# Programmation spatiale

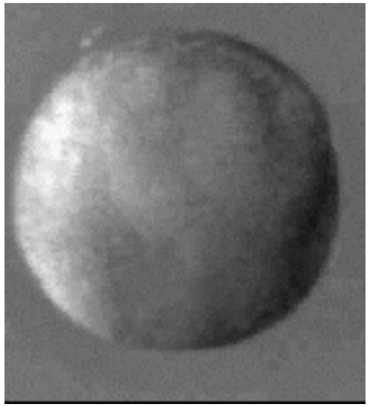
- Application à la biologie du développement,
- l'analyse des contes de fées
- et l'analyse musicale

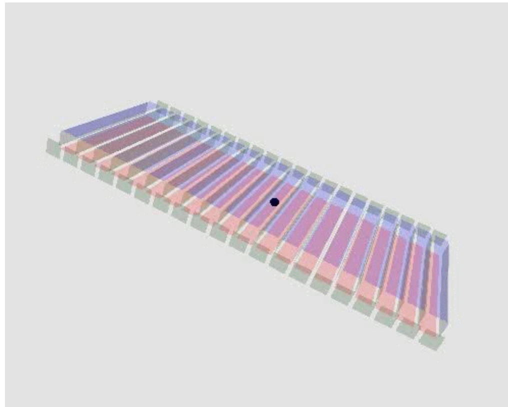
**Jean-Louis Giavitto**

**CNRS – IRCAM**

**Equipe représentations musicales**

**http://mgs.spatial-computing.org**





Plusieurs formalismes informatiques tentent de capturer la notion d'interaction dans un système. MGS, un langage de programmation expérimental, est fondé sur la constatation que l'ensemble des interactions possibles s'organise suivant une structure topologique qui permet de spécifier la description du système et son évolution. Le style de programmation qui en résulte, la programmation spatiale, s'appuie sur des relations topologiques (connexité, bord, obstruction...) pour renouveler la notion de structure de données et a trouvé des applications effectives dans la modélisation et la simulation en biologie des systèmes mais aussi en intelligence artificielle (représentation des connaissances et analogie) ainsi qu'en analyse musicale.

# Plan

**GRAME**
**CENTRE NATIONAL DE CRÉATION MUSICALE**

**10.11** MARS 2011

**RAPPEL ACTUALITÉ**

**Confluence des savoirs**
cycle de conférences 2010/2011

## Des Puces, des souris et des hommes

*ENS / Lyon (69)*
Mardi 15 mars à 18h30
Amphithéâtre Charles Mérieux
(Site Jacques Monod, 46 allée d'Italie, Lyon 7e)
Dans le cadre du cycle "Confluences des savoirs"

**Conférence de Jacques Samarut**, président de l'École Normale Supérieure (ENS) Lyon
**Écho sonore par Jacques Di Donato**, clarinettes, performance Omax
**Benjamin Lévy, Gérard Assayag**, performance musicale présentée par le Grame.
Cycle de conférence organisé par le Musée des Confluences de Lyon.

La biologie moderne subit actuellement une révolution épistémologiste. Pendant plus de 30 ans les biologistes ont cherché à identifier des gènes qui sont à la base du fonctionnement normal et pathologique de l'être vivant. A partir de situations pathologiques ils ont, bon an mal an, identifié environ 6000 gènes impliqués dans une démarche qui va de la fonction à l'identification du gène. Depuis moins de 10 ans, la révolution de la génomique a complètement changé l'approche. En effet le séquençage massif et automatisé des génomes de multiples espèces, dont l'homme, a conduit à identifier en quelques années plusieurs milliers de gènes jusqu'alors inconnus. Ainsi on estime que le génome humain renferme entre 25000 et 30000 gènes identifiés par leur nature mais dont la fonction reste à déterminer pour la majorité d'entre eux. Cette révolution épistémologique

*Jacques Samarut, Président de L'École Normale Supérieure (ENS) de Lyon*
*Jacques Samarut est également fondateur de rhône-alpes Génopôle et de l'institut de Génomique fonctionnelle de Lyon (iGfL). Docteur ès sciences de l'Université Claude-Bernard, Lyon 1, Jacques Samarut a suivi une formation de biologiste cellulaire et moléculaire et il dirige, actuellement à l'ENS, un groupe de recherche travaillant sur les cellules souches embryonnaires et sur la signalisation par les récepteurs nucléaires d'hormone.*

*Jacques Di Donato est un musicien interprète et improvisateur français. Clarinettiste, saxophoniste et batteur, il travaille dans des domaines variés allant du jazz à la musique contemporaine en passant par la musique classique et la musique improvisée. Il a été professeur de clarinette au CNSMDL entre 1984 et 2007.*

1. Modélisation informatique du développement : les systèmes dynamiques à structure dynamique
2. MGS et la programmation spatiale : réécriture de collections topologiques
3. La croissance du méristème
4. L'analyse des contes de fée et la résolution d'analogies
5. Programmation spatiale et représentation des objets et des processus musicaux

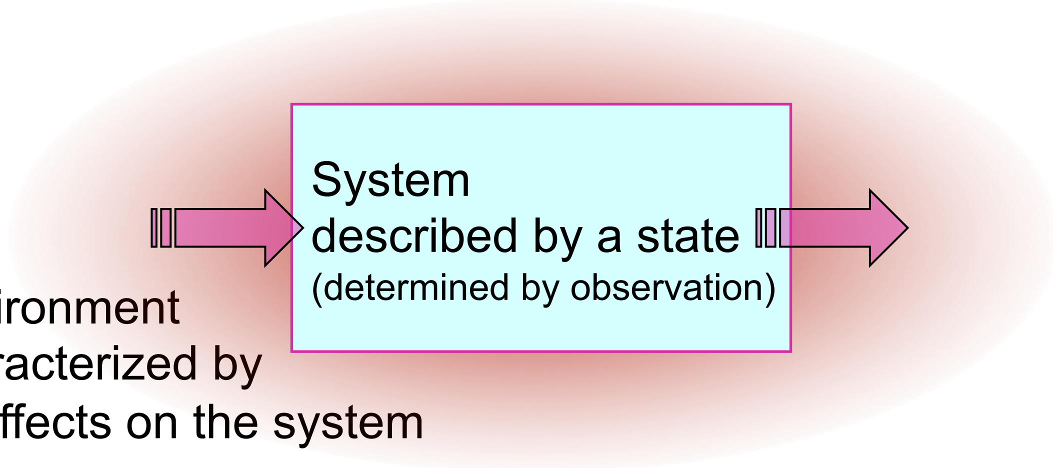
# Modelling morphogenesis: the approach of A. Turing

THE CHEMICAL BASIS OF MORPHOGENESIS

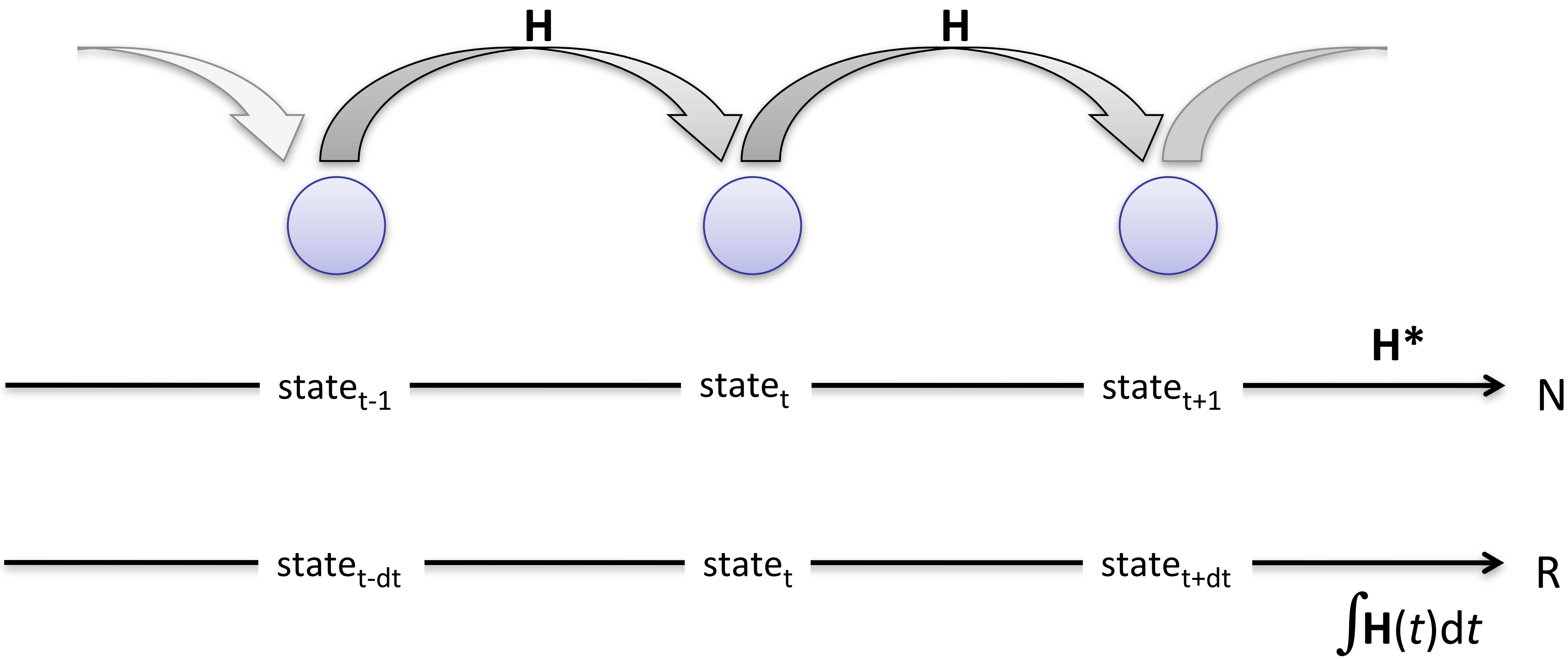
By A. M. TURING, F.R.S. *University of Manchester*

(*Received* 9 *November* 1951—*Revised* 15 *March* 1952)

With either of the models one proceeds as with a physical theory and defines an entity called 'the state of the system'. One then describes how that state is to be determined from the state at a moment very shortly before. With either model the description of the state consists of two parts, the mechanical and the chemical.

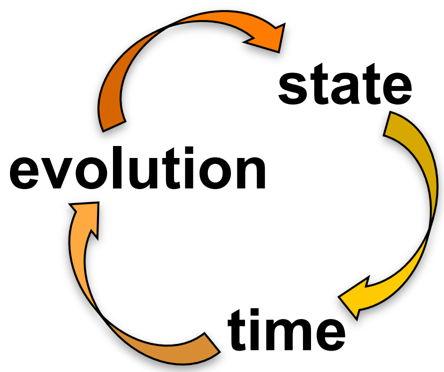


# Specifying a dynamical system (for simulation)




**Specification of**

- **structure of state**
- **structure of time**
- **evolution function**



MGS

## Modelling a dynamical system

- state, including space (e.g. fields)
- time
- evolution function

| C : continuous, D: discrete | PDE | Coupled ODE | Iteration of functions | Cellular automata | … |
|---|---|---|---|---|---|
| ***state*** | C | C | C | D | ... |
| ***time*** | C | C | D | D | ... |
| ***space*** | C | D | D | D | ... |

# Modelling morphogenesis: the approach of A. Turing

MGS

The model takes two slightly different forms. In one of them the cell theory is recognized but the cells are idealized into geometrical points. In the other the matter of the organism is imagined as continuously distributed. The cells are not, however, completely ignored, for various physical and physico-chemical characteristics of the matter as a whole are assumed to have values appropriate to the cellular matter.

- **Uniform matter, continuous-oriented system description**
  One choice is to ignore cells completely, e.g., Physiome models tissues as continua with bulk mechanical properties and detailed molecular reaction networks, which is computationally efficient for describing dense tissues and non-cellular materials like bone, extracellular matrix , fluids, and diffusing chemicals, *but not for situations where cells reorganize or migrate.*

*versus*

- **Cell-oriented discrete system description**
  Multi-cell simulations are useful to interpolate between single-cell and continuum-tissue extremes because cells provide a natural level of abstraction for simulation of tissues, organs and organisms.

  Treating cells phenomenologically reduces the millions of interactions of gene products to several behaviors: most cells can move, divide, die, differentiate, change shape, exert forces, secrete and absorb chemicals and electrical charges, and change their distribution of surface properties.

(CompuCell3D manual)

**Aggregate-** *vs.* **Entity-based** models

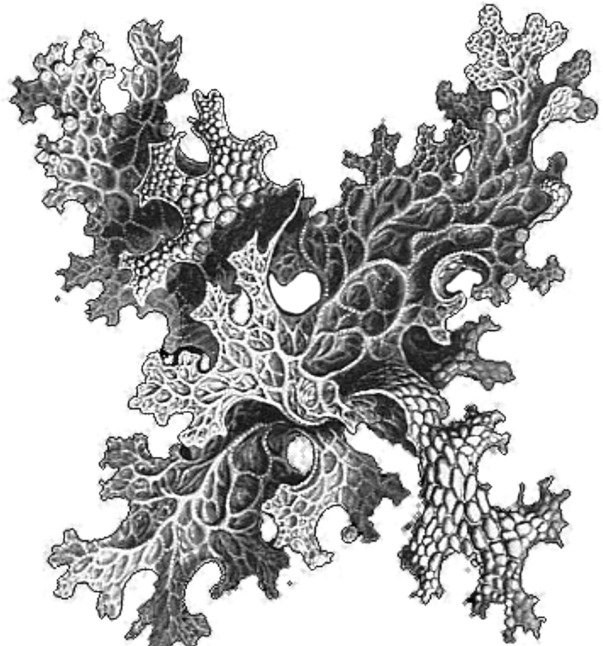
# Modelling morphogenesis: the predefined medium

> The interdependence of the chemical and mechanical data adds enormously to the difficulty, and attention will therefore be confined, so far as is possible, to cases where these can be separated.

> Suppose, for instance, that a 'leg-evocator' morphogen were being produced in a certain region of an embryo, or perhaps diffusing into it, and that an attempt was being made to explain the mechanism by which the leg was formed in the presence of the evocator. It would then be reasonable to take the distribution of the evocator in space and time as given in advance and to consider the chemical reactions set in train by it.

Compatible with
- the notion of morphogenetic field
- cell fate

But
- there is evidence for **feedback loops between the shape and the process inhabiting the shape**



from E. Haenkel (cited by C. Goodman-Strauss): example of a negative curvature surface. Curvature can be controlled while the surface is growing along a 'front'

# The medium/process problem

In determining the changes of state one should take into account

(i) The changes of position and velocity as given by Newton's laws of motion.

(ii) The stresses as given by the elasticities and motions, also taking into account the osmotic pressures as given from the chemical data.

(iii) The chemical reactions.

(iv) The diffusion of the chemical substances. The region in which this diffusion is possible is given from the mechanical data.

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MGS

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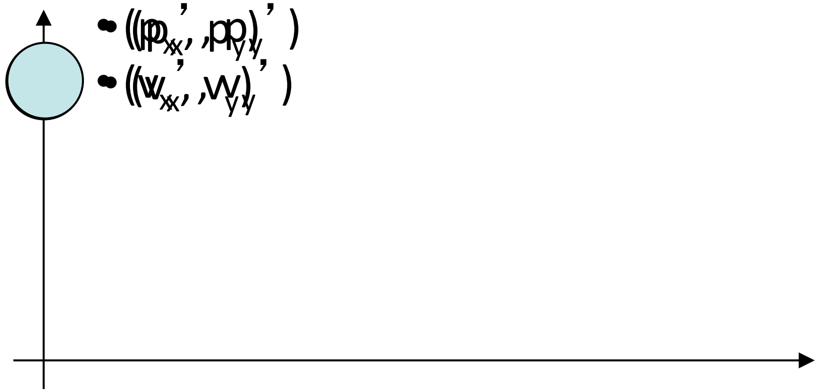
**a falling ball**




*at any time a state is a position and a speed*

A dynamical system (DS)

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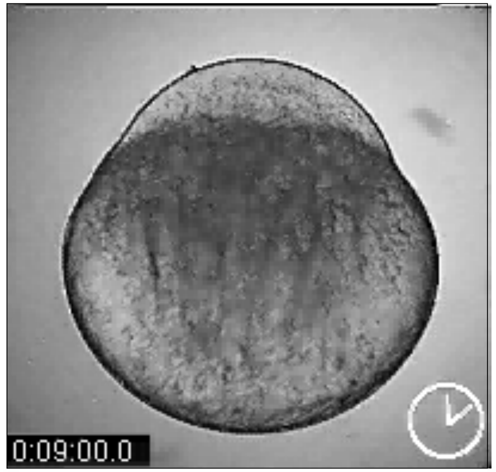
*at any time a state is a position and a speed*

A dynamical system (DS)

**a developing embryo**



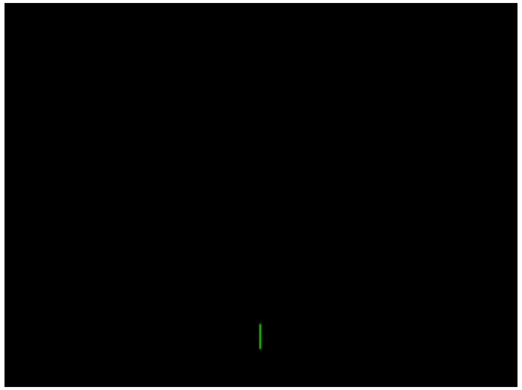

*the structure of the state*
*(chemical and mechanical state of each cell)*
*is changing in time*

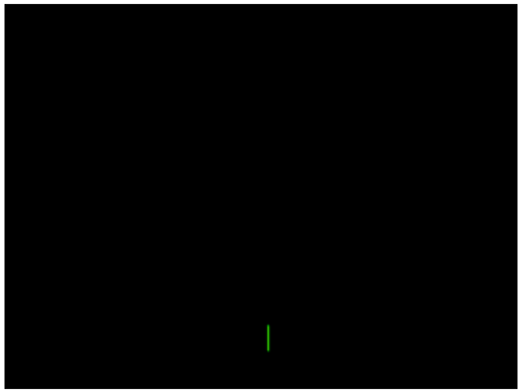
**A dynamical system with a dynamical structure $(DS)^2$**

# What has changed since Turing's time

MGS

It might be possible, however, to treat a few particular cases in detail with the aid of a digital computer. This method has the advantage that it is not so necessary to make simplifying assumptions as it is when doing a more theoretical type of analysis. It might even be possible to take the mechanical aspects of the problem into account as well as the chemical, when applying this type of method. The essential disadvantage of the method is that one only gets results for particular cases. But this disadvantage is probably of comparatively little importance.





P. Prusinkiewicz

A good example of **declarative** formalism: Lindenmayer systems

- The structure of a tree can be coded by a string of parenthetised symbols
- A symbol is an elementary part of the plant
- The symbol between [ and ] represents a sub-tree
- Additional conventions are used to represent main axis, orientation, depth, etc.
- A rule

  $s_0 \rightarrow s_1\ s_2\ s_3\ \ldots$

  represents the evolution of $s_0$

# Diffusion and reaction in a linear *growing medium*

M. Hammel and P. Prusinkiewicz (1996)

The following rules state that a differentiated cell (heterocyst) returns to a vegetative state if the concentration of the activator is too low. In addition, if the cell is large enough, it continues to grow.

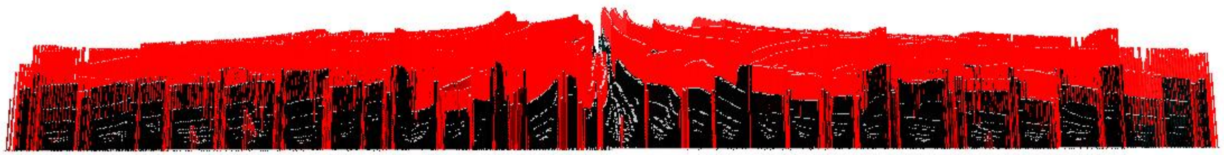
```
e / (D(e) & (e.a < thr) | (e.x >= shorter*gr))
=> {type ="C", a=e.a/gr, h=e.h/gr, x=e.x*gr, p=e.p};
```

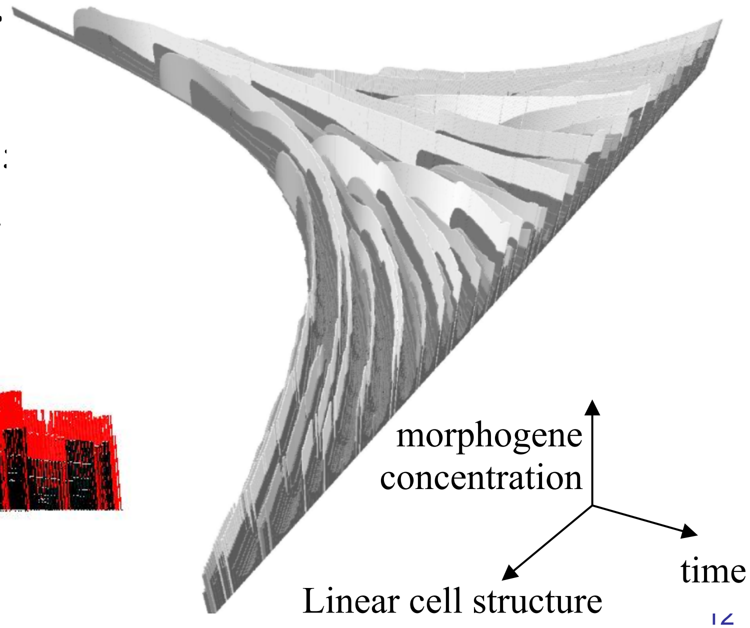
The following rule specifies when a cell with a left polarity divides.
Only vegetative cells can divide (hence the predicate C in the rule guard) and it must be large enough. The volume of the two daughter cells remains the same, so there is no variation in the concentration.

```
e / (C(e) & (e.x >= lm) & (e.p == L))

=> {type="C", a=e.a, h=e.h, x=e.x*shorte

   {type="C", a=e.a, h=e.h, x=e.x*longer
```





# Software as Science ?

M.-P. Cani & F. Bertails

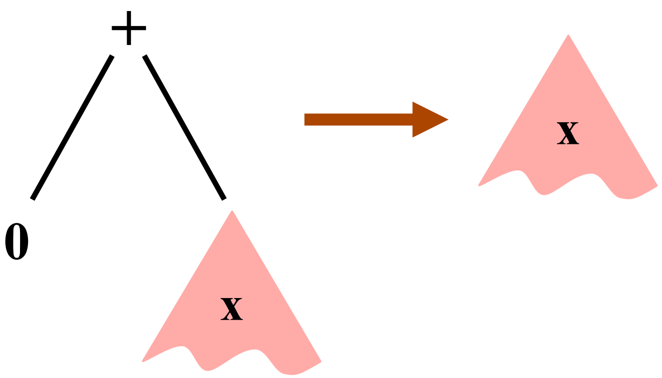
- Intelligibility
  *The entire process should be accessible for analysis into a finite, not very large number of stages, each stage being represented as a monotonic function of some definite initial conditions and a single variable such as time, or distance, etc.* (Gurwitsch, 1944)
  → compress behavior or shape in few rules
- Simulation is only a first step: models must enable reasoning
  → stay close to mathematical formalism

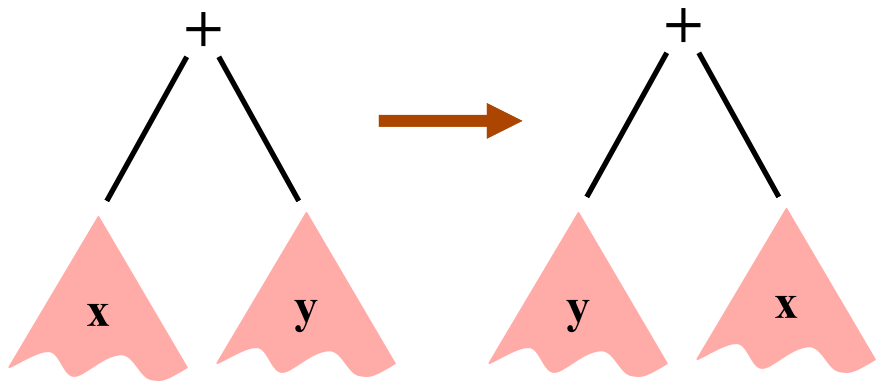
A program is a formal object (and some form of reasoning on it is possible) but a $10^6$ lines of codes is not an explanation !

# **Rewriting systems** (and abstract transition systems)

- Rewriting system
  - Used to formalize equationnal reasoning
  - A generative device (grammar)
  - Replace a sub-part of an entity by an other
  - Set of rewriting rules $\alpha \rightarrow \beta$
    - $\alpha$: pattern specifying a sub-part
    - $\beta$: expression evaluating a new sub-part

- Example: arithmetic expressions simplification



# A non-standard interpretation

$$e_1 + e_2 \rightarrow \ldots$$

$e_1$ can be a cell and $e_2$ a signal

$e_1$ and $e_2$ can be interacting cell

$+$ is the possibility of *interaction* between entities (or some other relationships)

$\rightarrow$ is the passing of time, a local evolution, a transition, the concretization of the interaction

**Examples:** if $e$ is a cell and $i$ a biochemical signal

| | |
|---|---|
| $e + i \rightarrow e'$ | growth (evolution of $e$ on signal $i$) |
| $e + i \rightarrow e + i'$ | quorum sensing |
| $e + i \rightarrow e' + e''$ | division |
| $e + i \rightarrow .$ | apoptose |

# Complex systems←→Rewriting techniques

| Modelling | Specification |
| --- | --- |
| **State (space)** | **Data structure** |
| hierarchical and tree organizations | formal trees (or terms) |
| *arbitrary complex organizations* | ***?*** |
| **Evolution function** | **Set of rules** |
| ***interactions*** → evolution | α : pattern → β : expression |
| ***local*** **evolution laws** | rewriting rules |

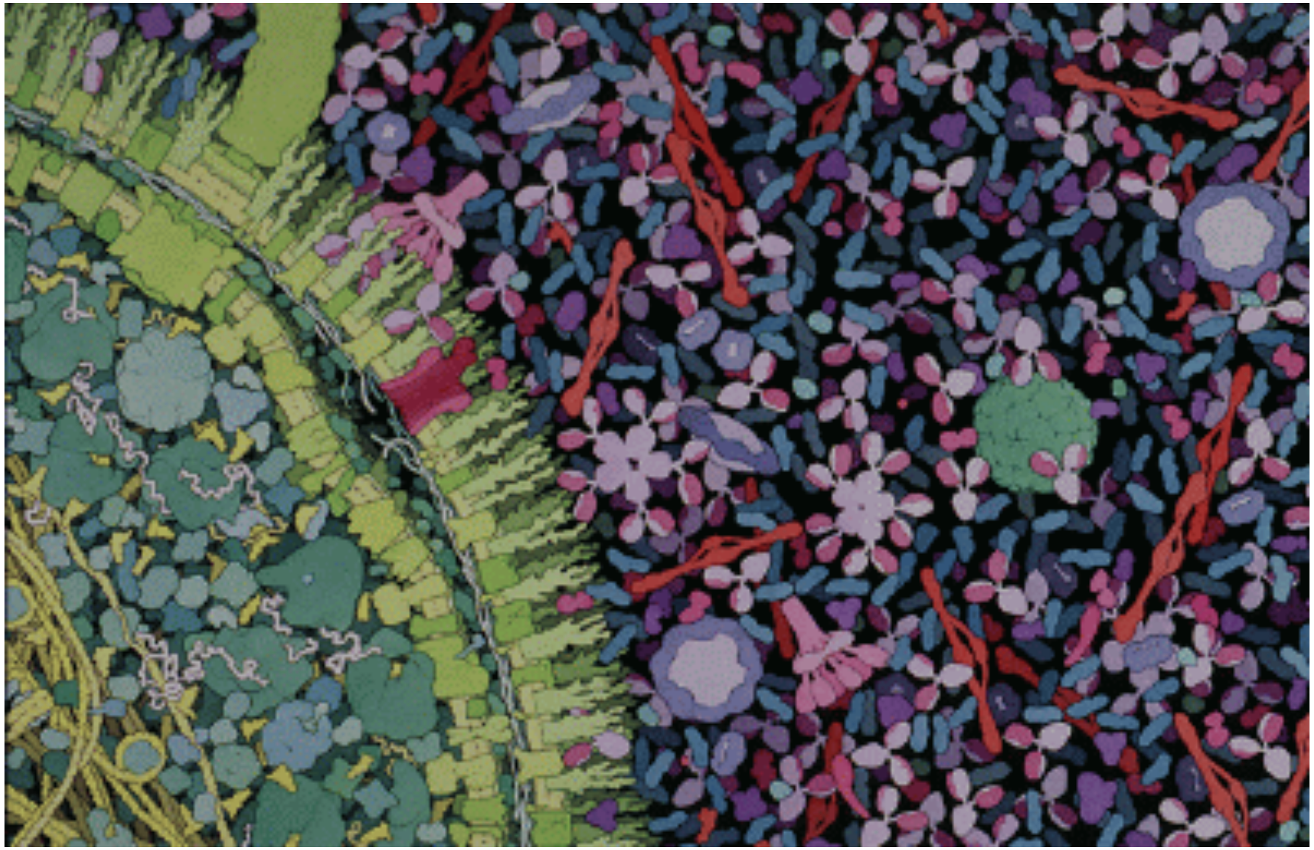
| Simulation | Application |
| --- | --- |
| Trajectories | Derivations |
| Time management | Rule application strategy |
| discrete, event-based,<br>synchronous vs. asynchronous | maximal parallel, sequential,<br>deterministic, stochastic |

# Properties

- *local evolution rules*
  mandatory when you cannot express a global function/relation because the domain of the function/relation is changing in time
- interaction based approach
  the l.h.s. of a rule specifies a set of elements in *interaction*, the r.h.s. the result of the interaction
- *the phase space is well defined but not well known*
  a generative process enumerates the elements but membership-test can be very hard
- *various kind of time evolution* (for the same set of rules)
- *demonstration by induction*
  on the rules or on the derivation (e.g. growth function in L system)

# How to extend to arbitrary spatial structure?

- Anabaena was « easy » because of the linear uniform structure
- How to handle the complex spatial structure of a cell?



David S. Goodsell

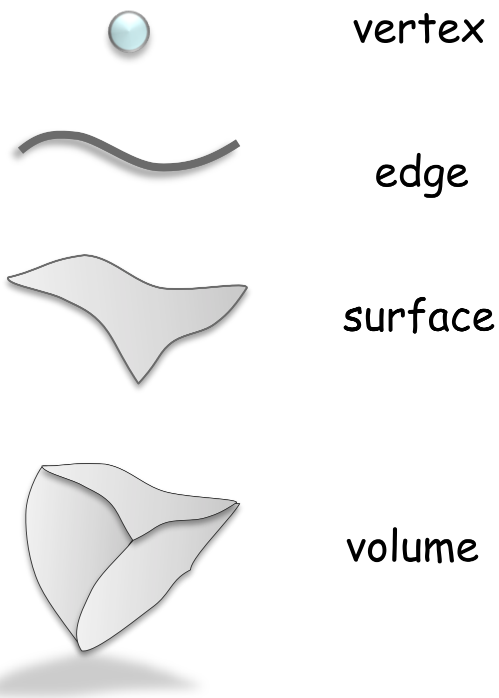
# The MGS project

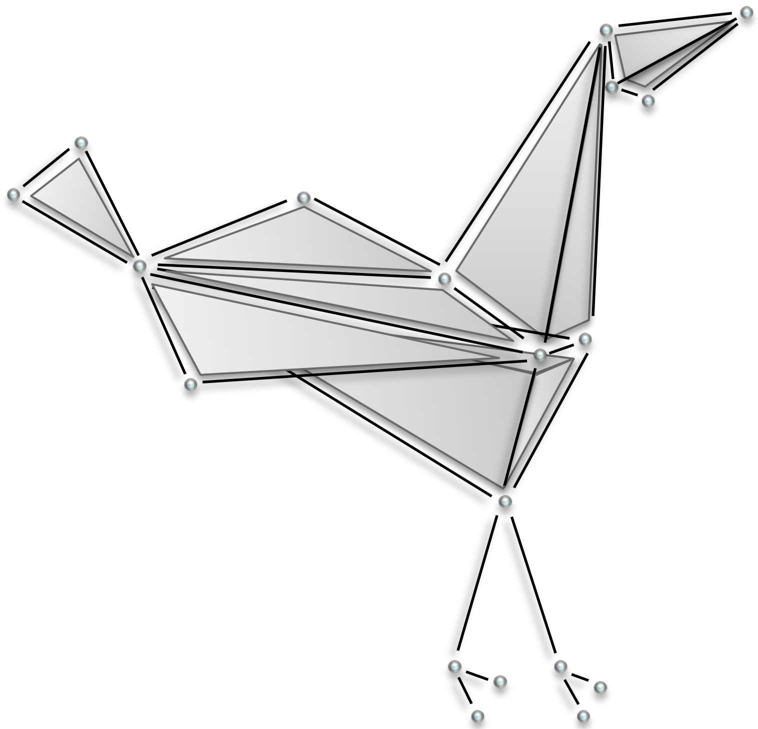
- Language dedicated to the simulation of $(DS)^2$
- Declarative (declarative simulation *vs* procedural)
- Abstract rewriting of complex spatial structures:
  - Data structure = topological collections

    sequence, generalized array, (multi-)set, arbitrary graph, Delaunay triangulation, g-map, …, <u>cell complexes</u>
  - Control structure = transformation
    - two powerful languages to specify sub-collections (elements in interaction)
    - Various rule application strategies: maximal parallel, asynchronous, stochastic, Gillespie-like, …

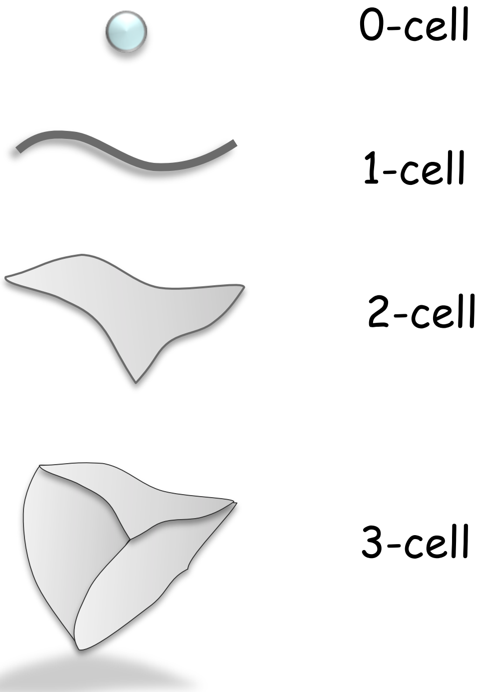
# MGS Proposition

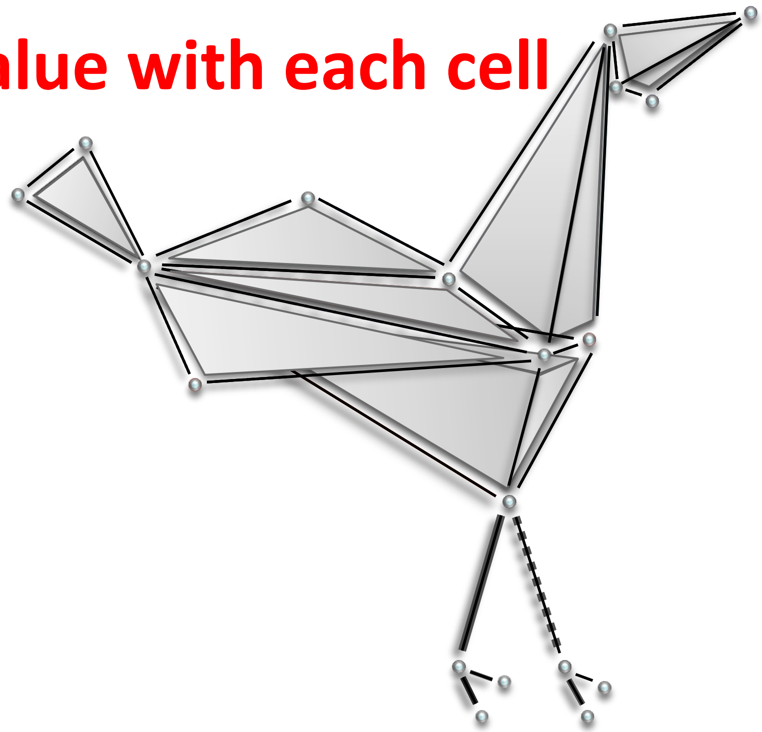
- Topological collections
  - Structure
    - A collection of topological cells
    - An ***incidence relationship***



- Topological collections
  - Structure
    - A collection of topological cells
    - An incidence relationship
  - Data: **association of a value with each cell**



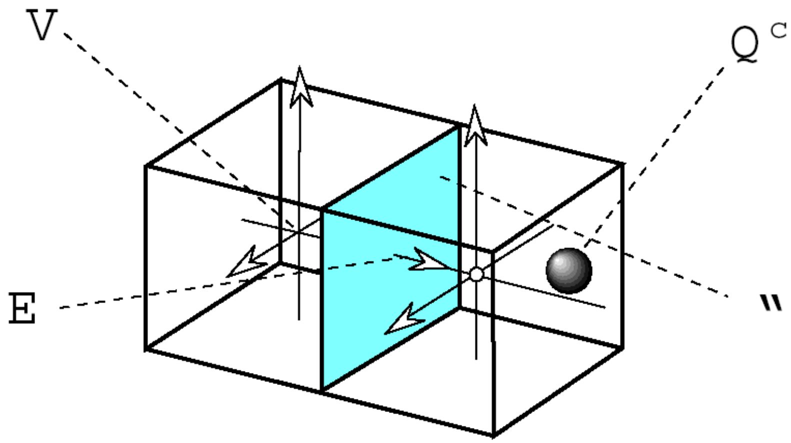



Example of electrostatic Gauss law [**Tonti** 74]

- Electric charge content ρ : dimension 3
- Electric flux $\boldsymbol{\Phi}$ : dimension 2
- Law available on a arbitrary complex domain

$$\phi = \oiint w \cdot dS = \frac{Q^c}{\varepsilon_0} = \iiint_{(V)} \frac{\rho}{\varepsilon_0} d\tau$$

*electric field in space*:
- V: electric potential (dim 0)
- E: voltage (dim 1)
- w: electric flux (dim 2)
- Qc: electric charge (dim 3)




