

TABLE DES PREMIERS CANONS

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ABSTRACT. Les tables suivantes présentent les premiers canons de \mathbb{Z}_n obtenus pour $n = 2$ à 16 ainsi que leurs formes duales. Les pavages de la ligne sont présentés en premier suivis des pavages du cercle. Tous les canons sont représentés par une somme directe $R \oplus S$ donnant la structure de la forme mélodique ou rythmique (R) et ses différents points d'entrée dans le temps (S). Chaque canon est caractérisé par un couple (R, S) qui peut être renversé dans sa forme duale (S, R). Les pavages sont considérés comme pavant la ligne ou le cercle selon que la somme est faite sur \mathbb{N} ou modulo n sur \mathbb{Z}_n . Les pavages de la ligne se construisent par produit tensoriel d'éléments horizontaux H_n formés par répétition de n unités ou d'éléments verticaux V_n représentés par une matrice unité à n lignes, n colonnes. Un pavage est singulier lorsqu'il ne peut pas se représenter en somme directe d'éléments de la forme $a\mathbb{Z}_b$.

1. CANONS SUR \mathbb{Z}_2

Il existe une seule solution de canons sur \mathbb{Z}_2 .

$$(1) R \oplus S = \mathbb{Z}_2 \oplus \mathbb{Z}_1 = \{0, 1\} \oplus \{0\} = H_2$$

1	1
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$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_2 = \{0\} \oplus \{0, 1\} = V_2$$

1	0
0	1

2. CANONS SUR \mathbb{Z}_3

Il existe une seule solution de canons sur \mathbb{Z}_3

$$(1) R \oplus S = \mathbb{Z}_3 \oplus \mathbb{Z}_1 = \{0, 1, 2\} \oplus \{0\} = H_3$$

1	1	1
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$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_3 = \{0\} \oplus \{0, 1, 2\} = V_3$$

1	0	0
0	1	0
0	0	1

3. CANONS SUR \mathbb{Z}_4

Il existe deux solutions de canons sur \mathbb{Z}_4

$$(1) R \oplus S = \mathbb{Z}_4 \oplus \mathbb{Z}_1 = \{0, 1, 2, 3\} \oplus \{0\} = H_4 = H_2 \otimes H_2$$

1	1	1	1
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$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_4 = \{0\} \oplus \{0, 1, 2, 3\} = V_4 = V_2 \otimes V_2$$

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

$$(2) R \oplus S = \mathbb{Z}_2 \oplus 2\mathbb{Z}_2 = \{0, 1\} \oplus \{0, 2\} = H_2 \otimes V_2$$

1	1	0	0
0	0	1	1

$$(2^*) R \oplus S = 2\mathbb{Z}_2 \oplus \mathbb{Z}_2 = \{0, 2\} \oplus \{0, 1\} = V_2 \otimes H_2$$

1	0	1	0
0	1	0	1

4. CANONS SUR \mathbb{Z}_5

Il existe une seule solution de canons sur \mathbb{Z}_5

$$(1) R \oplus S = \mathbb{Z}_5 \oplus \mathbb{Z}_1 = \{0, 1, 2, 3, 4\} \oplus \{0\} = H_5$$

1	1	1	1	1
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$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_5 = \{0\} \oplus \{0, 1, 2, 3, 4\} = V_5$$

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

5. CANONS SUR \mathbb{Z}_6

Il existe trois solutions de canons sur \mathbb{Z}_6

$$(1) R \oplus S = \mathbb{Z}_6 \oplus \mathbb{Z}_1 = \{0, 1, 2, 3, 4, 5\} \oplus \{0\} = H_6 = H_2 \otimes H_3$$

1	1	1	1	1	1
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$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_6 = \{0\} \oplus \{0, 1, 2, 3, 4, 5\} = V_6 = V_2 \otimes V_3$$

1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

$$(2) R \oplus S = \mathbb{Z}_3 \oplus 3\mathbb{Z}_2 = \{0, 1, 2\} \oplus \{0, 3\} = H_3 \otimes V_2$$

1	1	1	0	0	0
0	0	0	1	1	1

$$(2^*) R \oplus S = 3\mathbb{Z}_2 \oplus \mathbb{Z}_3 = \{0, 3\} \oplus \{0, 1, 2\} = V_3 \otimes H_2$$

1	0	0	1	0	0
0	1	0	0	1	0
0	0	1	0	0	1

$$(3) R \oplus S = \mathbb{Z}_2 \oplus 2\mathbb{Z}_3 = \{0, 1\} \oplus \{0, 2, 4\} = H_2 \otimes V_3$$

1	1	0	0	0	0
0	0	1	1	0	0
0	0	0	0	1	1

$$(3^*) R \oplus S = 2\mathbb{Z}_3 \oplus \mathbb{Z}_2 = \{0, 2, 4\} \oplus \{0, 1\} = V_2 \otimes H_3$$

1	0	1	0	1	0
0	1	0	1	0	1

6. CANONS SUR \mathbb{Z}_7

Il existe une solution de canons sur \mathbb{Z}_7

$$(1) R \oplus S = \mathbb{Z}_7 \oplus \mathbb{Z}_1 = \{0, 1, 2, 3, 4, 5, 6\} \oplus \{0\} = H_7$$

1	1	1	1	1	1	1	1
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$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_7 = \{0\} \oplus \{0, 1, 2, 3, 4, 5, 6\} = V_7$$

1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1

7. CANONS SUR \mathbb{Z}_8

Il existe six solutions de canons sur \mathbb{Z}_8

$$(1) R \oplus S = \mathbb{Z}_8 \oplus \mathbb{Z}_1 = \{0, 1, 2, 3, 4, 5, 6, 7\} \oplus \{0\} = H_8$$

1	1	1	1	1	1	1	1	1
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$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_8 = \{0\} \oplus \{0, 1, 2, 3, 4, 5, 6, 7, 8\} = V_8$$

1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1

$$(2) R \oplus S = \mathbb{Z}_4 \oplus 4\mathbb{Z}_2 = \{0, 1, 2, 3\} \oplus \{0, 4\} = H_2 \otimes H_2 \otimes V_2$$

1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1

$$(2^*) R \oplus S = 4\mathbb{Z}_2 \oplus \mathbb{Z}_4 = \{0, 4\} \oplus \{0, 1, 2, 3\} = V_2 \otimes V_2 \otimes H_2$$

