Riemann Hypothesis. Fractal Dynamics. Complex Time
Illustration of the question of the Rhythm?

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Summary

Canonical transfer function

Fractional transfert function
Canonical Dynamics in Fractal Media

Partition, Score and $\zeta(s)$

Physical meaning of Riemann's hypothesis

Physical meaning of Golbach's Conjecture.

Time. Rythm. Arithmetic

$\zeta(s)$

Connection between the following couple of talks based on

1. Categorical meaning of the Riemann's function
$\zeta(s)$ **: « *from arithmetics to Rythmics* »**

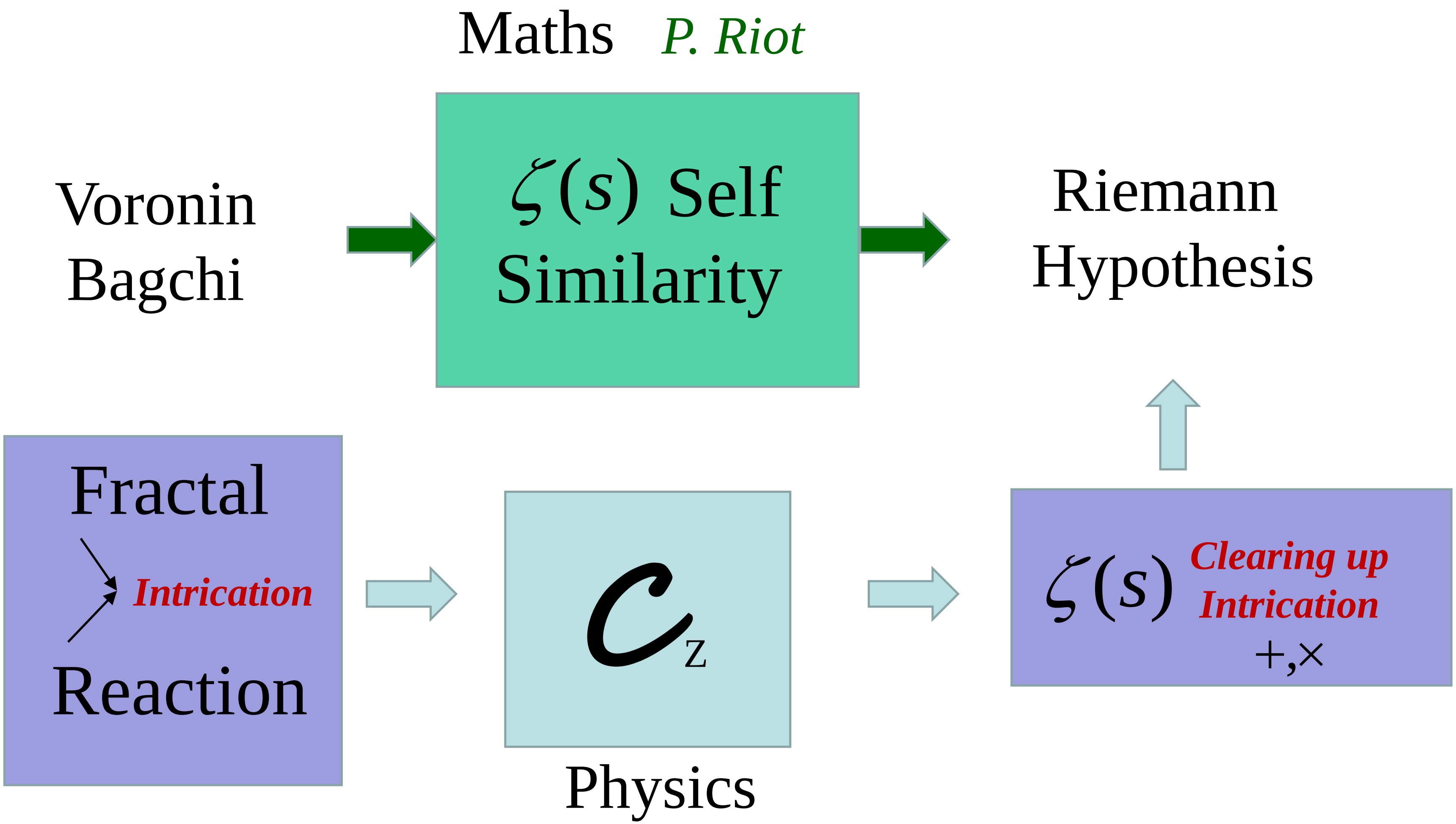
Philippe RIOT

2. Fractal Dynamics to understand Riemann Conjecture.
And, on the way, the meaning of the Rythm

Alain Le MEHAUTE
Dmitrii TAYURSKI

From Maths to Physics

In Lapidus' perspective



Yoneda Lemma

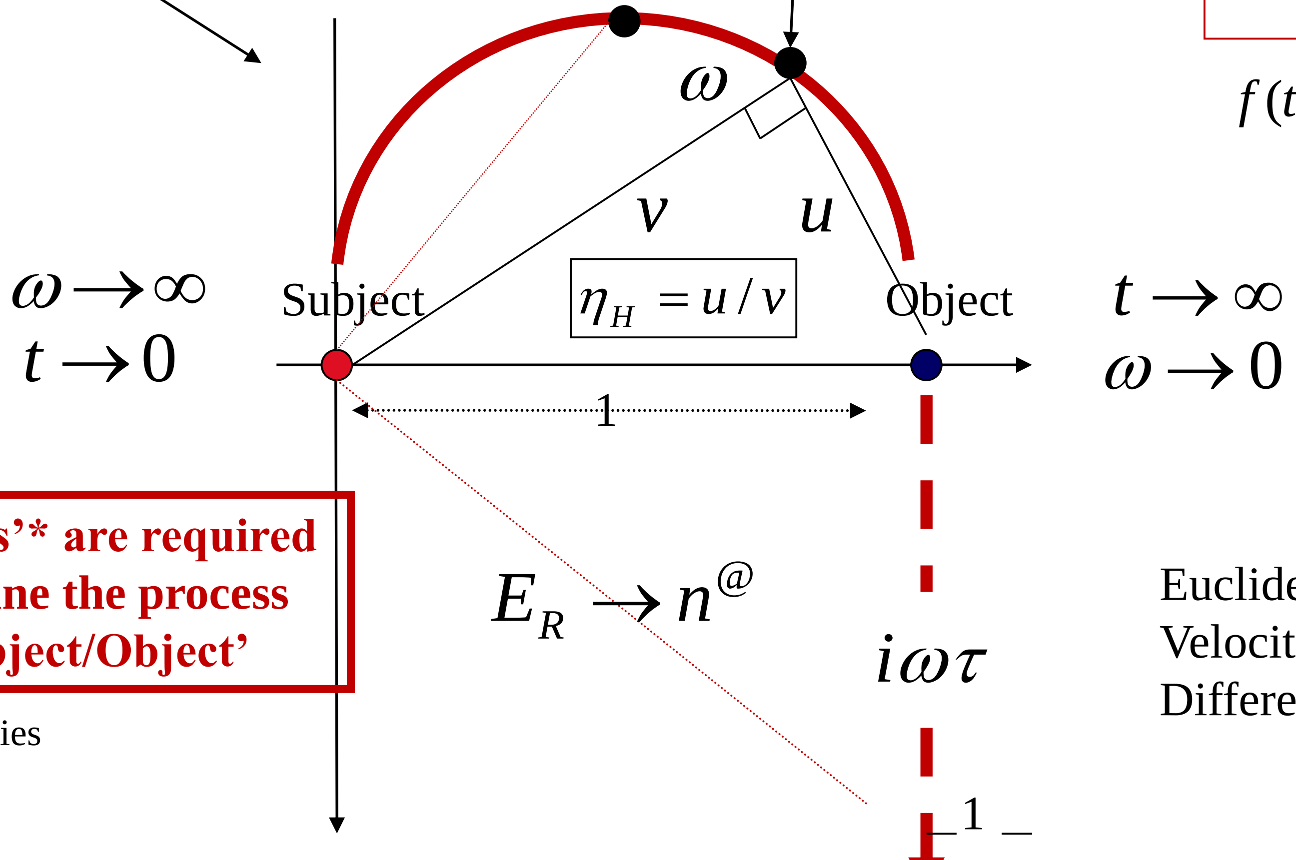
Category: {subject/object-Process/Vision}
in Fourier Space

Canonique first order equation

$$n@Z = u/v = \omega\tau$$

$$Z = \frac{1}{1 + i\omega\tau}$$

$$f(t) = \exp(t/\tau)$$



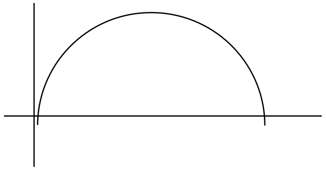
2 'points'* are required to define the process 'Subject/Object'