*A Direct Discrete Formulation of Field Laws: The Cell Method*

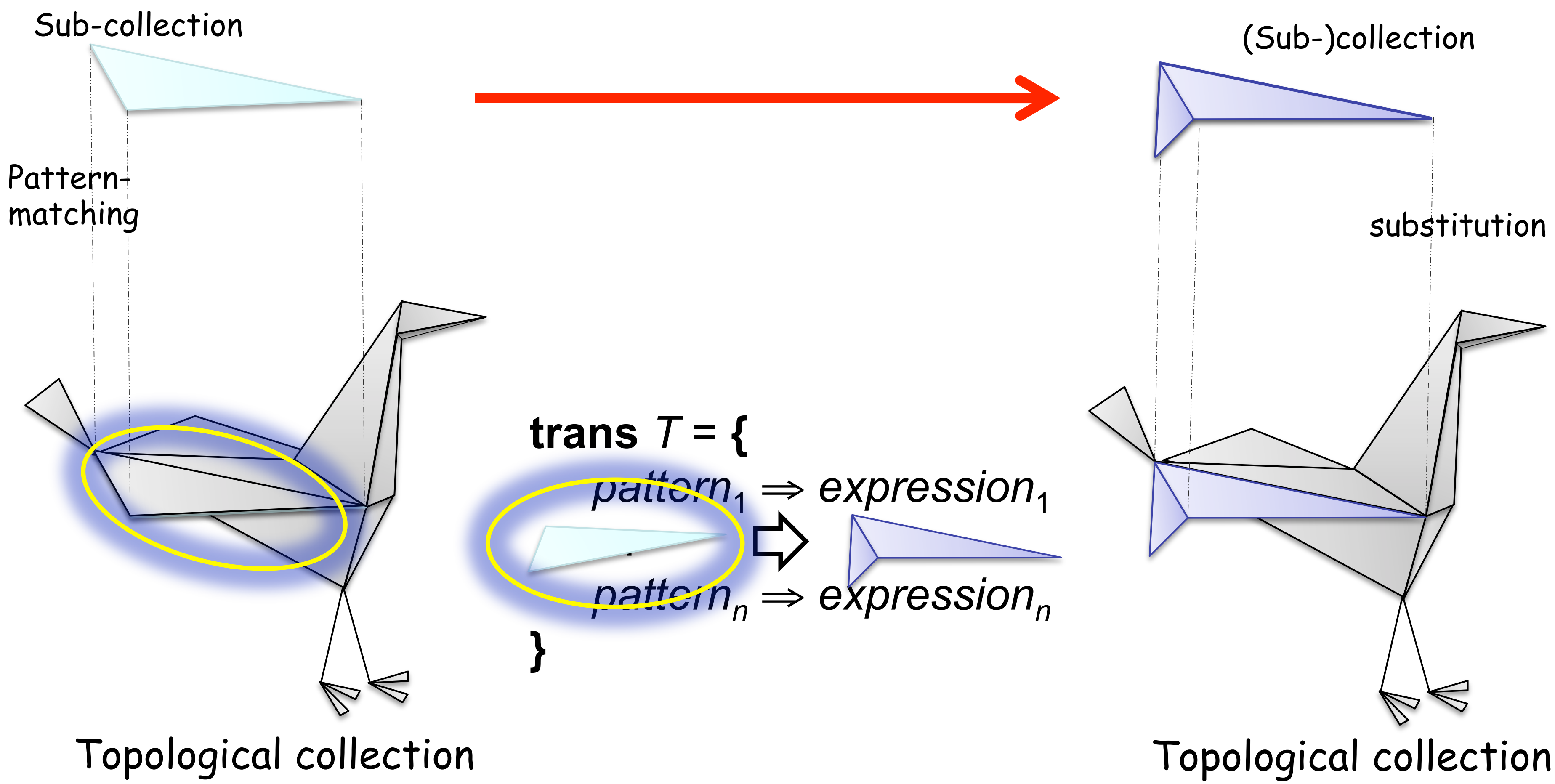
- Transformations
  - Functions defined by case on collections

    Each case (pattern) matches a sub-collection
  - Defining a rewriting relationship: ***topological rewriting***

**trans** *T* = **{**
  $pattern_1 \Rightarrow expression_1$
  …
  $pattern_n \Rightarrow expression_n$
**}**

# MGS Proposition

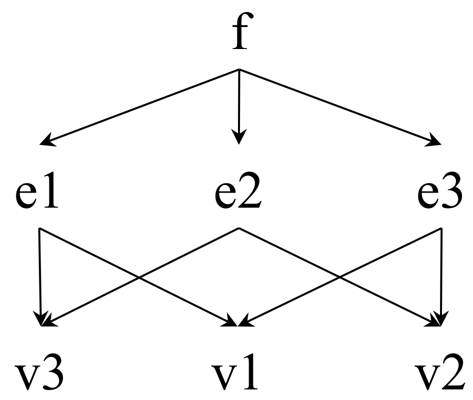
- Transformations

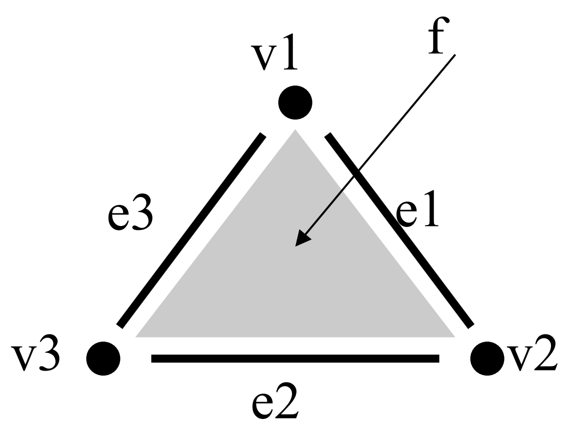


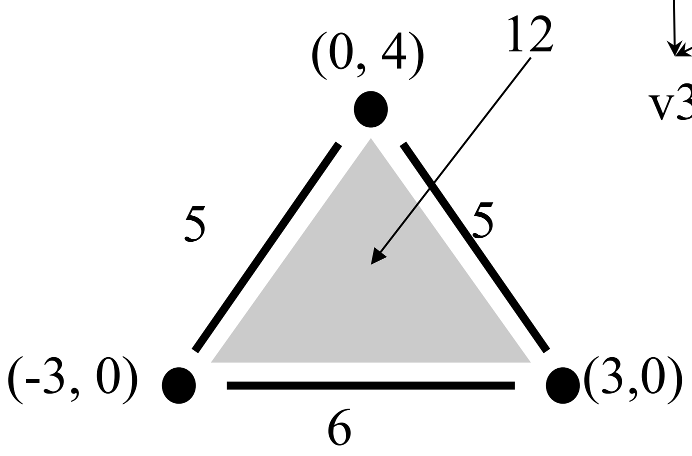
# Abstract Simplicial Complex and simplicial chains

*Incidence relationship and lattice of incidence:*

- boundary(f) = {v1, v2, v3, e1, e2, e3}
- faces(f) = {e1, e2, e3}
- cofaces(v1) = {e1, e3}




*Topological chain*

- coordinates with vertices
- lengths with edges
- area with f

$$\begin{pmatrix} 0 \\ 4 \end{pmatrix} . v_1 + \begin{pmatrix} 3 \\ 0 \end{pmatrix} . v_2 + \begin{pmatrix} -3 \\ 0 \end{pmatrix} . v_3 + 5.e_1 + 6.e_2 + 5.e_3 + 12.f$$

# Topological rewriting = transformation

$1 + 2 \rightarrow \ldots$ (arithmetic) term rewriting

arithmetic operation

$a \,.\, b \rightarrow \ldots$ string rewriting (~ L systems)

string concatenation: « . » is a formal associative operation

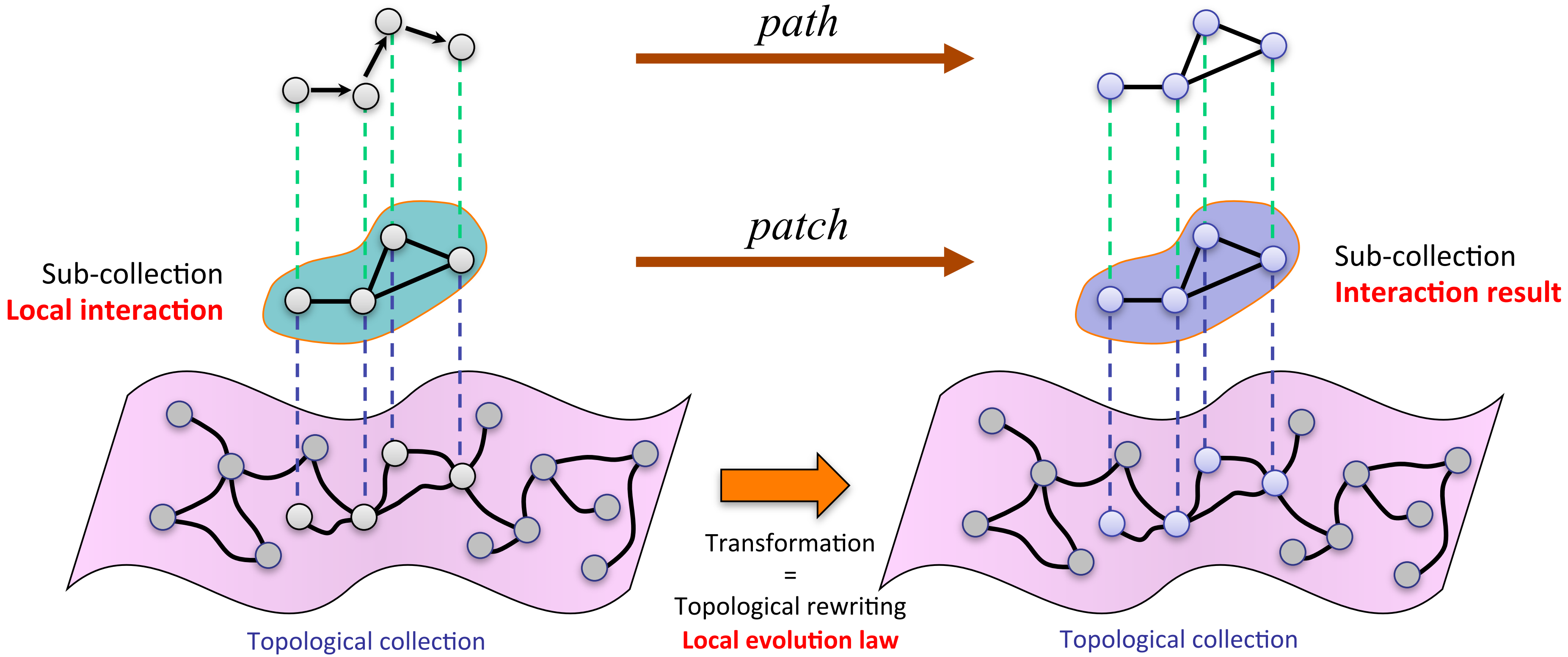
$2H + O \rightarrow H_2O$ multiset rewriting (~ chemistry)

multiset concatenation (= the chemical soup): « . » is AC

$v_1.\sigma_1 + v_2.\sigma_2 \rightarrow \ldots$ topological rewriting (MGS)

gluing cell in a cell complex: … (AC and algebraic machinery)

# Transformation




**Pattern matching : specifying a sub-collection of elements in interaction**

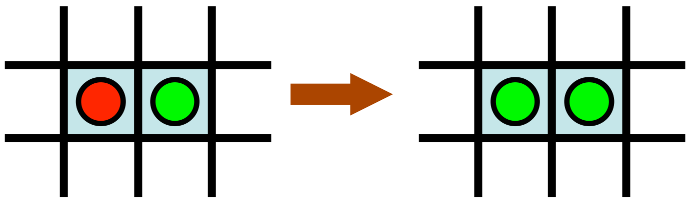
- *Path transformation* (path = sequence of neighbor elements)
  - Concise but limited expressiveness
- *Patch transformation* (arbitrary shape)
  - Longer but higher expressiveness

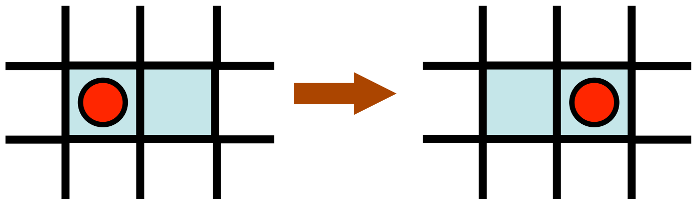
# Example: Diffusion Limited Aggregation (DLA)

- Diffusion: some particles are randomly diffusing; others are fixed
- Aggregation: if a mobile particle meets a fixed one, it stays fixed

```
trans dla = {
      `mobile , `fixed   => `fixed, `fixed ;
      `mobile , <undef> => <undef>, `mobile
}
```

