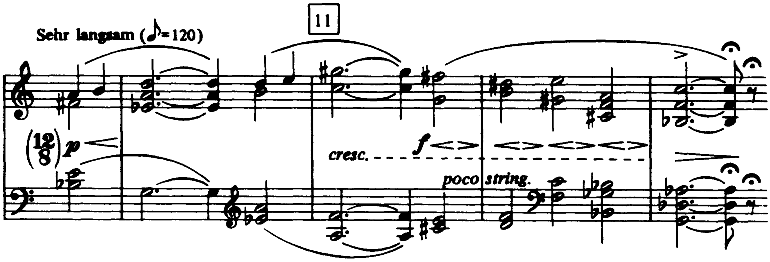
A TUTORIAL ON KLUMPENHOUWER NETWORKS, USING THE CHORALE IN SCHOENBERG'S OPUS 11, NO. 2

David Lewin

By "the chorale" I mean the passage from measures 9–13 reproduced in Example 1.[1]

The passage is clearly a single phrase, yet its harmonic structure sounds diffuse. That is a significant aspect of its aesthetic effect, and we shall take some time to explore more precisely some of its diverse features. Then we shall approach a question which arises naturally from this context: is there some way in which we can sense the harmonic field of the phrase as unified, rather than diverse? We shall see that Klumpenhouwer Networks provide a positive answer to that question.[2] We shall also see how they provide useful theoretical machinery for exploring further aspects of harmonic diversity in the passage.

The material that opens the chorale refers back to an earlier passage (in m. 4, to be discussed later). The verticalities in mm. 9–10, of Forte-types 4–Z15 (0146) and 4–16 (0157), can also be referred back to the opening melody of the piece, sketched in Example 2a.



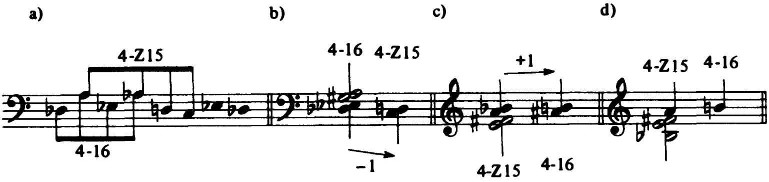
Example 1

The beams indicate a progression from a 4–16 set to a 4–Z15 set within the melody. Example 2b represents that progression schematically: the 2–1 dyad within the 4–16 remains fixed, representing the fixed 2–1 in the upper register of example 2a; meanwhile, the 2–2 dyad within the 4–16 of example 2b slips down a semitone to become the 2–2 dyad of the 4–Z15, symbolizing how {Db, Eb} in the lower register of example 2a slips down to {D,C}. Example 2c show a related progression of a 4–Z15 set to a 4–16 set: here, a common 2–2 dyad remains fixed, while the 2–1 dyad of the 4–Z15 slides up a semitone to become the 2–1 dyad of the 4–16. Example 2d shows how the abstract chords of 2c are realized by the harmonies that begin the chorale in measure 9.

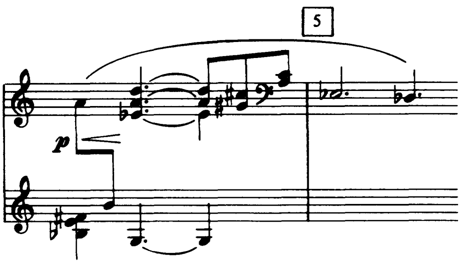
The third chord of the chorale is also of form 4–16. The three chords that open the chorale appear as the opening chords of the earlier passage mentioned above, which is sketched in example 3.

The upper staff of example 3 shows how the passage incorporates the ritornello hexachord of example 2a. The ritornello is typically heard above a low ostinato figure that alternates eighths on F2 and D2. The ostinato and the ritornello together form a heptachord comprising F and the melody notes. (D is already in the ritornello hexachord.) The lower staff of example 3 shows how the beginning of the passage presents the complement of the heptachord; the total chromatic is thereby completed in m. 4.

The contexts we have been examining so far make the harmony at the beginning of measure 11 particularly surprising. Instead of a 4–Z15 or 4–16, the chord provides a 4–17 (major-minor triad). In doing so, it also breaks the exact sequence that began in measure 10, thereby breaking off as well the obvious reference to earlier material.[3] Schoenberg writes "cresc." on the sustained harmony; this makes little sense for the mechanics of piano playing, but it is a perfectly clear psychological indication for the performer, who is to feel that the



Example 2



Example 3

harmony pushes onwards with growing tension, rather than subsiding into repose. The high G♯ of the chord reattains the highest note of the piece so far, heard 3 measures earlier at the loudest dynamic level so far. The effect is particularly strong because the G♯ of measure 11 is "too high," as the sequence makes us expect G natural.

Indeed, the {C,G♯} of the chord "should be" an octave lower, in the left hand, according to the sequence pattern. That draws particular attention to the augmented fifth {C5, G♯5} in the right hand. The effect is amplified by the mirroring minor sixth {A3, F4} of the left hand; the mirror sonority stands out as the first such in the phrase. Sensitized to vertical augmented fifths and minor sixths by the first chord of measure 11, we can retroactively pick up a progression of such intervals from the beginning of the phrase, as sketched in example 4. The {C4, A♭4} "expected" in the left hand at measure 11 is bracketed with an X above it; an arrow asserts its octave transfer to {C5, G♯5}.

{C4, G♯4} appears in its "correct" register within the second chord of measure 12; the minor sixth {G♯4, E5} appears above it, and the {C4, E5} dyad further emphasizes the sound of ic4 within the 4–19 sonority (0148). The next chord, at the end of measure 12, is also heavily saturated with vertical ic4 dyads; it embeds both a 4–17 set ({G♭, B♭, C♯, A}) and two 4–19 sets ({G♭, C♯, F, A}, {B♭, C♯, F, A}).

