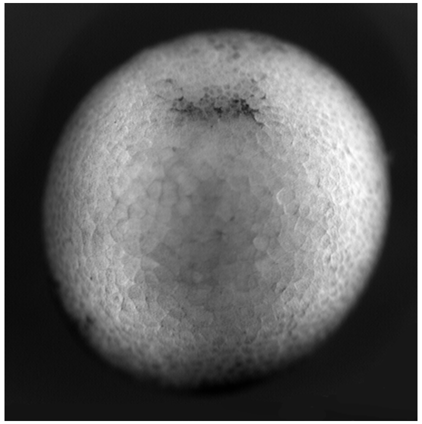
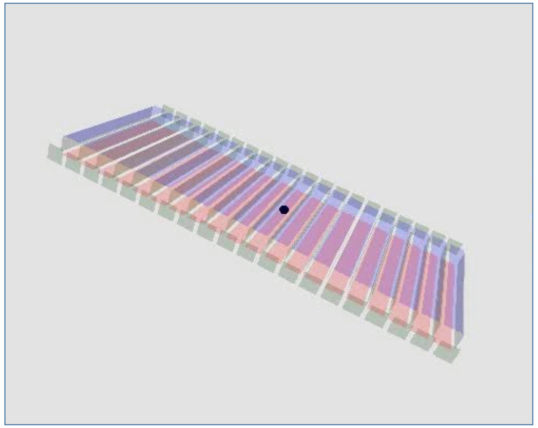
Interaction based simulation of dynamical systems with a dynamical structure $(DS)^2$ in MGS



mgs.spatial-computing.org



Jean-Louis Giavitto[1] & Antoine Spicher[2] & Olivier Michel[2]

[1]IRCAM, CNRS & UPMC

[2]LACL, University Paris Est Créteil

SCSC – Jun. 2011

Outline

- Motivations : modeling morphogenesis
  - $(DS)^2$ and their modeling
  - The topological structures of interactions
- Topological collection and their transformation in MGS
  - Topological collections
  - Transformation
  - Examples
- Applications
  - The flock of birds
  - The growth of an epithelial tissue
  - Modeling a synthetic bacteria

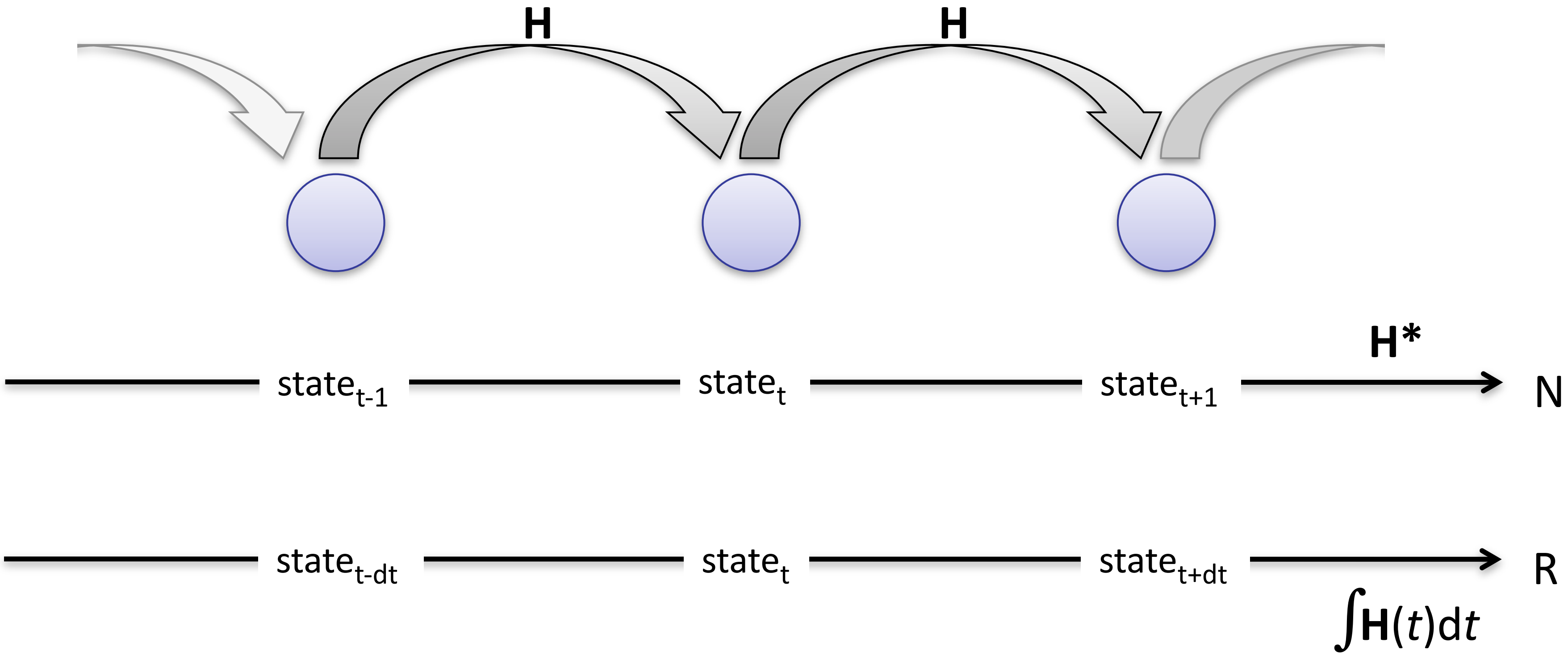
Motivations

- Dynamical Systems *and* dynamical structure
  - Morphogenesis = patterning formation + growth
- $(DS)^2$
- The modeling of $(DS)^2$
  - Locality
- The topological structure of interaction
- A general picture

Dynamical systems
and
Dynamical Structures

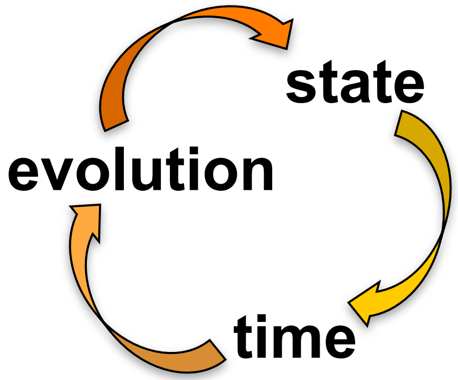
(a problem outlined by A. Turing)

Specifying a dynamical system (for simulation)



Specification of

- **structure of state**
- **structure of time**
- **evolution function**



Formalism for Dynamical System

- State : often structured by space (e.g. fields)
- Time
- Evolution function

C : continuous, D: discrete	PDE	Coupled ODE	Iteration of functions	Cellular automata	…
state	C	C	C	D	...
time	C	C	D	D	...
space	C	D	D	D	...

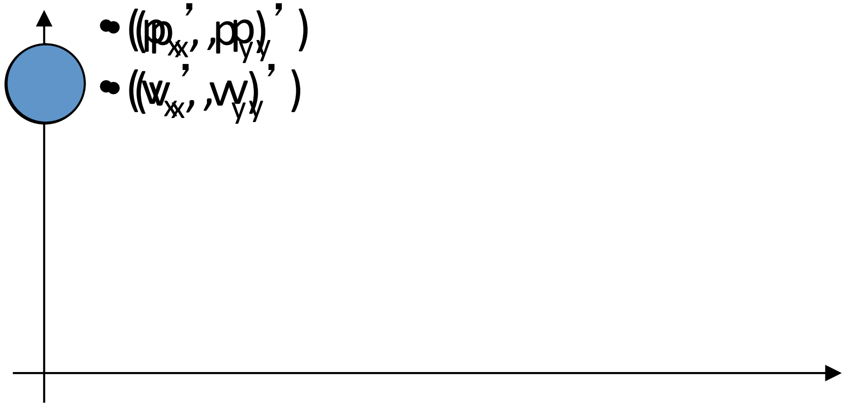
The medium/process problem

THE CHEMICAL BASIS OF MORPHOGENESIS

BY A. M. TURING, F.R.S. *University of Manchester*

(*Received* 9 *November* 1951—*Revised* 15 *March* 1952)

a falling ball



at any time a state is a position and a speed

A dynamical system (DS)

The medium/process problem

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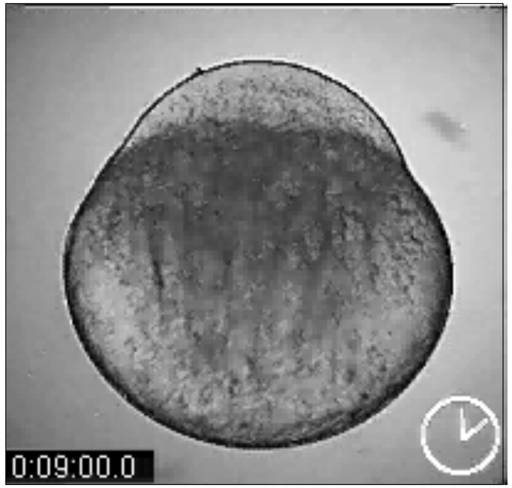
at any time a state is a position and a speed

A dynamical system (DS)

a developing embryo



the structure of the state is changing in time (chemical and mechanical state of each cell)

A dynamical system
with a dynamical structure
$(DS)^2$

Modelling morphogenesis: the predefined medium

The interdependence of the chemical and mechanical data adds enormously to the difficulty, and attention will therefore be confined, so far as is possible, to cases where these can be separated.

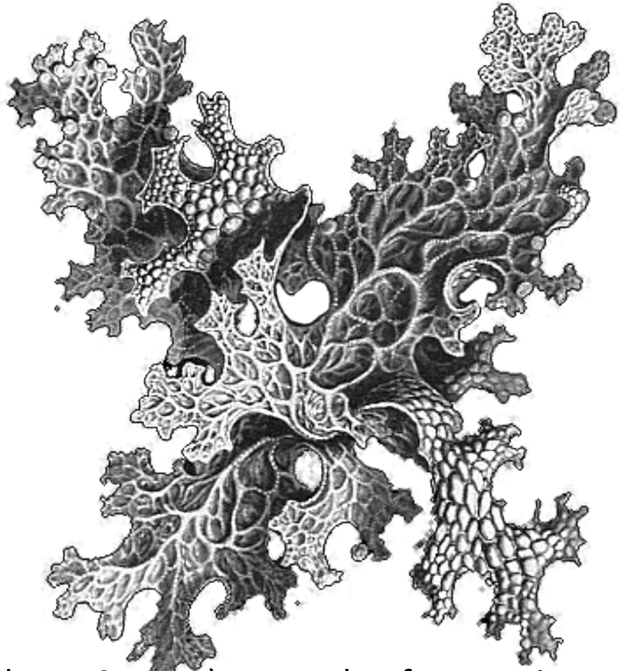
Suppose, for instance, that a 'leg-evocator' morphogen were being produced in a certain region of an embryo, or perhaps diffusing into it, and that an attempt was being made to explain the mechanism by which the leg was formed in the presence of the evocator. It would then be reasonable to take the distribution of the evocator in space and time as given in advance and to consider the chemical reactions set in train by it.

