The Multilevel Hypernetwork Dynamics of Complex Systems of Robot Soccer Agents

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A mathematical formalism is sketched for representing relational structure between agents. n-ary relations, $n > 2$, require hypernetworks, which generalize binary relation networks. n-ary relations on sets create structure at higher levels of representation to the elements in multilevel systems. The *state* of a system is represented by its multilevel relational structure. The *dynamics* of a system are represented by state changes through time. These can be continuous with no change in the hypernetwork topology, but often they are not. Controlling such systems involves taking actions intended to result in desirable state changes. The concept of multilevel hypernetwork can be applied to multiagent systems in general.

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1. INTRODUCTION

The emerging science of complex systems is giving new insights into a wide variety of systems across the physical, biological, human, and engineering sciences.

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In particular, we are developing new ideas of what it means to predict and control systems for which it is impossible to know their particular state at any particular time in the long-term future. A large class of systems has the property that system behavior emerges from the interaction of their parts. They are called *interaction-based systems*. Generally their class of parts is large and heterogeneous, and they have multilevel structure with dynamics at microlevels coupled with dynamics at macrolevels. The concept of *multiagent system* has proved to be a powerful paradigm for interaction-based systems [Wooldridge and Jennings 1995].

Systems of interacting robots make an excellent laboratory subject for complexity science. They are less complex than human systems because they can be studied objectively from the outside, and because it is possible to do experiments that would be impractical for human systems. Robot-robot interactions can be very rich and understanding the multilevel dynamics of interacting robot communities involves some deep and important research questions. For these reasons, we focus on robot soccer in which teams of small (about 10cm) autonomous mobile robots compete to score goals using a goal ball on a small pitch (about 3×2 meters). We also use data from the RoboCup simulation league which is published each year after competition [Asada et al. 1999].

This article begins by discussing some of the general issues of complex systems and explains why the agent-based approach is attractive. Following this a hypernetwork formalism is introduced for representing robot interactions. Networks allow relationships between pairs of robots to be represented and hypernetworks generalize this to relationships between many robots. This leads to a way of representing multilevel structure and multilevel multidimensional dynamics. Particular plays can be represented as multidimensional trajectories; we discuss how trajectories can be transformed into others, and related topological ideas. The representation allows individual robots to communicate through their perceptions of the environment, recognizing patterns of good or bad structures. Thus the robots all process their perception of the environment autonomously in parallel, and they communicate what they have perceived to the other robots through their actions. If a set of collaborating robots recognize the same good future trajectory and they move in ways consistent with that future, there is a massive visual positive feedback between them that the particular good trajectory is what they are working towards. The ideas are illustrated by a number of detailed examples for robot soccer, and the article includes a discussion of related areas of application that can inform and be informed by this research.

1.1 The Science of Complex Systems

Multiagent systems is a research paradigm that is beginning to make a significant contribution to the emerging science of complex systems. Although there is no widely accepted definition of complexity, there is consensus that the following are important characteristics:

—many heterogeneous parts, for example, a city, a company, the climate;

—complicated transition laws, for example, economic systems, disease transmission;
—unexpected or unpredictable emergence, for example, chemical systems, accidents;
—sensitive dependence on initial conditions, for example, weather systems, investments;
—path-dependent dynamics, for example, qwerty keyboard evolution, international relations;
—network connectivity and multiple subsystem dependencies, for example, ecosystems;
—dynamics that emerge from interactions of autonomous agents, for example, road traffic, parties;
—self-organization into new structures and patterns of behavior, for example, social groupings;
—nonequilibrium and far-from equilibrium dynamics, for example, combat aircraft, share prices;
—discrete dynamics with combinatorial explosion, for example, chess, communication systems;
—adaptation to changing environments, for example, biological systems, manufacturing design;
—coevolving subsystems, for example, land use and transportation, computer virus software;
—ill-defined boundaries, for example, genetically modified crops, pollution, terrorism;
—multilevel dynamics, for example, companies, armies, governments, aircraft, the Internet.

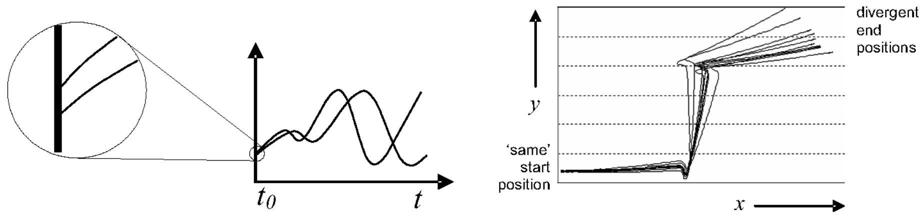
Many systems exhibit many of these characteristics. Any one of them can make systems appear complex, but together they can make systems very difficult to understand and control.

Emergence is one of the central ideas in the science of complex systems. Ashby [1956] writes:

> "The concept of "emergence" has never been defined with precision, but the following examples will probably suffice as a basis for discussion: (1) Ammonia is a gas, and so is hydrogen chloride. When the two gases are mixed, the result is a solid—a property not possessed by either reactant. (2) Carbon, hydrogen, and oxygen are all practically tasteless, yet the particular compound 'sugar' has a characteristic taste possessed by none of them. (3) The twenty (or so) amino-acids in a bacterium have none of them the property of being "self-reproducing", yet the whole, with some other substances, has this property."

Von Bertalanffy [1969] gives a similar insight:

> "The meaning of the somewhat mystical expression "The whole is more than the sum of its parts" is simply that constitutive



(a) measurement error makes it impossible to start from the same position (b) robot trajectories from the same start position diverge through time

Fig. 1. Many systems are sensitive to initial conditions with diverging trajectories.

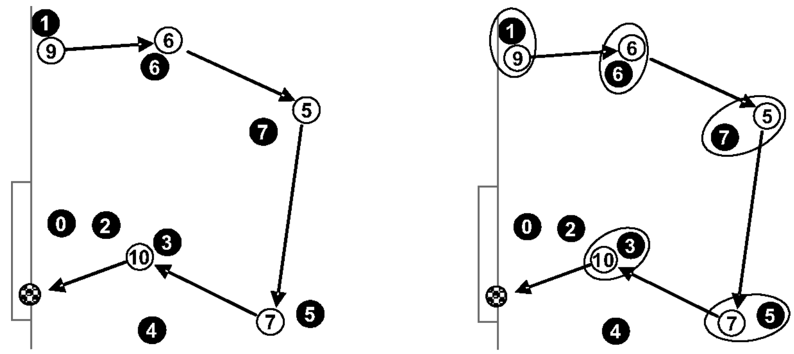
> characteristics are not explainable from the characteristics of isolated parts. The characteristics of the complex, therefore, compared to those of the elements, appear as "new" or "emergent". If, however, we know the total of parts contained in a system and the relations between them, the behavior of the system may be derived from the behavior of the parts. We can also say: While we can conceive of a sum as being composed gradually, a system as total of parts with its interrelations has to be conceived of as being composed instantly."

Laplace wrote in his 1814 *Essai philosophique sur les probabilités* that:

> "We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at any given moment knew all of the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit the data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom; for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes".

Today we know that for many systems, even if we had all the data and the right formula, inevitable limitations to measurement mean that some things will be uncertain in the future. Many systems are sensitive to initial conditions as illustrated in Figure 1(a) where an imperceptible difference in the measured value at t_0 is associated with two very different system trajectories. Figure 1(b) gives a practical example in which a robot is started from the same position many times, executing the same control instructions. As can be seen, the end positions diverge from each other through time because this system is sensitive to initial conditions such as the exact states of the gears, the wheels, the batteries and so on.

Systems that are bounded and sensitive to initial conditions are defined to be *chaotic*, and, for such systems, it is impossible to predict their states at precise times in the long-term future. Figure 1(b) shows divergent but not necessarily bounded dynamics so this system is not chaotic in the technical sense. Nonetheless it is sensitive to initial conditions and divergent, and long-term predictions of a robots' precise positions from the given starting position are impossible. Of course, most engineered systems are like this, and control



(a) the passing relationship (b) the marking relationship

Fig. 2. Robot soccer involves interactions between fast-moving agents.

theory attempts to keep them on the desired trajectory. But what does it mean to say, for example, that a robot soccer is on the right trajectory? That, and its generalization to other multilevel complex systems, is the subject of this article.

