

## A GEOMETRIC MODEL FOR THE ANALYSIS OF POP MUSIC

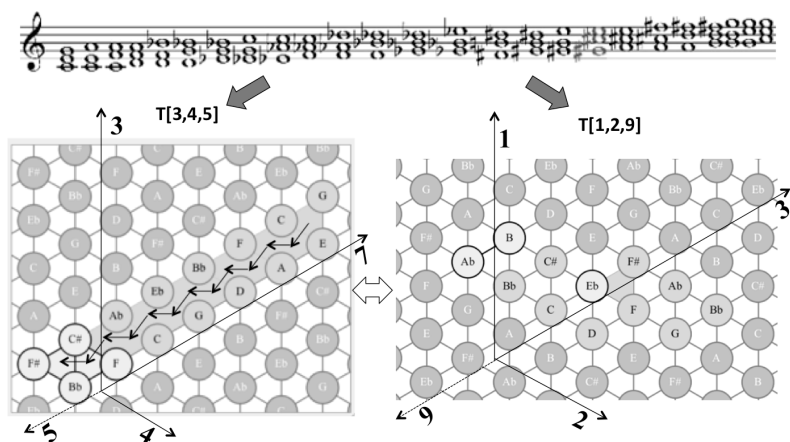
LOUIS BIGO, MORENO ANDREATTA

Pop music is unquestionably becoming the subject matter of a very rich field of study that is of concern to the music theory community and the computer-aided analysis community alike. This growing interest can partly be explained as a consequence of the flexibility of some of the formal tools and concepts that have been developed recently in music analysis and particularly within the transformational tradition—a branch that extends the set-theoretical paradigm as well as includes other orientations such as neo-Riemannian analysis, diatonic theory, voice-leading orbifold-based approaches and so on. In this essay, we will focus on some computational properties of neo-Riemannian music analysis, particularly once it is generalized by means of powerful computer-aided paradigms such as spatial computing and then applied to the repertoire of pop music.<sup>1</sup> This enables the extension of the classical *Tonnetz* structure to a family of generalized *Tonnetze* that can be selected according to the compactness property of a given musical grammar with respect to a possible geometrical space in which a musical piece will be represented. The underlying hypothesis is that every musical style has its own spatial character since it is linked to the more or less compact way in which the musical structure can be embedded in a given geometry.

As a starting point, let us take one of the most frequently quoted examples of a harmonic progression demonstrating the relevance of the traditional *Tonnetz* structure as a music-analytical tool, i.e. the “zigzag” chord sequence of the second movement of Beethoven’s Ninth Symphony. What makes the *Tonnetz* a good candidate for the analysis of this passage is clearly the fact that the trajectory of the harmonic progression within this geometrical space is a regular one. This no longer holds true when we change the intervals associated with the axes generating the space, taking for instance the *Tonnetz* generated by the minor second and the major second, which gives the major third as the diagonal axis. The different shapes afforded by this harmonic progression in the

traditional *Tonnetz*  $T[3,4,5]$  and in the “chromatic” *Tonnetz*  $T[1,2,9]$  are presented in example 1.