Compatible with

- the notion of morphogenetic field
- cell fate

But

- there is evidence for
  feedback loops between the shape and the process inhabiting the shape



from E. Haenkel (cited by C. Goodman-Strauss): example of a negative curvature surface. Curvature can be controlled while the surface is growing along a 'front'

Patterning vs. Growth

- **Dynamics <u>ON</u> form**

  patterning, motif formation in a predefined form

  - Diffusion, reaction-diffusion, transport (continuous models)
  - cellular automata (discrete models)

- **Dynamics <u>OF</u> form**

  growth, deformation of a shape

  - Deformation of elastic bodies (continuous models)
  - Lindenmayer systems (discrete models)

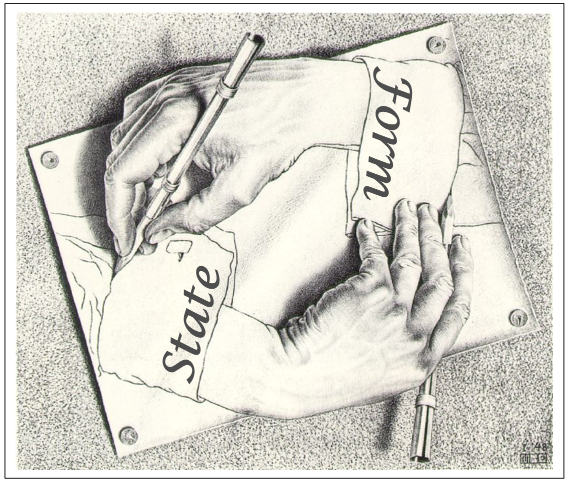
The interplay between state and form

a developing embryo

the state **as well as the structure of the state** is changing in time

*(chemical and mechanical state of each cell **as well as the arrangement of the cells**)*

Dynamics ON form



Dynamics OF form

A dynamical system with a dynamical structure

$(DS)^2$

$(DS)^2$ versus DS

- Processes in the form are topologically (geometrically) meaningful
  e.g. growth rate

- Topological (geometric) information is meaningful
  e.g. domain of diffusion, information transfert

OK: the coupling is important.

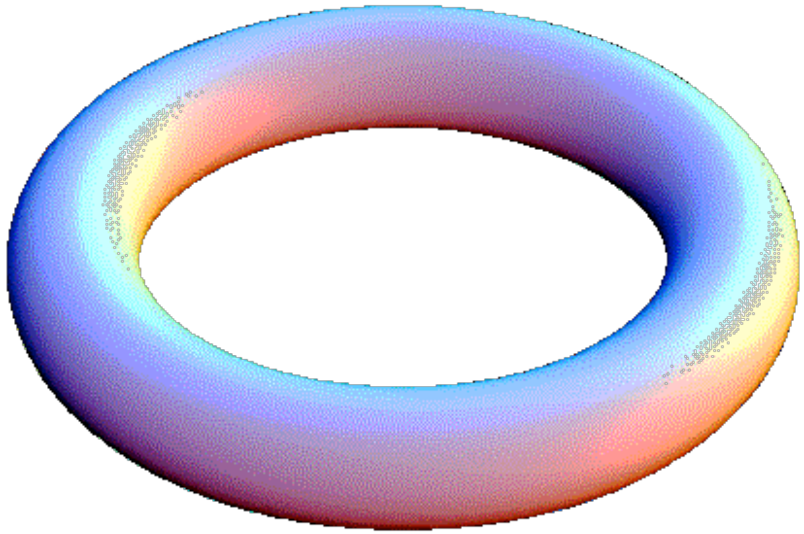
What can we do?

1. **"Dynamics ON form" toward DS^2**

   - parameterize (control) the shape by some quantities
     (e.g. curvature for a manifold
     or adjacency matrix for a graph)

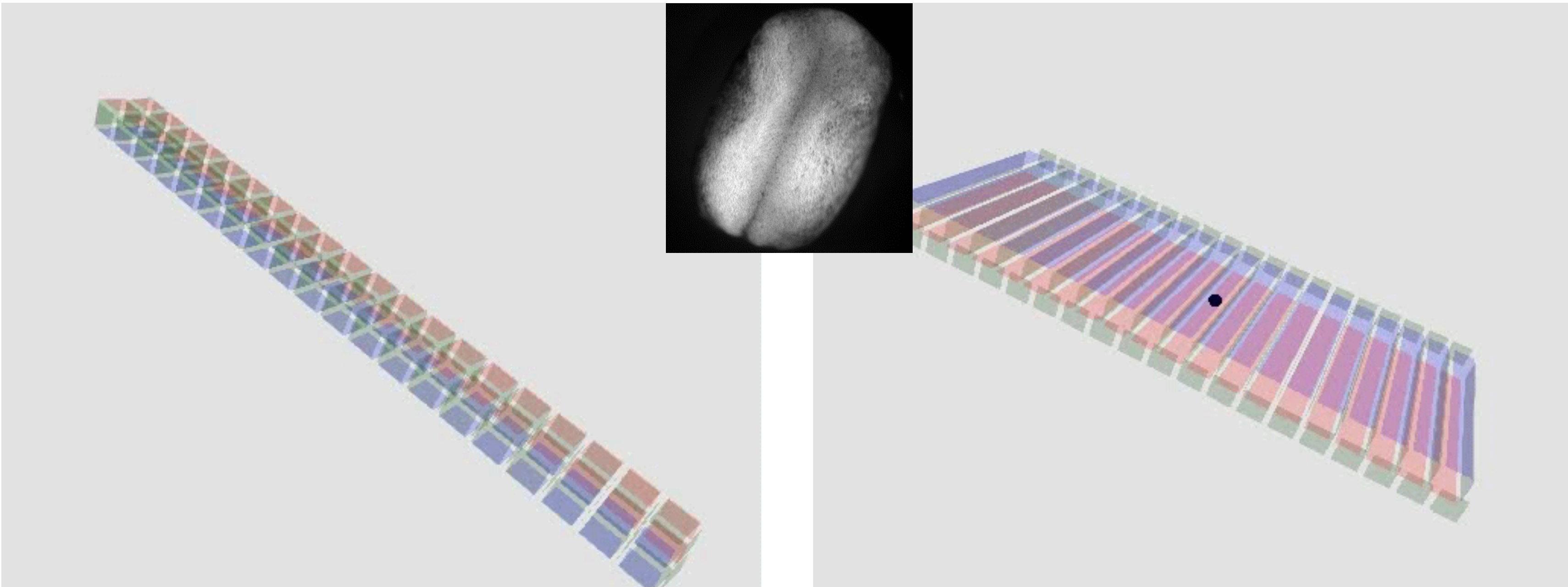
   - link these quantities with processes in the shape

     - growth depending on concentration



Differential geometry is not enough

- Encoding a shape into continuous parameters is difficult
- It does not handle topological changes very well



apical elongation depends of time

- Ok, sometimes they are some tricks (e.g. level set) but they are very smart tricks

OK: the coupling is important. What can we do?

1. **"Dynamics ON form" toward DS²**

   - parameterize (control) the shape by some quantities
     (e.g. curvature for a manifold
     or adjacency matrix for a graph)
   - link these quantities with processes in the shape
     - growth depending on concentration

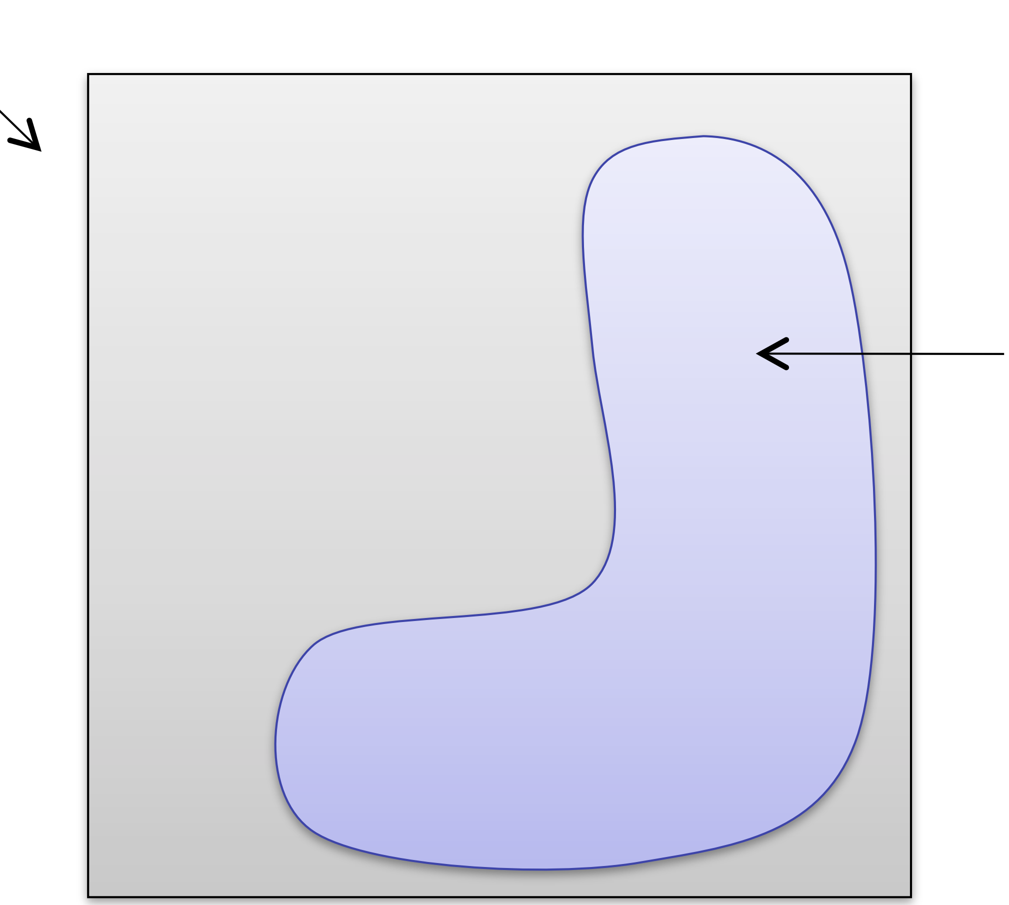
2. **"Dynamics OF form" toward DS²**

   - enhance the form by parameters
   - put a dynamic on these parameters
     example: module in Lindenmayer systems

The Topological Structure of Interactions

Decompose a system into subsystems following the elements in interaction

A system in some state

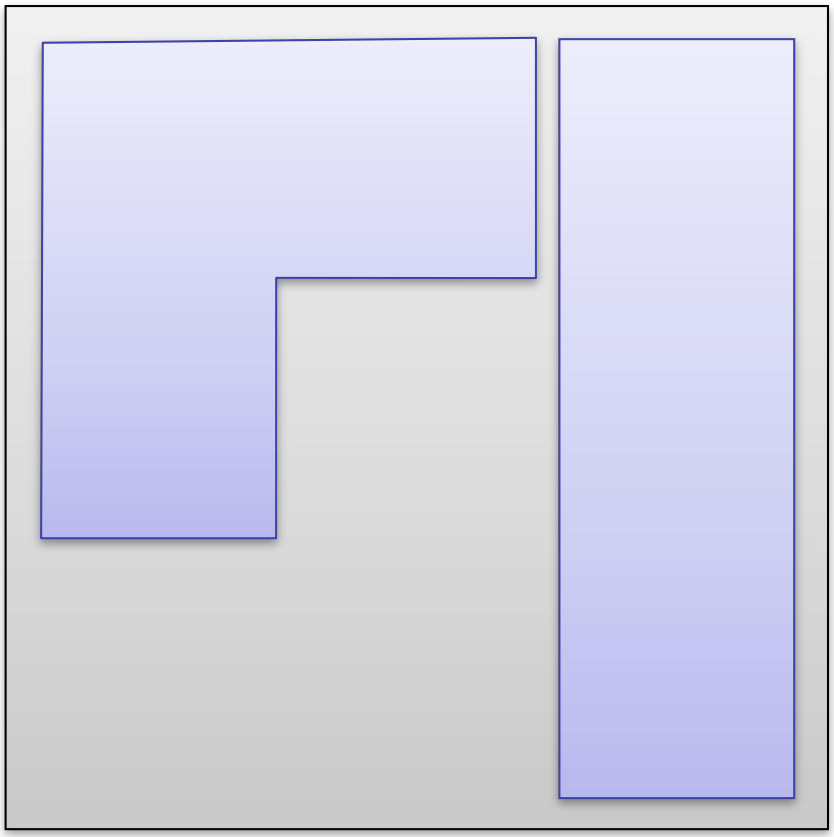


Part of a system that evolves.