2. INTERACTIONS OF AGENTS IN MULTIAGENT SYSTEMS

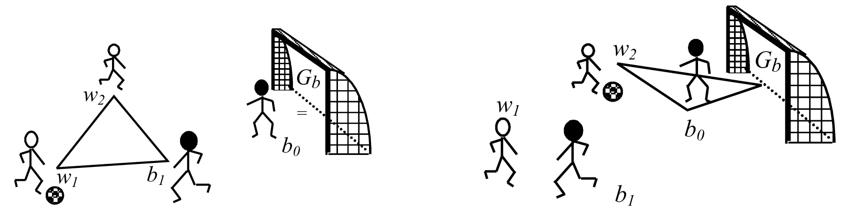
The dynamics of multiagent systems emerge from interactions between the agents, as illustrated by robot football. It is an excellent test-bed for multiagent research due to the complex interactions between fast-moving agents, each having its own view of the universe. It involves many research issues such as cooperation, communication, real-time actuation, noisy and multidimensional sensory-motor data, and partial information [Asada et al. 1999]. We view the robots as autonomous agents, interacting purposefully to achieve their objectives of scoring goals against, and not conceding goals to, an opponent.

Figure 2(a) shows a sequence of passes in a robot soccer game that concludes in a goal for the white team. These passes establish relationships between the players represented by the arrows. We use the notation w_i to represent white robot i and b_j, to represent black robot j. Then the sequence of arrows, $\langle w_9, w_6\rangle$, $\langle w_6, w_5\rangle$, $\langle w_5, w_7\rangle$, $\langle w_7, w_{10}\rangle$ is a *path* in a network. However, these are not the only relevant network structures. Figure 2(b) shows pairs of robots, $\langle w_9, b_1\rangle$, $\langle w_6, b_6\rangle$, $\langle w_5, b_7\rangle$, $\langle w_7, b_5\rangle$, $\langle w_{10}, b_3\rangle$, which are marking each other. Of course, many other important relationships also occur during robot soccer games.

Network theory is excellent for modeling the interactions of pairs of agents, but generally in multiagent systems there are interactions between many agents. In other words, the concept of binary link (oriented edge or arrow) between two vertices in a network has to be generalized to the notion of *n-ary* multidimensional edge (*simplex*) in what we will call a *hypernetwork* Johnson [2006, 2007]. This article investigates two interrelated ideas:

—hypernetwork theory applied to multiagents systems; and
—the multilevel relational dynamics of multiagent systems.

The second of these involves what could be called *hyperagents*, collections of agents forming entities with their own agent-like behavior. The term is taken



(a) the defender's dilemma triangular relation between w_1, w_2, and b_1

(b) the goalkeeper's dilemma triangular relation between w_2, b_0, and G_b

Fig. 3. n-ary relations between robot soccer agents.

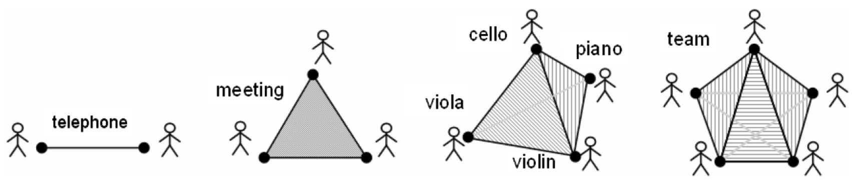


Fig. 4. Representing relationships by multidimensional polyhedra (Source Johnson [2006]).

from hypergraphs which generalize the idea of a two-vertex edge in a graph to an edge that can have any number of vertices [Berge 1989]. Examples of hyperagents can include teams with more than two members, departments made up of many people, and so on. Indeed, social organization may have hyper-hyperagents such as departments forming a company.

The purpose of this article is to show how the formalism of hypernetworks can represent multiagent dynamics at the level of the individual agent and investigate how it can represent the higher-level dynamics as sets of agents interact locally to form interacting hyperagents. We use robot soccer for illustration because it is easy to understand but sufficiently complex to illustrate issues common to agent-based systems in general.

3. ASSEMBLING AGENTS TO FORM MULTILEVEL STRUCTURES

The fundamental idea behind our research is that the elements of sets can be assembled under n-ary relations to form structures. Figure 3(a) shows a classic soccer configuration with two white players, w_1 and w_2, challenged by b_1. Generally b_1 has the choice between attempting to tackle w_1 in which case w_1 passes to w_2, or trying to intercept the pass to w_2 in which case w_1 dribbles past him. This is a 3-ary relation, involving all three players simultaneously. We give it the name *defenders dilemma*.

Similarly, once the ball is passed to w_2 (Figure 3(b)) the 3-ary relation between w_2, b_0, and the goal defended by black, G_b, becomes what we call the *goalkeeper's dilemma*: if the keeper moves to the left the right, of the goal is exposed, if he moves to the right, the left of the goal is exposed, and if he approaches the attacker to narrow the angle, the attacker may dribble round him to have an open goal.

As Figure 4 illustrates, many relationships between agents are n-ary relations, relations between n agents, rather than the usual binary relations

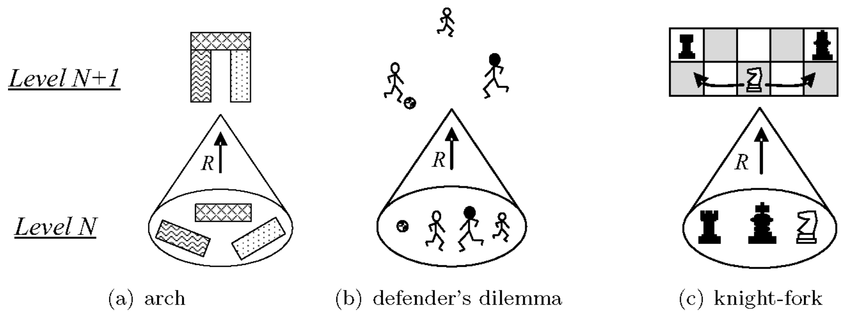


Fig. 5. Parts assembled into a structured whole at a higher level.

between pairs of agents represented by networks. For example, remove any of the players from a piano quartet and it ceases to be a piano quartet. The polyhedra in Figure 4 generalizes the concept of edge (link, arrow) in networks. A relation between two agents can be represented by a line (1-dimension), a relation between three agents can be represented by a triangle (2-dimensions), a relation between four agents can be represented by a tetrahedron (3-dimensions), and so on. In general, a relation between n agents can be represented by an $(n-1)$-dimensional polyhedron in multidimensional space. The polyhedra are also called simplices, where a p-dimensional simplex has $p+1$ vertices.

Viewed this way, football is a game in which n-ary relations between agents and their polyhedra are changing rapidly. The successful pass from w_1 to w_2 between Figure 3(a) and Figure 3(b) activates the triangular relation between w_2, b_0, and G_b which is a good structure for white, and a bad one for black.

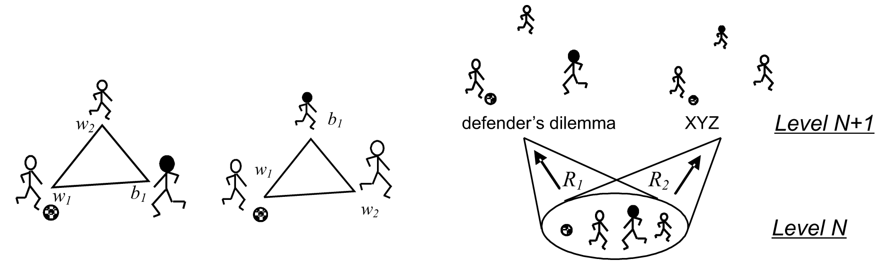
Structure exists at a higher level to the set in a *multilevel representation*. This is illustrated in Figure 5(a) where three blocks (*Level N*) are assembled into an arch (*Level N + 1*). The arch has the *gap* between the blocks as an emergent property, a property not possessed by the blocks themselves. The defenders dilemma is a structure (Figure 5(b)) which is a dynamic precursor to other structures as suggested in Figure 3. There is a strong parallel between robot football and computer chess. Figure 5(c) shows three chess agents assembled by a relation resulting in a *knight-fork structure*. Such structures are very important in chess, and they can be important in multiagent systems in general. Here the knight-fork is good for white and bad for black.

