Q-analysis and the structure of friendship networks†

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This is a study of a collection of social scientists. It examines the social context out of which they develop close friendship ties and the impact of an extended computer conference on such ties. The perspective of R. H. Atkin (1974) is used to uncover shifts in the structure of ties among the participants.

Introduction

The present paper is concerned with exploring some problems in analyzing the structure of friendship choices. The data come from a sociometric study of the impact of a computer conference on affective ties among some social scientists and mathematicians. They have been reported elsewhere (L. Freeman & S. Freeman, 1979; S. Freeman & L. Freeman, 1979). The analytic devices that will be brought to bear here were suggested by the work of an algebraic topologist, R. H. Atkin (Atkin, 1974). The specific problem to be explored is the degree to which some of the mathematical formalisms suggested by Atkin can help to reveal important structural properties of this set of sociometric data on friendship choice.

The data and the previous analysis

In January 1978 a group of social scientists and mathematicians, all specialists in the study of social networks, met for a day to be introduced to a system for computer conferencing. Some of these people were previously unacquainted and some were already intimate friends of long standing.

At the end of the meeting each participant filled out a questionnaire in which he or she reported the kinds of past and current relations he or she had with each of the others.

Among the questions were several that are important in the current context. Each participant indicated which, if any, of the others were considered "friends" and which, if any, were considered "close personal friends".

Since most of the participants were American speakers of English, it is difficult to determine whether their use of the term "friend" refers to an affect-based tie. Americans tend to use that term to describe anyone from the most superficial acquaintance to a trusted lifelong intimate.

In the present analysis, therefore, interest will be centered on ties that were reported as close and personal. Moreover, to rule out such relations as one-sided admiration, we shall look only at reciprocated—symmetrical—choices of close personal friends. It is

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hoped that in this way we can examine a more clearly defined, stable and meaningful human relationship.

The initial meeting was followed by seven months of participation in a computer conference. Participants were linked, by telephone lines, to a central computer that was programmed for the sole purpose of facilitating human communication. Any participant may call in at any time using a telephone and a keyboard computer terminal. Participants may send and receive messages to and from others, or read or write comments in conferences.

Communication is asynchronous; a message sent at a given time can be "picked up" immediately or at any convenient later time by the person to whom it is directed. And, at the same time, a conference mode is employed to facilitate group discussions. Conferences maintain complete records of all interaction by groups of participants so that a new participant can enter at any time and "catch up" by reviewing all, or any part, of the record of previous conversation.

One of the most interesting questions about this new mode of communication is what, if any, impact it might have on the formation and maintenance of ties of intimate friendship among participants. The questionnaire, therefore, was administered again after 7 months of electronic communication. All in all, 29 participants filled out questionnaires both times. They generated data for the analysis of changes in affect based linkages. Results are shown in Tables 1 and 2.

In the earlier analysis of both friendship and close personal friendship (L. Freeman & S. Freeman, 1979; S. Freeman & L. Freeman, 1979), several standard network analytic properties of affect based linkage patterns were reported. Comparative analyses of density, reachability, distance and symmetry were described. In addition, clique structure was examined according to Doreian's (1969) algorithm.

TABLE 1

Incidence matrix for "close personal friends" on the first questionnaire

P	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
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TABLE 2

Incidence matrix for "close personal friends" on the second questionnaire

P	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1																										1			
2		1																								1			
3	1			1																						1		1	
4			1					1	1			1			1														
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26	1												1						1		1								
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28		1	1																									1	
29		1	1			1	1																				1		

Overall, the results of these analyses were interesting enough. They showed a dramatic increase in the number of links based on affect—particularly at the level of friends—and an overall pattern of increased "closeness" for the group as a whole.

Although there were observable changes among the choices of close personal friends these were, in general, less dramatic. Moreover, most of the changes that were observed among these intimate ties were revealed only at the level of the analysis of pairs. There were so few links reported, it seemed, that the search for the potentially more complex and interesting clique structures could reveal very little.

