Time. Rhythm. Arithmetic

Connection between the following couple of talks based on $\varsigma(s)$

Categorical meaning of the Riemann's function $\zeta(s)$: « *from arithmetics to Rhythmics* »

Philippe RIOT

Riemann Conjecture. Fractal Dynamics. Complex Time Meaning of the Categorial Rhythm

Alain Le MEHAUTE

MAREX Network

Categorical meaning of the Riemann's function ς (s) :
« de l'arithmétique à la rythmique »

« Le nombre se révèle à l'art par le rythme, qui est le battement du cœur de l'infini »
Victor Hugo

Philippe RIOT

Rhythm

ρυθμοσ: any regular recurring motion, symmetry

« a movement marked by the regulated succession of strong and weak elements, or of opposite or different conditions » - Oxford English Dictionary 1971

Rhythm = movement or procedure with uniform or patterned recurrence of a beat, accent, or the like, a patterned repetition of a motif, formal element at regular or irregular intervals in the same or a modified form

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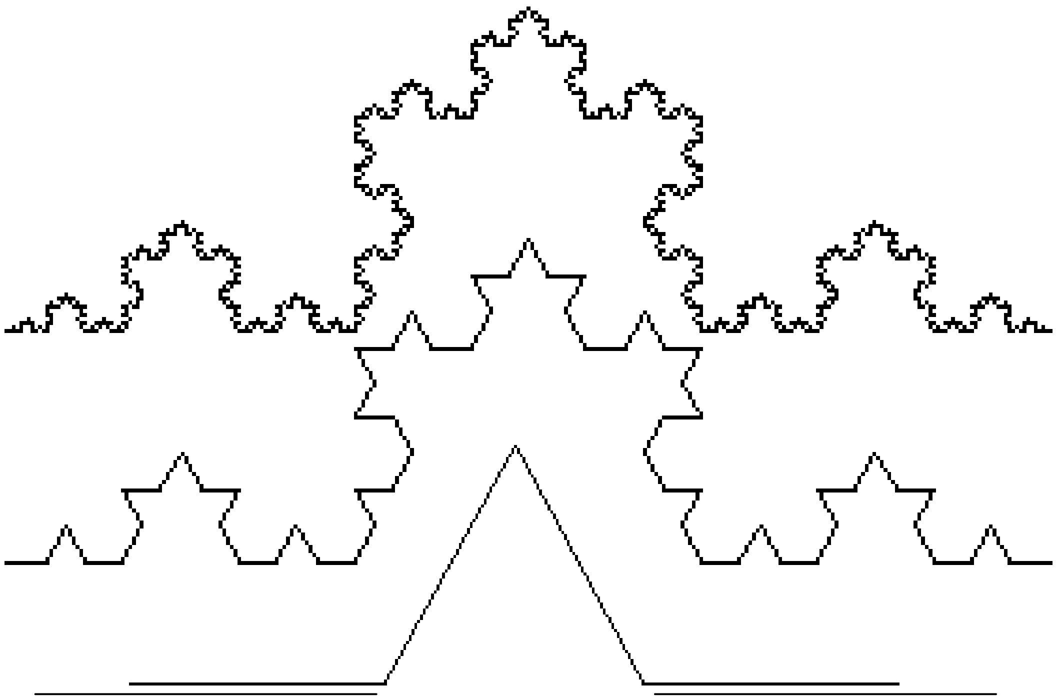
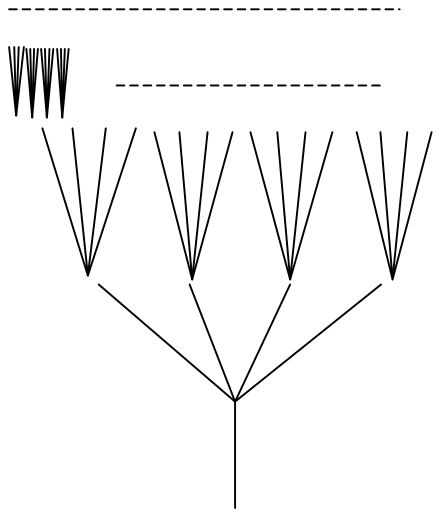
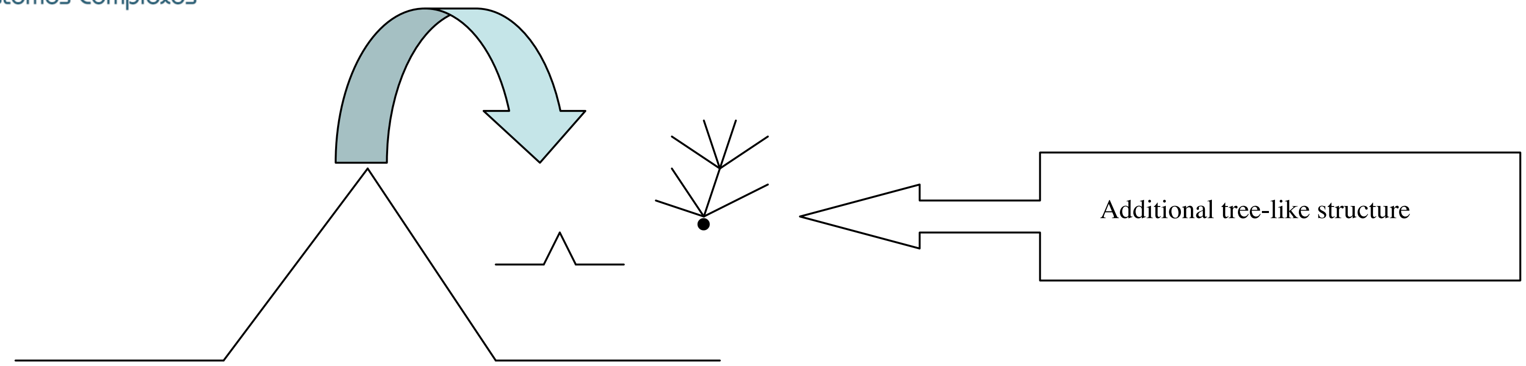
About Self Similar Structures and trees 1