1	0	0	0	1	0	0	0
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	1	0	0	0	1

$$(3) R \oplus S = \mathbb{Z}_2 \oplus 4\mathbb{Z}_2 \oplus 2\mathbb{Z}_2 = \{0, 1, 4, 5\} \oplus \{0, 2\} = H_2 \otimes V_2 \otimes H_2$$

1	1	0	0	1	1	0	0
0	0	1	1	0	0	1	1

$$(3^*) R \oplus S = 2\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus 4\mathbb{Z}_2 = \{0, 2\} \oplus \{0, 1, 4, 5\} = V_2 \otimes H_2 \otimes V_2$$

1	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0
0	0	0	0	1	0	1	0
0	0	0	0	0	1	0	1

$$(4) R \oplus S = 2\mathbb{Z}_4 \oplus \mathbb{Z}_2 = \{0, 2, 4, 6\} \oplus \{0, 1\} = V_2 \otimes H_2 \otimes H_2$$

1	0	1	0	1	0	1	0
0	1	0	1	0	1	0	1

$$(4^*) R \oplus S = \mathbb{Z}_2 \oplus 2\mathbb{Z}_4 = \{0, 1\} \oplus \{0, 2, 4, 6\} = H_2 \otimes V_2 \otimes V_2$$

1	1	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	1	1	0	0
0	0	0	0	0	0	1	1

$$(5) R \oplus S = 3\mathbb{Z}_4 \oplus 4\mathbb{Z}_2 = \{0, 3, 6, 9\} \oplus \{0, 4\}$$

1	0	1	0	0	1	0	1
0	1	0	1	1	0	1	0

$$(6) R \oplus S = 3\mathbb{Z}_2 \oplus 2\mathbb{Z}_4 = \{0, 3\} \oplus \{0, 2, 4, 6\}$$

1	0	0	1	0	0	0	0
0	1	0	0	0	0	1	0
0	0	1	0	0	1	0	0
0	0	0	0	1	0	0	1

8. CANONS SUR \mathbb{Z}_9

Il existe quatre solutions de canons sur \mathbb{Z}_9

$$(1) R \oplus S = \mathbb{Z}_9 \oplus \mathbb{Z}_1 = \{0, 1, 2, 3, 4, 5, 6, 7, 8\} \oplus \{0\} = H_9$$

1	1	1	1	1	1	1	1	1
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$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_9 = \{0\} \oplus \{0, 1, 2, 3, 4, 5, 6, 7\} = V_8$$

1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1

$$(2) R \oplus S = \mathbb{Z}_3 \oplus 3\mathbb{Z}_3 = \{0, 1, 2\} \oplus \{0, 3, 6\} = H_3 \otimes V_3$$

1	1	1	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	1	1	1

$$(2^*) R \oplus S = 3\mathbb{Z}_3 \oplus \mathbb{Z}_3 = \{0, 3, 6\} \oplus \{0, 1, 2\} = V_3 \otimes H_3$$

1	0	0	1	0	0	1	0	0
0	1	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0	1

$$(3) R \oplus S = 2\mathbb{Z}_3 \oplus 3\mathbb{Z}_3 = \{0, 2, 4\} \oplus \{0, 3, 6\}$$

1	0	1	0	0	0	0	1	0
0	1	0	0	0	0	1	0	1
0	0	0	1	0	1	0	1	0

$$(4) R \oplus S = 5\mathbb{Z}_3 \oplus 3\mathbb{Z}_3 = \{0, 5, 10\} \oplus \{0, 3, 6\}$$

1	1	0	0	0	1	0	0	0
0	0	1	0	0	0	1	1	0
0	0	0	1	1	0	0	0	1

9. CANONS SUR \mathbb{Z}_{10}

Il existe six solutions de canons sur \mathbb{Z}_{10}

$$(1) R \oplus S = \mathbb{Z}_{10} \oplus \mathbb{Z}_1 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \oplus \{0\} = H_{10}$$

1	1	1	1	1	1	1	1	1	1
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$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_{10} = \{0\} \oplus \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} = V_{10}$$

1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	1

$$(2) R \oplus S = 2\mathbb{Z}_5 \oplus \mathbb{Z}_2 = \{0, 2, 4, 6, 8\} \oplus \{0, 1\} = V_2 \otimes H_5$$

1	0	1	0	1	0	1	0	1	0
0	1	0	1	0	1	0	1	0	1

$$(2^*) R \oplus S = \mathbb{Z}_5 \oplus 2\mathbb{Z}_5 = \{0, 1\} \oplus \{0, 2, 4, 6, 8\} = H_2 \otimes V_5$$

1	1	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0
0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	0	0	1	1

$$(3) R \oplus S = \mathbb{Z}_5 \oplus 5\mathbb{Z}_2 = \{0, 1, 2, 3, 4\} \oplus \{0, 5\} = H_5 \otimes V_2$$

1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1

$$(3^*) R \oplus S = 5\mathbb{Z}_2 \oplus \mathbb{Z}_5 = \{0, 5\} \oplus \{0, 1, 2, 3, 4\} = V_5 \otimes H_2$$

1	0	0	0	0	1	0	0	0	0
0	1	0	0	0	0	1	0	0	0
0	0	1	0	0	0	0	1	0	0
0	0	0	1	0	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1

$$(4) R \oplus S = \{0, 1, 2, 4, 8\} \oplus \{0, 5\}$$

1	1	1	0	1	0	0	0	1	0
0	0	0	1	0	1	1	1	0	1

$$(5) R \oplus S = 7\mathbb{Z}_5 \oplus 5\mathbb{Z}_2 = \{0, 1, 4, 7, 8\} \oplus \{0, 5\}$$

1	1	0	1	1	0	0	1	0	0
0	0	1	0	0	1	1	0	1	1

$$(6) R \oplus S = 3\mathbb{Z}_2 \oplus 2\mathbb{Z}_5 = \{0, 3\} \oplus \{0, 2, 4, 6, 8\}$$

1	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0
0	0	1	0	0	1	0	0	0	0
0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	0	1	0	0	1