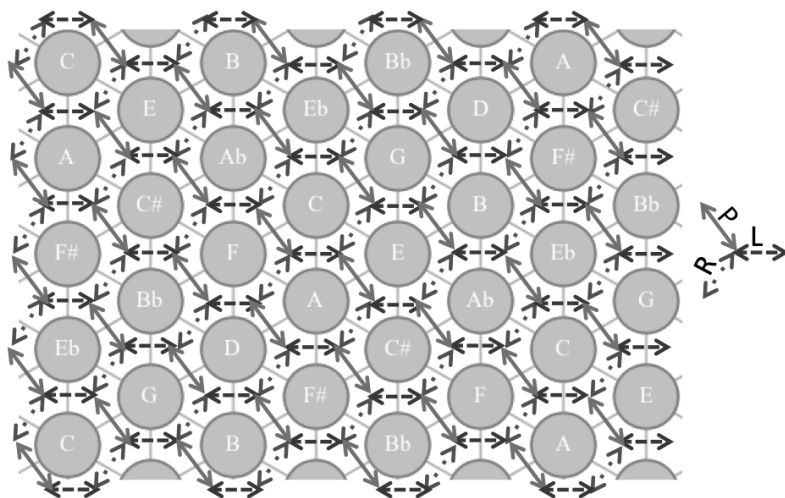
Ex. 1. Two *Tonnetze* representing the harmonic progression of the often-quoted passage of the second movement of Beethoven’s Ninth Symphony. The notation  $T[a,b,c]$  stands for the generic *Tonnetz* generated by two axes having respectively the intervals  $a$  and  $b$  as generators (where the interval corresponding to the diagonal axis is the inverse mod 12 of the sum of the two previous intervals). Note that this is equivalent to the notation  $T[a,b,c']$ , where  $c'$  is the sum of  $a$  and  $b$ , since the diagonal axis has two orientations, one of which corresponds to the interval  $c$  and the other, opposite one to the interval  $c'$ .



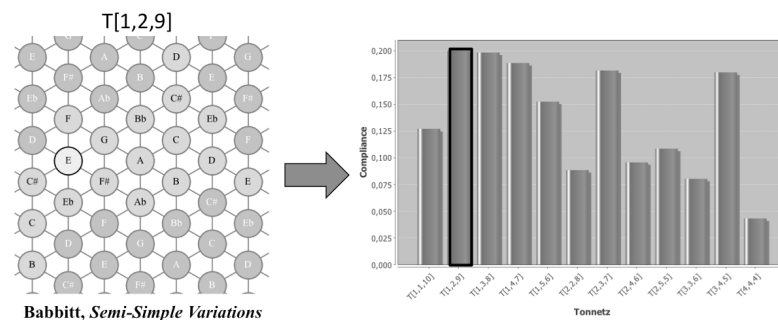
By mathematically defining the “compliance” of a space with respect to some given musical material as the maximization of connected trajectories corresponding to a harmonic progression,<sup>2</sup> it is easy to show that  $T[3,4,5]$  is the space which best fits the musical logic of Beethoven’s chord sequence. This is not surprising, considering that the chord progression results from the alternation between the R (as Relative) and L (as Leading-Tone) operators—which also explains the “zigzag” shape of the musical trajectory (see example 2).

By leaving the tonal realm of Beethoven’s Symphony and by analysing musical pieces such as Milton Babbitt’s *Semi-Simple Variations* that explore the atonal universe, one clearly sees the inadequacy of the traditional *Tonnetz* and the relevance of other potential geometric spaces, first of all what we call the “small-step” *Tonnetz*  $T[1,2,9]$ , which provides maximal compliance with respect to all the possible geometric spaces (see example 3).

Ex. 2. The complete structure of the traditional *Tonnetz*  $T[3,4,5]$  with the three elementary neo-Riemannian transformations P, L, and R.



