A Structural Theory of Rhythm Notation based on Tree Representations and Term Rewriting

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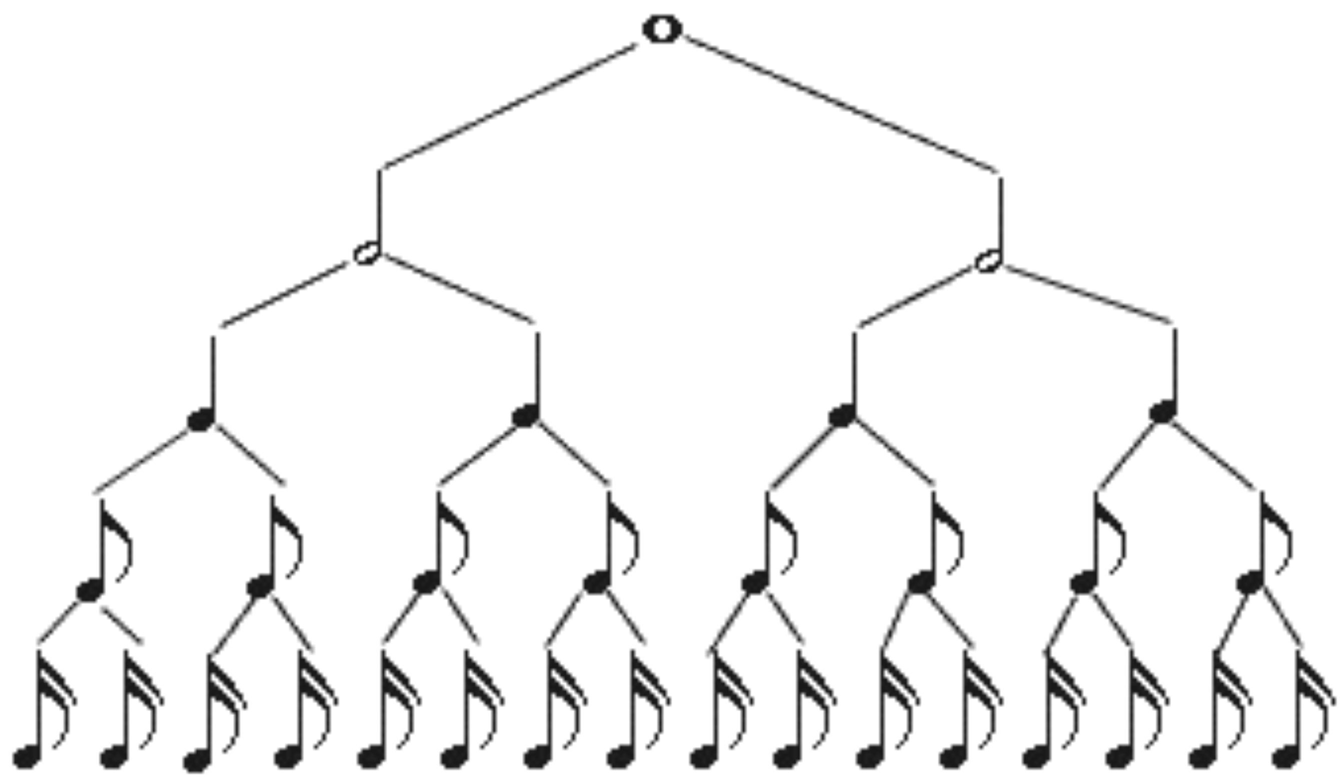
Objectives

- tree structured encoding of rhythm
- used for reasoning about rhythms
  with standard theoretical tools for tree processing
- for assisted algorithmic composition with OpenMusic

1. motivation and definition of a tree encoding for rhythm
2. tree languages (tree automata)
3. tree transformations by rewriting (equational theory),
   application to exploring equivalent rhythmic notations
4. properties, perspectives

Trees Encodings of Rhythm

natural representation of common western notations for rhythms
durations are defined hierarchically, by recursive subdivisions



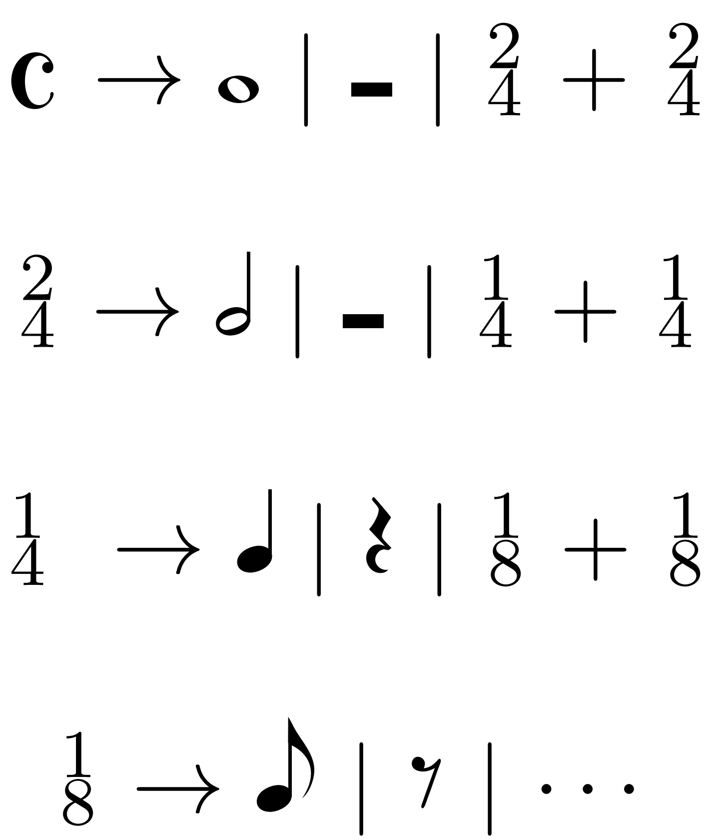
see survey in

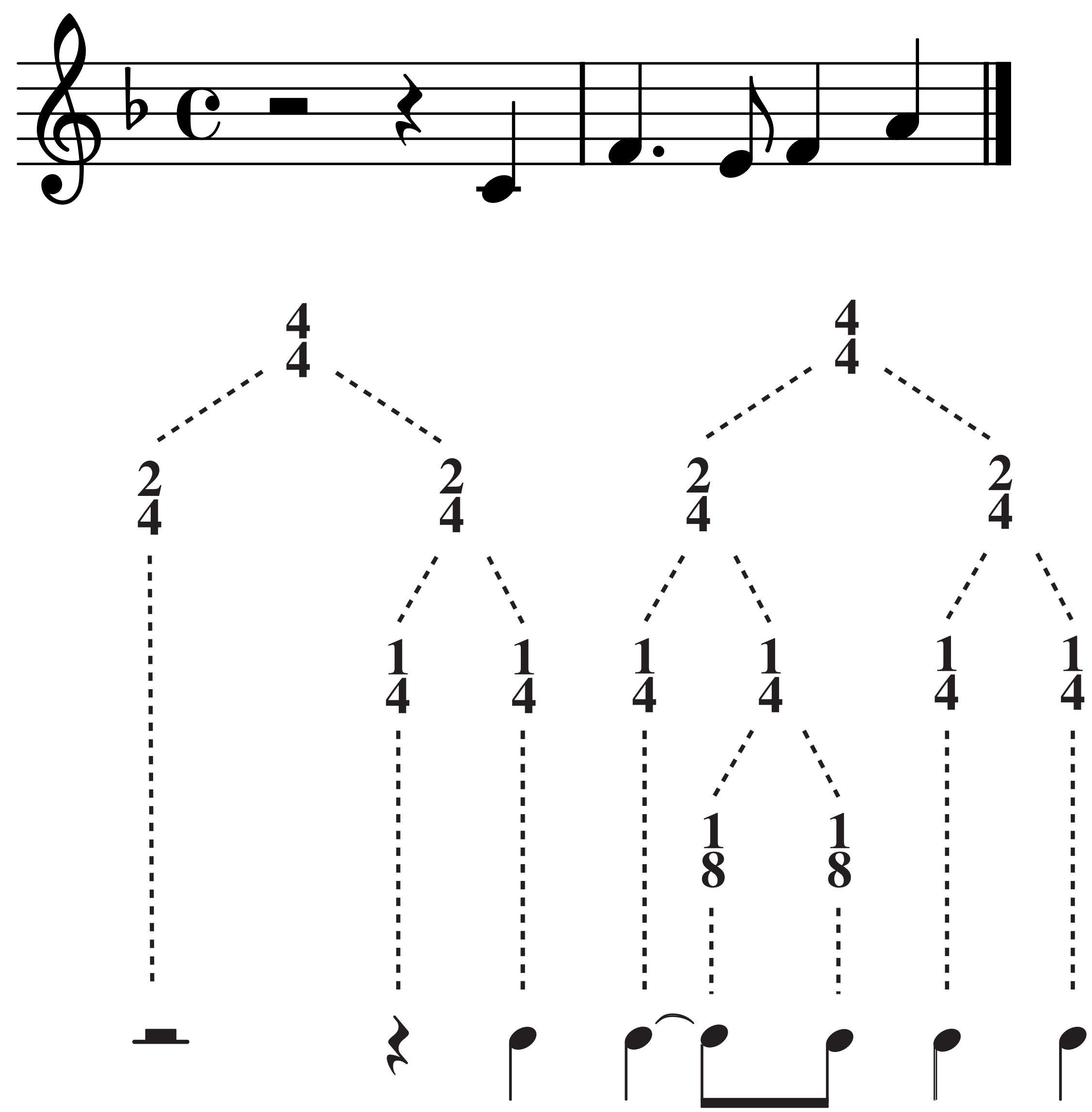
Rizo
Symbolic music comparison with tree data structures
PhD thesis U. Alicante, 2010