*NEIGHBOR OF*





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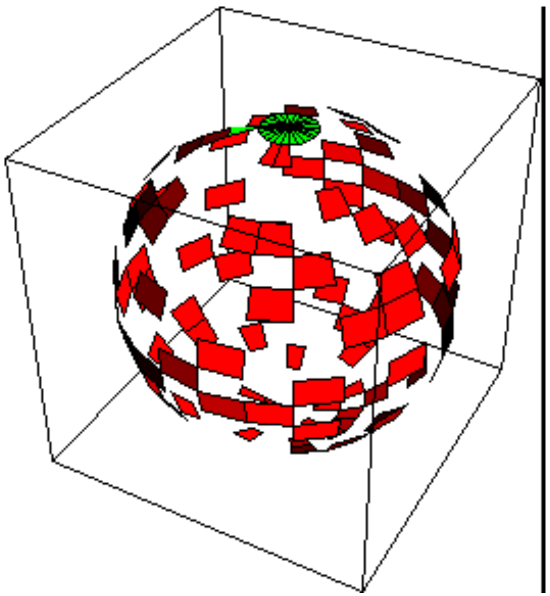
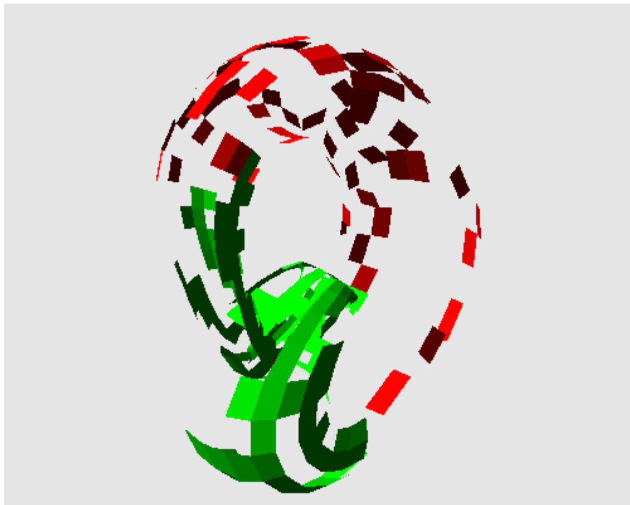
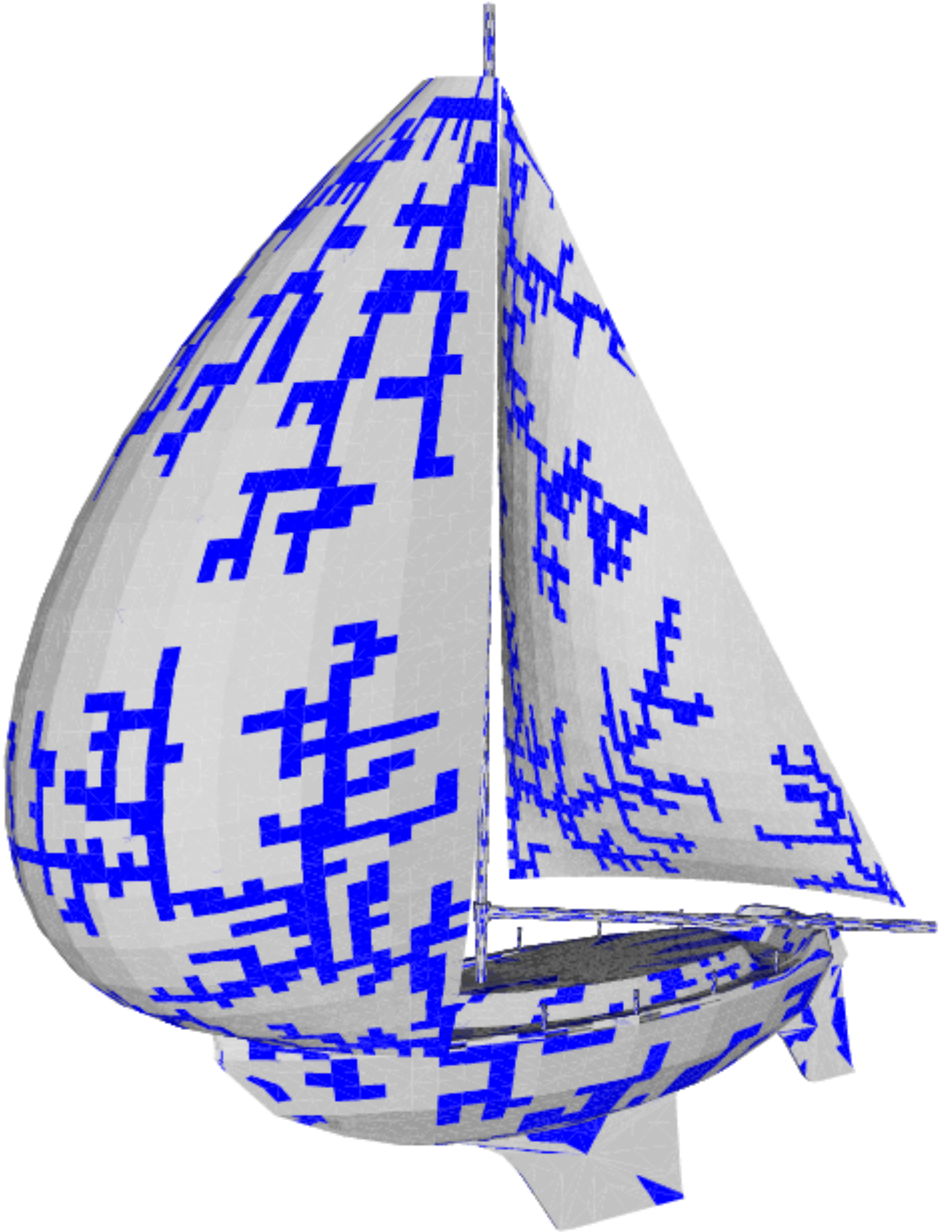
```

this transformation is an abstract process that can be applied to any kind of space

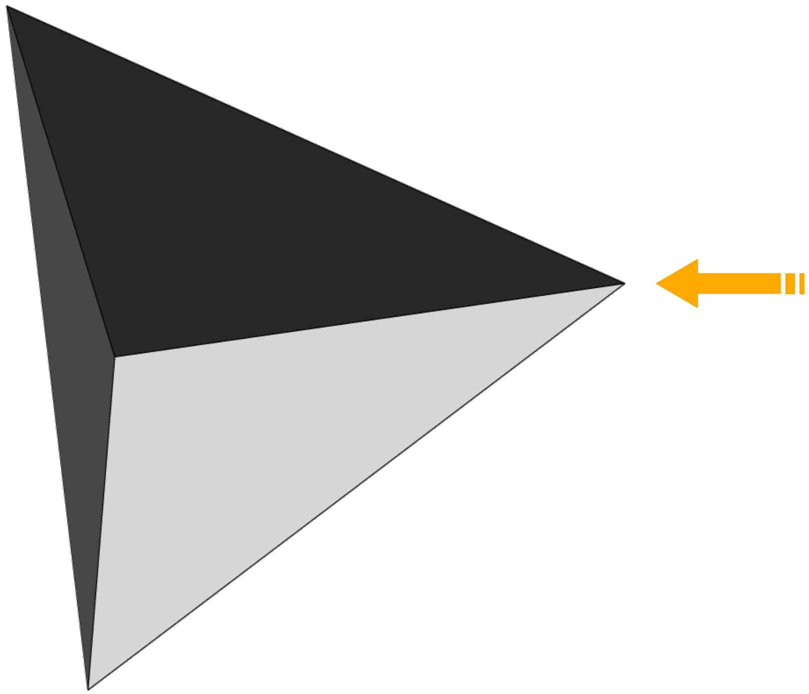




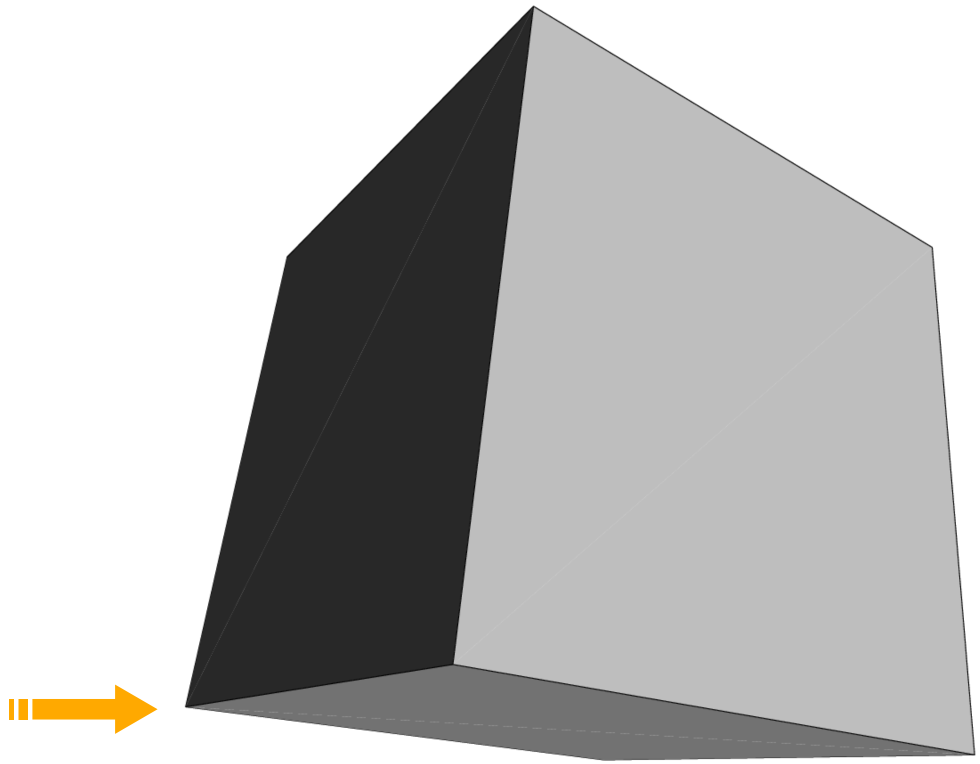
# Polytypisme



# Fractal construction by carving



Sierpinsky sponge (4 steps)



Menger sponge (2 steps)

# The Growth of a Meristem

## [PNAS 103(5), 1627-1632, 2006]

Pierre Barbier de Reuille

Mikaël Lucas

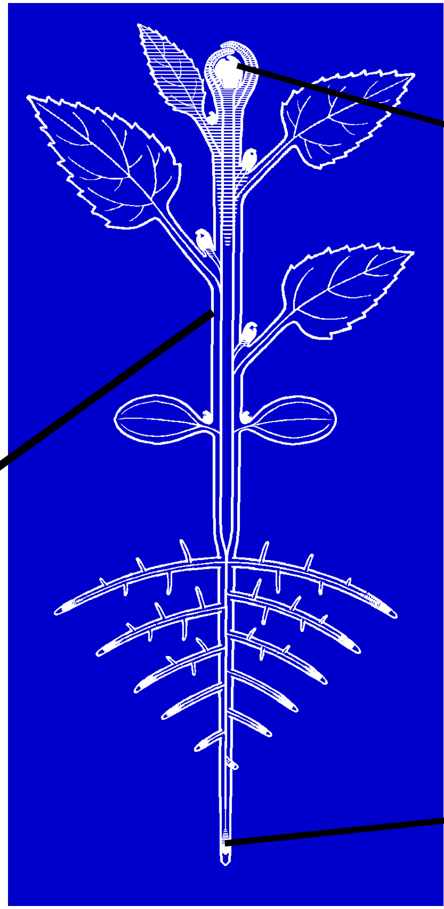
Jan Traas

Christophe Godin

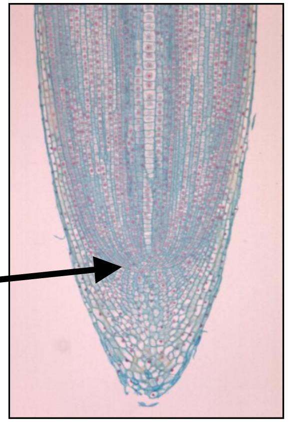
CIRAD/INRA/INRIA

Organs positionning at the shoot apex

Cambium

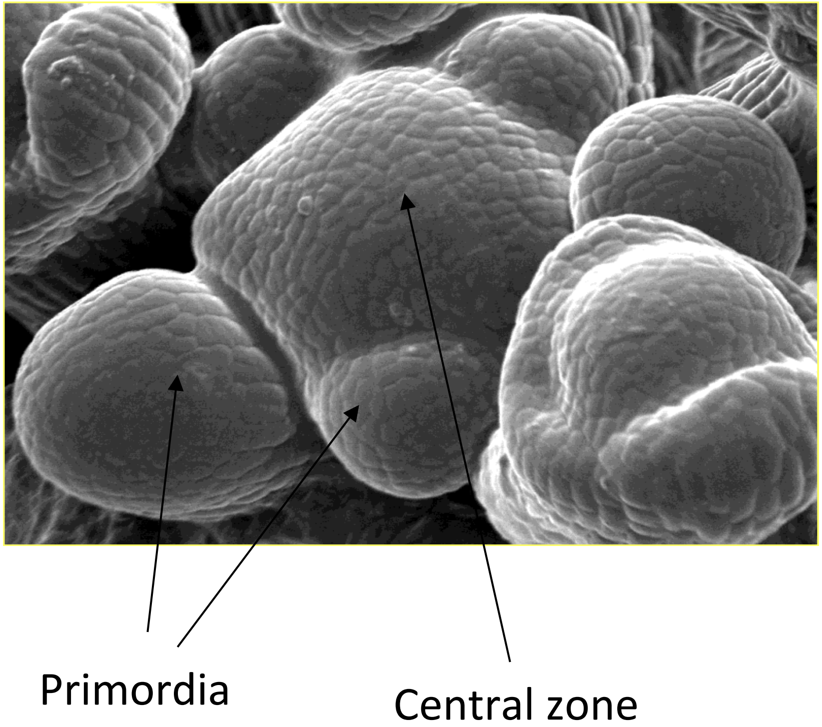


Shoot apical meristem



Root apical meristem

# A shoot apical meristem



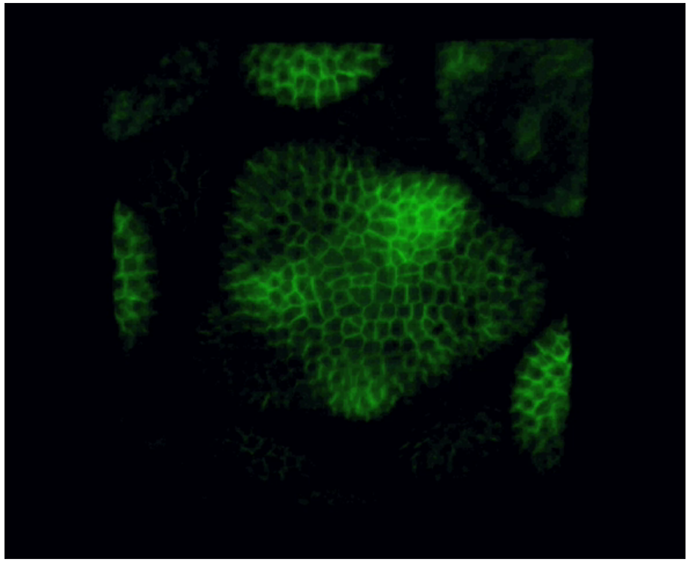
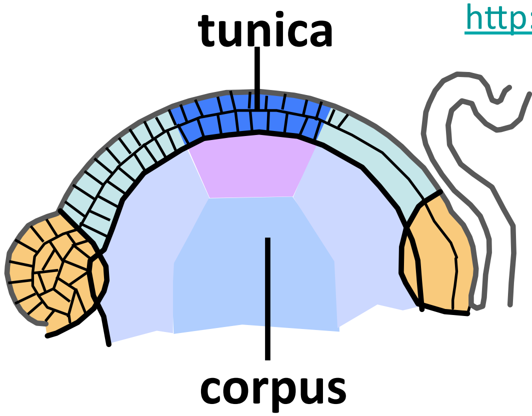



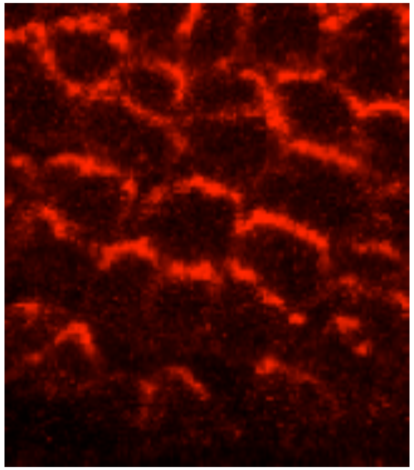
Image sequence showing cell division patterns via membrane-bound PIN1, in Shoot Apical Meristem (SAM), nearby floral meristems, and the boundaries between them (M. Heisler). http://computableplant.ics.uci.edu/ (E. Mjolness)



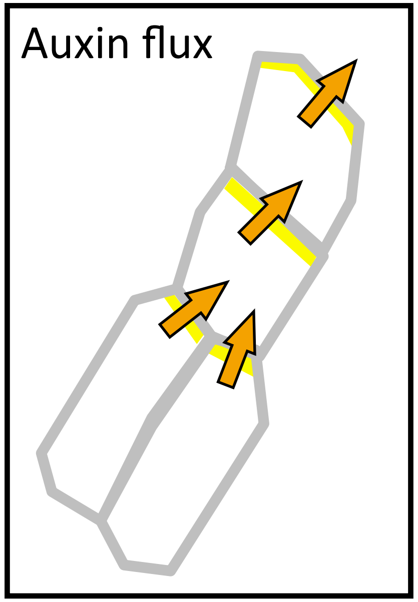
# Active transport of auxine

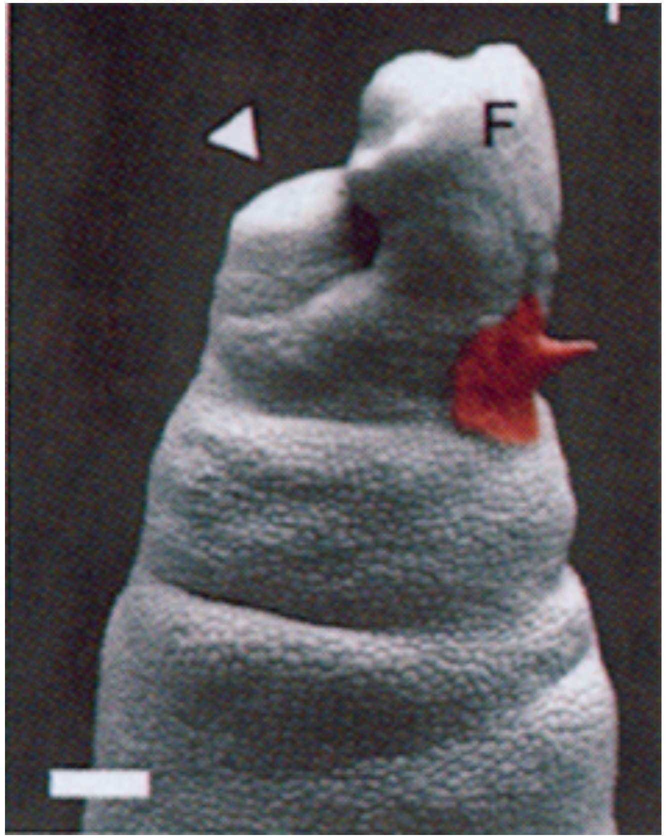
wild type

pin-1 mutant



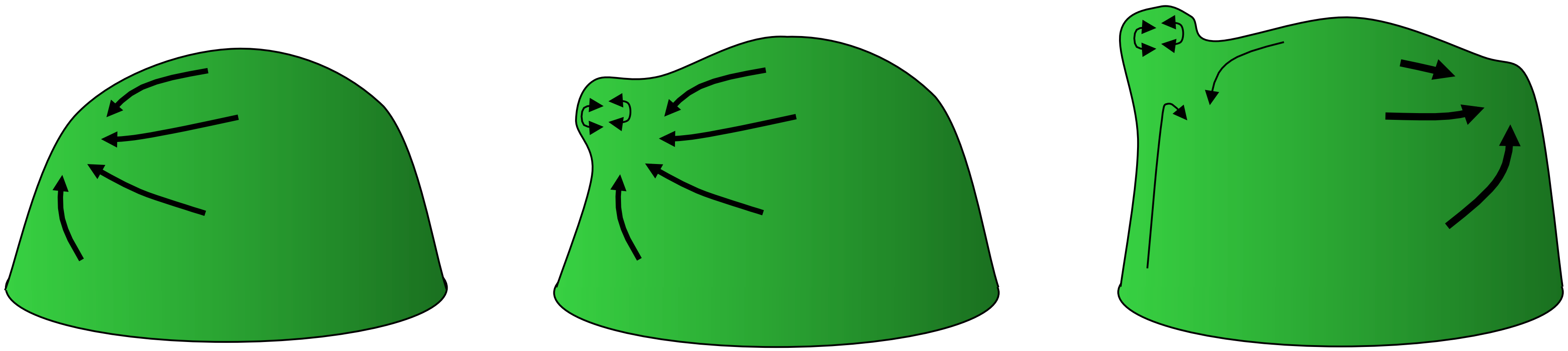
Immunolabelling of PIN-FORMED1 protein



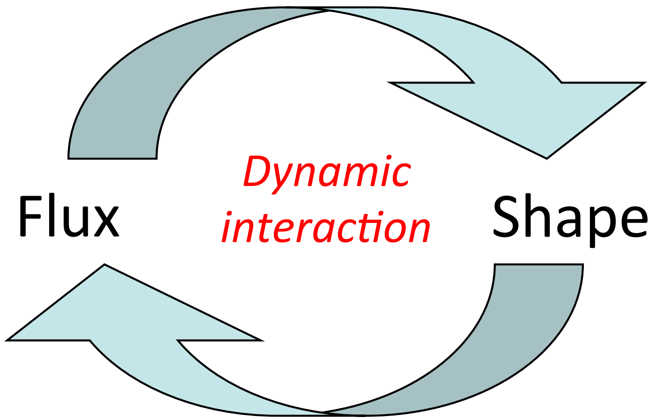




high concentration of
auxine induces organ initiation

# (DS)$^2$



flux… changes form… which changes flux…



# Model

- ***Cell internal state and processes***
  capacity of division, spring relaxed length,
  primordium/center,
  **concentration of auxin (inhibitor), saturation, auxin degradation / evacuation, promotion to primordium, “pump magnetism”**

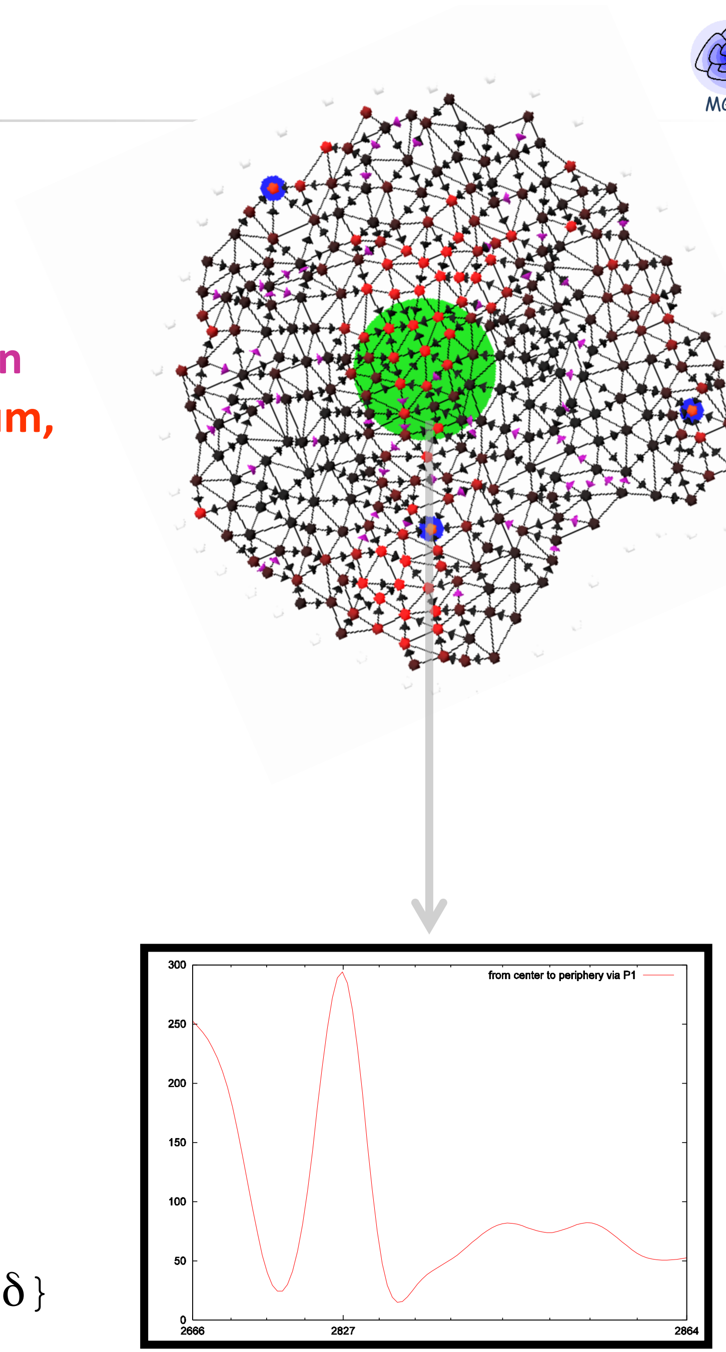
- ***Movement*** (due to cell growth)

- ***Growth***: increase of spring relaxed length

- ***Division***: when size > threshold

- ***Cell interaction***
  **Passive diffusion of auxin, active pumping of auxin**

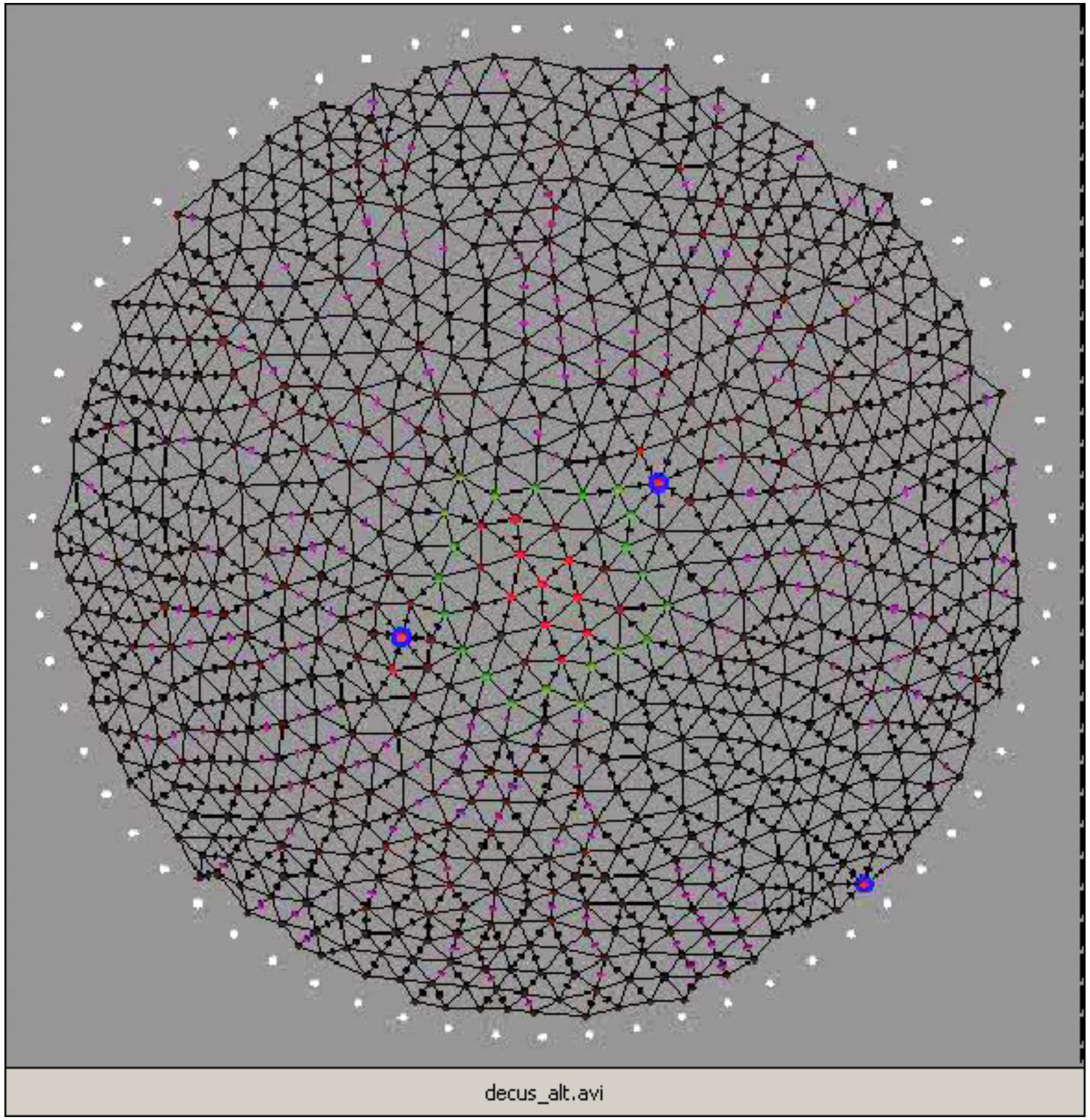
```
trans Auxin = {
        x, y / pump(x,y)
        → {x.auxin -= δ}, {y.auxin += δ}
}
```



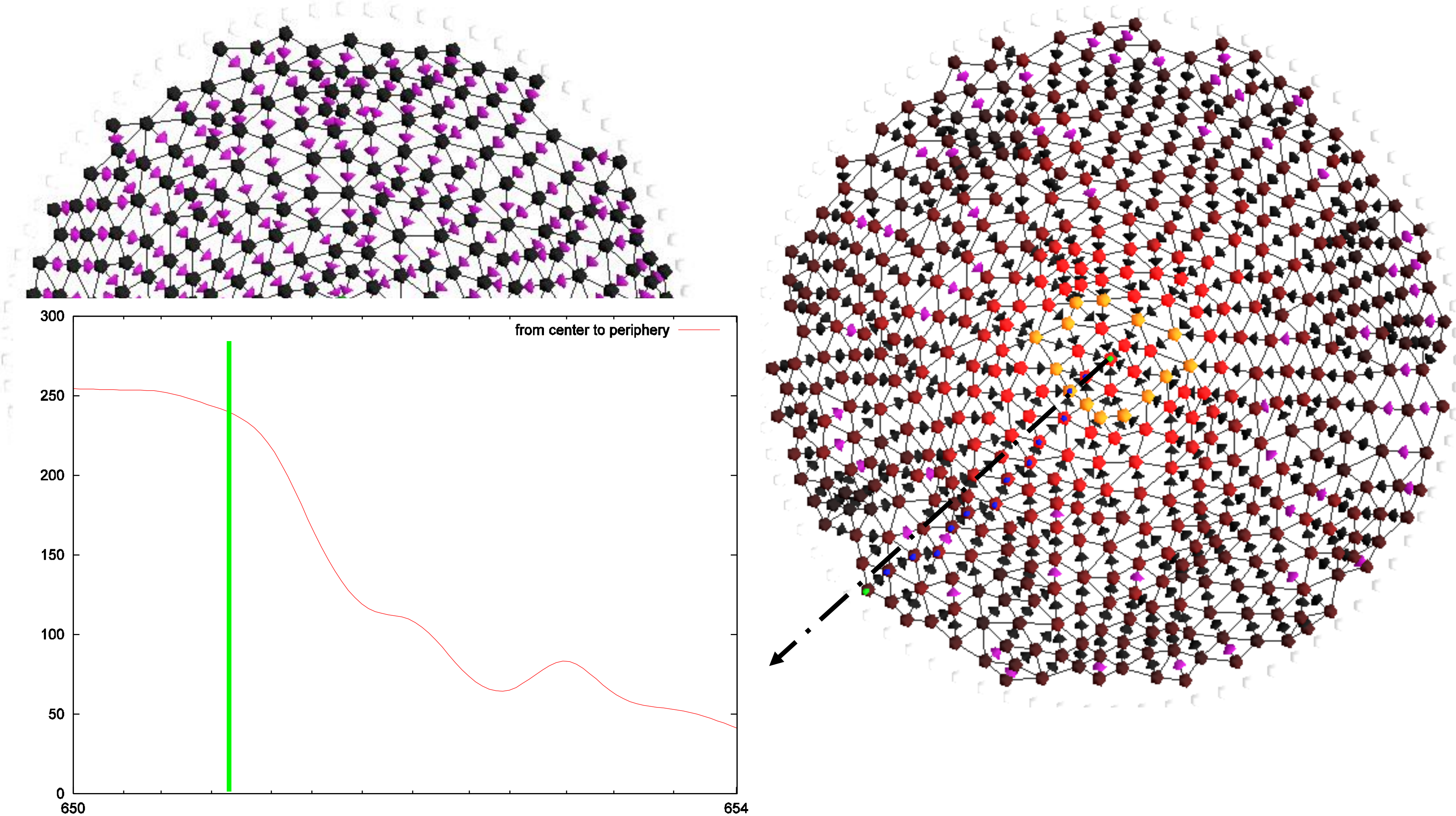

P. Barbier de Reuille

Auxin level

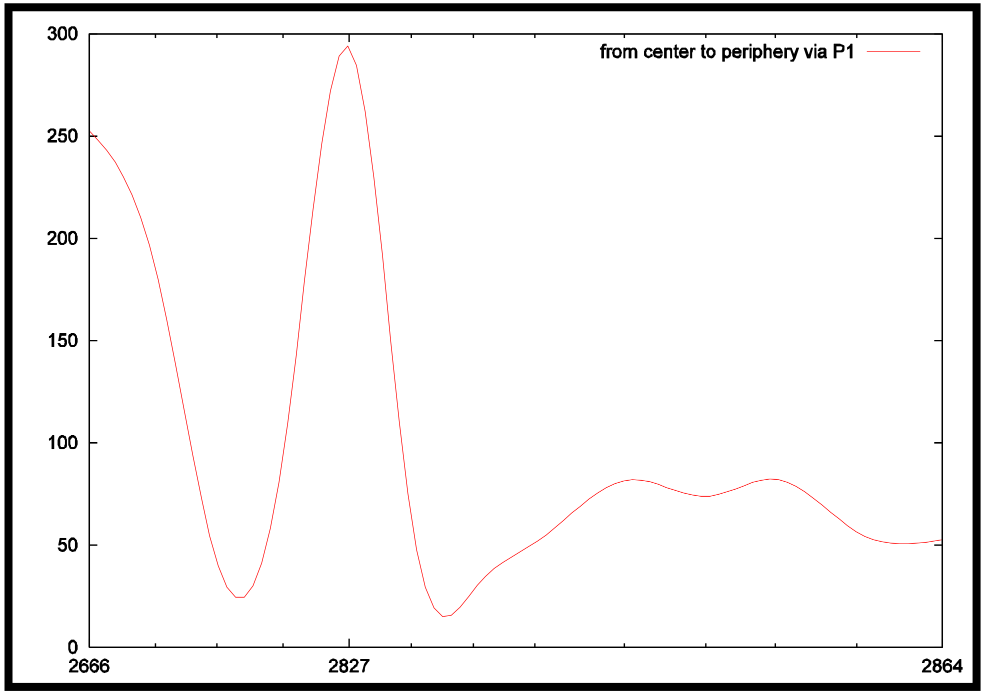
# Simulation



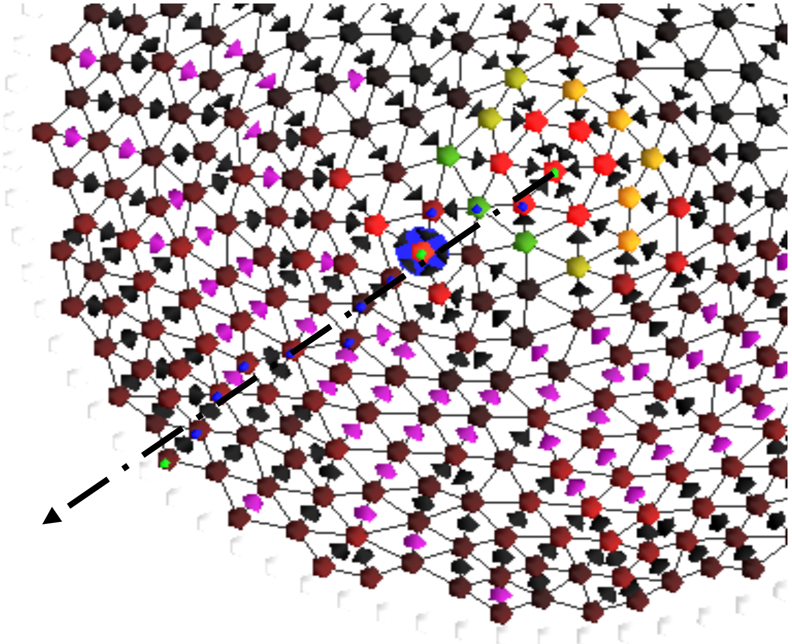
# Generation of a primordium



# Primordium local inhibition




Auxin level



- …
- Various models of Phage l
- Sperm crawling
- Neurulation
- 4 models for prototyping a « synthetic multicellular bacteria »
- …

# MGS drawbacks and successes

## Success

- Polytypisme is good
- Rule application strategies are good
- Patterns/rules are expressive and usually concise
- Clean semantics

## Shortcomings

- Rules may be heavy (e.g. 100 variables for the fractal sponge)
  *graphical drawing of rules*
  *look for better notations (e.g. path pattern)*
- Efficiency
  *well…*
- Implicit methods (solvers) are hairy
  *use explicit ones*

# Un « manifeste » topologique

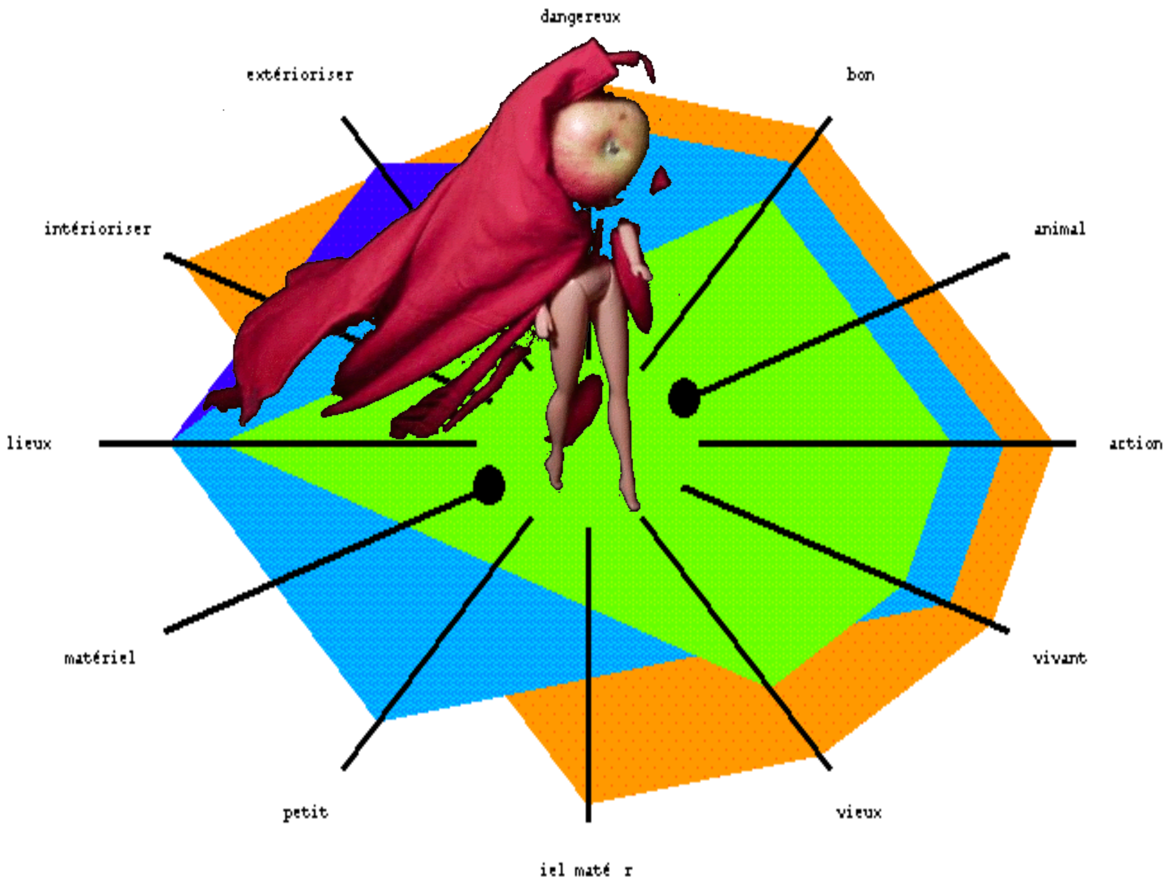
Spatial computing proposes to celebrate
corporeality of data rather than trying to deny it.
*Simon Greenworld* (MIT medialab)

- L'approche logique en informatique
  calculer = déduire
  (isomorphisme de Curry-Howard)

- D'autres paradigmes peuvent être fertiles
  calculer = se déplacer
  (?)

- Essayer de voir l'espace (et le temps) dans un programme (plutôt que des opérations logiques)
  visées : *heuristiques*, *techniques*, *pédagogiques*

# L’analyse des contes de fée
(sans Bettelheim)

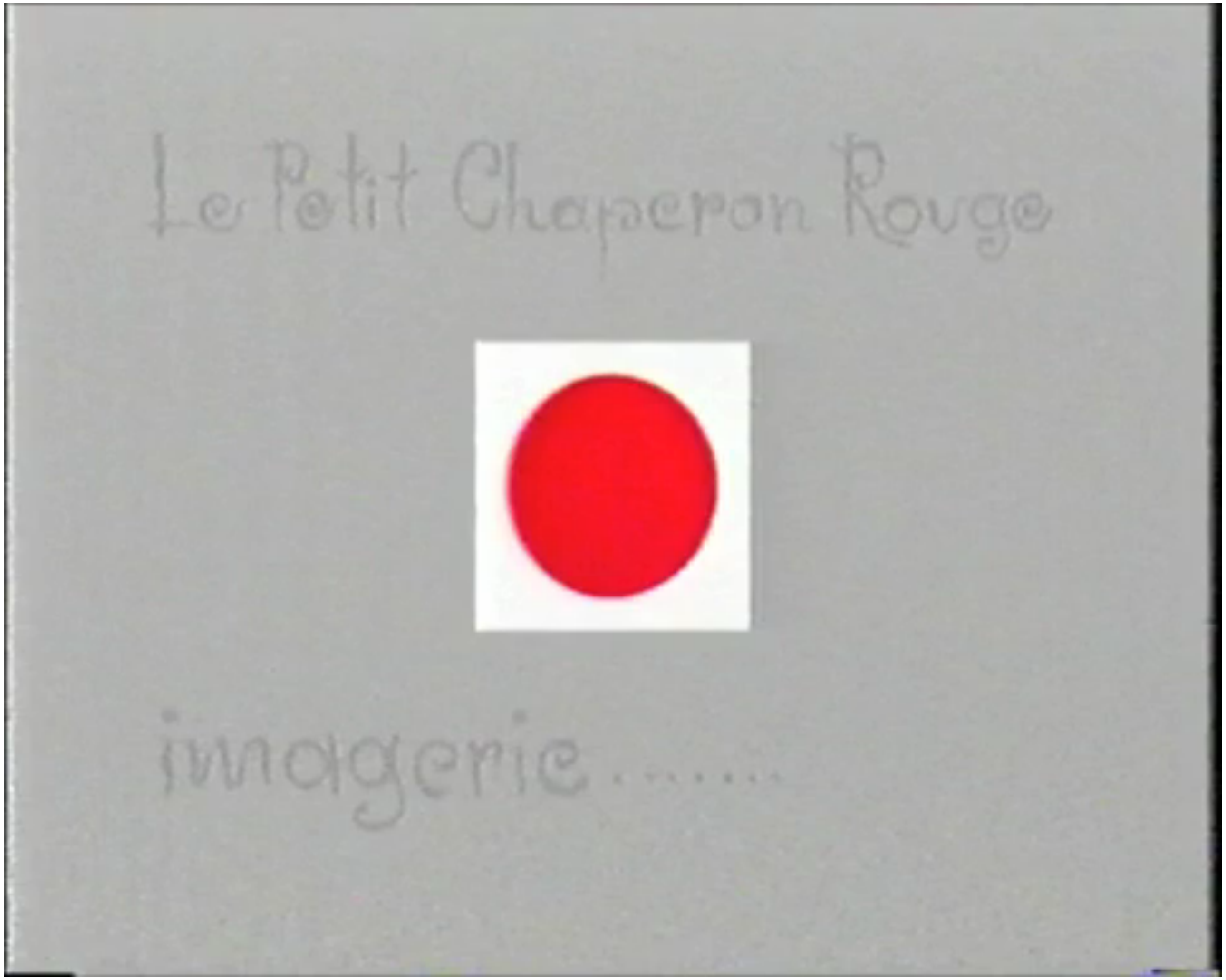


# L'analyse des contes de fées

La grand-mère se fait manger vers 2’30

Le PCR se fait manger vers 3’

Le chasseur tue le Loup vers 3’40



# L'analyse du petit chaperon rouge

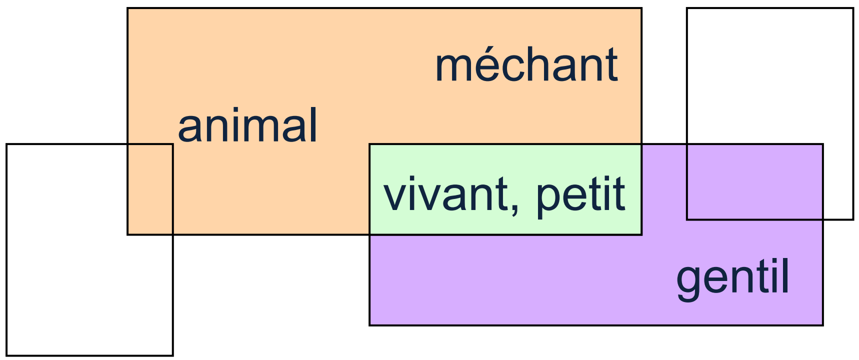
**Objectifs :**

- extraire d'une séquence d'ensembles d'objets (scène, image, etc.), les sous-ensembles (constituants, catégories, configurations, etc.) permettant de la décrire (et structure en treillis de ces constituants)
- Caractéristique ∈ vivant, animal, méchant, petit, gentil, moteur, extérieur, place
- Objet = { caractéristiques }

  le petit chaperon rouge = vivant, petit, gentil

  le loup = vivant, animal, méchant, petit
- Scène = { Objets}

  le loup et le PCR parle dans la forêt → vivant, animal, méchant, petit, gentil, moteur, extérieur, place



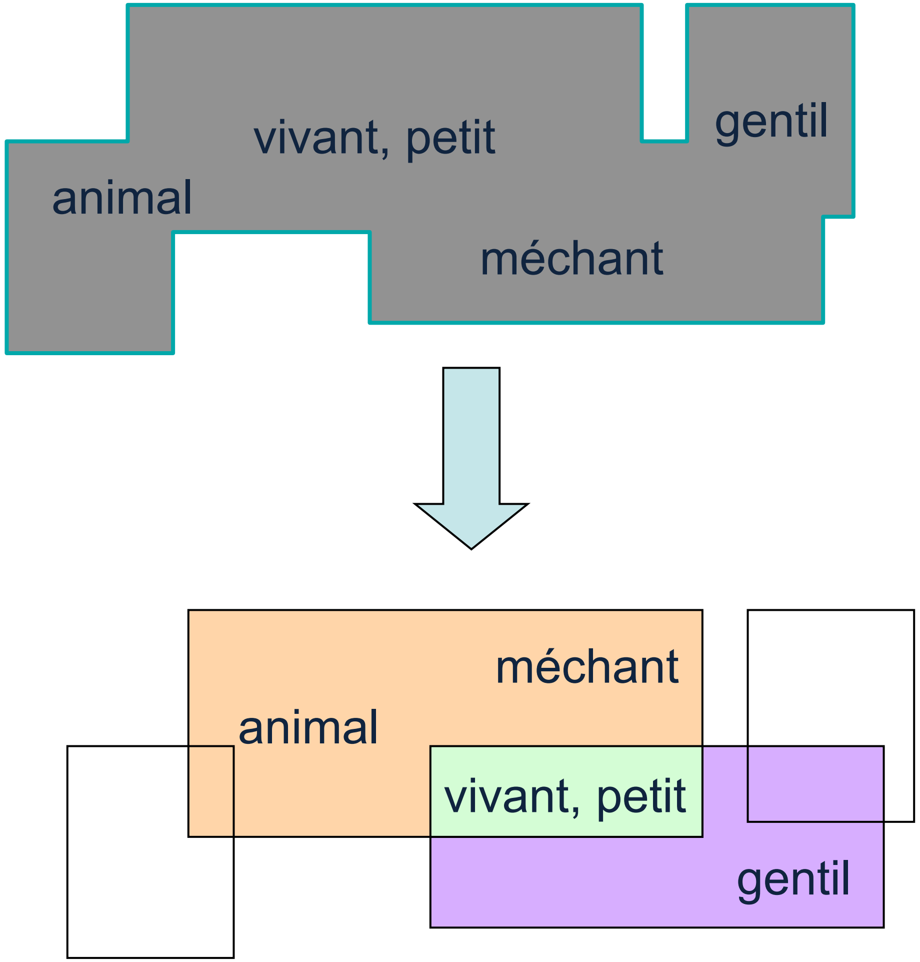
# Encodage

| Objets | Encodage (caractéristiques) |
|---|---|
| le Petit Chaperon Rouge | vivant, petit, bon |
| la mère | vivant, bon |
| la grand-mère | vivant, bon, vieux |
| le Loup | vivant, animal, dangereux |
| la forêt | vivant, lieux, dangereux |
| la maison | lieux |
| le panier | lieux, petit |
| donner | action, extérioriser, matériel |
| dormir | action, bon |
| manger | action, intérioriser, matériel |
| marcher | action, |
| parler | action, extérioriser |

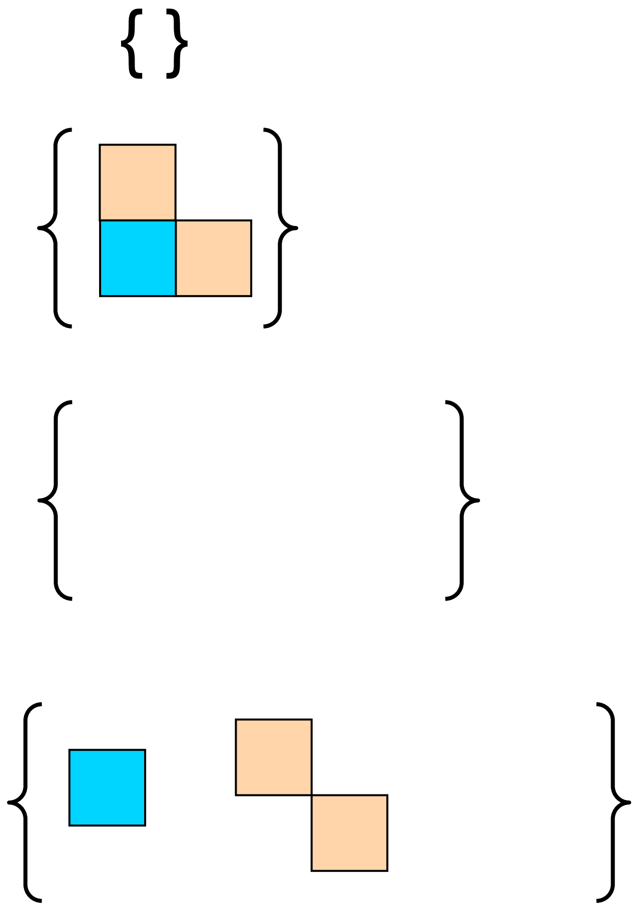
# Encodage

| Description | *Scène* (informel) |
| --- | --- |
| PCR, mère, parler, maison | La maman informe le PCR de sa GM malade |
| PCR, mère, donner, panier, maison | La maman donne le panier au PCR |
| PCR, marcher, forêt, panier | Le PCR marche dans la forêt |
| PCR, loup, parler, forêt, panier | Il rencontre le loup dans la forêt et lui raconte sa mission |
| loup, marcher, forêt | Le loup se dirige vers la maison de la GM |
| PCR, marcher, forêt, panier | Le PCR poursuit son chemin pour apporter le panier à la GM |
| GM, dormir, maison | La GM est à sa maison et dort |
| GM, loup, parler, maison | Le loup se présente à la maison de la GM et se fait passer pour PCR |
| loup, manger, GM | Le loup mange la GM |
