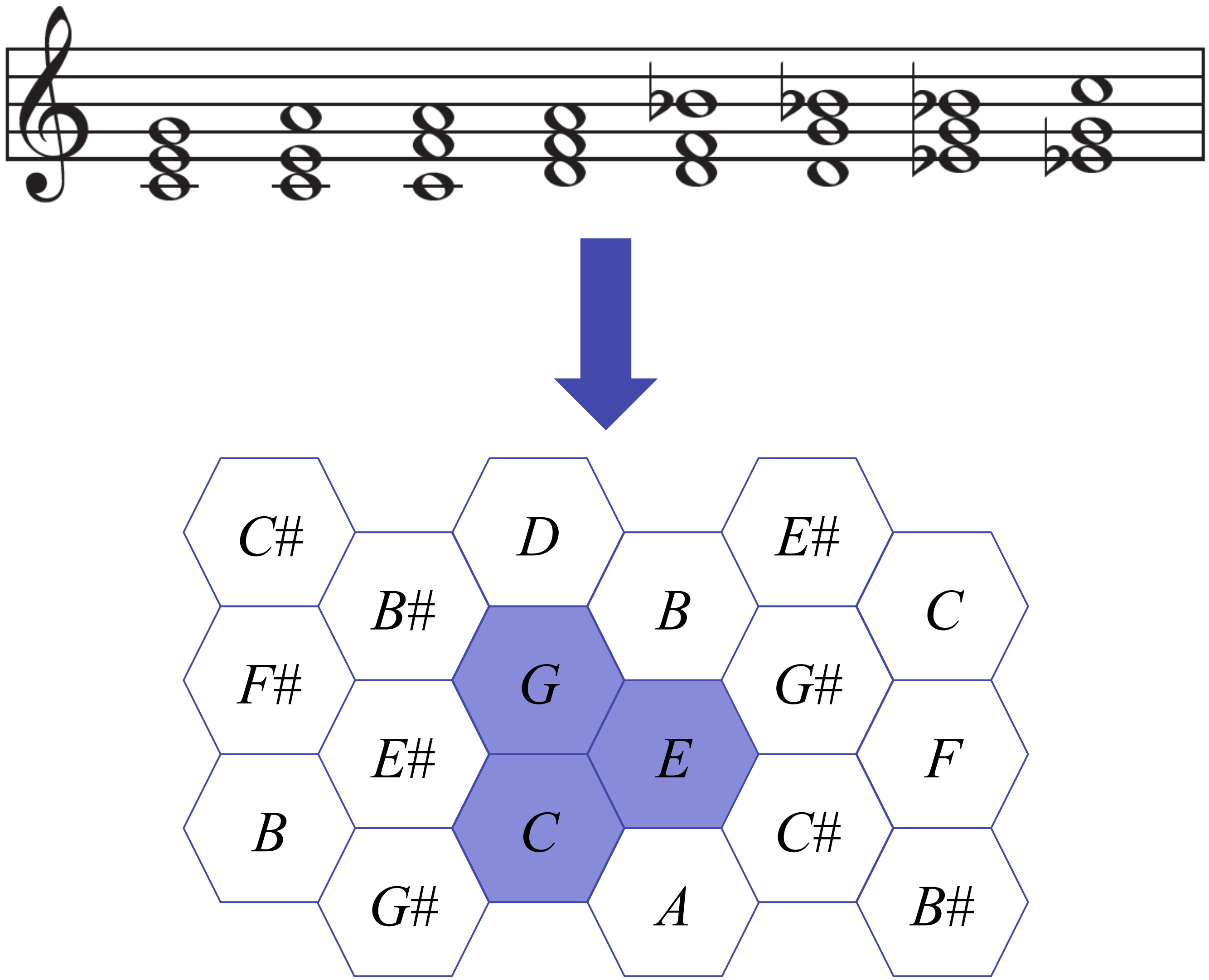
Applications

- Analysis example
  - Signature of a piece
  - Example : F. Chopin Prelude

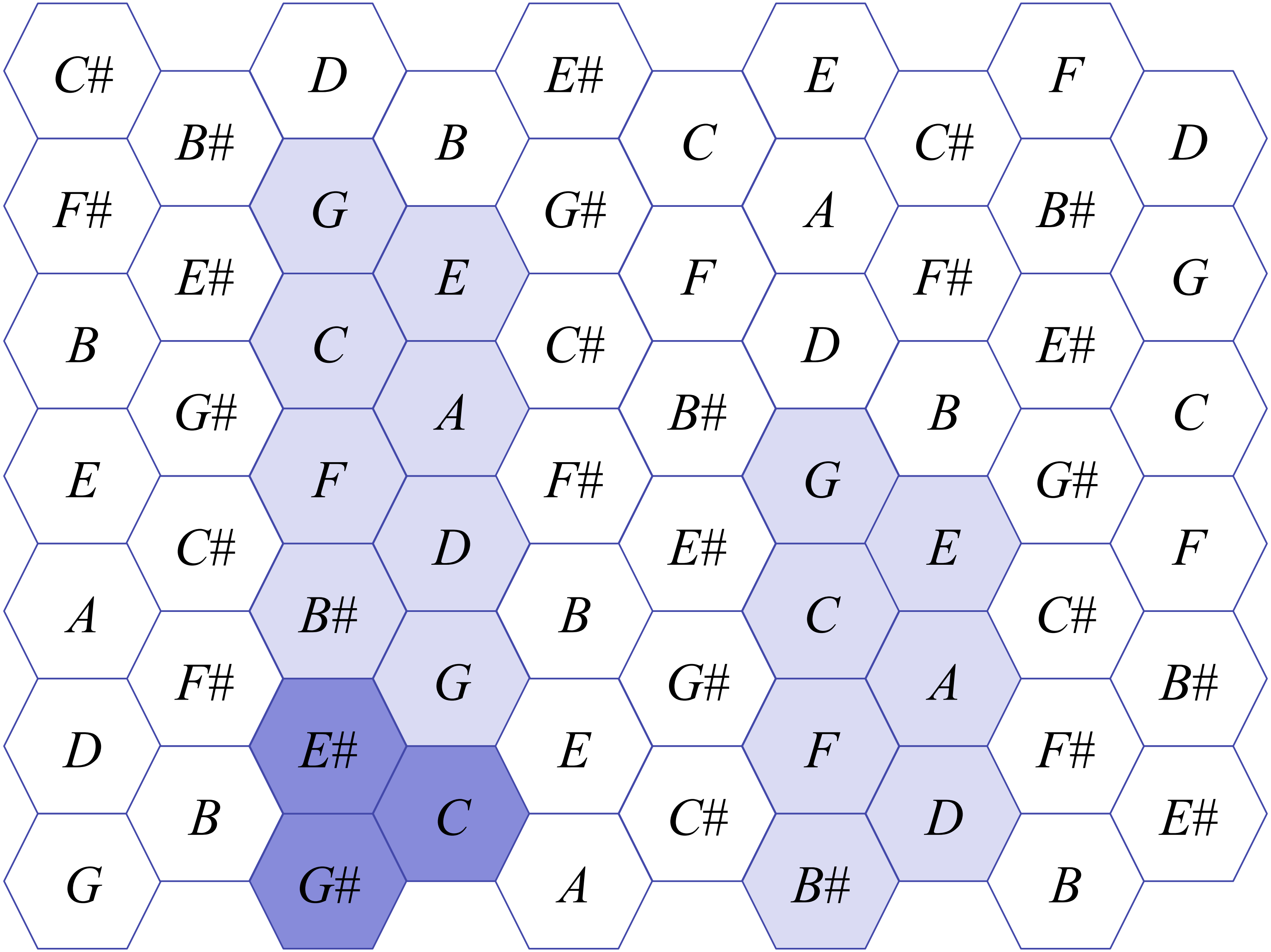
Extract of the 2nd movement
of the Symphony No. 9
L. van Beethoven

Extract of the Prelude N.4
Op28 of F. Chopin

Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

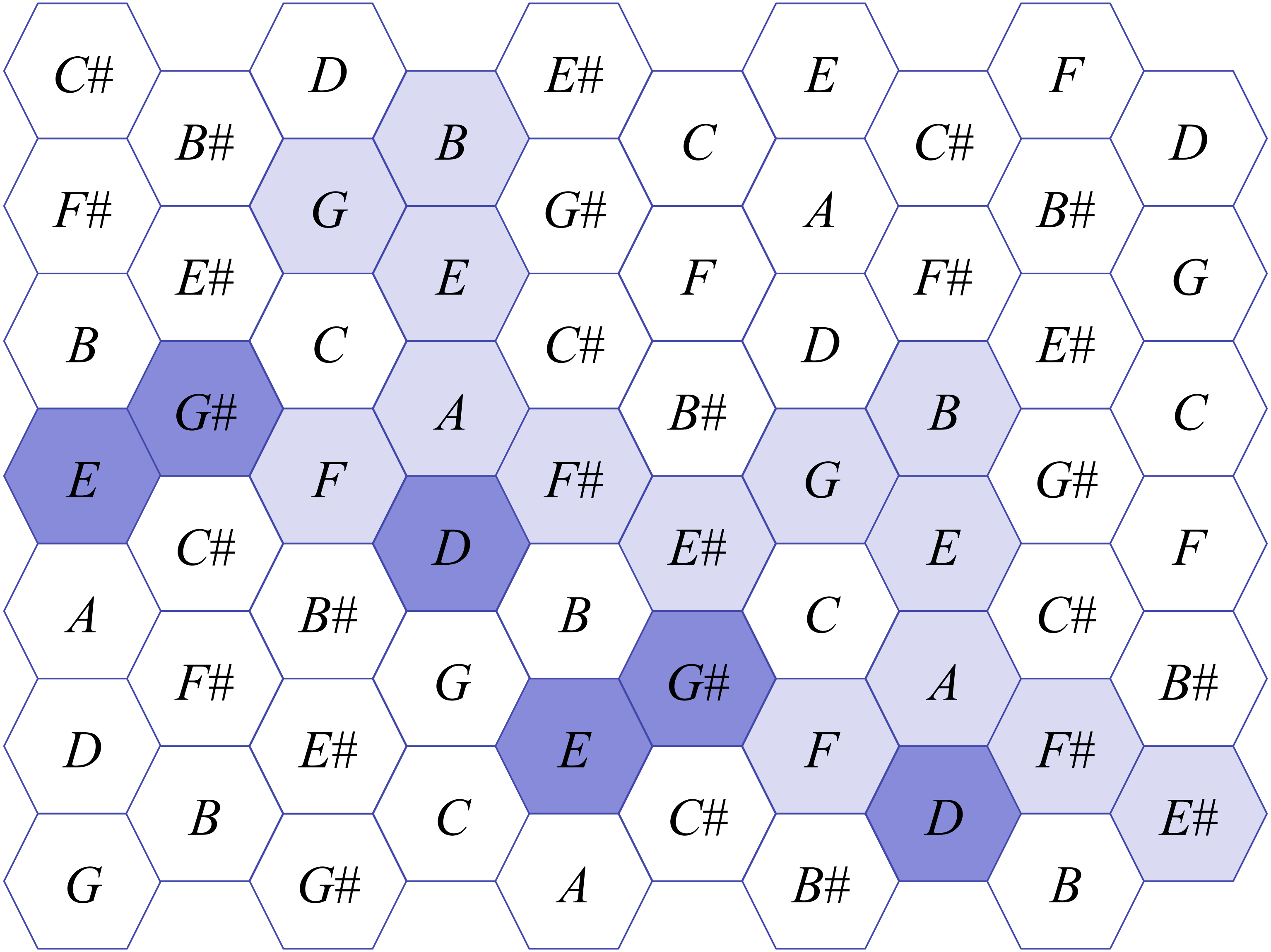


Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

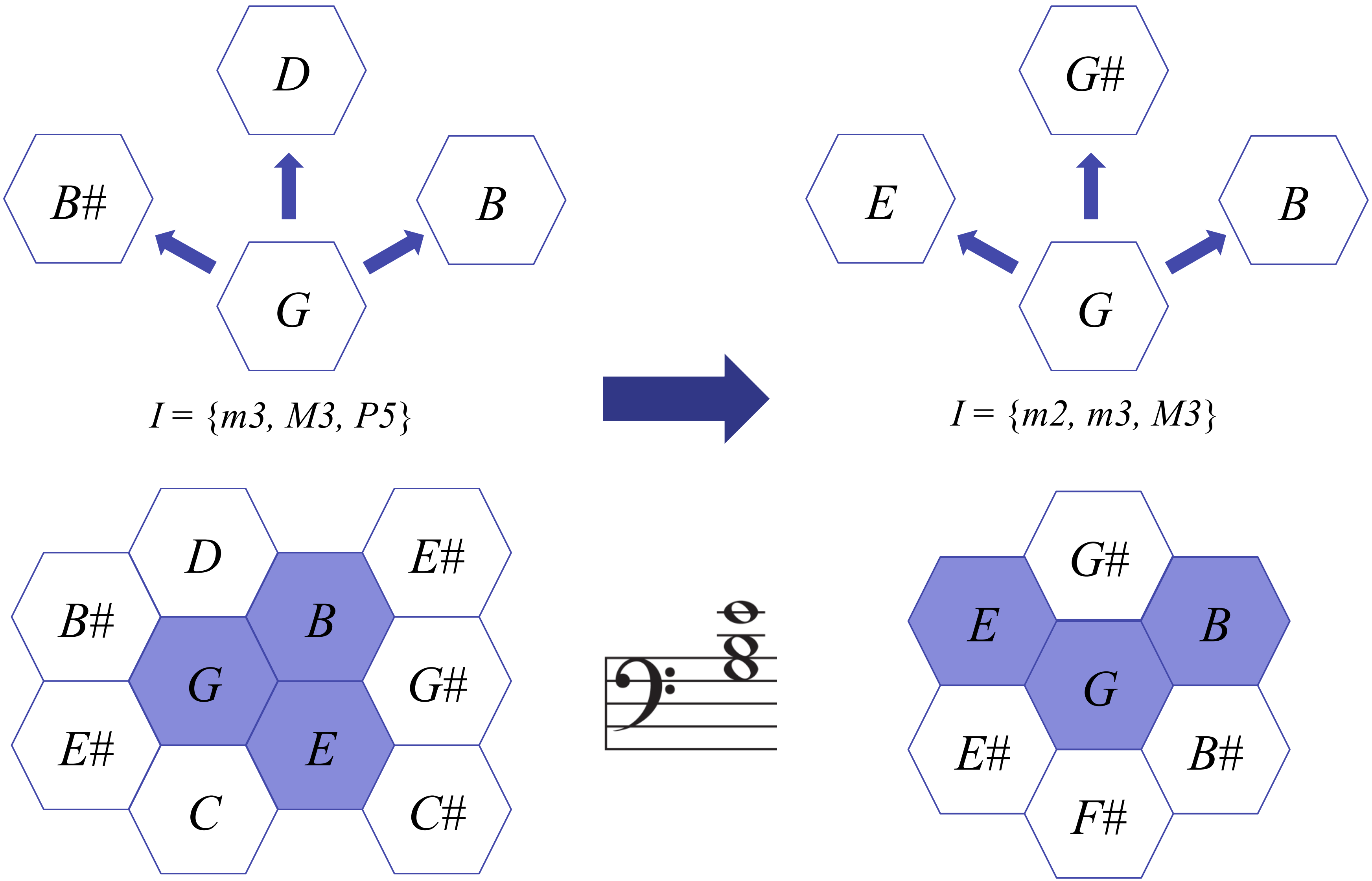


Extract of the Prelude Op.28 N.4
(F. Chopin)