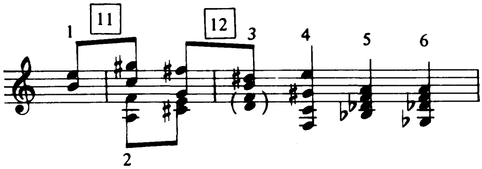
Example 4

But before we get to the middle of measure 12, our ears must first traverse the chords at the end of measure 11, and at the beginning of measure 12. The latter chord does contain a major third, and the {B4, D♯5} in its right hand does progress purposefully to the {G♯4, E5} and the {C♯4, A5} that continue the right-hand part through measure 12.[4] The chord at the end of m. 11, however, contains no vertical dyad of ic4, and a quite different principle groups it together strongly with the chord at the beginning of measure 12: both chords contain diminished triads. The disruptive effect of that apparently new principle is emphasized by the arrival of the forte dynamic, the indication "poco string.," and the sense of moving from a syncopated 6/4 meter into an explicit 3/2 hemiola.[5]

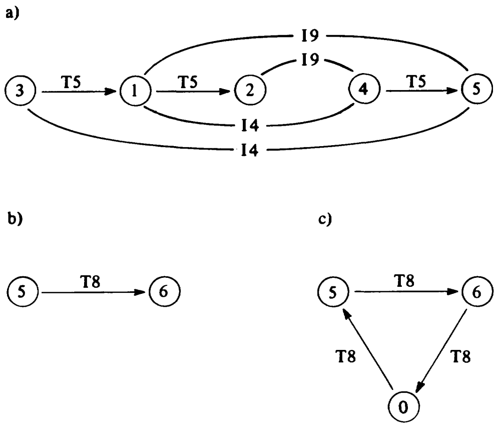
A more set-theoretic analytic ear, less rigidly oriented to vertical chord structure, can help us hear more harmonic continuity from the end of measure 10 through measure 12. Example 5 shows how we can hear 4–19 sets persisting through that segment of the music.

The example follows through on the idea of separating right-hand from left-hand material in measure 11, suggested by our observations so far about the chord at the beginning of that measure. The 4–19 sets are labeled on the example by numbers 1 through 6. T/I relations among these sets can can invoked to form variously structured transformational networks in the manner of Lewin 1987. Example 6 displays some such networks, using the numbers from example 5 to represent the various 4–19 sets of that example. The number 0, in example 6c, will be explained shortly.

Example 6a observes that set 5 is the T5-transpose of set 4, just as set 2 is the T5-transpose of set 1, just as set 1 is the T5-transpose of set 3; the I-labels then follow, given that sets 4 and 5 are inversions of sets 1–2–3. The left-to-right aspect of the visual display conflicts with the chronological order in which sets 1–2–3 are projected by the music. The particular display for example 6a was constructed so as to emphasize as strongly as possible the abstract triple proportion among sets 3, 1 and 2, making the T5-relation a consistent visual motif. Just so, one could arrange nodes marked "dominant," "tonic," and "subdominant" in left-to-right visual order on the page, even though some actual music which these categories addressed in an analysis might



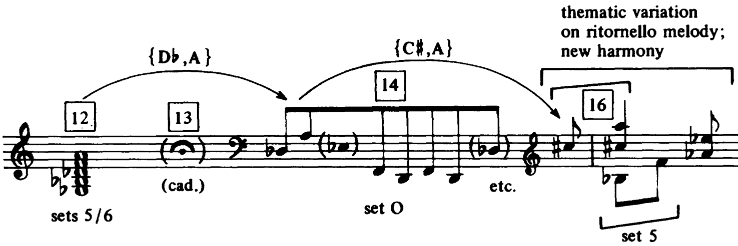
Example 5



Example 6

proceed I IV V in chronological time, rather than V I IV. The structuring effect of T5 here is suggestive in connection with the T5 interval for the aborted sequence within the chorale, a sequence which will later be unleashed as described in note 3.

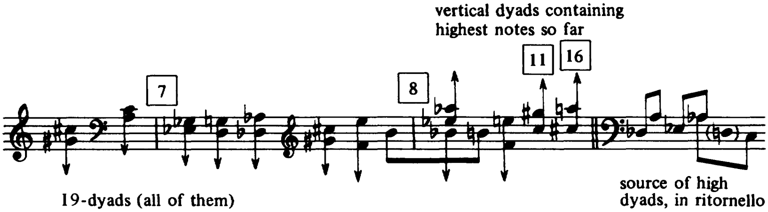
The strong T5 motif of example 6a engages all the 4–19 sets of example 5 except for set 6. Example 6b shows set 6 in T8-relation to set 5; both these sets are embedded within the same chord of the chorale. T8, characteristic of pc relations within 4–19 sets, is thus projected here as a relation *between* 4–19 sets, a T8-relation on a higher level. To complete the "augmented triad" structure of 4–19 on the higher level, we would require a "set 0" form of 4–19, a set which could participate in the network of example 6c. The required set 0 would be T8 of set 6, which is {G♭, C♯, F, A}; accordingly set 0 would be {D, F, C♯, A}. And we find just that set as the first 4 pcs of the piece, gathering the ostinato F–D figure together with the incipit C♯–A of the principal



Example 7

melody above the ostinato. At first, the observation seems too abstract. But if we examine the end of the chorale together with its immediate sequellae in the music, example 6c seems much more relevant to the composition.

Example 7 sketches some of the relevant events. At the left of the example we see the penultimate chord of the chorale, comprising 4–19 sets 5 and 6. The cadence chord follows, with a fermata afterward. The very next thing we hear is the incipit dyad D♭–A for the reprise of the ritornello melody. On example 7, an arrow labeled {D♭, A} connects the {D♭, A} of sets 5 and 6, with the incipit D♭–A of the ritornello; this arrow is "audible," with some aural focusing. The bass ostinato on F–D returns in measure 14, after the ritornello has gone on past D♭–A to the parenthesized Eb of the example. Despite the intervening E♭, we will link the D♭–A incipit to the ostinato F–D, forming a projection of set 0. That is because we recognize the reprise of the ritornello figure at the end of measure 13, and we know that the ostinato F–D competes its implied harmonization, even though the F–D has not yet entered physically. After the ritornello melody has ended (on the parenthesized D♭ of the example over the persisting F–D ostinato), there begins with the pickup to measure 16 what I have identified on example 7 as the "thematic variation on the ritornello melody," with the new texturing and harmonization. The large upper bracket at the right of the example shows the opening of the ritornello, D♭–A–E♭–A♭–, textured in a new form, and two octaves higher. The small upper bracket marks the incipit C♯–A in particular; an arrow labeled "{C♯, A}" connects this music to the ritornello incipit of measure 13. The new harmony for the incipit is provided by B♭ and F, rather than the F–D ostinato; the new harmony is symbolized on example 7 by the B♭–F at the lower right of the example, marked by a lower bracket. This B♭–F, together with the incipit C♯–A that it harmonizes, once more projects set 5. Once our attention has been drawn



Example 8

to that, it is not difficult to hear the "set 5" at the right of example 7 associating with the "set 5" at its left; thence we can hear the association in the music itself. Example 7 thus highlights compositionally, via marked continuities in the music of measure 12–16, the abstract relationships of example 6c.

