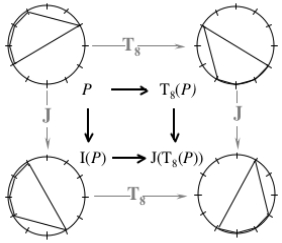
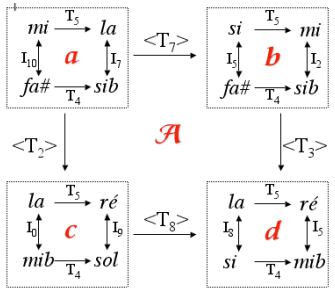
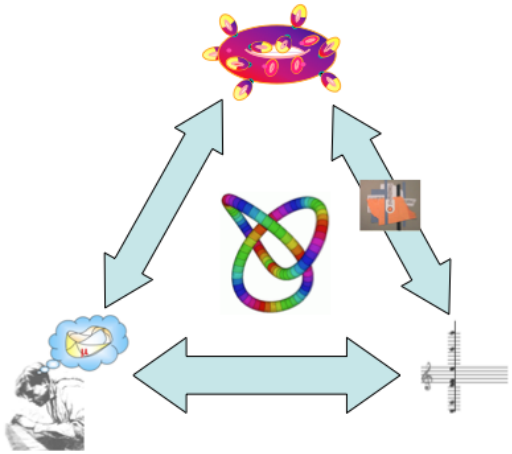
Mental and mathematical representations of music



EMPG2011

ParisTech, August 31, 2011





Moreno ANDREATTA
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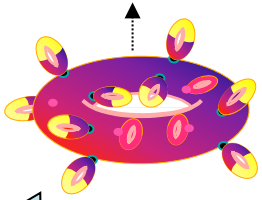
University of Turin

Maths/Music & Cognition / Geometry of Information & Music

- http://recherche.ircam.fr/equipes/repmus/mamux/Cognition.html
- http://recherche.ircam.fr/equipes/repmus/mamux/PEPS-GdIM.html

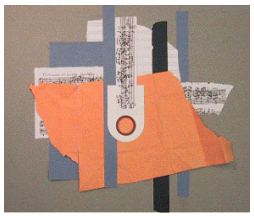
Mathematical representations

Mathematical Music Theory

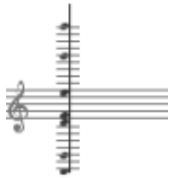


Maths & Cognition

Maths & Music



European Society for the Cognitive Sciences Of Music



Music & Cognition

Musical representations

Mental representations

Cognitive musicology

Outline of the talk

- Research context of our research
- Mental representations
  - In cognitive philosophy
  - In cognitive musicology : GTTM as an example
  - Levels of supervenience of musical structures
- Mathematical representations
  - Geometrical representations of musical structures
  - Transformational music analysis
  - Cognitive aspects of a category-oriented formalization in music

MR in cognitive philosophy

• A controversial and fundamental (foundational) concept for the philosophy of mind and the cognitive science.

Problems: format of MR (symbolic vs connectionist); syntax/semantics; non-conceptual content; disjunction problem, etc.

• In philosophy of mind MR means **mental entities with a content**; in cognitive science MR means **information structure coded in the mind with some role in cognitive tasks** (Paternoster, *Introduzione alla filosofia della mente*, 2002). The two definitions are compatible.

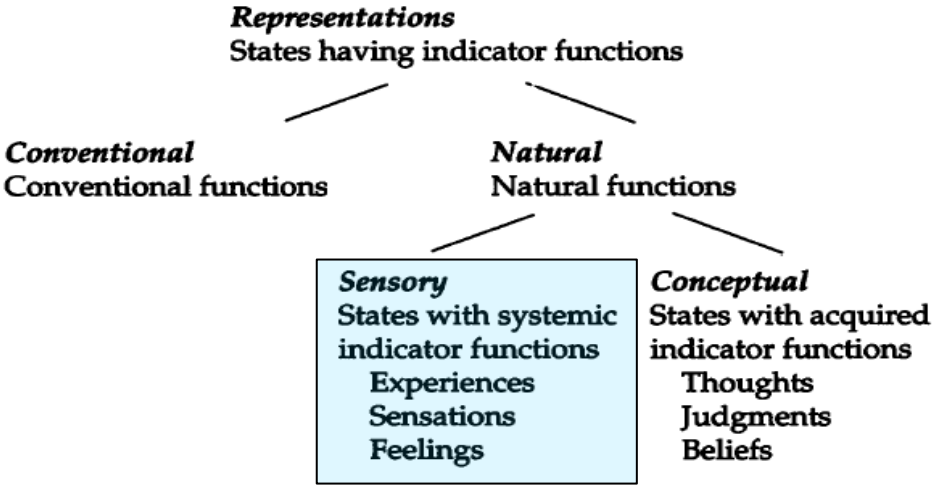
• A further distinction made by Bechtel (*Mental mechanisms*, 2008) is important here: cognitive science [*i.e* also cognitive philosophy] is more attentive to the **content** of representations; neuroscience is more attentive to the **vehicle** of representations.

MR in cognitive musicology

• The use of the concept is quite widespread but ambiguous and vague.

• The mainstream cognitive musicology maintains that mental representations of music are non-conceptual (in Dretske's language, sensory mental states: experiences, sensations, feelings).

• Musical representations do not lead ultimately to the construction of conceptual structures (Jackendoff, *Consciousness and the computational mind*, 1987).



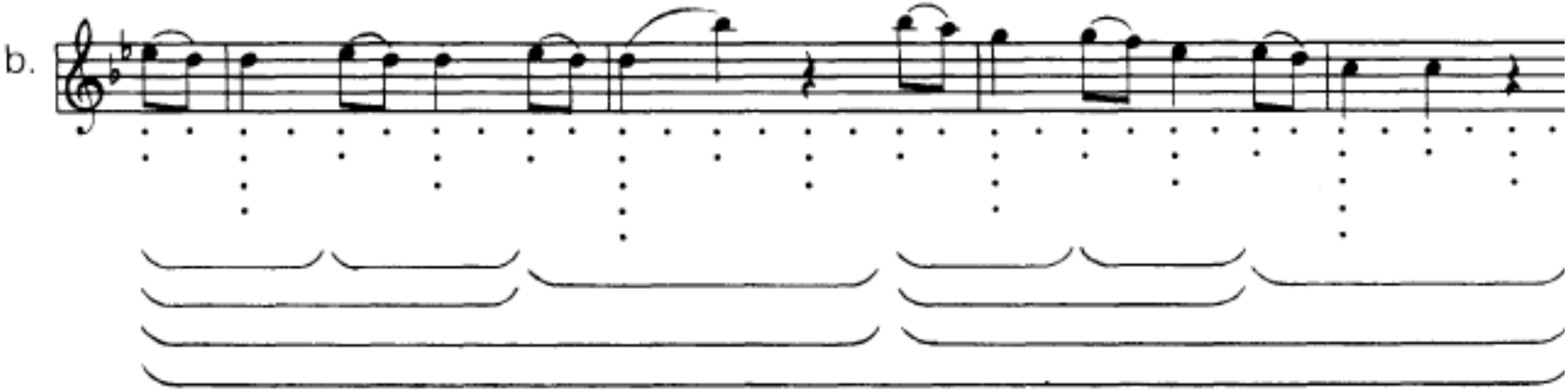
Generative Theory of Tonal Music (GTTM)

• In Lerdahl and Jackendoff's *Generative Theory of Tonal Music* (GTTM) - one of the most popular cognitive theories of music - the term '**representations**' is used in a large sense, with the perfect synonym of '**musical structure**'.

• In many cognitive theories of music, mental representations of music are considered to be (implicitely) construed by the mind according with the perception of musical flow. In GTTM the mental representations of music are considered in the framework of a **final-state theory** so the authors are not committed to explane musical cognition *online*: the symbolic representations of GTTM are alleged to represent the entire **static mental representation of a piece of music** (that's a kind of natural and spontaneous **musical analysis** made by the mind).

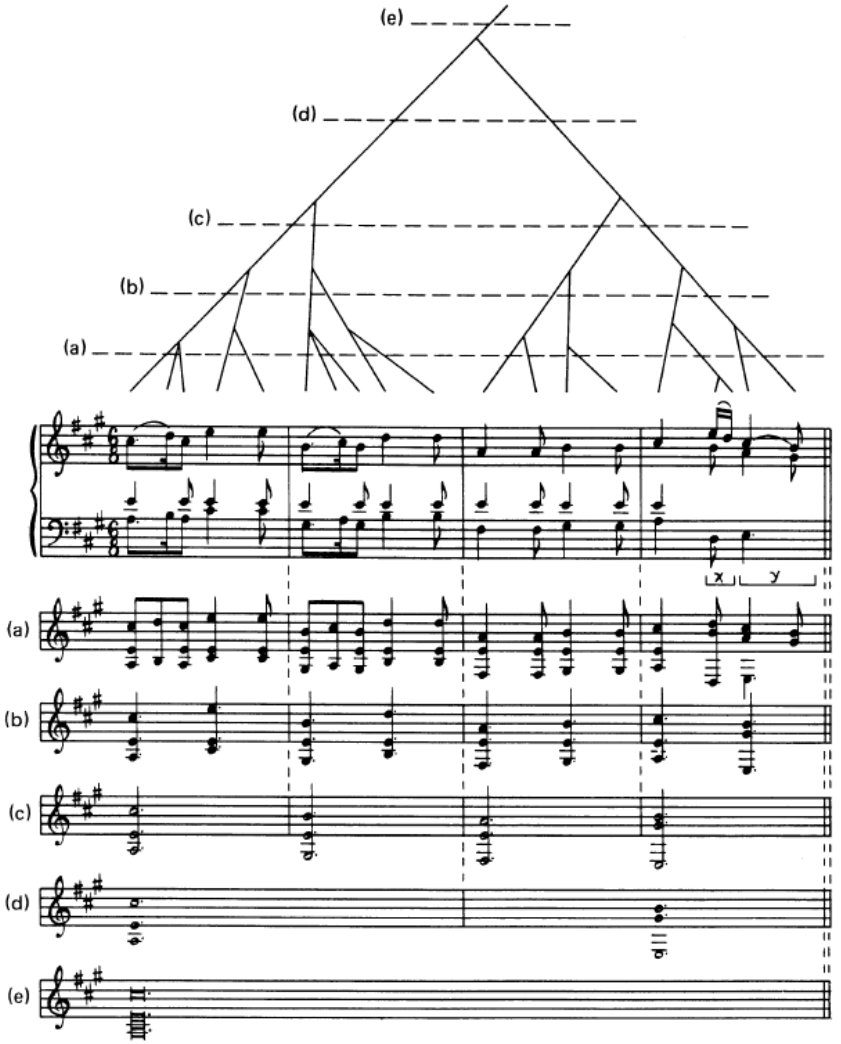
Cognitive structures of music

- Of course, GTTM has to assume that **symbolic representation of music has the same value of the mental representation of music**.



Cognitive structures of music

- The pitch-events of a piece are heard in a **hierarchy of relative importance**.
- Structurally less important events are heard as ornamentations or elaborations of events of greater importance.

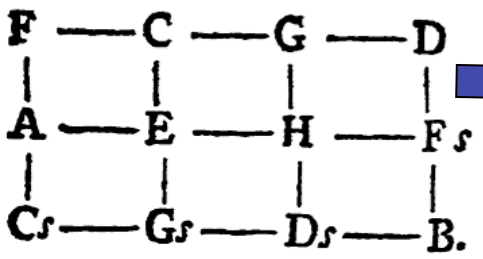


Figure|11.6
Time-span reduction of the opening of Mozart, K. 331

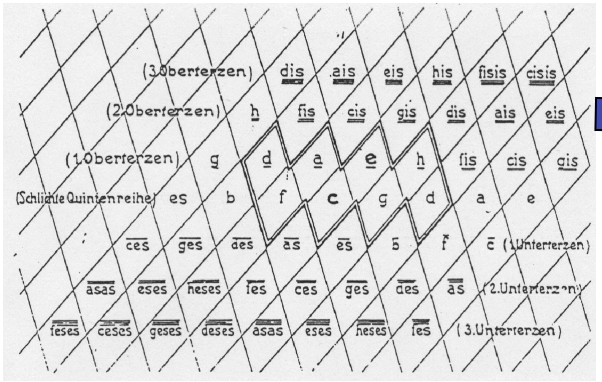
Sound, musical and mathematical objets

- Sound objects are intentional objects (Bullot-Egré 2010), but **sound perception** is not the entire **music perception**
- The status of the "musical object" has to be of higher level than sound object. The "musical object" *supervenes on* (= depends from/covaries with/is irreductible to) the sound object.
- Problem for "mathemusicological" point of view: is the mathematical representation of music of lower/ equal/higher level than the musical object? Does the musical object supervene on a mathematical representation of music?