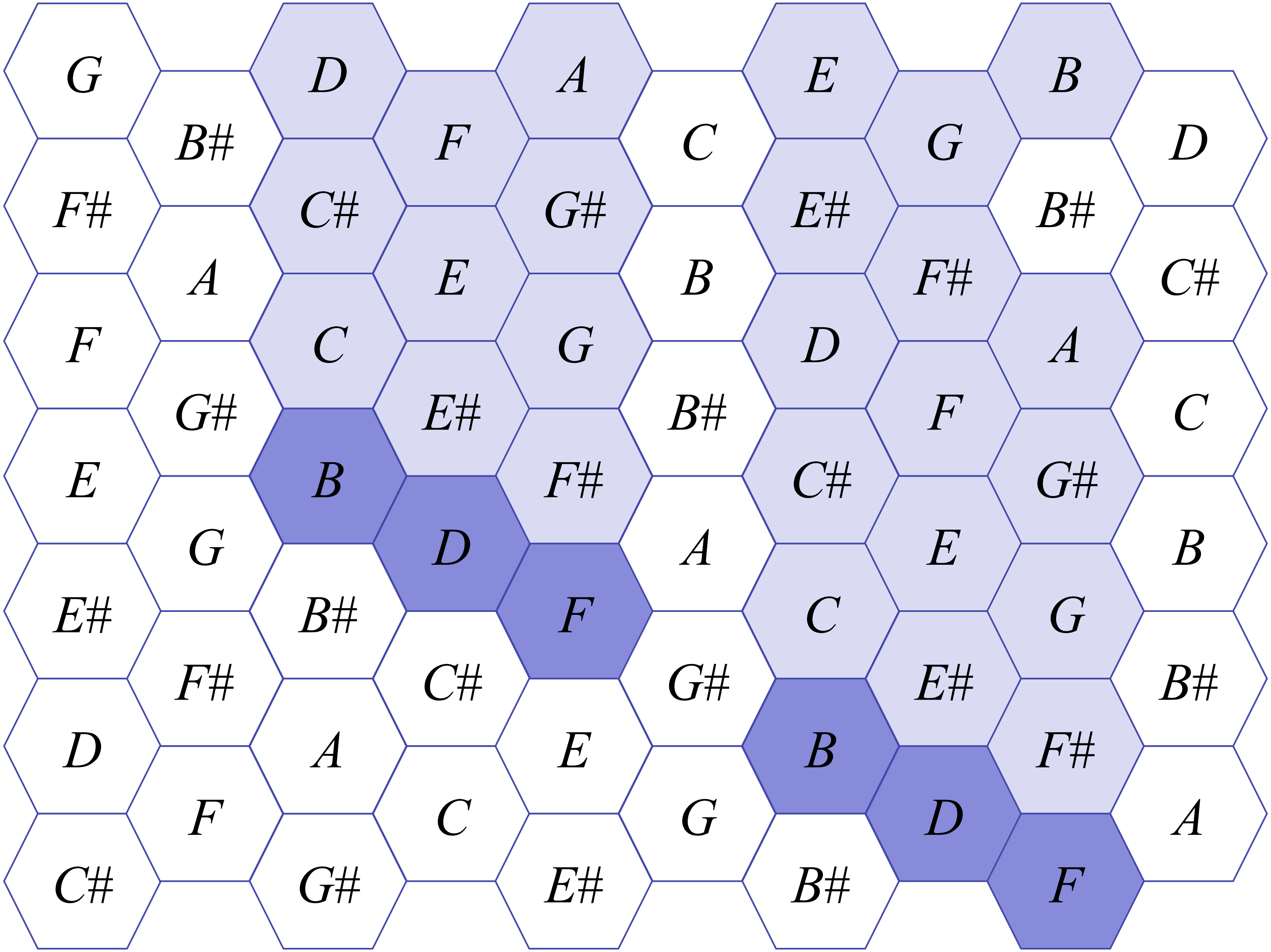
MGS



Extract of the Prelude Op.28 N.4 (F. Chopin)



Extract of the Prelude Op.28 N.4
(F. Chopin)



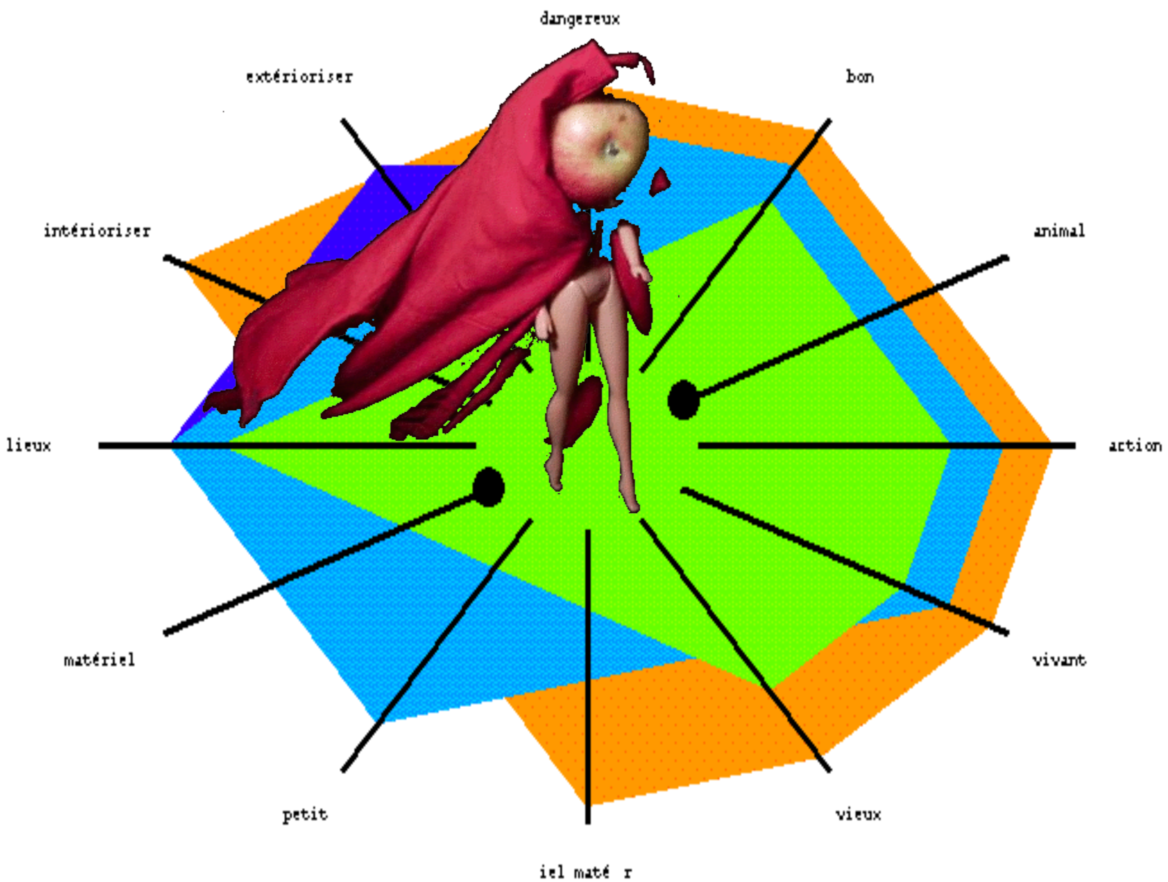
Music & spatial computing?

- Preliminary work
- Collaborations with composers / musicologists
- Extend the validation on more musical problems
- Extension to study musical styles
- Spatial properties ⇔ musical properties
- Acknowledgements

Louis Bigo, Antoine Spicher,
Moreno Andreatta, Carlos Agon,
Jean-Marc Chouvel

L'analyse des contes de fée

(sans Bettelheim)

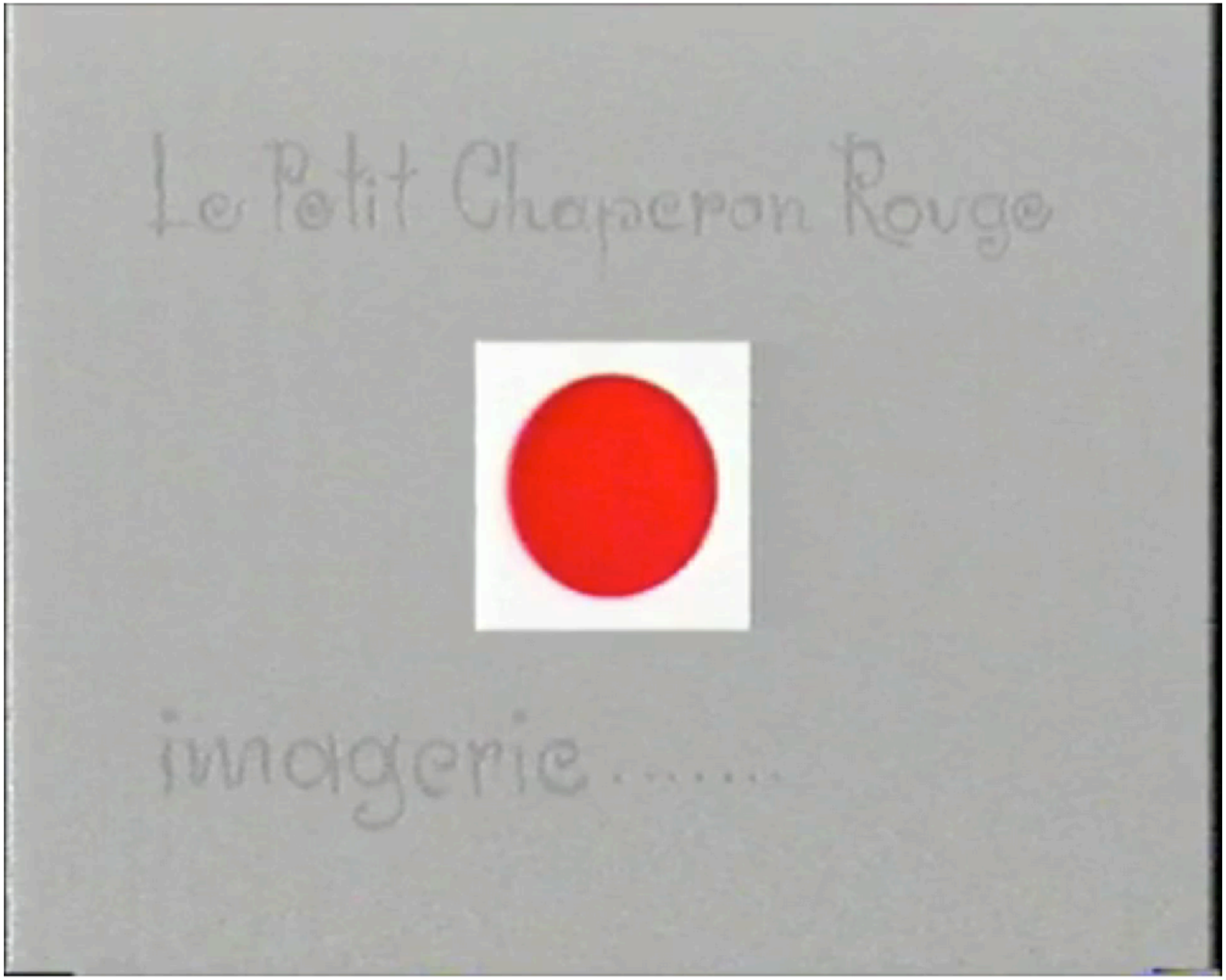


L’analyse des contes de fées

La grand-mère se fait manger vers 2’30

Le PCR se fait manger vers 3’

Le chasseur tue le Loup vers 3’40



L' analyse du petit chaperon rouge

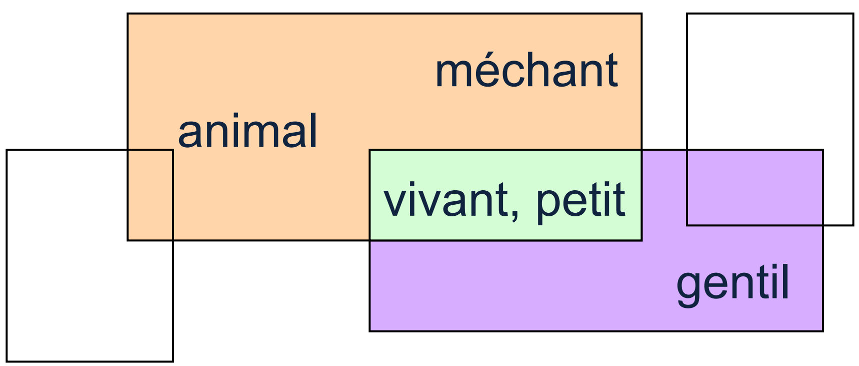
Objectifs :

- extraire d'une séquence d'ensembles d'objets (scène, image, etc.), les sous-ensembles (constituants, catégories, configurations, etc.) permettant de la décrire (et structure en treillis de ces constituants)
- Caractéristique $\in$ vivant, animal, méchant, petit, gentil, moteur, extérieur, place
- Objet = { caractéristiques }

  le petit chaperon rouge = vivant, petit, gentil

  le loup = vivant, animal, méchant, petit
- Scène = { Objets}

  le loup et le PCR parle dans la forêt → vivant, animal, méchant, petit, gentil, moteur, extérieur, place



Encodage

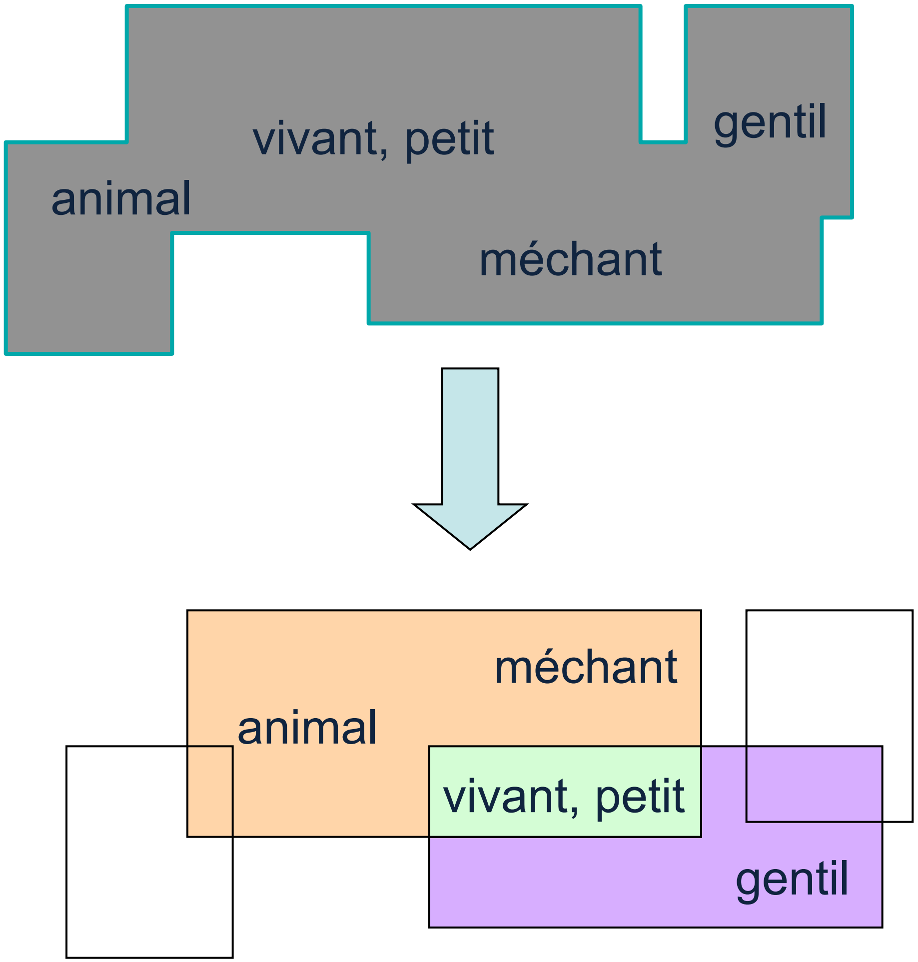
Objets	Encodage (caractéristiques)
le Petit Chaperon Rouge	vivant, petit, bon
la mère	vivant, bon
la grand-mère	vivant, bon, vieux
le Loup	vivant, animal, dangereux
la forêt	vivant, lieux, dangereux
la maison	lieux
le panier	lieux, petit
donner	action, extérioriser, matériel
dormir	action, bon
manger	action, intérioriser, matériel
marcher	action,
parler	action, extérioriser

