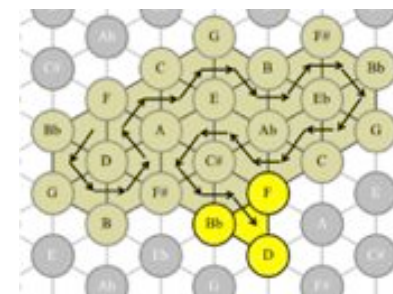
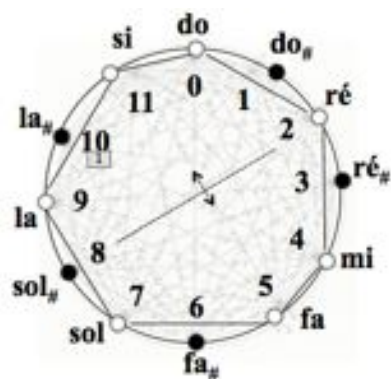
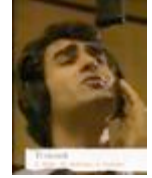


Musica contemporanea: la formalizzazione dell'organizzazione delle altezze nello spazio temperato e la teoria trasformatoriale

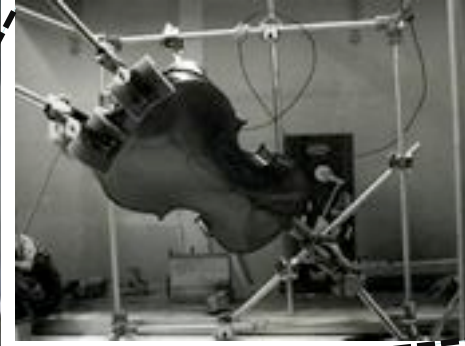
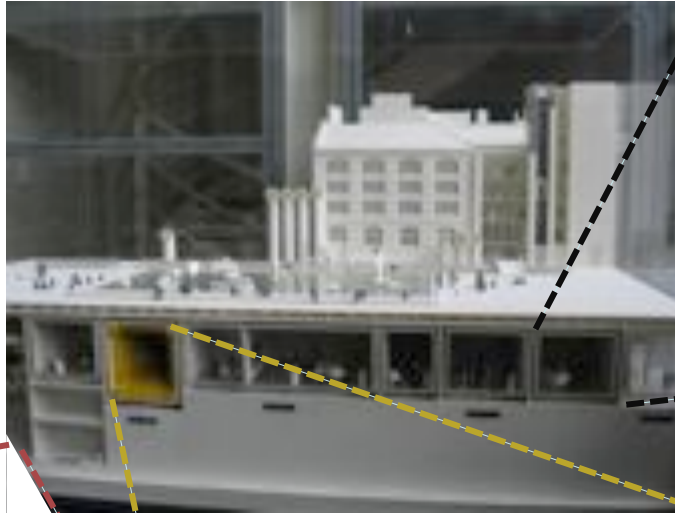


Il mio percorso personale fra la matematica e la musica

- **1990-1996** Laurea in matematica (Università di Pavia) – Studi di composizione e direzione d'orchestra (Laboratorio Permanente Nuova Musica di Pergine, M° F. Valdambrini)
- **1998** Diploma di pianoforte (Conservatorio di Novara)
- **1998-2003** DEA (Master 2) e tesi di dottorato in Musicologia computazionale (EHESS) e pianista di piano-bar (Bateaux Parisiens, fino al 2009)
- **Dal 2004** CR1 CNRS all'IRCAM (Institut de Recherche et Coordination Acoustique/Musique)
- **2005-2009** Pianista-cantante e direttore artistico del gruppo di canzone franco-italiana *Nationale91*
- **2010** HDR in matematica e le sue interazioni (IRMA, Strasburgo)
- **Dal 2012** Coordinatore del Master ATIAM (Acoustique, Traitement du signal et informatique appliqués à la musique) e responsabile dell'UE 'Musicologia Computazionale'
- **2013-2017** Comitato di pilotaggio del GDR ESARS (Esthétique, Arts et Science) – Resp. dell'asse « Math'n Pop »



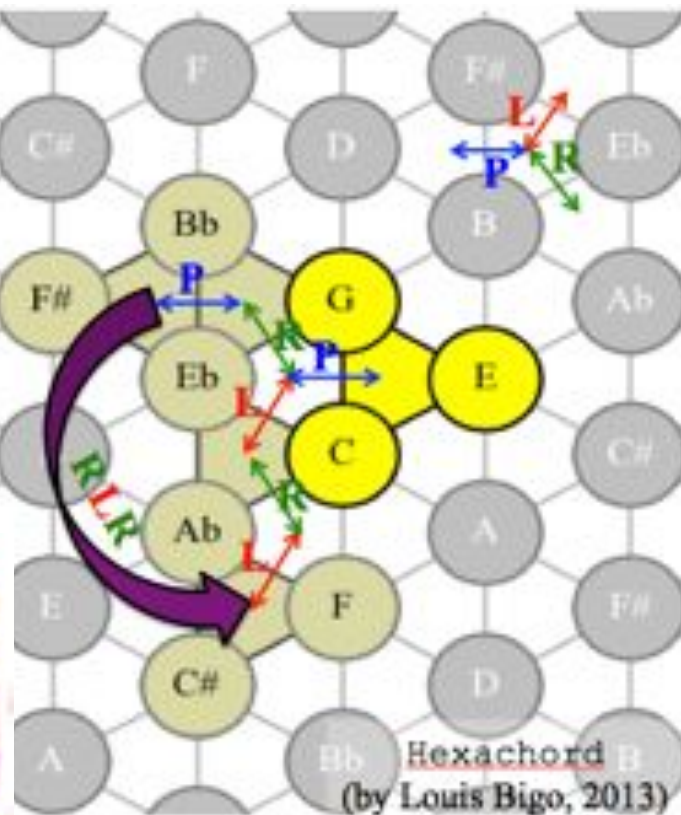
La ricerca musicale e scientifica all'IRCAM...



...fra musica contemporanea e *popular music*



MusiqueLab 2



OMAX



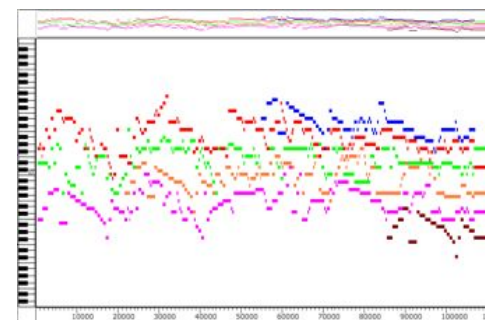
Var. 1



Var. 2



Orig.



Sommario

○ **Premessa sulla ricerca matematica**

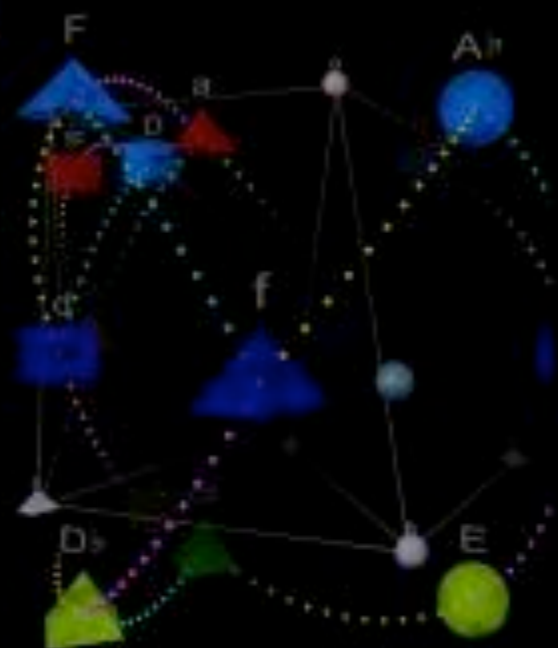
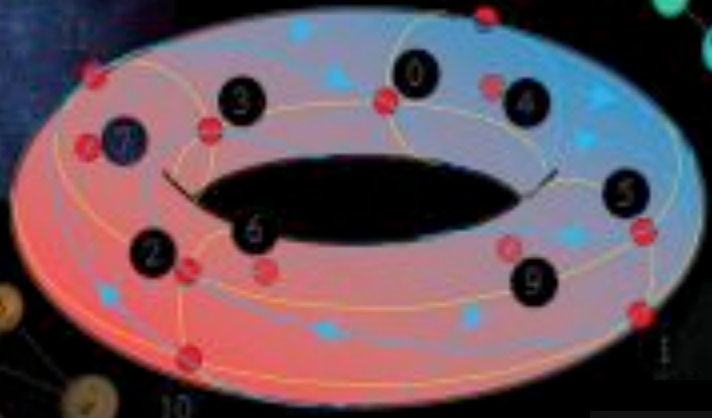
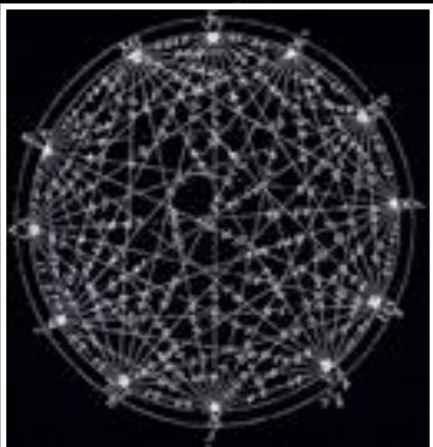
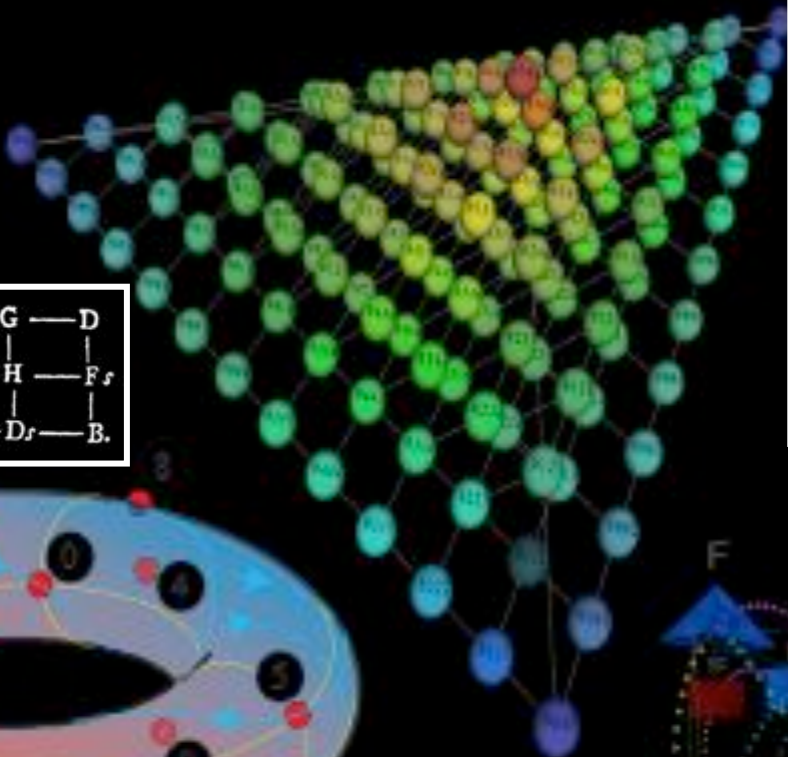
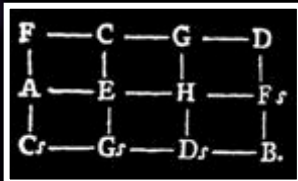
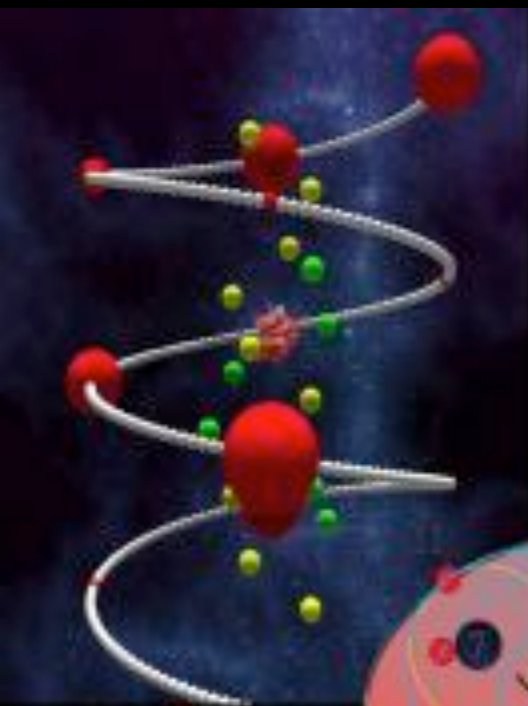
- Un esempio elementare di teorema matematico (Babbitt's Theorem)
- Tavola comparativa sugli sviluppi della matematica e della musica (Xenakis)
- La nascita della musicologia sistematica (Guido Adler, 1885)

○ **La rappresentazione circolare del sistema temperato equabile**

- La nascita della combinatoria musicale (Marin Mersenne, 1648)
- La simmetria del re come base dell'armonia (Durutte, 1855)
- Elementi di formalizzazione algebrica
- Enumerazione dei modi di Messiaen a trasposizione limitata
- Teoria dei setacci (Xenakis)
- La *Set Theory* e l'analisi insiemistica (Vieru, Forte, Carter, ...)
- Estensioni dell'approccio insiemistico: la *Transformational Theory* (Lewin)

○ **Elementi di analisi neo-riemanniana**

- Il *Tonnetz* e le sue generalizzazioni
- Le origini ramiste dell'analisi neo-riemanniana (Henri Pousseur)
- Verso una classificazione automatica dello stile musicale
- Applicazioni alla *popular music* (pop, rock, canzone, ...)



L'articolazione algebra/geometria nella musica



“Per quanto riguarda la **musica**, essa si realizza nel **tempo**, come l'**algebra**. In **matematica**, c'è una dualità fondamentale fra, da una parte, la **geometria** – che corrisponde alle arti visive, un'intuizione immediata – e d'altra parte l'**algebra**. Quest'ultima non è visiva ma ha una temporalità. L'algebra si realizza nel tempo, è un calcolo, un qualcosa che è estremamente vicino al linguaggio, del quale ha la stessa precisione diabolica. **E si percepisce lo sviluppo dell'algebra solo attraverso la musica**” (A. Connes).

➔ <http://videotheque.cnrs.fr/>



➔ <http://agora2011.ircam.fr>



La dinamica della ricerca 'matemusicale'

MATEMATICA

Enunciato
matematico

generalizzazione

Teorema
generale

formalizzazione



OpenMusic

applicazione

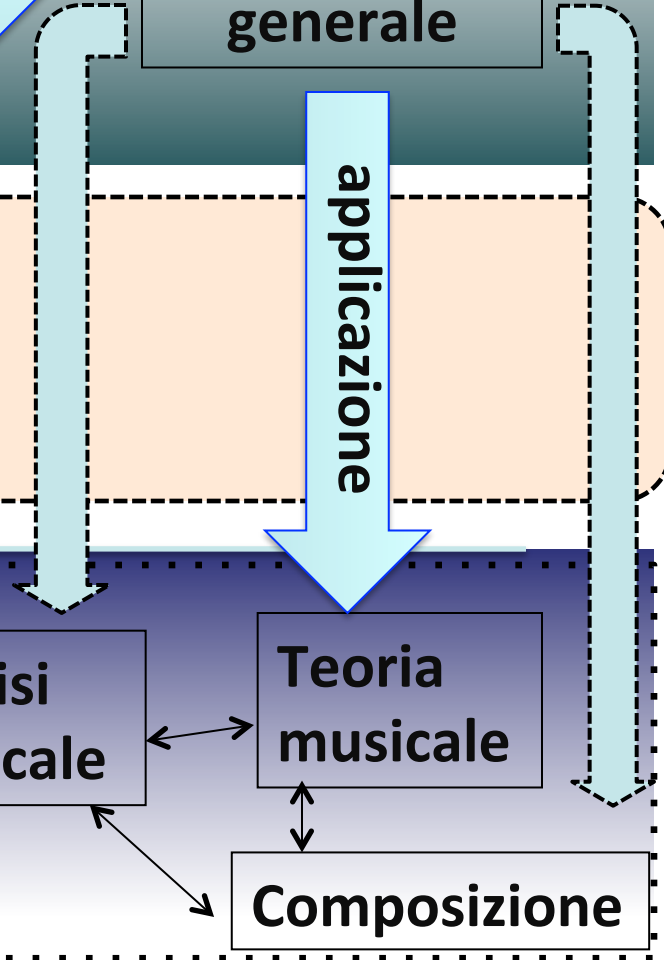
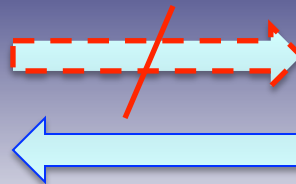
MUSICA

Problema
musicale

Analisi
musicale

Teoria
musicale

Composizione



La dinamica della ricerca 'matemusicale'

MATEMATICA

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M. Andreatta, *Méthodes algébriques en musique et musicologie du XX^e siècle : aspects théoriques, analytiques et compositionnels*, PhD, EHESS-IRCAM, 2003

La dinamica della ricerca 'matemusicale'

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Problema
musicale

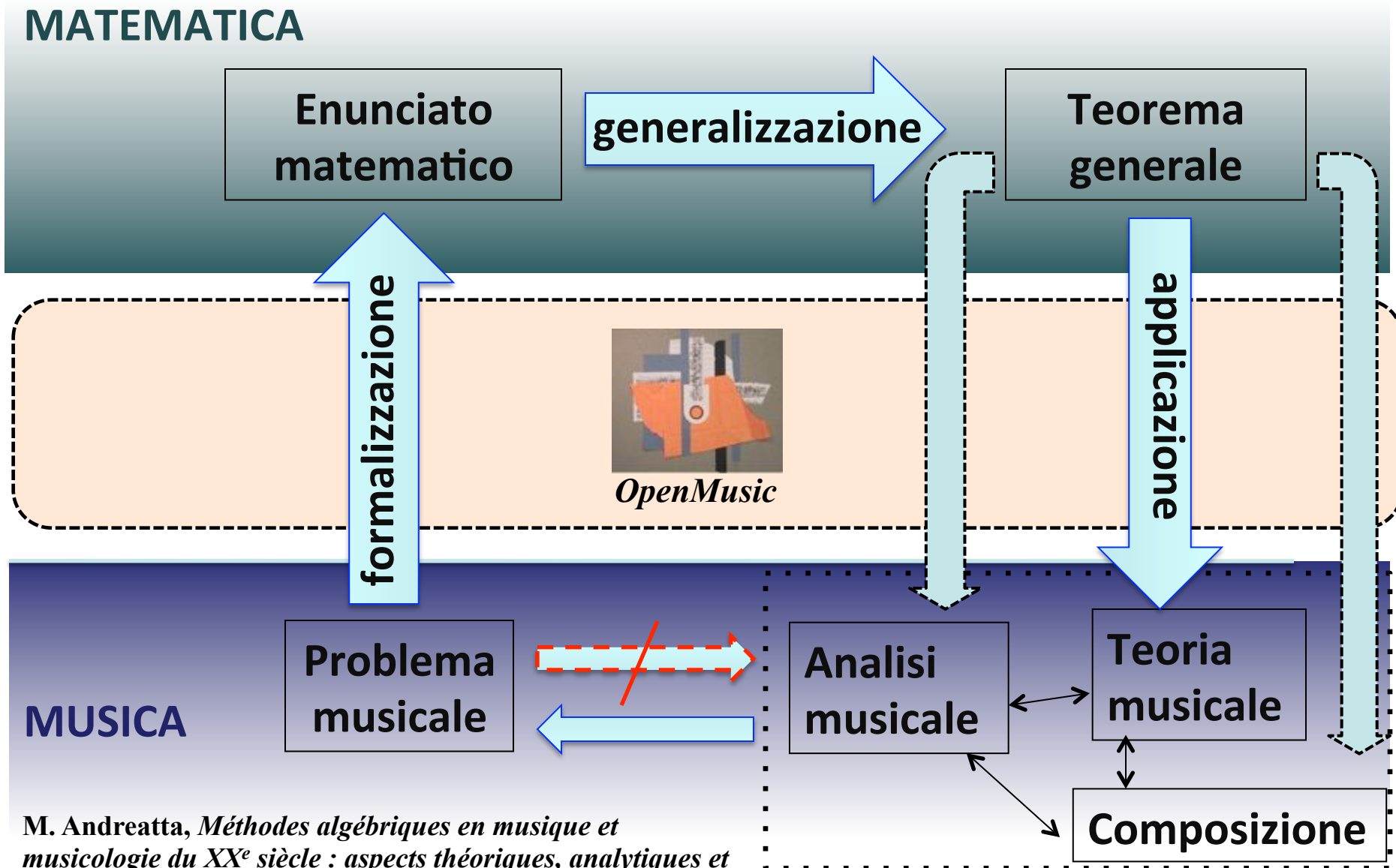
Analisi
musicale

Teoria
musicale

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OpenMusic, un linguaggio di programmazione visuale per la CAO

www.repmus.ircam.fr/openmusic/home

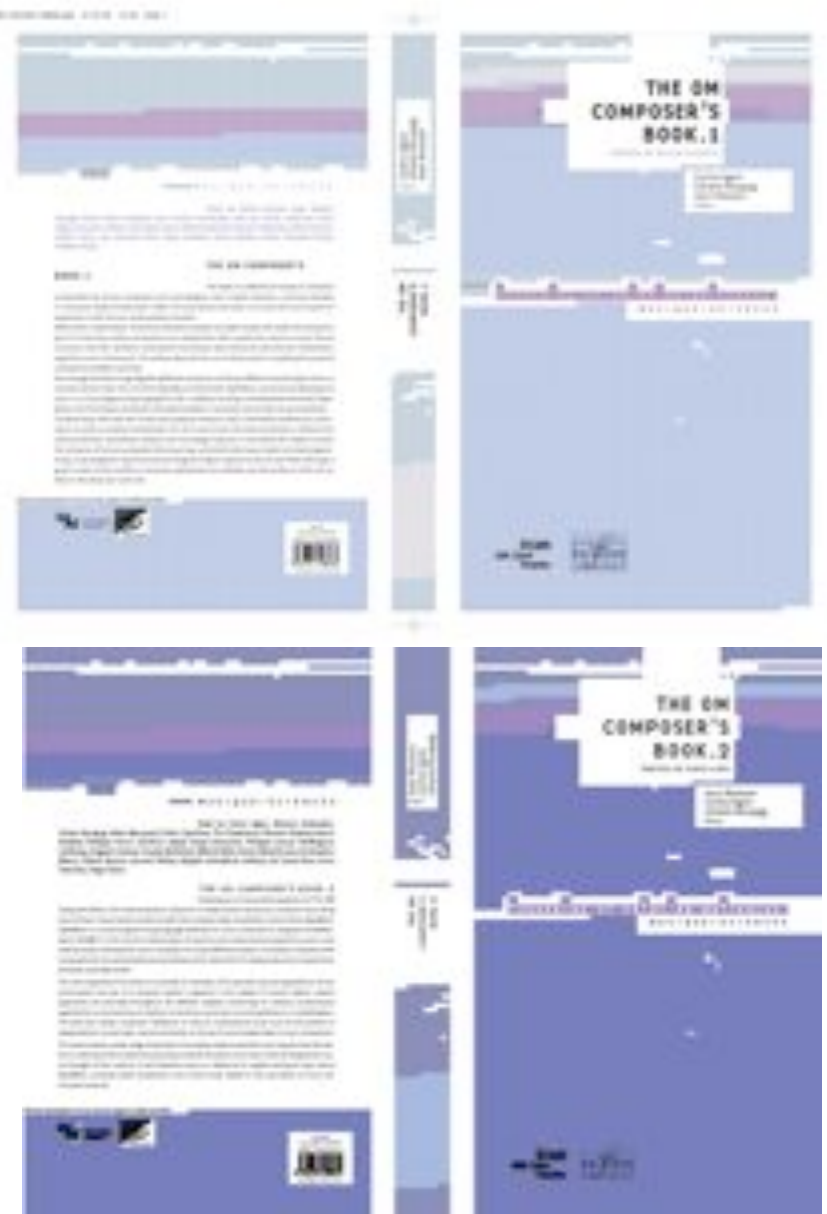
OpenMusic

(c) Ircam - Centre Pompidou



Dedicated to the memory of Gérard Grisey (French composer, 1940-1998)

Design and development : G. Assayag, A. Agon and J. Bresson
with help from C. Ronda, O. Delerue, Uwe Mülhausen (Grame)
Musical expertise by : M. Andreatta, J. Babou, J. Finsberg, K. Halbed,
C. Matherbe, M. Malt, T. Mural, O. Sandret, M. Stoppa, K. Tutschku.
Artwork : A. Mülhausen.

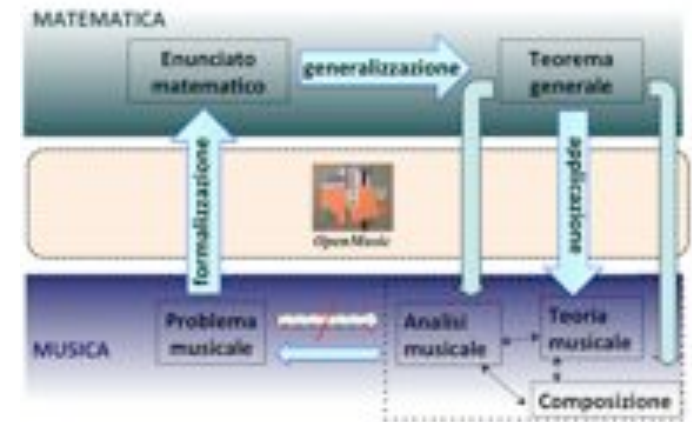


C. Agon, G. Assayag, J. Bresson, *The OM Composer's Book* (2 volumi), 2006-2007

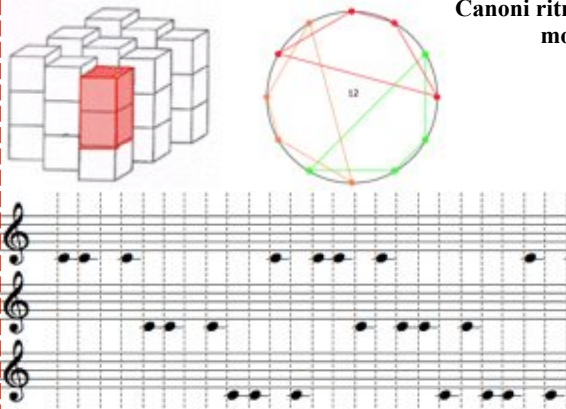
Alcuni esempi di problemi ‘matemusicali’

M. Andreatta: *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

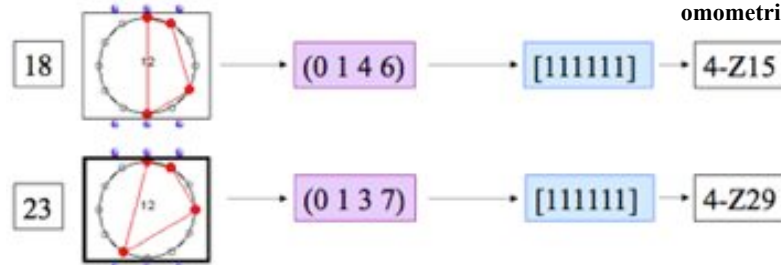
- La costruzione dei canoni ritmici a mosaico
- La relazione Z e la teoria dell'omometria
- La *Set Theory* e la *Transformational Theory*
- Teorie neo-riemanniane e programmazione spaziale
- Teorie diatoniche e *Maximally-Even Sets*
- Sequenze periodiche e calcolo delle differenze finite
- *Block-designs* e composizione algoritmica



Canoni ritmici a mosaico



Z-relazione e insiemi omometrici

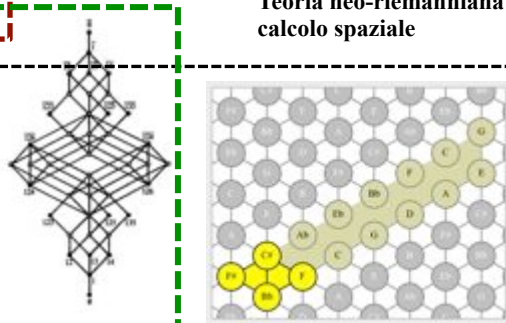


$$Df(x) = f(x) - f(x-1).$$

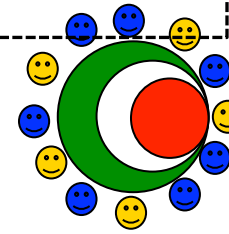
7 11 10 11 7 2 7 11 10 11 7 2 7 11...
 4 11 1 8 7 5 4 11 1 8 7 5 4 11...
 7 2 7 11 10 11 7 2 7 11 10 11...
 7 5 4 11 1 8 7 5 4 11 1 8...

Calcolo delle differenze finite

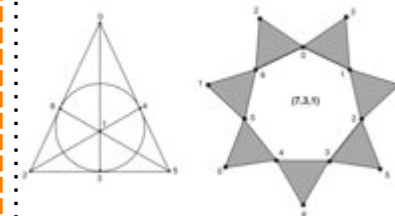
Teoria neo-riemanniana e calcolo spaziale



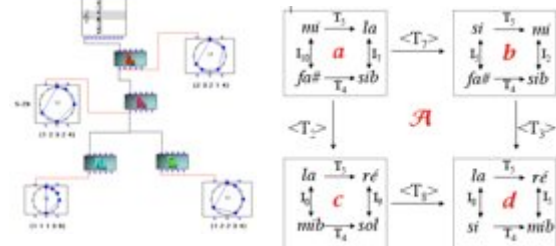
Teorie diatoniche e ME-sets



Block-designs

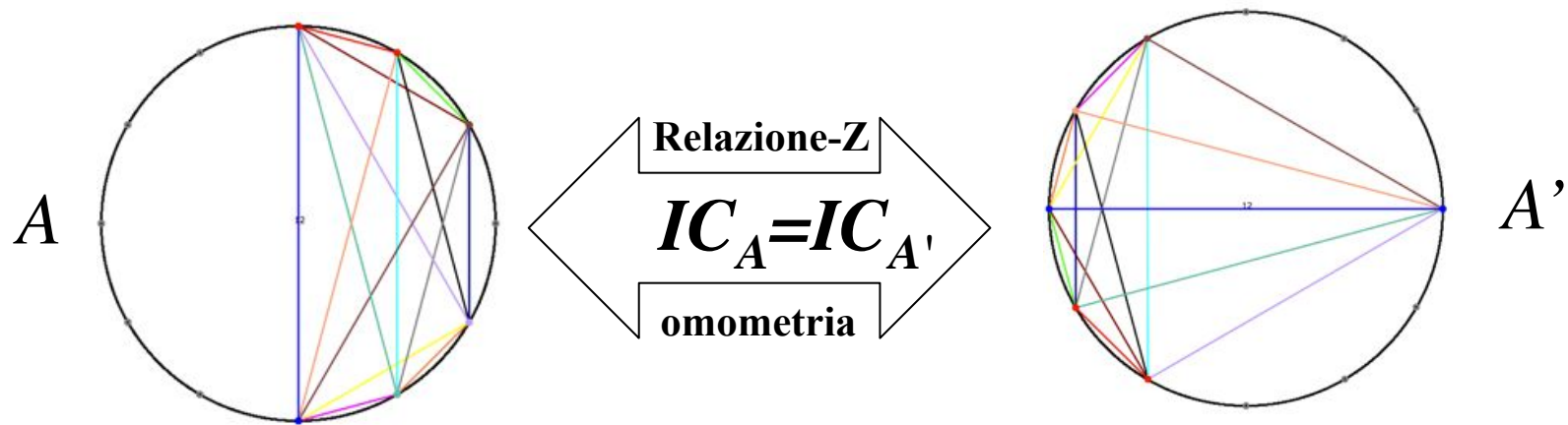


Set Theory e teoria trasformativa



Un esempio di teorema ‘matemusicale’

Il *contenuto intervallare* IC_A di un accordo è la molteplicità di apparizione dei suoi intervalli



$$IC_A = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = IC_{A'}$$

Teorema dell'esacordo (Milton Babbitt):
Un esacordo e il suo complementare hanno lo stesso contenuto intervallare.



Milton Babbitt

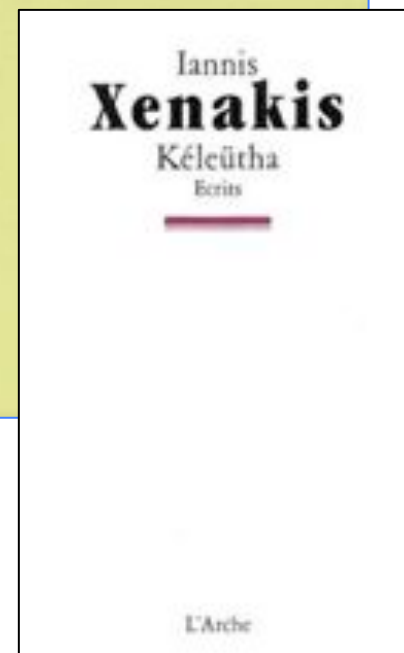
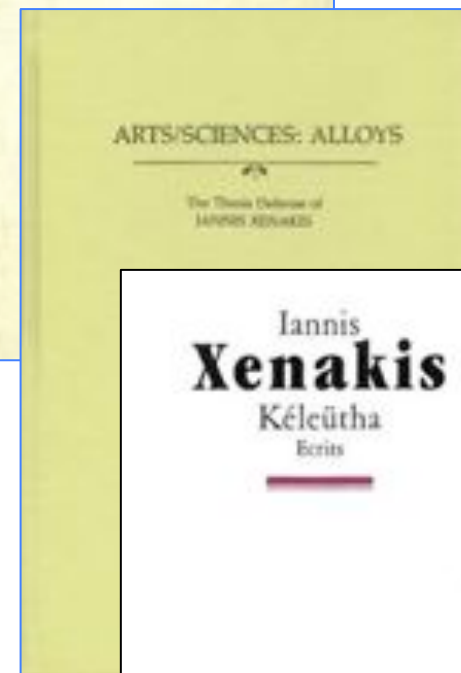
(Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, ..., Amiot, ...)