| PCR, loup, parler, maison, panier | Le PCR arrive et parle au loup |
| loup, manger, PCR, panier | Le loup mange le PCR |

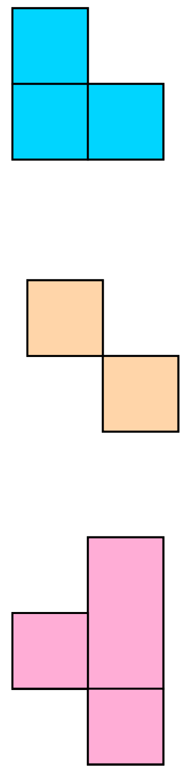
**Question** : si je connais seulement les caractéristiques des scènes, comment puis-je retrouver les objets (et en faire une hiérarchie) ?



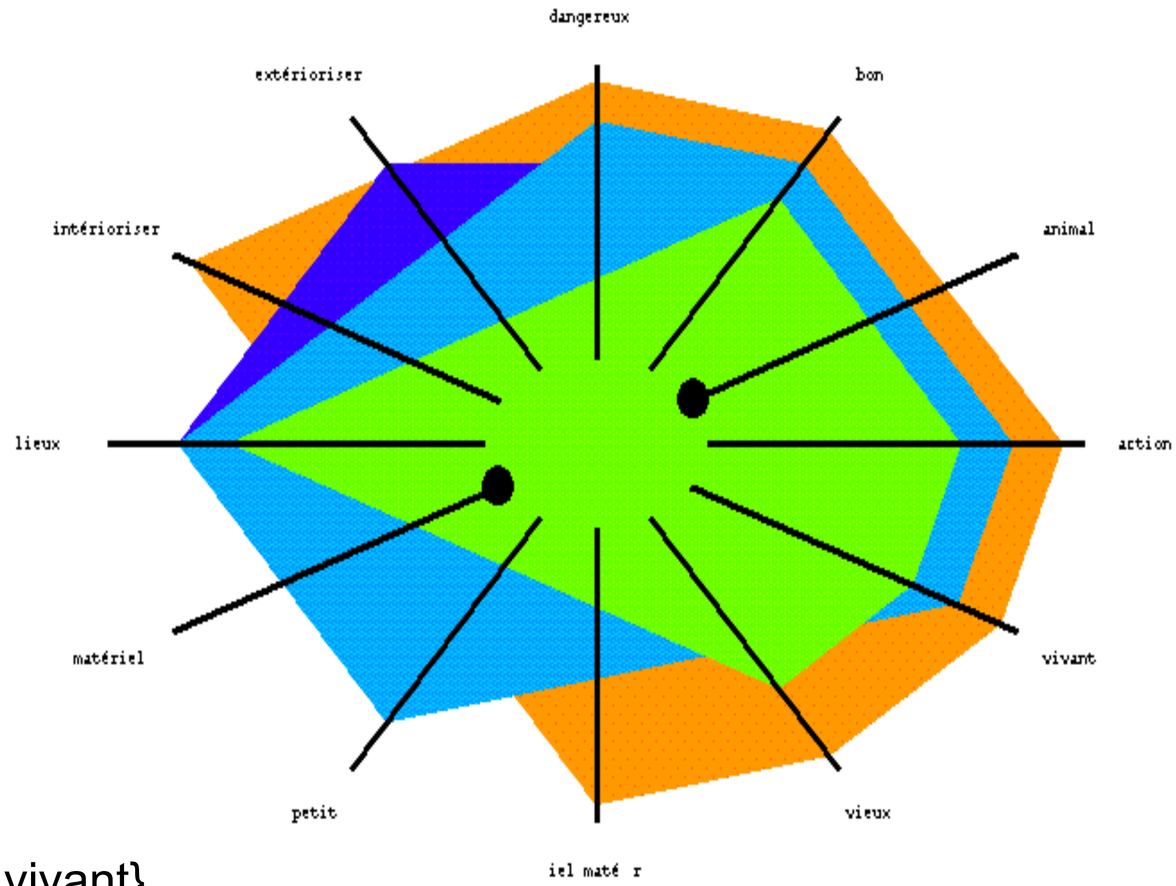
## Base courante d’objets



## Scène courante

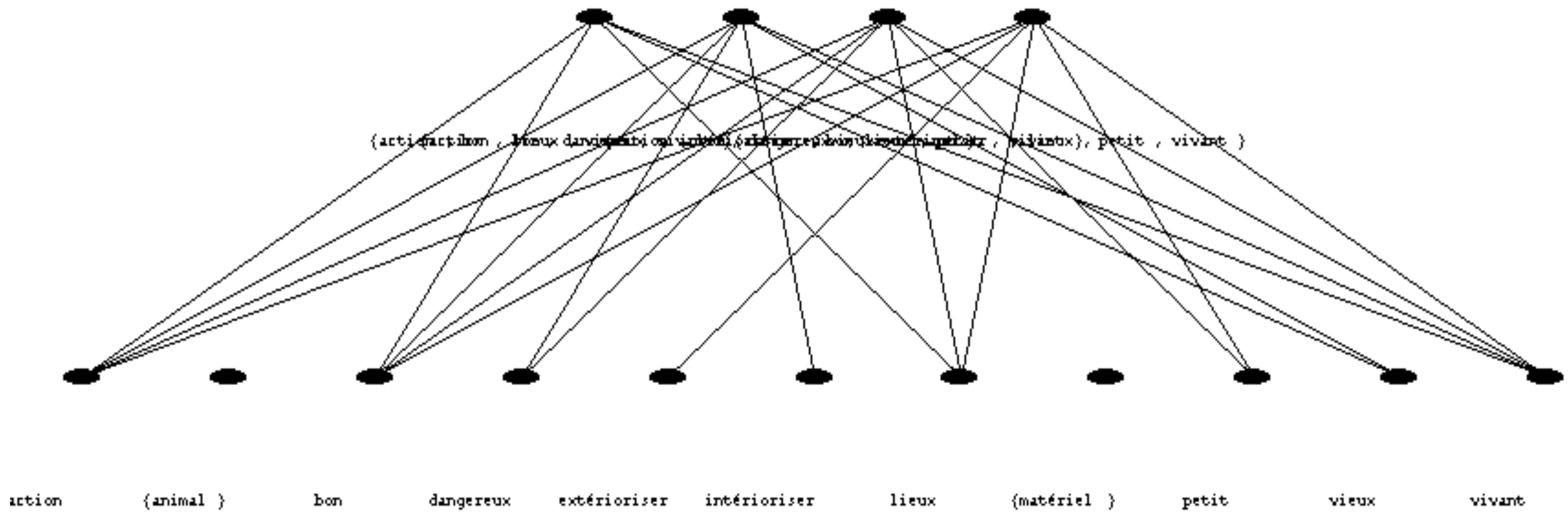


# Un ensemble de caractéristiques = un complexe simplicial




{
  {animal},
  {extérioriser},
  {intérioriser}, {matériel},
  {action, dangereux, lieux, vivant},
  {action, bon, lieux, vieux, vivant},
  {action, bon, dangereux, intérioriser, vieux, vivant},
  {action, bon, dangereux, lieux, petit, vivant},
  {action, bon, extérioriser, lieux, petit, vivant}
}

# Treillis des concepts formels




Treillis des concepts formels (Wille 97) pour une relation $\lambda \subset A \times P$,
on définit une opération § par :

$$B \subset A,\ \S B = \{p \in P \mid \forall b \in B,\ (b,p) \in \lambda\}$$

$$Q \subset P,\ \S P = \{a \in A \mid \forall p \in Q,\ (a,p) \in \lambda\}$$

§ établit une connexion de Galois.

(B,Q) *concept formel* ssi B=§Q et Q=§B : B *extension* et Q *intension* du concept.

## Simplexe et Complexe

**Un complexe simplicial abstrait : (V,C)**

- **V** ensemble de sommets,
- **C** est un ensemble de parties de V fermé pour l'inclusion :

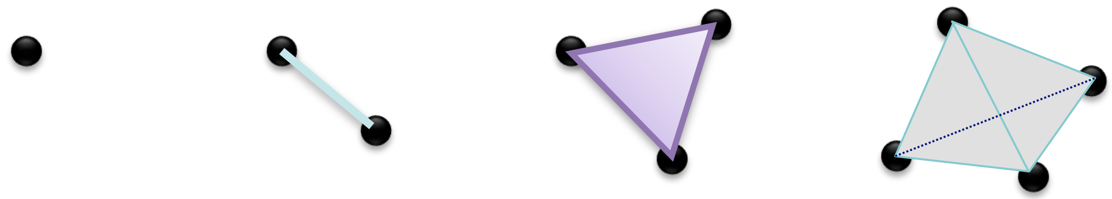
  si $s \in C$ et $s' \subset s$ alors $s' \in C$

- simplexe = élément de C
- dimension d'un simplexe $s = |s| - 1$
- dimension d'un complexe = dimension de son plus grand simplexe
- chemin $P$ = suite de simplexes $s_i$ tel que *si* $s_i \cap s_{i+1} \neq \varnothing$
-bord $\partial s$ d'un simplexe $s$ : $\{s',\ s' \subset s$ et $s' \neq s\}$

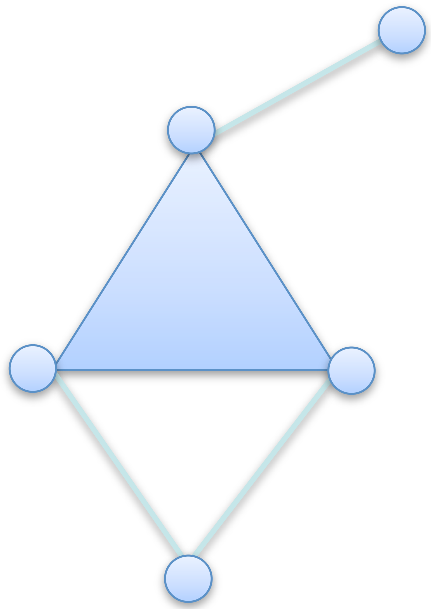
**Représentation graphique (généralise les graphes)**

| simplexe abstrait | {a} | {a,b} | {a,b,c} | {a,b,c,d} | |
|---|---|---|---|---|---|
| | point | arête | face | volume | … |

simplexe euclidien



# Complexe



{

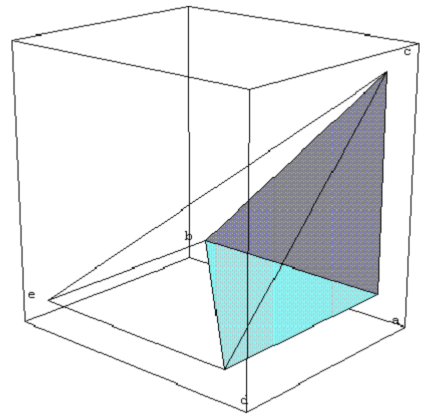
{a}, {b}, {c}, {d}, {e}, simplexe de dim 0

{a,b}, {b,c}, {b,d}, {a,d}, {a,e}, {c,d}, simplexe de dim 1

{a,b,d} simplexe de dim 2

}

# Topologie ↔ complexe simpliciaux ↔ treillis



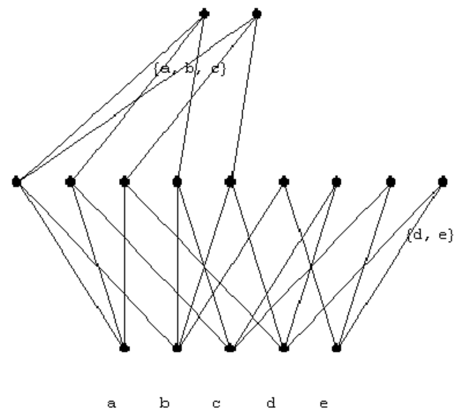
Topologie (ensemble des ouverts et des fermés)

↓ simplexes = fermés de la topologie ↑

Complexe simplicial (ensemble d'ensembles fermés par inclusion)

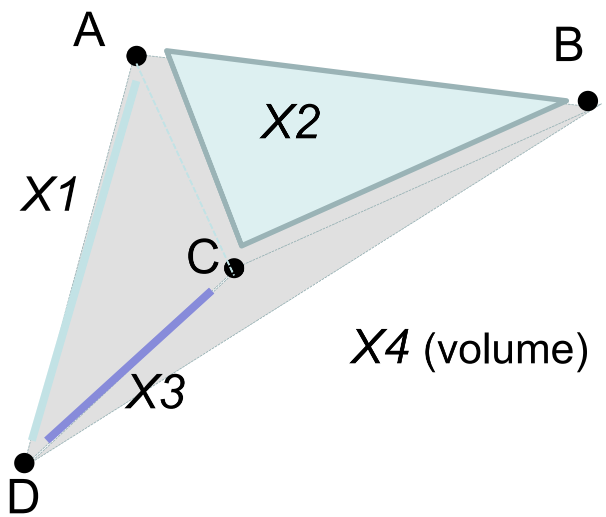
↓ simplexes = cônes du treillis ↑

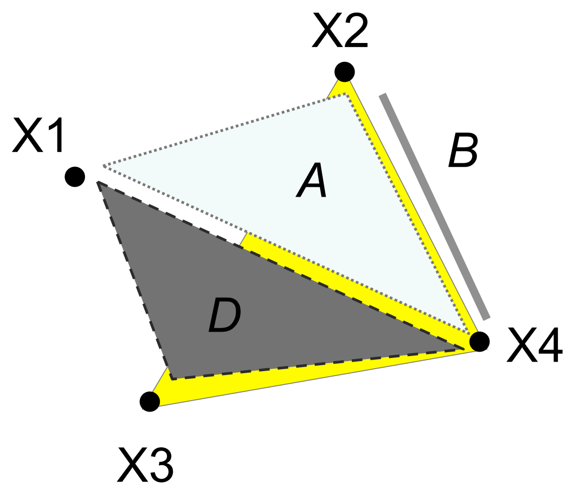
Treillis (relation d'ordre: ∧ , ∨)




- Reformulation élégante d'approches « classiques »
- Extension des outils

| | A | B | C | D |
|---|---|---|---|---|
| *X1* | 1 | 0 | 0 | 1 |
| *X2* | 1 | 1 | 1 | 0 |
| *X3* | 0 | 0 | 1 | 1 |
| *X4* | 1 | 1 | 1 | 1 |

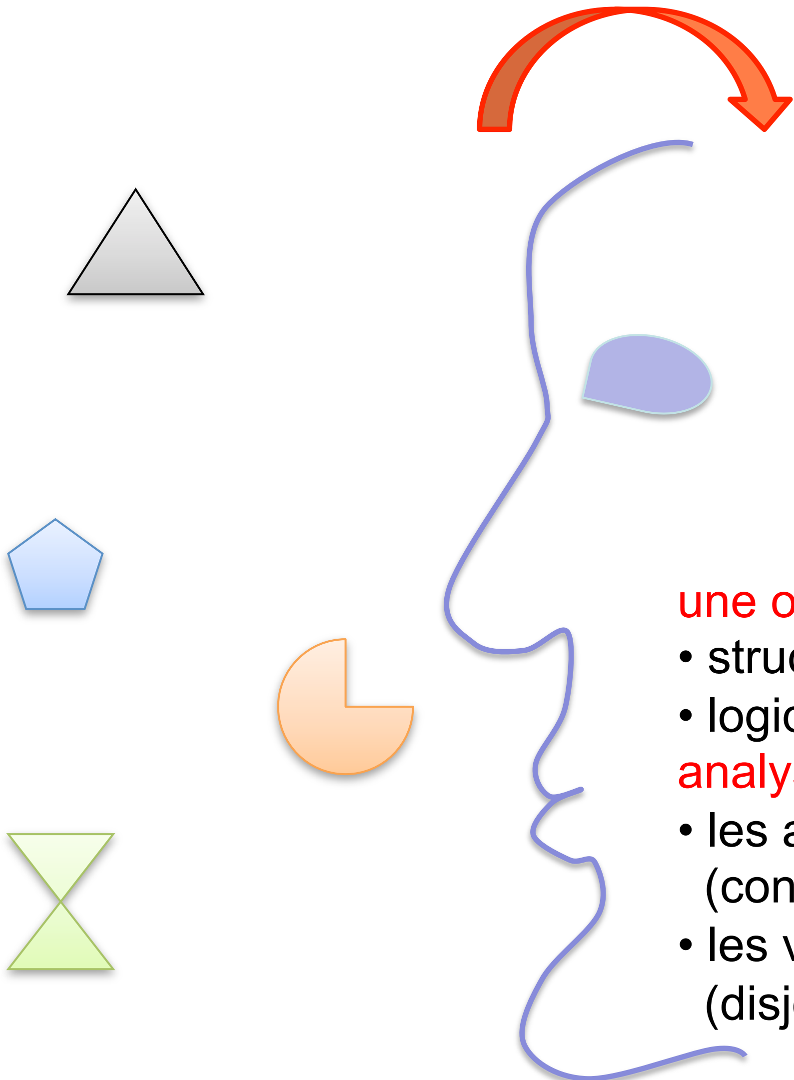


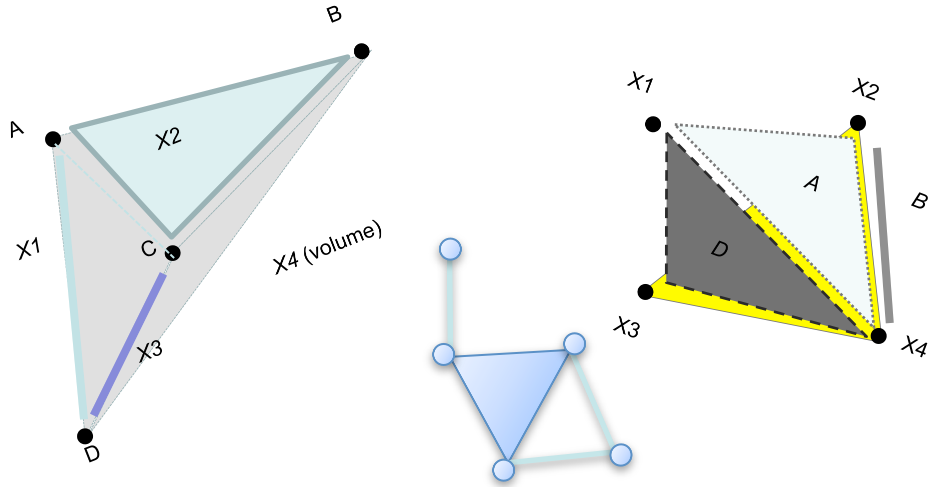




(le dual)

**Percepts** **Concepts**

la pompe à concept






une organisation spatiale :

- structure de la co-occurence des percepts
- logique des observations (= topologie des ouverts)

analyse moins naïve :

- les attributs se perçoivent simultanément (conjonction)
- les valeurs d'un attribut sont exclusives (disjonction)

## L’analogie aristotelitienne

# Un problème d’analogie

Etant donné A, B et C,

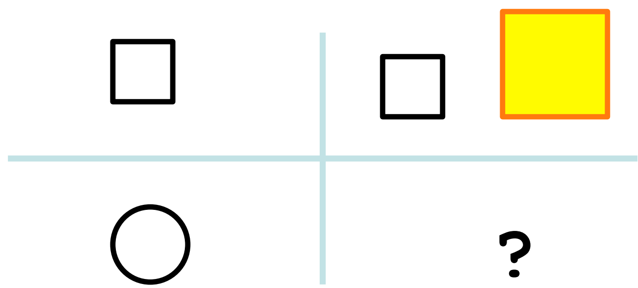
Trouver D qui est à C ce que B est à A

| | |
|---|---|
| A | B |
| C | ? |

Exemple numérique

| | |
|---|---|
| 3 | 6 |
| 7 | ? |

Exemple géométrique

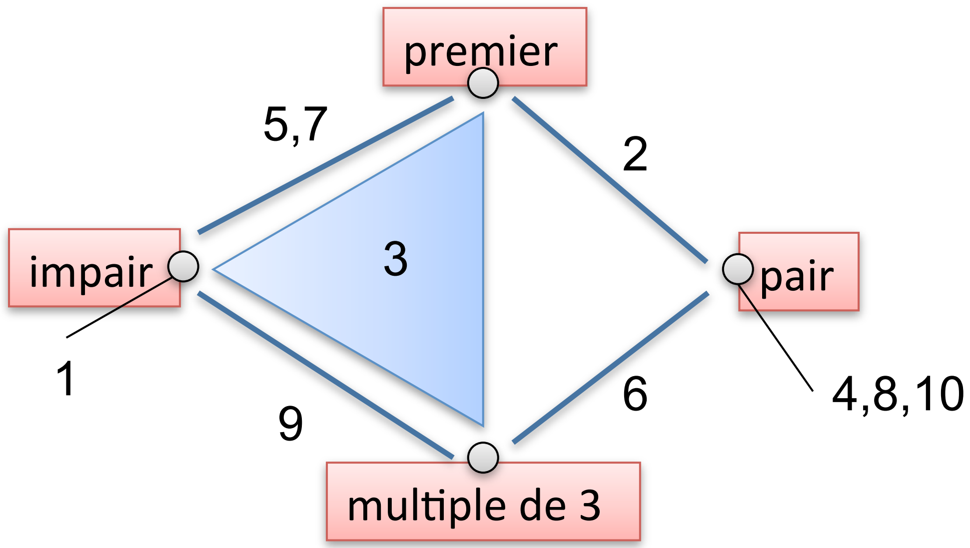


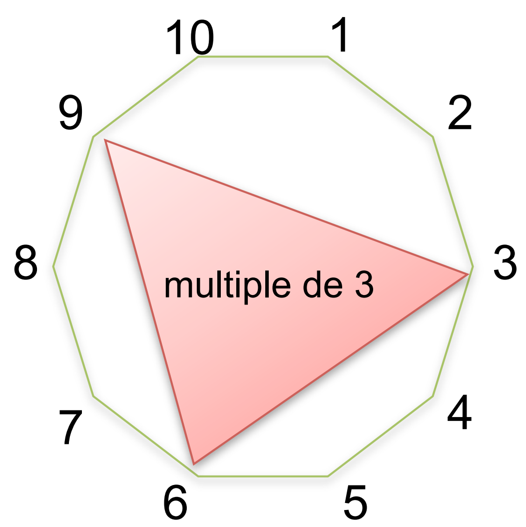
# Représentation d'un ensemble de prédicats

$\lambda \subset \text{Objets} \times \text{Predicats} : (o,p) \in \lambda \Leftrightarrow p(o)$

Objets = {1, 2, 3, …, 10}

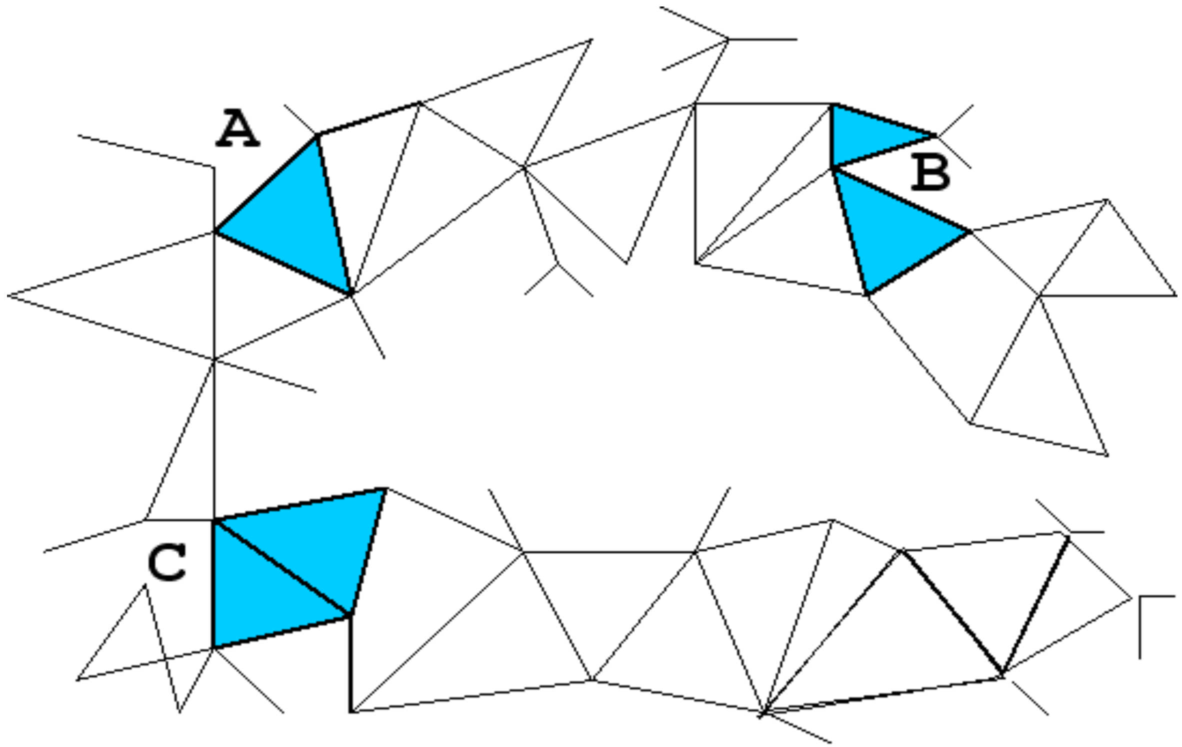
Predicats = {premier, pair, impair, multiple-de-3}



# Une méthode de résolution d'analogie

1. Représenter chaque figure comme une région de l'*espace des propriétés des figures*
2. Trouver un *chemin* $\gamma$ entre la région A et la région B
3. Généraliser ce chemin en l'interprétant comme une *transformation* $\Gamma$ *de l'espace (i.e. de propriétés)* via le chemin de A à C
4. Appliquer $\Gamma$ à $\gamma$ pour trouver D



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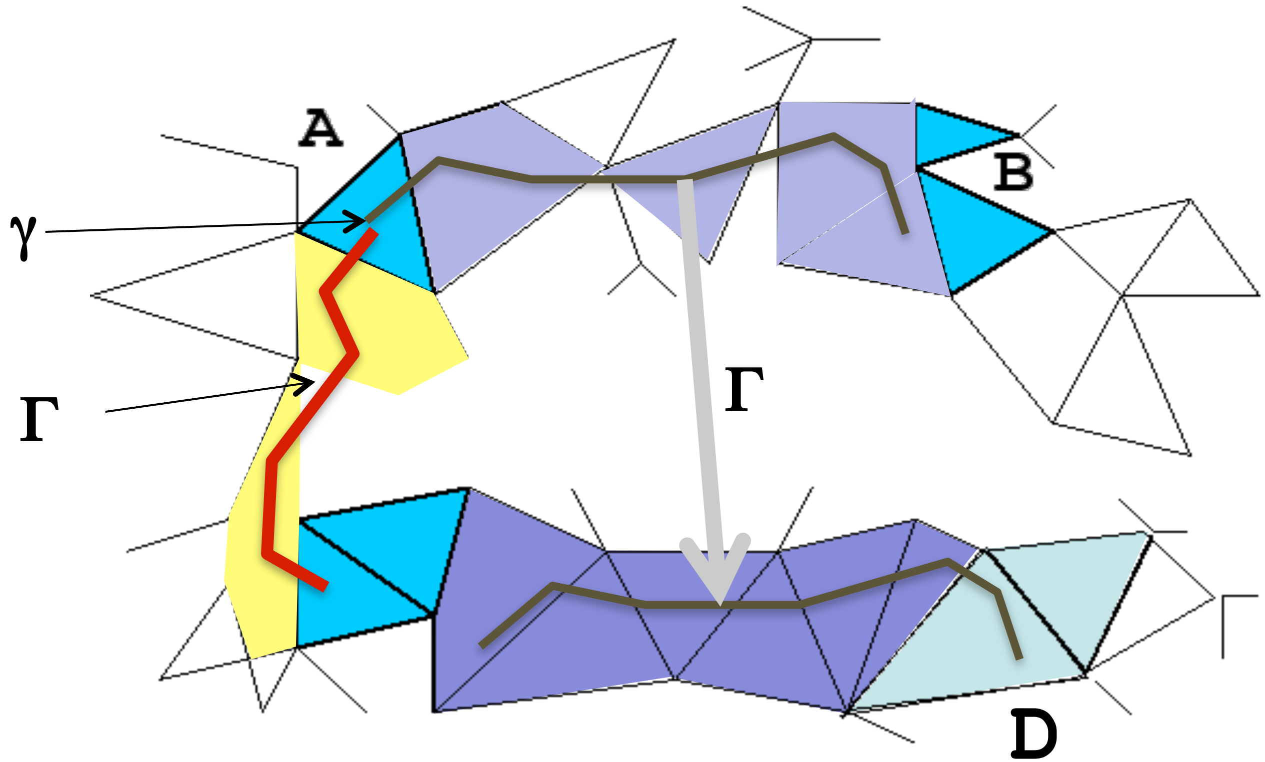
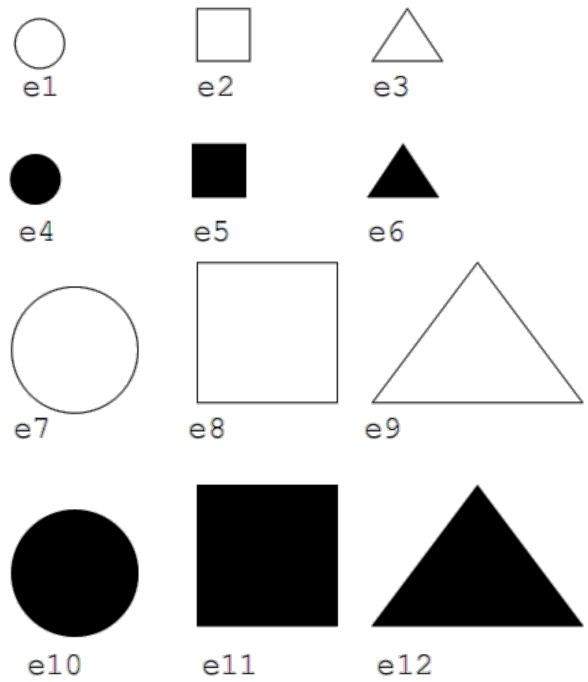
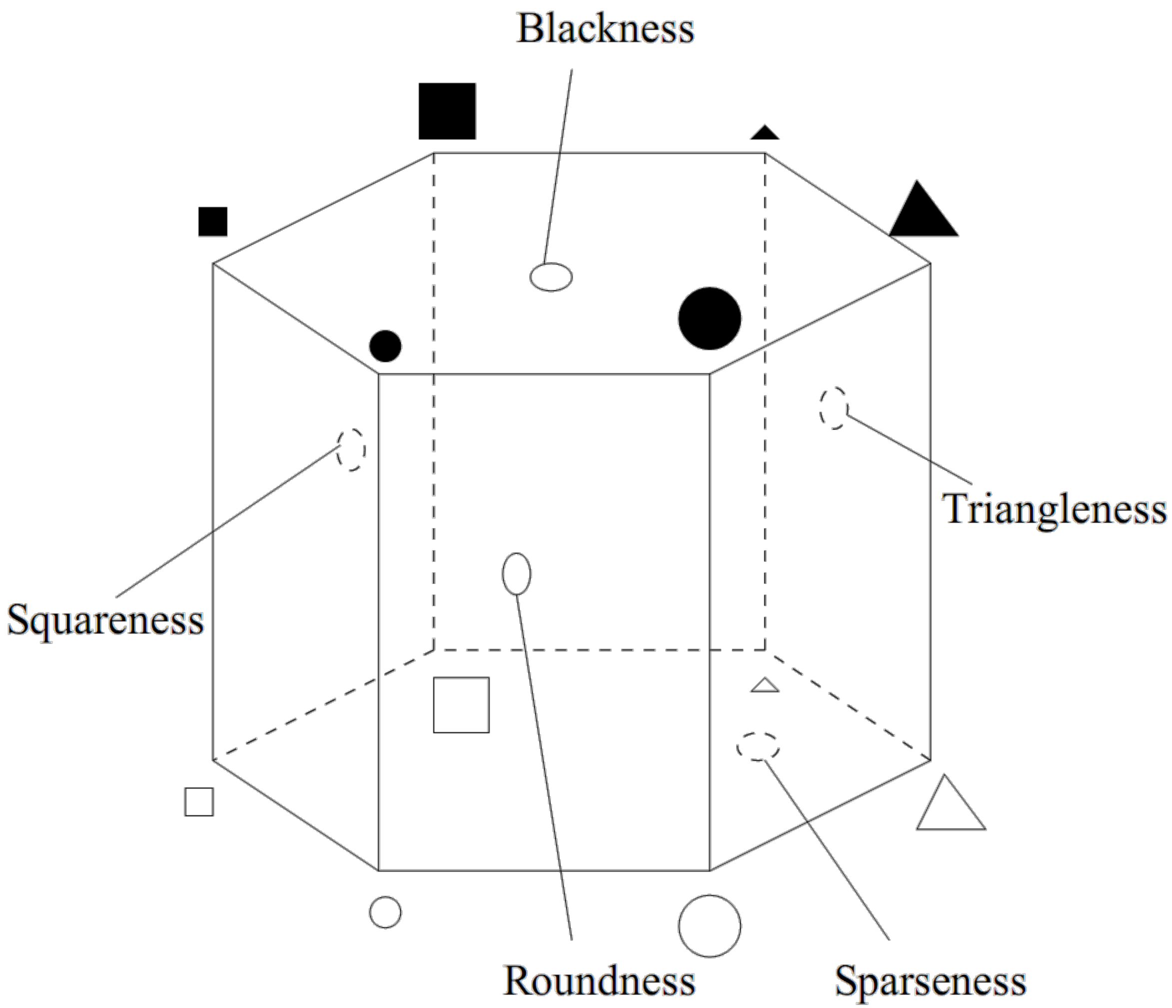
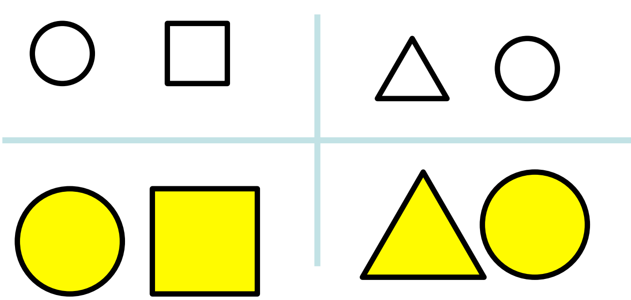


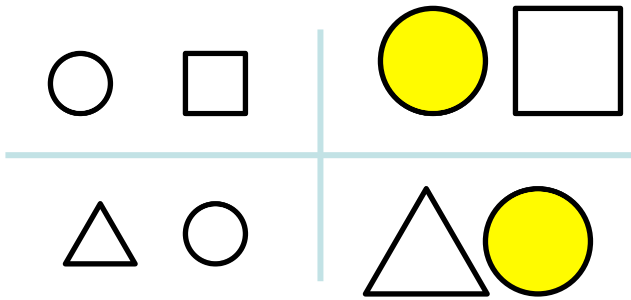
# Notre petit monde…

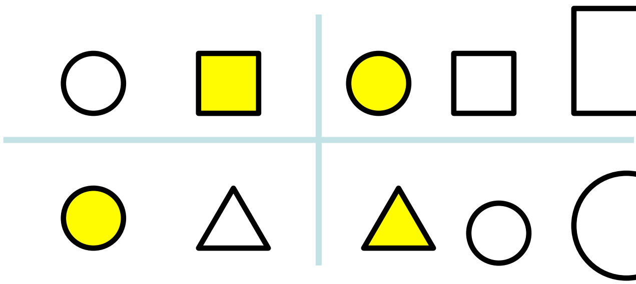


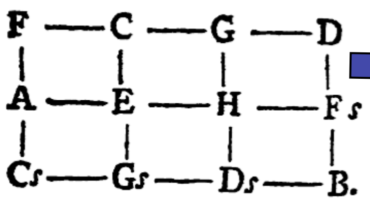
- Exemple de résolution avec *Esqimo*



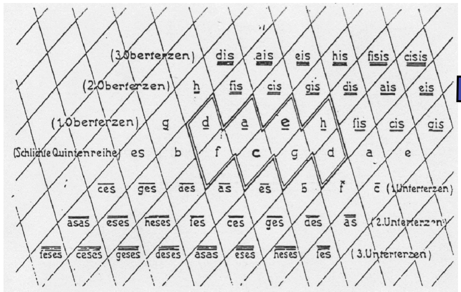




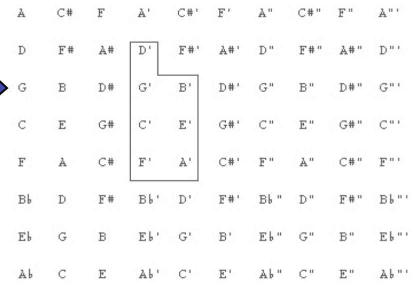
# Représentations topologiques de structures musicales



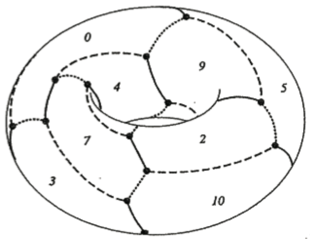

Euler : *Speculum musicum*, 1773



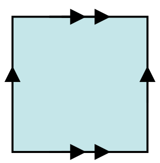
Hugo Riemann : « Ideen zu einer Lehre von den Tonvorstellung », 1914

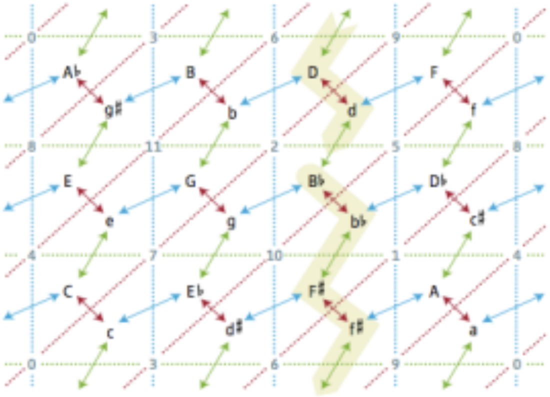


Longuet-Higgins (1962)

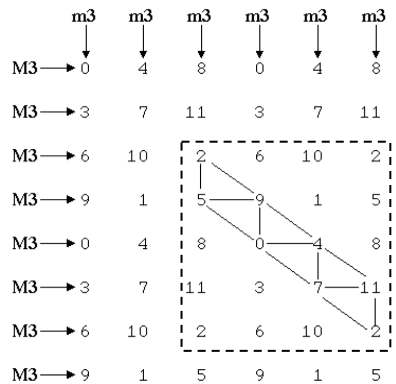


Douthett & Steinbach, *JMT*, 1998





J. Hook, « Exploring Musical Space », *Science, 2006*



Balzano (1980)

# Neo-Riemannian Problematic

- Traditional western music representation
  - Based on the use of a *staff*
  - Main drawbacks for visualization of harmony
    - *Contrapuntal* proximity

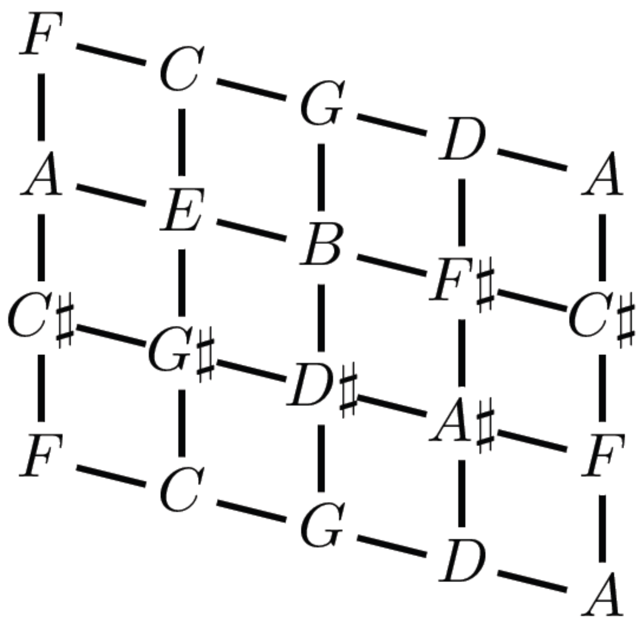
      The spatial distance between notes is not relevant for harmonic purpose
    - Spatially close patterns can sound very different

- Neo-Riemannian representation of music
  - Graphical representation of harmony rules
  - Consonance used as a graphical criterion

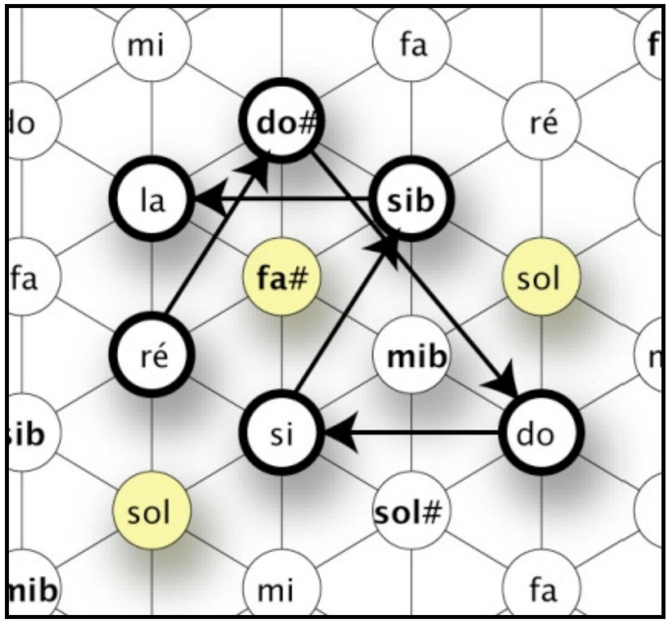