Encodage

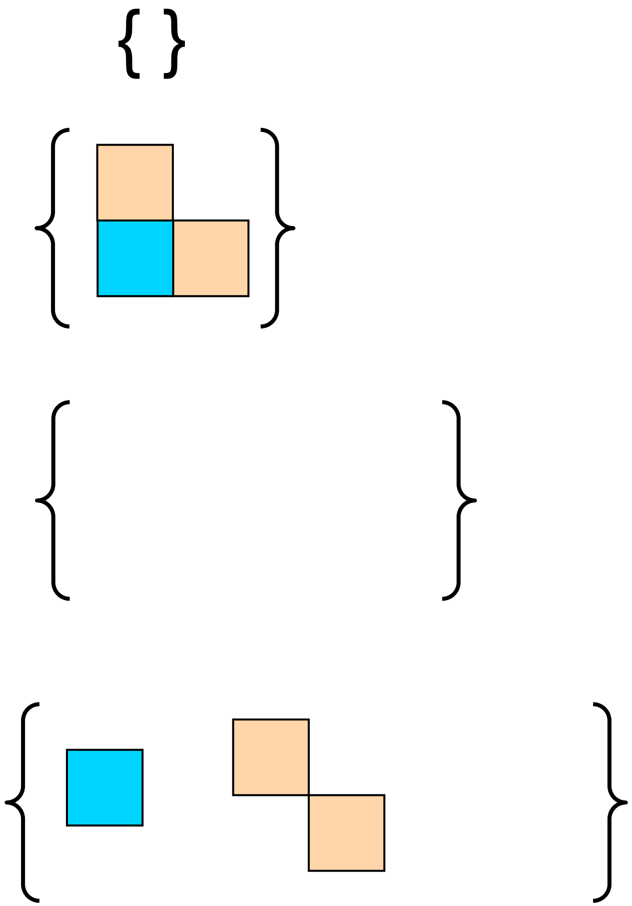
Description	*Scène* (informel)
PCR, mère, parler, maison	La maman informe le PCR de sa GM malade
PCR, mère, donner, panier, maison	La maman donne le panier au PCR
PCR, marcher, forêt, panier	Le PCR marche dans la forêt
PCR, loup, parler, forêt, panier	Il rencontre le loup dans la forêt et lui raconte sa mission
loup, marcher, forêt	Le loup se dirige vers la maison de la GM
PCR, marcher, forêt, panier	Le PCR poursuit son chemin pour apporter le panier à la GM
GM, dormir, maison	La GM est à sa maison et dort
GM, loup, parler, maison	Le loup se présente à la maison de la GM et se fait passer pour PCR
loup, manger, GM	Le loup mange la GM
PCR, loup, parler, maison, panier	Le PCR arrive et parle au loup
loup, manger, PCR, panier	Le loup mange le PCR

Tomographie

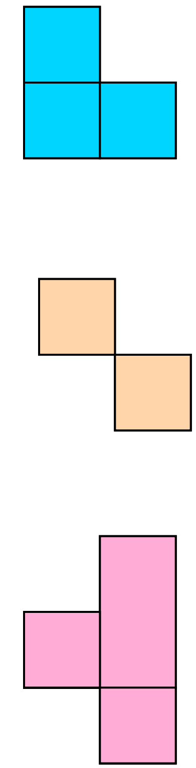
Question : si je connais seulement les caractéristiques des scènes, comment puis-je retrouver les objets (et en faire une hiérarchie) ?



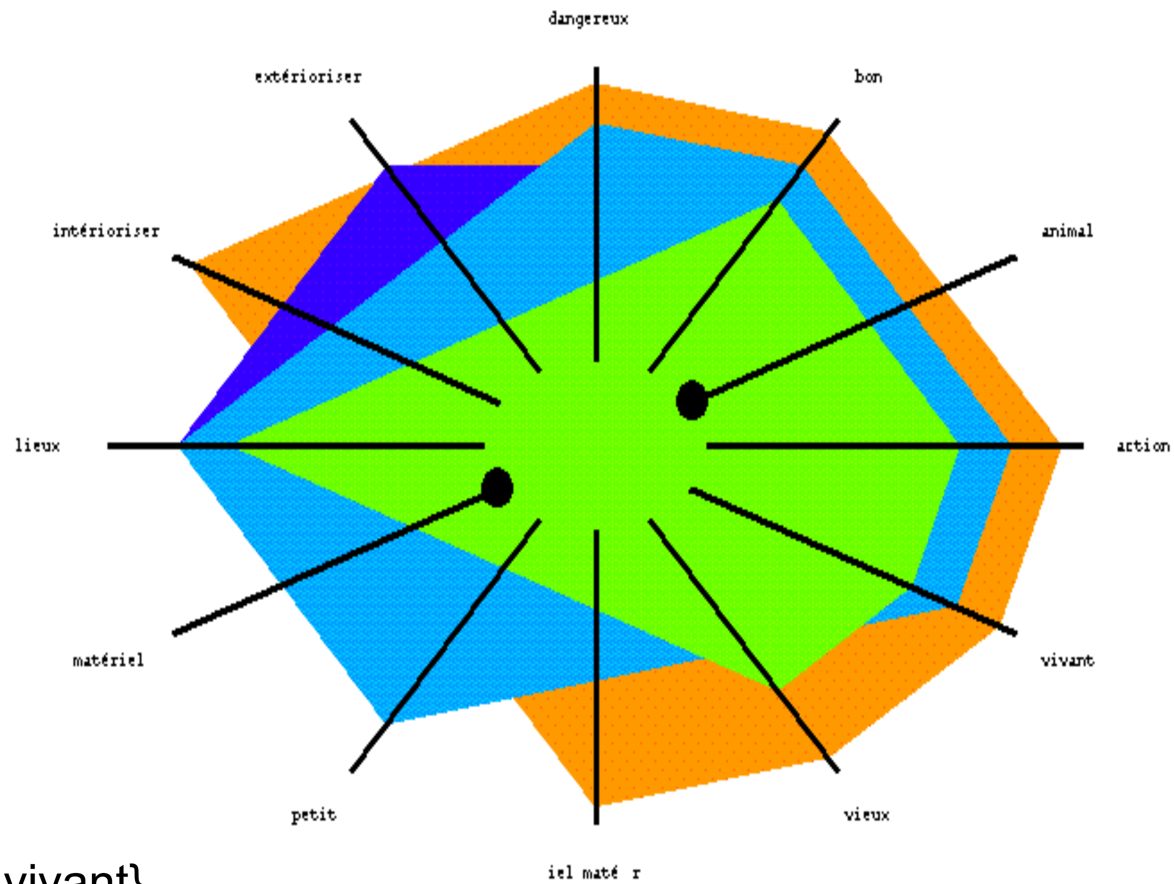
Base courante d’objets



Scène courante

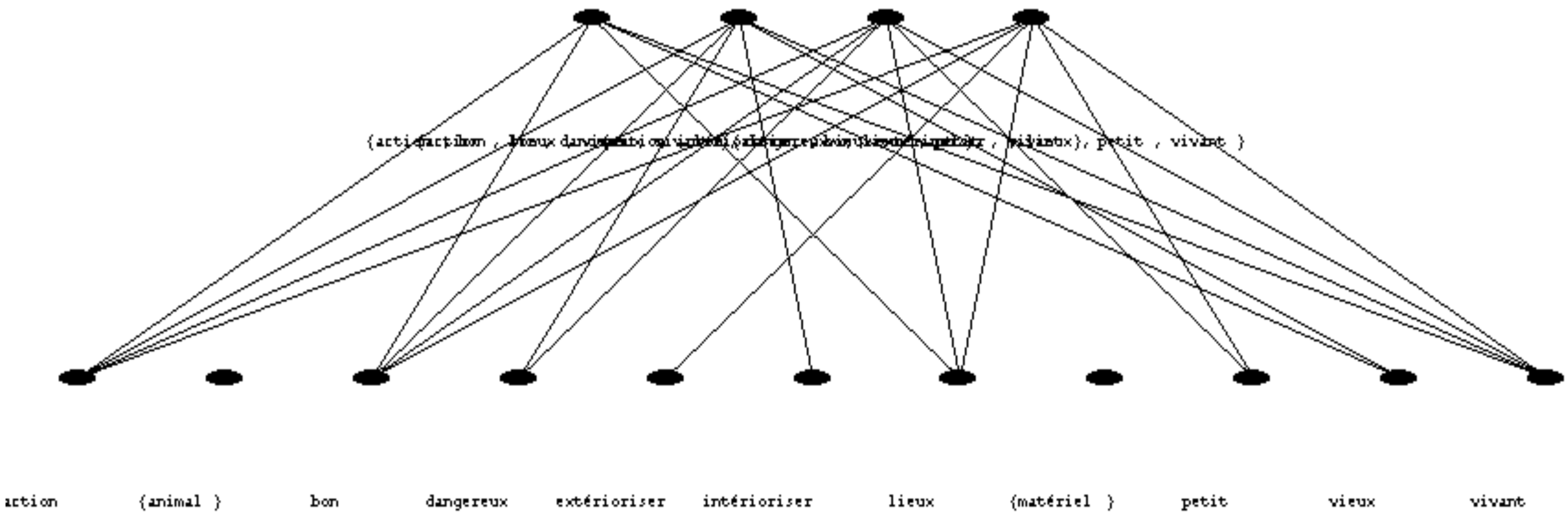


Un ensemble de caractéristiques = un complexe simplicial



{
  {animal},
  {extérioriser},
  {intérioriser}, {matériel},
  {action, dangereux, lieux, vivant},
  {action, bon, lieux, vieux, vivant},
  {action, bon, dangereux, intérioriser, vieux, vivant},
  {action, bon, dangereux, lieux, petit, vivant},
  {action, bon, extérioriser, lieux, petit, vivant}
}

Treillis des concepts formels



Treillis des concepts formels (Wille 97) pour une relation $\lambda \subset A \times P$, on définit une opération § par :

$$B \subset A,\ \S B = \{p \in P \mid \forall b \in B,\ (b,p) \in \lambda\}$$

$$Q \subset P,\ \S P = \{a \in A \mid \forall p \in Q,\ (a,p) \in \lambda\}$$

§ établit une connexion de Galois.

(B,Q) *concept formel* ssi B=§Q et Q=§B : B *extension* et Q *intension* du concept.

Simplexe et Complexe

Un complexe simplicial abstrait : (V,C)

- **V** ensemble de sommets,
- **C** est un ensemble de parties de V fermé pour l'inclusion :

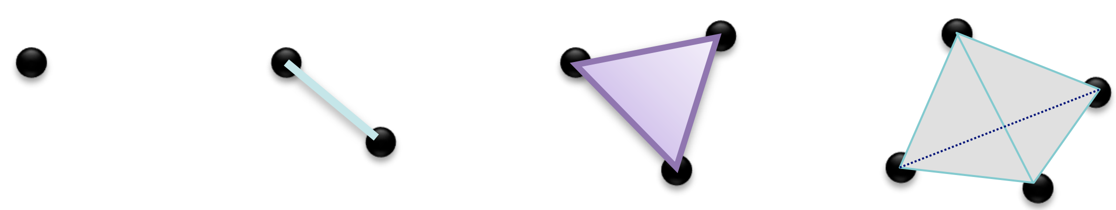
  si $s \in C$ et $s' \subset s$ alors $s' \in C$

- simplexe = élément de C
- dimension d'un simplexe $s = |s| - 1$
- dimension d'un complexe = dimension de son plus grand simplexe
- chemin P = suite de simplexes s_i tel que *si* $s_i \cap s_{i+1} \neq \varnothing$
- bord ∂s d'un simplexe s : $\{s', \; s' \subset s$ et $s' \neq s\}$

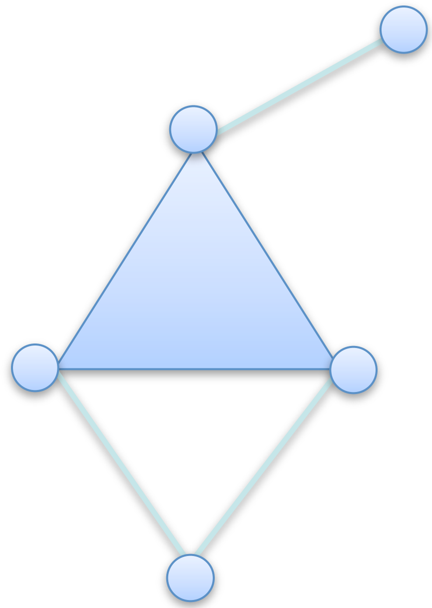
Représentation graphique (généralise les graphes)

simplexe abstrait	{a}	{a,b}	{a,b,c}	{a,b,c,d}	
	point	arête	face	volume	…

simplexe euclidien



Complexe



{

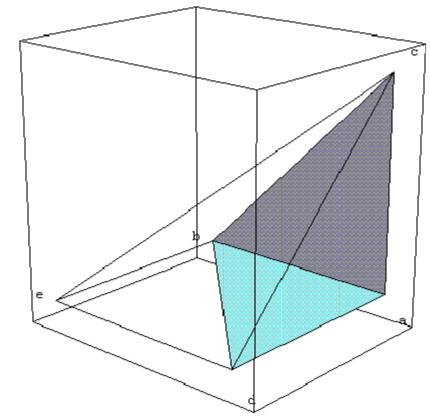
{a}, {b}, {c}, {d}, {e}, simplexe de dim 0