10. CANONS SUR \mathbb{Z}_{11}

Il existe une solution de canons sur \mathbb{Z}_{11}

$$(1) R \oplus S = \mathbb{Z}_{11} \oplus \mathbb{Z}_1 = H_{11}$$

1	1	1	1	1	1	1	1	1	1	1
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$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_{11} = V_{11}$$

1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	1

11. CANONS SUR \mathbb{Z}_{12}

Il existe 23 solutions de canons sur \mathbb{Z}_{12}

$$(1) R \oplus S = \mathbb{Z}_{12} \oplus \mathbb{Z}_1 = H_{12}$$

1	1	1	1	1	1	1	1	1	1	1	1
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$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_{12} = V_{12}$$

1	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	1

$$(2) R \oplus S = \mathbb{Z}_6 \oplus 6\mathbb{Z}_2 = \{0, 1, 2, 3, 4, 5\} \oplus \{0, 6\} = H_6 \otimes V_2$$

1	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1

$$(2^*) R \oplus S = 6\mathbb{Z}_2 \oplus \mathbb{Z}_6 = \{0, 6\} \oplus \{0, 1, 2, 3, 4, 5\} = V_6 \otimes H_2$$

1	0	0	0	0	0	1	0	0	0	0	0
0	1	0	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0	0	0	1

$$(3) R \oplus S = \mathbb{Z}_4 \oplus 4\mathbb{Z}_3 = \{0, 1, 2, 3\} \oplus \{0, 4, 8\} = H_4 \otimes V_3$$

1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1

$$(3^*) R \oplus S = 4\mathbb{Z}_3 \oplus \mathbb{Z}_4 = \{0, 4, 8\} \oplus \{0, 1, 2, 3\} = V_4 \otimes H_3$$

1	0	0	0	1	0	0	0	1	0	0	0
0	1	0	0	0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0	0	0	1	0
0	0	0	1	0	0	0	1	0	0	0	1

$$(4) R \oplus S = 2\mathbb{Z}_6 \oplus \mathbb{Z}_2 = \{0, 2, 4, 6, 8, 10\} \oplus \{0, 1\} = V_2 \otimes H_6 = V_2 \otimes H_2 \otimes H_3$$

1	0	1	0	1	0	1	0	1	0	1	0
0	1	0	1	0	1	0	1	0	1	0	1

$$(4^*) R \oplus S = \mathbb{Z}_2 \oplus 2\mathbb{Z}_6 = \{0, 1\} \oplus \{0, 2, 4, 6, 8, 10\} = H_2 \otimes V_6 = H_2 \otimes V_3 \otimes V_2$$

1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0
0	0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1

$$(5) R \oplus S = 3\mathbb{Z}_4 \oplus \mathbb{Z}_3 = \{0, 3, 6, 9\} \oplus \{0, 1, 2\} = V_3 \otimes H_4 = V_3 \otimes H_2 \otimes H_2$$

1	0	0	1	0	0	1	0	0	1	0	0
0	1	0	0	1	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0	1	0	0	1

$$(5^*) R \oplus S = \mathbb{Z}_3 \oplus 3\mathbb{Z}_4 = \{0, 1, 2\} \oplus \{0, 3, 6, 9\} = H_3 \otimes V_4 = H_3 \otimes V_2 \otimes V_2$$

1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1

$$(6) R \oplus S = \mathbb{Z}_2 \oplus 4\mathbb{Z}_3 \oplus 2\mathbb{Z}_2 = \{0, 1, 4, 5, 8, 9\} \oplus \{0, 2\} = H_2 \otimes V_2 \otimes H_3$$

1	1	0	0	1	1	0	0	1	1	0	0
0	0	1	1	0	0	1	1	0	0	1	1

$$(6^*) R \oplus S = \mathbb{Z}_2 \oplus 4\mathbb{Z}_3 \oplus 2\mathbb{Z}_2 = \{0, 2\} \oplus \{0, 1, 4, 5, 8, 9\} = V_2 \otimes H_2 \otimes V_3$$

1	0	1	0	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	1	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	1	0
0	0	0	0	0	0	0	0	0	1	0	1

$$(7) R \oplus S = \mathbb{Z}_2 \oplus 6\mathbb{Z}_2 \oplus 2\mathbb{Z}_3 = \{0, 1, 6, 7\} \oplus \{0, 2, 4\} = H_2 \otimes V_3 \otimes H_2$$

1	1	0	0	0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0	1	1	0	0
0	0	0	0	1	1	0	0	0	0	1	1

$$(7^*) R \oplus S = \mathbb{Z}_2 \oplus 6\mathbb{Z}_2 \oplus 2\mathbb{Z}_3 = \{0, 2, 4\} \oplus \{0, 1, 6, 7\} = V_2 \otimes H_3 \otimes V_2$$

1	0	1	0	1	0	0	0	0	0	0	0
0	1	0	1	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	1	0	1	0
0	0	0	0	0	0	0	1	0	1	0	1

$$(8) R \oplus S = \mathbb{Z}_3 \oplus 6\mathbb{Z}_2 \oplus 3\mathbb{Z}_2 = \{0, 1, 2, 6, 7, 8\} \oplus \{0, 3\} = H_3 \otimes V_2 \otimes H_2$$

1	1	1	0	0	0	1	1	1	0	0	0
0	0	0	1	1	1	0	0	0	1	1	1

$$(8^*) R \oplus S = \mathbb{Z}_3 \oplus 6\mathbb{Z}_2 \oplus 3\mathbb{Z}_2 = \{0, 3\} \oplus \{0, 1, 2, 6, 7, 8\} = V_3 \otimes H_2 \otimes V_2$$

1	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	1	0	0	1	0
0	0	0	0	0	0	0	0	1	0	0	1

$$(9) R \oplus S = \{0, 1, 2, 3, 5, 10\} \oplus \{0, 6\}$$

1	1	1	1	0	1	0	0	0	0	1	0
0	0	0	0	1	0	1	1	1	1	0	1

$$(10) R \oplus S = \{0, 1, 2, 4, 5, 9\} \oplus \{0, 6\}$$

1	1	1	0	1	1	0	0	0	1	0	0
0	0	0	1	0	0	1	1	1	0	1	1

$$(11) R \oplus S = \{0, 1, 2, 5, 9, 10\} \oplus \{0, 6\}$$

1	1	1	0	0	1	0	0	0	1	1	0
0	0	0	1	1	0	1	1	1	0	0	1

$$(12) R \oplus S = 5\mathbb{Z}_6 \oplus 6\mathbb{Z}_2 = \{0, 1, 3, 5, 8, 10\} \oplus \{0, 6\}$$

