SOUND AND FURY OF MODELING

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HOTEL IL CAVALIER D'ARPINO

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Titles and abstracts

Italo Capuzzo Dolcetta [Mini Course]

Differential games with a large number of players

Riassunto/Résumé : The mini course*Differential games with a large number of players* aims to introduce a non specialistic audience to some mathematical models of decision processes and to the technical tools employed for their validation, both theoretical and computational.
The first session is dedicated to the description from a dynamic programming-Hamilton Jacobi equations point of view of a classical single-agent optimal control problem.
The second session is an introduction to differential games with a "large" number of interacting agents. A similar dynamic programming-Hamilton Jacobi approach leads to the systems of pde's which forms the core of the theory of Mean Field Games.

Jean-Michel Coron [Mini Course]

A short history of feedback control: From the Ctesibios float regulator to the regulation of rivers

Résumé/Riassunto: The goal of feedback control is to allow a system to remain close to an equilibrium despite various errors and perturbations. There are very common in nature, as, for example, the temperature regulation of the human body by neural feedback mechanisms.

One of the earliest feedback device created by the human is the Ctesibios float regulator used in water clocks. The industrial revolution is a golden age for the construction of mechanical feedback controllers. They include float regulators, pressure regulators and centrifugal governors. These feedback controllers are motivated by practical issues and rely mainly on trial and error together with great engineering skills and intuitions. Buss, Farcot, Foucault, Galloway, Hick, Jenkin, Mead, Molinié, Porter, Périer brothers, Siemens, Thiollier, Villarceau, Watt, in particular, created innovative mechanical feedback controllers with important robustness properties.

One of the holy grails during the nineteen century was the construction of isochronous regulators (i.e. regulators with no offset). Various ingenious devices have been proposed. Unfortunately it was experimentally observed that regulators which were nearly isochronous have poor stability properties. In order to solve the offset problem, the now well-known solution is to use the "integral term" and very few controllers had this integral term at that time. Among these controllers having this integral term, there was the Jenkin controller studied by Maxwell in his 1868 paper. However there were older controllers having this integral term, as, for example, the regulator used by the Périer brothers on the steam engine they installed in île des Cygnes (Paris).

The mathematical studies of regulators started in the nineteen century with the pioneer works by Airy (in connection with the regulation of the motion of telescopes), by Maxwell (in connection with the testing of Ohm's law) and by Wischnegradski.

Related to this notion of feedback control is the key notion of asymptotic stability. Many great mathematicians proposed different notions of stability, as, for example Dirichlet, Lagrange, Laplace, Poisson. However it is only in 1892 that the "good" definition, at least for the purpose of regulation despite errors and perturbations, of asymptotic stability was given by Lyapunov in his thesis. Still in his thesis, Lyapunov gave two fundamental results, which are

now called the first and second Lyapunov's theorem. It is interesting to point out that the first Lyapunov theorem was already used by Maxwell in his 1868 paper.

Feedback control then develops as part of "modern control" with an increasing use of computers and mathematics and the stabilization of complex systems as, for example, systems modelled by means of partial differential equations. As an illustration, recent applications to the regulation of rivers will be presented.

Debora Di Giacchino [Talk]

Modelling Economic Policy in a (representative) democracy

Riassunto/Résumé : In the lecture, after introducing economic policy as a field of study, I will start with the welfare economics' normative" benchmark (economic policy as the choice of a "benevolent dictator") and majority voting in a direct democracy (median-voter theorem). I will then move to model economic policy in a representative democracy using redistribution policy as an example. In the political economy approach, the chosen (economic) policy is the outcome of a political process, modelled as a "game" between, voters, parties, politicians and interest groups. In Downs' (1957) model of electoral competition, the chosen policy is the policy preferred by the median voter. One of the assumptions of the model is that the policy space is unidimensional, otherwise an equilibrium might fail to exist. Several solutions have been put forward for the non-existence of a solution when the policy space is multidimensional. I will discuss probabilistic voting and issue salience as a way to understand "why the poor do not expropriate the rich". Directions for further study will be indicated.