P. Riot : MAMUX - 6 décembre 2013

About self-similar structures and trees 2



Two operations carried out in parallel:
-Composition/analytic continuation
-Decomposition/duplication

-In between a tree-like structure is step by step set up as a driver
-Process iterated up to infinity (compactification)

=> mono/epi-morphisms exchangeable as well as input/output (stabilization)

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Natural numbers set as categoral object (Lawvere)

$$\begin{tikzcd}
 & N \arrow[r, "\sigma"] \arrow[d, "x"] & N \arrow[d, "x"] \\
1 \arrow[ur, "0"] \arrow[r] & X \arrow[r, "\xi"] & X
\end{tikzcd}$$

Axiom Dedekind-Peano:

there exist $(N,0,\sigma)$ such that for any diagram (X,ξ),

there exists a unique sequence x for which both

$x\,0=x_o$; $\quad x\,\sigma=\xi\,x$

Recursion: **The successor map σ** is injective but not surjective; therefore N is Dedekind infinite.

$$\begin{tikzcd}
1 \arrow[dr, "i_1"] \arrow[drr, "0"] & & \\
 & 1+N \arrow[r, "f"] & N \\
N \arrow[ur, "i_N"] \arrow[urr, "\sigma"'] & &
\end{tikzcd}$$

Universal mapping property of

coproduct:unique f

$$\begin{tikzcd}
 & & N \\
N \arrow[r, "x"] \arrow[urr, "u"] \arrow[drr, "g"'] & N \times A \arrow[ur, "p_A"] \arrow[dr, "p_N"'] & \\
 & & A
\end{tikzcd}$$

Inverse g for f by recursion through

product scheme

Corollary: there is a unique map N to N called the **predecessor** for which

$p(0)=0;\ p\sigma=1_N$

In other words $p(n+1)=n$; hence σ is injective since it has a **retraction**.

Therefore $1+N\cong N$

Arithmetic of N: Addition and Multiplication

Given $\alpha:A\to A$ there is a sequence $\alpha^{()}:N\to A^A$ such that $\alpha^0=1_A;\ \alpha^{n+1}=\alpha\alpha^n$

For any A map $iter_A:A^A\to(A^A)^N$ that assigns to any α the name of the sequence of iterates of α

Example $A=N$ $iter_N:N^N\to(N^N)^N\cong N^{NxN}$ which assigns to each $\alpha:N\to N$ a binary operation $\alpha°$ on N:

$$\alpha°(0,m)=m \text{ for all } m$$

$$\alpha°(n+1,m)=\alpha(\alpha°(n,m)) \text{ for all } n,m$$

If we take $\alpha=\sigma$ *then* $\sigma°(0,m)=m;\sigma°(n+1,m)=\sigma°(n,m)+1$

This proves the existence of the operation of addition

Given any set A equipped with an addition assumed associative $+:A\times A\to A$ and a « *zero* »: $1\to A$

We can apply the iterator to the transpose of + and follow the result by evaluation at 0:

$$A\to A^A\to(A^A)^N\to A^N$$

Obtaining a map whose transpose is the map « . « : $N\times A\to A$ which is usually called multiplication

In other words the multiplication is defined as iteration of addition

Recall: Functor

See : René Guitart and Andrée Erhesmann…

Categories

$$C \xrightarrow{F} C'$$

Functor Between Categories

$$f \in \hom_C(M', M)$$

$$M \xrightarrow{f} M' \qquad F(M) \xrightarrow{F(f)} F(M')$$

Requirement

$$F(g \circ f) = F(f) \circ F(g)$$

$$\begin{tikzcd} \arrow[d, "f"'] \arrow[r, "F(g)"] & \arrow[d, "F(f)"] \\ \arrow[r, "g"'] & \end{tikzcd}$$

Algebraically compact category (P.Freyd)
Successor and Predecessor

Functor: $TX=1+X$

T-algebra structure on object A: « point » $1 \rightarrow A$ and ***endomorphism*** $A \rightarrow A$ (or map $1+A \rightarrow A$)

*Lawvere definition of natural numbers **object as <u>initial</u> T-algebra** for this functor*

Dual property: **T-coalgebra** is a map of the form a: $A \rightarrow TA$

Lambek lemma: if $f: F \rightarrow TF$ is ***a <u>final</u> T-coalgebra***, then f is an ***isomorphism***

if $f: T\,F \rightarrow F$ is ***a <u>initial</u> T-algebra***, then f is an ***isomorphism***

In the category of sets co-algebra structure for $TX=1+X$ viewed as partial endomorphism

the final coalgebra is a universal partial endomorphism:

