

Algebraic and topological models in computational music analysis



IRCAM = Institut de Recherche et Coordination Acoustique/Musique



■ UMR STMS

www.ircam.fr

The mixed research lab UMR9912 brings together the CNRS, the UPMC, the French Ministry of Culture, and IRCAM around the theme of multidisciplinary research on sciences and technologies for music and sound.

The lab is associated with the CNRS Institutes for Information Sciences and Technologies ([INSI](#)), for Engineering and Systems Sciences ([INSE](#)), for Humanities and Social Sciences ([INSHS](#)) and of Biological Sciences ([INSB](#)). It is also a part of the UPMC's faculty of engineering (JFR 912) in the [Research Pole for Modeling and Engineering](#).

Director: Gérard Assayag (IRCAM)

Deputy Director: Hugues Vinet (IRCAM)

As of January 1, 2012, the laboratory consisted of the following teams:

- [Instrumental Acoustics](#)
- [Acoustic and Cognitive Spaces](#)
- [Perception and Sound Design](#)
- [Analysis/Synthesis](#)
- [Music Representation](#)
- [Analysis of Musical Practices](#)
- [Real-Time Musical Interactions](#)
- [IRCAM Resource Center](#)



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Acoustics, Signal Processing and Computer Sciences



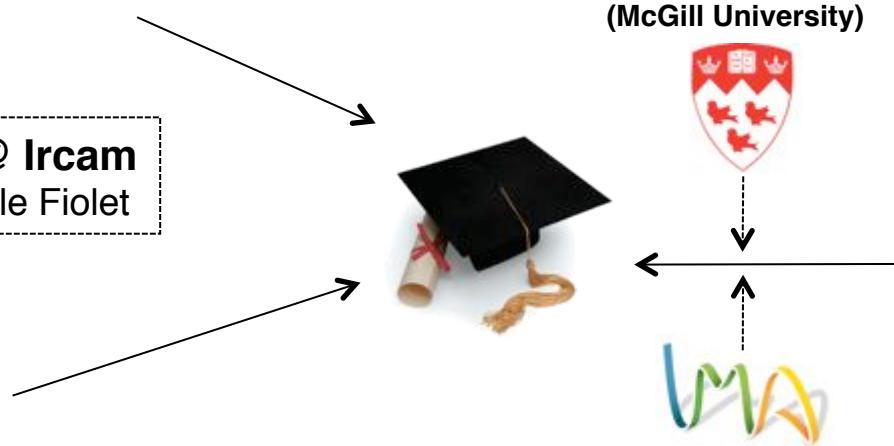
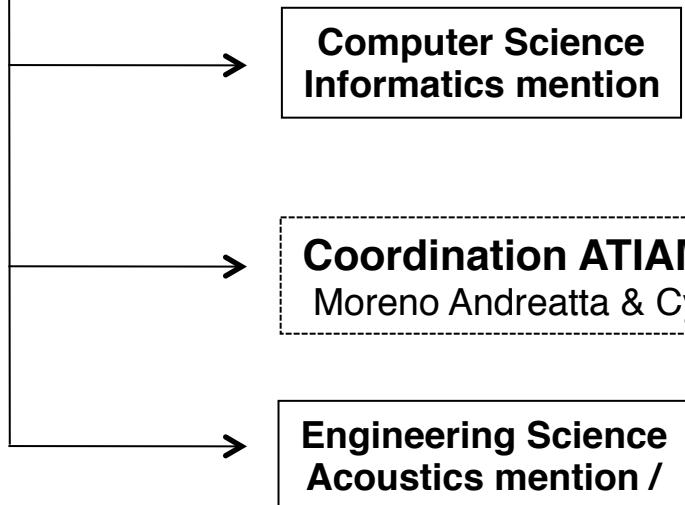
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PARCOURS MASTER 2



Parcours multi-mentions du master Sciences et technologies
Université Pierre et Marie Curie
en collaboration avec l'Ircam et Télécom ParisTech



(McGill University)

Laboratory of Mechanics
and Acoustics / Marseille

The interplay between algebra and geometry in music

MATH / MUSIC MEETINGS

Creativity in Music and Mathematics

Pierre Boulez & Alain Connes

Encounter with two major figures of musical creation and contemporary mathematical research: Pierre Boulez and Alain Connes.

What is the role of intuition in mathematical reasoning and in artistic activity? Is there an aesthetic dimension to mathematical activity? Does the notion of elegance of a mathematical demonstration or of a theoretical construction in music play a role in creativity?



Gérard Assayag, director of the CNRS/IRCAM Laboratory for The Science and Technology of Music and Sound, will lead this dialogue on invention in the two disciplines.

Photo: Pierre Boulez © Jean Racine

Wednesday, June 15, 2011, 8:30pm / IRCAM, Espace de projection

→ <http://agora2011.ircam.fr>

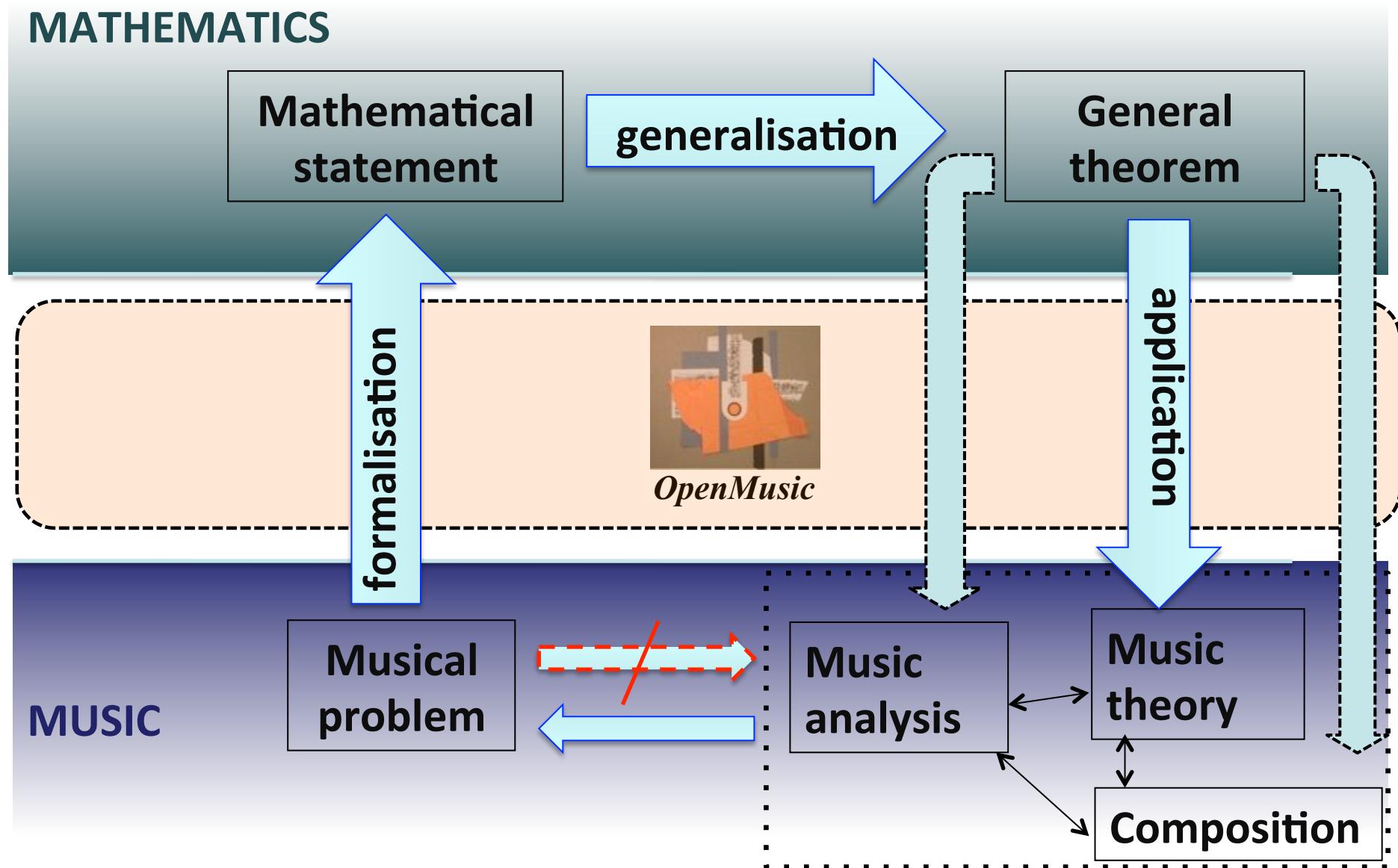


“Concerning **music**, it takes place in **time**, like **algebra**. In **mathematics**, there is this fundamental duality between, on the one hand, **geometry** – which corresponds to the visual arts, an immediate intuition – and on the other hand **algebra**. This is not visual, it has a temporality. This fits in time, it is a computation, something that is very close to the language, and which has its diabolical precision. [...] **And one only perceives the development of algebra through music**” (A. Connes).

→ <http://videotheque.cnrs.fr/>



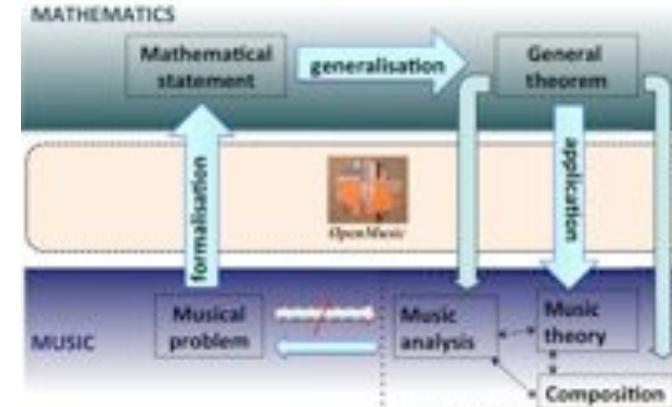
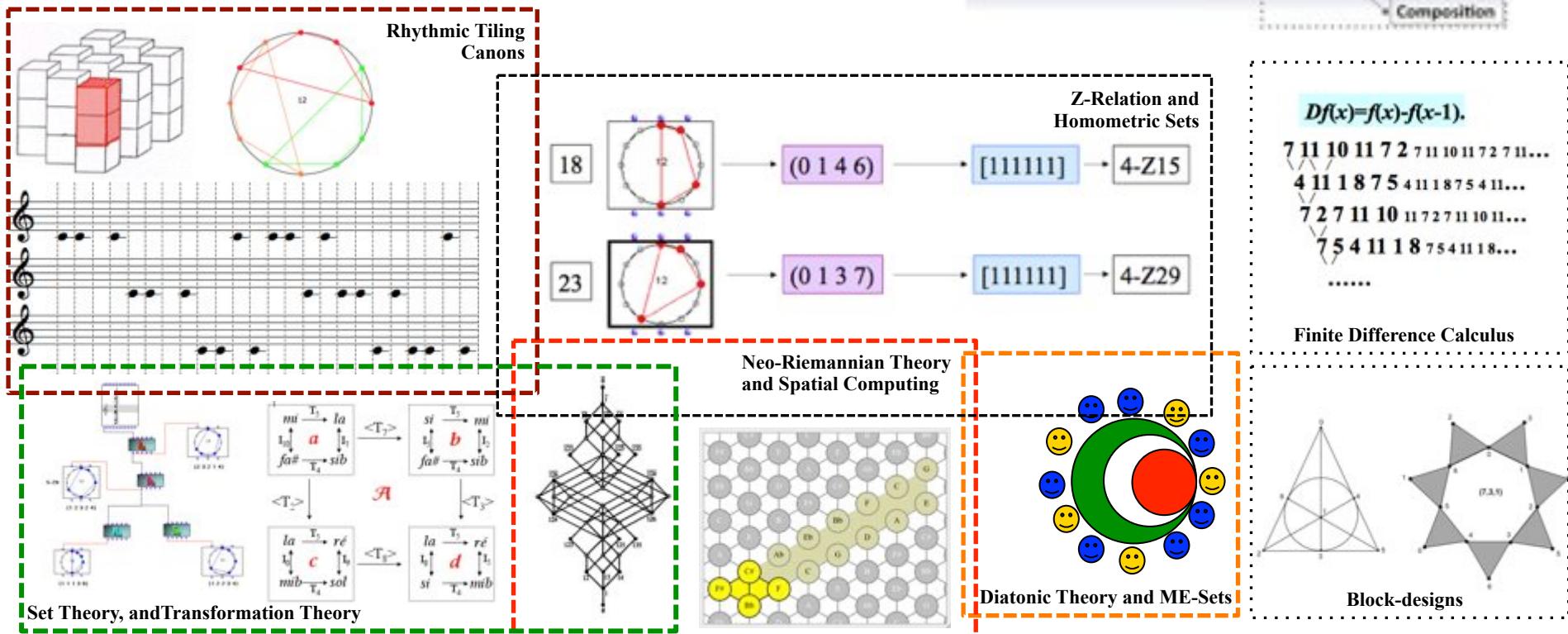
The double movement of a ‘mathemusical’ activity



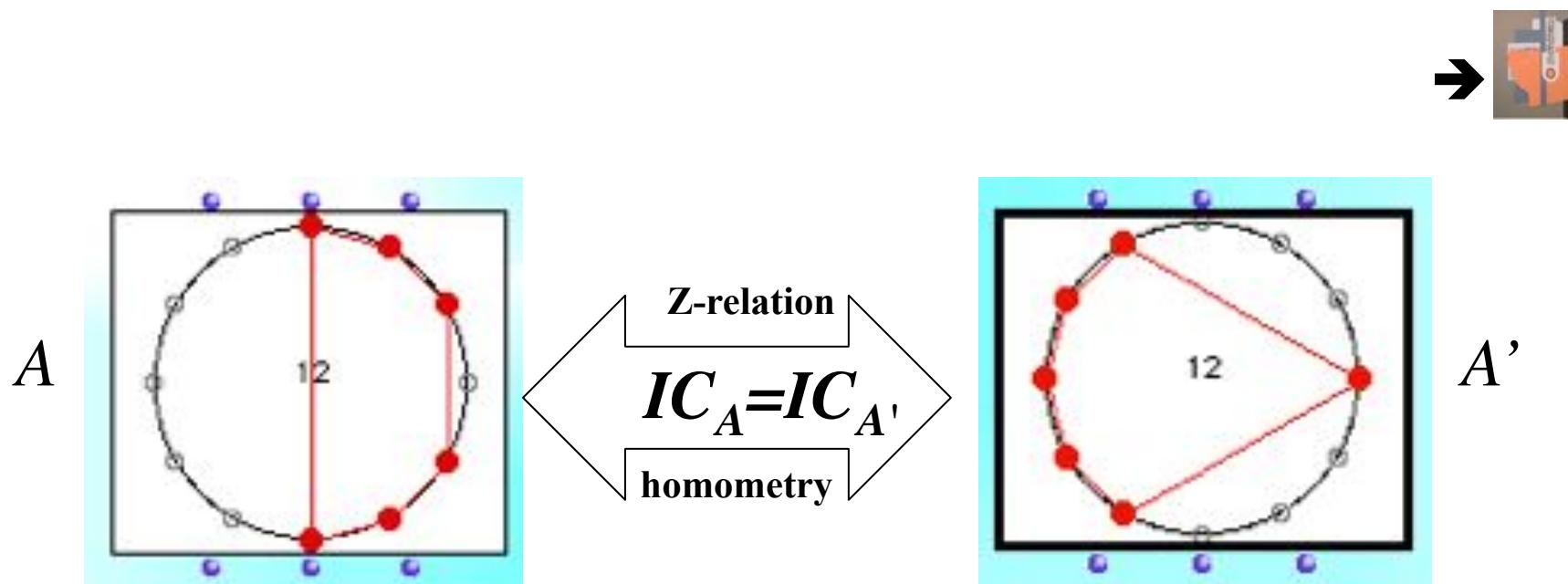
Some examples of ‘mathemusical’ problems

M. Andreatta : *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

- The construction of Tiling Rhythmic Canons
- The Z relation and the theory of homometric sets
- *Set Theory* and Transformational Theory
- Neo-Riemannian Theory, Spatial Computing and FCA
- Diatonic Theory and Maximally-Even Sets
- Periodic sequences and finite difference calculus
- Block-designs and algorithmic composition



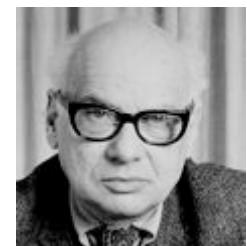
A ‘Mathemusical’ Theorem



$$IC_A = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = IC_{A'}$$

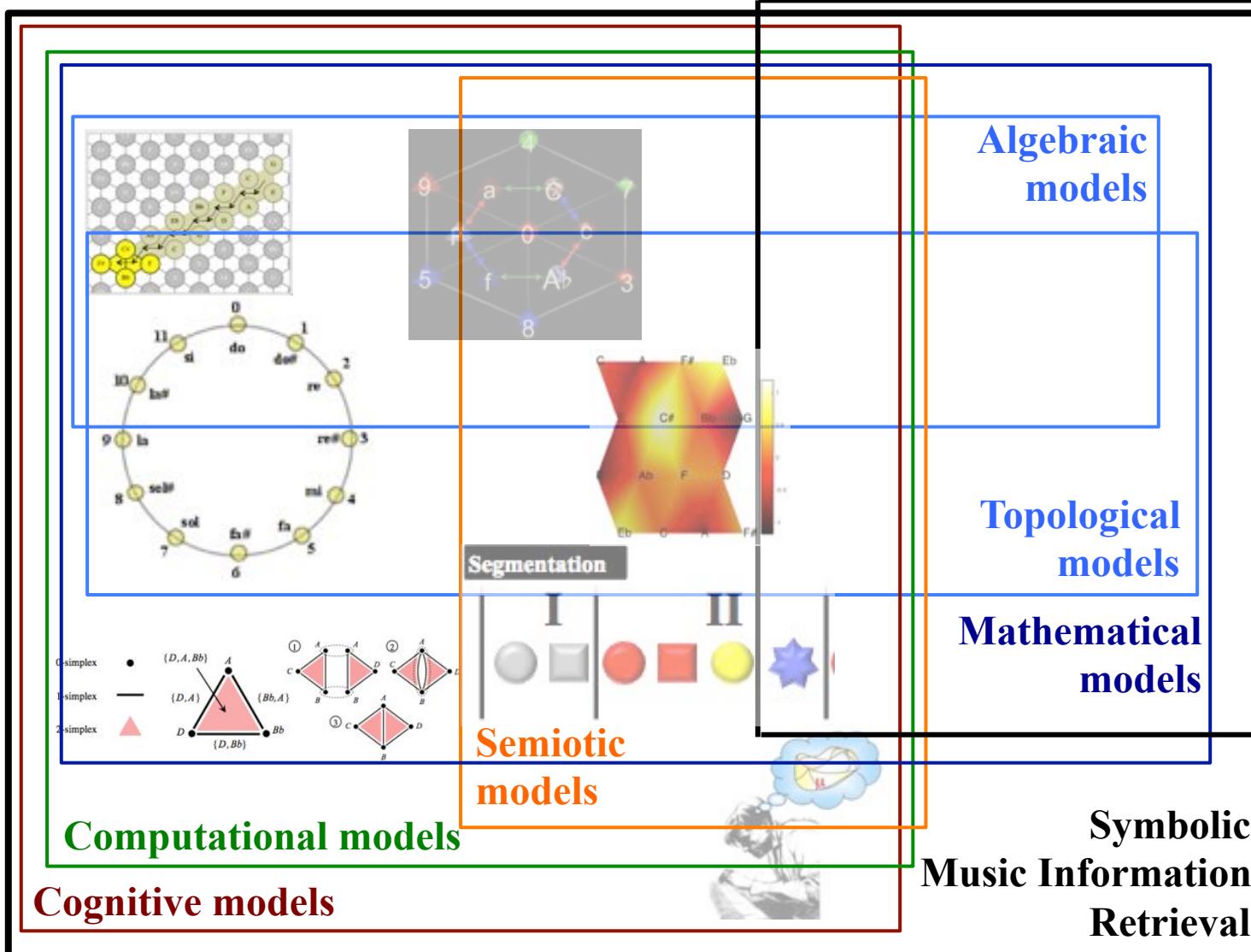
Babbitt's Hexachord Theorem:

A hexacord and its complement have the same interval content

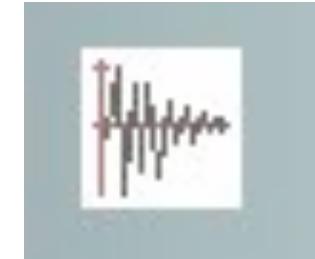


(Proofs by Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, ..., Amiot, ...)

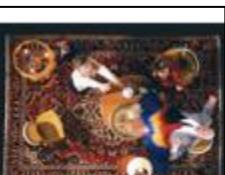
Math'n Pop Project: formal models for and within popular music



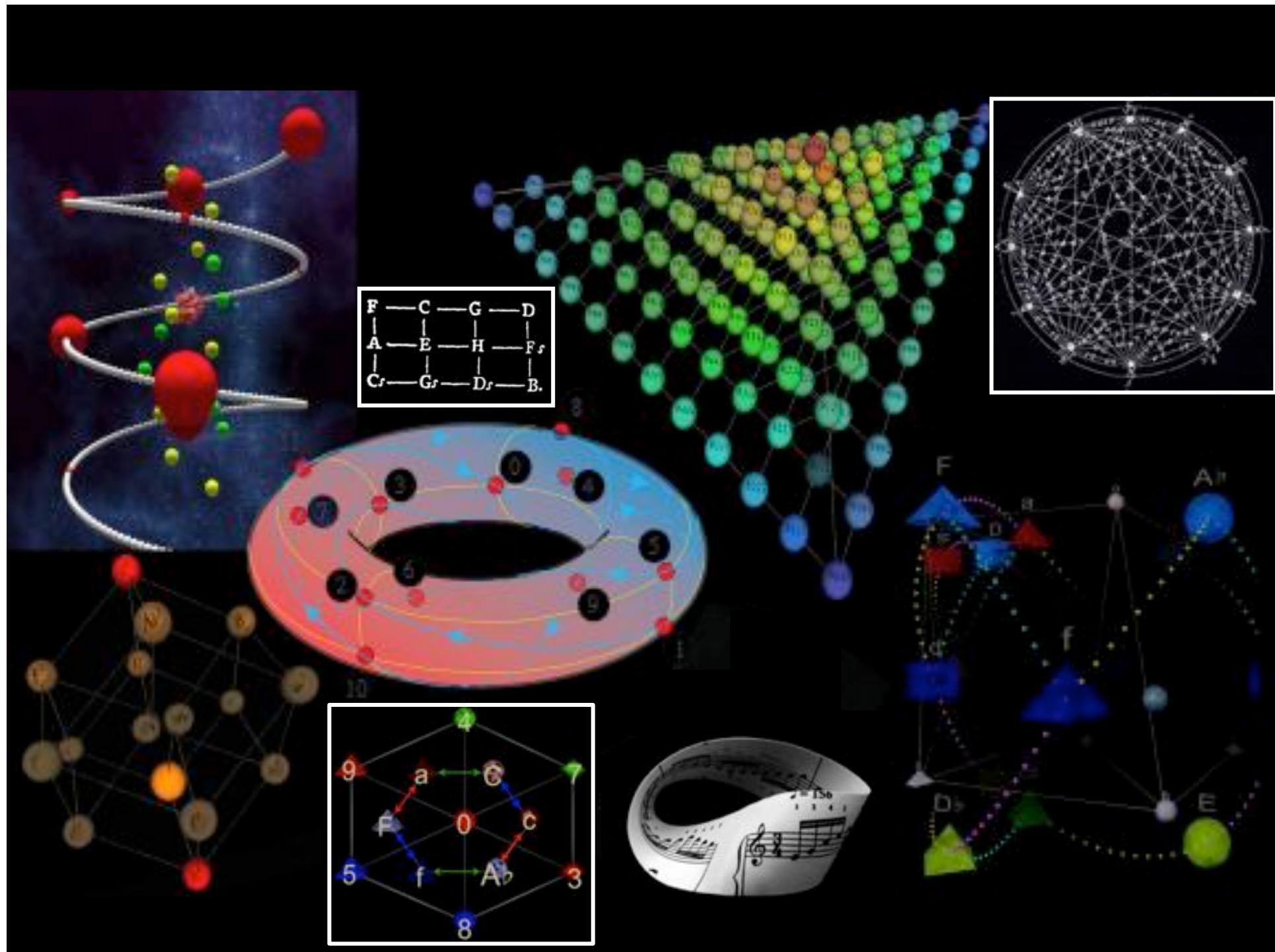
Signal-based
Music Information
Retrieval



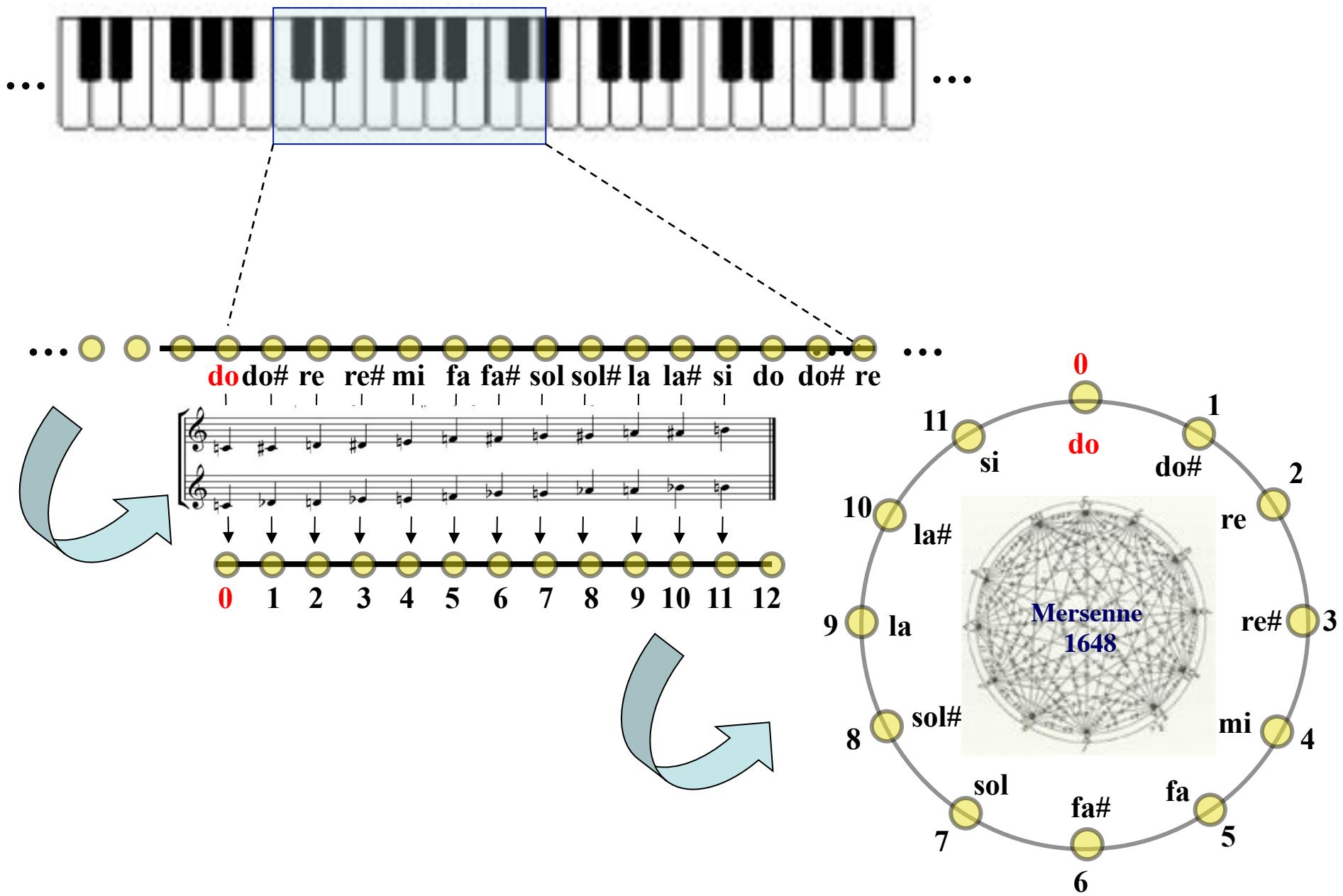
- Andreatta M. (2014), « Modèles formels dans et pour la musique pop, le jazz et la chanson : introduction et perspectives futures », dans *Esthétique & Complexité : Neurosciences, Philosophie et Art*, Z. Kapoula, L.-J. Lestocart, J.-P. Allouche éds., éditions du CNRS, 2014