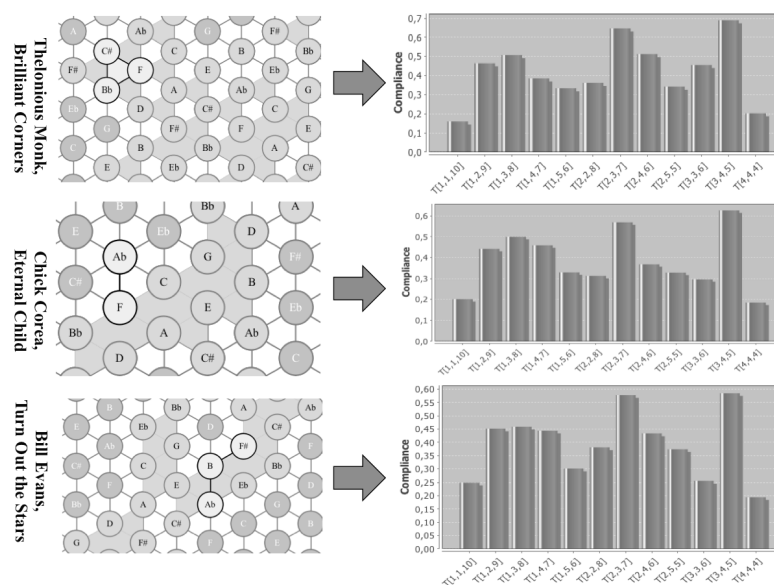
Ex. 3. The *Tonnetz* distributions within Milton Babbitt's *Semi-Simple Variations*, showing the relevance of the “chromatic” *Tonnetz*  $T[1,2,9]$  for capturing the logical structure of the piece (always according to a criterion of compactness of trajectory).



The distribution of the compliance measures of the twelve possible *Tonnetze* also shows that there are many geometric spaces that provide a good description of the inner logics of a musical piece in accordance with the compliance criterion. This can be interpreted as a sign of the “complexity” of the piece, which can “live” equally (or almost equally) well in conceptually different geometric spaces. We can observe the same multiplicity of *Tonnetze* associated with three jazz standards, although the high value obtained

for the traditional  $T[3,4,5]$  and the “pentatonic”  $T[2,3,7]$  *Tonnetze* suggests that, in the case of Thelonious Monk’s *Brilliant Corners* as well as of Chick Corea’s *Eternal Child* and Bill Evans’s *Turn Out the Stars*, the underlying musical logic is deeply rooted in the tonal idiom although it also extends towards more modal regions, particularly pentatonicism (example 4).

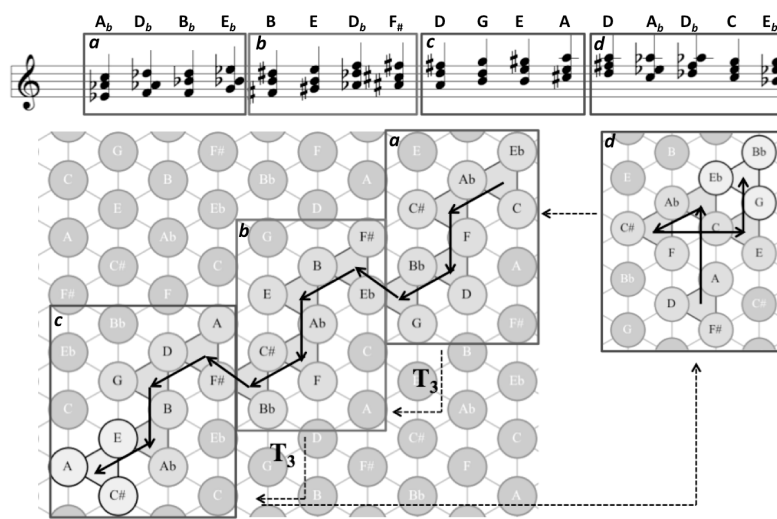
Ex. 4. The *Tonnetz* statistics for three jazz standards.



The previous examples show the flexibility of a spatial computing model together with a compliance measure of the trajectories associated with a given harmonic structure, with respect to the analysis of style. In the special case of the analysis of pop music, there is rarely any ambiguity about which geometric space best fits a musical passage. Nevertheless, there are interesting questions that arise: firstly, from representing the structure and the evolution of a musical piece within a given geometric space and, secondly, from transforming either the trajectory of the piece or the underlying musical space. We will discuss these geometric properties by taking as a case study a song by the Italian “chansonnier” Paolo Conte, *Madeleine*, which is almost entirely based on a highly symmetric harmonic progression. This progression (see example 5) can be decomposed in four blocks, the first three of which are related by minor-third transposition. The fourth block shows a

different trajectory on the *Tonnetz*; its function is basically that of “closing the loop” by forcing the musical piece to return to the initial chord (and hence to the initial key).<sup>3</sup>

Ex. 5. The overall structure of the main harmonic progression of the song *Madeleine* by Paolo Conte. The letters *a* to *d* indicate the four blocks of the progression, the first three of which are related by minor third transposition.

