Mental and mathematical representations of music

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Mental representations

Musical representations

Mathematical representations

Mathematics & Cognition

Maths & Music

Musical representations

Cognitive musicology

Mathematical Music Theory

http://recherche.ircam.fr/equipes/repmus/mamux/Cognition.html

http://recherche.ircam.fr/equipes/repmus/mamux/PEPS-GdIM.html
Outline of the talk

• Research context of our research

• Mental representations
  – In cognitive philosophy
  – In cognitive musicology: GTTM as an example
  – Levels of supervenience of musical structures

• Mathematical representations
  – Geometrical representations of musical structures
  – Transformational music analysis
  – Cognitive aspects of a category-oriented formalization in music
MR in cognitive philosophy

- A controversial and fundamental (foundational) concept for the philosophy of mind and the cognitive science.

**Problems: format of MR (symbolic vs connectionist); syntax/semantics; non-conceptual content; disjunction problem, etc.**

- In philosophy of mind MR means **mental entities with a content**; in cognitive science MR means **information structure coded in the mind with some role in cognitive tasks** (Paternoster, *Introduzione alla filosofia della mente*, 2002). The two definitions are compatible.

- A further distinction made by Bechtel (*Mental mechanisms*, 2008) is important here: cognitive science [*i.e. also cognitive philosophy*] is more attentive to the **content** of representations; neuroscience is more attentive to the **vehicle** of representations.
MR in cognitive musicology

• The use of the concept is quite widespread but ambiguous and vague.

• The mainstream cognitive musicology maintains that mental representations of music are **non-conceptual** (in Dretske's language, sensory mental states: experiences, sensations, feelings).

  • Musical representations do not lead ultimately to the construction of conceptual structures (Jackendoff, *Consciousness and the computational mind*, 1987).
In Lerdahl and Jackendoff's *Generative Theory of Tonal Music* (GTTM) - one of the most popular cognitive theories of music - the term 'representations' is used in a large sense, with the perfect synonym of 'musical structure'.

In many cognitive theories of music, mental representations of music are considered to be (implicitly) construed by the mind according with the perception of musical flow. In GTTM the mental representations of music are considered in the framework of a final-state theory so the authors are not committed to explain musical cognition online: the symbolic representations of GTTM are alleged to represent the entire static mental representation of a piece of music (that's a kind of natural and spontaneous musical analysis made by the mind).
Cognitive structures of music

• Of course, GTTM has to assume that symbolic representation of music has the same value of the mental representation of music.
Cognitive structures of music

• The pitch-events of a piece are heard in a hierarchy of relative importance.

• Structurally less important events are heard as ornamentations or elaborations of events of greater importance.

Figure 11.6
Time-span reduction of the opening of Mozart, K. 331
Sound, musical and mathematical objects

• Sound objects are intentional objects (Bullot-Egré 2010), but sound perception is not the entire music perception.

• The status of the “musical object” has to be of higher level than sound object. The “musical object” supervenes on (depends from/covaries with/is irreductible to) the sound object.

• Problem for “mathemusicological” point of view: is the mathematical representation of music of lower/equal/higher level than the musical object? Does the musical object supervene on a mathematical representation of music?
Geometrical representations of musical structures

Euler: *Speculum musicum*, 1773

Hugo Riemann: « Ideen zu einer Lehre von den Tonvorstellung », 1914

Longuet-Higgins (1962)


Balzano (1980)
Extract of the 2nd movement of the Symphony No. 9
(L. van Beethoven)

Louis Bigo, Ircam / Université Paris 12
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Graphs and hamiltonian paths in the torus
The *Tonnetz* as a Lewin’s Generalized Interval System

\[ \rho = \langle L, R | L^2 = (LR)^{12} = 1 ; LRL = L(LR)^{-1} \rangle \]

\[ \Rightarrow \rho \text{ acts in a simply transitive way on the set } S \text{ of the 24 consonant triads} \]
Transformational theory and music perception
(In collaboration with Steve McAdams, University of McGill)

« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special group of transformations is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges all pentachord forms of the music into a spatial configuration that illustrates network structure, for this particular phenomenon, over the entire piece. »

D. Lewin, « Making and Using a Pcset Network for Stockhausen's Klavierstück III »
“[…] the sequence of events moves within a clearly defined world of possible relationships, and because
-in so moving - it makes the abstract space of such a world accessible to our sensibilities. That is to say
that the story projects what one would traditionally call form.”

“Rather than asserting a network that follows pentachord relations one at a time, according to the
chronology of the piece, I shall assert instead a network that displays all the pentachord forms
used and all their potentially functional interrelationships, in a very compactly organized little
spatial configuration.”

Generalized Interval Systems structure and Category Theory

Every diagram commutes
\( \forall f, g \in < T, J > \)

Every diagram commutes
\( \forall f \in D_{12} \forall g \in \rho \)

[Crans, Fiore & Satyendra, 2008]
Cognitive aspects of a category-oriented formalization in music

« De même qu’en mathématique le structuralisme des Bourbaki est déjà doublé par un mouvement faisant appel à des structures plus dynamiques (les « catégories » […] de même toutes les formes actuelles du structuralisme […] sont certainement grosses de développements multiples… »

J. Piaget: Le structuralisme, 1968

« La théorie des catégories est une théorie des constructions mathématiques, qui est macroscopique, et procède d’étage en étage. Elle est un bel exemple d’abstraction réfléchissante, cette dernière reprenant elle-même un principe constructeur présent dès le stade sensori-moteur. Le style catégoriel qui est ainsi à l’image d’un aspect important de la genèse des facultés cognitives, est un style adéquat à la description de cette genèse »

Jean Piaget, Gil Henriques et Edgar Ascher, Morphismes et Catégories. Comparer et transformer, 1990

• A. Ehresmann, J.-P Vanbremeerch, Memory Evolutive Systems, Hierarchy, Emergence, Cognition, 2007
• MES and computational musicology (cf. PhD thesis by John Mandereau)
Thank you for your attention!