{a,b}, {b,c}, {b,d}, {a,d}, {a,e}, {c,d}, simplexe de dim 1

{a,b,d} simplexe de dim 2

}

Topologie ↔ complexe simpliciaux ↔ treillis

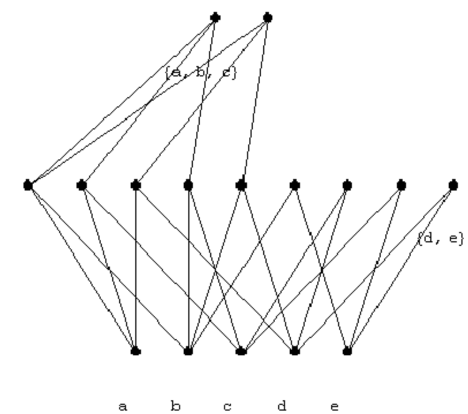


Topologie (ensemble des ouverts et des fermés)

simplexes
=
fermés de la topologie

Complexe simplicial (ensemble d'ensembles fermés par inclusion)

simplexes
=
cônes du treillis



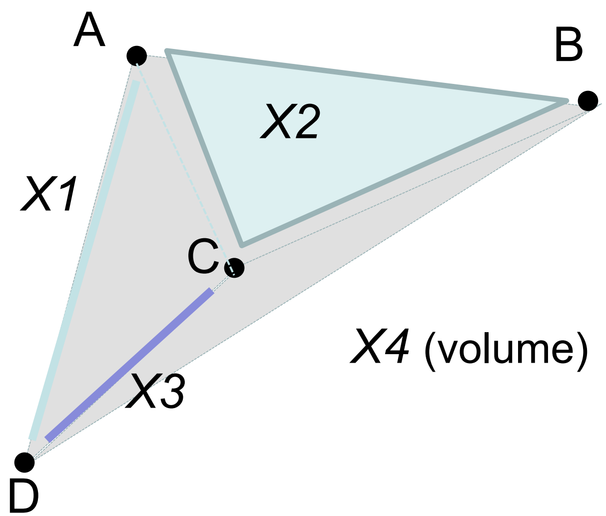
Treillis (relation d'ordre: ∧ , ∨)

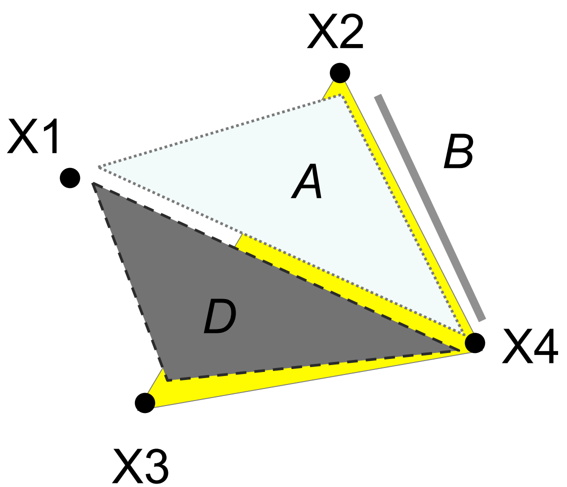
- Reformulation élégante d'approches « classiques »
- Extension des outils

• Une représentation topologique des relations : la Q-analyse

MGS

	A	B	C	D
X1	1	0	0	1
X2	1	1	1	0
X3	0	0	1	1
X4	1	1	1	1





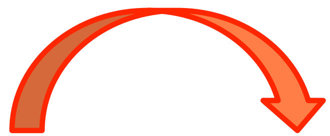
(le dual)

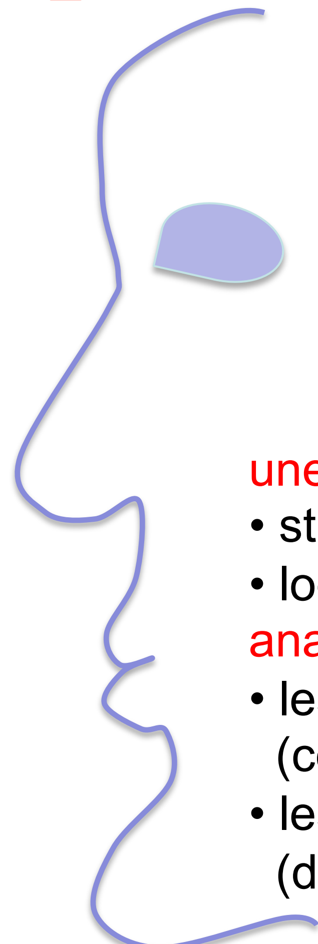
Du percept au concept (~Jean Nicod)

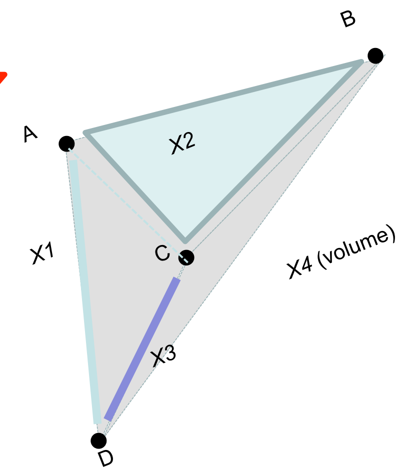
Percepts

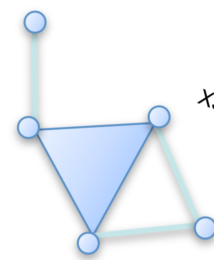
Concepts

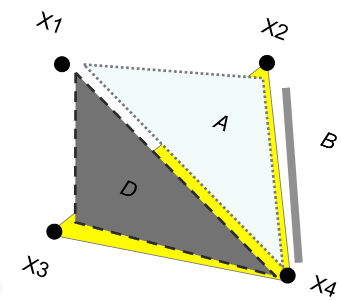
la pompe à concept

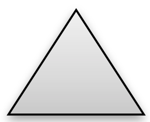


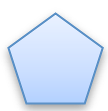


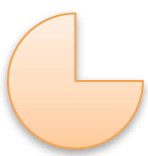


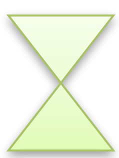












une organisation spatiale :

- structure de la co-occurence des percepts
- logique des observations (= topologie des ouverts)

analyse moins naïve :

- les attributs se perçoivent simultanément (conjonction)
- les valeurs d'un attribut sont exclusives (disjonction)

L'analogie aristotelitienne

Un problème d'analogie

Etant donné A, B et C,

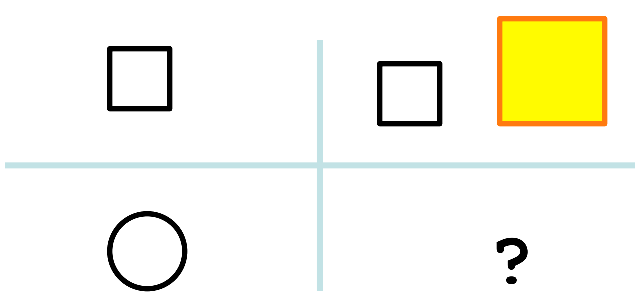
Trouver D qui est à C ce que B est à A

A	B
C	?

Exemple numérique

3	6
7	?

Exemple géométrique

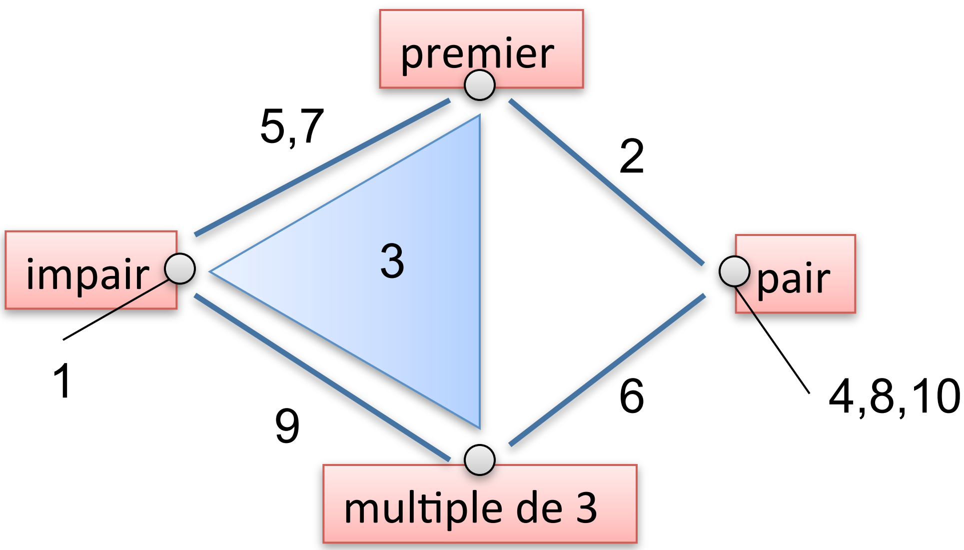


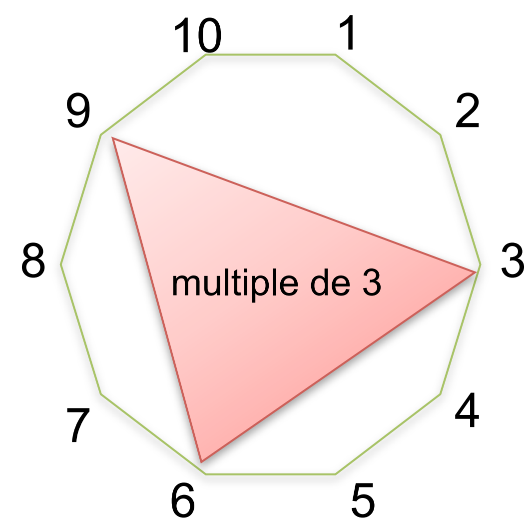
Représentation d’un ensemble de prédicats

$\lambda \subset \text{Objets} \times \text{Predicats} : (o,p) \in \lambda \Leftrightarrow p(o)$

Objets = {1, 2, 3, …, 10}

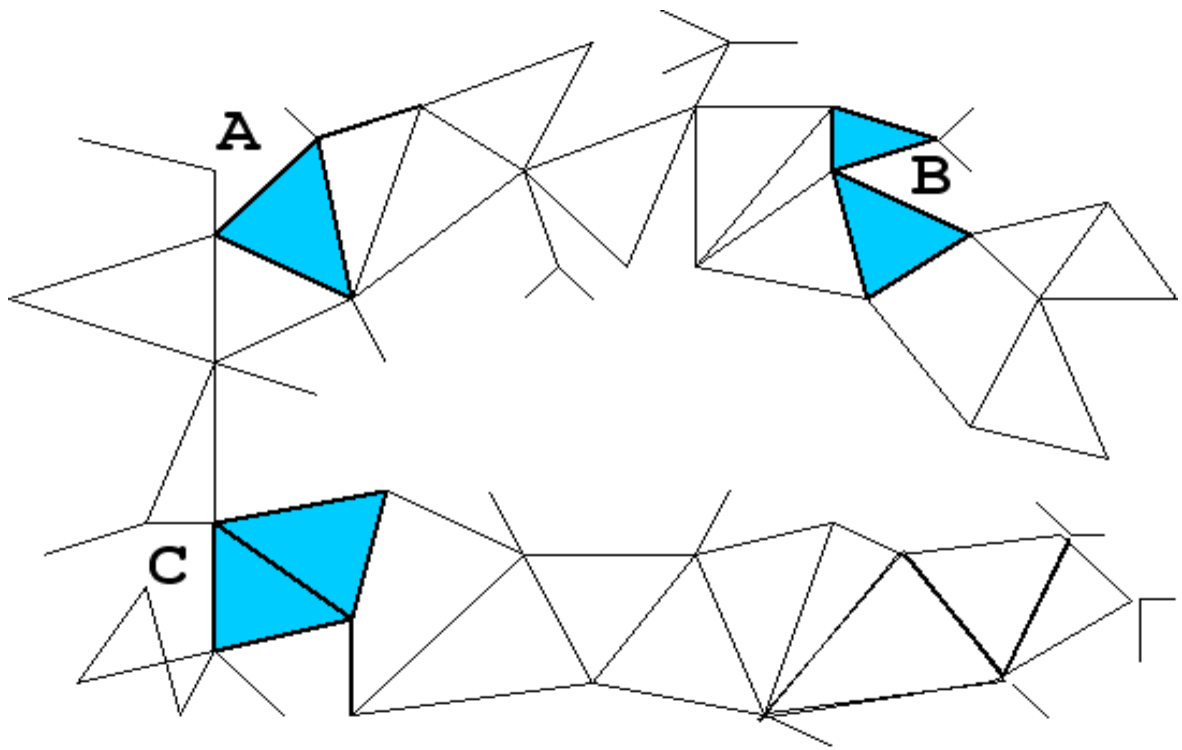
Predicats = {premier, pair, impair, multiple-de-3}





Une méthode de résolution d'analogie

1. Représenter chaque figure comme une région de l'*espace des propriétés des figures*
2. Trouver un *chemin* γ entre la région A et la région B
3. Généraliser ce chemin en l'interprétant comme une *transformation* Γ *de l'espace (i.e. de propriétés)* via le chemin de A à C
4. Appliquer Γ à γ pour trouver D