The high A5 at the opening of measure 16, at the top of the {C♯5, A5} dyad, is the highest note so far in the piece. Before this, the highest note has been the G♯5 of measure 11, already discussed in connection with the {C5, G♯5} dyad there. As mentioned in the earlier discussion, the high G♯5 was also attained at the beginning of measure 8, on top of the {E♭5, A♭5} dyad there. Example 8 collates these observations, and other material, for further commentary.

The high dyads just discussed appear in the middle of the example, with arrows pointing up from their stems. At the right of the example, after a double bar, the source of the three high dyads is seen in the ritornello series. Earlier discussion (around examples 4 and 7) has indicated why it is reasonable to separate out from the surrounding textures the high {C, G♯} of m. 11 and the high {C♯, A} of m. 16. As for the high {E♭, A♭} of measure 8, example 8 gives a rationale for separating that dyad, too, from its surrounding texture. That texture, below and to the left of the {E♭, A♭} dyad on the example, shows that the surrounding material in the right hand is engaged in another project, to which the {E♭, A♭} does not belong. The project is to present all the various I9-dyads. The {B, B♭} dyad, below the high {E♭, A♭}, is in fact the last of the I9-dyads to put in its appearance.

The three stems-up dyads in the middle of the example, like the three beamed dyads of the ritornello at the right, manifest T5 and T8 relations between lower and higher notes of the dyads. Once again we note the strong constructive aspect of those T-relations in the present contexts.

Reviewing our work so far, let us formulate an agenda for further work.

Agenda 1: To formulate an overall view of the chorale, we must somehow relate the 4–19 sets of its middle to the 4–16s and 4–Z15s of its opening.

Agenda 2: The cadence chord of the chorale must be integrated into that view.

Agenda 3 : So must the verticalities at the end of measure 11 and the beginning of measure 12, chords that strongly project diminished triads. Example 5 showed to some extent how the left and right hands of the chords fit into a scheme of 4–19 sets, but the effect of the chords as vertical totalities cannot be ignored. Furthermore, the {D4, F4} of the chord at the beginning of m. 12 was not addressed by example 5; this dyad has yet to be integrated into any overall harmonic view of the chorale.[6]

We shall take up the agenda issues in turn.

Agenda 1: To formulate an overall view of the chorale, we must somehow relate the 4–19 sets of its middle to the 4–16s and 4–Z15s of its opening.

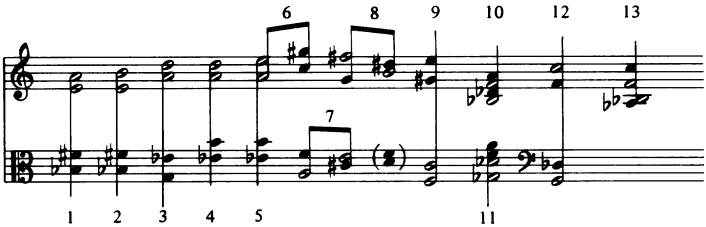
We can take example 4 as a point of departure; there we hear how the T8-dyads of the 4–19s continue a pattern of T8-dyads already present within the 4–16 and 4–Z15 verticalities. T8, as we observed (in examples 6b, 6c, and 7), is also involved in larger-scale transformational structures. So is T5, as demonstrated by example 6a, along with the material referenced in notes 3 and 4. Our interest in T5 and T8 as structural transformations suggests that we listen to the chorale as indicated by example 9.

The example parses most of the chorale into T5-dyads (indicated with open noteheads) and T8-dyads (indicated with fillled-in noteheads). In particular, every 4–16 set of the chorale, and every 4–Z15 set, parses into a T5-dyad plus a T8-dyad. One hears in particular how the first five structures of example 9 fill in the verticalities to the left of example 4, adjoining T5-dyads thereto.

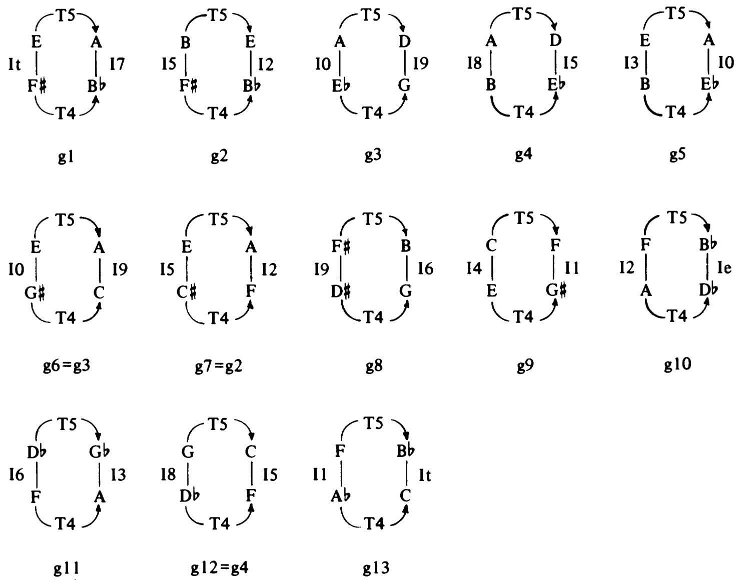
In general, the structures of example 9 subsume as well the 4–19 sets of example 5. Structure 6 of example 9 does not address set 1 of example 5. But structure 6 of example 9 does project a 4–19 set, and structure 6 is a more logical way of subsuming the high {C5, G♯5} into example 9, whose "orchestration" puts {A4, E5}, not {B4, E5}, into the "T5-playing instruments" of the fifth chord.