1	1	0	1	0	1	0	0	1	0	1	0
0	0	1	0	1	0	1	1	0	1	0	1

$$(13) R \oplus S = \{0, 1, 2, 7\} \oplus \{0, 4, 8\}$$

1	1	1	0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	1	1	1	0
0	0	0	0	1	1	1	0	0	0	0	1

$$(14) R \oplus S = \{0, 1, 3, 6\} \oplus \{0, 4, 8\}$$

1	1	0	1	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	1	1	0	1
0	0	0	0	1	1	0	1	0	0	1	0

$$(15) R \oplus S = \{0, 1, 3, 10\} \oplus \{0, 4, 8\}$$

1	1	0	1	0	0	0	0	0	0	1	0
0	0	1	0	1	1	0	1	0	0	0	0
0	0	0	0	0	0	1	0	1	1	0	1

(16) $R \oplus S = \{0, 1, 7, 10\} \oplus \{0, 4, 8\}$

1	1	0	0	0	0	0	1	0	0	1	0
0	0	1	0	1	1	0	0	0	0	0	1
0	0	0	1	0	0	1	0	1	1	0	0

(17) $R \oplus S = 2\mathbb{Z}_2 \oplus 5\mathbb{Z}_2 \oplus 4\mathbb{Z}_3 = \{0, 2, 5, 7\} \oplus \{0, 4, 8\}$

1	0	1	0	0	1	0	1	0	0	0	0
0	1	0	1	0	0	0	0	1	0	1	0
0	0	0	0	1	0	1	0	0	1	0	1

(18) $R \oplus S = \{0, 1, 5\} \oplus \{0, 3, 6, 9\}$

1	1	0	0	0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	1	1	0
0	0	0	1	1	0	0	0	1	0	0	0
0	0	0	0	0	0	1	1	0	0	0	1

(19) $R \oplus S = \{0, 1, 8\} \oplus \{0, 3, 6, 9\}$

1	1	0	0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	1	1	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	1	1	0

(20) $R \oplus S = 2\mathbb{Z}_3 \oplus 3\mathbb{Z}_4 = \{0, 2, 4\} \oplus \{0, 3, 6, 9\}$

1	0	1	0	1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	1	0	1
0	0	0	1	0	1	0	1	0	0	0	0
0	0	0	0	0	0	1	0	1	0	1	0

(21) $R \oplus S = 7\mathbb{Z}_3 \oplus 3\mathbb{Z}_4 = \{0, 2, 7\} \oplus \{0, 3, 6, 9\}$

1	0	1	0	0	0	0	1	0	0	0	0
0	1	0	0	0	0	1	0	1	0	0	0
0	0	0	1	0	1	0	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	0	1

(22) $R \oplus S = 3\mathbb{Z}_2 \oplus 2\mathbb{Z}_6 = \{0, 3\} \oplus \{0, 2, 4, 6, 8, 10\}$

1	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1	0
0	0	1	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	1	0	0	0	0
0	0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0	0	1

(23) $R \oplus S = 5\mathbb{Z}_2 \oplus 2\mathbb{Z}_6 = \{0, 5\} \oplus \{0, 2, 4, 6, 8, 10\}$

1	0	0	0	0	1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	1

12. CANONS SUR \mathbb{Z}_{13}

Il existe une solution de canons sur \mathbb{Z}_{13}

$$(1) R \oplus S = \mathbb{Z}_{13} \oplus \mathbb{Z}_1 = H_{13}$$

1	1	1	1	1	1	1	1	1	1	1	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---

$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_{13} = V_{13}$$

1	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	1

13. CANONS SUR \mathbb{Z}_{14}

Il existe 13 solutions de canons sur \mathbb{Z}_{14}

$$(1) R \oplus S = \mathbb{Z}_{14} \oplus \mathbb{Z}_1$$

1	1	1	1	1	1	1	1	1	1	1	1	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---

$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_{14} = V_{14}$$

1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1

$$(2) R \oplus S = \mathbb{Z}_7 \oplus 7\mathbb{Z}_2 = \{0, 1, 2, 3, 4, 5, 6\} \oplus \{0, 7\} = H_7 \otimes V_2$$

1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	1	1

$$(2^*) R \oplus S = \mathbb{Z}_7 \oplus 7\mathbb{Z}_2 = \{0, 7\} \oplus \{0, 1, 2, 3, 4, 5, 6\} = V_7 \otimes H_2$$

1	0	0	0	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0	0	0	0	0	1

$$(3) R \oplus S = 2\mathbb{Z}_7 \oplus \mathbb{Z}_2 = \{0, 2, 4, 6, 8, 10, 12\} \oplus \{0, 1\} = V_2 \otimes H_7$$

1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	1	0	1	0	1	0	1	0	1	0	1	0	1

$$(3^*) R \oplus S = 2\mathbb{Z}_7 \oplus \mathbb{Z}_2 = \{0, 1\} \oplus \{0, 2, 4, 6, 8, 10, 12\} = H_2 \otimes V_7$$

1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1

$$(4) R \oplus S = \{0, 1, 2, 3, 4, 6, 12\} \oplus \{0, 7\}$$

1	1	1	1	1	0	1	0	0	0	0	0	1	0
0	0	0	0	0	1	0	1	1	1	1	1	0	1

$$(5) R \oplus S = \{0, 1, 2, 3, 5, 6, 11\} \oplus \{0, 7\}$$

1	1	1	1	0	1	1	0	0	0	0	1	0	0
0	0	0	0	1	0	0	1	1	1	1	0	1	1

$$(6) R \oplus S = \{0, 1, 2, 3, 6, 11, 12\} \oplus \{0, 7\}$$

1	1	1	1	0	0	1	0	0	0	0	1	1	0
0	0	0	0	1	1	0	1	1	1	1	0	0	1

$$(7) R \oplus S = \{0, 1, 2, 4, 5, 6, 10\} \oplus \{0, 7\}$$

1	1	1	0	1	1	1	0	0	0	1	0	0	0
0	0	0	1	0	0	0	1	1	1	0	1	1	1

$$(8) R \oplus S = \{0, 1, 2, 4, 6, 10, 12\} \oplus \{0, 7\}$$

1	1	1	0	1	0	1	0	0	0	1	0	1	0
0	0	0	1	0	1	0	1	1	1	0	1	0	1

$$(9) R \oplus S = 5\mathbb{Z}_7 \oplus 7\mathbb{Z}_2 = \{0, 1, 2, 5, 6, 10, 11\} \oplus \{0, 7\}$$