Musica e matematica: “prima la musica”!



Iannis Xenakis

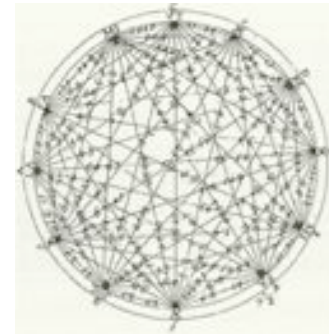
MUSIQUE	MATHS
500 av. J. C. Relation hauteur/longueur corde. La musique est source d'inspiration pour la théorie des nombres et la géométrie.	Nombres naturels et rationnels.
<i>Pas de correspondance musicale.</i>	Nombres irrationnels, théorème de Pythagore.
300 a.J. Invention (théorique) de la gamme chromatique tempérée égale par Aristoxénos de Tarente) et prémonition de la théorie des groupes . Isomorphismes entre les logarithmes (intervalles musicaux) et les exponentiels (longueur d'une corde).	Les mathématiques ne réagissent pas.
1000 ap. J.C. Invention de la représentation bidimensionnelle des hauteurs.	<i>Aucune correspondance.</i>
1500 <i>Aucune reprise des concepts précédents.</i>	Nombres négatifs. Construction des rationnels.
1600 <i>Aucune relation.</i>	Nombres réels et les logarithmes. Invention des repères cartésiens.
1700 La fugue comme un automate abstrait. Manipulation inconsciente du groupe de Klein.	Nombres complexes (Euler, Gauss), les quaternions (Hamilton), continuité (Cauchy), structure de groupe (Galois, Abel).
1900 Libération de la prison de la tonalité (Loquin, Hauer, Schoenberg).	Nombres infinis et transfinis (Cantor). Axiomatique de Peano. Théorie de la mesure (Lebesgue, Borel).
1920 Formalisation radicale des macrostructures à travers le système sériel (Schoenberg).	<i>Aucun développement de la théorie des nombres. Logique (contradictions de la théorie des ensembles).</i>



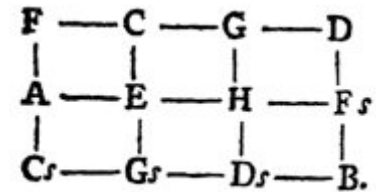
- *Musique. Architecture*, Casterman, 1971/1976
- *Arts/Sciences Alliages*, Casterman, 1979 (tr. *Arts/Sciences. Alloys*, Pendr. Press, 1985)
- « Les chemins de la composition musicale » (tr. Française E. Gresset, in *Musique et ordinateur*, Les Ulis, 1983)
- « Music Composition Treks », in *Composers and the Computer*, edited by C. Roads, MIT Press, Cambridge, Mass, 1985

Musica e matematica: alcune dimenticanze...

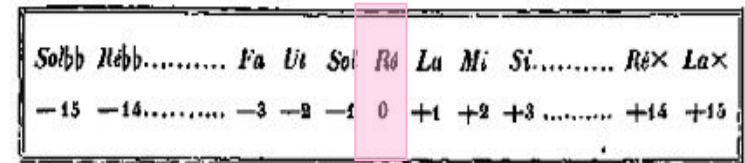
MUSIQUE	MATHS
500 av. J. C. Relation hauteur/longueur corde. La musique est source d'inspiration pour la théorie des nombres et la géométrie.	Nombres naturels et rationnels.
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1500 Aucune reprise des concepts précédents.	Nombres négatifs. Construction des rationnels.
1600 Aucune relation.	Nombres réels et les logarithmes. Invention des repères cartésiens.
1648 Marin Mersenne : invention de la combinatoire musicale (<i>Harmonicorum Libri</i>)	Systématisation du calcul des probabilités par Bernoulli (<i>Ars Conjectandi</i> , 1713)
1700 La fugue comme un automate abstrait. Manipulation inconsciente du groupe de Klein.	Nombres complexes (Euler, Gauss), les quaternions (Hamilton), continuité (Cauchy), structure de groupe (Galois, Abel).
1773 Leonhard Euler : représentation géométrique des hauteurs (<i>Speculum Musicum</i>)	Invention de la théorie des graphes
1855 Camille Durutte : analyse harmonique, rythmique et mélodique	Développement en série d'une fonction (Wronski)
1900 Libération de la prison de la tonalité (Loquin, Hauer, Schoenberg).	Nombres infinis et transfinis (Cantor). Axiomatique de Peano. Théorie de la mesure (Lebesgue, Borel).
1920 Formalisation radicale des macrostructures à travers le système sériel (Schoenberg).	Aucun développement de la théorie des nombres. Logique (contradictions de la théorie des ensembles).
1937-1939 Ernst Krenek : les axiomes en musique	David Hilbert, <i>Les fondements de la géométrie</i> (1899)
1946 Milton Babbitt : théorie des groupes et système dodécaphonique	Rudolf Carnap, <i>The Logical Syntax of Language</i> (1937)



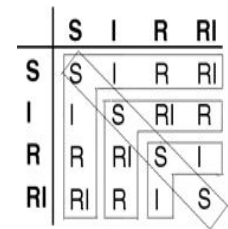
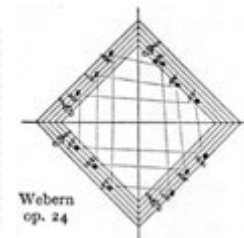
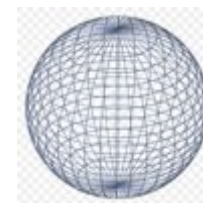
Mersenne,
*Harmonicorum
Libri XII*, 1648



Euler, *Speculum
musicum*, 1773



Durutte, *Technie, ou lois générales du
système harmonique* (1855)



Krenek e Babbitt, tecnica dodecafonica,
assiomatica e gruppo di Klein

Il ruolo della matematica nella musicologia sistematica

Guido Adler: „Umfang, Methode und Ziel der Musikwissenschaft“ (1885)

II. Systematisch.
Aufstellung der in den einzelnen Zweigen der Tonkunst zuhöchst stehenden Gesetze.

A. Erforschung und Begründung derselben in der			B. Aesthetik der Tonkunst.	C. Musikalische Pädagogik und Didaktik	D. Musikologie
1. <i>Harmonik</i> (tonal od. tonlich).	2. <i>Rhythmik</i> (temporär oder zeitlich).	3. <i>Melik</i> (Cohärenz von tonal und temporär).	1. Vergleichung und Werthschätzung der Gesetze und deren Relation mit den apperzipirenden Subjecten behufs Feststellung der <i>Kriterien des musikalisch Schönen</i> .	(Zusammenstellung der Gesetze mit Rücksicht auf den Lehrzweck) 1. Tonlehre, 2. Harmonielehre, 3. Kontrapunkt, 4. Compositionslehre,	(Untersuchung und Vergleichung zu ethnographischen Zwecken).
			2. Complex unmittelbar und mittelbar damit zusammenhängender Fragen.	5. Instrumentationslehre, 6. Methoden des Unterrichtes im Gesang und Instrumentalspiel.	

Hilfswissenschaften: Akustik und Mathematik.

Physiologie (Tonempfindungen).

Psychologie (Tonvorstellungen, Tonurtheile und Tongefühle).

Logik (das musikalische Denken).

Grammatik, Metrik und Poetik.

Pädagogik

Ästhetik etc.

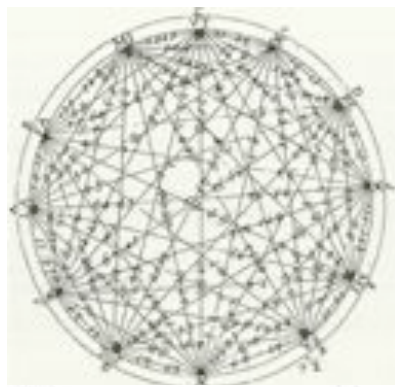


G. Adler (1855-1941)

“La seconda grande parte della musicologia è la parte sistematica, basata sulla dimensione storica. (...) L’accento dell’osservazione risiede nell’analogia fra metodo musicologico e metodo scientifico”

I cataloghi musicali da Mersenne a Forte passando per Costère

Edmond Costère, *Lois et Styles des Harmonies Musicales*. Paris: Presses Universitaires de France, 1954.



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LIBER SEPTIMVS.
DE CANTIBVS, SEV CANTILENIS,
EARVMO: NVMERO, PARTIBVS, ET SPECIEBVS.

Tabella pulcherrima & utilissima Combinationis duodecim Cantilenarum.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
1	1	1	1	1	1	1	1	1	1	1	1
2	5	4	3	2	1	1	2	3	4	5	6
3	6	10	15	20	25	30	35	40	45	50	55
4	10	20	35	54	78	108	143	183	228	278	333
5	15	35	70	126	210	330	495	715	1001	1365	1820
6	21	56	126	252	462	792	1287	2002	3003	4368	6083
7	28	84	210	462	924	1716	3003	5005	8005	12376	18564
8	36	120	330	792	1716	3432	6435	11440	19448	31824	50188
9	45	165	495	1287	3003	6435	12870	24110	43738	75181	125970
10	55	220	715	2002	5005	11440	24110	43738	81710	149614	270415
11	66	286	1001	3003	8005	19448	43738	91378	185476	351716	646646
12	78	378	1365	4368	12870	34320	81710	185476	437380	1001001	2257920
13	91	495	1848	6188	18480	50050	125970	295930	646646	1551078	3551078
14	105	636	2430	8168	24300	67200	167960	401775	913780	2101001	4810010
15	120	816	3003	10010	30030	85800	210100	500500	1144000	2600000	5900000
16	136	1001	3780	12870	37800	106260	270415	646646	1551078	3551078	8166600
17	153	1287	4950	17160	50050	143000	355107	816660	1944800	4510000	10200000
18	171	1596	6188	22579	64350	184800	451000	1020000	2357920	5510000	12579200
19	190	1848	7920	28600	81660	235792	551000	1257920	2959300	7000000	15900000
20	210	2184	9900	35100	100100	286000	700000	1590000	3780000	9137800	20700000
21	231	2520	11970	42840	123000	343200	858000	2001000	4730000	11000000	25000000
22	253	2860	14300	52140	153000	430000	1062600	2500000	5900000	13600000	30800000
23	276	3246	17160	63600	184800	521400	1259700	3000000	7000000	15900000	35510000
24	300	3780	20020	77220	225792	643500	1551078	3551078	8166600	19448000	45100000
25	325	4284	24300	92400	270415	772200	1944800	4510000	10200000	23579200	55100000

THESAURUS

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(0 1 3 7)

Interval vector: +1 1 1 1 1 1

Triad type: (0 1 3 7)

complementary: (0 2 3 4 6 7 8 9)

element: (0 1 4 6) (0 2 5 8)

Tri-roughness: 4.08 factor: 1.18

tenacity: 9.38

phenoty: 33.75

Costive number: 21 13 14 1+0

Forte number: 4-229

asym: -

-87.21°

0 1 2 3 4 5 6 7 8 9 10

root: 15 11 (2) 15 (0 8 5) 10 (8 5 1 4)

vertex: 10 12 (6) 11 (3 6 0) 10 (5 1 7 5)

cardinal: 2 2 (3) 1 (1 1 2) 2 (3 0 1 1)

tonal M: (6 0 5 4 5 4 5 6 7 3 5 4)

tonal m: 6 (6 4 4 4 7 4 6 5 4 4 6)

transpositional: 8 (8 5 5 5 8 6 5 5 8)

inversional: (5 0 8 10 4 6 5 8 8 4 6)

Tri invariance: 4 1 1 1 1 1 2 1 1 1 1

Tri invariance: 1 2 2 2 2 0 1 2 2 0 2 0

4
forms

Asymmetrical

relative: (0 4 6 7)

integrally total-invariant

(0 1 2 3 4 5 6 7 8 9 10 11)

Cardinally Transitive

with balanced cardinal pole

cardinal poles: 0 (2) (5)

Tonal Minor

Tonal Major

Tonal Minor

Tonal Major

total poles: (0 1) (5 6)

intrinsic tonic poles: (0 1)

COMMONALITY

0 1 2 3 4 5 6 7 8 9 10 11

Tri: 100 20 20 20 20 20 20 20 20 20 20 20

Tri: 30 47 47 31 44 24 35 37 34 30 49 23

(0 4 7): 62 31 31 31 31 31 31 31 31 31 31 31

(0 1 3 7): 79 23 10 28 25 34 28 31 31 23 25 12

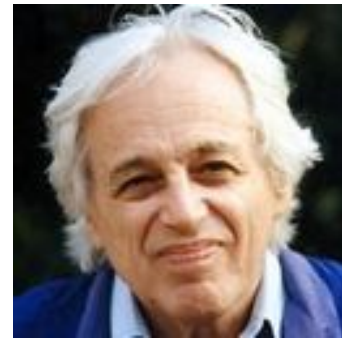
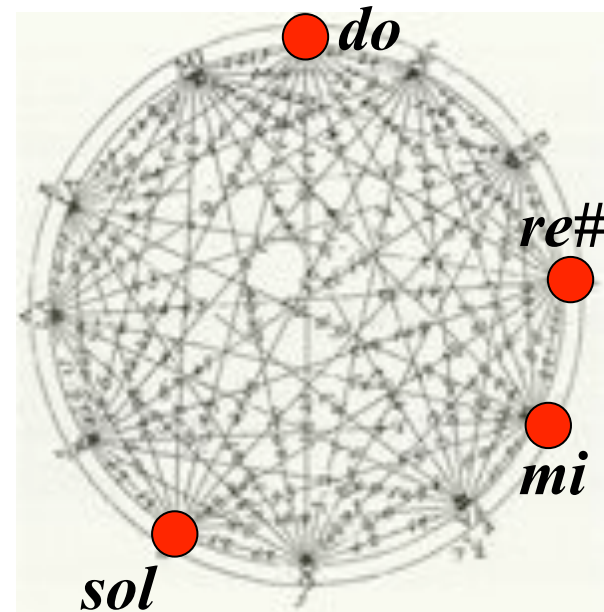
Mersenne e la nascita della combinatoria musicale

II 4. Marin Mersenne, *Harmonicorum Libri XII*, 1648

LIBER SEPTIMVS. DE CANTIBVS, SEV CANTILENIS, EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 12.

I	1
II	1
III	6
IV	24
V	110
VI	710
VII	3040
VIII	40310
IX	361880
X	3618800
XI	39916800
XII	479001600
XIII	6127010800
XIV	87178191000
XV	1307674368000
XVI	20911789888000
XVII	313687418096000
XVIII	6401373705718000
XIX	11164100408310000
XX	2431301008176640000
XXI	5090941871709440000
XXII	1114000717777607680000



Six Bagatelles
(G. Ligeti, 1953)



Approcci 'permutazionali' nella canzone

Se telefonando, 1966 (Maurizio Costanzo/
Ennio Morricone). Interprete: Mina



Je changerais d'avis, 1966 (Françoise
Hardy/Ennio Morricone)

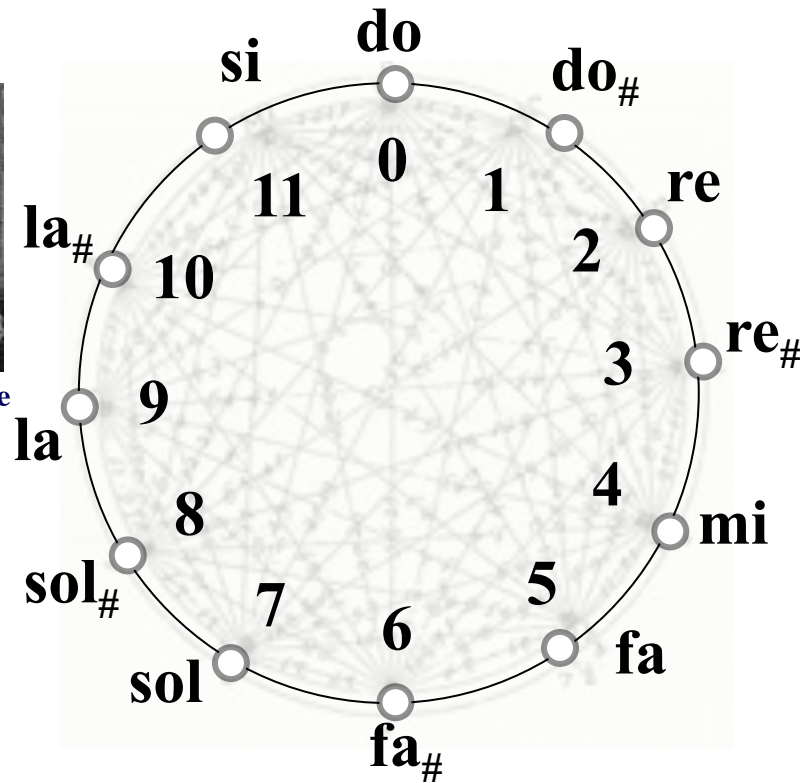


Ennio Morricone

La rappresentazione circolare delle note musicali



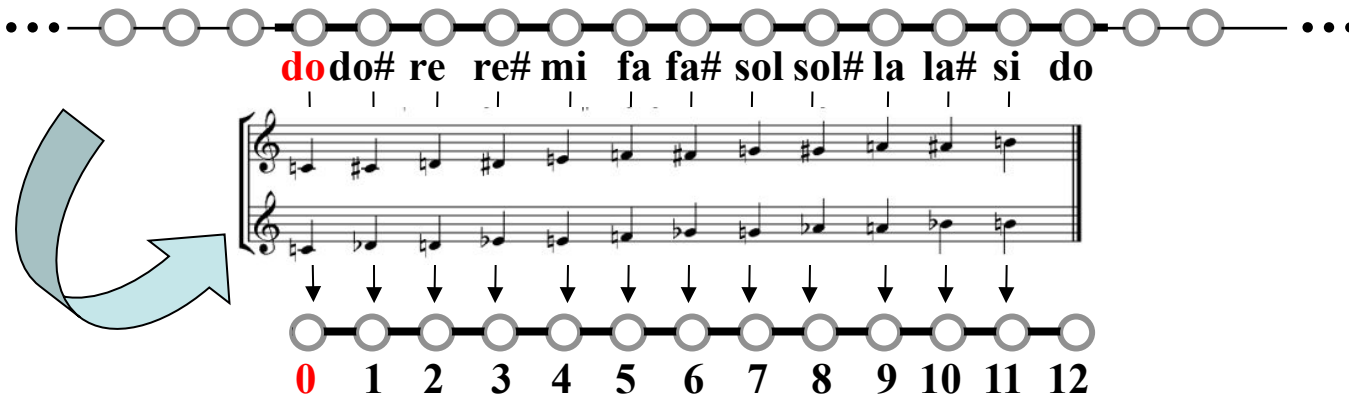
Marin Mersenne



Harmonicorum Libri XII, 1648



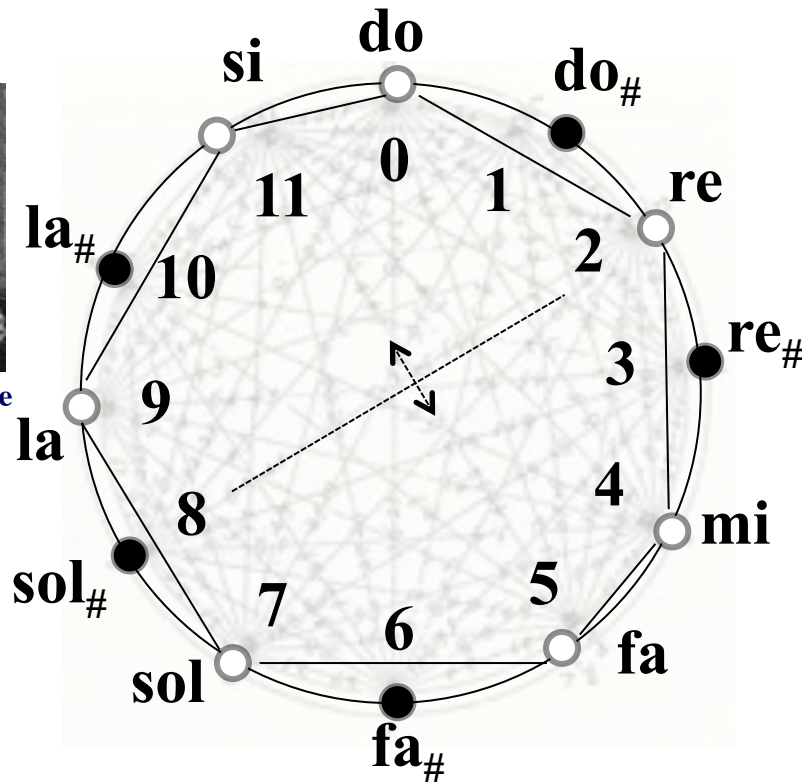
M. Andreatta, C. Agon,
« La musique mise en algèbre »,
Pour la Science, 2008



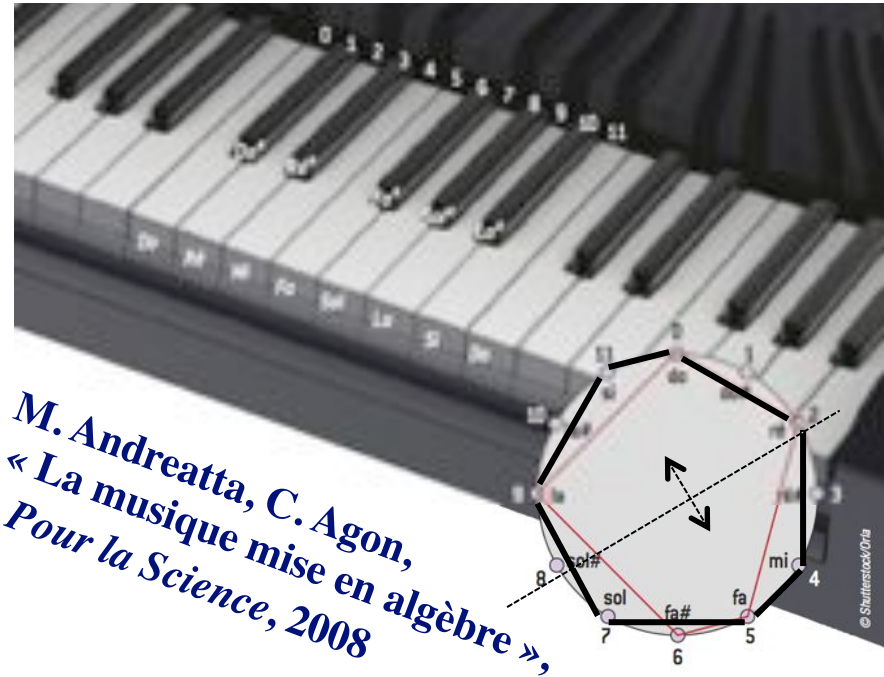
La rappresentazione circolare delle note musicali



Marin Mersenne

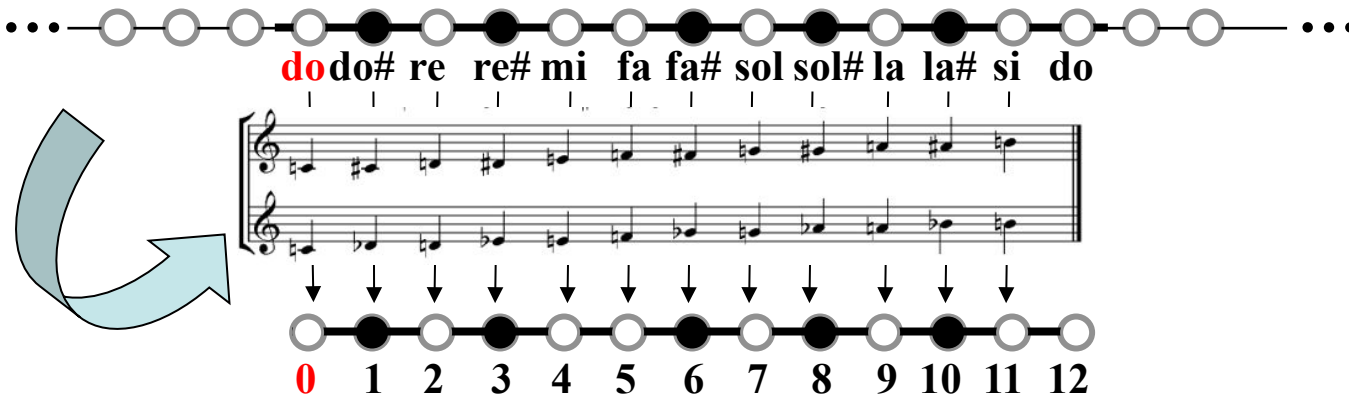


Harmonicorum Libri XII, 1648



M. Andreatta, C. Agon,
« La musique mise en algèbre »,
Pour la Science, 2008

© Shutterstock/Orla

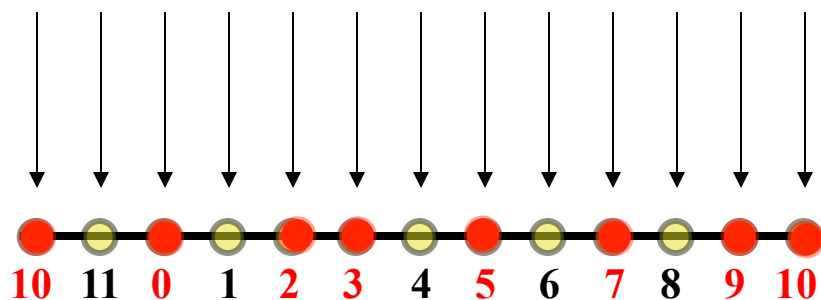


La simmetria del re come base dell'armonia

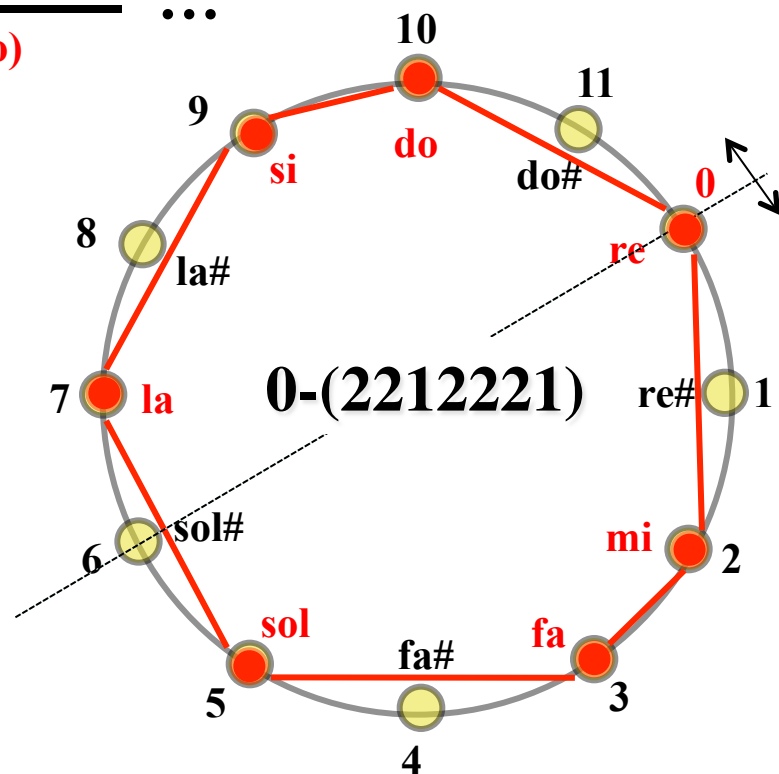
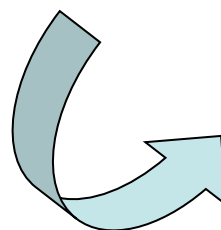


Camille Durutte

Sol $\flat\flat$	Re $\flat\flat$		Fa	Ut	Sol	Re	La	Mi	Si		Re $\times$	La $\times$
-15	-14		-3	-2	-1	0	+1	+2	+3		+14	+15



- *Technie, ou lois générales du système harmonique* (1855)
- *Résumé élémentaire de Technie harmonique, et complément de cette Technie* (1876)



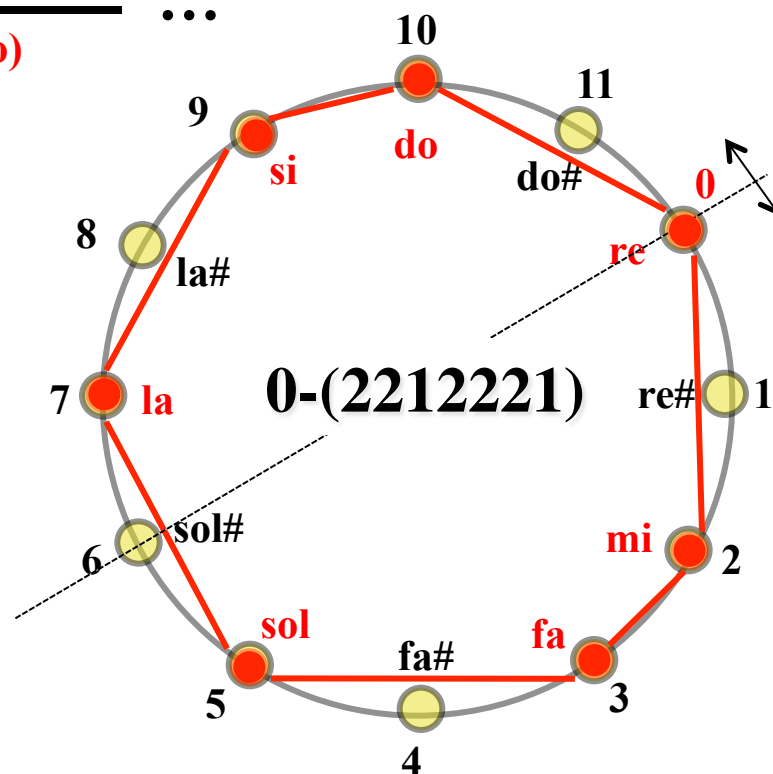
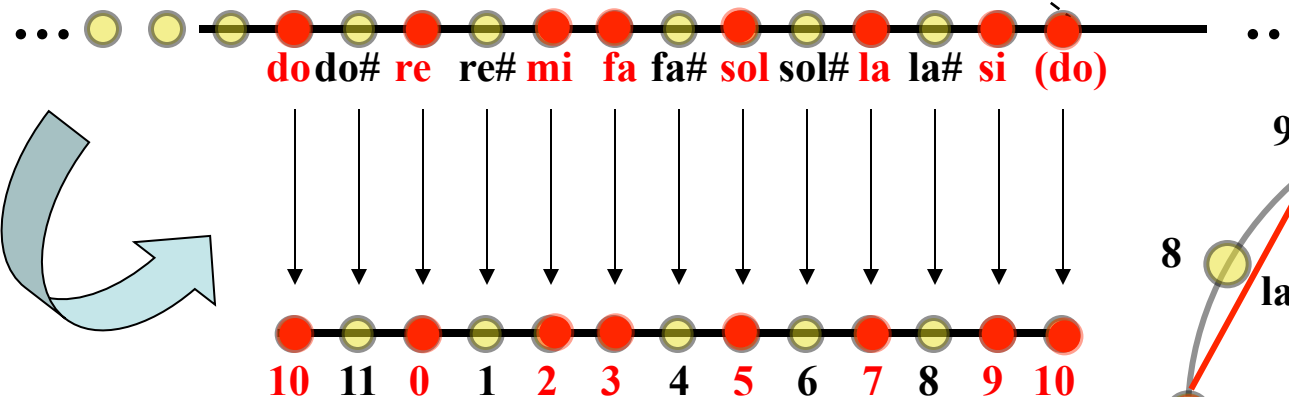
La simmetria del re come base dell'armonia



Sol $\flat\flat$	Re $\flat\flat$		Fa	Ut	Sol	Re	La	Mi	Si		Re $\times$	La $\times$
-15	-14		-3	-2	-1	0	+1	+2	+3		+14	+15



