

# An Introduction on Formal and Computational Models in Popular Music Analysis and Generation

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**Abstract** This article provides a first introduction to some formal and computational models applied in the analysis and generation of popular music (including rock, jazz, and *chanson*). It summarizes the main philosophy underlying the project entitled “Modèles formels *dans et pour* la musique pop, le jazz et la *chanson*”, which constitutes one of the research axes of the GDR ESARS (Esthétique, Art & Science). Initially conceived as an extension of the MISA project carried on by the Music Representation Team at IRCAM, this research axis aims at bringing together researchers from different horizons, from the traditional MIR community of Music Information Retrieval to the most sophisticated approaches in mathematical music theory and computational musicology. It also includes an epistemological and critical evaluation of the relations between music and mathematics, together with some programmatic reflections on the possible cognitive and perceptual implications of this research.

## Introduction

There is an increasing interest within the computational musicological community for formal and computational models applied not only in the analysis but also in the generation of popular music. With this label, one generally includes repertoires—such as rock, jazz, and *chanson*—which are not considered as belonging to the art or contemporary music.<sup>1</sup> The common point among all formal and computational methods

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<sup>1</sup>This paper summarizes some aspects of this project that have been described in details in Andreatta (2014a). For a pedagogical and large-public introduction to mathematical models in popular music, also see Andreatta (2014b). A more technical presentation of the main concepts described in this paper and addressed to the community of researchers working on computational musicology is given in Bigo and Andreatta (2015).

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described in this article relies on the relevance of the interplay between geometric and algebraic approaches in music theory, analysis and composition.<sup>2</sup> This postulate applies equally well to contemporary art music and popular music repertoires, which opens interesting questions about the possible articulations between these two study domains.<sup>3</sup> Moreover, the tools described in this paper also apply to the field of folk or traditional music, which is—according to a programmatic article by Philip Tagg on theoretical, methodological and practical aspects of popular music studies (Tagg 1982)—one of the three possible kinds of music (together with classical or art music and popular music).<sup>4</sup> This is possible thanks to the flexible nature of geometric representations, which enable to grasp equally well the logic behind the songs and “chansons”, from The Beatles to Paolo Conte, as well as the harmonic construction of rock/pop pieces (from Frank Zappa’s to the songs by Depeche Mode).<sup>5</sup> After briefly describing some theoretical aspects underlying the geometric representations used in the field of computational (popular) music analysis, we will show some new visualisations of musical structures and processes

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<sup>2</sup>According to the Field-medallist Alain Connes, “concerning music, it takes place in time, like algebra. In mathematics, there is this fundamental duality between, on the one hand, geometry—which corresponds to the visual arts, an immediate intuition—and on the other hand algebra. This is not visual, it has a temporality. This fits in time, it is a computation, something that is very close to the language, and which has its diabolical precision. [...] And one only perceives the development of algebra through music” (Connes 2004). This duality constitutes a major common point between music and mathematics, allowing proposing a common basis for the creative processes in both fields of music and mathematics, as suggested by Alain Connes in his dialogue with Pierre Boulez on creativity in mathematics and music (Boulez and Connes 2011). See Andreatta (2010) for a detailed description of the “mathemusal” research that has been carried on in the last ten years within the MISA project (*Modélisation Informatique des Structures Algébriques en musique*), with a special emphasis on the interplay between algebra and geometry. See Andreatta et al. (2013) for a description of a category-oriented framework for describing the creative process in music and mathematics.

<sup>3</sup>This question has been explicitly addressed in the conference “Musique savante/musiques actuelles: articulations” (Contemporary art music/popular music: articulations), hosted by IRCAM and organised under the auspices of the French Society of Music Analysis, in collaboration and with the financial support of the IReMus (Institute of Research in Musicology, UMR 8223, Paris-Sorbonne) and the BPI of the Centre Georges Pompidou and with the participation of the French component of IASPM (International Association for the Study of Popular Music). The Proceedings are forthcoming in a special issue of the multimedia online journal *Musimédiane* (Andreatta 2016). For a first attempt at analysing the necessity of substituting this typology with a finer taxonomy based on computational models focusing on musical objects and making use of different theoretical approaches in order to carry on computer-aided music analysis, see Bergomi et al. (2015).

<sup>4</sup>This typology constitutes what Tagg calls an *axiomatic triangle* of musical genres, each of which being characterized by criteria such as the usual or unusual mass distribution, the existence of a circle of professionals or a circle of amateurs who produces and transmits it, the principle modality of storage and distribution (ranging from oral transmission, in the case of folk music, to the recorded sound, in the case of popular music), the anonymous versus authorial character of the underlying compositional process, and so on.