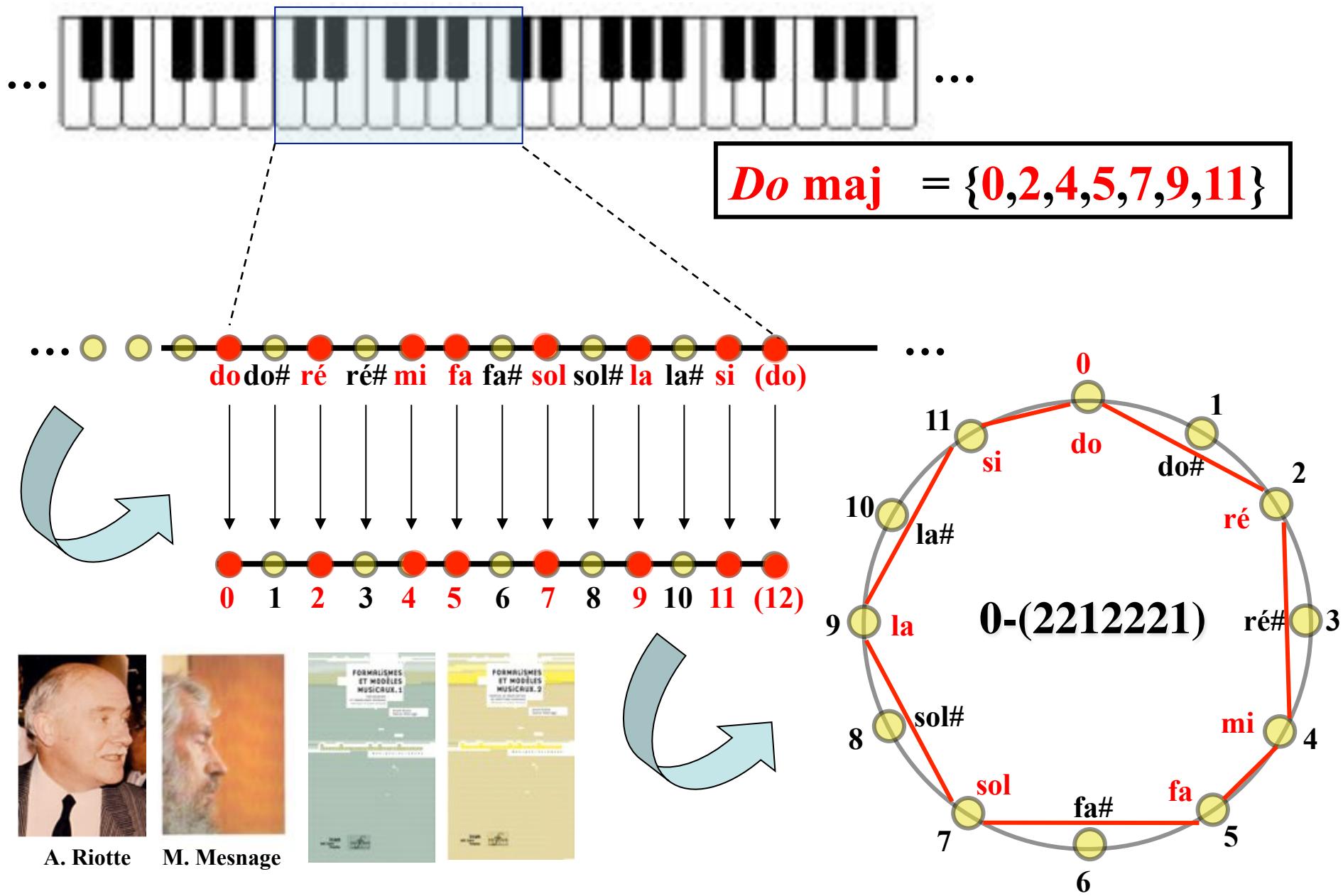
Esthétique et complexité – II
Neurosciences, évolution,
épistemologie, philosophie
Sous la direction de Zeki Kapoula,
Louis-Jean Lestocart et Jean-Paul Allouche



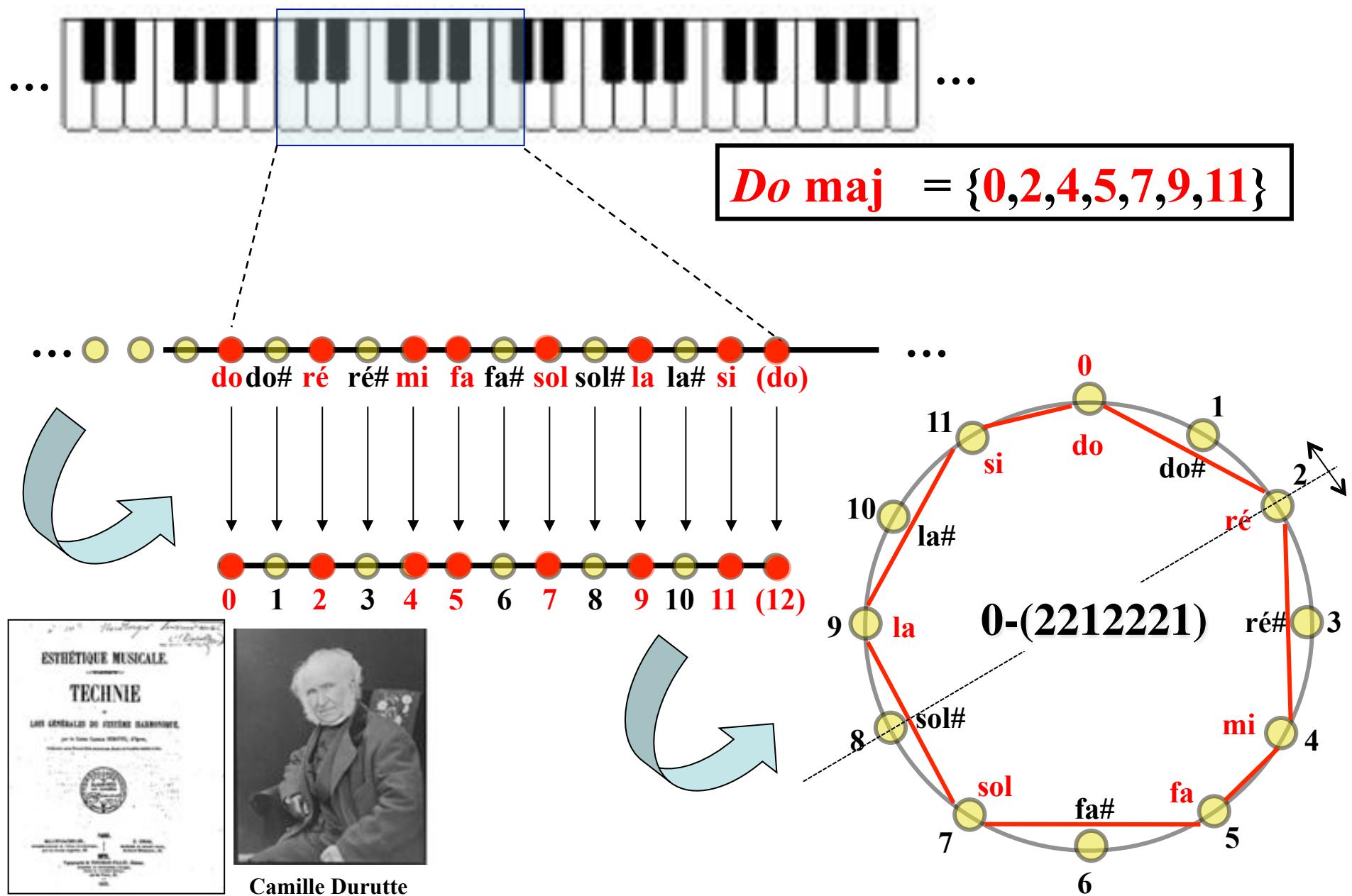
Octave reduction and mod 12 congruence



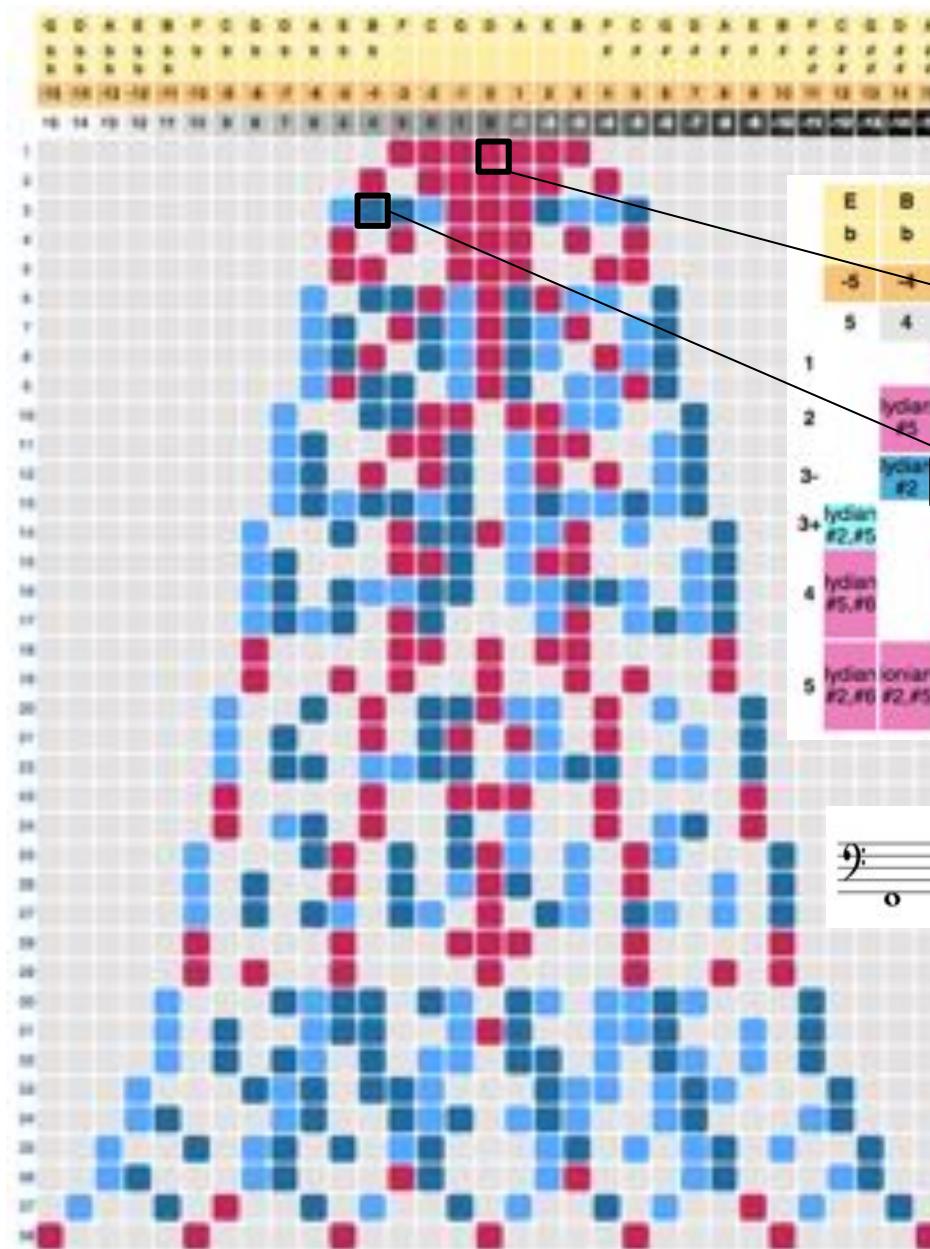
A musical scale as a polygon in a circle



Scales and symmetry



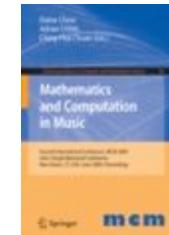
The diatonic bell (P. Audétat & co.)



E	B	F	C	G	D	A	E	B	F	C
b	b							#	#	
-5	-4	-3	-2	-1	0	1	2	3	4	5
5	4	3	2	1	0	-1	-2	-3	-4	-5
1		lydian	ionian	mixolydian	dorian	aeolian	phrygian	locrian		
2		lydian #5	lydian b7	ionian b3	mixolydian b6 or aeolian 3	phrygian b5 locrian 2			locrian b4	
3		lydian #2,5	lydian #2	ionian b6	dorian #4 aeolian 7 phrygian 3 locrian 6			locrian b6,bb7	locrian	minor harmonic
3+		lydian #5, #6	lydian #5,b7	lydian b3	ionian b2,b3 or phrygian 6,7	dorian b5	phrygian b4 locrian bb7	locrian 2,b4	locrian b6,bb7	major harmonic
4		lydian #2,6		lydian b6,b7	ionian b2,b3 or phrygian 6,7	locrian 2,3		locrian b4	locrian bb3,b4	unitonic
5		lydian ionian #2,6 #2,5			aeolian #4,7	ionian b2,b6 or phrygian 3,7	mixolydian b2,b5	phrygian b4,bb7	locrian bb6,bb7	double harmonic



Junod, J., Audétat, P., Agon, C., Andreatta, M.,
« A Generalisation of Diatonicism and the
Discrete Fourier Transform as a Mean for
Classifying and Characterising Musical Scales »,
Second International Conference MCM 2009,
vol. 38, New Haven, 2009, pp. 166-179



The pitch-rhythm *cognitive* isomorphic correspondence

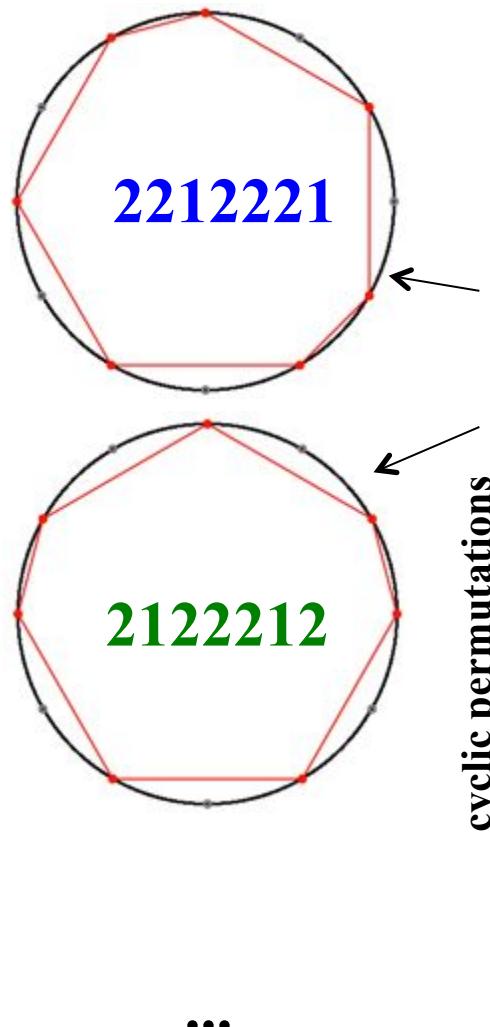


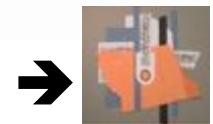
TABLE I
Comparison of M = 7, L = 12 patterns for pitch (scales) and rhythm (time-lines)

pattern	pitch domain name and notation (in C)	rhythm domain notation	examples from West Africa	references
1. 2212221	major scale (Ionian) CDEFGAB	↓↓↑↑↑↑↑	Ewe (Atsiabek, Sogba, Atsia) also Yoruba	Jones (1959), C. K. Ladzekpo, S. K. Ladzekpo and Pantaleoni, Locke
2. 2122212	Dorian CDE↑FGAB↑	↓↑↓↑↓↑↓	Bemba—Northern Rhodesia	Jones (1965). (Ekwueme)
3. 1222122	Phrygian CD↑E↑FGA↑B↑	↑↓↑↓↑↓↑	—	—
4. 2221221	Lydian CDEF#GAB	↓↓↑↑↑↓↑	Ga-Adangme (common) also common Haitian pattern, Akan (Ab fo)	C. K. Ladzekpo, Combs (1974), R. Hill, Asiamah
5. 2212212	Mixolydian CDEFGAB↑	↓↓↑↓↑↓↑	Yoruba sacred music from Ekiti	King
6. 2122122	Aeolian CDE↑FGA↑B↑	↓↑↓↑↓↑↓	Ashanti (Ab fo , Mpre)	Koetting
7. 1221222	Locrian CD↑E↑FG↑A↑B↑	↑↓↑↓↑↓↑	Ghana*	Nketia (1963a)
8. 2121222	(#2 Locrian) CDE↑FG↑A↑B↑	↓↑↓↑↓↑↓	Ashanti (Asedua)	C. K. Ladzekpo
9. 2112123	— CDD#EF#GA	↓↓↑↓↑↓↑	Akan (juvenile song)	Nketia (1963b)

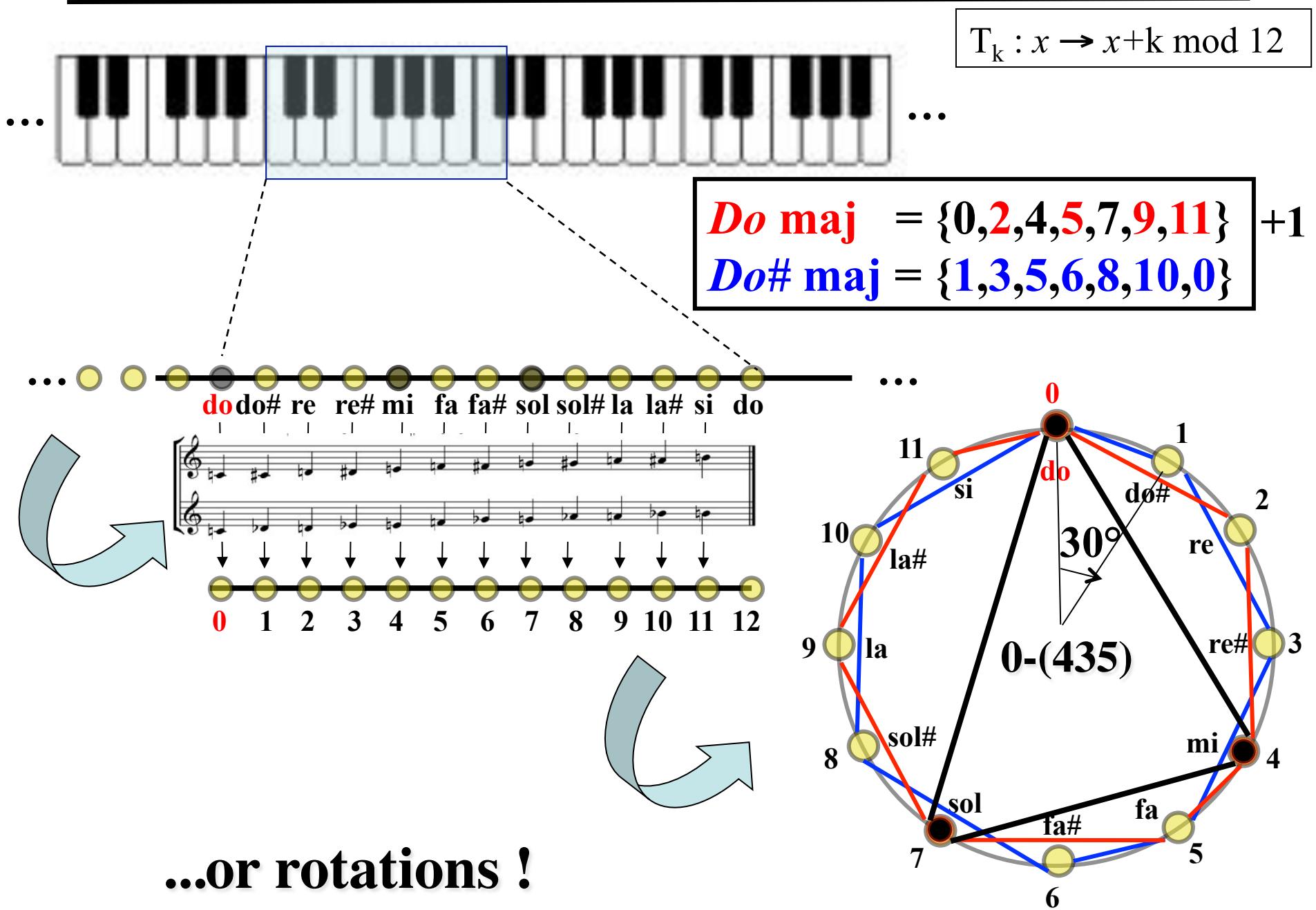
* clap pattern

† mute stroke on bell

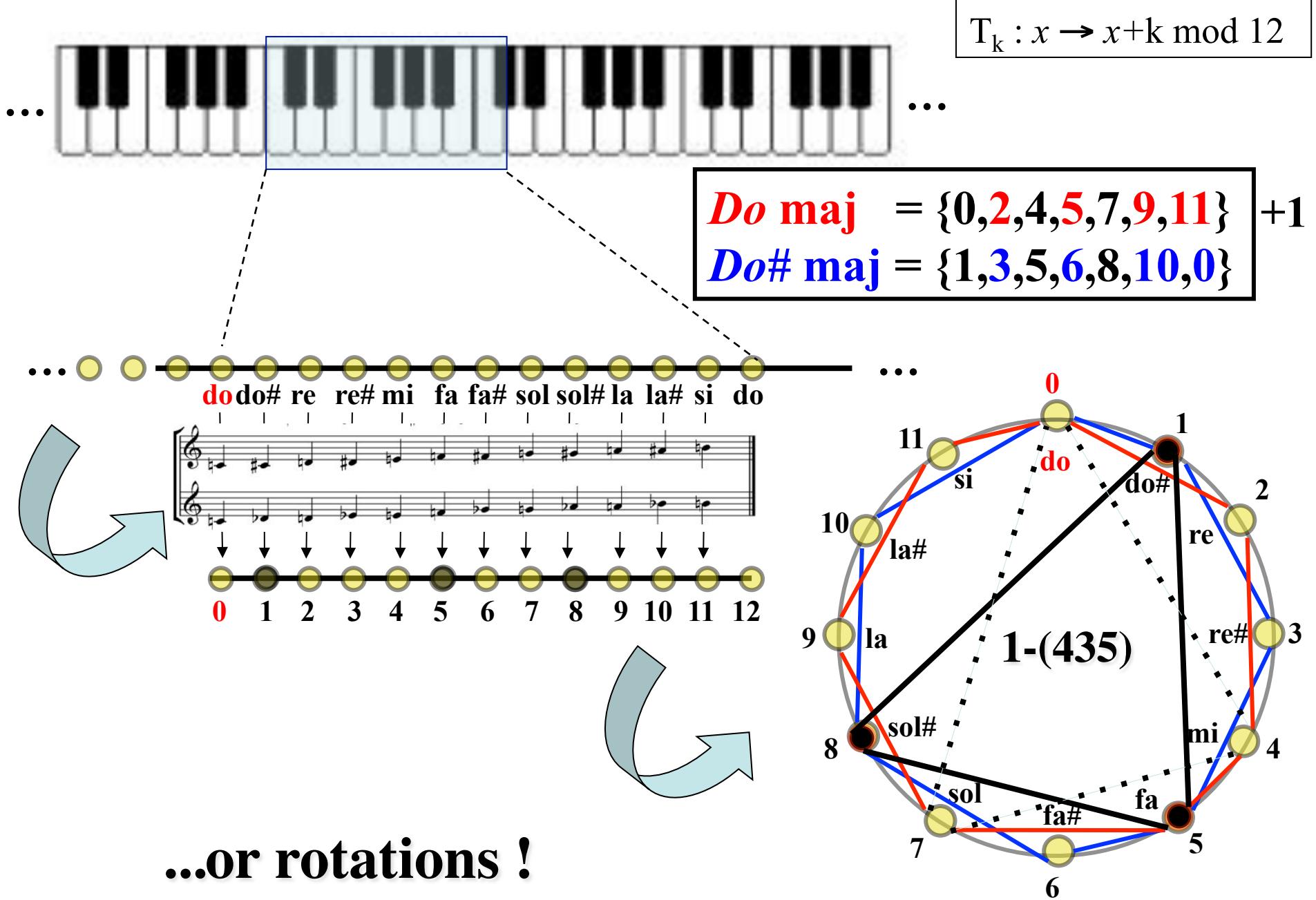
J. Pressing, “Cognitive isomorphisms between pitch and rhythm in world musics: West Africa, the Balkans and Western tonality”, *Studies in Music*, 17, p. 38-61



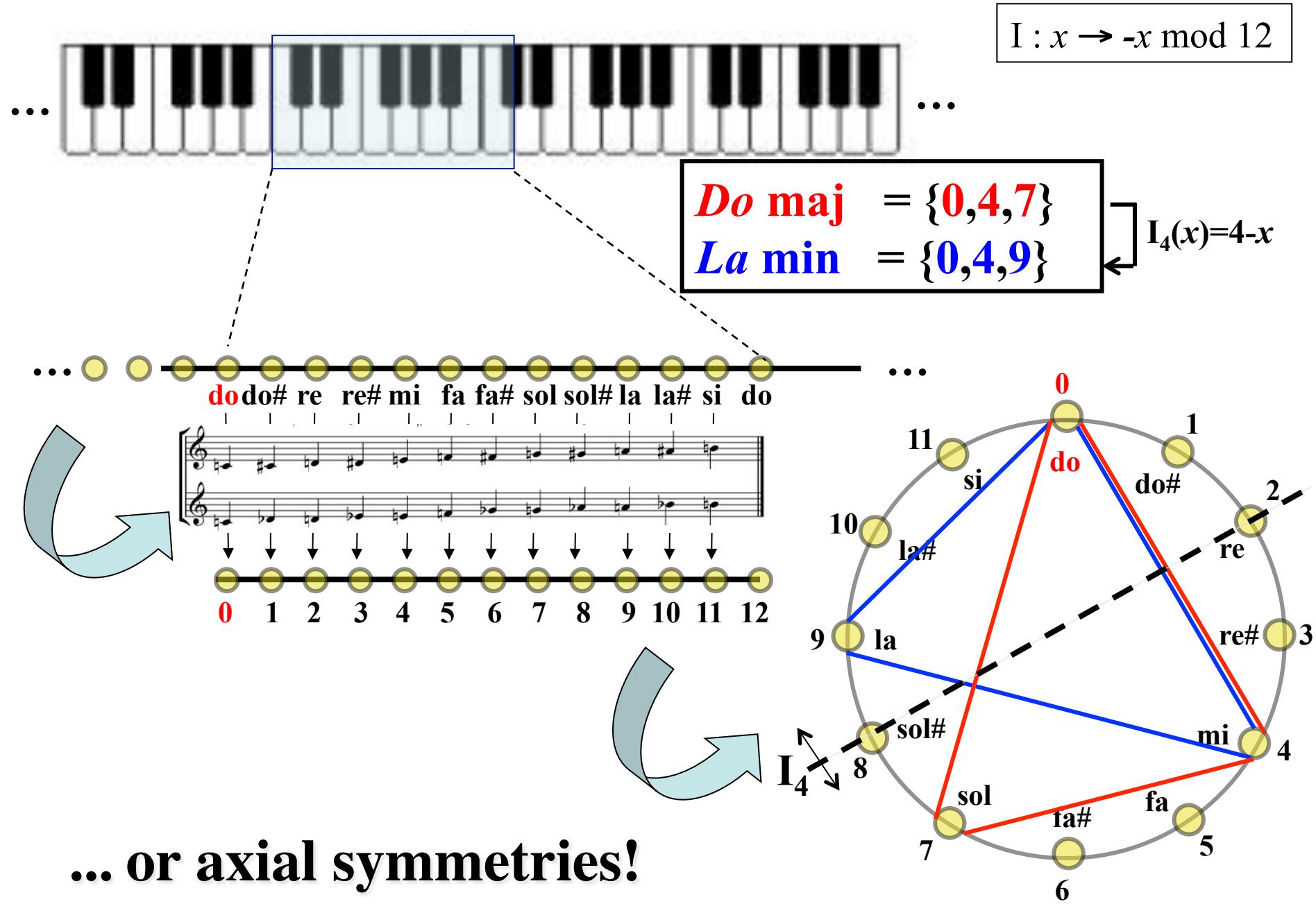
Musical transpositions are additions...