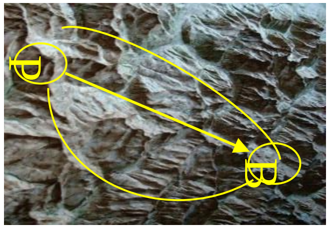
(*) categories

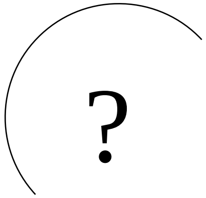
Euclidean Geometry
Velocity/Time Constant
Differential Equations

The point at issue

$$\frac{d}{dt}$$

EUCLIDEAN GEOMETRY → 

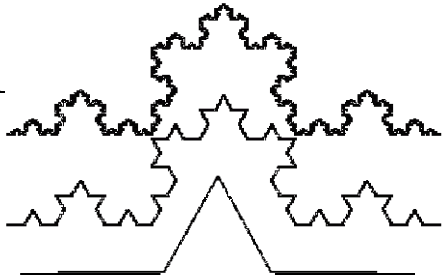


FRACTAL GEOMETRY
HYPERBOLIC GEOMETRY → 

$$\frac{d^{\alpha}}{dt^{\alpha}}$$

$$\eta^{d} N \sim 1$$

$$F_r$$



Arithmetic/Distribution

Citation

‘L’évidence des objets c’est le dehors comme substance’

‘*The evidence of object is the substance of its outside*’

René Guitart*

In’ *Le lemme de Yoneda : enjeu pour une conjecture philosophique?’* Charles Alunni
« *A lumière des mathématiques et à l’ombre de la philosophie* » Ed. Delatour Paris 2012

Experimental method

and TEISI Model (1977-1983)

-X=X approch by epimormism

Immersion of fractal media in reactive environment

Energy *U* is a key factor

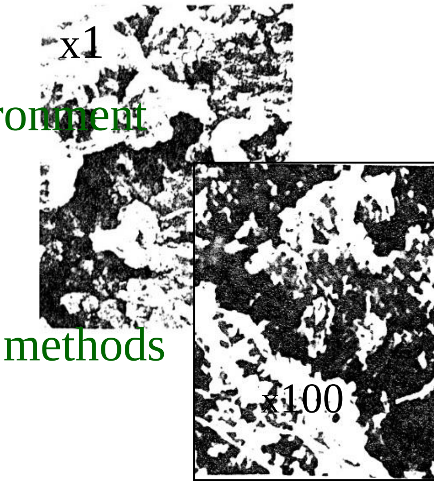
$$U : [ML^2 t^{-2}]$$
$$TF(U) : [ML^2 t^{-1}]$$

Spectral analysis (filtrering): look like IRM methods

Geodesic Representation in Complex Plan

Pointing out universality:

- Electrochemical (>10 decades of scale and multi reactions)
- Mechanical (3-4 decade of scale. and one standard method)
- Electrodynamics (3-4decade of scale and multi methods)



PbO_2
Self similar
Electrode

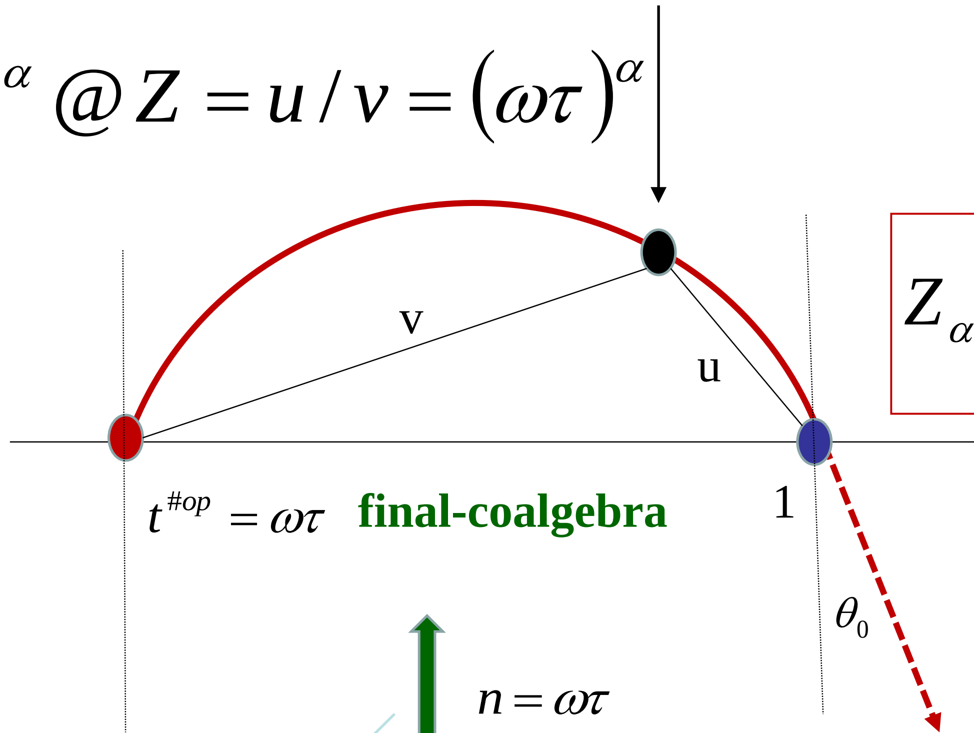
Related to Kan Extension method
Distribution Theory

Universality and Canonical form $\dfrac{\partial^{\alpha}}{\partial t^{\alpha}} - \Delta_{t,\infty}$

$$n^{\alpha} @ Z = u / v = (\omega\tau)^{\alpha}$$

$$\eta = (i\omega\tau)^{-\alpha}$$

$$Z_{\alpha} = \frac{1}{1 + (i\omega\tau)^{\alpha}}$$



$t^{\#op} = \omega\tau$ **final-coalgebra**

$n = \omega\tau$

$(i\omega\tau)^{\alpha}$

3 'points'* are required to define the process
~~**Subject/Object**~~
Phenomenology

$t + 1 \rightarrow t$

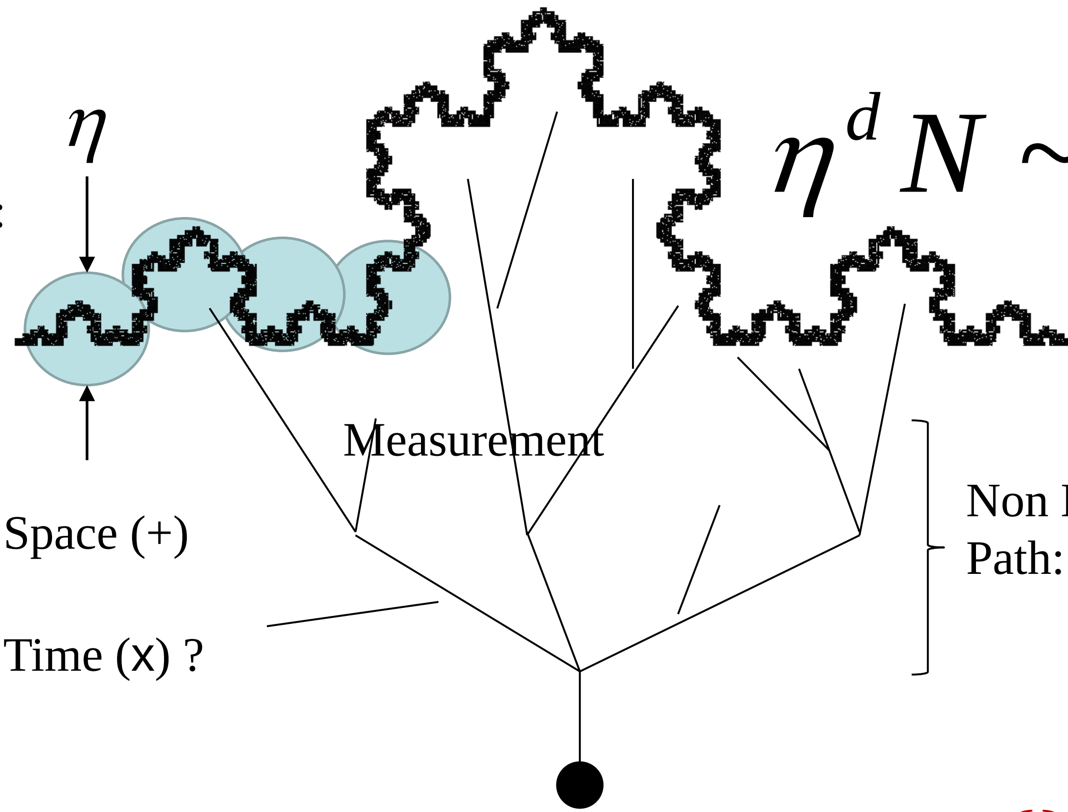
U

t-initial algebra
~ space algebra

Dialectic :Chance and Necessity

TEISI Experiment and Model: Chemical Reaction/ Fractal Interface

Final
Self similar object:
Necessity



$$\eta^d N \sim 1$$

Non Deterministic
Path: Chance

$n+1 \rightarrow n$ Space (+)

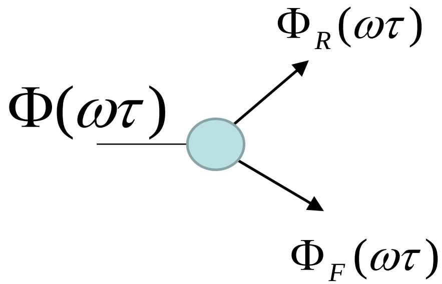
$\omega\tau \rightarrow n$ Time (x) ?

