How far can Q-analysis go into social systems understanding ?

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Abstract:
Q-analysis as a whole is a grey area. The long way from algebraic topology to metaphorical discourses is carefully inspected. The implicit semantic choices are studied in a self-contained development to clearly circumscribe the underlying assumptions, hence highlighting their importance. The major addressed issue is to separate the syntactic perspective from the semantic perspective.
Speaking technically, the point is to determine which interpretations make sense to identify potential for a future systemic development. The limits of Q-analysis are explored from a systemic point of view and personal examples are suggested to separate artefact exploration from systems dynamics understanding. So, a social science methodology may shift to a branch of social systems theory.

Keywords:
systems theory, algebraic topology, structural properties

1. Introduction

Q-analysis[1], introduced by Ronald Atkin [Atkin (1972), (1974), (1977)] in the early seventies is referred to as a language of structure in many writings. Whereas systems are aggregate of many constituents which are interrelated in complex ways, the structural description of linkage among components may fall into the scope of Q-analysis.
We aim at an appraisal of existing and potential applications of Q-analysis in the field of systems theory and, subsequently, at an outlook as plans for future research.
The progressive elaboration of systems modelling with Q-analysis methodology is a long way from general topology to metaphors where fairly abstract and affected philosophical lectures are mixed together.
Faced with a metaphorical discourse heavily flavoured by the methods of algebraic topology, abstract methodology, practical applications and their relationships are to be inspected in order to separate the syntactic perspective from the semantic perspective.
The implicit semantic choices are to be studied in an attempt to clearly circumscribe the underlying assumptions, hence highlighting their importance.
The present paper aims at scrutinizing and questioning Q-analysis methodology. A review of concurrent methodologies which purport to represent systems is not within its scope.
The rigorous structure of paragraphs in the paper acts as a guarantee of success in enlightening the shift from mathematical tools to metaphorical discourses.
In Section 2, after a review of the barest essentials, the necessary topological definitions are introduced. Then, Q-analysis is summarised as a strictly mathematical tool. Section 3 inspects some of the applications of the methodology from a critical point of view to formulate the right questions. Some of the personal answers are eventually in Section 4. The conclusion of

[1] This methodology is not to be confused with an algorithm that groups objects (from variables) and called Q-analysis (or Q-factor analysis) in opposition to R-analysis that groups variables.

this paper highlights the shortcomings and indicates possible directions for future developments.

2. From topology to Q-analysis

2.1 Basic notations

It is assumed that the reader is familiar with capital letters standing for sets and lower case indexed letters standing for elements. A binary relation λ is a rule to decide whether an element a_i of A and an element b_j of B are related according to a specified condition. Only certain pairs of $A \otimes B$ satisfy the condition, λ determines a subset of $A \otimes B$. These pairs are ordered, $< a_i, b_j >$ is distinguished from $< b_j, a_i >$. The inverse relation of λ is denoted λ^{-1}.
A graph pictures a set of points connected by edges which represent a list of pairs. The edges are directed to represent a binary relation.
A hypergraph edge no longer connects pairs of points but rather encloses subsets thereof. The underlying relation needs not to be of fixed arity. The representative graph of a hypergraph connects two edges when their intersection is non-empty.

2.2 Algebraic topology

Topology studies spatial relationships and continuous deformations of paths. Combinatorial topology shifts from a topological space seen as a continuum to a finite discrete space. Algebraic topology adds algebraic tools.
This paragraph avoids unduly mathematical writings. The remaining notational conventions are necessary for further understanding. The non-mathematician reader can quickly grasp through intuitive visualizations while the mathematician reader is invited to use the referenced books [Hocking (1988)], [Kinsey (1991)], [Spanier (1966)] for further investigations.