Can be identified by comparaison with the previous global state

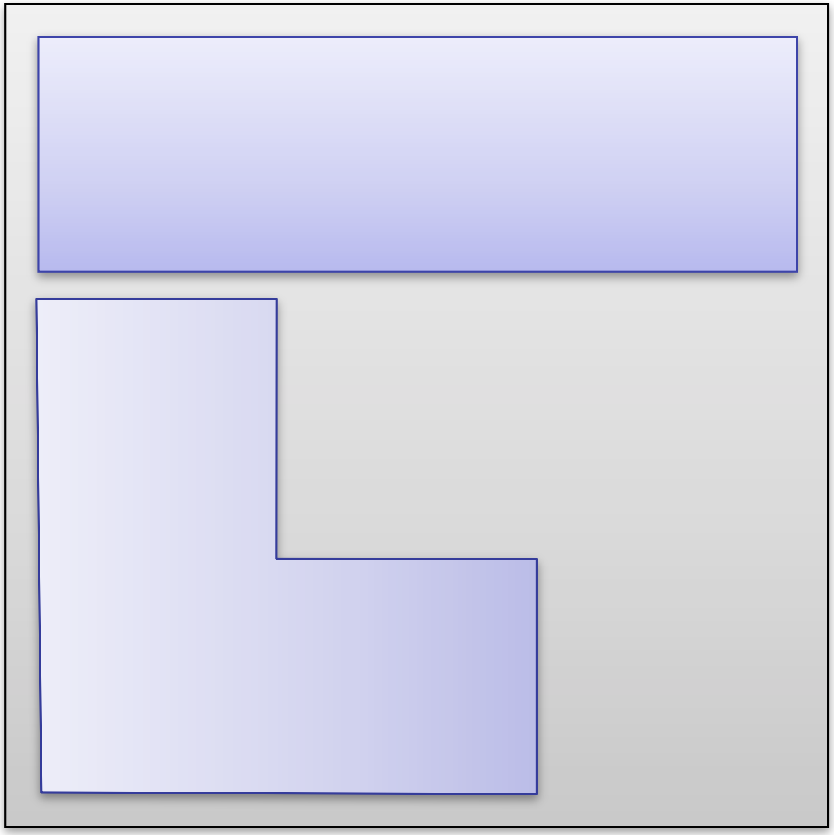
Decompose a system into subsystems following the elements in interaction

$t = 1$



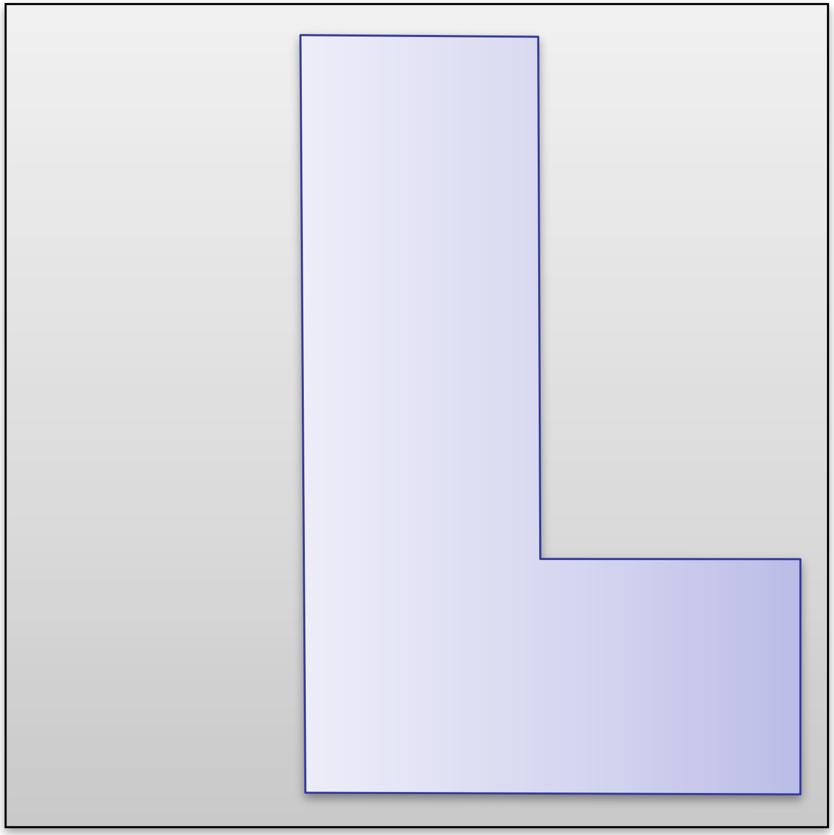
Decompose a system into subsystems following the elements in interaction

$t = 2$



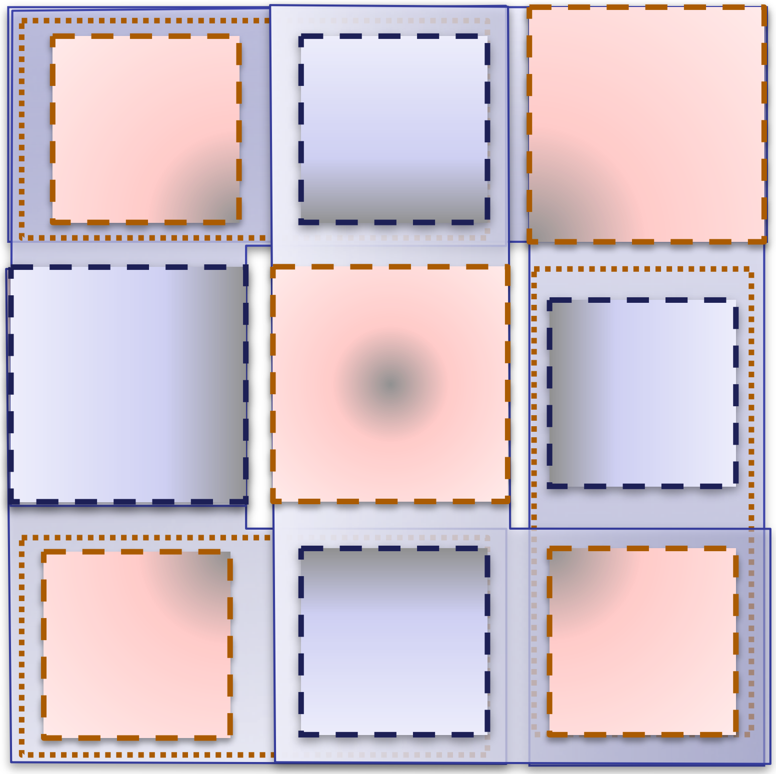
Decompose a system into subsystems following the elements in interaction

t = 3

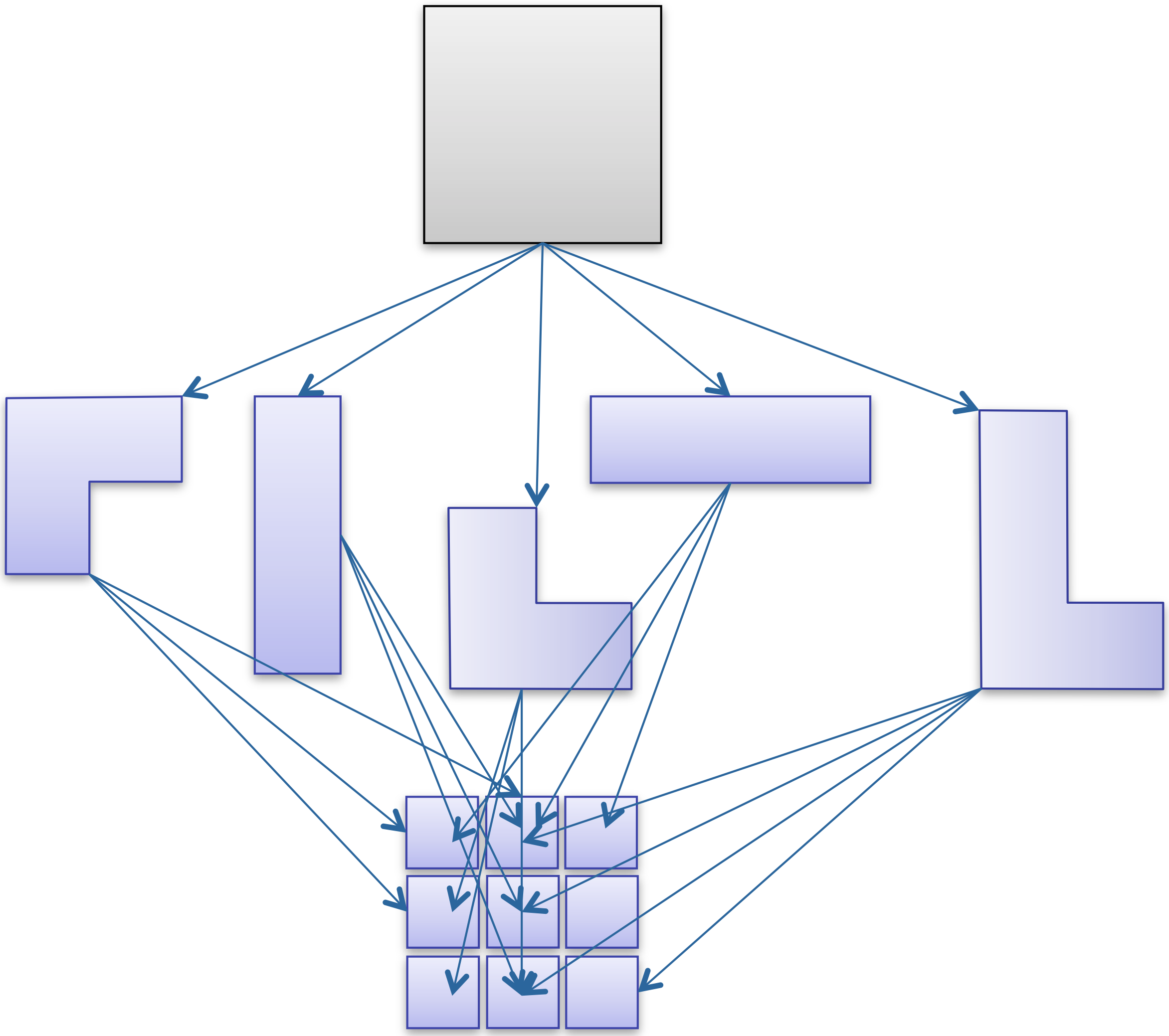


Decompose a system into subsystems following the elements in interaction

The interactions decomposes the systems into elementary parts.
An interaction implies one or several elementary parts.