ω-tempo
(Fourier Space)

Deterterminism/Indeterminism
Geodesic = sum of the whole combinaison between the chance and the necessity.

RESERVES *VS* TEISI MODEL

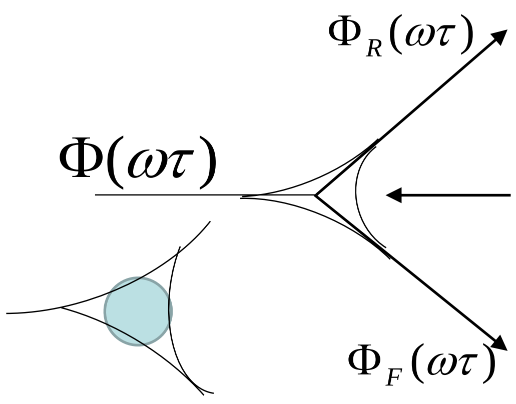
Excellente modelization of experimental data
but fundamental questions stay open concerning
the closure of the Energy and the balance between
Energy and Entropy



TEISI IS BASED ON 'BALANCE' EQUATION

Due to the absence of Velocity leading
the Action
the main invariant. $\left[L\, t^{-\alpha}\right]$

According to HYPERBOLIC GROUP (M. Gromov)



Fine curved triangle

Entropic Hyperbolic Coupling
ignored in TEISI model
may be considered

Required
Additive factor

$\left[L\, t^{1-\alpha}\right]$

It points out the need
for Complex Time

Representation at issue

Unbalanced Energy through Z_α

(invalidation of Noether requirements)

TEISI Model

$$\eta(\omega\tau) = (i\omega\tau)^{-\alpha}$$

$$[\eta(t)]^d = \gamma_t$$

Reaction = Folding of the interface of exchange ↑

No more proportional

Variation of local curvature (acceleration!) ↑

REPRESENTATION

of the accelarated world (in N) ?

$$\alpha = \frac{1}{d}$$

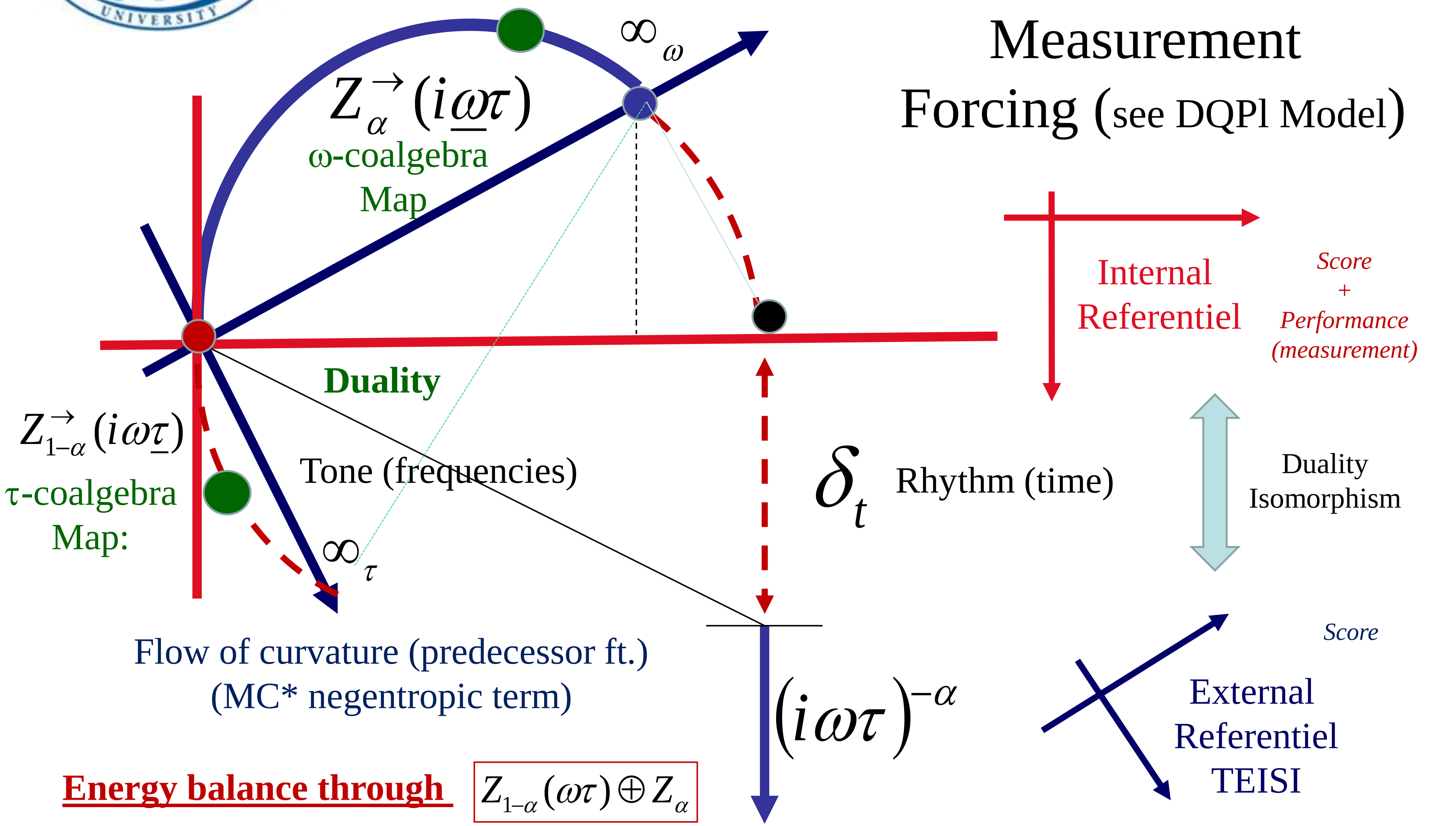
Canonical Case

Well know example: Coriolis force

External vision

Internal Vision

MC* Negentropic term Kan Extension Measurement Forcing (see DQPl Model)