2.2.1 Simplicial complexes

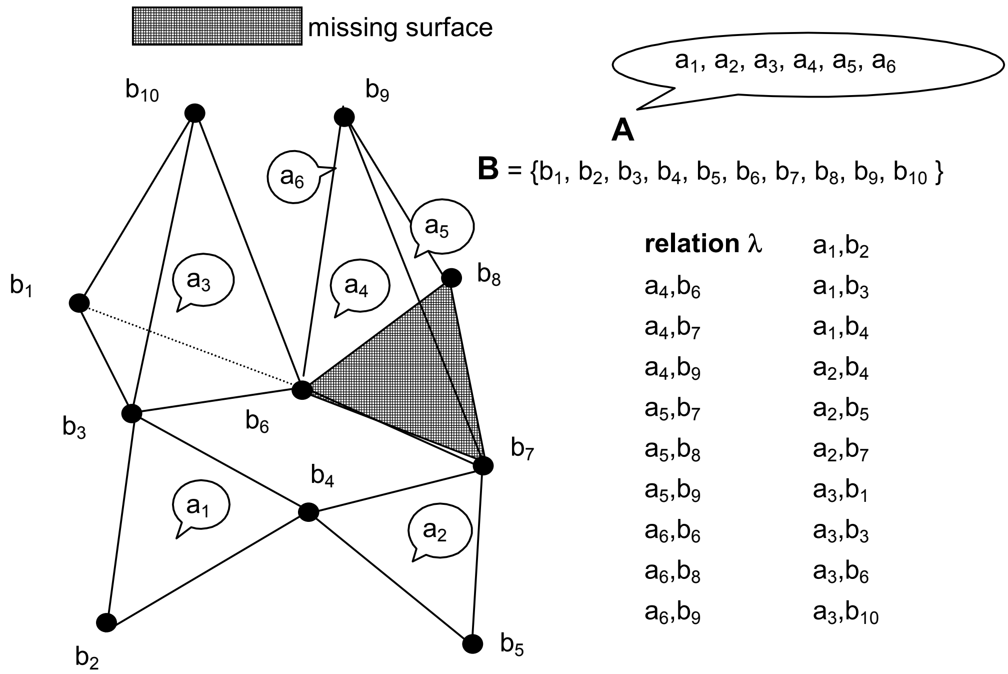
A finite simplicial complex K is a set of vertices, $B = \{b_1, \dots , b_n\}$ and a set of subsets of B. The subset σ_p^i with $p+1$ vertices is called a p-simplex. σ_p^i is said to have dimension p (one less than the number of vertices). The superscript i is an index (more than one simplex has dimension p). A simplex σ_q is said to be a q-dimensional face of σ_p, if and only if every vertex of σ_q is also a vertex of σ_p. K satisfies the condition that all the faces of its simplices are also in K. The dimension of K is the largest value of p for which there exists σ_p^i.

The simplices can be represented with a spatial structure usually shown as a polyhedral one. Gluing such polyhedra of mixed dimension forms the complex.
The following Figure 1, pictures the geometrical visualization and will be used for illustration purposes in further paragraphs.

2.2.2 Homotopy

A path in a space X is a continuous mapping from $[0,1]$ onto X. The values for 0 and 1 are the ends of the path. Two paths are homotopic if one mapping can be deformed continuously (while preserving the ends) into the other one.
One may imagine stripes on a waving flag (topologically equivalent at all times) but one has to keep in mind that the objects at hand are functions.
A loop in X based at x is a path which begins and ends at x .
Since homotopy is an equivalence relation, the set of all loops in X based at x can be decomposed into disjoint classes, each class being a collection of homotopic loops[2].

[2] With the definition of an arithmetic (juxtaposition, constant and inverse loops) the subspace becomes the fundamental Poincaré group of X at x. The homotopy groups are topological invariant features.



simplices	<b_2,b_3,b_4>	<b_4,b_5,b_7>	<b_6,b_7,b_9>	**complex**
dimension 2	<b_7,b_8,b_9>	<b_6,b_8,b_9>		$K_A(B,\lambda)$
dimension 3	<b_1,b_3,b_6,b_{10}>			dimension 3

0-faces		**1-faces**		**2-faces**	
simplex a_1	<b_2>	simplex a_4	<b_6,b_7>	simplex a_3	<b_1,b_3,b_{10}>
	<b_3>		<b_7,b_9>		<b_3,b_6,b_{10}>
	<b_4>		<b_6,b_9>		<b_1,b_6,b_{10}>
					<b_1,b_3,b_6>

Speaking intuitively, the definition of homotopy in terms of shrinking loops suggests a geometrical visualization. One can imagine how a space may be described with inserted bootlaces. The ability to retract a lace (as a snare) reveals in part the structure of that space.