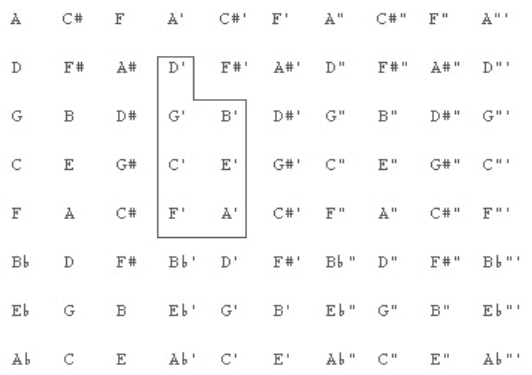
Geometrical representations of musical structures



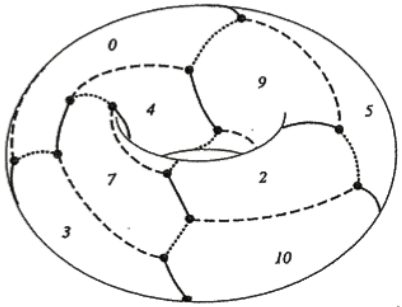
Euler : ***Speculum musicum*****, 1773**



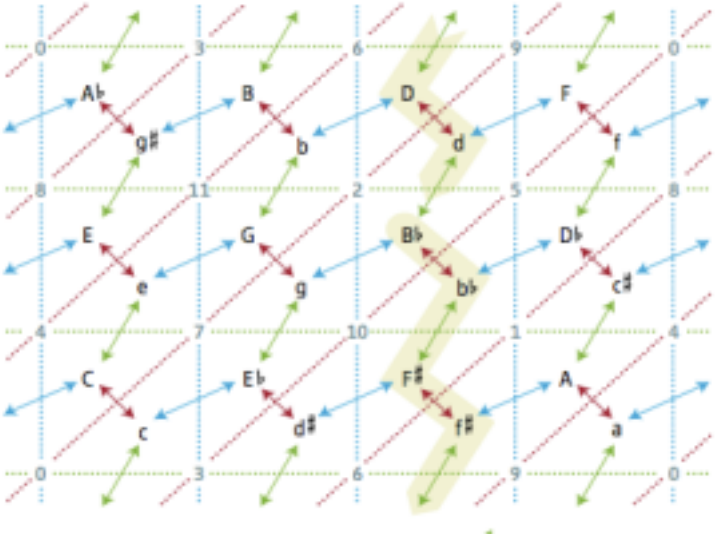
Hugo Riemann : « Ideen zu einer Lehre von den Tonvorstellung », 1914



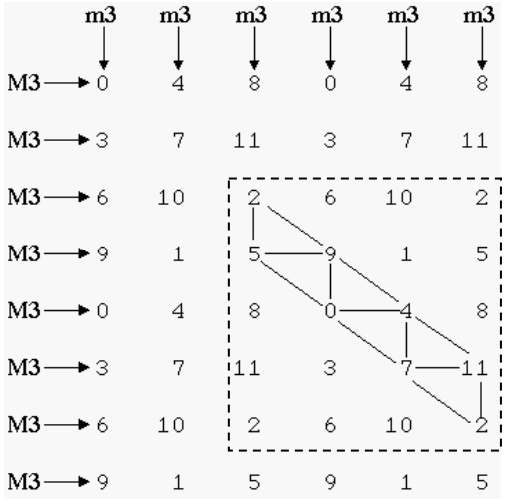
Longuet-Higgins (1962)



Douthett & Steinbach, ***JMT*****, 1998**

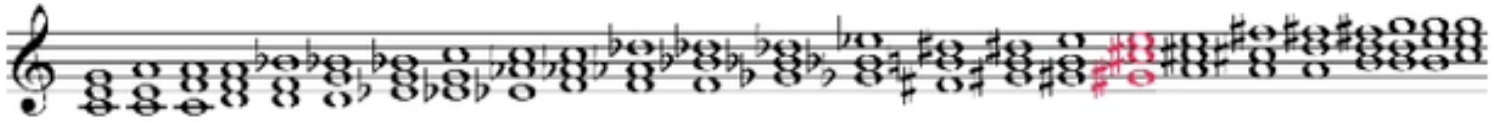


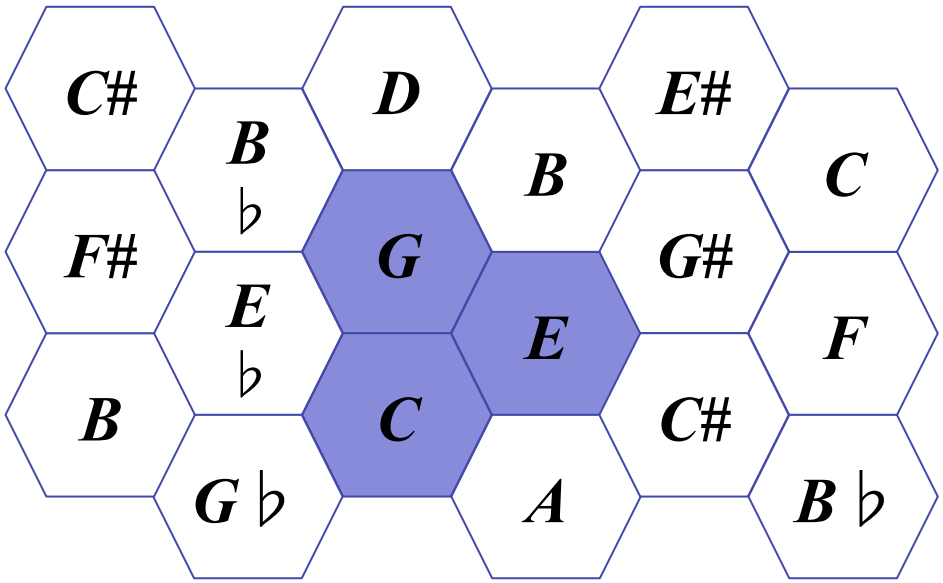
J. Hook, « Exploring Musical Space », ***Science, 2006***



Balzano (1980)

Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

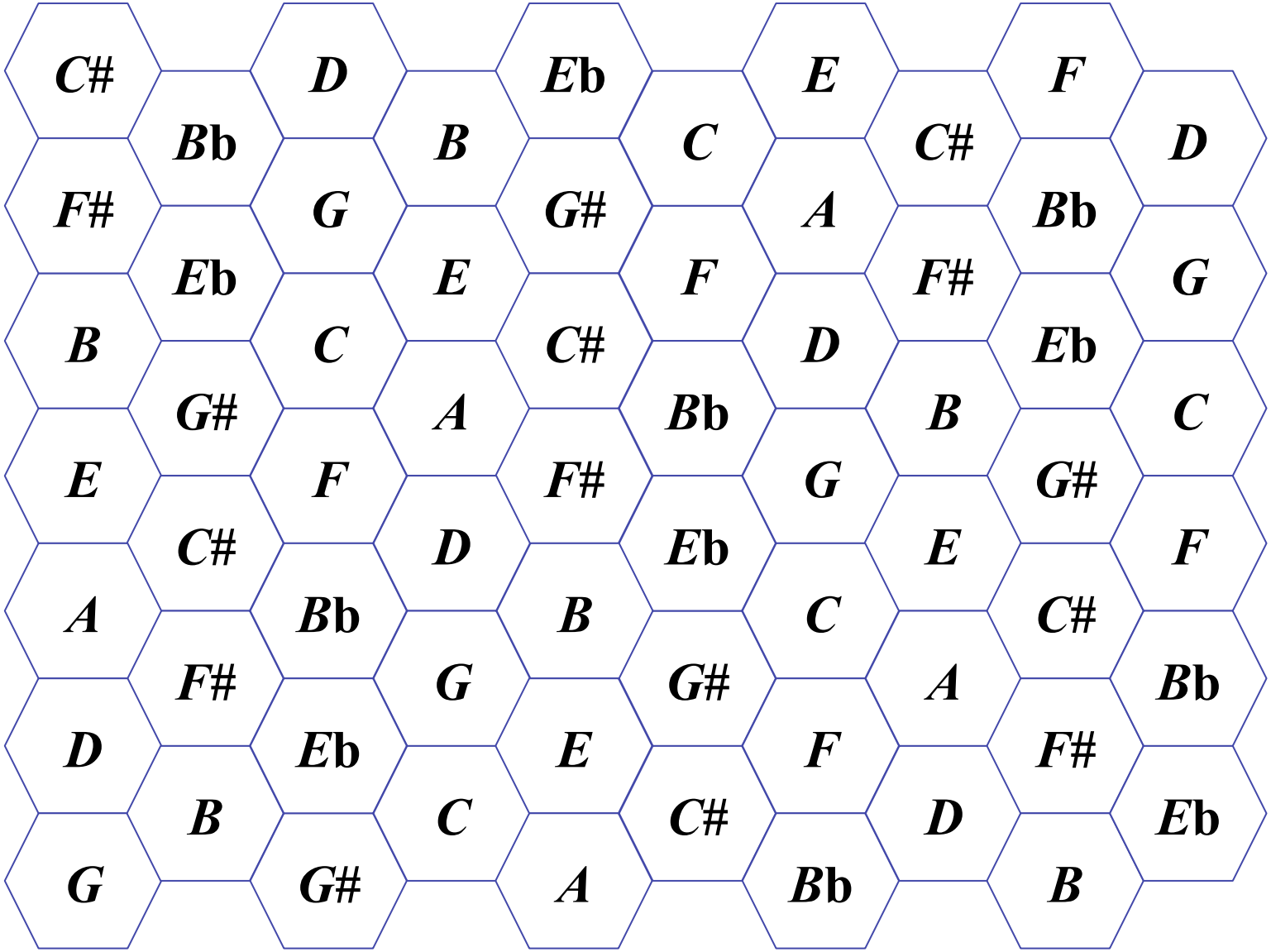




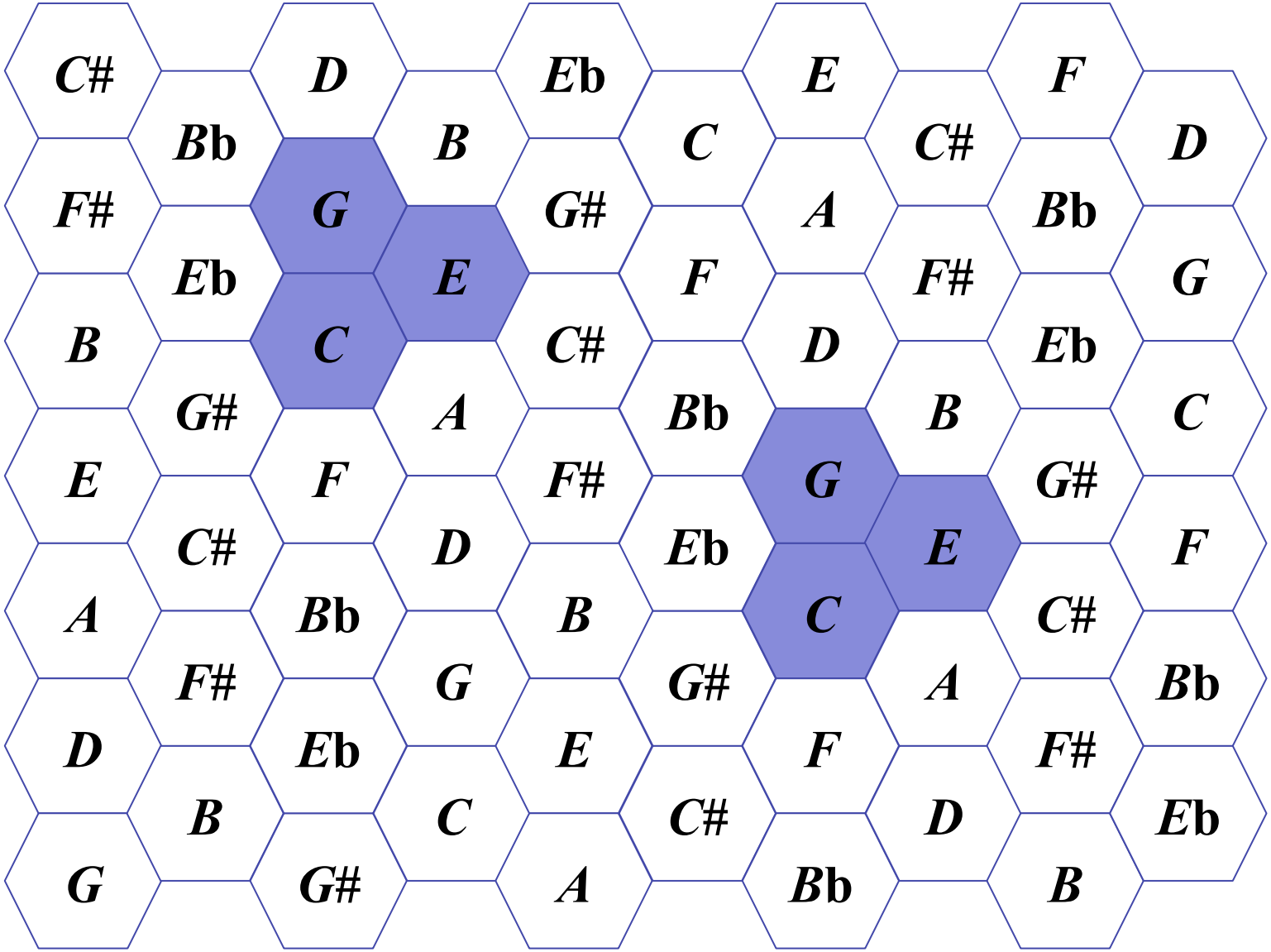
b

Louis Bigo, Ircam / Université Paris 12

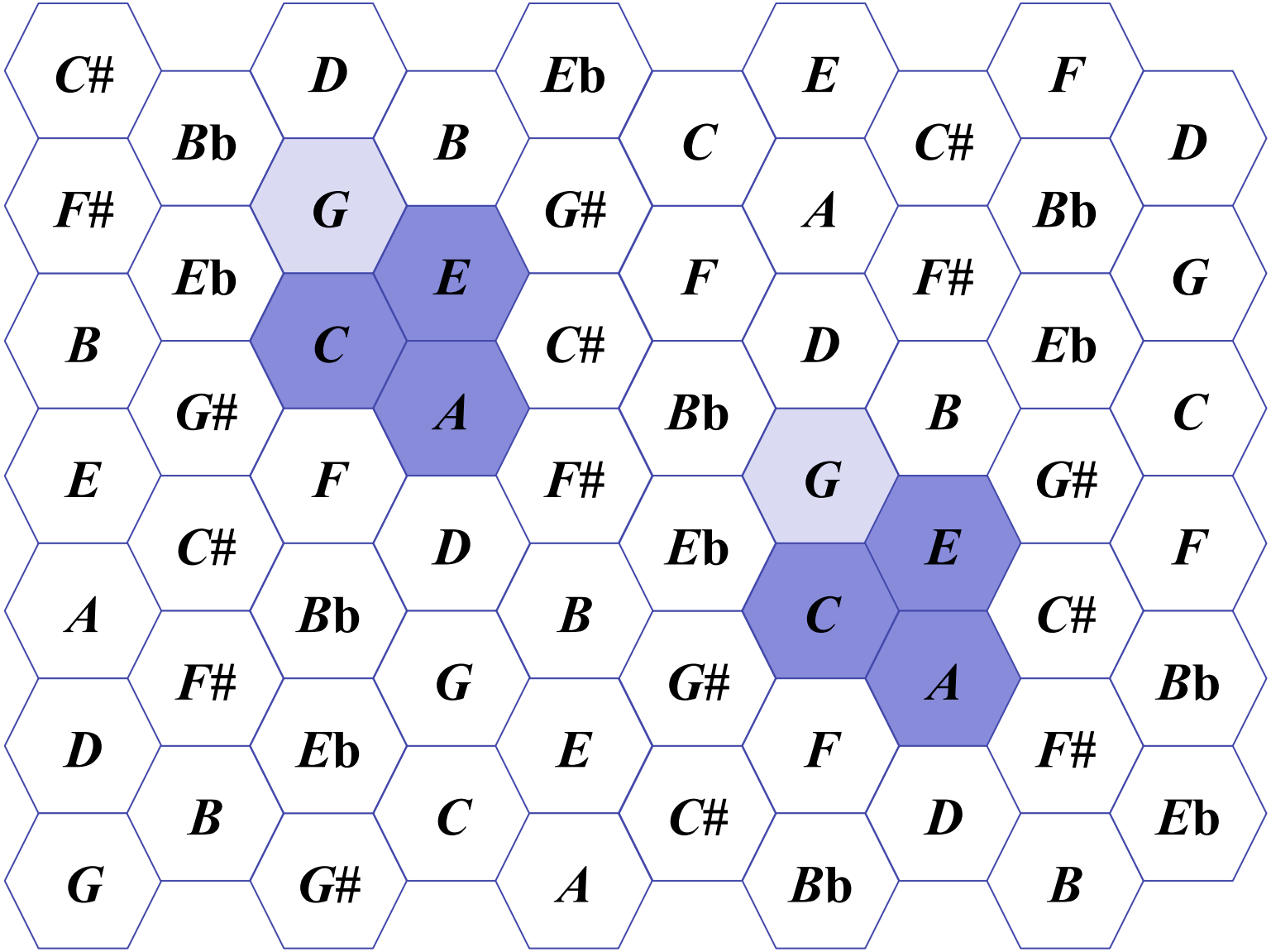
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



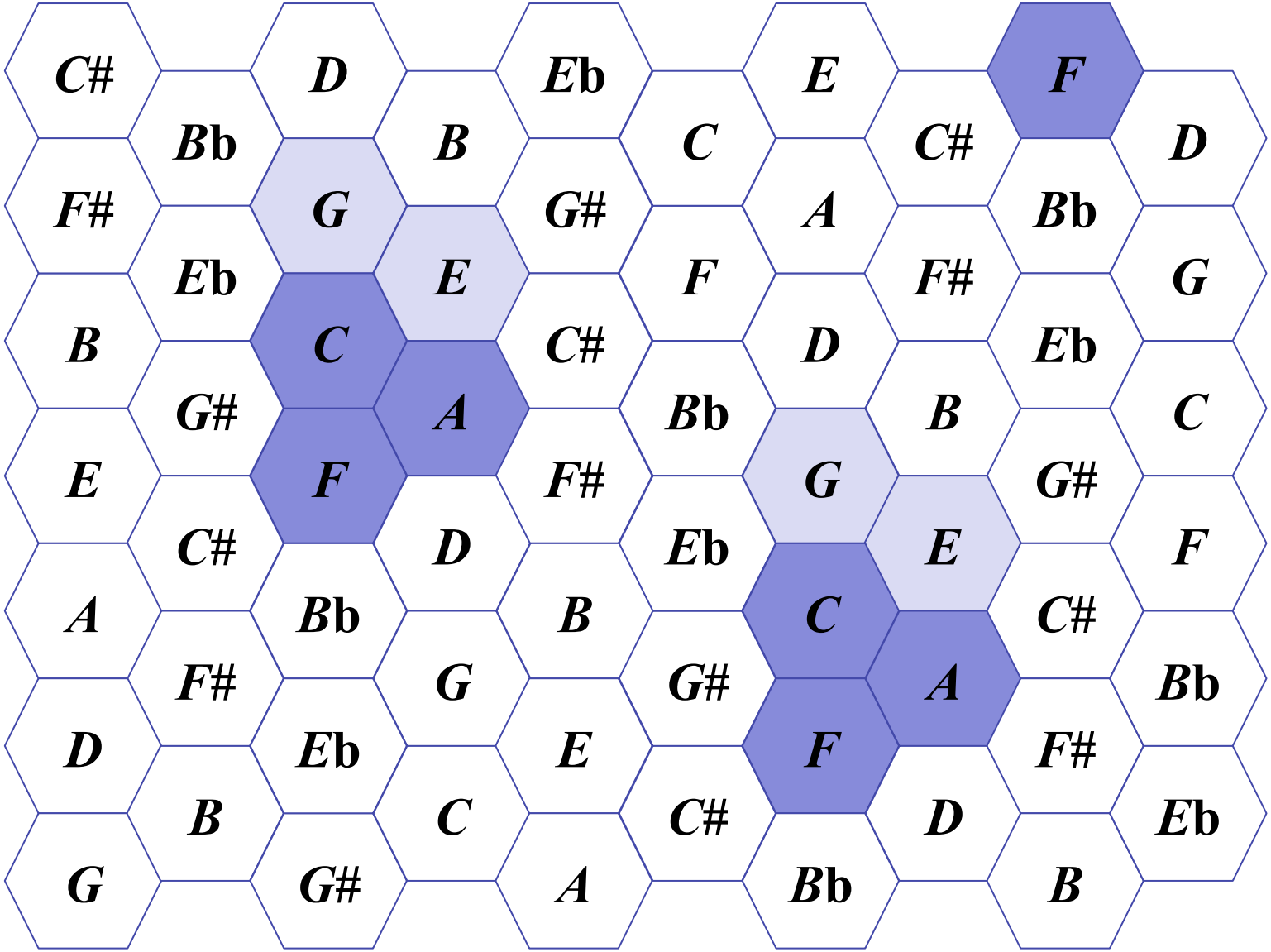
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



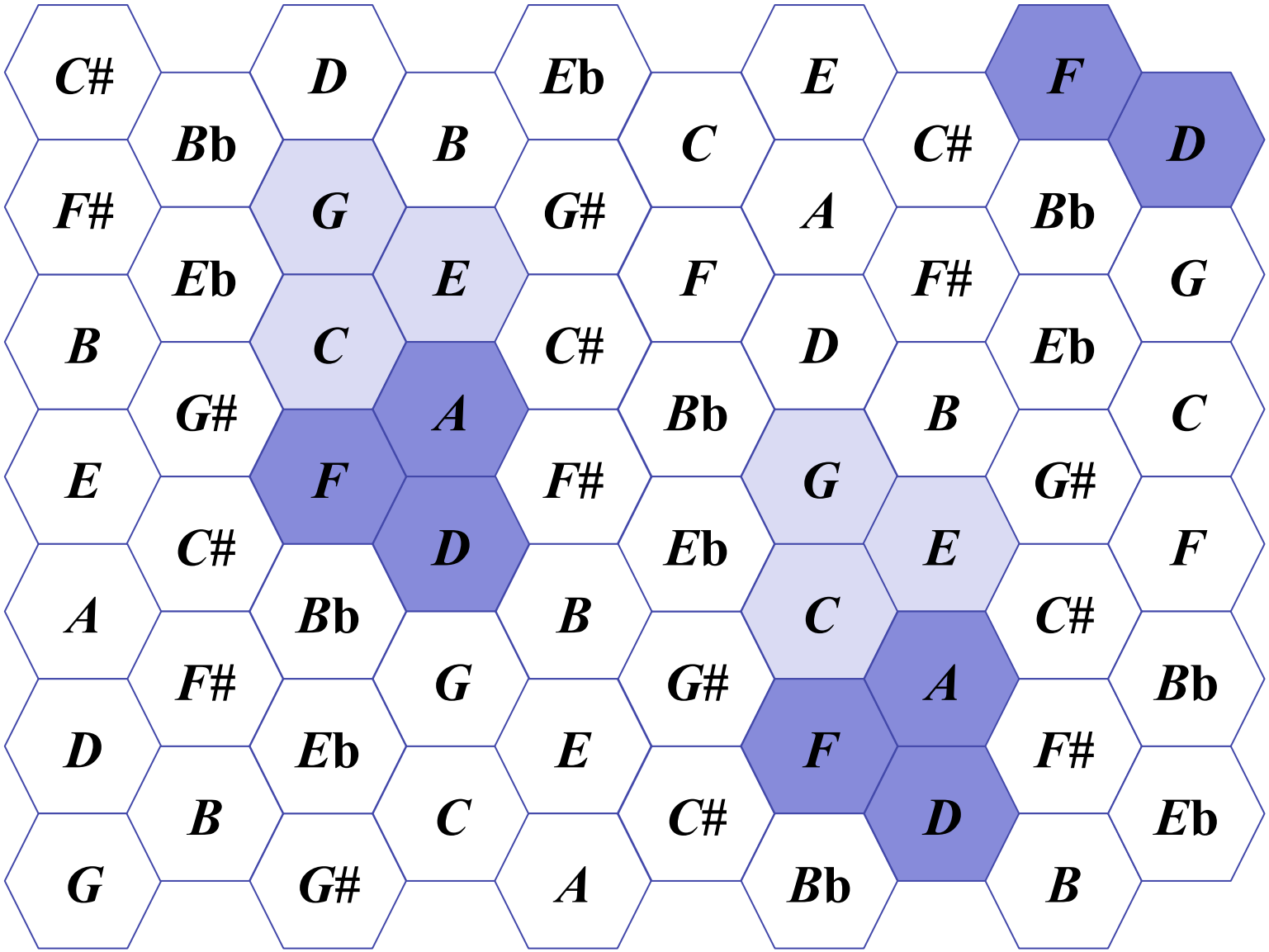
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