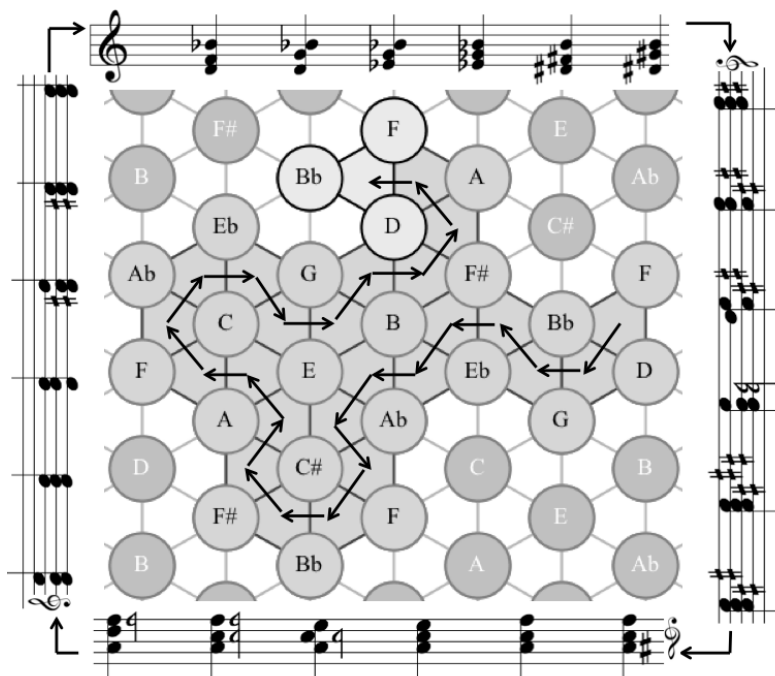


As observed and discussed by one of the present authors in an introductory article on formal methods for the study of popular music,<sup>4</sup> the passage is also intriguing from a theoretical point of view since it almost realizes a *covering* of a proper subspace of the traditional *Tonnetz*, which is only constituted by the twelve major chords. There is a kind of “maximal variety principle” acting in this piece, together with a clear directionality in the chord progression, which draws the listener into a permanent “translational” state. The same holds for the piece *Easy Meat* by Frank Zappa, in which the harmonic sequence can also be decomposed in elementary blocks related by descending minor-third transpositions.<sup>5</sup> The main structure of the chord progression, with all its inner symmetries, is given in example 6.

Both musical examples resonate with those contemporary compositional techniques that avoid repetition and maximize the variety of the musical material such as are found in the integral serialism