Decompose a system into subsystems following the elements in interaction



the inclusion structure between the elementary and interacting parts is **a lattice**

a (simplicial) complex is a better (topological) equivalent representation

The grand picture

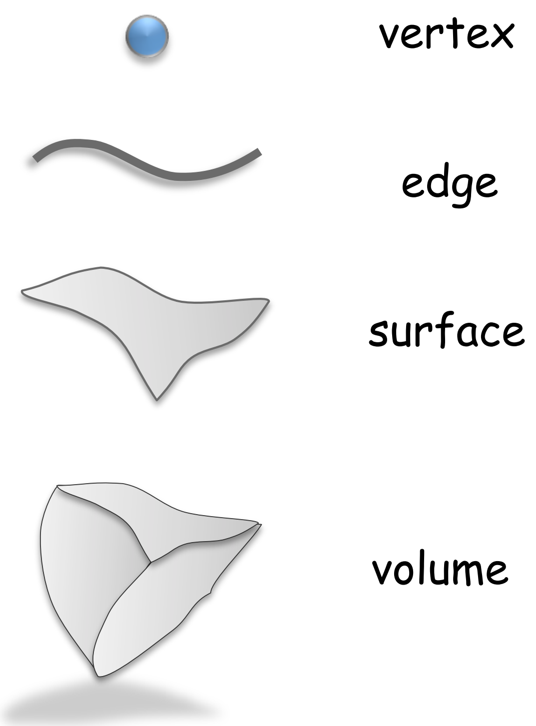
1. Describe a dynamical system following the interaction of its parts
2. Each part is characterized by a (local) state
3. The global state of the system is the “sum” of its local state and their topological organization
4. An interaction makes evolve a (small) subset of local states
5. An interaction potentially changes the topological organization of state

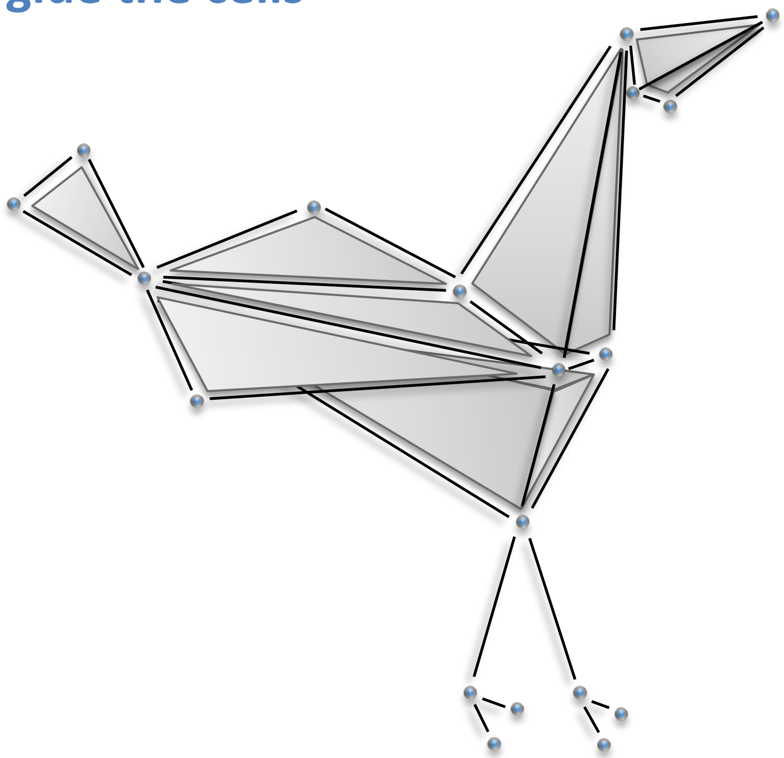
Topological Collections and their Transformation in MGS

- Topological collections
- Topological Rewriting
- MGS examples

MGS Proposition

- Topological collections
  - Structure
    - A collection of topological cells
    - An ***incidence relationship*** **to glue the cells**



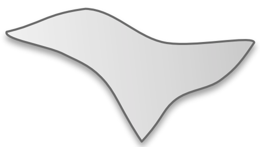


MGS Proposition

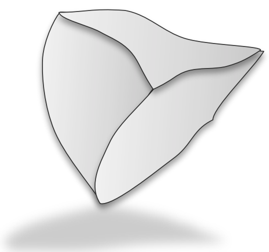
- Topological collections
  - Structure
    - A collection of topological cells
    - An incidence relationship
  - Data: **association of a value with each cell**

0-cell

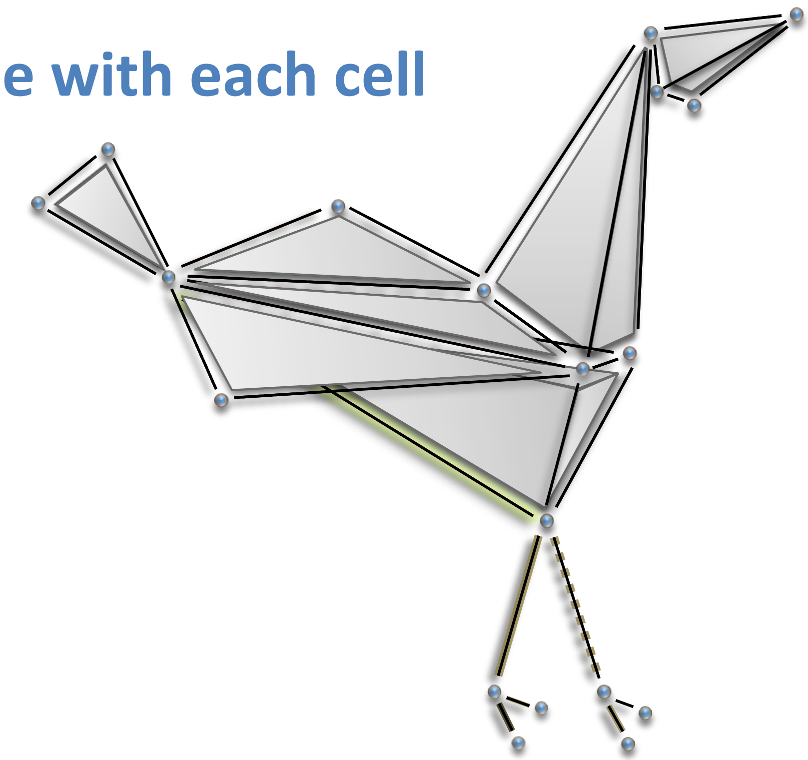
1-cell



2-cell



3-cell



MGS Proposition

- Transformations
  - Functions defined by case on collections

    Each case (pattern) matches a sub-collection
  - Defining a rewriting relationship: ***topological rewriting***