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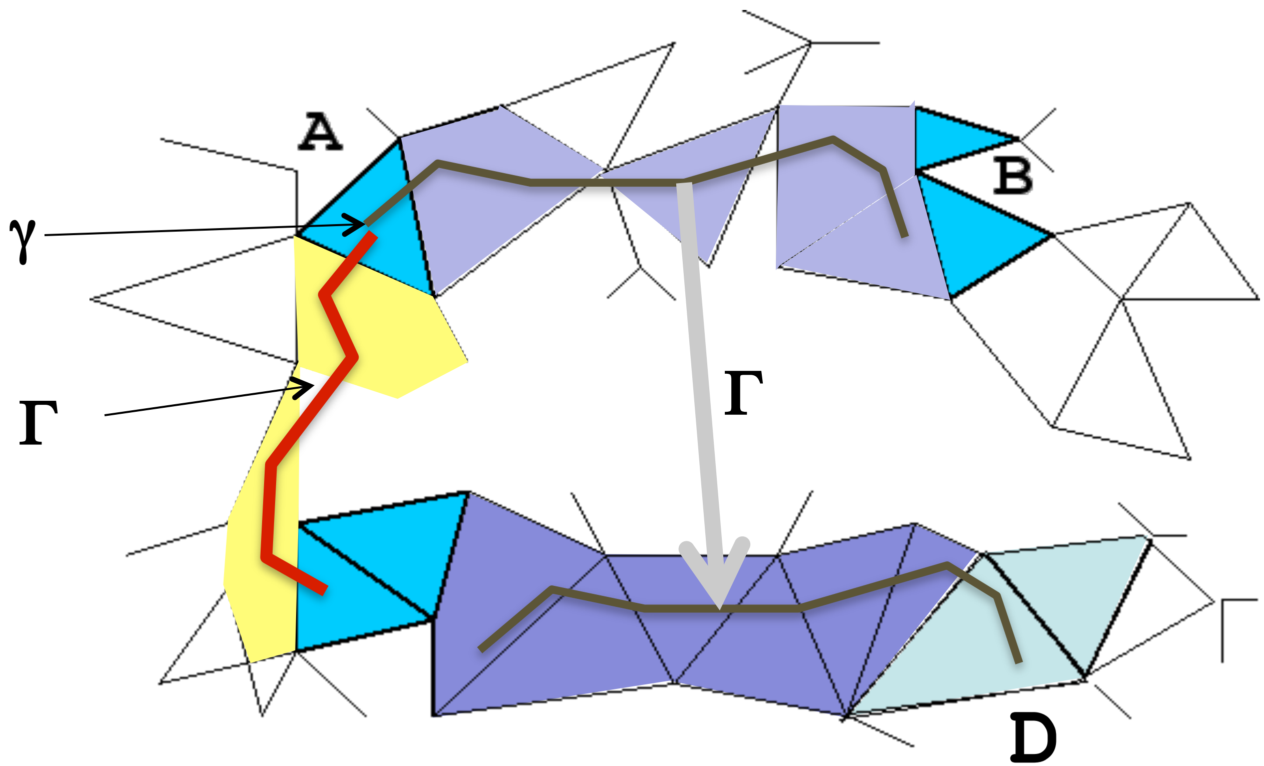
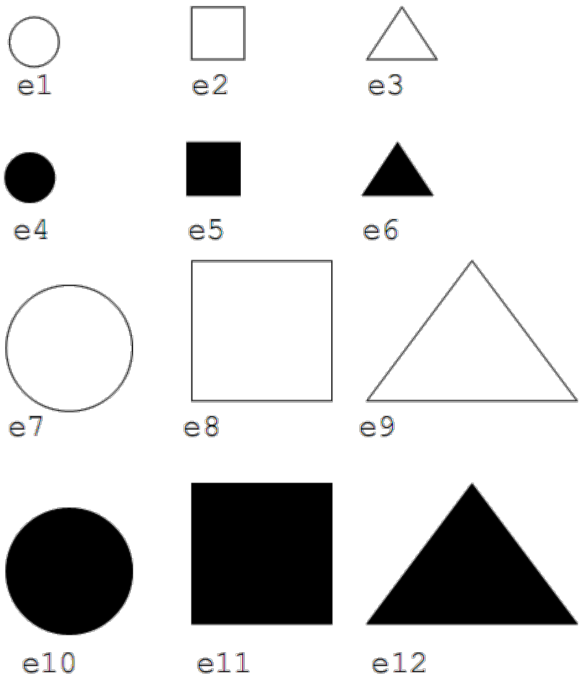
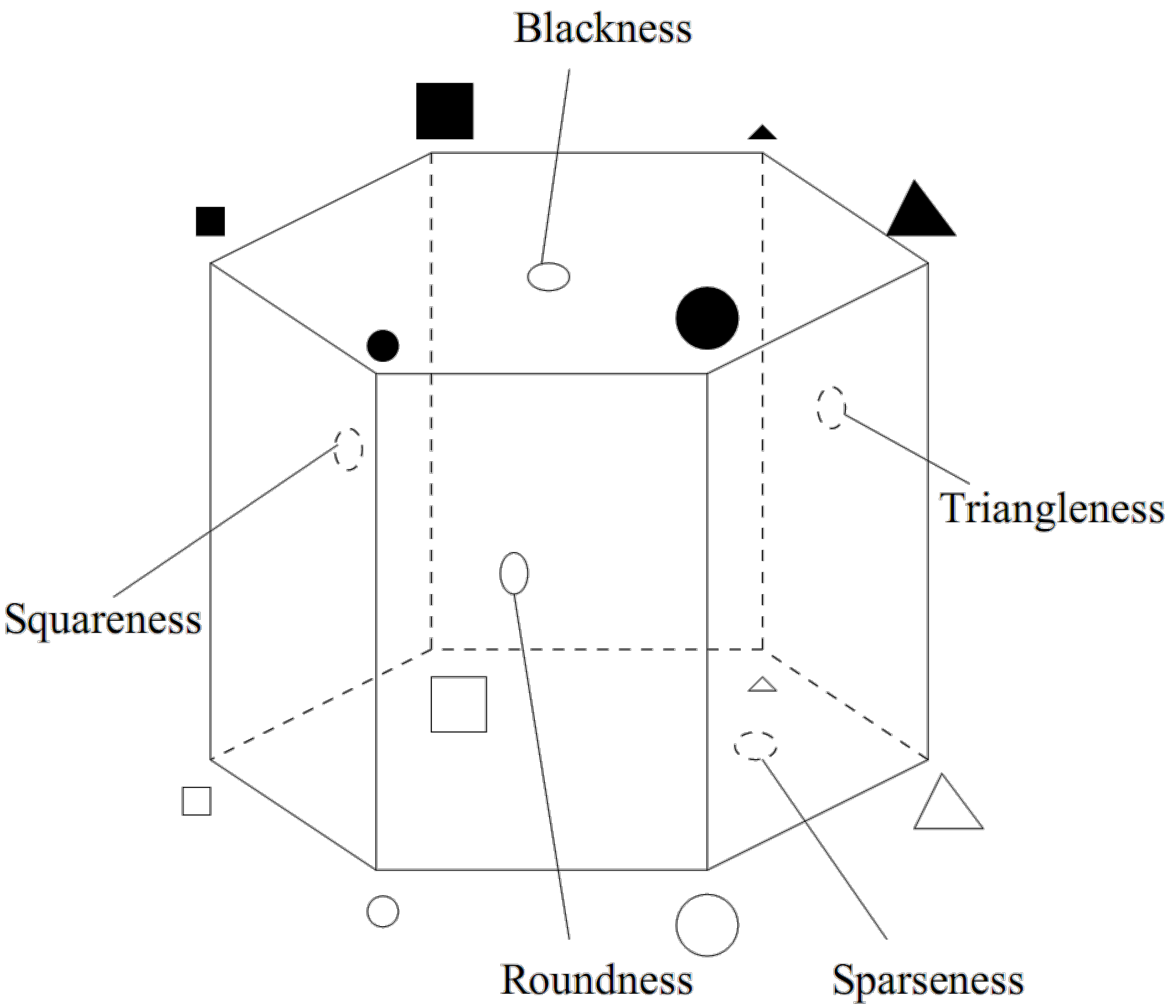
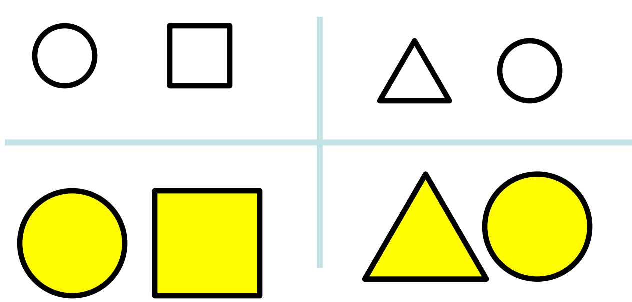


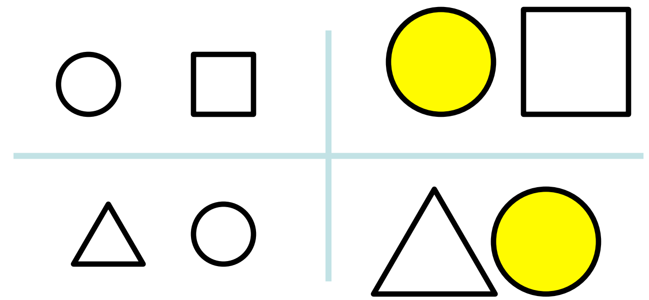
Notre petit monde…

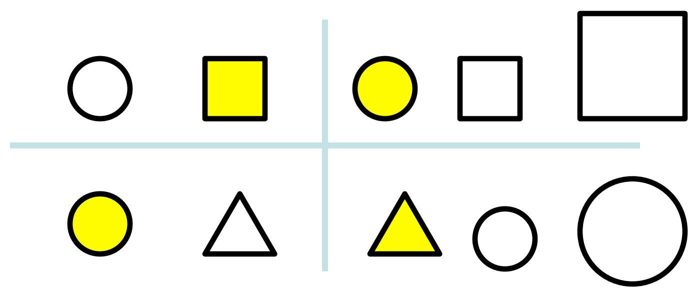




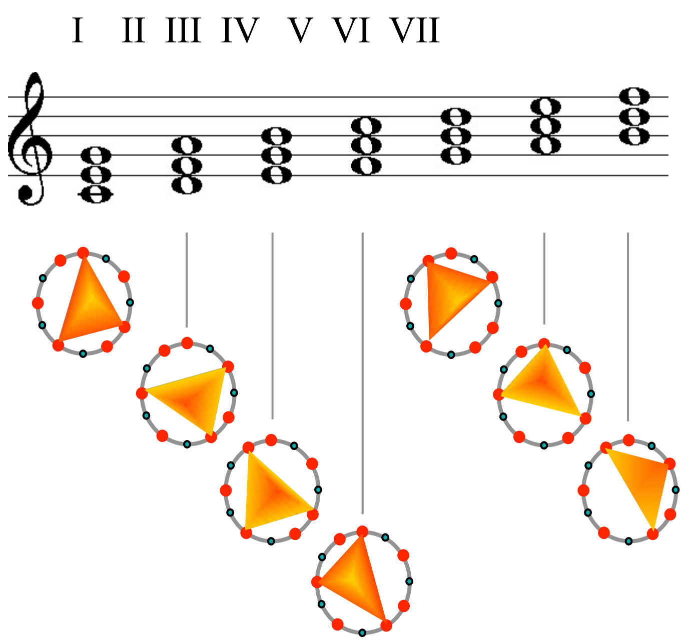
- Exemple de résolution avec *Esqimo*

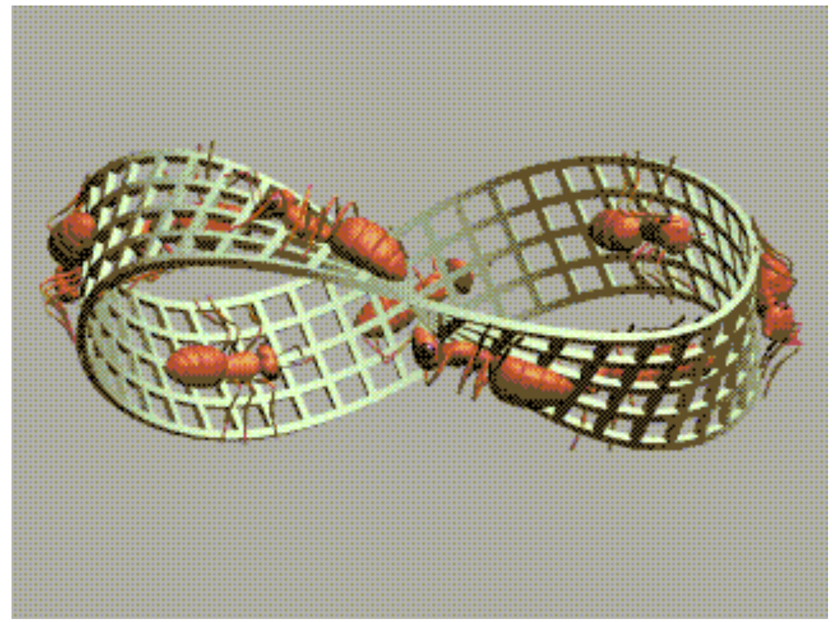






Après la pause : le ruban de Moebius dans les degrées de la gamme majeur



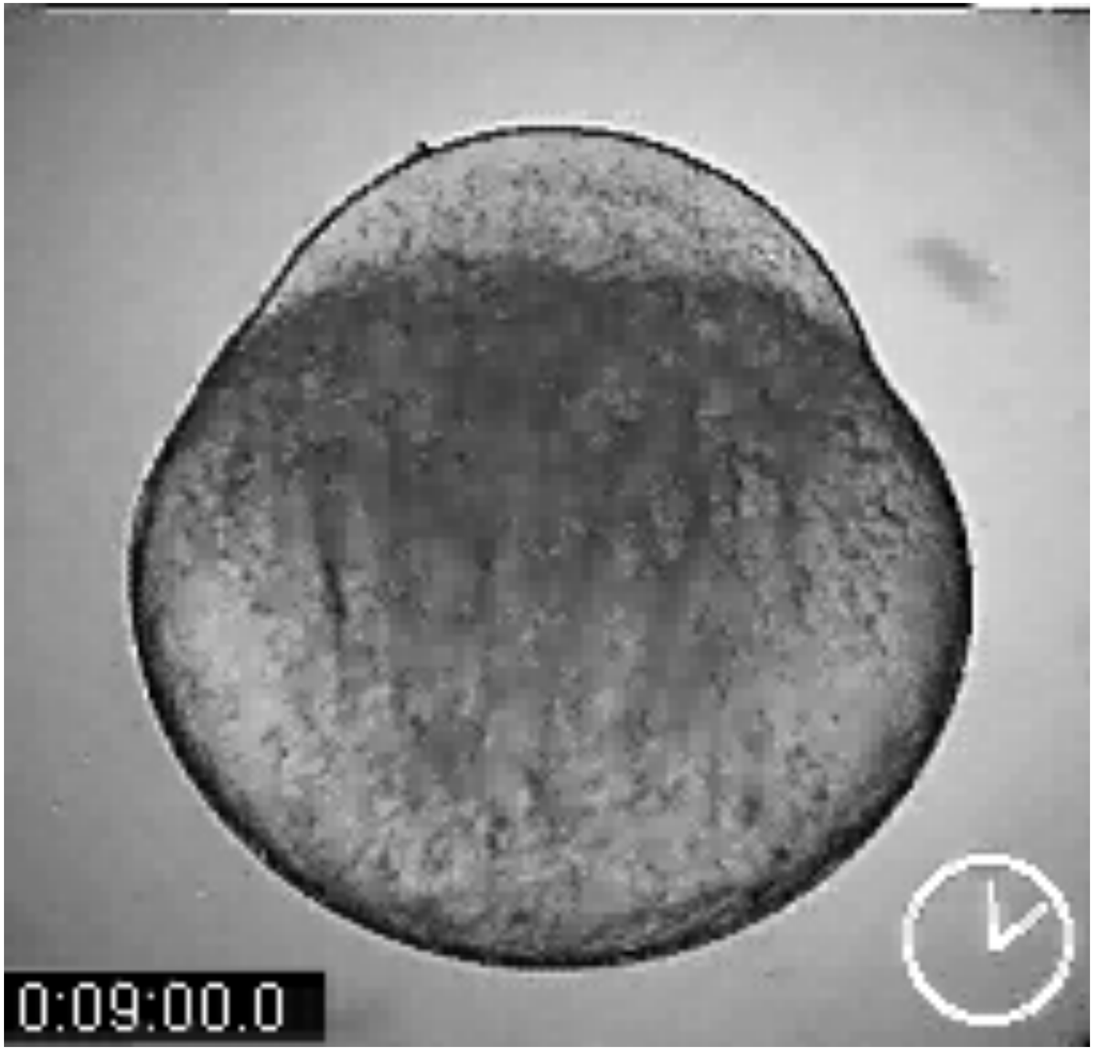


The MGS project

- Language dedicated to the simulation of $(DS)^2$
- Declarative (declarative simulation *vs* procedural)
- Abstract rewriting of complex spatial structures:
  - Data structure = topological collections

    sequence, generalized array, (multi-)set, arbitrary graph, Delaunay triangulation, g-map, …, <u>cell complexes</u>
  - Control structure = transformation
    - two powerful languages to specify sub-collections (elements in interaction)
    - Various rule application strategies: maximal parallel, asynchronous, stochastic, Gillespie-like, …

A dynamical system with a dynamical structure $(DS)^2$



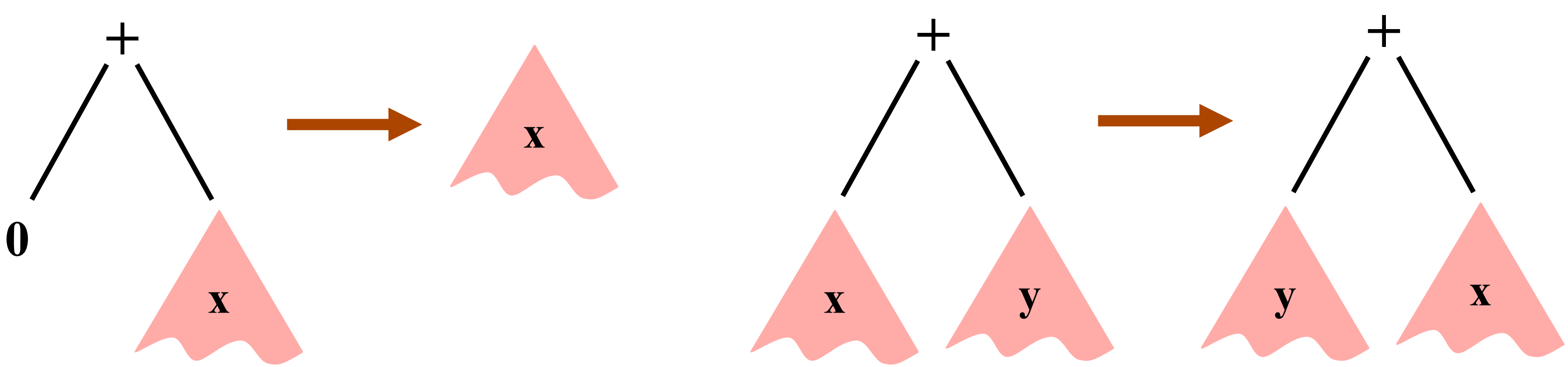
- *You don't know the whole structure in advance (it is* computed*)*
- the structure *of the state (chemical and mechanical state of each cell)* is changing in time
- *You may specify* local evolution
- *the* global *evolution is achieved by "integrating" the local ones*

How to model and simulate?