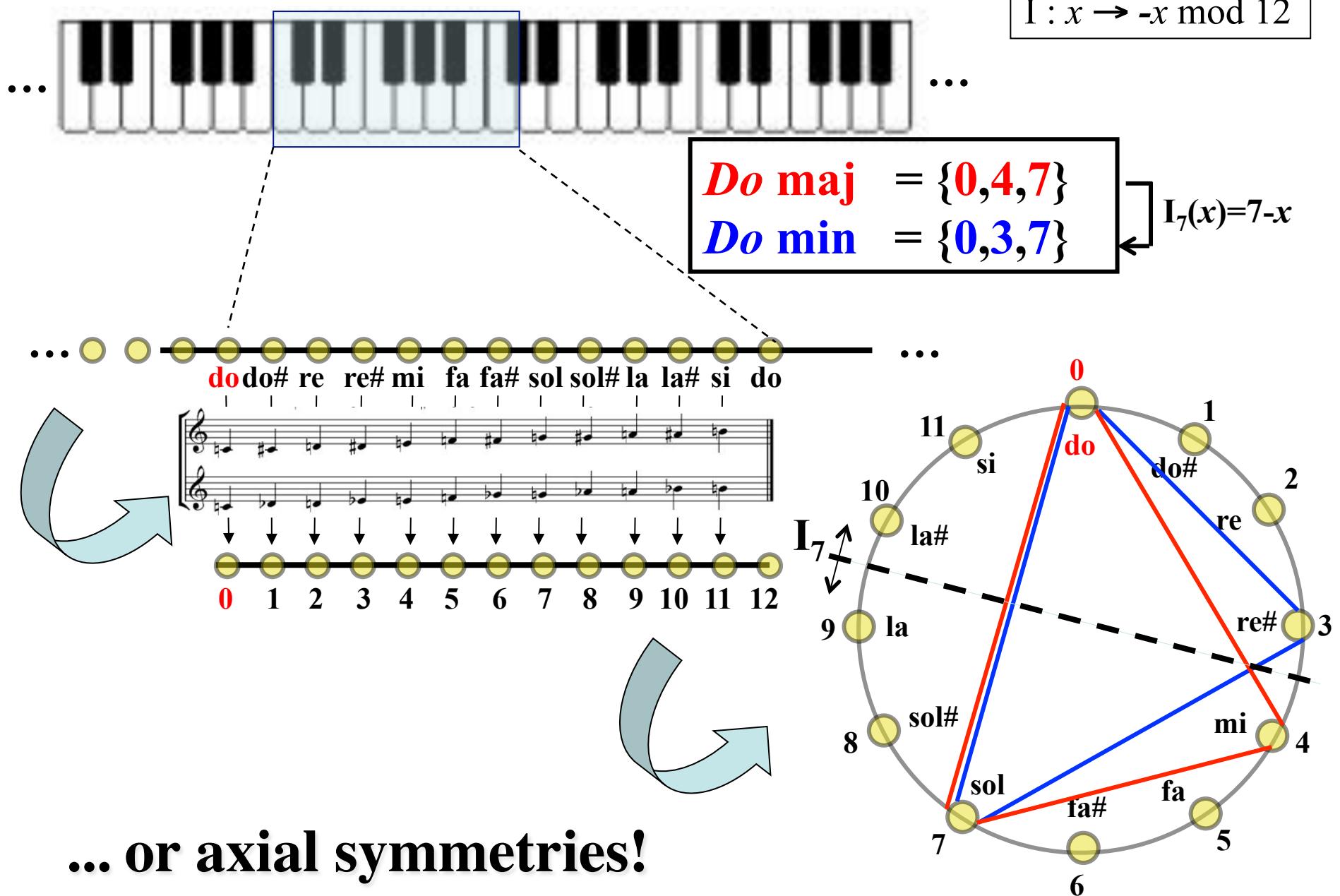
Musical transpositions are additions...



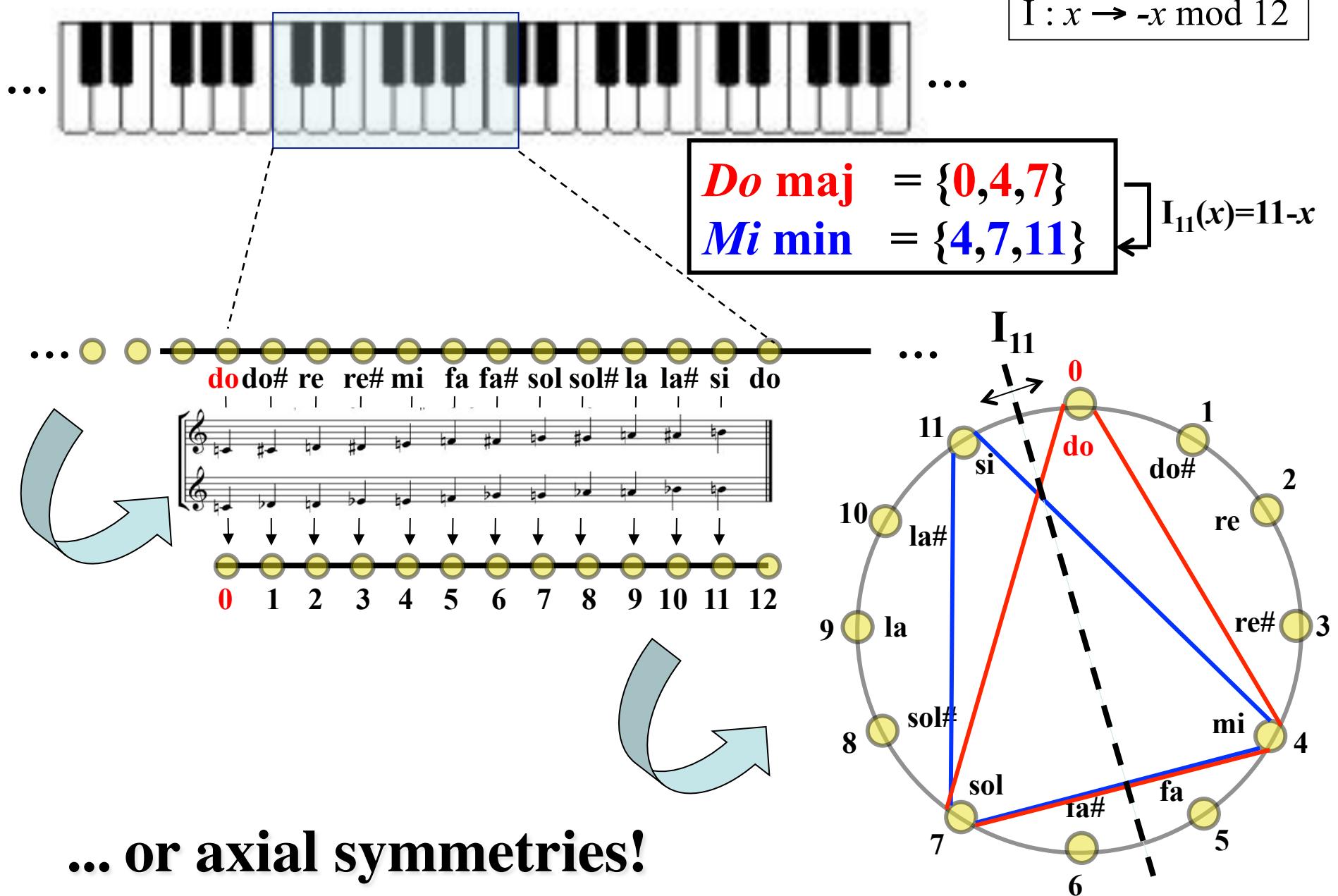
Musical inversions are differences...



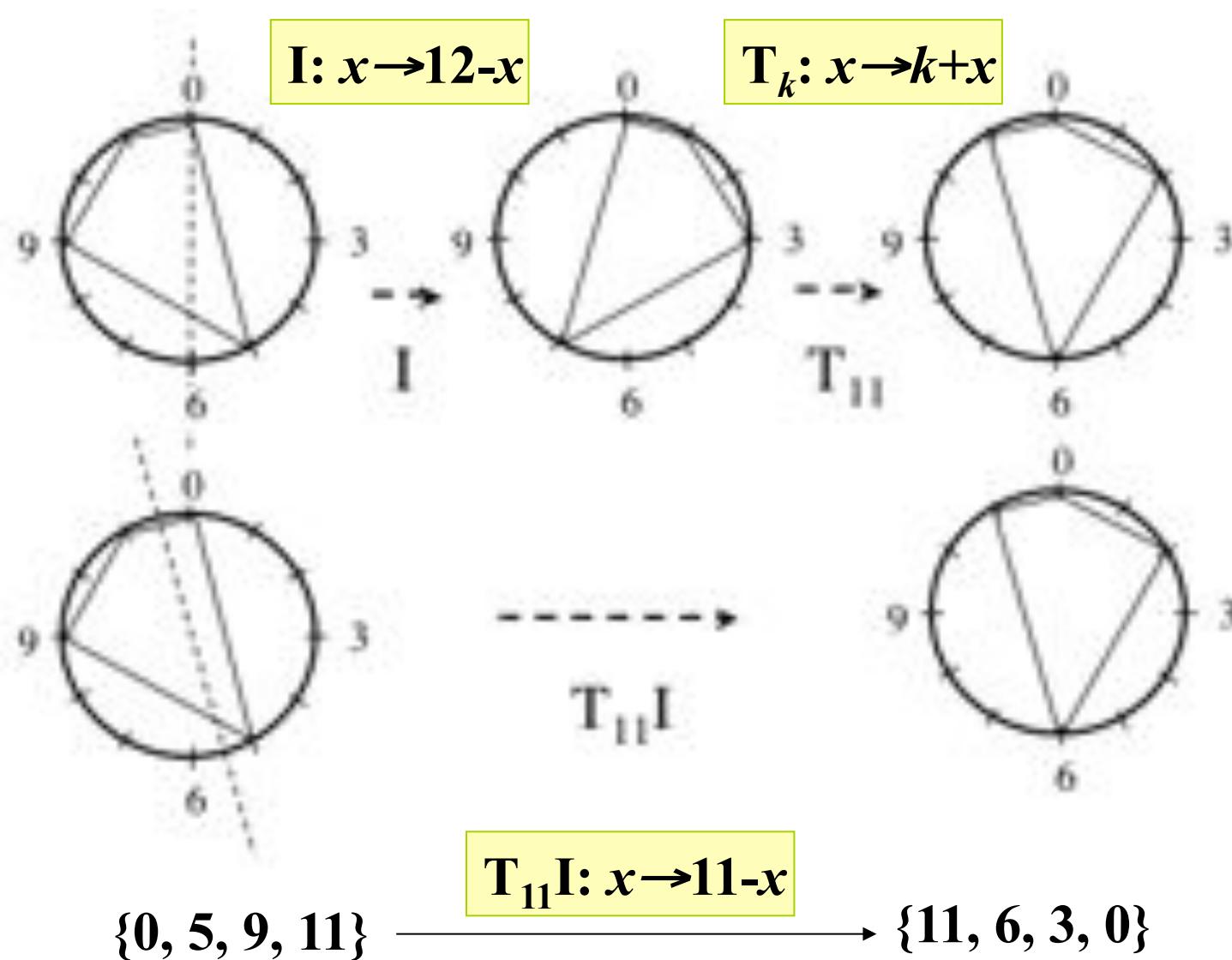
Musical inversions are differences...



Musical inversions are differences...



Composing Transposition and Inversion operators





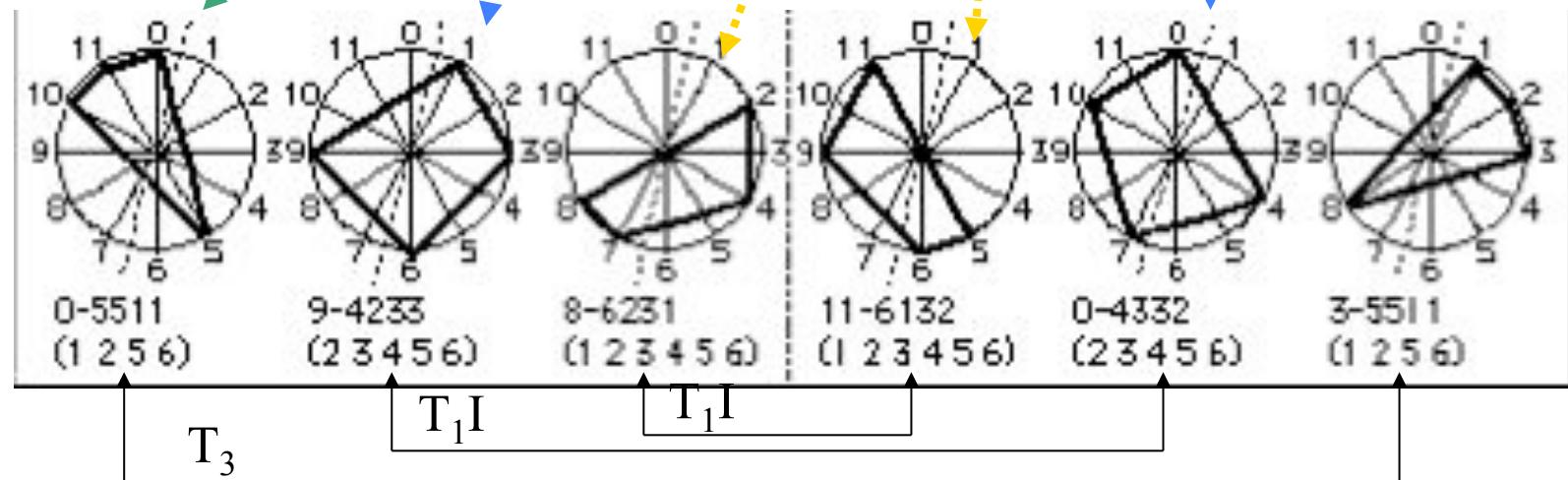
« Entités formelles pour l'analyse musicale »

Marcel Mesnage (1998)

A. Schoenberg : *Klavierstück Op. 33a, 1929*

The musical score shows two staves. Several musical entities are highlighted with colored boxes and arrows pointing to corresponding circle graphs below:

- A blue box highlights a cluster of notes in the upper staff.
- An orange diamond highlights a cluster of notes in the upper staff.
- A green dashed box highlights a cluster of notes in the lower staff.
- A blue box highlights a cluster of notes in the lower staff.
- A yellow diamond highlights a cluster of notes in the upper staff.



Equivalence relation between musical structures

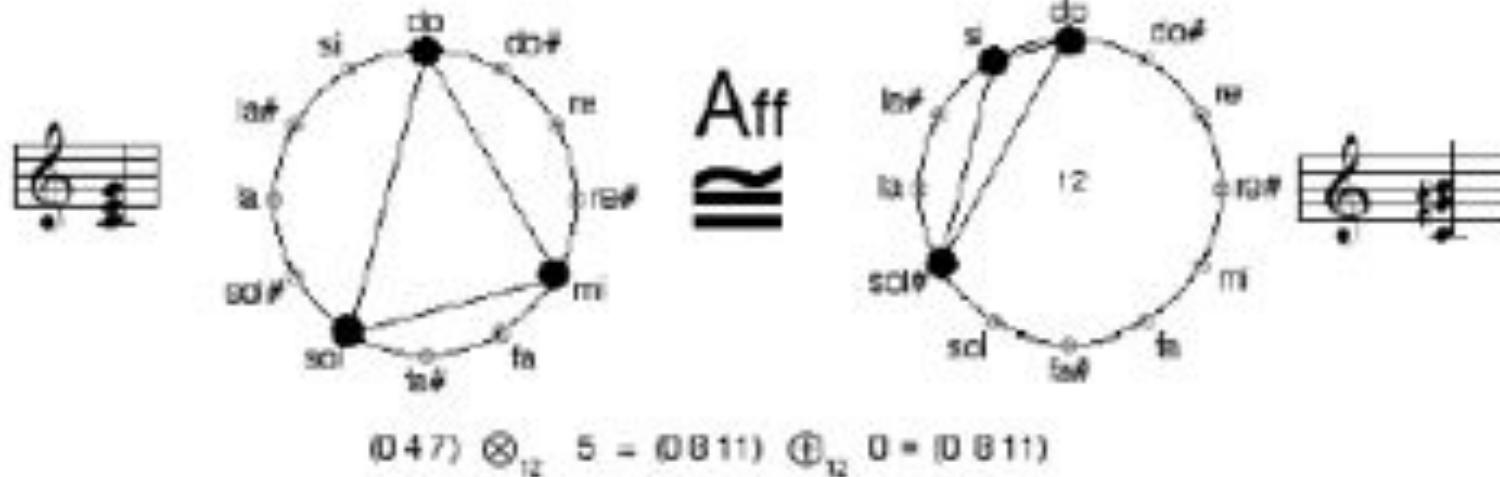


Transposition

$$T_3\{0, 4, 7\} = 3 + \{0, 4, 7\} = \{3, 7, 10\}$$

Transposition and/or inversion

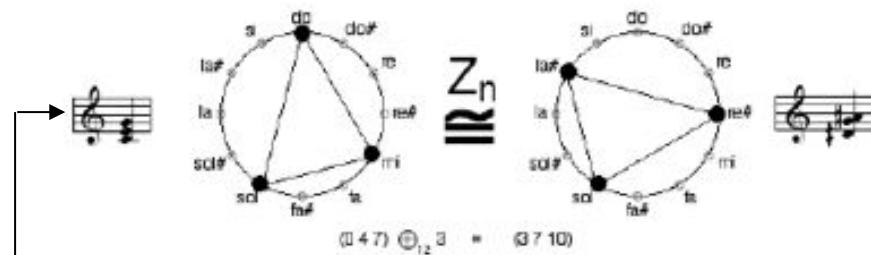
$$T_3I\{0, 4, 7\} = 3 + \{0, -4, -7\} = \{3, 11, 8\}$$



Multiplication (or affine transformation)

$$M_5\{0, 4, 7\} = 5 \times \{0, 4, 7\} = \{0, 8, 11\}$$

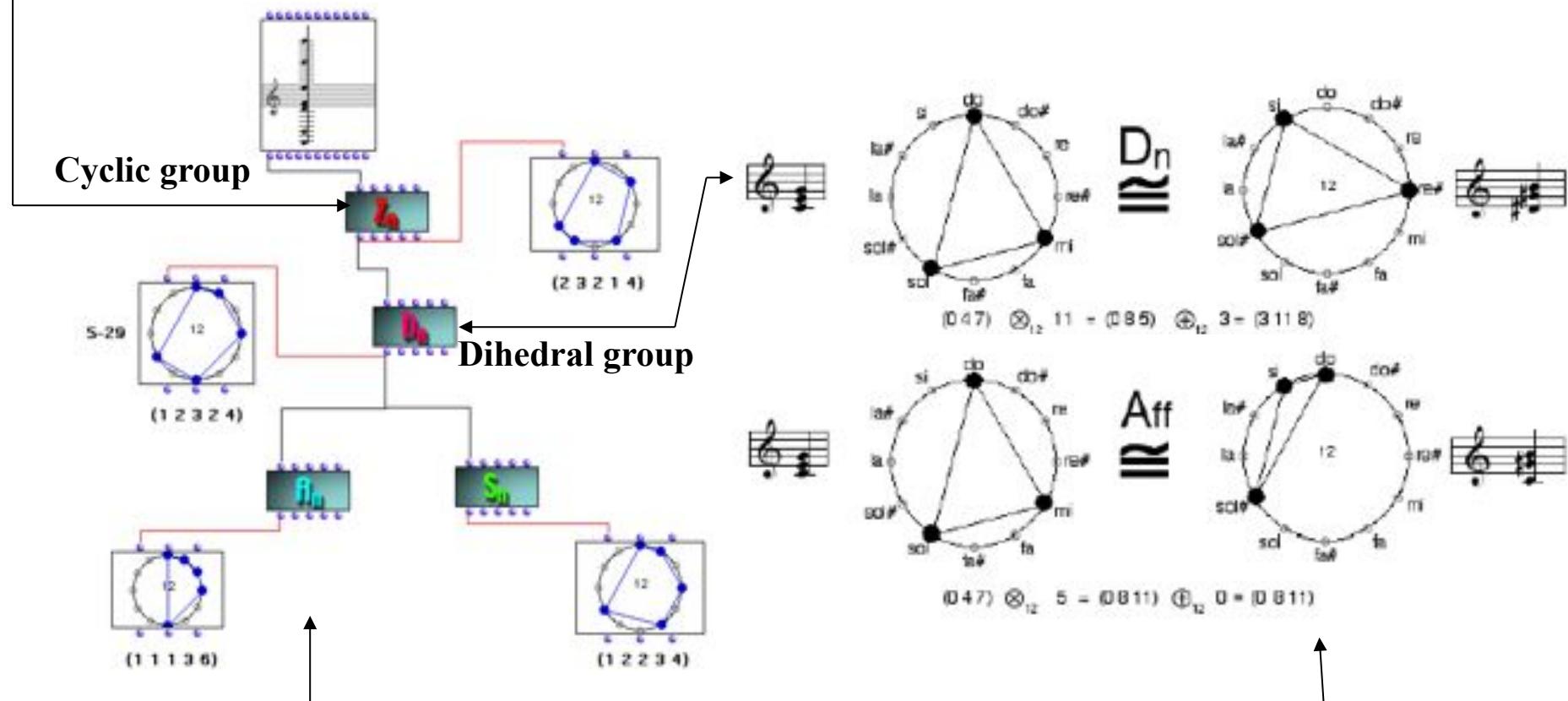
Equivalence classes of chords (up to a group action)



$$Z_{12} = \langle T_k \mid (T_k)^{12} = T_0 \rangle$$

$$D_{12} = \langle T_k, I \mid (T_k)^{12} = I^2 = T_0, ITI = I(IT)^{-1} \rangle$$

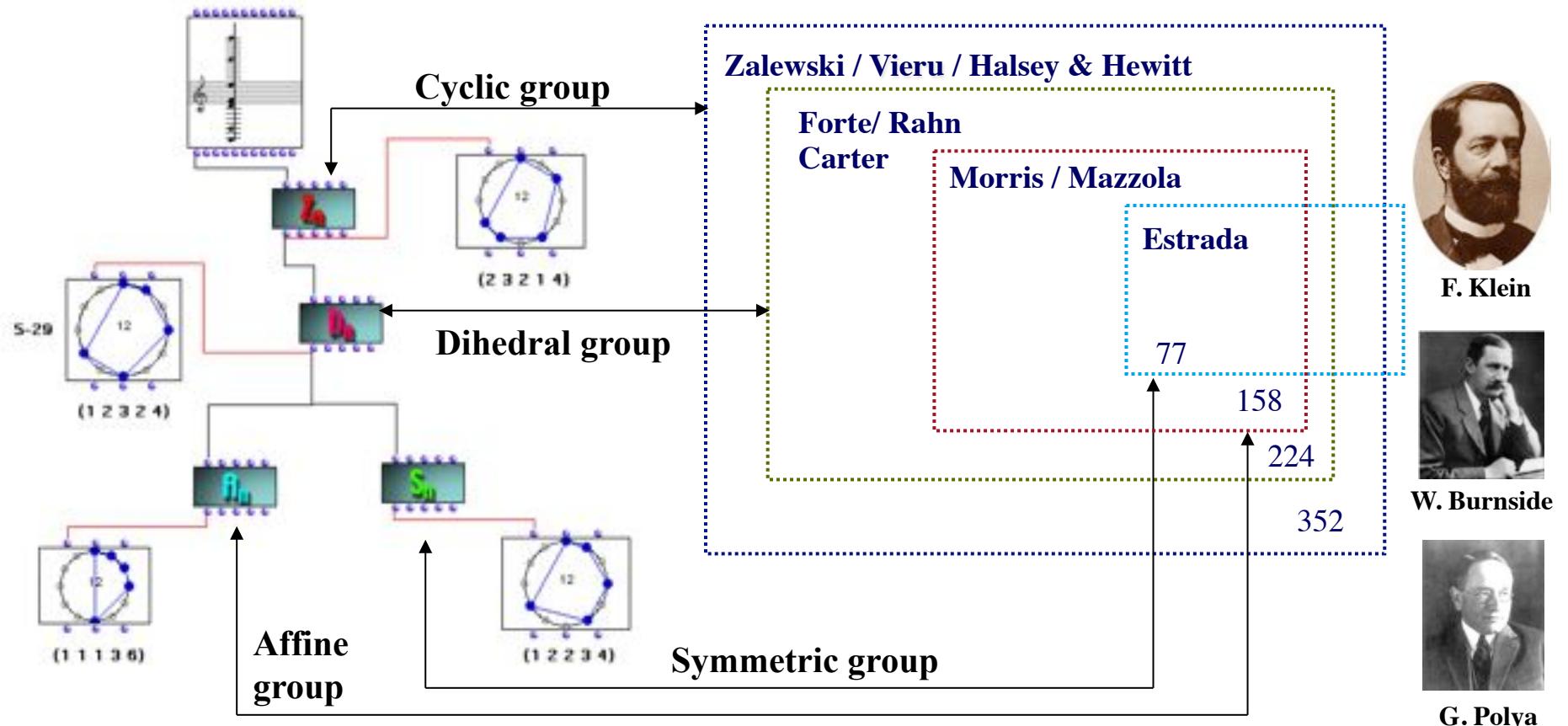
$$\text{Aff} = \{f \mid f(x) = ax + b, a \in (Z_{12})^*, b \in Z_{12}\}$$