    Two notes that sound well must be spatially close

- Examples

Neo-Riemannian approach for tonal music



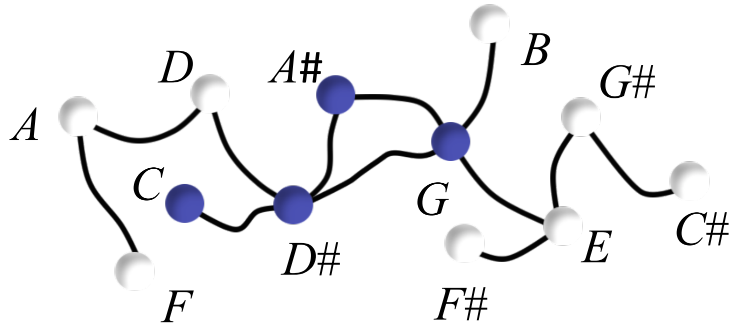

Euler's tonnetz



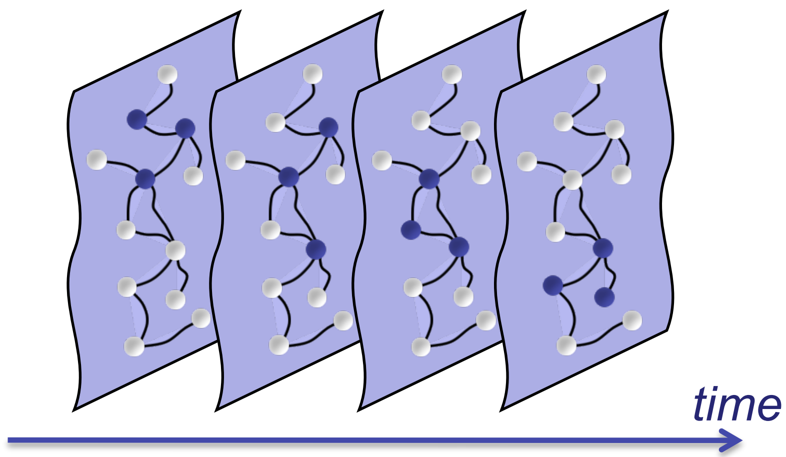

Hexagonal network of notes
(J.-M. Chouvel)

# Spatial Representation of Musical Sequences

- Temporal succession of musical event
- Musical event as a *topological collection*
  - Positions are notes
  - Labels represent played notes



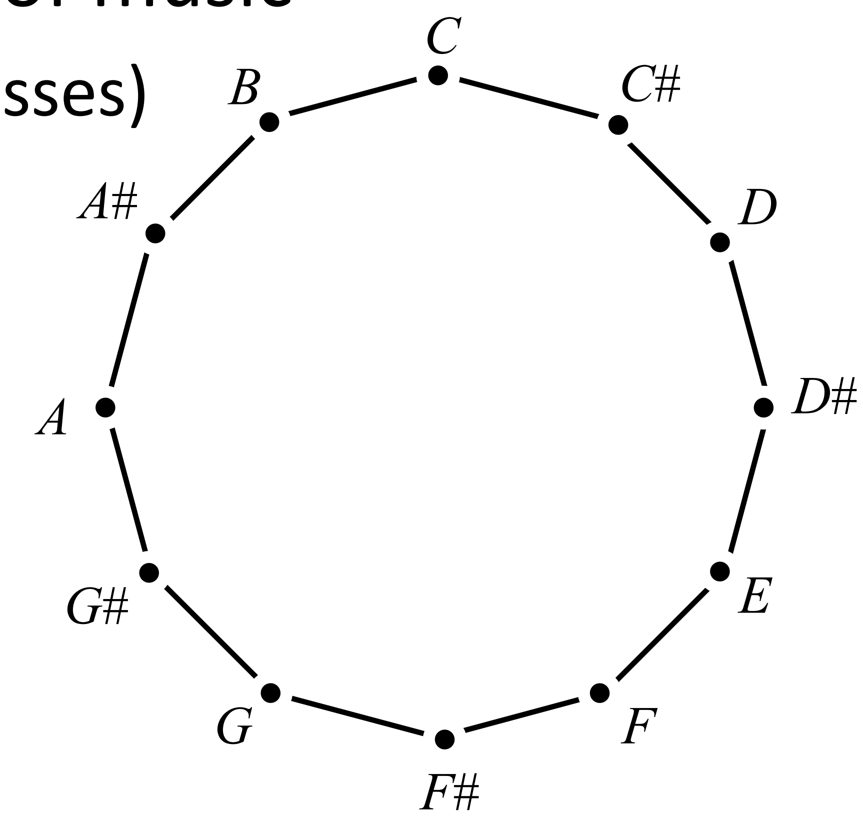

- Succession of events as a *stream of collections*



# Formalization of Notes Neighborhoods

- Which neighborhoods for significant visualization?
- Strong algebraic structure of music
  - Set N of notes (i.e. pitch classes)

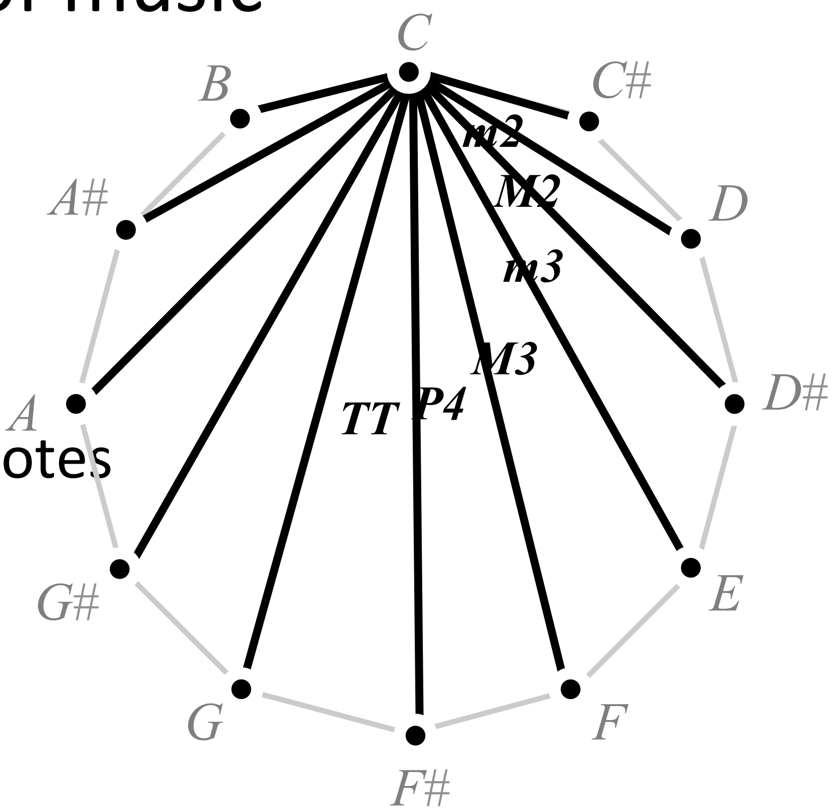
We do not consider octaves




$$N = \{ C, C\#, D, D\#, E, F, F\#, G, G\#, A, A\#, B \}$$

# Formalization of Notes Neighborhoods

- Which neighborhoods for significant visualization?
- Strong algebraic structure of music
  - Set N = { *C*, *C#*, …, *A#*, *B* }
  - Group (I,+) of intervals
    - Relative difference between notes

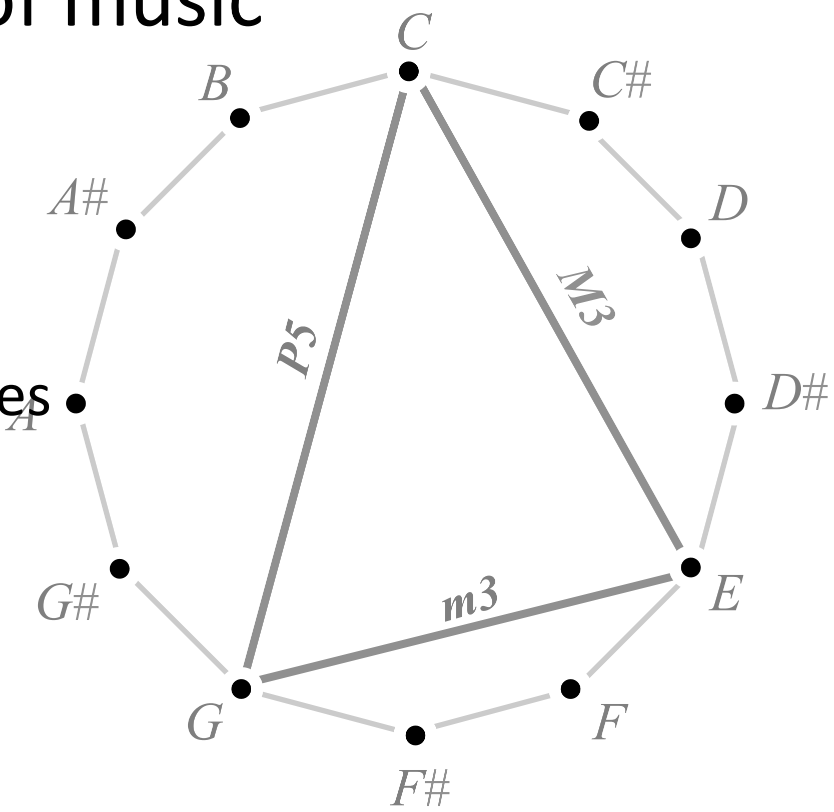



I = { *P1*, *m2*, *M2*, *m3*, *M3*, *P4*, *TT*, *P5*, *m6*, *M6*, *m7*, *M7* }

# Formalization of Notes Neighborhoods

- Which neighborhoods for significant visualization?
- Strong algebraic structure of music
  - Set N = { *C, C#, …, A#, B* }
  - Group (I,+) of intervals
    - Relative difference of two notes
    - (I,+) ≅ ($Z_{12}$,+) (isomorphism)
    - Example

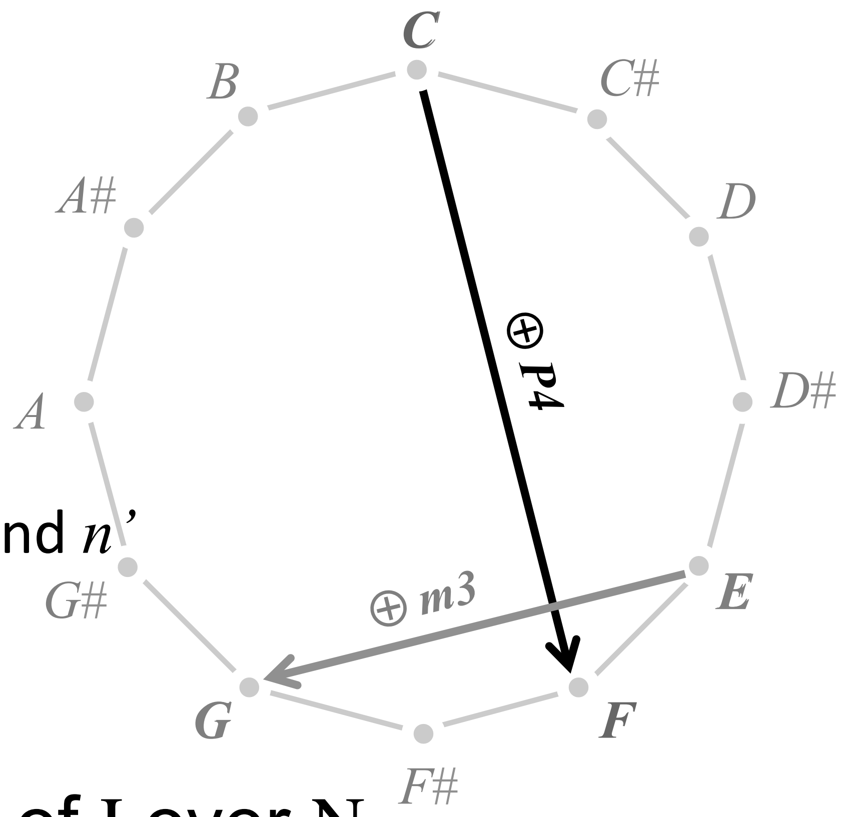
      *M3 + m3 = P5*




I = { *P1, m2, M2, m3, M3, P4, TT, P5, m6, M6, m7, M7* }

# Formalization of Notes Neighborhoods

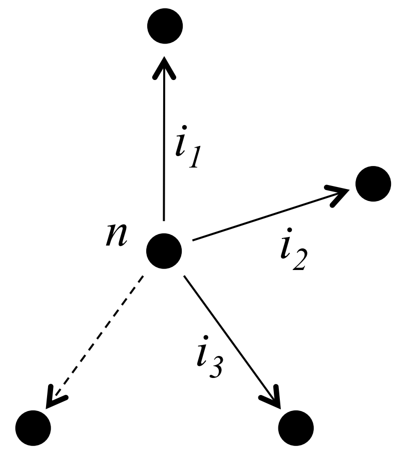
- Which neighborhoods for significant visualization?
- Strong algebraic structure of music
  - Set $N = \{ C, C\#, \ldots, A\#, B \}$
  - Group $(I,+) = \{ P1, \ldots, M7 \}$
  - Transposition $\oplus$ of notes
    - $n \oplus i = n'$
      if $i$ is the interval between $n$ and $n'$
    - Example
      - $C \oplus P4 = F$
      - $E \oplus m3 = G$




$\oplus$ action of I over N

# Formalization of Notes Neighborhoods

- Consonance as a neighborhood relationship
  - $S \subset \mathrm{N} \times \mathrm{N}$
  - $(n_1, n_2) \in S$ if $n_1$ "sounds well" with $n_2$
- Assumptions on $S$
  - $S$ is symmetric
    - $(n_1, n_2) \in S \Rightarrow (n_2, n_1) \in S$
  - $S$ is defined up to a transposition
    - $\forall i \in \mathrm{I}, \quad (n_1, n_2) \in S \;\Rightarrow\; (n_1 \oplus i, n_2 \oplus i) \in \mathrm{S}$
    - $(C, G)$ sounds well $\Rightarrow$ $(E = C \oplus M3, B = G \oplus M3)$ sounds well
- $S$ characterized by a subset $I$ of $\mathrm{I}$
  - $\mathrm{I} = \{ i_1, i_2, \ldots, i_n \}$
  - $\forall n \in \mathrm{N}, \forall \mathrm{i} \in I, \quad (n, n \oplus i) \in S$

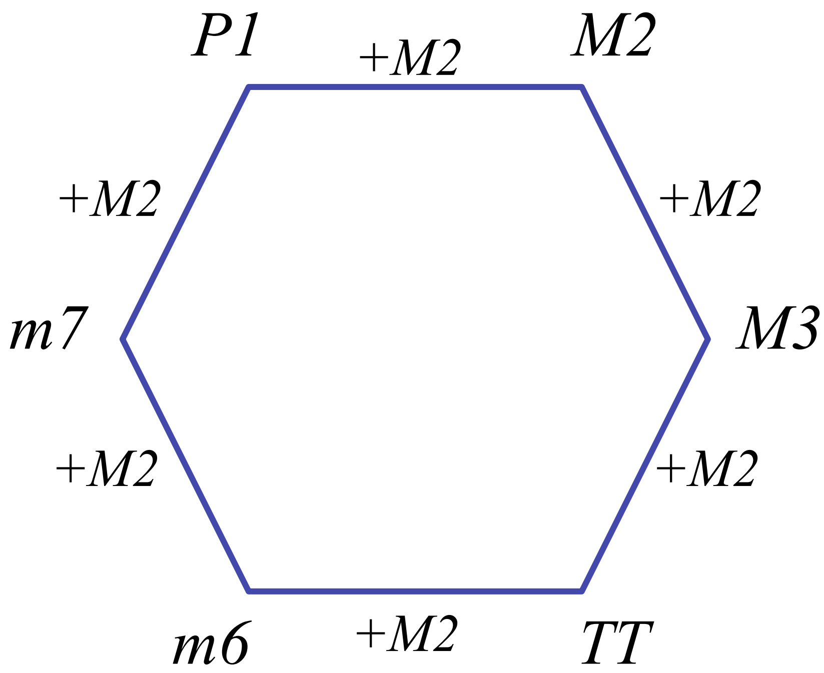


# Formalization of Notes Neighborhoods

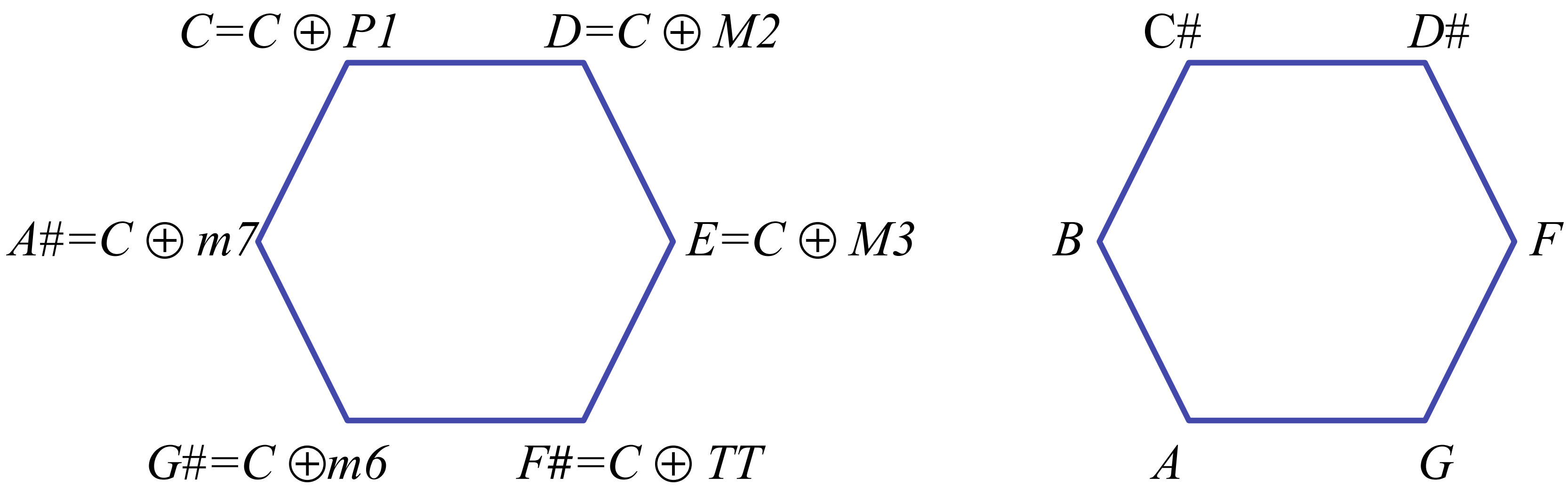
- Spatial representation of $S$
  - $I$ as a set of group generators
    - $< I >$ subgroup of I generated by the elements of $I$
    - Example with $I = \{ M2 \}$

$$
\begin{aligned}
< I > \quad &= \{ P1, P1 + M2, P1 + 2.M2, \ldots \} \\
&= \{ P1, M2, M3, TT, m6, m7 \}
\end{aligned}
$$

- Spatial representation of $S$
  - $I$ as a set of group generators
  - Graph representation of $< I >$
    - Cayley's graph
      - Vertices: intervals of $< I >$
      - Edges: generators of $I$
    - Example with $I = \{ M2 \}$

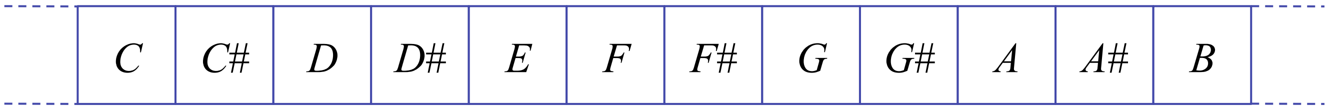


- Spatial representation of $S$
  - $I$ as a set of group generators
  - Graph representation of $< I >$
  - Representation of $S$ based on Cayley graph
    - Action of $< I >$ on $\mathrm{N}$
    - Example with $I = \{ M2 \}$

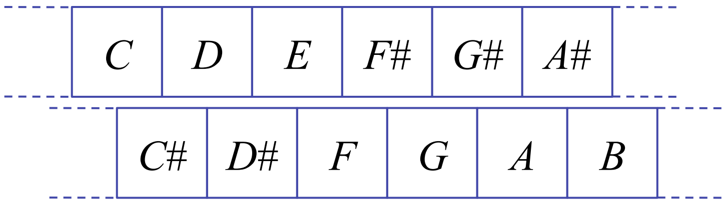


# Applications

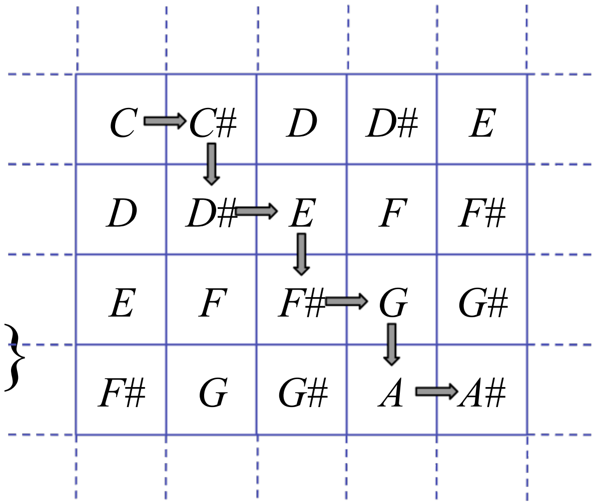
- Scale representations
  - Chromatic scale $I = \{ m2 \}$




  - Whole-tone scale $I = \{ M2 \}$

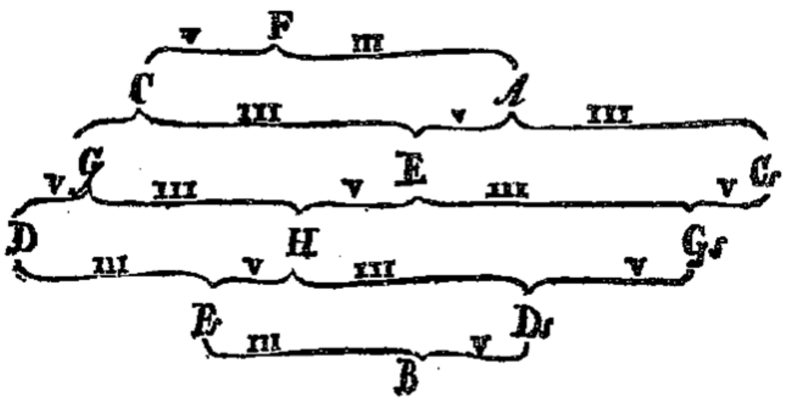


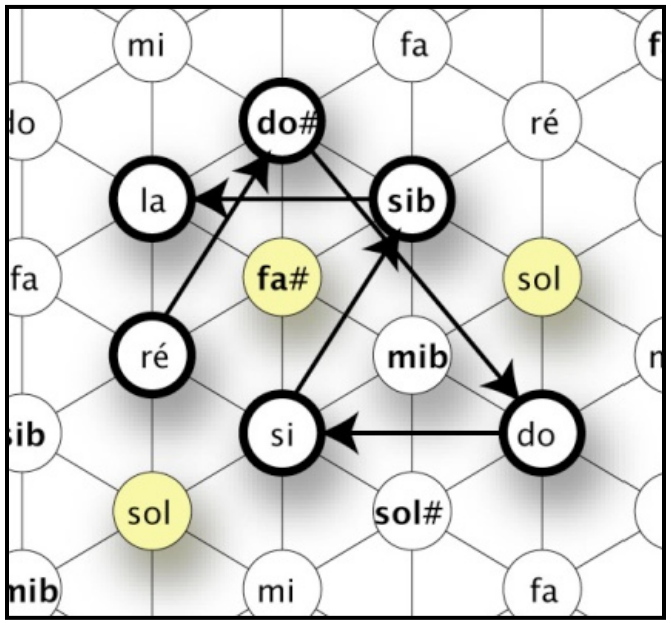

  - Diminished scale $I = \{ m2, M2 \}$



# Applications

- Traditional harmony representation
  - $I = \{ M3, P5 \}$ (Euler's Tonnetz)
  - $I = \{ m3, M3, P5 \}$ (Harmonic table)




(J.-M. Chouvel)

- Instruments conception

$I = \{ m2, P4 \}$ (Guitar)

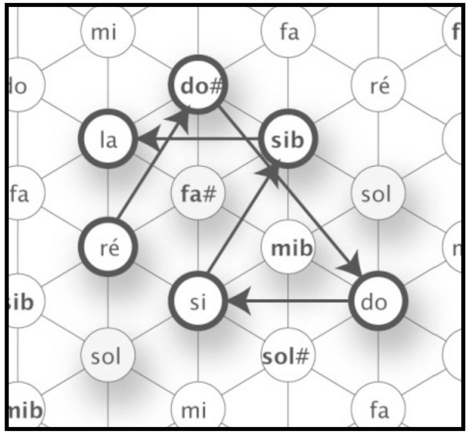
$I = \{ m2, P5 \}$ (Violin)


$I = \{ m2, M2, m3 \}$ (Accordion)

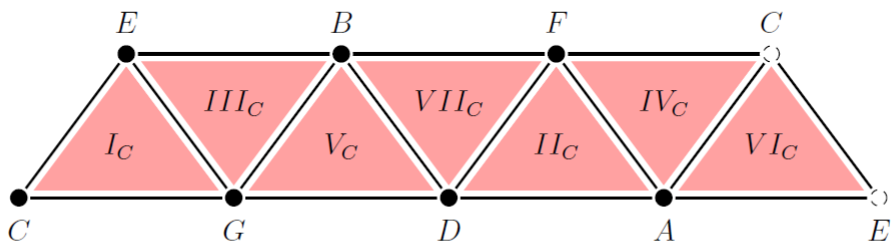
- Analysis example
  - Signature of a piece
  - Example : F. Chopin Prelude

Extract of the 2nd movement of the Symphony No. 9
L. van Beethoven

Extract of the Prelude N.4 Op28 of F. Chopin

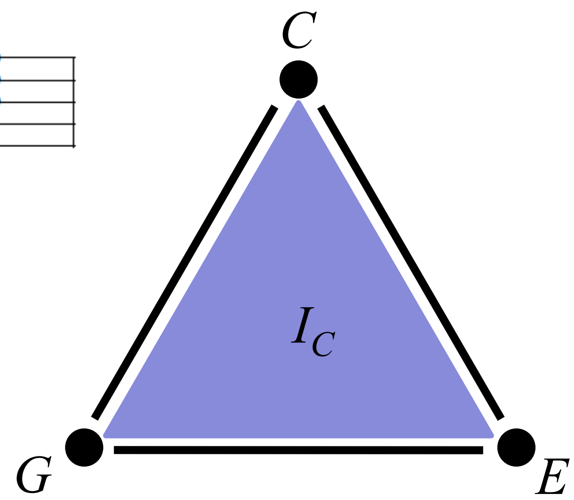



Music in a space

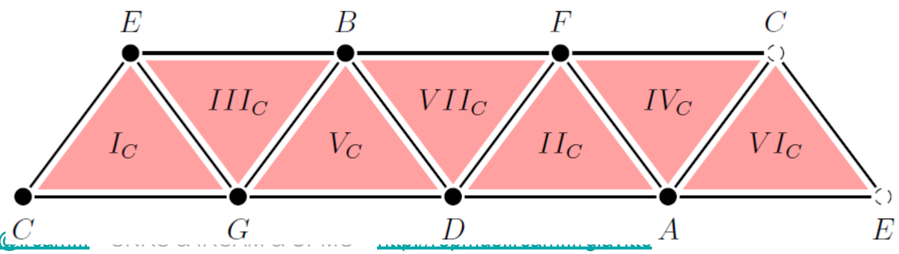


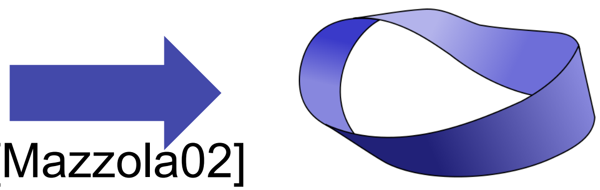

**Computing the space of music**

- Motivation: spatial visualization of tonality
- Association of a chord set with the tonality: the *degrees*
  - Example: 



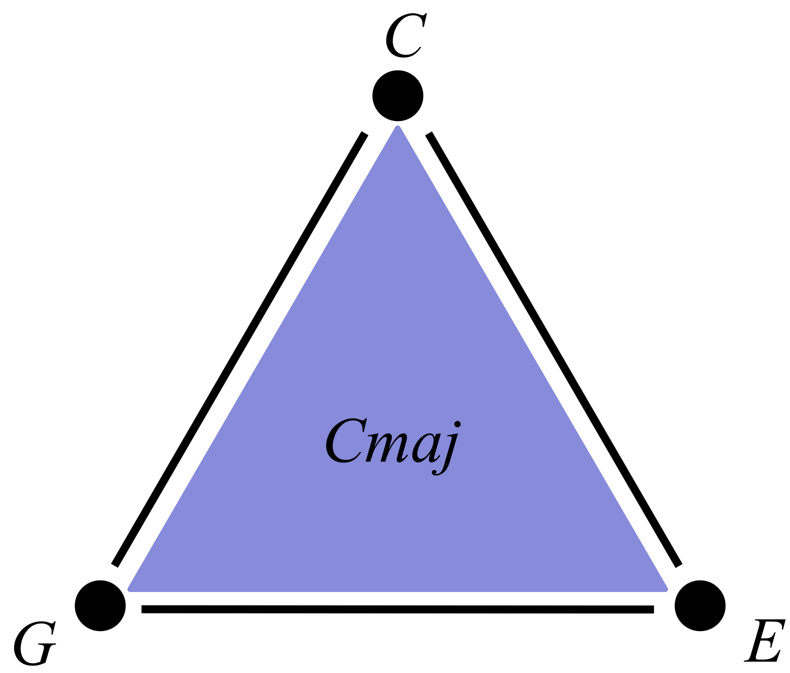

- Spatial representations
  - Note = vertex
  - Chord = surface
- Fusion of the common notes for the 7 degrees



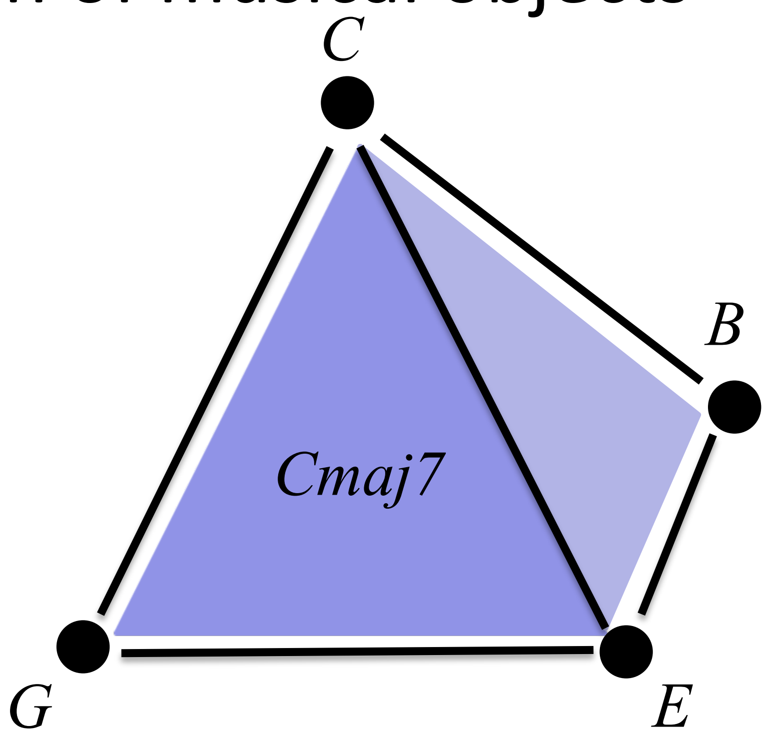



[Mazzola02]

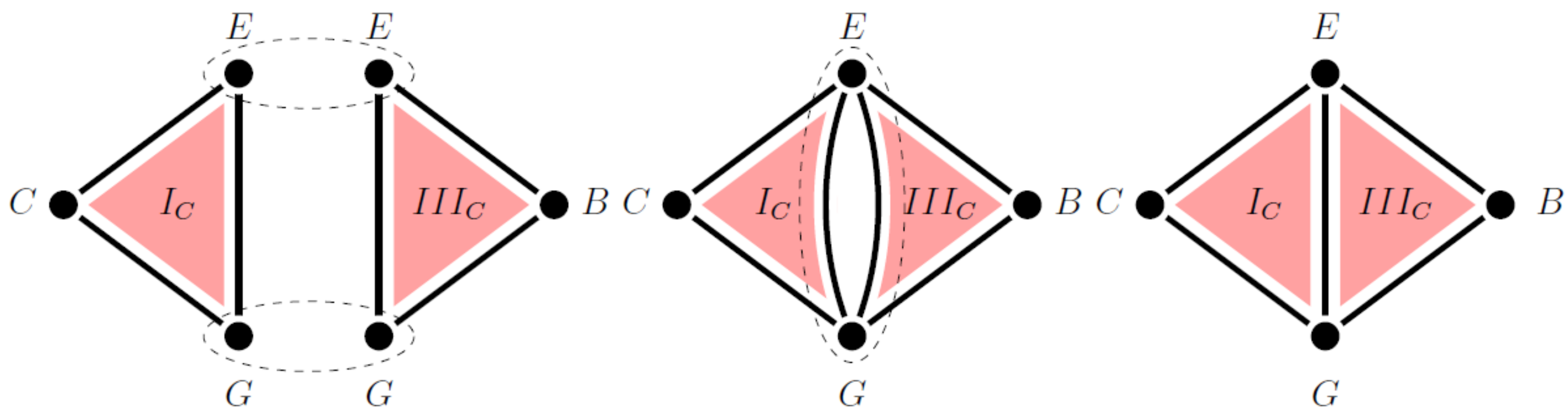
- Automation of the process for the analysis of other chords sequences
- *Reaction* of the chords between themselves
- Simplicial representation of musical objects
  - Note: 0-simplex
  - 2-note chord: 1-simplex
  - 3-note chord: 2-simplex



- Automation of the process for the analysis of other chords sequences
- *Reaction* of the chords between themselves
- Simplicial representation of musical objects
  - Note: 0-simplex
  - 2-note chord: 1-simplex
  - 3-note chord: 2-simplex
  - 4-note chord: 3-simplex



- MGS transformation for self-assembly process

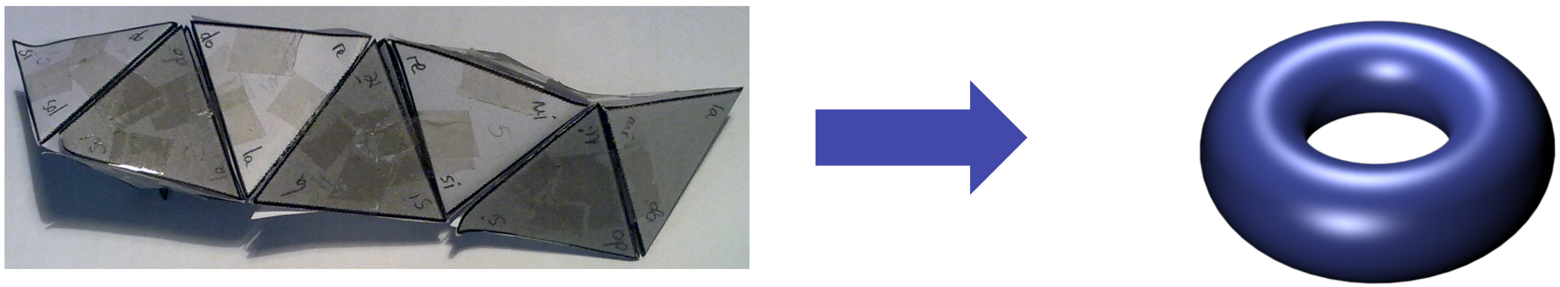