Natural numbers with a point at infinity adjoined

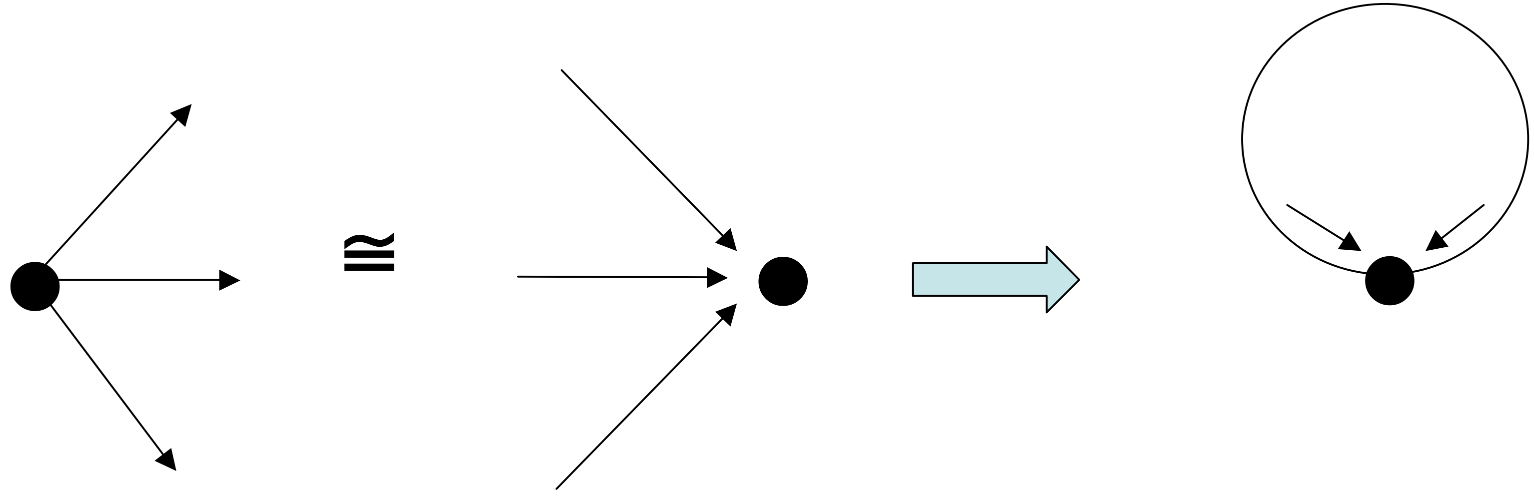
*The universal partial endomorphism is the **predecessor function**,*

*undefined at zero and with **infinity as a fixed point***

Input/Output equivalence (Principle of versality: every universal mapping definition is equivalent to a dual definition)

A category is said to be <u>algebraically compact</u> if every covariant endofunctor has an initial algebra and a final coalgebra and they are canonically isomorphic

Symbolic representation

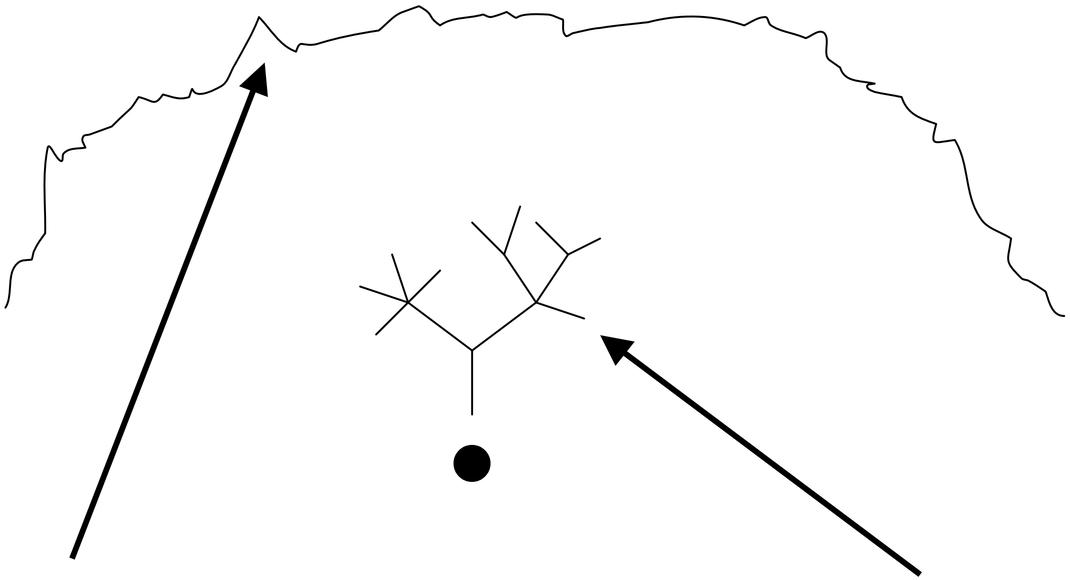


Initial object
Initial T-Algebra
Successor(+)

Final object
Final T-coalgebra
Predecessor(×)

Self-similar stabilized object

Dialectic of chance and necessity



Self-similar final object = necessity

Non deterministic path = chance

Relevance in computer science

$$\begin{array}{ccc} \text{objects} & & \text{acts} \\ \updownarrow & & \updownarrow \\ \text{Data types} & & \text{processes} \\ \updownarrow & & \updownarrow \\ \text{algebra} & & \text{coalgebra} \end{array}$$

$$\text{program} \underset{\text{execution}}{\overset{\text{programming}}{\rightleftarrows}} \text{file}$$

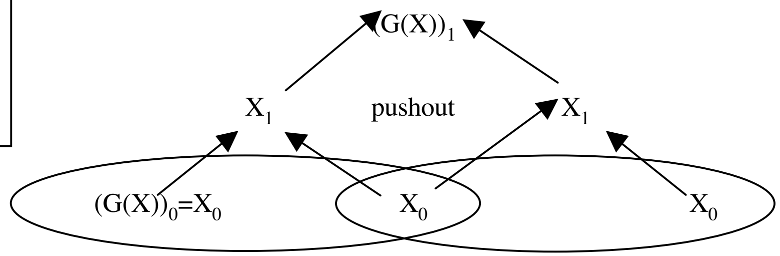
Relevant computer science frame =fixed-point categorical object = self-similar structure

Characterization of the real interval [0,1] - P.Freyd

[0,1] interval is isomorphic to two copies of itself joined end to end; that it is universal as such

$$\xi \in G(X), x_0 \in X_1 : \xi(x_0) \in \big(G(X)\big)_1$$

Given $X \in C$ New object $G(X)$ of C By gluing two copies of X end to end



$$(X,\xi) \rightarrow (I,i)$$

[01]
[0,0.5][0.5,1]
[0,0.25][0.25,0.5][0.5,0.75][0.75,1]
…

Let a G - coalgebra (X,ξ) and an element $x_0 \in X_1$, $\xi(x_0) \in (G(X))_1$ either the left-hand or the right-hand copy of X_1, so **gives rise to a binary digit $m_1 \in [0,1]$** **and** **a new element $x_1 \in X_1$**

Iterating gives a binary representation $0.m_1m_2\ldots$ of an element of [0,1]
This is the image of x_0 under the unique coalgebra map $(X,\xi) \rightarrow (I,i)$