Paradigmatic architecture

Affine group

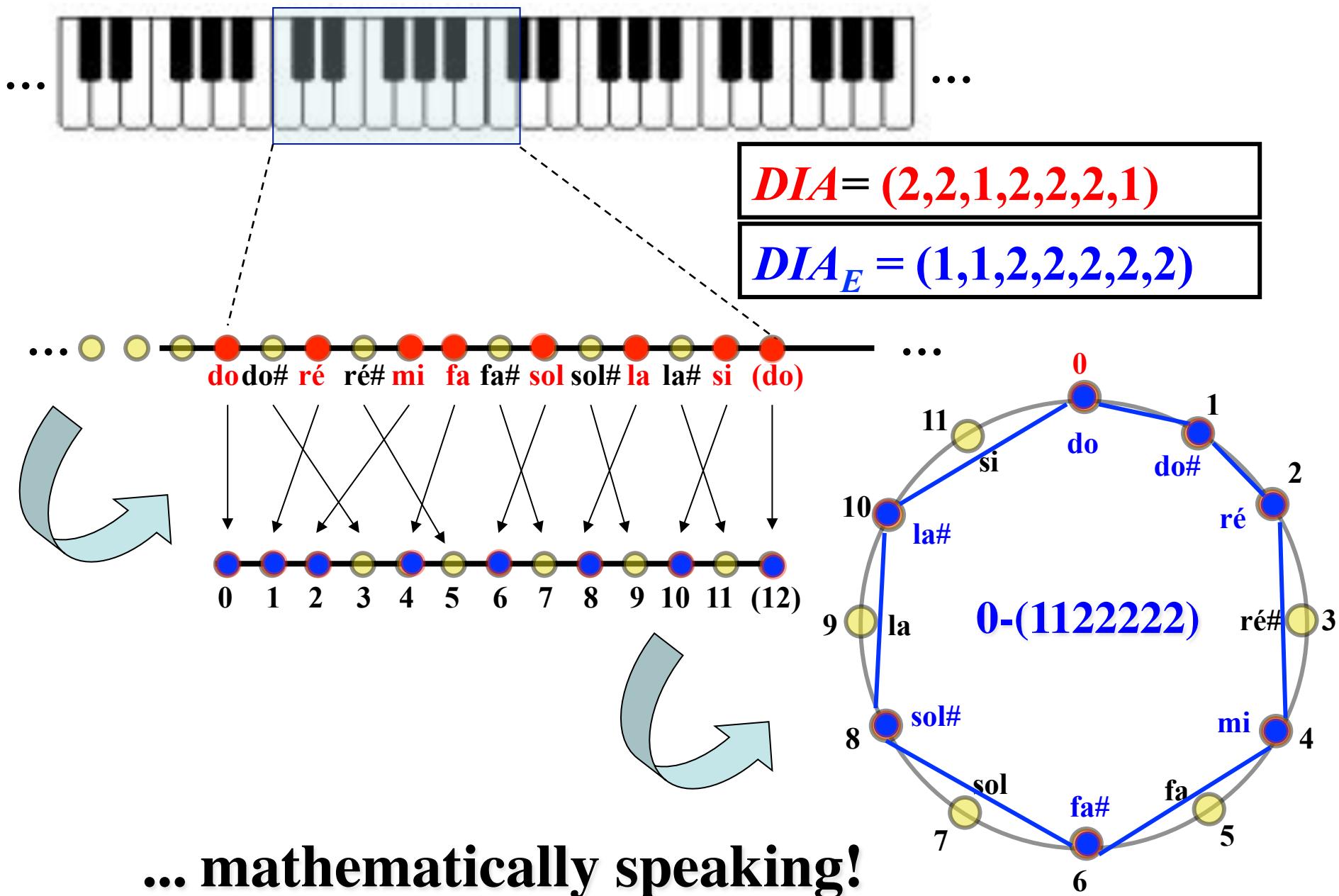
Group actions and the classification of musical structures



	1	2	3	4	5	6	7	8	9	10	11	12
\mathbb{Z}_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1
S_n	1	6	12	15	12	11	7	5	3	2	1	1



Permutations are ‘partitions’...



The permutohedron as a combinatorial space

Julio Estrada, *Théorie de la composition : discontinuum – continuum*, université de Strasbourg II, 1994

ILLUSTRATION III. REPRESENTATION EN NOTATION MUSICALE DE L'ENSEMBLE DE PARTITIONS DE L'ECHIÈLE DE HAUTEURS D12 :
12 NIVEAUX DE DENSITÉ, 77 IDENTITÉS.

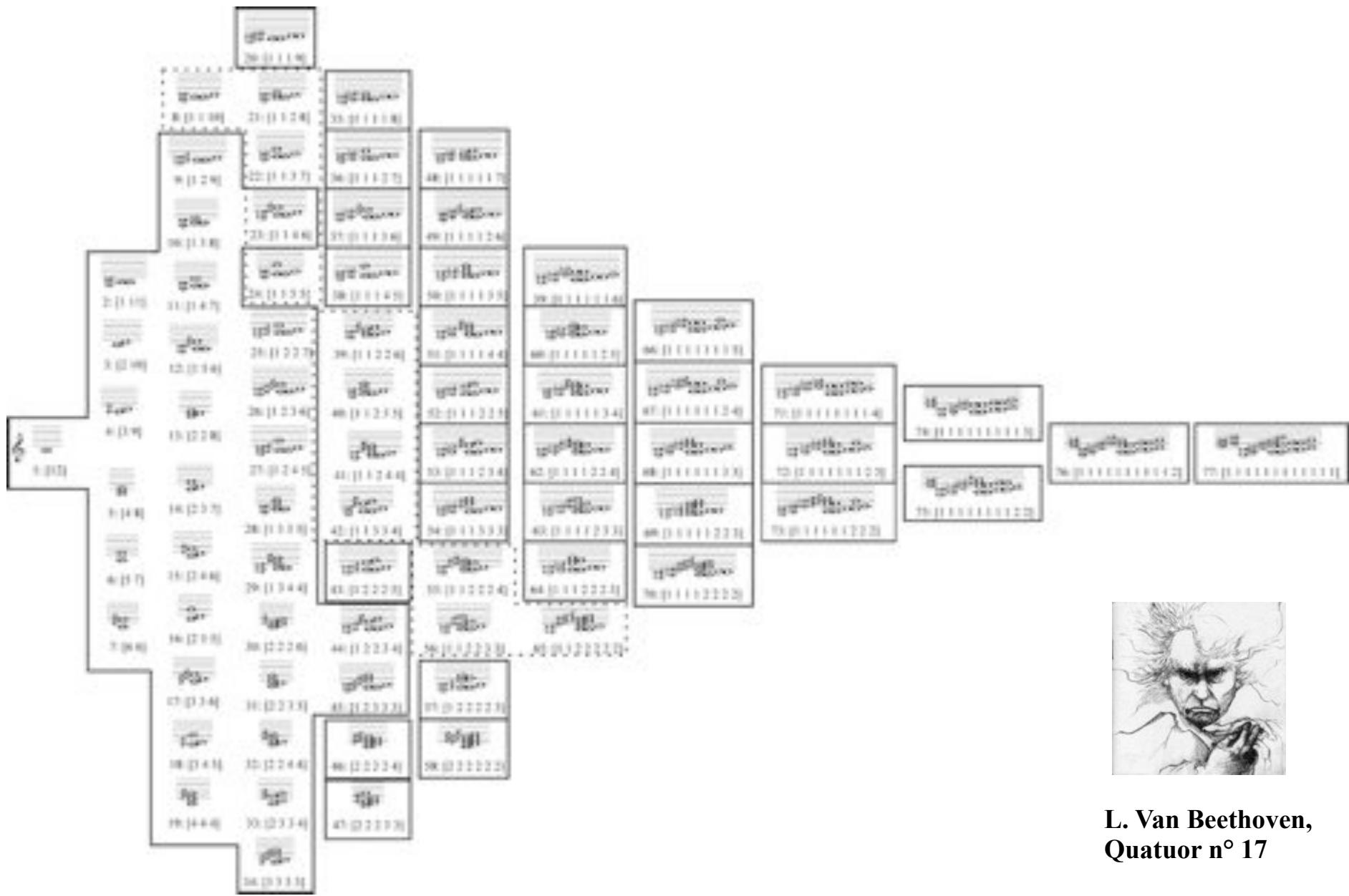
$DIA_E = (1,1,2,2,2,2,2)$

THEORIE DE LA COMPOSITION :
DISCONTINUUM - CONTINUUM
Julio ESTRADA
—
Thèse en Musicologie
Nouveau Doctorat
Département de Musicologie
Université de Strasbourg II
Sciences Humaines
Directeur de thèse : François-Bernard Mâche
Membres du Jury :
M. François-Bernard Mâche, Ecole d'Hautes Etudes
Mme. Marta Grabsz, Université de Strasbourg
M. Hartmut Möller, Université de Potsdam
Rapporteur de thèse :
Mme. Evelyne Andreani, Université de Paris VIII
M. Daniel Charles, Université de Nice



J. Estrada

The permutohedron as a musical conceptual space



The permutohedron as a musical conceptual space

B. Bartok, Quartet n° 4 (3^d movement)



A. Schoenberg, *Six pieces* op. 19



Towards an OpenMusic computational model

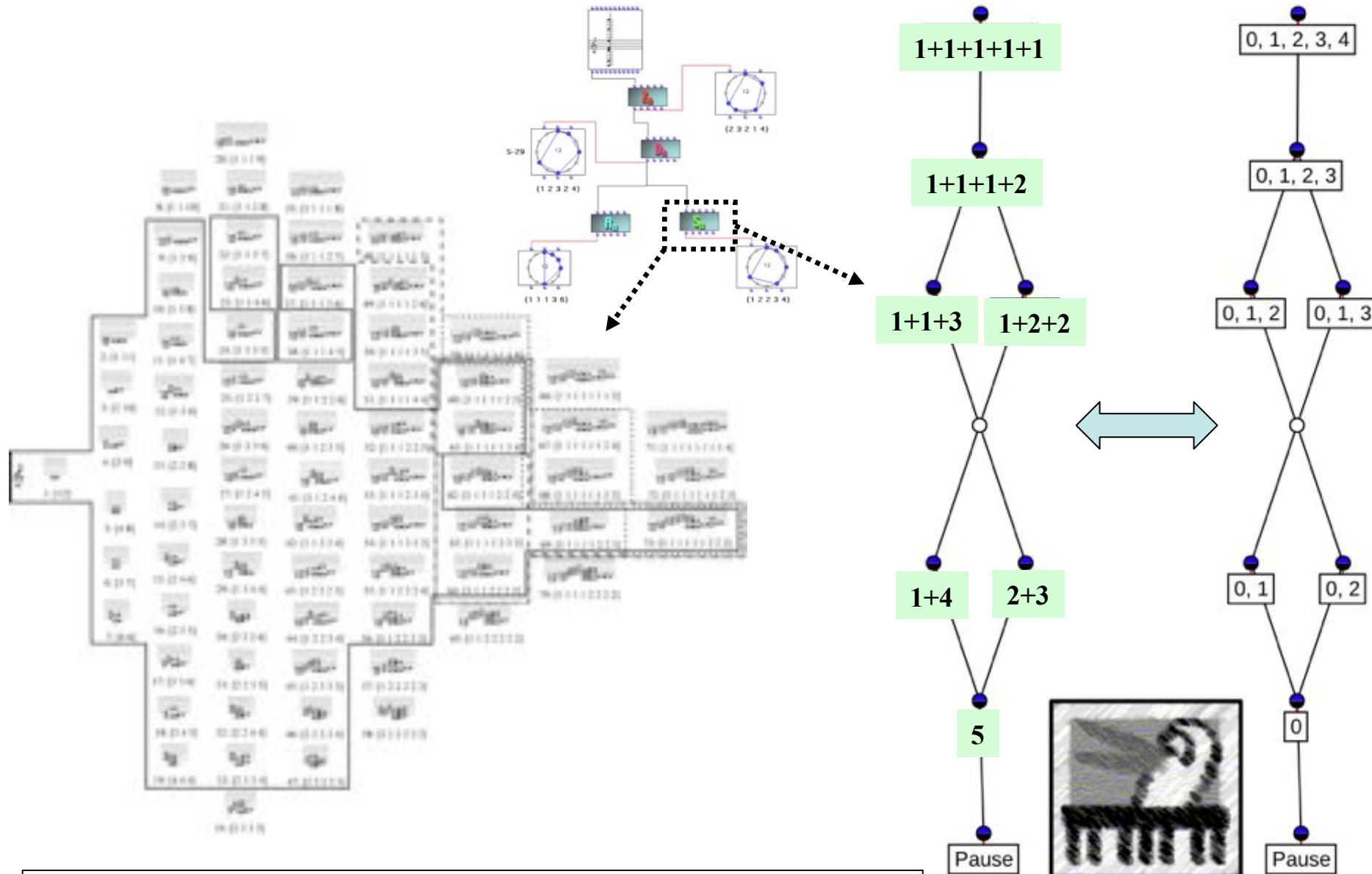
The figure shows a screenshot of the MuSIIC-Win software. At the top, there's a menu bar with options like File, Edit, View, Tools, Help, and About. Below the menu is a toolbar with various icons. The main window features a musical score at the top left with a staff and notes. To the right of the score is a large grid of numbered boxes. The grid consists of 10 rows and 8 columns. The numbers in the boxes range from 1 to 99. Some boxes are empty. The numbers are arranged as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99
--	---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

Below the grid, there's a photo of four people smiling. The software title 'MuSIIC-Win' is displayed prominently at the top right, along with 'Theory d1' and 'Music: Interactive System for Research and Composition'. The version information 'Version 3.2 (Windows®)' is also visible.

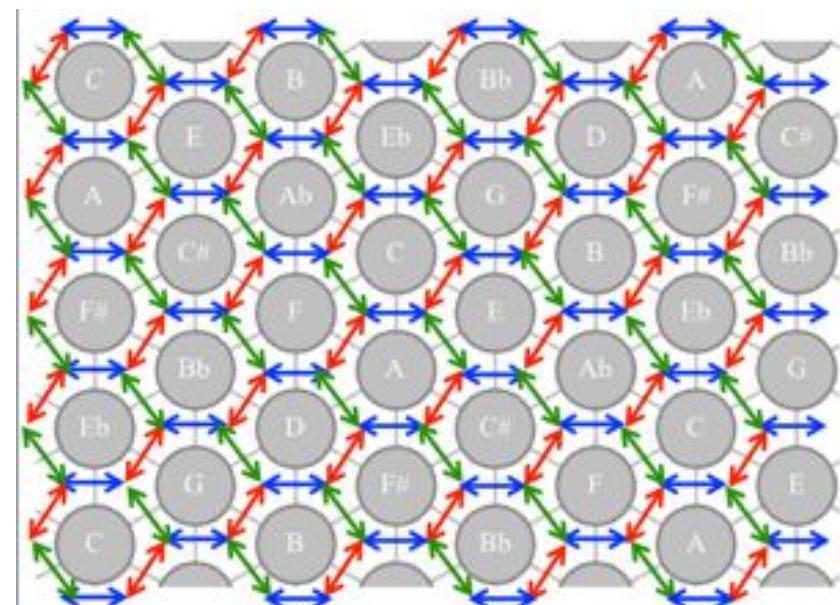
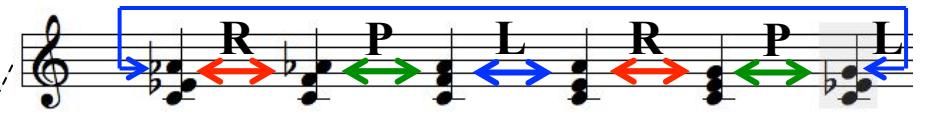
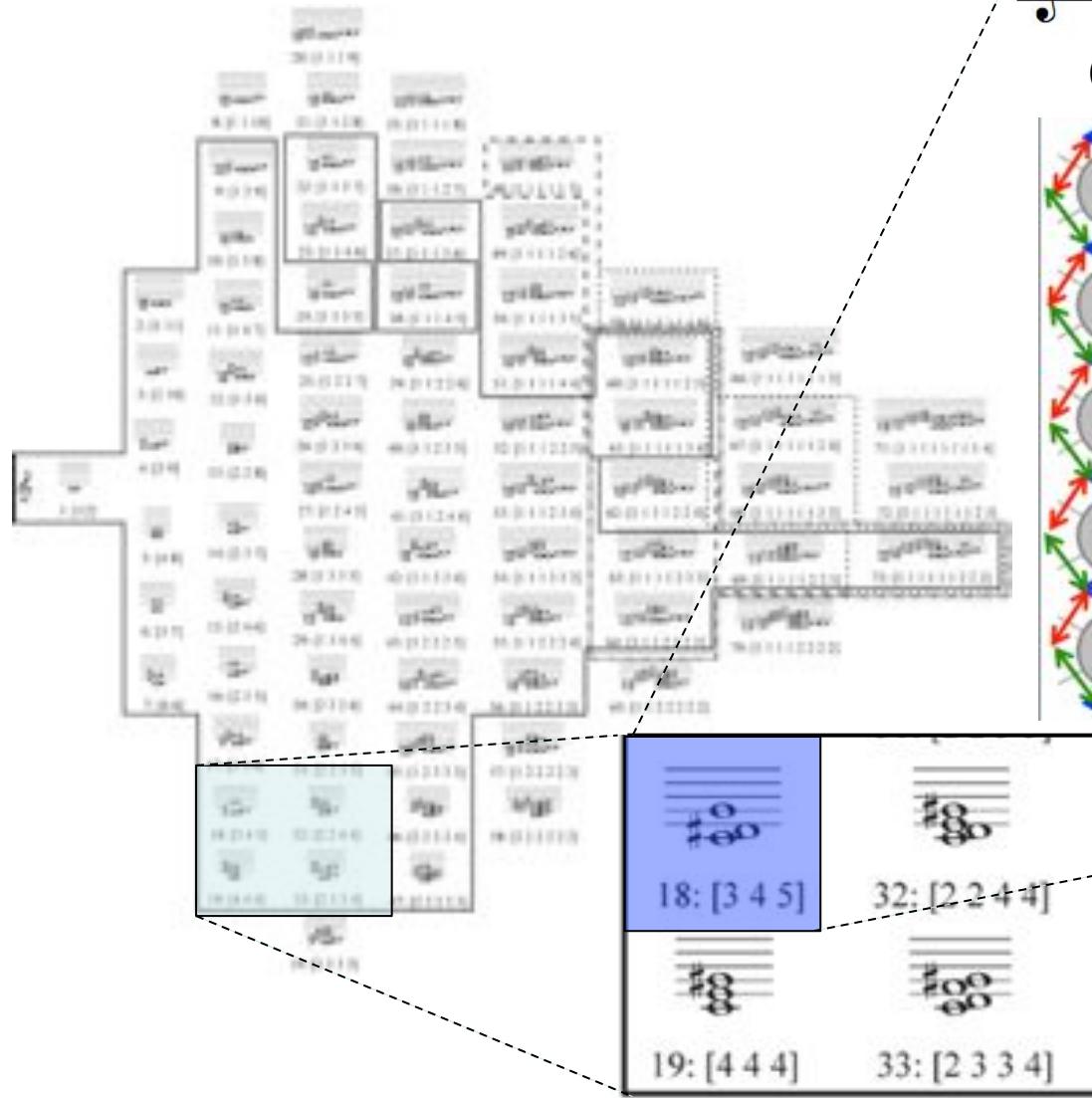


The permutohedron as a lattice of formal concepts

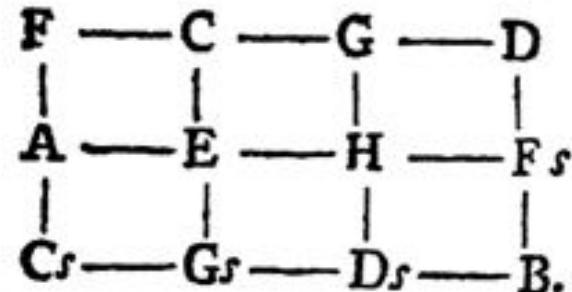
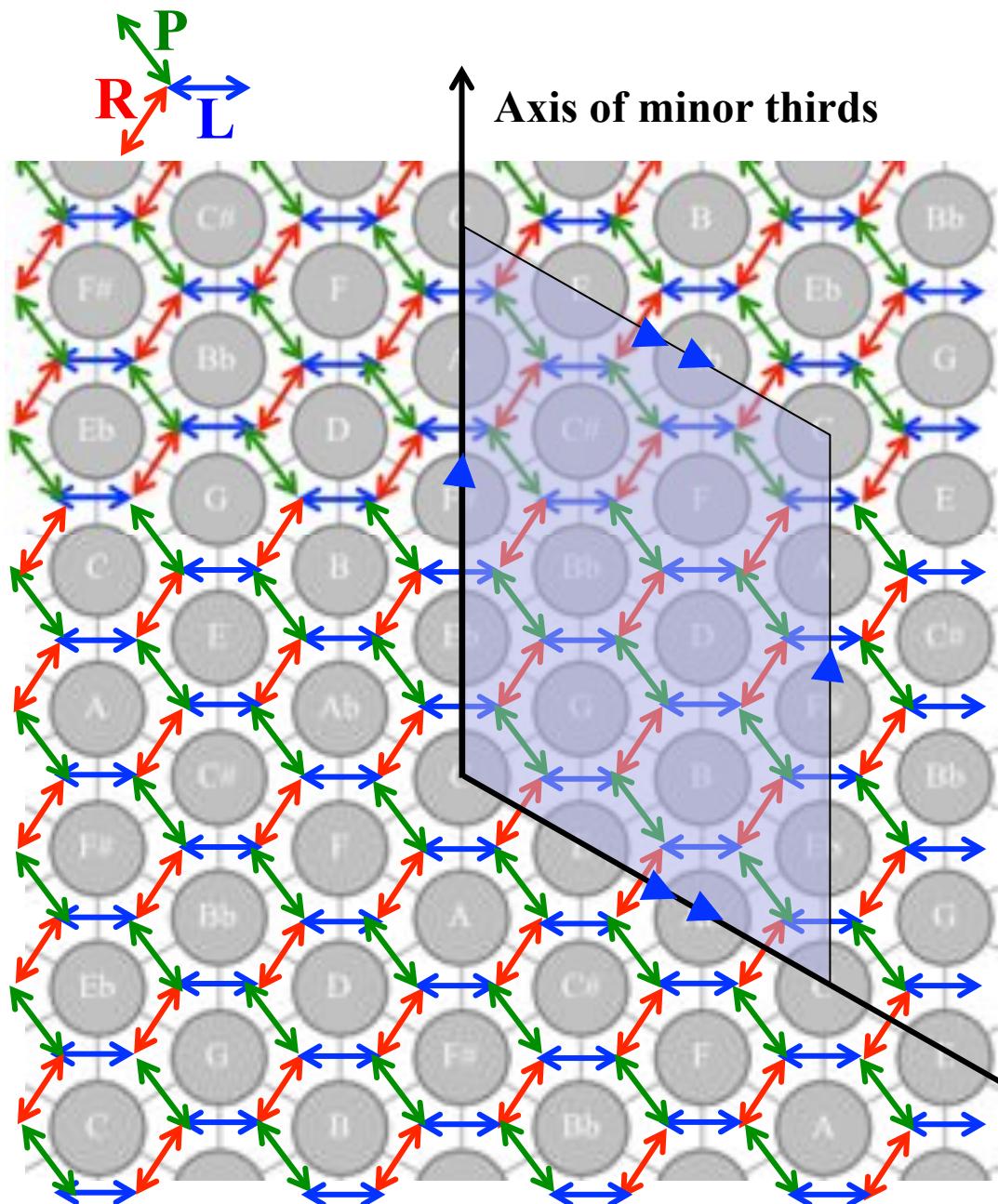


- T. Schlemmer, M. Andreatta, « Using Formal Concept Analysis to represent Chroma Systems », MCM 2013, McGill Univ., Springer, LNCS.