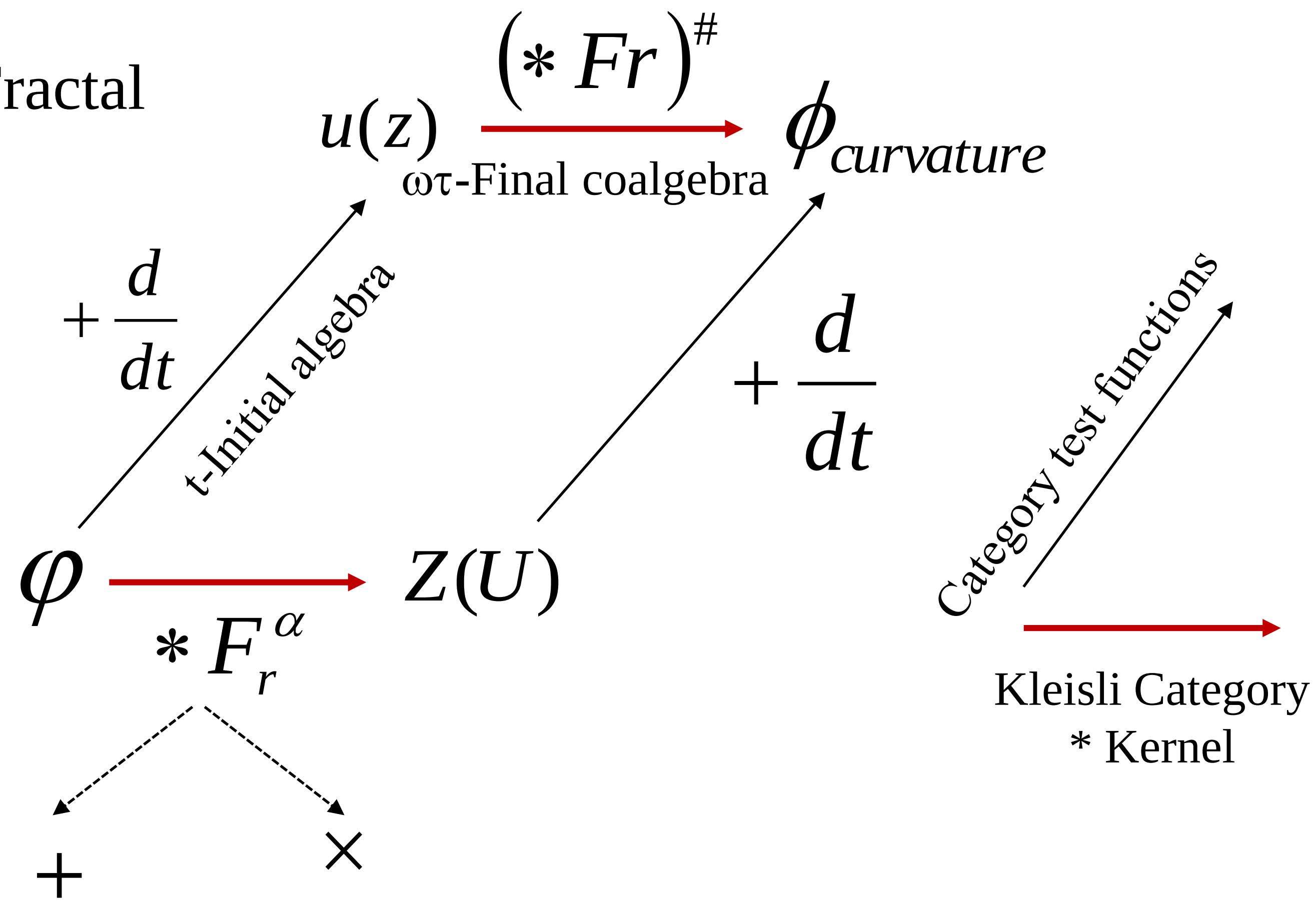


Energy balance through $Z_{1-\alpha}(\omega\tau) \oplus Z_{\alpha}$

(*) MC Mandelbrot-Coriolis

Kleisli Category and TEISI model

F_r^{α} : d-Fractal



Toward Riemann Function

$$n \in N : \zeta(s) = \sum_{n=1,2,\ldots}^{\infty} \frac{1}{n^s} \xrightarrow{F_{\theta},\tau} Z_{\alpha}$$

« L'hypothèse de Riemann est probablement le problème le plus basique
Des mathématiques, au sens où il sagit d'un entrelacement de l'addition
Et de la multiplication. C'est le trou béant dans notre compréhension… »*

Alain Connes

The Riot contribution to this question is the resolution via the lifting in Kleisli category
With emergence of the self similarité by closure (Fractality)

(*) cités dans K. Sabbagh, Dr. Riemann's zeros (Atlanti, 2002) ,p.208
Traduction: Yves André dans « le problème de l'orientation dans la pensée mathématique
Et l'art des conjectures » in « à la lumière des mathématiques et à l'ombre de la philosophie »p. 119

Riemann Function
Riemann Hypothesis

+

$$\zeta(s) = \sum_{n=1,2,\ldots}^{\infty} \frac{1}{n^s}$$

$$\frac{1}{\zeta(s)} = \sum_{n=1,2,\ldots}^{\infty} \frac{\mu(n)}{n^s}$$

Complex Power Law

$$\zeta(s) = \prod_{p_i \in \mathrm{Prime}} \frac{1}{1-p^{-s}}$$

×

Riemann Hypothesis

$$s = \alpha + i\theta$$

$$\zeta(s) = 0$$

$$\alpha = 1/2$$

$$\pi^{-s/2}\Gamma(\frac{s}{2})\zeta(s) = \pi^{-(1-s)/2}\Gamma(\frac{1-s}{2})\zeta(1-s)$$

$$\xi(s) = \pi^{-s./2}\Gamma(s/2)\zeta(s)$$

$$\xi(s) = \xi(1-s)$$

$$\Gamma(s) = \int_0^\infty e^{-t} t^{s-1} dt$$

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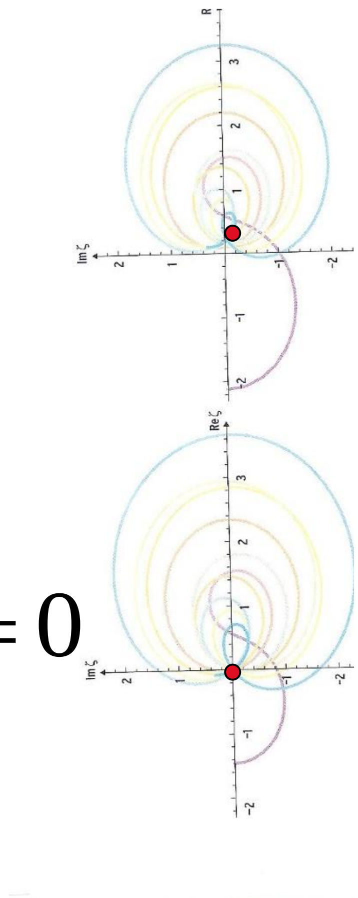
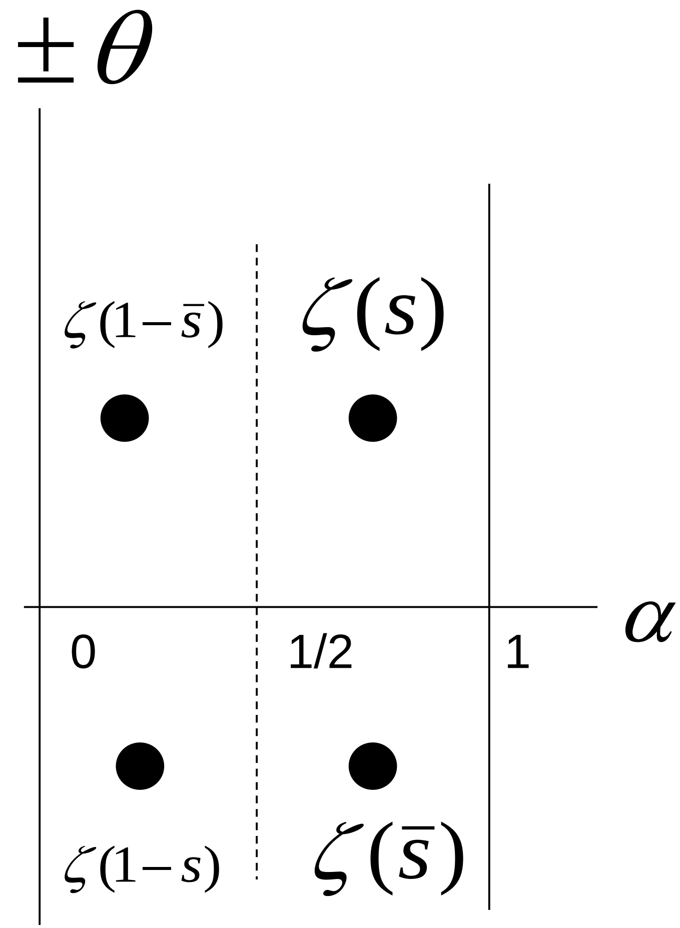