<sup>5</sup>For a recent analytical application of the formal tools discussed in this paper from the perspective of a geometric-based automatic classification, see Bergomi et al. (2015).

making use of a recent model providing some additional tonal information with respect to the traditional *Tonnetz* representation.<sup>6</sup>

## Geometric Representations of Musical Structures and Processes

Although sometimes very far from a stylistic point of view, there are pieces belonging to the rock, pop and “chansons” repertoire which somehow share the same “musical logics” concerning the harmonic organization. More precisely, if one restricts the analysis to consonant chords (major and minor), it is possible to find interesting common points between stylistically-different pieces in the way in which the chord progressions are constructed. In order to make these similarities evident, the (computational) music analyst can use several geometric representations of harmonic spaces, including the circular representation, the different types of *Tonnetze*, the orbifolds, the spiral array and many others.<sup>7</sup>

In this paper we will focus on the *Tonnetz*, a geometric representation of the pitch space originally proposed by Euler (1774) in the second half of the XVIII<sup>e</sup> century as an alternative to the well-established circular representation previously introduced by Marin Mersenne in his *Harmonie universelle* (Mersenne 1636).<sup>8</sup>

The *Tonnetz* is a symbolic organization of pitches in the Euclidean space defined by infinite axes associated with particular musical intervals. Although these graph-theoretical representations have been rediscovered later by music theorists, musicologists, and composers (including Arthur von Oettingen, Hugo Riemann and Henri Pousseur), the interest of the computational musicology community for this type of structure is very recent. The model is currently used to represent chord

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<sup>6</sup>Two main models, the “Polarized Tonnetz” and the “Spinnen Tonnetz”, originally conceived by Hugo Seress and Gilles Baroin, represent a very interesting way of integrating some tonality-based constructions within transformational music analysis. For a critical presentation of these two models and their comparison with other tools belonging to the transformational musical analysis tradition, see Seress and Baroin (2016).

<sup>7</sup>See Bigo (2013) and Bigo and Andreatta (2015) for a historical description of the main geometric representations in computational music analysis. Algebraic topology has provided a very elegant theoretical framework for describing all these representations, as shown by Bergomi (2015) in his recent doctoral thesis.

<sup>8</sup>The reader interested to learn more about the three main contributions of Leonhard Euler (as a mathematician, physicist and music theorist) can refer to Hascher and Papadopoulos (2015).

progressions within the so-called neo-Riemannian transformational approach,<sup>9</sup> whose application includes post-romantic tonal music (Cohn 2012) but also rock, jazz and pop music repertoires (Capuzzo 2004; Hascher 2007; Briginshaw 2012; Bigo and Andreatta 2015). From a generative perspective, this model has also been used in contemporary music (for example by the French composer Jean-Marc Chouvel), as well as in popular music contexts, leading to a geometrically constrained-based series of Hamiltonian Songs (Andreatta 2014b; Bigo and Andreatta 2014).<sup>10</sup>

Mathematically-speaking, the circular representation and the *Tonnetz* are equivalent ways of formalizing in an algebraic way the structural properties of the equal-tempered system (i.e. the division of the octave into twelve equal intervals, as in the piano). The main computational property is the possibility of generating the system by using combinations of major third (i.e. four semitones) and minor third (i.e. three semitones) intervals, as depicted in Fig. 1.

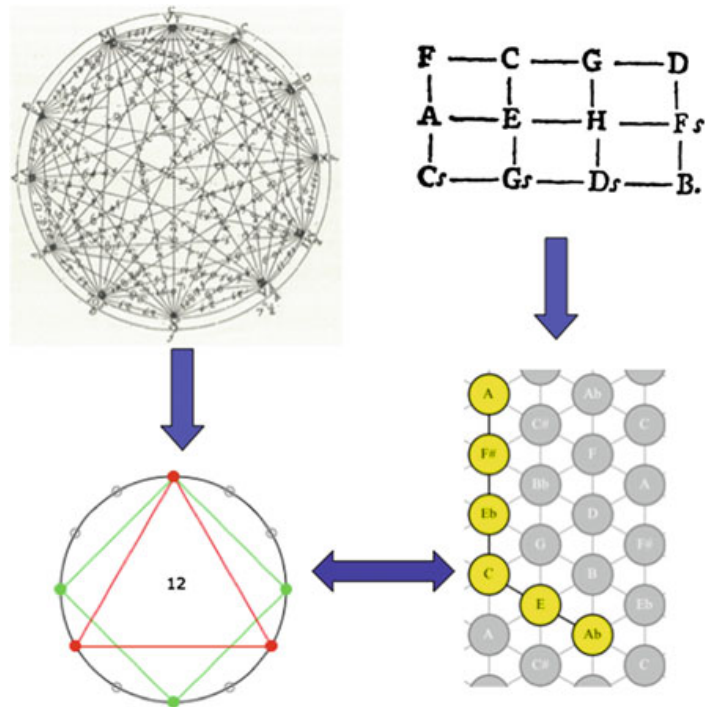
As the previous figure suggests, we are interested in the computational aspects of the geometric representations, and in particular in the fact that they can be implemented in programming languages for computer-aided music analysis and composition. For example, to compute the compactness of harmonic trajectories in different automatic-generated *Tonnetze*, the computational musicologist has a new geometric way of handling the problem of style classification, which is one of the most interesting research areas in Music Information Retrieval. We will not enter here into this aspect of our research, which has been largely addressed in several recent contributions (Bigo et al. 2013; Bigo and Andreatta 2015; Bergomi 2015; Bergomi et al. 2015), but we will focus on visualisation techniques as applied, in particular, to popular music repertoires. Figure 2 shows the *Tonnetz* as generated by three musical operators (P, R and L), corresponding to the three possible ways of transforming a major chord into the corresponding minor chord having two notes in common with the initial chord. These operators are respectively called the “parallel” (indicated by P and transforming, for example, the C major chord into the C minor chord, and vice versa), the “relative” (indicated by R and transforming, for