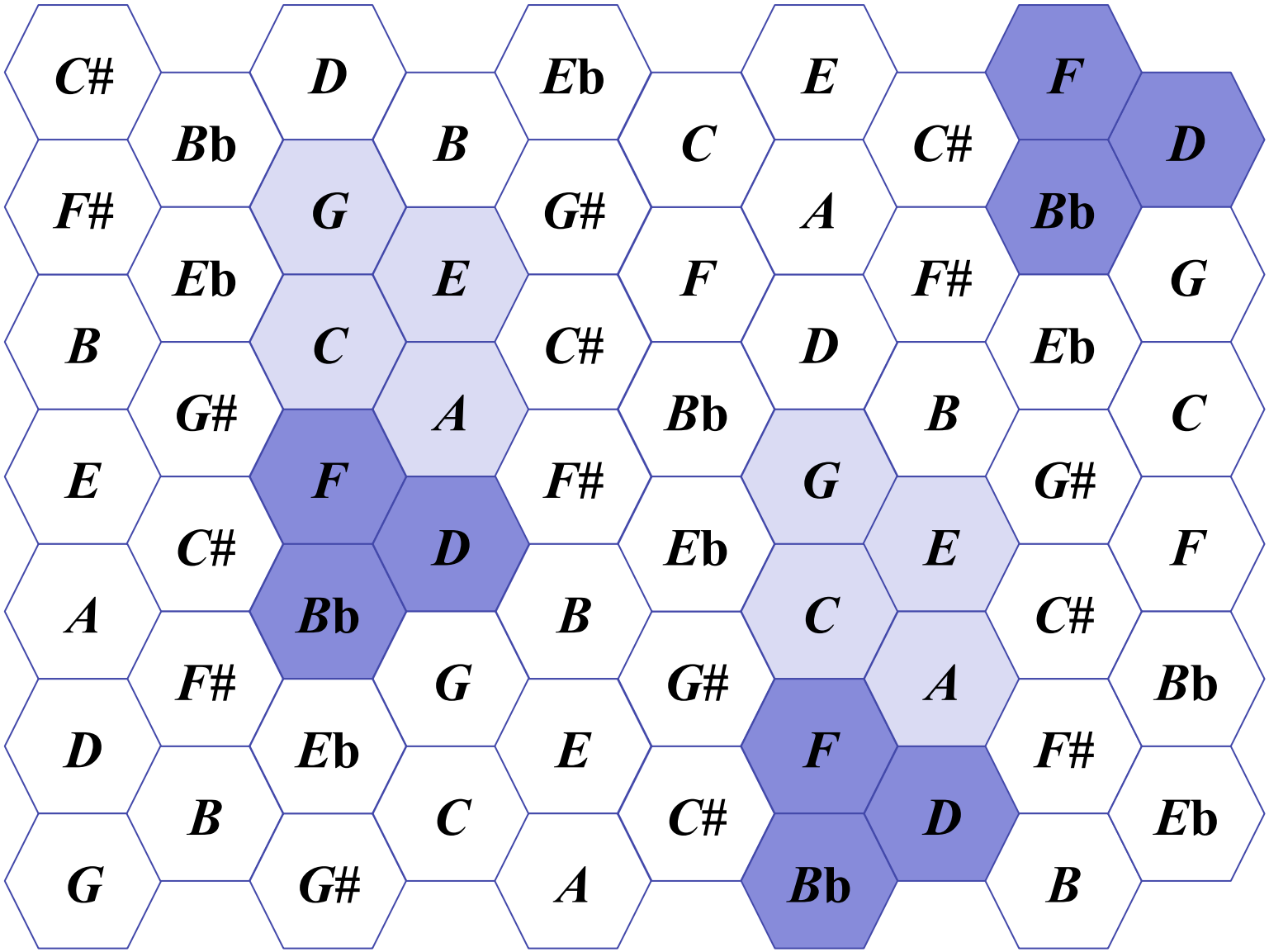
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



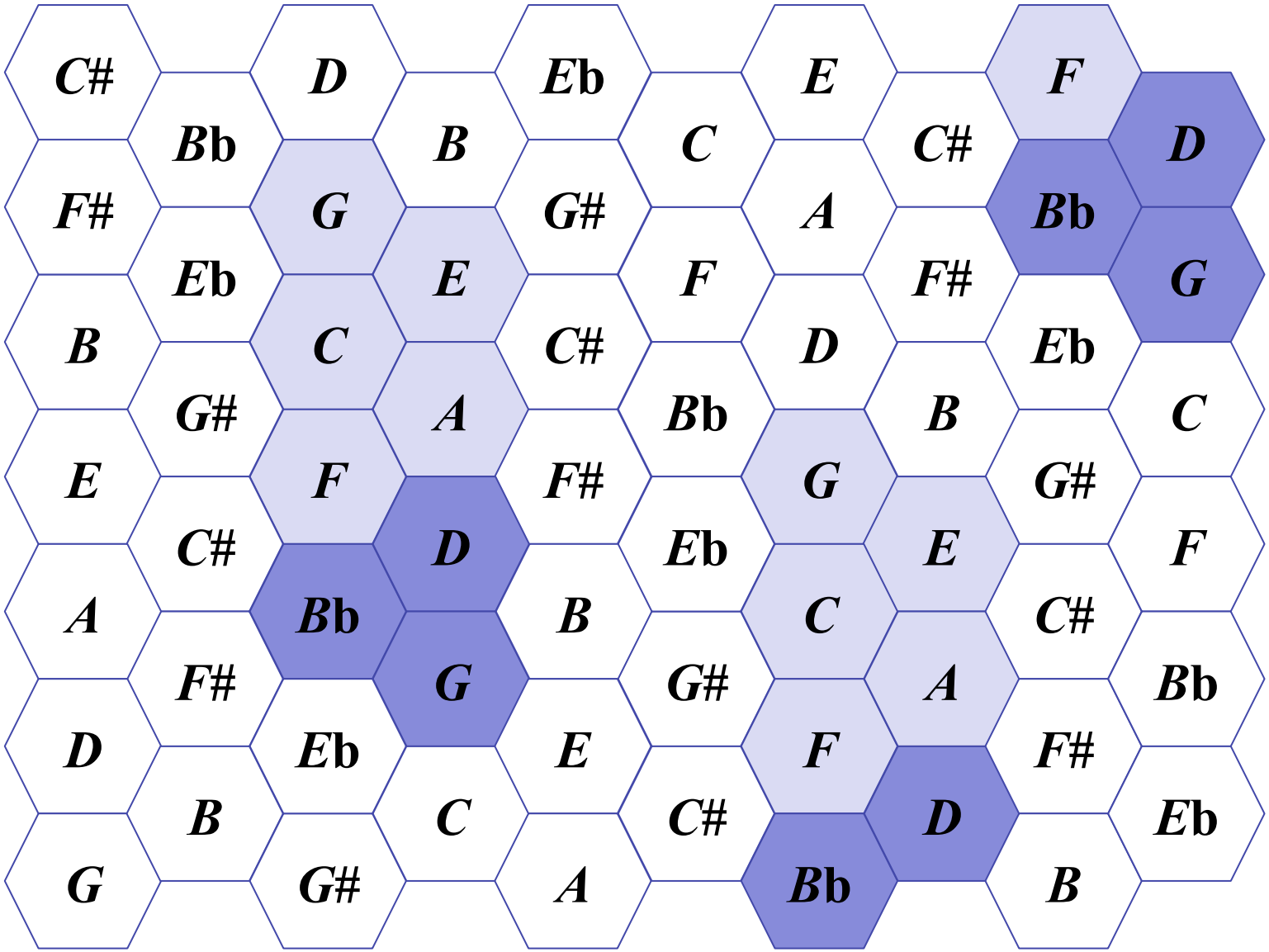
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



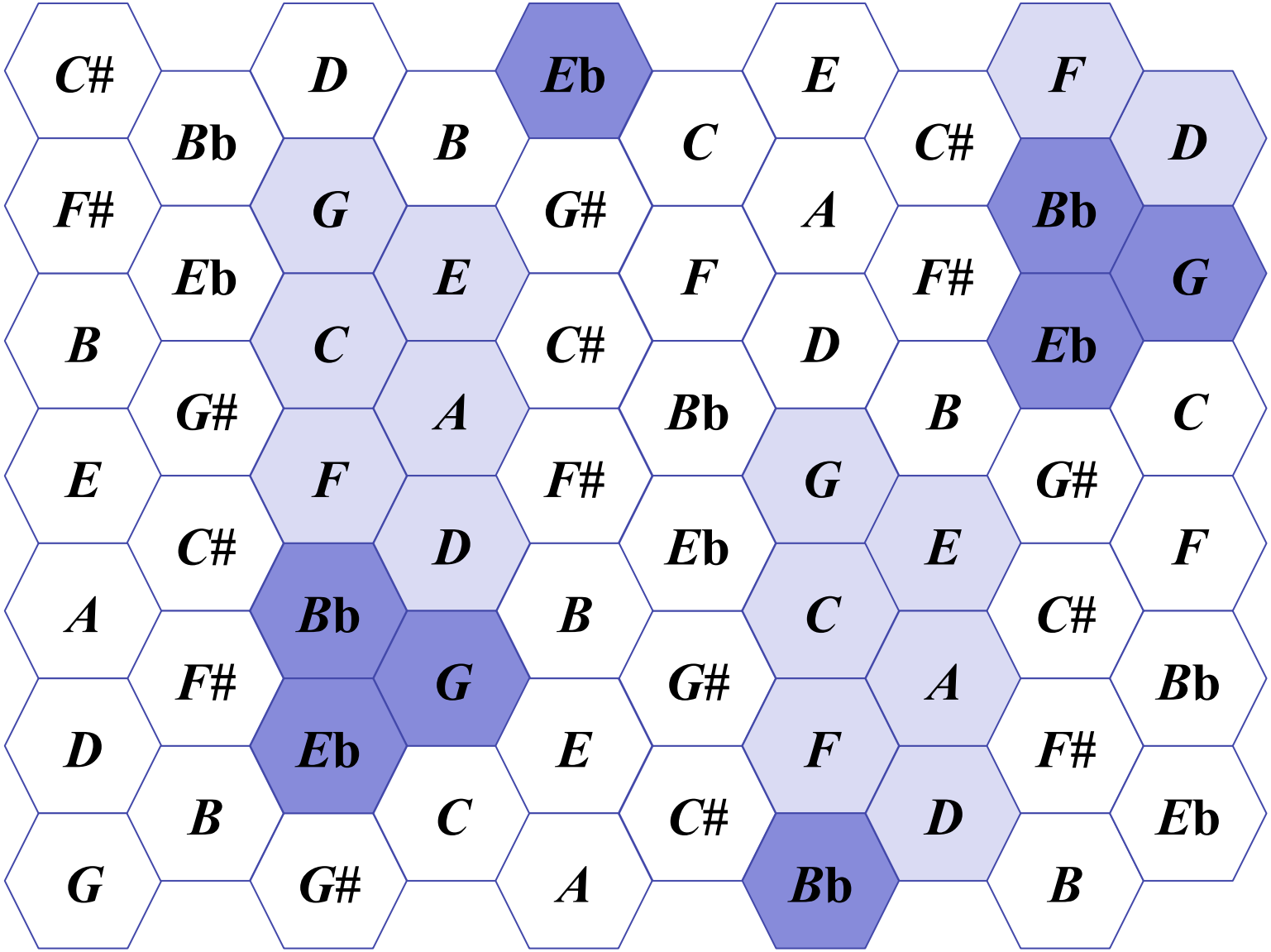
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



