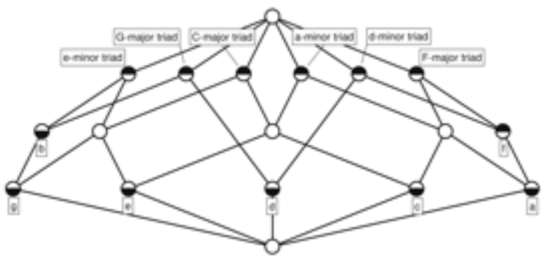
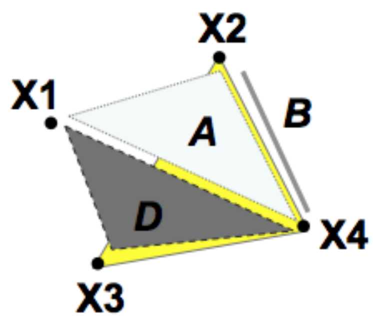
The interplay between algebra and geometry in computational musicology

I





Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS/UPMC
http://www.ircam.fr/repmus.html

PARCOURS DU MASTER SCIENCES ET TECHNOLOGIES
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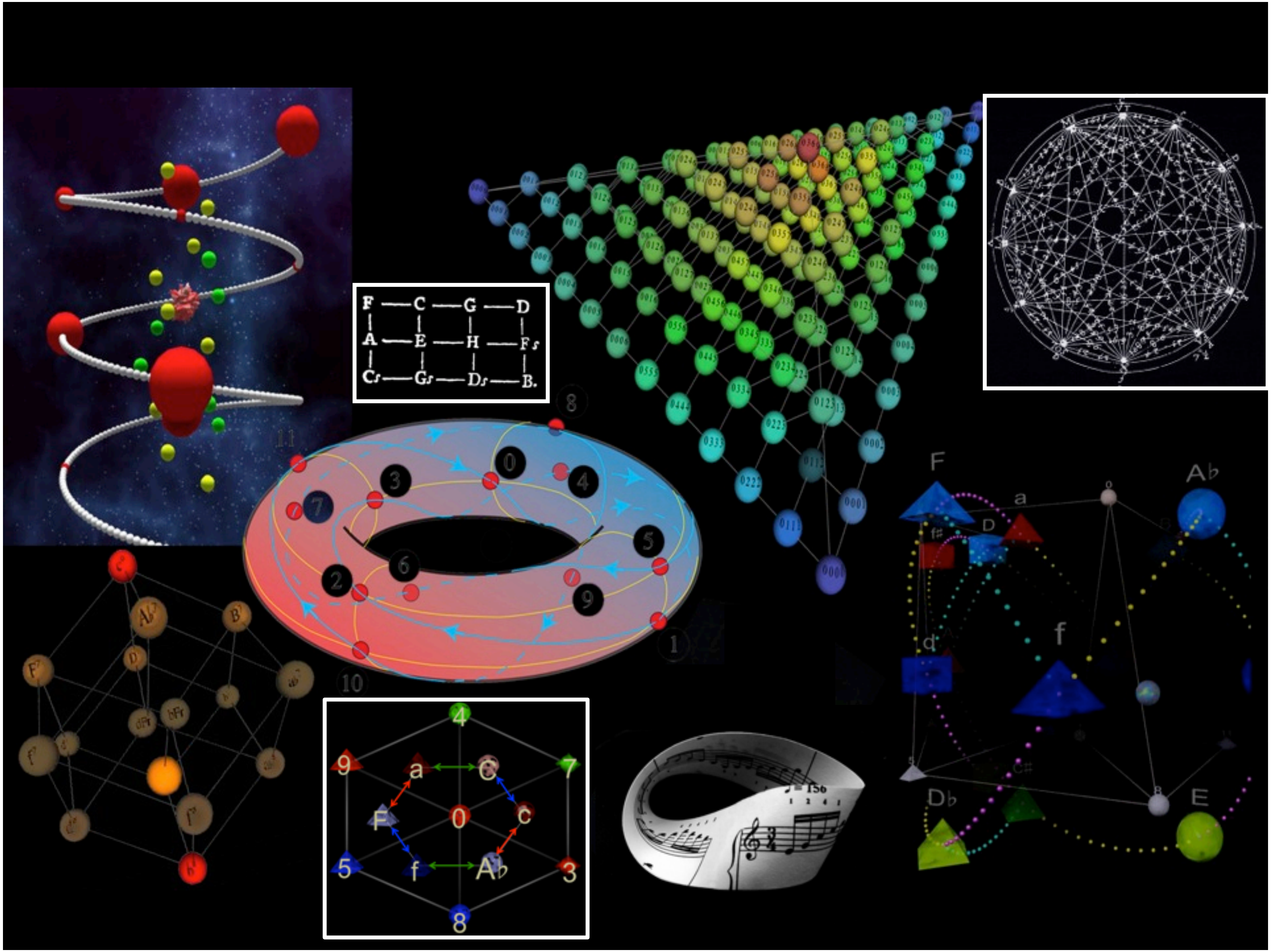
F — C — G — D
A — E — H — Fs
Cs — Gs — Ds — B.
F
a
D
f#
A♭
d
f
D♭
E
C#
4
9
7
a
C
F
0
c
5
f
A♭
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8
0
3
4
7
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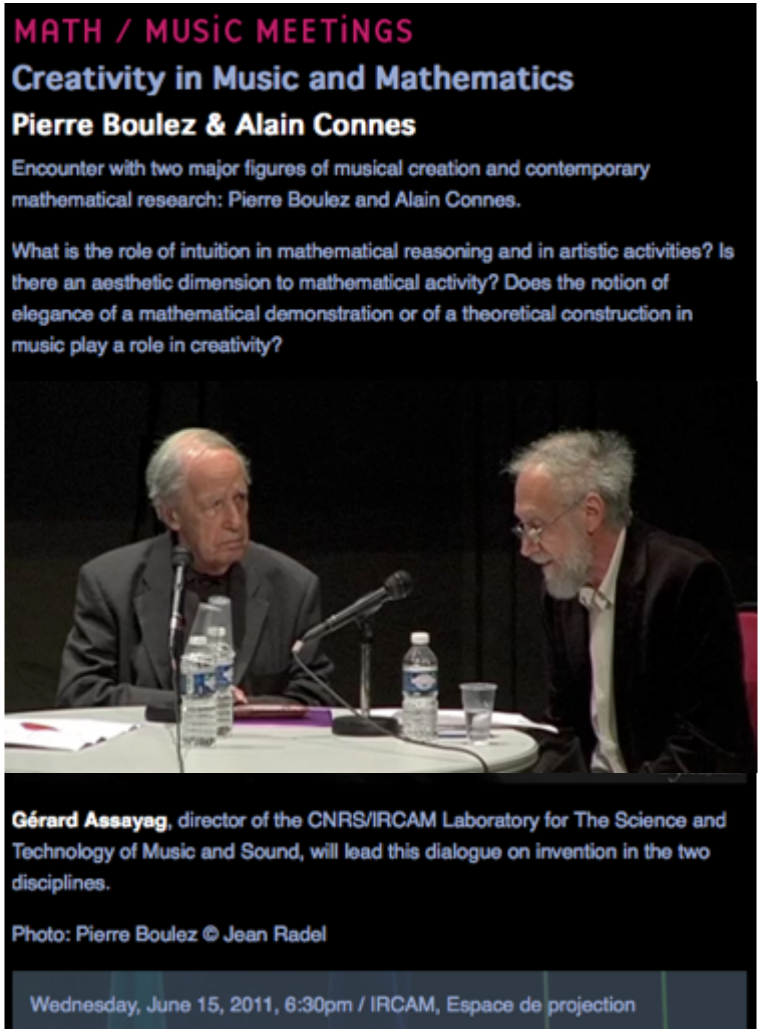
6
2
9
1
10
11