trans *T* = **{**

    pattern$_1$ ➔ *expression*$_1$

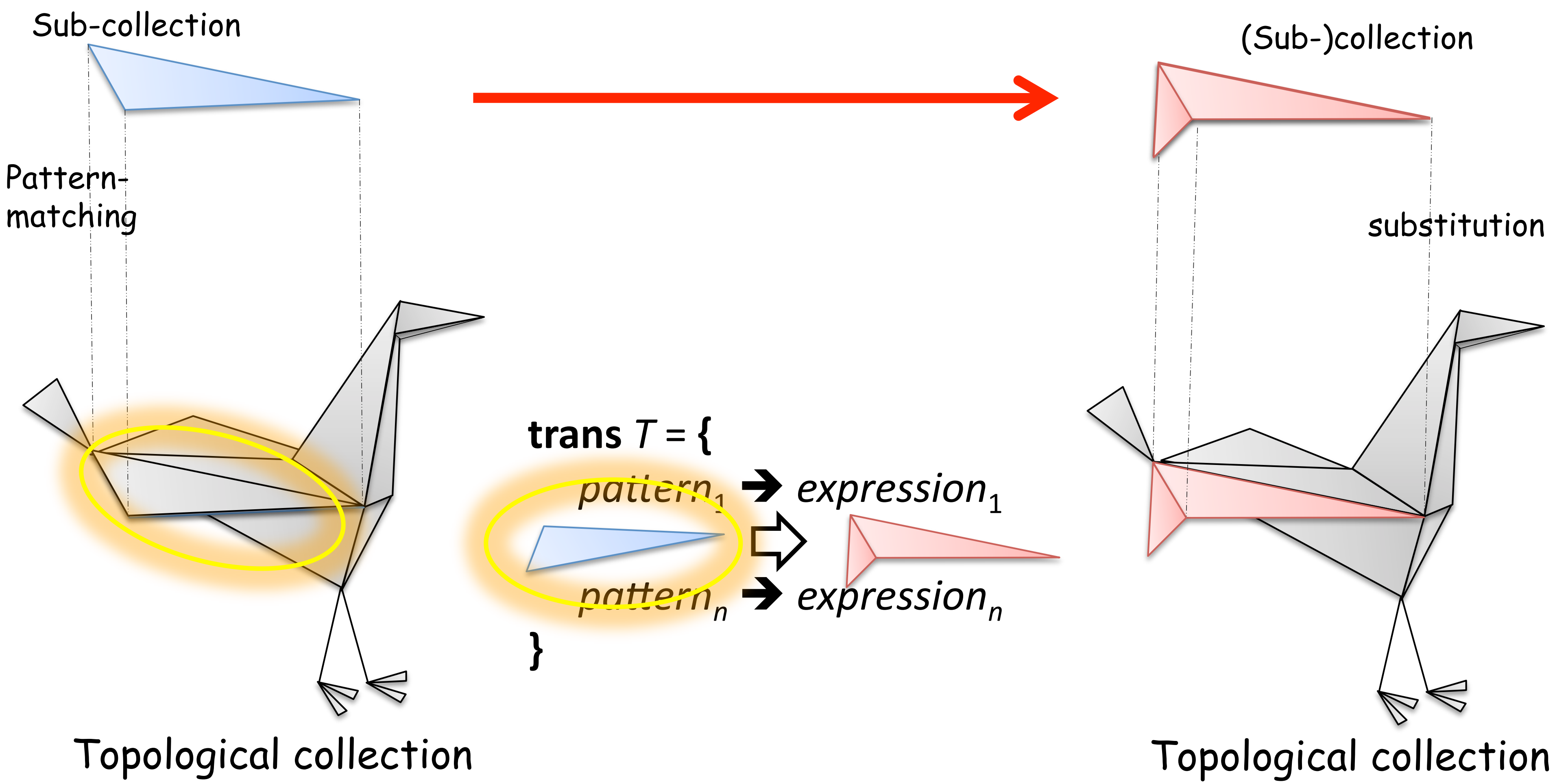
    …

    pattern$_n$ ➔ *expression*$_n$

}

MGS Proposition

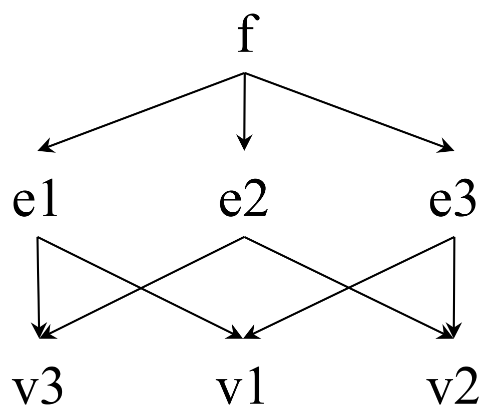
- Transformations

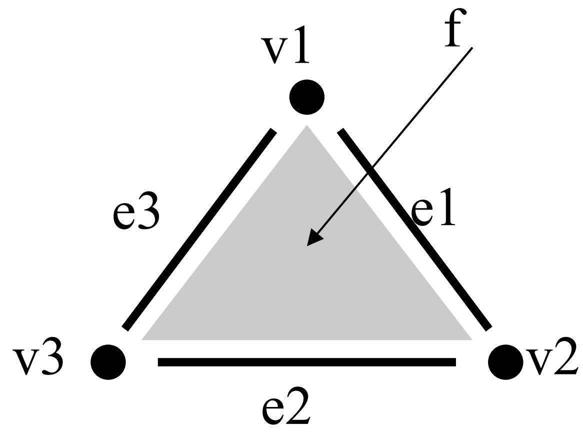


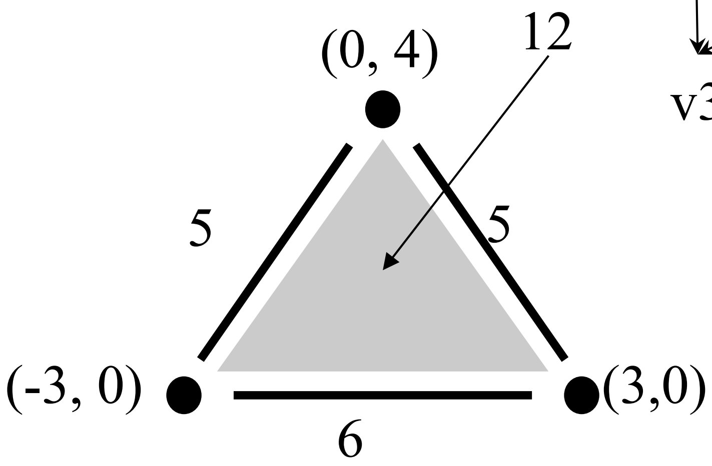
Abstract Simplicial Complex and simplicial chains

Incidence relationship and lattice of incidence:

- boundary(f) = {v1, v2, v3, e1, e2, e3}
- faces(f) = {e1, e2, e3}
- cofaces(v1) = {e1, e3}







Topological chain

- coordinates with vertices
- lengths with edges
- area with f

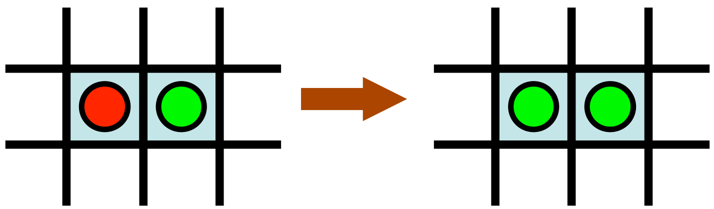
$$\begin{pmatrix} 0 \\ 4 \end{pmatrix}.v_1 + \begin{pmatrix} 3 \\ 0 \end{pmatrix}.v_2 + \begin{pmatrix} -3 \\ 0 \end{pmatrix}.v_3 + 5.e_1 + 6.e_2 + 5.e_3 + 12.f$$

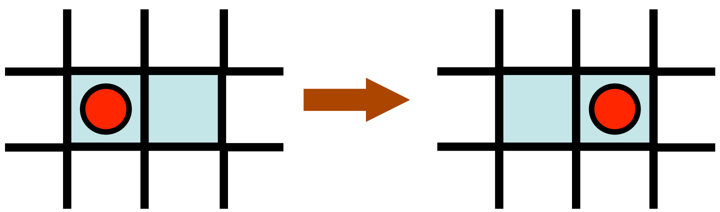
Example: Diffusion Limited Aggregation (DLA)

- Diffusion: some particles are randomly diffusing; others are fixed
- Aggregation: if a mobile particle meets a fixed one, it stays fixed

```
trans dla = {
      `mobile , `fixed  => `fixed, `fixed ;
      `mobile , <undef> => <undef>, `mobile
}
```