Rewriting systems (and abstract transition systems)

- Rewriting system
  - Used to formalize equationnal reasoning
  - A generative device (grammar)
  - Replace a sub-part of an entity by an other
  - Set of rewriting rules $\alpha \rightarrow \beta$
    - α: pattern specifying a sub-part
    - β: expression evaluating a new sub-part

- Example: arithmetic expressions simplification



A non-standard interpretation

$$e_1 + e_2 \rightarrow \ldots$$

e_1 can be a cell and e_2 a signal

e_1 and e_2 can be interacting cell

$+$ is the possibility of *interaction* between entities (or some other relationships)

$\rightarrow$ is the passing of time, a local evolution, a transition, the concretization of the interaction

Examples: if *e* is a cell and *i* a biochemical signal

$e + i \rightarrow e'$	growth (evolution of *e* on signal *i*)
$e + i \rightarrow e{+}i'$	quorum sensing
$e + i \rightarrow e' + e''$	division
$e + i \rightarrow .$	apoptose

MGS

Complex systems←→Rewriting techniques

Modelling	Specification
State (space)	**Data structure**
hierarchical and tree organizations	formal trees (or terms)
arbitrary complex organizations	***?***
Evolution function	**Set of rules**
interactions → evolution	α : pattern → β : expression
local **evolution laws**	rewriting rules

Simulation	Application
Trajectories	Derivations
Time management	Rule application strategy
discrete, event-based, synchronous vs. asynchronous	maximal parallel, sequential, deterministic, stochastic

Properties

- *local evolution rules*
  mandatory when you cannot express a global function/relation because the domain of the function/relation is changing in time

- interaction based approach
  the l.h.s. of a rule specifies a set of elements in *interaction*, the r.h.s. the result of the interaction

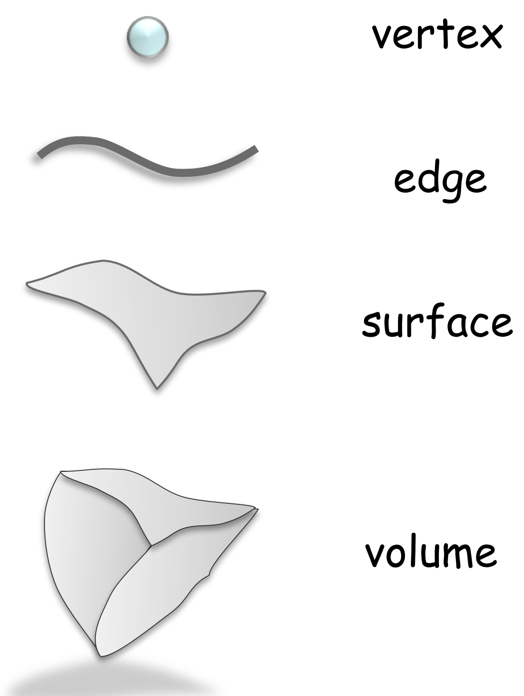
- *the phase space is well defined but not well known*
  a generative process enumerates the elements but membership-test can be very hard

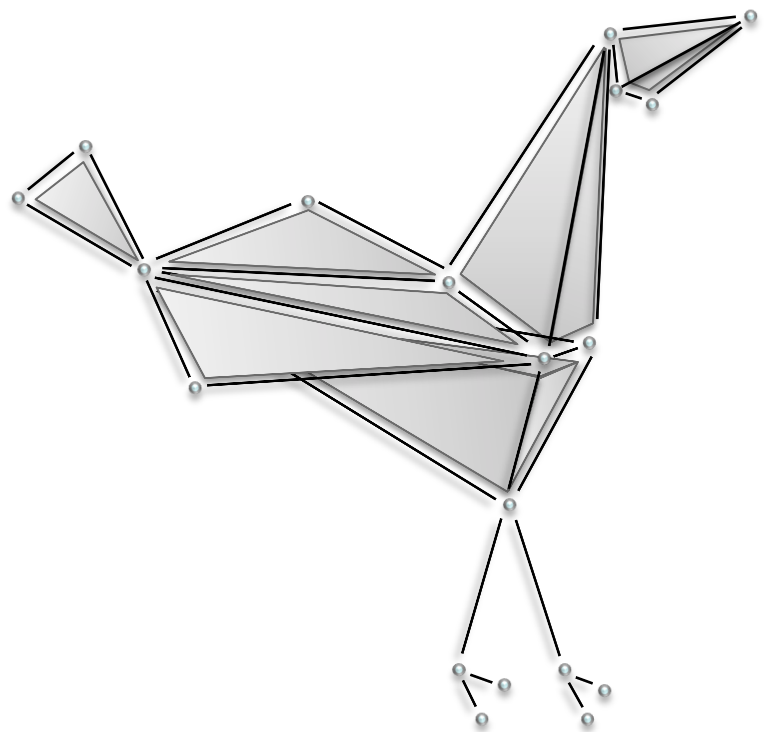
- *various kind of time evolution* (for the same set of rules)

- *demonstration by induction*
  on the rules or on the derivation (e.g. growth function in L system)

MGS Proposition

- Topological collections
  - Structure
    - A collection of topological cells
    - An ***incidence relationship***



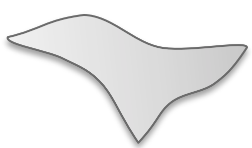


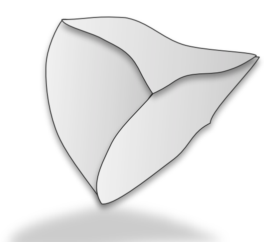
MGS Proposition

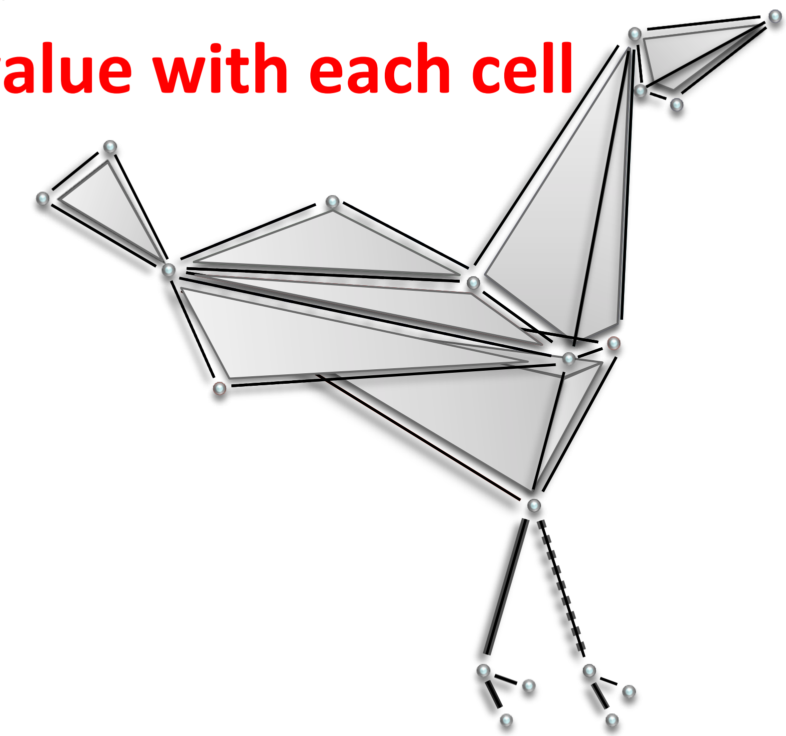
- Topological collections
  - Structure
    - A collection of topological cells
    - An incidence relationship
  - Data: **association of a value with each cell**


0-cell


1-cell


2-cell


3-cell



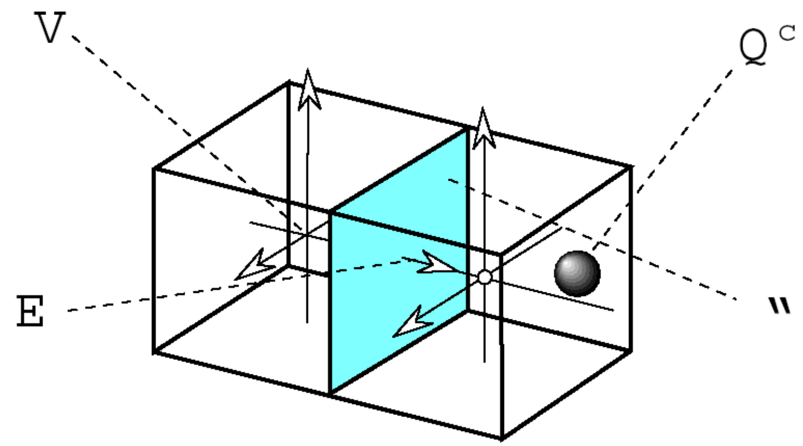
Higher dimensional objects for complex simulations

Example of electrostatic Gauss law [**Tonti** 74]

- Electric charge content ρ : dimension 3
- Electric flux $\boldsymbol{\Phi}$: dimension 2
- Law available on a arbitrary complex domain

$$\phi = \oint\!\!\!\oint w \cdot dS = \frac{Q^c}{\varepsilon_0} = \iiint_{(V)} \frac{\rho}{\varepsilon_0} d\tau$$

electric field in space:
- V: electric potential (dim 0)
- E: voltage (dim 1)
- w: electric flux (dim 2)
- Qc: electric charge (dim 3)



A Direct Discrete Formulation of Field Laws: The Cell Method

MGS Proposition

- Transformations
  - Functions defined by case on collections

    Each case (pattern) matches a sub-collection
  - Defining a rewriting relationship: ***topological rewriting***

trans T = **{**

$pattern_1 \Rightarrow expression_1$

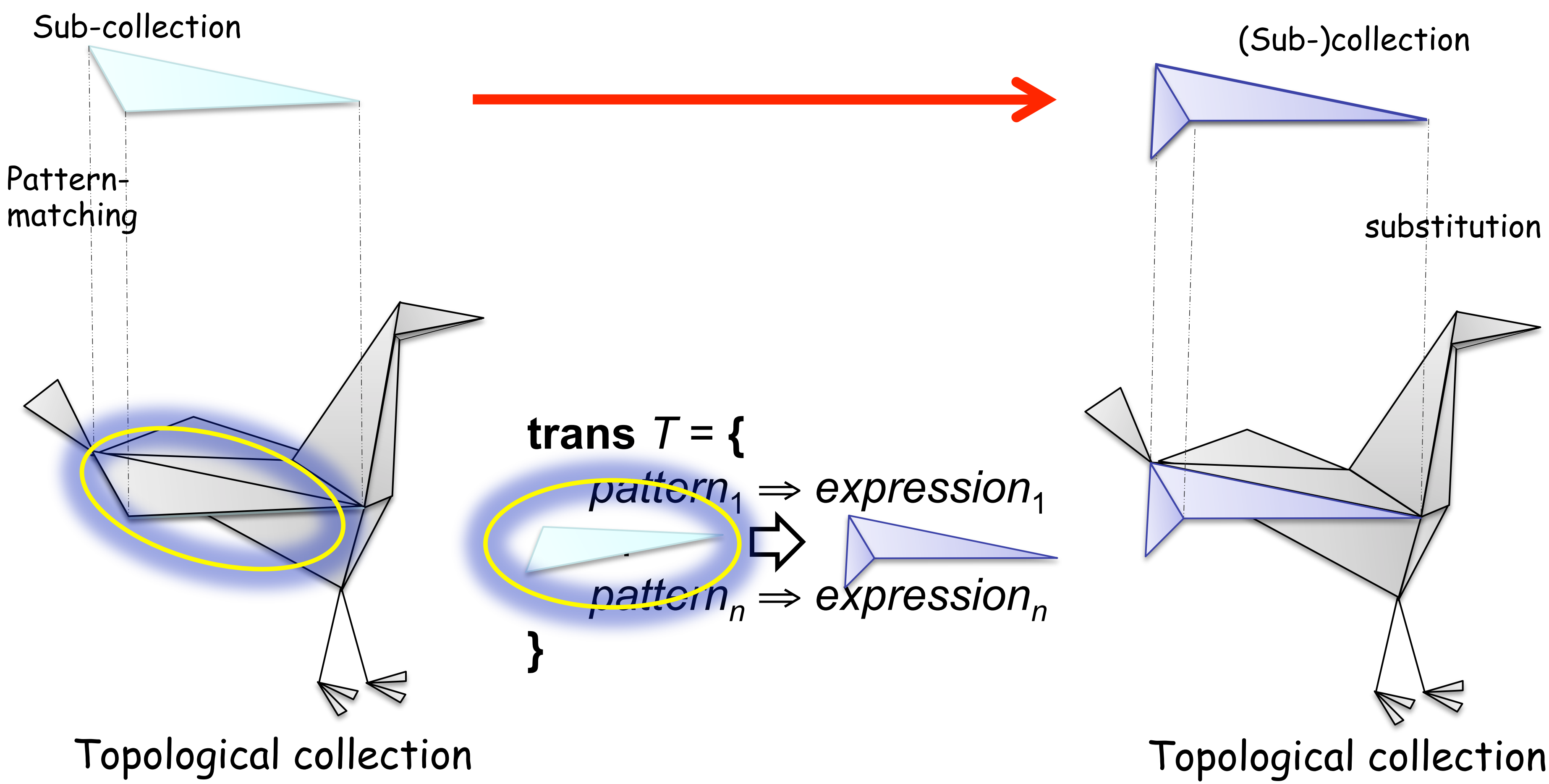
…

$pattern_n \Rightarrow expression_n$

}

MGS Proposition

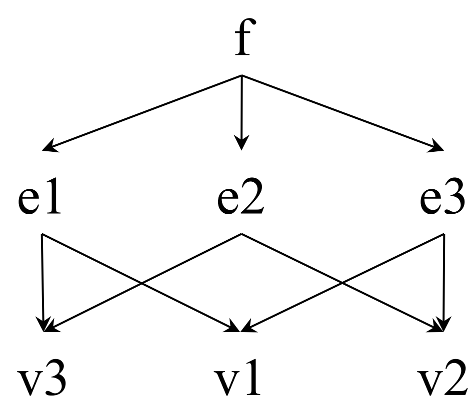
- Transformations

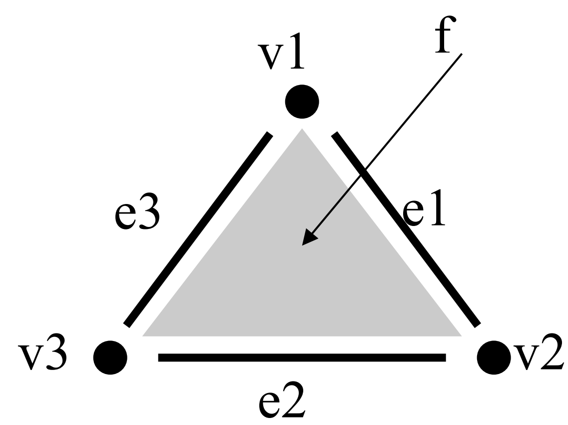


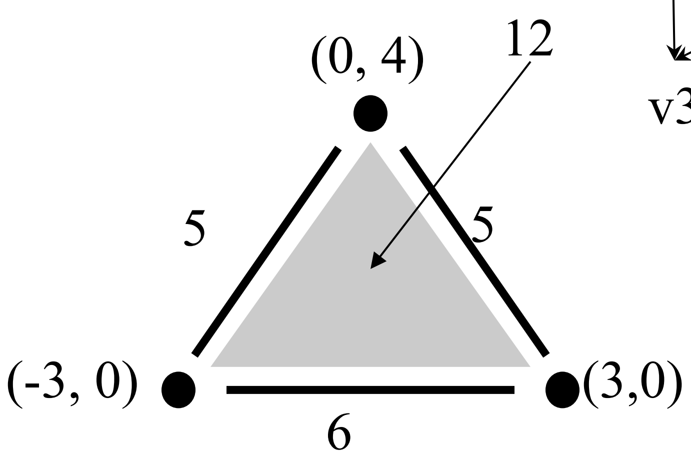
Abstract Simplicial Complex and simplicial chains