Canonical example of coalgebra for an endofunctor

Coalgebras for an endofunctor describe systems of formal <u>recursive equations</u>.
The semantics is given by the final coalgebra

Fixed set A: set A^ω <u>of infinite</u> sequences of elements of A (streams parametrized with ω)

<u>« zip up » two streams</u>: $zip[(a_0,a_1,\ldots),(b_0,b_1,\ldots)]=(a_0,b_0,a_1,b_1,\ldots)$

<u>System of recursive equations</u>: zip(a,b)=(head(a),zip(b,tail(a))

With $head(a_0,a_1,\ldots)=a_0, tail(a_0,a_1,\ldots)=(a_1,a_2,\ldots)$

Equation encoded by a map $e:A^\omega x A^\omega \to A x A^\omega x A^\omega$

$(a,b) \to (head(a),b,tail(a))$

Endofunctor	$\Phi : X \to A \times X$
Coalgebra for Φ	$e : X \to \Phi X = A \times X$
Final coalgebra	$\tau : TA \to A \times TA$ if any
For any coalgebra	$c : Z \to A \times Z$

$$\begin{tikzcd} Z \arrow[r, "c"] \arrow[d, "c\#"'] & A \times Z \arrow[d, "A \times c^{\#}"] \\ TA \arrow[r, "\tau", "\cong"'] & A \times TA \end{tikzcd}$$

$$A^\omega \times A^\omega \to A \times A^\omega \times A^\omega$$

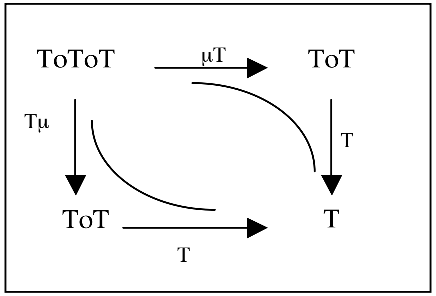
$TA \cong A^\omega$ $\tau : a \in A^\omega \to (head(a),tail(a))$

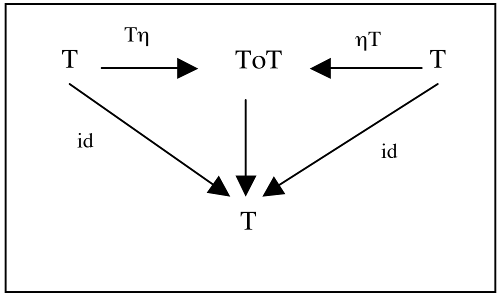
For c=e $e\# : A^\omega x A^\omega \to A^\omega$ satisfies zip equation ($e^\# \equiv zip$)

Monad

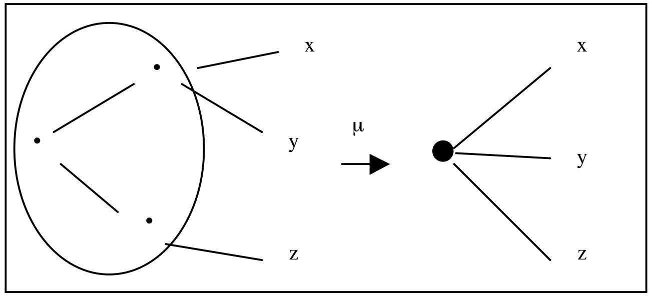
Underlying idea: **monoidal structure (iteration) attached to an endofunctor**

(T,μ,η) T endofunctor, μ acting as multiplication, η as unity





Example 1: Graph (μ: patching of paths)



Example 2: $\wp$ powerset

$\eta : x \to \{x\}$

$\mu = \cup : \wp\wp X \to \wp X$

$\{\{x,y\},z\} \to \{x,y,z\}$

Kleisli category

Kleisli category attached to a functor T in category C:
- objects = objects in C
- Arrow $A \to B$ in Kl(T) = arrow $A \to TB$ in C

-Composition of $f: A \to B$ and $g: B \to C$ in Kl(T) is transposed in C as:

$$\begin{tikzcd}
 & & TTC \arrow[d, "\mu_C"] \\
 & TB \arrow[ur, "Tg"] & TC \\
A \arrow[ur, "f"] & B \arrow[ur, "g"] &
\end{tikzcd}$$

Adjunction: $\mathrm{Set} \underset{\longleftarrow}{\overset{\longrightarrow}{\perp}} \mathrm{Kl}(T)$ $J:(f:X \to Y)$ in Set$\to$ $(\eta_Y \circ f : X \to Y)$ in Kl(T)

Relevant application: $\mathrm{Kl}(\wp) \cong \mathrm{Rel}$ category of sets and binary relations

Lifting:

$$\begin{tikzcd}
\mathrm{Kl}(\wp) \arrow[r, "T^{\#}"] & \mathrm{Kl}(\wp) \\
\mathrm{Set} \arrow[u] \arrow[r, "T"] & \mathrm{Set} \arrow[u, "J"']
\end{tikzcd}$$

Kleisli category (Plotkin/Smyth)

Self duality Op: $Kl(\wp) \rightarrow Kl(\wp)$ f: $X \rightarrow \wp Y$ in Set get $f^{\#}: Y \rightarrow \wp X$ in Set with $f^{\#}(y) = \{x \in X / y \in f(x)\}$

$$
\begin{tikzcd}
\mathrm{Set} \arrow[r, shift left] \arrow[r, phantom, "\perp"] & Kl(\wp) \arrow[l, shift left] \arrow[r, "\cong"] & Kl(\wp)^{Op} \\
F & F^{\#} & F^{\#Op}
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathrm{Alg}(F) \arrow[r, shift left] \arrow[r, phantom, "\perp"] \arrow[d] & \mathrm{Alg}(F^{\#}) \arrow[l, shift left] \arrow[r] \arrow[d] & \mathrm{Alg}(F^{\#Op}) \arrow[r, "\cong"] \arrow[d] & \mathrm{Coalg}(F^{\#})^{Op} \\
\mathrm{Set} \arrow[r, shift left] \arrow[r, phantom, "\perp"] & Kl(\wp) \arrow[l, shift left] \arrow[r] & Kl(\wp)^{Op} &
\end{tikzcd}
$$