$\zeta(s) = 0$

$\alpha \neq 1/2$

$\alpha = 1/2$



$$\pi^{-(s/2)}\Gamma(s/2)\zeta(s) = \pi^{-((1-s)/2)}\Gamma((1-s)/2)\zeta(1-s)$$

Zeta Function As Discrete Path Integral in Hyperbolic Space 1

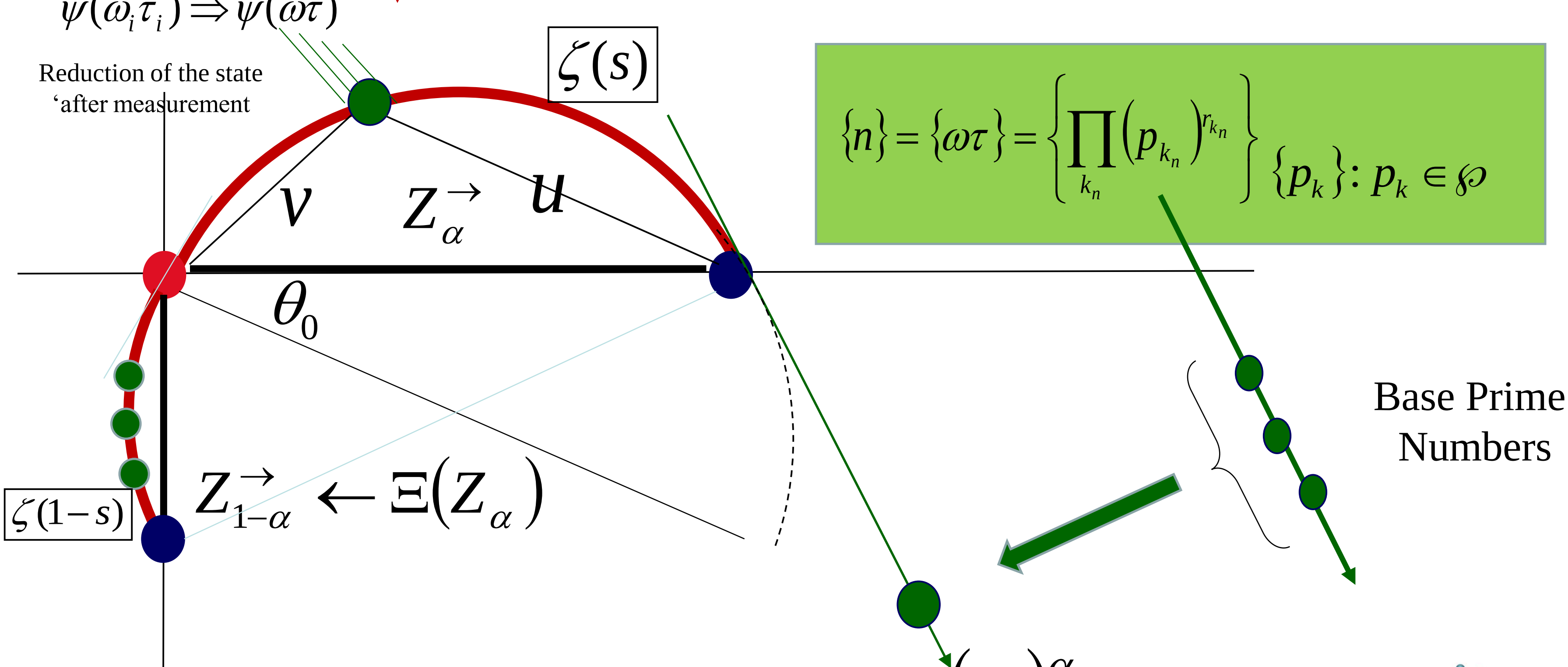
$$\eta_{H,\theta} \sim \frac{1}{n^s}$$

$$\zeta(s) = \zeta(\theta_0, \theta) = \sum_{n=1,2,\ldots}^{\infty} \frac{1}{n^{\alpha(\theta_0)+i\theta}}$$

$$\frac{v}{u} = \frac{1}{(\omega\tau)^\alpha} = \frac{1}{n^\alpha}$$

$n \in N$

Building of the series step by step along the arc $\vec{Z_\alpha}$



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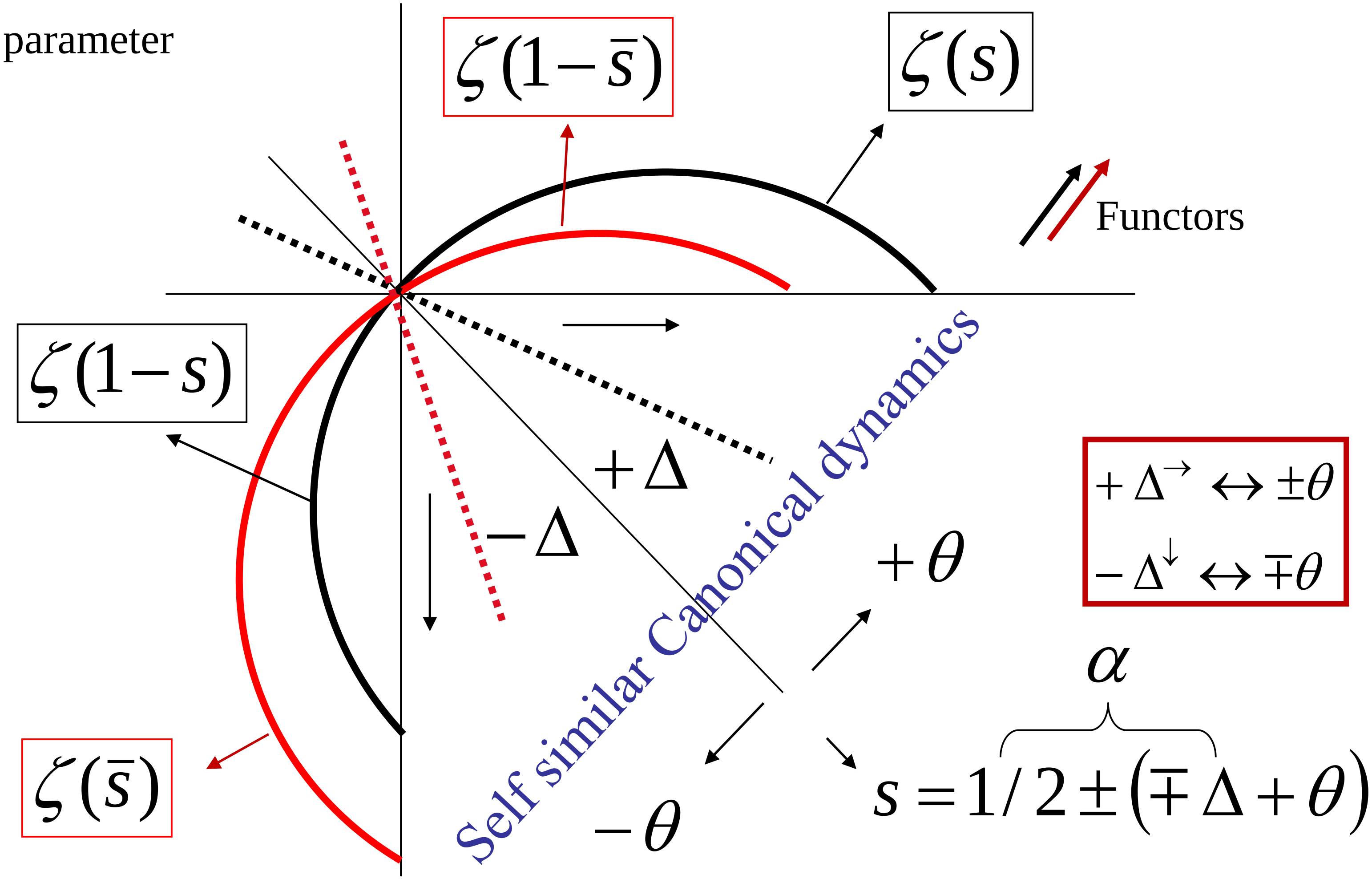
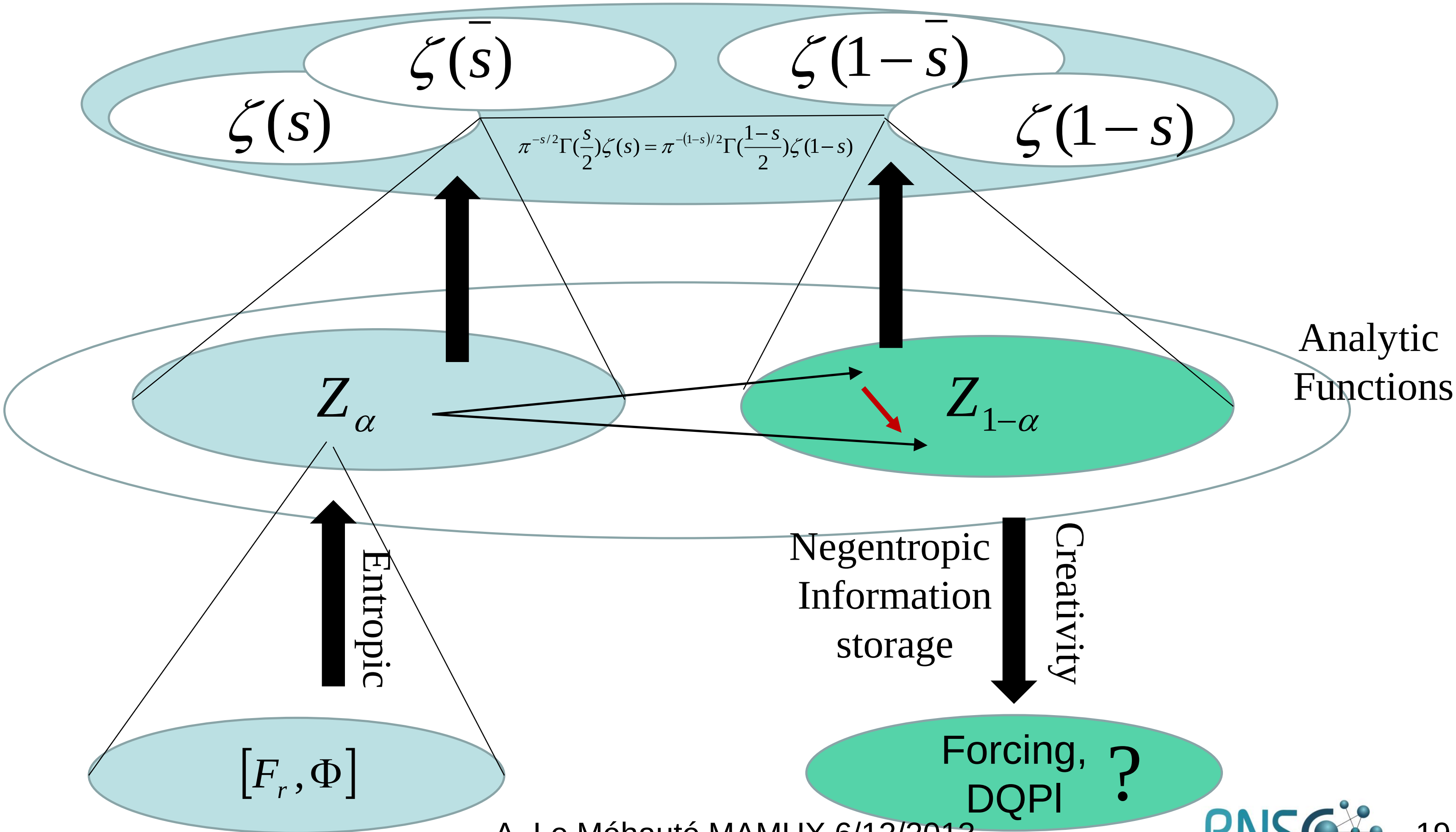
Relationship between Transfert Functions and Zeta Riemann Functions