Jean-Stéphane Dhersin [Talk]

Spatial evolution of an epidemic and ≪ social ≫ networks

Résumé/Riassunto: One of the criticisms that can be leveled at the modeling of the epidemic is the lack of consideration of spatial structure. A reasonable approach is to plot the epidemic on a graph. We will see what happens on theoretical graphs. However, understanding the true structure of the underlying graph is often complicated. One method is to look at a random branching walk on the graph to find it out.We will look at the information we can get from it. This method, called 'Respondent-driven' study have been applied in Paris to discover the social network of people who inject drugs (PWIDs).

Antoine Falaize [Mini Course]

The port-Hamiltonian formalism for the passive guaranteed modeling and simulation of multiphysical systems with applications in numerical sound synthesis

Résumé/Riassunto: The numerical analysis of acoustic and audio systems involves the modeling of complex open multi-physical systems, i.e. systems that interact by exchanging energy, matter or information through their boundary and involve subsystems arising from multiple physical domains (mechanic, electromagnetic, thermodynamic, etc.). Besides their apparent complexity, all such physical systems share a common property: they are passive, in the sense that the increase in the internal energy is less than or equal to the energy supplied through the boundary. Preserving this property in the modeling is of great importance since it ensures the stability of the system, otherwise difficult to achieve, and can be exploited to construct unconditionally stable discretization and control schemes. Among the numerous formalisms available in the literature, we focus on that of Port-Hamiltonian Systems (PHS) to address the aforementioned objective. This class of systems has been introduced in the 90's as an extension of Hamiltonian systems defined on Poisson manifolds that includes external (or interface) variables. This construction provides a systematic state-space formulation of open multi-physical systems as models structured according to energy flows, thus guarantying the system's passivity. The PHS theory combines several paradigms, namely (i) port-based modeling, (ii) geometric mechanics and (iii) control theory. We shortly introduce the PHS formalism along with

examples from musical acoustics starting from the simple damped linear oscillator (EDO) to the nonlinear interaction of a bow with a string (coupled PDE and ODE), and then present the open-source CAE software PyPHS that exploits these features to provide a highly systematic toolchain for passive guaranteed numerical analysis.

Jean-Louis Giavitto [Mini Course]

Modeling as programming: the contribution of declarative and domain-specific languages

Résumé/Riassunto: William Thurston emphasized that a computer program is a formalized text whose requirements for rigor exceed those expected of a mathematical proposition. But can we reason about programs in the same way as we reason about propositions? This question is important in the field of modeling, where an increasing number of models cannot be expressed concisely and are specified solely by a computer program, which at the very least allows simulation of the models.

Declarative programming has developed to address this question. And while this programming approach has not become dominant in general-purpose languages, it is indeed prevalent in Domain-Specific Languages (DSLs, programming languages tailored to a specific problem domain or application).

In this mini-course, we will present the expectations and methods of declarative programming. We will illustrate this approach in two very different application areas: the modeling of morphogenesis processes involving both differentiation and development simultaneously, and the modeling of musical time in an application that allows a computer to calculate an electronic voice in real-time synchronization with a live musician.

Francesco dell'Isola [Mini Course]

Principle of Virtual Work as Foundational Framework for Metamaterial Discovery and Rational Design

Riassunto/Résumé : Novel theories are needed for the discovery of innovative and exotic metamaterial and for their rational design. The current practice of mechanical analyses based upon moribund classical theories and experimental trial-error campaigns is caught in an inescapable vortex and illusion of inductive reasoning. The needed novel research paradigm is one in which the formulation of theoretical concepts precede their experimental validation. In the absence of theoretical understanding, the design experiments and collection of experimental evidence will remain unavoidably circumscribed. History of science can provide us guidance in the search for the needed powerful tools required for discovery. The principle of virtual work provides the necessary framework for development of theories that can lead to novel metamaterials, as it was the unifying principle which allowed the French-Italian School, headed by D'Alembert, Lagrange and Gabrio Piola, to found modern continuum mechanics. Based upon this framework we have conceived a metamaterial synthesis schema that exploits micro-macro identification traceable to the early days of the formulation of continuum theories for deformable solids. The schema is illustrated with application to metamaterials with pantographic and granular motifs based upon higher-gradient and higher-order theories.