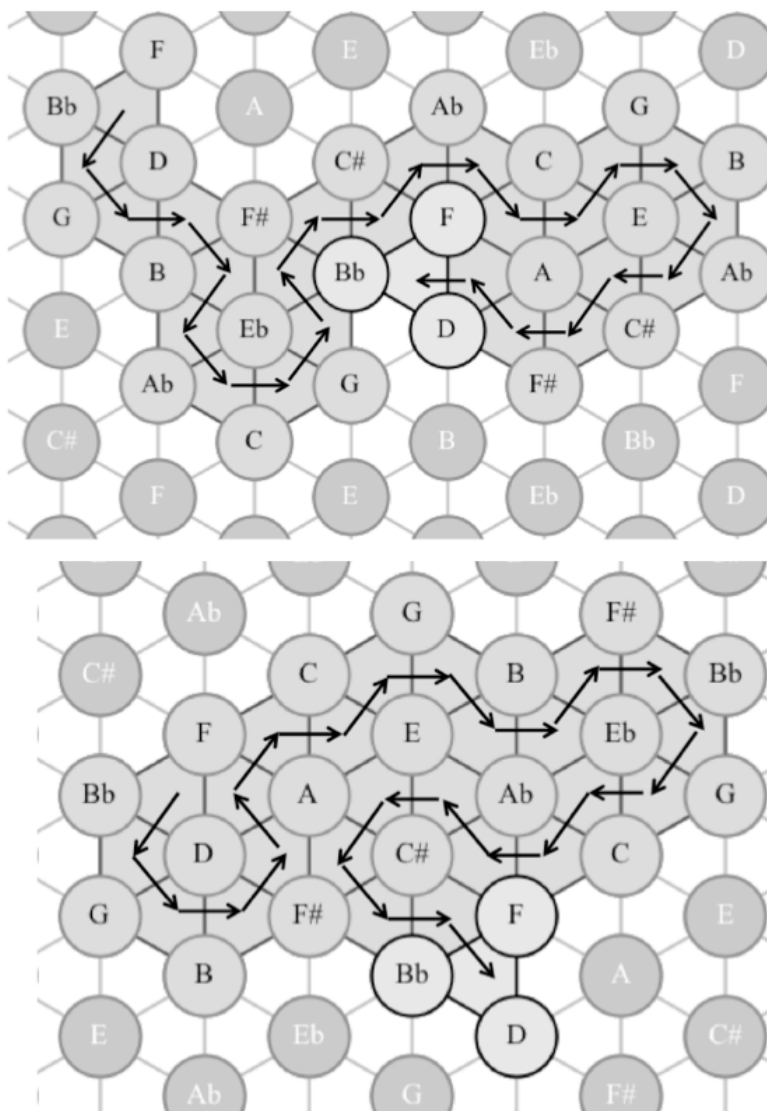
Ex. 7. A non-redundant Hamiltonian cycle which is used to structure the harmonic material of the song *Aprile*, by Moreno Andreatta (lyrics by Gabriele D'Annunzio).



This is clearly not the only possible non-redundant trajectory in this *Tonnetz* to pass without repetition through all major and minor chords. Example 8 shows the other two Hamiltonian cycles that can be found in the same song. We leave to the interested reader the task of checking that these two harmonic progressions are also indecomposable into sub-patterns and that they are structurally different from each other and from the progression shown in example 8 (meaning that they are associated with essentially different words of length 24 in the P, R and L neo-Riemannian alphabet).<sup>9</sup>

We conclude this short investigation of the use of the *Tonnetz* model and of its possible generalization to the analysis of popular music by presenting some transformational techniques that are now available to the analyst (and potentially to the composer interested in the exploration of the creative ramifications of these

Ex. 8. The two remaining non-redundant Hamiltonian cycles used in the instrumental part of the song *Aprile*.



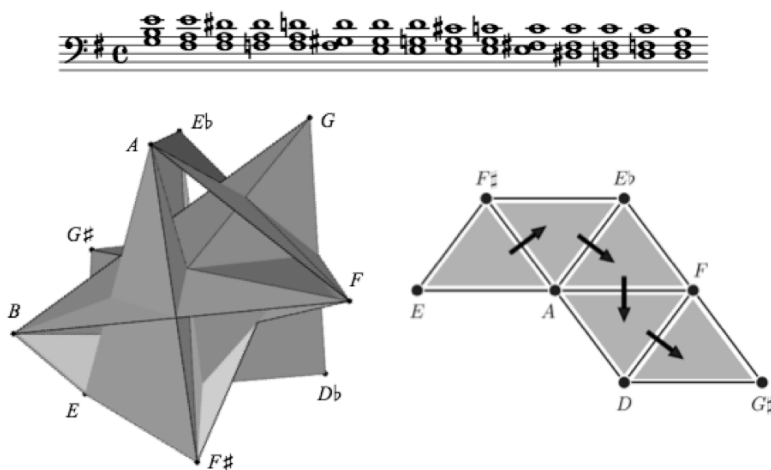
analytical tools). We are far from understanding all the potential musical meanings of these transformations, of which only a small part is well defined in the music-theoretical literature. As we have seen, our analytical model rests on the hypothesis that musical style is endowed with a spatial character. In other words, in order