Syntax Trees

Longuet-Higgins
The perception of music
I.S.R., 1978

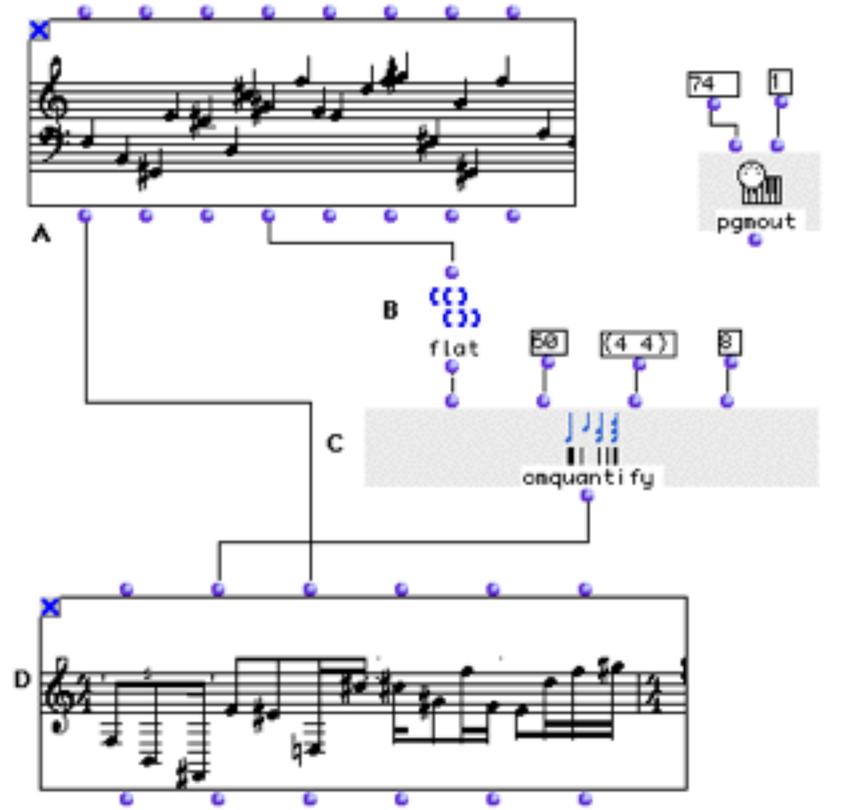
Lee
The rhythmic interpretation of simple musical sequences
Musical Structure and Cognition, 1985





OpenMusic Rhythm Trees

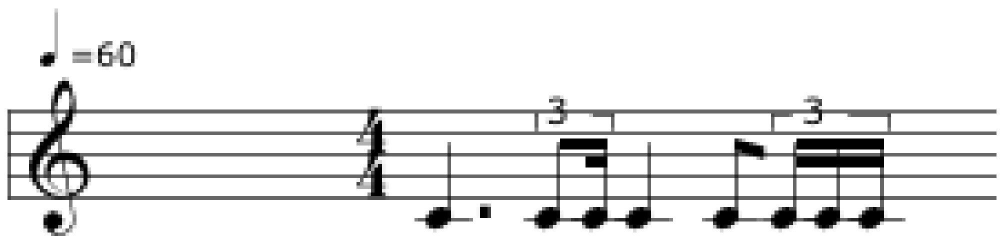
OpenMusic: graphical programming environment for algorithmic composition developed at Ircam

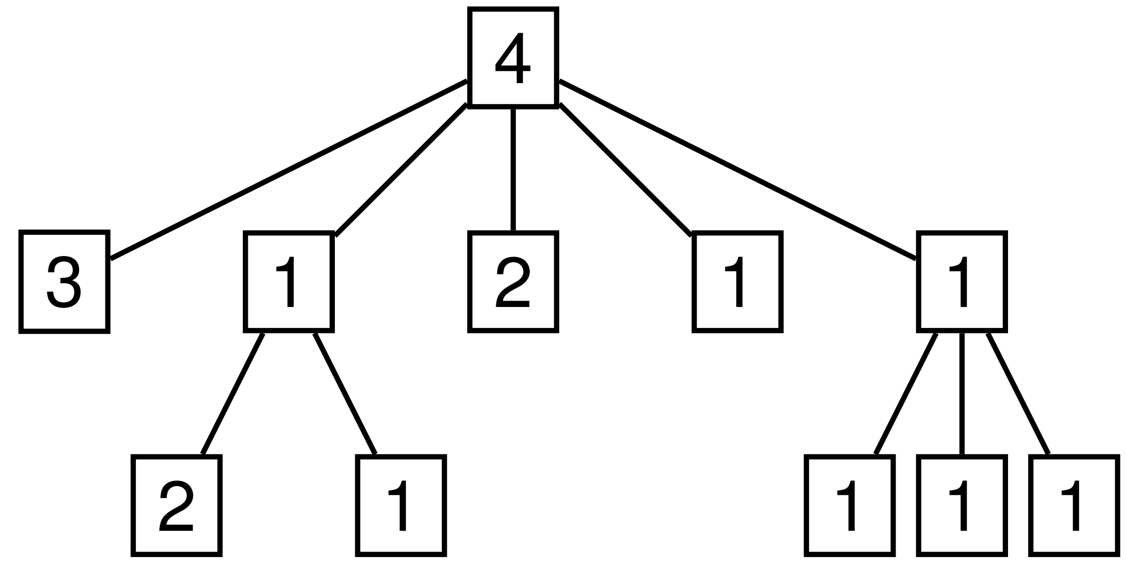


OM RT (nested lists) are a first class data structure for the representation of rhythms in OM

Laurson
Patchwork: A Visual Programming Language
Helsinki: Sibelius Academy, 1996

OpenMusic Rhythm Trees





- infinite alphabet (integers)
- processing require arithmetics

Objective to use Term Rewriting tools:

➡ purely syntactic processing

➡ labeling with finite alphabet

Durations in Semi-Structured Music Encodings

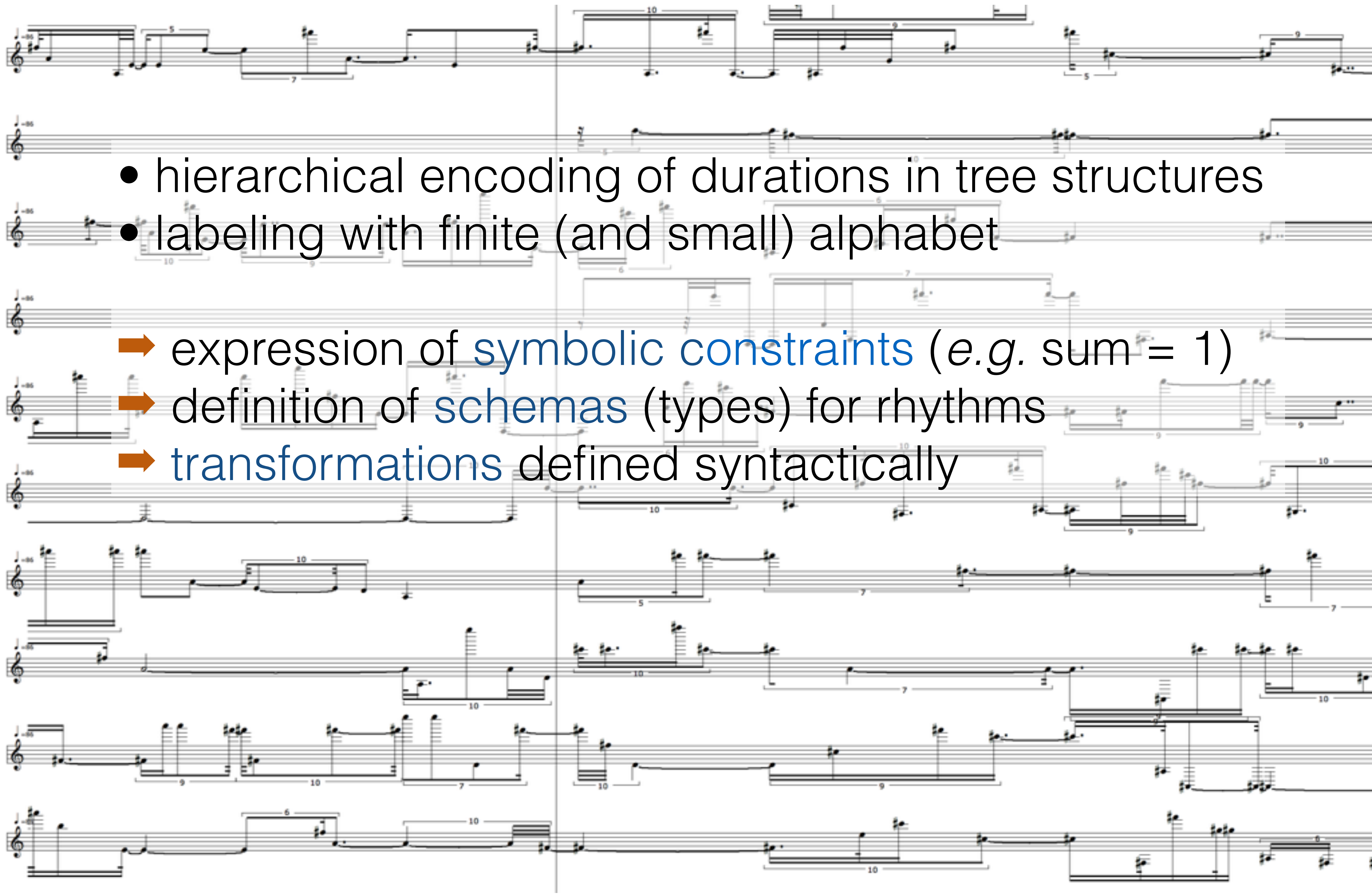
in MEI, MusicXML, *etc* a score is a tree (XML doc)
durations are attributes of notes

```
<mei:note pname="c" oct="5" dur="4"/>
```

score transformations can be defined using (tree) patterns but not for rhythms…

➡ encode durations in the tree structure

Rhythm Trees (sum-up)