Chris Hundertmark [Short talk]

Necessity of quantum corrections for semi–classical bounds on the number of bound states for Schrödinger and generalized Schrödinger operators

Résumé/Riassunto: In three or more space dimensions, the famous Cwikel–Lieb–Rosenblum (CLR) bound gives an upper bound on the number of bound (or trapped) states of a quantum system. Such a system is described by the classical Schrödinger operator $P^2 + V$ (P is the momentum operator, V the potential). The CLR bound is in terms of the phase–space volume of the set $\{(\eta, x) \in \mathbb{R}^d \times \mathbb{R}^d : H(\eta, x) < 0\}$, where the classical energy $H(\eta, x) = |\eta|^2 + V(x)$ is negative. I.e., the volume of the set of "classical negative energies" in the phase space $\mathbb{R}^d \times \mathbb{R}^d$. Hence the name 'semi–classical bound'. Even though the volume of this set is usually also finite in dimensions one and two, it is well–known that the semi–classical CLR bound cannot hold in one and two dimensions.

For generalized Schrödinger operators of the form $T(P) + V$, where V is a potential, P is again the momentum operator, and $T : \mathbb{R}^d \to [0, \infty)$ is a measurable function (the momentum–kinetic energy relation of the free particles), the question of existence of quantitative bounds for the number of bound states is even harder than for the usual Schrödinger operator, where $T(\eta) = |\eta|^2$ for $\eta \in \mathbb{R}^d$. It turns out that there is a simple necessary and sufficient criterion for a quantitative bound to hold: Under mild regularity assumptions on the kinetic energy symbol T, a quantitative bound for the number of bound states of a generalized Schrödinger operator $T(P) + V$ exist, if and only if the inverse function $1/T$ is locally integrable, in particular, integrable near the zero set of T, given by $\{\eta \in \mathbb{R}^d : T(\eta) = 0\}$.

Marta Menci [Short Talk]

Modelling and simulations of cell dynamics across the scales

Riassunto/Résumé : I will present a quite general class of hybrid mathematical models of collective motions of cells under the influence of chemical stimuli. The models are hybrid in the sense that cells are discrete particles driven by ODE, while the chemoattractant is considered as a continuous signal which solves a diffusive equation. For these models it is possible to prove the mean-field limit in the Wasserstein distance to a system given by the coupling of a Vlasov-type equation with the chemoattractant equation. Moreover, in the monokinetic case, a new pressureless nonlocal Euler-type model with chemotaxis has been rigorously derived. Far from the monokinetic assumption, a numerical study has been performed, as a first step in the study of the model at different scale. Two main aspects will be debated: the difference with respect to Euler system of equation accounting for pressure, and the role of the non-local integral term. Numerical simulations, investigating the role of key parameters of the model in different scenarios, will be shown.

Thierry Paul [(mini)Mini Course]

How macro models micro, and conversely

Résumé/Riassunto: Understanding how a vision of he macroscopic world, namely the one in which we live, can be obtained out of a microscopic one, namely the one of "elementary" particles, is one of the oldest task in sciences, It relies in particular on how a continuous paradigm of the matter such as the one we actually perceive can be related to a discrete one like the one, say, of atoms or molecules. The scope of this equation is quite universal, as it covers physics, biology, social sciences and more, all of these fields being hatted by a needed mathematical setting We will treat both bottom-up and up-bottom cases: we will first consider systems of a large number of particles (indistinguishable) or agent (distinguishable) subject to two-by-two interactions with a so-called "mean-field" scaling (bottom-up). Both cases of particles (cells) in presence of chemiotaxis and the so-called graph limit of agent systems will be considered. As a by-product we will then show how to approximate partial differential equations by a system of agents with an interaction becoming singular when the number of agents diverges (up-bottom).