NEIGHBOR OF





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```
trans dla = {
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      `mobile , <undef> => <undef>, `mobile
}
```
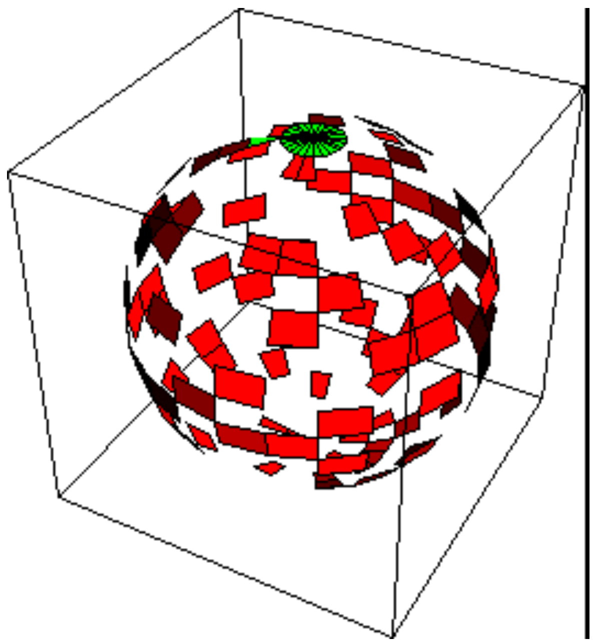
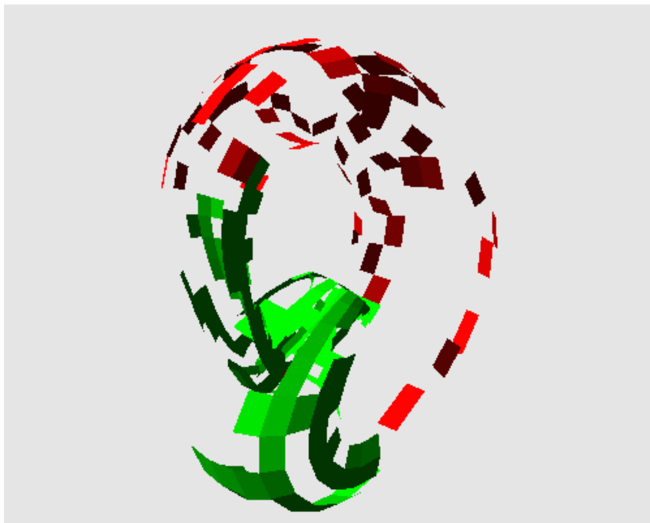
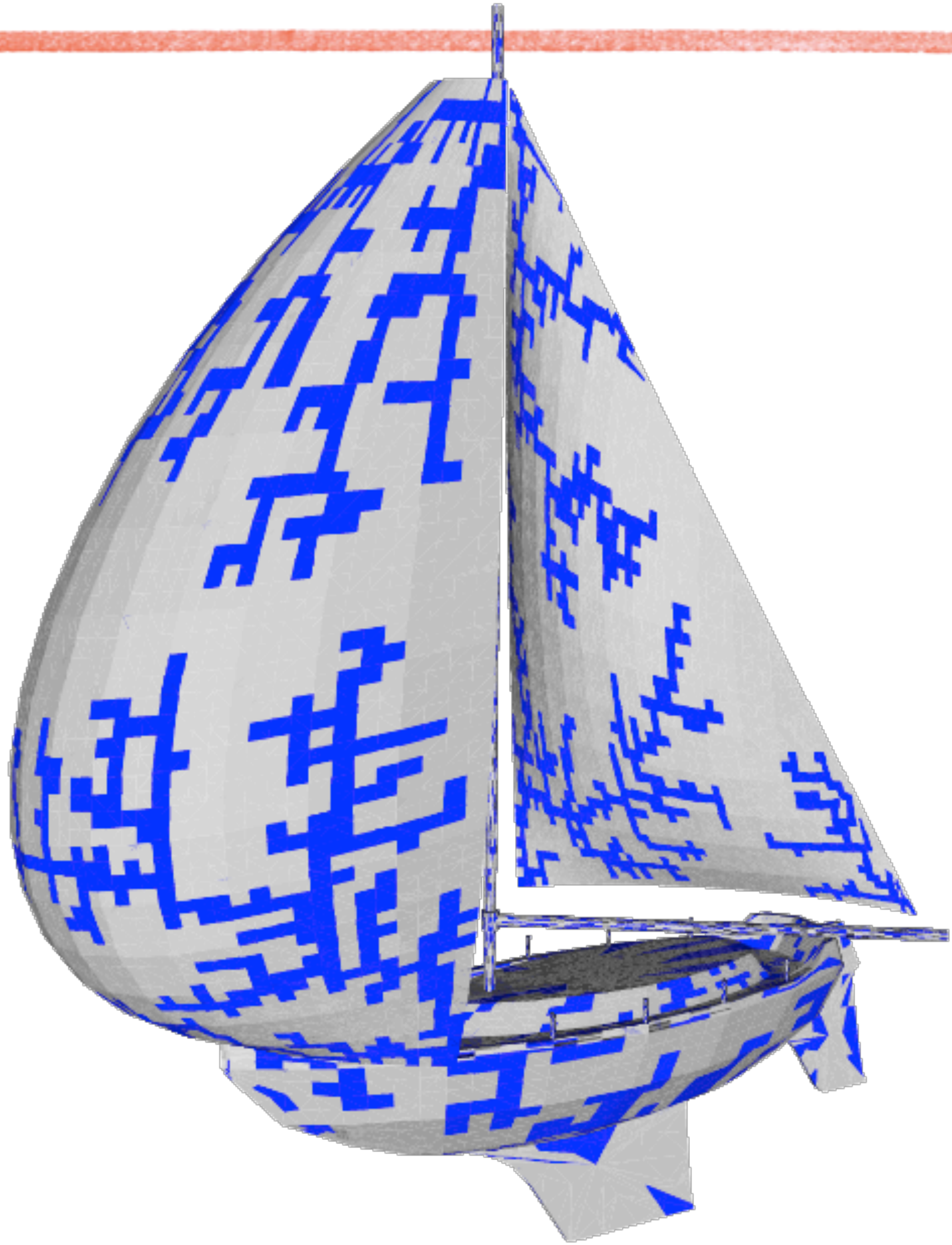

NEIGHBOR OF

this transformation is an abstract process that can be applied to any kind of space

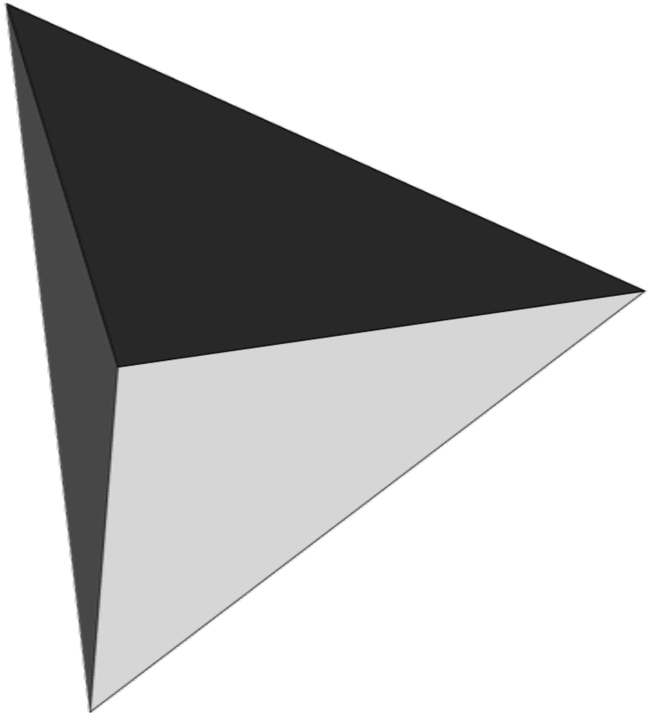




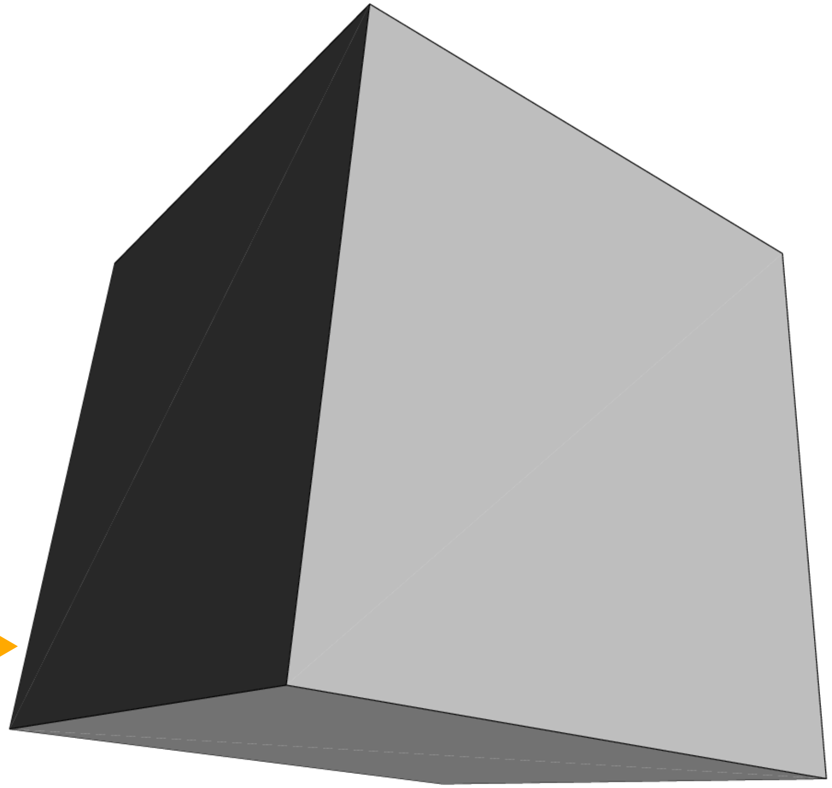
Polytypisme



Fractal construction by carving



Sierpinsky sponge (4 steps)



Menger sponge (2 steps)

The Spatial Approach

- Use space (topology) to unify the various collection structures
  - space as as a resource
  - space as a constraint
  - space as an input/output

- Neighborhood relationships:
  - the structure of the collection
  - the structure of the subcollection
  - the computation dependencies

- Substitution (replacement)
  topological surgery

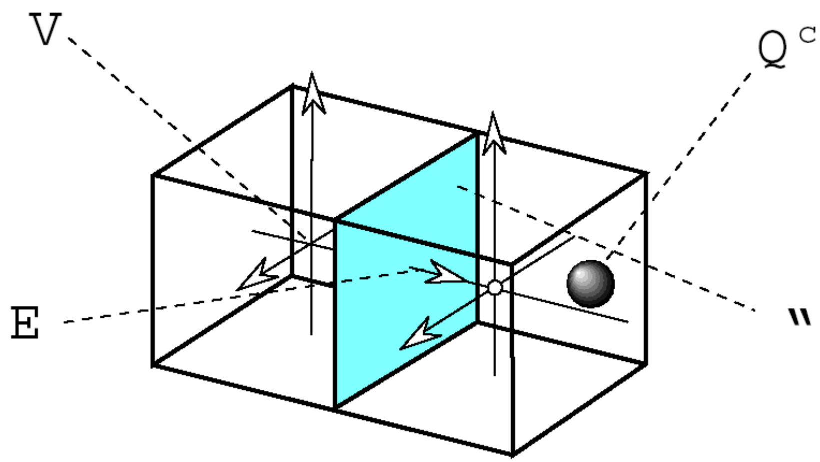
Why higher dimensional objects and not just graphs?

Example of electrostatic Gauss law [**Tonti** 74]

- Electric charge content ρ : dimension 3
- Electric flux Φ : dimension 2
- Law available on a arbitrary complex domain

$$\phi = \oint\!\!\oint w \cdot dS = \frac{Q^c}{\varepsilon_0} = \iiint_{(V)} \frac{\rho}{\varepsilon_0} d\tau$$

electric field in space:
- V: electric potential (dim 0)
- E: voltage (dim 1)
- w: electric flux (dim 2)
- Qc: electric charge (dim 3)

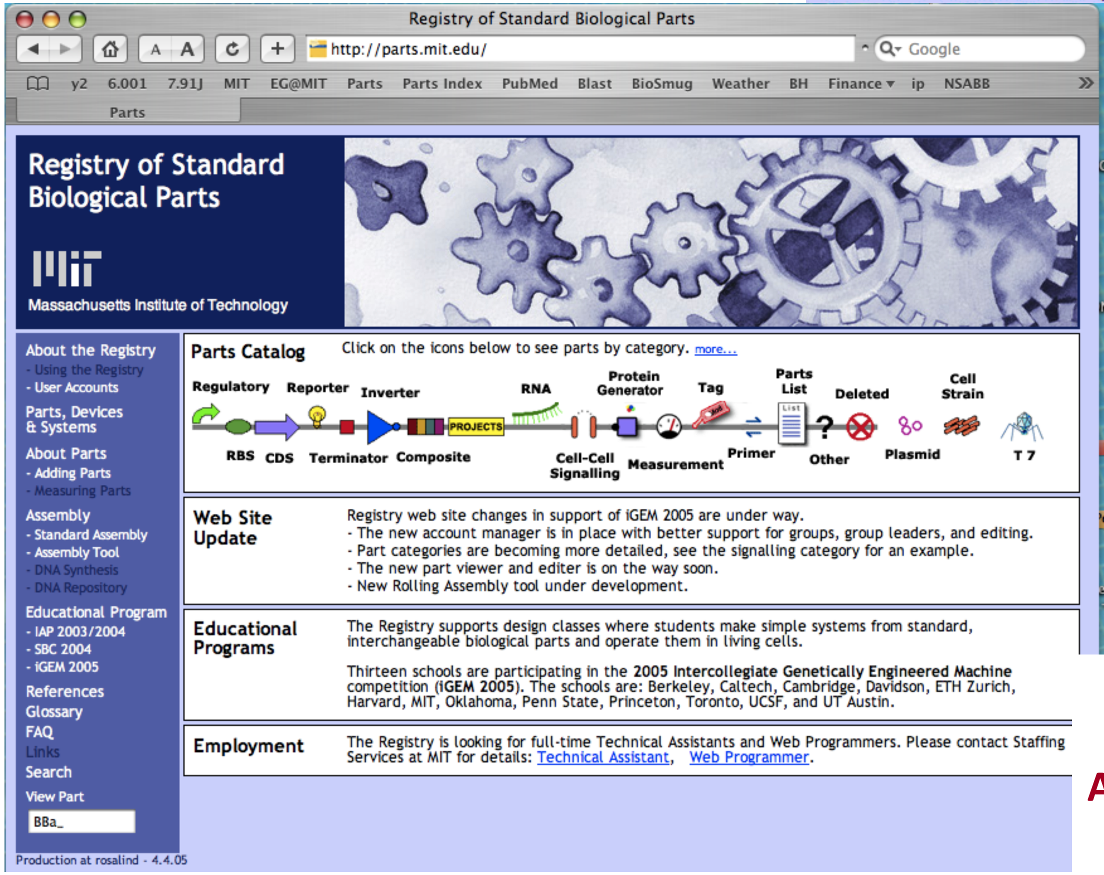


<u>*A Direct Discrete Formulation of Field Laws: The Cell Method*</u>

Properties *w.r.t.* (DS)^2

- *local evolution rules*
  mandatory when you cannot express a global function/relation because the domain of the function/relation is changing in time
- interaction based approach
  the l.h.s. of a rule specifies a set of elements in *interaction*, the r.h.s. the result of the interaction
- *the phase space is well defined but not well known*