Incidence relationship and lattice of incidence:

- boundary(f) = {v1, v2, v3, e1, e2, e3}
- faces(f) = {e1, e2, e3}
- cofaces(v1) = {e1, e3}







Topological chain

- coordinates with vertices
- lengths with edges
- area with f

$$\begin{pmatrix} 0 \\ 4 \end{pmatrix}.v_1 + \begin{pmatrix} 3 \\ 0 \end{pmatrix}.v_2 + \begin{pmatrix} -3 \\ 0 \end{pmatrix}.v_3 + 5.e_1 + 6.e_2 + 5.e_3 + 12.f$$

Topological rewriting = transformation

$1 + 2 \rightarrow \ldots$ (arithmetic) term rewriting

arithmetic operation

$a \,.\, b \rightarrow \ldots$ string rewriting (~ L systems)

string concatenation: « . » is a formal associative operation

$2H + O \rightarrow H_2O$ multiset rewriting (~ chemistry)

multiset concatenation (= the chemical soup): « . » is AC

$v_1.\sigma_1 + v_2.\sigma_2 \rightarrow \ldots$ topological rewriting (MGS)

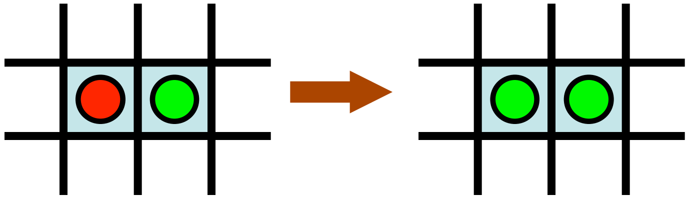
gluing cell in a cell complex: … (AC and algebraic machinery)

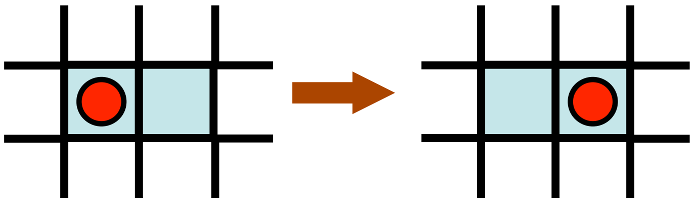
Example: Diffusion Limited Aggregation (DLA)

- Diffusion: some particles are randomly diffusing; others are fixed
- Aggregation: if a mobile particle meets a fixed one, it stays fixed

```
trans dla = {
    `mobile , `fixed  => `fixed, `fixed ;
    `mobile , <undef> => <undef>, `mobile
}
```

NEIGHBOR OF





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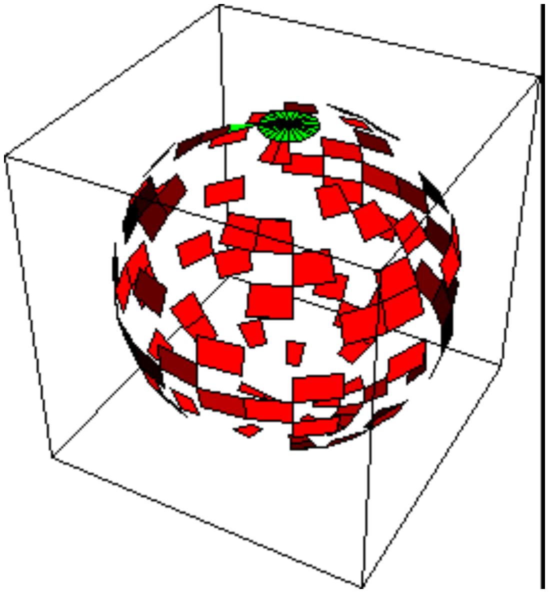
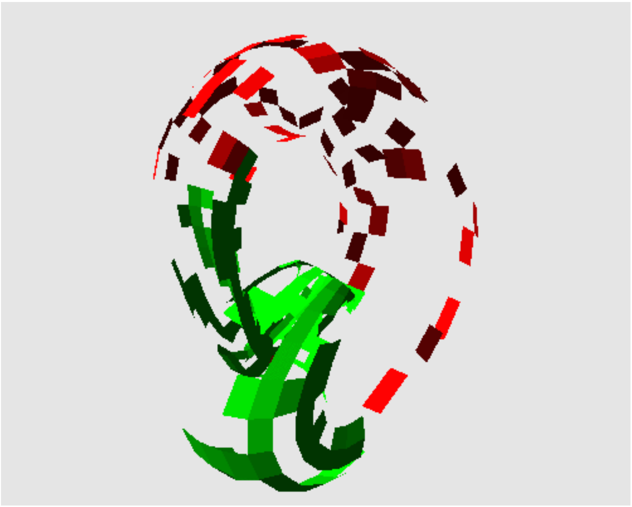
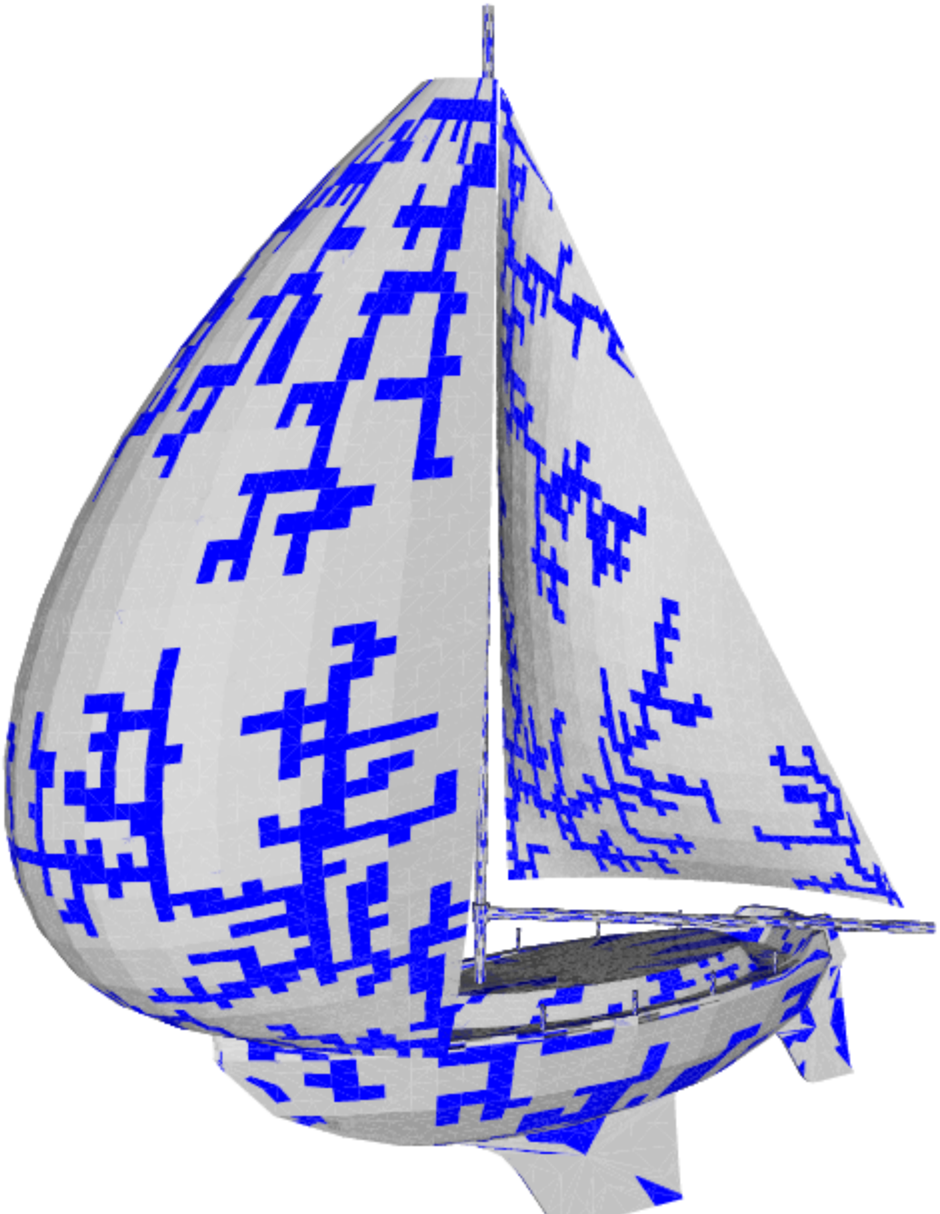
}
```

this transformation is an abstract process that can be applied to any kind of space

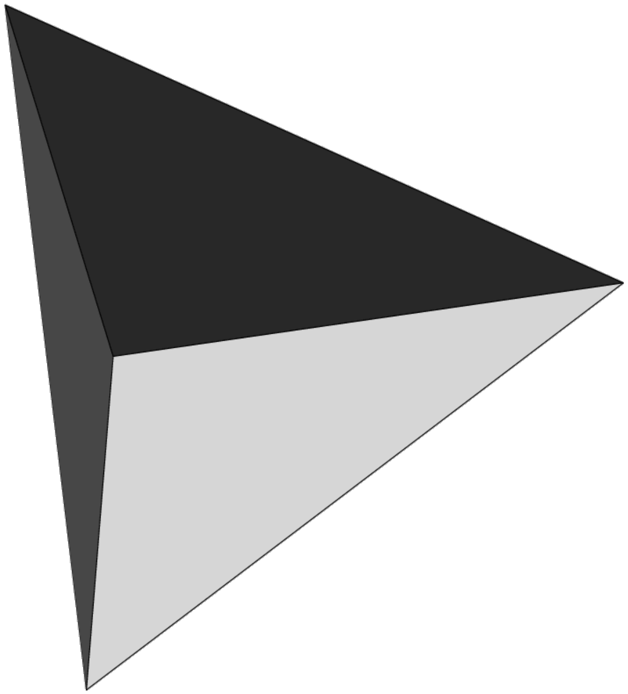




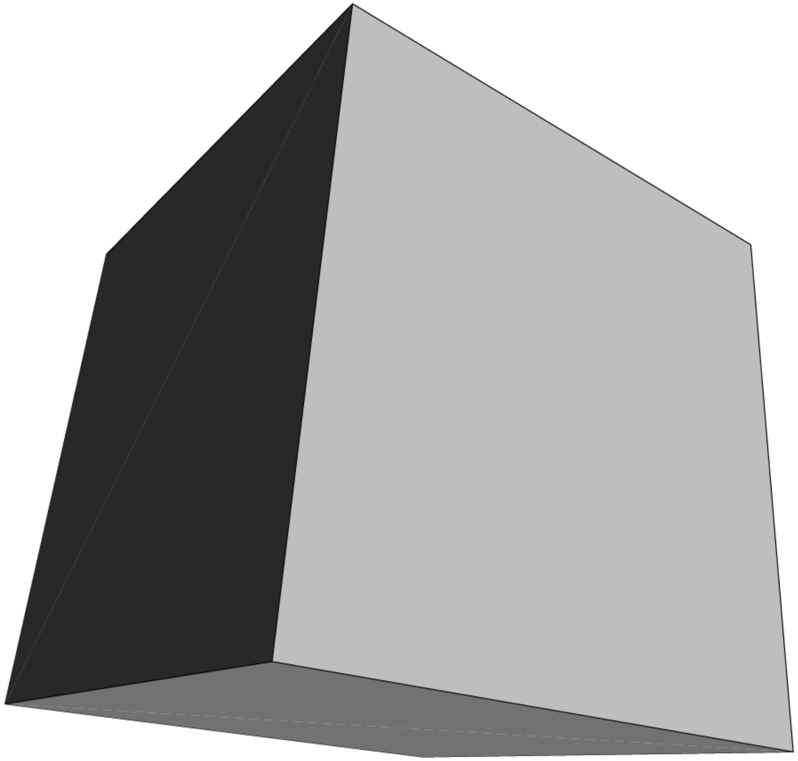
Polytypisme



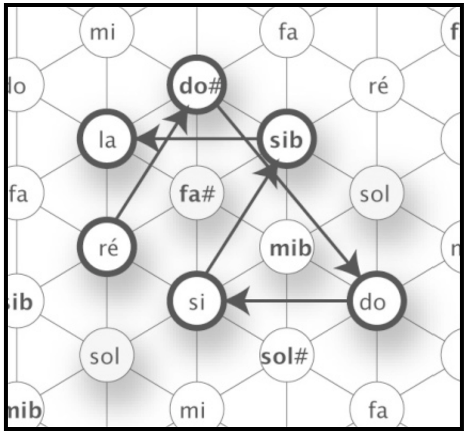
Fractal construction by carving



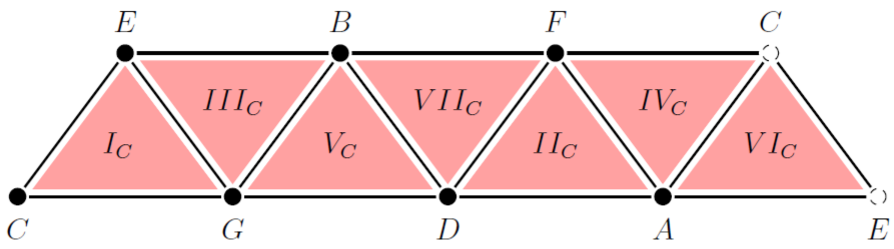
Sierpinsky sponge (4 steps)



Menger sponge (2 steps)



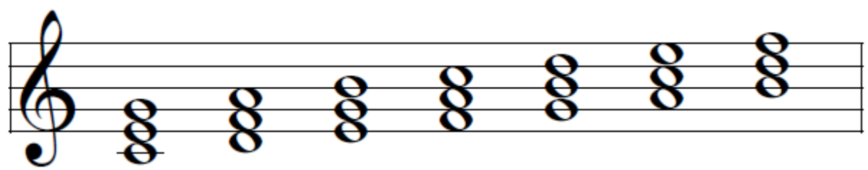
Music in a space

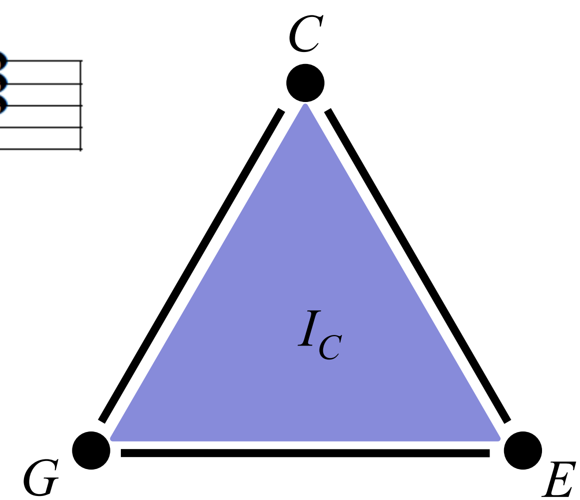


Computing the space of music

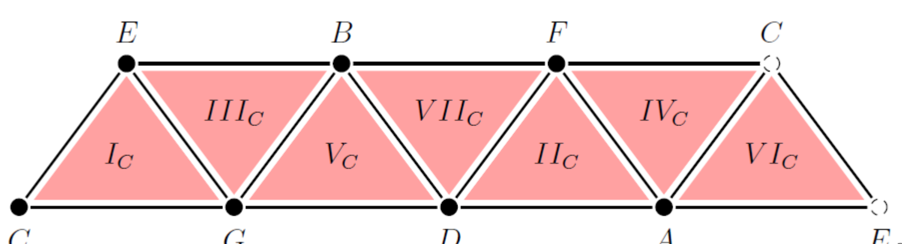
Tonality and Möbius Strip

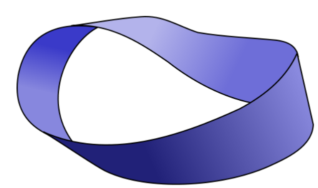
- Motivation: spatial visualization of tonality
- Association of a chord set with the tonality: the *degrees*
  - Example:





- Spatial representations
  - Note = vertex
  - Chord = surface
- Fusion of the common notes for the 7 degrees

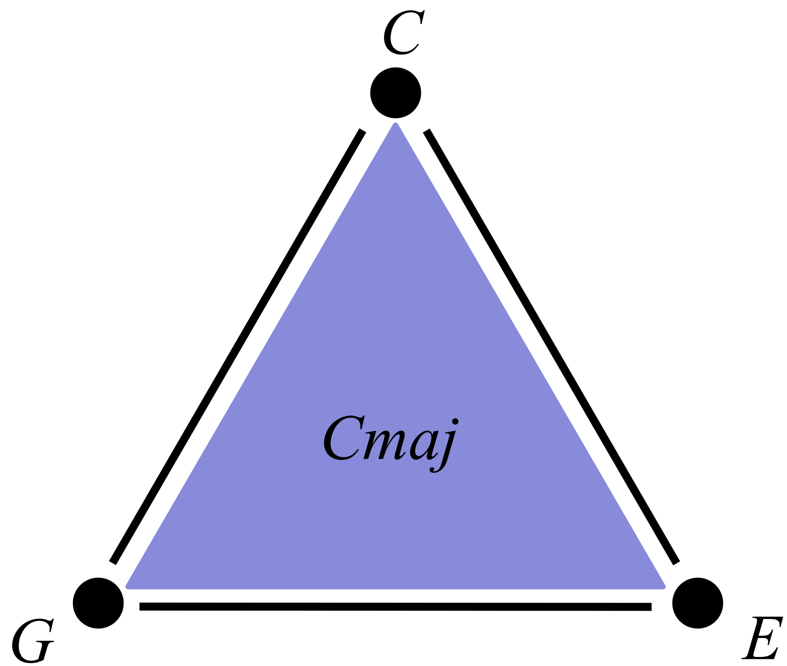




[Mazzola02]

Self-Assembly of Chords

- Automation of the process for the analysis of other chords sequences
- *Reaction* of the chords between themselves
- Simplicial representation of musical objects
  - Note: 0-simplex
  - 2-note chord: 1-simplex
  - 3-note chord: 2-simplex



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  - Note: 0-simplex
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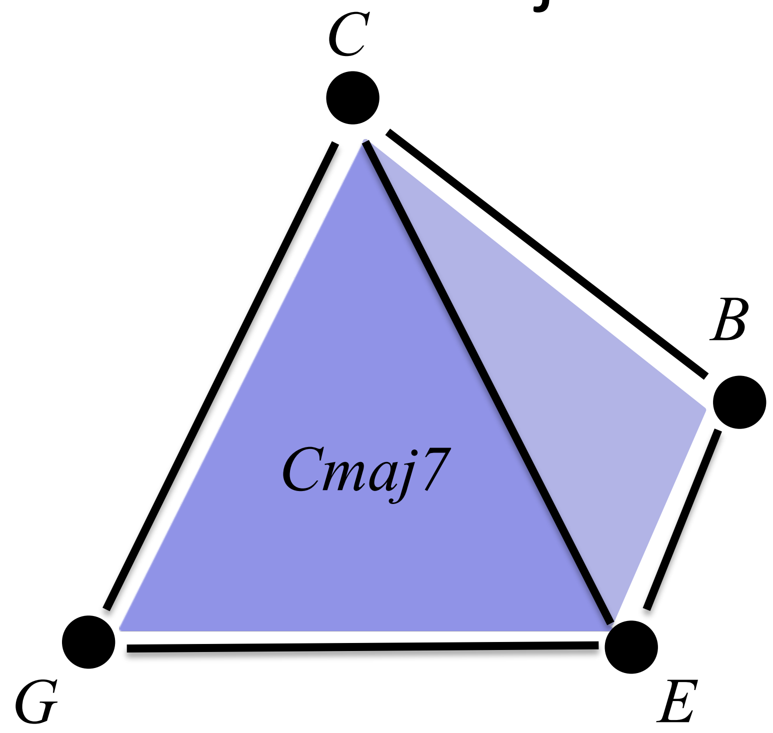
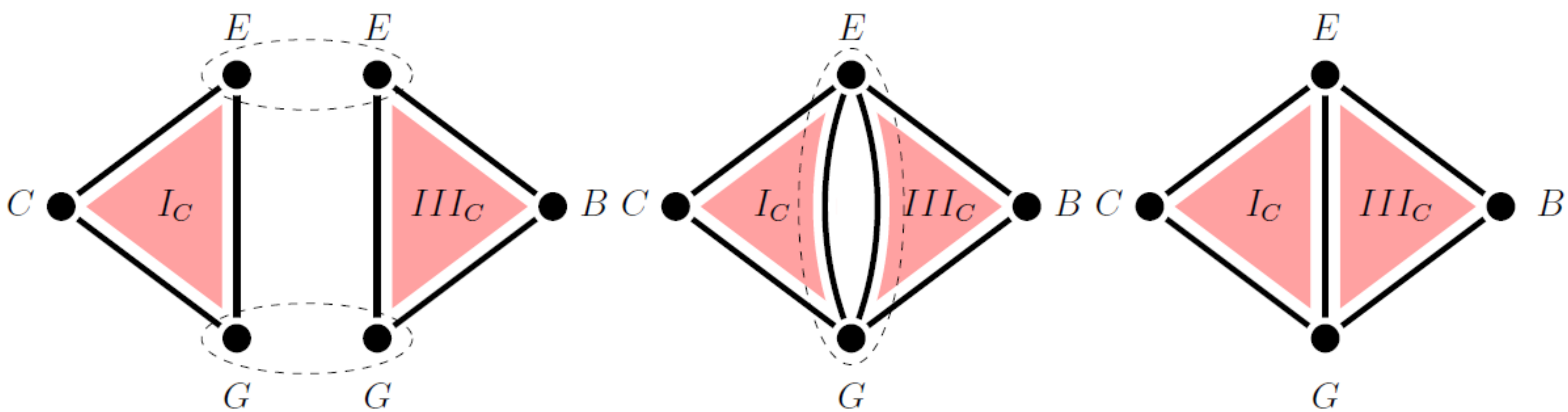
  - 3-note chord: 2-simplex
  - 4-note chord: 3-simplex



- MGS transformation for self-assembly process

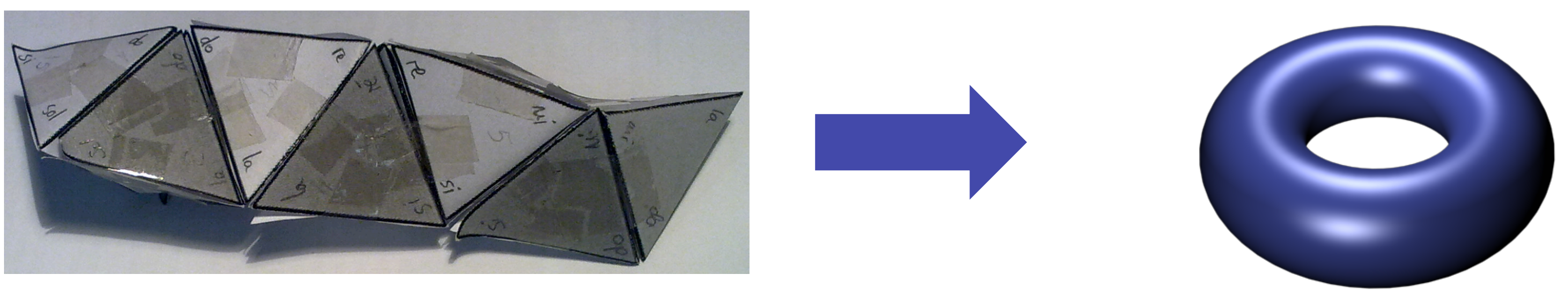