Camille Durutte

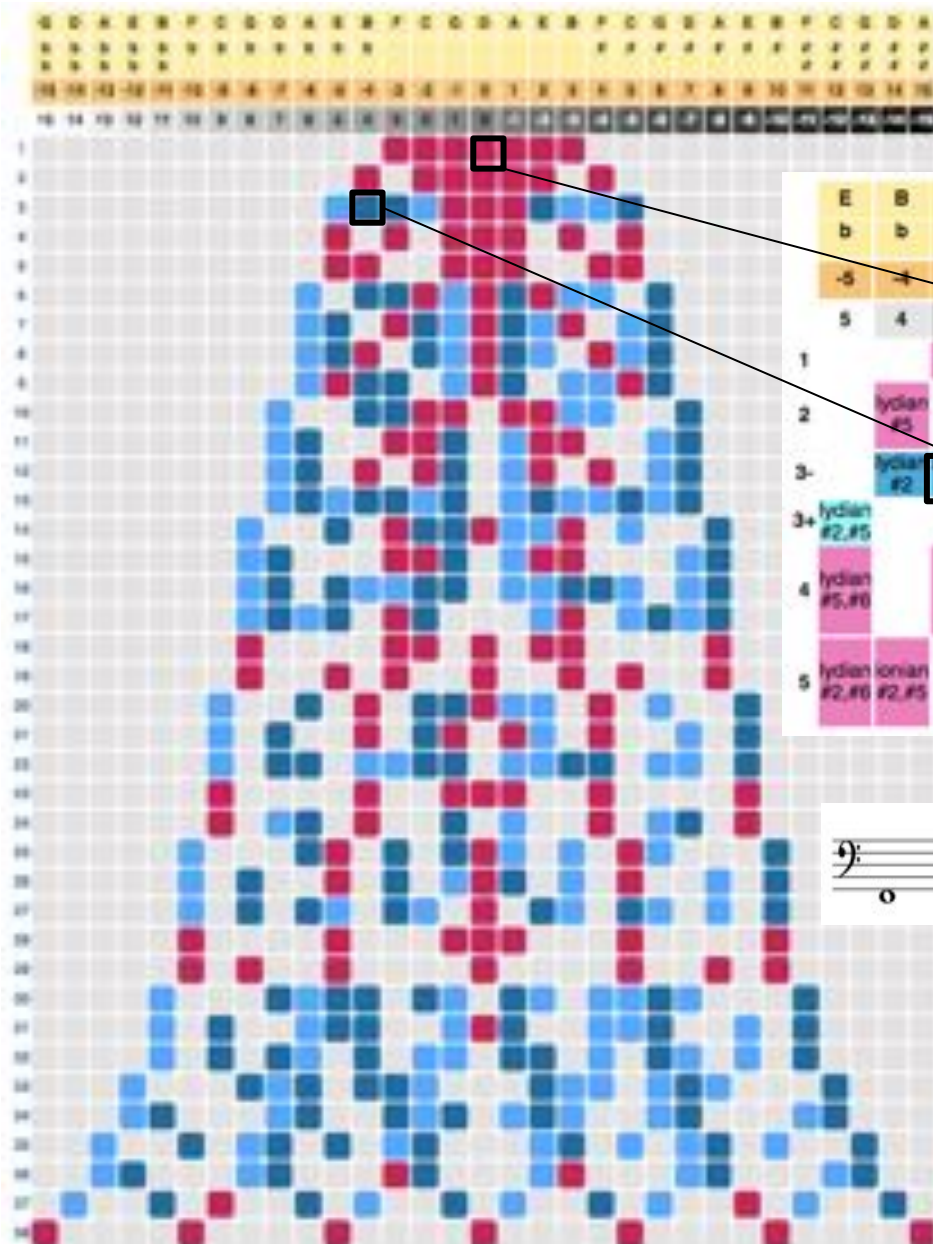


- *Technie, ou lois générales du système harmonique* (1855)
- *Résumé élémentaire de Technie harmonique, et complément de cette Technie* (1876)

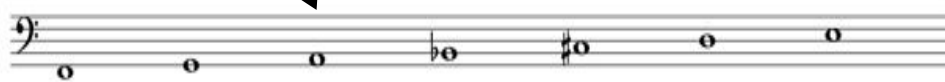
La campana diatonica (o eptatonica)



HE MU
UNIVERSITÉ DE LAUSANNE
conservatoire
de lausanne



	E	B	F	C	G	D	A	E	B	F	C	
	b	b								#	#	
	-5	-4	-3	-2	-1	0	1	2	3	4	5	
	5	4	3	2	1	0	-1	-2	-3	-4	-5	
1			lydian	ionian	mixolydian	dorian	aeolian	phrygian	locrian			diatonic
2		lydian #5		lydian b7	ionian b3	mixolydian b5 or aeolian 3	phrygian 6	locrian 2		locrian b4		minor melodic
3		lydian #2	ionian		dorian #4	aeolian 7	phrygian 3	locrian 5			locrian b4, bb7	minor harmonic
3+	lydian #2, #5			lydian b3	ionian b6	mixolydian b2	dorian b5		phrygian b4	locrian bb7		major harmonic
4	lydian #5, #6		lydian #5, b7		lydian b6, b7	ionian b2, b3 or phrygian 6, 7	locrian 2, 3		locrian 2, b4		locrian bb3, b4	unisonic
5	lydian ionian #2, #6				aeolian #4, 7	ionian b2, b6 or phrygian 3, 7	mixolydian b2, b5			phrygian b4, bb7	locrian bb3, bb7	double harmonic



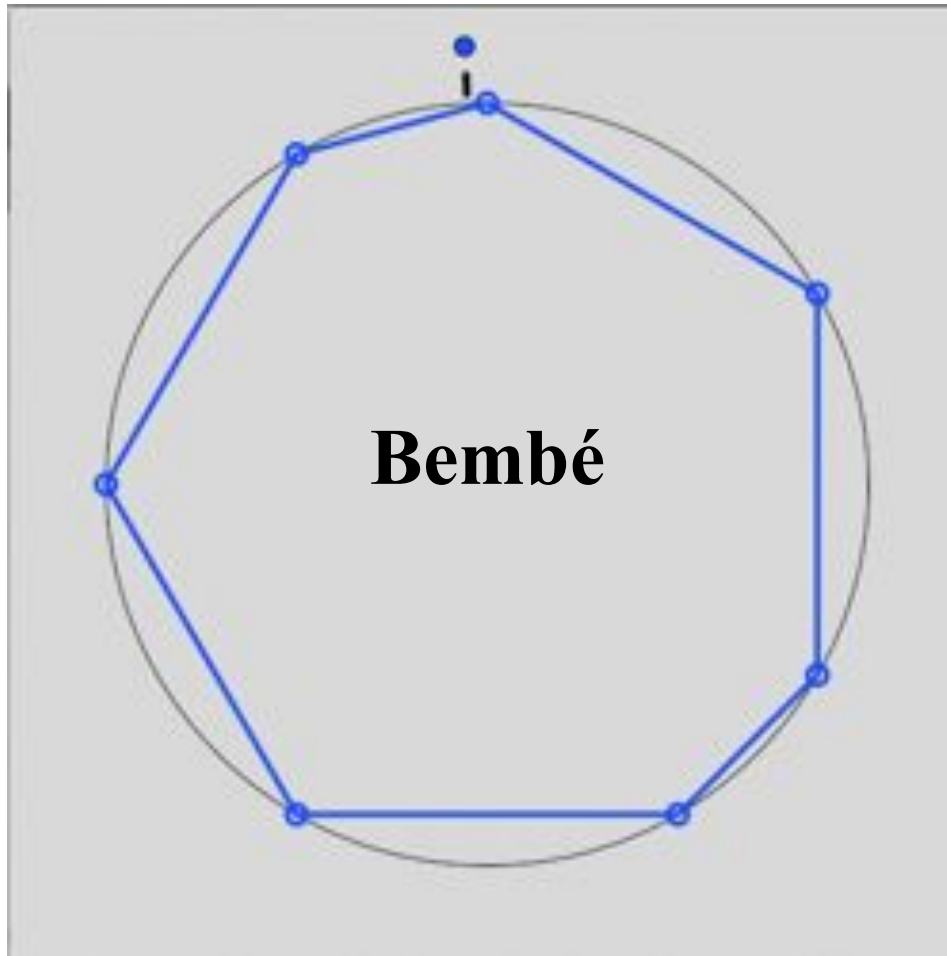
Julien Junod, *Etude combinatoire et informatique du caractère diatonique des échelles à sept notes*, Master ATIAM, Ircam/UPMC, 2008

<http://articles.ircam.fr/>

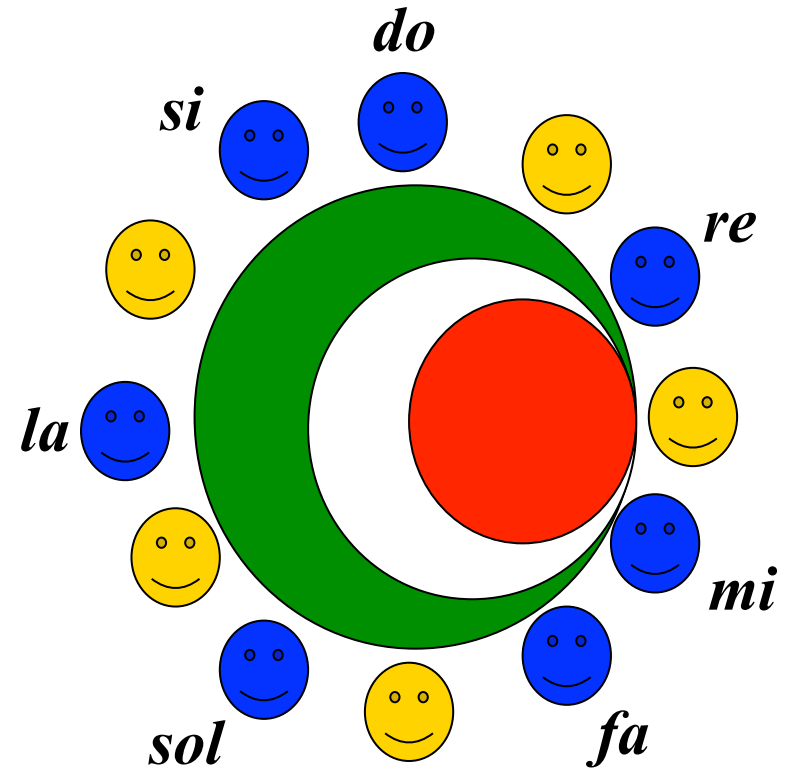


<http://www.cloche-diatonique.ch/>

Insiemi ben ripartiti (*ME-Sets*) e diatonicità



Dinner Table Problem

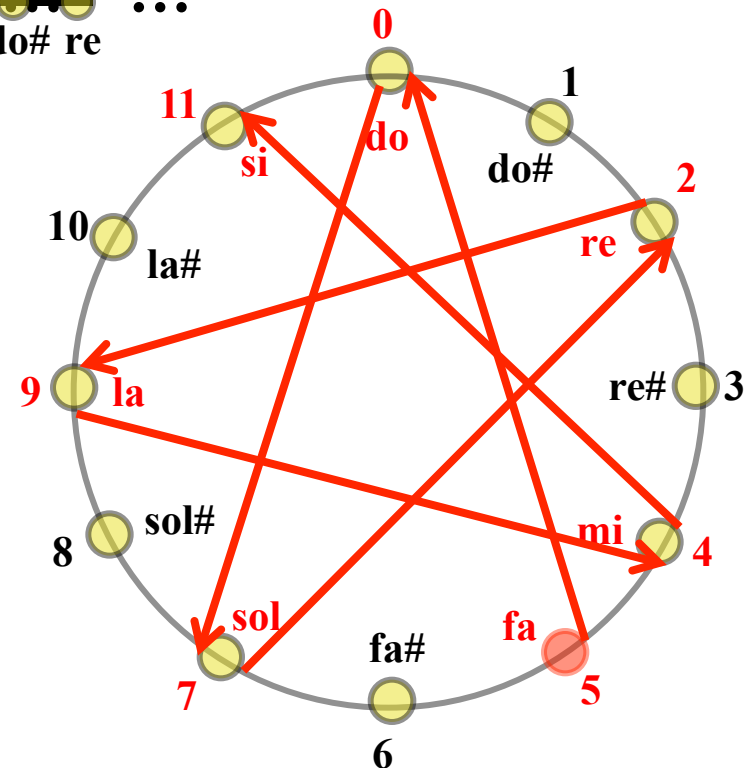
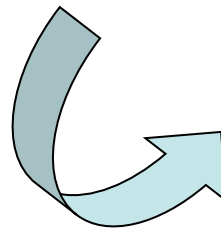
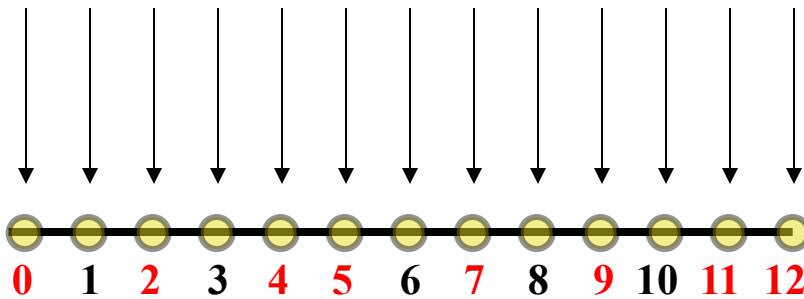
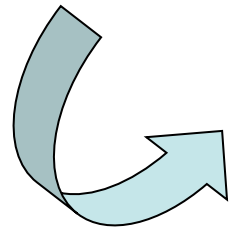


Jack Douthett & Richard Krantz, "Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction", *J. Math. Phys.* 37 (7), July 1996

Il circolo delle quinte e la scala diatonica



... do do# re re# mi fa fa# sol sol# la la# si do do# re ...



La riduzione all'ottava e l'aritmetica modulare



7

... ...

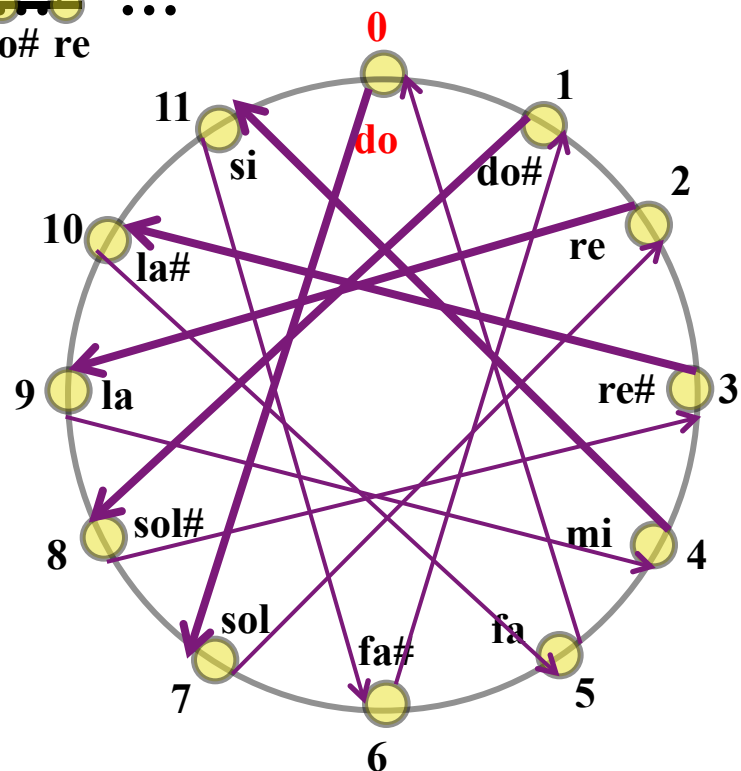
do do# re re# mi fa fa# sol sol# la la# si do do# re



...

0 1 2 3 4 5 6 7 8 9 10 11 12

Circolo delle quinte



La riduzione all'ottava e l'aritmetica modulare



5

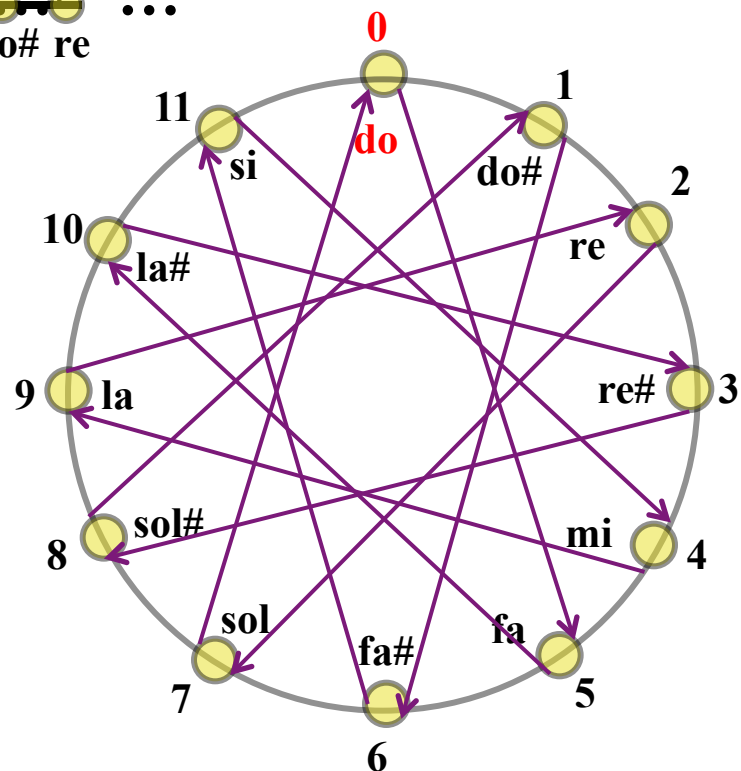
... ...

do do# re re# mi fa fa# sol sol# la la# si do do# re



0 1 2 3 4 5 6 7 8 9 10 11 12

Circolo delle quarte



La riduzione all'ottava e l'aritmetica modulare



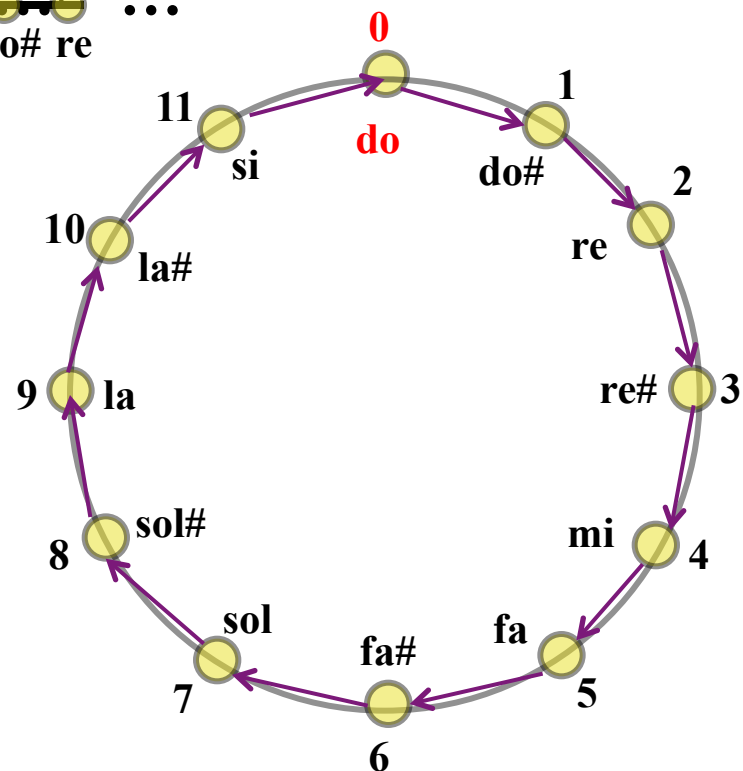
1

... ..
do do# re re# mi fa fa# sol sol# la la# si do do# re

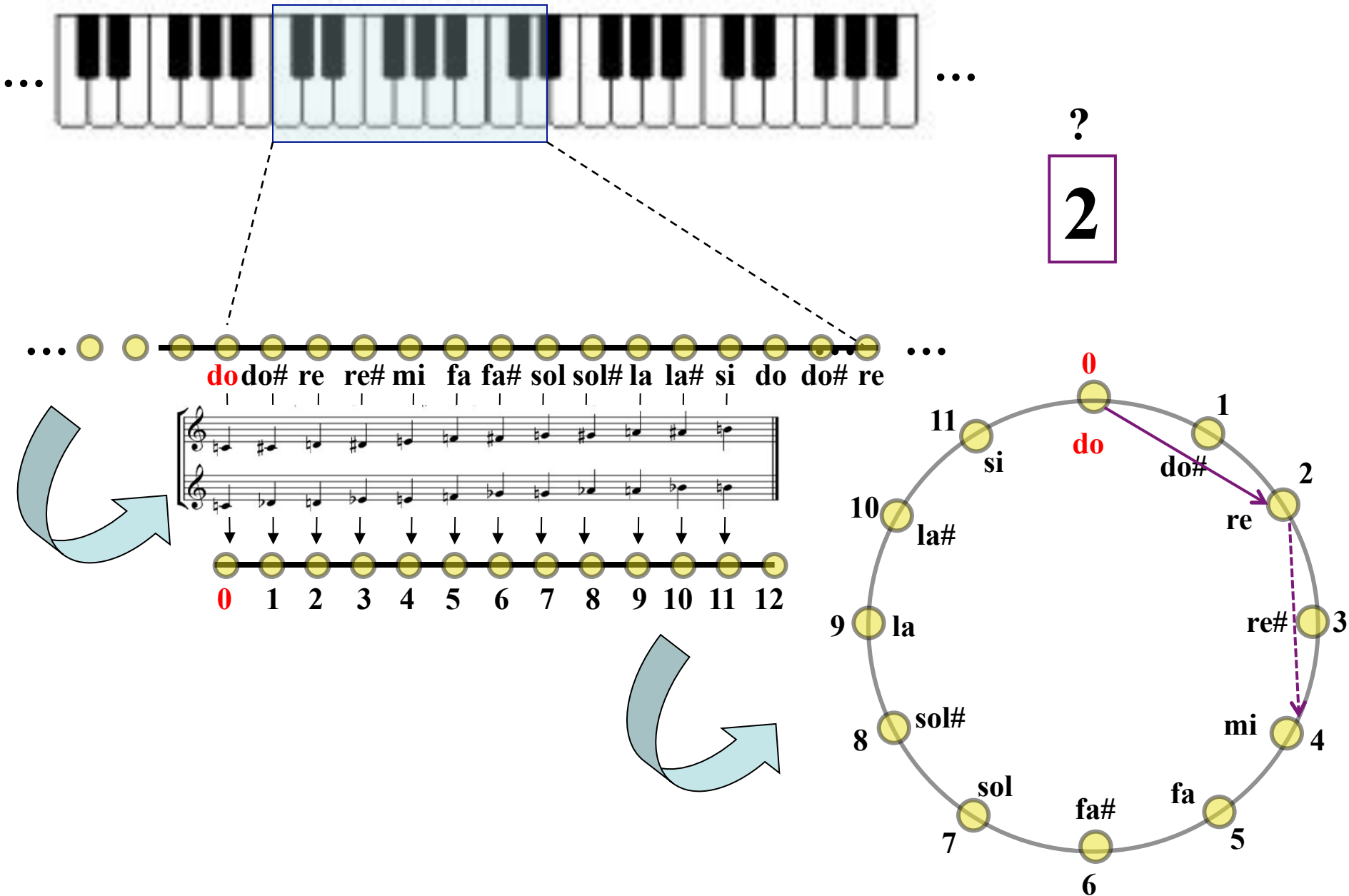


0 1 2 3 4 5 6 7 8 9 10 11 12

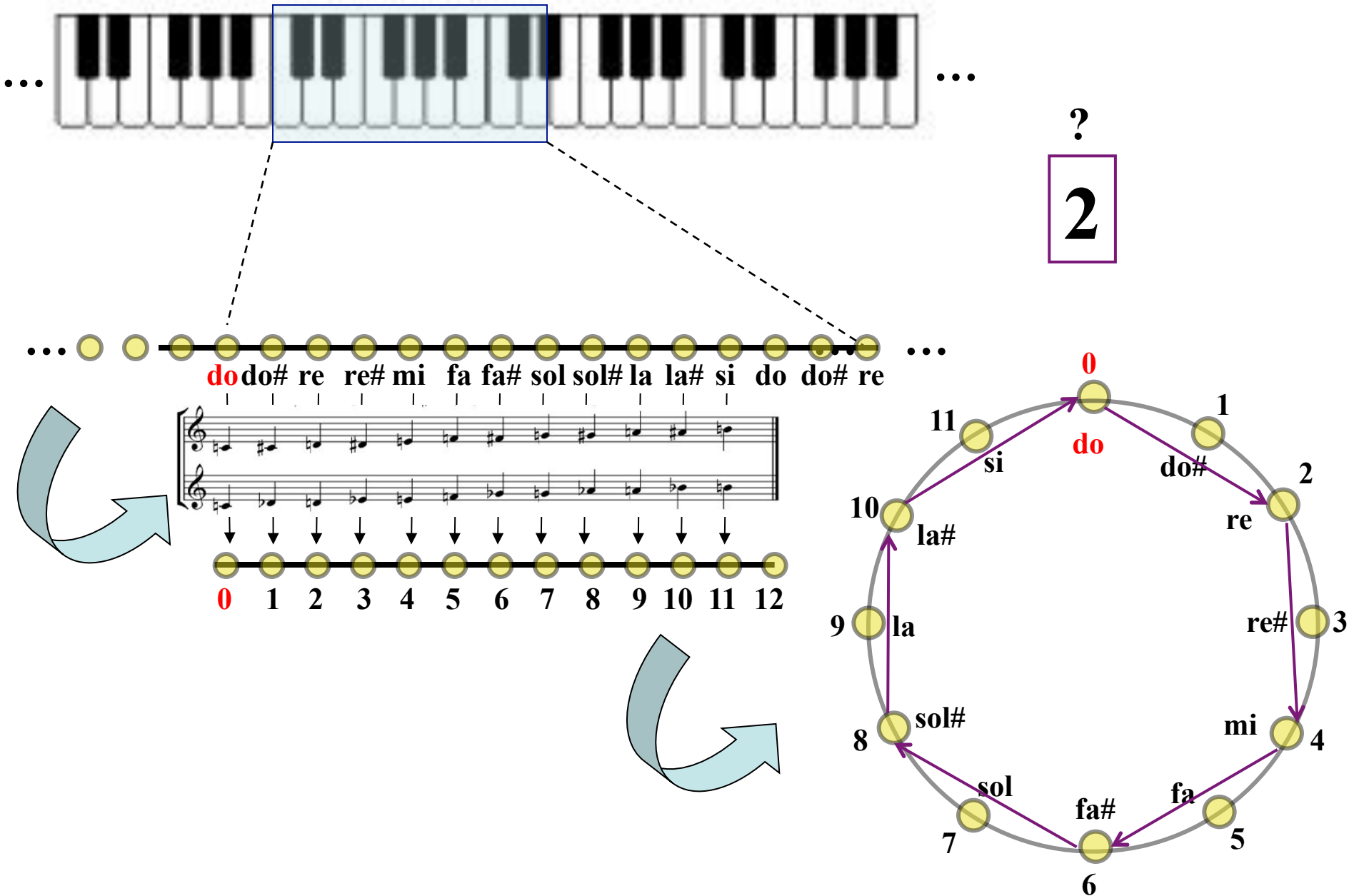
➔ Quali sono gli intervalli
che generano il totale
cromatico?



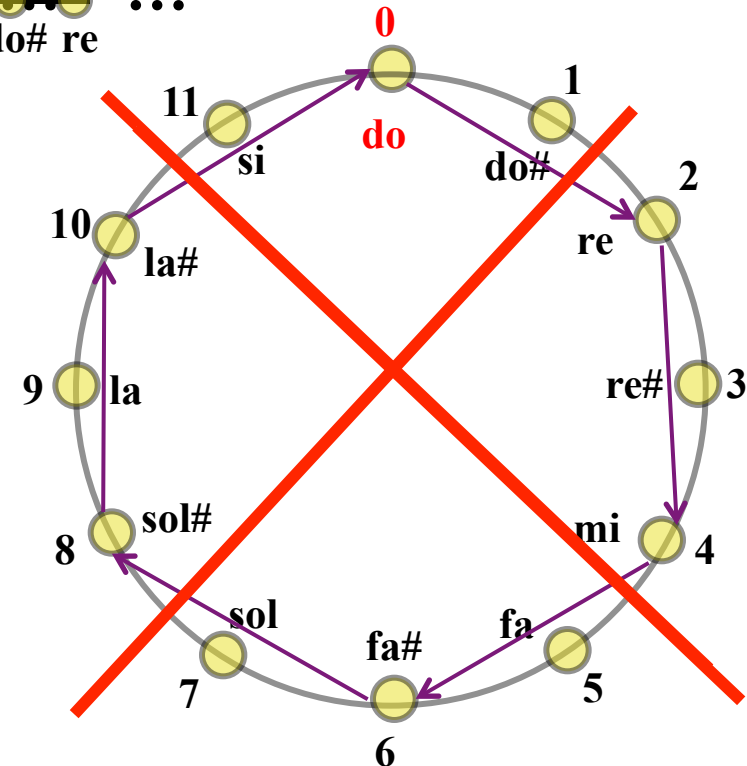
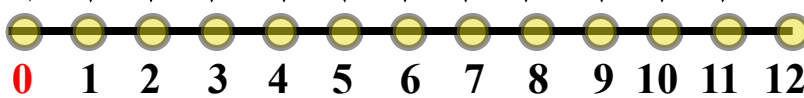
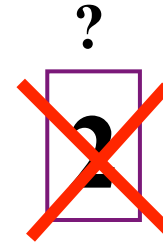
La riduzione all'ottava e l'aritmetica modulare



La riduzione all'ottava e l'aritmetica modulare



La riduzione all'ottava e l'aritmetica modulare



La riduzione all'ottava e l'aritmetica modulare



11

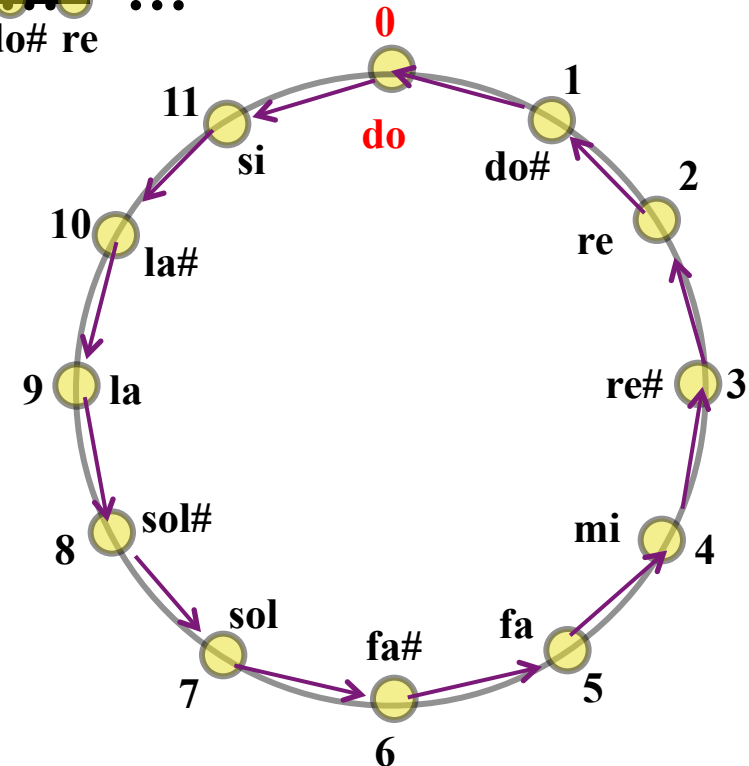
... ...

do do# re re# mi fa fa# sol sol# la la# si do do# re

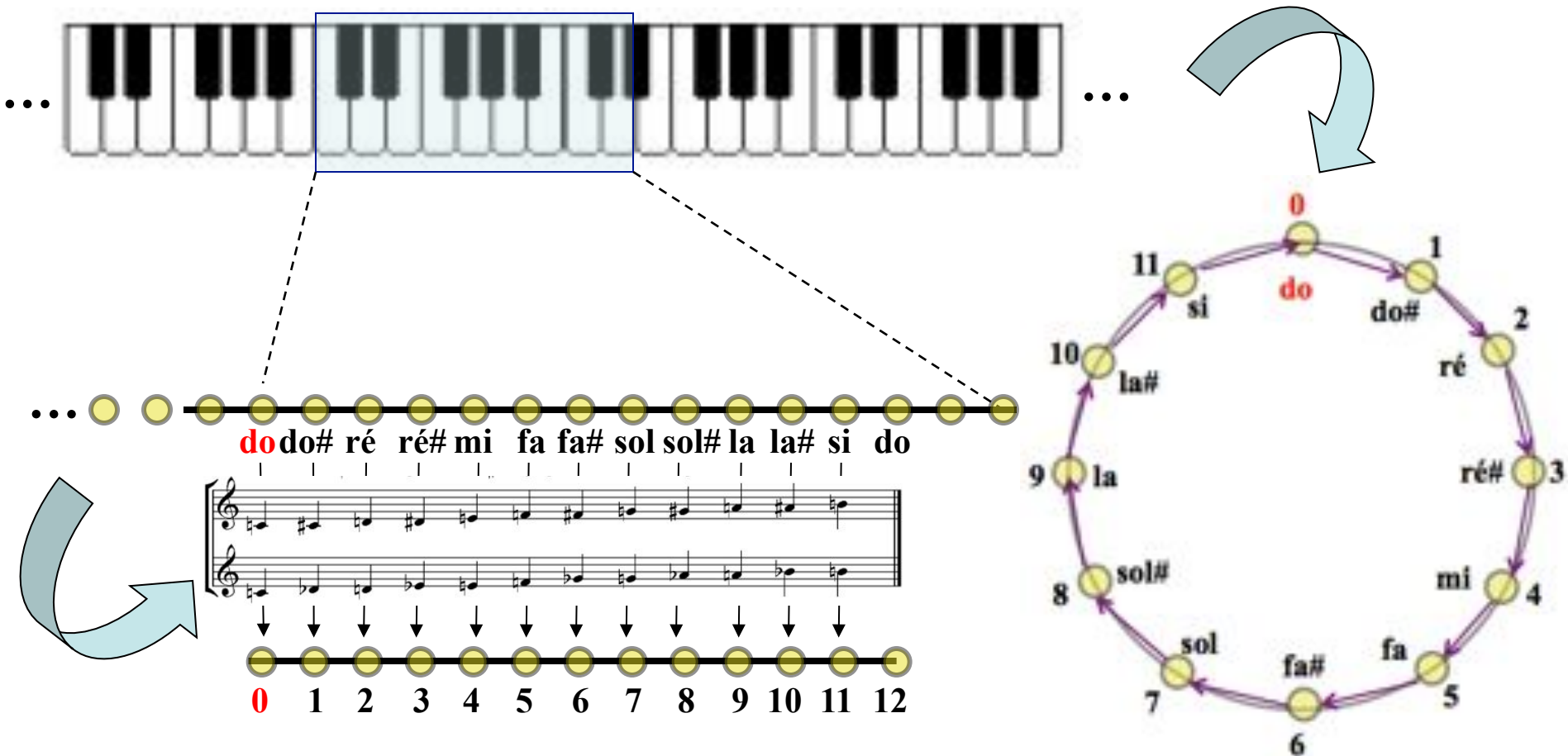


...