The initial object in Alg(F) is carried to that of in $(\mathrm{Coalg}(F^{\#}))^{Op}$, hence the final object in $\mathrm{Coalg}(F^{\#})$

In a kleisli category initial/final coalgebra coincidence or, equivalently, limit/colimit coincidence

Exchange of relevant mono/epimorphisms

Interpretation as state-based branching system

The monad T specifies the branching type of the system
The powerset monad $\wp$ is modelling a non-deterministic or possibilistic branching

The functor F specifies the transition type of the system
Model of a system by a coalgebra $X \rightarrow F^{\#}$ in the Kleisli category Kl (T) where $F^{\#}$ is the suitable lifting of F in it

$T = \wp$

Ultrafilter $\Rightarrow$ Stone-Cech compactification

Decomposition into primes + scale factor

Non-deterministic/possibilistic branching

Riemann's Function $\varsigma(s)$

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$$ ($\Re e(s)>1$) additive law $\Rightarrow$ algebraic structure

$$\zeta(s) \prod_{p\in\{prime\}} \left(1 - p^{-s}\right)$$ ($\Re e(s)>1$) multiplicative law $\Rightarrow$ coalgebraic structure

Initial algebraic/final coalgebraic coincidence would be relevant

Appropriate frame: algebraically compact and complete category as a Kleisli category

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} \qquad \sum_{n=1}^{\infty} n^{-X} \qquad -X \longrightarrow [n] \qquad ?$$

$$f(s) = \sum_{n=1}^{\infty} a_n s^n \qquad \sum_{n=1}^{\infty} a_n X^n \qquad [n] \longmapsto X$$

Analytical series | Formal series | Exponentiation

Question: is it possible to give a meaning for these analogies?

Analytic functor (Joyal)

Cocone

$$\begin{tikzcd} & A_1 \arrow[d, "f_1"'] \\ A_2 \arrow[r, "f_2"] & A \end{tikzcd}$$

Pullback

$$\begin{tikzcd} P \arrow[r] \arrow[d] & A_1 \arrow[d] \\ A_2 \arrow[r] & A \end{tikzcd}$$

Generalization with
Many f_i: $A_i \to A$
Wide pullback

When f_i mono P finite intersection

Trnkova: every set functor preserves non void finite intersections

Accessible set functor if it preserves all filtred colimits:

$FX = \cup_{U \subseteq X} \cup_{|U|<\lambda} Fi[FU]$ i:$U \subseteq X$ λ infinite cardinal

An accessible set functor weakly preserves wide pullbacks if and only if it is analytic

An analytic functor is defined as an Kan extension:

$$F(A) = \int^n A^n \times F[n]$$

$$\begin{tikzcd} \mathrm{Set} & \\ \text{Finite Set} \arrow[u, "F"] \arrow[r, "\subseteq"'] \arrow[ur, phantom, "\Rightarrow"] & \mathrm{Set} \arrow[ul, "F"'] \end{tikzcd}$$

The F[n] as Taylor's coefficients
of a formal series

Virtual species (Joyal)

Virtua species = formal difference V = F - G between finite species (or sets)

Equality between species F - G = M - N means F + N = M + G

Transitivity obtained through simplification of finite species:

$$F + M = G + M \Rightarrow F = G$$

Simplification based on involution principle (Garsia – Milne):

A, B, and C finite sets with bijection φ: $A + C \cong B + C$, then residual bijection can be build

ψ: $A \cong B$ depending only on φ

X finite: finite species **-X is defined as couple of finite sets (H,K)** within a bijection $K \cong X + H$

Accordingly an epimorphism always exists $K \rightarrow X$

Integer sets in Kleisli category (Joyal - Street - Verity)

In the category Rel (objects = sets, arrows = relations), monoidal structure with addition X + Y (disjoint union) as tenbsor product

Arrows in Rel between multiple tensor products written as matrices:

Relation R: X + V → Y + U written as a matrix:

$$R = \begin{vmatrix} A & B \\ C & D \end{vmatrix} : X + V \to Y + U$$

New subcategory IntRel of integer sets. The cardinality of the integer set (X,U) defined by the difference #(X,U) = #X - #U of the cardinalities of X,U

Explicit description of IntRel: objects = pairs (X,U) and arrows R: (X,U) → (Y,V) depicted as:

$$\begin{tikzcd} X \arrow[r, "C"] \arrow[d, "A"'] & U \\ Y & V \arrow[u, "D"'] \arrow[l, "B"] \end{tikzcd}$$

Composition in IntRel:

$$\begin{tikzcd} X \arrow[r, "C"] \arrow[d, "A"'] & U \\ Y \arrow[r, shift right, "G"'] \arrow[d, "E"'] & V \arrow[u, "D"'] \arrow[l, shift right, "B"'] \\ Z & W \arrow[u, "H"'] \arrow[l, "F"] \end{tikzcd} = \begin{tikzcd} X \arrow[r, "C \cup D(GB)^*GA"] \arrow[d, "E(BG)^*A"'] & U \\ Z & W \arrow[u, "D(GB)^*H"'] \arrow[l, "F \cup E(BG)^*BH"] \end{tikzcd}$$

A new natural transformation

In IntRel arrow - X = (K,H) $\rightarrow$ (n,0) defined

$$\Xi(X) = \int^{n \in \text{Finite Set}} \mathrm{Hom}(-X,n) \times n$$

$$Y \in \mathrm{rel} \quad \left[\int^{n} \mathrm{Hom}(-X,n), Y\right] = \int_{n} [\mathrm{Hom}(-X,n),Y] \text{ (covariance)}$$

$$= \int_{n}[\mathrm{Hom}(-X,n),[n,Y]] \text{ (Curry's rule from } \lambda \text{ - calculus)}$$

$$\cong \mathrm{Nat}\,(\mathrm{Hom}(-X,-),[-,Y]) \text{ (coend definition)}$$

$$\cong [-X,Y] \text{ (Yoneda)}$$

In other words $[\Xi(X),Y] \cong [-X,Y]$ then $\Xi(X) \cong -X$

Interpretation: the virtual species -X is completely described by epimorphisms, or equivalently by partitions