The interplay between algebra and geometry in music



"Concerning **music**, it takes place in **time**, like **algebra**. In **mathematics**, there is this fundamental duality between, on the one hand, **geometry** – which corresponds to the visual arts, an immediate intuition – and on the other hand **algebra**. This is not visual, it has a temporality. This fits in time, it is a computation, something that is very close to the language, and which has its diabolical precision. [...] **And one only perceives the development of algebra through music**" (A. Connes).

➔ `http://videotheque.cnrs.fr/`

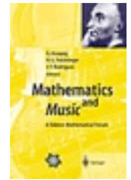
➔ `http://agora2011.ircam.fr`

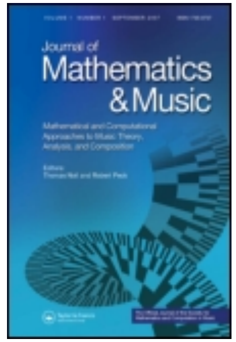
ircam Centre Pompidou AGORA FESTiVAL 8-18 JUiN 2011

Société Mathématique de France SMF

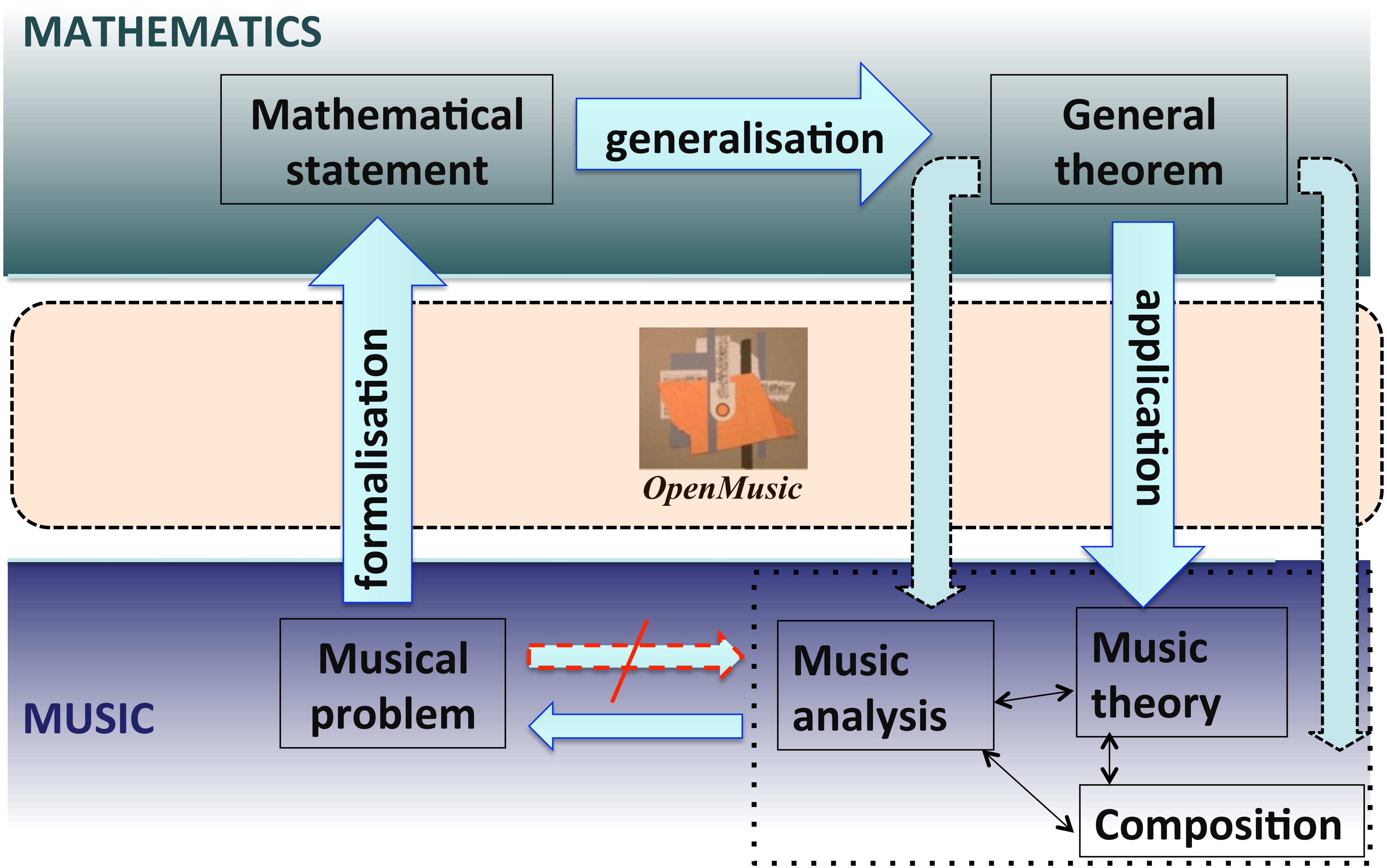
Maths/music: a (very) recent perspective

- **1999: 4e Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (G. Assayag, H.G. Feichtinger, J.F. Rodrigues, Springer, 2001)**
- **2000-2001: *MaMuPhi Seminar, Penser la musique avec les mathématiques ?* (Assayag, Mazzola, Nicolas éds., Coll. 'Musique/Sciences', Ircam/Delatour, 2006)**
- **2000-2003: International Seminar on *MaMuTh* (*Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Luis-Puebla eds, epOs, 2004)**
- **2003: *The Topos of Music* (G. Mazzola et al.)**
- **2001-…: *MaMuX* Seminar at Ircam**
- **2004-…: *mamuphi* Seminar (Ens/Ircam)**
- **2006: Collection 'Musique/Sciences' (Ircam/Delatour France)**
- **2007: Journal of Mathematics and Music (Taylor & Francis) and SMCM**
- **2007: First MCM 2007 (Berlin) and Proceedings by Springer**
- **2007-…: AMS Special Session on Mathematical Techniques in Musical Analysis**
- **2009: *Computational Music Science* (eds: G. Mazzola, M. Andreatta, Springer)**
- **2009: MCM 2009 (Yale University) and Proceedings by Springer**
- **2010: Mathematics Subject Classification : 00A65 Mathematics and music**
- **2011: MCM 2011 (Ircam, 15-17 June 2011) and Proceedings LNCS Springer**
- **2013: MCM 2013 (McGill University, Canada, 12-14 June 2013) - Springer**
- **2015: MCM 2015 (Queen Mary University, Londres, 22-25 June 2013) - Springer**