Data from the first questionnaire showed only one 3-person clique (involving persons P_2, P_3 and P_{28}) of close personal friends. Those from the second, revealed three overlapping cliques (involving P_2, P_3 and P_{28}; P_3, P_{28} and P_{29}; P_3, P_8 and P_{29}) each containing three persons, and centered on the original triple.

Clearly, this sort of result is pretty dull stuff. What is needed is a more sensitive approach to analysis that can show some more sociologically interesting structural aspects of the data of Tables 1 and 2. In the next section of this paper, an attempt will be made to find more sensitive analytic instruments in the simplicial complexes of Atkin (1974).

Simplicial complexes

The algebraic topology of Atkin is based on a fundamental distinction between what he calls the "backcloth" of social action and the "traffic" on that backcloth. The backcloth is the structure of the "space" in which objects are located and events take place. The traffic consists of the objects and events that are defined in terms of that backcloth.

Thus, a physicist, for example, can discuss planets as objects and events such as planetary motion in the context of the usual three dimensional representation of space.

Atkin's discussion of backcloth and traffic is embedded in a good deal of fairly abstract and heavy philosophical discourse, but for present purposes the distinction alone is enough. His advice—somewhat simplified—is that in examining social interaction it is useful, first to specify a basic social structure as a backcloth, then to examine the activities that can and do occur within the context of that structure.

In thinking about relations of trust and affect among human beings, it is perhaps almost trivial to suggest that such relations cannot develop without some significant sort of interaction among the people involved. Thus, one would expect that the reported close personal friendships between pairs of persons shown in Table 1 must have developed in the context of regular face-to-face contacts at some point among the persons involved. It is not proposed that such contacts are *sufficient* for the development of intimate ties, only that they are *necessary*. In Atkin's terms, what is being suggested is that current *traffic* in interpersonal trust and intimacy between people can only occur in the context of a stable *backcloth* based on current or prior regular and recurrent interaction between those people.

In practical terms, it would be impossible to determine all the prior interpersonal contacts between pairs of the 29 people under examination in this study. We have no data on actual events that may have linked these people. We did, however, collect data in the questionnaire about certain kinds of extended and potentially important kinds of pair-wise contacts that might provide the context for the development of close personal friendships. We know which pairs of these people are, or have been, together in the same locations as professional colleagues, as fellow students and/or as teacher–student pairs. The data are shown in Table 3. It is not entirely unreasonable to suspect that the

TABLE 3
Incidence matrix of persons who are or have been linked together as colleagues, fellow students or teacher–student pairs

P	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
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28	1	1		1																									
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union of these three kinds of colleagueship might, for academics, provide the sort of setting in which friendships could be developed. These, then, are the data from which we shall construct a backcloth for the traffic of friendship.

From the data of Table 3 we can, using perfectly standard network analysis procedures, extract all the Luce–Perry (1949) cliques. A Luce–Perry clique is a maximal complete subgraph. Starting with a point, we find the largest set of other points that are directly connected to that point and to each other. We repeat this operation for each point, eliminate duplicate sets and we have the set of Luce–Perry cliques.

Clique-detection has always been a major preoccupation of network analysts. There are literally dozens of ways to define and find them. Once they have been located, however, the network analyst is often somewhat at a loss as to what to do with them. Typically, cliques are located, using one procedure or another, and that is it. Seldom are they fit into any further analysis of social structure.

Atkin, however, has proposed a radical departure from this stance. He uses cliques—which he calls *simplexes*—as the basic building blocks of his social backcloth. In the present instance, for example, each clique or simplex of two or more people represents a "linking event", a situation in which people got together in an academic setting for a period of time either as professional colleagues, co-students or as student–teacher pairs. Each can be labelled and described—each refers to an actual historical or contemporary event. Thus we are able to reconstruct the history of linking events through analysis of pair-wise relations.