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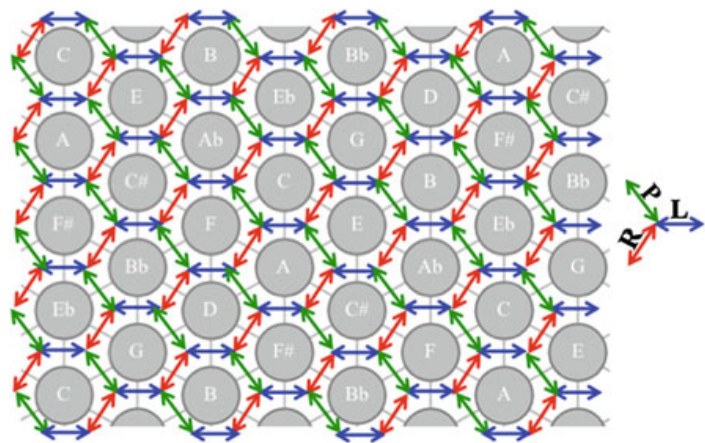
<sup>9</sup>Neo-Riemannian music analysis is a formal methodology developed after the writings by the German music theorist Hugo Riemann (1849–1919). Following David Lewin’s transformational turn in music theory and analysis (Lewin 1987/2007; 1993/2007), which integrates neo-Riemannian techniques within a much more general approach, one may speak about neo-Riemannian transformational music analysis as a structural methodology combining the two independent approaches. See Gollin and Rehding (2014) for a comprehensive textbook on Neo-Riemannian analysis.

<sup>10</sup>Hamiltonian Songs are so-called after the Irish physicist, astronomer, and mathematician Sir William Rowan Hamilton (1805–1865). In graph theory, a Hamiltonian cycle is a path passing through all possible nodes of a graph and ending precisely where it started. It is well known that there are exactly 124 Hamiltonian cycles in the *Tonnetz* (Albini and Antonini 2009) which can be classified by using their inner symmetries (i.e. the possibility of decomposing a given cycle into sub-sequences that repeat identically in order to generate the entire cycle). The complete list of Hamiltonian cycles with some examples of Hamiltonian Songs is available at: <http://repmus.ircam.fr/moreno/music>.

**Fig. 1** Two mathematically equivalent representations of the family of pitch-classes within the equal-tempered system: the circular representation, whose origin goes back to Mersenne (*on the left*) and the “Speculum Musicum” by Euler, which is the ancestor of the *Tonnetz* (*on the right*). The circular representations and the *Tonnetz* are obtained respectively by using the OpenMusic visual programming language (see Agon 2004) and the *Hexachord* software (see Bigo 2013)



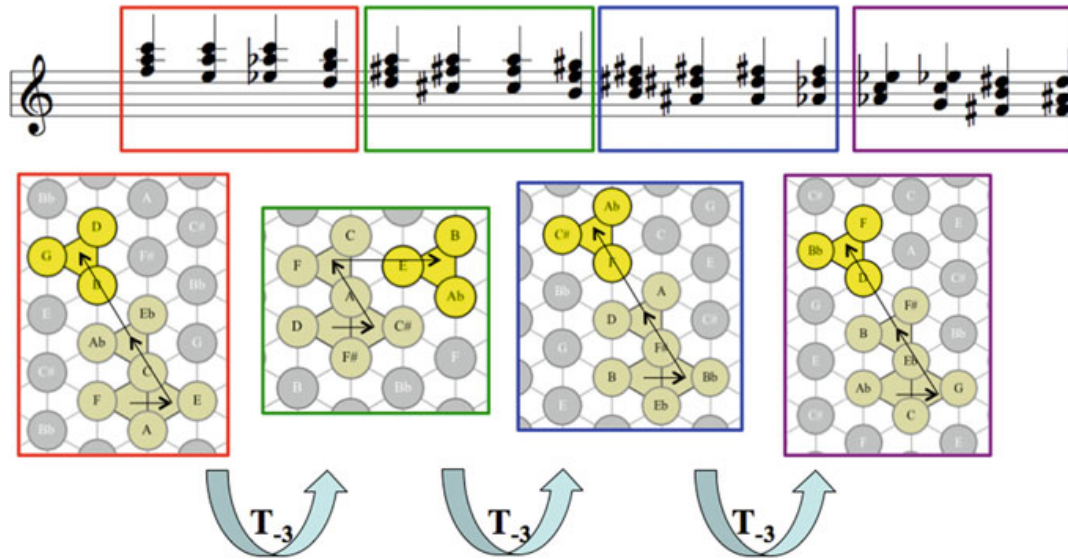
**Fig. 2** The *Tonnetz* as a hexagonal tiling of the plane where each major (respectively minor) chord is transformed into three minor (respectively major) chords via the three P, R L operators preserving two of the three notes of the initial chord