Figure 1

2.2.3 Homology

Homology theory is a combinatorial approach of the connectivity properties of a space. It can be used to codify how the different connected pieces fit together to generate the entire complex. *It is not to be confused with the above homotopy*.

Let an unordered sequence of p-dimensional simplices of a complex K be a p-dimensional chain. For example, on Figure 1, $(a_1 + a_2 + a_5)$ is a 2-chain (defined as a linear combination where + stands for a kind of union). The addition of k-chains is defined naturally *modulo*[3] *2*.

Speaking intuitively, the coefficients in a chain are either *0* or *1*. For brevity, if a simplex is present twice, the addition taken *modulo 2* (*1+1=0*) "cancels" the simplex. For example, on Figure 1, $(a_1 + a_2 + a_5) + (a_4 + a_2 + a_5)$ is the 2-chain $(a_4 + a_1)$.

The boundary (called δ) of a p-dimensional chain is the *(p-1)*-dimensional chain that forms a linear combination of all *(p-1)*-faces of each p-simplex. For example, on Figure 1, the boundary of a_3 ($\delta(a_3)$) is the 2-chain $(\langle b_1,b_3,b_{10}\rangle + \langle b_3,b_6,b_{10}\rangle + \langle b_1,b_6,b_{10}\rangle + \langle b_1,b_3,b_6\rangle)$.

[3] In general, homology theory defines positive and negative orientations for the simplices by choosing an arbitrary order on the vertices. The use of an integer *modulo 2* as coefficient avoids the orientation.

A p-chain is called a p-cycle if its boundary is a *zero* p-chain (each face is counted an even number of times since the simplices share $(p-1)$-faces two by two). For example, on Figure 1, the 2-chain $(a_4 + a_5 + a_6 + a_4)$ is not a 2-cycle. Conversely, the boundary of a_1 is the 1-cycle $(<b_2,b_4> + <b_3,b_2> + <b_4,b_3>)$ with $(<b_2> + <b_3> + <b_4> + <b_2> + <b_3> + <b_4>)$, a *zero* 0-chain, as boundary.
The boundary of a boundary is zero and therefore every boundary is a cycle.
The p-cycles are of especial importance because they enclose either a $(p+1)$-chain or a cavity. For example, on Figure 1, the 1-cycle $(<b_6,b_7> + <b_7,b_4> + <b_4,b_{3\,+}<b_3,b_6>)$ is a 1-hole since the cycle encloses a cavity (e.g. : the 2-chain $(<b_6,b_7,b_4> + <b_6,b_3,b_4>)$ does not exist).
Homology (*modulo 2* homology in that case) comes as an equivalence relation used to decompose the set of all p-cycles into disjoint classes[4] (to deal with redundant information).
Two p-cycles are homologous if their sum is a bounding cycle (equal to $\delta(D)$ for some $p+1$-chain D). For example, on Figure 1 $(<b_6,b_7> + <b_7,b_4> + <b_4,b_2> + <b_2,b_3> + <b_3,b_6>)$ is a 1-cycle homologous to the 1-cycle $(<b_6,b_7> + <b_7,b_4> + <b_4,b_3> + <b_3,b_6>)$ since the sum is the 1-chain $(<b_4,b_2> + <b_2,b_3> + <b_4,b_3>)$ bounding cycle of the simplex a_1.

2.3 Complexes on a relation

2.3.1 Simplicial complex construction

Atkin's aim[5], in associating a simplicial complex K with a binary relation λ, is to explicit some of the knowledge content of λ with properties of K (a multidimensional representation).
Let λ be a binary relation λ on $A \otimes B$. Let B be the vertex set. A simplex σ_p is constructed with $p+1$ distinct vertices b_j which are λ-related to a_i.
The set of simplices is a simplicial complex denoted $K_A(B,\lambda)$. Since an arbitrary element a_i is λ-related to exactly $(k+1)$ b_j, $\sigma_k(a_i)$ is distinguished as a named simplex (naming is a many-one mapping from A onto $K_A(B,\lambda)$). One has to bear in mind that some $\sigma_k(a_m)$ may be equal to $\sigma_k(a_i)$ (two simplices are identified whenever they have the same vertex set) and that the faces of $\sigma_k(a_i)$ have no name. Figure 1, is an example of such a simplicial complex.
The same procedure associates a conjugate complex with λ^{-1}. It is called $K_B(A,\lambda^{-1})$.