——————— [(mini)Talk]

Quantum mechanics is intuitive outside the classical modeling

Résumé/Riassunto: In this brief and by no means technical talk, I would like to show that the fake quantum paradoxes, considered as a trace on nonintuitivity, arise in fact when quantum mechanics is modeled on the classical paradigm, and only in this case. Choosing the right modeling, namely the one directly linked to th fundamental axioms of the theory non only crosses out any inconsistency, but, even more important, that makes quantum perfectly intuitive. The argument relies on the fact the axiomatic mathematical quantum paradigm can be modeled, without

any reference to classical physics, to everyday life situations such as music, 3D movies, and even, unexpectively, the very practical experience of facing a traffic light.

Luca Placidi [Talk]

Granular-based continuum elasto-plastic–damage variational formulation for strain gradient solids

Riassunto/Résumé : This work is devoted to the presentation of a continuum theory for materials having granular microstructure. It accounts for tension–compression asymmetry of grain interactions and for dissipative phenomena like damage and plasticity. The continuum description is constructed by assuming expressions of elastic and dissipation energies as well as postulating a hemi-variational principle. Granular micromechanics is connected kinematically to the continuum scale through Piola's ansatz. Karush–Kuhn–Tucker (KKT)-type conditions, providing evolution equations for damage and plastic variables associated with grain–grain interactions, are derived solely from the fundamental postulates. Numerical experiments have been performed to investigate the applicability of the model. Results show: (i) damage and plastic induced anisotropy evolution including the emergence of a type of chiral behavior. (ii) formation of finite localization zones. (iii) loading–unloading histories have been considered to elucidate the material hysteretic features of the continuum. (iv) The interaction between damage and plasticity, each having an effect on the other, shows the fatigue behavior of the material. (v) A particular expression for the dissipation energy results in numerical simulations of the experimental behavior of the Ultra High Performance Concrete. (vi) Emergence of the critical state of a granular assembly has been derived with a systematic parametric analysis.

Mario Pulvirenti [Talk]

The Boltzmann equation as limit of mechanical particle systems

Riassunto/Résumé : I will discuss how to derive rigorously the (irreversible) Boltzmann equation as a low-density limit of (reversible) Hamiltonian systems.

Tobias Ried [Short talk]

Domain branching in materials: regularity theory in action!

Résumé/Riassunto: The highly complex patterns at the boundary of type-I superconductors (in the intermediate phase) are believed to be the manifestation of domain branching between the normal and superconducting regions in the interior of the sample. This is a common phenomenon in systems where short-range interfacial energies compete with long-range field energies, and can also be observed in strongly uniaxial ferromagnets and as twinning in shape-memory alloys. I will show how these phenomena, through local energy bounds, are related to interesting questions from elliptic regularity theory.

Alice Rixte [Short Talk]

Structural representation of media

Résumé/Riassunto: This presentation proposes a to symbolically represent any media content in a structured manner. It focuses on understanding the role of multiscaling, which is pervasive in both the analysis of piece of art as well as in the creation process. In particular, we will focus on multicale representation of a very simple tonal piece of music, in order to illustrate our model.

Stefano Rossi [Short Talk]

Mean-field limit and propagation of chaos for topologically interacting particle systems

Riassunto/Résumé : In this talk I will consider N-particle systems with "topological" interactions: in this case the interaction between two agents does not depend on the euclidean distance between them but rather on the proximity rank. These topological models has proven to be very efficient in describing the dynamics of flocks of birds, locust swarms, fish schools, etc.