to understand the “logic” of a musical piece, we must first understand how this piece moves within a given geometry. This is not entirely different from what other music-theorists and analysts have suggested when highlighting the relevance of generalized manifolds such as orbifolds with regard to the voice-leading problem.<sup>10</sup> The main difference concerns the fact that, for a given musical piece, we can easily switch between “intrinsic” and “extrinsic” notions of space. The twelve possible *Tonnetze* define an “extrinsic” universe of possible spatial interpretations of a piece that “intrinsically” has its own natural space. This “intrinsic” space is given by the simplicial complex generated by the piece itself—a topological structure that is essentially multi-dimensional and which we can only visualize by projecting it in low dimensions.

As an example, let us consider Chopin’s Prelude, Op. 28, No. 4 whose harmonic progression has been largely discussed in the context of orbifold-based voice-leading theory.<sup>11</sup> In the same way that all the possible *Tonnetze* are obtained by spatially pasting together generic trichords—such as major and minor chords in the special case of  $T[3,4,5]$ —it is possible to “naturally” associate with a specific harmonic collection the simplicial complex generated by that collection. In the case of Chopin’s piece, we obtain a topological structure that corresponds to the “intrinsic” space generated by the chord progression (example 9).

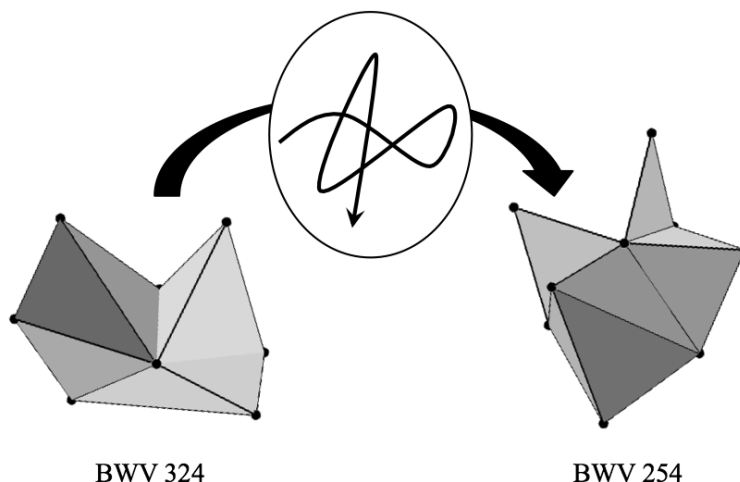
Ex. 9. The multi-dimensional simplicial complex associated with the harmonic chord progression of Chopin’s Prelude, Op. 28, No. 4.



Note that this simplicial complex is an abstract structure that neglects the temporal character of the harmonic progression. The temporal dimension of the analysis is recovered by considering all possible trajectories within a given simplicial complex. It can be demonstrated that there are 120 potential Hamiltonian paths for this particular topological structure and, astonishingly, Chopin made use of the only Hamiltonian path that exhibits the strong property of involving the shortest motion of the remaining tone in the transition between two consecutive trichords that retain two common tones.

