Methodology

Developments in the techniques employed in research and in the practice of forecasting

Q-ANALYSIS

A hard language for the soft sciences

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The relevant data sets in a 'soft' science can be manipulated and analysed using topology, an exercise which also reveals the 'backcloth' which limits or modifies such interrelationships. The method is currently being used in many fields: eg industrial relations, medicine, and architecture. An example of a university's committee structure is used to show how the underlying, and often unnoticed, geometry can frustrate the aims of an organisation.

SOFT SCIENCE requires mathematical refurbishing. Almost all of our daily lives falls under the spell of soft science: eg such disciplines (if that is not too strong a word) as social science, politics, economics, industrial relations, community studies, planning (by governments and other bodies), organisational analysis, decision making, and general systems analysis.

Hard science constitutes the highly mathematicised disciplines of mechanics and physics—and their 19th-century spin-offs to be found in chemistry, biology, and all the engineering fields. It is hard science which has taken man to the moon, and which has created the technological world we are trying to live in: it is soft science which is grappling with all those difficult problems which arise on the social scene—problems which seem to have become more acute as the technological advances shrink both the space and the time of our world. This shrinking is no mere illusion either, but a reality expressed by our experiences of the objects which generate our sense of space and of the events which generate our sense of time.

It is not surprising that some 'hard' scientists (and others) find it irresistible to sneer at the soft sciences, on the grounds that they are 'not science', and this criticism is sound (as a conclusion). But there was a time (the 12th century) when mechanics was just as soft as sociology is today. It is now time for us to transform our soft science into hard science—and the sooner the better. But how is this to be done?

The role of mathematics

It is certainly not just a matter of introducing some mathematical symbolism into the subject—although this is an illusion commonly, but often unconsciously, accepted

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by many academics working in this field. For after all mathematics is only an esoteric language, even though it is the best we have for carrying and expressing the notions of logical deduction. Behind mathematics, preceding it in the development of any hard science, lies a scientific *method* which enables us to marshal facts into disciplined sets of data—sets which are *relevant* to the study in hand. And this idea of relevance, seeming to beg the question, is quite unavoidable in any research enterprise.

No scientist worth his salt ever confuses data relevant to measuring masses with data relevant to observing electric charges, and this distinction precedes any description of these entities in an esoteric mathematical language. In the end there has to be a common ground among scientists about this very point of relevance, and this ground is an expression of the human sensibilities to external phenomena—sensibilities which appear to be tuned to intuition rather than to rational discourse.

We cannot hope to make soft science hard merely by transporting the concepts and words of (say) physics into some new area like social studies. The assumption, in general systems theory, that any system (however soft) must be modelled by a collection of differential equations is a case in point. Such systems (of differential equations) refer to a backcloth of data sets which might well have been acceptable in some field of mechanics—where 'particles' constitute the objects and 'vector fields' provide the dynamics—but in a multidimensional company manufacturing various products via the exploitation of a thousand different skills we are likely to need concepts which are more complex than that of 'particle', and relations which are more relevant and awkward than the convenient mathematical functions contained in such a model. Furthermore, when these functions are taken as probability functions (when the description becomes the so-called stochastic model) we seem to be abandoning any hope of a deterministic scientific theory.

Nor should we assume that, in making social science into a hard discipline, we shall find physics-type laws of nature—indeed the search for them could well be an intellectual straitjacket which inhibits discovery. It might be truer to expect *meta-laws*, in the sense of 'laws which tell us how the (apparent) laws are allowed to change' —or even meta-meta-laws?

This would be like finding that the law of gravity is not some fixed distance function (as we believe it to be in physics) but rather some algorithm which tells us how and when to find just one such distance function, among many. So in this sense we would expect the current model-building in the soft sciences to be replaced by *meta-modelling*—a do-it-yourself kit for building relevant models and the relations between them as they change. This point of view also emphasises that the beginnings of this whole process must be with the study of mathematical *relations* between sets of relevant data.

Such relations have their own geometrical structures which are far removed from the usual metrical and euclidean geometries of the school textbooks. Indeed there is no necessity, in the structures we have in mind, for the introduction of distance (or of angle)—those entities which mathematicians refer to as metrical—for these concepts are often an obstructive imposition on that underlying geometrical structure which is naturally defined by a relation. And the strange connectivities of these structures lift them into multidimensional spaces where only a well-programmed computer can find its way about.