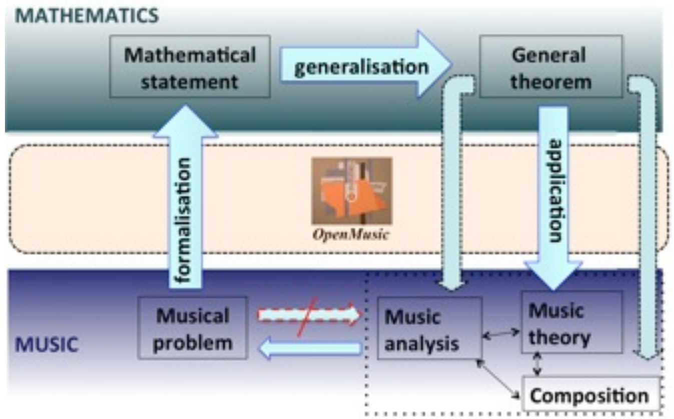
The double movement of a ‘mathemusical’ activity

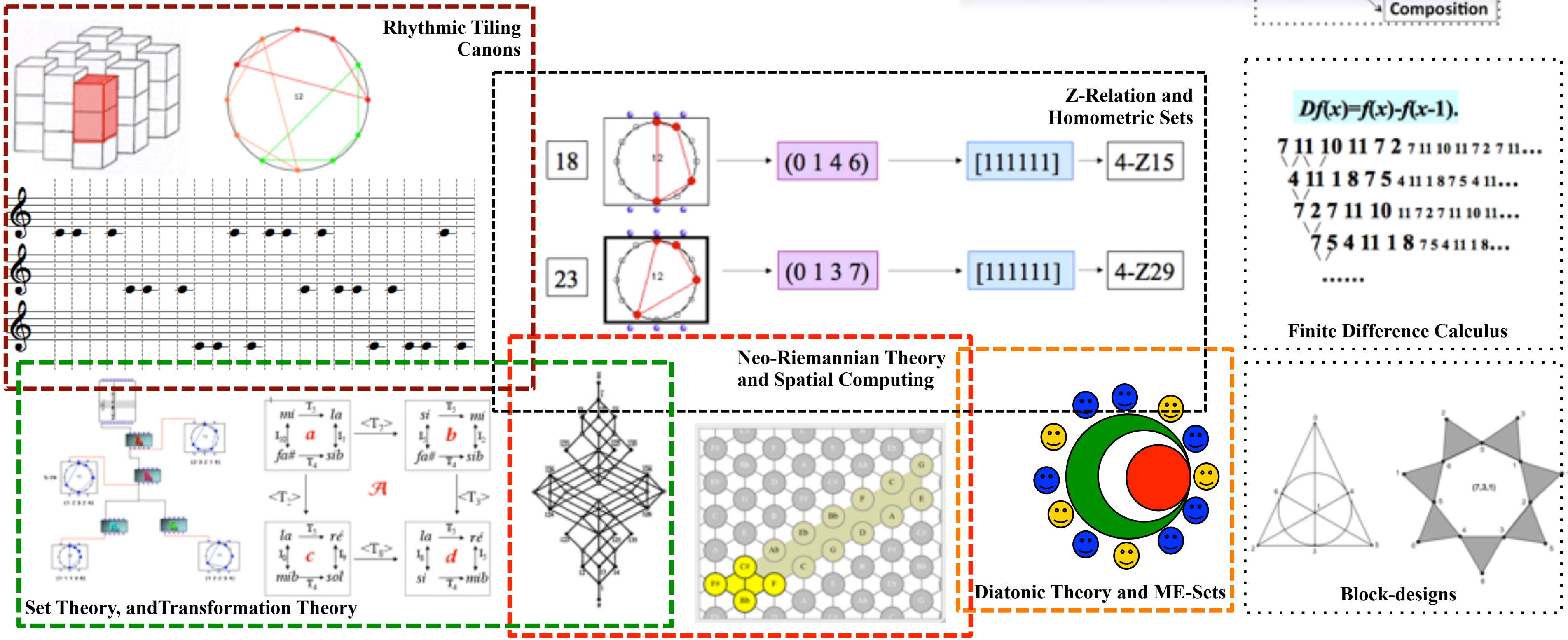


Some examples of ‘mathemusical’ problems

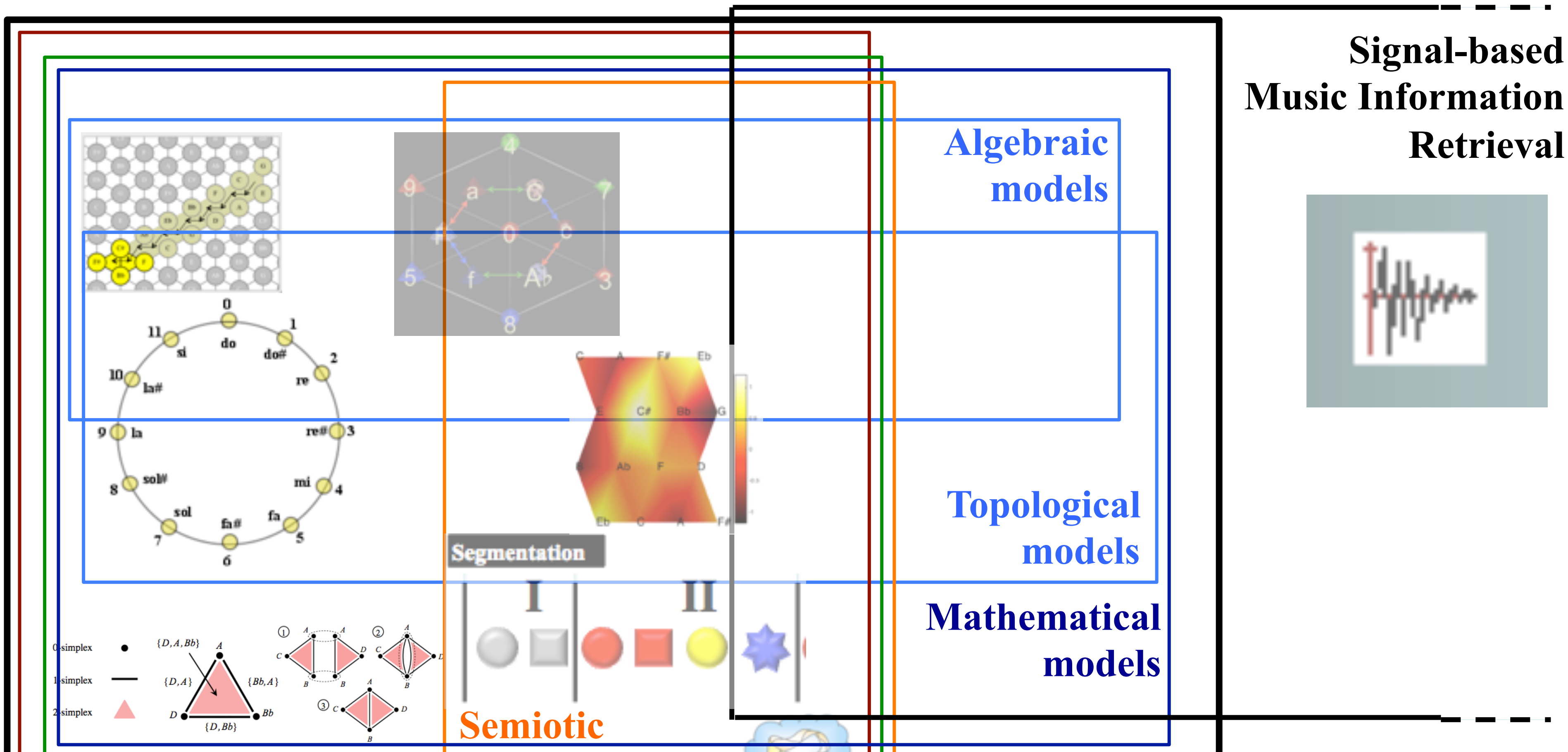
M. Andreatta : *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

- The construction of Tiling Rhythmic Canons
- The *Z* relation and the theory of homometric sets
- *Set Theory* and Transformational Theory
- Neo-Riemannian Theory, Spatial Computing and FCA
- Diatonic Theory and Maximally-Even Sets
- Periodic sequences and finite difference calculus
- Block-designs and algorithmic composition





Math'n Pop Project: formal models *for* and *within* popular music



- Andreatta M. (2014), « Modèles formels dans et pour la musique pop, le jazz et la chanson : introduction et perspectives futures », dans Esthétique & Complexité : Neurosciences, Philosophie et Art, Z. Kapoula, L.-J. Lestocart, J.-P. Allouche éds., éditions du CNRS, 2014

Math'n Pop Workshop

Formal and Computational Models in Popular Music
An Introductory Tutorial with Pedagogical Demonstrations

Tuesday September 16
13:30-17:30

LUNDI 15 ET MARDI 16 DÉCEMBRE, 10H-18H
IRCAM ET MAISON DE LA RECHERCHE

MUSIQUE SAVANTE ET MUSIQUES ACTUELLES : ARTICULATIONS

COLLOQUE

Cette double journée, organisée en collaboration étroite entre l'Ircam et l'université Paris-Sorbonne, sous l'égide de la SFAM, vise à mettre en discussion l'opposition habituelle entre musique savante et musiques actuelles en montrant la richesse des articulations et passerelles réciproques.