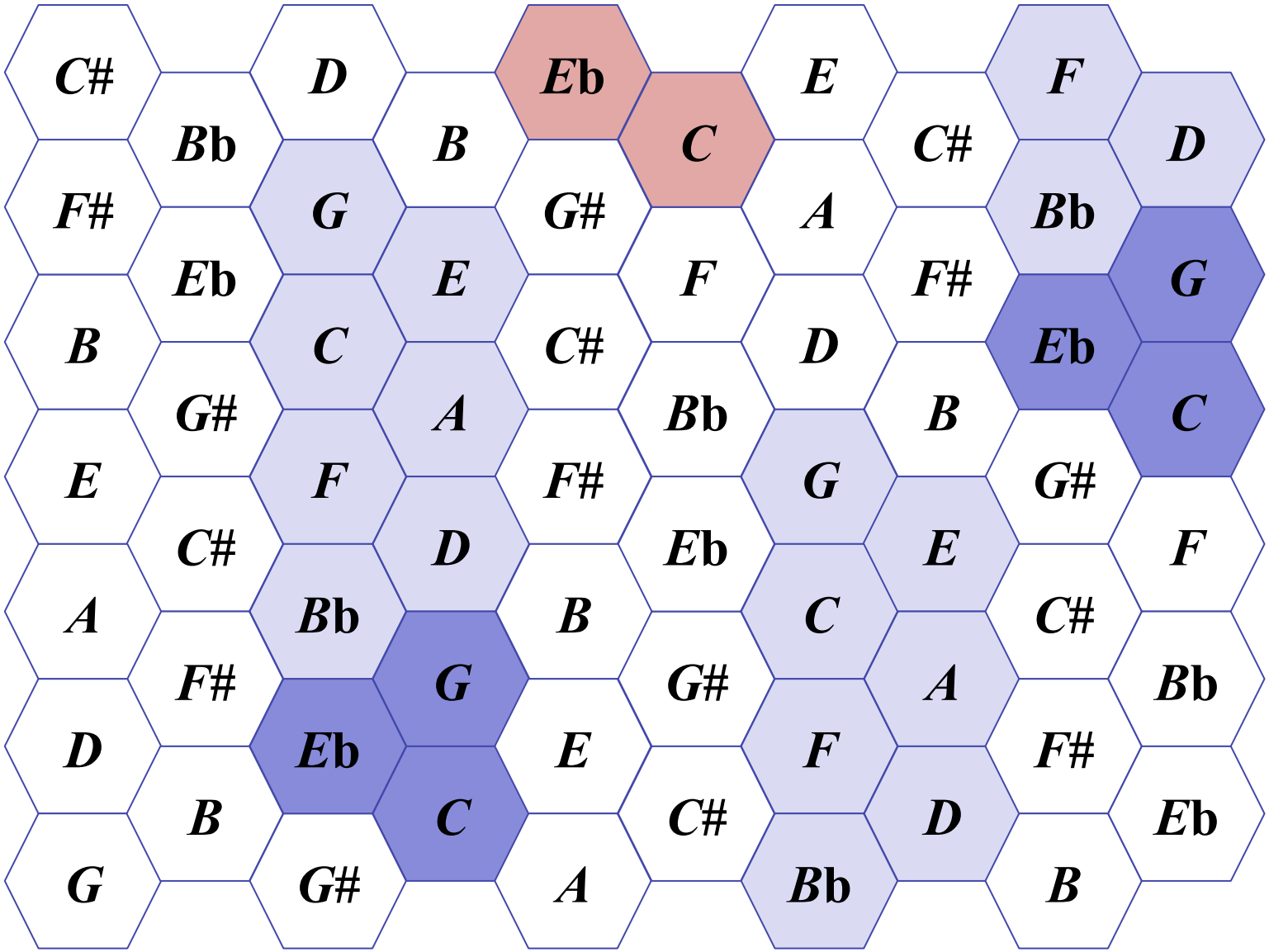
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



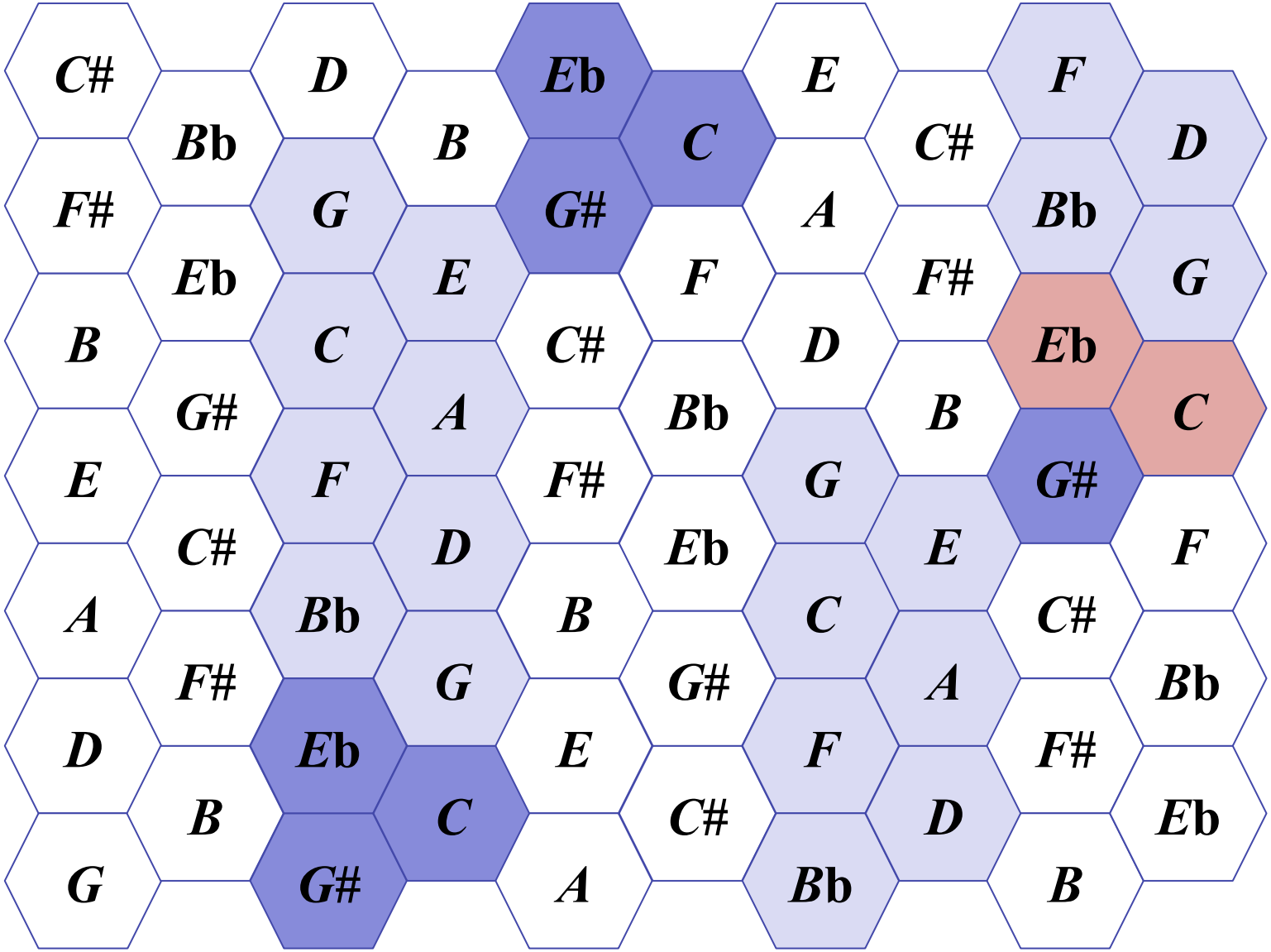
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



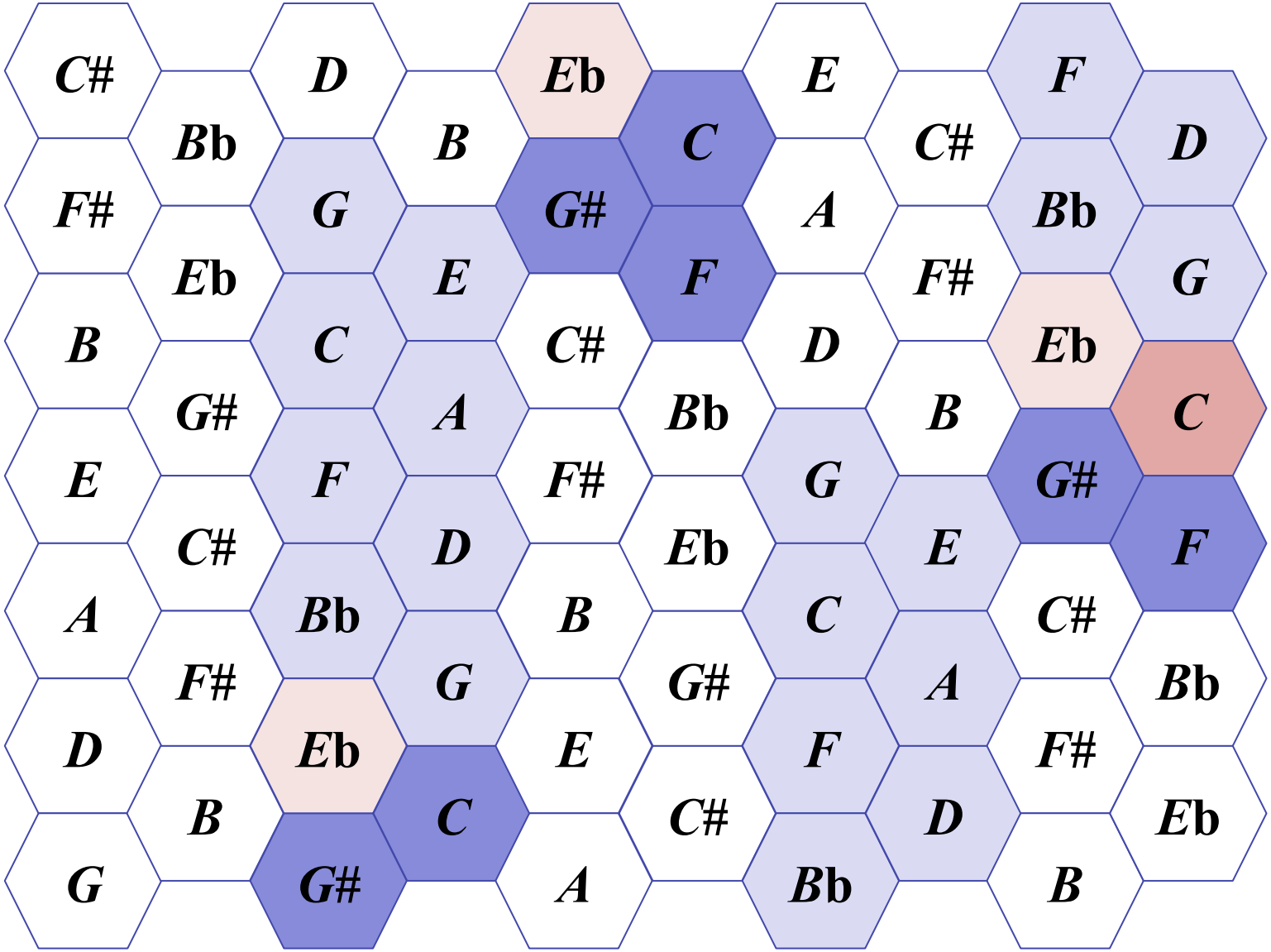
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