4. MULTILEVEL HYPERNETWORKS

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set. A *hypergraph* on X, $H = \{E_1, E_2, \ldots, E_m\}$, is a family of subsets of X, such that $E_i \neq \emptyset$ and $\cup E_i = X$, $i = 1, \ldots, m$ [Berge 1989]. The E_i are called the *edges*.

As an example, let X be a set of football players. Let $\{x_i\}$ be the singleton set containing just footballer x_i. Then let H contain all the $\{x_i\}$ and all sets of players that interact during a game. Then H is a hypergraph. For example, the defender's dilemma introduced in the previous section involves the hypergraph edge $\{w_1, w_2, b_1\}$.

As illustrated in Figure 6, the hypergraph edge $\{w_1, w_2, b_1\}$ can be configured in different ways by different 3-ary relations, R_1 and R_2. Of these, only



(a) $\langle w_1, w_2, b_1; R_1\rangle$ (b) $\langle w_1, w_2, b_1; R_2\rangle$ (c) $base$(defender's dilemma) $=$ $base$(XYZ)

Fig. 6. The hypergraph edge $\{w_1, w_2, b_1\}$ configured into two different structures.

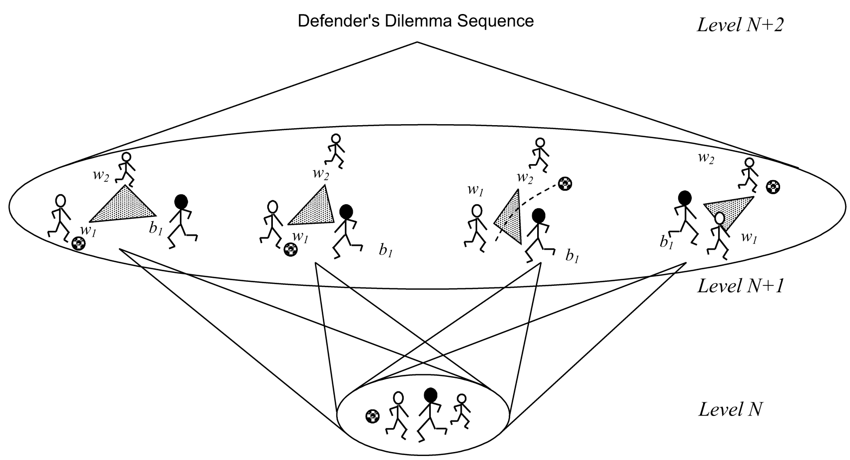


Fig. 7. The same set of players assembled in different ways forms a structure at *Level N + 2*.

R_1 assembles the players into a defenders dilemma. To make the distinction between set and structure clear, we use the notation $\langle w_1, w_2, b_1; R_i\rangle$ to mean the structure created by imposing the relation R_i on the set of vertices $\{w_1, w_2, b_1\}$. Then R_1 assembles $\{w_1, w_2, b_1\}$ to the defender's dilemma, while R_2 assembles them to another configuration which we have arbitrarily called XYZ. Then we can write *base*(defender's dilemma) $=$ *base*(XYZ) even though defender's dilemma $\neq$ XYZ, where base means the set making up the base of the cone.

Most of the interesting structures in robot football involve sequences of positions, and part of the interest in football is to watch the choreography as the sequence progresses. A *move* in football can be thought of as an assembly of structured sets of players through time. Thus a move is an *N + 2 Level* simplex in a multilevel hypernetwork that combines both spatial and temporal structure as illustrated in Figure 7.

Hypernetworks have a richer connectivity structure than networks. Let $\sigma_1 = \langle a_1, a_2, \ldots; R_1\rangle$, and let $\sigma_2 = \langle \alpha_1, \alpha_2, \ldots; R_2\rangle$. Then σ_1 and σ_2 are said to be *q-near* if their polyhedras share a q-dimensional face. Since a q-dimensional simplex has $(q+1)$ vertices, two simplices are q-near if they share at least $q+1$ vertices. In other words, σ_1 and σ_2 are q-near if the intersection of their vertex sets, $\{a_1, a_2, \ldots\} \bigcap \{\alpha_1, \alpha_2, \ldots\}$, contains at least $q+1$ vertices.

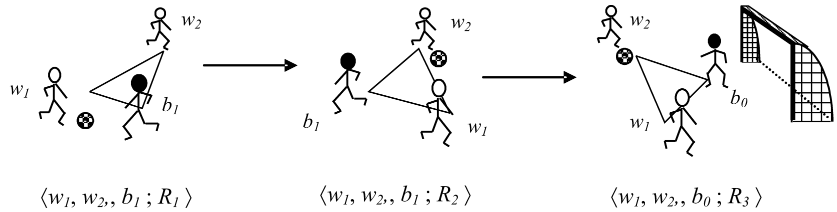


Fig. 8. A move evolving through a sequence of connected simplices.

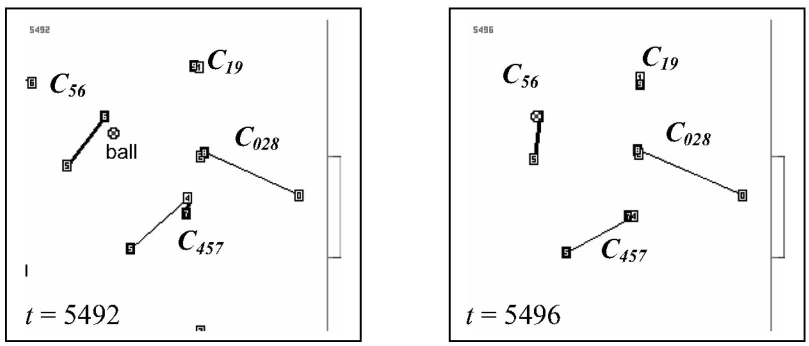


Fig. 9. Robots connected by lines are closest to each other and form structures.

Figure 8 gives an example in which the simplex $\langle w_1, w_2, b_1; R_1\rangle$ becomes the 2-near simplex $\langle w_1, w_2, b_1; R_2\rangle$, which in turn becomes the 1-near simplex $\langle w_1, w_2, b_0; R_3\rangle$. The simplices $\langle w_1, w_2, b_1; R_1\rangle$ and $\langle w_1, w_2, b_1; R_2\rangle$ are 2-near because they share all three vertices. The simplices $\langle w_1, w_2, b_1; R_2\rangle$ and $\langle w_1, w_2, b_0; R_3\rangle$ are 1-near because they share two vertices, the 1-dimensional face $\langle w_1, w_2\rangle$.

5. EXAMPLE: AGENT DYNAMICS IN A ROBOT SOCCER GAME

When robot soccer agents interact, they form structures such as those seen in the previous section. To illustrate the dynamics, we use data from the RoboCup Simulation League which is available on the internet (http://sserver.sourceforge.net).

The dynamics of a simulated football game depend on many factors including the agents' behaviors, perceptions, actions, team strategies and the underlying physical properties of the system. How is it possible to analyze a system as complex as multiagent simulated football? This article proposes a multilevel multidimensional approach to address this question. It involves defining *elements* and *relation*s at low levels of description and more abstract constructs resulting in higher levels of a hierarchical organization.

To illustrate our approach, we consider a sequence of snapshots between time $t = 5492$ and $t = 5539$ in which b_6 takes the ball down the field to the left of the opponents' goal, and passes the ball to player b_7 who is well placed in front of the goal (goalkeeper's dilemma) and scores (see Figure 9).