example, the C major chord into the A minor chord, and vice versa) and the “leading-tone” (indicated by L and transforming, for example, a C major chord into a E minor chord, and vice versa).

### Circular Representations and Tonnetze for Popular Music

In order to show how the circular representation and the *Tonnetz* constitute two complementary approaches in the analysis of harmonic progressions, let us stress a little bit more the relevance of the notion of symmetry in music. An interesting starting point is provided by two stylistically different pieces having the same



**Fig. 3** Harmonic progression in Zappa’s piece *Easy Meat*, seen as a series of transpositions (of a minor descending third  $T_{-3}$ ) of a same cell (the first one, in red). The four cells generate therefore the same trajectory in the *Tonnetz* (where apparently different shapes correspond in fact to the same trajectory because of the toroidal structure of the *Tonnetz*) (color figure online)

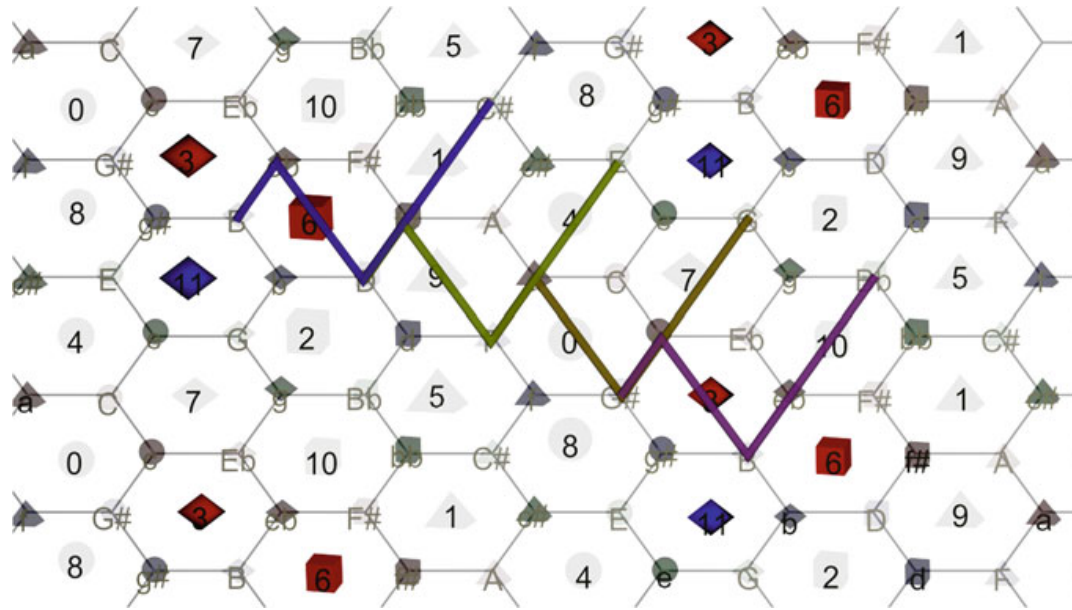
“spatial” logics with respect to the harmonic organisation: *Easy Meat* by Frank Zappa and *Madeleine* by Paolo Conte.

Let start with Zappa’s piece *Easy Meat* and one of the most recurrent harmonic progression in the piece.<sup>11</sup> This progression, shown in Fig. 3, contains sixteen chords and can be decomposed as a repetition (via the transposition operation) of a given cell of four chords. Each cell contains the same series of neo-Riemannian operators, as Fig. 4 shows.