0 1 2 3 4 5 6 7 8 9 10 11 12



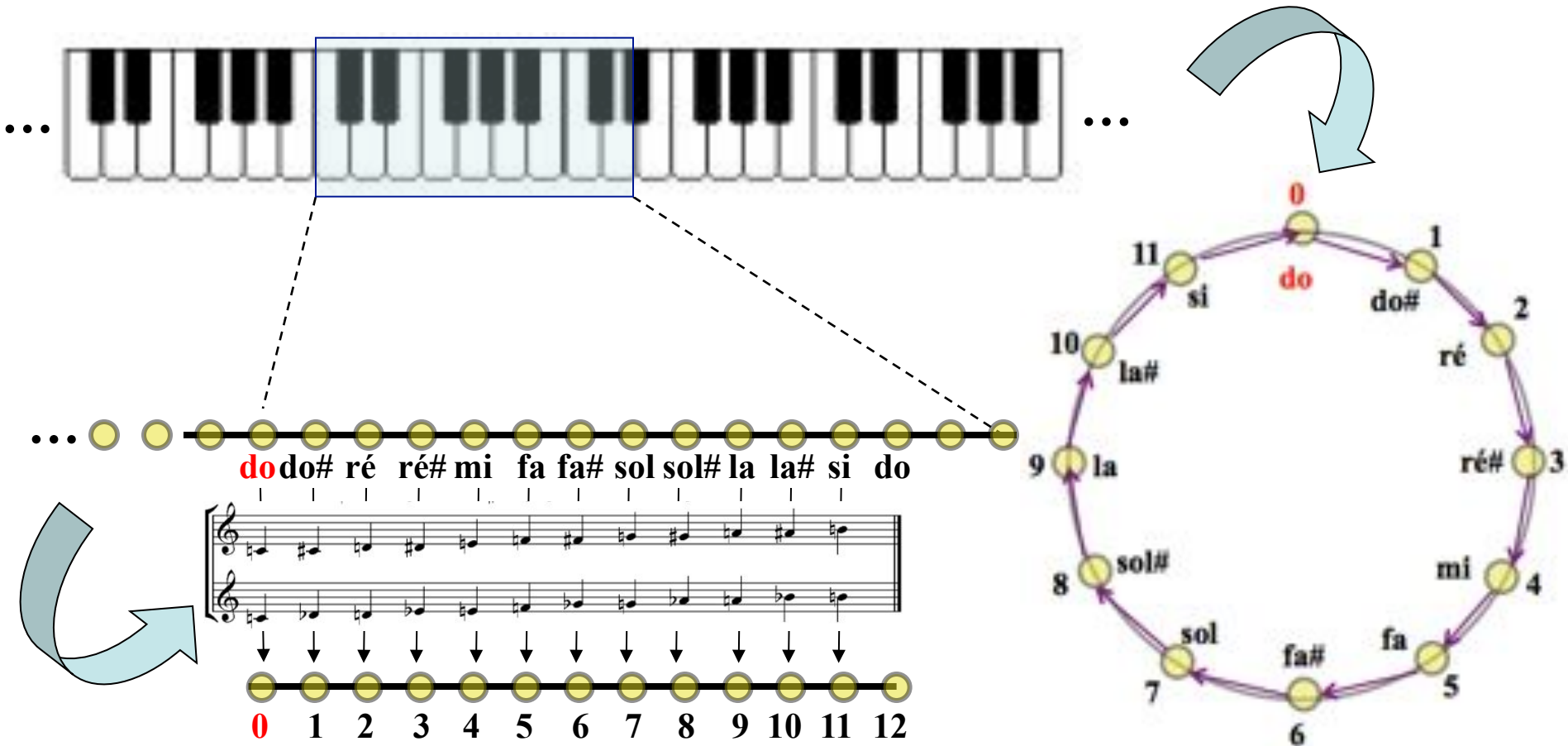
Lo spazio temperato (equabile) è un gruppo (ciclico)



Iannis Xenakis

“L’insieme degli intervalli melodici è munito di una **struttura di gruppo** che ha come legge di composizione interna l’addizione. Si tratta di una struttura che non è caratteristica esclusivamente delle **altezze** ma che si ritrova nell’insieme delle **durate, intensità, densità** e altre qualità sonore o musicali, quali per esempio, il **grado di ordine o di disordine**” (“La voie de la recherche et de la question”, *Preuves*, 1965).

Lo spazio temperato (equabile) è un gruppo (ciclico)



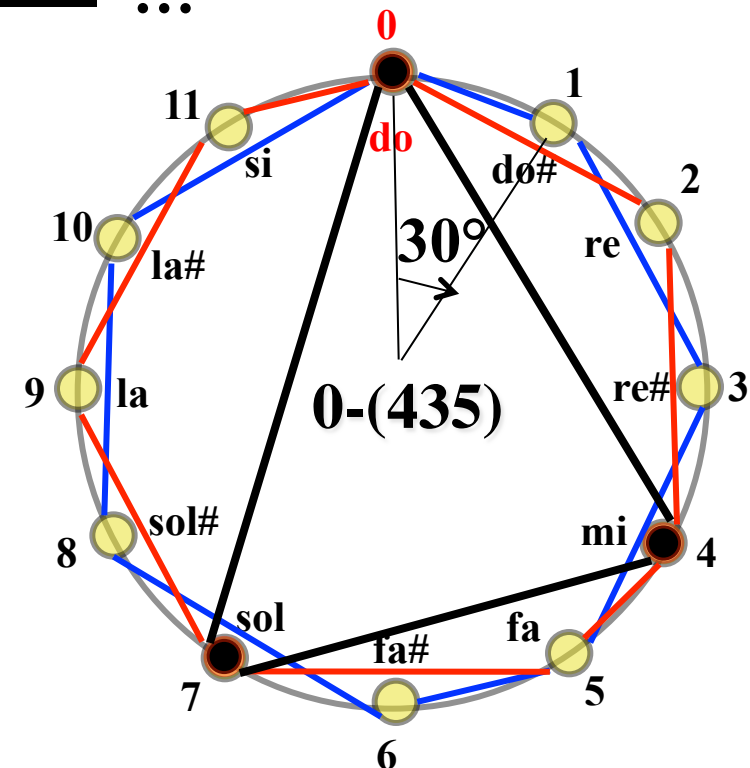
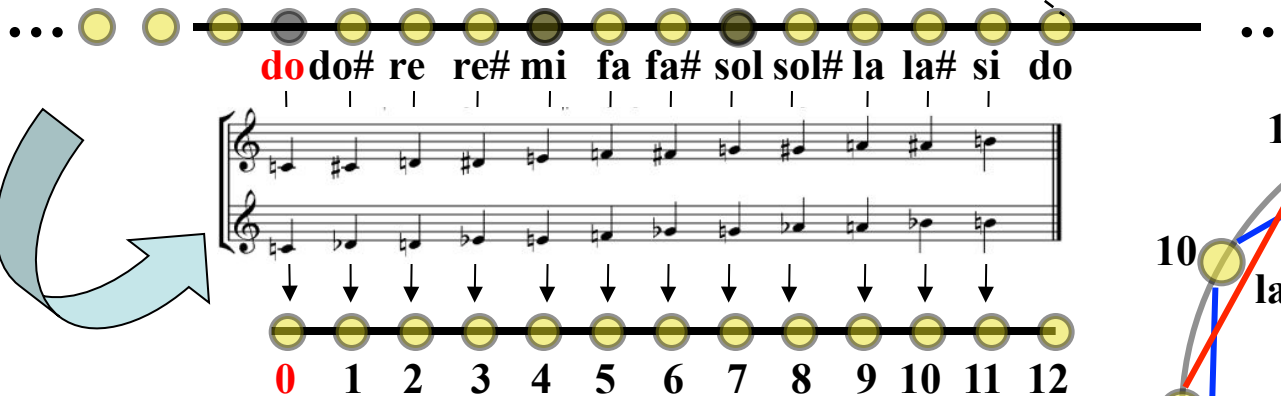
“[...] È necessario scavare più nel profondo del nostro **pensiero musicale**. Ed è là che si scopre l'importanza della **simmetria**, le proprietà dei suoni o le operazioni che l'ascoltatore o il musicista effettuano inconsciamente. La musica, come d'altra parte tutto l'universo, è immersa nell'idea della ricoerenza, della ripetizione più o meno fedele, della simmetria “*en temps*” e “*hors-temps*”. Ed è per questo che emergono incessantemente delle strutture di gruppo in ogni situazione.” (“Problèmes actuels en composition musicale”, Conférence à Saclay, 1983).

Le trasposizioni sono addizioni...

$$T_k : x \rightarrow x+k \bmod 12$$



$$\begin{aligned} \text{Do maj} &= \{0, 2, 4, 5, 7, 9, 11\} \\ \text{Do\# maj} &= \{1, 3, 5, 6, 8, 10, 0\} \end{aligned} +1$$



...o rotazioni!

Le trasposizioni sono addizioni...

$$T_k : x \rightarrow x+k \bmod 12$$

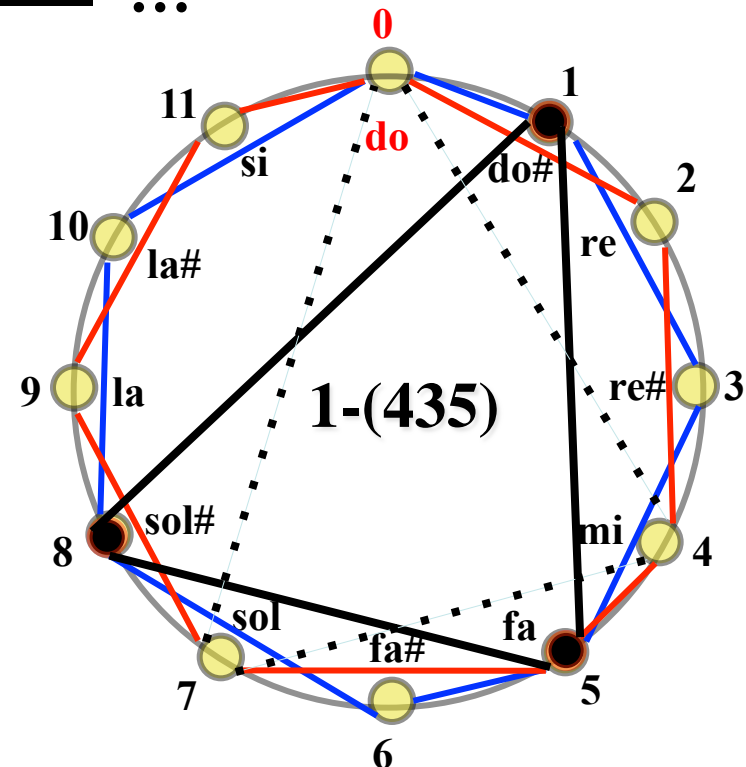


$$\begin{aligned} \text{Do maj} &= \{0, 2, 4, 5, 7, 9, 11\} \\ \text{Do\# maj} &= \{1, 3, 5, 6, 8, 10, 0\} \end{aligned} +1$$

... do do# re re# mi fa fa# sol sol# la la# si do ...



0 1 2 3 4 5 6 7 8 9 10 11 12



...o rotazioni!

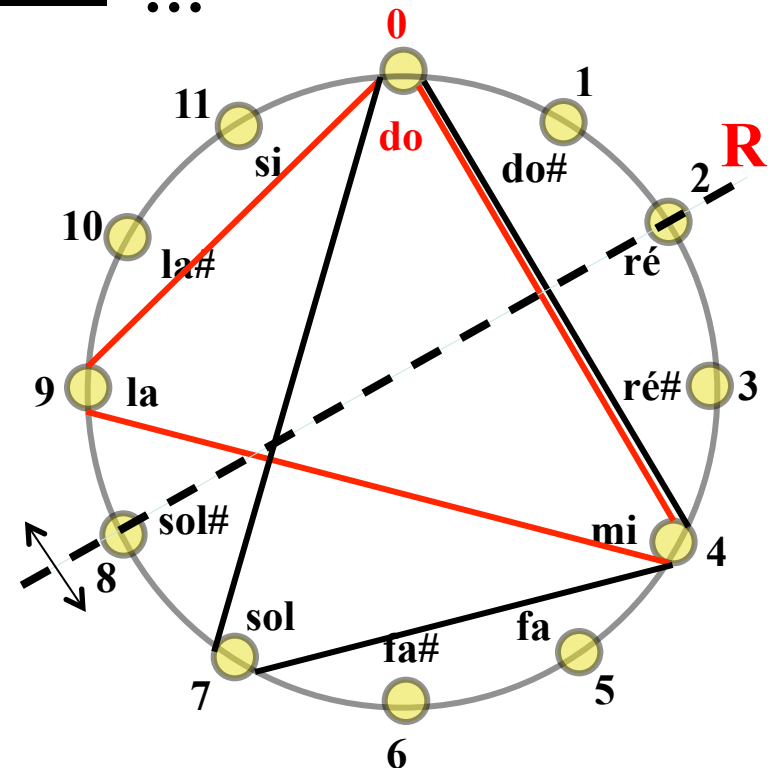
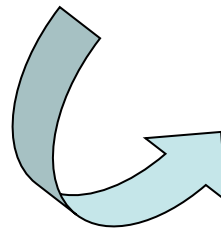
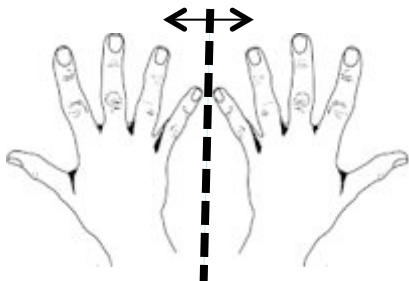
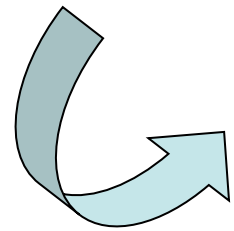
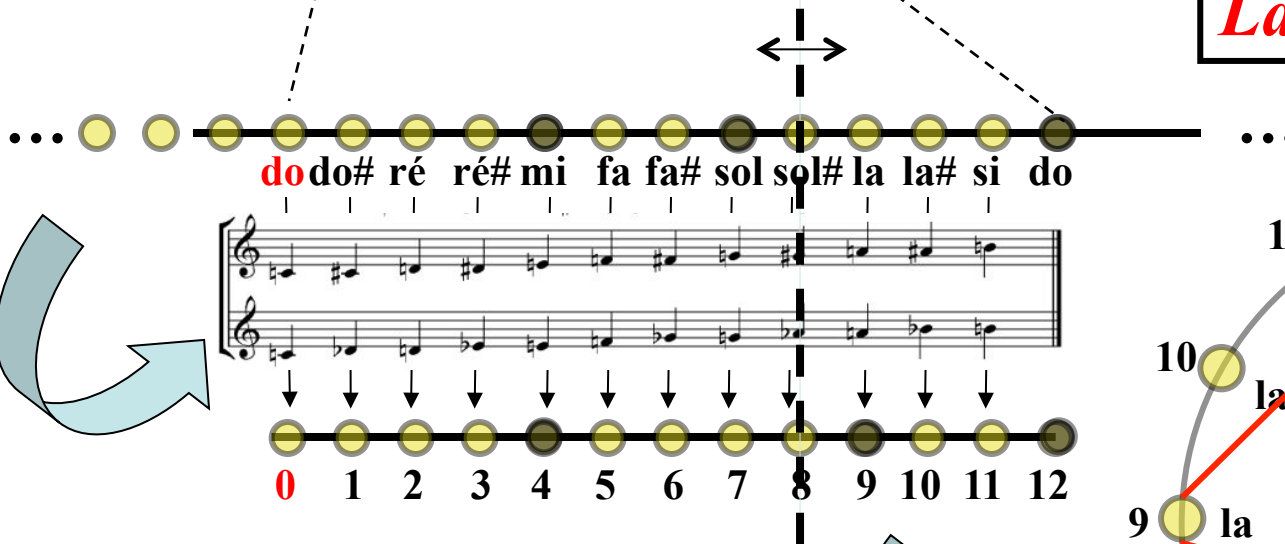
Le inversioni sono simmetrie assiali



R come **relativo**

Do maj = {0,4,7}

La min = {0,4,9}



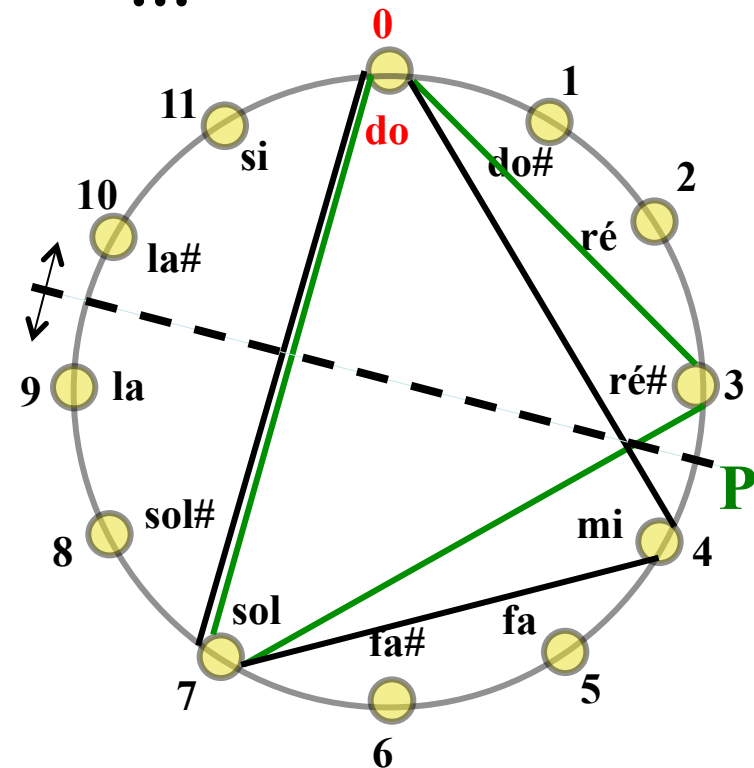
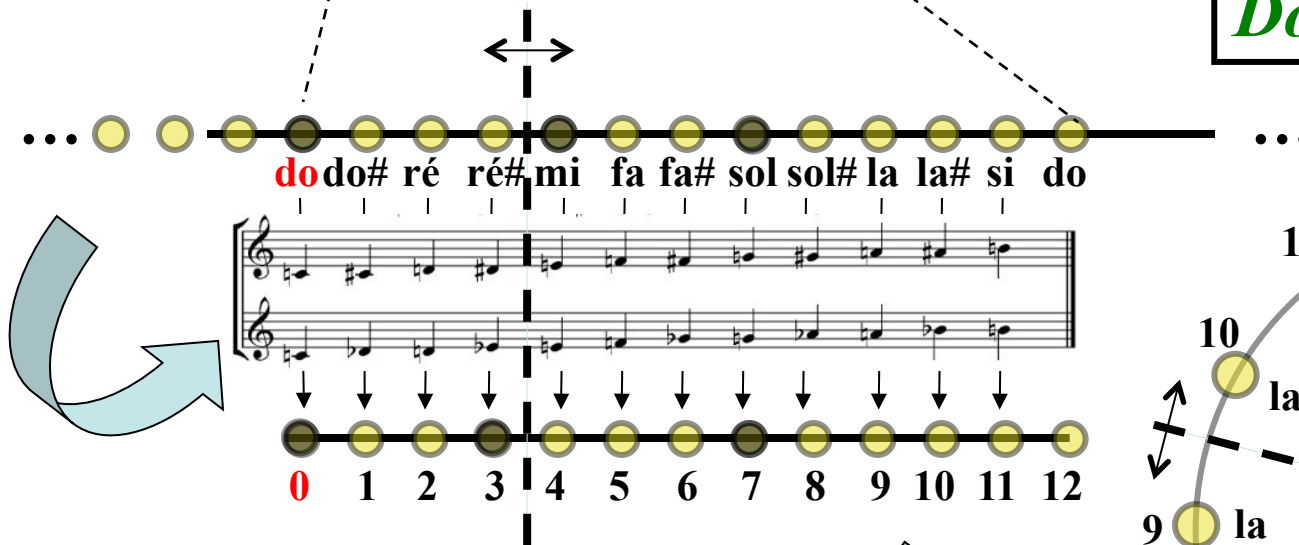
Le inversioni sono simmetrie assiali



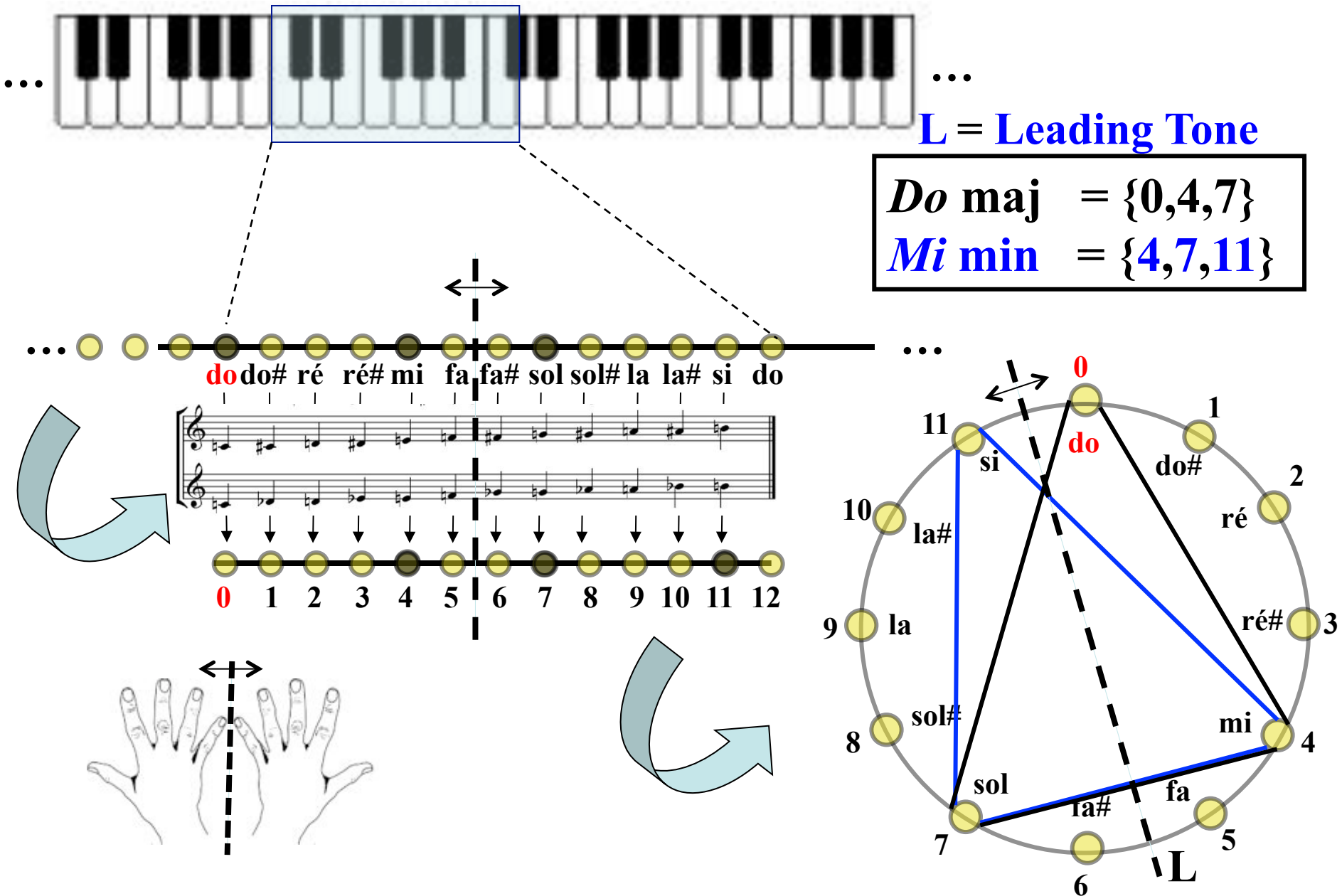
P come **parallelo**

Do maj = {0,4,7}

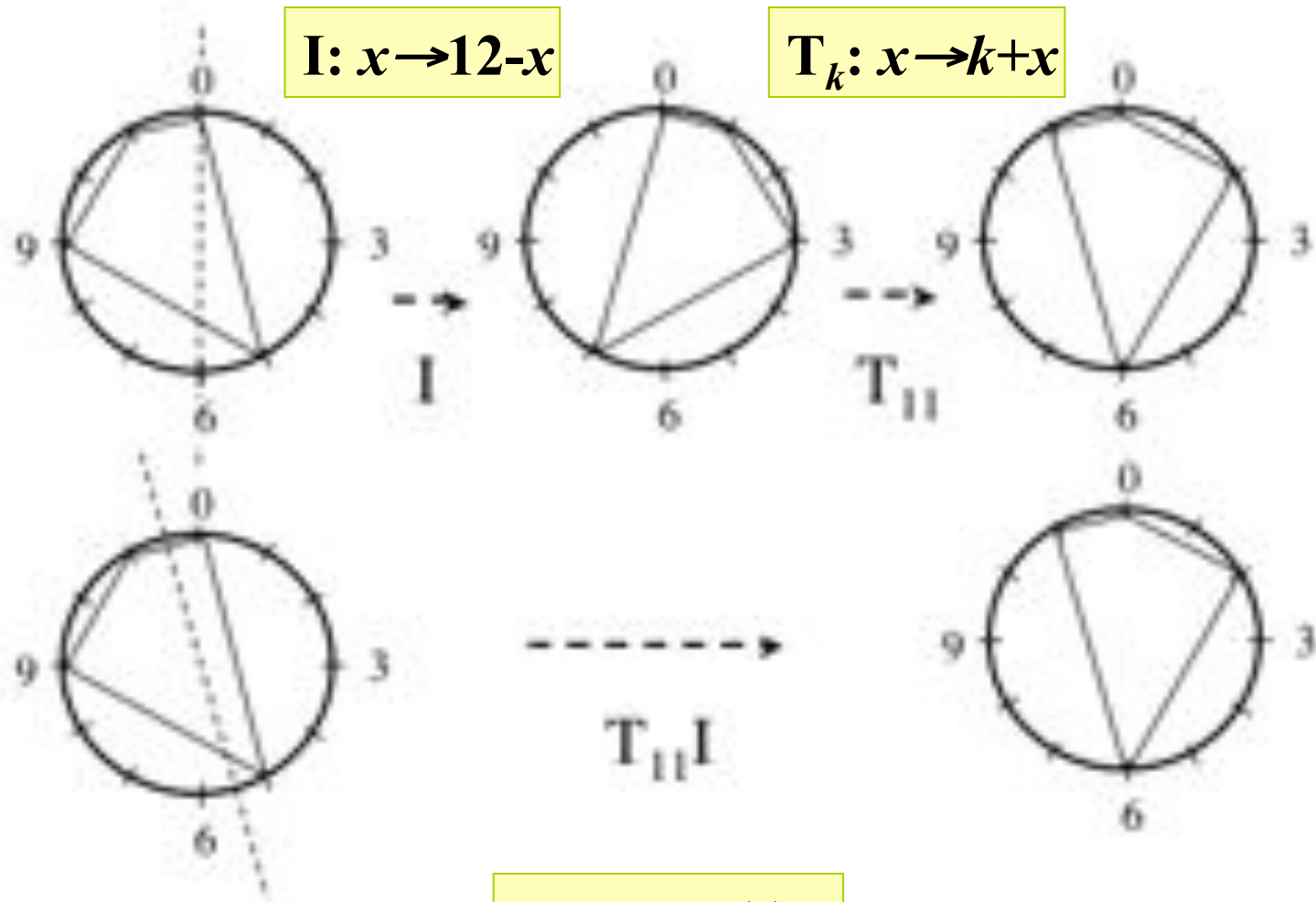
Do min = {0,3,7}



Le inversioni sono simmetrie assiali



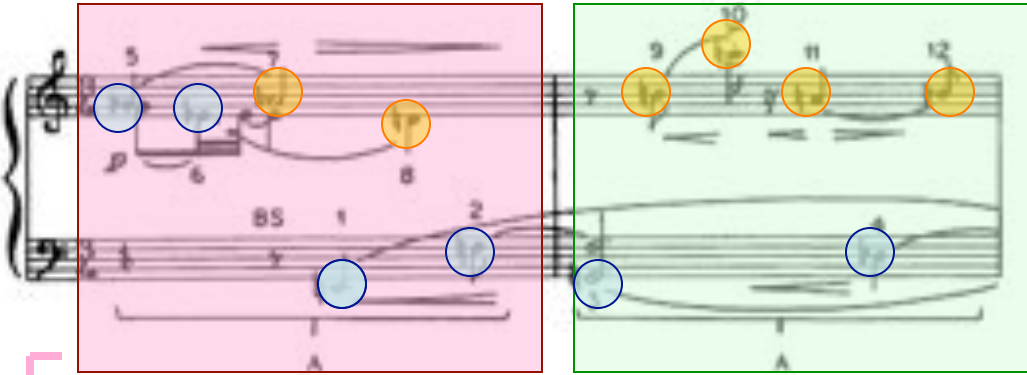
Una simmetria assiale = trasposizione + inversione



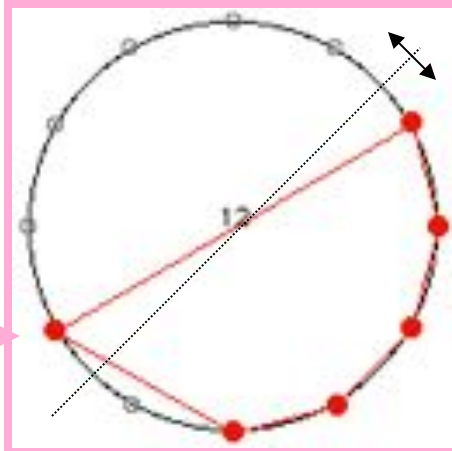
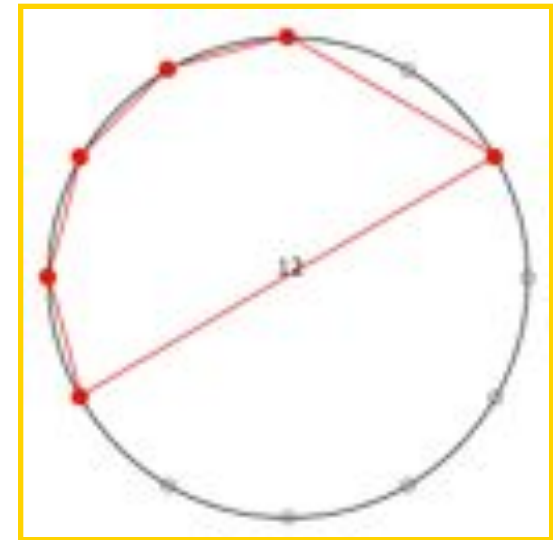
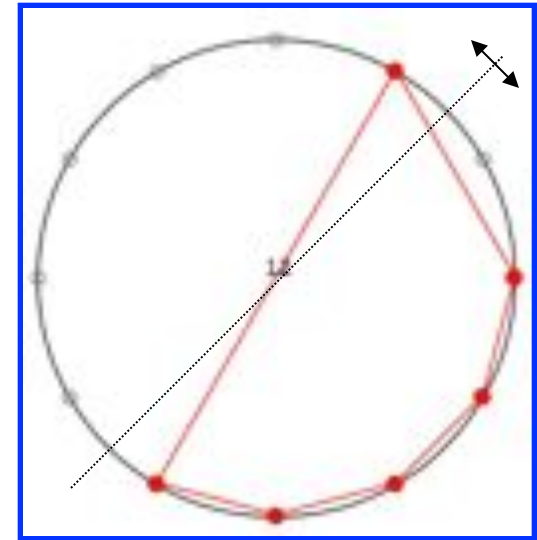
$$\{0, 5, 9, 11\} \xrightarrow{T_{11}I: x \rightarrow 11-x} \{11, 6, 3, 0\}$$

Serialismo e 'combinatorialità' esacordale

Schoenberg: Suite Op.25, Minuetto



Bi-combinatorialità



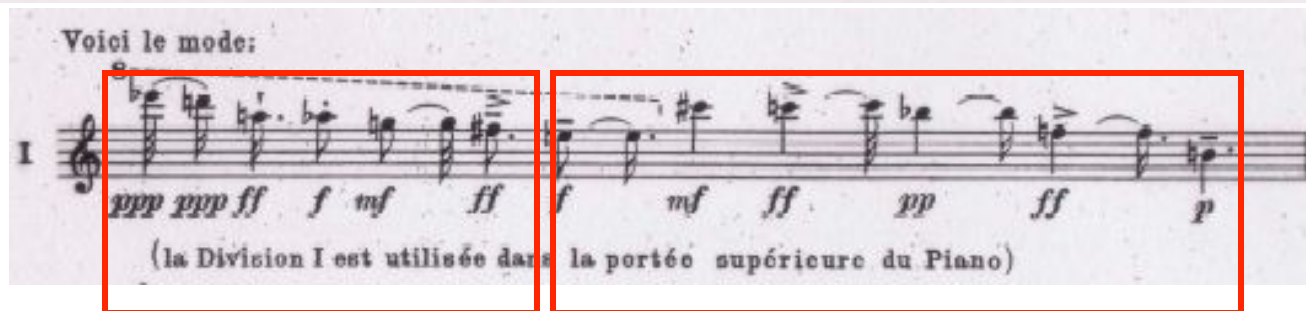
Combinatorialità esacordale e teoria dei tropi (Hauer)



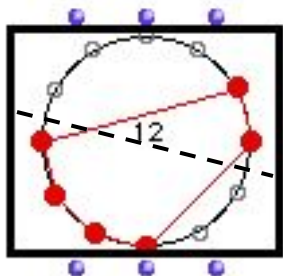
TAFEL I

Combinatorialité esacordale in Messiaen

- Mode de valeurs et d'intensités (1950)