1	1	1	0	0	1	1	0	0	0	1	1	0	0
0	0	0	1	1	0	0	1	1	1	0	0	1	1

$$(10) R \oplus S = 3\mathbb{Z}_7 \oplus 7\mathbb{Z}_2 = \{0, 1, 3, 4, 6, 9, 12\} \oplus \{0, 7\}$$

1	1	0	1	1	0	1	0	0	1	0	0	1	0
0	0	1	0	0	1	0	1	1	0	1	1	0	1

$$(11) R \oplus S = \{0, 1, 3, 5, 6, 9, 11\} \oplus \{0, 7\}$$

1	1	0	1	0	1	1	0	0	1	0	1	0	0
0	0	1	0	1	0	0	1	1	0	1	0	1	1

$$(12) R \oplus S = 3\mathbb{Z}_2 \oplus 2\mathbb{Z}_7 = \{0, 3\} \oplus \{0, 2, 4, 6, 8, 10, 12\}$$

1	0	0	1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	1	0
0	0	1	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	1

$$(13) R \oplus S = 5\mathbb{Z}_2 \oplus 2\mathbb{Z}_7 = \{0, 5\} \oplus \{0, 2, 4, 6, 8, 10, 12\}$$

1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	1

14. CANONS SUR \mathbb{Z}_{15}

Il existe 25 solutions de canons sur \mathbb{Z}_{15}

$$(1) R \oplus S = \mathbb{Z}_{15} \oplus \mathbb{Z}_1 = H_{15}$$

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_{15} = V_{15}$$

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

$$(2) R \oplus S = \mathbb{Z}_5 \oplus 5\mathbb{Z}_3 = \{0, 1, 2, 3, 4\} \oplus \{0, 5, 10\} = H_5 \otimes V_3$$

1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1

$$(2^*) R \oplus S = \mathbb{Z}_5 \oplus 5\mathbb{Z}_3 = \{0, 5, 10\} \oplus \{0, 1, 2, 3, 4\} = V_5 \otimes H_3$$

1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
0	1	0	0	0	0	1	0	0	0	0	1	0	0	0
0	0	1	0	0	0	0	1	0	0	0	0	1	0	0
0	0	0	1	0	0	0	0	1	0	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	0	0	0	0	1

$$(3) R \oplus S = \mathbb{Z}_3 \oplus 3\mathbb{Z}_5 = \{0, 3, 6, 9, 12\} \oplus \{0, 1, 2\} = V_3 \otimes H_5$$

1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
0	1	0	0	1	0	0	1	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0	1	0	0	1	0	0	1

$$(3^*) R \oplus S = \mathbb{Z}_3 \oplus 3\mathbb{Z}_5 = \{0, 1, 2\} \oplus \{0, 3, 6, 9, 12\} = H_3 \otimes V_5$$

1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1

$$(4) R \oplus S = \{0, 1, 2, 3, 9\} \oplus \{0, 5, 10\}$$

1	1	1	1	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	1	1	1	1	0
0	0	0	0	0	1	1	1	1	0	0	0	0	0	1

$$(5) R \oplus S = \{0, 1, 2, 4, 8\} \oplus \{0, 5, 10\}$$

1	1	1	0	1	0	0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	1	1	1	0	1
0	0	0	0	0	1	1	1	0	1	0	0	0	1	0

$$(6) R \oplus S = \{0, 1, 2, 4, 13\} \oplus \{0, 5, 10\}$$

1	1	1	0	1	0	0	0	0	0	0	0	0	1	0
0	0	0	1	0	1	1	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	1	1	1	0	1

$$(7) R \oplus S = 8\mathbb{Z}_5 \oplus 5\mathbb{Z}_3 = \{0, 1, 2, 8, 9\} \oplus \{0, 5, 10\}$$

1	1	1	0	0	0	0	0	1	1	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	1	1	1	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0	1	1

$$(8) R \oplus S = \{0, 1, 2, 9, 13\} \oplus \{0, 5, 10\}$$

1	1	1	0	0	0	0	0	0	1	0	0	0	1	0
0	0	0	1	0	1	1	1	0	0	0	0	0	0	1
0	0	0	0	1	0	0	0	1	0	1	1	1	0	0

$$(9) R \oplus S = \{0, 1, 3, 4, 7\} \oplus \{0, 5, 10\}$$

1	1	0	1	1	0	0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	1	1	0	1	1
0	0	0	0	0	1	1	0	1	1	0	0	1	0	0

(10) $R \oplus S = \{0, 1, 3, 4, 12\} \oplus \{0, 5, 10\}$

1	1	0	1	1	0	0	0	0	0	0	0	1	0	0
0	0	1	0	0	1	1	0	1	1	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	1	1	0	1	1

(11) $R \oplus S = \{0, 1, 3, 7, 9\} \oplus \{0, 5, 10\}$

1	1	0	1	0	0	0	1	0	1	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	1	1	0	1	0
0	0	0	0	0	1	1	0	1	0	0	0	1	0	1

(12) $R \oplus S = \{0, 1, 3, 9, 12\} \oplus \{0, 5, 10\}$

1	1	0	1	0	0	0	0	0	1	0	0	1	0	0
0	0	1	0	0	1	1	0	1	0	0	0	0	0	1
0	0	0	0	1	0	0	1	0	0	1	1	0	1	0

(13) $R \oplus S = \{0, 1, 4, 7, 8\} \oplus \{0, 5, 10\}$

1	1	0	0	1	0	0	1	1	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	1	1	0	0	1
0	0	0	0	0	1	1	0	0	1	0	0	1	1	0

(14) $R \oplus S = \{0, 1, 4, 7, 13\} \oplus \{0, 5, 10\}$

1	1	0	0	1	0	0	1	0	0	0	0	0	1	0
0	0	1	0	0	0	0	0	1	0	1	1	0	0	1
0	0	0	1	0	1	1	0	0	1	0	0	1	0	0