At $t = 5492$, b_6 approaches the ball and takes possession of it at $t = 5496$. At this time, there are four distinct configurational simplices at what we will

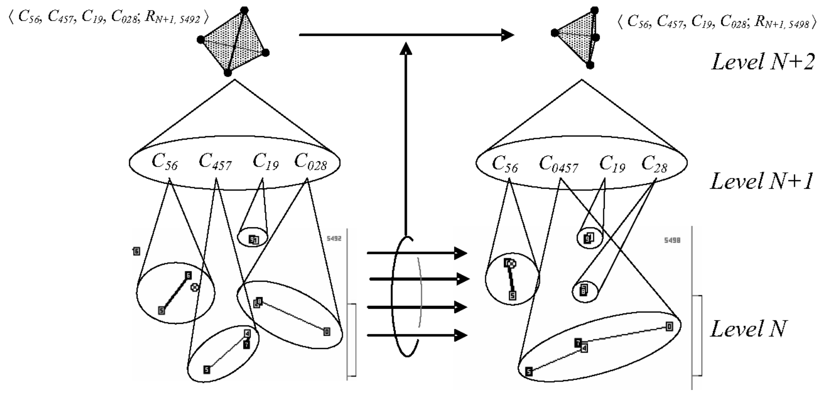


Fig. 10. Multilevel dynamics.

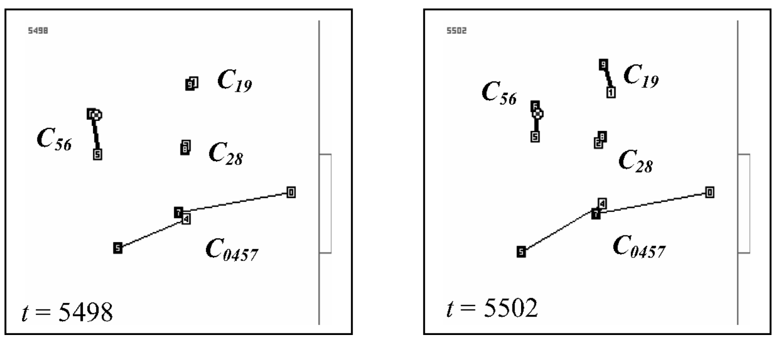


Fig. 11. The *Level N + 1* topology remains constant between $t = 5498$ and $t = 5502$.

call *Level N + 1*:

$$\langle w_5, b_6 \rangle \rightarrow C_{56} \qquad \langle w_1, b_9 \rangle \rightarrow C_{19} \qquad \langle w_0, w_2, b_8 \rangle \rightarrow C_{028} \qquad \langle w_4, b_5, b_7 \rangle \rightarrow C_{457}$$

These structures consist mostly of pairs of players related by marking each other. Under the closest-to relation, they are mutually related. These structures at *Level N + 1* formed from sets of players can themselves be assembled into structures at *Level N + 2* as illustrated by the simplex $\langle C_{56}, C_{457}, C_{19}, C_{028}; R_{N+1,5492} \rangle$ in Figure 10. The relation defining this *Level N + 2* structure is closeness to the goal defended by white. As the game progresses, there are relational changes at both *Level N + 1* and *Level N + 2* within the multilevel representation.

At $t = 5498$, there is a change in the topology, illustrated in Figure 10, as b_7 comes closer to the goalkeeper than b_8, who remains closely marked by w_2. Thus we have the simplices:

$$\langle w_5, b_6 \rangle \rightarrow C_{56} \qquad \langle w_1, b_9 \rangle \rightarrow C_{19} \qquad \langle w_2, b_8 \rangle \rightarrow C_{28} \qquad \langle w_0, w_4, b_5, b_7 \rangle \rightarrow C_{0457}$$

This structure persists until $t = 5502$ (see Figure 11). Player b_9 is weakening the relationship with marker, w_1, by moving further out to the wing. w_1 remains where it was. Note that b_7 is changing its relationship with w_4. This will have significant consequences later in the game.

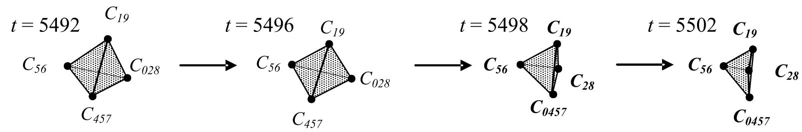


Fig. 12. The evolution of the *Level N* + *2* structure.

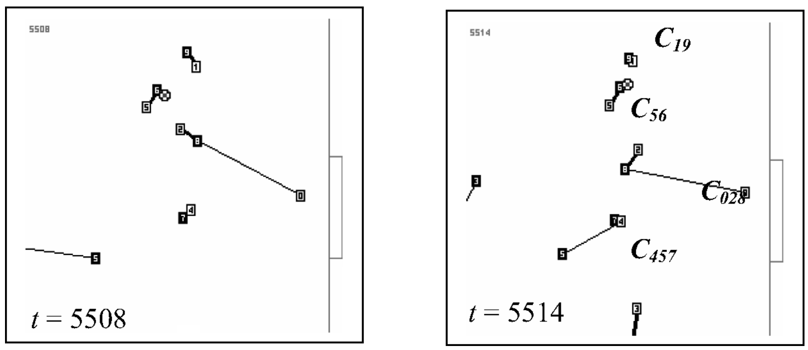


Fig. 13. Player b_6 begins to break away from its marker, w_5.

The *Level N* + *2* structure, $\langle C_{56}, C_{19}, C_{028}, C_{457}; R_{N+1,t}\rangle$ at $t = 5496$ becomes $\langle C_{56}, C_{19}, C_{28}, C_{0457}; R_{N+1,t}\rangle$ at $t = 5498$. Although there is the change from $C_{028} + C_{457}$ to $C_{28} + C_{0457}$, the geometric relationship between the configurations remains relatively stable as illustrated in Figure 12.

Between $t = 5504$ and $t = 5508$, there is little change at *Level N* + *1*. The two-player configurations, C_{19} and C_{56} remain almost unchanged. The structures C_{28}, C_{0457} revert to C_{028}, C_{457}. Their mutually-marking paired-player substructures C_{28} and C_{56} remain very tightly coupled. However, something important is happening for C_{56}. Player b_6 is breaking away from its marker, w_5 (Figure 13).

At $t = 5508$, b_6 is making its break between C_{19} and C_{028}. While C_{19} is a very static structure in this sequence, C_{028} changes, with b_8 on the goal side of w_2 at $t = 5508$ with w_2 taking a position between b_8 and b_6 advancing with the ball. The pair C_{19}, although close to the action, remains almost unchanged. The structure C_{457} forms at $t = 5510$, and this remains a persistent difficulty for White. At $t = 5514$ the defender, w_4 is closely marking attacker b_7, and is the closest defender to b_5 (Figure 14).

At $t = 5520$, player w_2 reverts to closely marking b_8 in C_{028} rather than continuing on its intercept trajectory with b_6. The structure C_{028} may explain why w_2 remains at the center rather than pursuing the dangerous b_6, since b_8 is otherwise unmarked except by the goalkeeper. The topology changes at $t = 5526$, with b_6 the closest opponent to w_0, the goalkeeper, with w_5 still in pursuit in C_{056}. The structure C_{457} persists throughout this period, but the relational structure between w_4 and b_7 changes so that b_7 is now closest to the goal (Figure 15).

The sequence between $t = 5532$ and $t = 5534$ shows a very dangerous situation emerging for White (Figure 16). Player w_4 seems to have been distracted

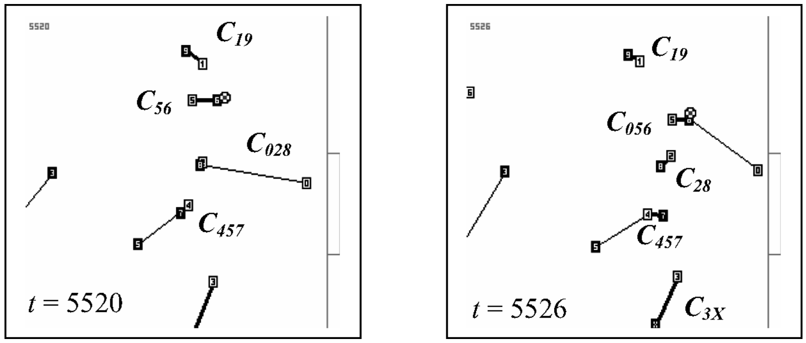


Fig. 14. A topological change as the goalkeeper becomes closer to b_6 than b_8.