```
Trans identification = {
  s1 s2 / (s1 == s2 & (faces s1) == (faces s2))
    =>
      let c = new_cell (dim s1)
                       (faces s1)
                       (union (cofaces s1)
                              (cofaces s2))
      in s1 * c
}
```

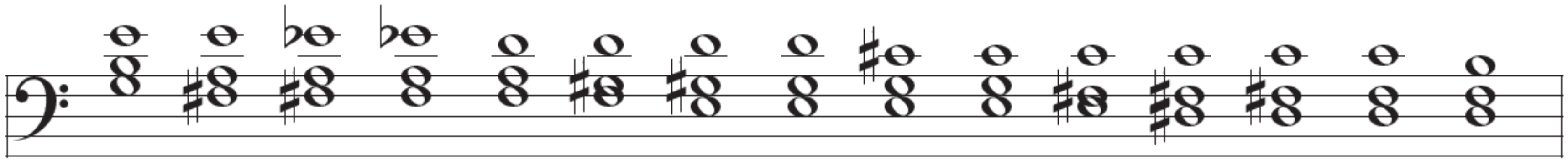
# Applications

- Four-note degrees of C-major tonality

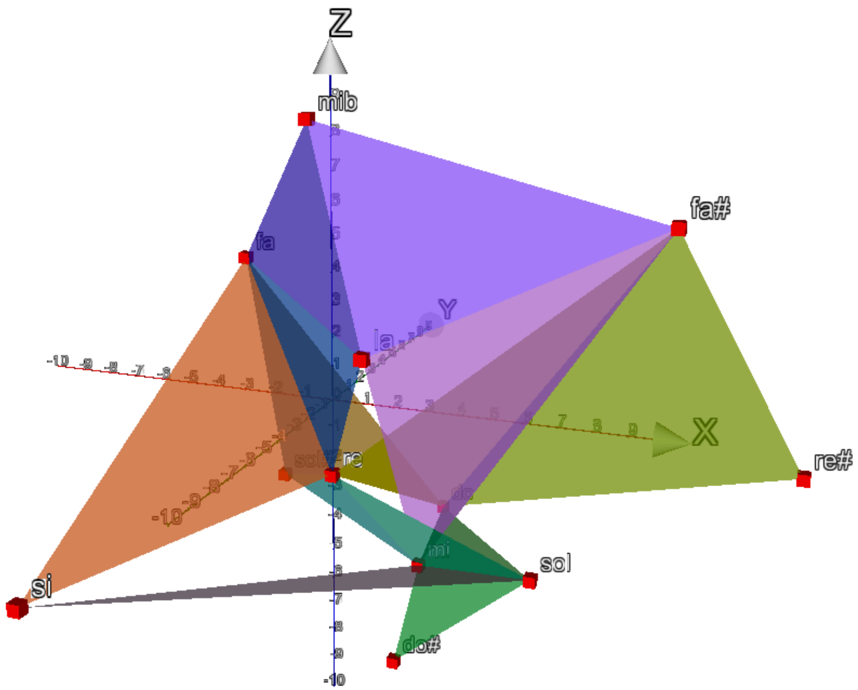
- Chord = 3-simplex (tetrahedrons)
- Self-assembly



- Extract of the Prelude No. 4 Op. 28 of F. Chopin



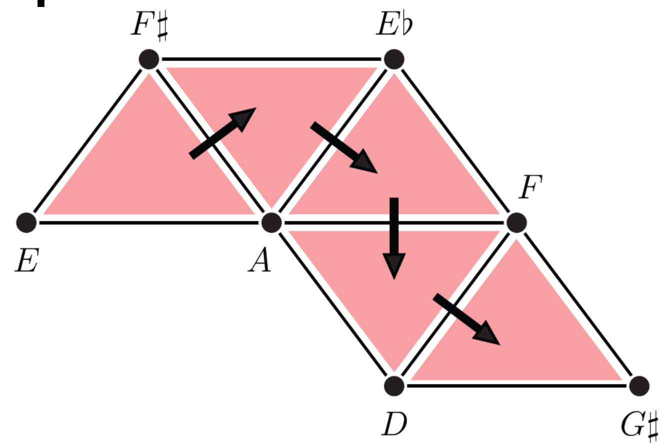
- Simplicial complex



## Applications

- Extract of the Prelude No. 4 Op. 28 of F. Chopin

- Analysis of the path under the chords




- The path chosen by F. Chopin is associated with the smallest movements on the chords

## Music & spatial computing ?

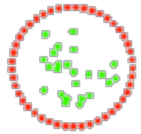
- Preliminary work
- Strong collaborations with composers / musicologists
- Extend the validation on more musical problems
- Extension to study musical styles
- Spatial properties ⇔ musical properties
- Acknowledgements

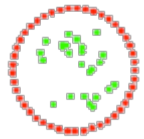
Louis Bigo, Antoine Spicher,
Moreno Andreatta, Carlos Agon,
Jean-Marc Chouvel

# Thanks

- **Antoine Spicher**
- **Olivier Michel**

http://mgs.spatial-computing.org

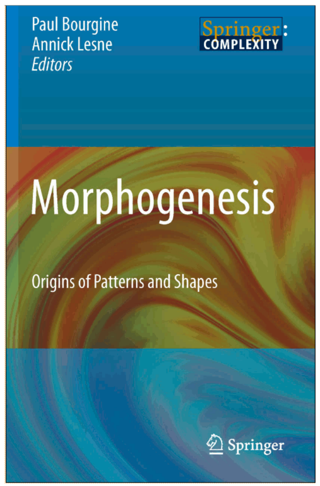


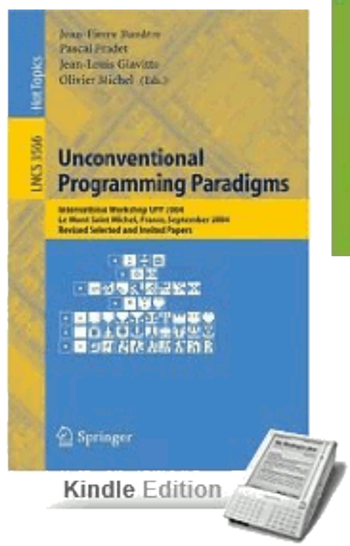


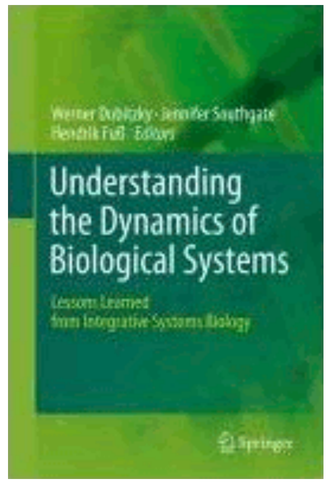
- **PhD and other students**

  **Louis Bigo**
  J. Cohen, P. Barbier de Reuille,
  E. Delsinne, V. Larue, F. Letierce, B. Calvez,
  F. Thonerieux, D. Boussié *and the others...*

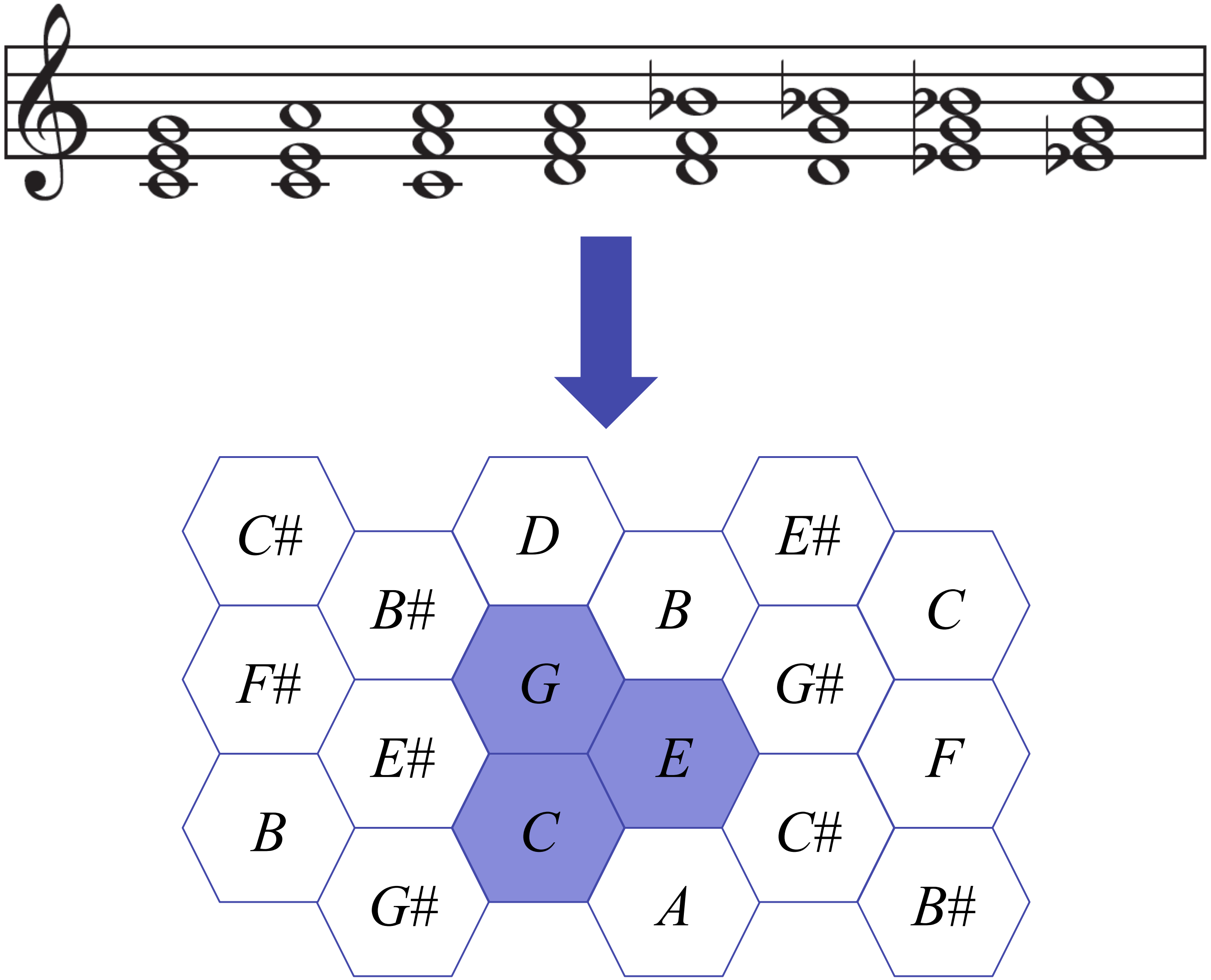
- **Past and presents Collaborations**
  - A. Lesne (IHES, stochastic simulation)
  - P. Prusinkiewicz (UoC, declarative modeling)
  - P. Barbier de Reuille (meristeme model)
  - C. Godin (CIRAD, biological modeling)
  - H. Berry (INRIA, stochastic simulation)
  - G. Malcolm (Liverpool, rewriting)
  - J.-P. Banâtre (IRISA, programming)
  - F. Delaplace (IBISC, synthetic biology)
  - P. Dittrich (Jena, chemical organization)
  - F. Gruau (LRI, language and hardware)
  - P. Liehnard (Poitier, CAD, Gmap and quasi-manifold)



# Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



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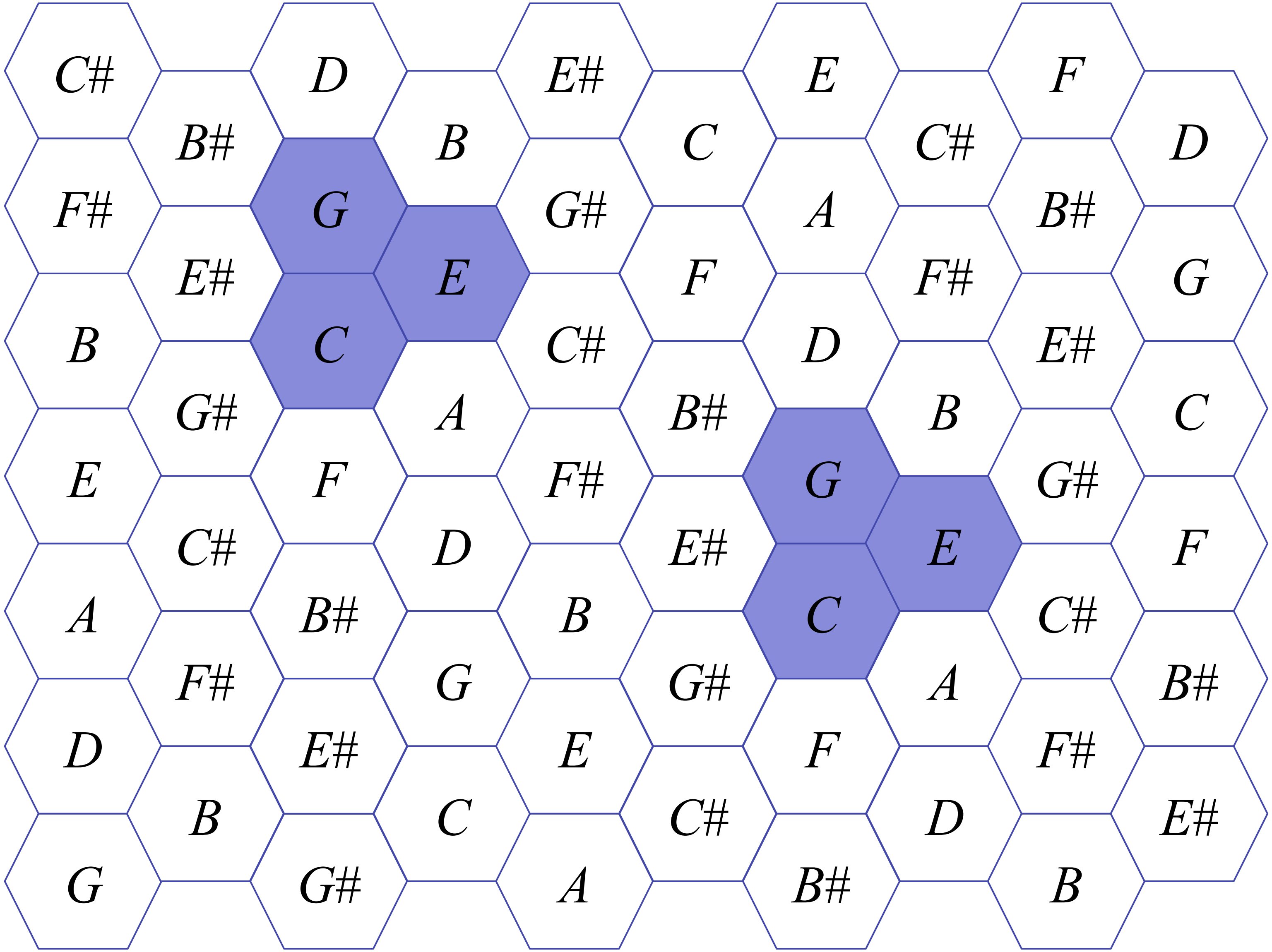

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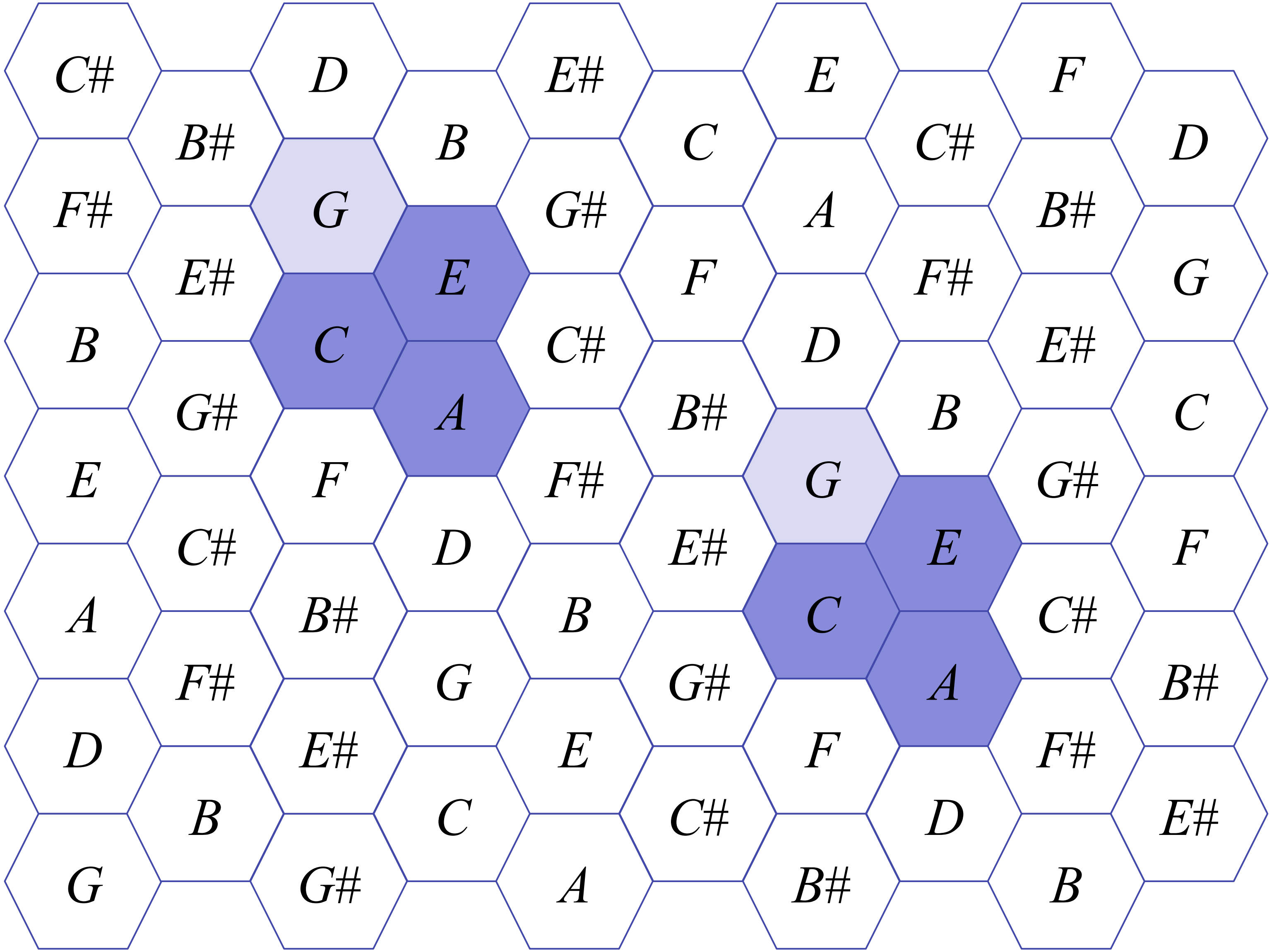

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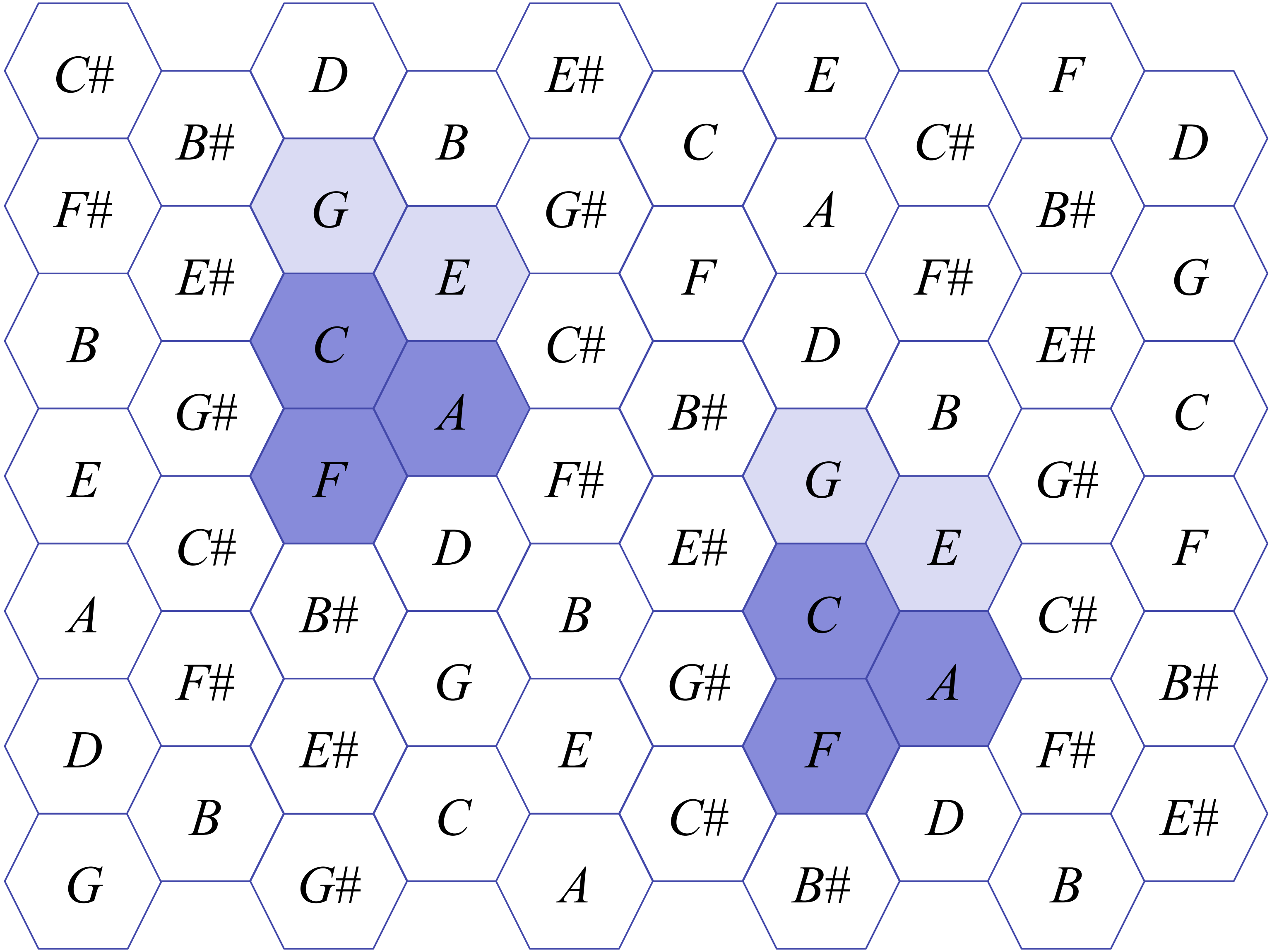


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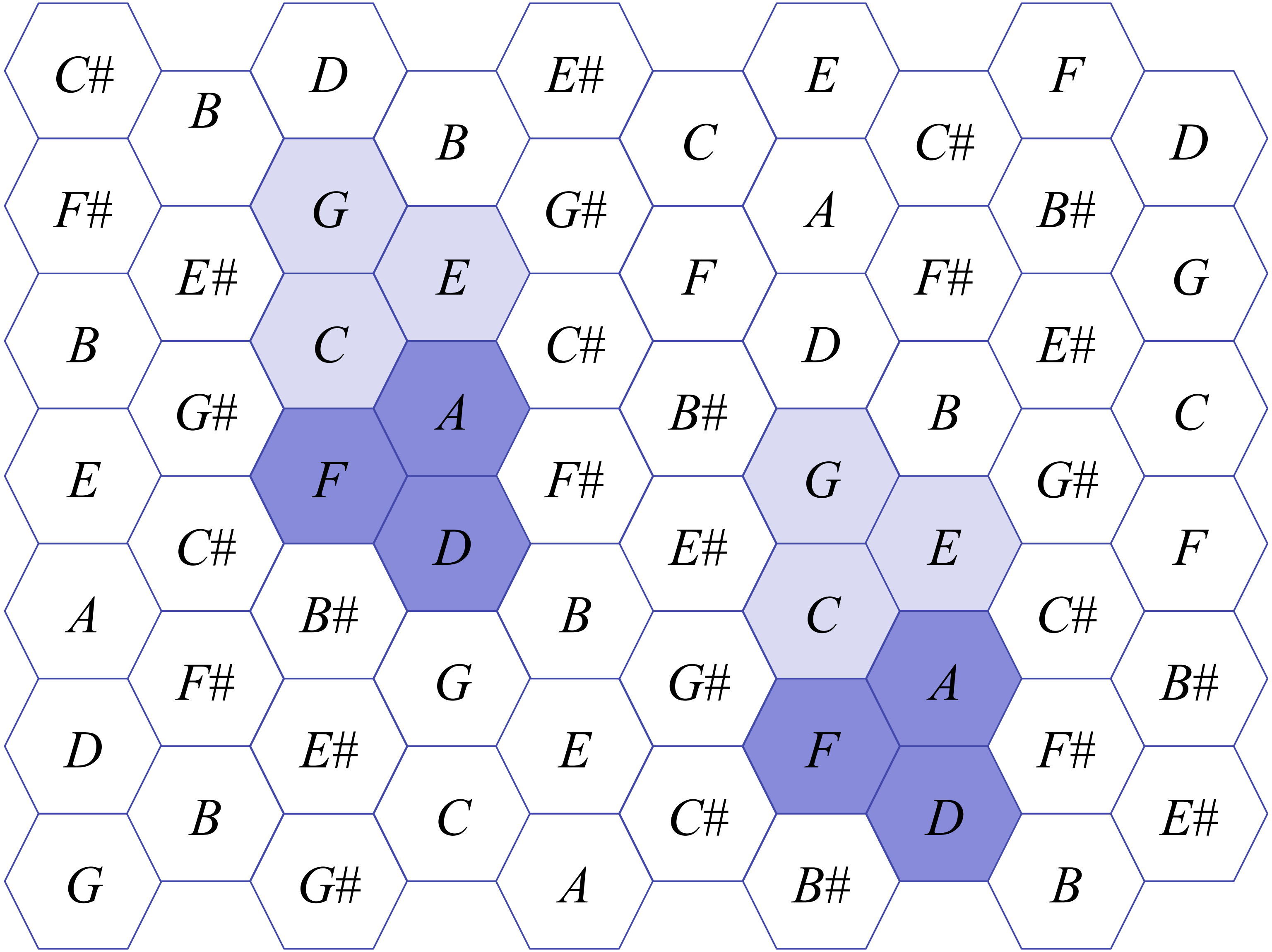

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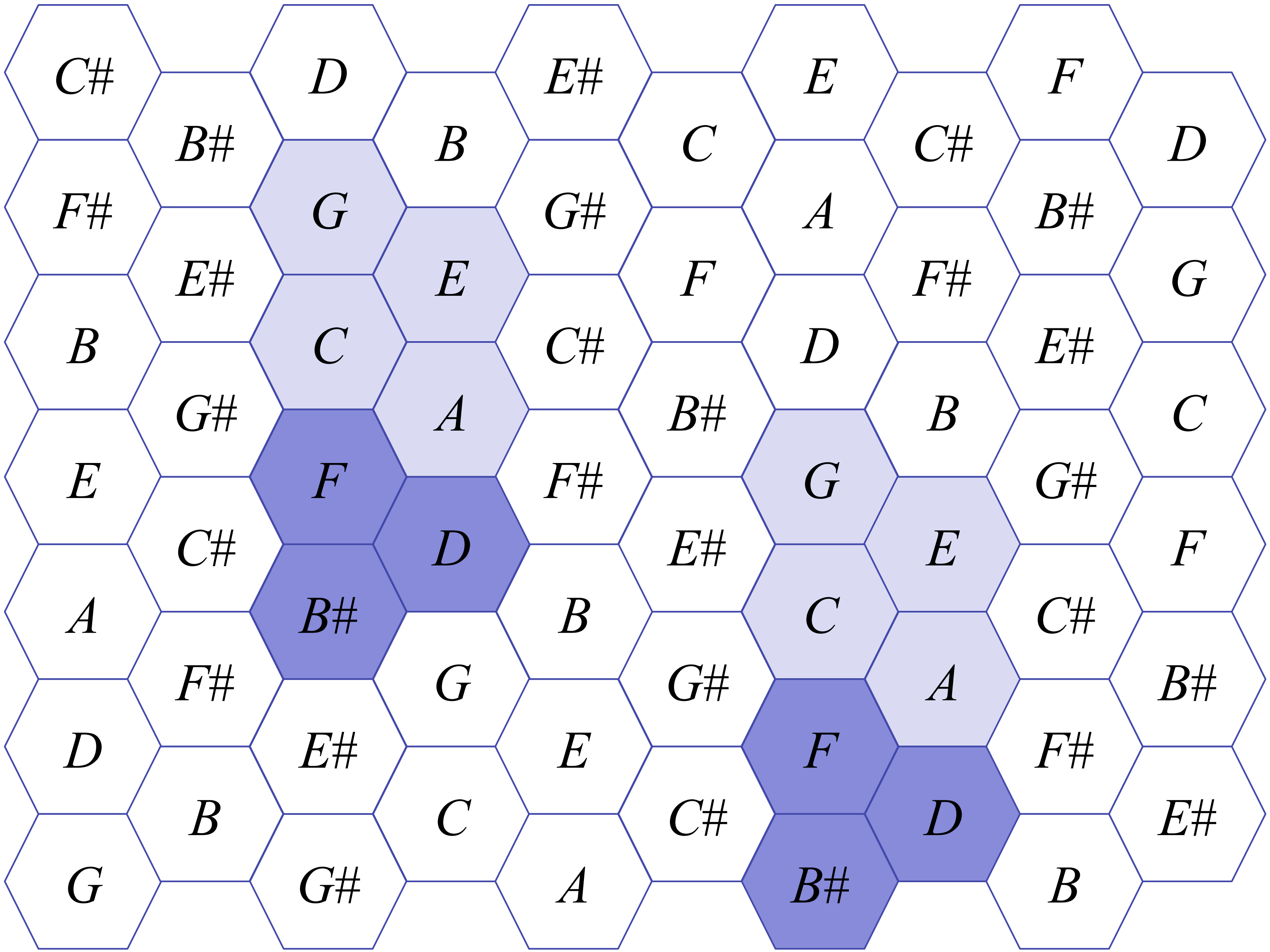

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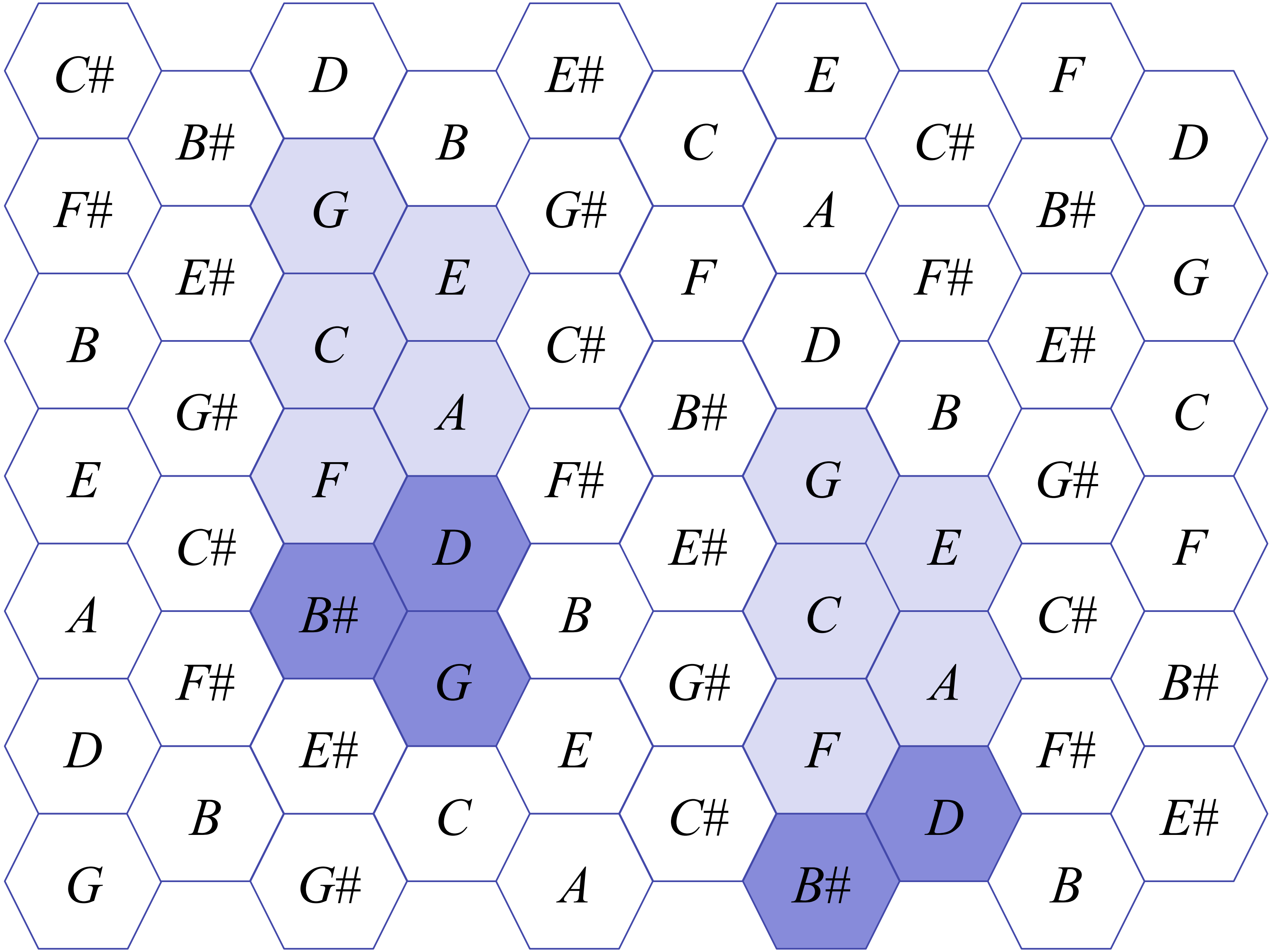


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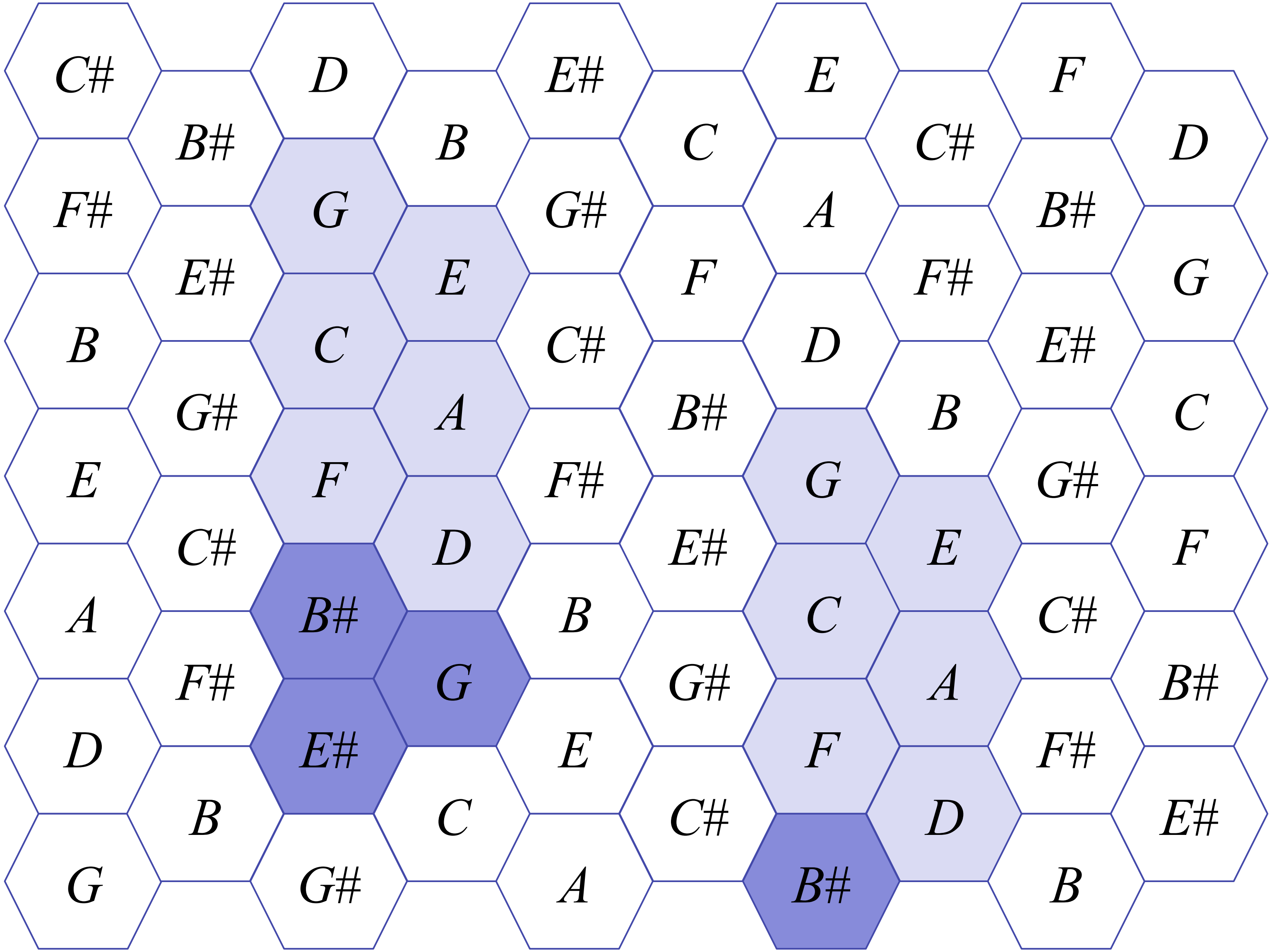

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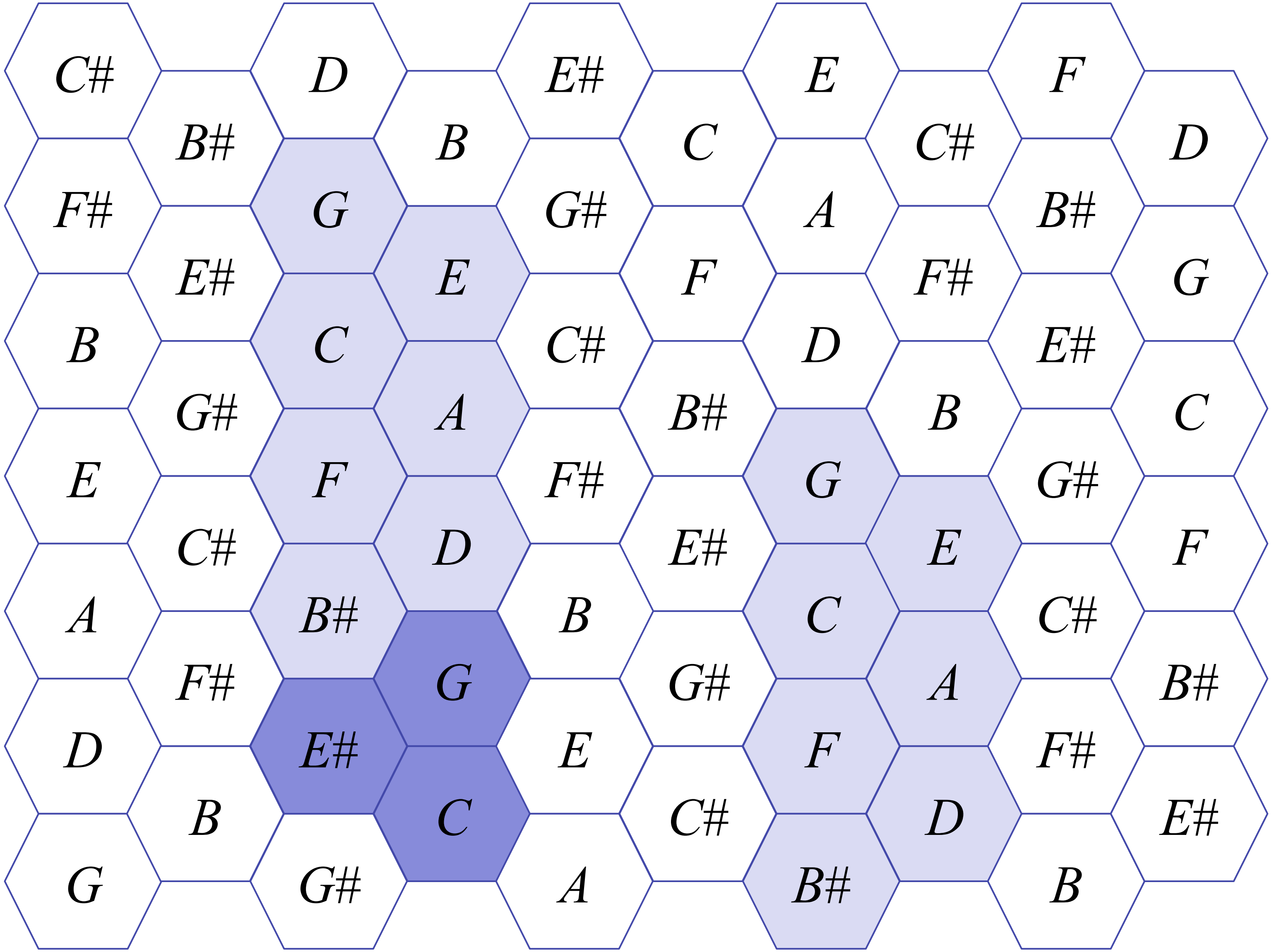


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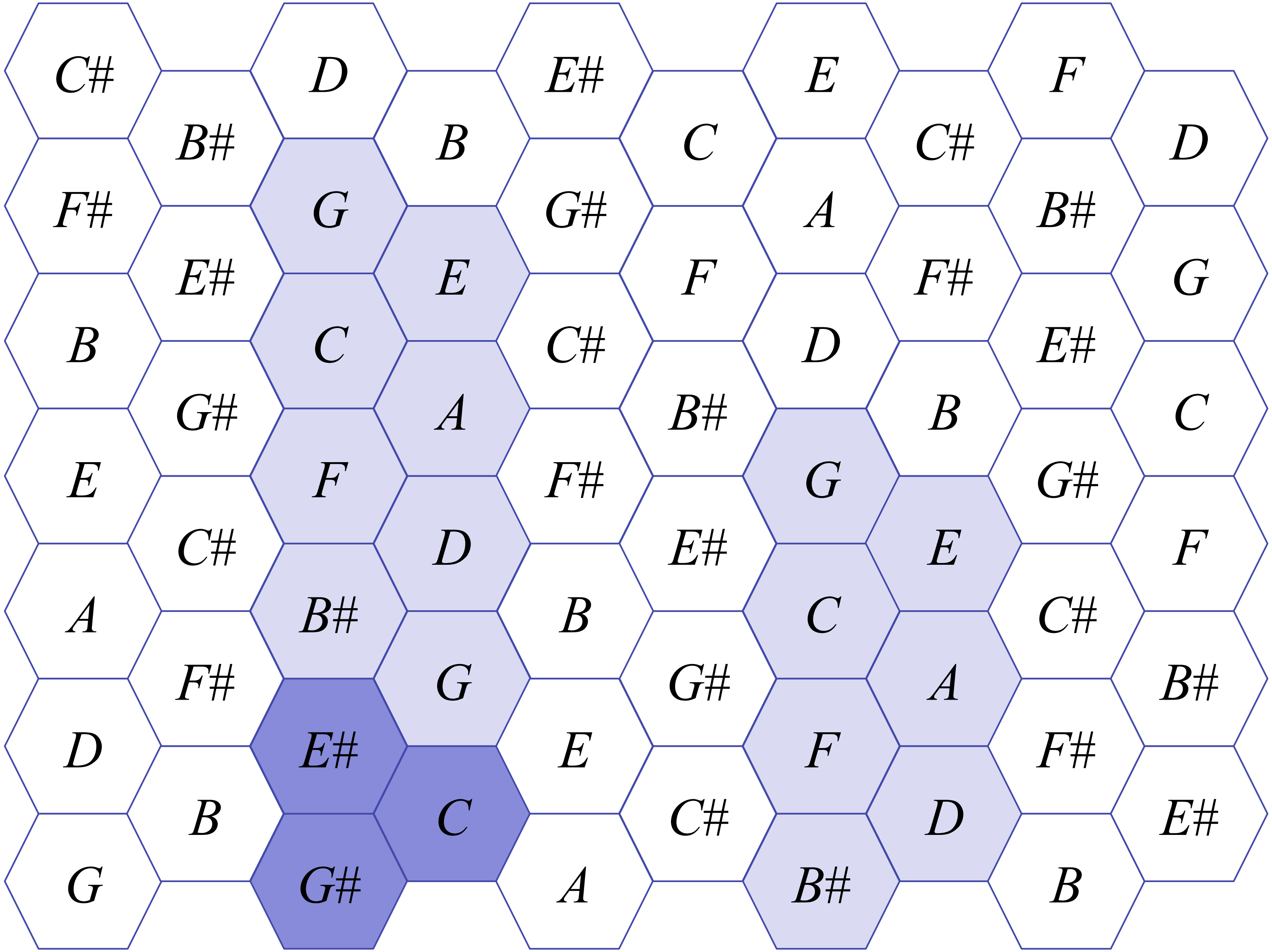
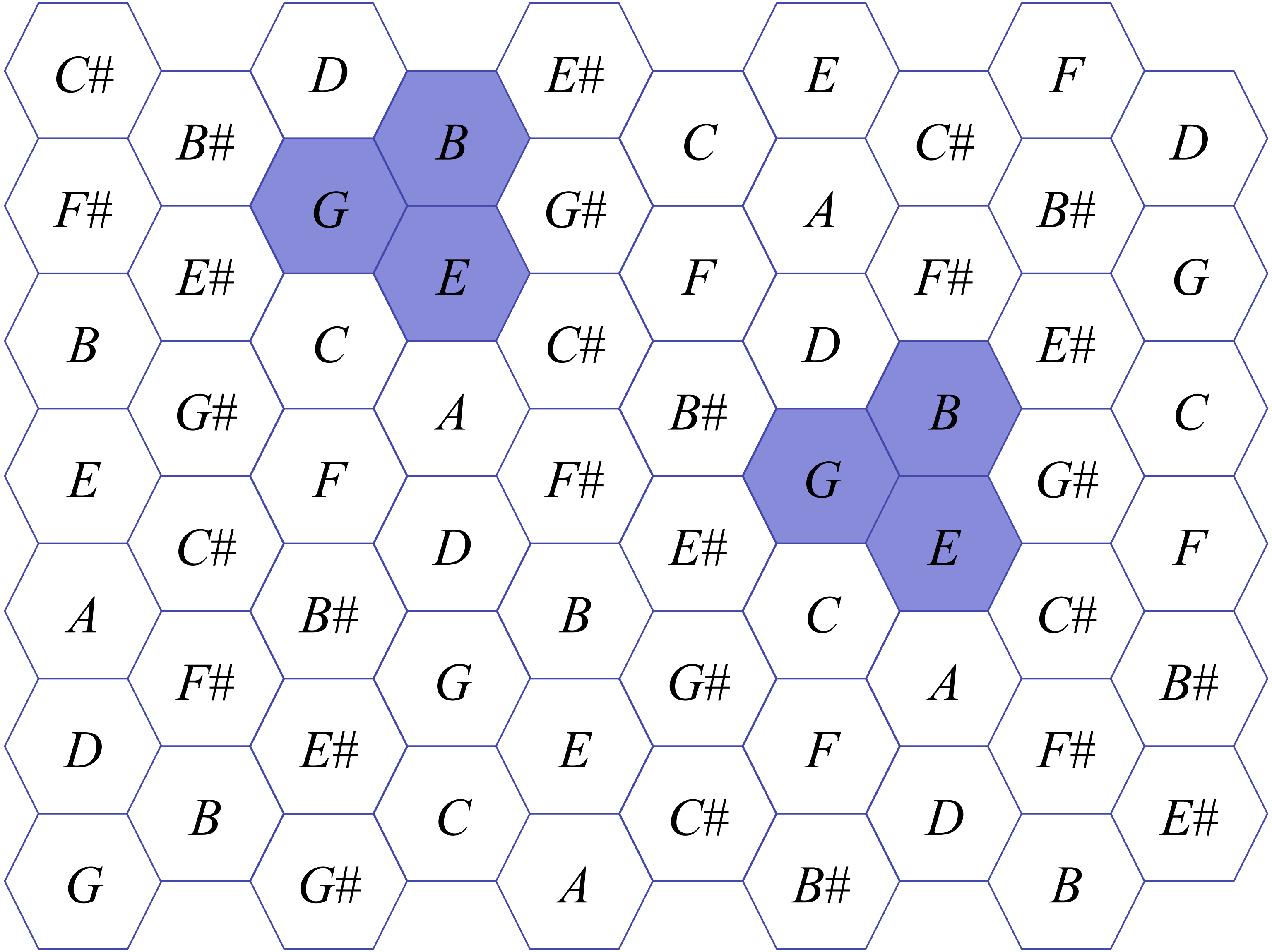


# Extract of the Prelude Op.28 N.4 (F. Chopin)

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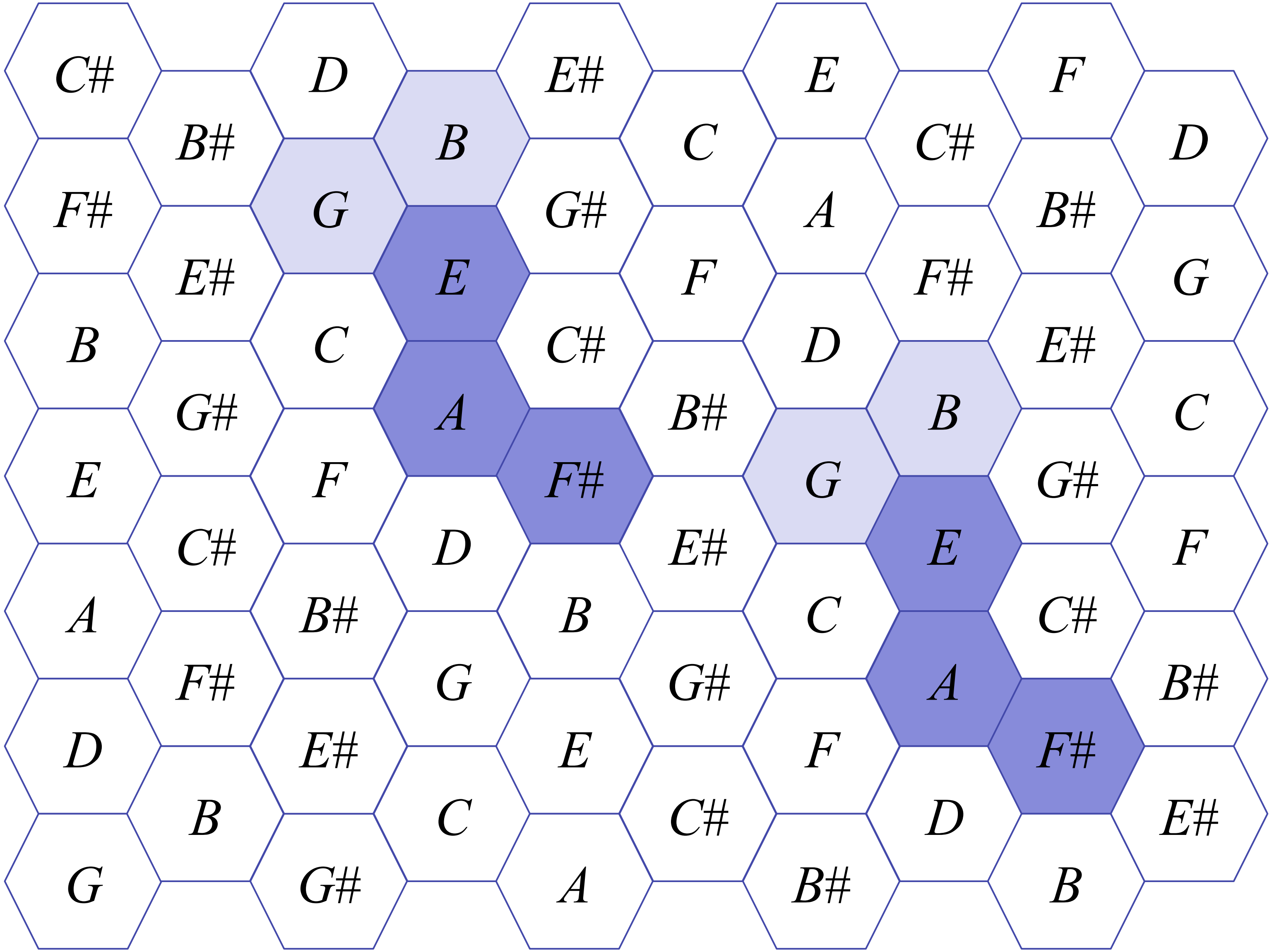
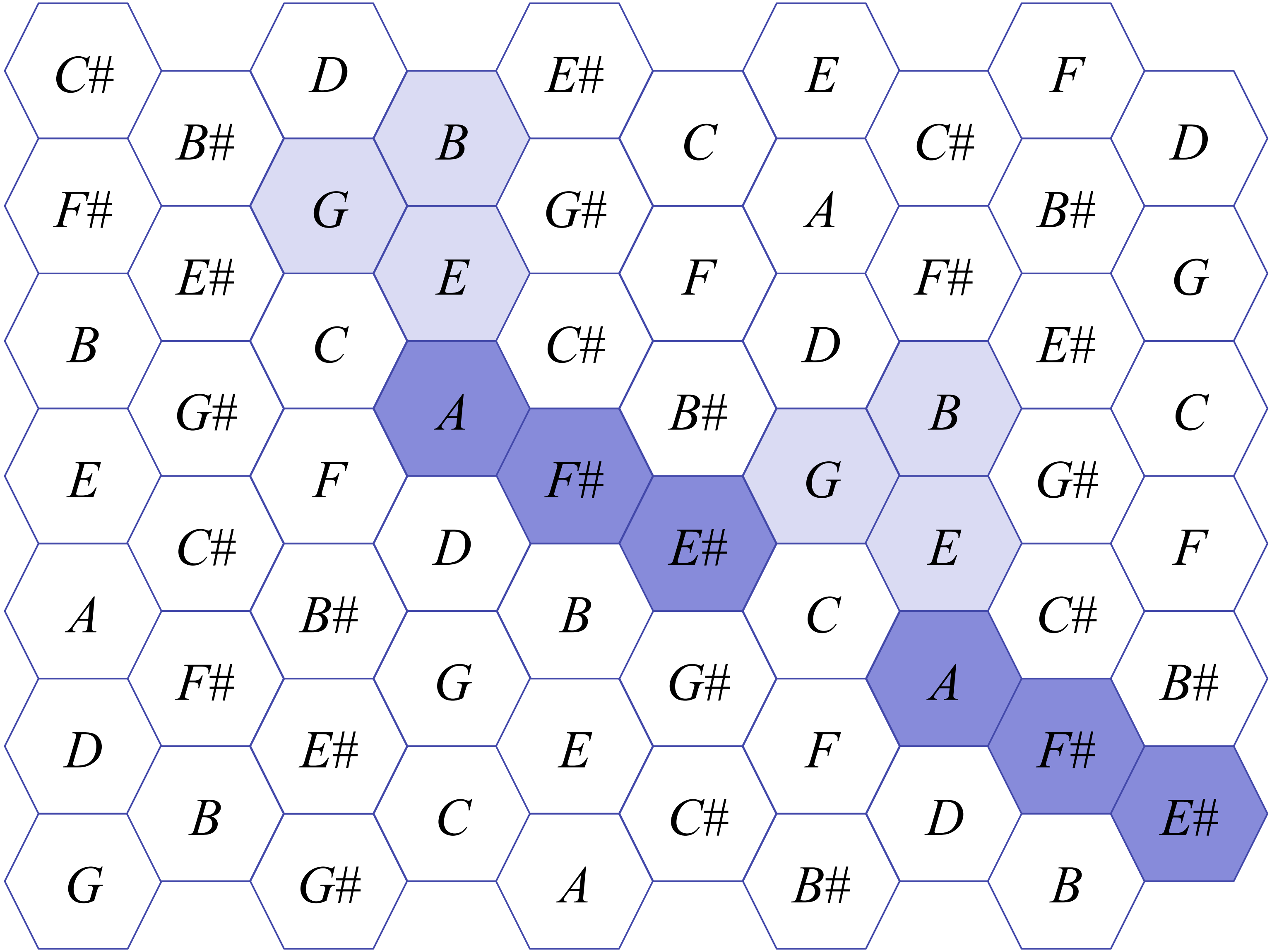
# Extract of the Prelude Op.28 N.4
# (F. Chopin)



# Extract of the Prelude Op.28 N.4
# (F. Chopin)



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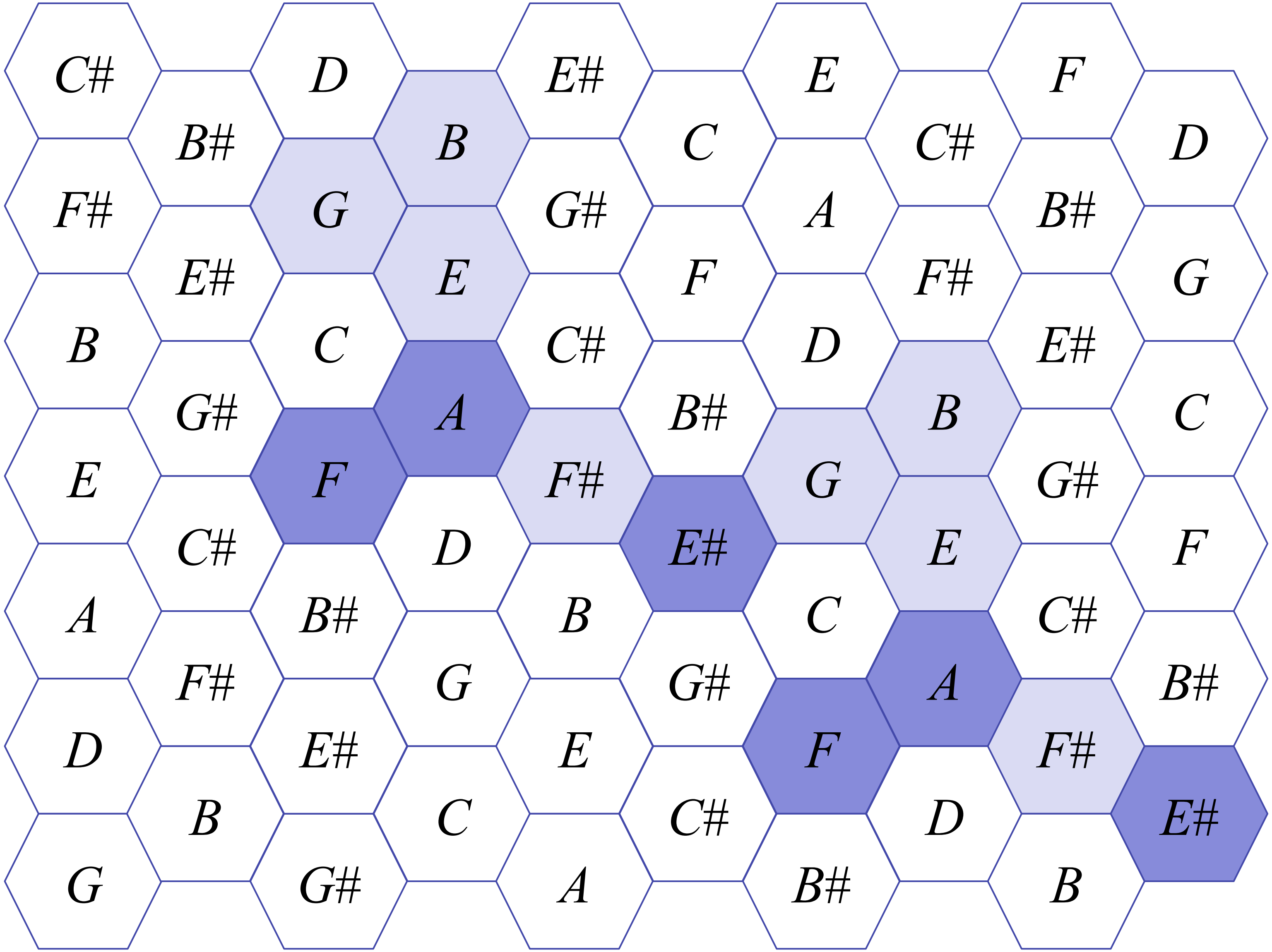


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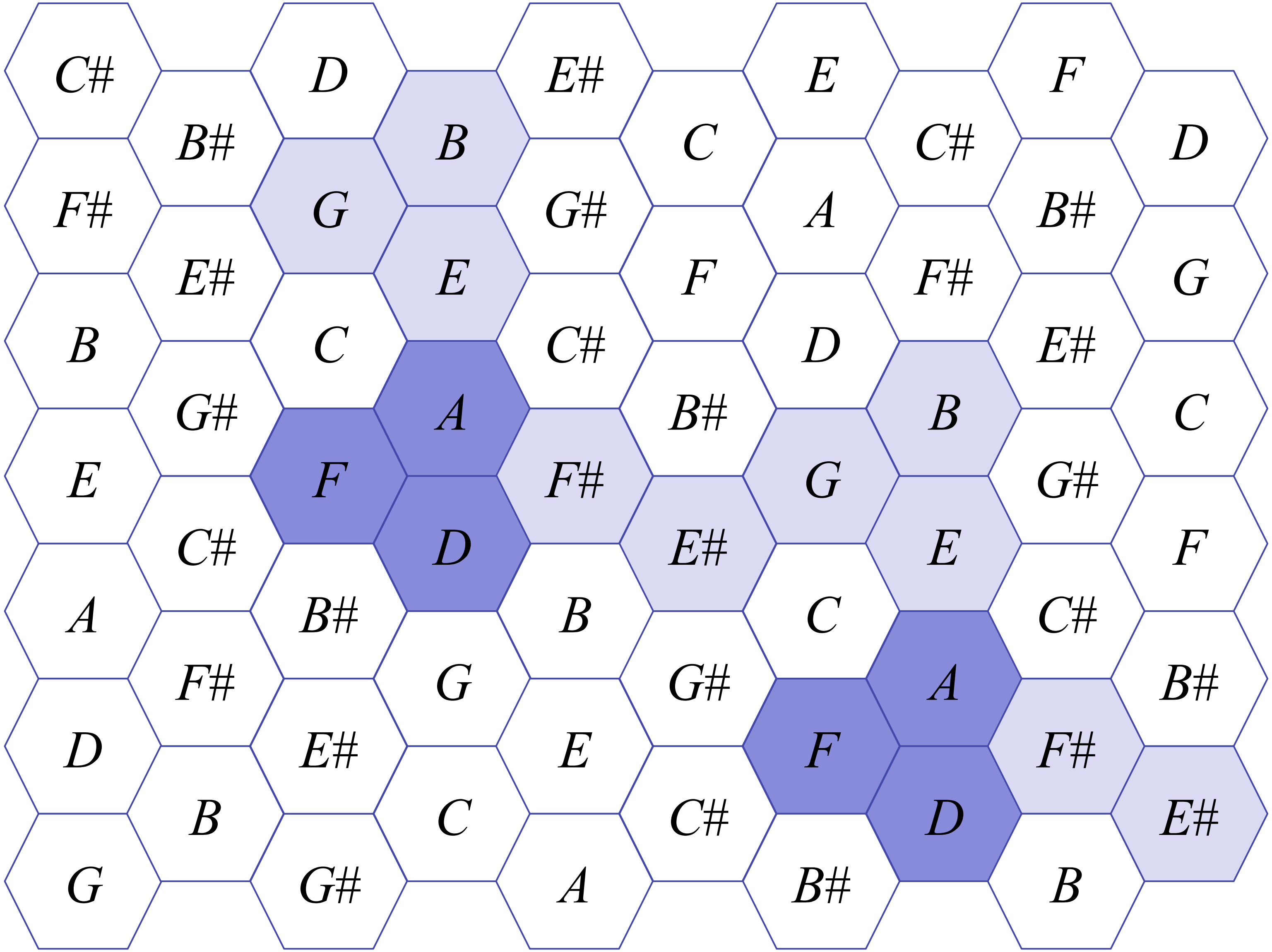

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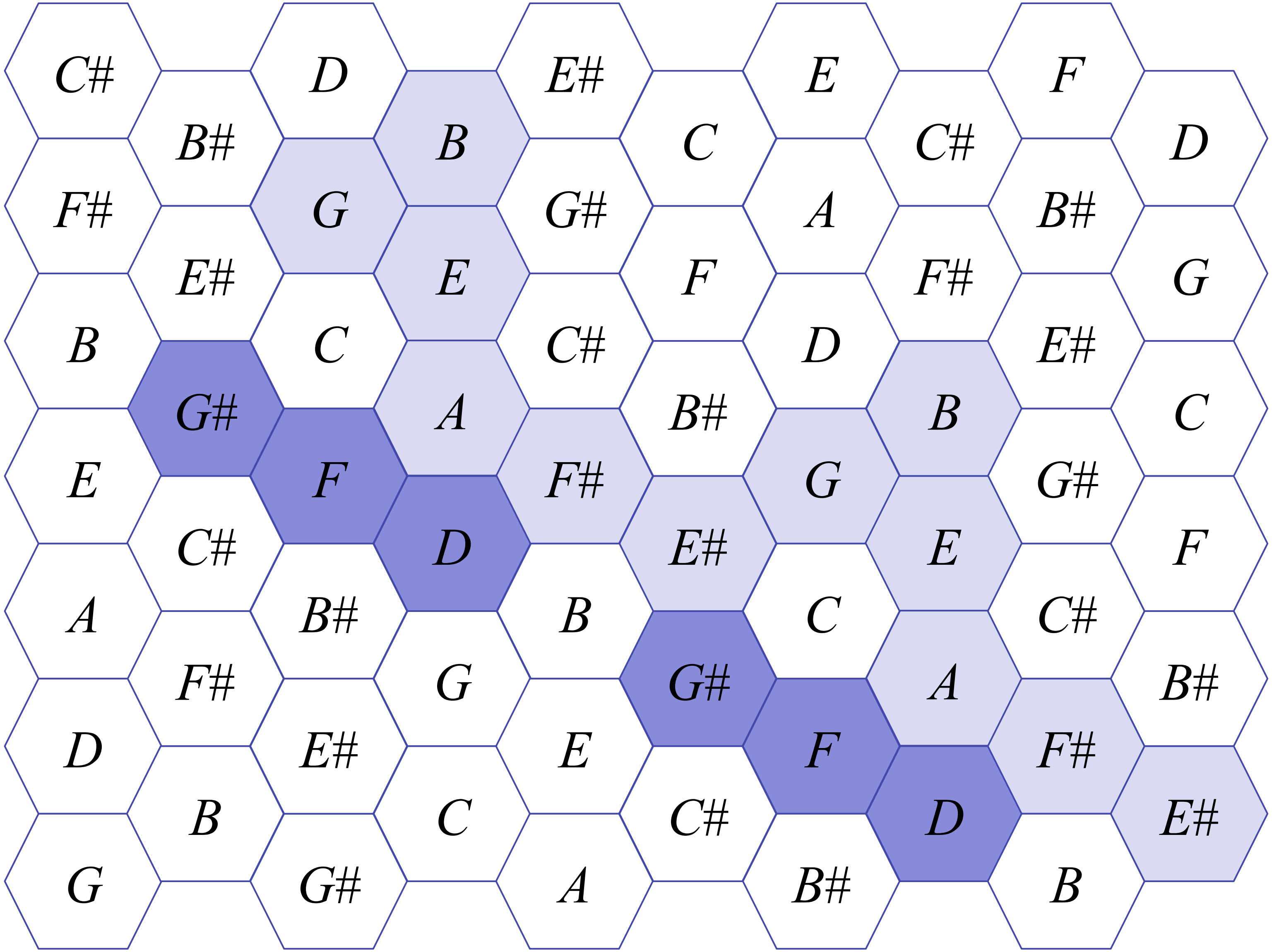

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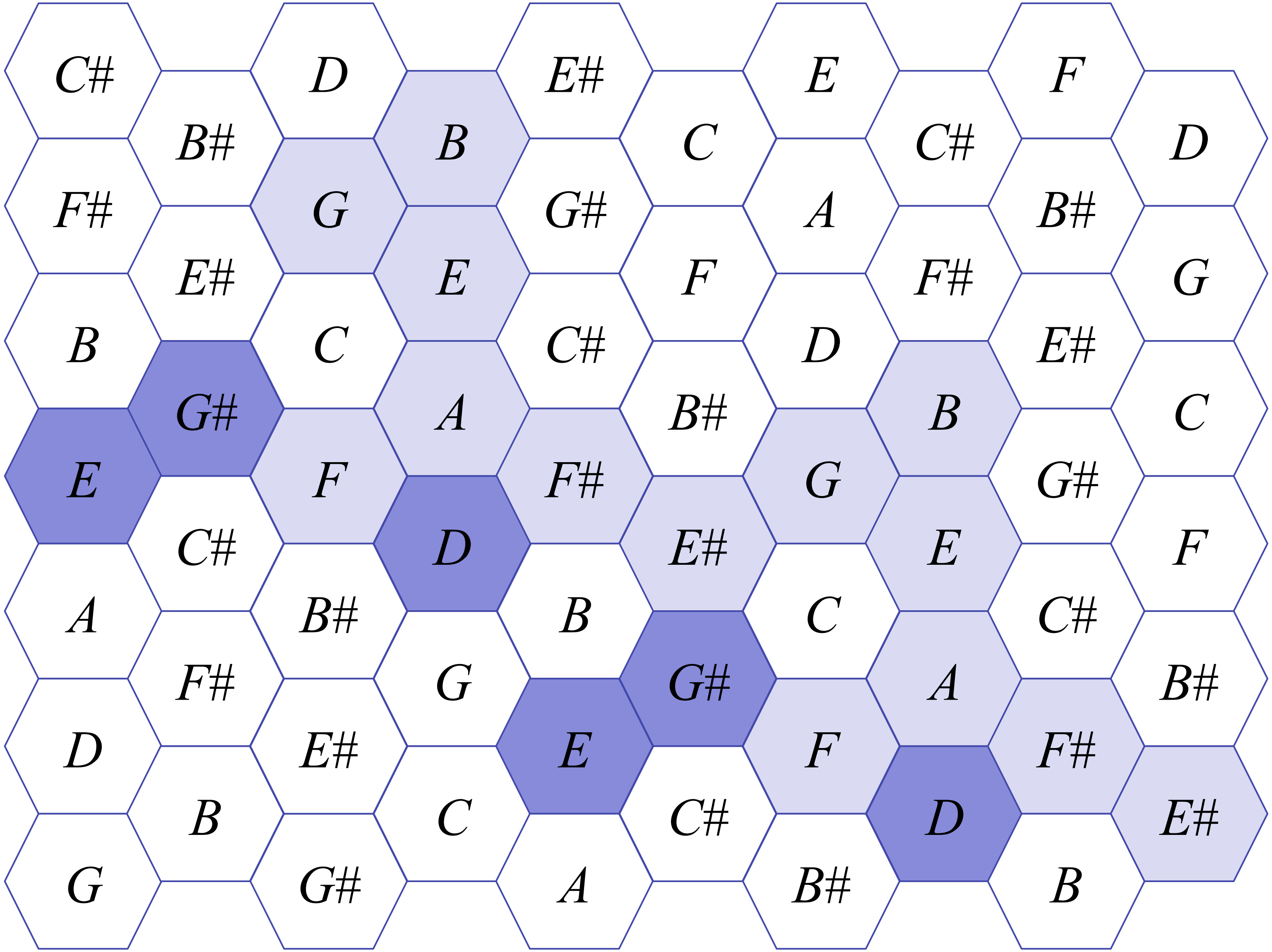


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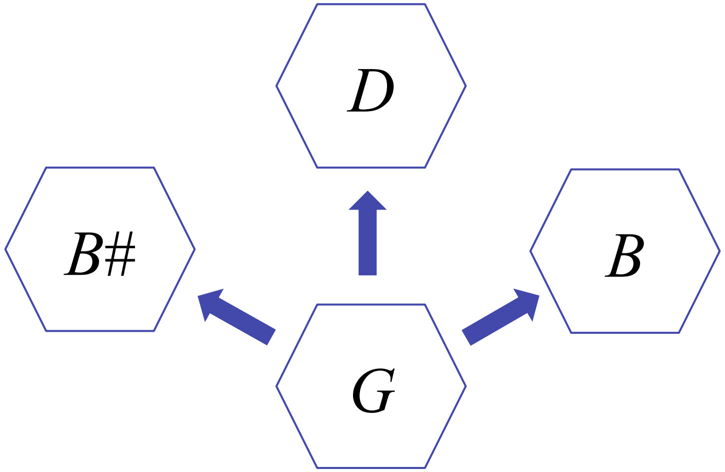
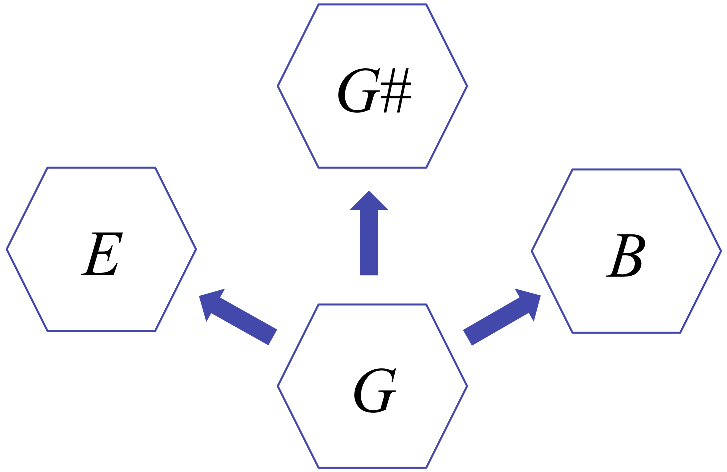
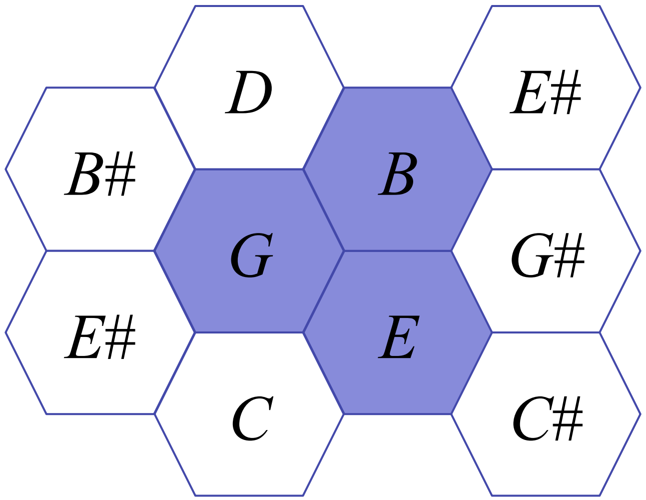
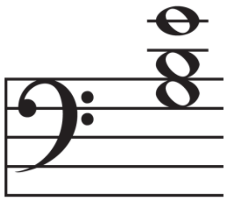


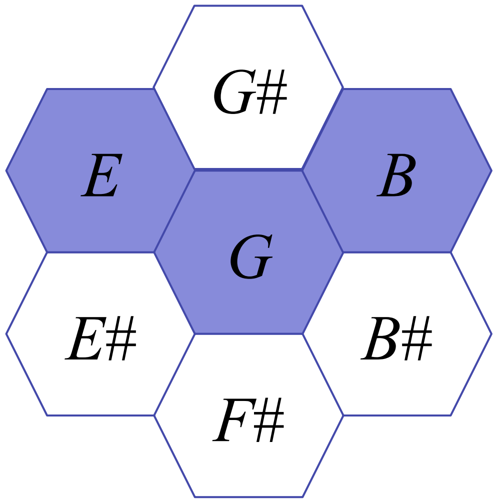

$I = \{m3, M3, P5\}$




$I = \{m2, m3, M3\}$





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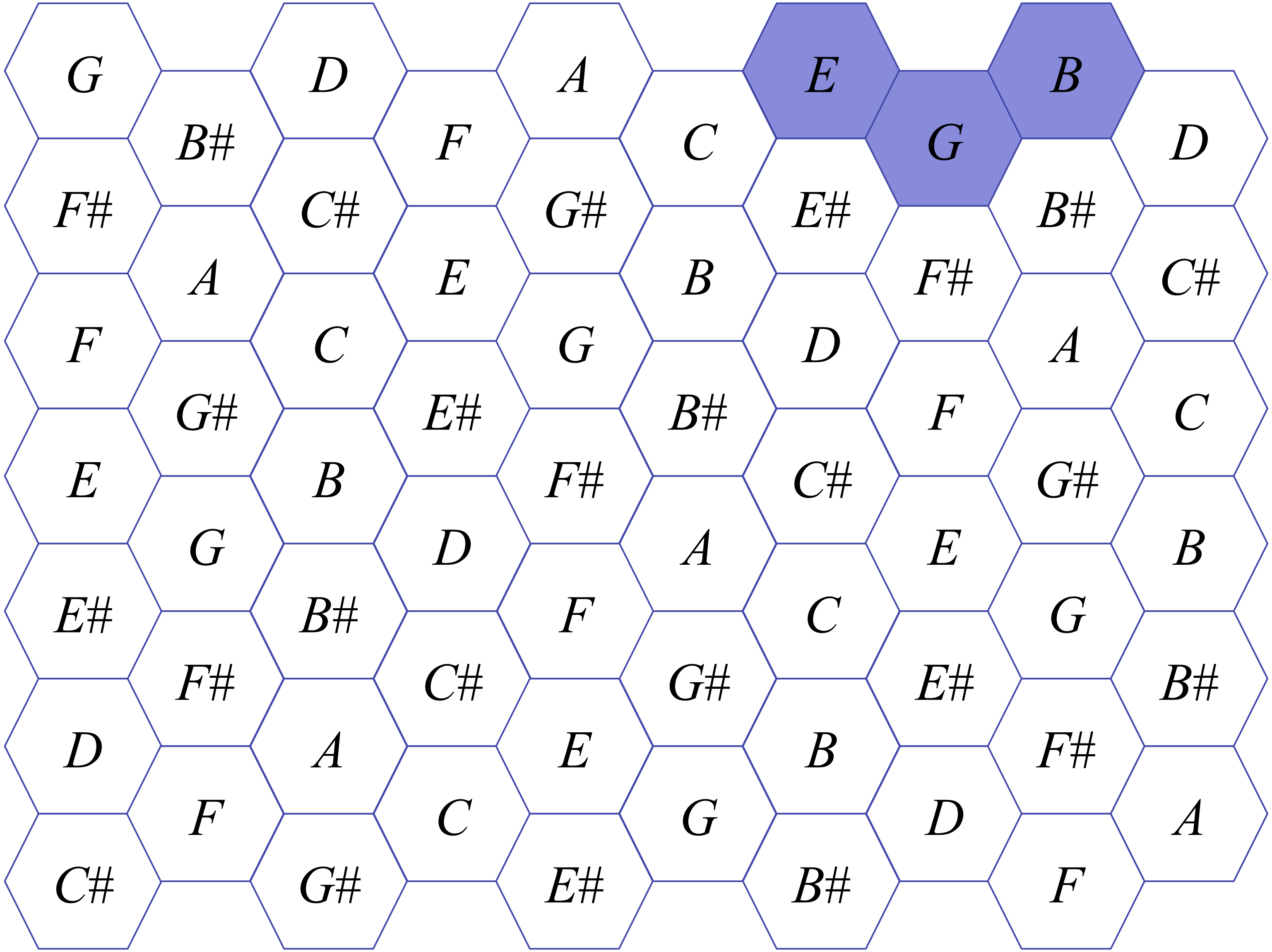

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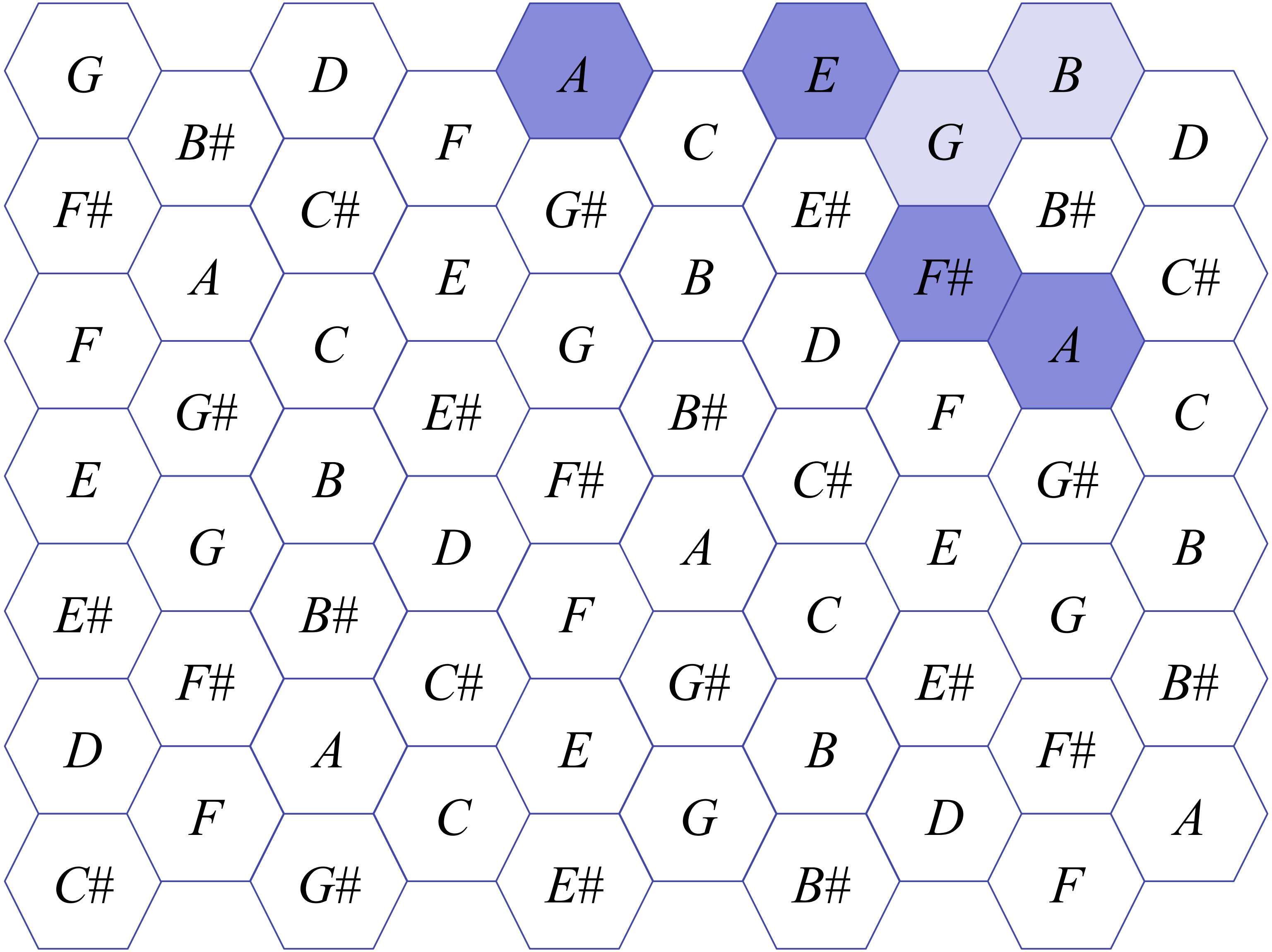