Approximation of a set by monomorphisms or by epimorphisms

Approach by epimorphisms

$$\begin{tikzcd} \Xi(X) \cong -X & X \arrow[l, "\cong"'] \arrow[d] \arrow[dl, phantom, "\perp"] \\ & F(X) \end{tikzcd}$$

Approach by monomorphisms

Orthogonality in relation with the factorization of maps

Best approximation of shape (Cordier - Porter/Leinster)

Arbitrary functor $G : B \to A$

Right Kan extension:

$$T^G(a) = \lim_{b\in B,\, f:A\to G(b)} G(b)$$

$$\begin{tikzcd}
B \arrow[r, "G"] \arrow[dr, "G"'] & A \arrow[d, "T^G"] \arrow[dl, phantom, "\Leftarrow"] \\
 & A
\end{tikzcd}$$

First application: **Finite Set → Set codensity monad = ultrafilter monad (Stone-Cech compactification)**

Second application: **Field → Ring** $T^G(Z) = Q\times\Pi_{p \text{ prime}} Z/pZ$

(correspondence between prime ideals and proper ultrafilters)

Spec(T^G(Z)) is the Stone-Cech compactification of the discrete space Spec(Z)

Consequence: the compactification process introduces prime decomposition and a supplementary parameter introduced via Q for the characteristic zero field

Third application: Finite Dimensional Vector Space → Vector Space

$T^G(\mathrm{Vect})$ = double dualization monad $\wp\wp$ on Vect

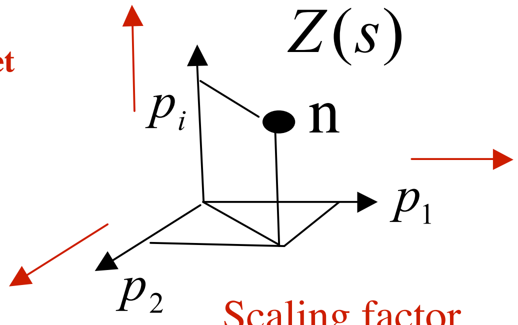
Application of Category Theory to render an account to Riemann and Golbach hypotheses

Riemann

Goldbach à Euler

A formal calculus with function $\varsigma(s)$

Infinite dimensional vector space Γ with a basis in bijection with the prime numbers set
Points with natural numbers as coordinates (all ccordinates null but a finite number)
Bijective correspondence between these points and the natural numbers set N



Scaling factor

$n \rightarrow n^{-s}$

Point with coordinates $(r_1, r_2, \ldots, r_k, \ldots)$, numeral function

$h: \Gamma \rightarrow N \; h(r_1, r_2, \ldots, r_k, \ldots) = n = \Pi_k p_k^{rk} \in N$

By scaling through the factor -*s* the same point takes new coordinates $(-sr_1, -sr_2, \ldots, -sr_k, \ldots)$ and the image by h is now n^{-s} ⇒ definition of a special space distribution called Z(s) which is a scale operator

Total Measure μ of Z(s):

$$\boxed{\mu(Z(s)) = \zeta(s)}$$

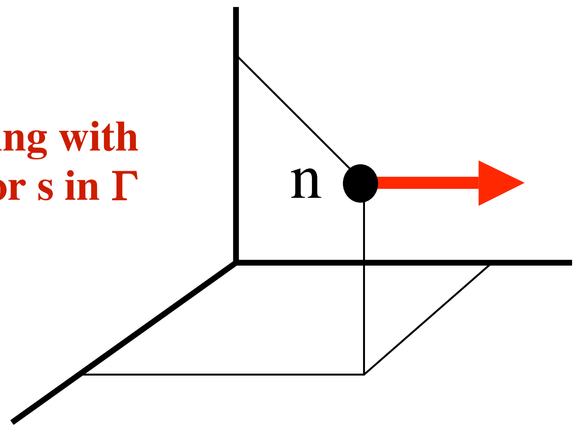
Weighting factors $a_n s^n$ at each point corresponding to n, the total measure equals $f(s) = \Sigma_n a_n s^n$
⇒ Definition of another type of space distribution called F(s) which is a complex function with discrete support

$$\boxed{F(s) : f(s) = \sum_{n=1}^{\infty} a_n s^n}$$

Interpretation of the Riemann's function $\varsigma(s)$

The Riemann function = total measure of a space distribution which is a simple scale operator in the infinite dimensional space Γ belonging to a relevant category of topological spaces E

Scaling with factor s in Γ

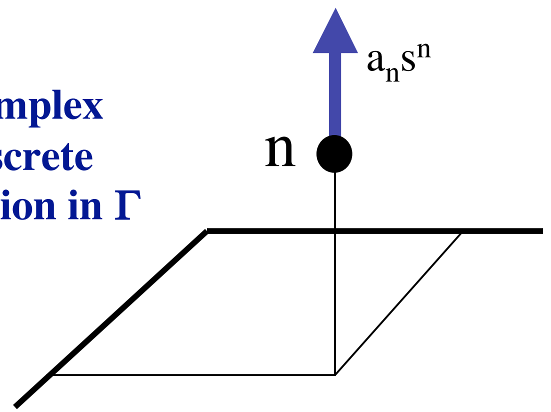


$$\xrightarrow{\int_\Gamma}$$

$$\boxed{\mu\big(Z(s)\big) = \zeta(s) \in 1}$$

(where 1 is a terminal object of E)