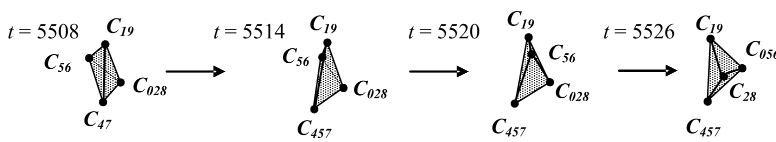


Fig. 15. Geometric changes at *Level N + 2* and topological changes in the *Level N + 1* dynamics.

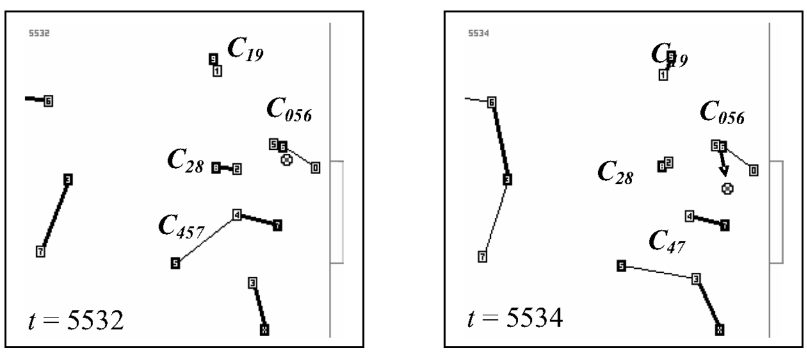


Fig. 16. Black has forced a defender's dilemma configuration with b_6 passing to b_7.

by b_5 and allowed b_7 to move into a position in front of an open goal (Figure 17). Player w_3, who might have played a useful role, seems to have been preoccupied with marking b_5 and b_x. w_5 is gamely chasing b_6 who has the ball and who could shoot at goal if it was not for the keeper, w_0. $C_{056} + C_{47}$ contain a version of the defender's dilemma mentioned in Section 3.

Thus, at $t = 5534$, b_6 successfully passes the ball to b_7 in front of an open goal. b_7 controls the ball at $t = 5536$ and shoots for goal at $t = 5538$. At $t = 5539$, Black has scored a goal.

Was the goal scored by Black inevitable as early as $t = 5492$? Was there something about the structure then that guaranteed Black would score the goal? Or did White do something wrong? And if so, what could have White done differently to prevent this undesirable outcome?

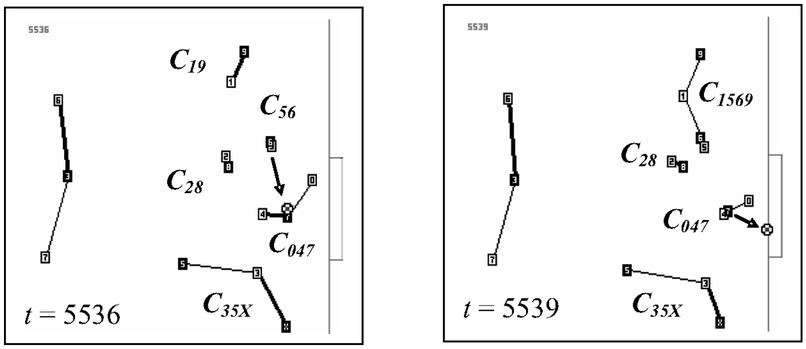


Fig. 17. b_7 controls the ball and shoots into an open goal to score.

Tracing the game back from the goal, the pass at $t = 5534$ made a goal inevitable. Although w_5 was close to b_6 when the pass was made, it was not close enough. After b_6 slipped past w_5 at $t = 5502$, there appears to have been little w_5 could have done to prevent the goal.

The only other players who could have intercepted b_6 were w_1 and w_2 at $t = 5498$ or before. Indeed, at $t = 5514$, w_2 made a belated attempt to intercept, and then appeared to give up in favor of marking b_8 more closely. In contrast, w_1 made no attempt, being closely bound to b_9 in C_{19} throughout.

If w_2 had continued in its challenge to b_6, b_8 would have been unmarked in front of goal which is undesirable. However, although w_5 was unable to catch up with b_6 it is possible that it could have moved itself into a relationship with b_8, possibly lessening the threat.

6. MULTILEVEL DYNAMICS

Relational structure is clearly relevant in robot soccer. Thus objects of the form $\langle w_{i1}, w_{i2}, w_{i3}, b_{j1}, b_{j2}, b_{j3}, \ldots; R_t\rangle$ certainly exist, and some are named as objects at a higher level of representation. If the relation R were not decomposable, at each time t, there would be a simplex of the form:

$$\langle w_0, w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8, w_9, w_{10}, b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}; R_t\rangle$$

and the action would be played out on a 21-dimensional space. However, in robot soccer, and most other multiagent systems, R_t is decomposable. Here some agents are sufficiently far from the ball to have no affect on the local dynamics. For example, when a goal is being scored at one end of the pitch, the goalkeeper at the other end of the pitch is usually not involved. Thus robot soccer is characterized by the existence of subrelations between the agents and simplices such as $\langle w_{i1}, w_{i2}, \ldots, b_{j1}, b_{j2}, ..; R_t\rangle$. The dynamics of the agents are expressed, at least in part, by the dynamics of these simplices. To simplify the notation $\langle w_{i1}, w_{i2}, w_{i3}, \ldots, b_{j1}, b_{j2}, b_{j3}, \ldots; R_t\rangle$, let $W_i = w_{i1}, w_{i2}, w_{i3}, \ldots$ and $B_i = b_{i1}, b_{i2}, b_{i3}, \ldots$ so that the simplex can be rewritten as $\langle W_i, B_i; R_t\rangle$.

Let f be the mapping that transforms the state of system at time t to its state at time $t + 1$. Then there are four possibilities.

(1) A given simplex is transformed into another simplex with the same vertices $f : \langle W_i, B_i, R_t\rangle \rightarrow \langle W_i, B_i, R_{t+1}\rangle$. In this case, the vertices do not change,

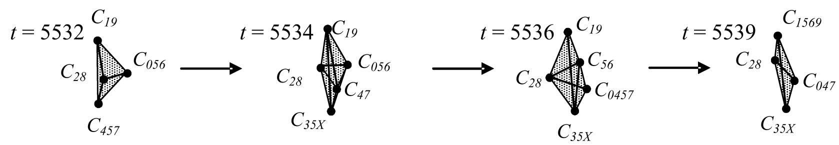


Fig. 18. The *Level N + 2* dynamics leading to the goal.

only the relation between them changes, and we will say that the transformation f is *continuous* for $\langle W_i, B_i, R_t \rangle$.
For example, in Figure 11:
$f : \langle w_5, b_6; R_{56,t} \rangle \rightarrow \langle w_5, b_6; R_{56,t+1} \rangle$,
$f : \langle w_1, b_9; R_{19,t} \rangle \rightarrow \langle w_1, b_9; R_{19,t+1} \rangle$,
$f : \langle w_2, b_8; R_{28,t} \rangle \rightarrow f : \langle w_2, b_8; R_{28,t+1} \rangle$ and
$f : \langle w_0, w_4, b_5, b_7; R_{0457,t} \rangle \rightarrow \langle w_0, w_4, b_5, b_7; R_{0457,t+1} \rangle$.