  a generative process enumerates the elements but membership-test can be very hard
- *various kind of time evolution* (for the same set of rules)
- *demonstration by induction*
  on the rules or on the derivation (e.g. growth function in L system)

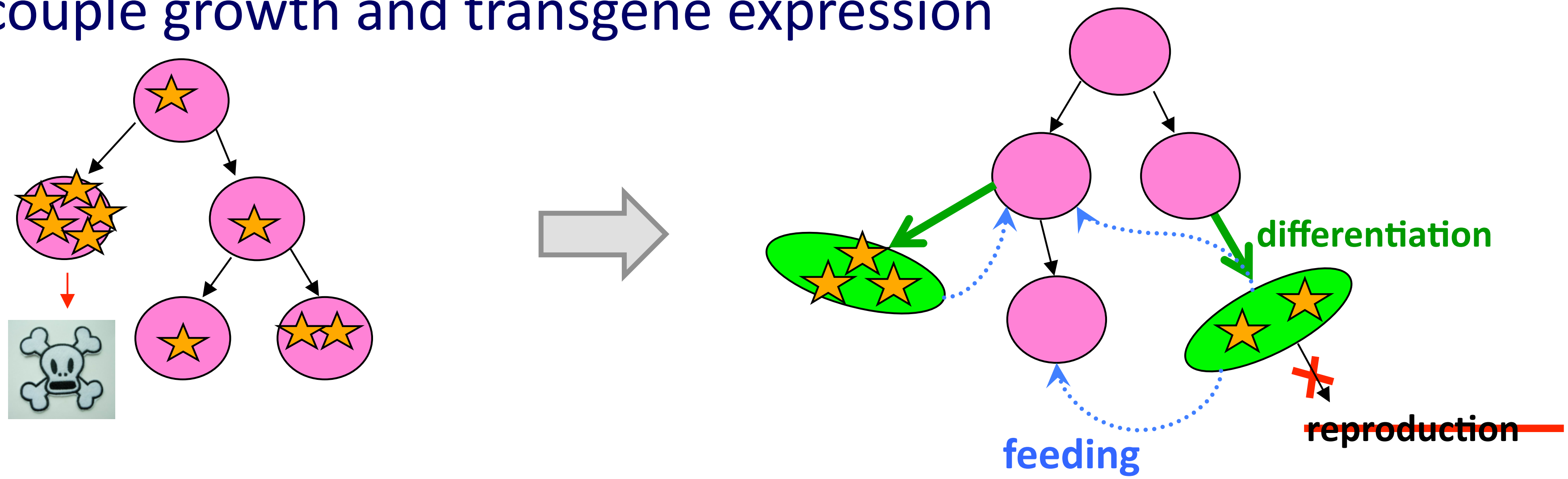
A Synthetic « Multicellular Bacterium »

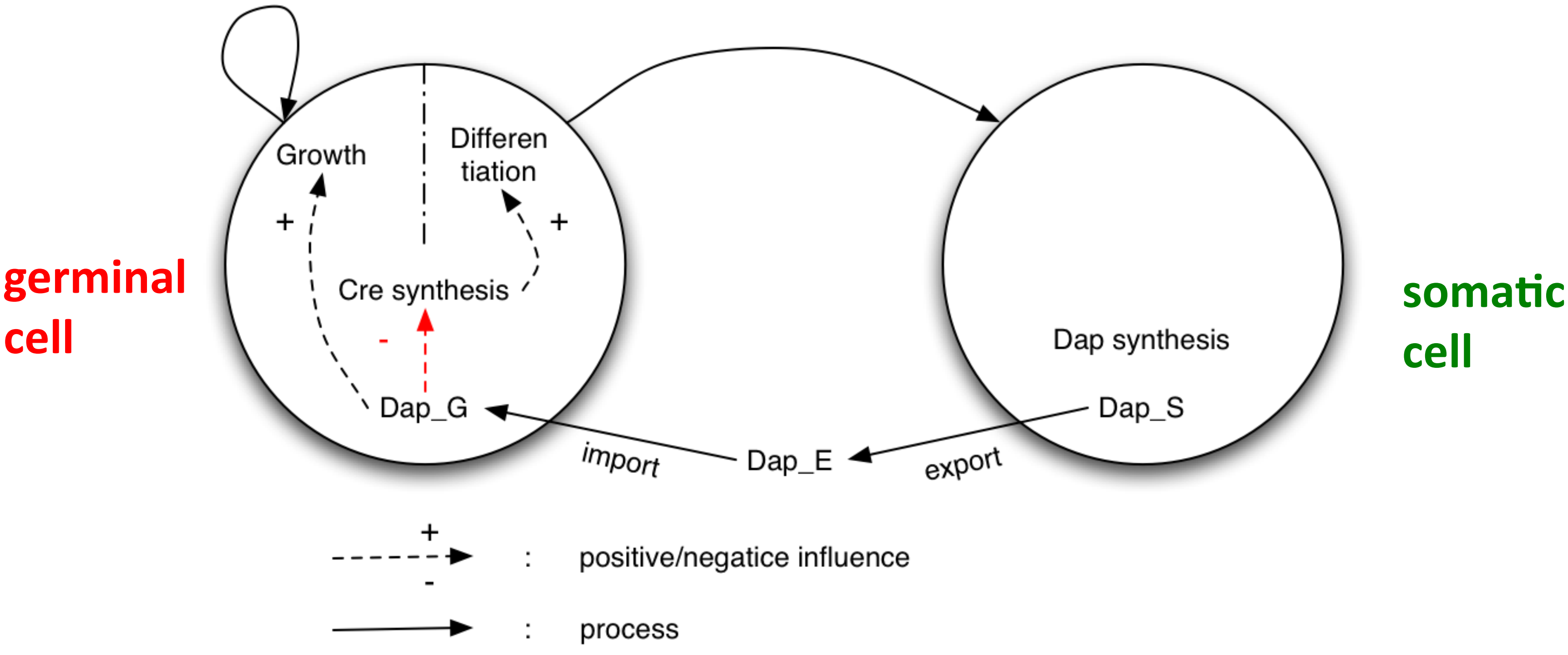


David Bikard, Thomas Landrain, David Puyraimond, Eimad Shotar, Gilles Vieira, Aurélien Rizk, David Guegan, Nicolas Chiaruttini, Thomas Clozel, Thomas Landrain

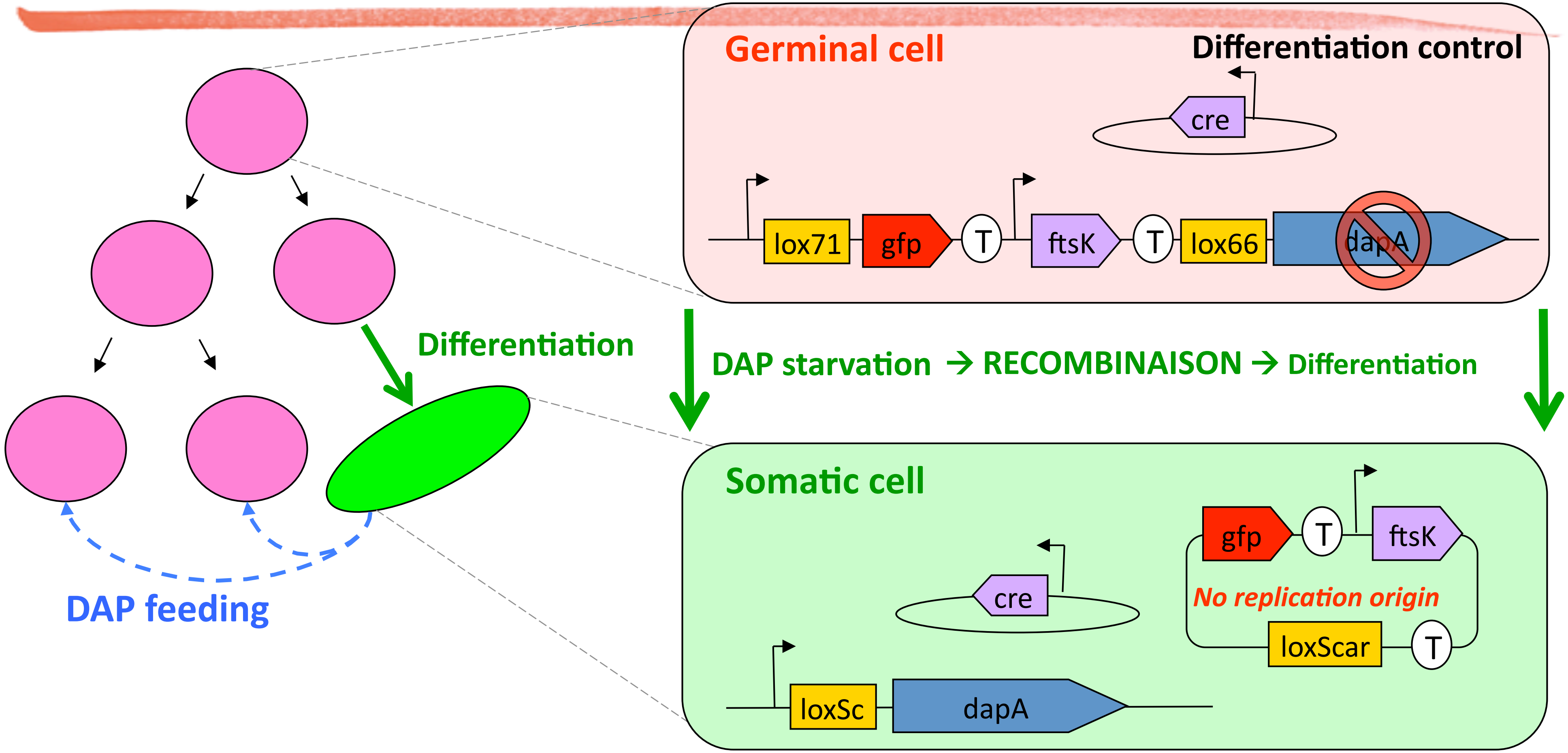
The Paris iGEM project: a « multicellular bacteria »

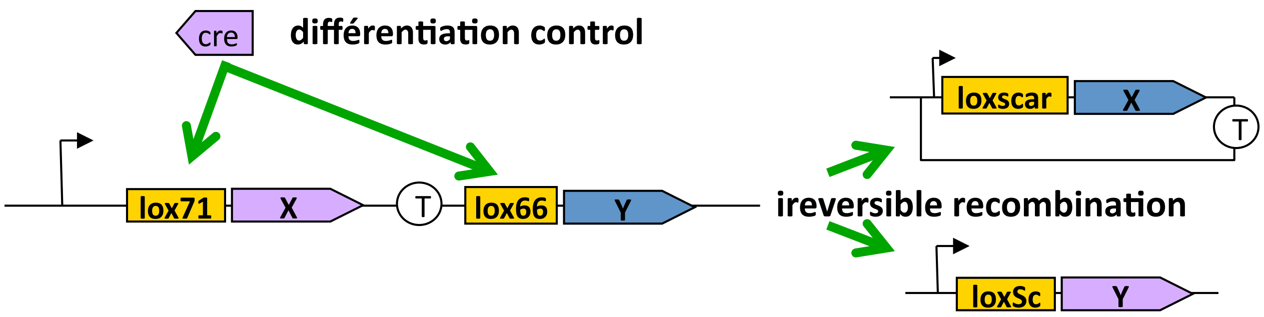
to decouple growth and transgene expression

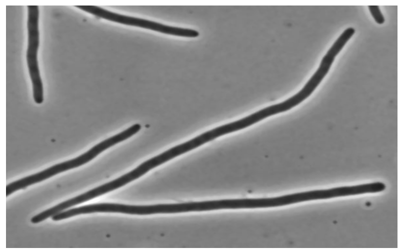




Implementation using BioBricks



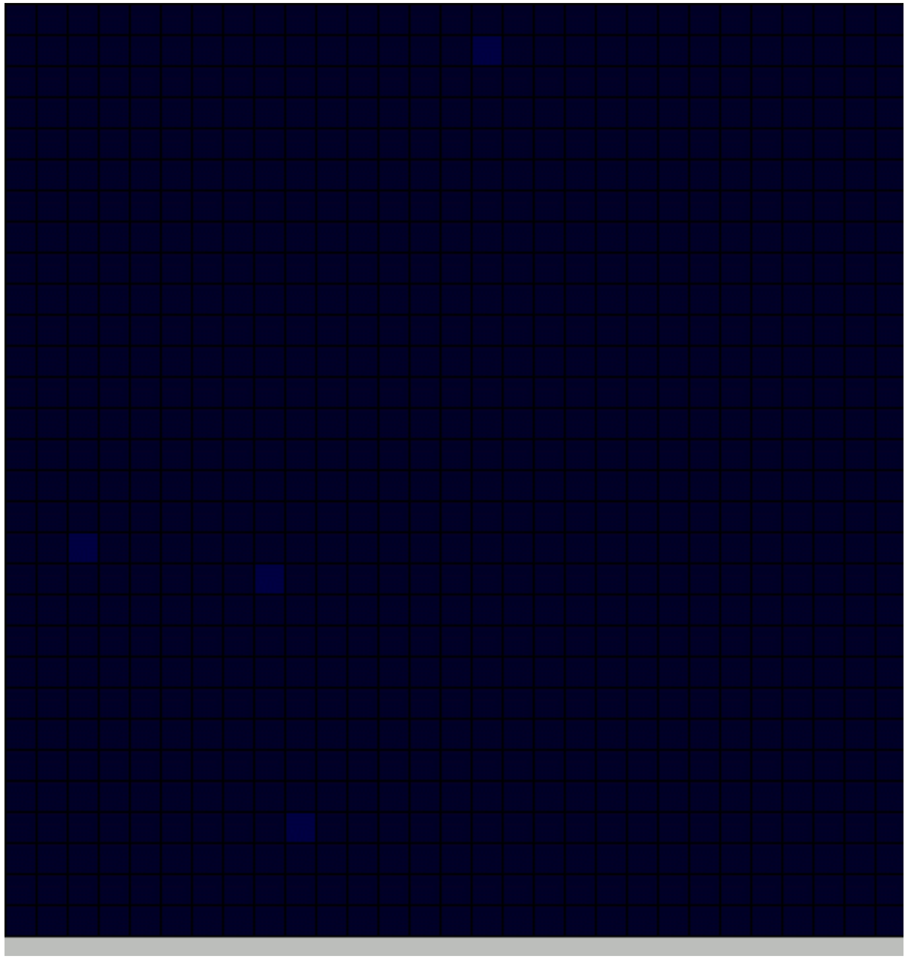




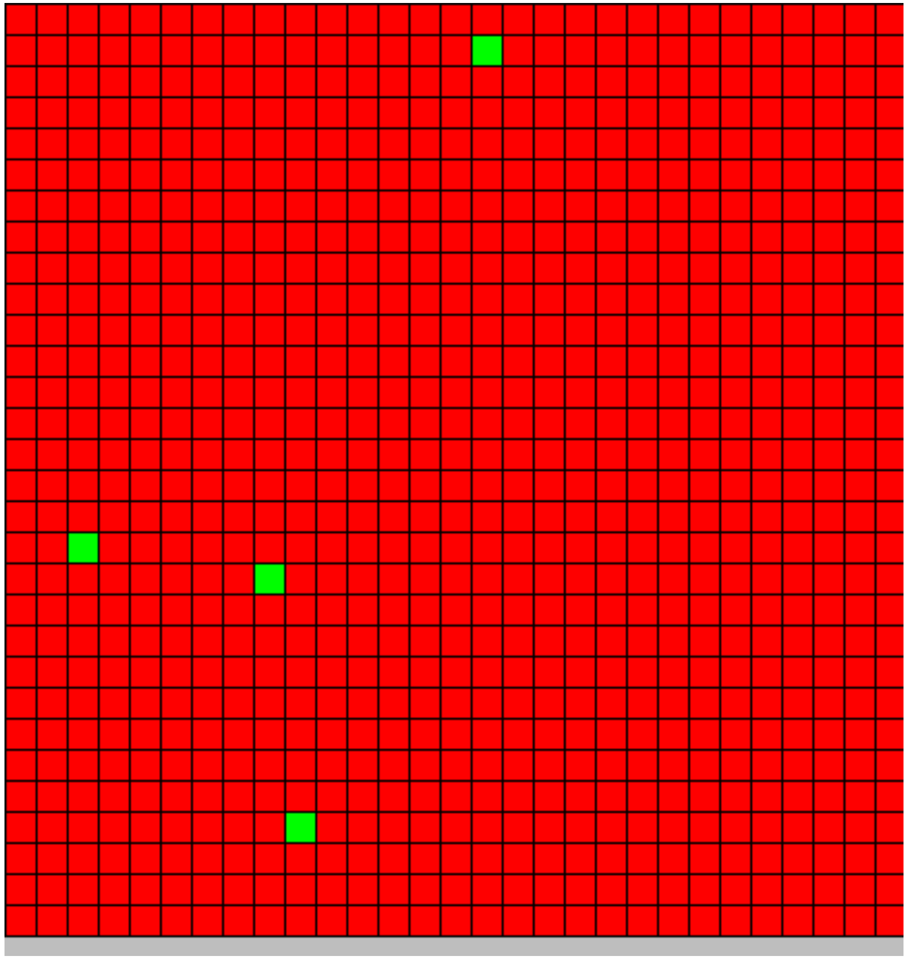
ftsK
needed for
cellular
division

Proof of Concept: Simulation to answer 4 questions

- **How does differentiation induces feeding?** (proof of concept)
  cellular automaton (in MGS)



diffusion of DAP



somatic and germ cell

Proof of Concept: Simulation to answer 4 questions

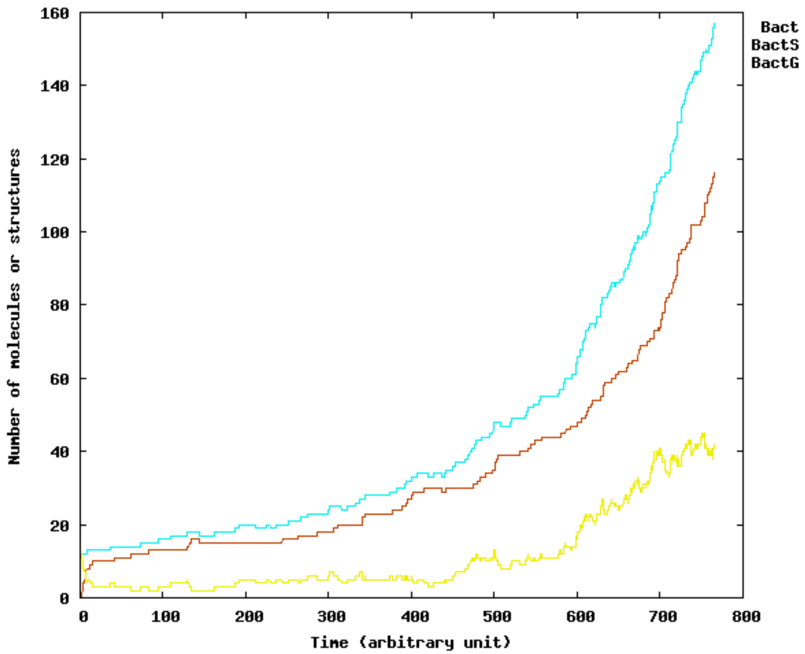
- **How does differentiation induces feeding?** (proof of concept) cellular automaton (in MGS)
- **How do spatial organization and distribution evolve?**
  (diffusion in a growing medium) agents based system (in MGS)

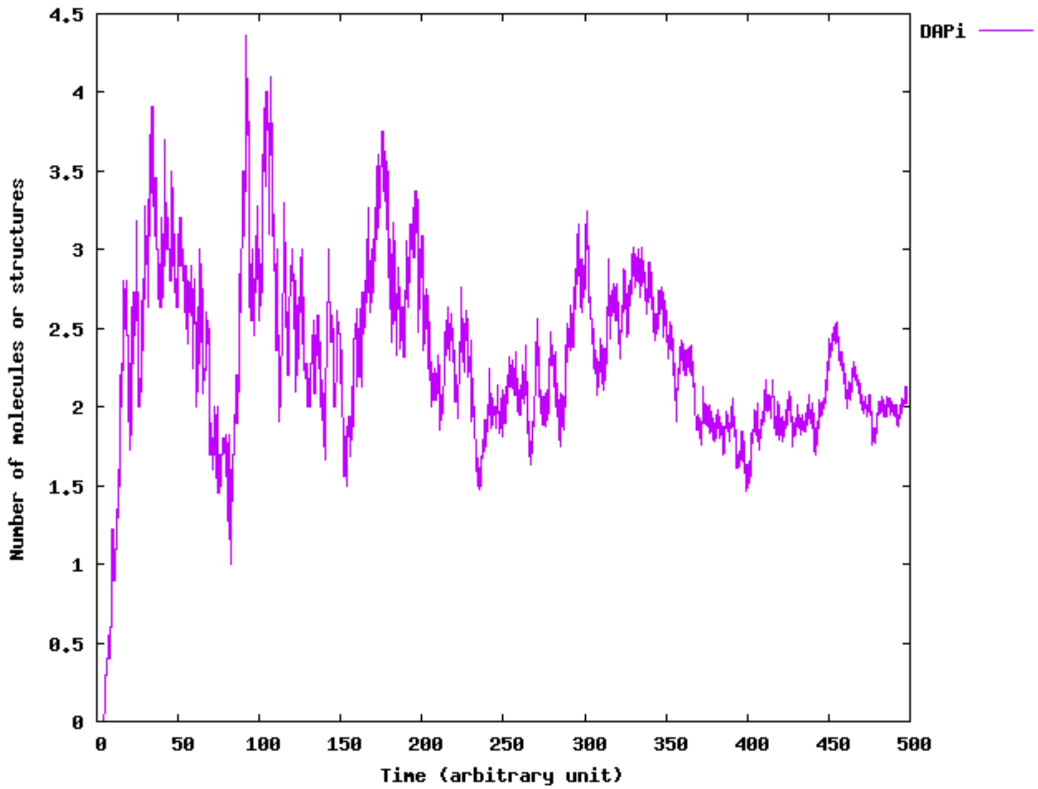
Proof of Concept: Simulation to answer 4 questions

- **How does differentiation induces feeding?** (proof of concept)
  cellular automaton (in MGS)
- **How do spatial organization and distribution evolve?**
  agents based system (in MGS)
- **How robust and tunable is the model?**
  ODE kinetics (matlab)

Proof of Concept: Simulation to answer 4 questions

- **How does differentiation induces feeding?** (proof of concept)
  cellular automaton (in MGS)