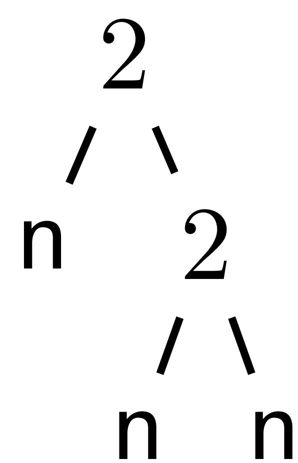
- hierarchical encoding of durations in tree structures
- labeling with finite (and small) alphabet

➡ expression of symbolic constraints (*e.g.* sum = 1)
➡ definition of schemas (types) for rhythms
➡ transformations defined syntactically

Rhythm Trees (RT)

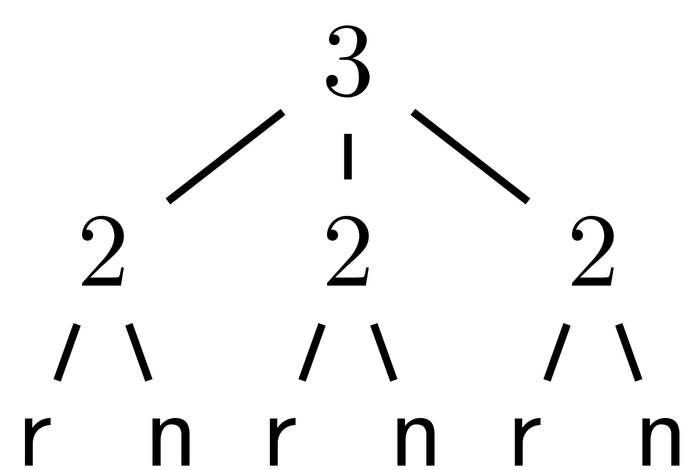
ordered ranked trees over a signature:

- inner nodes labeled by prime numbers (= arity)
- leaves labeled by **n**, **r**, **s**, **d**, **o**



denoted: 2(n(2(n,n))

Distinguished Nodes and Node Relations



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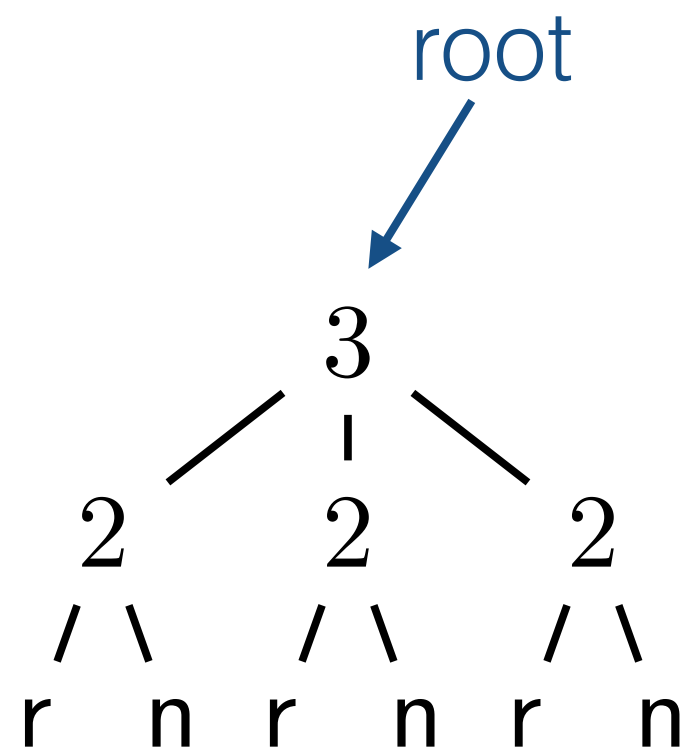

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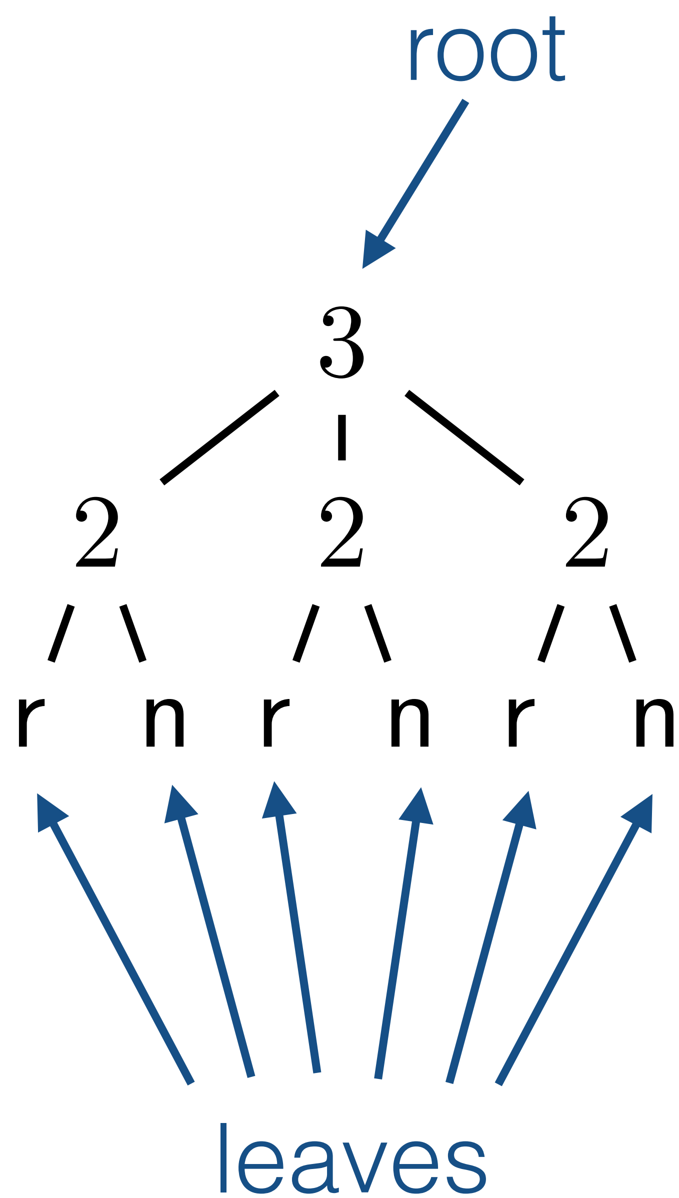

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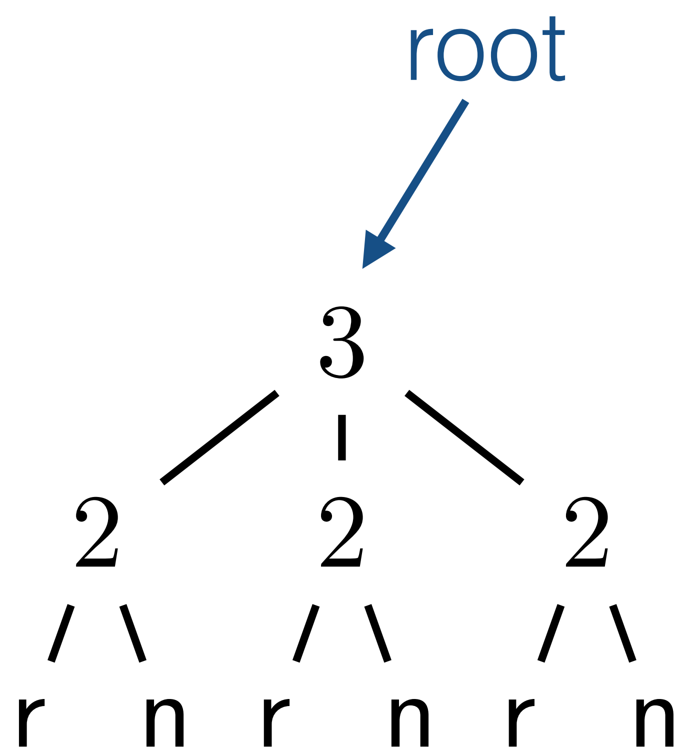