(2) If the simplex is transformed into two or more simplices, then we will say that f *decomposes* $\langle W_i, B_i; R_t \rangle$.
$f : \langle W_i, B_i; R_t \rangle \rightarrow \langle W_j, B_j; R_{j,t+1} \rangle + \langle W_k, B_k; R_{k,t+1} \rangle + \cdots$

(3) If two or more simplices are transformed into one simplex, then we will say that f *combines* $\langle W_i, B_i; R_{i,t} \rangle$, $\langle W_j, B_j; R_{j,t} \rangle$, etc.
$f : \langle W_i, B_i; R_{i,t} \rangle + \langle W_j, B_j; R_{j,t} \rangle + \cdots \rightarrow \langle W_k, B_k; R_{k,t+1} \rangle$
For example, in Figure 17:
$f : \langle w_1, b_9; R_{19,t} \rangle + \langle w_5, b_6; R_{56,t} \rangle \rightarrow \langle w_1, w_5, b_6, b_9; R_{1569,t+1} \rangle$.

(4) If two or more simplices are transformed into two or more different simplices, then we will say that f *recombines* $\langle W_i, B_i; R_{i,t} \rangle$, $\langle W_j, B_j; R_{j,t} \rangle$, etc.
$f : \langle W_i, B_i; R_{i,t} \rangle + \langle W_j, B_j; R_{j,t} \rangle + \cdots \langle W_k, B_k; R_{k,t+1} \rangle + \langle W_l, B_l; R_{l,t+1} + \ldots$
For example, in Figure 14:
$f : \langle w_0, w_2, b_8; R_{028,t} \rangle + \langle w_5, b_6; R_{56,t} \rangle \rightarrow \langle w_2, b_8; R_{28,t+1} \rangle + \langle w_0, w_5, b_6; R_{056,t+1} \rangle$

At *Level N + 2*, the configurations of players are treated as vertices as illustrated in Figure 18. The relation in this case means that the configurations are interacting in front of the goal being defended by White. Generally the dynamics are continuous at *Level N + 1*, but at $t = 5534$, the simplex $\langle C_{457}, C_{19}, C_{28}, C_{056}; R \rangle$ and vertex $\langle C_{35X} \rangle$ are combined. Before this, the players of C_{35X} are not explicitly part of the goal-scoring structure, although b_5 and b_6 keep w_3 away from participating in the defense of the goal.

Until the very end of the match, C_{19} remains unchanged with neither player w_1 nor b_9 participating in the action. Was it remiss of w_1 not to engage sooner in defending the goal, or was it a good tactic to ensure the b_9 was marked and safely out of the action? Was it remiss of b_9 not to have participated more actively, possibly giving b_6 a teammate on the wing?

This illustrates that simplices coevolve through time. In order to win the game, each team is trying to dynamically evolve their current simplices into ones with a higher probability of scoring. As we observe in current games, this means passing the ball to simplices that are closer to the opponent's goal. The evolution of simplices through time describes paths or trajectories in the space of all possible simplices.

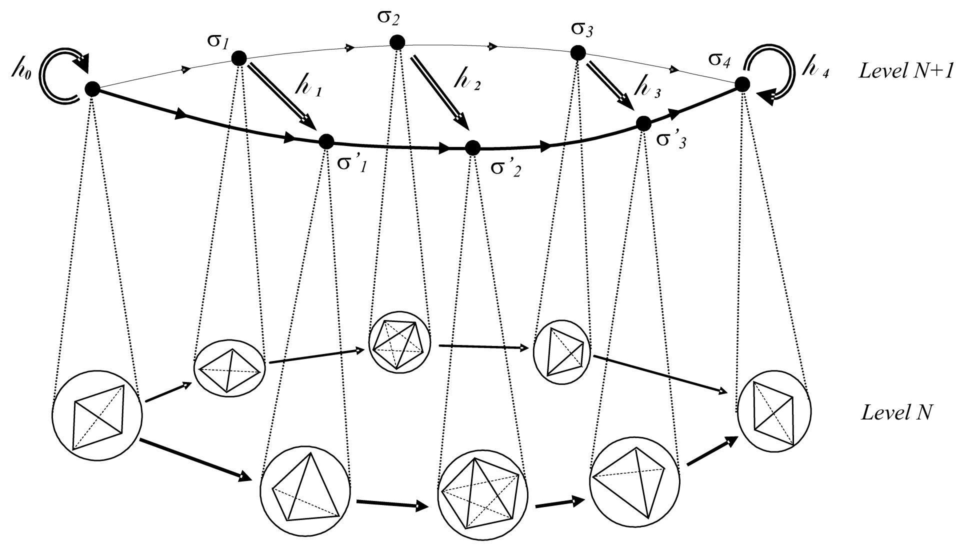


Fig. 19. Discrete pseudohomotopy at *Level N* and dihomotopy at *Level N + 1*.

7. TRANSFORMING TRAJECTORIES

The trajectories of the teams in soccer evolve dynamically, and they are very sensitive to the interventions of the other side. For example, a trajectory that might have led to a goal may collapse if a player fumbles the ball or an opponent mounts an effective tackle to capture the ball. Adherence to a particular trajectory represents a tactic, but it is more strategic to think in terms of classes of trajectories that are connected so that disruption to one good trajectory may simply nudge it into another good trajectory.

Figure 19 illustrates this in a multilevel representation in which *Level N + 1* represents the trajectory as a path of points in multidimensional space, while *Level N* gives the detailed structure of those points. In this example, let a *path* be defined as an order set of points, $\sigma_1, \sigma_2, \sigma_3, \sigma_4$, and let the mappings h_i be defined as $h_i : \sigma_i \rightarrow \sigma_i'$. Under appropriate conditions, the mappings h_i can be considered to be a *homotopy* [Hilton and Wylie 1965].

Atkin [1977] developed a concept of *pseudohomotopy*, or *shomotopy*, applicable to *Level N* in Figure 19 for the discrete case of simplicial complexes. The details are beyond the scope of this article, but the idea is that chains of connection can be transformed into other chains of connection by face-saving maps (a discrete analogue to continuity).

If the *Level N + 1* trajectories are continuous, then a continuous transformation from one trajectory to the other can be considered to be *homotopic*. Moreover, as the evolution of simplices follows a direction, that of simplices with higher probability of scoring, then *directed homotopy* (dihomotopy) could be used as a tool to study the state space of possible simplices and their evolution [Goubault and Raussen 2002; Fajstrup et al. 2006].

The interlevel relationship between dihomotopy and discrete pseudohomotopy is a new idea of great potential relevance to complex systems science in general[1]. This area remains to be investigated and will be further studied in our future work.

8. VISUAL COMMUNICATION IN SWARMS OF ROBOT AGENTS

The hypernetwork structures developed in the previous sections support a theory of robot interaction that does not require communication of symbolic information [Johnson 2001].

The idea is that each robot perceives the robot soccer field using their vision system to get information on the position of themselves, the ball, and the other players. The robots each process autonomously the information available to them to obtain an individual world view. This information processing is done in parallel, and the individual perceptions will sometimes be inconsistent. The world view may involve memory of previous world view, for example even though a robot cannot see another robot out of the field of view, they can know that robot is in the universe because it was seen previously.

Thus each robot can see other objects and relationships between them as a simplex (or set of simplices). Some simplices may be known to be part of good trajectories by all the robots. Thus if all the robots recognize a configurational simplex at a given time, they are all in a position to recognize it as being a precursor to another good position related to the good trajectory. If all the robots move to the next-good position, there are now two recognized simplices of the trajectory. In this way, there is massive positive feedback between the robots just by observing each others' relative positions that they are all on message to pursue the good trajectory.

Of course, given the many random events in a soccer game, a good trajectory may fail. Some robots may not move fast enough or move too fast. The ball may move unexpectedly, possibly due to grit on the pitch or unpredictable spinning. An opponent may unexpectedly move to spoil the trajectory, and so on.