The numbered structures of the example are not exactly sets; they are rather *interpretations* of sets. As such, they are particularly amenable to representation by Klumpenhouwer networks (K-networks). Example 10 gives pertinent K-networks to represent them.

The network labeled g1 symbolizes the ideas that A is the T5-transpose of E, that B♭ is the T4-transpose of F♯, that E and F♯ are



Example 9



Example 10

It-inversions, each of the other, and that A and B♭ are I7-inversions, each of the other. Network g1 symbolizes structure 1 of example 9.[7]

And so forth for the other networks of example 10. The symbols g1, g2, etc. refer to the *graphs* of the various networks, that is the configurations of nodes and arrows, together with the various T/I transformations that label the arrows. The I-lines are abbreviations for two-headed I-arrows. The annotation "g6 = g3" means that the *graphs* for networks 6 and 3 are the same; that is the two networks have the same configuration of nodes and arrows, and their corresponding arrows are labled by the same transformations.

Equality of graphs is a strong abstract relation between networks. A weaker but more useful relation is *isomorphism of graphs*. Two graphs are "isomorphic" when they share the same structure of nodes-and-arrows, and when also the operations labeling corresponding arrows correspond under a particular sort of mapping f among T/I operations. To generate isomorphic graphs, the mapping f must be what is called an *automorphism* of the T/I system. Networks that have isomorphic graphs are called *isographic*. The specifics of these notions are spelled out carefully in Lewin 1990. For present purposes, it will suffice to reproduce here some of the results.

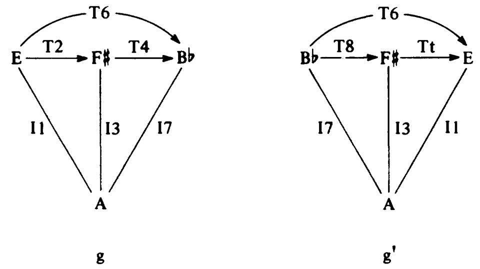
Two networks are *positively isographic* when they share the same configuration of nodes and arrows, when the T-numbers of corresponding arrows are equal, and when the I-numbers of corresponding arrows differ by some fixed number j mod 12. So for example networks g1 and g2 of example 10 are positively isographic: they share the same configuration of nodes and arrows; each T-number of network g2 is the same as the corresponding T-number of network g1; each I-number of network g2 is exactly 7 more than the corresponding I-number of network g1. The graph isomorphism will be denoted by <T7>, following the orthography of Klumpenhouwer (1991a, 1991b). We can write "g2 = <T7> (g1)'"; to get graph g2 from graph g1, leave the T-numbers alone and add 7 to each of the I-numbers.

Similarly, we can write "g4 = <T8>(g3)"; to get graph g4, leave the T-numbers of g3 alone and add 8 to each of the I-numbers. In similar fashion, we see that ALL the networks of example 10 are (positively) isographic, each to any other.

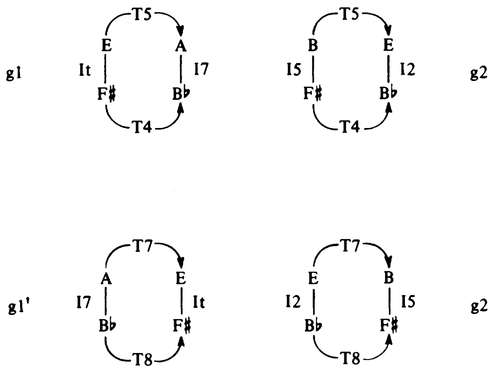
The notation "<Tj>" is suggestive because the system of <Tj> operations on graphs behaves exactly the same way as does the system of Tj operations on pitch classes (or sets or rows or whatever). That is, <Tj><Tk> = <T(j+k)>, meaning that for any graph g, the <Tj>-transform-of-the-<Tk>-transform of g is the same graph as the <T(j+k)>-transform of g. That is, <Tj>(<Tk>(g)) = <T(j+k)>(g). The <Tj> transformations combine under the same law as do the corresponding Tj transformations: <Tj><Tk> = <T(j+k)> just as TjTk = T(j+k). One says that the group of <Tj> operations is "isomorphic with" the group of Tj operations, under the correspondence of each <Tj> with the similar-numbered Tj.[8]

Any network can be *retrograded* by reversing all arrows and adjusting the transformations accordingly. Example 11 gives a sample K-network g and its retrograde g′.

Example 12 reproduces networks g1 and g2 from example 10, and then gives their respective retrogrades, the networks g1′ and g2′.



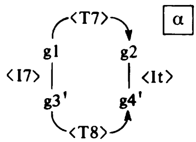
Example 11



Example 12

Network g1′ is *negatively isographic* with network g1. The two networks share the same configuration of nodes and arrows; each T-number in g1′ is the negative (mod 12) of the corresponding T-number in g1; each I-number in G1′ sums to 5 (mod 12) with the corresponding I-number in g1. Since corresponding I-numbers sum to 5, we shall write g1′ = <I5> (g1), saying that "g1′ is the <I5>-transform of g1." Likewise, g2′ is the <I7>-transform of g2: they share the same configuration of nodes and arrows; corresponding T-numbers are mod-12 negatives (complements); corresponding I-numbers sum to 7 (mod 12). We can write "g2′ = <I7> (g2)." In similar fashion, we see that g1′ is the <I0>-transform of g2, and that g2′ is the <I0>-transform of g1.[9]

<Ij> transformations of graphs behave just like Ij transformations of pitch classes (sets, rows, etc.). That is, <Tj> <Ik> = <I (j+k)>



Example 13

just as TjIk = I(j+k), and <Ij> <Tk> = <I(k−j)> just as IjTk = I(k−j), and <Ij> <Ik> = <T(k−j)> just as IjIk = T(k−j). One says that the group of <Tj>-and-<Ij> operations is "isomorphic with" the group of Tj-and-Ij operations, under the correspondence of each <Tj> with the similar-numbered Tj, and each <Ij> with the similar-numbered Ij.