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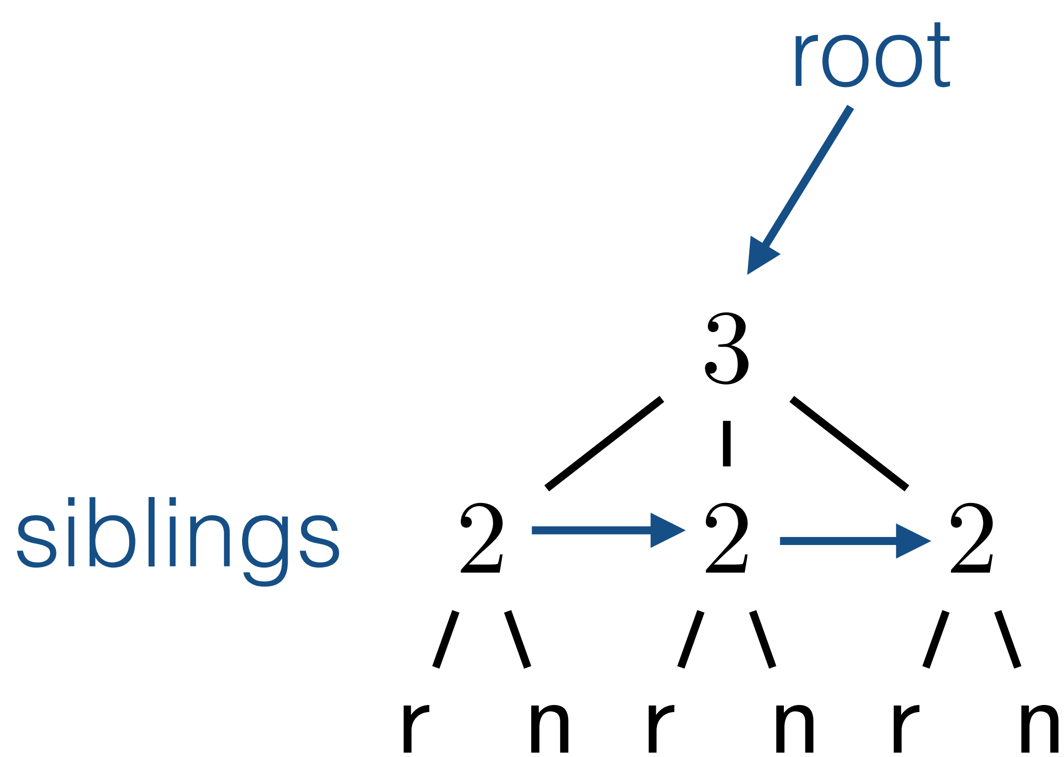

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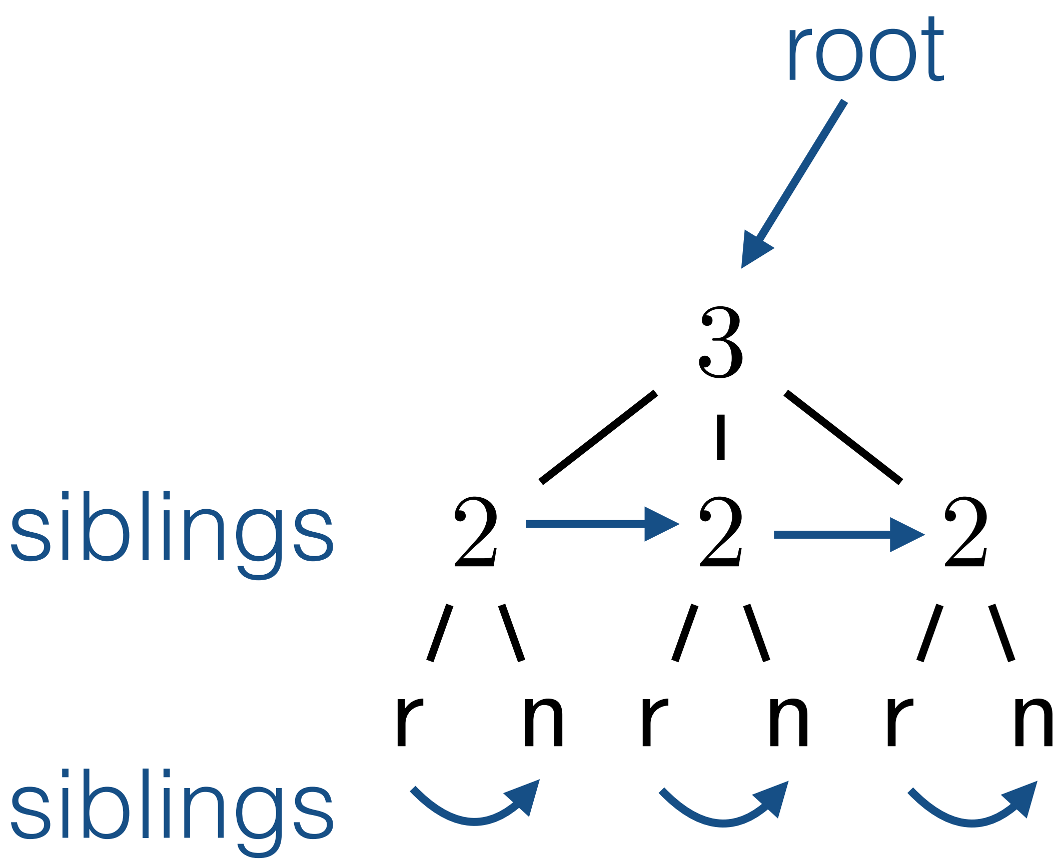

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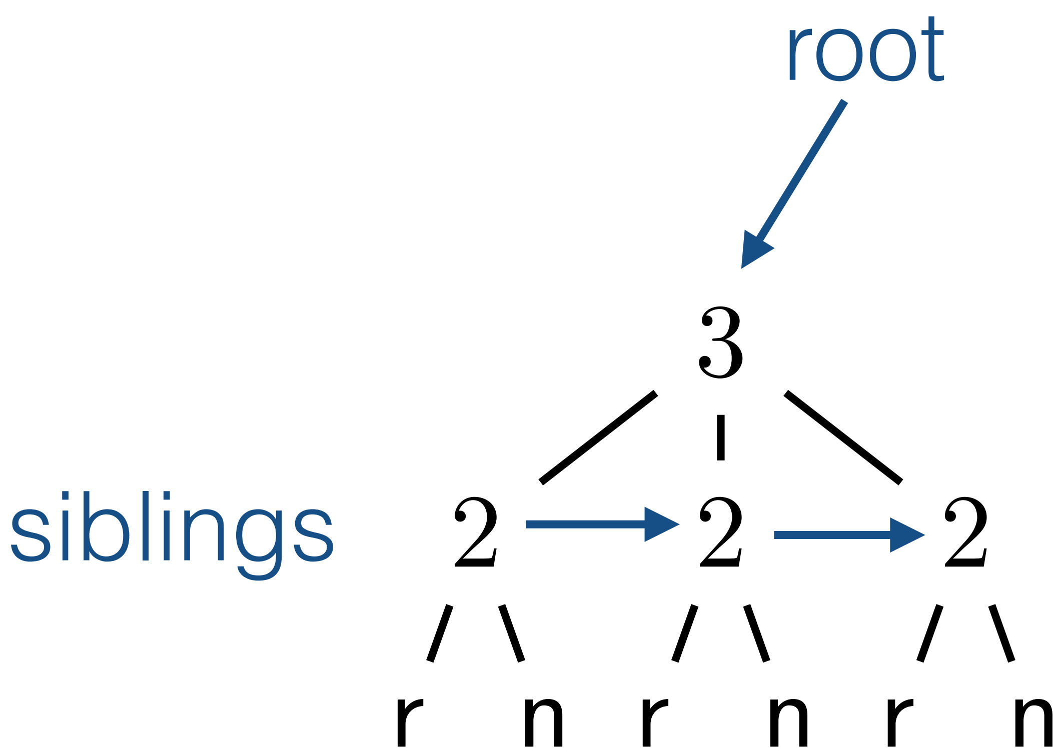

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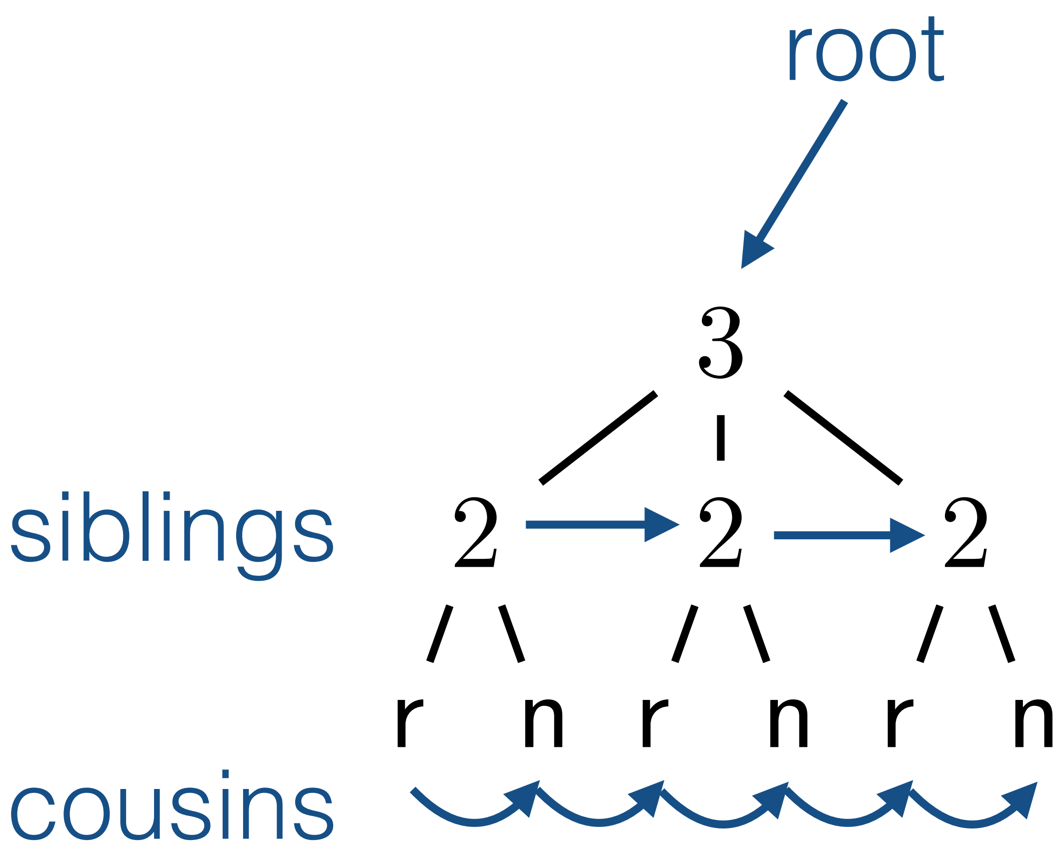