Complex discrete function in Γ



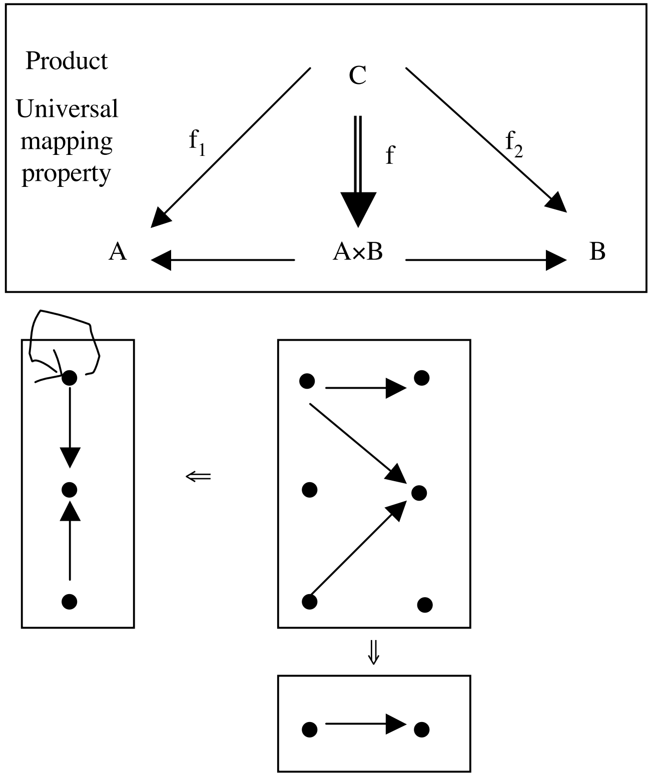
$$\xrightarrow{\int_\Gamma}$$

$$\boxed{F(s) : f(s) = \sum_{n=1}^{\infty} a_n s^n}$$

f(s) = total measure (F(s)) $\in$ 1

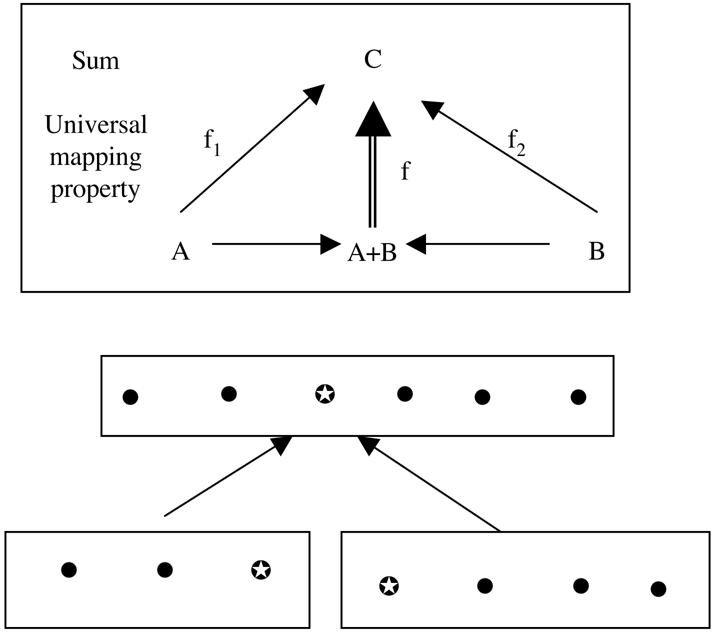
Extensive and intensive quantities (Lawvere)

Extensity



Distributive category: product is distributive on coproduct:
$a\times(b + c) = a\times b + a\times c$

Intensity



Linear category: possible isomorphism between product and coproduct (direct sum)

Extensive/intensive duality (Kock)

Distributive category: continuous space, measurable derivable space
Category of space = distributive category

Linear category: vector space, abelian group, functional space
Category of quantity = linear category

$$\begin{tikzcd}
\text{E} & \text{Q} \\
\text{Distributive category} \arrow[r, shift left, "\text{Quantity of space}"] \arrow[d] & \text{Linear category} \arrow[l, shift left, "\text{Space of quantity}"] \arrow[d] \\
\text{Space distribution} & \text{measure}
\end{tikzcd}$$

Double dualization

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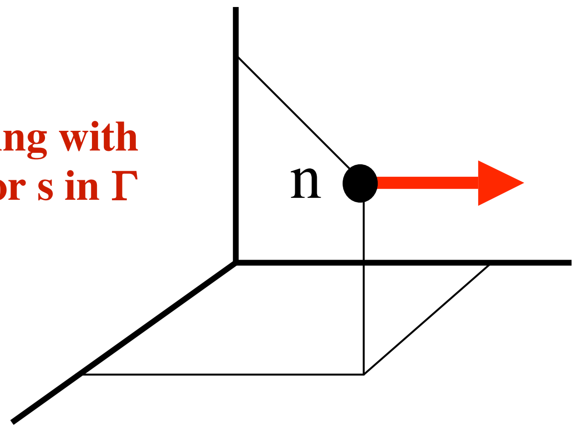
Scaling with factor s in Γ

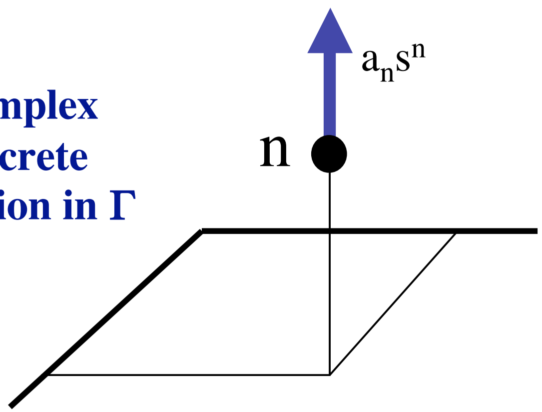


$$\xrightarrow{\int_\Gamma}$$

$$\boxed{\mu\big(Z(s)\big) = \zeta(s) \in 1}$$

(where 1 is a terminal object of E)