2.3.2 Simplicial complex interpretation

Thus far, the simplicial complex is a special type of family of sets.
Yet, the grey area extending between pure topology and an application-devoted formalism includes the choice of an interpretation for the achievement of the simplicial complex.

The above complexes are abstract structures that generalize graphs into hypergraphs. Consequently, every vertices of a simplex is adjacent to all other vertices in that simplex. The "traces" of the relation λ in the vertex set induce a new n-ary relation. This is a critical bridge and any application ought to precise the meaning of this new relation.

The unsettled actual meaning of the simplex may be illustrated with an alternative construction of a simplicial complex [Vainshtein (1977)] that skips the §-2.3.1 previous step.
Let a system be considered parts standing in interaction because the state of each part is dependent on the state of other parts via a directed influence/dependence linkage.
When a part is replaced by a simplex whose vertices are its influence/dependence contacts and when the vertices of the simplices corresponding to linked contacts are identified, a complex can be constructed. Face sharing represents the parts linkage.
The above complex represents a composite graph in which a directed graph (linkage) on the vertices (contacts) is associated with a hypergraph (clusters of contacts).

[4] Under operation on chains the subspace becomes the p^{th} homology group of K. Its number of generators is called the p^{th} Betti number (number of p-dimensional holes or missing $(p+1)$-chains).
[5] Atkin's idea is based on mathematical work of C. H. Dowker, 1952 (Homology groups of relations, *Annals of Mathematics*, **56**, 84-95).

It is important to note that the actual interaction stands outside the hypergraph edges.

2.4 Q-analysis

The topological notion of connectivity is the keystone of Q-analysis methodology.
Q-analysis, inviting data to be inspected without distorting, contrasts with the conventional metric approach, dealing with a manipulation of data involving some loss of information.

2.4.1 Connectivity

Two simplices of $K_A(B,\lambda)$ (resp. $K_B(A,\lambda^{-1})$), are said to be q-near simplices when they share a $(q+1)$-face (connected to each other by the sharing of vertices). For example, on Figure 1, a_1 and a_2 are 0-near ; a_4 and a_5 are 1-near. A finite sequence of q-near simplices defines a sequence of q-connectivity. For example, on Figure 1, a_3 is 0-connected to a_2 (via a_1) ; a_4 is 1-connected to a_5 (via a_6). Since q-connectivity is an equivalence relation, the simplices may be classified into classes of q-connected simplices.

We emphasize that q-connectivity is the transitive closure of q-nearness. The meaning of "near" lets the reader think that "similarity" is the purpose in mind. Now, "to be similar" is not a transitive relation when facing with empirical data. So, the length of the sequences should have been a key indicator. According to the semantic interpretation of the simplices, any application ought to keep the "nearness weakening" in mind.

The q-nearness graphs (the edges join points representing simplices when they are q-near) may be used together with the list of q-connected components. These graphs generalize the representative graph of the hypergraph. Consequently, the induced n-ary relation (§-2.3.2) induces a family of binary relations identifying a linkage between simplices at each q-level. This is a new critical bridge and a semantic interpretation should come before any application.

Several structure indicators can be computed to feature the global structure (complexity) and local structure (concentricity, eccentricity) of a complex. The main basis is the structure vector Q which lists the number of "pieces" existing in the complex at level q for each q.

2.4.2 Pseudo-homotopy

Shomotopy, or pseudo homotopy, is Atkin's attempt at developing a discrete analogue of homotopy [Atkin (1976), (1977)].
Q-loops, when a simplex is q-connected to itself, are the basis for defining the q-adjacency (an equivalence relation on the q-loops) and for constructing the classes of shomotopic (*modulo* q-adjacent) loops of a complex. Intuitively speaking, q-loops of one class can be deformed into shorter q-adjacent q-loops in that class. For example, on Figure 1, $(a_1, a_3, a_4, a_5, a_6, a_4, a_2, a_1)$ is a 0-loop. and is shomotopic to $(a_1, a_2, a_4, a_3, a_1)$. According to the density of connections, the loop can be contracted in a single simplex (shomotopicaly trivial). For example, on Figure 1, (a_4, a_5, a_6, a_4) is a 1-loop and also a 0-loop shomotopic to the 0-face $<b_9>$. Should it be otherwise, the "contraction" stops with a minimum sequence length. For example, on Figure 1, the previous 0-loop $(a_1, a_2, a_4, a_3, a_1)$ is not 0-adjacent to a shorter 0-loop. The analogy with the bootlace of §-2.2.2 is attractive. Some missing simplices seem to be an obstacle to a free shrinking ; Atkin called them shomotopic q-objects.