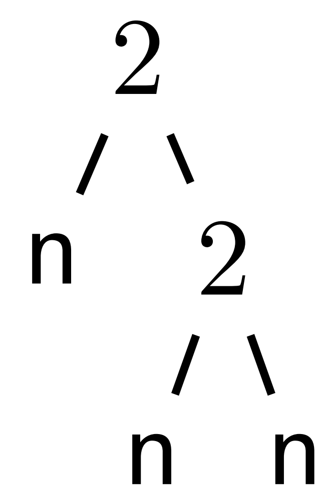
Semantics: Rhythmic Value

we associate durations to nodes:

dur(root) = 1 beat or 1 measure

$$\text{dur(node)} = \frac{\text{dur(parent)}}{\text{arity(parent)}}$$

when previous cousin is not **o**



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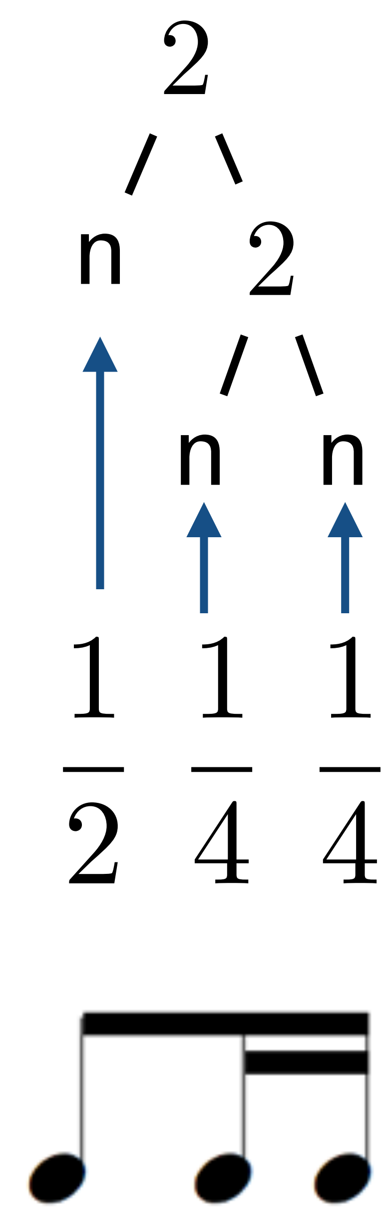
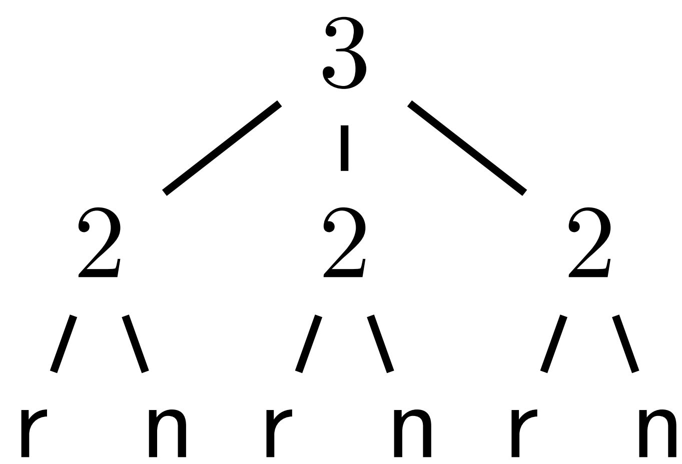
rhythmic value = sequence of ratios
= duration of leaves (in dfs traversal)
in the case of **n** and **r**

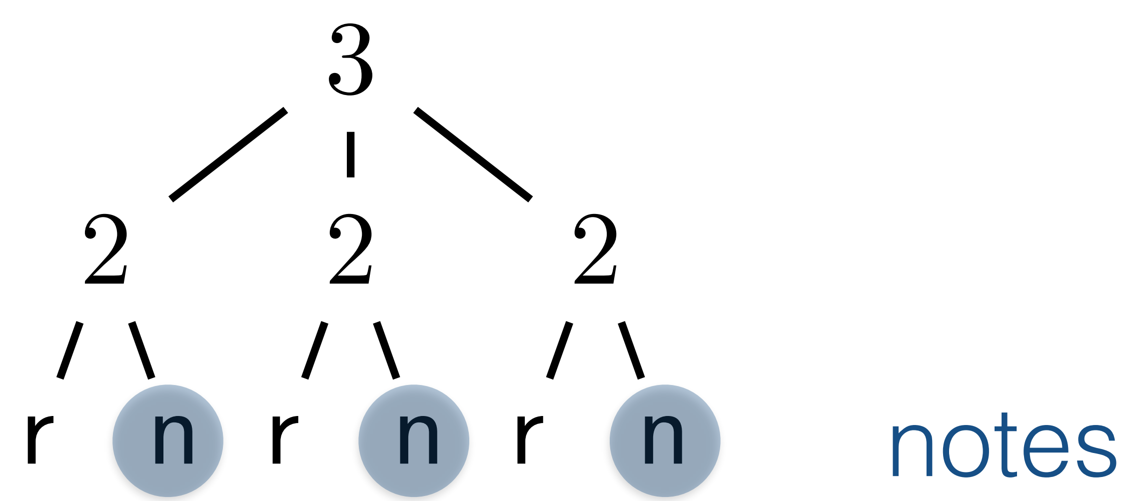


Rhythmic Value



rhythmic value $\left[\frac{1}{6}\right] \frac{1}{6} \left[\frac{1}{6}\right] \frac{1}{6} \left[\frac{1}{6}\right] \frac{1}{6}$

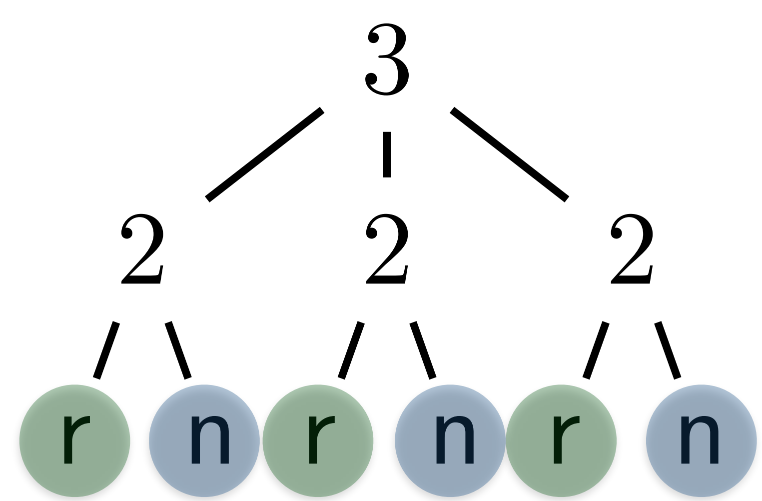
Rhythmic Value



rhythmic value $\left[\frac{1}{6}\right] \; \frac{1}{6} \; \left[\frac{1}{6}\right] \; \frac{1}{6} \; \left[\frac{1}{6}\right] \; \frac{1}{6}$

Rhythmic Value

rests

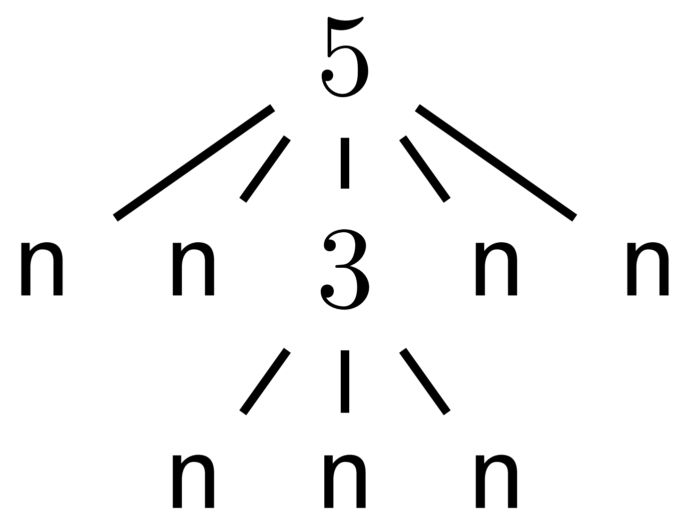


notes

rhythmic value

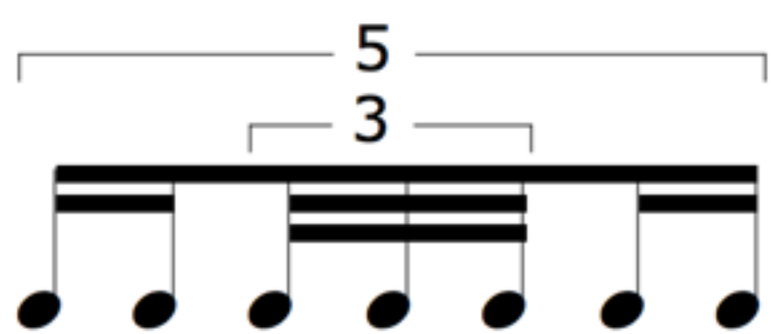
$$\left[\frac{1}{6}\right] \frac{1}{6} \left[\frac{1}{6}\right] \frac{1}{6} \left[\frac{1}{6}\right] \frac{1}{6}$$