with equivalence of the sign of rotation

Δ : order parameter



$\{\zeta(s)\}$ as colimit



Riemann Hypothesis… and final coalgebra

$$\{n\} = \{\omega\tau\} = \left\{ \prod_{k_n} \left(p_{k_n}\right)^{r_{k_n}} \right\}$$

…Let us consider the case 'n' reduced solely to one Prime Number

$$\downarrow \qquad \{p_k\} : p_k \in \wp$$

$$\{n_\wp\} = \{p_k\}$$

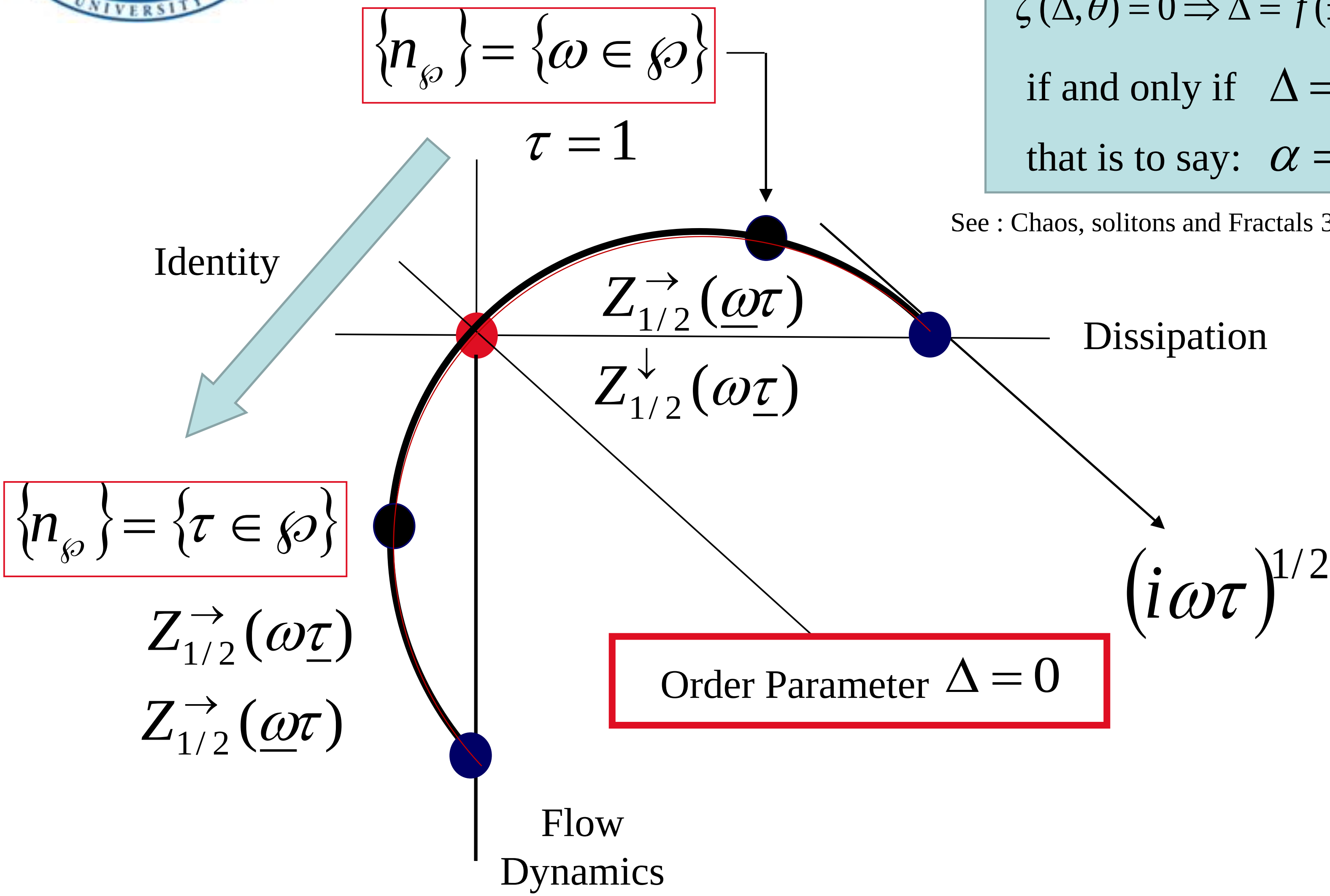
Interpretation of Riemann hypothesis

$\zeta(\Delta,\theta)=0 \Rightarrow \Delta = f(\pm\theta)$

if and only if $\Delta = 0$

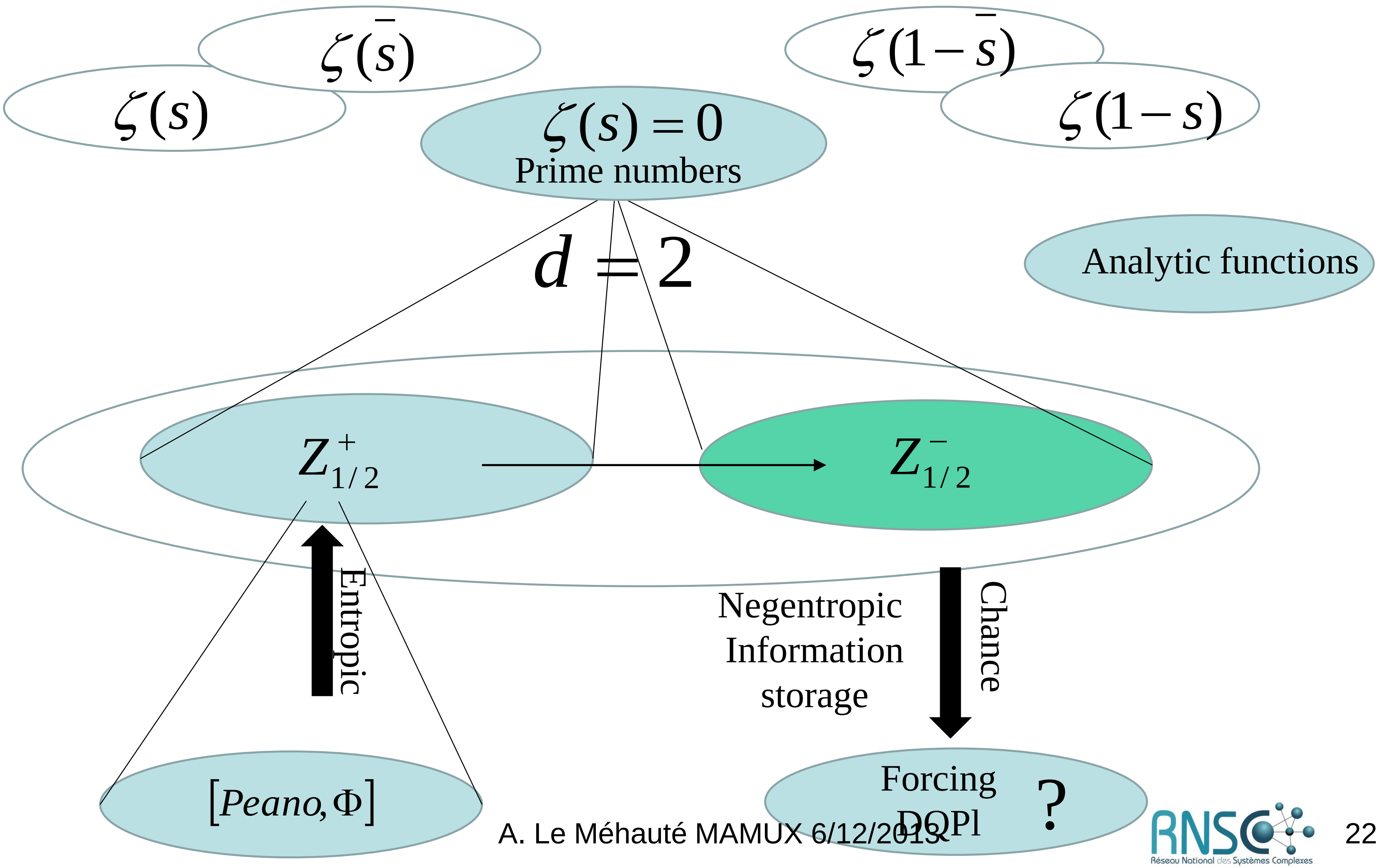
that is to say: $\alpha = 1/2$

See : Chaos, solitons and Fractals 35 (2008) 659-663



$$\zeta(\theta_0 - \frac{\pi}{2}, \theta) = \zeta(\theta'_0, \theta)$$

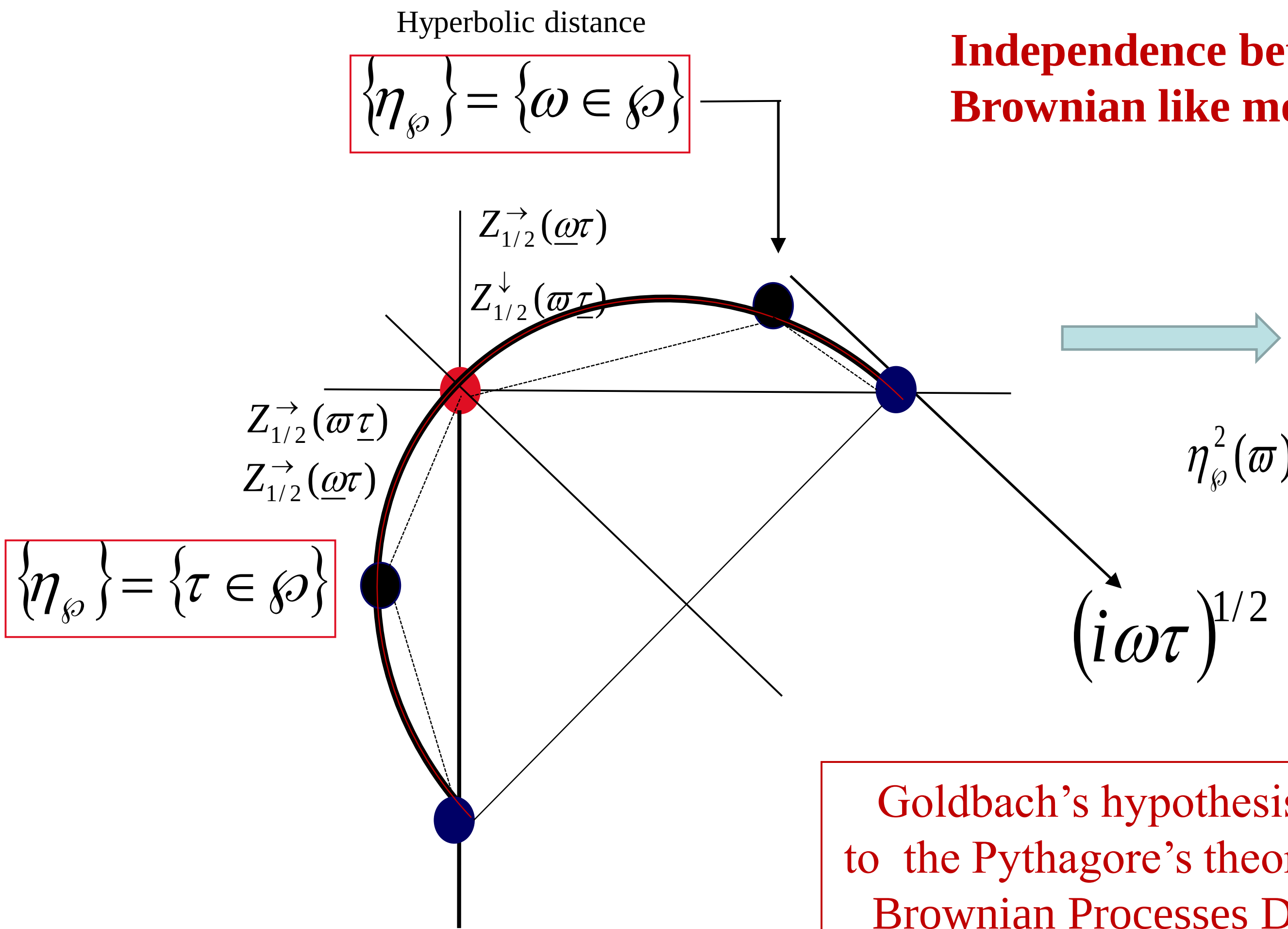
$\zeta(s) = 0$ as 'stochastic' colimit

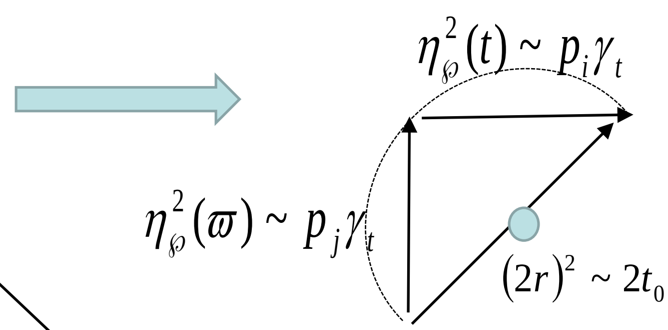


Goldbach hypothesis and diffusive like process