Coordination : **Moreno Andreatta** (Ircam-CNRS-UPMC), **Jean-Michel Bardez** (SFAM), **Philippe Cathé** (université Paris-Sorbonne).

Entrée libre.

LUNDI 15 DÉCEMBRE, 19H
CENTRE POMPIDOU, PETITE SALLE

MATH'N POP

En complément du colloque « Musique savante et musiques actuelles : articulation » et dans le cadre d'un cycle de conférences/débats « les mathématiques pour comprendre le monde » organisés par la Bpi, une soirée grand public consacrée à l'application d'outils formels dans la musique pop et la chanson.

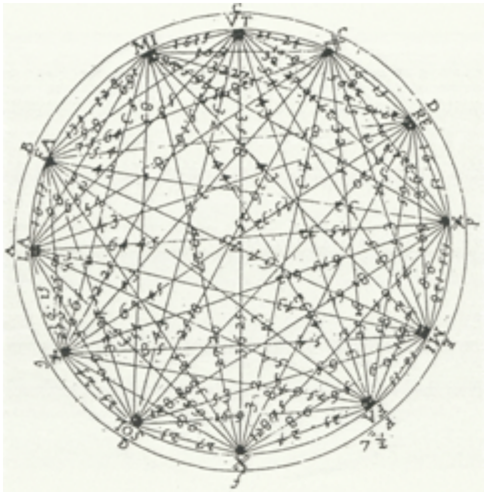
Coordination : **Jean Dhombres** (CNRS), **Caroline Raynaud** (BPI-Centre Pompidou).

Entrée libre.

Combinatorics and axiomatic methods in music

Tabella pulcherrima & vtilißima Combinationis duodecim Cantilenarum.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
1	1	1	1	1	1	1	1	1	1	1	1
2	3	4	5	6	7	8	9	10	11	12	13
3	6	10	15	21	28	36	45	55	66	78	91
4	10	20	35	56	84	120	165	220	286	364	455
5	15	35	70	126	210	330	495	715	1001	1365	1820
6	21	56	126	252	462	792	1287	2002	3003	4368	6188
7	28	84	210	462	924	1716	3003	5005	8008	12376	18564
8	36	120	330	792	1716	3432	6435	11440	19448	31824	50388
9	45	165	495	1287	3003	6435	12870	24310	43758	75582	125970
10	55	220	715	2002	5005	11440	24310	48620	92378	167960	293930
11	66	286	1001	3003	8008	19448	43758	92378	184756	352716	646646
12	78	364	1365	4368	12376	31824	75582	167960	352716	705432	1352078
13	91	455	1820	6188	18564	50388	125970	293930	646646	1352078	2704156
14	105	560	2380	8568	27132	77520	203490	497420	1144066	2496144	5200300
15	120	680	3060	11628	38760	116280	319770	817190	1961256	4457400	9657700
16	136	816	3876	15504	54264	170544	490314	1307504	3268760	7726160	17383860
17	153	969	4845	20349	74613	245157	735471	2042975	5311735	13037895	30421755
18	171	1140	5985	26334	100947	346104	1081575	3124550	8436285	21474180	51895935
19	190	1330	7315	33649	134596	480700	1562275	4686825	13123110	34597290	86493225
20	210	1540	8855	42504	177100	657800	2220075	6906900	20030010	54627300	141120525
21	231	1771	10626	53130	230230	888030	3108105	10015005	30045015	84672315	225792840
22	253	2024	12650	65780	296010	1184040	4292145	14307150	44352165	129024480	354817320
23	276	2300	14950	80730	376740	1560780	5852925	20160075	64512290	193536720	548354040
24	300	2600	17550	98280	475020	2035800	7888725	28048800	92561040	286097760	834451800
25	325	2925	20475	118755	593775	2629575	10518300	38567100	131128140	417225900	1251677700



Marin Mersenne,
Harmonicorum Libri XII, 1648

Josef-Mathias Hauer

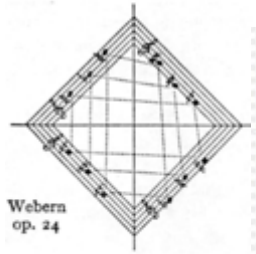
Combinatorics

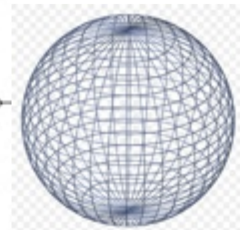
Axioms

Physicists and mathematicians are far in advance of musicians in realizing that their respective sciences do not serve to establish a concept of the universe conforming to an ***objectively existent nature.***

As the study of axioms eliminates the idea that ***axioms*** *are something absolute, conceiving them instead as* ***free propositions of the human mind****, just so would this musical theory free us from the concept of major/minor tonality […] as an irrevocable law of nature.*