Rhythmic Value

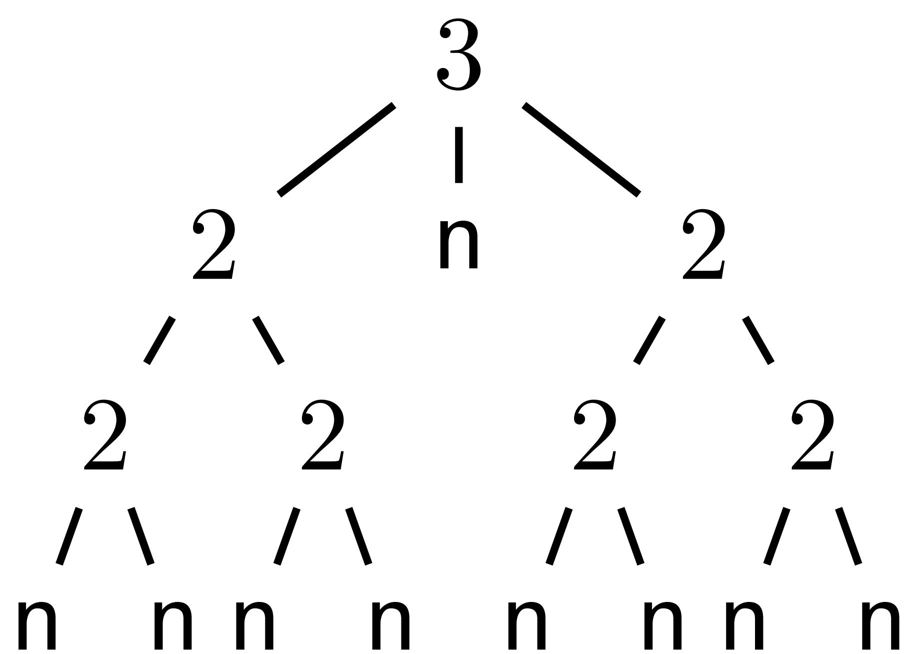


rhythmic value

$$\frac{1}{5}\;\frac{1}{5}\;\frac{1}{15}\;\frac{1}{15}\;\frac{1}{15}\;\frac{1}{5}\;\frac{1}{5}$$



Rhythmic Value



rhythmic value

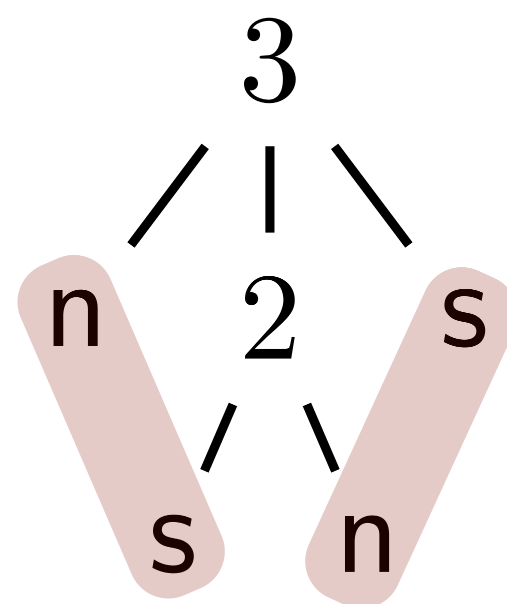
$$\frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{3}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}$$

Ties and Dots

we sum durations for subsequences
of leafs of the form **n s … s** or **n d** or **n d d**

$\frac{1}{4}$ $\frac{3}{4}$ rhythmic value $\frac{1}{4}$ $\frac{3}{4}$

Simplifiable RT with Ties



$$\frac{1}{2}\ \frac{1}{2}$$

Summation with Symbol **o** (a)

$$\mathrm{dur}(\mathrm{node}) = \frac{\mathrm{dur}(\mathrm{parent})}{\mathrm{arity}(\mathrm{parent})} + \mathrm{dur}'(\mathrm{node})$$

where d’(node) = dur(previous cousin)
when the previous cousin labelled with **o**
and 0 otherwise

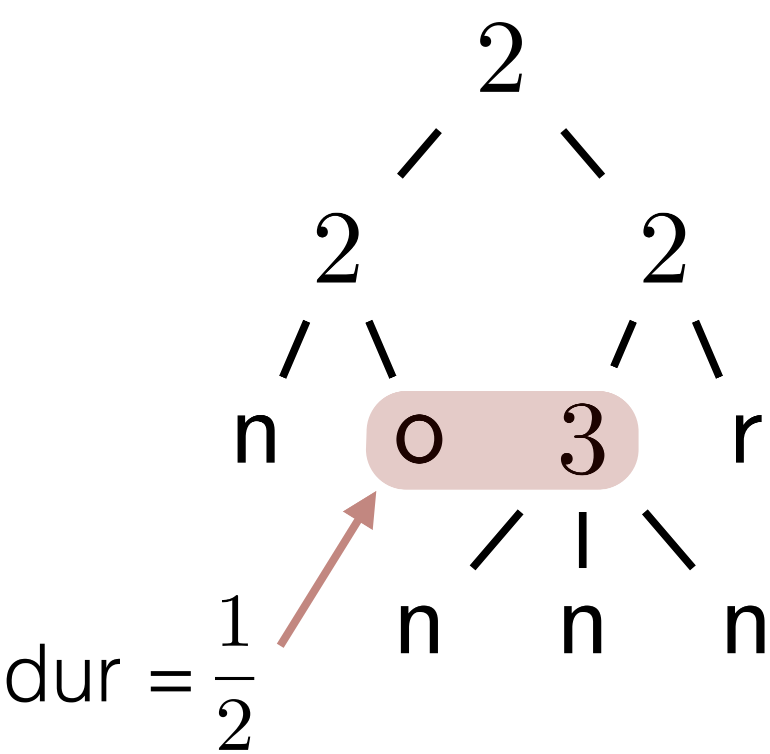
we ignore the leaves labeled with **o** in the computation of rhythmic value

```
      2
     / \
   2     2
  / \   / \
 n   o n   n
```

$$\frac{1}{4} \; \frac{1}{2} \; \frac{1}{4}$$

Ratios with Symbol **o**

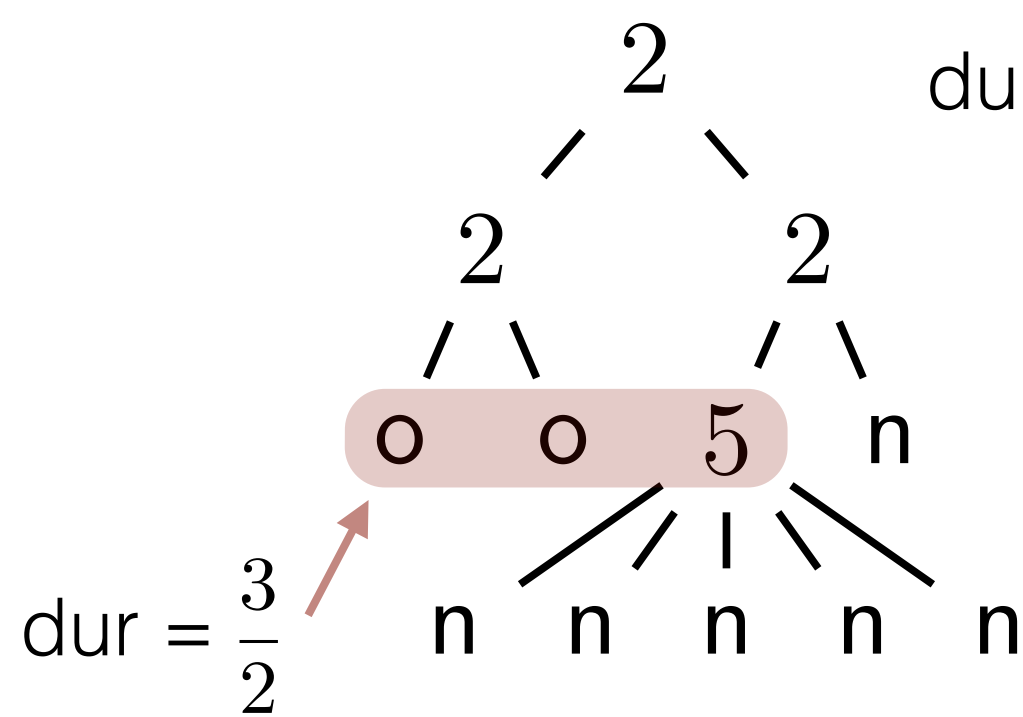
3 in the time of 2



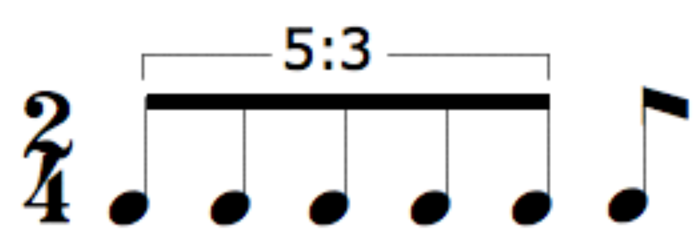
$$\frac{1}{4}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \left[\frac{1}{4}\right]$$