Example 13 displays a network whose graph is called "alpha." The network display collates the following facts: graph g2 (from example 10) is the <T7>-transform of graph g1; graph g4′, the retrograde of graph g4, is the <T8>-transform of graph g3′; graphs g1 and g3′ are <I7>-transforms, each of the other; graphs g2 and g4′ are <It>-transforms, each of the other. To verify that g1 and g3′ are <I7>-related, we observe that the I-number to the left of g1 is t, and that the I-number to the left of g3′ (to the right of g3) is 9; the sum of the corresponding I-numbers t and 9 is the <I>-number 7 on graph alpha. Likewise, the I-number to the right of g1 is 7, and the I-number to the right of g3′ (to the left of g3) is 0; the sum of the corresponding I-numbers 7 and 0 is the <I>-number 7 on graph alpha.

Example 13 illustrates the *recursive* potential of K-network analysis. Compare graph alpha to graph g1′ in example 12. The two networks address different entities—graphs g1, g2, g3′, g4′ in the one case and pitch classes A, E, B♭, F♯ in the other. But if we focus *only* on the graph structure itself, we observe that the two graphs are "the same." That is, they have the same abstract configuration of nodes and arrows; furthermore, corresponding arrows are labeled by "the same" transformations. The transformations, more exactly, correspond exactly under the identification of <Tj> with Tj, <Ij> with Ij. We can thus say that the network of example 13 is (positively) isographic to network g1′ of example 12. It is then also positively isographic to network g2′ of example 12, and negatively isographic to networks g1 and g2 in example 12.

We can appreciate the power of the isography here if we review what the networks are doing on the scene. Example 9 interprets each of chords 1–2–3–4 in the chorale as comprising an ic5 dyad and an ic4 dyad. Networks g1 through g4 on example 10 give isographic

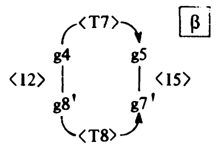
K-networks to represent those interpretations of the four chords. The networks g1 through g4 themselves interrelate as in example 13; that example thus interprets the *progression* of chords 1-2-3-4 by a K-structure that exactly reproduces, on a higher level, the structure that interpreted each chord.[10]

Example 13 organizes the networks of example 10 whose graphs have order-numbers 1 through 4. Example 14, in like manner, organizes the networks whose graphs have order-numbers 4 through 8 except for g6 (which has the same graph as g3). The network of example 14 is labeled "beta"; it is positively isographic to graph alpha of example 13, hence negatively isographic (in the recursive sense) to g1, g2, etc. Looking back on example 9, we see that the segment of the chorale comprising events there labeled 4 through 8 is being organized by example 14, which interprets the *progression* of those events in a manner analogous to the way example 13 interpreted the progression of events 1–2–3–4. As before, the interpretation of the progression-of-events reproduces, on a higher structural level, the interpretation of each event itself. The reason for putting g8′ to the left of g7′ on graph beta will be addressed later by analogous commentary for a later graph.

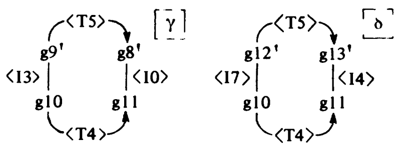
Example 15 continues the analysis of progressions in the same vein. It presents networks with graphs labeled "gamma" and "delta," organizing events 8–9–10–11 and events 10–11–12–13 respectively. Events 10 and 11 occur at the same time; consequently there is no strong temporal implication, on networks gamma and delta, about the left-to-right order of the symbols g10 and g11. The same obtains for events 12 and 13. That being so, why should we put g9′ to the left of g8′ on the gamma network? Why not use the retrogrades of gamma and delta, which do not violate the temporal order of events in the music? Those retrogrades would be positively isographic with networks alpha and beta.

There is a good structural reason for using gamma and delta as they stand, nevertheless. The reason involves recursion of network structure at a *yet higher* level. Example 16 illustrates what I mean.

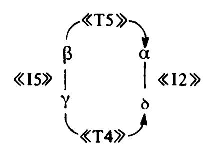
The example collates various observations. Graph alpha (example 13) is the <<T5>>-transform of graph beta (example 14). That is: corresponding <T>-numbers on the two graphs are the same; each <I>-number on alpha is 5 more than the corresponding <I>-number of beta. Likewise, delta is the <<T4>>-transform of gamma. Furthermore, gamma is the <<I5>>-transform of beta. That is: each <T>-number of gamma complements the corresponding <T>-number of beta; each <I>-number of gamma sums, with the corresponding <I>-number of beta, to form the number 5 mod 12—the 5 of "<<I5>>." Likewise, delta is the <<I2>>-transform of alpha.



Example 14



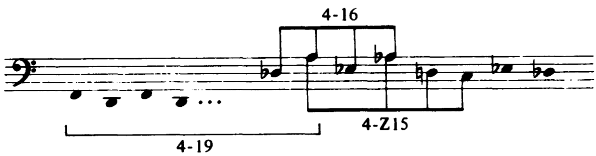
Example 15



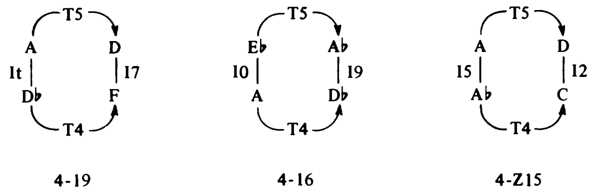
Example 16

Because of these relationships, the network of example 16 *is isographic* (at a higher structural level) to all the other networks studied since example 10. One sees this very clearly in comparing the graph of example 16 to graph g2 in particular, or to the retrograde of graph beta.