The method of *Q*-analysis, which I have developed over recent years, concerns itself with this central problem of finding, analysing and interpreting the *structures* of relations in those areas of our experience we are describing as 'soft'. The whole field can in a certain sense be treated as that of decision making—for that encompasses organisations (of workforces, of committees, of assembly lines, of management, and unions), planning (by taking cognisance of structural changes and by monitoring hypothetical or actual policies), and social and political affairs (which depend upon the choice of relations between people and institutions, and on what the resulting geometry can carry in the way of traffic). In any event the claim is that we are dealing with a structure which is capable of being defined in precise mathematical terms (it is technically known as a *simplicial complex*), and in any particular study, there is a structure which acts the part of a (relatively) static *backcloth* (say, S) which replaces the conventional three-space of the physical scientist. This backcloth, S, must carry the 'traffic' of whatever kind—just as the euclidean space of the engineer must carry the traffic of 'motion particles'. By 'carry' I mean only that the connectivities of the geometry, at any appropriate dimensional level (and the letter Q in the theory merely serves as a parameter for these dimensions), must be the determining features (the so-called *topology*) for allowed flow of traffic (or changes in that flow).

Structure and traffic

For example, there might be 250 skills (job types) needed to manufacture a range of products in 150 locations (factories or parts of factories), in some large company. The distribution of these skills (whose names, we suppose, form a set Y) among the locations (whose names form a set X) defines a mathematical relation, and this identifies two geometrical structures (simplicial complexes) which we denote by $\mathrm{KY}(X)$ and $\mathrm{KX}(Y)$.

In $\mathrm{KY}(X)$ the skills are represented by convex polyhedra (in a suitable multidimensional space) whose vertices are various selections from the locations—so if skill Y_1 is employed at four locations it is represented by a tetrahedron (a 3-dimensional entity with four vertices), and so on. The connectivity between different skills is now expressed by the polyhedra (or faces thereof) they share—that is to say, by the locations they share.

In $\mathrm{KX}(Y)$ we get a conjugate situation, where the locations (the X's) are represented by polyhedra whose vertices are various selections of the skills. So, for example, the factory X_1 might be a 30-dimensional polyhedron (or simplex) if there are 31 skills located there. If one is primarily interested in the skills (or the people who possess them), as the trades unions might well be, then the relevant backcloth S is the structure $\mathrm{KY}(X)$. If one is primarily interested in the locations, as the company directors might well be, then the relevant backcloth S is the structure $\mathrm{KX}(Y)$—and these are not mutually exclusive but then they are unlikely to be identical.

What is the traffic which can be observed on this kind of backcloth?

When $\mathrm{KY}(X)$ is the backcloth geometry we can have for example the distribution of employees per skill throughout all the locations, as possible traffic (which possibly changes over a period of time); or the wages bill per job types; or the fringe benefits, or other working conditions, per job types throughout the company. Such traffic might well be the dynamics generated by trades unions: indeed a particular trade union might well be regarded mathematically as being defined by such traffic on $\mathrm{KY}(X)$.

When $\mathrm{KX}(Y)$ is the backcloth, we need traffic to be defined on the locations (in their relation to the skills). So recurrent costs per factory could be traffic in this case; or the goods produced in each location; or the productivity per factory; or the costs of raw materials per factory; and so on. Whenever any of these things *change*, either in fact or hypothetically by way of policy proposals, the changes must adjust themselves so as to be compatible with the underlying connectivities of the backcloth, S. So, for example, production of some goods at a certain factory might fall, due to three kinds of workers staging a go-slow action. The goods affected are naturally those which constitute traffic on those polyhedra, in $\mathrm{KX}(Y)$, which have the three skills as vertices, and the *spread* of such changes throughout the company depends exactly on the connectivity between the locations (the X's) *vis-à-vis* these particular skills.

Such *changes* in traffic constitute the presence of *structural forces* throughout the backcloth—and such forces will be felt as keenly by the traffic (production of goods) as gravity is felt by falling bodies. Controlling such forces obviously involves being able to control the topology of these backcloth structures—and this can only be done by first discovering what they are.