5 in the time of 3

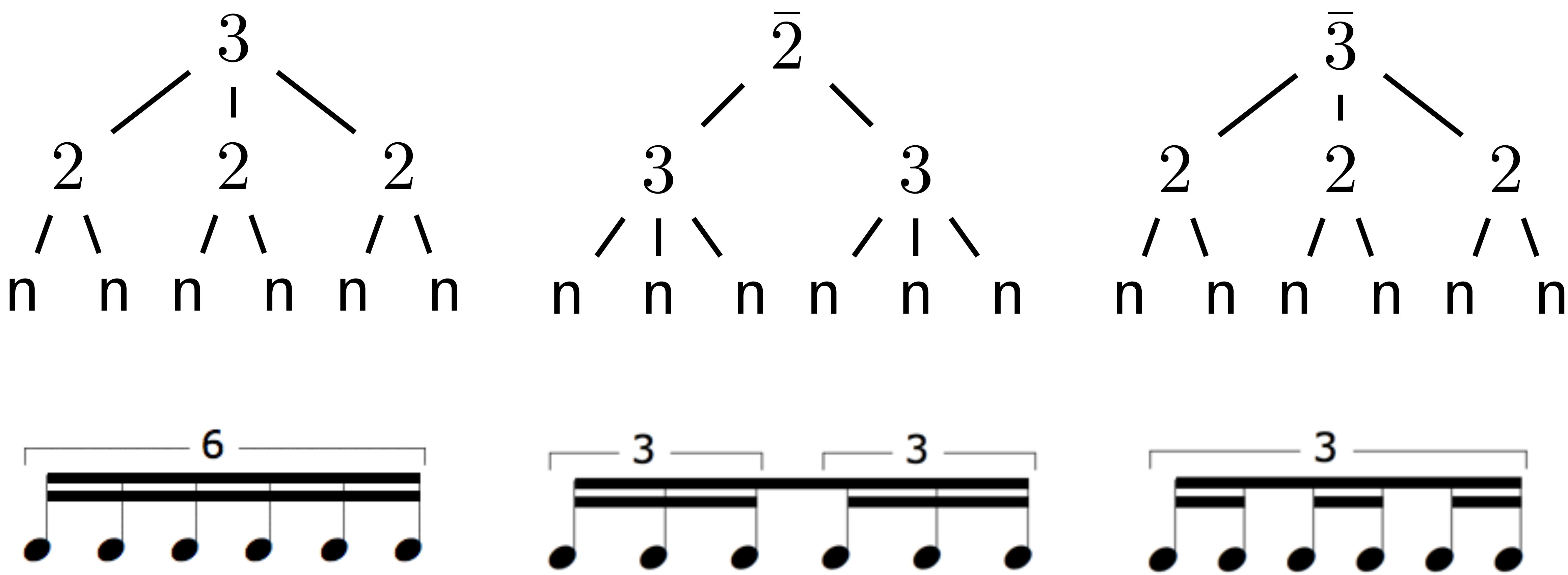
dur(root) = 2



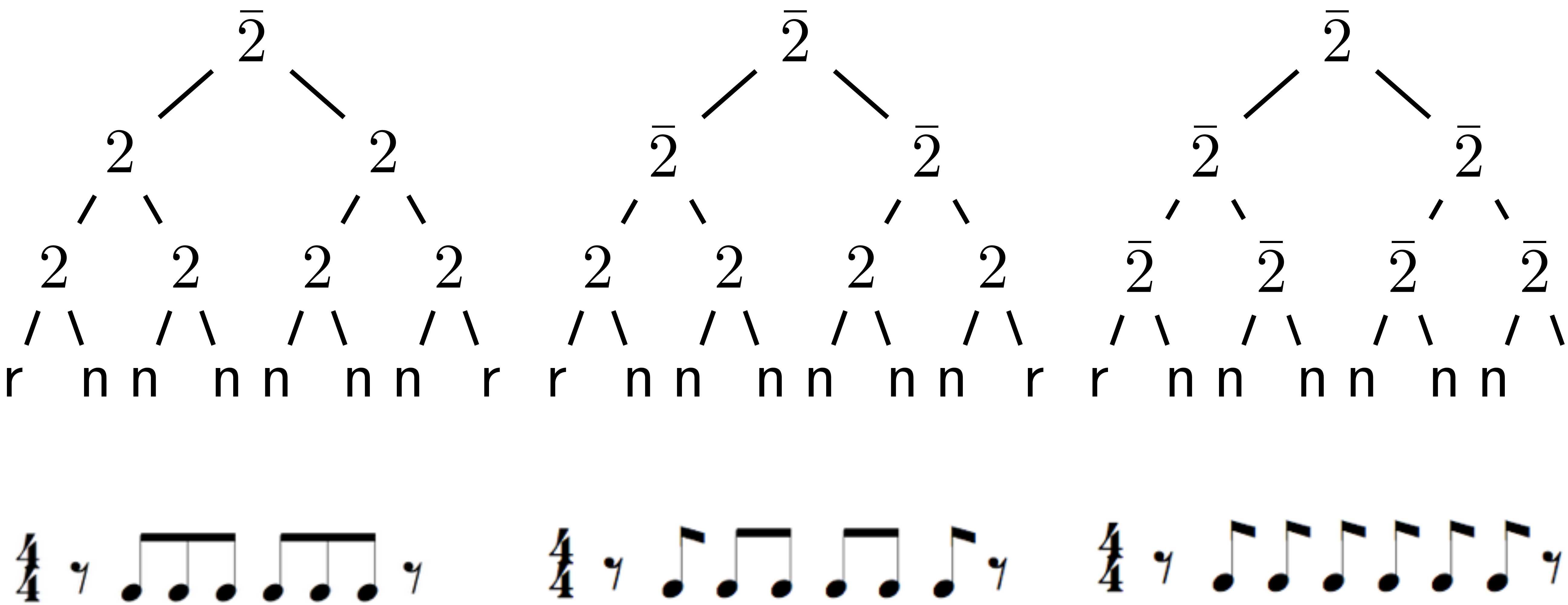
$$\frac{3}{10}\ \frac{3}{10}\ \frac{3}{10}\ \frac{3}{10}\ \frac{3}{10}\ \frac{1}{2}$$



Tuplet Beaming: one beat



Tuplet Beaming: one bar



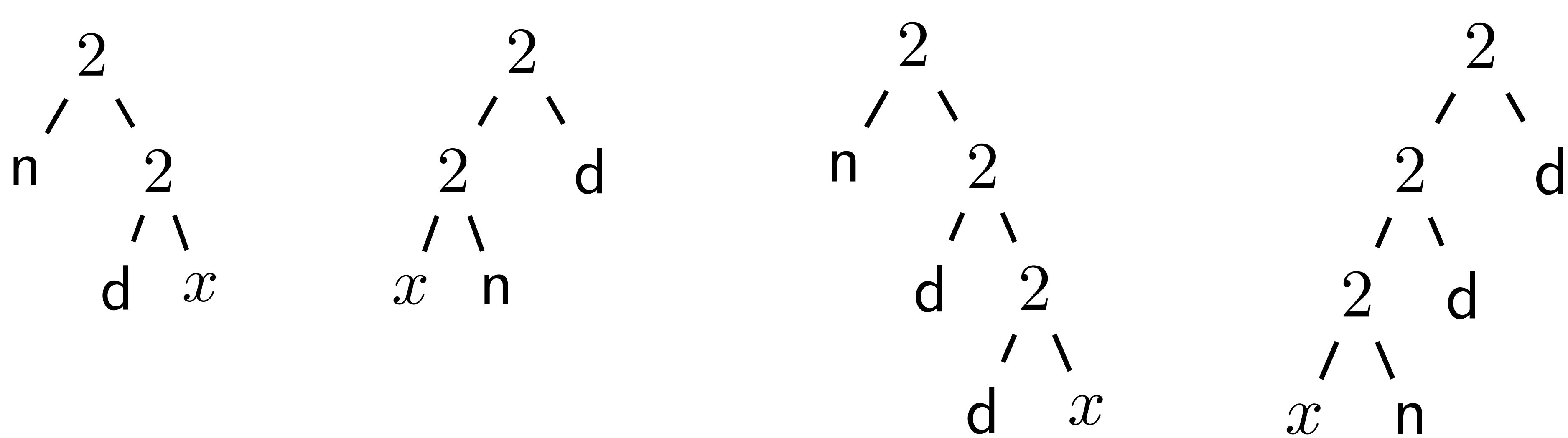
Regular Tree Languages

defined by tree automata
(embed all current XML schema languages)

Murata et al
Taxonomy of XML schema languages using formal language theory
ACM Trans. Internet Technol., 5:660–704, 2005

definition of well-formed trees

e.g. every **d** must occur in one of the following patterns



definition of user preferences

Rewrite Rules

local transformations of RT

$\boxed{\text{symbols with same semantics}}$

$$\mathsf{d} \rightarrow \mathsf{s} \tag{1}$$

$$\bar{p}(x_1, \ldots, x_p) \rightarrow p(x_1, \ldots, x_p) \quad p \in \mathbb{P} \tag{2}$$

replacement of a subtree (matching the left pattern)
by a subtree

Rewrite Rules

$\boxed{\text{addition of rests}}$

$$p(\underbrace{\mathsf{r}, \ldots, \mathsf{r}}_{p}) \to \mathsf{r} \quad p \in \mathbb{P} \tag{3}$$

$$\mathsf{r}; \mathsf{s} \to \mathsf{r}; \mathsf{r} \tag{4}$$

$$\mathsf{o}; \mathsf{r} \to \mathsf{r}; \mathsf{r} \tag{5}$$

; denotes the cousin relation
replacement of a sequence of cousins
by a sequence of cousins of same length