(15) $R \oplus S = 4\mathbb{Z}_5 \oplus 5\mathbb{Z}_3 = \{0, 1, 4, 8, 12\} \oplus \{0, 5, 10\}$

1	1	0	0	1	0	0	0	1	0	0	0	1	0	0
0	0	1	0	0	1	1	0	0	1	0	0	0	1	0
0	0	0	1	0	0	0	1	0	0	1	1	0	0	1

(16) $R \oplus S = \{0, 1, 7, 9, 13\} \oplus \{0, 5, 10\}$

1	1	0	0	0	0	0	1	0	1	0	0	0	1	0
0	0	1	0	1	0	0	0	1	0	1	1	0	0	0
0	0	0	1	0	1	1	0	0	0	0	0	1	0	1

(17) $R \oplus S = 2\mathbb{Z}_5 \oplus 5\mathbb{Z}_3 = \{0, 2, 4, 6, 8\} \oplus \{0, 5, 10\}$

1	0	1	0	1	0	1	0	1	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0	1	0	1	0	1
0	0	0	0	0	1	0	1	0	1	0	1	0	1	0

(18) $R \oplus S = \{0, 2, 4, 8, 11\} \oplus \{0, 5, 10\}$

1	0	1	0	1	0	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	1	0	1	0	0	0	1	0
0	0	0	1	0	0	1	0	0	0	1	0	1	0	1

$$(19) R \oplus S = \{0, 1, 5\} \oplus \{0, 3, 6, 9, 12\}$$

1	1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	1	1	0
0	0	0	1	1	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	1	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	1	1	0	0	0	1

$$(20) R \oplus S = 8\mathbb{Z}_3 \oplus 3\mathbb{Z}_5 = \{0, 1, 8\} \oplus \{0, 3, 6, 9, 12\}$$

1	1	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	1	1	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	1	1	0
0	0	0	0	0	0	1	1	0	0	0	0	0	0	1

$$(21) R \oplus S = \{0, 1, 11\} \oplus \{0, 3, 6, 9, 12\}$$

1	1	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	1	1	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	1	1	0

$$(22) R \oplus S = 2\mathbb{Z}_3 \oplus 3\mathbb{Z}_5 = \{0, 2, 4\} \oplus \{0, 3, 6, 9, 12\}$$

1	0	1	0	1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	1	0	1
0	0	0	1	0	1	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	1	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	1	0	1	0

$$(23) R \oplus S = \{0, 2, 7\} \oplus \{0, 3, 6, 9, 12\}$$

1	0	1	0	0	0	0	1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	1	0	1	0	0	0
0	0	0	1	0	1	0	0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	1	0	1	0	0	0	0	1	0

$$(24) R \oplus S = \{0, 2, 10\} \oplus \{0, 3, 6, 9, 12\}$$

1	0	1	0	0	0	0	0	0	0	1	0	0	0	0
0	1	0	0	0	0	1	0	1	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	0	1	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	1	0	1

$$(25) R \oplus S = 4\mathbb{Z}_3 \oplus 3\mathbb{Z}_5 = \{0, 4, 8\} \oplus \{0, 3, 6, 9, 12\}$$

1	0	0	0	1	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	0	1	0	0	0	1	0
0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1	0	0	0	1

15. CANONS SUR \mathbb{Z}_{16}

Il existe 49 solutions de canons sur \mathbb{Z}_{16}

$$(1) R \oplus S = \mathbb{Z}_{16} \oplus \mathbb{Z}_1 = H_{16}$$

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

$$(1^*) R \oplus S = \mathbb{Z}_1 \oplus \mathbb{Z}_{16} = V_{16}$$

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

$$(2) R \oplus S = \mathbb{Z}_8 \oplus 8\mathbb{Z}_2 = \{0, 1, 2, 3, 4, 5, 6, 7\} \oplus \{0, 8\} = H_8 \otimes V_2$$

1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

$$(2^*) R \oplus S = 8\mathbb{Z}_2 \oplus \mathbb{Z}_8 = \{0, 8\} \oplus \{0, 1, 2, 3, 4, 5, 6, 7\} = V_8 \otimes H_2$$

1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

$$(3) R \oplus S = 8\mathbb{Z}_2 \oplus \mathbb{Z}_4 \oplus 4\mathbb{Z}_2 = \{0, 1, 2, 3, 8, 9, 10, 11\} \oplus \{0, 4\} = H_4 \otimes V_2 \otimes H_2$$

1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1

$$(3^*) R \oplus S = 8\mathbb{Z}_2 \oplus \mathbb{Z}_4 \oplus 4\mathbb{Z}_2 = \{0, 4\} \oplus \{0, 1, 2, 3, 8, 9, 10, 11\} = V_4 \otimes H_2 \otimes V_2$$

1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1

$$(4) R \oplus S = \mathbb{Z}_4 \oplus 4\mathbb{Z}_4 = \{0, 1, 2, 3\} \oplus \{0, 4, 8, 12\} = H_4 \otimes V_4$$

1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1

$$(4^*) R \oplus S = \mathbb{Z}_4 \oplus 4\mathbb{Z}_4 = \{0, 4, 8, 12\} \oplus \{0, 1, 2, 3\} = V_4 \otimes H_4$$

1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0
0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1

$$(5) R \oplus S = \mathbb{Z}_2 \oplus 4\mathbb{Z}_4 \oplus 2\mathbb{Z}_2 = \{0, 1, 4, 5, 8, 9, 12, 13\} \oplus \{0, 2\} = H_2 \otimes V_2 \otimes H_4$$

1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1

$$(5^*) R \oplus S = \mathbb{Z}_2 \oplus 4\mathbb{Z}_4 \oplus 2\mathbb{Z}_2 = \{0, 2\} \oplus \{0, 1, 4, 5, 8, 9, 12, 13\} = V_2 \otimes H_2 \otimes V_4$$

1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1

$$(6) R \oplus S = \mathbb{Z}_2 \oplus 4\mathbb{Z}_2 \oplus 2\mathbb{Z}_2 \oplus 8\mathbb{Z}_2 = \{0, 1, 4, 5\} \oplus \{0, 2, 8, 10\} = H_2 \otimes V_2 \otimes H_2 \otimes V_2$$