The structure would not emerge, if we used the retrogrades of networks gamma and delta. Example 16 makes a very strong statement about the *overall progression* of the chorale, or rather the *progression-of-progressions* therein, as interpreted by our work so far. Each of the events 1 though 13 (on example 9) is interpreted (on example 10) by a K-network that uses our basic graph. The progression of interpretations for events 1–2–3–4 is itself interpreted by a network of relations that manifests the same basic graph, in form alpha. The same is true for the progression of interpretations for events 4–5–7–8, via the graph beta. The same is true for the progression of interpretations for events 8–9–10–11, via the graph gamma. And the same is true for the



Example 17



Example 18

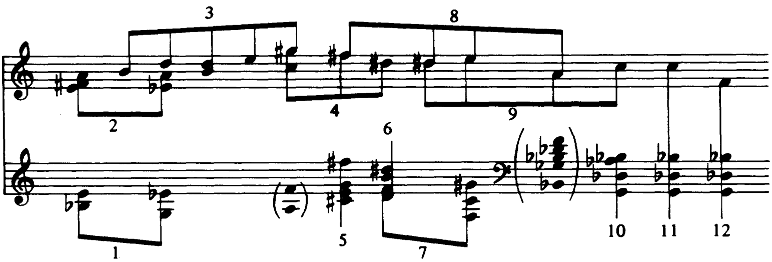
progression of interpretations for events 10–11–12–13, via the graph delta. Finally, and almost incredibly, the progression of interpretations beta, alpha, gamma, delta is *itself* interpreted by yet another manifestation of the same basic graph.

Example 17 shows the Ur-forms of sets 4–19, 4–16, and 4–Z15 that are projected by material at the opening of the piece. We have already discussed these sets. Example 18 interprets them by K-networks that use our basic graph. The three graphs of example 18 are exactly graphs g1, g3, and g2 of example 10. That is, a <T0>-relation obtains between the 4–19 graph of example 18 and graph g1; a <T0>-relation obtains between the 4–16 graph of example 18 and graph g3; a <T0>-relation obtains between the 4–Z15 graph of example 18 and graph g2. The opening of the chorale is thus strongly related to the opening of the entire piece, via our K-networks.

The foregoing analysis has responded to agenda item 1: we have formulated an overall view of the chorale that relates the 4–19 sets of its middle to the 4–16s and 4–Z15s of its opening. Indeed the analysis has done a great deal more. Let us proceed with the rest of the agenda.

Agenda 2: The cadence chord of the chorale must be integrated into the overall view.

Our work so far has already responded to this demand. Events 12 and 13 on example 9 address aspects of the cadence chord, including



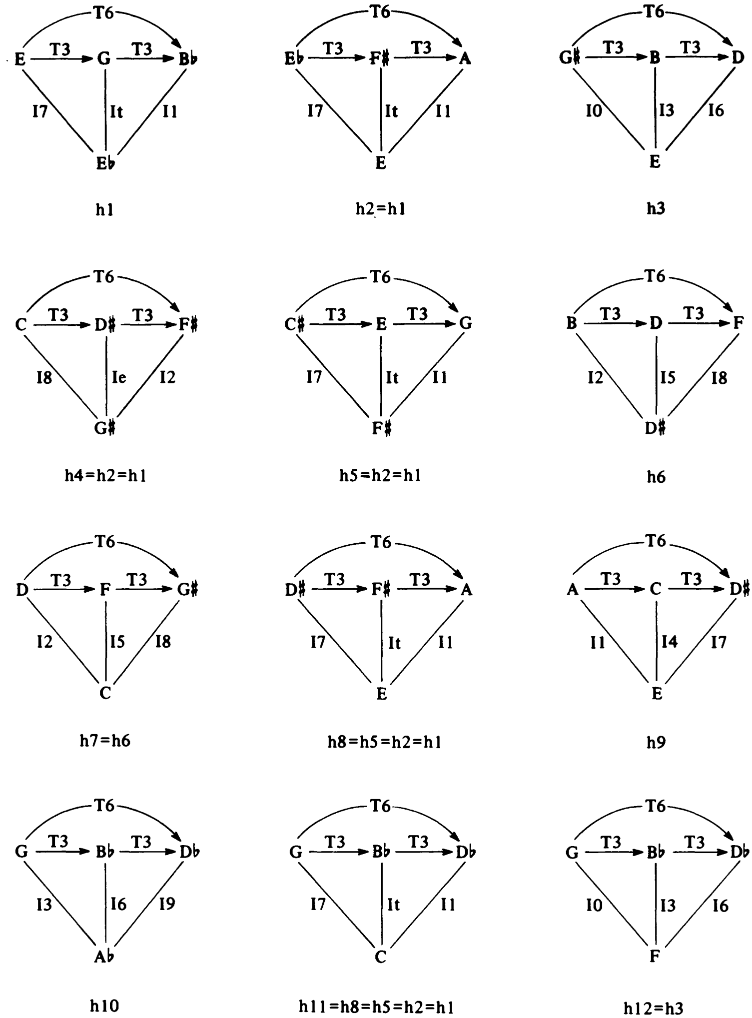
Example 19

all notes of the chord; networks g12 and g13 on example 10 interpret those events using our basic graph; graphs g12 and g13 participate in network delta, whose graph participates—at a yet higher level—in the network of example 16. The whole hierarchic system of the analysis closes with the network of example 16, which in turn closes with the completion of graph delta, which in turn closes with the manifestation of graphs g12 and g13, graph interpreting events projected by the cadential chord. All this closure is certainly appropriate to the notion of a structural (and not just textural) "cadence."

Event 12 is of particular interest in this connection. It projects the pcset {G, D♭, F, C}, a 4–16 set. That bind the event to the earlier 4–16s at the beginning of the chorale, reinforcing the sense of closure. The specific pcset {G, D♭, F, C} also recurs significantly later in the piece. It does so for instance as the first verticality within measure 25, where it is strongly marked as the incipit for the material of that measure. More vehemently yet, the pcset is featured in measures 46–47 as the insistently repeated ultimate goal of the unleashed sequence in the left hand.

Having responded to agenda item 2, let us now turn our attention to item 3. We must incorporate into our overall picture of the chorale the verticalities at the end of measure 11 and the beginning of measure 12, chords that strongly project diminished triads. We have now seen how the left and rights hands of the chords mostly fit into our K-network picture, but—as we observed earlier—the effect of the chords as vertical totalities cannot be ignored. And remarks we made earlier, about the {D4, F4} of the chord at the beginning of m. 12, are still pertinent. This dyad does not participate in any event of example 9; hence it does not participate in our K-network analysis so far.