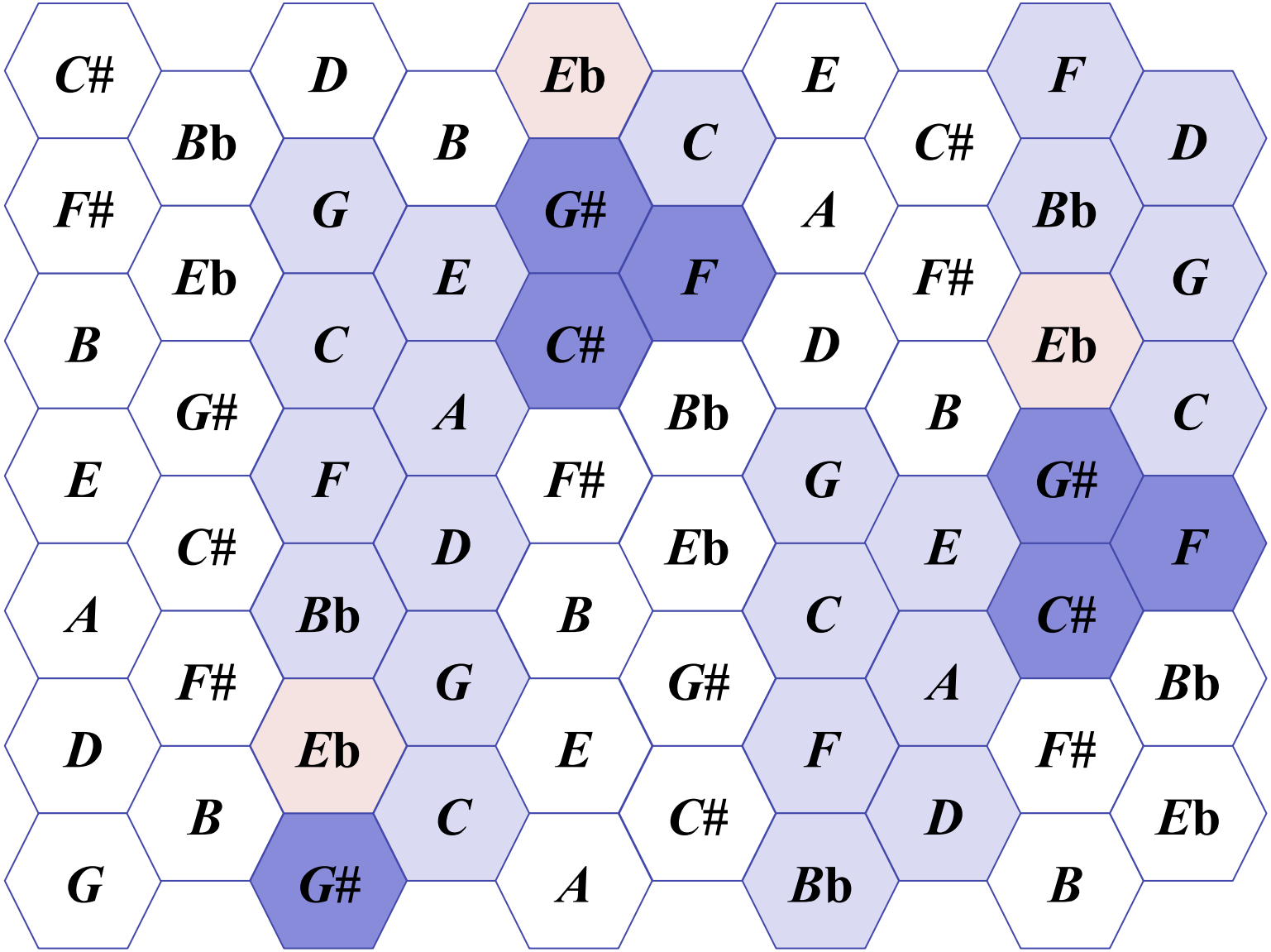
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



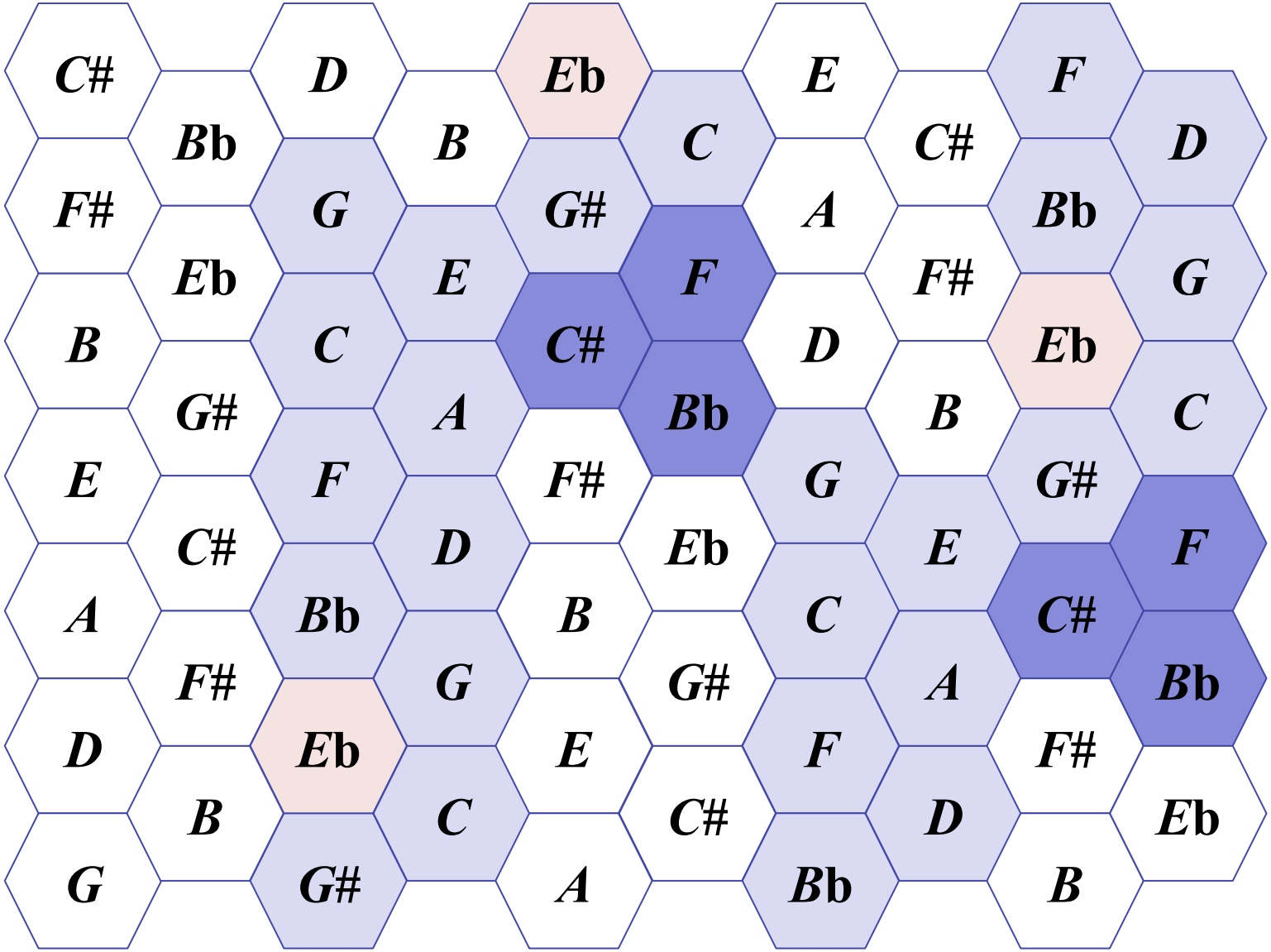
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



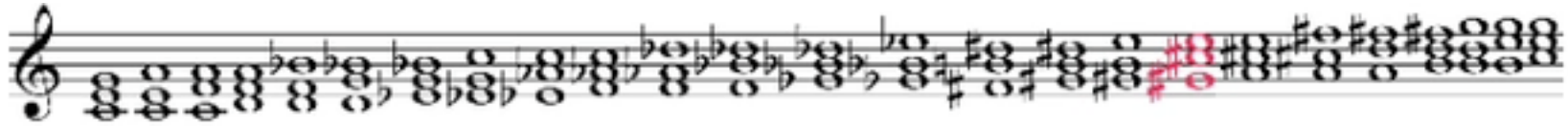
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

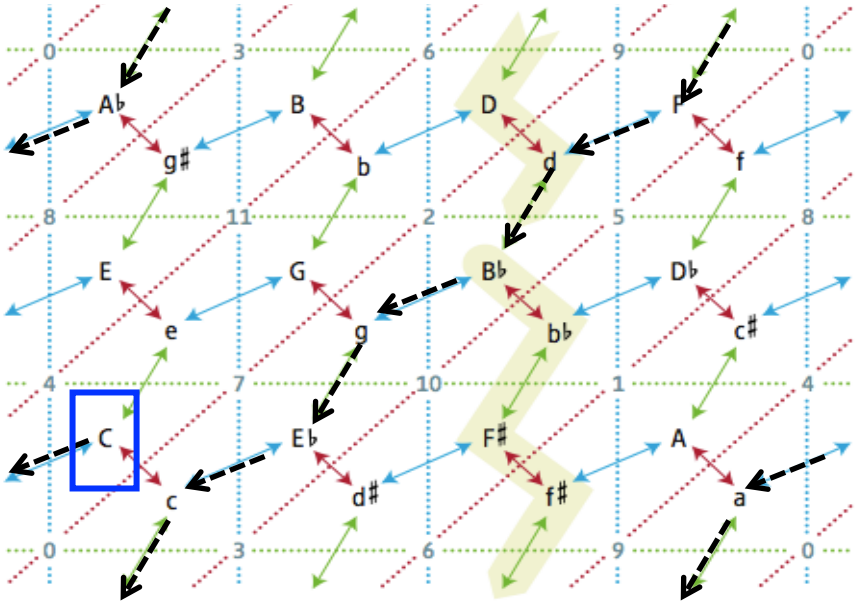


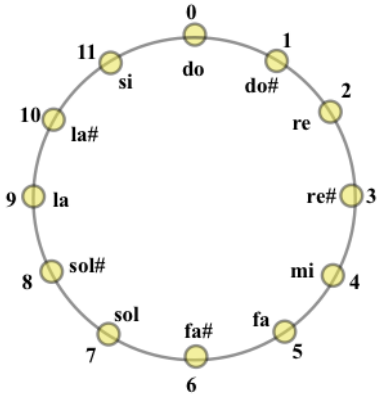
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

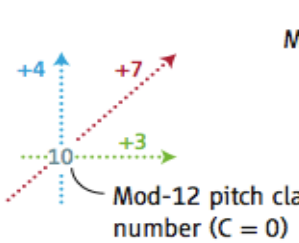


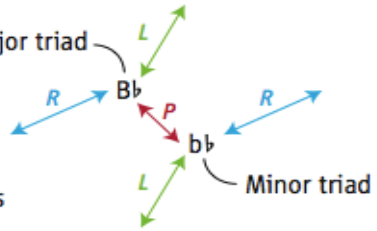
Graphs and hamiltonian paths in the torus

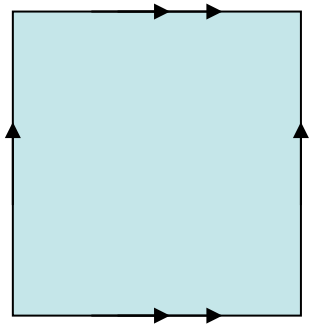


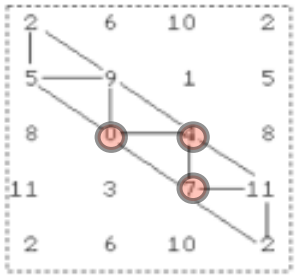


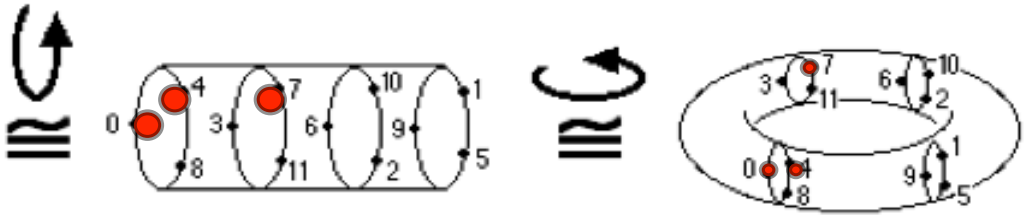




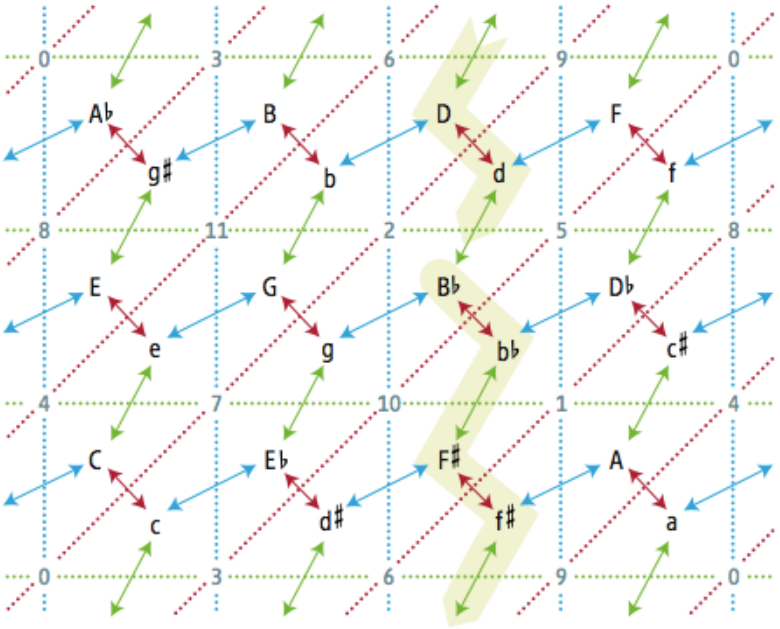


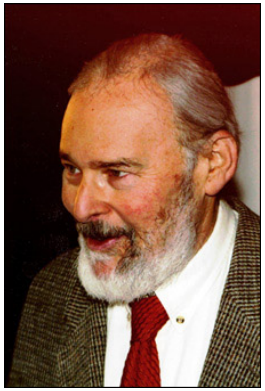


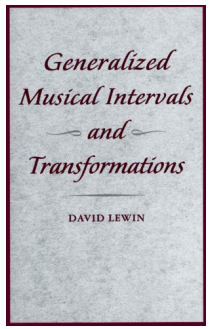


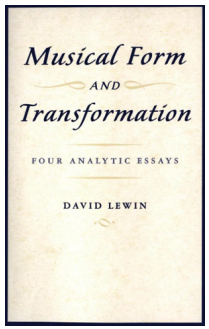


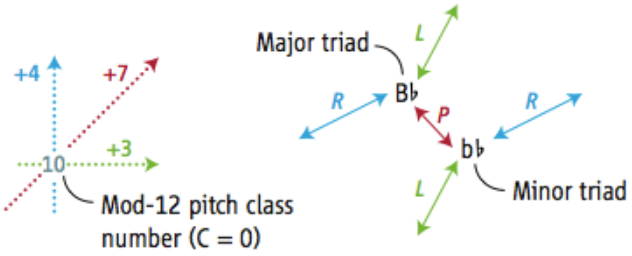
The *Tonnetz* as a Lewin's Generalized Interval System