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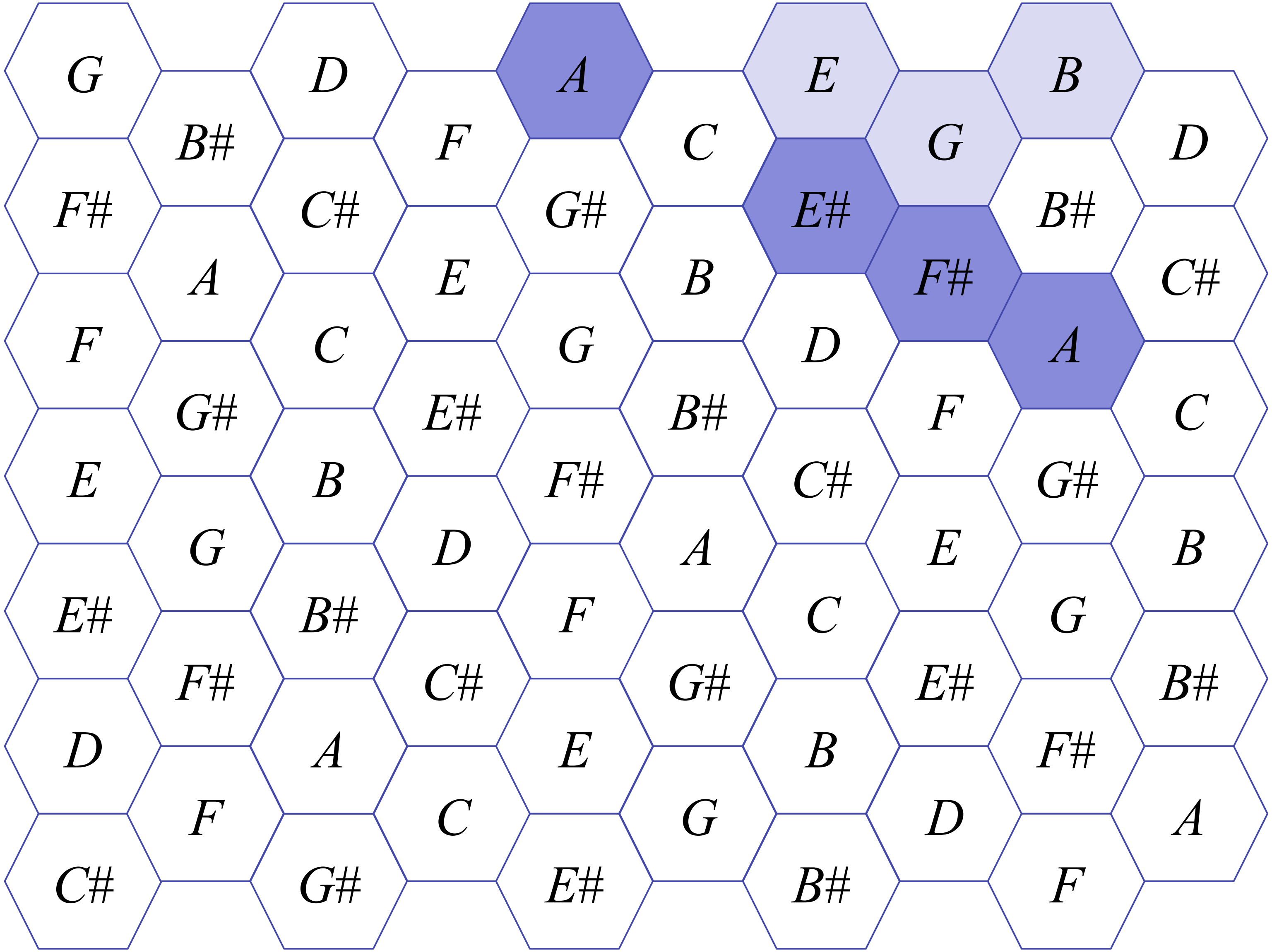


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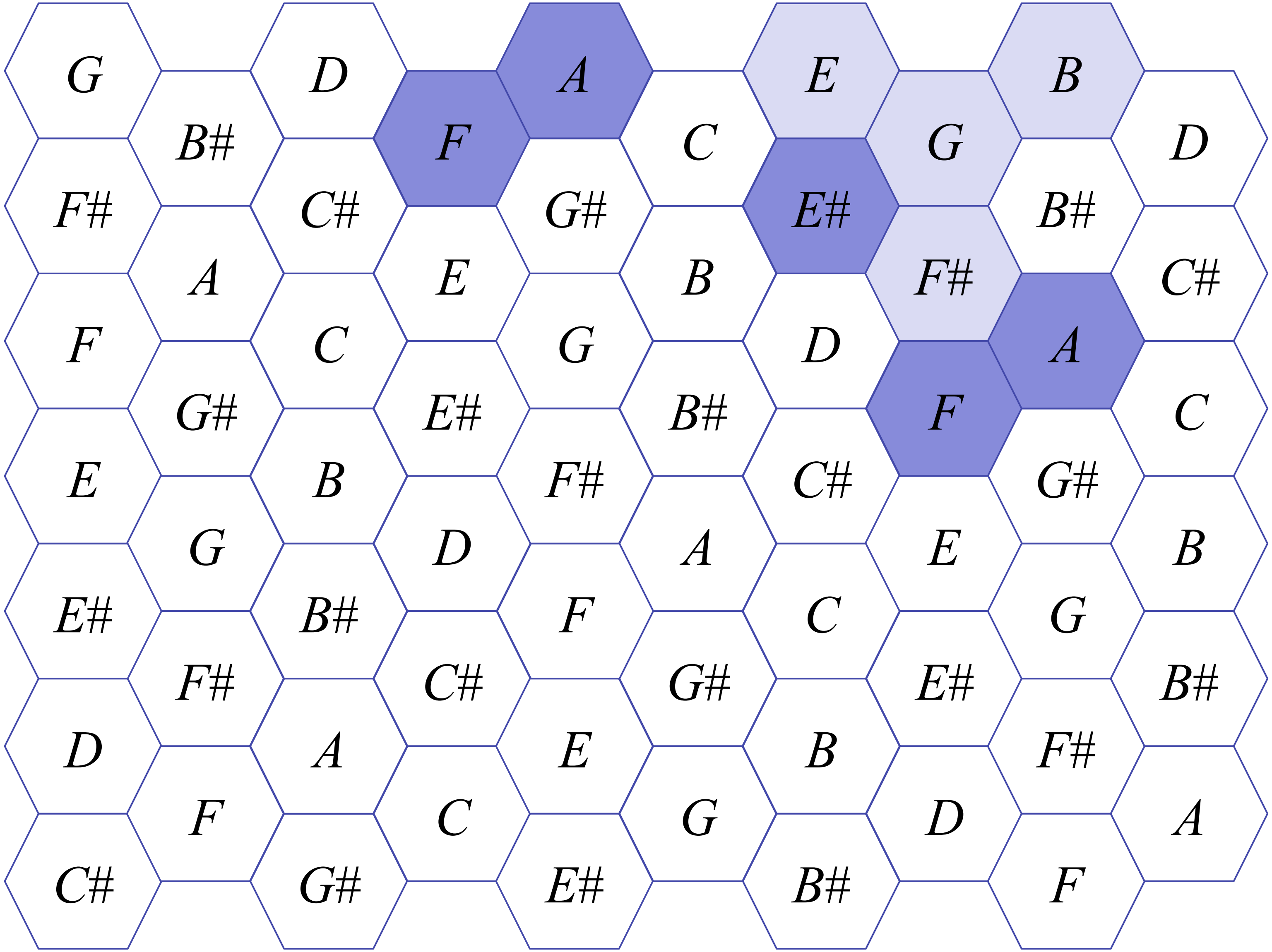


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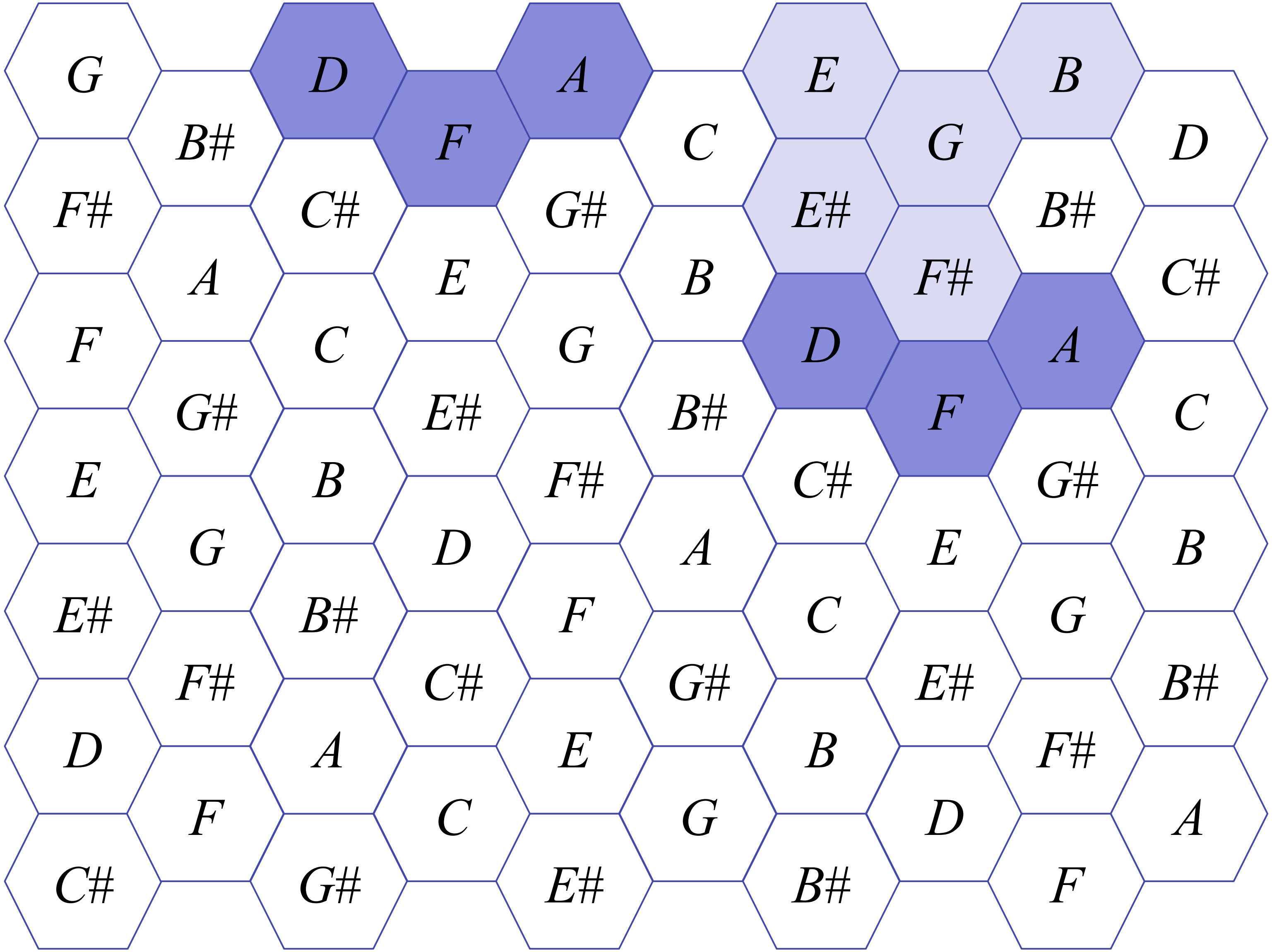

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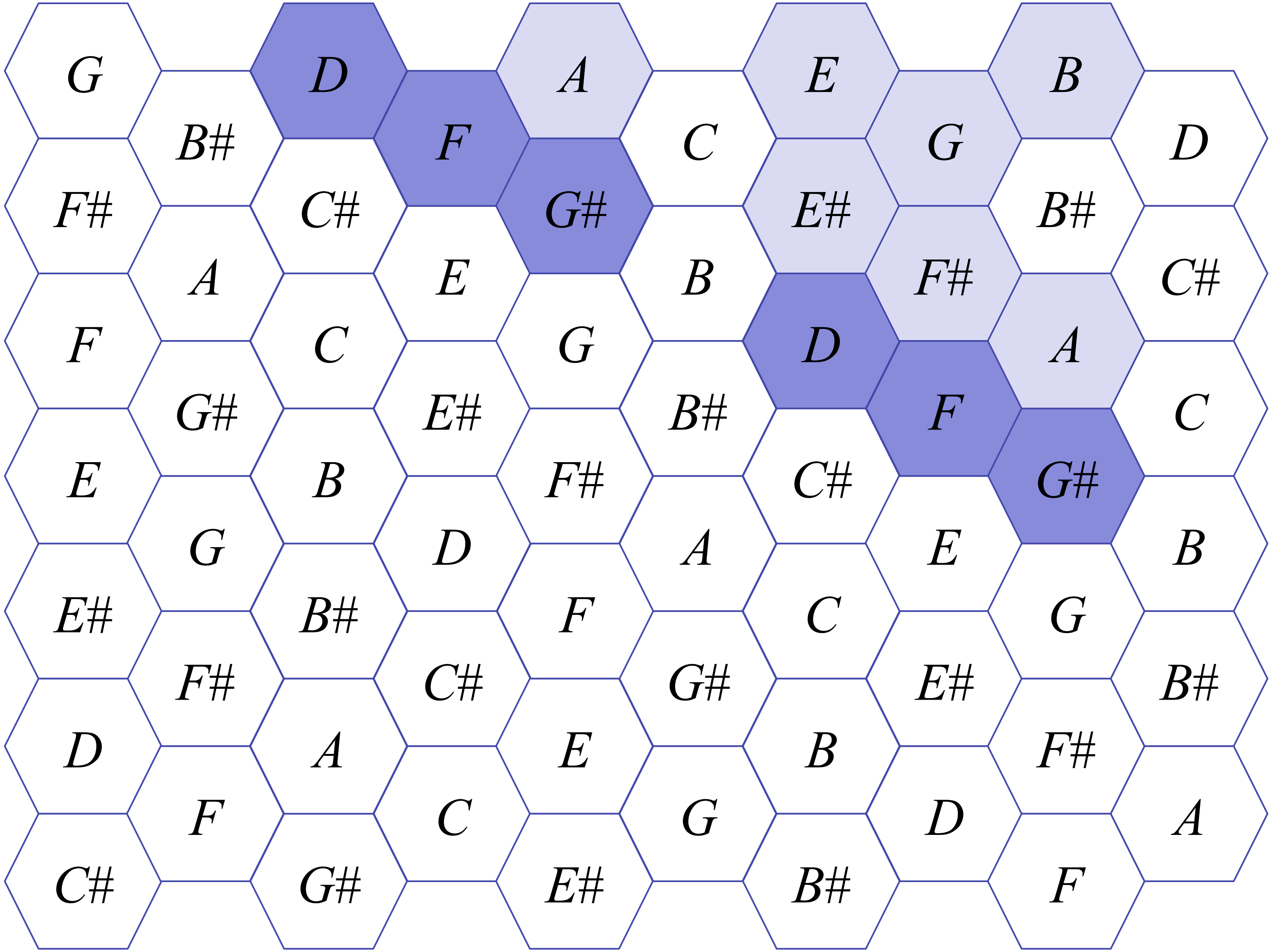

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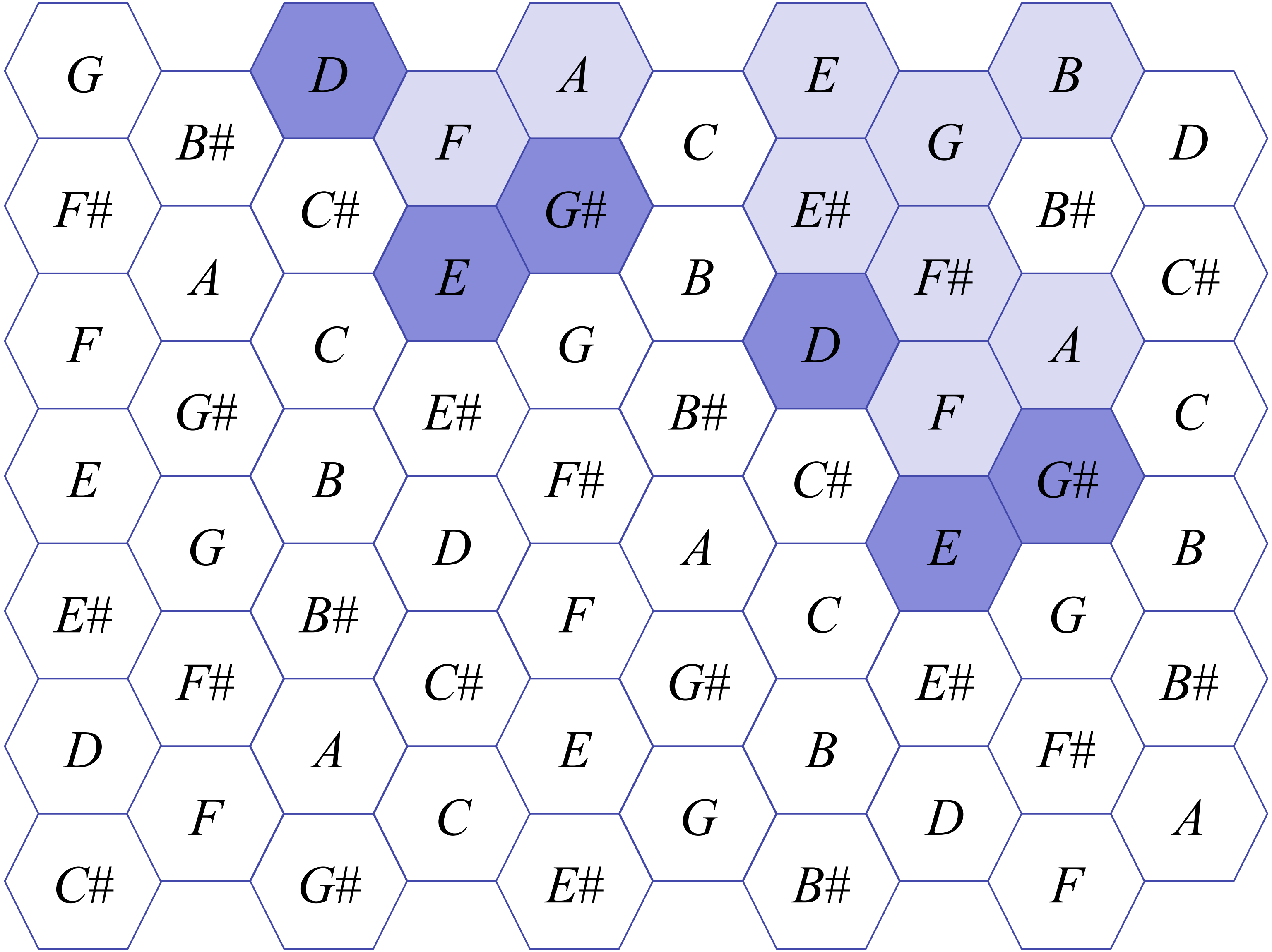

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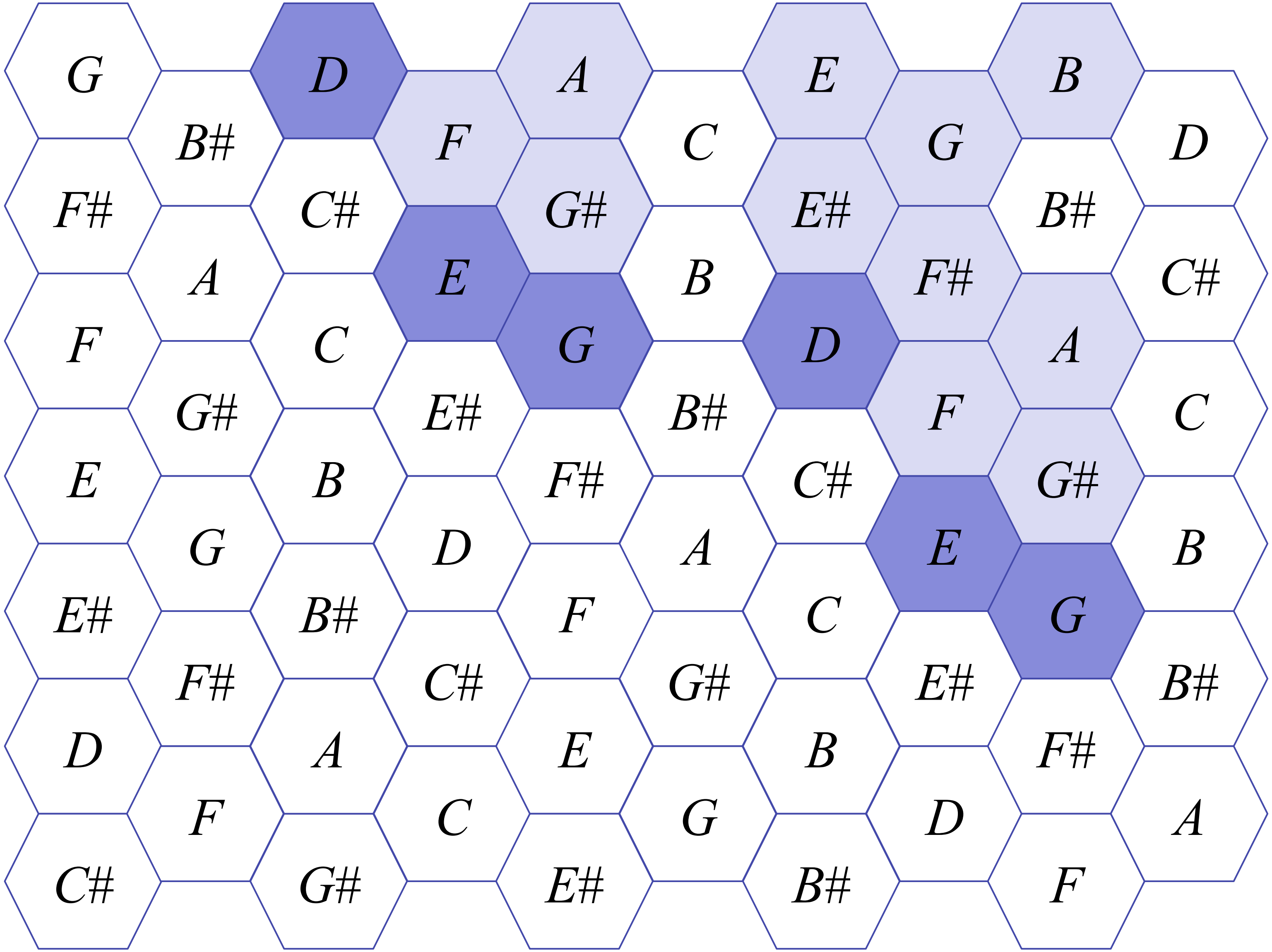


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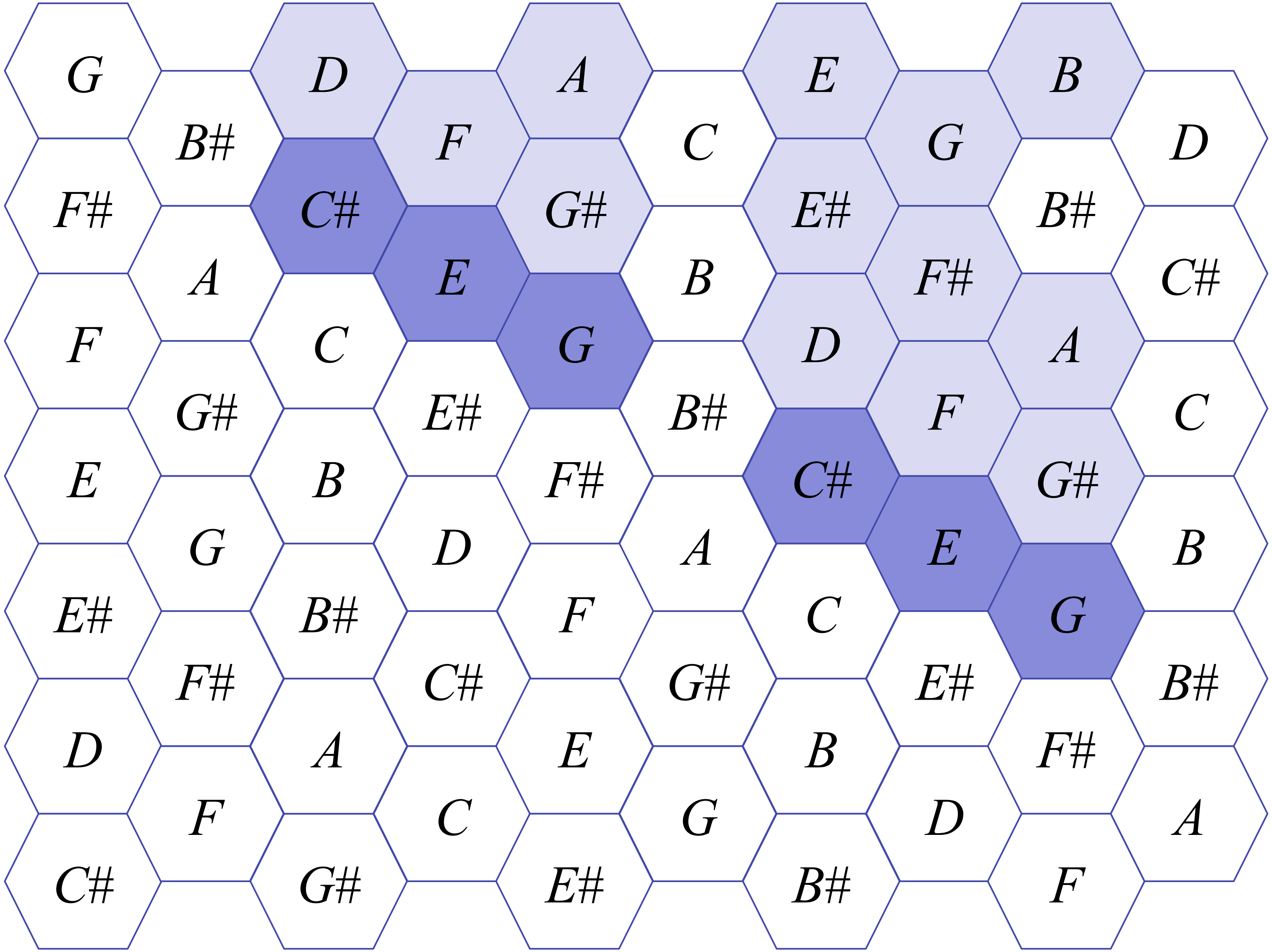

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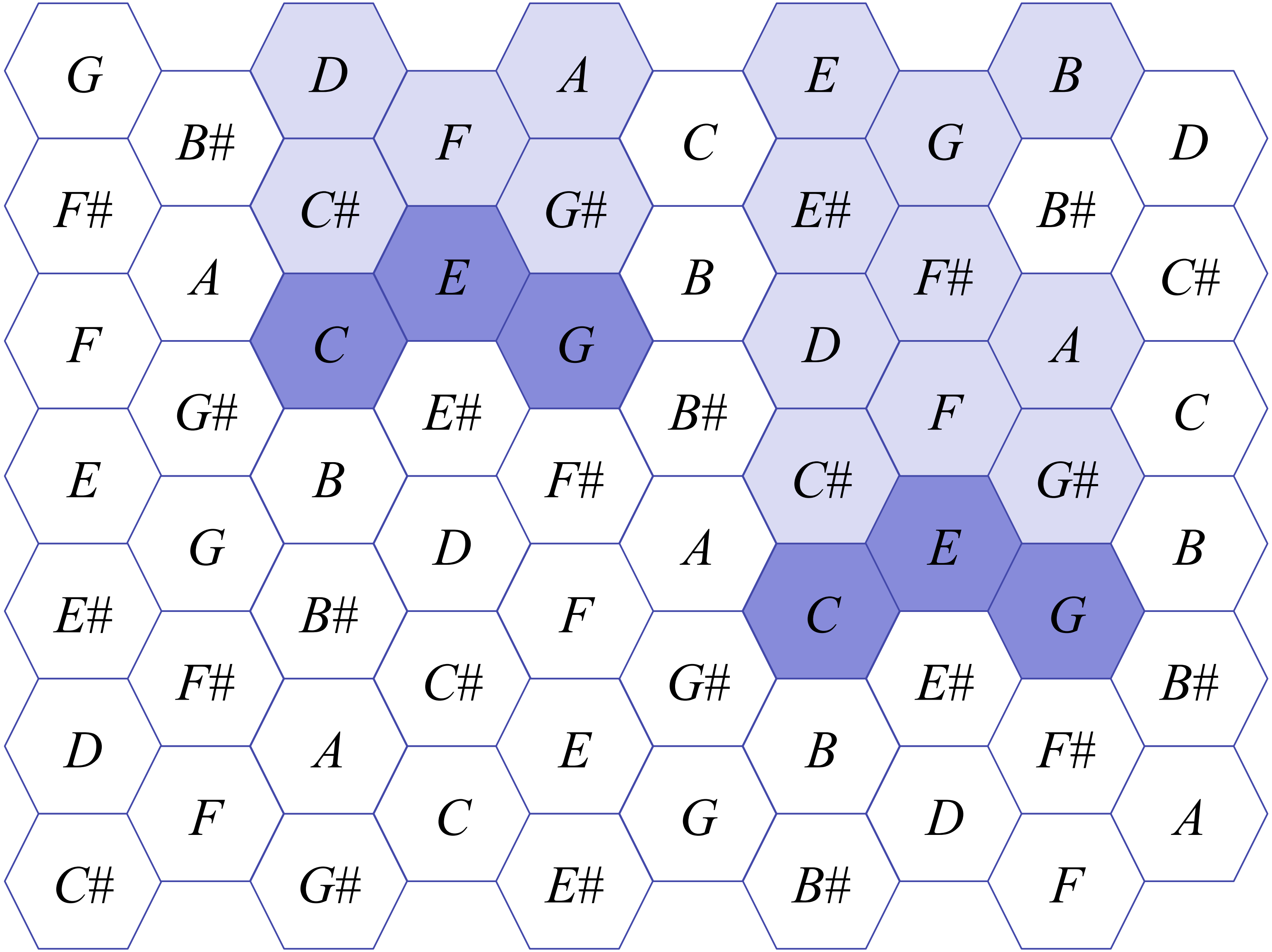


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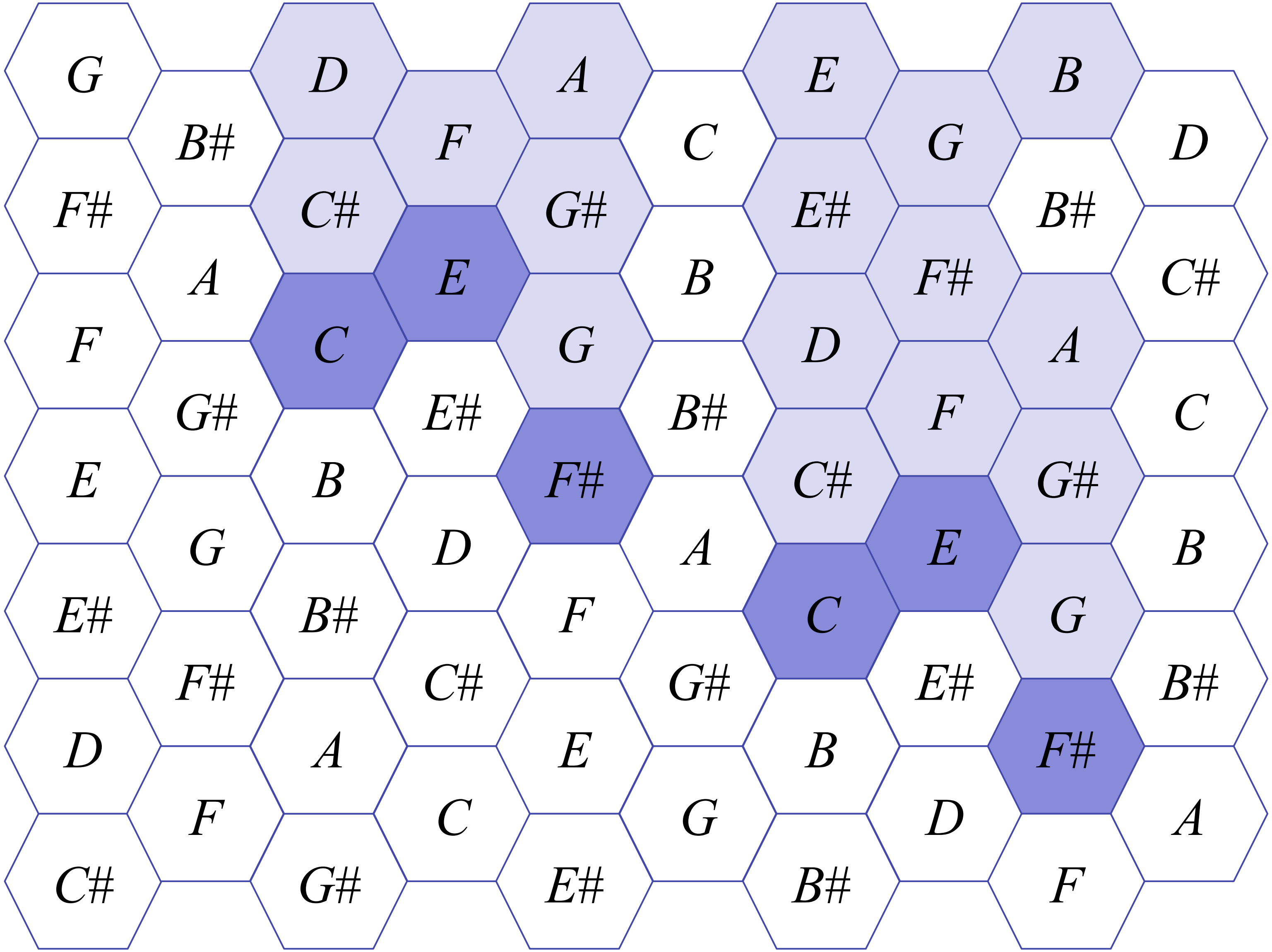

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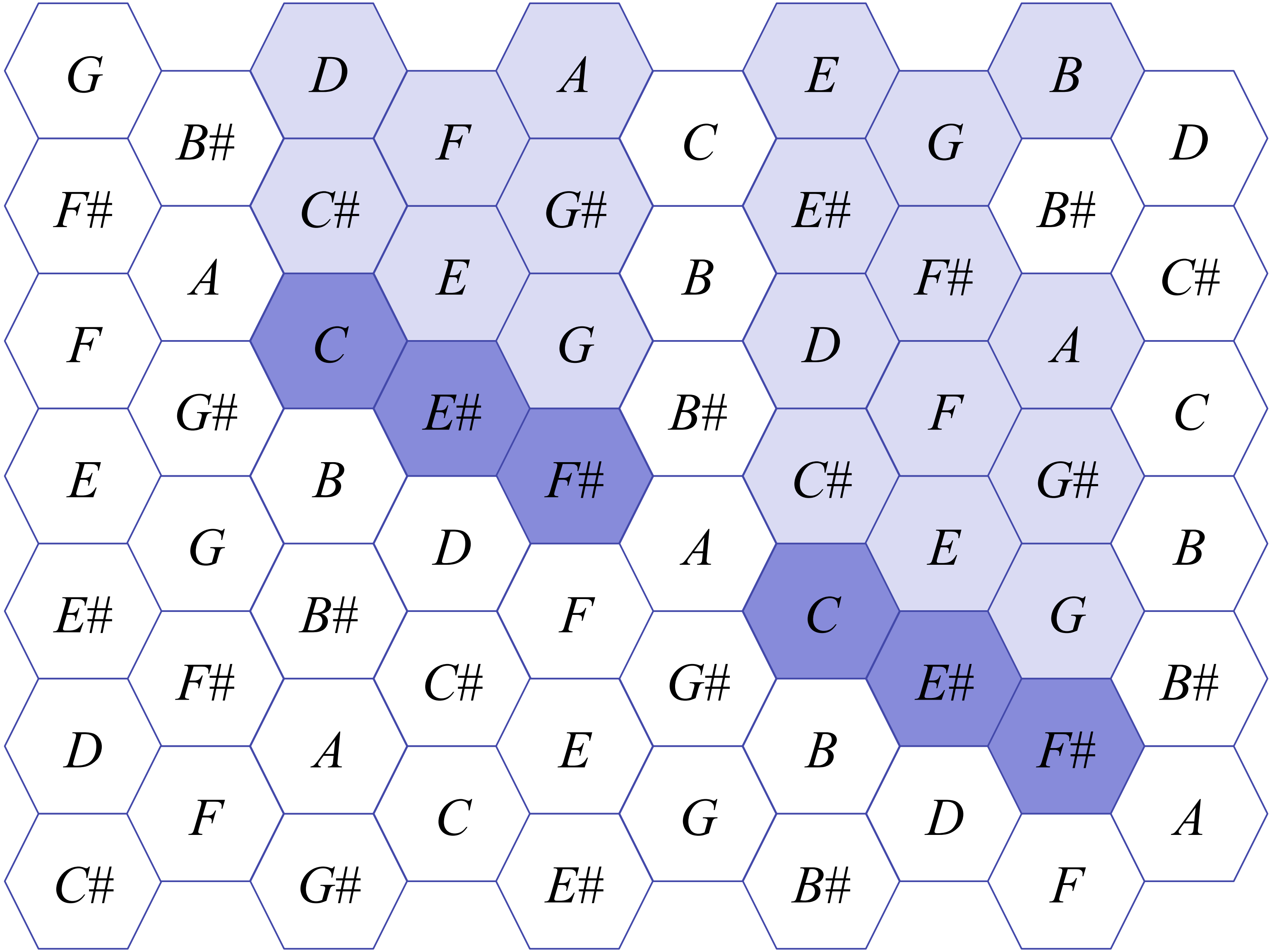

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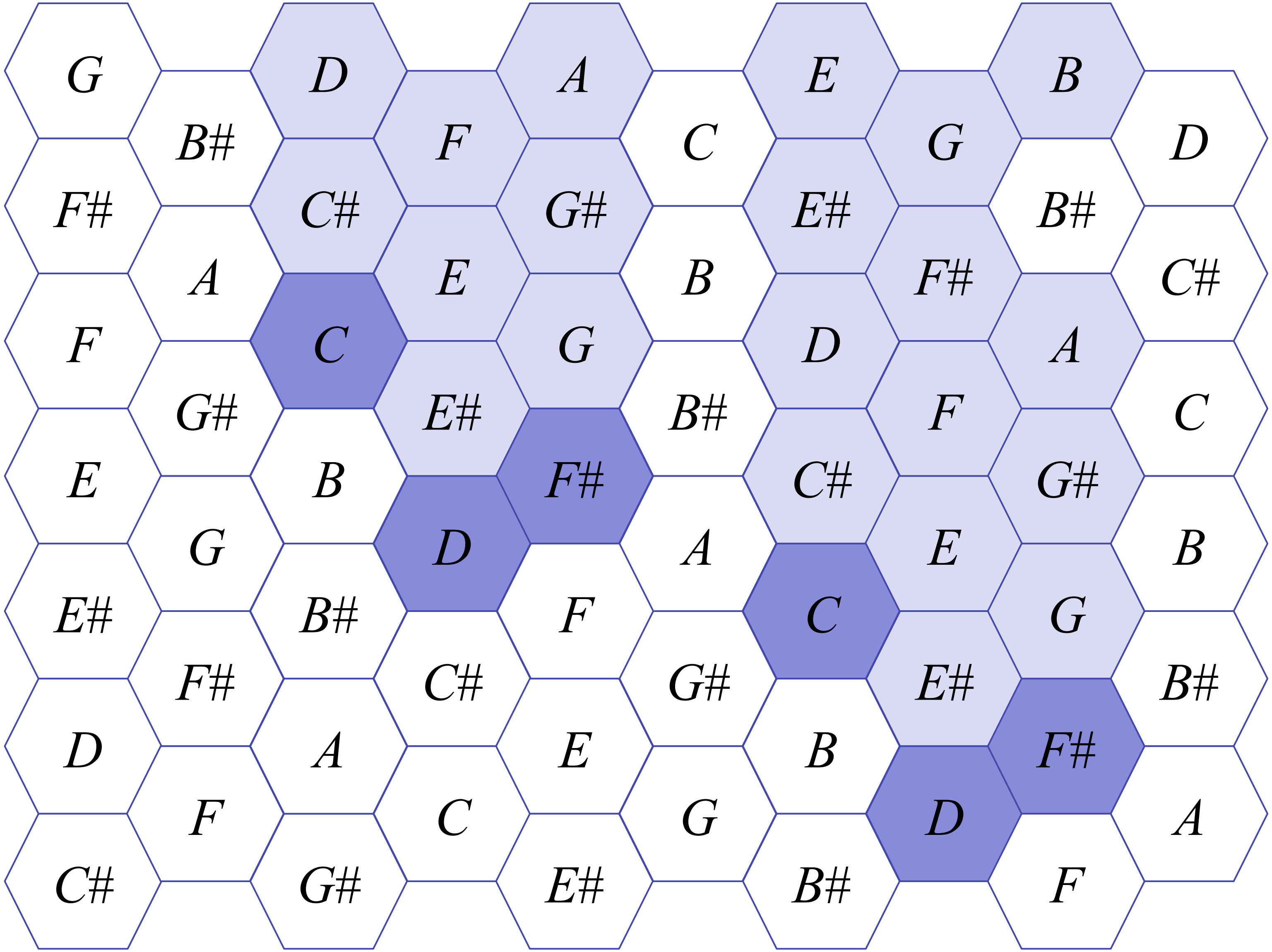

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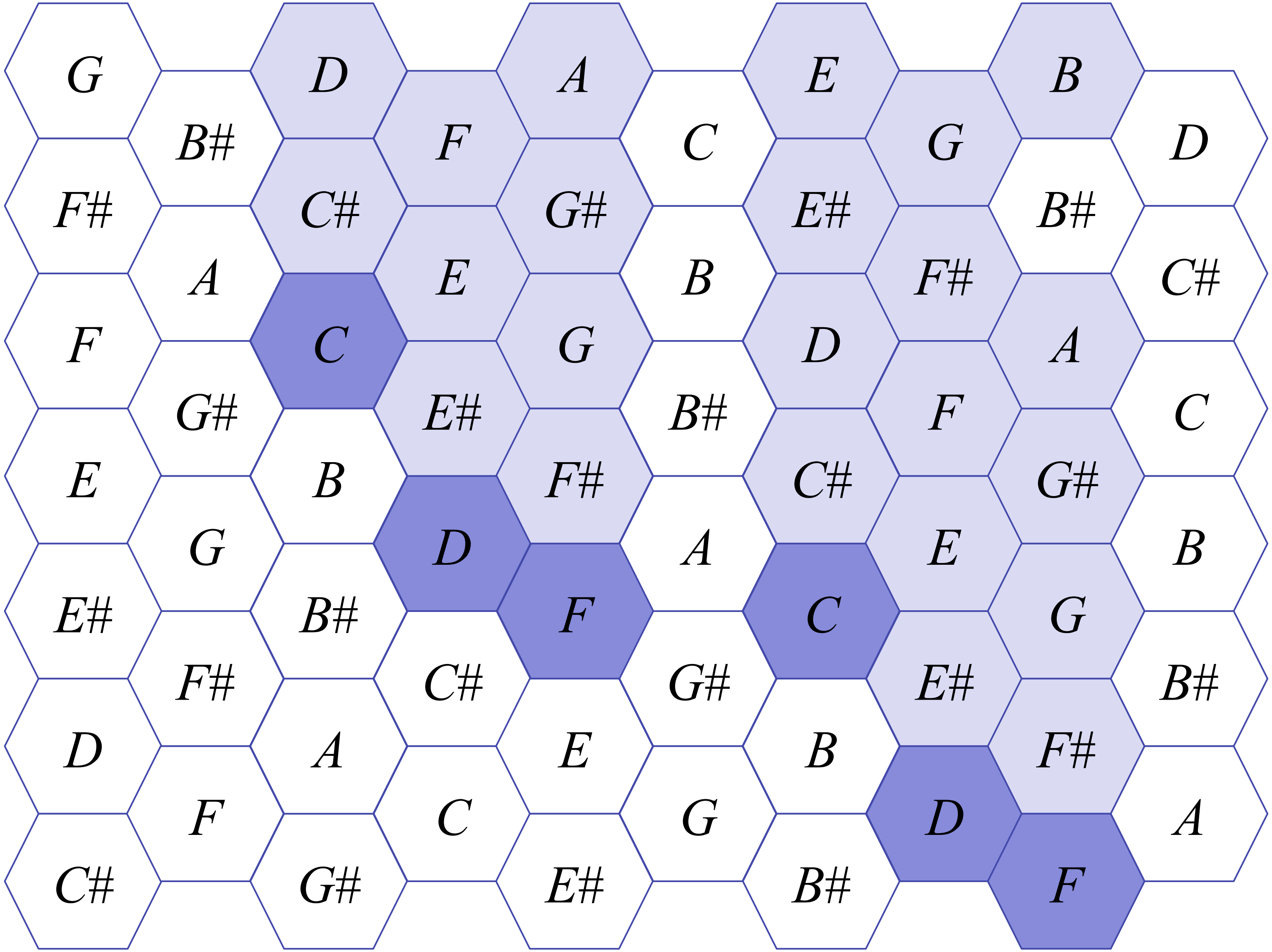


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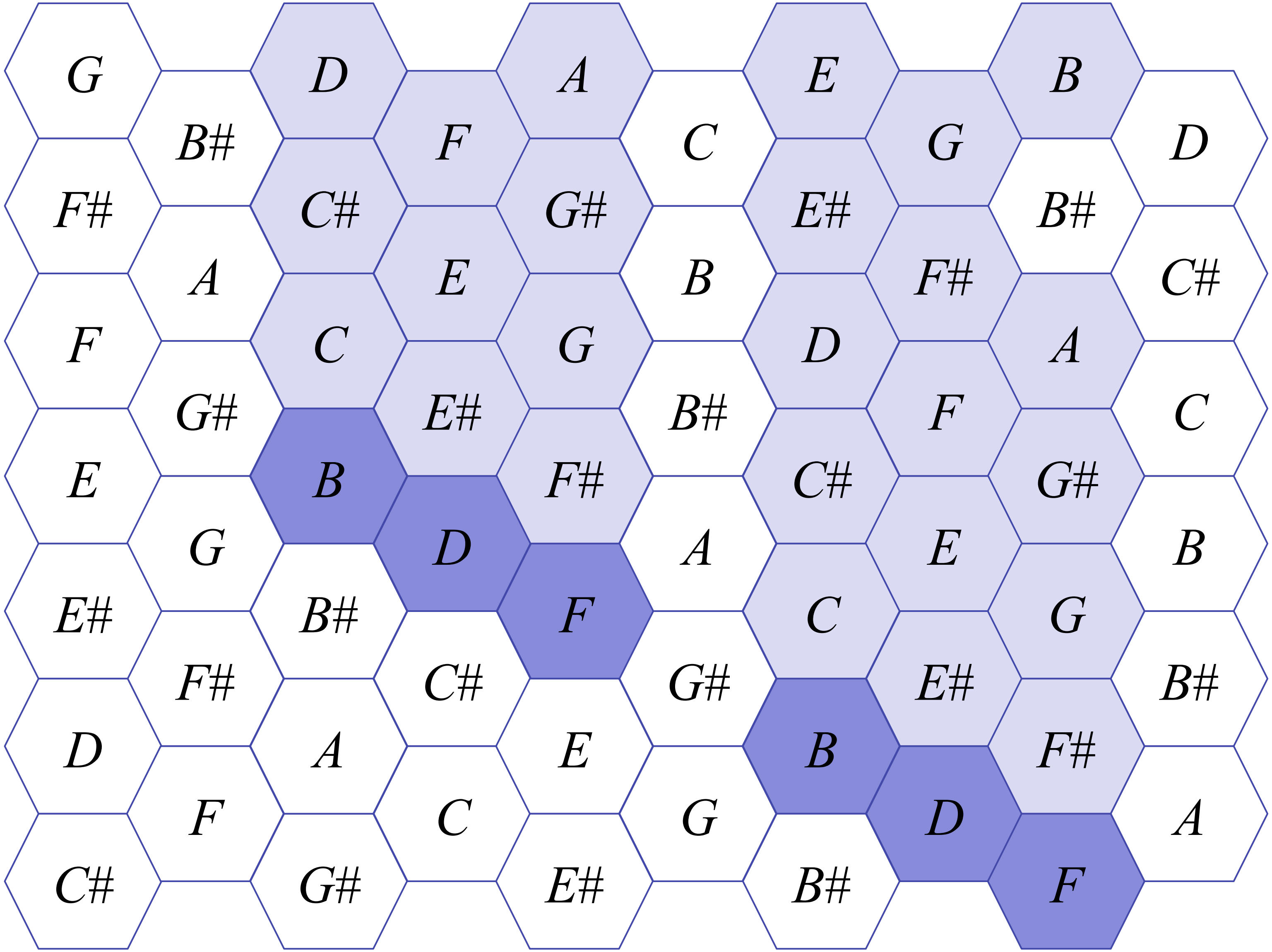


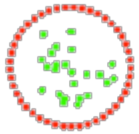
# Thanks

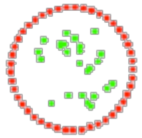
- **Antoine Spicher**
- **Olivier Michel**

http://mgs.spatial-computing.org

- **PhD and other students**

  **Louis Bigo**
  J. Cohen, P. Barbier de Reuille,
  E. Delsinne, V. Larue, F. Letierce, B. Calvez,
  F. Thonerieux, D. Boussié *and the others...*





- **Past and presents Collaborations**
  - A. Lesne (IHES, stochastic simulation)
  - P. Prusinkiewicz (UoC, declarative modeling)
  - P. Barbier de Reuille (meristeme model)
  - C. Godin (CIRAD, biological modeling)
  - H. Berry (INRIA, stochastic simulation)
  - G. Malcolm (Liverpool, rewriting)
  - J.-P. Banâtre (IRISA, programming)
  - F. Delaplace (IBISC, synthetic biology)
  - P. Dittrich (Jena, chemical organization)
  - E. Goubault (CEA, topological formalization)
  - F. Gruau (LRI, language and hardware)
  - P. Liehnard (Poitier, CAD, Gmap and quasi-manifold)