Unfortunately, a naive feeling for these forms coming from picturing simplices as polyhedra may be prejudicial. All the simplices in the sequence may have the same q-face. A sequence may go there and back since any simplex is connected to itself. The q-loops stands in a q-connected component (miscellaneous k-simplices ($k \geq q$) of the complex) where p-simplices ($p < q$) do not appear. Q-loops *are not to be confused* with p-chains in homology terminology.
The point is the tiny link between homoloy and shomotopy.
Since a 1-cycle is included in a 0-loop (and *may* be a 1-hole) [Griffiths (1983), p.421], a q-loop implies the existence of a 1-cycle. No additional implication is valid.

Shomotopic objects may exist without homological holes and holes may exist without shomotopic objects. Shomotopy says nothing about the cavities but shomotopy suggests that some simplices are missing. However, when homological holes are present, the addition of the corresponding higher level simplices changes connectivity (the faces of a $(q+2)$-chain form a bounding $(q+1)$-cycle and the involved simplices are q-connected in a q-loop) and may increase the *degree* of connectivity. For example, on Figure 1, the addition of the 2-simplex $<b_6,b_7,b_8>$ creates a 2-hole and the addition of the 3-simplex $<b_6,b_7,b_8,b_9>$ suppresses the hole and introduces 2-connectivity in the complex (2-near from a_4, for example).

Once more, we face with a semantic choice. What is "more discerning" ? Atkin works with "free movement" considered "grade by grade" in mind.

Jeffrey Johnson, translates the "free movement" into analytical terms (q-transmission [Johnson (1982)]). Much of the relevance of q-transmission is based on the assumptions that one can find examples of real systems where q-connected components are readily interpretable and q-nearness is necessary while $(q \pm 1)$-nearness is disregarded.

For a quick grasp, imagine the greedy, and yet wise, worm in the doughnut. It has an interest in shomotopy (it moves round the 2-loops). When the doughnut is in a circular box, our worm has an interest in homology (the core 2-hole) because it can move round a shorter 1-loop.

3. Applied Q-analysis

Q-analysis came to be used to solve problems ranging from failure diagnosis of large-scale systems [Ishida (1985)] to crossroad's ability to carry flows [Johnson (1981)] over a wide field of inquiry including organization of a rule-based system [Duckstein (1988)] or multicriterion decision-making [Chin (1991)]. We aim at extracting from a careful review of past Q-analyses[6] the reality which the above formalisms purport to model.

3.1 Objects and features

In Q-analyses the named simplex σ_p (a_i) of the complex $K_A(B,\lambda)$ is an object represented by a collection of features (b_j) to which it is λ-related. The meaning of "object" and "feature" is afforded several interpretations and allows interchangeability of A and B (complex $K_B(A,\lambda^{-1})$).

The objects may be either persons (patients, farmers, readers, community members) or concepts (social needs, scientific subjects, social events) or things. The things may be either concrete (shopping centres, spatial areas, manufacturing parts, databases) or abstract. The abstract things may be either representations (bungalow plans, routes design in a road system, failures of fault-units) or symbols (geometrical figures).

The features are either attributes of the objects (measures, properties, scale of descriptors) or "comparable" objects (junction in routes, technology requirements for needs, machines, databases hosts, types of retail outlet for shopping, plant species in areas, squares of the chess board, clinical symptoms, diagnostic categories) or the objects themselves (relationships among people, distance between markets).

When objects and features are involved in a predicative relationship, q-connectivity depicts a between-object similarity (computed by counting shared features).

The meaning of the n-ary relation induced by λ (see §-2.3.2) is part of the application.

It is crucial to distinguish between simplices in which the elements are only λ-related to a vertex and those in which they are linked to each other. When routes are related to the sets of physical segments that constitute them, the vertices are the *parts* of the simplices. When people are related to events in which they are engaged, the connected vertices of $K_A(B,\lambda)$ *are* the simplices of $K_B(A,\lambda^{-1})$ because a social event is a reificated and named meeting of people.