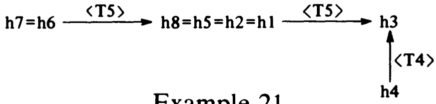
Example 19 articulates the chorale into new "events," or better "configurations," each of which comprises a diminished triad plus an odd-note-out. The configurations address the {D,F} dyad just dis-



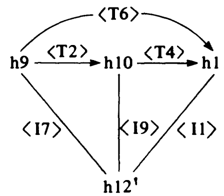
Example 20

cussed; they also address virtually the entire melody of the chorale, integrating it with a new reading for the opening five chords, and for the cadence chord.

Example 20 interprets configurations 1, 2, . . . , 12 of example 19 by isographic K-networks of a new sort. The graphs of the networks



Example 21



Example 22

Example 23

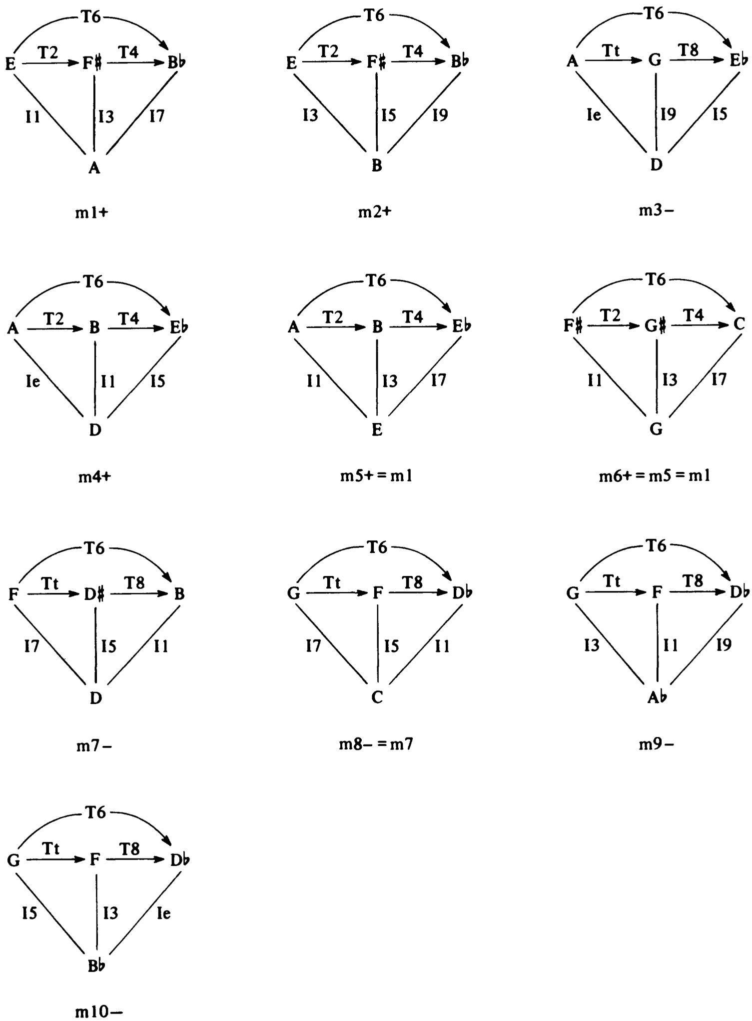
are labeled h1, h2, . . . , h12. We shall say that these networks are (all) "in MODE II," distinguishing them from the earlier K-networks of examples 10 through 18, which are (all) "in MODE I."

Example 21 arranges graphs h1 through h8 in a supernetwork that projects inter alia a triple proportion of <T5>-relations. We earlier observed a triple proportion of T5-relations among 4–19 sets 3, 1, and 2 of example 6. The unleashed sequence mentioned in note 3 projects a triple proportion of T5-relations; so does the progression of dyads {B, D♯}, {G♯, E}, and {C♯, A} in the right hand of measure 12.

Example 21 also analyzes h4 as a "<T4>-accessory" to h3; it thus projects <T4> and <T5> in the same context, somwhat in the spirit of mode I.

Example 21 does not address the mode-II graphs h9 though h12. Example 22 arranges them in a supernetwork whose graph is of a new type. We shall say that this graph is MODE III.

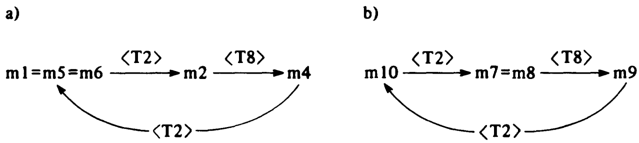
Example 23 shows the pertinence of mode III to the foreground of the chorale. Each stemmed or beamed event comprises some (026) trichord, displayed with open noteheads, and another note displayed with a filled-in notehead. Events 1 through 5 of the example address very strongly the melody/accompaniment texture at the beginning of the chorale; event 7 addresses the {F, D} dyad of agenda item 3; events



Example 24

8 through 10 address the cadence chord. Each of the ten events can be interpreted by a K-network in mode III, as illustrated by networks m1 through m10 on example 24.

Networks m1, m2, m4, m5, and m6 are *positively* isographic to the network of example 22; that is why they have plus signs after their



Example 25

names on example 24. Networks m3 and m7-though-m10 have minus signs after their names; they are *negatively* isographic to the network of example 22. The plus-signed networks of example 24 are all positively isographic, each to any other; any plus-signed network and any minus-signed network are negatively isographic, each to the other.[11]

Networks m6 and m5 have identical graphs; they are <T0>-related. This links event 6 strongly with event 5 in mode III, connecting the incipit section of the chorale with the apparent disruption in measure 11. Networks m7 and m8 also have identical graphs. This links the problematic chord containing the {D, F} verticality to the 4–16 set within the cadence chord.