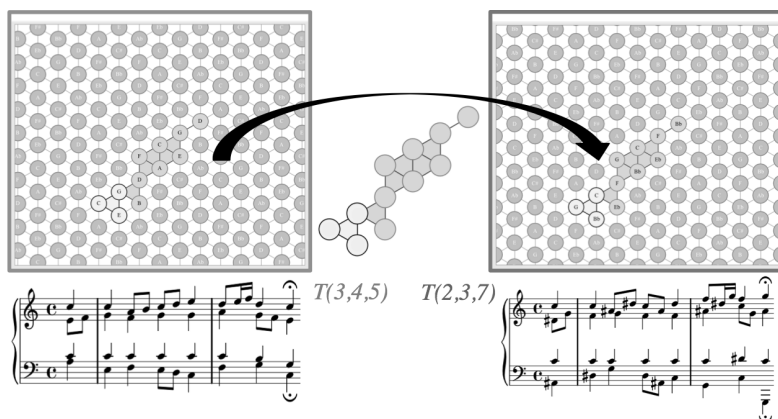
Starting with these simple topological constructions associated with a given harmonic progression, one can try to understand what it means to work at the simplicial level. Example 10 displays simplicial complexes representing the chorales BWV 324 and BWV 254 by J.S. Bach. The first structure has the strong property of being linked to the second by an *injective morphism*. Intuitively, this can be understood as the possibility of mapping the trajectory of BWV 324 into the structure representing BWV 254. This rare particularity enables the generation of a new musical sequence that results from embedding the progression of the first chorale in the “intrinsic” space of the second one.

Ex. 10. Two Chorales by J.S. Bach and their simplicial representations. The left simplex is structurally included in the right one.



Morphisms between simplicial complexes constitute powerful tools for capturing style intersections within a set of musical pieces. The *Tonnetz* can be used to achieve similar transformations on trajectories owing to the generic topology shared by the different *Tonnetze* used in this study, which all appear as triangular tessellations. Thus, two *Tonnetze* are related by several isomorphisms and a trajectory drawn in one space can easily be embedded in the second one. Example 11 shows on the left-hand side the first measures of J.S. Bach's Choral BWV 256. The sequence is represented as a trajectory in the *Tonnetz*  $T[3,4,5]$ . On the right-hand side, the trajectory is embedded in the *Tonnetz*  $T[2,3,7]$ . This latter space has the property of being generated by the predominant intervals of the pentatonic scale (major second, minor third and fifth), which is typically associated with traditional Asian music. On the bottom right of the example, the same measures are reproduced after the pitches have been updated according to the new underlying space. The resulting sequence has unquestionably undergone a switch to a different style, which may be perceived as close to Asian music.

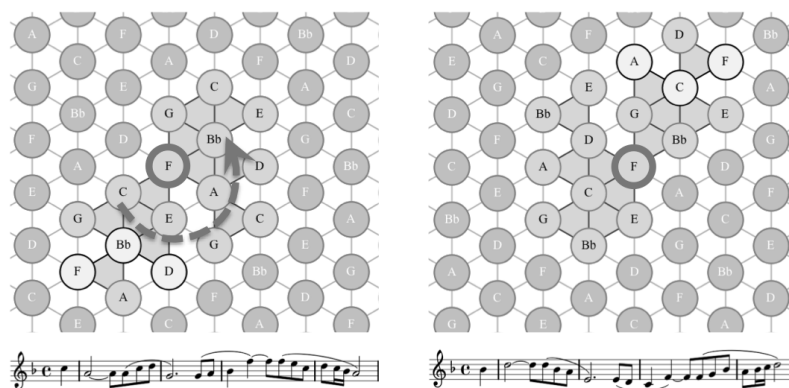
Ex. 11. On the left-hand side, the trajectory representing the first measures of Bach's Chorale BWV 256 in  $T[3,4,5]$ . On the right-hand side, the trajectory has been embedded in  $T[2,3,7]$  and the notes of the score have been updated.