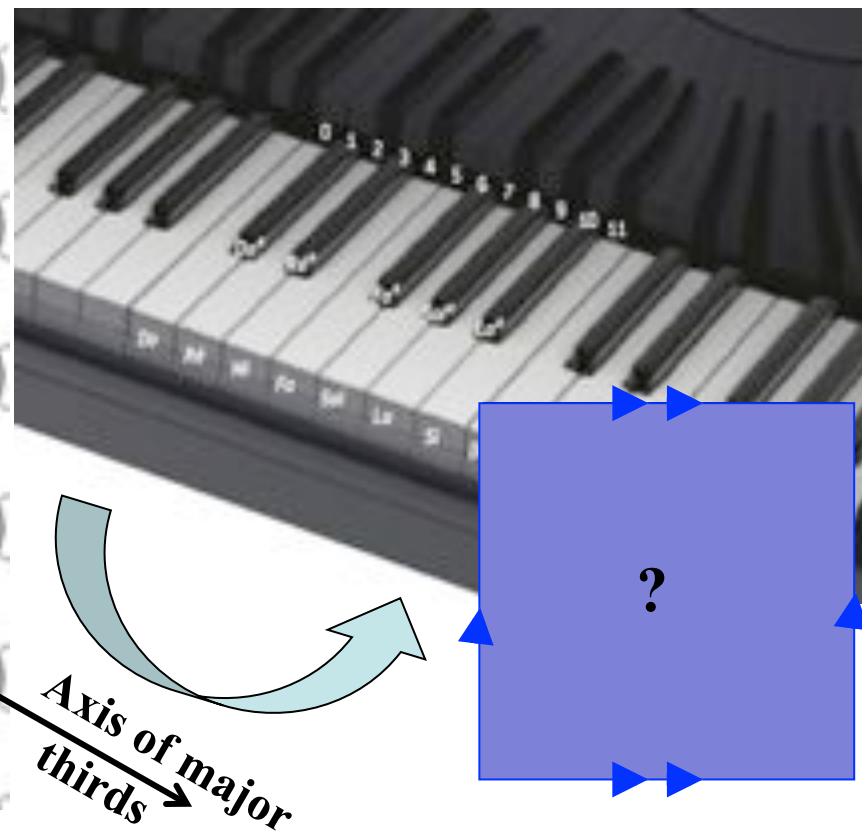
Permutohedron and Tonnetz: a structural inclusion



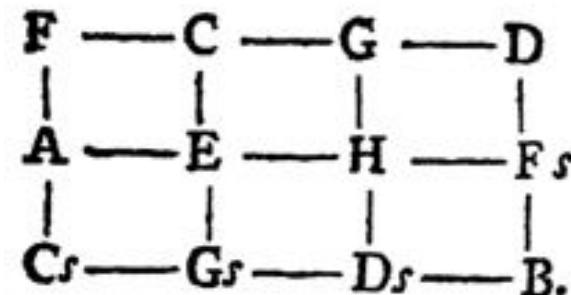
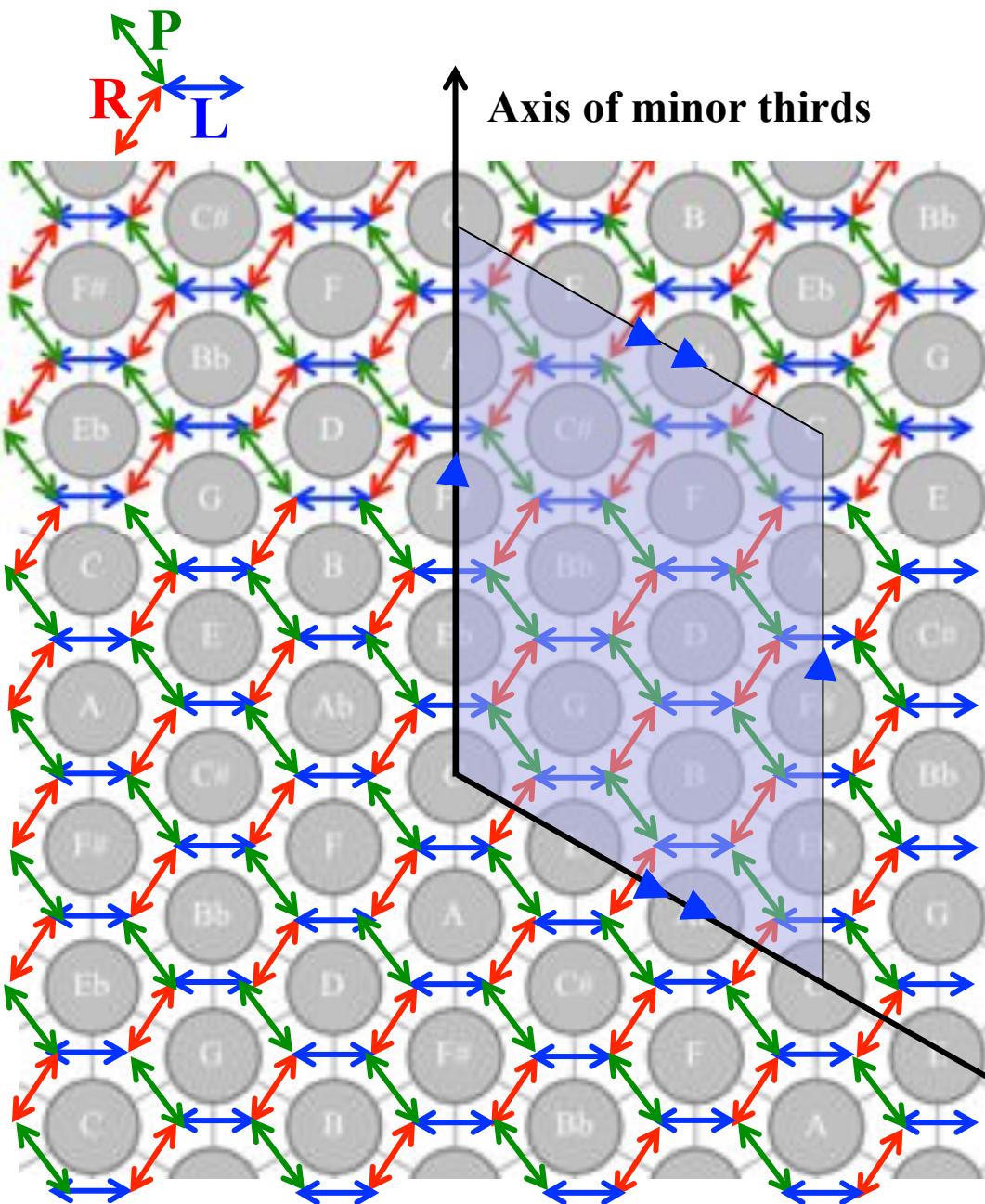
The Tonnetz as a topological structure



Speculum Musicum (Euler, 1773)



The *Tonnetz* as a topological structure: a torus

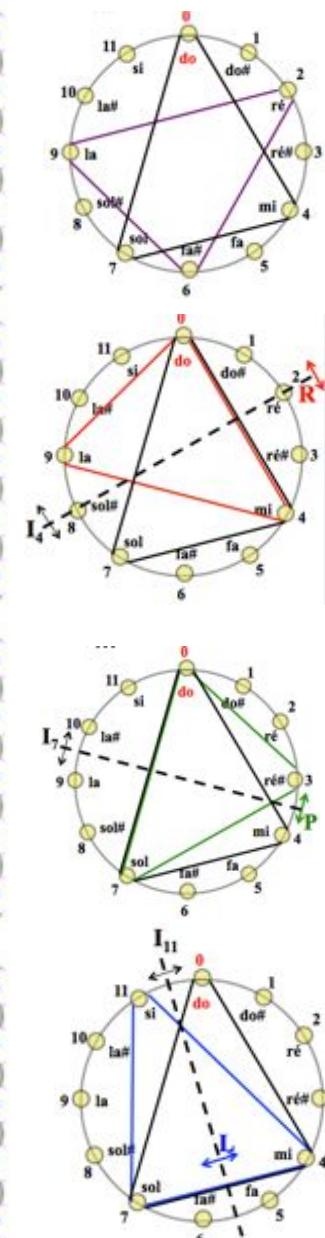
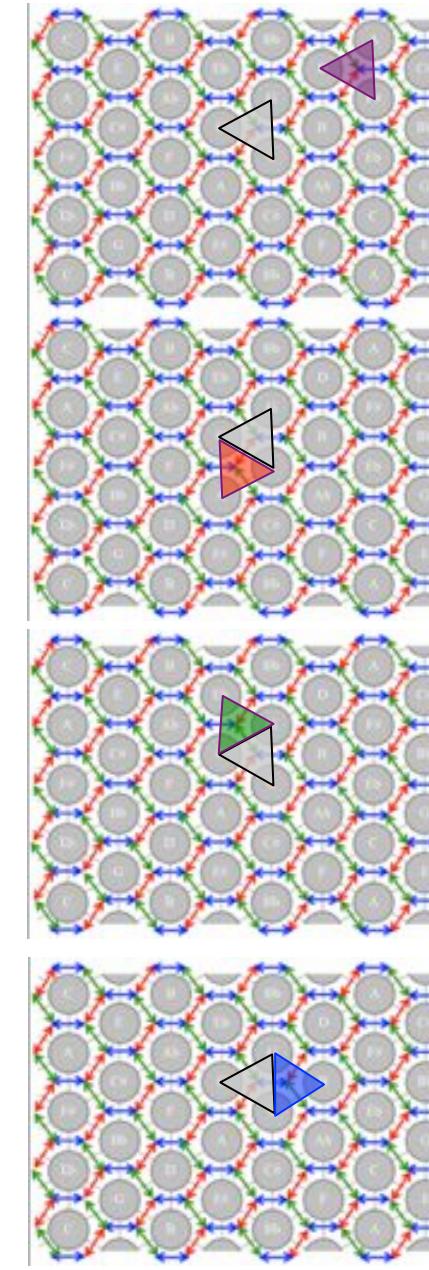
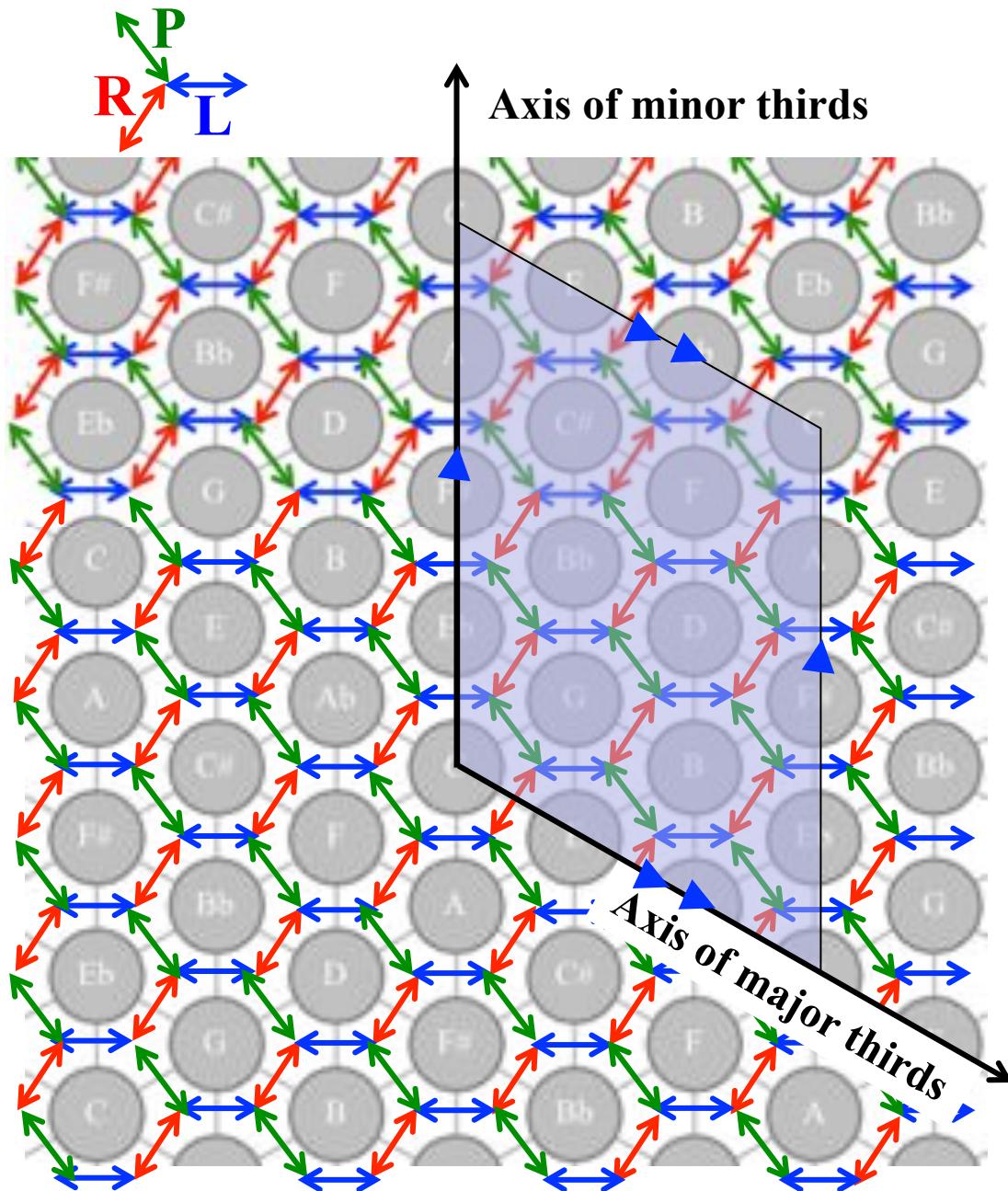


Speculum Musicum (Euler, 1773)



Axis of major thirds

The Tonnetz and its symmetries



Symmetries in Frank Zappa's music

Fa la_m La_b Sol Ré fa#_m Fa Mi Si la#_m Ré Ré_b La_b do_m Si Si_b

<http://www.mathemusic.net>

« Easy Meat » - 1981 (Frank Zappa)
min. 1'44" – 2'39"

Symmetries in Paolo Conte's *Madeleine*

Lab → Réb/Fa → Sib⁷ → Mib⁷/Réb



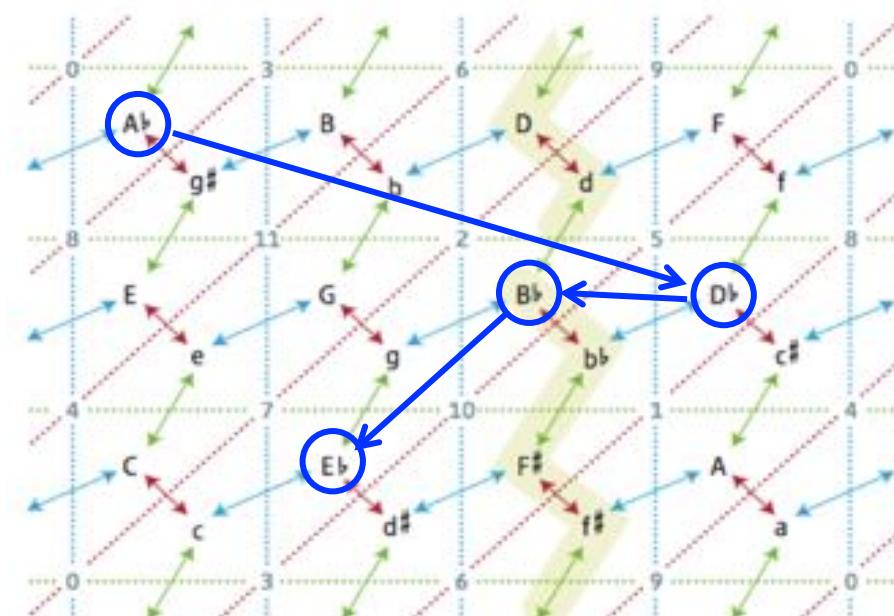
S. La Via, *Poesia per musica e musica per poesia.*
Dai trovatori a Paolo Conte, Carocci, 2006

→ Lab → Réb/Fa → Sib⁷ → Mib⁷/Réb

→ Si/Ré# → Mi → Do# → Fa#

→ Ré/La → Sol → Mi⁷ → La⁷

→ Ré → Lab⁷ → Réb → Do⁷ → Mib



Symmetries in Paolo Conte's *Madeleine*

Lab → Réb/Fa → Sib⁷ → Mib⁷/Réb



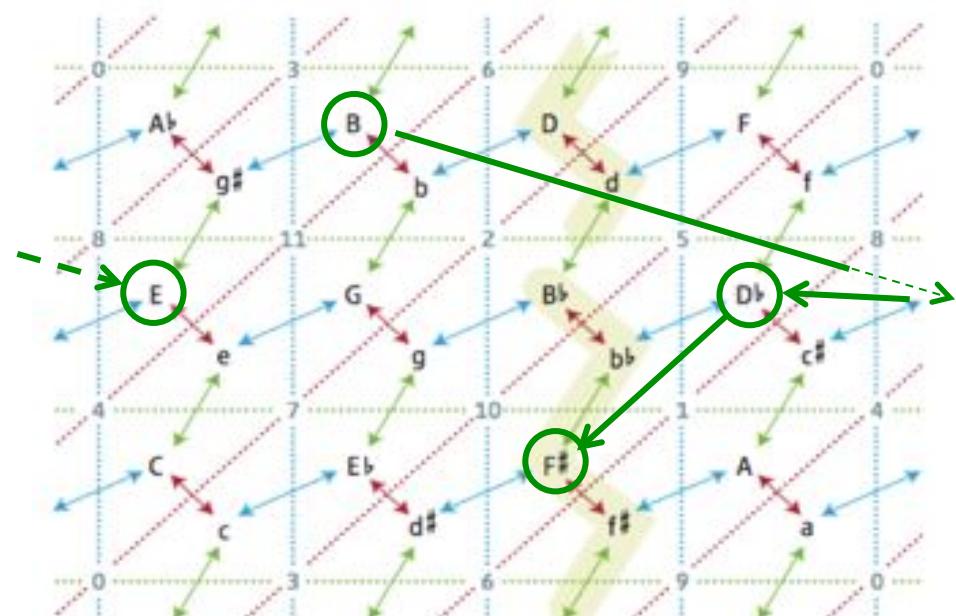
S. La Via, *Poesia per musica e musica per poesia.*
Dai trovatori a Paolo Conte, Carocci, 2006

→ Lab → Réb/Fa → Sib⁷ → Mib⁷/Réb

→ Si/Ré# → Mi → Do# → Fa#

→ Ré/La → Sol → Mi⁷ → La⁷

→ Ré → Lab⁷ → Réb → Do⁷ → Mib



Symmetries in Paolo Conte's *Madeleine*

Lab → Réb/Fa → Sib⁷ → Mib⁷/Réb



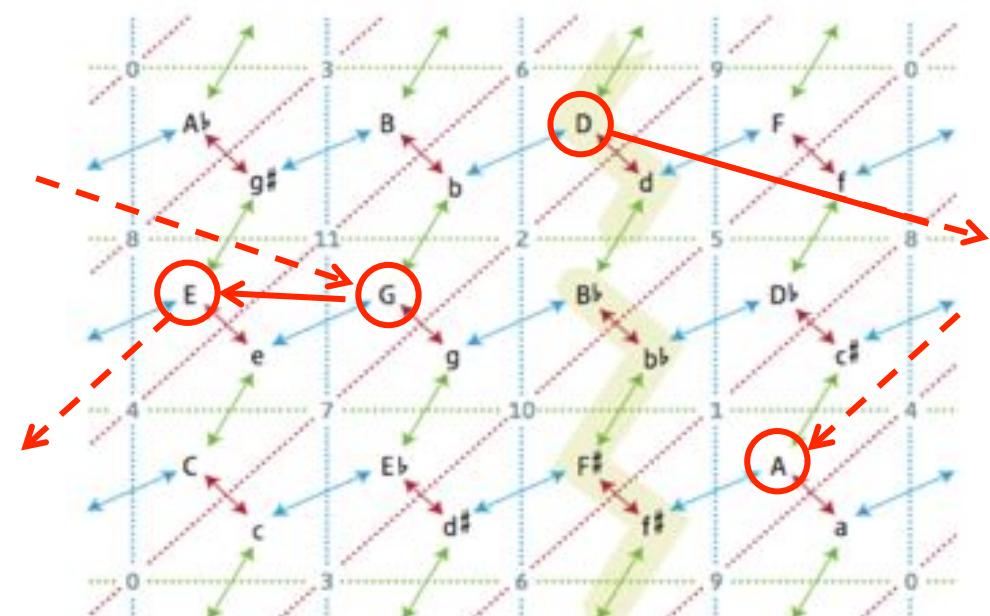
S. La Via, *Poesia per musica e musica per poesia.*
Dai trovatori a Paolo Conte, Carocci, 2006

→ Lab → Réb/Fa → Sib⁷ → Mib⁷/Réb

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→ Ré/La → Sol → Mi⁷ → La⁷

→ Ré → Lab⁷ → Réb → Do⁷ → Mib



Symmetries in Paolo Conte's *Madeleine*

Lab → Réb/Fa → Sib⁷ → Mib⁷/Réb



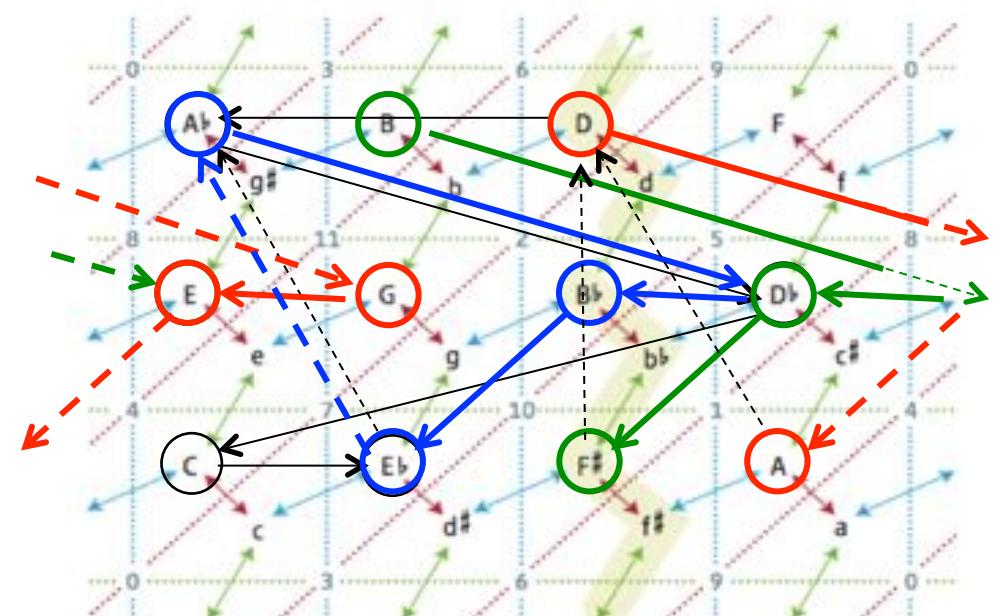
S. La Via, *Poesia per musica e musica per poesia.*
Dai trovatori a Paolo Conte, Carocci, 2006

→ Lab → Réb/Fa → Sib⁷ → Mib⁷/Réb

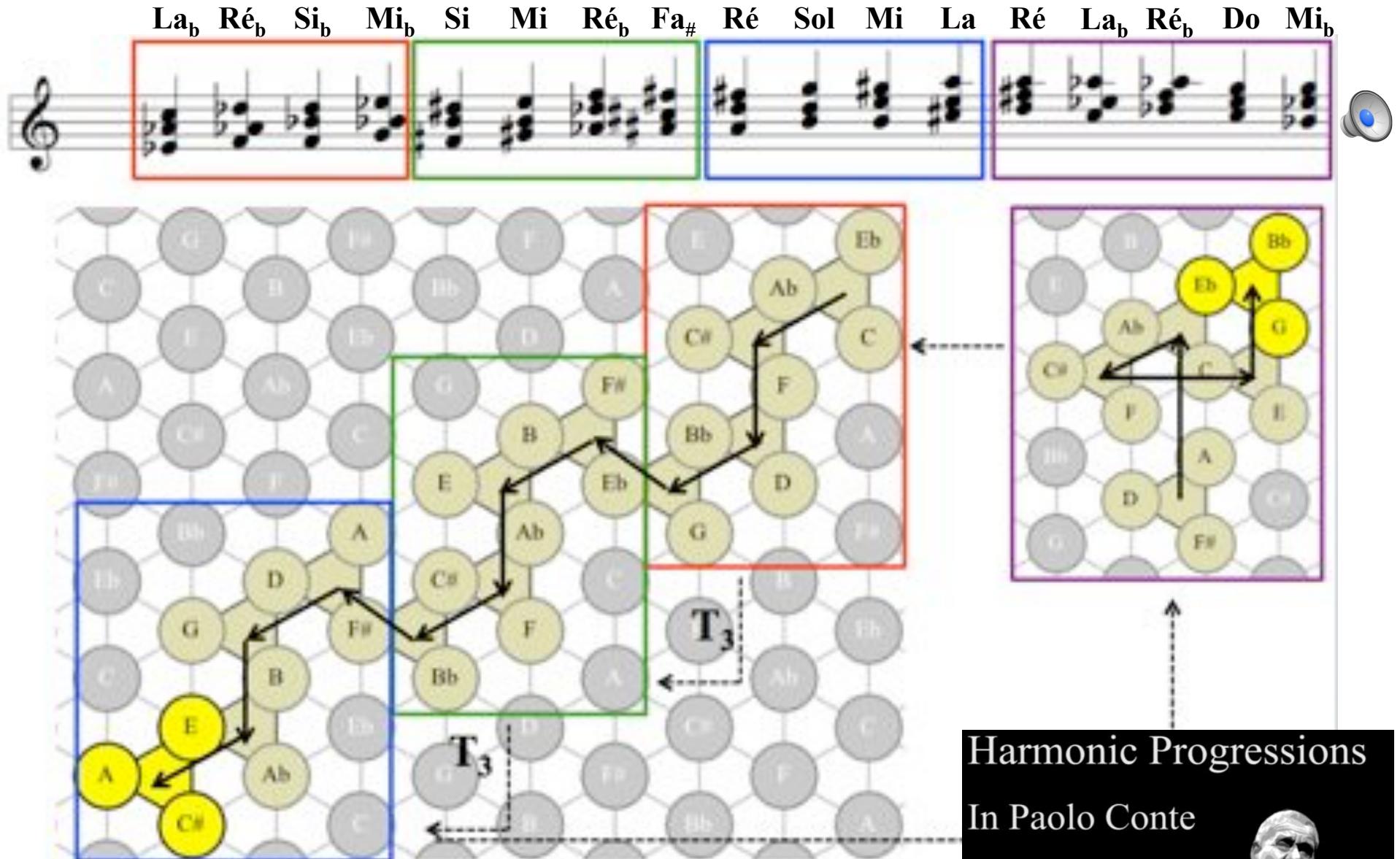
→ Si/Ré# → Mi → Do# → Fa#

→ Ré/La → Sol → Mi⁷ → La⁷

→ Ré → Lab⁷ → Réb → Do⁷ → Mib



Madeleine's spatial trajectory



Harmonic Progressions
In Paolo Conte
Madeleine



<http://www.mathemusic.net>

Partial covering of the Tonnetz

Prelude Adagio

Lab → Réb/Fa → Sib⁷ → Mib⁷/Réb



Missing major chord

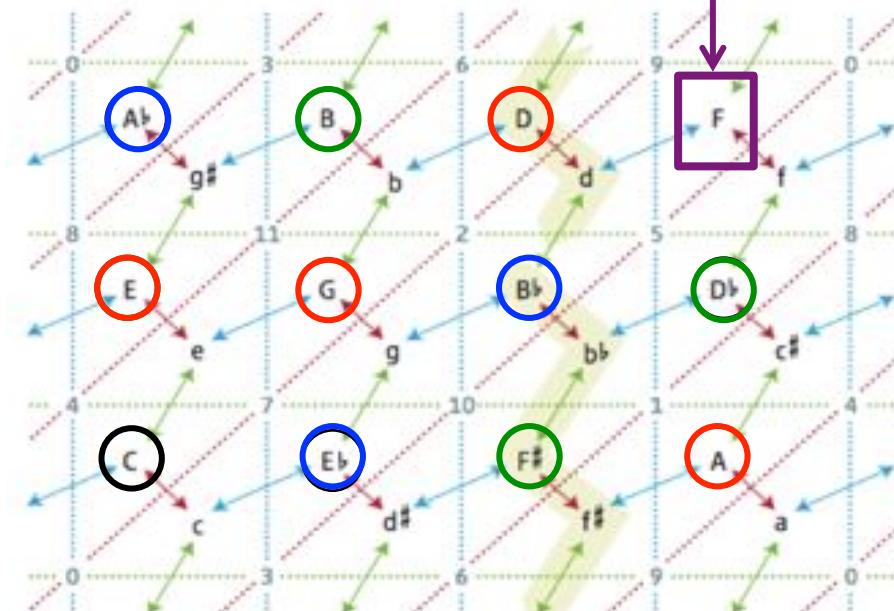
S. La Via, *Poesia per musica e musica per poesia.*
Dai trovatori a Paolo Conte, Carocci, 2006