I will briefly present various works in collaboration with D. Benedetto, E. Caglioti, P. Degond, T. Paul and M. Pulvirenti where we study the validity, in a scale regime called "mean-field" regime, of the statistical description of the evolution of these systems by means of effective partial differential equations, in the limit in which the number N of agents is large.

In the topological case, the rigorous proof of this limit procedure is made challenging precisely because of the topological nature of the interaction, being in general discontinuous and depending on the positions of all the N particles.

Michael Schmidt [Talk]

On Michel Foucault's "Naissance de la clinique"

Résumé/Riassunto: The birth of the clinic may be considered as a reflexion on the changes within the thought itself. This means on the changes of the status of time and space, and the impact of them on the human expression itself. The clinic is for Foucault the concrete actualization of these processes. It is at the same time model and realization.

Giovanni Seminara [Talk]

A long journey wandering through Fluid Mechanics: Bio- Morpho- Eco-

Riassunto/Résumé : I will briefly illustrate my attempts to express the "transversal capacities to reach multi-disciplinary fields" in my research activity.

Marco Stroppa [Talk]

Elements of cognitive composition and sensory thought

Riassunto/Résumé : In the realm of music composition, there exists a fascinating relationship between the initial spark of inspiration—the driving force behind a composer's creative endeavour—and the ultimate manifestation of that vision, often represented by a meticulously notated score, particularly within the classical tradition I belong to. This creative process involves a series of steps, each uniquely nuanced and sometimes even subconsciously traversed.

While every composer possesses a distinctive approach, their creative methodologies can be categorized into general models: combinatory, procedural, constraint-based, morphological, gestural approaches, etc. Drawing inspiration from cognitive psychology and Eleanor Rosch's prototype theory, I will put forward a morphological model developed in the 80s. This model has not only profoundly influenced my own compositional work, but has also found resonance in the creative processes of other composers. During my presentation, I will provide a concise analysis of some of my instrumental pieces, and will delve into the profound implications they have on the design of sound synthesis by a computer. Through this exploration, I aim to shed light on the intricate interplay between abstract models and written music, offering insights into the creative process and its transformative impact on musical expression.

Niccolò Tassi [Short Talk]

Asymptotic Behaviour of a Nonlocal Fokker-Planck Equation

Riassunto/Résumé : We present a nonlocal Fokker-Planck equation $\partial_t u = \frac{1}{\varepsilon^2}[J_\varepsilon u - u] + \operatorname{div}(xu)$ on $\mathbb{R}^d$, in which we can estimate the speed of convergence to equilibrium independent of the non-locality of the equation. This uniform estimate cannot be easily obtained with standard inequalities or entropy methods, but can be obtained through the use of Harris's theorem, a tool coming from probability. We point out some interesting links to quantitative versions of the central limit theorem in probability. We also carry out a study of the properties of the stationary states of the problem, especially the behavior of its tails using various techniques.

Cristóbal Villalobos Guillén [Short Talk]

Different numerical approaches for the Magnetization Inverse Problem in 2D

Résumé/Riassunto: The problem to be presented is the one of inverse scanning magnetic microscopy, which aims to recover magnetization distributions of thin rock samples. It is based on the Poisson equation $\Delta\phi = \boldsymbol{\mu}$, where the vector filed $\boldsymbol{\mu}$ is to be recovered from measurements of one or more components of $\nabla\phi$. This inverse problem is ill-posed since the forward operator has a kernel so extra assumptions are needed to ensure uniqueness of solutions. We will start by exploring the theoretical limitation of the inverse problem given by this kernel. Then, we will focus on the planar case, where we have found two particular cases where we could theoretically recover the original magnetization. This is done by taking the group LASSO regularization technique letting the regularizing parameter to zero. Unfortunately, this method relies in zero noise, which is not the case when working with real data and we will show what the problems of the naive use of the group LASSO technique. Then, we will show different techniques to overcome this issues, including extension of the data or changing the how the measurements are taken, together with different machine leaning techniques.