Furthermore, the notion of a matching relation between a trajectory and a space reaches to other transformations of musical pieces. In particular, a large variety of geometric operations on a trajectory drawn within a *Tonnetz* are possible as a result of the regularity of this structure. For example, the trajectory illustrated

in example 1 could be translated along one of the axes, thus generating a new musical sequence. In the case of a translation, it is easy to see that the musical result will amount to an exact transposition of the original sequence. However, this musical interpretation is less obvious for alternative operations such as rotations or homothetic transformations. Example 12 illustrates the case of two trajectories related by a  $180^\circ$  rotation. The left side of the example represents the beginning of the song *Hey Jude* by The Beatles in a *Tonnetz* that includes only the pitch-classes of the F-major tonality. The three axes are respectively associated with the diatonic intervals of a second, a third and a fifth. The transformation produces a new piece, also in the key of F major. The two score excerpts display the first measures of the melody before and after the transformation.

Ex. 12. On the left-hand side, a trajectory represents the first measures of the song *Hey Jude* by The Beatles in a diatonic *Tonnetz*. On the right-hand side, a rotation is applied to the trajectory. The two scores correspond to the beginning of the melody before and after the transformation.



Because of its  $180^\circ$  angle, this rotation is musically equivalent to a diatonic inversion. However, other transformations such as a  $60^\circ$  rotation or a  $120^\circ$  rotation would raise additional questions with regard to their musical implications and necessitate further music-theoretical research.

## NOTES

- <sup>1</sup> See Louis Bigo, Moreno Andreatta, Jean-Louis Giavitto, Olivier Michel and Antoine Spicher, “Computation and Visualization of Musical Structures in Chord-based Simplicial Complexes,” in J. Yust, J. Wild and J.A. Burgoyne (eds.), *Mathematics and Computation in Music: Proceedings of the 4th International Conference, MCM 2013* (Berlin: Springer, 2013), pp. 38–51.
- <sup>2</sup> See Bigo et al., “Computation and Visualization of Musical Structures,” for the precise definition of the compliance measure of a piece with respect to a given geometric space.
- <sup>3</sup> For a multi-dimensional visualization of this chord progression, from the standard *Tonnetz* to the “Any Set Hypersphere Model” of Gilles Baroin, see the video posted on the MatheMusic4D YouTube channel: <http://www.youtube.com/user/MatheMusic4D?feature=watch>.
- <sup>4</sup> See Moreno Andreatta, “Modèles formels dans et pour la musique pop, le jazz et la chanson: introduction et perspectives futures,” in Z. Kapoula, L.-J. Lestocart and J.-P. Allouche (eds.), *Esthétique et complexité, II: neurosciences, évolution, épistémologie et philosophie* (Paris: Éditions du CNRS, forthcoming in 2014).
- <sup>5</sup> See Guy Capuzzo, “Neo-Riemannian Theory and the Analysis of Pop-Rock Music,” *Music Theory Spectrum* 26 (2004): 177–99.
- <sup>6</sup> Note that zigzags generated by other pairs of operators (LP or RP) do not generate Hamiltonian paths. RL is the only pair to verify this property.
- <sup>7</sup> See Giovanni Albin and Samuele Antonini, “Hamiltonian Cycles in the Topological Dual of the Tonnetz,” in Elaine Chew, Adrian Childs and Ching-Hua Chuan (eds.), *Mathematics and Computation in Music: Proceedings of the 2nd International Conference, MCM 2009* (Berlin: Springer, 2009), pp. 1–10.
- <sup>8</sup> We leave it as an exercise to the reader to verify that the harmonic progression is non-redundant, meaning that it is not possible to find a combination of the three neo-Riemannian operators P, R and L that provides a sub-pattern generating the entire sequence by simple repetition.
- <sup>9</sup> A computer-aided visualization of this song, together with some other combinatorial and geometrical song-writing experiences directly influenced by the use of generalized *Tonnetze*, are also available from Gilles Baroin’s MatheMusic4D YouTube channel at [http://www.youtube.com/watch?v=oazDu9t\\_DTk](http://www.youtube.com/watch?v=oazDu9t_DTk).
- <sup>10</sup> See Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York, Oxford University Press, 2011).
- <sup>11</sup> See *ibid.*