Notice too that there is often a confusion between traffic on the two conjugate complexes $KY(X)$ and $KX(Y)$. For in labour relations we can find a trade union being described by forces on $KY(X)$, via its interest in job rewards throughout all locations, although the shop steward (sited at one location) is described by forces on $KX(Y)$, via some specific factory and its various skills. And when management is accused of being indifferent to the fate of the workers it is an expression of the apparent conflict between those who see only the backcloth $KY(X)$ and those who see only $KX(Y)$.

In the latter case we can understand how the workers (who represent the skills in the set Y) become 'only vertices' (or identical units) subordinate to the locations (the set X), whilst in the former case we get the exact opposite (people count more than factories). But the truth is that both backcloths are generated by the *same relation*; the real backcloth is the union of both $KY(X)$ and $KX(Y)$; traffic on one part is subtly dependent on traffic on the other. Salvation in industrial relations lies entirely within our grasps if we can only understand this interplay of traffic on the two conjugate backcloth structures. A study currently underway with a large British company has demonstrated that a new insight can be gained into that total structure on which manufacture is actually based—and the roles of unions and management can be discussed in a common language (and understanding) which augurs well for bringing the two sides together.

Although the relations which we examine in finding a backcloth must be between relevant and fundamental data sets, there are other relations (with similarly defined structures) which naturally arise via the need to avoid logical difficulties connected with the properties of mathematical sets. These are logical difficulties which centre around well-known paradoxes and which are avoided by appealing to the Russell theory of types. Essentially this means that we must be careful to distinguish between elements or members of a set X and subsets of those elements. All the subsets of a set X constitute another logically distinct set, which mathematicians call the power set, $P(X)$, of X. In this language of structure these distinctions are maintained by regarding the data sets as forming a *hierarchical* scheme, say H, in which the levels are referred to as N, $(N \pm 1)$, $(N \pm 2)$, etc.

Table 1 illustrates some areas where the method has been (or currently is being) applied, and indicates some of the sets (hard data) and relations which help to define the relevant backcloths.

The point is that data sets at, for example, the $(N + 1)$ level consist of elements which represent subsets (or collections) of the elements at the N level; and so on. But such an arrangement means, once more, that the whole of H is represented by a collection of relations—with their consequent geometrical structures. For example, P might be a set of individual people and this set could be regarded as being at the N level in the hierarchy H of data. Then at the $(N + 1)$ level we could have a set $\overline{P}$ whose members are the names of groups of these people (eg societies they belong to, or committees they sit on, or clubs they join, or area they live in). Any person, P_1, in the set P is then clearly related to some of the group-names in $\overline{P}$; the resulting relation naturally defines the simplicial complex structures $KP(\overline{P})$ and $K\overline{P}(P)$, as above. Such a sort-out of the relevant data sets into a number of hierarchical levels

TABLE 1. CURRENT APPLICATIONS OF *Q*-ANALYSIS

Area of study	Data sets	Relation between sets	Area of study	Data sets	Relation between sets
Urban planning	City streets, commercial activities	Defined by locating activities in streets	Design problems	Available hardware, functional requirements	Defined by dependence of one on the other
Regional planning	Towns and villages, land-use activities	Defined by locating activities in towns etc	Architecture	Visual features of hardware, portions of buildings	Defined by association of features and structure
Clinical psychology	Individual people, list of psychological conditions	Defined by associating conditions with the individuals	Medical diagnosis	Medical symptoms, patients	Defined by patients exhibiting symptoms
Industrial relations	Manufacturing sites, list of job types	Defined by locating jobs in factory sites	Politics	Political proposals, political parties, political groups	Defined by associating proposals with groups
Large organisations	Functional activities, executive employees	Defined by responsibilities of executives			
International flow of television programmes	Programme descriptors, television productions	Defined by describing the programmes	Transport	Traffic routes between towns, list of roads/streets	Defined by routes containing streets

plays a major role in any application of Q-analysis—and seems to constitute a novel obstacle at the beginning of any specific study. Now the backcloth, S, needs to be described at each of these levels—and is consequently denoted by $S(\mathcal{N})$ or $S(\mathcal{N}+1)$ etc. Furthermore the traffic on that backcloth will be describable in a similar way.

What shape is an organisation in?