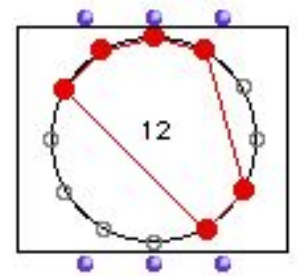


(la Division I est utilisée dans la portée supérieure du Piano)



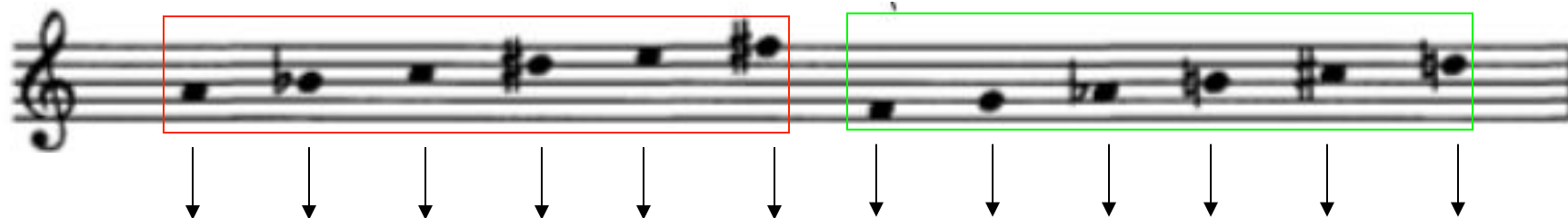
$$\{3,2,9,8,7,6\} \longrightarrow \{4,5,10,11,0,1\}$$

$$T_7I : x \rightarrow 7-x$$

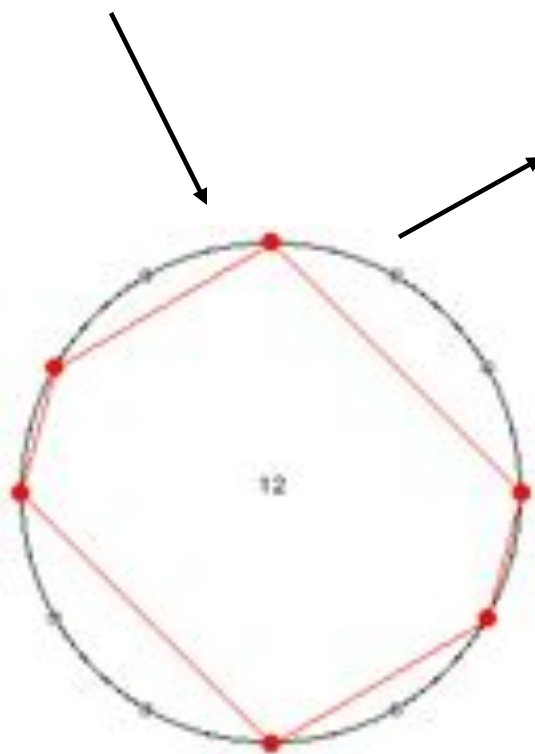


Combinatorialità esacordale e modi di Messiaen

Schoenberg: Serenade Op.24, Mouvement 5



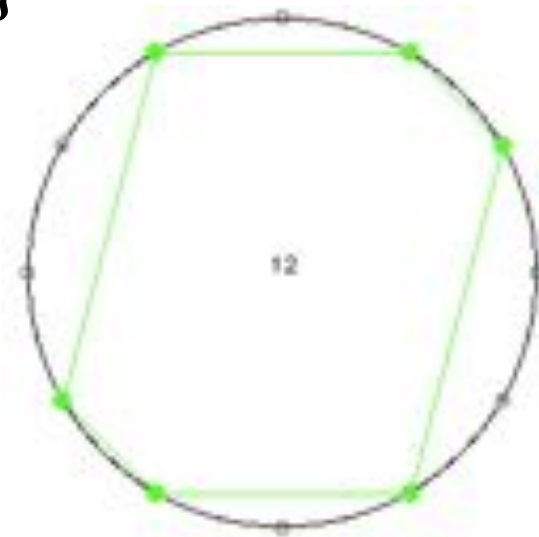
$A = \{9, 10, 0, 3, 4, 6\}$ $\{5, 7, 8, 11, 1, 2\}$



$(3, 1, 2, 3, 1, 2)$

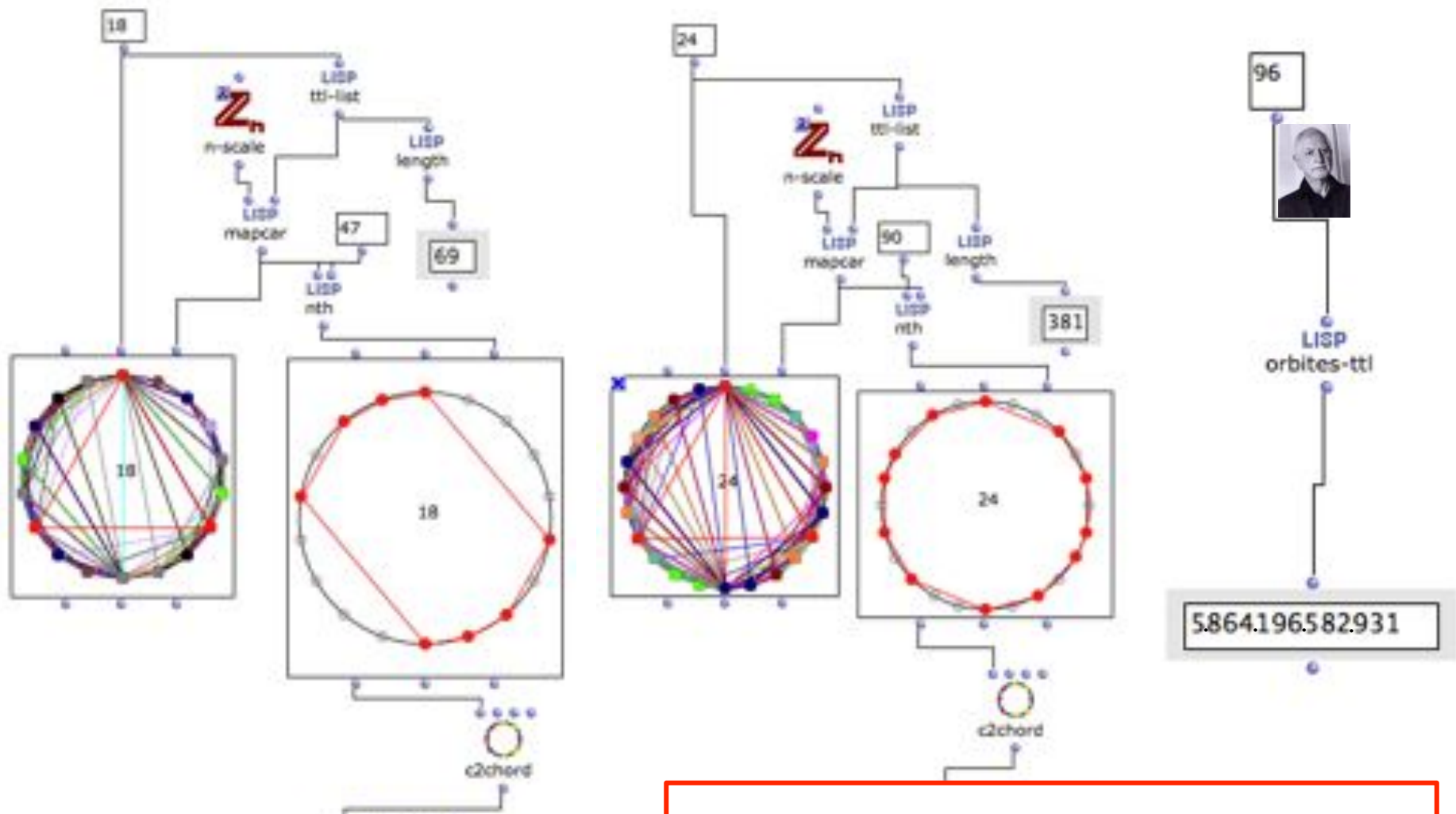
$$\begin{aligned} T6\{9,10,0,3,4,6\} &= \\ &= \{6+9, 6+10, 6, 6+3, 6+4, 6+6\} = \\ &= \{3, 4, 6, 9, 10, 0\} \end{aligned}$$

$$T6(A) = A$$

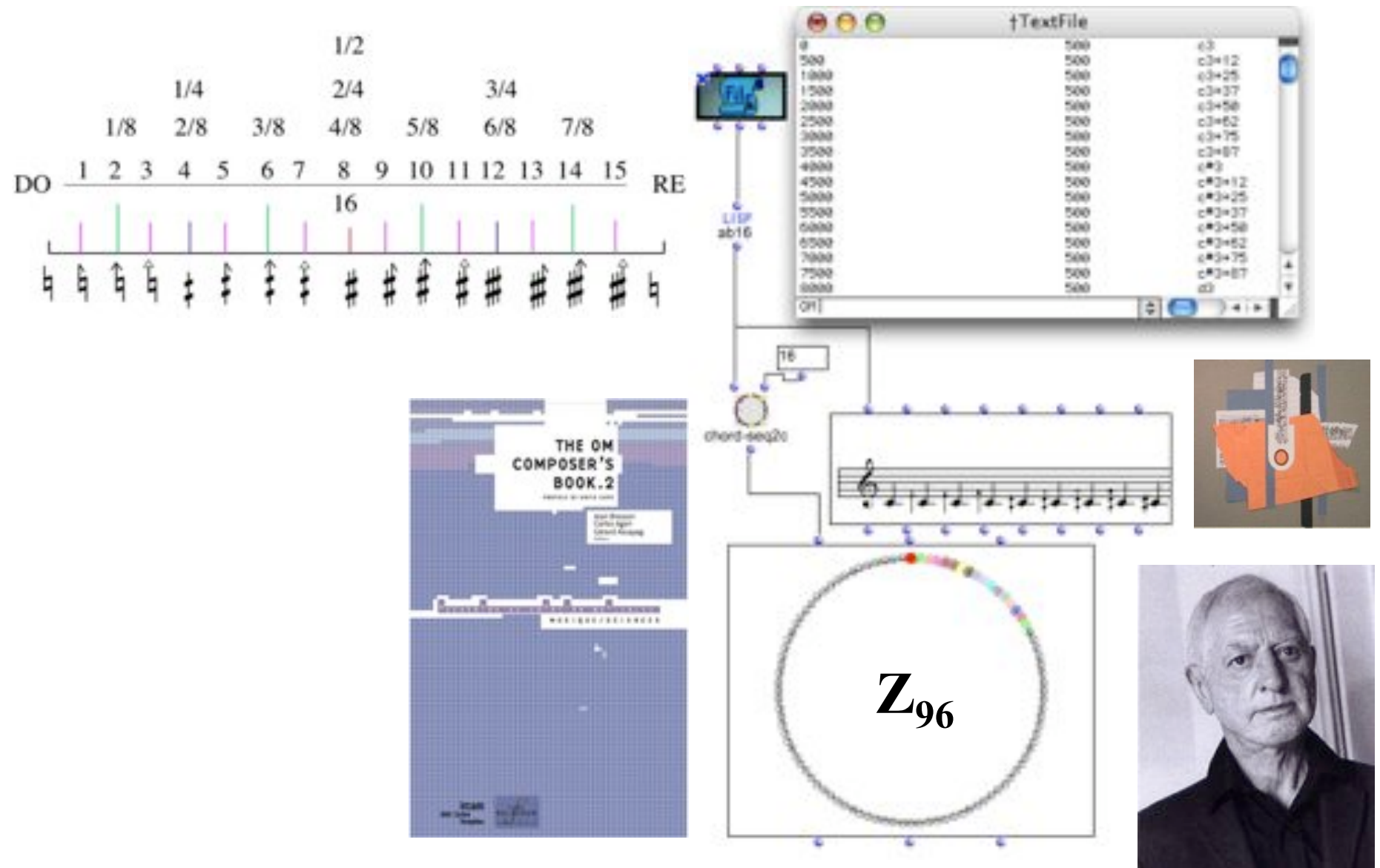


$(2, 1, 3, 2, 1, 3)$

Estensioni microtonali del catalogo dei modi a trasposizione limitata di Messiaen



Potenziale combinatorico della composizione microtonale



A. Bancquart, M. Andreatta, et C. Agon, « Microtonal Composition », The OM Composer's Book 2, éd. Jean Bresson, Carlos Agon, Gérard Assayag (Ircam/Delatour France, Sampzon), 2008, p. 279-302.

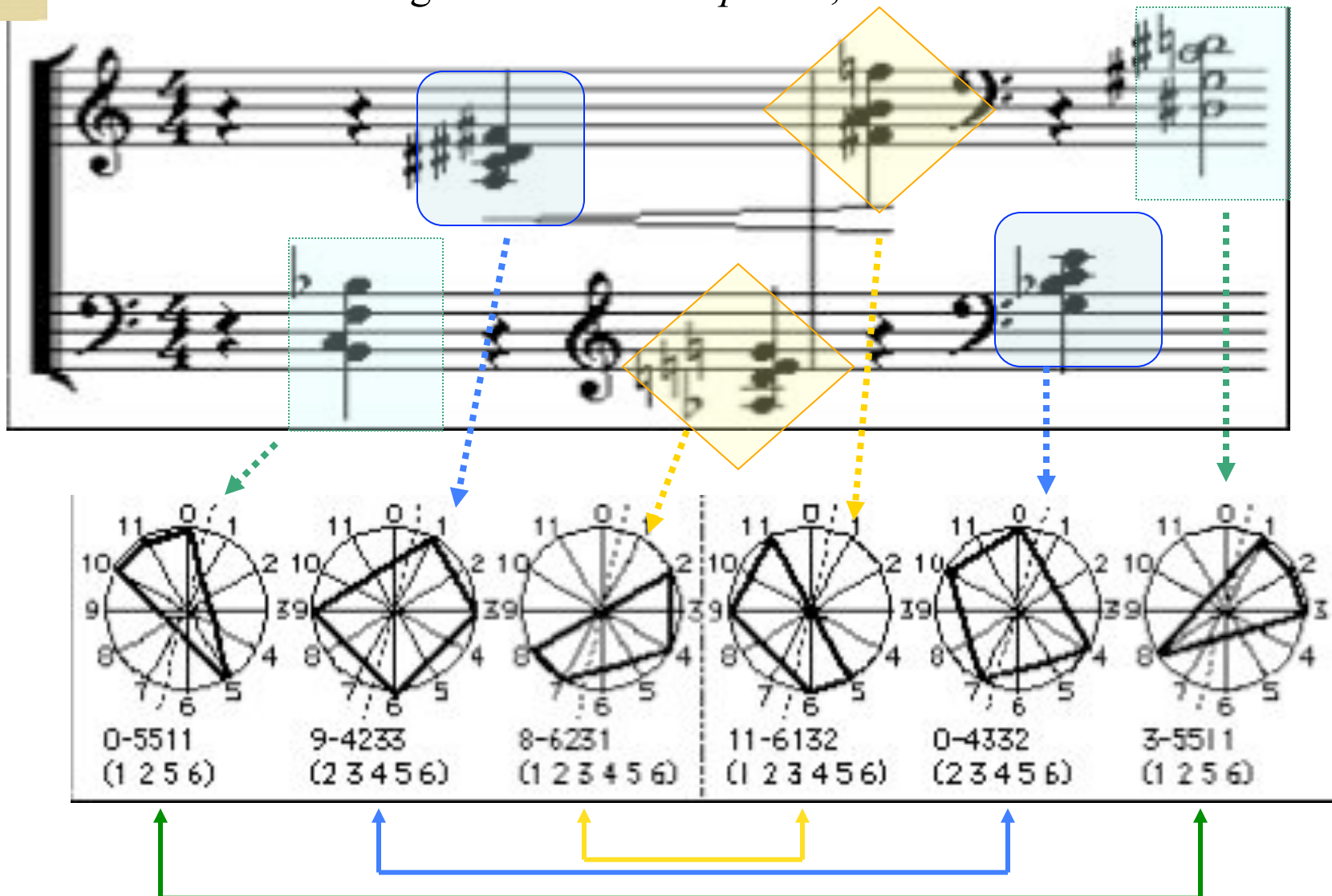
Alain Bancquart

Primi esempi di analisi musicale computazionale

A. Riotte & M. Mesnage, *Formalismes et modèles musicaux* (in 2 volumes),
Collection « Musique/Sciences », Ircam/Delatour France, 2006



A. Schoenberg : *Klavierstück Op. 33a*, 1929



Relazione d'equivalenza fra accordi

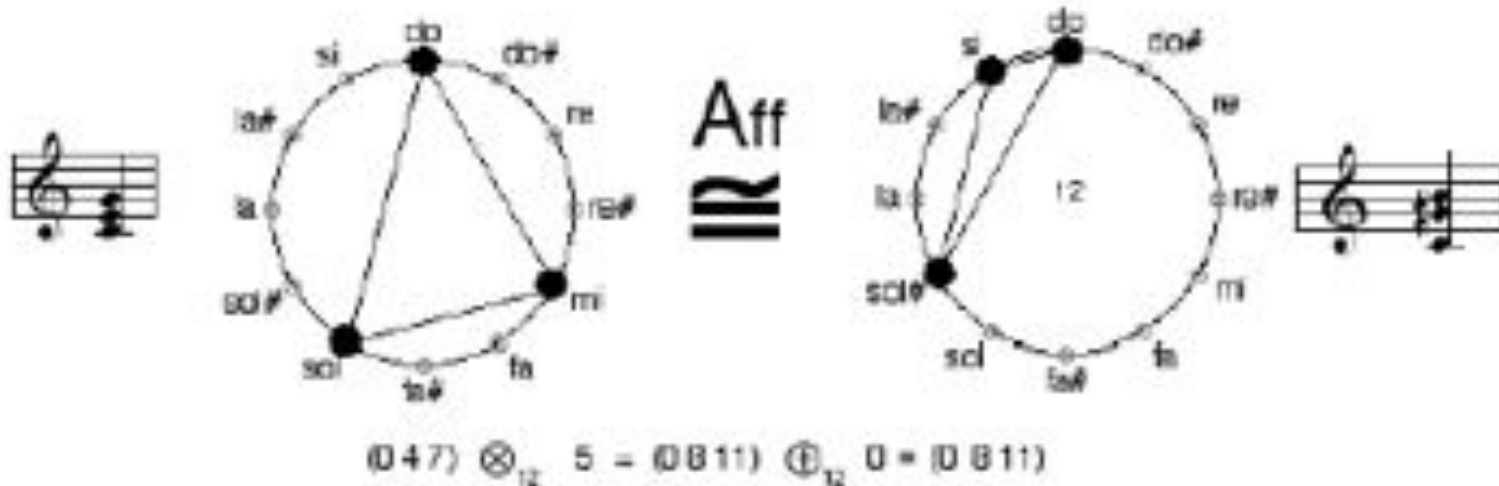


Trasposizione

$$T_3\{0, 4, 7\} = 3 + \{0, 4, 7\} = \{3, 7, 10\}$$

Trasposizione e/o inversione

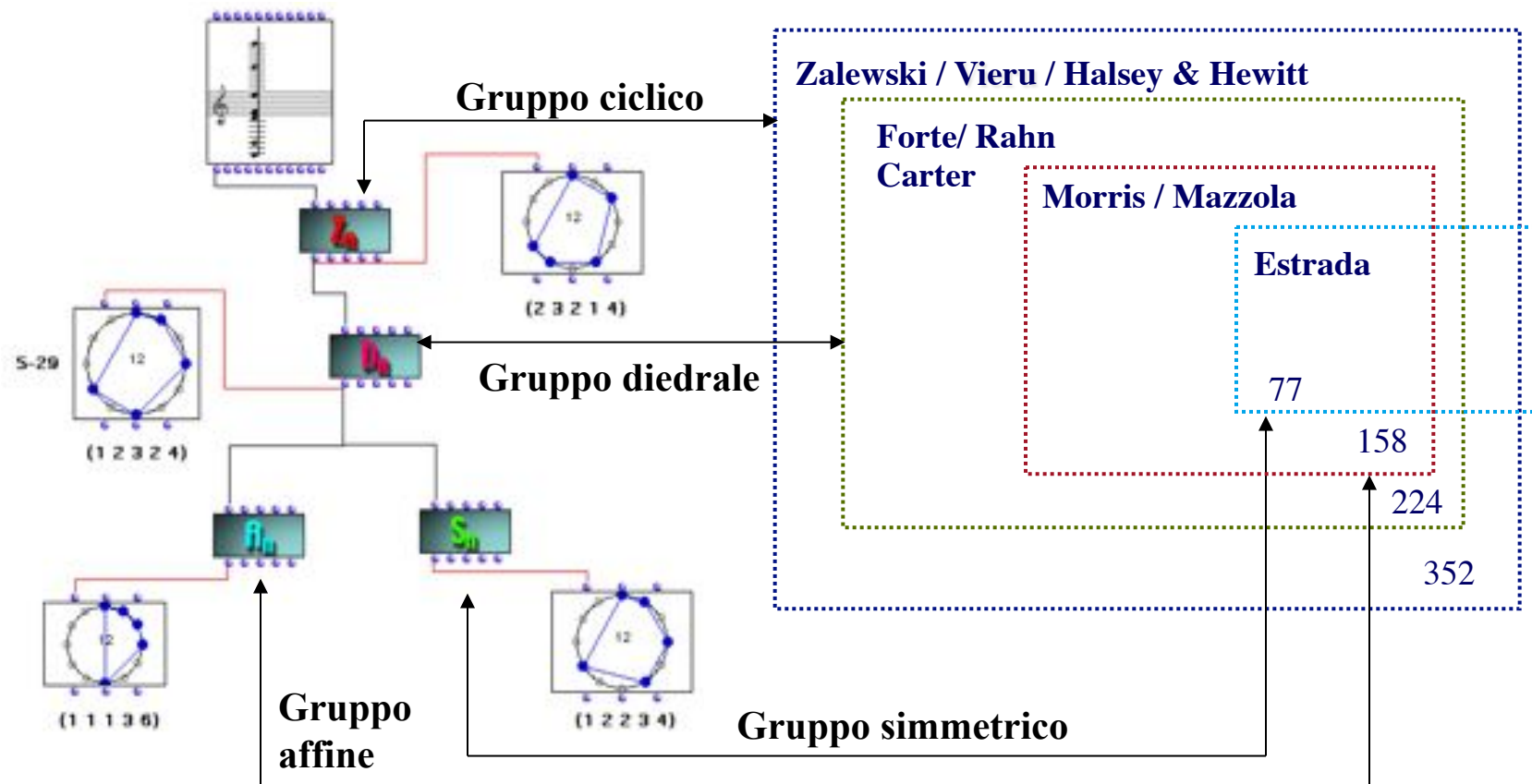
$$T_3I\{0, 4, 7\} = 3 + \{0, -4, -7\} = \{3, 11, 8\}$$



Moltiplicazione (o applicazione affine)

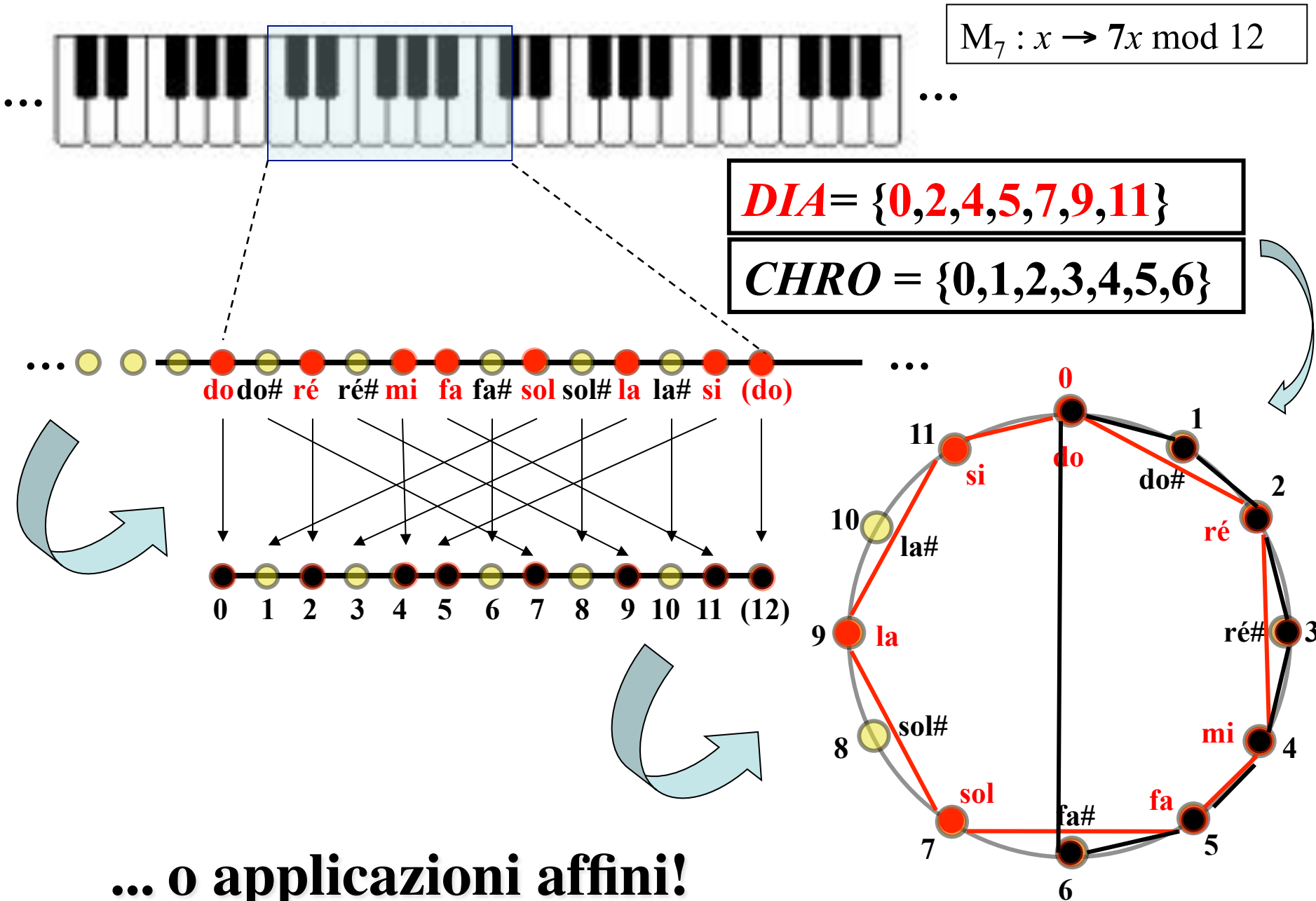
$$M_5\{0, 4, 7\} = 5 \times \{0, 4, 7\} = \{0, 8, 11\}$$

Classificazione e enumerazione delle strutture musicali



	1	2	3	4	5	6	7	8	9	10	11	12
Z_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1
S_n	1	6	12	15	12	11	7	5	3	2	1	1

Le aumentazioni sono moltiplicazioni...



Significato musicale di un'applicazione affine

$$M_7 : x \rightarrow 7x \bmod 12$$

$$DIA = \{0, 2, 4, 5, 7, 9, 11\}$$

$$CHRO = \{0, 1, 2, 3, 4, 5, 6\}$$

...  ...

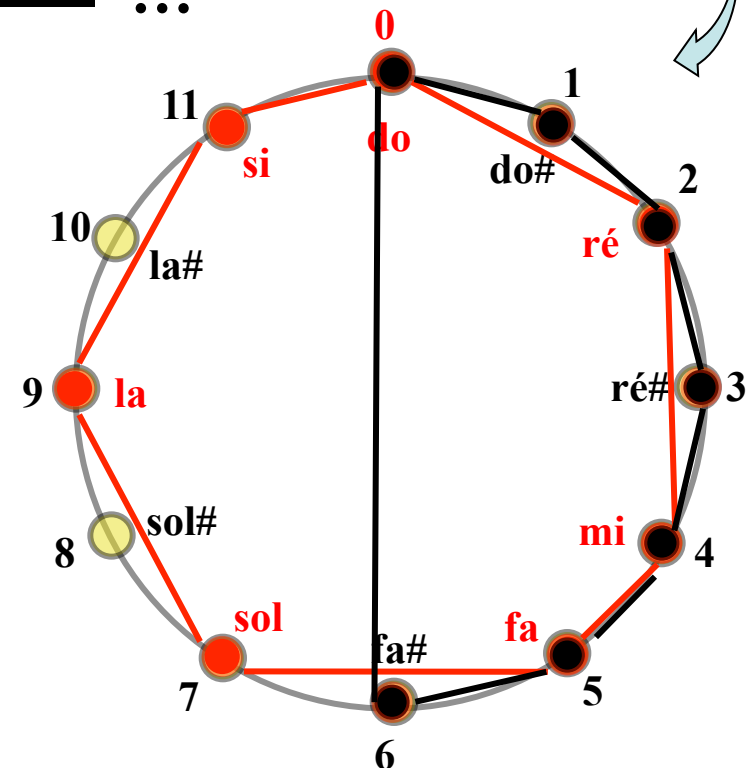
...  ...

do do# ré ré# mi fa fa# sol sol# la la# si (do)

0 1 2 3 4 5 6 7 8 9 10 11 (12)

“Il **diatonismo** e il **cromatismo** non possono essere studiati in termini di semplicità o di complessità, come si credeva nel passato. Si tratta, piuttosto, di una questione d'**unità dei contrari** nel gruppo ciclico $\mathbb{Z}/12\mathbb{Z}$ ”.

(A. Vieru, “The Musical Signification of Multiplication by 7. Diatonicity and Chromaticity”, *Muzica*, 1995)



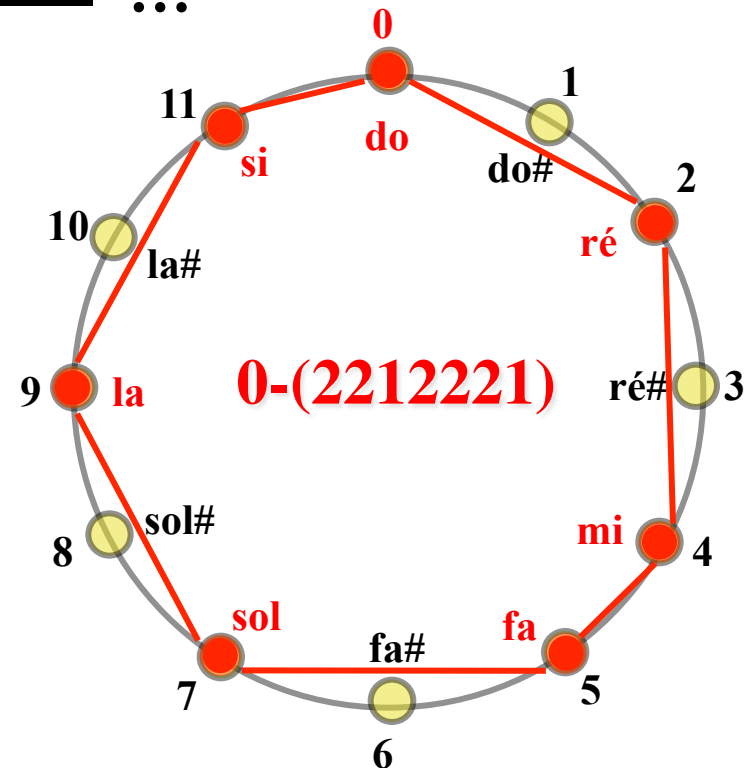
Le permutazioni sono ‘partizioni’...



$DIA = (2, 2, 1, 2, 2, 2, 1)$

... ...

0 1 2 3 4 5 6 7 8 9 10 11 (12)



Le permutazioni sono ‘partizioni’...