When the vertices stand for uncorrelated variables (used for example to explain dependent concepts), the hypergraph is abstract and the vertices are not actually connected.

[6] We apologize in advance to the authors for the unforgivable reference cuts.

3.2 Backcloth and traffic

The existence of q-nearness graphs (§-2.4.1) and shomotopic objects (§-2.4.2) put forward the meaning of connectivity for examination. Thus far, the methodology looks irredeemably static, and yet, it just purports to be the opposite (John Casti terms Q-Analysis "polyhedral dynamics" [Casti (1982)]). The answer to this questioning comes with a suggestive metaphor : Q-analysis builds a space of descriptions of "things" (the backcloth) over which dynamics will run (the traffic) and the "action space" influences "activity".
Q-connectivity expresses the way changes can be propagated over the backcloth. It turns the q-nearness graph edges into "channels" or "tunnels" of transmission.
According to the applications, the shomotopic objects (the "empty space") become either an "obstacle" (to bypass for the traffic) or a "driving force" (traffic generator). This remark stresses the need for a systemic-oriented interpretation of the prejudicial "free movement" metaphor. The stability may come from structural constraints. The key point is illustrated with structural analysis [Ténière (1988)] of a business or administrative organization. The shomotopic objects are the very meaning of the system. When any factor influences any factor (trivial shomotopy for loops), one may argue that there is no organization at all.

4. Backcloth and traffic in system theory

Before some systemic personal extensions, it is important to note that the translation (of the suggestive metaphor) into topological terms is by no means completed.
Firstly, the distinction between structural features and traffic features is relative. As a non-exhaustive list of examples, we can list : quantity of goods, monetary expenditure, traffic congestion, travels time, electrical current). The implicit agreement seems to be the "less-permanent" characteristic of the attributes.
Secondly, there are disagreements over the formal linkage between traffic and backcloth structure. Atkin' s work conceives traffic as functions (from the set of simplices to values) called patterns on the complex [Atkin (1976)]. Unfortunately, it turns to be a topological burden as soon as the values on a simplex are not determined by the values on its faces.
An alternative way of considering the mapping consists in constructing the simplicial complexes corresponding to *two* binary relations [Gaspar (1981)]. The first one stands for the backcloth (objects, features), the second one stands for the traffic (objects, values). Much remains to be done during the comparative study of those complexes.

Last, but not the least, the relationship between structural form and structural change is not actually formalized. When the "space" changes, the connectivity changes and alters the transmission of "activity" ; when "activity" changes, it may produce "space" change … and so on. And yet, the methodology does not allow structural variations. Moreover, q-connectivity plays a crucial part, the traffic patterns are q-graded. And yet, the methodology does not allow changes in grade during the transmission of "activity" (remember §-2.4.2 and the worm's story). Consequently, the interaction between traffics of different grades should be considered.

4.1 A systemic assessment

The backcloth claims to be is a qualitative representation of structural features but Q-analysis uses the homogenizing notion of dimension. The q-connectivity, as well as the structure indicators (§.2.4.1), says "how many" regardless of the actual meaning of particular objects. The specificity of vertices may affect the traffic. It may seem somewhat intriguing to stress the numbers when the grounding sets and relations are arbitrarily settled (they cannot be improved interactively). Whereas "hard" data of Q-analysis rest on a strict set-membership precious rule, the pitfalls of sets definition are encountered [Couclelis (1983)].

When an indicator measures the intersections between alternatives and when the length of a sequence of q-connected simplices measures the similarity, the backcloth remains abstract and the traffic is a computational search. Polyhedral dynamics rests on external algorithms just as any algorithm depends on the underlying data structure.

Up to that point we are not convinced that Q-analysis is a branch of systems theory.

With a general definition for a system in terms of interacting parts, the alternative construction of a simplicial complex of §-2.3.2 may be used to go further.