Example 25a is a higher-level network organizing the plus-signed graphs; example 25b organizes the minus-signed graphs, except for m3. The two networks have identical graphs. (They are <<T0>>-related.) The (a)-graph addresses the incipit section of the chorale and the transition into measure 11; the (b)-graph addresses the chord containing {D, F} and the cadence chord. The arrow from "m10" to "m7 = m8," on graph (b), does not violate musical chronology: events 10 and 8 happen at the same time in the piece. The <T2>-progression of m1 to m2 arises from the rising whole-tone in the melody of events 1 and 2, while the accompaniment remains fixed. The <T2>-progression of m4 to m5 has an analogous source in events 4 and 5. The <T2> and <T8> relations within graph (b) arise from the analysis of the cadence chord in mode III: within events 8, 9, and 10, the (026) trichord is consistently {G, F, D♭}; the odd-notes-out are C, A♭, and B♭ respectively. As the odd-notes-out vary, the I-numbers of the graphs with constant {G, F, D♭} vary accordingly. From C to A♭ is T8, so from m8 to m9 is <T8>; from A♭ to B♭ is T2, so from m9 to m10 is <T2>; from B♭ to C is T2, so from m10 to m8 is <T2>. The identity of graphs (a) and (b), in example 25, thus arises from a relation in the chorale, between the melody/accompaniment structure of measures 9–10 and the harmonic structure of the cadence chord. The melody of measure 9–10 involves whole-tone rises over steady (026) trichords in the accompaniment; the cadence chord is articulated by mode III into the various whole-tone related notes A♭, B♭, and C, as they combine with the steady (026) trichord {G, F, D♭}. Example 25 makes this overall relation precise, and shows just how strong it is.

Mode III, we recall, also organized a supernetwork of mode II configurations, as they approached the cadence of the chorale (example 22).

One could investigate yet other modes of K-network structuring. Examples 2b and 2c, for instance, suggest that we investigate articulating 4–Z15 and 4–16 into a 2–1 dyad and a 2–2 dyad. That would give rise to K-networks "in mode IV." Or we could articulate each 4-Z15 and 4-16 into an (016) trichord and an odd-note-out; the cadence chord would then produce three "mode V" events, in which the left-hand {G, A♭, D♭} was a constant (016) component, and the odd-notes-out would give rise to <T7>-relations among the three cadence networks in mode V. That would relate nicely to the T5-chain of relations among the ic4 dyads {B, D♯}, {G♯, E}, and {C♯, A} in the right hand of measure 12. It would also relate nicely to the T5 structure of the sequence (to be unleashed later on).

And so forth. Our "overall view" is no longer simply the mode-I picture; the view rather involves a family of motivic transformational "filters" using the triple T5-proportion, modes I–II–III, etc. Through those filters we perceive various aspects of large-scale continuity and organization over the chorale as a whole.

NOTES

1. I have taken the term "chorale" from Natalie Boisvert, who recently—in an interesting octatonic analysis—drew my attention to the issue of its harmonic unity.
2. Klumpenhouwer Networks are described and explored in Lewin 1990, in Klumpenhouwer 1991a, chapter 8 (pages 8:1–8:29), and in Klumpenhouwer 1991b. For the present article, necessary theoretical background on the networks will be provided as occasions arise for their application.
3. Later on, at the climax of the piece (mm. 45–47), the sequence will be allowed to continue exactly, and to continue thereafter for one more exact stage. That will lead to extended subsiding passages that prepare the coda of the movement.
4. The chain of T5 relations here, which relates transformationally to the T5 interval for the sequence, will recur in later discussion of 4–19 sets within the chorale.

   It is not difficult to hear T5 relations interacting with impressions of tonality in the chorale. Much of the material in the right hand specifically suggests a local key of A minor, particularly in connection with T5 relations. The melody in itself suggests A minor. We shall not pursue any farther this important and complex issue, which deserves thorough investigation for its own sake.
5. It is not so obvious why the chorale can be heard in 6/4 (rather than 3/2) up to measure 12. That involves its relation to the earlier model within measure 4. But it is not so obvious, either, why the model of measure 4 is heard in 6/8 groups rather than 3/4 groups. Or—for that matter—what the audible (rather than notational) sense of the 12/8 time signature is for the music of measures 1–4. Reluctantly, I sidestep these issues for present purposes.

   On some pianos, the dynamic hairpins of mm. 11–12 can be executed as follows: strike the keys (forte) and then, holding them down, immediately afterwards depress the damper pedal; the open strings, resonating the chord and its overtones, can give a sense of added volume (acoustic power), if not exactly added dynamic (amplitude). I doubt, though, that this will be audible to anybody but the pianist, at least in a concert hall rather than a living room.
6. It is of course the ostinato dyad of the ritornello, 2 octaves higher. The observation is perhaps suggestive, but it still does not sufficiently address the internal harmonic coherence of the chorale.
7. The reasons for this particular configuration, e.g. for the T4-arrow rather than the T8-arrow from B♭ to F♯, will become clear later on.

   For g1 to be well-formed, the system of arrows-cum- T/I-labels must obey certain restrictions. For instance, it is crucial that the T5-transpose of the It-inversion of *any* pitch class is the same as the I7-inversion of the T4-transpose of that given pitch-class. It is not sufficient that those T/I relations obtain (only) among the four *particular* pcs involved in network g1. The theoretical background is provided in Lewin 1987, 193–95.
8. To illustrate the relation between ⟨Tj⟩ and Tj, we can focus on the T7-relation between {E, A}, the top dyad of network g1, and {B, E}, the analogous dyad of network g2. To be sure, the surface of the music enforces a different hearing of the dyads, E remaining fixed while A moves "stepwise" to B. Still, we can

focus on the T7-relations between the dyads if we wish; hearing {B, E} as the "dominant" of {E, A} is an obvious expedient. When {E, A} moves to {B, E} via T7, while {F♯, B♭} remains fixed, the I-numbers of network g1 perforce increase by 7 within network g2. The T7 relation between the dyads thus generates the ⟨T7⟩-relation between the graphs.

9. Not every network is negatively isographic to its retrograde. In example 11, for instance, the retrograde networks g and g′ are not negatively isographic. It is a special feature of graphs g1, g2,, etc, that they are "retrograde-inversionally symmetrical."
10. Here, it is important that g1 has a T5-arrow and T4-arrow, rather than a T5-arrow and a T8-arrow. The recursion would not work out in the latter event.
11. The retrogrades of m1+ etc. are *not* isographic with m3− etc. here. That is, graphs in mode III are *not* their own retrograde-inversions. Example 11, whose "g" is m1+, makes that clear: the g′ of example 11 is not isographic with m3−.

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