The data of Table 3 generate 19 cliques or simplexes. They are shown in Fig. 1. There were 19 linking events, L_1–L_{19}, each of which brought two or more of our 29 people together in a common setting.
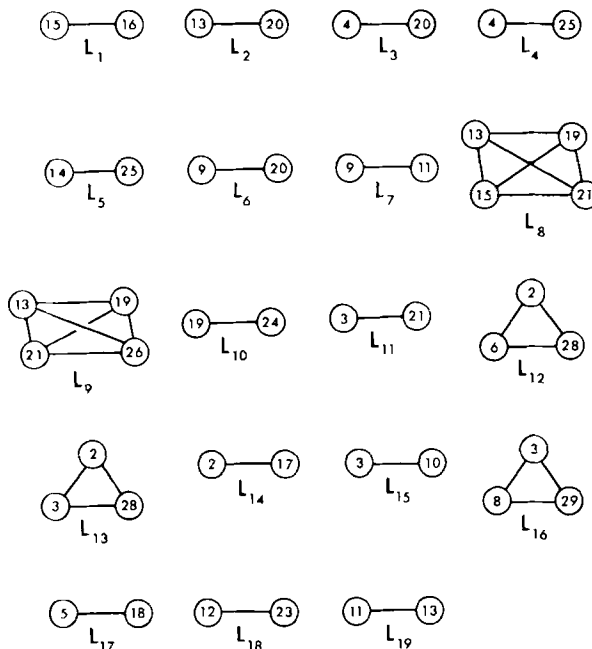
FIG. 1. Portraits of 19 linking events as simplexes.

The simplexes vary in complexity—in the number of people they link together. Each is represented as a polyhedron with a "spatial" dimension appropriate to its complexity. An event that links two persons together is represented by a line in 1 dimension. One that links three is represented by a trihedron in 2-space, and so on. The two highest order simplexes here are the 3-dimensional quadrahedrons, L_8 and L_9. They represent two different time periods at Harvard University. In view of Mullins' (1973) conclusions about the primacy of Harvard as the focal point in the development of contemporary social network studies, this result is not surprising.

In any case, determining cliques permits the construction of the person by linking event matrix shown in Table 4. It is the matrix that permits the procedures that Atkin calls q-analysis.

TABLE 4

Incidence matrix of 29 persons by their participation in 19 linking events

P \ L	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
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2												1	1	1		1			
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Q-analysis requires the specification of two sets of objects (here they are people and the linking events that bring sets of people together) and a relation defined in Cartesian product of these sets. Let

$P = P_i | P_i$ is a person who completed the questionnaire twice in this study,

$L = L_j | L_j$ is a linking event that brought two or more P_i together for an extended time as fellow students, colleagues or as faculty-student groups,

$\lambda \supset P \times L$ is a set of ordered pairs each mapping a person, P_i, into a linking event, L_j, at which he or she was present.

The relation, λ, defined over these sets, permits us to specify two collections of simplexes. One, that was used in constructing the linking events, has already been

shown in Fig. 1. It is embodied in the pattern of non-zero entries in the columns of Table 4.

The other is expressed in the rows of Table 4. A set of non-zero entries in a row serves to define the simplicial representation of the person specified in that row. Thus, row P_2 is a 2-simplex or trihedron,