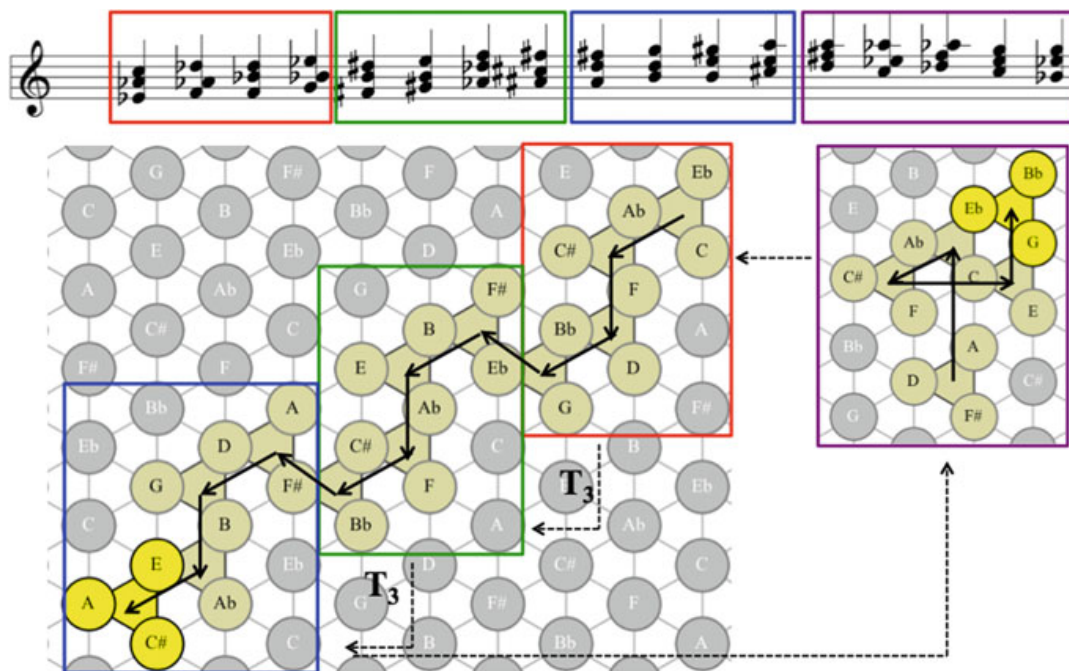
The figure shows the progression represented in an unfolding *Tonnetz* representation conceived by Gilles Baroin, corresponding to the two-dimensional projection of his *Hypersphere of Chords* (Baroin 2011). In this case the trajectory of a cell is rigorously translated in space, metaphorically providing a kind of composer’s “signature” for the piece.

It is interesting to compare this type of chord progression with a different harmonic progression used by the Italian “chansonnier” Paolo Conte in his piece entitled *Madeleine*. In this song, the harmonic progression of the verse, repeated several times all along the piece, is also constructed in a similar way. There are four blocks, the first three of which are obtained by transposing an initial cell by an ascending third. They therefore correspond to a same trajectory in the *Tonnetz*, whereas the symmetry breaking due to the fourth block, structurally different from the three previous ones, enables the chord progression to come back to the initial chord. This progression is given in Fig. 5.

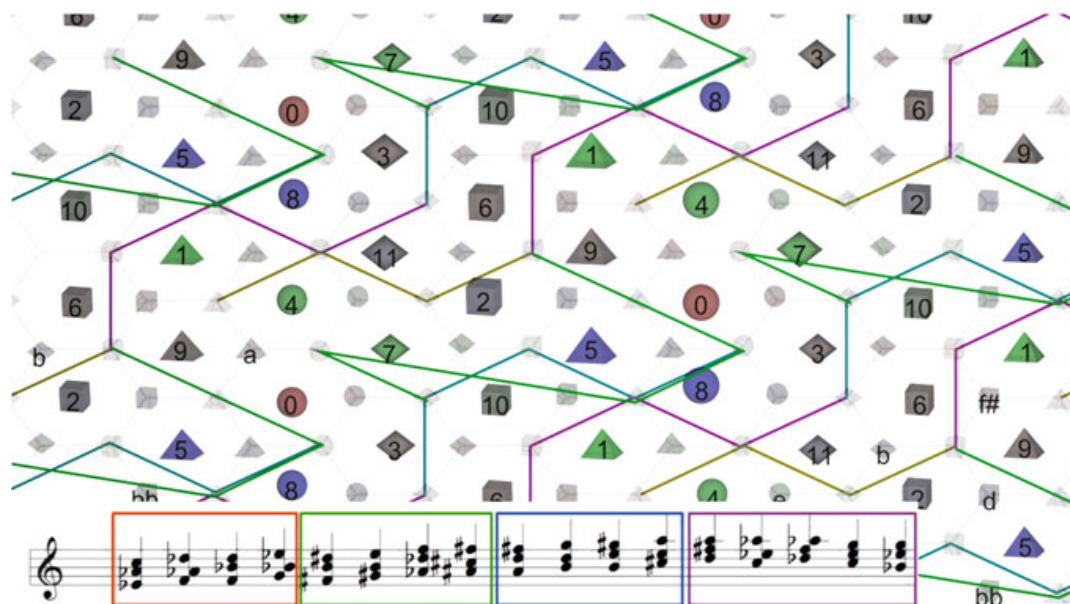
<sup>11</sup>The interest of using Neo-Riemannian techniques to analyse this passage has been originally pointed out by Capuzzo (2004).