- **How do spatial organization and distribution evolve?**
  agents based system (in MGS)
- **How robust and tunable is the model?**
  ODE kinetics
- **How sensitive is the system to noise?**
  Gillespie based simulation (in MGS)





MGS drawbacks and successes

Success

- Polytypisme is good
- Patterns/rules are expressive and usually concise
- Clean semantics

Shortcommings

- Rules may be heavy (e.g. 100 variables for the fractal sponge)
  graphical drawing of rules
  look for better notations (e.g. path pattern)
- Efficiency
  well…
- Implicit methods (solvers) are hairy
  use explicit ones

Conclusion & Perspectives

- A spatial & declarative approach to (DS)2 modeling & simulation
- Generic, formal, expressive & concise language
  - Implementation (mgs.spatial-computing.org)
  - Validation in many fields
    - Self-assembly processes
    - Systems biology
    - Protocol verification
    - …
- Future work
  - Typing, optimizing, compiling transformations
  - Refinement of chemical programs (Gamma, P systems)
  - Music(ologic) representations & analysis

Thanks

http://mgs.spatial-computing.org

- **Jean-Louis**
- **Antoine Spicher**
- **Olivier Michel**

Jean-Louis

Antoine

Olivier

- **PhD and other students**

  Julien, Antoine, P. Barbier de Reuille,
  T. Louail, E. Delsinne, V. Larue, F. Letierce,
  B. Calvez, F. Thonerieux, D. Boussié,
  iGem'07 Paris team, *and the others...*

- **Some past and present collaborations**
  - A. Lesne (IHES, stochastic simulation)
  - P. Prusinkiewicz (Calgary, declarative modeling)
  - C. Godin (CIRAD, biological modeling)
  - H. Berry (LRI, stochastic simulation)
  - G. Malcolm (Liverpool, rewriting)
  - J.-P. Banâtre (IRISA, programming)
  - P. Fradet (InriaAlpes, programming)
  - F. Delaplace (IBISC, synthetic biology)
  - D. Pumain (Geographie-Cité, city growth)
  - R.Doursat (ISC, morphogenetic engineering)
  - F. Gruau (U. PXI, language and hardware)
  - P. Liehnard (Poitier, CAD, Gmap and quasi-manifold)