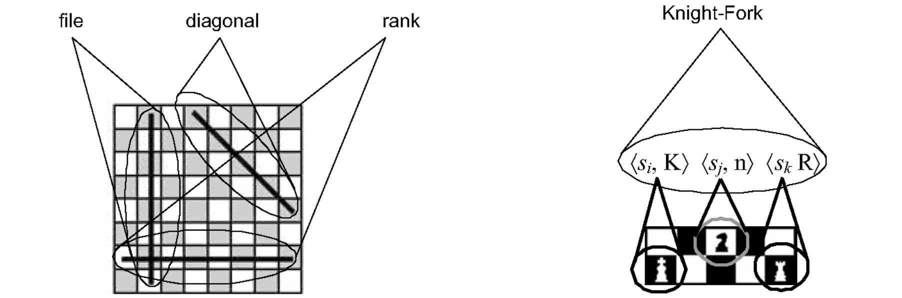
These considerations lead to the idea of *culture* in robot soccer teams where that culture is stored as patterns of trajectories that all the robots have experienced before and can recognize as being good or bad. Human soccer player train together, presumably for the very reason that they can share experiences and learn common structures. Thus one can imagine teams of soccer playing robots playing many training games together and learning new trajectories.

In a competitive game like soccer, a successful strategy may become unsuccessful as the opponents learn to counter it. Thus one can image a coevolution between robot soccer teams as each develops new strategies based on new trajectories and learns how to spoil strategies which the opponents have developed.

9. APPLICATIONS IN OTHER DOMAINS

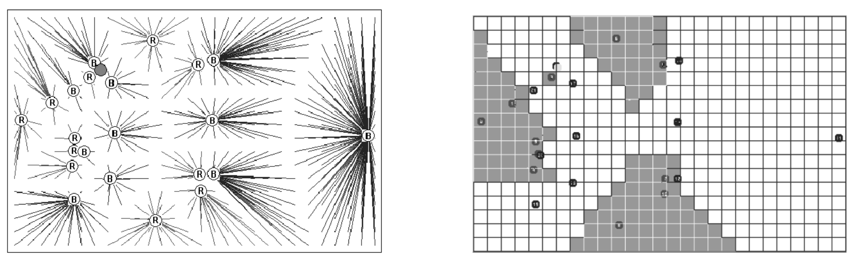
The ideas in this article have been developed in many domains over the years. This section will discuss a few of these to show how the ideas might be applied in particular, and to show where the approach is leading in general.

[1]We are grateful to an anonymous referee for making the connection between our trajectories and dihomotopy.



(a) squares aggregating into ranks and files (b) a multilevel knight-fork structure

Fig. 20. Hypernetwork structure in the game of chess.



(a) relationship between players and grid squares (b) the winning team owns most of the pitch

Fig. 21. Structuring space in robot soccer.

9.1 Multidimensional Chess

In the nineteen seventies Atkin [1974] used simplicial complexes to model the game of chess. Figure 20(a) shows how features such as the ranks, files, and diagonals can be considered to be structured sets of squares. At the lower-structural levels, Atkin defined relations between the pieces and the squares of the board. For example, if a square is attacked by a rook, a queen, a pawn, and a bishop, that square becomes associated with the simplex $\langle rook, queen, pawn, bishop \rangle$. Clearly the dimension of the simplex is important as are the actual pieces attacking. Figure 20(b) shows a well known structure in chess, the knight-fork. $\langle s_i, K \rangle$ is the substructure of the King on its square, $\langle s_j, n \rangle$ that of the knight, and $\langle s_k, R \rangle$ that of the Rook. Atkin showed how the multilevel simplices evolve during a game of chess, and his work is the inspiration for the approach we have taken to robot soccer.

9.2 The Space-Time Possession Game

In robot soccer, the relationship between the players and positions on the pitch is clearly important. Figure 21(a) shows a relation between points on the soccer pitch and the nearest in an analysis of a game by Johnson and Price [2003]. Here the player owns the parts of the pitch that they are closest to, and the team owns all the points of its players. Figure 21(b) shows this with the squares coded grey and white. In this case, the white team owns much more of the

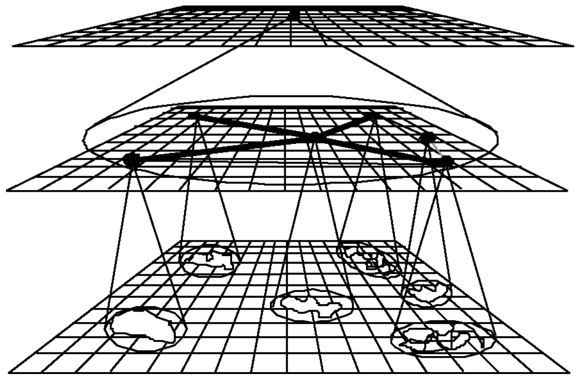


Fig. 22. Multilevel hypernetworks in machine vision.

pitch than the grey team, and this was related to a goal being scored shortly afterwards.

To analyze this kind of structure without the distraction of the rules of soccer, Law and Johnson [2004] devised what they called the *space-time possession game*. In this game, a set of players can move one square at a time, and the aim of the game is to own as much of the pitch as possible. In the terms of this article, the squares owned by a given player form a simplex, and the team owns the space defined by all its players' simplices. As with robot soccer, the game follows a trajectory through state space. The possibility of transformations between trajectories, as suggested in the previous section, remains to be investigated.

9.3 Machine Vision

Machine vision begins with images as sets of pixels at the lowest level, *Level N*. Typically, low-level features are extracted by local processing operators such as filters. In our terms, such operators assemble the pixels into intermediate-level objects. These intermediate-level features are then assembled into objects (Figure 22).

In our work on machine vision, we have defined *gradient runs* as particularly useful primitives. For example, a left-to-right-down gradient run is a horizontal set of pixels where each pixel is darker than its left neighbor. Similarly, a left-to-right-up gradient run is a set of pixels where each pixel is darker than its right neighbor. The horizontal runs of an image are thus partitioned into up- and down- gradient runs (similar structures exist for vertical and diagonal runs). Where there are edge features, these runs stack up vertically to form *gradient polygons*. Because no parameters are involved, gradient runs and gradient polygons are very robust features in images, and they are very useful as intermediate objects in multilevel vision architectures [Johnson and Simon 2001].

10. FUTURE WORK

A central issue for the control of multiagent systems is that of action-coordination. The action-coordination problem consists in finding the best individual action for each agent in order to maximize the team's outcome. In other words, what should each robotic player do in order to achieve a good team performance? For example, trying to get ball possession may be individually a good action to take but all players running for the ball is a bad tactic. Finding the joint actions (the set of best individual actions) is an exponentially increasing problem with the number of agents and becomes impractical even with few agents when these have a large action spaces (set of possible actions) [Durfee 2001].

Often in multiagent systems an assumption can be made that simplifies this problem. Agents are often situated in different physical, task, or time spaces, for example, robotic football players are positioned in different locations on the field, they are assigned different roles or task (goalkeep, attack, defend, cover, etc) and in this case, share a same time. When their situation in space, task, or time is considered farenough, then it can be assumed that their interactions will be low or zero. For example, the goalkeeper's actions have small or zero relevance when the team is attacking on the other side of the field (physical separation). In this way, it is possible to define subgroups of agents within the multiagent system that need to coordinate their actions and those that perform actions that are irrelevant.

Hypernetworks have been used in this article to represent relations among agents. A possible extension and application of this work would be to use hypernetworks to discover and represent relations between agents that need to coordinate their actions, in other words, agents that closely interact for example, the agents interacting in a defender's dilemma. Coordination graphs have been recently used to coordinate agent actions in simulated robotic soccer [Jelle et al. 2005]. This work defines graphs with agents as their nodes and interactions as their links. Then a variable elimination algorithm is applied to find the joint actions for the subgroups of agents (nodes) of a particular graph. This work relies on the previous assumption that a multiagent system can be decomposed into independent agent subgroups and that the multiagent's global outcome is the addition of the subgroup outcomes.