**Fig. 4** Zappa’s “signature” for the piece *Easy Meat*, represented in Gilles Baroin’s visualisation of the *Tonnetz*



**Fig. 5** Harmonic progression used by Paolo Conte in his song *Madeleine* represented as a series of spatial translations of an initial cell containing four chords. The fourth cell, containing five chords, functions as a new trajectory “forcing” the progression to come back to the initial tonality

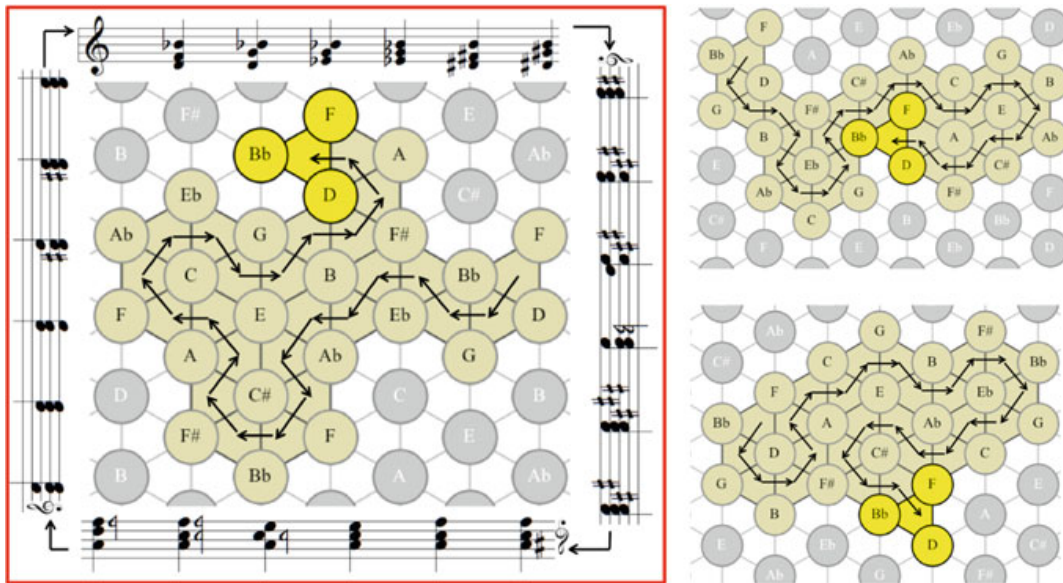


**Fig. 6** The “almost perfect” covering of the harmonic chromatic space by major chords and their transpositions in Paolo Conte’s *Madeleine*

Despite this superficial analogy between the two pieces, the compositional process in *Madeleine* has a remarkable property which makes the chord progression “qualitatively” very different from that used by Zappa. In fact, up to a single chord, which is missing, it constitutes a *covering* of the chromatic space by major chords and their transpositions. This covering property is much more evident in the following *Tonnetz* representation provided by Gilles Baroin (Fig. 6).

In other words, the harmonic progression of the piece corresponds to a trajectory which passes through (almost) all twelve major chords (with repetitions). This property admits a natural mathematical generalization by considering the traditional *Tonnetz* as a graph whose vertices consist of all major and minor chords and by studying trajectories passing only once through all major and minor chords and eventually coming back to the starting point. In this case, such paths are called “Hamiltonian cycles” and have been enumerated and classified (Albini and Antonini 2009) according to their inner symmetries. There are in fact Hamiltonian cycles which are “redundant” (meaning that they are generated by the repetition of a given pattern) and other cycles which are “maximal” (meaning that they are not obtained as a concatenation of a same pattern of P, L and/or R transformations). Such maximal Hamiltonian cycles have been used by one of the authors in the instrumental parts of the song *Aprile*, inspired by a text from the Italian decadent poet Gabriele D’Annunzio (1863–1938). More precisely, three structurally different Hamiltonian cycles have been used, with the goal of systematically frustrating the





**Fig. 7** The three maximal Hamiltonian cycles used in the instrumental part of the song *Aprile* by Moreno Andreatta, inspired by a Gabriele D’Annunzio poem (1863–1938)