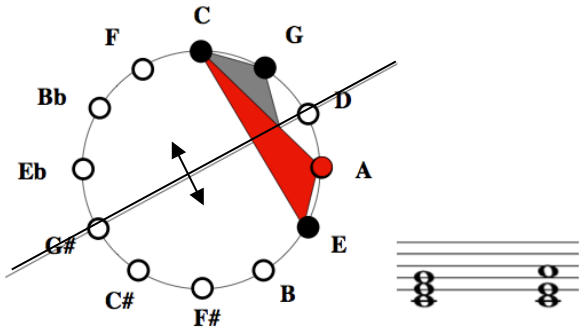




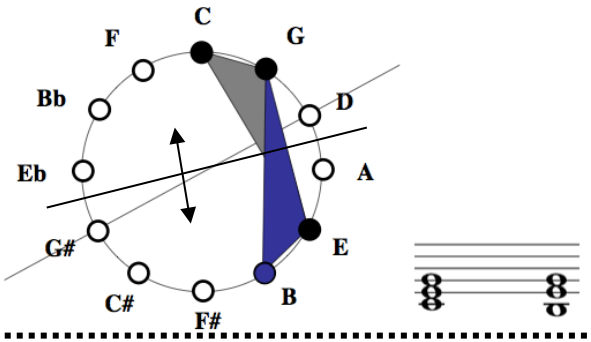
$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 ;\ LRL = L(LR)^{-1} \rangle$

⇒ ρ acts in a simply transitive way on the set *S* of the 24 consonant triads

(Neo-)Riemannian Operation **R** = „**Relative**“



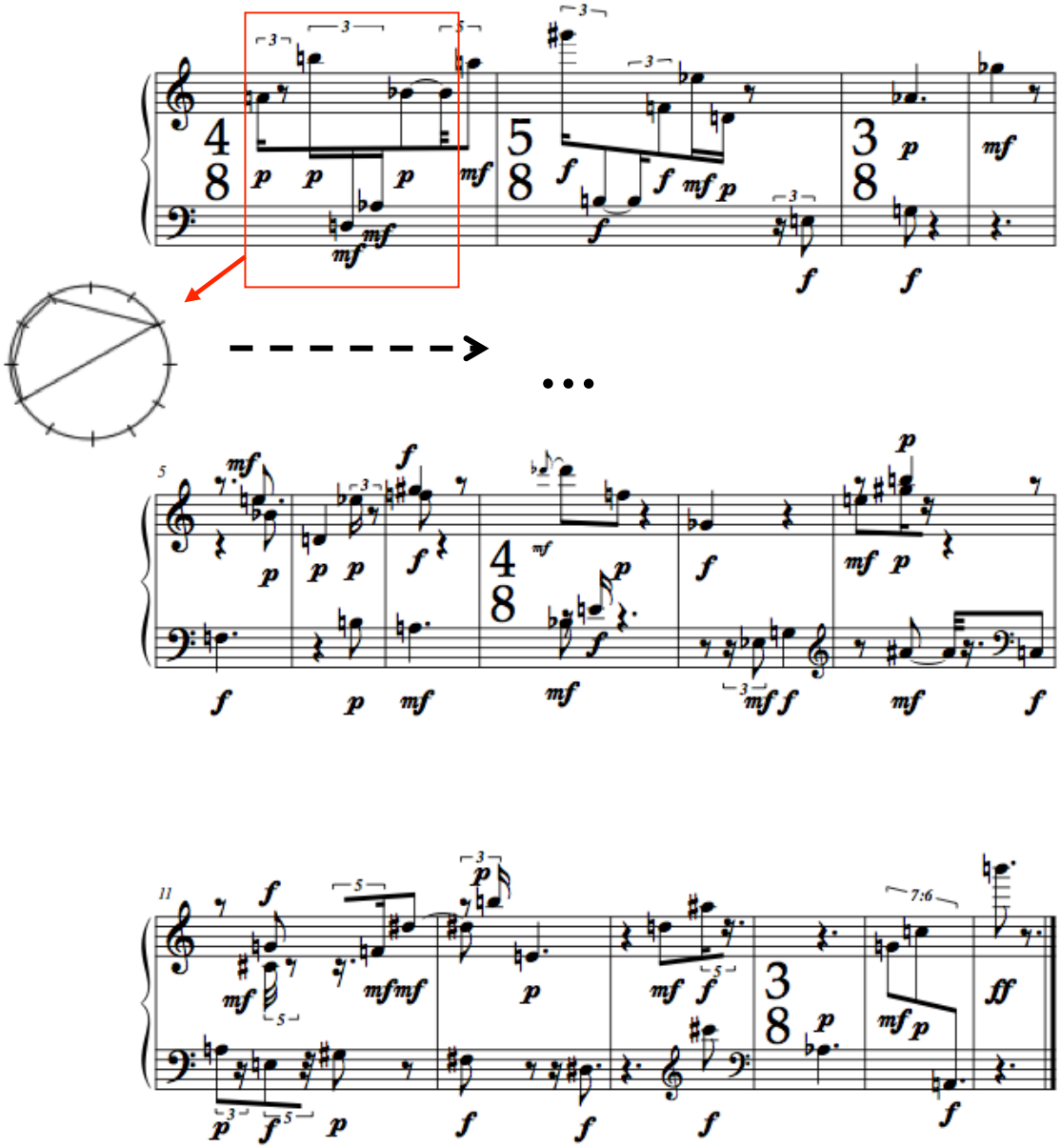
(Neo-)Riemannian Operation **L** = „**Leading-Tone**“



[Noll04]

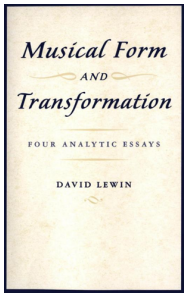
Transformational theory and music perception

(In collaboration with Steve McAdams, University of McGill)



Trois interprétations :