```
Trans identification = {
  s1 s2 / (s1 == s2 & (faces s1) == (faces s2))
    =>
      let c = new_cell (dim s1)
                       (faces s1)
                       (union (cofaces s1)
                              (cofaces s2))
      in s1 * c
}
```

Applications

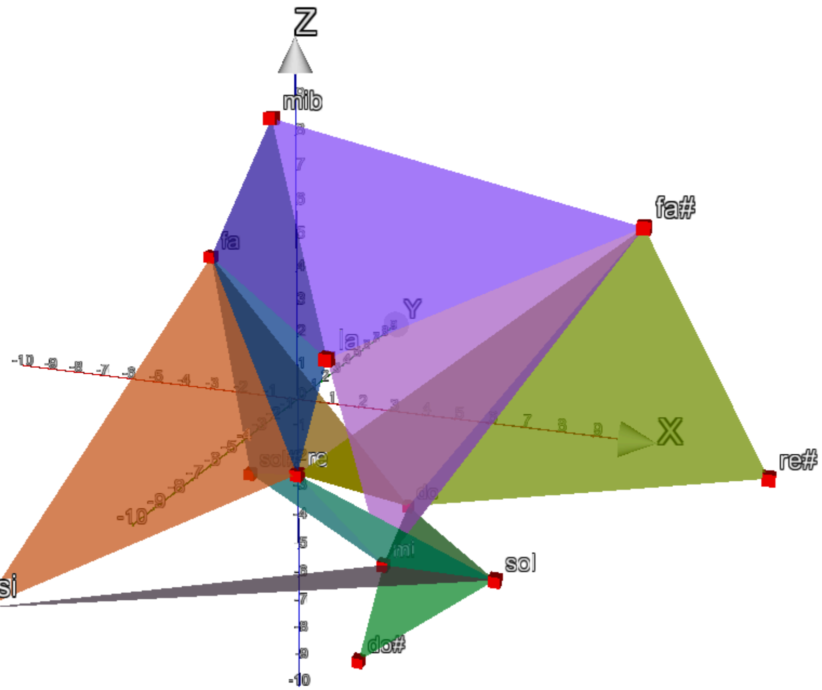
- Four-note degrees of C-major tonality

- Chord = 3-simplex (tetrahedrons)
- Self-assembly



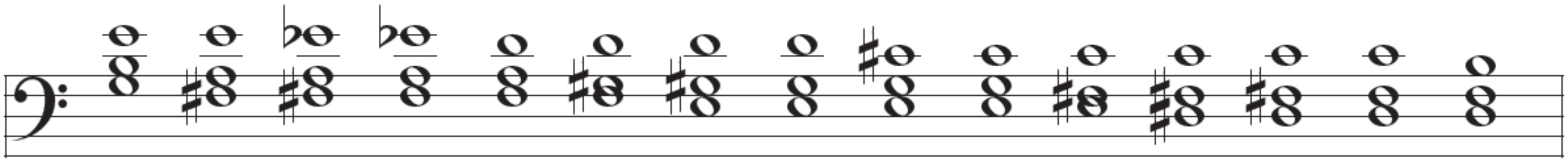
- Extract of the Prelude No. 4 Op. 28 of F. Chopin

- Simplicial complex

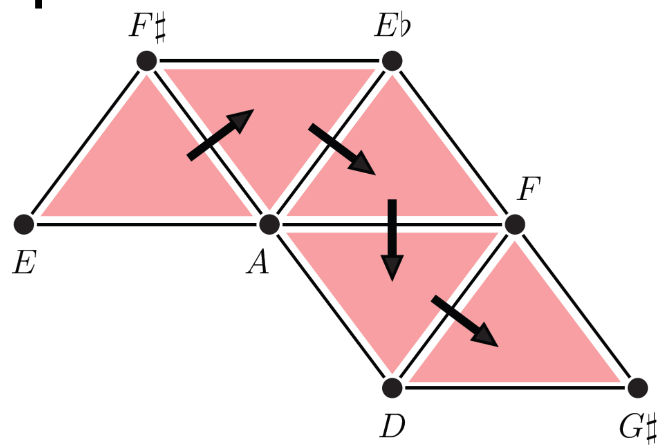


Applications

- Extract of the Prelude No. 4 Op. 28 of F. Chopin



- Analysis of the path under the chords



- The path chosen by F. Chopin is associated with the smallest movements on the chords

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- **Antoine Spicher**
- **Olivier Michel**

http://mgs.spatial-computing.org

http://repmus.ircam.fr/giavitto

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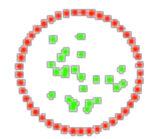
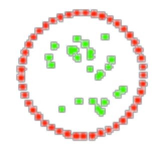
  E. Delsinne, V. Larue, F. Letierce, B. Calvez,
  F. Thonerieux, D. Boussié *and the others…*





- **Past and presents Collaborations**
  - A. Lesne (IHES, stochastic simulation)
  - P. Prusinkiewicz (UoC, declarative modeling)
  - P. Barbier de Reuille (meristeme model)
  - C. Godin (CIRAD, biological modeling)
  - H. Berry (INRIA, stochastic simulation)
  - G. Malcolm (Liverpool, rewriting)
  - J.-P. Banâtre (IRISA, programming)
  - F. Delaplace (IBISC, synthetic biology)
  - P. Dittrich (Jena, chemical organization)
  - F. Gruau (LRI, language and hardware)
  - P. Liehnard (Poitier, CAD, Gmap and quasi-manifold)