→ Lab → Réb/Fa → Sib⁷ → Mib⁷/Réb

→ Si/Ré# → Mi → Do# → Fa#

→ Ré/La → Sol → Mi⁷ → La⁷

→ Ré → Lab⁷ → Réb → Do⁷ → Mib

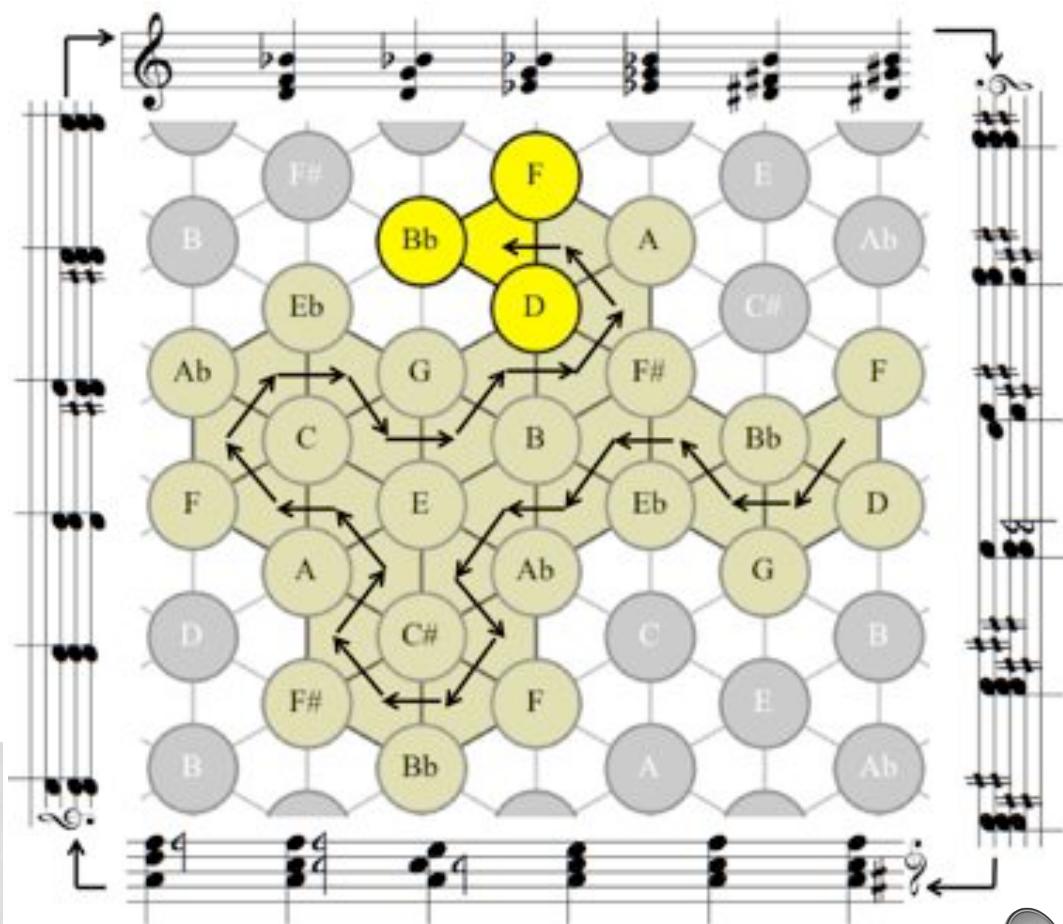


Aprile, a Hamiltonian « decadent » song

*Socchiusa è la finestra, sul giardino,
Un'ora passa lenta, sonnolenta.
Ed ella, ch'era attenta, s'addormenta
A quella voce che già si lamenta,
Che si lamenta in fondo a quel giardino.*

*Non è che voce d'acque su la pietra:
E quante volte, quante volte udita!
Quell'amore e quell'ora in quella vita
S'affondan come ne l'onda infinita
Stretti insieme il cadavere e la pietra.*

*Ella stende l'angoscia sua nel sonno.
L'angoscia è forte, e il sonno è così lieve!
(Par i' luce d'aprile quasi una neve
che sia tiepida.)
Ed ella certo deve soffrire,
Vagamente, anche nel sonno.*



G. D'Annunzio (1863-1938)

Aprile (d'après Gabriele D'Annunzio)**Tangente**
L'aventure mathématiquepour mieux comprendre
le monde

M. Andreatta, « Math'n pop : symétries et cycles hamiltoniens en chanson », *Tangente*

Aprile

4D & 2D Visualizations
Hamiltonian Cycles
M. Andreatta, G. Baroin 2013

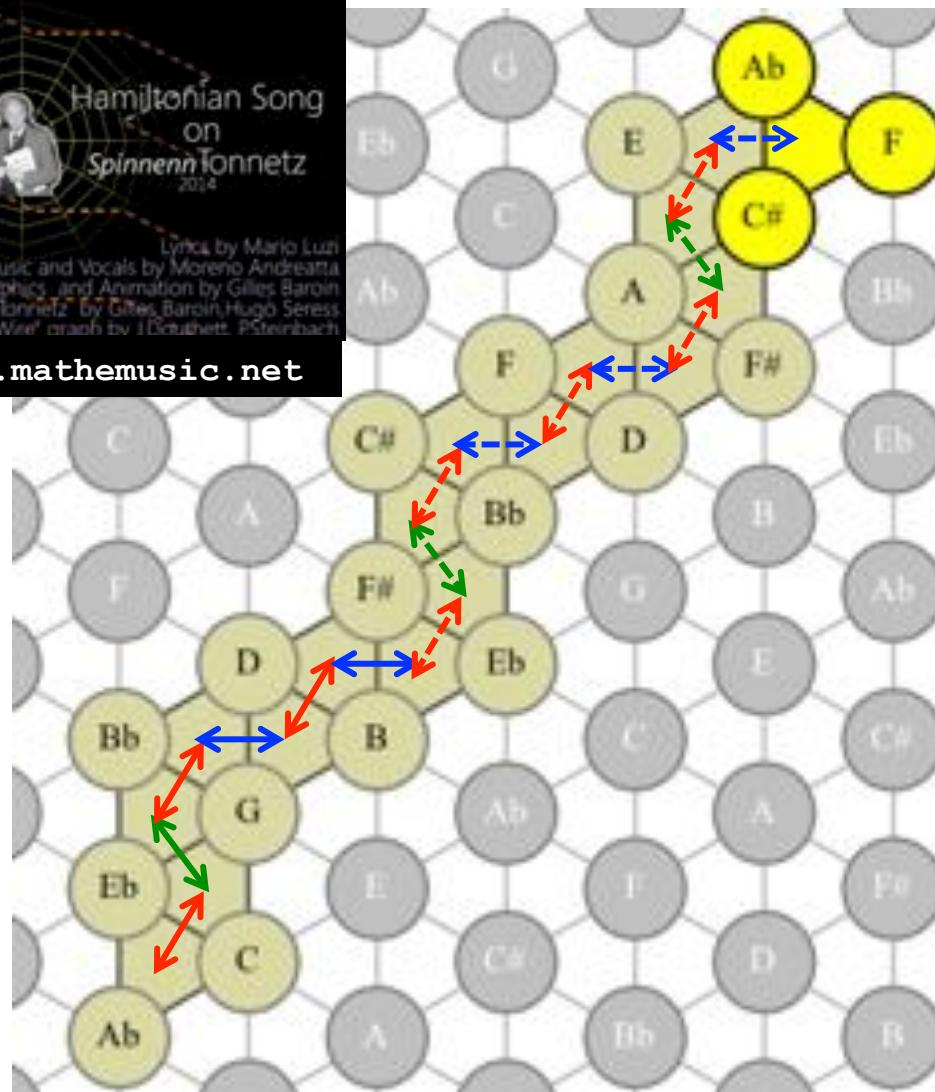
Lyrics: Gabriele d'Annunzio
Music and Vocals: Moreno Andreatta
Hypersphere and Ideogramms: Gilles Baroin
Original "Chicken Wire" graph: J. Douthett, P. Steinbach

<http://www.mathemusic.net>

Hamiltonian Cycles with inner periodicities

#41	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R
#42	E	P	E	P	E	P	E	P	E	P	E	P	E	P	E	P	E	P	E	P	E	P	E	P
#43	P	L	R	P	L	R	P	L	R	P	L	R	P	L	R	P	L	R	P	L	R	P	L	R
#44	P	L	P	L	R	P	E	R	P	L	P	H	P	E	P	R	E	P	L	P	H	P	E	P
#45	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R

L P L P L R L P L P L R ...
 → P L P L R L ...
 L P L R L P ...
 PL R L P L ...
 ↙ L R L P L P ...
 R L P L P L ...



La sera non è più la tua canzone
(Mario Luzi, 1945, tratto da *Poesie sparse*)

La sera non è più la tua canzone,
è questa roccia d'ombra traforata
dai lumi e dalle voci senza fine,
la quiete d'una cosa già pensata.

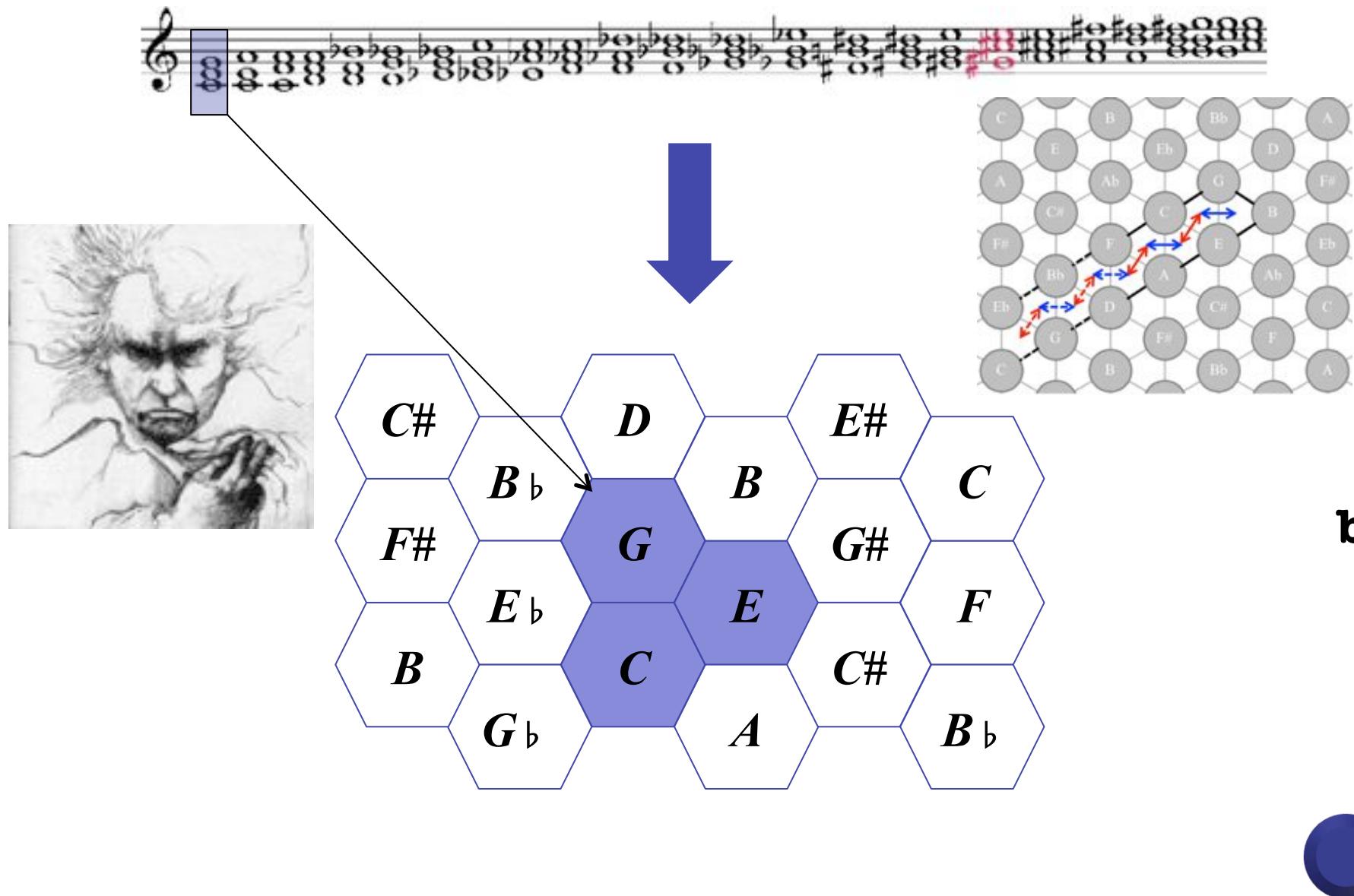
Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d'una cosa conosciuta,
per nome hai una parola ch'è passata
nell'intimo del cuore e s'è perduta.

Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell'essere, (compiuta).

L'aria tace ed il tempo dietro a te
si leva come un'arida montagna
dove vaga il tuo spirito e si perde,
un vento raro scivola e ristagna.



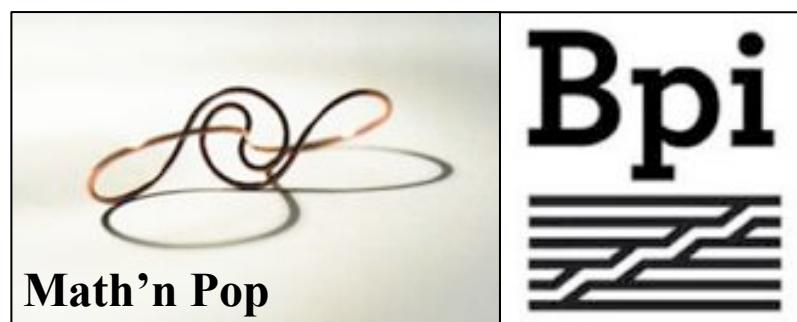
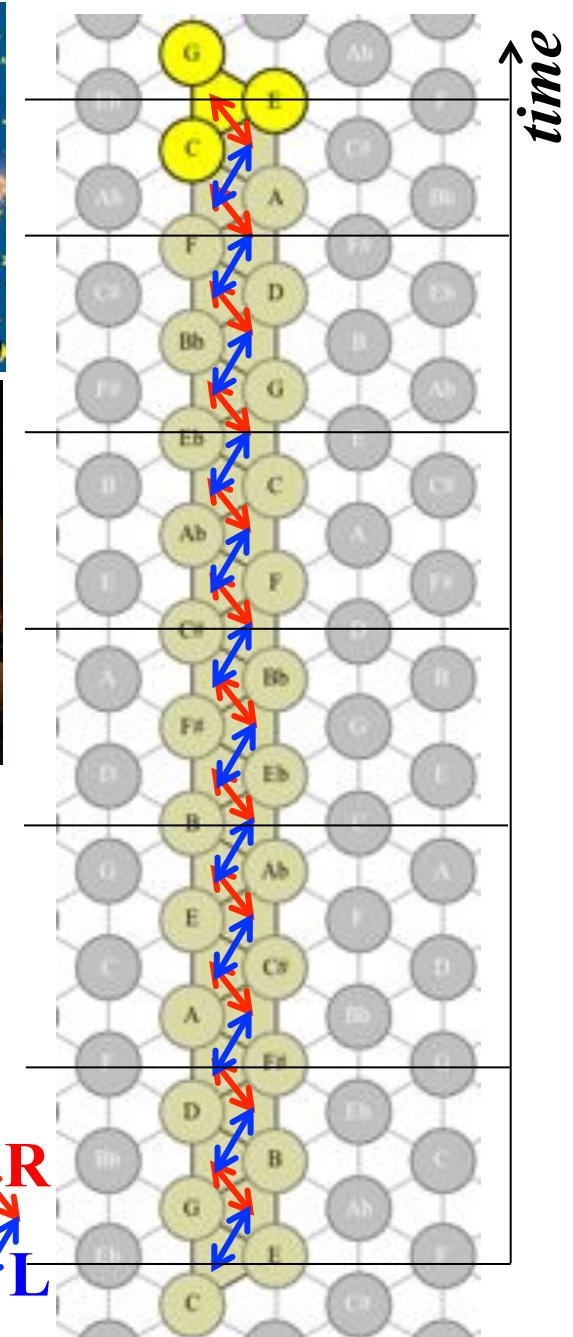
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



Le blé en herbe

(Pola/Morena/Dieu)

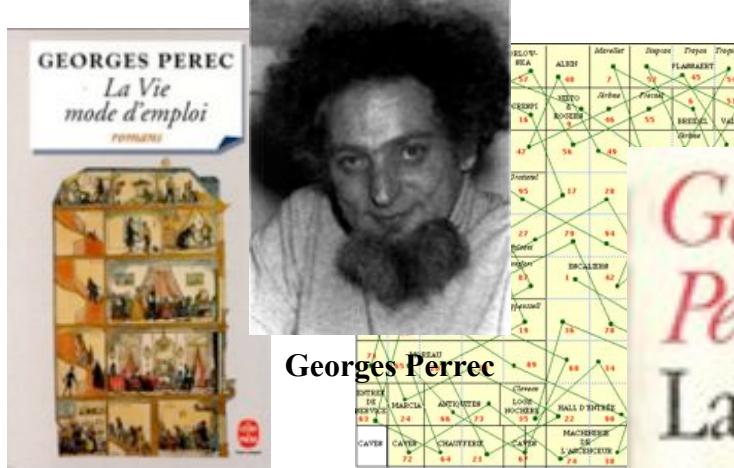
Plonger comme un enfant, cheveux au vent	Croiser matin dans l'herbe folle
Sous l'océan du blé en herbe	Deux tourterelles qui s'envolent
Marée d'épis couleur d'amande	Suivre les jeux des hirondelles
Qui tendent à caresser le ciel	Sur le paysage éternel
Algues tendres de mille plages	Nager comme un enfant, cheveux au vent
Frôlant le ventre des nuages	Sous l'océan
Cheveux de pluie, dos de poissons	Du blé en herbe
Qui frissonnent à l'unisson	Marée de fruits au goût amer
Suivre le bord des continents	Acide et salée comme la mer
Dans l'océan du blé en herbe	Vers l'îlot d'un petit village
Pêcher le corail du pavot	Vers un château d'eau sur la plage
Dans le sang des coquelicots	Quand tout s'éteint avant l'orage
	Quand se lève le vent du large
	Sur le blé vert



The use of constraints in arts



Cent mille milliards de poèmes, 1961

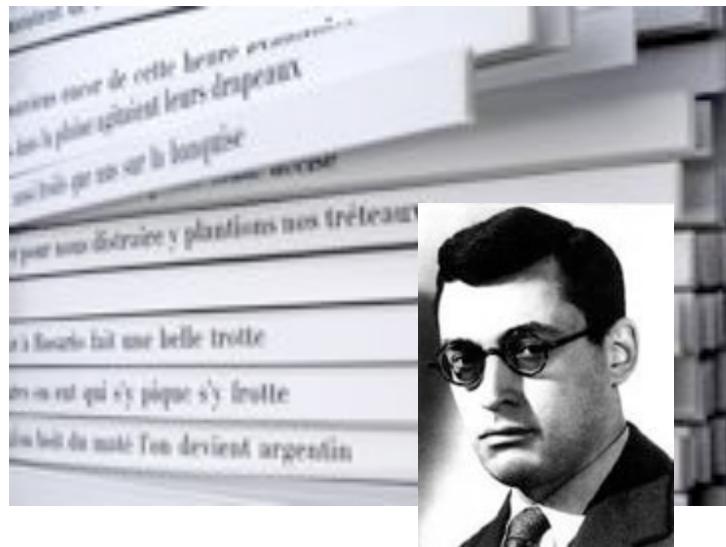


La vie mode d'emploi,



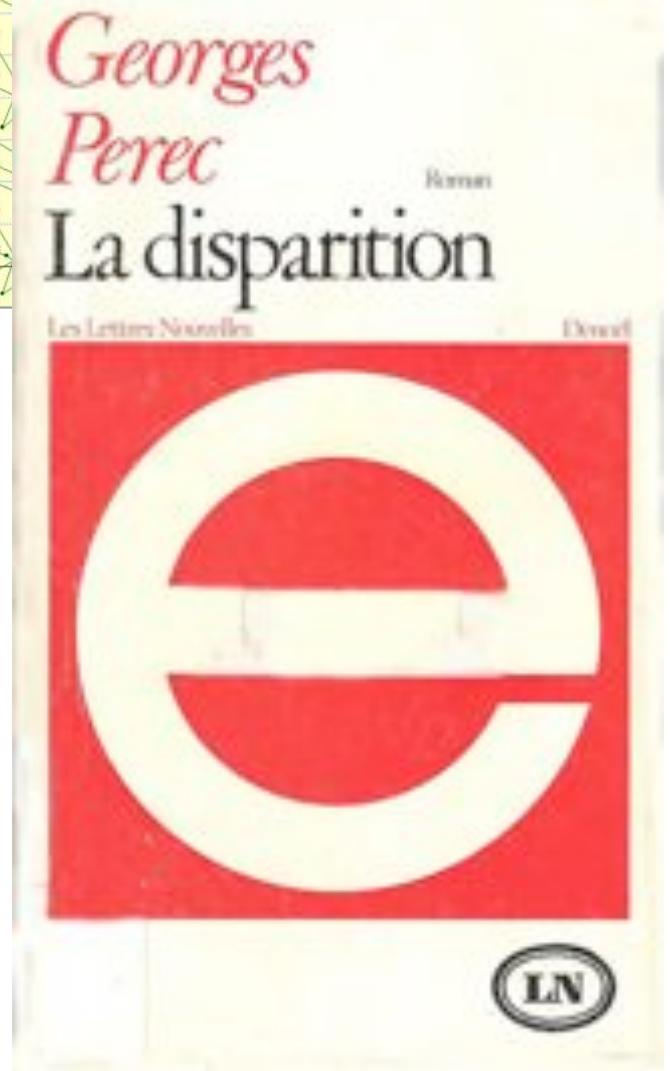
Italo Calvino

Il castello dei destini incrociati, 1969



Raymond Queneau

OuLiPo (Ouvroir de Littérature Potentielle)



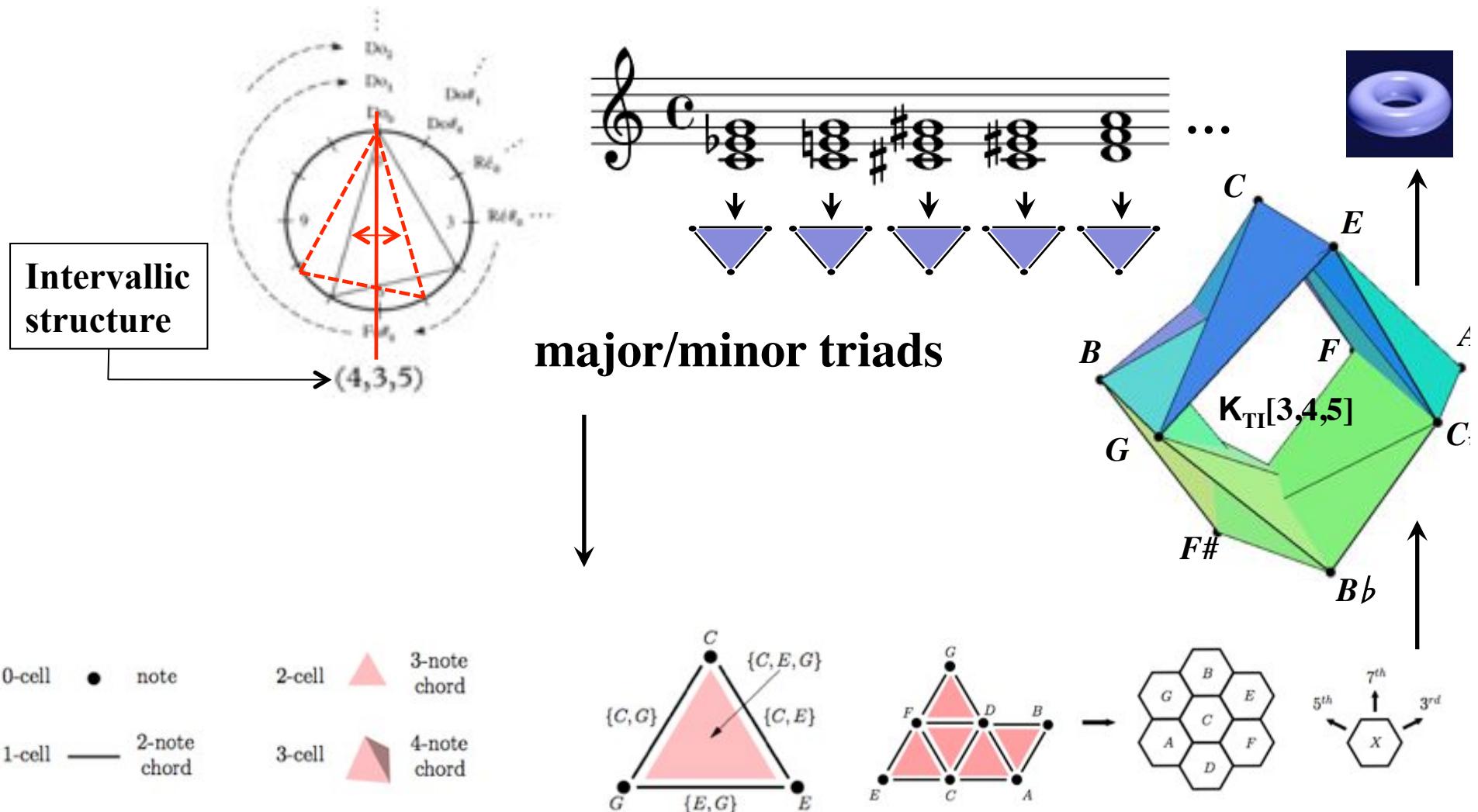
LN

The Tonnetz as a simplicial complex

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013



- Assembling chords related by some equivalence relation
 - Transposition/inversion: Dihedral group action on $\mathbb{P}(\mathbb{Z}_n)$

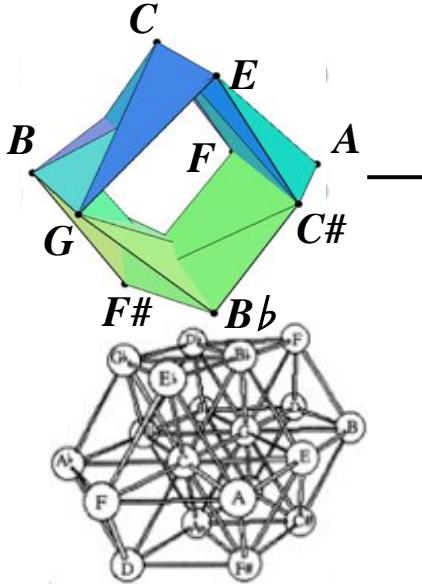


Classifying Chord Complexes

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- Complexes enumeration in the chromatic system