4.2 Traffic as external processes

When the simplices are parts and the vertices are interaction contacts, the q-connectivity results from common faces which result from identified vertices. Since a linkage (influence/dependence) justifies the merging, the q-connectivity has a semantic and substantial existence. If the environment interacts with one part (a k-simplex), the spreading interaction among other parts is permitted (or restricted) by q-connectivity. The part is present in some q-connected component ($k \geq q$) of the simplicial complex and may be involved in a sequence of q-connected parts. The transitive property of influence/dependence is thus represented. It gives information about the ability to change and moreover reveals a q-graded variety. The study of q-loops generalizes conventional feedback and the shomotopics objects (obstruction) give information about heavy coupling.

4.3 Traffic as internal dynamics

Empirical simplices do not always fit in collections of data but they may correspond to coupling structures. We argue that when the simplices are not only a static geometry inherited from a descriptive relation but result from a strong interaction mechanism, the backcloth should describe the action. It may be illustrated with an example.

Let a system be considered an aggregate of subsystems, the definition of whose corresponds to the above example (§-4.2) . The subsystems stand in interaction because they share parts.

A subsystem is replaced by a simplex whose vertices are its constituent parts. The vertices of the simplices corresponding to common parts are identified. Nothing establishes that every vertex is connected to all other vertices in a simplex but the state of each part (vertex) is dependent on the state of some other parts. When a subsystem state changes (the environment may interact with one part), the spreading interaction in a subsystem is required within the vertices of a simplex. The q-connectivity represents the intersections between subsystems. Consequently, some other subsystems may be affected.

Our proposition may be drawn closer to structural analysis[7] [Ténière (1988)]. The involved study of relationships between variables/factors may be seen as a particular case of our complex (reduced to a single influence/dependence relationship among parts). Since the parts have only one influence/dependence contact, parts and links may be identified. Nevertheless, the influence $\otimes$ dependence plane cannot be fruitfully reformulated in a simplicial complex (connectivity is meaningless). But, either there may be grounds for viewing Ténière's "influence" as multiple links or the existence of "influence" may be detected between the vertices in a simplex. Actually, in this particular case, the connection with structural analysis may reveal a typology of parts and, therefore, critical factors to the dynamics.

The distinction between backcloth and traffic vanishes. In our example dynamics is defined in terms of the relation that makes up the backcloth itself.

At the risk of generating a mathematical recreation, the degree to which our constructions differ mathematically from Atkin's ones is to be reduced. A binary relation may be substituted for any simplicial complex (in terms of the information they contain) [Earl (1981)]. The above complexes are not Atkin's complexes stricto sensu but Q-analysis methodology may be used.

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[7] English text is available in the 4th chapter (Identifying the key variables : structural analysis) of Michel Godet's book, 1993 (*From anticipation to action*. UNESCO Publishing).

5. Conclusion

As declared, the paper shed light on the semantic achievements in Q-analysis and the rethinking of traffic separates artefact exploration from systems dynamics understanding.

What emerges from the study is that the semantic interpretation of the simplicial complex depends on what the conclusion aims at. When Q-analysis concerns interaction of elements with their environment, original constructions of the complexes are suggested.

An accurate reading may reveal that our systems representation is unduly impoverished. Q-analysis is still not sufficient to fully describe complex interaction in systems (for example, connecting sequences are not directed). That generates difficulties in translating the propositions into a general operational form. Although the finding of operational tools may be the subject of further work by the author, we urge systems theorists to penetrate the terminology in the hope of achieving significant progress toward. Until its forthcoming, enthusiasm should be moderated.

From a more general point of view, our investigation yields useful insights concerning substantive technology at the law school.

On the one hand, classical Q-analyses may be undertaken, within the set out limits, to reveal latent structures in legal databases (terms), in judicial decision process (factors) or in prison-life relationships and second offences.

On the other hand, backcloth structure is metaphorically referred to as permitting or restricting. It is a potential basis for opening up the "legal fabric" line of research. Social rules differ from nature laws in the no functional way of their enforcement. The social rule allows or forbids but is not a functional description of behaviour. Infringement is a distinctive feature in social sciences as (wo)man's will or as preservation of his(her) integrity.

The major conclusion of this paper is that the gap between metaphorical discussion and woolliness is narrow. The understanding of some of all Atkin's ideas has been too intuitive in the past. However, the use of graphics as a language is a powerful thinking tool and Atkin has delivered a framework for thought. We hope our contribution will open up new lines of systemic research for social systems understanding.

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