Ernst Krenek : *Über Neue Musik*, 1937 (Engl. Transl. *Music here and now*, 1939)



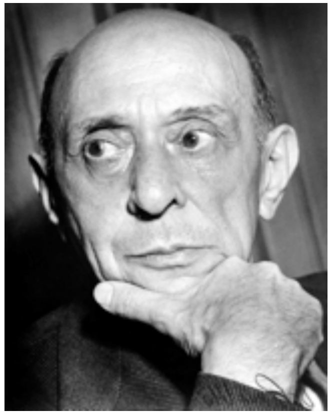


Ernst Krenek

David Hilbert

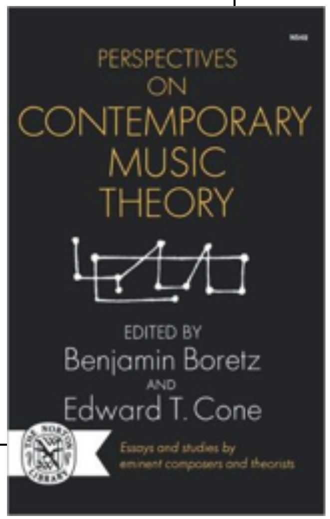
« In Schoenberg's theoretical quest, one can discern the spirit of what might be termed the *Bauhaus* mentality, which in turn was reflected […] in the [conceptualizations of] the Vienna Circle….and the writings of Schlick, Neurath, Carnap, and Wittgenstein »

« The recognition of music-theoretical questions as critical compositional ones is not, of course, unique to the twentieth century, nor to composers. But the uniquely explicit, uniquely consequential, and uniquely exposed contemporary involvement of composers in theory as writers and system builders has given the theoretical-compositional connection unprecedently wide, if not always benign or even accurate, publicity : we live, as every reader of the public musical print knows, in an age of **'theoretical composition'**. »



A. Schoenberg

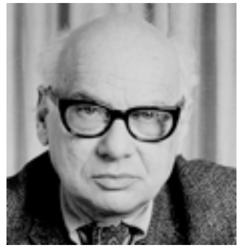
M. Babbitt



« Milton Babbitt, in particular, was the first to suggest that the force of any 'musical **system**' was not as universal constraints for all music but as alternative theoretical constructs, rooted in a communality of shared empirical principles and assumptions **validated** by **tradition**, **experience**, and **experiment** »

B. Boretz et E.T. Cone, *Perspectives on Contemporary Music Theory*, W.W. Norton and Company, New York, 1972.

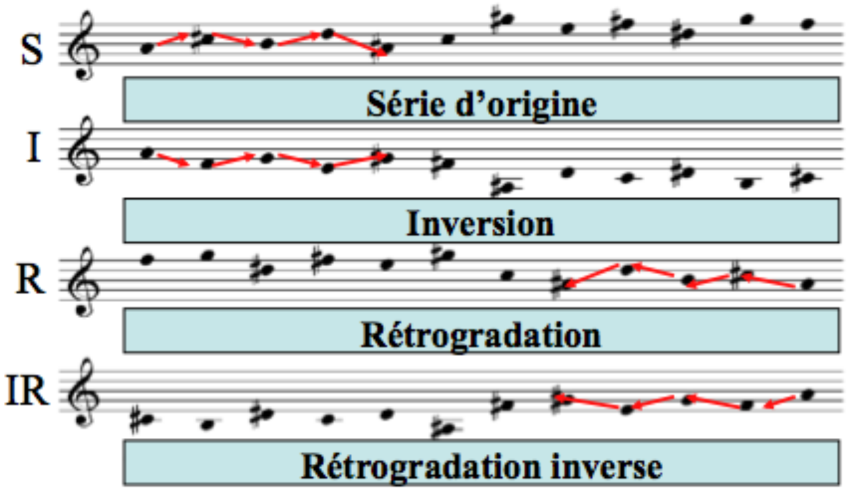
Le système dodécaphonique est « *un ensemble d'éléments, **relations** entre les éléments et **opérations** sur les éléments.* […] *Une vraie mathématisation aurait besoin d'une formulation et d'une présentation dictées par le fait que le système dodécaphonique est un **groupe de permutations** qui est façonné* [shaped] *par la structure de ce modèle mathématique* »

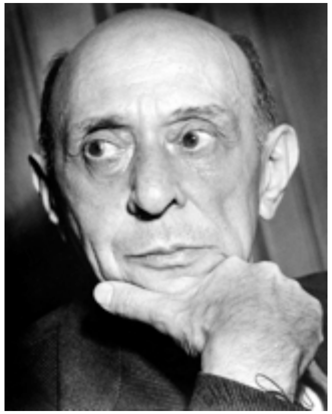


Milton Babbitt

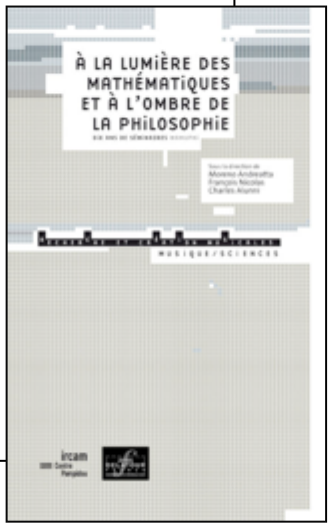
	S	I	R	RI
S	S	I	R	RI
I	I	S	RI	R
R	R	RI	S	I
RI	RI	R	I	S