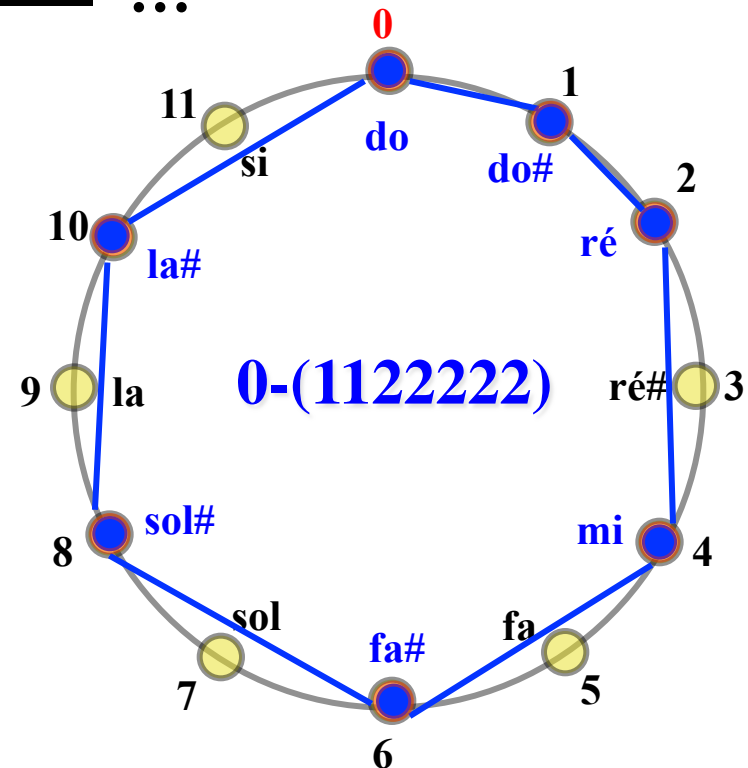


$$DIA = (2, 2, 1, 2, 2, 2, 1)$$

$$DIA_E = (1, 1, 2, 2, 2, 2, 2)$$

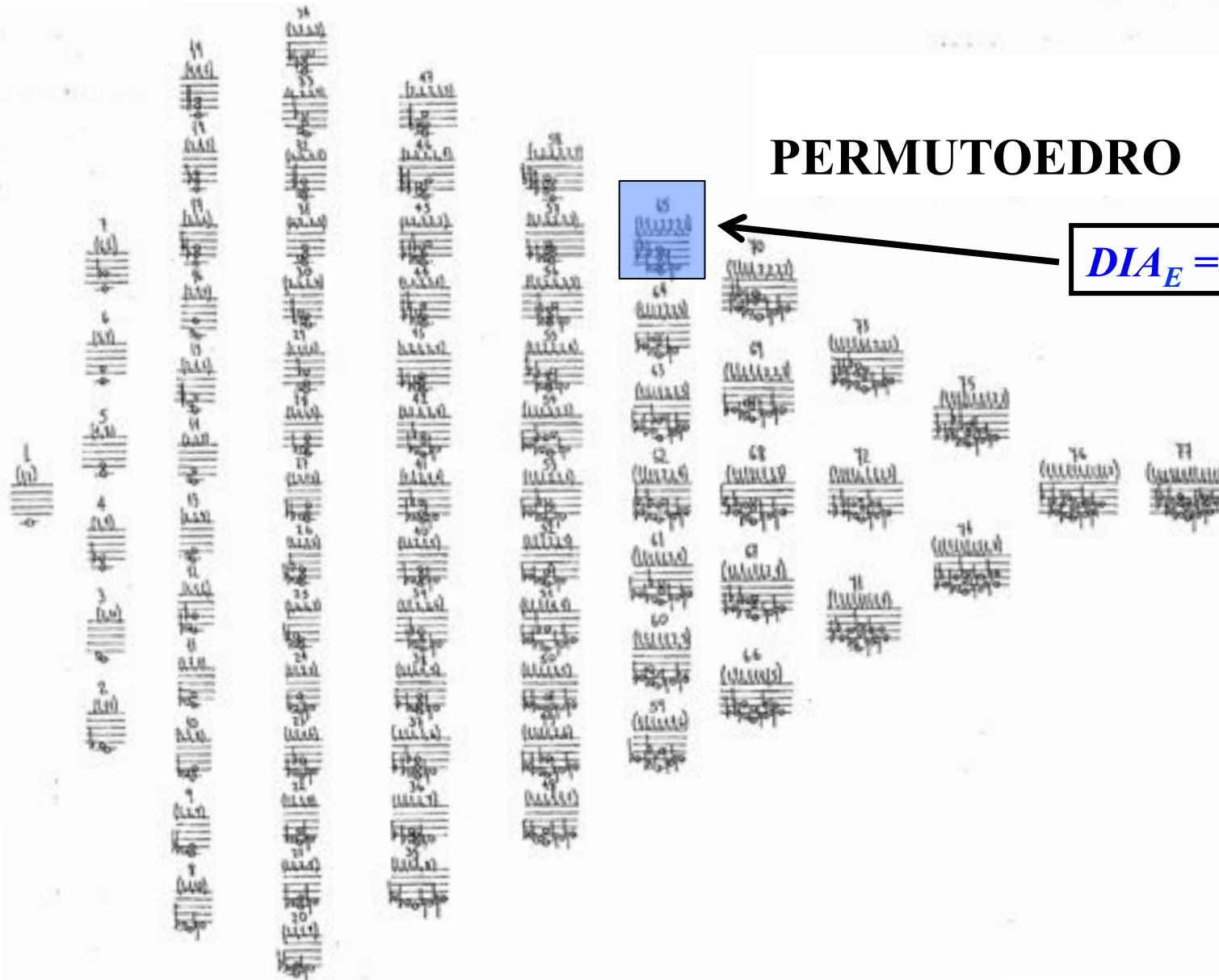
... **do do# ré ré# mi fa fa# sol sol# la la# si (do)** ...

0 1 2 3 4 5 6 7 8 9 10 11 (12)

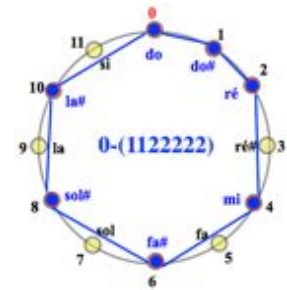


Il catalogo delle 77 partizioni di 12 (in somma di interi)

Julio Estrada, *Théorie de la composition : discontinuum – continuum*, université de Strasbourg II, 1994



$$DIA_E = (1,1,2,2,2,2,2)$$



J. Estrada

La formalizzazione algebrica del sistema temperato



anatol vieru

THE
BOOK
OF
MODES



Anatol Vieru



André Riotte



Julio Estrada



THEORIE DE LA COMPOSITION :
DISCONTINUUM - CONTINUUM

Julio ESTRADA

Thèse en Musicologie
Nouveau Doctorat
Département de Musicologie
Université de Strasbourg II
Sciences Humaines

Directeur de thèse :
François-Bernard Mâche

Membres du Jury :

M. François-Bernard Mâche, Ecole d'Hautes Etudes
Mme. Marta Grabóts, Université de Strasbourg
M. Hartmut Müller, Université de Rostock

Rapports de thèse :

Mme. Evélyne Andreani, Université de Paris VIII
M. Daniel Charles, Université de Nice

La formalizzazione algebrica del sistema temperato



3.43. The catalogue of modal structures
(in the tempered system modulo 12)

(1)(00)	—(1,1,1,1,1,1,1,1,1,1,1,1)	Yale structure — total structure
(200)	—(2,1,1,1,1,1,1,1,1,1,1,1)	zero structure
(3011)	—(1,1,1,1,1,1,1,1,1,1,1,1)	Perfect chromatic modes
(4010)	—(1,1,1,1,1,1,1,1,1,1,1,1)	Major second
(5000)	—(1,1,1,1,1,1,1,1,1,1,1,1)	Minor third
(6000)	—(1,1,1,1,1,1,1,1,1,1,1,1)	Major third
(7000)	—(1,1,1,1,1,1,1,1,1,1,1,1)	Perfect diatonic modes
(8000)	—(1,1,1,1,1,1,1,1,1,1,1,1)	Tritone
(9000)	—(1,1,1,1,1,1,1,1,1,1,1,1)	Perfect chromatic modes
(1000)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(1100)	—(2,1,1,1,1,1,1,1,1,1,1,1)	
(1200)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(1300)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(1400)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(1500)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(1600)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(1700)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(1800)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(1900)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(2000)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(2100)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(2200)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(2300)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(2400)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(2500)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(2600)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(2700)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(2800)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(2900)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(3000)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(3100)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(3200)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(3300)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(3400)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(3500)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(3600)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(3700)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(3800)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(3900)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(4000)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(4100)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(4200)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(4300)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(4400)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(4500)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(4600)	—(1,1,1,1,1,1,1,1,1,1,1,1)	
(4700)	—(1,1,1,1,1,1,1,1,1,1,1,1)	

M_1^0

 (2 2 2 2 2 2)
 $2_0 = 4_0 \cup 4_2$
 2 transpositions

M_2^0

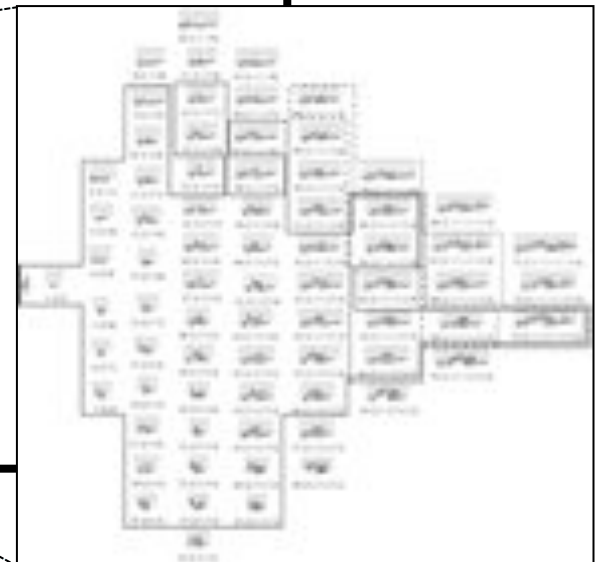
 (1 2 1 2 1 2 1 2)
 $3_0 \cup 3_1 = \overline{3_2}$
 3 transpositions

M_3^0

 (2 1 1 2 1 1 2 1 1)
 $4_0 \cup 4_2 \cup 4_3 = \overline{4_1}$
 4 transpositions

M_4^0

 (1 1 3 1 1 1 3 1)
 $6_0 \cup 6_1 \cup 3_2 = \overline{6_3 \cup 6_4}$
 6 transpositions



Composizione di strutture modali e modi di Messiaen

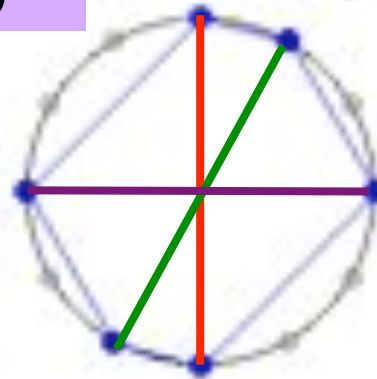


$$(6\ 6) \bullet (1\ 2\ 9) = (6\ 6) \bullet \{0, 1, 3\}$$

$$= ((6\ 6) \bullet \{0\}) \cup ((6\ 6) \bullet \{1\}) \cup ((6\ 6) \bullet \{3\}) =$$

$$= \{0, 6\} \cup \{1, 7\} \cup \{3, 9\} =$$

$$= \{0, 1, 3, 6, 7, 9\}.$$



**Moltiplicazione
d'accordi (Boulez)**



$$6_0 \cup 6_1 \cup 6_3$$

setaccio



Composizione di strutture modali e modi di Messiaen

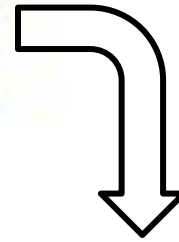
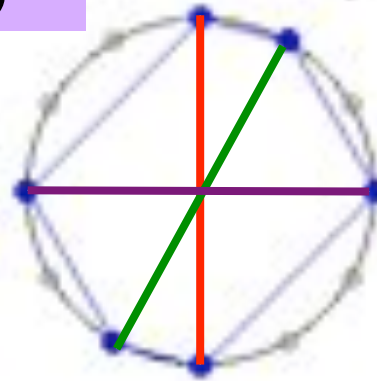


$$(6\ 6) \bullet (1\ 2\ 9) = (6\ 6) \bullet \{0, 1, 3\}$$

$$= ((6\ 6) \bullet \{0\}) \cup ((6\ 6) \bullet \{1\}) \cup ((6\ 6) \bullet \{3\}) =$$

$$= \{0, 6\} \cup \{1, 7\} \cup \{3, 9\} =$$

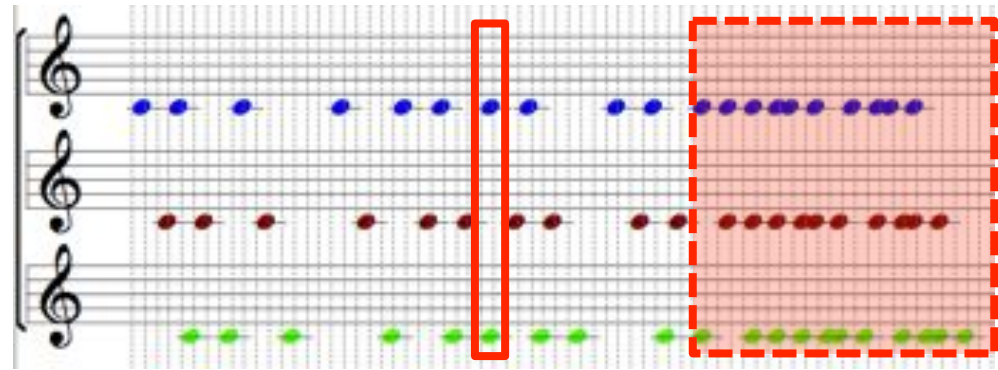
$$= \{0, 1, 3, 6, 7, 9\}.$$



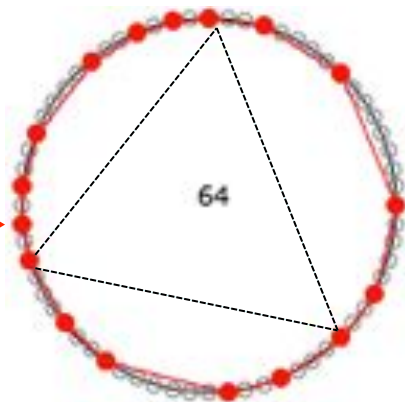
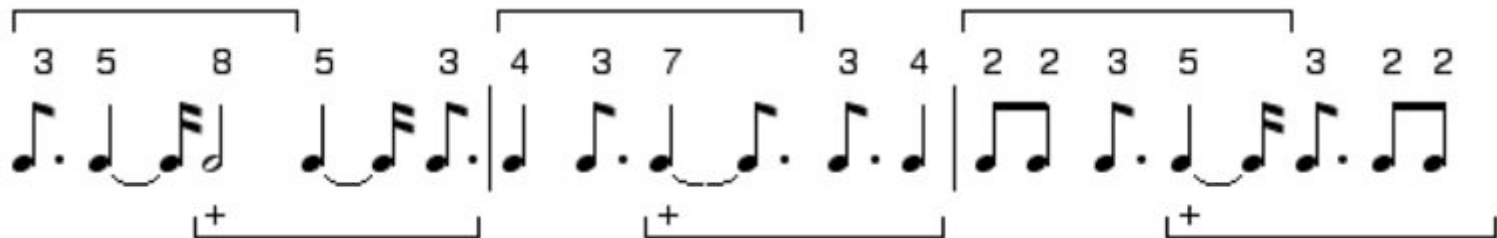
Ritmi non-retrogradabili e canoni a mosaico



 *Harawi (1945)*



Harawi: riduzione ritmica 



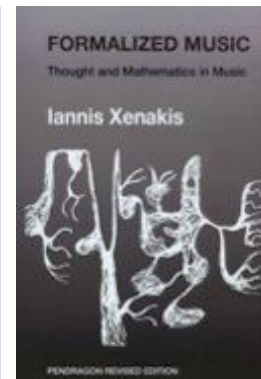
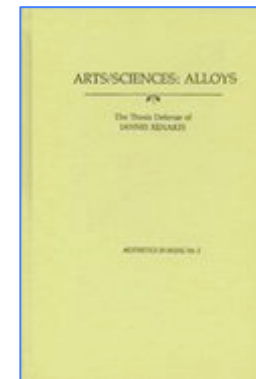
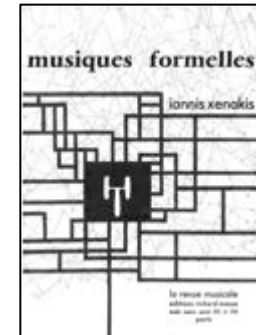
I tre ritmi non-retrogradabili dividono le durate in 5+5+7 durate [...]. Si noti inoltre che le durate sono molto diverse. Come risultato, le diverse sonorità si mescolano o contrastano l'una con l'altra in maniera molto diversa, senza alcuna coincidenza temporale fra le diverse voci...
È un sistema caotico organizzato



O. Messiaen

Altri sviluppi del rapporto matematica/musica (1930-1970)

- **1930** Microtonalità, ma in uno spirito tonale (Wischnegradsky, Haba, Carrillo).
- **1950** Seconda formalizzazione radicale delle macrostrutture (Messiaen).
- **1953** Introduzione della scala continua delle altezze (Xenakis, con il calcolo delle probabilità, calcolo logico e diverse **strutture di gruppo**).
- **1957** processi stocastici e catene di Markov (Hiller / Xenakis).
- **1960** Formalizzazione delle scale musicali attraverso la **teoria dei setacci** (via l'assiomatica di Peano)
- **1970** Nuove proposte nella microstruttura dei suoni (movimenti browniani).

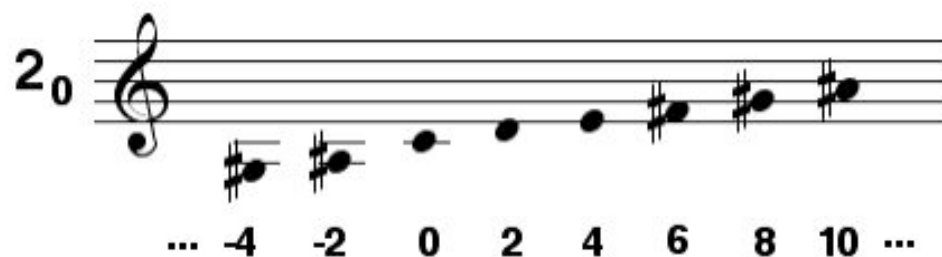
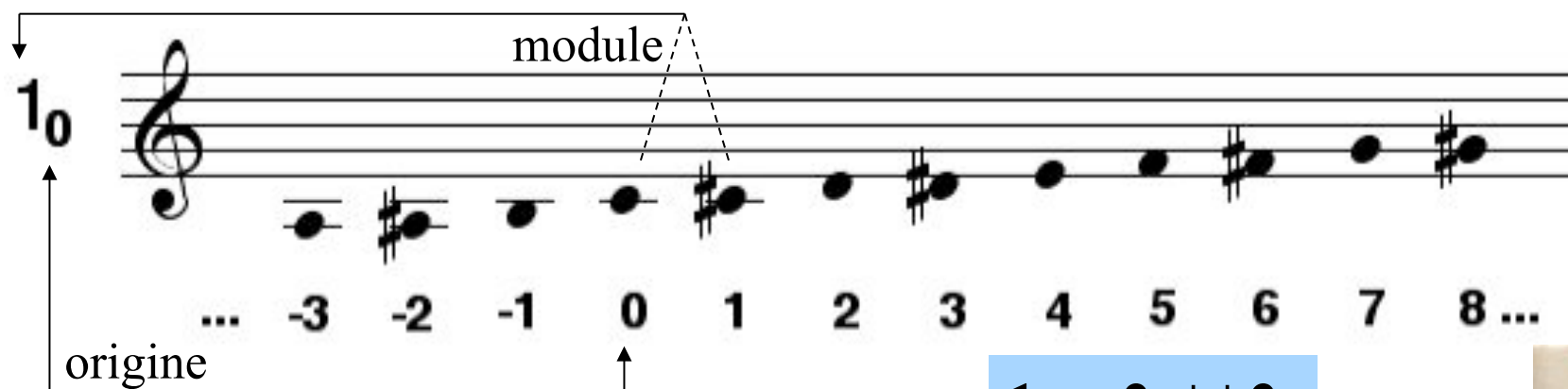


- *Musiques formelles: nouveaux principes formels de composition musicale*, LRM, 1963
- *Musique. Architecture*, Casterman, 1971/1976
- *Arts/Sciences Alliances*, Casterman, 1979 (tr. *Arts/Sciences. Alloys*, Pendr. Press, 1985)
- *Formalized Music: Thought and Mathematics in Music*, Pendragon Revised Edition 1990

La teoria dei setacci come teoria generale della periodicità

IN MUSIC, the question of symmetries (spatial identities) or of periodicities (identities in time) plays a fundamental role at all levels, from the sample, in sound synthesis by computer, to the architectures of a piece.¹ It is thus necessary to formulate a theory permitting the construction of symmetries which are as complex as one might want, and inversely, to retrieve from a given series of events or objects in space or time the symmetries that constitute the series. We shall call these series “sieves.”

I. Xenakis, “Sieves”, *Perspectives of New Music* 28(1), 1992, p. 58–78

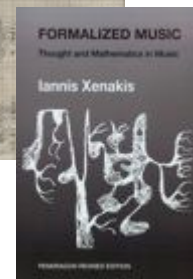
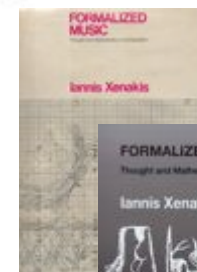


$$1_0 = 2_0 \cup 2_1$$

$$2_0 \cap 2_1 = \emptyset$$

$$(2_0)^c = 2_1$$

$$(2_1)^c = 2_0.$$

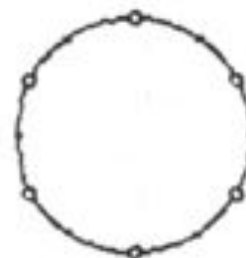


Teoria dei setacci e modi a trasposizione limitata



$$2_0 = 4_0 \cup 4_2$$

2 transpositions



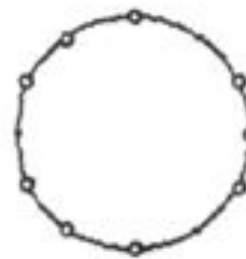
$$3_0 \cup 3_1 = \overline{3_2}$$

3 transpositions



$$4_0 \cup 4_2 \cup 4_3 = \overline{4_1}$$

4 transpositions



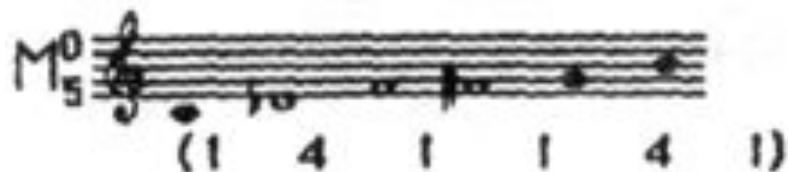
$$6_0 \cup 6_1 \cup 3_2 = \overline{6_3 \cup 6_4}$$

6 transpositions



A. Riotte, "L'utilisation de modèles mathématiques en analyse et en composition musicales", Quadrivium musiques et sciences, éditions ipmc, Paris, 1992.

Teoria dei setacci e modi a trasposizione limitata



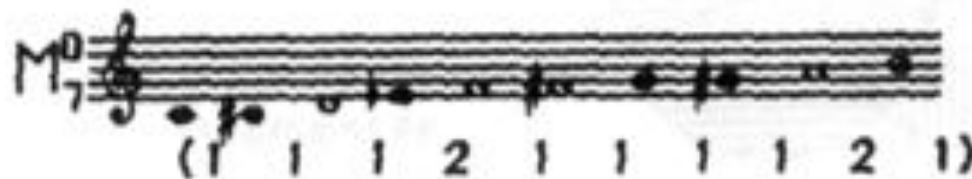
$$6_0 \cup 6_1 \cup 6_5 = \overline{M_5^3}$$

6 transpositions



$$2_0 \cup 6_5 = \overline{6_1 \cup 6_3}$$

6 transpositions



$$\overline{6_4} = 6_0 \cup 6_2 \cup 2_1$$

6 transpositions



“Il catalogo è questo”

(1 ₀)	(3 ₀)
(2 ₀)	(4 ₀)
	(6 ₀)

$6_0 \cup 6_1$

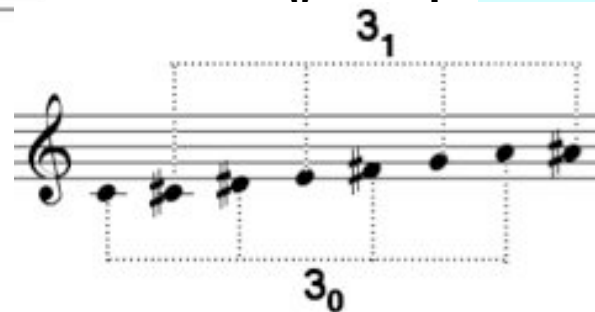
$6_0 \cup 6_2$

$6_0 \cup 6_1 \cup 6_5$

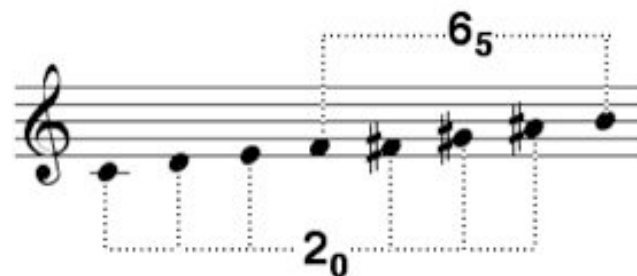
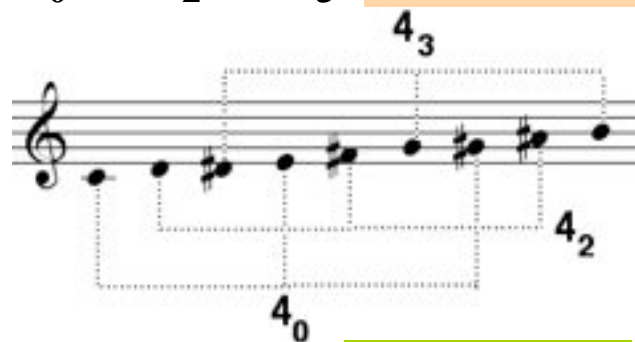
Mode n.5



$3_0 \cup 3_1$ *Mode n.2*

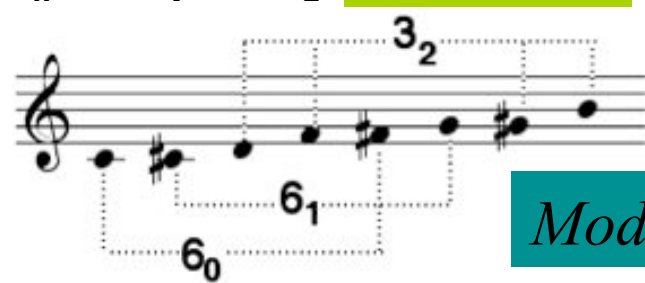


$4_0 \cup 4_2 \cup 4_3$ *Mode n.3*



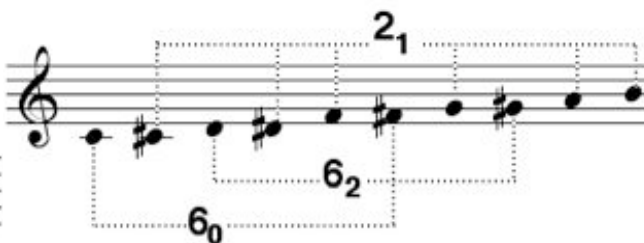
$2_0 \cup 6_5$ *Mode n.6*

$6_0 \cup 6_1 \cup 3_2$ *Mode n.4*



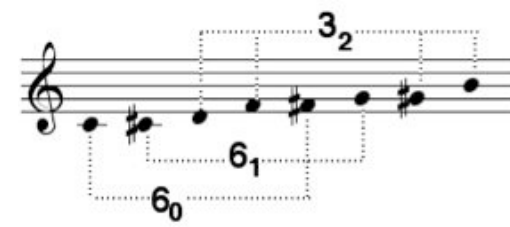
Mode n.7

$2_1 \cup 6_0 \cup 6_2$



Mode n.8

$6_0 \cup 6_1 \cup 3_2$

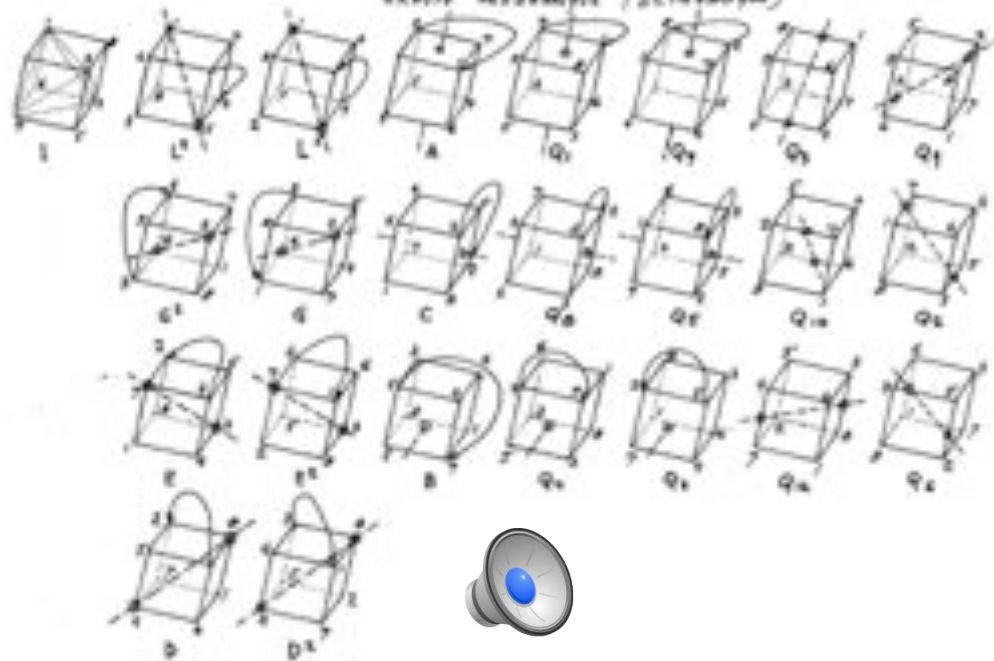


Dai modi di Messiaen alle scale non ottavianti

Nomos alpha (1965-66) : groupe des rotations du cube

Source : Iannis Xenakis, *Musicus Architecture*, Tournai, Castorini, 1976, p. 98

groupe des rotations du cube (octaédrique)



“Musica simbolica per violoncello solo che possiede un’architettura *hors-temps* fondata sui **gruppi di trasformazione**. Vi è fatto uso della **teoria dei setacci**, una teoria che annette le congruenze modulo z e che deriva da un’**assiomatica della struttura universale della musica**”

Nomos alpha pour violoncelle solo (1965-66)
(brano dedicato a Aristosseno di Taranto, Évariste Galois e Felix Klein)

VIOLONCELLE SOLO

NOMOS α

ἁποσπασμένο από το ἔργο "Ἐκείνη ἡ νύχτα" του Ἰωάννη Χανιάκη

per Siegfried Palm

1965 Iannis Xenakis

Handwritten musical score for Violoncelle Solo, titled "NOMOS α". The score is written on two staves with various musical notations, including notes, rests, and dynamic markings. The title is written in Greek and French. The composer's name, Iannis Xenakis, is also present.

La teoria dei setacci in quanto teoria della periodicità

In a similar fashion, a periodic rhythm, for example $(3,2,2) = \text{♩} . \text{♩} \text{♩}$ can be notated as $L = 7_0 \cup 7_3 \cup 7_5$. In both of these examples the sign $\cup$ is a logical union (and/or) of the points defined by the moduli and their shifts.

I. Xenakis, “Sieves”, *Perspectives of New Music* 28(1), 1992, p. 58–78

$$L(11, 13) = (A \cap B) \cup (C \cap D) \cup E$$

where:

$$A = (13_3 \cup 13_5 \cup 13_7 \cup 13_9)^c$$

$$B = 11_2$$

$$C = (11_4 \cup 11_8)^c$$

$$D = 13_9$$

$$E = 13_0 \cup 13_1 \cup 13_6$$

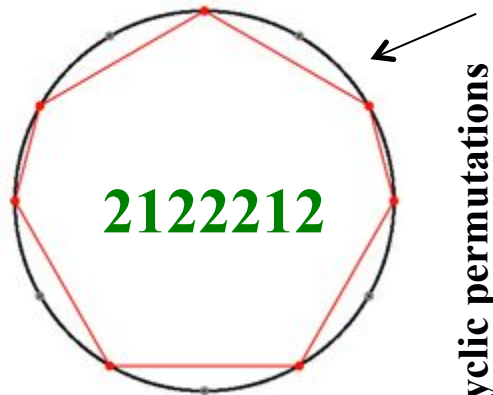
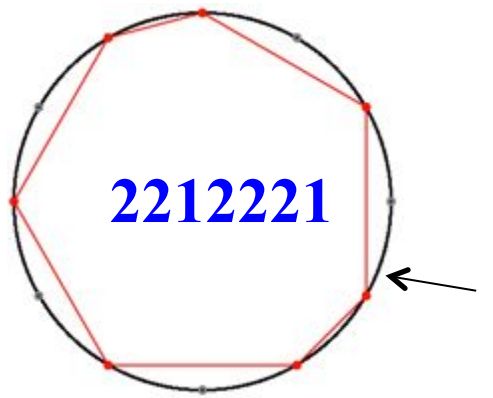


Interval structure 1 1 4 3 4 1 5 3 2 2 1 5 ...

“È possibile costruire architetture ritmiche che sono così complesse da essere percepite come distribuzioni pseudo-aleatorie di attacchi (*time-points*) nell’asse del tempo se il periodo è sufficientemente lungo”.

I. Xenakis, « Redécouvrir le temps », 1988

L'isomorfismo 'cognitivo' altezza-ritmo



cyclic permutations

...