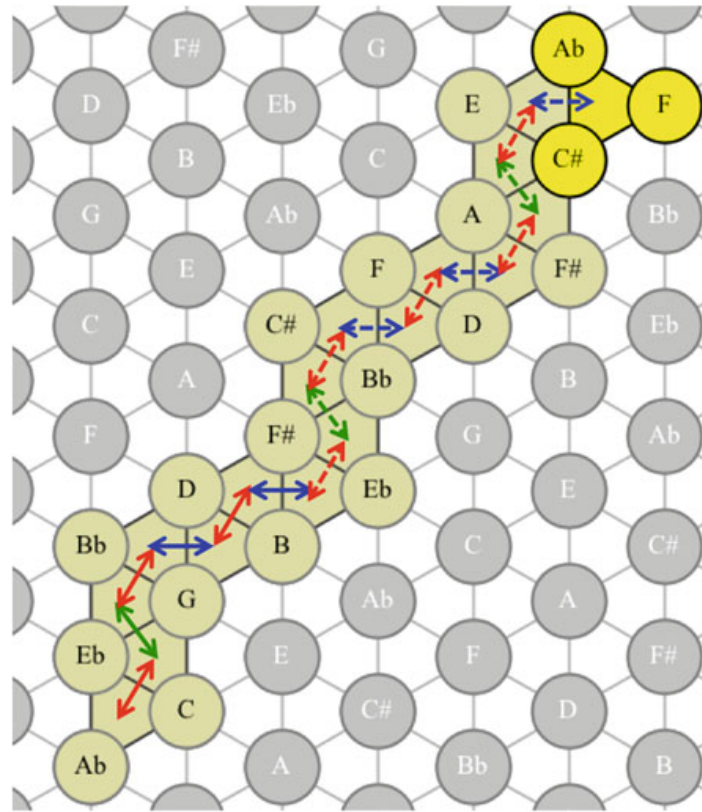
expectation of the listener, whose perception cannot find the logic in the three selected progressions of chords (Fig. 7).<sup>12</sup>

The previous example shows the interest of using Hamiltonian properties of chord progressions in a popular music context. Despite their intriguing character, Hamiltonian cycles are challenging objects for music perception and cognition. One may question their capability of providing harmonic material that the musical mind can process, without getting lost in the underlying maximal variety principle.<sup>13</sup> Since evidences of the perceptual relevance of these geometric and combinatorial structures are still lacking, it seems reasonable to try to add some inner symmetries in the Hamiltonian cycles used in song writing. Redundancy in the inner structure of the Hamiltonian harmonic progressions has been used by one of the authors (Moreno Andreatta) in the song entitled *La sera non è più la tua canzone* and based on a poem by Mario Luzi (1914–2005). Hamiltonian cycles are not only used in the instrumental parts, but—more challenging—in the verse, which obliges to create a melody capable of supporting a continuously changing harmony. Due to its inner

<sup>12</sup>The Hamiltonian trajectories of the song have been visualised by Gilles Baroin by mixing his *Hypersphere of Chords* representation and the traditional *Tonnetz*. It is available online at the address: ([www.mathemusic.net](http://www.mathemusic.net)).

<sup>13</sup>Note that “hamiltonicity” does not only concern popular music strategies, but it plays an important role in contemporary art music. The history of Twentieth-Century music shows that Hamiltonian properties have been implicitly used by composers such as Pierre Boulez or Milton Babbitt, who developed combinatorial strategies as natural extensions of the twelve-tone compositional system. Both composers and music theorists claimed the necessity of having a “maximal variety principles” in composition, in order to precisely question the notion of expectation in the musical listening process.

**Fig. 8** The redundant Hamiltonian cycle used in the song *La sera non è più la tua canzone* (music by Moreno Andreatta, based on a poem by Mario Luzi)

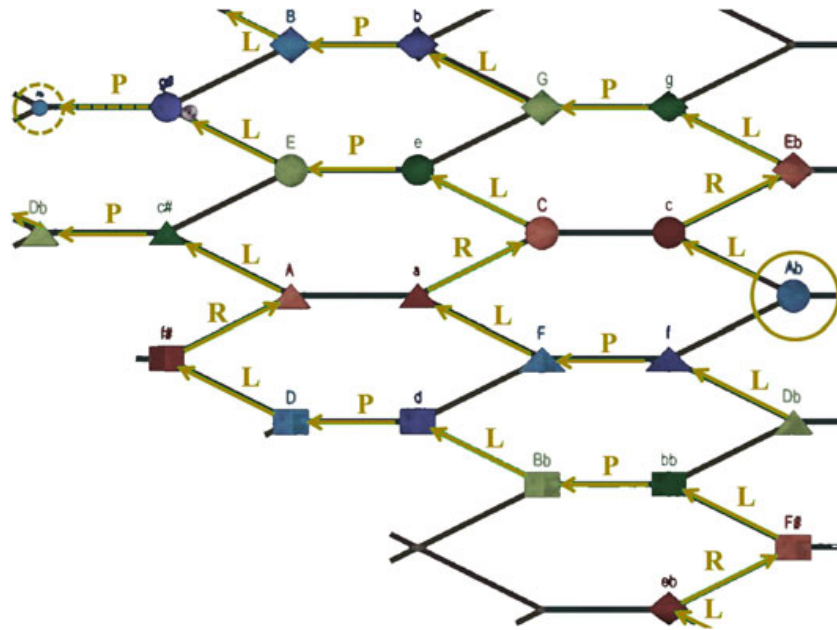


symmetry, the cycle of length 24 is obtained by repeating four times the pattern LRLPLP of six transformations, as shown in Fig. 8.