Using this method we can set about analysing any complex organisation—whether it be British Leyland, the civil service, or some specific community. For example, in a recently published study of the University of Essex it was argued that a minimum of five hierarchical levels were needed for the backcloth and associated traffic.[1] These were conveniently referred to as the $(\mathcal{N}-2)$ level, $(\mathcal{N}-1)$ level, $\mathcal{N}$ level, $(\mathcal{N}+1)$ level and $(\mathcal{N}+2)$ level. The level at $(\mathcal{N}-2)$ was taken as the one at which the individual (eg person, room, lecture topic, or restaurant menu item) ceases to act as a cover for anything else. The highest level, $(\mathcal{N}+2)$, was the level of the university senate (in the set of committees) and this acted as a cover of $(\mathcal{N}+1)$-level committees (school boards and other subcommittees or senate). At the $\mathcal{N}$ level we were able to place departments and various other entities of an administrative nature, whilst at $(\mathcal{N}-1)$ we would find research units/groups—being collections of individuals in the department.

A major task was to find a mutually compatible hierarchical placing of all the university's activities, either as backcloth or as traffic, but the result was a genuine view and analysis of the university as a community. It was a community study which is relevant to any large organisation. Any individual in that community presumably experiences all these hierarchical levels; when a chairman of a department attends a meeting and wears his departmental hat then he is operating in the structure as an

$\mathcal{N}$-level entity. In a similar way the items of business or scholarship which circulate around the community and through its committees and bars will be expressible in this hierarchical scheme, constituting traffic on $S(\mathcal{N}-2)$, $S(\mathcal{N}-1)$, etc.

One of the interesting features of the university was its committee structure (in 1974) and the analysis of the traffic and backcloth found a characteristic near-complete disconnection between the structures $S(\mathcal{N}+1)$ and $S(\mathcal{N}+2)$, on the one hand, and $S(\mathcal{N})$, $S(\mathcal{N}-1)$ and $S(\mathcal{N}-2)$ on the other.

By disconnection I mean that there was an almost total blockage of traffic flow between these two parts of S(H), and by traffic flow is here meant all the official and formal flow of business through the committee structure (commonly called the 'official channels'). There was then a strong analogy with the notion of a medieval citadel—formed by $S(\mathcal{N}+1)$ and $S(\mathcal{N}+2)$ in the total geometry of S(H). This had far-reaching effects for the personal experience in the democratic process, for the sense (or lack of it) of participation in the decision making scheme—which at the time of the study was much occupying the thoughts and emotions of the more revolutionary of the young scholars.

Another intriguing feature of this committee structure was the discovery of what came to be called local q-holes (where the q is the dimension parameter) or *loops* formed by a few committees, in the structure $K_C(P)$—here C denotes the set of committees and P is the set of people sitting on them. An example was the 4-hole with the following members: the academic planning committee, the committees on chairs, the board of maths studies, the committee on computing, and the senate, in that order, looping round to the academic planning committee again. This meant that the academic planning committee and the committee on chairs shared a 4-face (five people in common as members of their separate committees). Similarly there were at least five people common to the other neighbouring pairs in this 4-hole. Now this hole must provide a certain kind of boundary to the traffic which can move on the backcloth $K_C(P)$.

Thus an item of business which interests (not less than) five people on two committees on opposite sides of the loop (eg the academic planning committee and the board of maths studies) cannot find a *single* home (committee) in which it can be discussed (for decision making)—because the local geometry does not offer a place for it. This item of business must exist in at least two places round the loop, and the class of such items of business can therefore only find their 'place' in a geometry which consists of the *whole loop*. Thus this class of business becomes identified with traffic which *circulates around the 4-hole*. So this kind of traffic must 'bounce off' any such q-hole it encounters in the structure—because it cannot go through it—and so it 'sees' the q-hole as an opaque, solid object. This is why it is also legitimate to describe a q-hole as a *q-object*: it is a q-hole when viewed as a property of the local geometry of the backcloth, but a q-object when encountered by dynamic traffic which is moving on that backcloth.