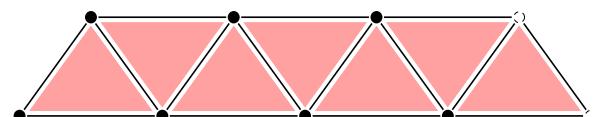
$K_{TI}[3,4,5]$
[Cohn – 1997]



$K_{TI}[2,3,3,4]$
[Gollin - 1998]



$K_T[2,2,3]$
[Mazzola – 2002]



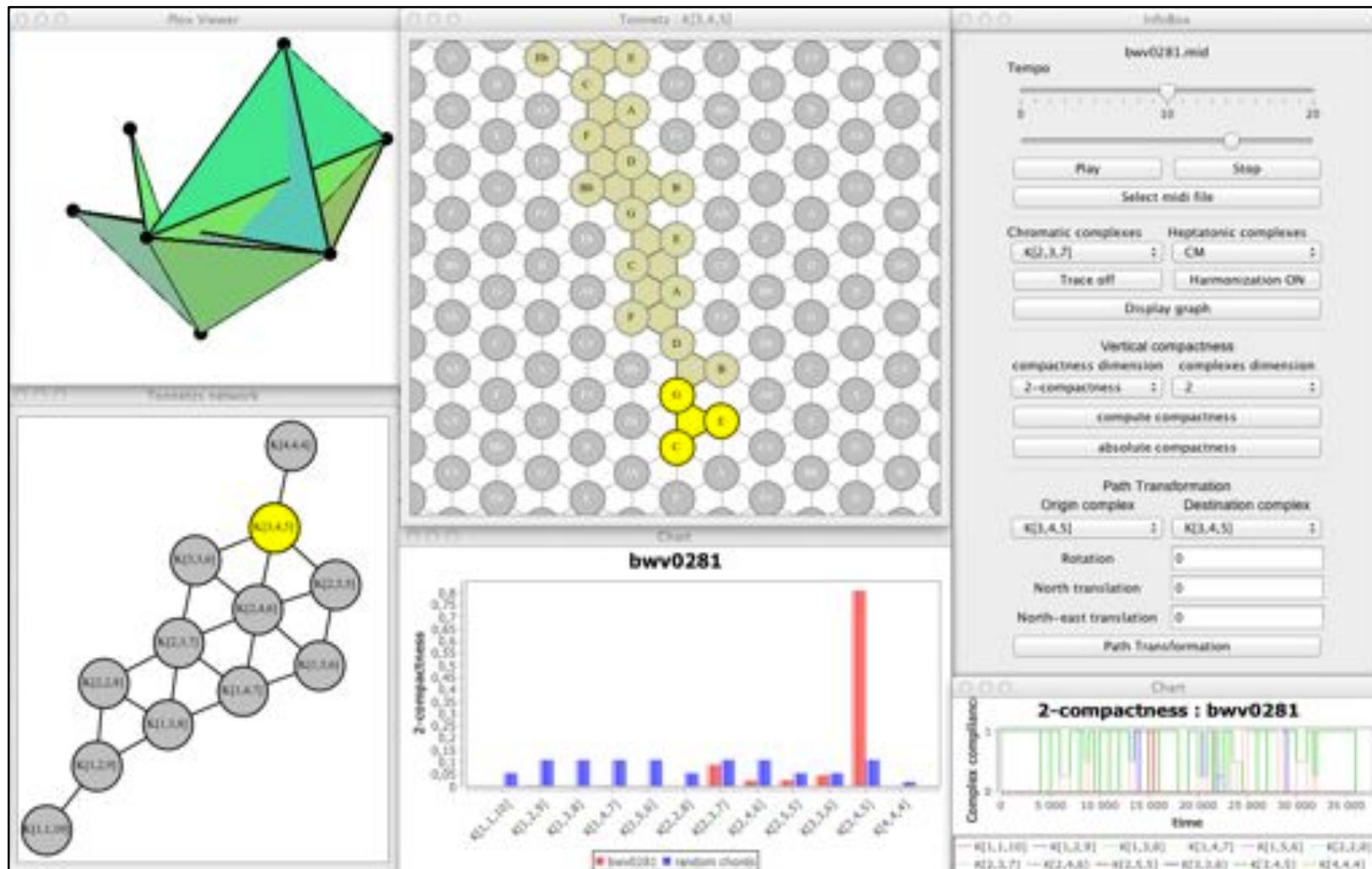
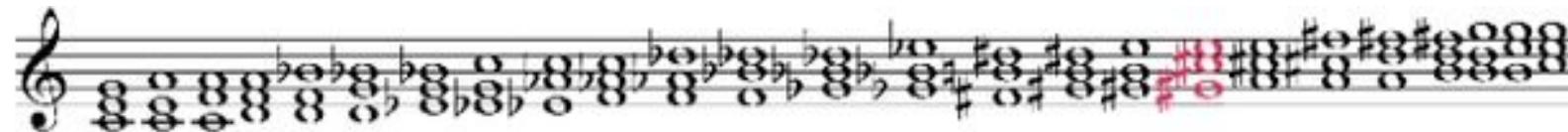
...

d	complexe	taille	b_n	p-v	χ
-	K_0	0	0		0
0	$K_{TI}[0]$	0	[0]		0
	$K_{TI}[1, 1]$	12	[1, 1]	x	0
	$K_{TI}[2, 10]$	12	[2, 2]		0
	$K_{TI}[3, 9]$	12	[3, 3]		0
	$K_{TI}[4, 8]$	12	[4, 4]		0
	$K_{TI}[5, 7]$	12	[1, 1]	x	0
	$K_{TI}[6, 6]$	6	[6, 0]		6
	$K_{TI}[1, 1, 10]$	12	[1, 1, 0]	x	0
	$K_{TI}[1, 2, 9]$	24	[1, 2, 1]	x	0
	$K_{TI}[1, 3, 8]$	24	[1, 2, 1]	x	0
	$K_{TI}[1, 4, 7]$	24	[1, 2, 1]	x	0
	$K_{TI}[1, 5, 6]$	24	[1, 1, 6]		6
	$K_{TI}[2, 2, 8]$	12	[2, 2, 0]		0
	$K_{TI}[2, 3, 7]$	24	[1, 2, 1]	x	0
	$K_{TI}[2, 4, 6]$	24	[2, 2, 6]		6
	$K_{TI}[2, 5, 5]$	12	[1, 1, 0]	x	0
	$K_{TI}[3, 3, 6]$	12	[3, 0, 3]		6
	$K_{TI}[3, 4, 5]$	24	[1, 2, 1]	x	0
	$K_{TI}[4, 4, 4]$	4	[4, 0, 0]		4
	$K_{TI}[1, 1, 1, 9]$	12	[1, 1, 0, 0]	x	0
	$K_{TI}[1, 1, 2, 8]$	24	[1, 1, 12, 0]		12
	$K_{TI}[1, 1, 3, 7]$	24	[1, 2, 13, 0]		12
	$K_{TI}[1, 1, 4, 6]$	24	[1, 1, 18, 0]		18
	$K_{TI}[1, 1, 5, 5]$	12	[1, 1, 6, 0]		6

Analyzing harmonic progressions as paths in a generic *Tonnetz*

- L. Bigo, M. Andreatta, J.-L. Giavitto, O. Michel, A. Spicher, « Computation and Visualization of Musical Structures in Chord-based Simplicial Complexes », MCM 2013, McGill University, Springer, LNCS.
 - Bigo L., D. Ghisi, A. Spicher, M. Andreatta (2014), Proceedings ICMC|SMC|2014, 14-20 Sept. 2014, Athens (revised and enlarged version forthcoming in *Computer Music Journal*, 39(3), 2015)
 - Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer (in press)

Analyzing harmonic progressions as paths in *Hexachord*

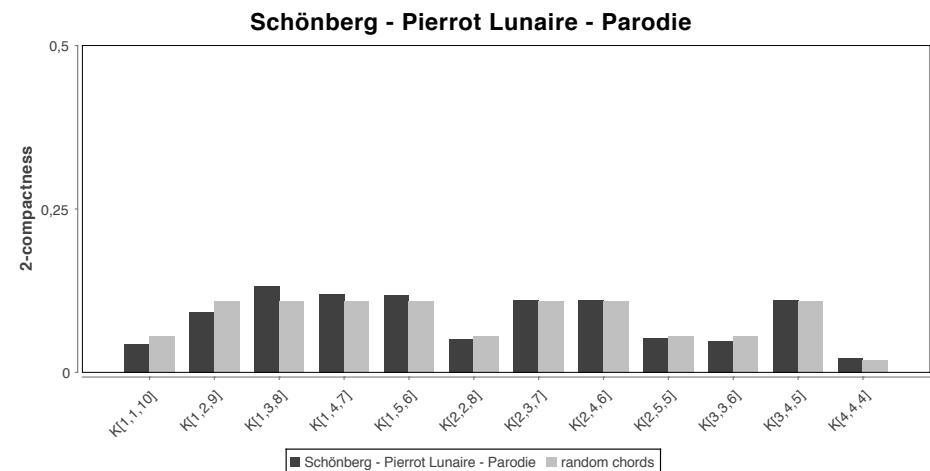
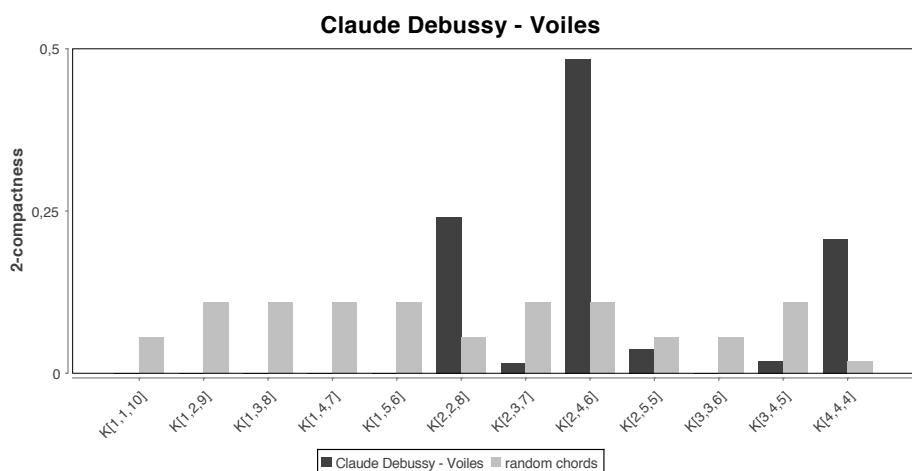
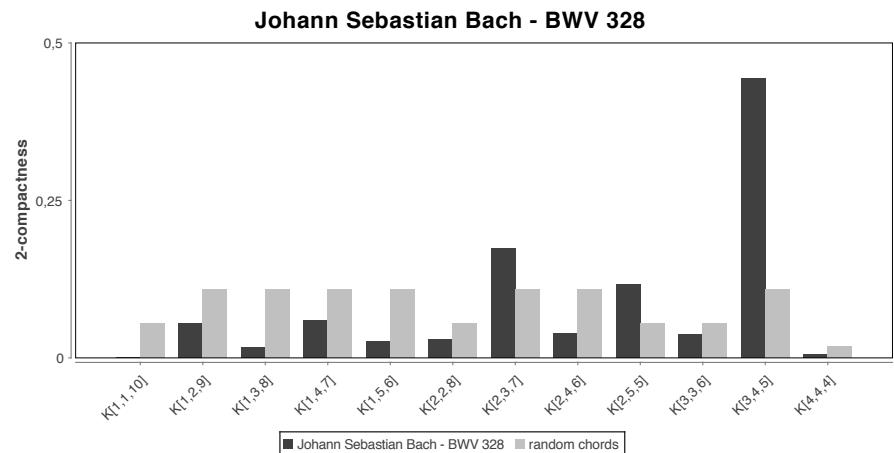
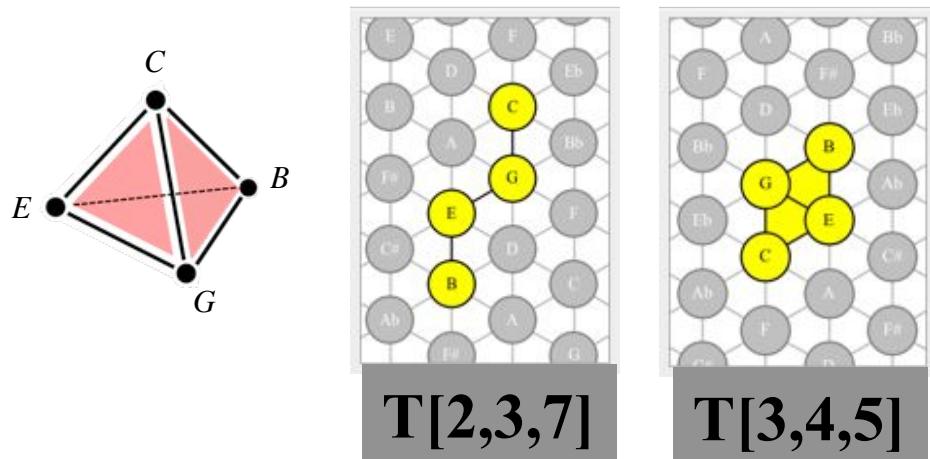


<http://www.lacl.fr/~lbigo/hexachord>

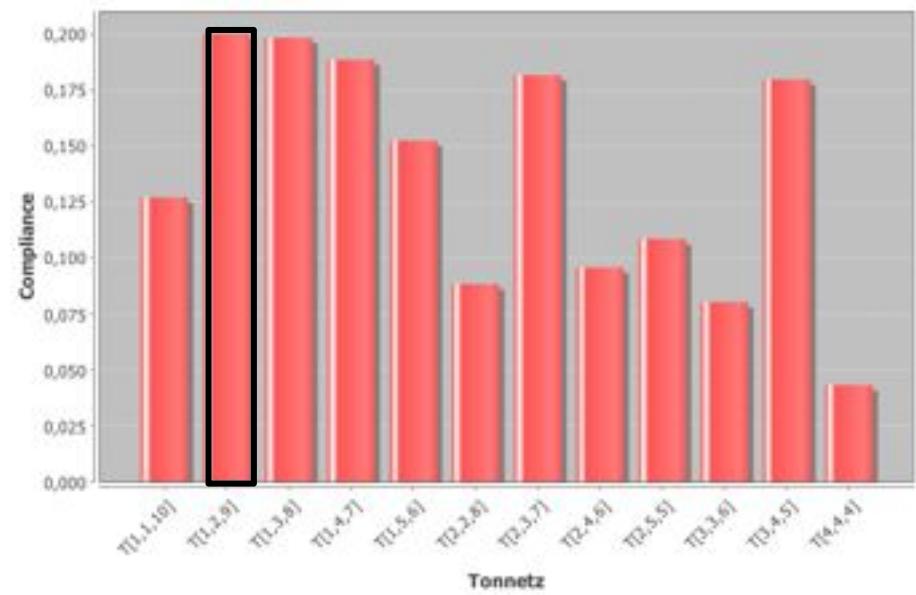
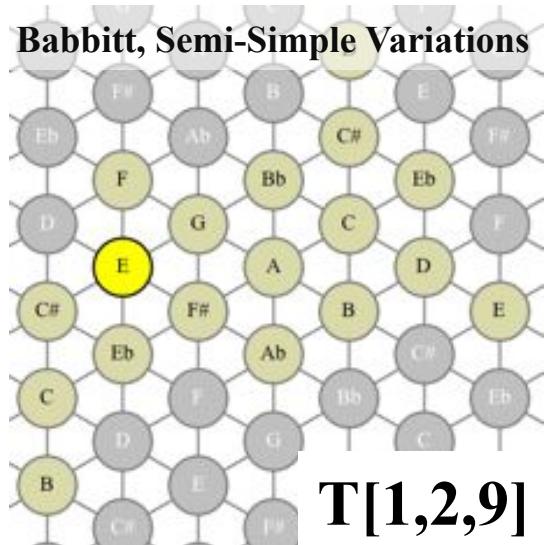
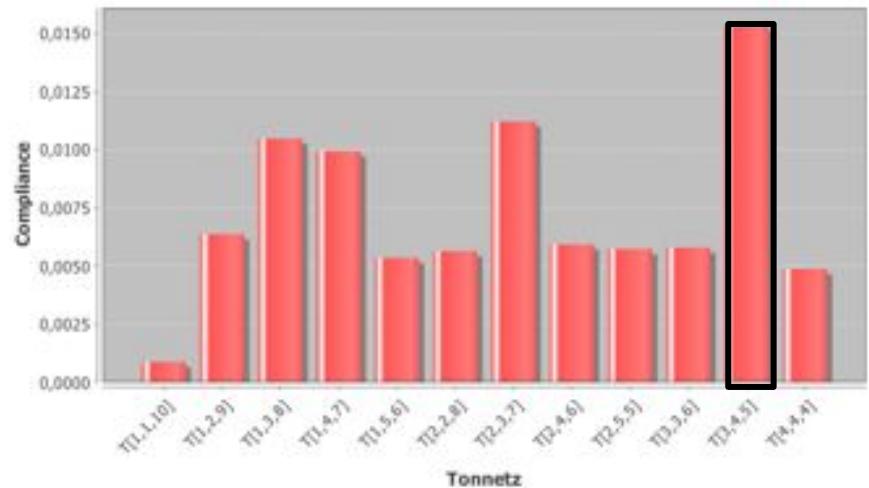
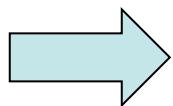
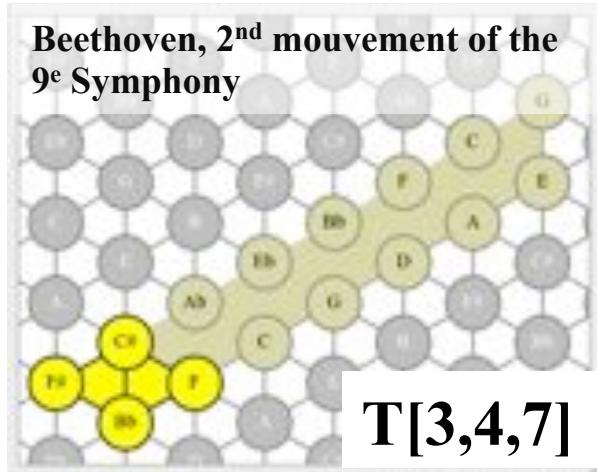
→ demo

The spatial character of the « musical style »

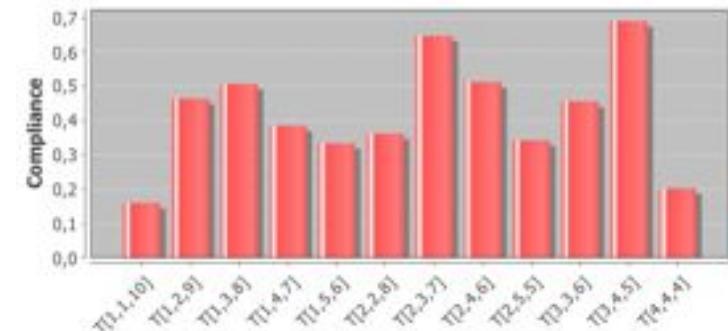
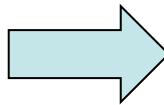
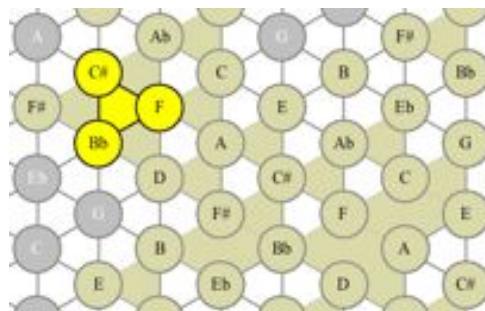
Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer (in press)



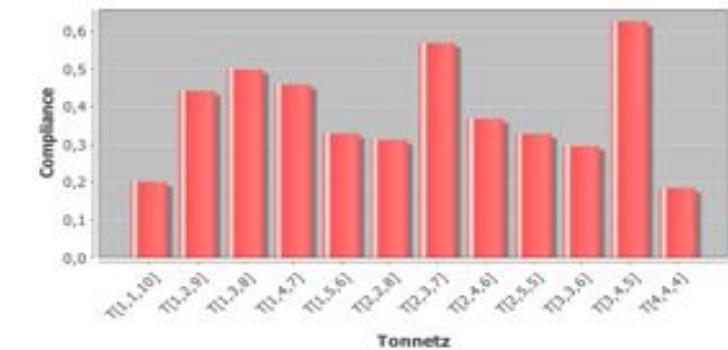
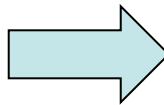
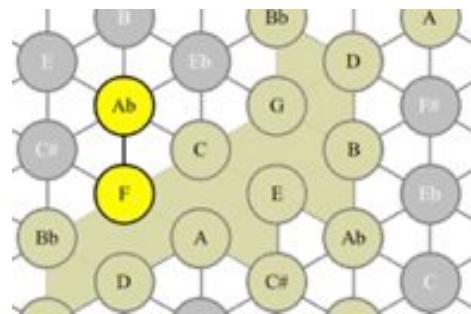
The spatial character of the « musical style »



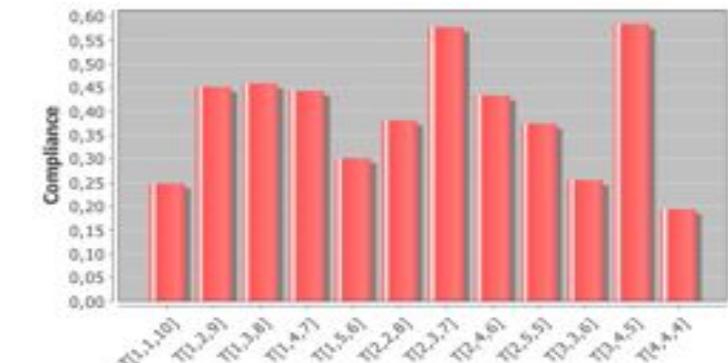
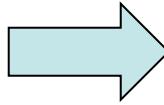
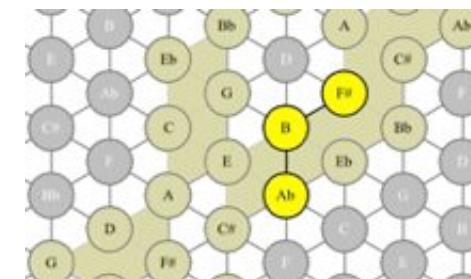
The « shape » of space distributions in jazz standards



Thelonious Monk, *Brilliant Corners*



Chick Corea, *Eternal Child*

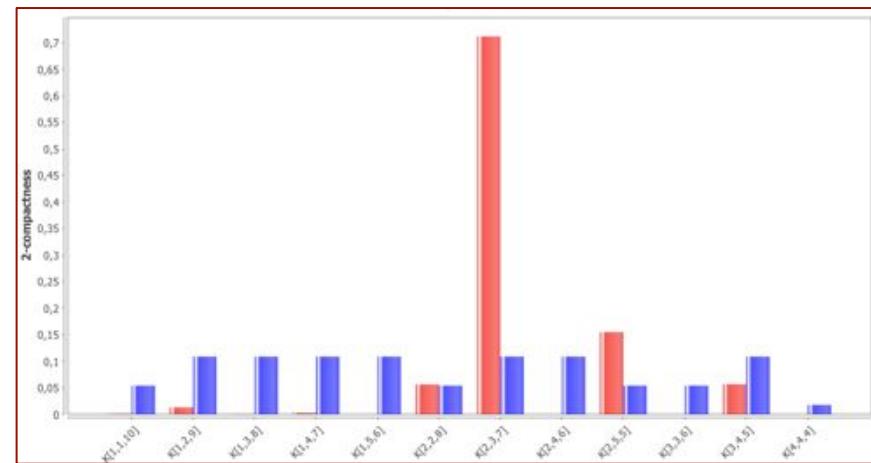
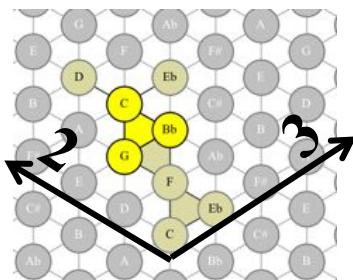
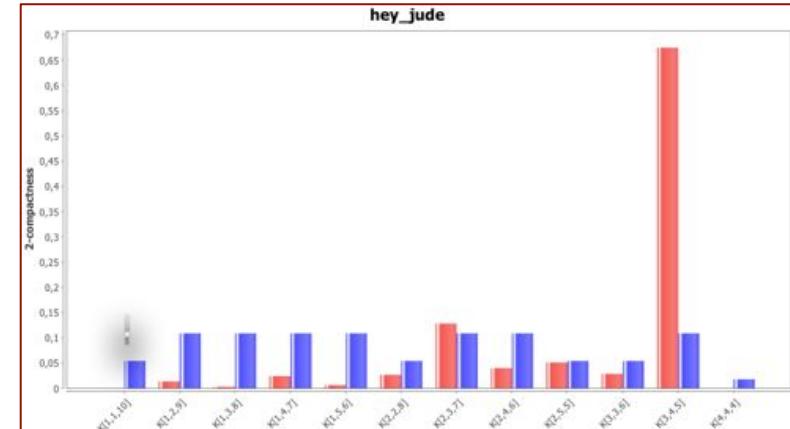
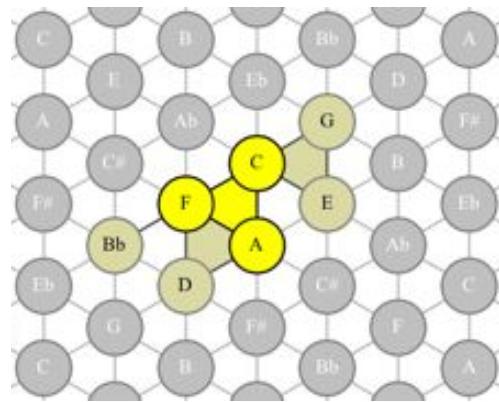