The Fig. 9 shows the visualisation of the redundant Hamiltonian cycle utilized in the song *La sera non è più la tua canzone* in a new *Tonnetz* representation called the *Spinnen-Tonnetz*.

Although one of the main features of the *Spinnen-Tonnetz* is to provide a tonal centre to a harmonic progression, hamiltonicity makes the recognition of a tonality impossible in the case of the previous song. This fact opens interesting questions about the capability, for the musical mind to grasp these mathematical representations and to follow the logics of continuous modulations. One of the objectives of the “Math’n Pop” project, which is carried on within the GDR ESARS, is precisely to go deeper into the connections between cognitive neurosciences and algebraic/geometric formalisations of musical structures and processes. As shown by Zatorre and Krumhansl (2002), the mental key maps are related to the way in which a major (resp. minor) chord is surrounded by minor (resp. major) chords having two notes in common. Although the authors do not make any reference to the neo-Riemannian transformations, the geometric space they suggest to use is precisely the traditional *Tonnetz*.<sup>14</sup>

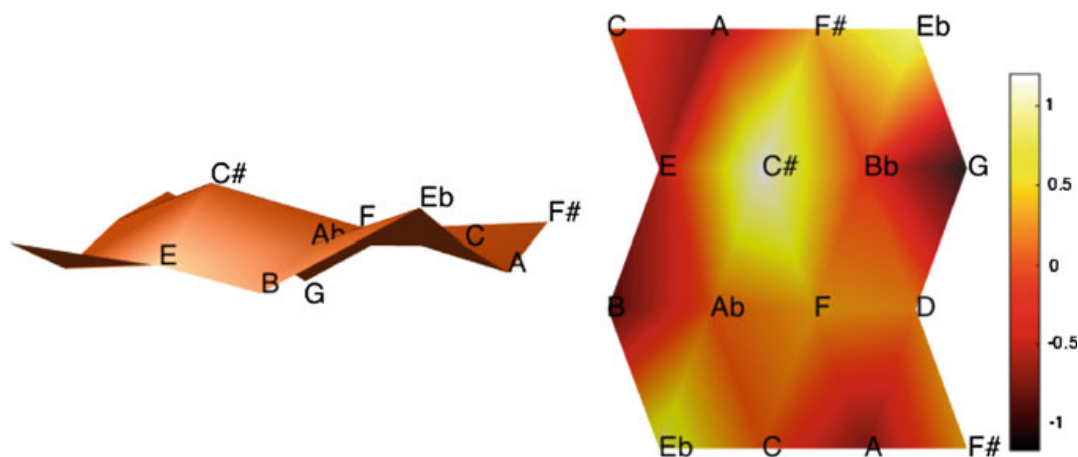
<sup>14</sup>We analysed the relations between mental and mathematical representations of music in Acotto and Andreatta (2012).



**Fig. 9** The visualisation of the trajectory corresponding to the redundant Hamiltonian cycle used in the song *La sera non è più la tua canzone* in the *Spinnen-Tonnetz*. The *circle* shows the initial chord of the Hamiltonian progression (which—because of the cyclic character of the path—is the same as the final chord, indicated with a *dotted circle*)

## Conclusions and Perspectives for Future Research

Starting from the analytical examples presented in this paper, together with the compositional applications that we have briefly sketched, it is clear that the popular music repertoire (including pop music, jazz, rock and *chanson*) can largely benefit from the use of formal and computational models. Although we have focused our attention on symbolic approaches and, in particular, on algebraic and geometric models, one interesting research area is precisely the interaction between symbolic approaches and different techniques based on signal processing within the field of Music Information Retrieval (MIR). A first attempt at filling the gap between these two main components of MIR has been carried on by using dissonance functions and advanced tools in algebraic topology in order to deform the original *Tonnetz* into an anisotropic structure (Bergomi and Andreatta 2015; Bergomi 2015; Bergomi et al. 2015). An example of deformation of the vertices of the *Tonnetz*



**Fig. 10** The anisotropic *Tonnetz* whose vertices are deformed by a dissonance function (in this special case it is the dissonance induced by a C major chord. The figure is taken from Bergomi and Andreatta 2015)

leading to an anisotropic geometric space is shown in Fig. 10. This new structure might be an excellent case study in order to fill the gap between the computational musicological community and the neuroscientists working on the cognitive aspects of the geometric formalisations of musical structures and processes.<sup>15</sup>

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<sup>15</sup>A special issue of the *Journal of Mathematics and Music* has been devoted to this specific problem with precisely the aim of bridging the Gap between Computational/Mathematical and Cognitive Approaches in Music Research. See Volk and Honing (2002).

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