TABLE I
Comparison of M = 7, L = 12 patterns for pitch (scales) and rhythm (time-lines)

pattern	pitch domain name and notation (in C)	rhythm domain notation	examples from West Africa	references
1. 2212221	major scale (Ionian) CDEFGAB	♩ ♩ ♩ ♩ ♩ ♩ ♩	Ewe (Atsiabek, Sogba, Atsia) also Yoruba	Jones (1959), C. K. Ladzekpo, S. K. Ladzekpo and Pantaleoni, Locke
2. 2122212	Dorian CDE [♭] FGAB [♭]	♩ ♩ ♩ ♩ ♩ ♩ ♩	Bemba—Northern Rhodesia	Jones (1965), (Ekwueme)
3. 1222122	Phrygian CD [♭] E [♭] FGA [♭] B [♭]	♩ ♩ ♩ ♩ ♩ ♩ ♩	—	—
4. 2221221	Lydian CDEF [♯] GAB	♩ ♩ ♩ ♩ ♩ ♩ ♩	Ga-Adangme (common) also common Haitian pattern, Akan (Ab fo)	C. K. Ladzekpo, Combs (1974), R. Hill, Asiamah
5. 2212212	Mixolydian CDEFGAB [♭]	♩ ♩ ♩ ♩ ♩ ♩ ♩	Yoruba sacred music from Ekiti	King
6. 2122122	Aeolian CDE [♭] FGA [♭] B [♭]	♩ ♩ ♩ ♩ ♩ ♩ ♩	Ashanti (Ab fo , Mpre)	Koetting
7. 1221222	Locrian CD [♭] E [♭] FG [♭] A [♭] B [♭]	♩ ♩ ♩ ♩ ♩ ♩ ♩	Ghana*	Nketia (1963a)
8. 2121222	(#2 Locrian) CDE [♭] FG [♭] A [♭] B [♭]	♩ ♩ ♩ ♩ ♩ ♩ ♩	Ashanti (Asedua)	C. K. Ladzekpo
9. 2112123	— CDD [♯] EF [♯] GA	♩ ♩ ♩ ♩ ♩ ♩ ♩	Akan (juvenile song)	Nketia (1963b)

* clap pattern

† mute stroke on bell

J. Pressing, "Cognitive isomorphisms between pitch and rhythm in world musics: West Africa, the Balkans and Western tonality", *Studies in Music*, 17, p. 38-61, 1983

Verso una formalizzazione algebrica del sistema dodecafonico

*I fisici e i matematici sono molto in anticipo rispetto ai musicisti nel fatto di realizzare che le loro scienze rispettive non hanno per scopo di stabilire una visioone dell'universo come conforme ad una **natura esistenze oggettiva**.*

*Come lo studio degli **assiomi** elimina l'idea que questi sono un qualcosa di assoluto, considerandoli invece come **libere proposizioni dello spirito umano**, allo stesso modo una **teoria musicale** ci libererebbe dal concetto di tonalità maggiore/minore [...] come una legge irrevocabile della natura*

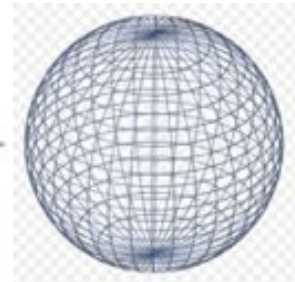
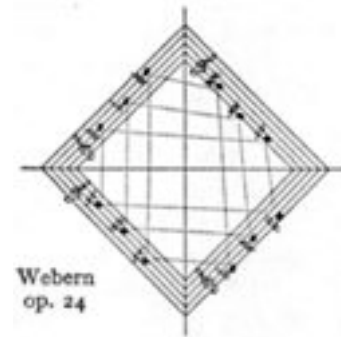
Ernst Krenek : *Über Neue Musik*, 1937 (Engl. Transl. *Music here and now*, 1939)



Ernst Krenek



David Hilbert



*Il sistema dodecafonico è “un insieme di elementi, **relazioni** fra gli elementi e **operazioni** sugli elementi. [...] Una vera matematizzazione avrebbe bisogno di una formulazione e di una presentazione dettate dal fatto che il sistema dodecafonico è un **gruppo di permutazioni** determinato dalla struttura di questo modello matematico”.*

M. Babbitt: *The function of Set Structure in the Twelve-Tone System*, PhD (1946/1992)

Il transfert delle idee del positivismo logico in musica

« There is no field of experience which cannot, in principle, be brought under some form of **scientific law**, and no type of speculative knowledge about the world which is, in principle, beyond the power of science to give. [...] **The propositions of philosophy are not factual, but linguistic in character** – that is, they do not describe the behavior of physical, or even mental, objects ; they express definitions, or the formal consequences of definitions. Accordingly, we may say **that philosophy is a department of logic.** »

Alfred J. Ayer : *Language, Truth & Logic*, Penguin Modern Classics, 1952



« For the essential elements of the above characterizations, involving the correlations of the syntactic and semantic domains, the notion of analysis, and – perhaps most significantly – the requirements of **linguistic formulation** and the differentiation among predicate types, beyond strongly suggesting that the proper object of our assigned investigation may be – in the light of these criteria – a vacuous class, and strongly reminding us of the systematic obligations attending our own necessarily verbal presentation and discussion of the presumed subject, provide the important reminder that **there is but one kind of language, one kind of method for the verbal formulation of ‘concepts’ and the verbal analysis of such formulations: ‘scientific’ language and ‘scientific’ method** »



Milton Babbitt

M. Babbitt : « Past and Present Concepts of the Nature and Limits of Music », *Congress Report of the International Musicological Society*, 1961.

➡ M. Andreatta, « Mathématiques, Musique et Philosophie dans la tradition américaine: la filiation Babbitt/Lewin », in M. Andreatta, F. Nicolas, Ch. Alunni (dir.), *A la lumière des mathématiques et à l'ombre de la philosophie, Dix ans de séminaire mamuphi* , Collection "Musique/Sciences", Ircam-Delatour France, 2012.



Linguaggi formalizzati nella tradizione analitica americana

A SKETCH OF THE USE OF FORMALIZED LANGUAGES FOR THE ASSERTION OF MUSIC

MICHAEL KASSLER

Perspectives of New Music, Vol. 1, No. 2 (Spring, 1963), pp. 83-94

Abstract. After a brief explanation of formalized languages, a formalized language adequate for the assertion of a few twelve-note-class compositions is presented. Elementary set-theoretic concepts are introduced to specify properties of the formalized language and particular extensions of it. The paper concludes with discussion of applications of other formalized languages to various musical situations.

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1. Halmos, Paul R., *Naive set theory*. Princeton, Van Nostrand, 1960.
2. Church, Alonzo, *Introduction to Mathematical Logic*, V. 1., Princeton University Press, 1956
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4. Kassler, Michael, « A system for the automatic reduction of musical scores ». In Papers presented at the seminar in mathematical linguistics, V.6, 1960
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 Indeed any proof, in a formalized language for the assertion of music, may well be viewed as a “derivational” analysis: of the composition(s) denoted by the theorem which is the last wff in the proof; from the composition(s) denoted by the wffs in the proof which are axioms; by means of the musical “techniques” specified by a suitable interpretation of the rules of inference by which wffs in the proof have been inferred from preceding wffs in the proof. And the class of theorems, of a specific formalized language for the assertion of statements about music, may well be viewed as a formalization of a specific informal theory of music.

Dalla teoria musicale alla teoria della musica (*Music Theory*)

Df. 3.1: $\perp(t) =_{df} (: Q) (\exists i \exists j ((i, j) \in Q \supset Ts(i, j) \wedge t = j - i)).$

("The timespan interval t is the Q such that there exists an i and there exists a j such that if (i, j) is a member of Q then (i, j) is a timespan and t is $(j - i)$."

B. Boretz, *Meta-Variations: Studies in the Foundations of Musical Thought*. Red Hook, NY: Open Space.

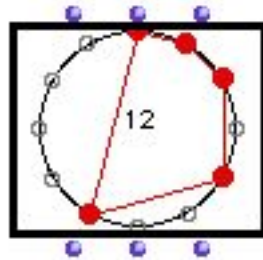
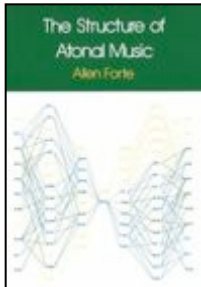
“Progressivamente, dal concetto di legge (generalità sintetica) si arriva ad un **sistema imbricato di leggi deduttive** che costituisce una **teoria**, esprimibile come un **insieme connesso di assiomi**, definizioni e teoremi le cui dimostrazioni sono ottenute attraverso un opportuno ragionamento logico. Una teoria musicale si riduce, o dovrebbe ridursi, ad una teoria formale una volta che predicati e operazioni non ancora interpretati si sostituiscono ai termini e alle operazioni che designano gli **osservabili musicali**”

M. Babbitt : « Past and Present Concepts of the Nature and Limits of Music », *Report of the Int. Mus. Soc.*, 1961

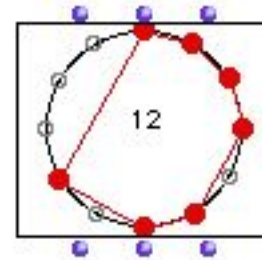
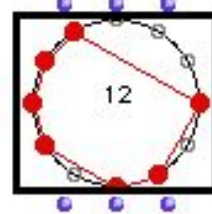
“[La struttura e la funzione della teoria della musica] sono di rendere possibile, da un lato, lo studio della struttura dei **sistemi musicali** [...] e la **formulazione** dei vincoli di questi sistemi in una prospettiva compositiva [...] ma anche, come tappa prealabile, una **terminologia adeguata** [...] per rendere possibile e stabilire un **modello** che autorizzi degli **enunciati ben determinati e verificabili sui brani musicali**”

M. Babbitt : « The Structure and Function of Music Theory », 1965

Il catalogo dei *pcs* d'Allen Forte (1973) e la relazione Z



Passaggio al complementare



Allen Forte

5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140

5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211

5-Z36 0,1,2,4,7 222121

6-Z4(12)	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

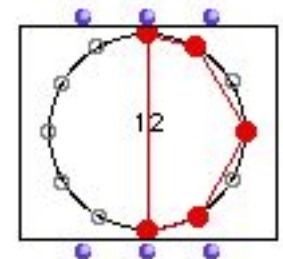
7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361

7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442

7-Z36 0,1,2,3,5,6,8 444342

6-Z37(12)	0,1,2,3,4,8	
6-Z38(12)	0,1,2,3,7,8	

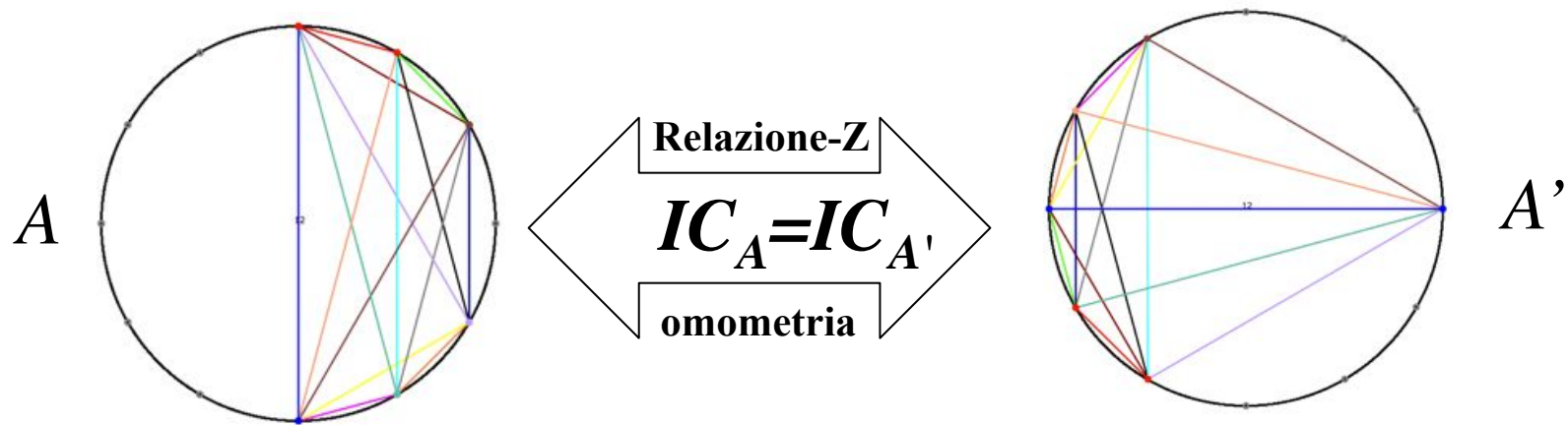
6-Z39	0,2,3,4,5,8	
6-Z40	0,1,2,3,5,8	
6-Z41	0,1,2,3,6,8	
6-Z42(12)	0,1,2,3,6,9	



5-Z12

Il teorema dell'esacordo

Il *contenuto intervallare* IC_A di un accordo è la molteplicità di apparizione dei suoi intervalli



$$IC_A = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = IC_{A'}$$

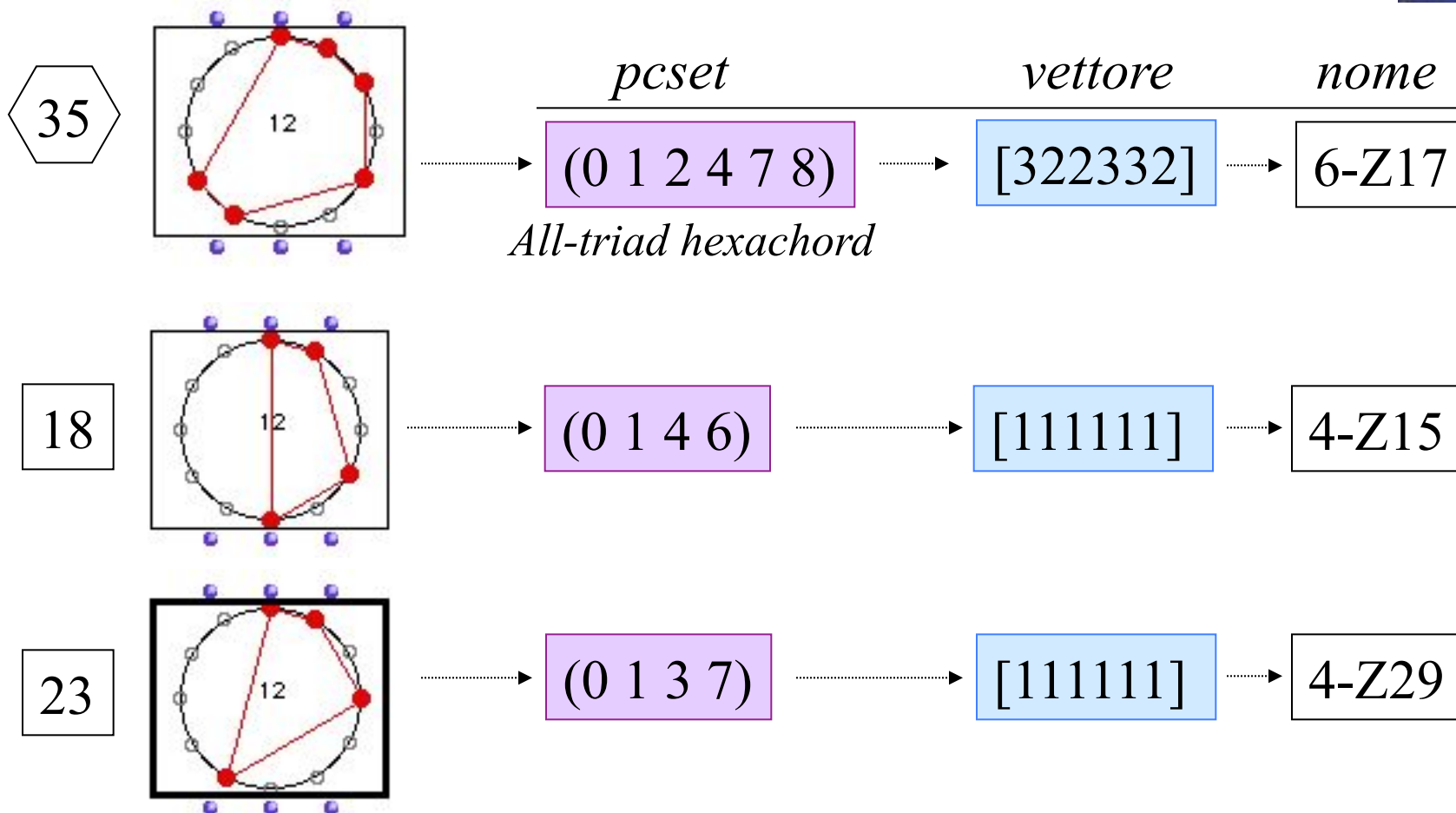
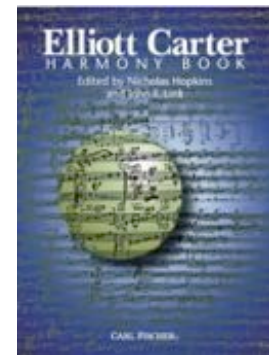
Un esacordo e il suo complementare hanno lo stesso contenuto intervallare.



Milton Babbitt

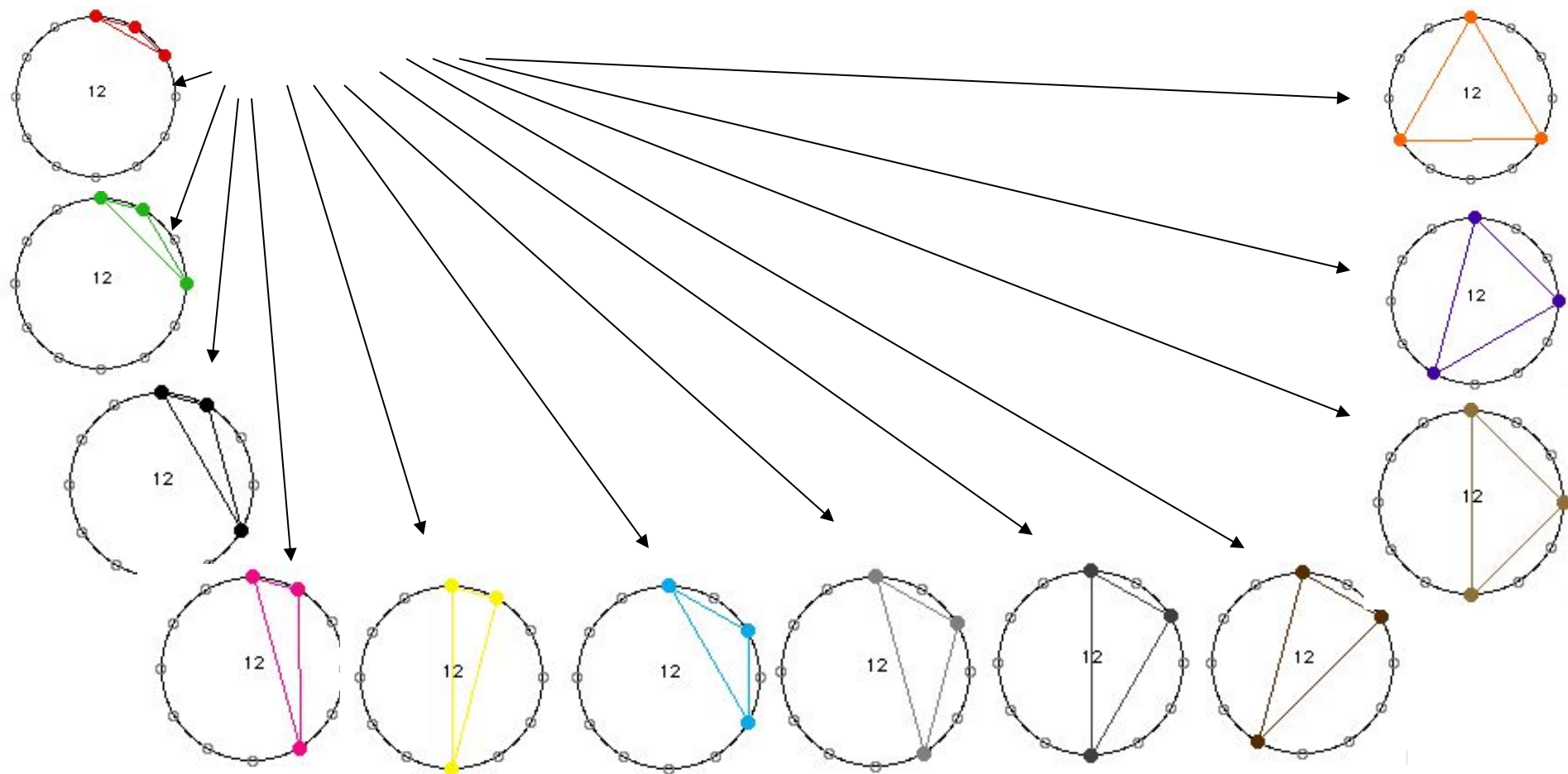
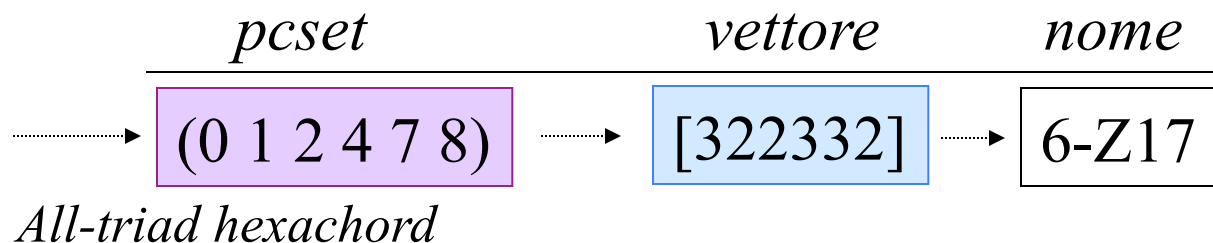
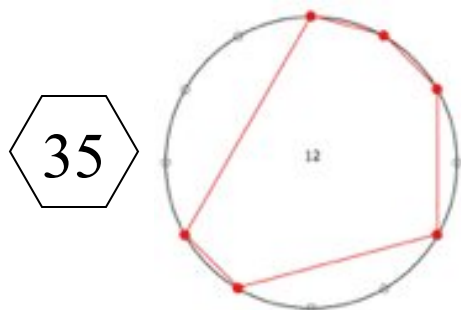
Carter e il catalogo compositivo della *Set Theory*

“A partire dal 1990 ho ridotto progressivamente il mio vocabolario di accordi alla struttura esacordale n°35 e ai due tetracordi n°18 e n°23, che contengono tutti gli intervalli” (*Harmony Book*, 2002, p. ix)



L'esacordo 'omnitriadico'

	1	2	3	4	5	6	7	8	9	10	11	12
Z_0	1	6	19	43	66	80	66	43	19	6	1	1
D_0	1	6	12	29	38	50	38	29	12	6	1	1
A_0	1	5	9	21	25	34	25	21	9	5	1	1
S_0	1	6	12	15	12	11	7	5	3	2	1	1



Elliott Carter: 90+ (1994)



• Combinatoria accordale

- Esacordi
- Tetracordi
- Triadi
- Z-relazione
- Link-chords (caso particolare di una serie 'all-intervals')



(piano: John Snijders)

90+ Elliott Carter (1994)

musica e sonando sempre il coro Goffredo

♩ = 96

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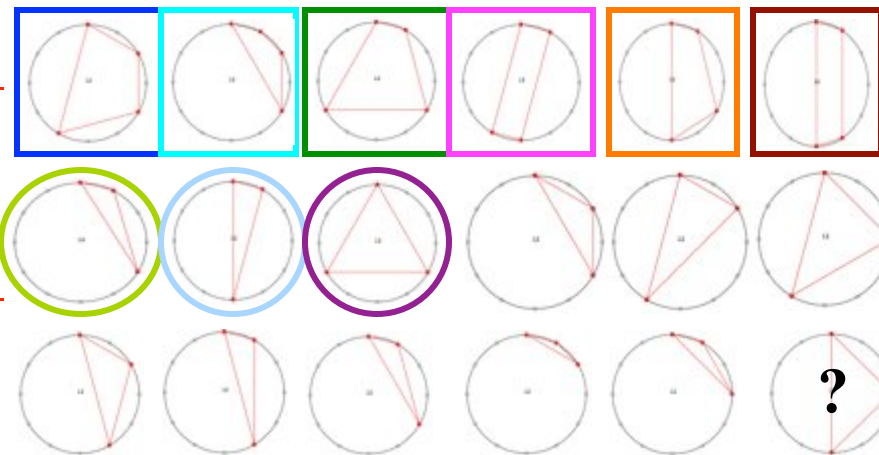
1442

Elliott Carter : 90+ (1994) : combinatoria tetra-tricordale

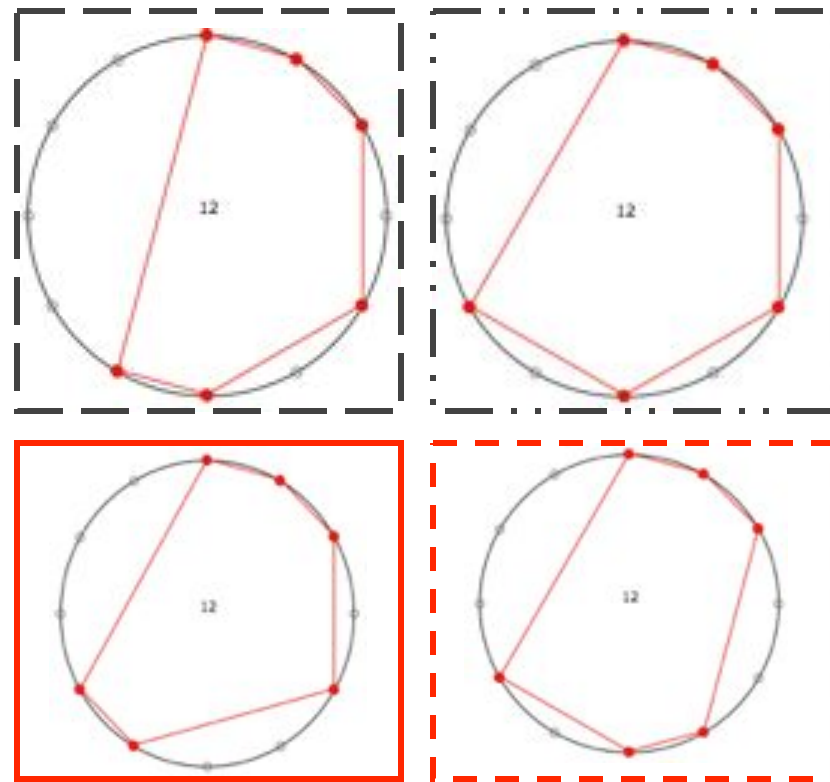
musica e armonia ispirati a John Coltrane

90+

Elliott Carter
(1994)



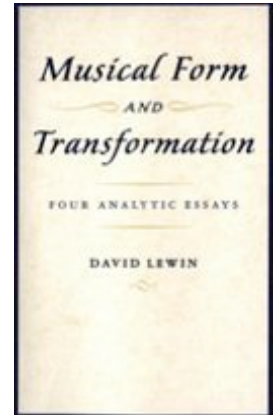
Musical score for Elliott Carter's '90+'. The score is divided into four systems, each with two staves. Various sections are highlighted with colored boxes (blue, green, orange, yellow, magenta, cyan, purple, brown) and dashed lines, indicating specific harmonic or structural elements.



Estensioni trasformativazionali della *Set Theory*



David Lewin



« Making and Using a Pcset Network for Stockhausen's *Klavierstück III* »



Tre interpretazioni:



Henck



Kontarsky

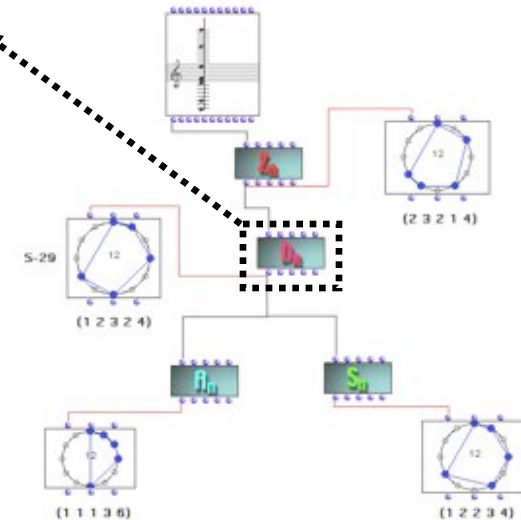
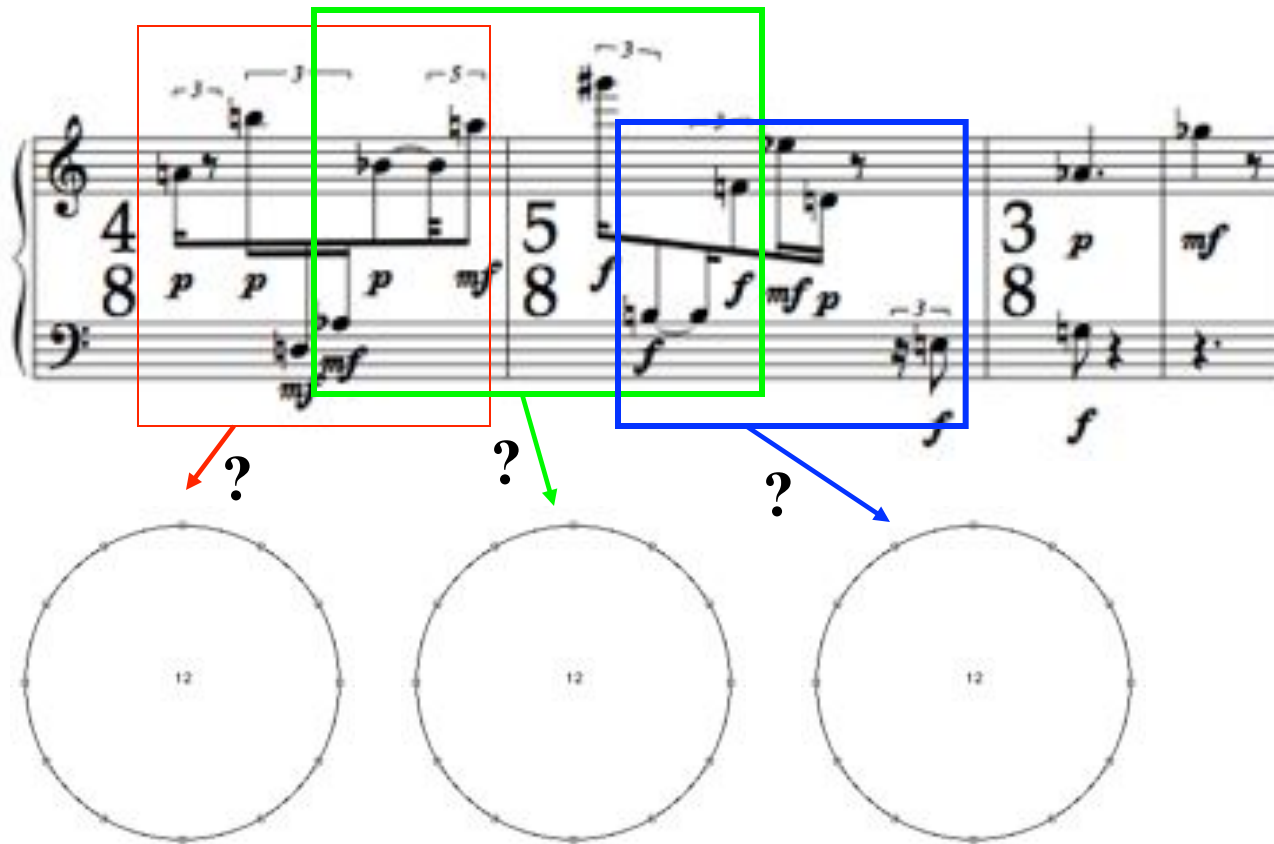


Tudor



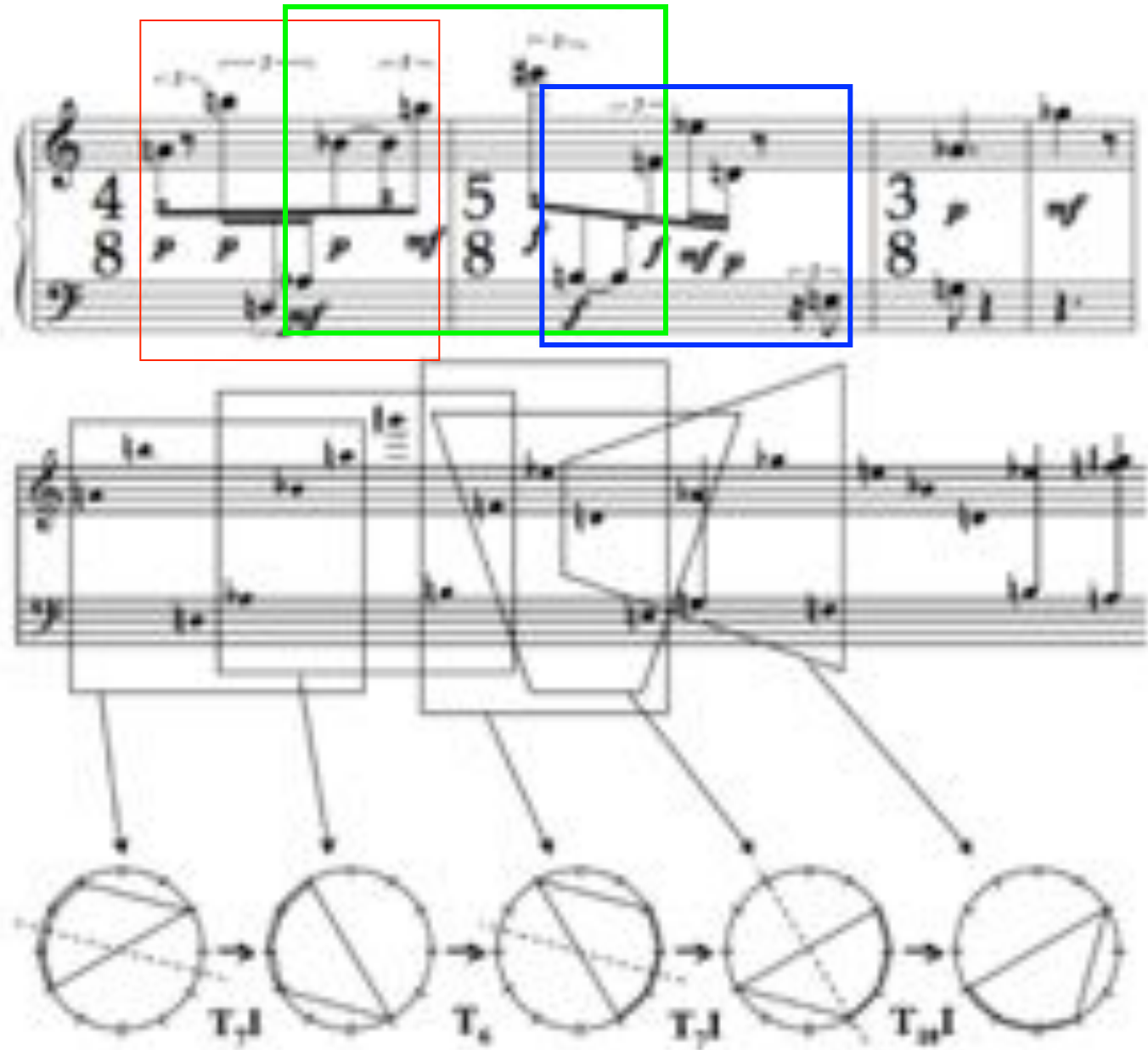
K. Stockhausen

« Making and Using a Pcset Network for Stockhausen's Klavierstück III »

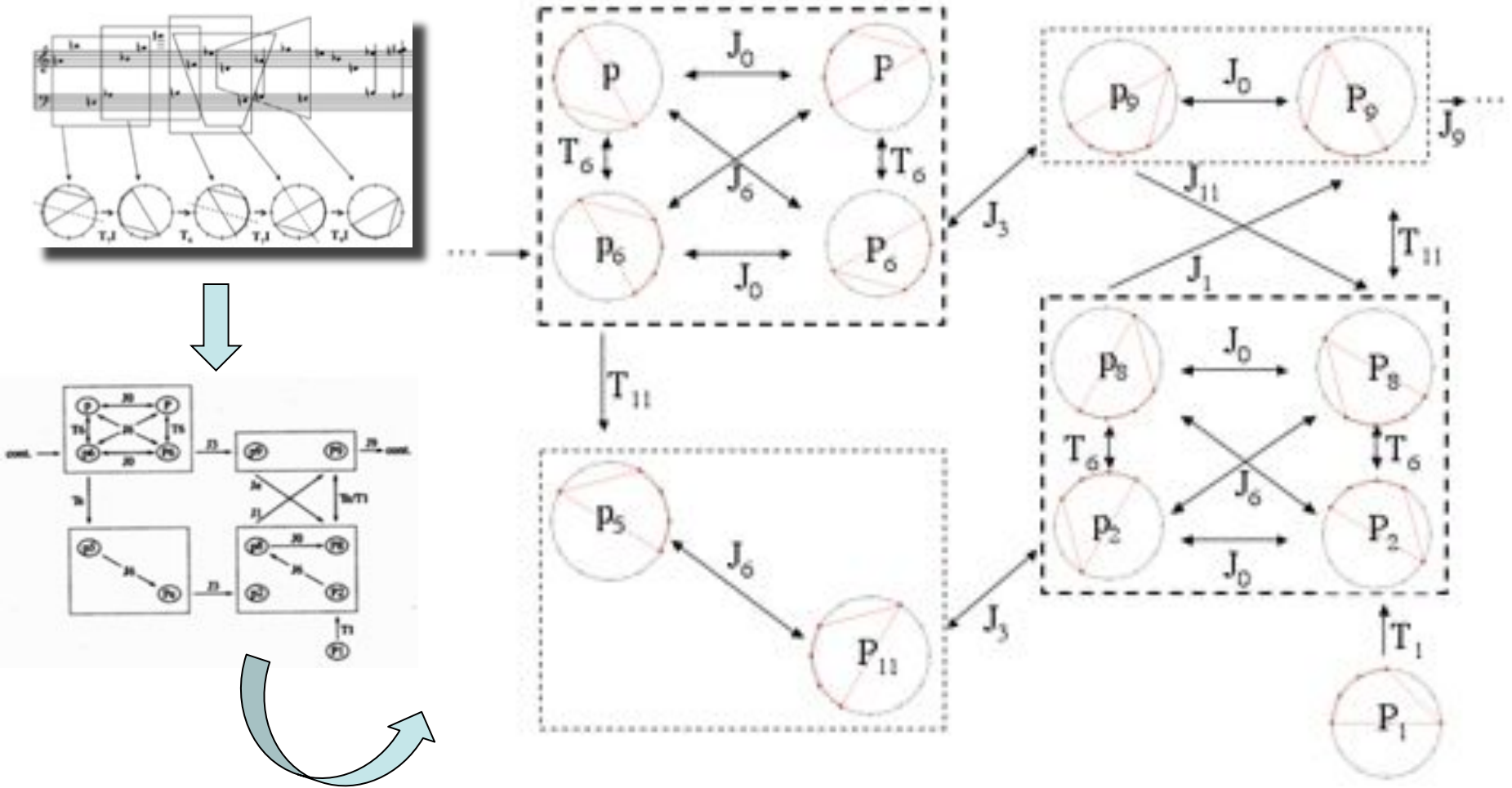


“Fra i quattro saggi, questo è sicuramente il più ‘teorico’, e si concentra sulle diverse forme di un pentacordo relativamente mobile all’interno del brano. Un gruppo di trasformazioni è messo in evidenza a partire dalle relazioni musicali delle diverse forme del pentacordo. Attraverso questa famiglia di trasformazioni, il saggio dispone le forme del pentacordo in una configurazione spaziale che illustra la struttura reticolare per questo particolare fenomeno all’interno dell’intero brano”.