$$P_2 = \{L_{12}, L_{13}, L_{14}\},$$

and row P_3 is a 3-simplex or quadrihedron,

$$P_3 = \{L_{11}, L_{13}, L_{15}, L_{16}\}.$$

Portraits of the entire set of 29 persons as simplexes are shown in Fig. 2.
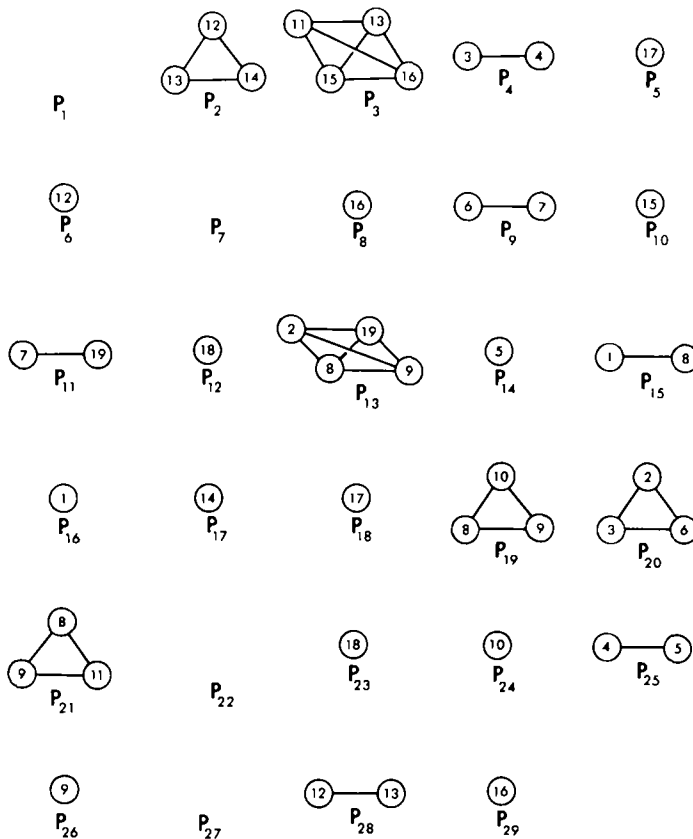
FIG. 2. Portraits of 29 persons as simplexes.

In a sense, these persons are no more than collections of linking events in which they have been involved. In the context of the present analysis, the only relevant characteristics of a person—say P_2—are the three linking events in which he or she has been involved. But the important thing from Atkin's perspective is not the sets of simplexes as such, but the ways in which they are woven together into a larger structure—into a

simplicial complex. According to Atkin (1974, pp. 26–27) simplicial complexes are important social objects. He says:

> "The sense we have of the quality of our lives, the richness or barrenness, the complexities of our mutual dependencies, the opportunities for thought or action, the rewards or punishments which might come our way, all these intangibles are our intuitively apprehended sense of mathematical relations which exist between us and others, between us and physical things— and surely these relations can well include relations between (well-defined) sets of relations. And it is the *structure of a relation*—of the *simplicial complexes* which represent that relation—which contains an expression of its *quality*."

A simplicial complex, then, is a linked set of simplexes. In the present instance, for example, if two people, P_i and P_j, share one or more common linking events, they are themselves linked. And in general, whenever 2 simplexes share a common "face" they are somehow "glued together" into a larger connected chain.

The simplicial complex of people glued together by common linking events, $K_P(L; \lambda)$, is shown in Fig. 3. In the figure, person P_{25}, for example, is linked to person P_4 through their common experience in one linking event, L_4. Thus they are glued together by a point—they are linked at dimension 0.

Person P_{13}, however, is linked to person P_{21} by two events, L_8 and L_9. They are glued together by a 1-dimensional line. The degree overlap between 2 simplexes can be indexed by its dimension. Simplexes that share a point are 0-connected, those that share a line are 1-connected, and so on.
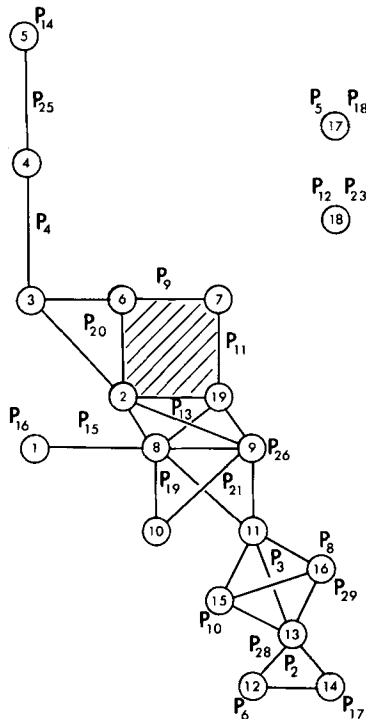
FIG. 3. Simplicial complex, $K_P(L; \lambda)$, showing the pattern of links between persons (P_1–P_{29}) through their shared linking events (L_1–L_{18}).