Bill Evans, *Turn Out the Stars*

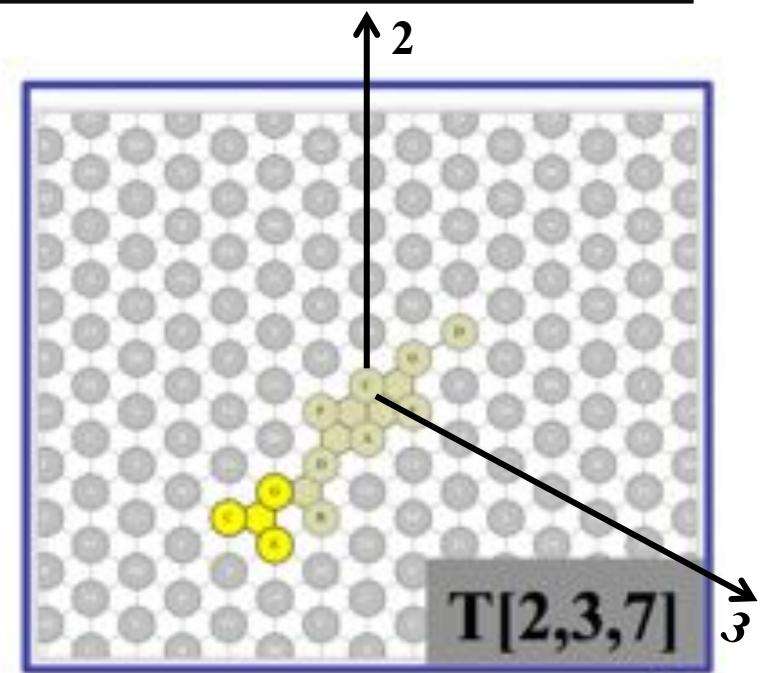
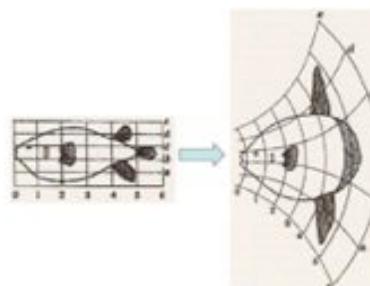
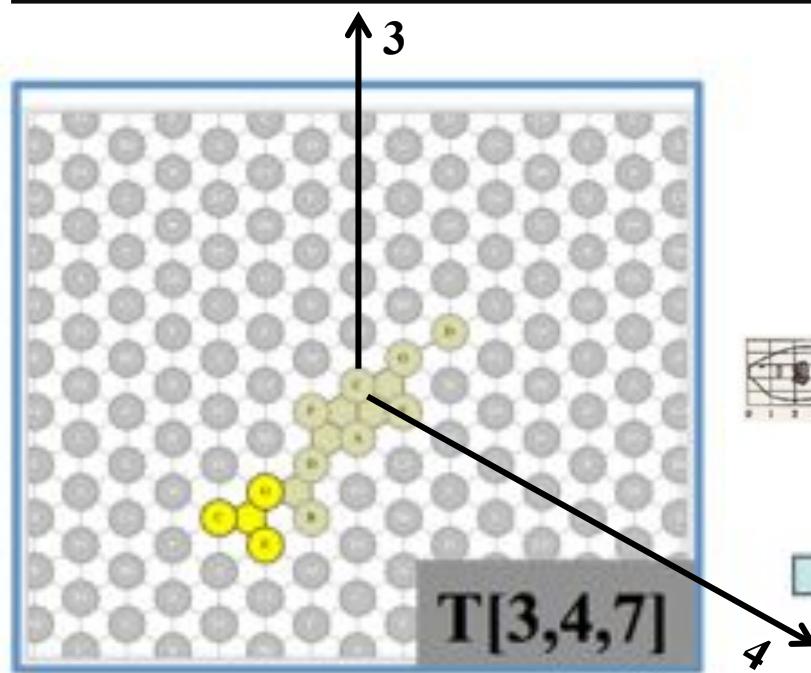
Beatles' 'natural' Space and stylistic embeddings



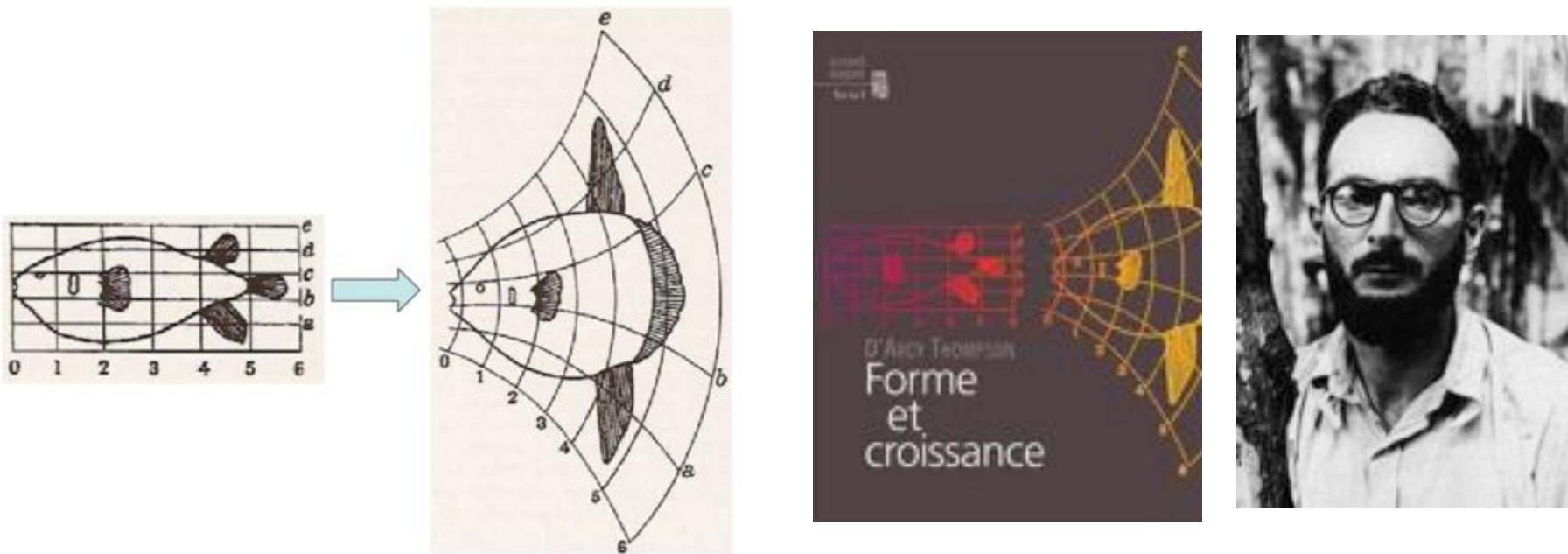
The Beatles,
Hey Jude



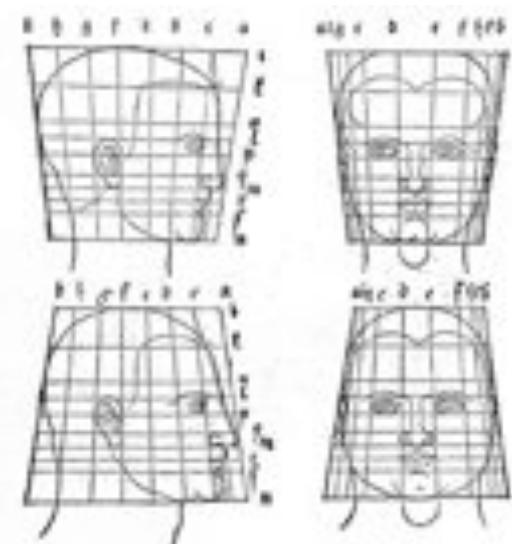
A trajectory realized in different support spaces



The morphological vs the mathematical genealogy of the structuralism

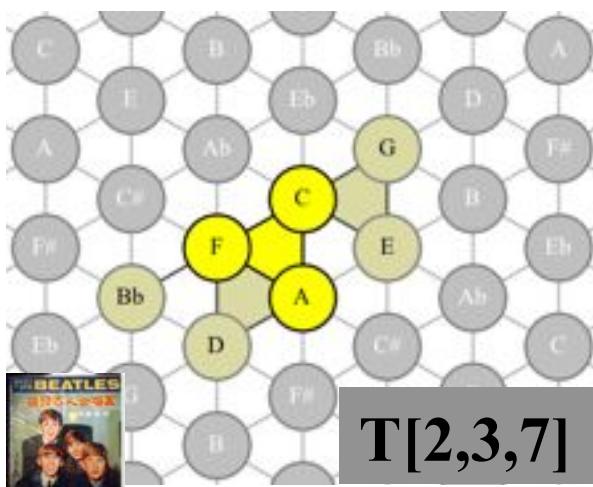
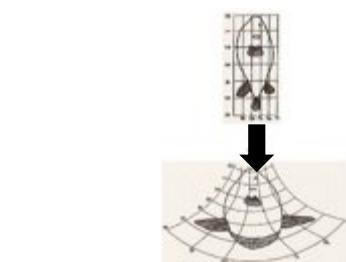
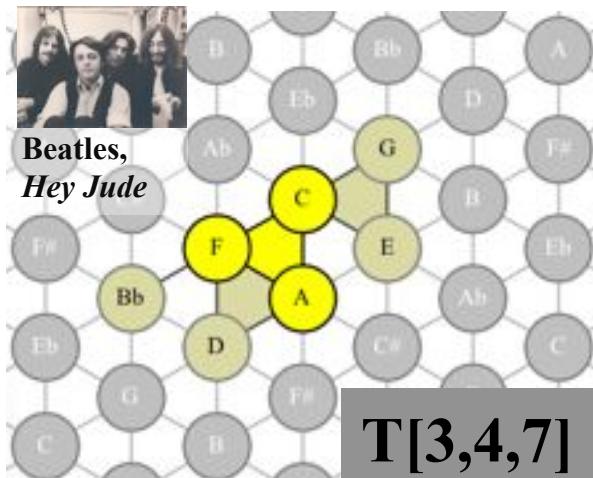


“[The notion of **transformation**] comes from a work which played for me a very important role and which I have read during the war in the United States : *On Growth and Form*, in two volumes, by **D'Arcy Wentworth Thompson**, originally published in 1917. The author (...) proposes an interpretation of the visible transformations between the species (animals and vegetables) within a same gender. This was fascinating, in particular because I was quickly realizing that this perspective had a long tradition: behind Thompson, there was **Goethe's** botany and behind Goethe, **Albert Dürer** with his *Treatise of human proportions*”
(Lévi-Strauss, conversation with Eribon, 1988).



Musically interesting Trajectory Transformations

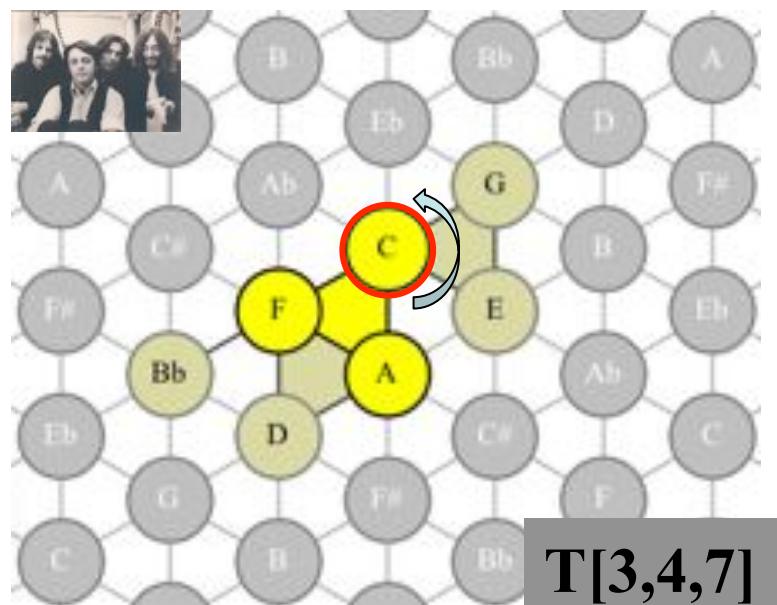
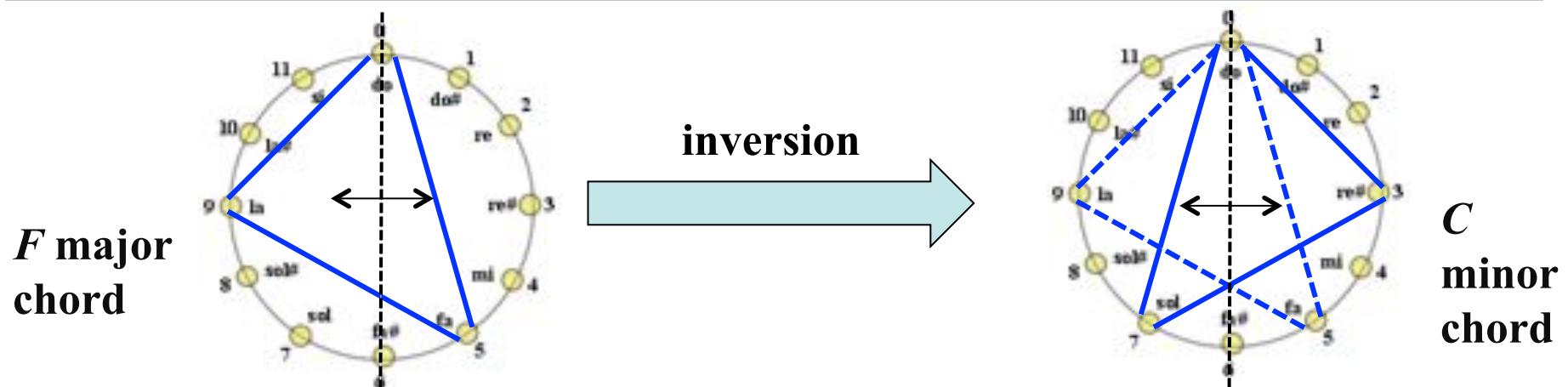
Isomorphism from a support space to a different one



Transformation sur l'espace	Transformation sur la trajectoire	Transformation musicale
$K^{\circ}_{\text{TL}}[X_{\text{chro}}] \rightarrow K^{\circ}_{\text{TL}}[X'_{\text{chro}}]$	Translation	Transposition chromatique
	Symétrie centrale	Inversion chromatique
	Rotation d'angle π Symétrie axiale	?
	Homothétie ($\Leftrightarrow K^{\circ}_{\text{TL}}[X_{\text{chro}}] \rightarrow K^{\circ}_{\text{TL}}[X'_{\text{chro}}]$)	?
$K^{\circ}_{\text{TL}}[X_{\text{mod}}]_T \rightarrow K^{\circ}_{\text{TL}}[X'_{\text{mod}}]_T$	Translation	Transposition modale
	Symétrie centrale	Inversion modale
	Rotation d'angle π Symétrie axiale	?
	Homothétie ($\Leftrightarrow K^{\circ}_{\text{TL}}[X_{\text{mod}}]_T \rightarrow K^{\circ}_{\text{TL}}[X'_{\text{mod}}]_T$)	?
$K^{\circ}_{\text{TL}}[X_{\text{chro}}] \rightarrow K^{\circ}_{\text{TL}}[X'_{\text{chro}}]$	Plongement	?
$K^{\circ}_{\text{TL}}[X_{\text{mod}}]_T \rightarrow K^{\circ}_{\text{TL}}[X'_{\text{mod}}]_T$	Plongement	?
$K^{\circ}_{\text{TL}}[X_{\text{mod}}]_T \rightarrow K^{\circ}_{\text{TL}}[X_{\text{chro}}]_T$	Plongement	Transposition chromatique (+ transposition modale)
Trace \rightarrow Trace	Isométrie	Permutation dans le temps des ensembles de notes
$K \rightarrow K'$	Isométrie	?
$K \rightarrow K'$	Plongement	?

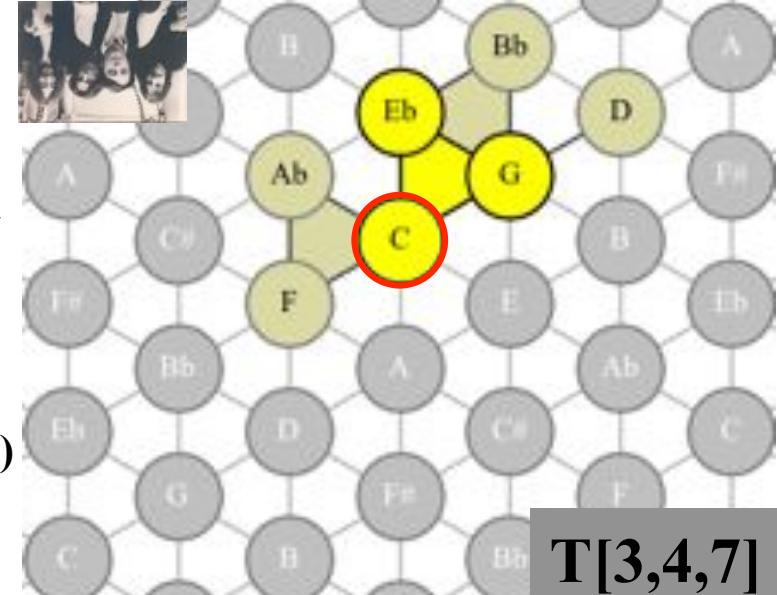
Musically interesting Trajectory Transformations

Automorphism of the support space



Beatles, Hey Jude
(orig. version)

Rotation
(autour du *do*)

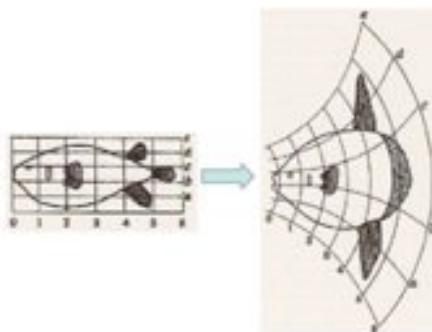
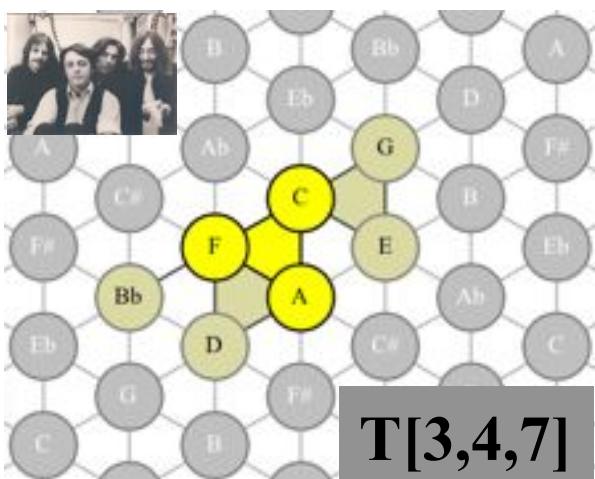


Beatles, Hey Jude
(transformed version)

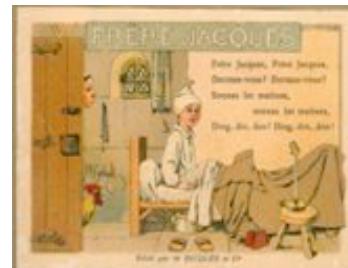
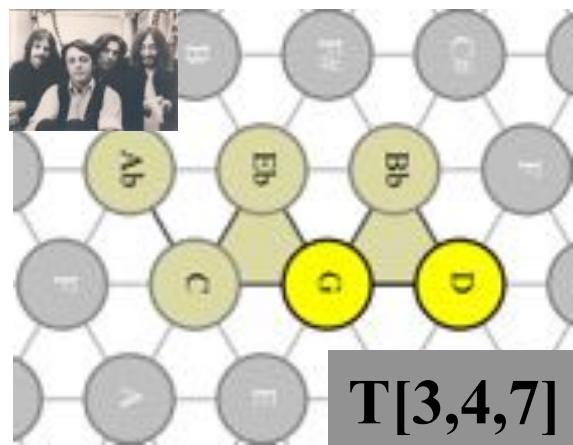
→ Hexachord

Musically interesting Trajectory Transformations

The “M” transformation



M



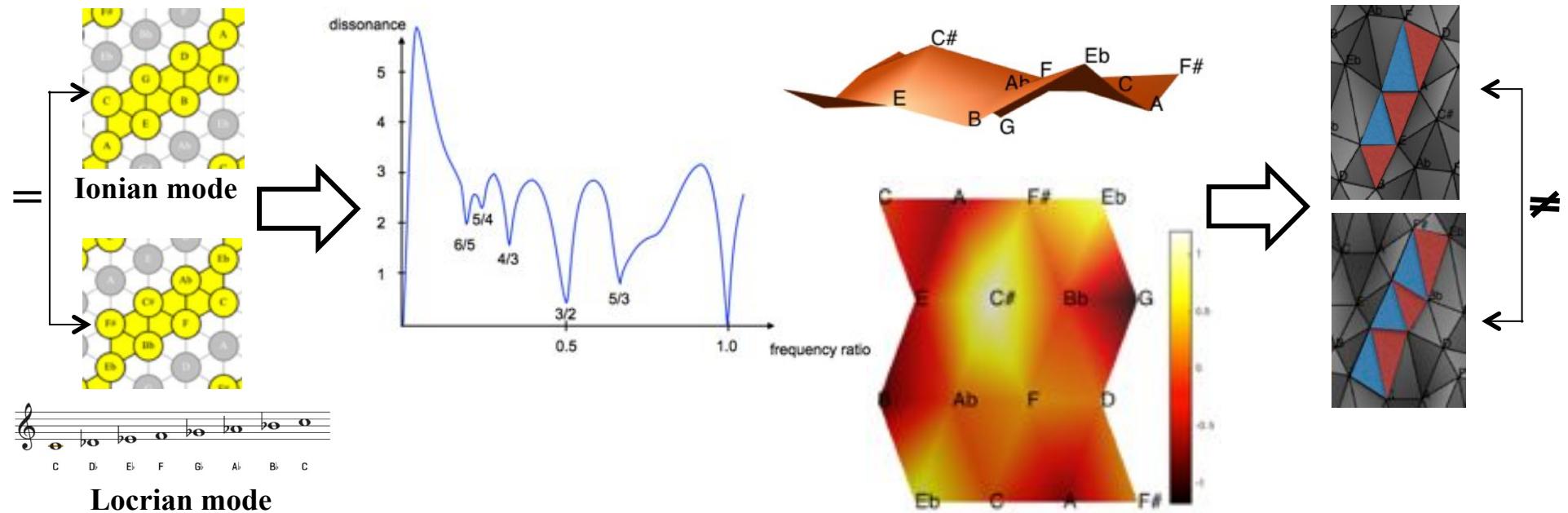
M



M



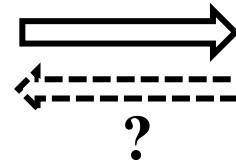
Signal/Symbolic articulation in MIR



M. Bergomi, *Dynamics and Algebraic Topology Tools for Music in the Symbolic/Signal interaction domain*, ongoing PhD

→ Towards a geometric dynamic modeling of a musical piece ?

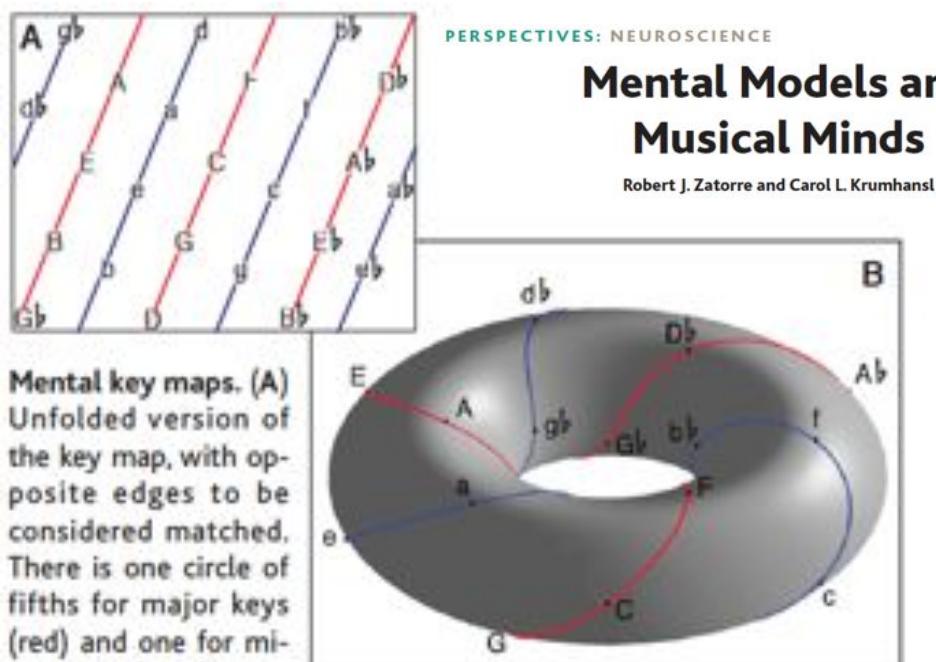
SPACE



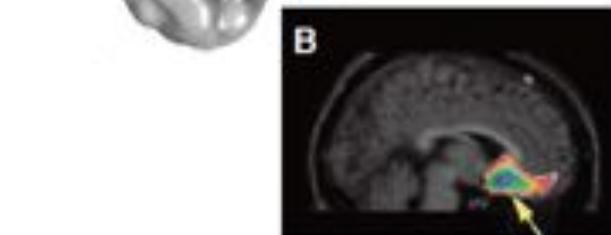
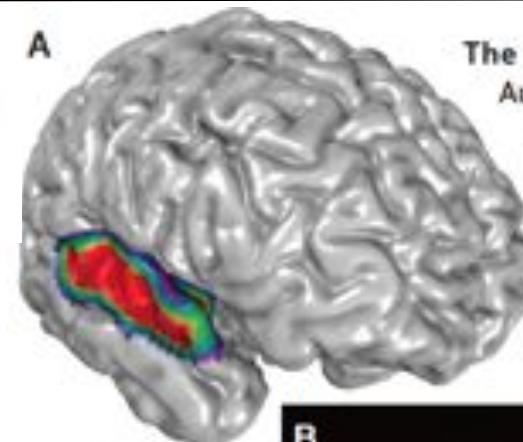
MUSIC

→ Towards a topological signature of a musical piece?

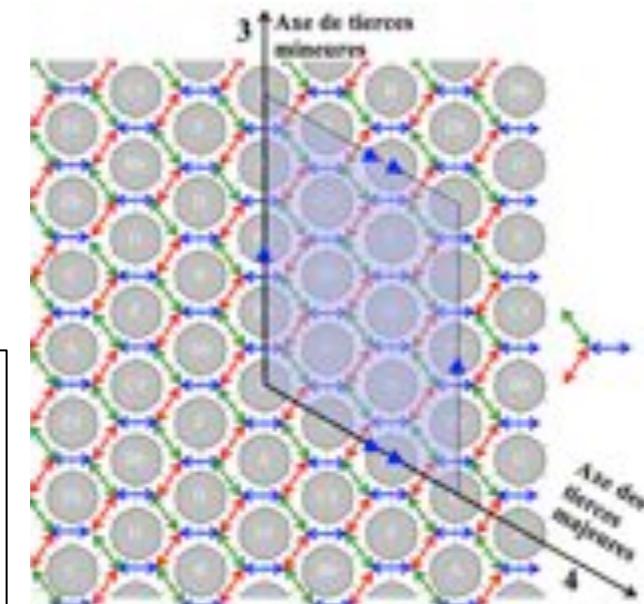
Neurosciences and Tonnetz



Acotto E. et M. Andreatta (2012),
« Between Mind and Mathematics.
Different Kinds of Computational
Representations of Music »,
Mathematics and Social Sciences, n° 199,
2012(3), p. 9-26.



The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.



Thank you for your attention!

Hexachord (by Louis Bigo, 2013)