Complex discrete function in Γ



$$\xrightarrow{\int_\Gamma}$$

$$\boxed{F(s) : f(s) = \sum_{n=1}^{\infty} a_n s^n}$$

f(s) = total measure (F(s)) ∈ 1

Voronin's universality theorem

Let K a compact subset of the critical strip $1/2 < \sigma < 1$ with connected complement, and let f(s) a non-vanishing continuous function on K which is analytic in the interior of K. Then for any $\varepsilon > 0$
$\operatorname{Lim\,inf}_{T\to\infty} 1/T \text{ meas } \{\tau\in[0,T];\ \max |\varsigma(s+i\tau)-f(s)| < \varepsilon\} > 0$

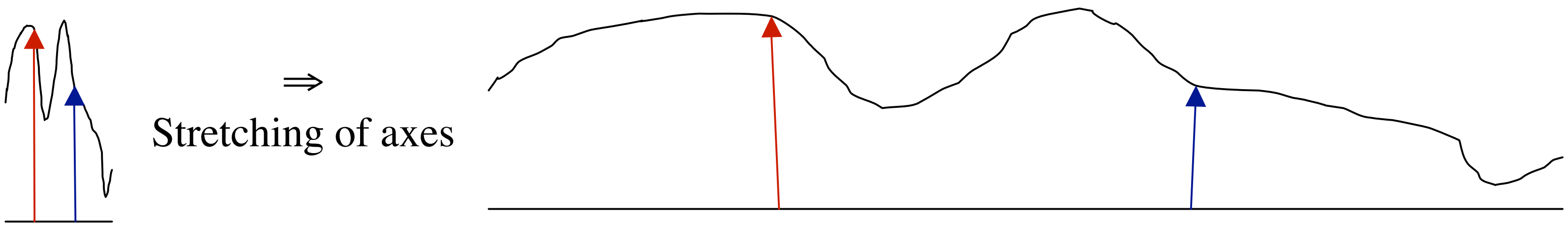
Considering in the space Γ the above defined distributions Z and F whose respective total measures are $\varsigma(s)$ and $f(s)$.

When the scale of the space is changed through the Z operator, the F distribution is converging to Z (intuitively graph flattening by stretching of axes)

By double dualizing the effect on the relevant total measures is exaxtly expressed by the Voronin's theorem

This process is similar to the scaling process defined by M.Gromov for building the tangent bundle of a manifold:

$$TM = \lim_{\lambda\to\infty} \lambda M \text{ with } \lambda \text{ scale factor}$$



The Riemann hypothesis is true if and only if for every compact subset on the critical strip $1/2 < \sigma < 1$ with connected complement and for any $\varepsilon>0$, $\liminf_{T\to\infty}$ meas $\{\tau\in[0,T];\ \mathrm{Max}\,|\varsigma(s+i\tau) - \varsigma(s)|<\varepsilon\}>0$

(demonstration, among many others, based on the classical Rouché's theorem)

The distribution Z as scale operator is self-similar; accordingly its total measure clearly verifies the Bagchi's criterium.

Corollary:The Riemann's hypothesis is true

Categorical field — Analytic field

Algebraic/coalgebraic coincidence $\longleftrightarrow$ Self-similarity $\longleftrightarrow$ Distribution Of non-trivial zeros

Extensive quantity — Intensive quantity

Goldbach's conjecture is true

$2n = p + q$; p,q primes

« + » $\Rightarrow$ algebraic structure; primes $\Rightarrow$ coalgebraic structure

Accordingly the relevant frame for solving this conjecture is based on algebraic/coalgebraic coincidence

$2n = p + q$ with $p = n - r$ and $q = n + r$, thus $pq = n^2 - r^2 \leq n^2$

Natural numbers $n^2 - k$ with $k < n$, the $n^2 - k = pq \Rightarrow \exists\, r \,/\, k = r^2$

Thus $\forall\, n\, \exists\, p, q\;\; 2n = p + q \Leftrightarrow \exists\, p < n, \exists\, q > n$ p,q primes su!ch $pq < n^2$

Inverse of the function ς: $1/\varsigma(s) = \Sigma_{n\geq 1}\, \mu(n)/n^s$ with μ Möbius function

In the space Γ this function is represented by the hypercube with diameter 1

If the conjecture fails, by considering the natural numbers inn the interval $[1,n^2]$, the points corresponding to pq (p,q primes $p < n$, $q > n$ are missing

These points are corresponding to the 2 - faces of the hypercube

By scaling effect, choice of the parameter s such as $\Sigma_{l\leq n\times n}\, \mu(l)/l^s \cong 1/\varsigma(s)$

If all these points are missing with a non null density, a contradiction is rising

The density of this family of points is always non null; it is proved by comparing the growth ratio of 2 - faces in the hypercube of dimension l $\boxed{\binom{l}{2}2^{l-2}}$ and the growth ratio of prime numbers up to n^2 ($n^2 \log n^2 = 2n^2 \log n$) and by taking $l = [2n^2 \log n]$ (entire part of l)

Conclusion

Two structures within N: ● addition = coproduct; algebra (composition process)

● multiplication = product; coalgebra (decomposition process)

Through a lifting into a Kleisli category both structures are well fitted together for. The powerset monad $\wp$ is used for obtaining it and by the way a compactification process is rising with scale factor emergence.

The representation of the natural numbers set N by an space in an relevant extensive category is accordingly provided with a scale distribution. The total measure of this distribution is identified with the function $\varsigma(s)$.

The universal property of the function $\varsigma(s)$ (Voronin's theorem) is simply obtained as a differentiation operation by homogeneously changing the scale of the axes.

The self-similarity of the above mentioned extensive structure allows to infer the Riemann conjecture is true through the Bagchi's theorem; therefor the Goldbach's conjecture is equally verified.

This new approach could likely be fruitful for analyzing the polyzeta functions and the role of shuffle algebras for composing it.

More generallythe a fixed point structure attached to a functor seems a relevant framefor modelling the rhythm concept as examplified by the Freyd's result (wide generalization performed by T.Leinster)

Réseau National des Systèmes Complexes

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