Atkin's q-analysis is designed to capture these patterns of relations among relations. It is based not only on direct connections between simplexes, but on chains of connections as well. In Fig. 3, for example, P_{25} is 0-connected to P_4 and P_4 is 0-connected to P_{20}. Thus, these three simplexes are linked in a chain of connection of order 0, and P_{25} is (perhaps weakly) connected to P_{20} via P_4. Both P_{25} and P_{20} are linked to P_4, though by different events, but they are viewed as linked to each other in terms of their ties to that intermediary. It is this set of chains of connection and their dimensions that is described by q-analysis.

In the present case, the q-structure of the complex, $K_P(L, \lambda)$, is shown in Table 5. The first column lists the dimensions of components (made up of chains of simplexes) in the

TABLE 5

Q-analysis of the simplicial complex, $K_P(L; \lambda)$

q	Q_q	Components
3	2	$\{P_3\}, \{P_{13}\}$
2	6	$\{P_2\}, \{P_3\}, \{P_{13}\}, \{P_{19}\}, \{P_{20}\}, \{P_{21}\}$
1	9	$\{P_{13}, P_{19}, P_{21}\}, \{P_2, P_{28}\}, \{P_3\}, \{P_4\}, \{P_9\}, \{P_{11}\}, \{P_{15}\}, \{P_{20}\}, \{P_{25}\}$
0	3	$\{P_2, P_3, P_4, P_6, P_8, P_9, P_{10}, P_{11}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{19}, P_{20}, P_{21}, P_{24}, P_{25}, P_{26}, P_{28}, P_{29}\}, \{P_5, P_{18}\}, \{P_{12}, P_{23}\}$

complex. In effect, as we look down the table, we are led to look successively at chains from high to low dimension. The second column is the structure vector. It records the number of chains at each level. And the third column lists the names of the simplexes making up each chain.†

This simplicial complex, $K_P(L; \lambda)$, is the backcloth of interpersonal contacts upon which the traffic on intimate affective relations must flow. All in all, it is a rather meagre structure. The highest order persons, P_3 and P_{13}, have been involved in only four linking events and have a dimension of 3. The overall complex, therefore, requires no more than 3 dimensions for its structural description.

No links between simplexes—no shared faces—are observed at any dimension greater than 1, where P_{13}, P_{19} and P_{21} share a pair of events, as do P_2 and P_{28}. But this linkage is not impressive—the 12 simplexes that appear at q = 1 are so loosely linked that they form nine separate components.

It is not until q = 0 that we begin to see the simplexes link together into a large chain. There, 21 are linked together, and four others form two pairs. But four, P_1, P_7, P_{22} and P_{27} never appear at all. They were never involved in any linking events with any of the others.

Another curious structural feature of $K_P(L; \lambda)$ is evident from looking back at Fig. 3. The shaded area surrounded by P_9, P_{11}, P_{13} and P_{20} is a 0-dimensional *q-hole* in the structural backcloth. A q-hole is any area with four or more edges enclosed by a *circuit* that begins and ends at the same point. Its presence indicates some sort of obstruction in the free flow of traffic along the backcloth.

† For certain kinds of analyses it is useful to perform a q-analysis on the transpose of the incidence matrix (Table 4). This leads to the construction of the conjugate simplicial complex, $K_L(P; \lambda^{-1})$ and shows the chains of linking events. Such an analysis would be important in an attempt to reconstruct, for example, the historical importance of various linking events, but it is not relevant to the present analysis.