Fractal substructure dimension $d = 2$

Independence between $\omega - \tau$
Brownian like motion: therefore





Goldbach's hypothesis is equivalent
to the Pythagore's theorem in the frame
Brownian Processes Dynamics (d=2)

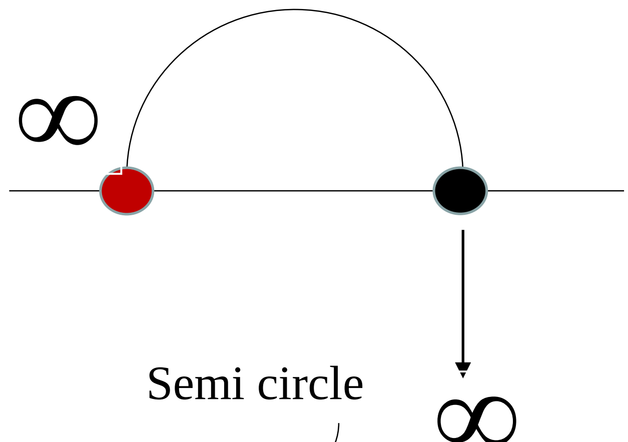
UNIVERSAL 'FRACTIONAL' GEODESIC

…wie die Zeit Vergeht…*

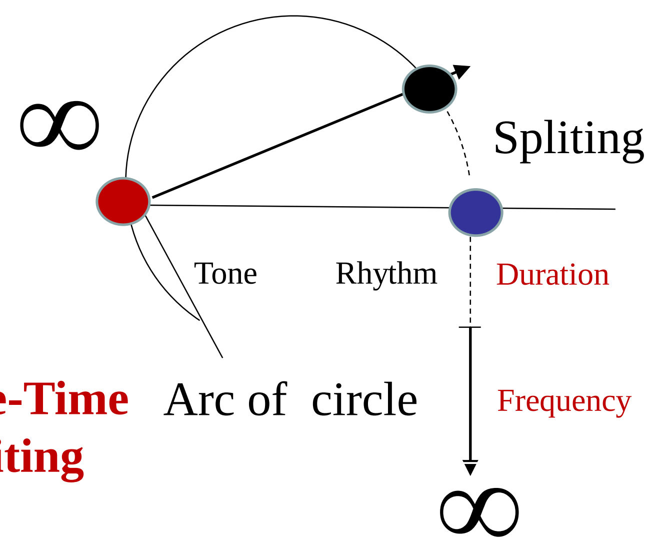
Subject ∪ Zeit Objekt

Euclidean

Euclidean
With compactification



Fractional
Hyperbolic



Straight line

Semi circle

Space ≈ Time

Space-Time spliting

Arc of circle

(*) Karleheinz Stockhausen
Refered by Thierry Paul (EPFL)

…from above analysis

The rhythm might be the measure δt of long range symmetries, related to the MC internal flows of 'curvature' (Negentropic flow due to long range Correlations).

The rhythm arose from a closure of energy as Noether's invariant, to rebuilt within self similar structure, a 'mental' linear causality.