Segmentazione per imbricazione e progressione ‘crono-logica’



Reticolo trasformatoriale come spazio concettuale del Klavierstück

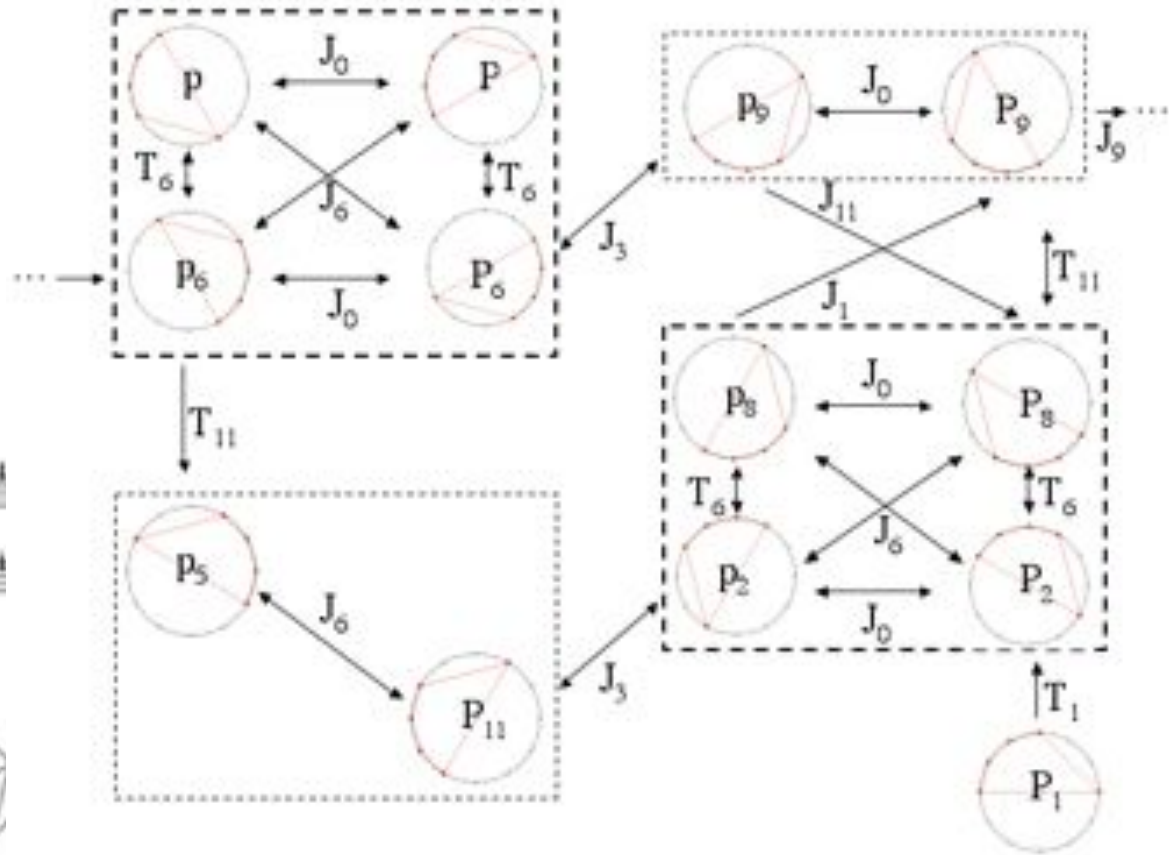


“Piuttosto che affermare l’esistenza di un reticolo che mette in evidenza le relazioni pentacordali una ad una, seguendo la cronologia del brano, il saggio proone un reticolo che dispone le diverse forme del pentacordo utilizzato e le loro relazioni potenziali funzionali in una configurazione spaziale estremamente compatta”

Analisi trasformazionale e *rational reconstruction*: alla ricerca di relazioni strutturali (o ‘isografie’)

SH: (1, 1, 1, 3, 6) (6, 3, 1, 1, 1) (6, 3, 1, 1, 1)
 IFUNC: (5 3 2 2 1 1 1 1 2 2 3) (5 3 2 2 1 1 1 1 2 2 3) (5 3 2 2 1 1 1 1 2 2 3)
 VR: (3 2 2 1 1 1) (3 2 2 1 1 1) (3 2 2 1 1 1)

The diagram shows a musical score in 4/8 time with three systems. Below each system are pitch class diagrams (circles with lines) representing the pitch classes of the notes. The first system has three diagrams, the second has three, and the third has five. Arrows indicate transformations between these diagrams, labeled T_1 , T_6 , T_7 , and T_9 .



“Una **ricostruzione razionale** di un brano, in quanto **teoria** del brano analizzato, è una spiegazione (*explanation*) non tanto del processo reale di costruzione del brano ma di come il brano può essere costruito da un ascoltatore, di come ciò che è dato può essere assunto (*how the ‘given’ may be ‘taken’*)”



M. Babbitt

Klumpenhouwer Networks e isografie

D. Lewin, A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2, JMT, 1994



D. Lewin



H. Klumpenhouwer

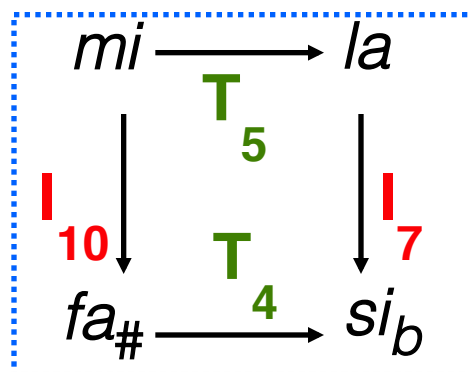
$$\langle T_k \rangle : T_m \rightarrow T_m$$

$$I_m \rightarrow I_{k+m}$$

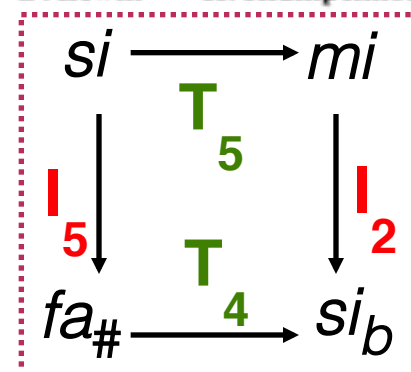
$$T_m : x \rightarrow x+m$$

$$I_m : x \rightarrow m-x$$

$$\langle T_k \rangle \cdot \langle T_m \rangle = \langle T_{k+m} \rangle$$

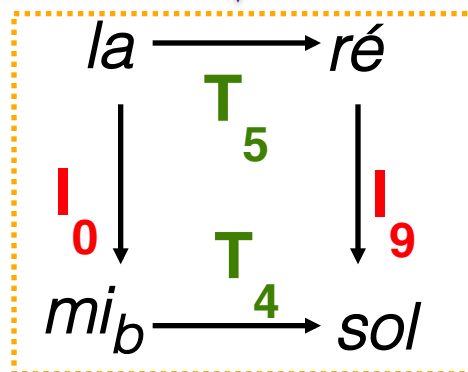


$$\langle T_7 \rangle$$

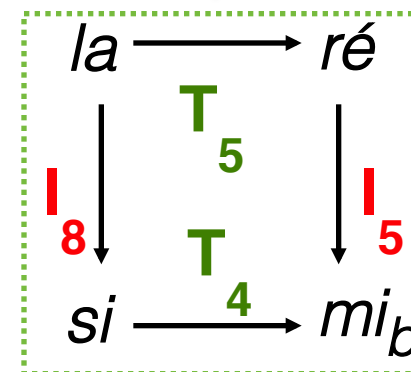


$$\langle T_{10} \rangle$$

$$\langle T_3 \rangle$$



$$\langle T_8 \rangle$$



Klumpenhouwer Networks e isografie

D. Lewin, A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2, JMT, 1994



D. Lewin



H. Klumpenhower

$$\langle T_k \rangle : T_m \rightarrow T_m$$

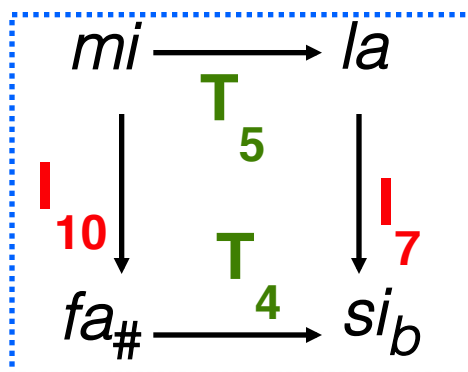
$$I_m \rightarrow I_{k+m}$$

$$\langle I_k \rangle : T_m \rightarrow T_{-m}$$

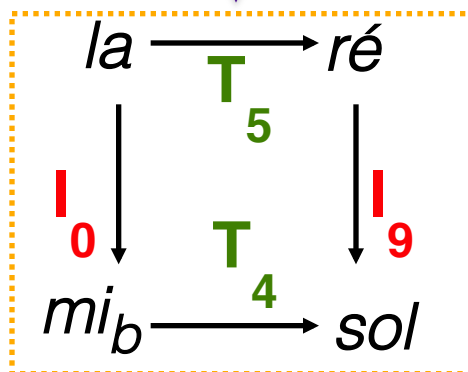
$$I_m \rightarrow I_{k-m}$$

$$\langle T_k \rangle \cdot \langle T_m \rangle = \langle T_{k+m} \rangle$$

$$\langle I_k \rangle \cdot \langle I_m \rangle = \langle T_{m-k} \rangle$$



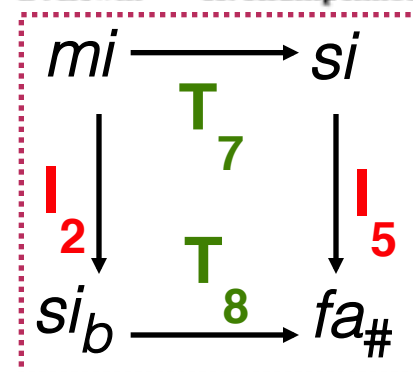
$\langle T_2 \rangle$



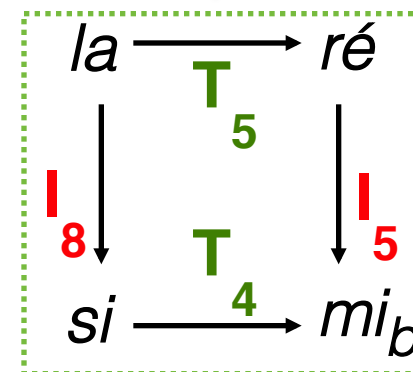
$\langle I_0 \rangle$

$\langle T_{10} \rangle$

$\langle T_8 \rangle$



$\langle I_{10} \rangle$



Klumpenhower Networks e isografie

D. Lewin, A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2, JMT, 1994



D. Lewin



H. Klumpenhower

$$\langle T_k \rangle : T_m \rightarrow T_m$$

$$I_m \rightarrow I_{k+m}$$

$$\langle I_k \rangle : T_m \rightarrow T_{-m}$$

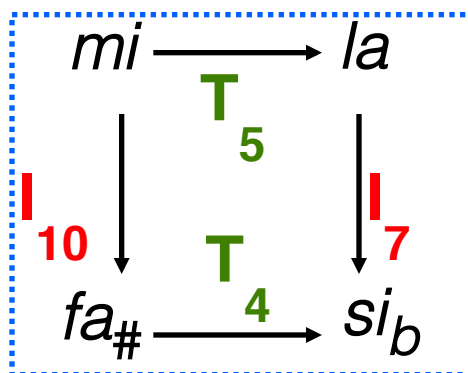
$$I_m \rightarrow I_{k-m}$$

$$\langle T_k \rangle \cdot \langle T_m \rangle = \langle T_{k+m} \rangle$$

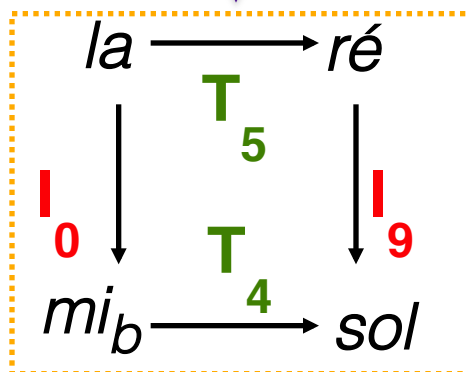
$$\langle T_k \rangle \cdot \langle I_m \rangle = \langle I_{m-k} \rangle$$

$$\langle I_m \rangle \cdot \langle T_k \rangle = \langle I_{k+m} \rangle$$

$$\langle I_k \rangle \cdot \langle I_m \rangle = \langle T_{m-k} \rangle$$



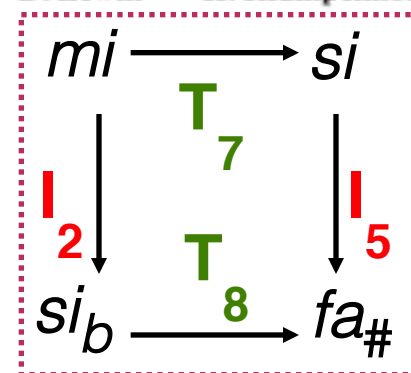
$\langle T_2 \rangle$



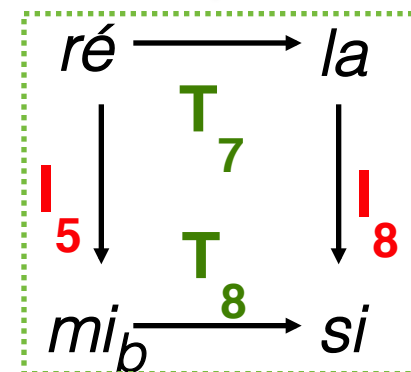
$\langle I_0 \rangle$

$\langle I_3 \rangle$

$\langle I_5 \rangle$

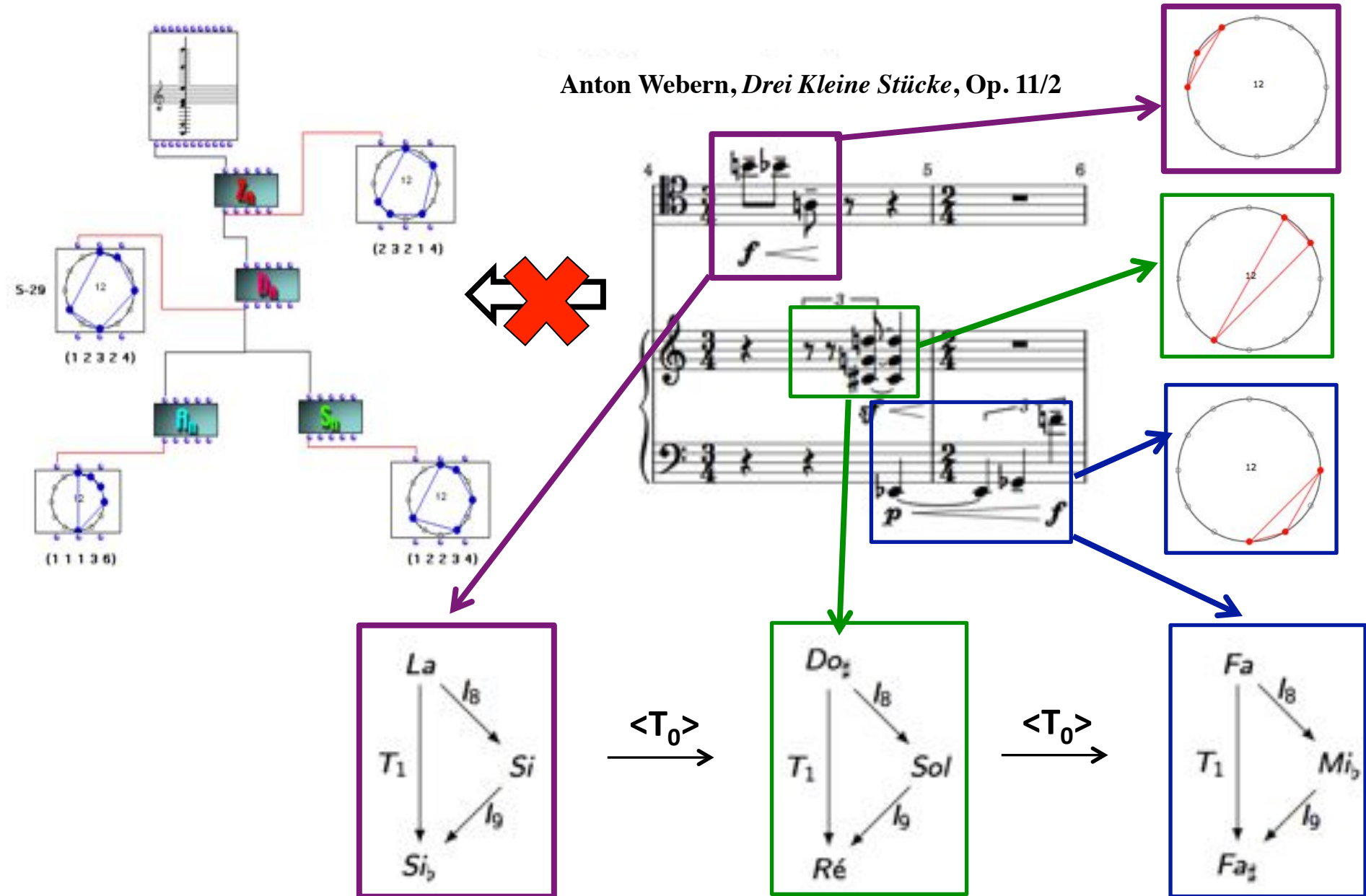


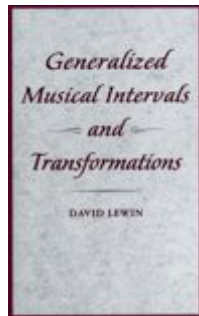
$\langle T_3 \rangle$



I Klumpenhouwer Networks e l'approccio paradigmatico

Anton Webern, *Drei Kleine Stücke*, Op. 11/2





D. Lewin

La generalizzazione della nozione di intervallo

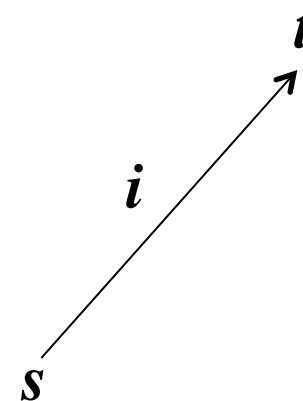
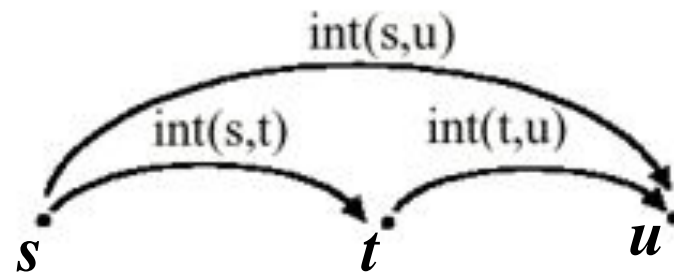
$$\text{GIS} = (S, G, \text{int})$$

S = insieme

$(G, \bullet)$ = gruppo d'intervalli

int = distanza intervallare

$$S \times S \xrightarrow{\text{int}} G$$



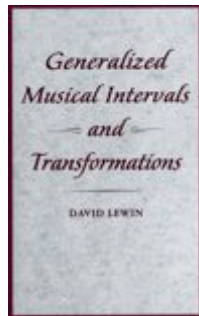
Azione
semplicemente
transitiva

1. Dati tre oggetti musicali s, t, u in S :

$$\text{int}(s, t) \bullet \text{int}(t, u) = \text{int}(s, u)$$

2. Dato un oggetto musicale s in S e un intervallo i in G , vi è un solo oggetto musicale t in S tale che $\text{int}(s, t) = i$

- $S = \{\dots, do, do_{\#} = ré_b, ré, \dots, si, do', \dots\}$, $G = \mathbf{Z}$, $\text{int}(do, ré) = 2$, $\text{int}(fa, do) = -5$ etc.
- $S = \{\dots, do, ré, mi, fa, sol, la, si, do', \dots\}$, $G = \mathbf{Z}$, $\text{int}(do, ré) = 1$, $\text{int}(fa, do) = -3$ etc.
- $S = G = \mathbf{Z}_{12} = \{do, do_{\#} = ré_b, ré, \dots, si\}$, $\text{int}(do, ré) = 2$, $\text{int}(fa, do) = 7$ etc.



D. Lewin

La generalizzazione della nozione di intervallo

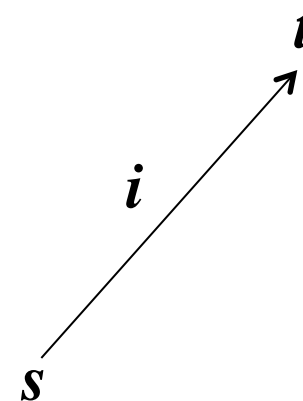
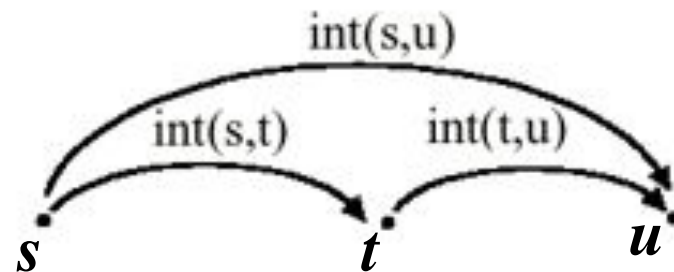
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$$S \times S \xrightarrow{\text{int}} G$$



Azione
semplicemente
transitiva

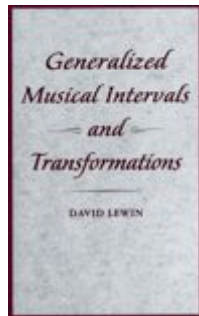
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2. Dato un oggetto musicale s in S e un

intervallo i in G , vi è un solo oggetto musicale t in S tale che $\text{int}(s, t) = i$

- $S = \{(a, x) \in \mathbf{R} \times \mathbf{R}^+\}$, $G = (\mathbf{R}, +) \times (\mathbf{R}^+, \times)$, $(s, x) \bullet (t, y) = (s+t, xy)$
 $\text{int}((s, x), (t, y)) = (t-s, y/x) \rightarrow (S, G, \text{int})$ è un GIS commutativo
- $S = \{(a, x) \in \mathbf{R} \times \mathbf{R}^+\}$, $G = (\mathbf{R}, +) \times (\mathbf{R}^+, \times)$, $(s, x) \bullet (t, y) = (s+xt, xy)$
 $\text{int}((s, x), (t, y)) = ((t-s)/x, y/x) \rightarrow (S, G, \text{int})$ è un GIS non commutativo



D. Lewin

La generalizzazione della nozione di intervallo

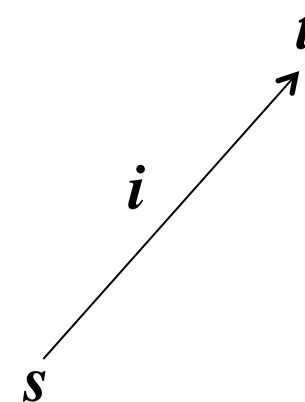
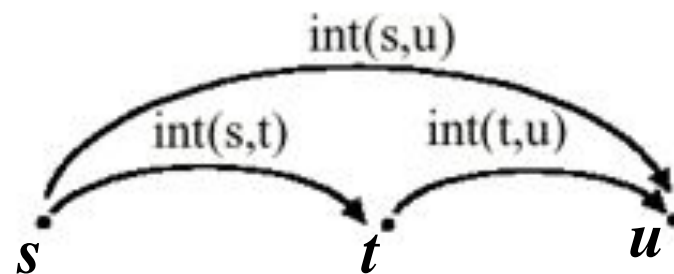
$$\text{GIS} = (S, G, \text{int})$$

S = insieme

$(G, \bullet)$ = gruppo d'intervalli

int = distanza intervallare

$$S \times S \xrightarrow{\text{int}} G$$



Azione
semplicemente
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$$\text{int}(s, t) \bullet \text{int}(t, u) = \text{int}(s, u)$$

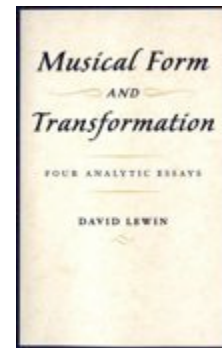
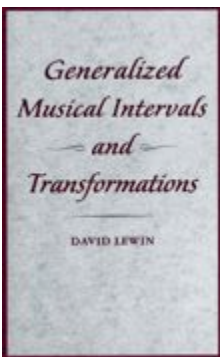
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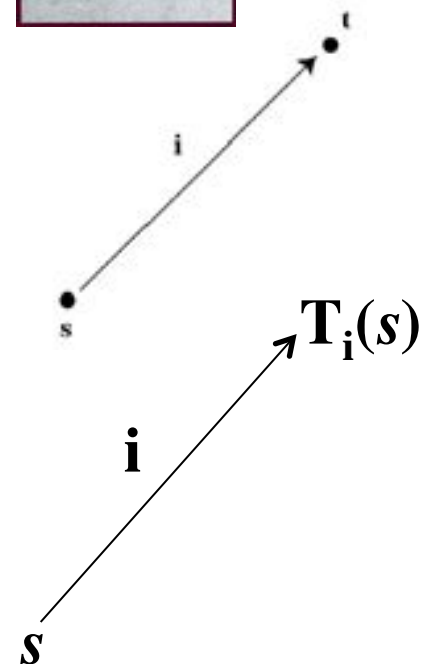
- $S = \{(a, x) \in \mathbb{R} \times \mathbb{R}^+\}$, $G = (\mathbb{R}, +) \times (\mathbb{R}^+, \times)$, $(s, x) \bullet (t, y) = (s+t, xy)$
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 $\text{int}((s, x), (t, y)) = ((t-s)/x, y/x) \rightarrow (S, G, \text{int})$ è un GIS non commutativo

Implicazioni filosofiche dell'approccio trasformatzionale

Cartesianismo vs Fenomenologia

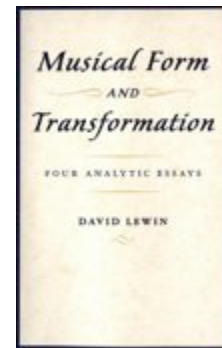
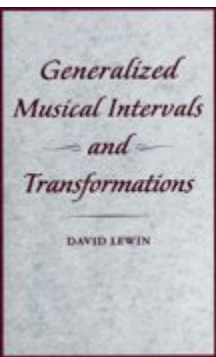


“Anziché prendere come punto di partenza la struttura di GIS (= sistema d’intervalli generalizzati) e derivare da questo alcune trasformazioni caratteristiche, è possibile considerare come punto di partenza una famiglia di trasformazioni caratteristiche e derivare da questa una struttura di GIS. Questo significa che, anziché osservare la freccia ‘intervallo’ i come la misura di un’*estensione* fra due punti s e t osservati in modo passivo come “out there”, in una sorta di *res extensa* cartesiana, è possibile osservare la situazione musicale in modo *attivo*, come farebbe un cantante, uno strumentista o un compositore che si chiede: “A partire dal punto s , quale trasformazione particolare devo operare [*perform*] per arrivare nel punto t ?” [È in questa diversa attitudine che risiede un’] “intricazione concettuale [*conceptual interrelation*] fra l’intervallo inteso come estensione e la trasposizione in quanto movimento caratteristico all’interno di uno spazio musicale”.

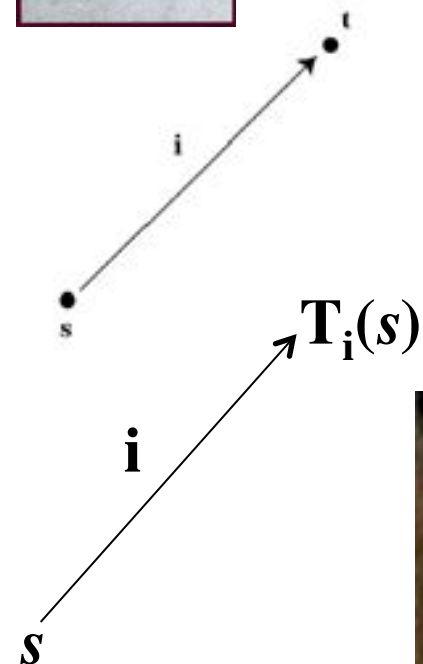


Implicazioni filosofiche dell'approccio trasformatzionale

Generalizzazione dello spazio musicale



“Non abbiamo l'intuizione di ciò che potrebbe essere definito lo “spazio musicale”. Ma piuttosto abbiamo l'intuizione di una molteplicità e una varietà di spazi musicali allo stesso tempo. La struttura di GIS e i reticoli trasformationali possono aiutarci a esplorare queste diverse intuizioni e studiare la maniera attraverso la quale queste intuizioni interagiscono, da un punto di vista logico e all'interno di un brano musicale particolare”.



F	C	G	D
A	E	H	F _s
C _s	G _s	D _s	B.