Atkin suggests that the q-holes in complexes are analogue of objects in ordinary space. One might not think of a tree as a hole in space—until, not looking, one tries to walk through it. Such holes, then, are conceived of by Atkin as somehow "generating" traffic—like walking around a tree.

This reasoning seems fairly unconvincing in the context of applications to social structure. Nevertheless, the presence of a q-hole in our social backcloth is worth noting as another kind of barrier to the uniform flow of traffic.

This, then, is the structural backcloth in the context of which we might expect affect-based relations to be expressed. If, indeed, this is the proper backcloth—the structure that constrains the pattern of close personal friendship—we will observe some gross restrictions in that pattern. Of the 406 unordered pairs of persons here, only 31 pairs are adjacent even at the 0-level in this backcloth, and only 212 (including the adjacent pairs) are linked by 0-level chains of connection. At the level of 1-conductivity, there are only four linked pairs, and there are none that are 2-connected. The question, then, is the degree to which the data on choices of close personal friends may be viewed simply as traffic on this backcloth—conforming to its restrictions.

There are 12 symmetrical pairs of close personal friends reported in the data collected at the start of this experiment and recorded in Table 1. They are shown, graphically, in Fig. 4. *Not a single observed pair of close personal friends was among the 194 pairs prohibited by the obstructions in $K_P(L; \lambda)$.* Even more impressive is the fact that *11 of the 12 pairs of mutual close personal friends are among the 31 adjacent pairs in the $K_P(L; \lambda)$.*†
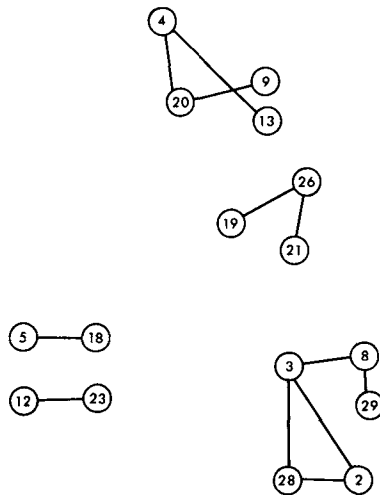
FIG. 4. Graph of mutual close personal friend choices from the first questionnaire.

Only one friendship pair was formed between non-adjacent simplexes on the backcloth. That linked points P_4 and P_{13} that were separated by one intermediary in $K_P(L; \lambda)$. Moreover, it's worth noting that all three of the points contained in the

† It is worth noting here that the one intimate link that was formed between non-adjacent simplexes in the backcloth was formed between two individuals who had recently spent a year as neighbors in the same geographic locale but since they were associated with different institutions, were not tabulated as sharing a linking event.

component of the friendship graph that included point P_4 were simplexes that surrounded the q-hole in the simplicial complex. The simplicial complex based on previous linking events, it seems, is indeed the backcloth upon which close personal friendships are formed. Friendship seems simply to be traffic on that backcloth.

Another question involves the impact of the computer hook-up on these friendships. The electronic computer conference may simply have provided an opportunity for participants who were so disposed to express warmth and support to their friends. There may, in effect, have been an increase in affective traffic on the already established backcloth or, since all 29 participants were easily accessible to each other, the computer conference may have modified the structure of the backcloth itself.

We have, of course, no direct information on the question of whether the computer linkages constituted a change in the backcloth for friendship. Indirect evidence, however, may be established by looking at the results of the second administration of the questionnaire shown in Table 2.
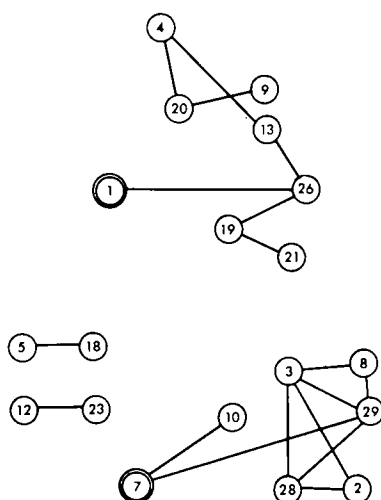
FIG. 5. Mutual close personal friend choices from the second questionnaire.