Benoit Mandelbrot
in memoriam

This measure is shown to be related to the properties of Riemann Zeta function through a functorial relation with a category defined from canonical non integer fractional differential transfer function.

The set of non trivial Zeros of Riemann Zeta function are related to a degeneration of this alpha transfer function to stochastic one (geometric phase transition $\alpha = 1/d \Rightarrow \alpha = 1/2$).

The relationship between Riemann and Goldbach hypotheses, *that is between trivial and non trivial zeros of zeta function*, lighten from 1/2 transfer functions opens probably two main opportu

1. Opportunities in cryptography (introduction of fractional transfer function constraint)
2. Opportunity in quantum gravity analysis (Initial Algebra ~ Final Coalgebra via the flow of curvature for $d_{x,t}=2$ and $d_{t,x}=1/2$)

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Acknowledgements

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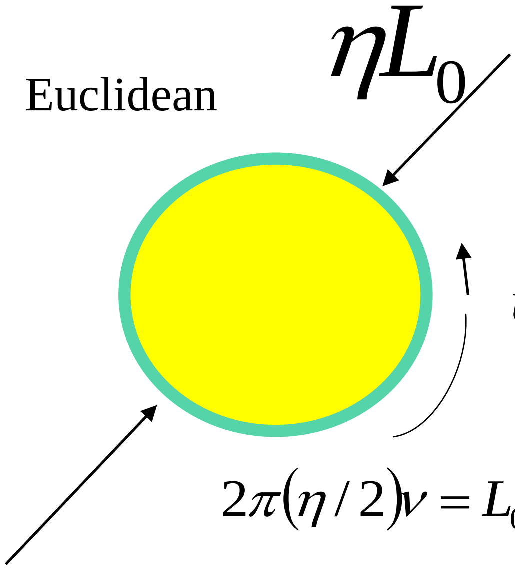
TECHNICAL ADDENDUM

Understanding fractional dynamics

Non Linear coupling between space and time scales

$$\varpi_0 = \frac{L_0}{\tau}$$

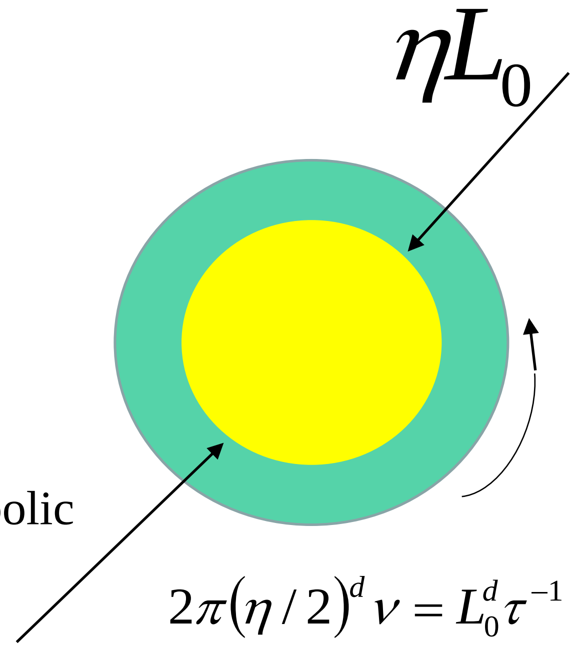
Euclidean



$$U = \frac{1}{2} m \varpi_0^2$$

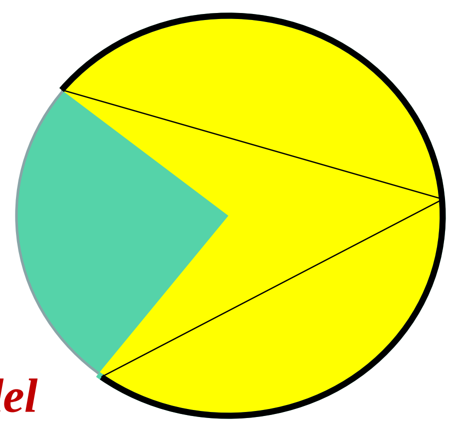
$$2\pi(\eta/2)\nu = L_0/\tau = \varpi_0$$

Hyperbolic



$$2\pi(\eta/2)^d \nu = L_0^d \tau^{-1} \neq velocity$$

$$(2\pi)^\alpha (\eta/2)\nu^\alpha = L_0 \tau^{-\alpha}$$

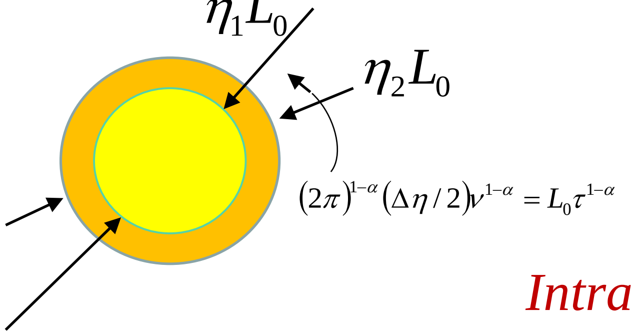


$$\equiv$$

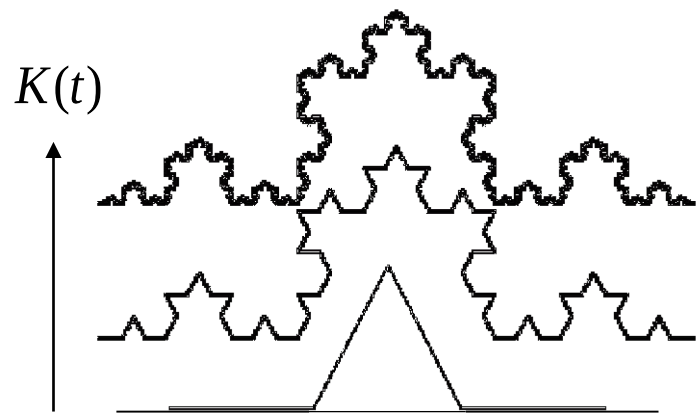
$$d > 1$$

$$\alpha = 1/d$$

Basis of DQPl Model



Completude via the Flow of Curvature $K(t)$

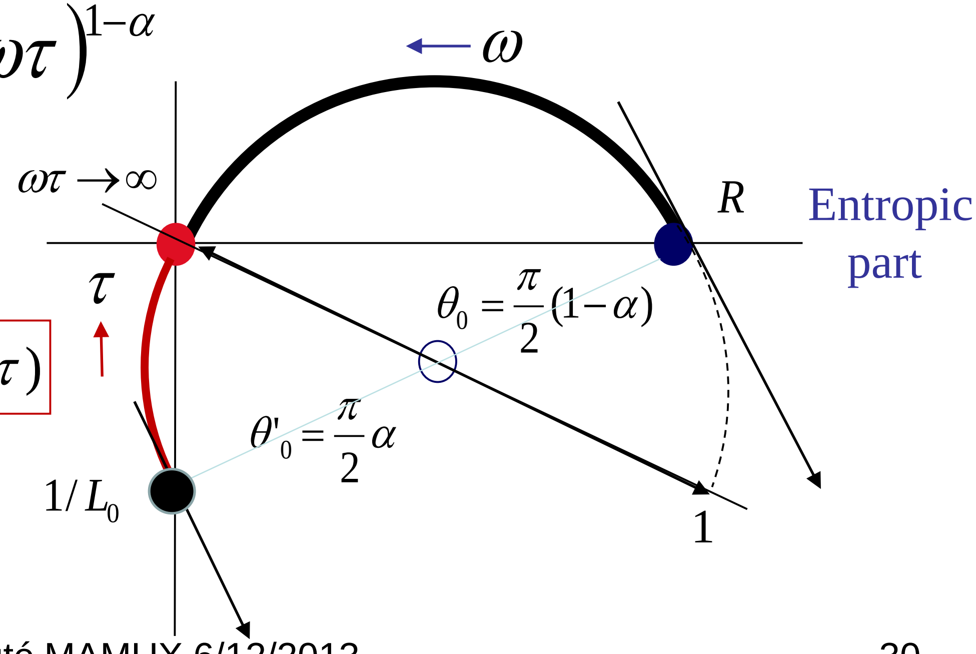


Intra levels correlations

$$\frac{\partial}{\partial t}(\eta) \rightarrow i\omega\tau.\eta(i\omega\tau) \sim L(i\omega\tau)$$

$$\rho = \frac{1}{K(i\omega\tau)} \cong \frac{1}{L(i\omega\tau)} \sim \frac{1}{(i\omega\tau)^{1-\alpha}}$$

Compactification
Rectification → $Z^{\downarrow}_{1-\alpha}(\omega\tau)$



$$[U(\omega)] \equiv [M\,\underline{L^2 t^{-1}}] \equiv [h]$$
$$\equiv [MLt^{-\alpha}][Lt^{-(1-\alpha)}]$$