- **Henck**
- **Kontarsky**
- **Tudor**

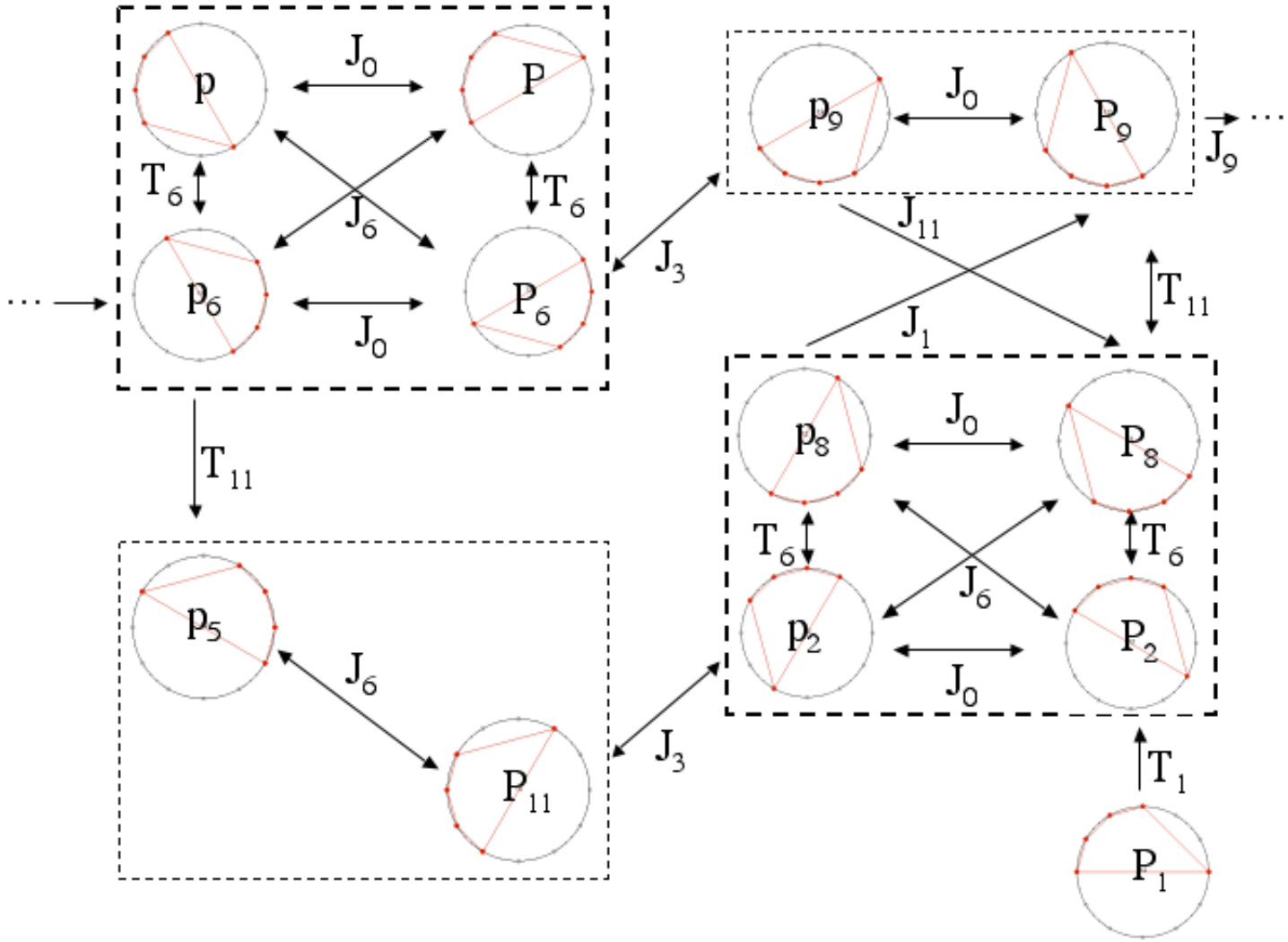


« The most 'theoretical' of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

D. Lewin, « *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »

Transformational network as a spatial configuration

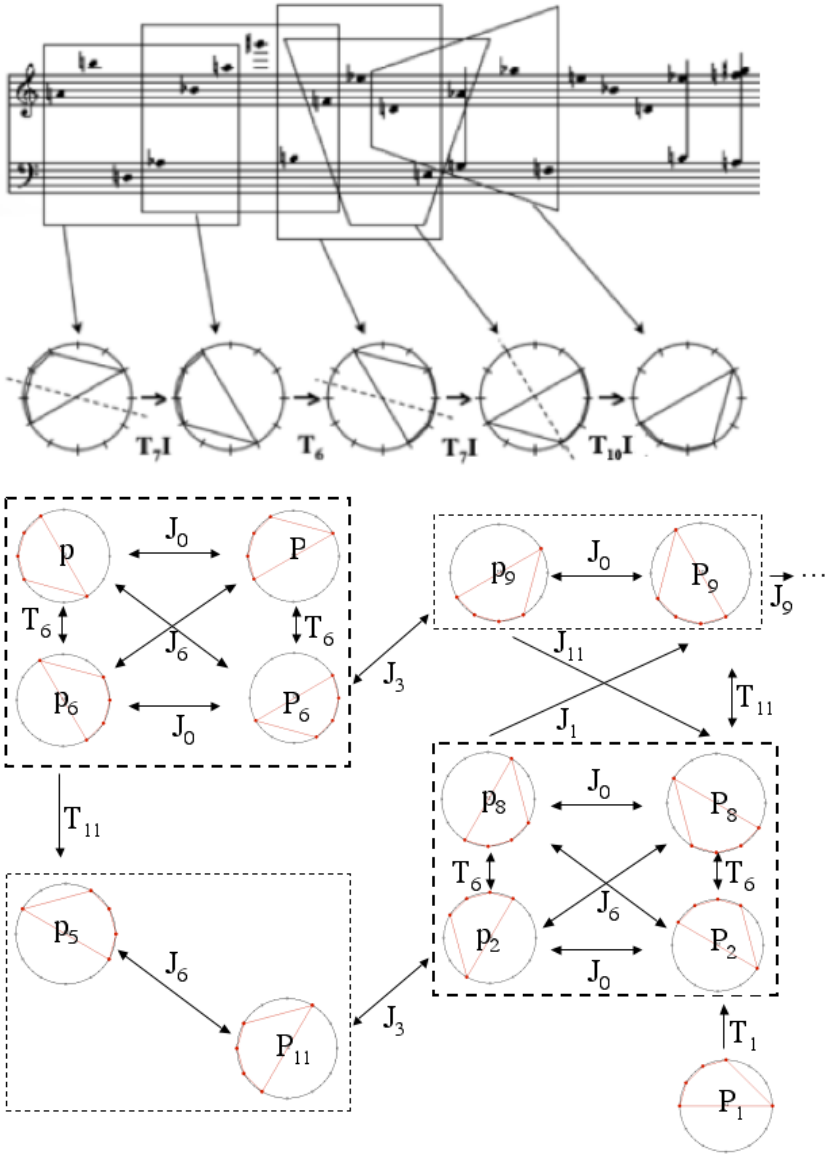
« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their potentially functional interrelationships, in a very compactly organized little spatial configuration. »

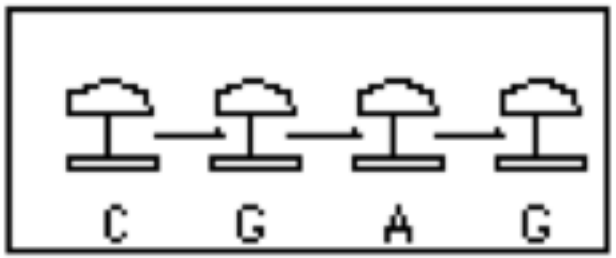


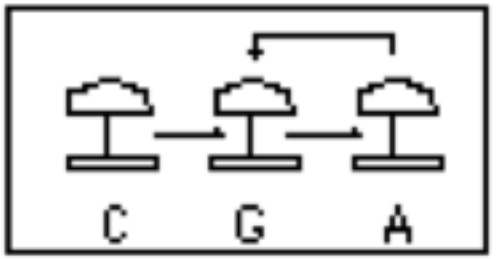
« […] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - it makes the abstract space of such a world accessible to our sensibilities. That is to say that the story projects what one would traditionally call form. »

Transformational network and music cognition

Bamberger, J. (1986). Cognitive issues in the development of musically gifted children. In *Conceptions of giftedness (eds., R. J. Sternberg, & J. E. Davidson), pp. 388-413. Cambridge University Press, Cambridge*

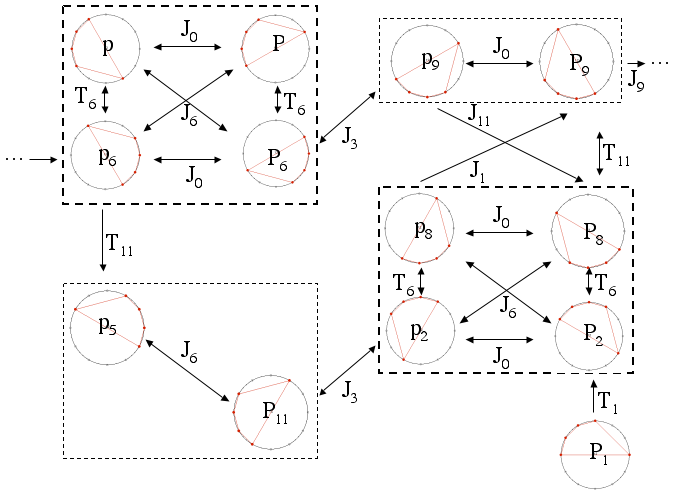


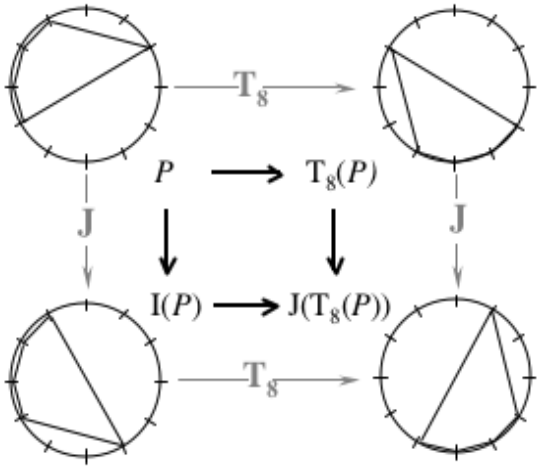


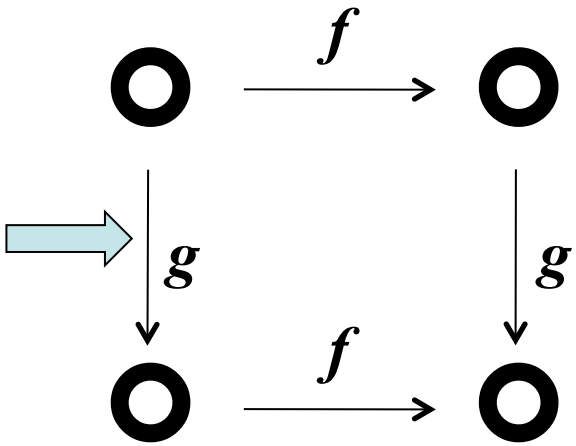


Bamberger, J. (2006). "What develops in musical development?" In G. MacPherson (ed.) *The child as musician: Musical development from conception to adolescence.* Oxford, U.K. Oxford University Press.

Generalized Interval Systems structure and Category Theory

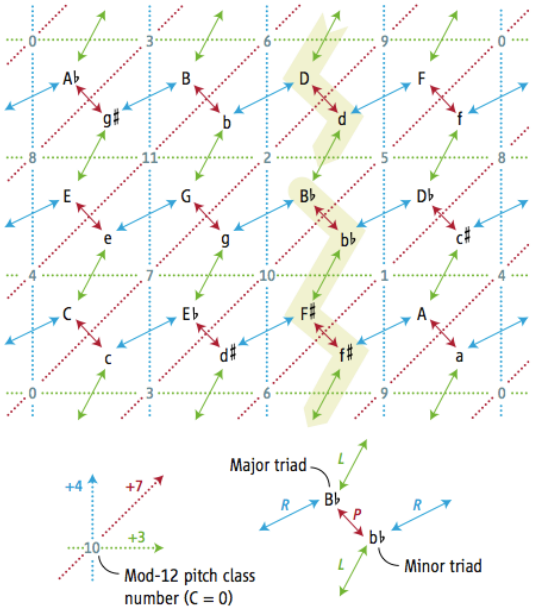


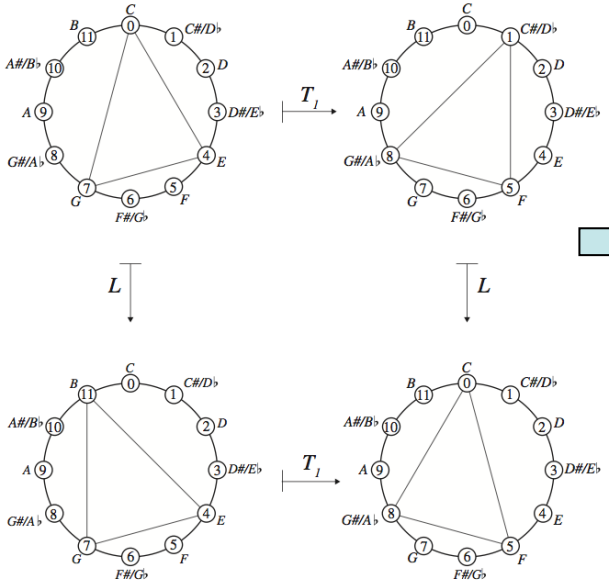




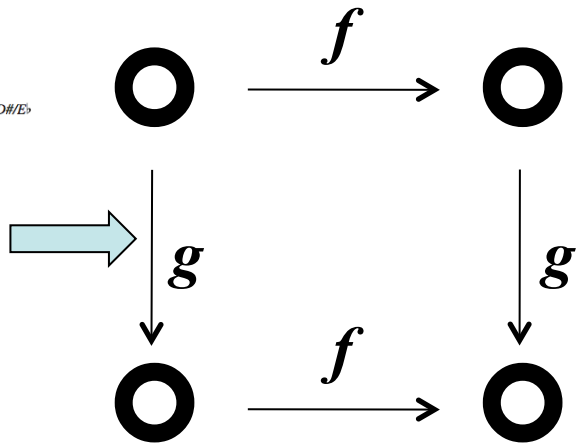
Every diagram commutes

$\forall f, g \in \langle T, J \rangle$





[Crans, Fiore & Satyendra, 2008]



Every diagram commutes

$\forall f \in D_{12} \forall g \in \rho$

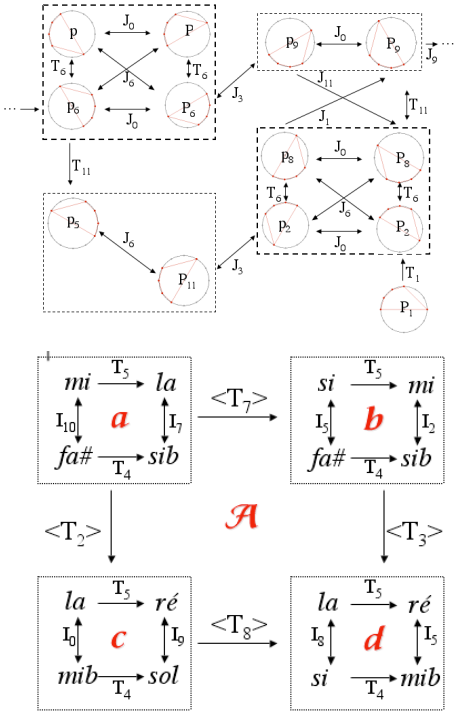
Cognitive aspects of a category-oriented formalization in music

*« De même qu'en mathématique le structuralisme des Bourbaki est déjà doublé par un mouvement faisant appel à des **structures plus dynamiques** (les « catégories » […]) de même toutes les formes actuelles du structuralisme […] sont certainement grosses de développements multiples… »*

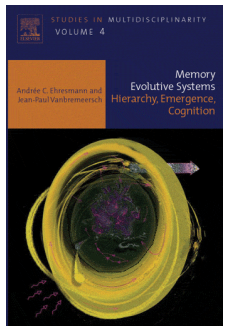
J. Piaget: *Le structuralisme*, 1968

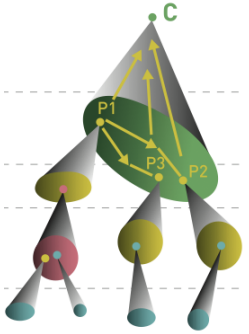
*« La théorie des catégories est une théorie des constructions mathématiques, qui est macroscopique, et procède d'étage en étage. Elle est un bel exemple d'abstraction réfléchissante, cette dernière reprenant elle-même un principe constructeur présent dès le stade sensori-moteur. Le **style catégoriel** qui est ainsi à l'image d'un aspect important de la **genèse des facultés cognitives**, est un style adéquat à la description de cette genèse »*

Jean Piaget, Gil Henriques et Edgar Ascher, *Morphismes et Catégories. Comparer et transformer*, 1990



- G. S. Halford & W. H. Wilson, "A Category Theory Approach to Cognitive Development", *Cognitive Psychology*, 12, 1980
- J. Macnamara & G. E. Reyes, *The Logical Foundation of Cognition*, OUP, 1994
- A. Ehresmann, J.-P Vanbremeerch, *Memory Evolutive Systems, Hierarchy, Emergence, Cognition*, 2007
- MES and computational musicology (*cf.* PhD thesis by John Mandereau)





Thank you for your attention!