1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1

$$(6^*) R \oplus S = \mathbb{Z}_2 \oplus 4\mathbb{Z}_2 \oplus 2\mathbb{Z}_2 \oplus 8\mathbb{Z}_2 = \{0, 2, 8, 10\} \oplus \{0, 1, 4, 5\} = V_2 \otimes H_2 \otimes V_2 \otimes H_2$$

1	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0	0	1	0	1	0	0	0	0
0	0	0	0	1	0	1	0	0	0	0	0	1	0	1	0
0	0	0	0	0	1	0	1	0	0	0	0	0	1	0	1

$$(7) R \oplus S = \mathbb{Z}_2 \oplus 8\mathbb{Z}_2 \oplus 2\mathbb{Z}_4 = \{0, 1, 8, 9\} \oplus \{0, 2, 4, 6\} = H_2 \otimes V_4 \otimes H_2$$

1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1

$$(7^*) R \oplus S = \mathbb{Z}_2 \oplus 8\mathbb{Z}_2 \oplus 2\mathbb{Z}_4 = \{0, 2, 4, 6\} \oplus \{0, 1, 8, 9\} = V_2 \otimes H_4 \otimes V_2$$

1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0
0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1

$$(8) R \oplus S = 2\mathbb{Z}_8 \oplus \mathbb{Z}_2 = \{0, 2, 4, 6, 8, 10, 12, 14\} \oplus \{0, 1\} = V_2 \otimes H_8$$

1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

$$(8^*) R \oplus S = 2\mathbb{Z}_8 \oplus \mathbb{Z}_2 = \{0, 1\} \oplus \{0, 2, 4, 6, 8, 10, 12, 14\} = H_2 \otimes V_8$$

1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1

(9) $R \oplus S = \{0, 1, 3, 6, 8, 9, 11, 14\} \oplus \{0, 4\}$

1	1	0	1	0	0	1	0	1	1	0	1	0	0	1	0
0	0	1	0	1	1	0	1	0	0	1	0	1	1	0	1

(10) $R \oplus S = \{0, 1, 2, 3, 4, 5, 7, 14\} \oplus \{0, 8\}$

1	1	1	1	1	1	0	1	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	1	1	1	1	1	1	0	1

(11) $R \oplus S = \{0, 1, 2, 3, 4, 6, 7, 13\} \oplus \{0, 8\}$

1	1	1	1	1	0	1	1	0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	1	1	1	1	1	0	1

(12) $R \oplus S = \{0, 1, 2, 3, 4, 7, 13, 14\} \oplus \{0, 8\}$

1	1	1	1	1	0	0	1	0	0	0	0	0	1	1	0
0	0	0	0	0	1	1	0	1	1	1	1	1	1	0	1

(13) $R \oplus S = \{0, 1, 2, 3, 5, 6, 7, 12\} \oplus \{0, 8\}$

1	1	1	1	0	1	1	1	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	1	1	1	1	0	1	1	1

(14) $R \oplus S = \{0, 1, 2, 3, 5, 7, 12, 14\} \oplus \{0, 8\}$

1	1	1	1	0	1	0	1	0	0	0	0	1	0	1	0
0	0	0	0	1	0	1	0	1	1	1	1	0	1	0	1

(15) $R \oplus S = \{0, 1, 2, 3, 6, 7, 12, 13\} \oplus \{0, 8\}$

1	1	1	1	0	0	1	1	0	0	0	0	1	1	0	0
0	0	0	0	1	1	0	0	1	1	1	1	0	0	1	1

(16) $R \oplus S = \{0, 1, 2, 3, 7, 12, 13, 14\} \oplus \{0, 8\}$

1	1	1	1	0	0	0	1	0	0	0	0	1	1	1	0
0	0	0	0	1	1	1	0	1	1	1	1	0	0	0	1

$$(17) R \oplus S = \{0, 1, 2, 4, 5, 7, 11, 14\} \oplus \{0, 8\}$$

1	1	1	0	1	1	0	1	0	0	0	1	0	0	1	0
0	0	0	1	0	0	1	0	1	1	1	0	1	1	0	1

$$(18) R \oplus S = \{0, 1, 2, 4, 6, 7, 11, 13\} \oplus \{0, 8\}$$

1	1	1	0	1	0	1	1	0	0	0	1	0	1	0	0
0	0	0	1	0	1	0	0	1	1	1	0	1	0	1	1

$$(19) R \oplus S = \{0, 1, 2, 4, 7, 11, 13, 14\} \oplus \{0, 8\}$$

1	1	1	0	1	0	0	1	0	0	0	1	0	1	1	0
0	0	0	1	0	1	1	0	1	1	1	0	1	0	0	1

$$(20) R \oplus S = 2\mathbb{Z}_2 \oplus 5\mathbb{Z}_4 \oplus 8\mathbb{Z}_2 = \{0, 1, 2, 5, 6, 7, 11, 12\} \oplus \{0, 8\}$$

1	1	1	0	0	1	1	1	0	0	0	1	1	0	0	0
0	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1

$$(21) R \oplus S = \{0, 1, 2, 5, 7, 11, 12, 14\} \oplus \{0, 8\}$$

1	1	1	0	0	1	0	1	0	0	0	1	1	0	1	0
0	0	0	1	1	0	1	0	1	1	1	0	0	1	0	1

$$(22) R \oplus S = 3\mathbb{Z}_4 \oplus 12\mathbb{Z}_2 \oplus 8\mathbb{Z}_2 = \{0, 1, 3, 4, 6, 7, 10, 13\} \oplus \{0, 8\}$$

1	1	0	1	1	0	1	1	0	0	1	0	0	1	0	0
0	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1

$$(23) R \oplus S = 7\mathbb{Z}_8 \oplus 8\mathbb{Z}_2 = \{0, 1, 3, 5, 7, 10, 12, 14\} \oplus \{0, 8\}$$

1	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0
0	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1

$$(24) R \oplus S = \{0, 1, 3, 6, 7, 10, 12, 13\} \oplus \{0, 8\}$$

1	1	0	1	0	0	1	1	0	0	1	0	1	1	0	0
0	0	1	0	1	1	0	0	1	1	0	1	0	0	1	1

$$(25) R \oplus S = \{0, 2, 4, 6\} \oplus \{0, 3, 8, 11\}$$

1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	1
0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0

$$(25^*) R \oplus S = \{0, 3, 8, 11\} \oplus \{0, 2, 4, 6\}$$

1	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0
0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0
0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1
0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0