In this particular context one would suppose that the q-objects for higher q-values would be relevant to the more significant traffic in the community—in so far as it refers to business items which are of interest and concern to a larger number of people. So the filling of a q-hole (if that is possible) is likely to be appreciated by more people if q is large than if q is small. And this filling is highly relevant to the question of making decisions on university business items. For such a process involves a considerable rearrangement of priorities or rankings over many matters of policy (what is often unkindly known as horse-trading) and this requires all the relevant traffic to be accommodated in *one place* in the geometry (that is to say, in *one* committee). If the traffic is constantly having to move around a q-hole then decision making becomes impossible in that geometry.

For example, here, the 4-hole was located in $S(\mathcal{N}+1)$ (subcommittees of senate) and so any $(\mathcal{N}+2)$ traffic (referring to $(\mathcal{N}+2)$ business items) will make a contribution to this category of business which is circulating around. After all, the so-called policy matters are usually at a higher hierarchical level, and so $(\mathcal{N}+1)$ committees discussing $(\mathcal{N}+1)$ business items will often find themselves invoking $(\mathcal{N}+2)$ policy in their quest for decision. But these $(\mathcal{N}+2)$ matters usually *cover* many items of $(\mathcal{N}+1)$ business: they therefore appear (disguised) in more than one of the members of our 4-hole. This is why there is a great deal of 'circulating traffic' around the q-holes in a structure such as the one of this context. Because of this I have called such traffic merely *noise*—since it gets in the way of the 'signal', or decision making. Noise can be recognised as traffic on the structure which is (often) obviously trying to avoid the decision-making net: passing the buck is what noise is all about. And noise must be expensive; it needs a lot of energy and resources to keep it going.

Of course, all noise is not going to be consciously buck passing. Some of it will arise out of simple ignorance of the local geometrical structure (accident rather than design); and then there is the problem of how decisions are made in practice—when they relate to traffic which is circulating because of the mere presence of various q-holes in the backcloth. This is when it is necessary to 'fill' in the q-hole. In our committee example this will require either an official or an unofficial committee with suitable q-connections to join up the sides of the 4-hole. If it cannot be done officially (which requires a meta-hierarchy from which the structure of the hierarchy can be modified) then it is done unofficially by creating a *pseudocommittee* (Figure 1). This

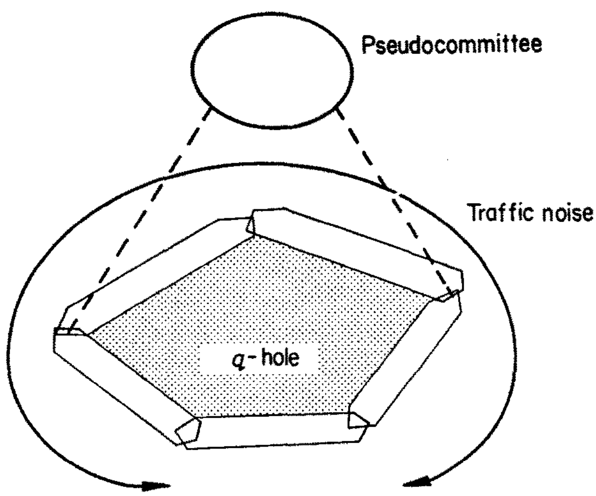

Figure 1. Pseudocommittee and q-hole

often turns out to be the vice-chancellor, in an English university, or the president, in an American university, . . . and so on for other kinds of organisations.

Of course the pseudocommittee is naturally blamed for pulling strings and wielding undemocratic power—and I think this charge must be true. However, it requires a re-examination of what is democratic (or what is regarded as virtuous in the context) in this light—because we can hardly 'blame' the traffic for circulating around q-holes if that is what the backcloth contains.

The essential point about this lies in the fact that in any complex organisation (whether political, economic, or social) the underlying structure of the backcloth is an unavoidable determining factor in the behaviour of those people (traffic) who are trying to run it. So even though the 'authorities' have the best will in the world, are full of the best of intentions, when they invite people to take an active part in decision making, yet they are powerless to transcend the topological connections of the backcloth—connections which subtly control the effectiveness of that participation. Transcending those constraints can only begin after we have unearthed them and laid them bare.

Democracy in the sense that 'all men are equal' goes out of the window when the structure of the backcloth gets punched full of q-holes!

References

1. R. H. Atkin, *Combinatorial Connectivities in Social Systems* (Basel, Birkhäuser, 1977).
2. R. H. Atkin, *Multidimensional Man* (Harmondsworth, Penguin 1979).