The multilevel hypernetwork approach described in this article could be used to relax the previous assumption. Given that hierarchical levels of hypernetworks can be defined, these could be used to represent coordination requirements among agents at different levels, starting with highly-coupled agents at low-levels of the hierarchy and moving into lower-coupled subgroups. Figure 23 illustrates this idea. Figure 23(a) illustrates the game at time $t = 154$ in which the black team is attacking. At this particular point in time, player b_8 is passing the ball to b_9 (upper right-hand side in the figure). It also illustrates possible hypernetwork configurations at *Level N + 1*, representing the strongest coordination requirements for the agents at this point in time. For example, players b_8 and b_9 are passing the ball, a process which requires the coordination of actions (passing and receiving), thus the simplex $C_{89} = \langle b_8, b_9; R_{N+1,t=154}\rangle$ can be used to describe this necessity for action coordination. In the same way, players b_1, b_2, and b_4 need to be coordinated to play the attacker's offside rule; this can be represented by simplex $C_{124} = \langle b_1, b_2, b_4; R_{N+1,t=154}\rangle$. Still, in *Level N + 1*, there are three other simplices $C_0 = \langle b_0; R_{N+1,t=154}\rangle$, $C_{36} = \langle b_3, b_6; R_{N+1,t=154}\rangle$,

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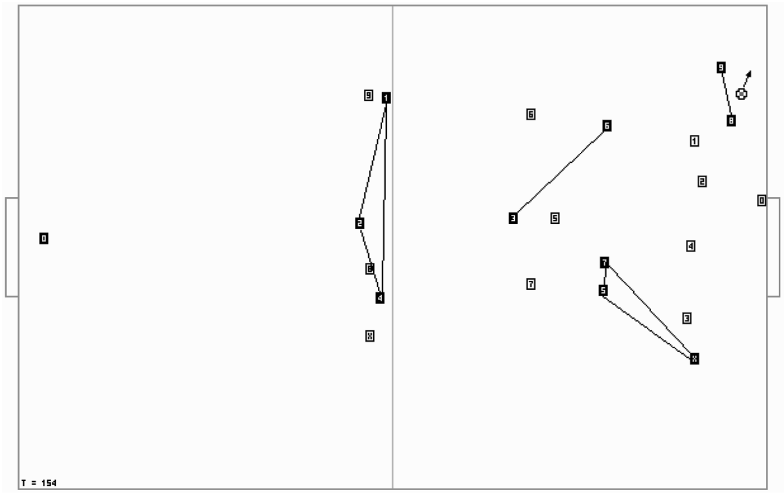
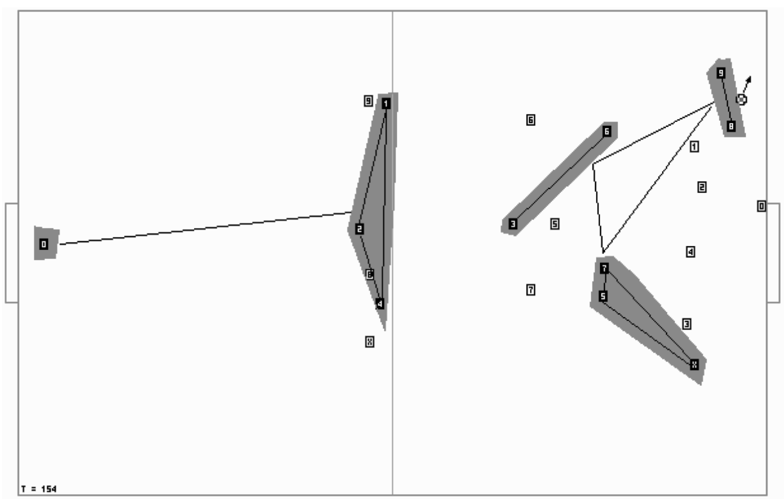


(a) Snapshot at $t = 154$ and Hypernetworks at *Level N+1*



(b) Hypernetworks at *Level N+2*

Fig. 23. Coordination hypernetworks at various hierarchical levels.

and $C_{75x} = \langle b_7, b_5, b_x; R_{N+1,t=154}\rangle$ used to describe the coordination among the rest of the back team.

In a second level of this hierarchy (*Level* $N + 2$), structures at *Level* $N + 1$ are reassembled into attacking and defending structures illustrated in Figure 23(b). The attacking one is $A = \langle C_{89}, C_{36}, C_{75x}; R_{N+2,t=154}\rangle$ and the defending one is $D = \langle C_{124}, C_0; R_{N+2,t=154}\rangle$. At this level, coordination requirements will have to be described using a different language as coordination may not be expressible in terms of individual agent actions but as subtasks for the agents forming the subgroup. In other words, coordination is at the level of subgroups.

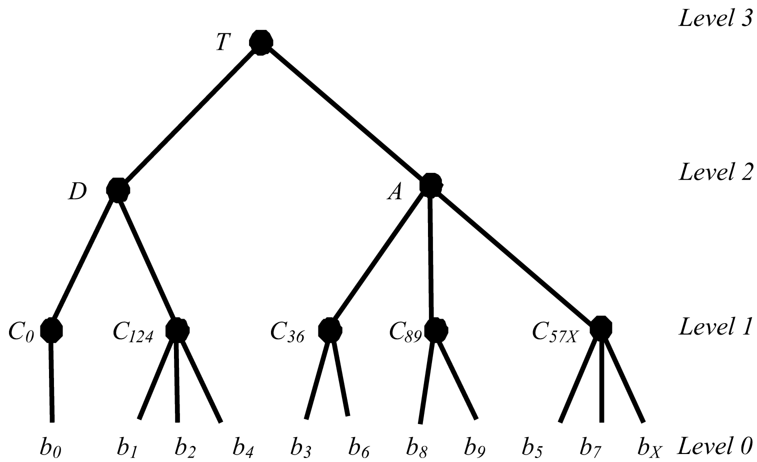


Fig. 24. Hypernetwork levels representing coordination requirements.

Finally, structures representing coordination requirements at *Level* $N + 2$ can be assembled into a team coordination level (*Level* $N + 3$) and represented by simplex $T = \langle A, D, R_{N+3t=154}\rangle$. Again, coordination at this level may need to be in term of group subtasks. Figure 24 illustrates the hypernetwork hierarchy representing the coordination requirements in this example.

11. CONCLUSIONS

This article has investigated the application of multilevel hypernetworks in team robotics as an example of a complex interaction-based system. We have shown how hypernetworks can represent multilevel relational dynamics by the in-depth analysis of a robot soccer simulation game.

In this article, we have sketched a mathematical formalism for representing, the relational structure between agents. We have shown that some important structures are associated with n-ary relations, where $n > 2$, requiring hypernetworks rather than networks to represent them. Imposing an n-ary relation on a set creates structures at higher levels of representation. Thus it is possible to discriminate levels in a multilevel system.

At any instance, the state of the system is represented by its multilevel relational structure. The dynamics of the system are represented by state changes. Sometimes these are continuous with no change in the topology of the hypernetworks, and sometimes they are not. Controlling such systems involves taking actions intended to result in desirable state changes at all levels.

In general, the game is portrayed as a sequence of passes, where for both teams the game seems to be characterized by the heuristic of "try to pass safely to a teammate closer to the goal". Both teams are very good at this as befits the 2003 RoboCup finalists. However, there is little evidence of the teams working towards good intermediate structures where the players make space for themselves, putting themselves into good combinatorial positions as human soccer players do.

The analysis of this sequence suggests that structural opportunities may have been lost. In other words, the multilevel structures and their names form a vocabulary that can be used to reason about the game. Some structures are desirable for one team and undesirable for the other, for example, the defender's dilemma. Recognizing these structures as intermediate goals is an important part of controlling a robot soccer system.

Our group is investigating these multilevel structures in the belief that mastering them will give our soccer-playing robots an advantage. An important aspect of the structures is where players position themselves relative to each other, and we have developed a space-time possession game in which we investigate how players can move to command the greatest areas of the pitch. Iravani et al. have shown that certain configurations of players are associated with successful passes [Iravani 2005; Iravani and Johnson 2005; Iravani et al. 2005]. Identifying simplices at all levels involves pattern recognition, effectively defining relevant relations, R_i, and seeing whether they hold for interacting subsets of players and higher level structures.

Although these ideas have been illustrated for robot soccer, we believe that they can usefully be applied to a wide range of multiagent systems.

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