(26) $R \oplus S = 6\mathbb{Z}_4 \oplus \mathbb{Z}_2 \oplus 8\mathbb{Z}_2 = \{0, 2, 6, 12\} \oplus \{0, 1, 8, 9\}$

1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0
0	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	1	0	1	0	0	0	1	0
0	0	0	0	0	1	0	0	0	1	0	1	0	0	0	1

(27) $R \oplus S = \{0, 2, 6, 12\} \oplus \{0, 3, 8, 11\}$

1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	1	0	0	0	1	0	1	0	0
0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	1
0	0	0	0	1	0	0	0	1	0	1	0	0	0	1	0

(28) $R \oplus S = \{0, 3, 4, 7\} \oplus \{0, 2, 8, 10\}$

1	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0
0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	1
0	0	0	1	0	1	0	0	0	0	0	1	0	1	0	0
0	0	0	0	1	0	1	0	0	0	0	0	1	0	1	0

(28*) $R \oplus S = \{0, 1, 4, 13\} \oplus \{0, 2, 8, 10\}$

1	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0
0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	1
0	0	0	0	0	1	0	0	1	1	0	0	1	0	0	0
0	0	0	0	0	0	0	1	0	0	1	1	0	0	1	0

(29) $R \oplus S = \{0, 1, 5, 12\} \oplus \{0, 2, 8, 10\}$

1	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	1	1	0	0	0	1	0	0	0	0	0	0	1	0
0	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0
0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	1

(30) $R \oplus S = \{0, 3, 7, 12\} \oplus \{0, 2, 8, 10\}$

1	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0
0	1	0	0	0	0	1	0	0	0	1	0	0	1	0	0
0	0	1	0	0	1	0	0	0	1	0	0	0	0	1	0
0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	1

(31) $R \oplus S = \{0, 1, 2, 7\} \oplus \{0, 4, 8, 12\}$

1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0
0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1

$$(32) R \oplus S = \{0, 1, 2, 11\} \oplus \{0, 4, 8, 12\}$$

1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	0

$$(33) R \oplus S = \{0, 1, 3, 6\} \oplus \{0, 4, 8, 12\}$$

1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	1
0	0	0	0	1	1	0	1	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	0	1	0	0	1	0

$$(34) R \oplus S = \{0, 1, 3, 10\} \oplus \{0, 4, 8, 12\}$$

1	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	1	1	0	1	0	0	0	0
0	0	0	0	1	1	0	1	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0	0	0	0	1	1	0	1

$$(35) R \oplus S = \{0, 1, 3, 14\} \oplus \{0, 4, 8, 12\}$$

1	1	0	1	0	0	0	0	0	0	0	0	0	0	1	0
0	0	1	0	1	1	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	1	1	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	1

$$(36) R \oplus S = \{0, 1, 6, 7\} \oplus \{0, 4, 8, 12\}$$

1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	0	0	1	1	0	0	0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1

$$(37) R \oplus S = \{0, 1, 6, 11\} \oplus \{0, 4, 8, 12\}$$

1	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	1	0	0	0	0	1	1	0	0
0	0	0	1	0	0	0	0	1	1	0	0	0	0	1	0
0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	1

$$(38) R \oplus S = \{0, 1, 7, 10\} \oplus \{0, 4, 8, 12\}$$

1	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	1
0	0	0	1	0	0	1	0	0	0	0	0	1	1	0	0
0	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0

(39) $R \oplus S = \{0, 1, 7, 14\} \oplus \{0, 4, 8, 12\}$

1	1	0	0	0	0	0	1	0	0	0	0	0	0	1	0
0	0	1	0	1	1	0	0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	0	0	1	0	1	1	0	0
0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	1

(40) $R \oplus S = \{0, 1, 11, 14\} \oplus \{0, 4, 8, 12\}$

1	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0
0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	1	0	1	1	0	0

(41) $R \oplus S = \{0, 2, 5, 7\} \oplus \{0, 4, 8, 12\}$

1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0	0	0	1	0	1	0
0	0	0	0	1	0	1	0	0	1	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	1	0	0	1	0	1

(42) $R \oplus S = \{0, 2, 5, 11\} \oplus \{0, 4, 8, 12\}$

1	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0
0	1	0	0	0	0	0	1	0	0	0	0	1	0	1	0
0	0	0	1	0	0	0	0	1	0	1	0	0	1	0	0
0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	1

(43) $R \oplus S = 7\mathbb{Z}_4 \oplus 4\mathbb{Z}_4 = \{0, 2, 7, 9\} \oplus \{0, 4, 8, 12\}$

1	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	1	0	0	0	0	1
0	0	0	1	0	1	0	0	0	0	0	0	1	0	1	0
0	0	0	0	1	0	1	0	0	0	0	1	0	1	0	0

(44) $R \oplus S = \{0, 2, 7, 13\} \oplus \{0, 4, 8, 12\}$

1	0	1	0	0	0	0	1	0	0	0	0	0	1	0	0
0	1	0	0	1	0	1	0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	0	1	0	0	1	0	1	0
0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	1

(45) $R \oplus S = 3\mathbb{Z}_4 \oplus 4\mathbb{Z}_4 = \{0, 3, 6, 9\} \oplus \{0, 4, 8, 12\}$

1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	1
0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0

$$(46) R \oplus S = 3\mathbb{Z}_2 \oplus 2\mathbb{Z}_8 = \{0, 3\} \oplus \{0, 2, 4, 6, 8, 10, 12, 14\}$$

1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1

$$(47) R \oplus S = 5\mathbb{Z}_2 \oplus 2\mathbb{Z}_8 = \{0, 5\} \oplus \{0, 2, 4, 6, 8, 10, 12, 14\}$$

1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1

$$(48) R \oplus S = \{0, 6\} \oplus \{0, 1, 4, 5, 8, 9, 12, 13\}$$

1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1

$$(49) R \oplus S = 7\mathbb{Z}_2 \oplus 2\mathbb{Z}_8 = \{0, 7\} \oplus \{0, 2, 4, 6, 8, 10, 12, 14\}$$

1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1