The data on symmetrical choices are shown in Fig. 5. There are 18 mutual choices. Thirteen are from the 31 adjacencies in $K_P(L; \lambda)$. They are similar to the choices made in the first data set. Two are in the connected set in the complex but the other three are unconnected. Two of these involve point P_7 and one includes point P_1. Both these points were isolates in $K_P(L; \lambda)$, but after the computer hook-up they have been tied into connected chains of close personal friends.

This result seems to suggest that the computer conference is not merely providing an opportunity for the flow of affect, but it—or some other event that took place during the same time period—has resulted in a shift in the backcloth of linking events. As a matter of fact, anecdotal evidence indicates that one of the new links shown in Fig. 5 ties two people together who have *never* met face-to-face. Clearly then, at least for some people, computer communication is a satisfactory substitute for the kind of direct personal contacts upon which close friendships can be built. There has been a shift, not

only in the pattern of friendship reported here, but in the underlying skeleton upon which such patterns are displayed.

Summary and conclusions

Data from a study of close personal friendship in the context of an extended computer conference have been examined. The procedures for examining these data were suggested by the algebraic topology of Atkin (1974). The Atkin approach stresses the need for analysis on two levels: (1) the structural backcloth of simplicial complexes, and (2) the traffic or pattern exhibited on that backcloth.

In the present analysis, evidence for some history prior colleagueship was used to establish the backcloth. It was reasoned that mutual choice as close personal friends between pairs of people should grow out of the context of prior collegial ties.

Results show that for these people this is indeed the case. More interesting, however, is the result that shows that linking these people together electronically serves not only to increase the flow of "traffic" in friendship, but it seems to produce a change in the structure of the backcloth itself. This suggests that an extended computer conference can, at least to some degree, substitute for extended face-to-face contacts in providing the kinds of ties between people out of which friendship can be built.

With respect to the power of the tools provided by Atkin for an analysis of this sort, my final reaction is mixed. On one hand, there is nothing in this analysis that could not have been expressed—often more simply—using the terms and concepts of graph theory. On the other hand, Atkin has stressed a number of ideas—like the distinction between backcloth and traffic—that would probably not have occurred to me had I been using ordinary network analytic tools. On balance, I suspect that Atkin has some important things to say to network analysts. We can, perhaps, produce richer descriptions of structural phenomena if we draw on Atkin's insights and formal procedures.†

References

ATKIN, R. H. (1974). *Mathematical Structure in Human Affairs.* New York: Crane, Rusak.
DOREIAN, P. (1969). A note on the detection of cliques in valued graphs. *Sociometry,* **22**, 237–242.
DOREIAN, P. (1980). On the evolution of group and network structure. *Social Networks* (in press).
FREEMAN, L. C. & FREEMAN, S. C. (1979). A semi-visible college: structural effects of seven months of EIES participation by a social networks community. M. HENDERSON, Ed., *The Future of Electronic Communication*: AAAS Symposium Series. New York: Westview Press (in press).
FREEMAN, S. C. & FREEMAN, L. C. (1979). The networkers network: a study of the impact of a new communications medium on sociometric structure (submitted for publication).
LUCE, R. D. & PERRY, A. (1949). A method of matrix analysis of group structure. *Psychometrika,* **14**, 94–116.
MULLINS, N. C. (1973). *Theories and Theory Groups in Contemporary American Sociology.* New York: Harper & Row.

† A similar attempt to employ q-analysis to enrich structural descriptions of social interaction has recently been made by Doreian (1980).