Rewrite Rules

$\boxed{\text{normalization of ties}}$

$$p(\mathsf{s}, \ldots, \mathsf{s}) \to \mathsf{s} \quad p \in \mathbb{P} \tag{6}$$

$$p(\mathsf{n}, \mathsf{s}, \ldots, \mathsf{s}) \to \mathsf{n} \quad p \in \mathbb{P} \tag{7}$$

Rewrite Rules

$\boxed{\text{elimination of } \mathbf{o}}$

$$\mathsf{o};\mathsf{s} \to \mathsf{s};\mathsf{s} \tag{8}$$

sum and division by 1 $\quad \mathsf{o};\mathsf{n} \to \mathsf{n};\mathsf{s}$ (9)

sum and division by 2 $\quad \mathsf{o}; 2(x_1, x_2) \to x_1; x_2$

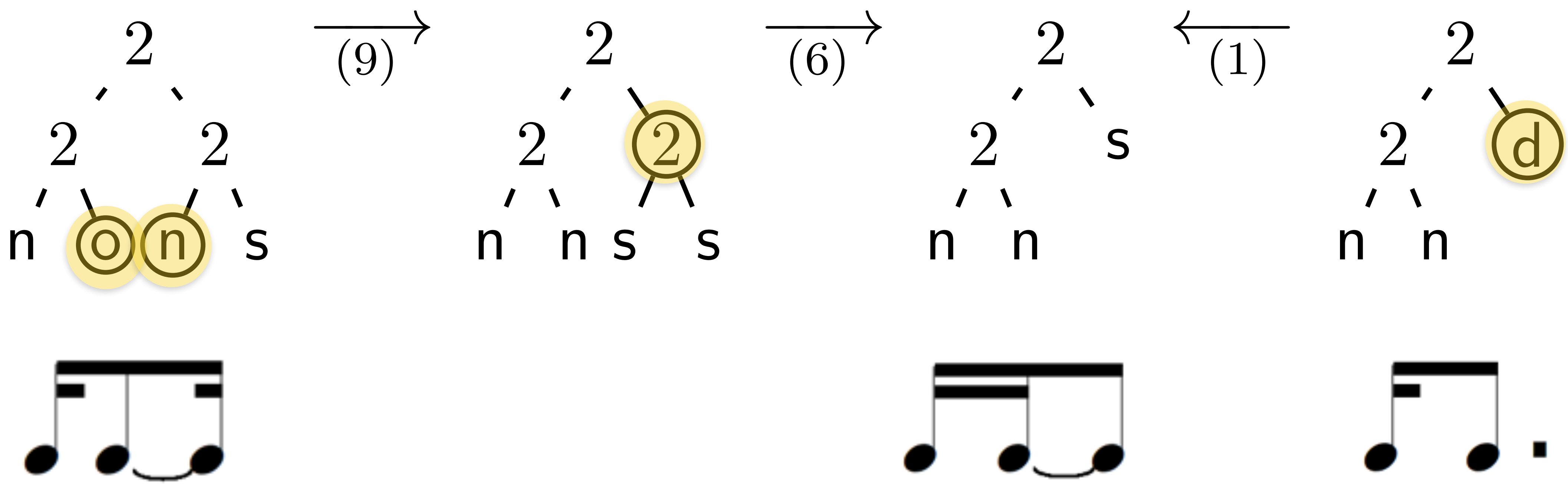
$$\mathsf{o};\mathsf{o};\mathsf{o}; 2(x_1, x_2) \to \mathsf{o}; x_1; \mathsf{o}; x_2$$

sum and division by 3 $\quad \mathsf{o};\mathsf{o}; 3(x_1, x_2, x_3) \to x_1; x_2; x_3$

$$\underbrace{\mathsf{o}; \ldots; \mathsf{o}}_{kp-1}; p(x_1, \ldots, x_p) \to \underbrace{\mathsf{o}; \ldots; \mathsf{o}}_{k-1}; x_1; \underbrace{\mathsf{o}; \ldots; \mathsf{o}}_{k-1}; x_2; \ldots; \underbrace{\mathsf{o}; \ldots; \mathsf{o}}_{k-1}; x_p \tag{10}$$

simulated with intermediate rules and auxiliary symbols

Rewriting Equivalent Rhythms



Rewrite Rules

$\boxed{\text{equivalent subdivisions}}$

$$\begin{aligned}
2(x_1, x_2) &\to 3(2(\circ, \circ), 2(x_1, \circ), 2(\circ, x_2)) \\
2(x_1, x_2) &\to 5(2(\circ, \circ), 2(\circ, \circ), 2(x_1, \circ), 2(\circ, \circ), 2(\circ, x_2)) \\
3(x_1, x_2, x_3) &\to 2(3(\circ, x_1, \circ), 3(x_2, \circ, x_3)) \ldots
\end{aligned}$$

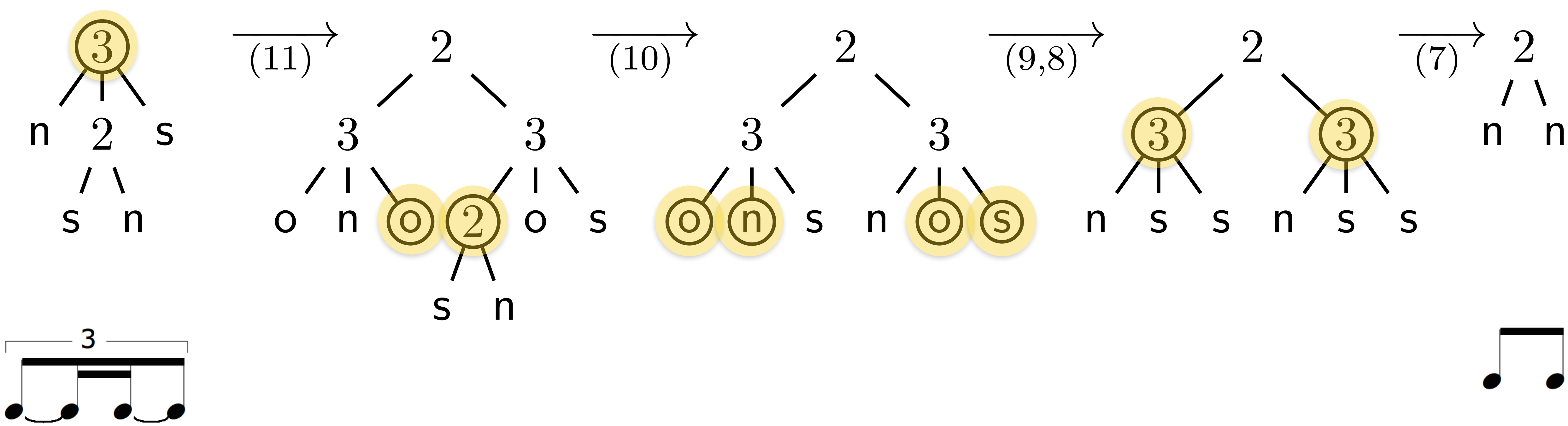
$$p(x_1, \ldots, x_p) \to p'\big(p(u_{1,1}, \ldots, u_{1,p}), \ldots, p(u_{p',1}, \ldots, u_{p',p})\big)$$

where $p, p' \in \mathbb{P}$, $p \neq p'$, (11)
for all $1 \leq i \leq p'$, $1 \leq j \leq p$, $u_{i,j} \in \{\circ, x_1, \ldots, x_p\}$
and the sequence $u_{1,1}, \ldots, u_{1,p}, \ldots, u_{p',1}, \ldots, u_{p',p}$
has the form $\underbrace{\circ, \ldots, \circ, x_1}_{p'}, \ldots, \underbrace{\circ, \ldots, \circ, x_p}_{p'}$.

Reduction Sequence

(simplification)

$$3(x_1, x_2, x_3) \quad \rightarrow \quad 2(3(\mathsf{o}, x_1, \mathsf{o}), 3(x_2, \mathsf{o}, x_3)) \qquad (11)$$



Properties

for *well-formed* trees

Every two trees in relation by rewriting have the same rhythmic value (equivalent)

➡ explore the space of rhythms with same value as a given rhythm

➡ suggest alternative notations

Properties (perspectives)

under restriction for termination (*bounded depth*)

- confluence

$$\begin{tikzcd}
& T \arrow[dl] \arrow[dr] & \\
T_1 \arrow[dr, dotted] & & T_2 \arrow[dl, dotted] \\
& T' &
\end{tikzcd}$$

➡ canonical representation of equivalence classes of rhythms

- rewrite strategies e.g. top-down

➡ for efficiency

➡ prove completeness?

Conclusion

- tree structured encoding of rhythm
- defining well formed tree languages (schemas)
- tree rewriting rules defining rhythm equivalence

Applications and Perspectives

➡ framework for rhythm transcription (by quantization) in OpenMusic, based on RT

➡ conversions

- RT → OMRT for rendering
- RT ↔ standard encodings

➡ alternative:
rewriting and tree automata with build-in arithmetic