M. Babbitt: *The function of Set Structure in the Twelve-Tone System*, PhD (1946/1992)





A. Schoenberg



M. Andreatta, « Mathématiques, Musique et Philosophie dans la tradition américaine: la filiation Babbitt/Lewin », dans *A la lumière des mathématiques et à l'ombre de la philosophie, Dix ans de séminaire mamuphi*, Ircam-Delatour France, 2012.

From permutations to algebraic combinatorics

114 Marin Mersenne, *Harmonicorum Libri XII*, 1648

LIBER SEPTIMVS.

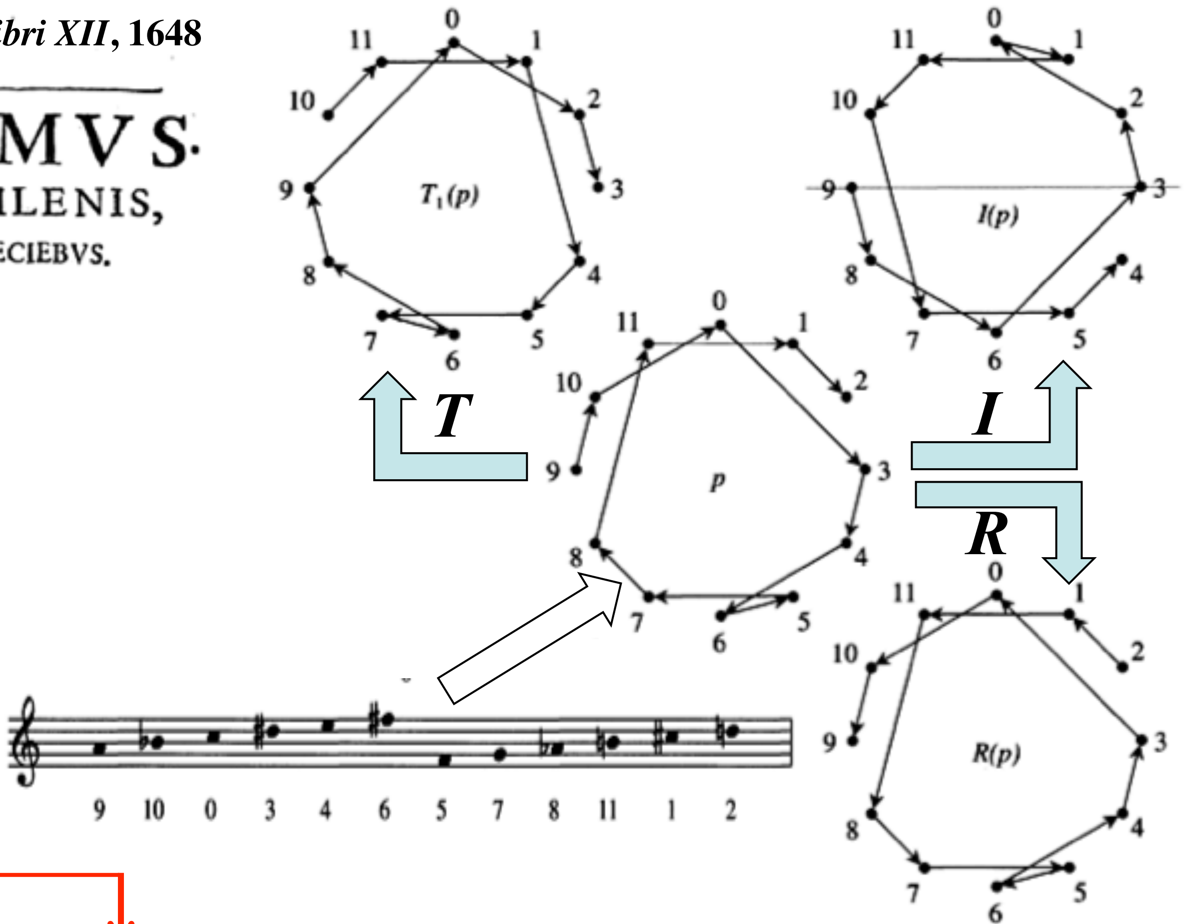
DE CANTIBVS, SEV CANTILENIS,

EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 22.

I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6227020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000

?



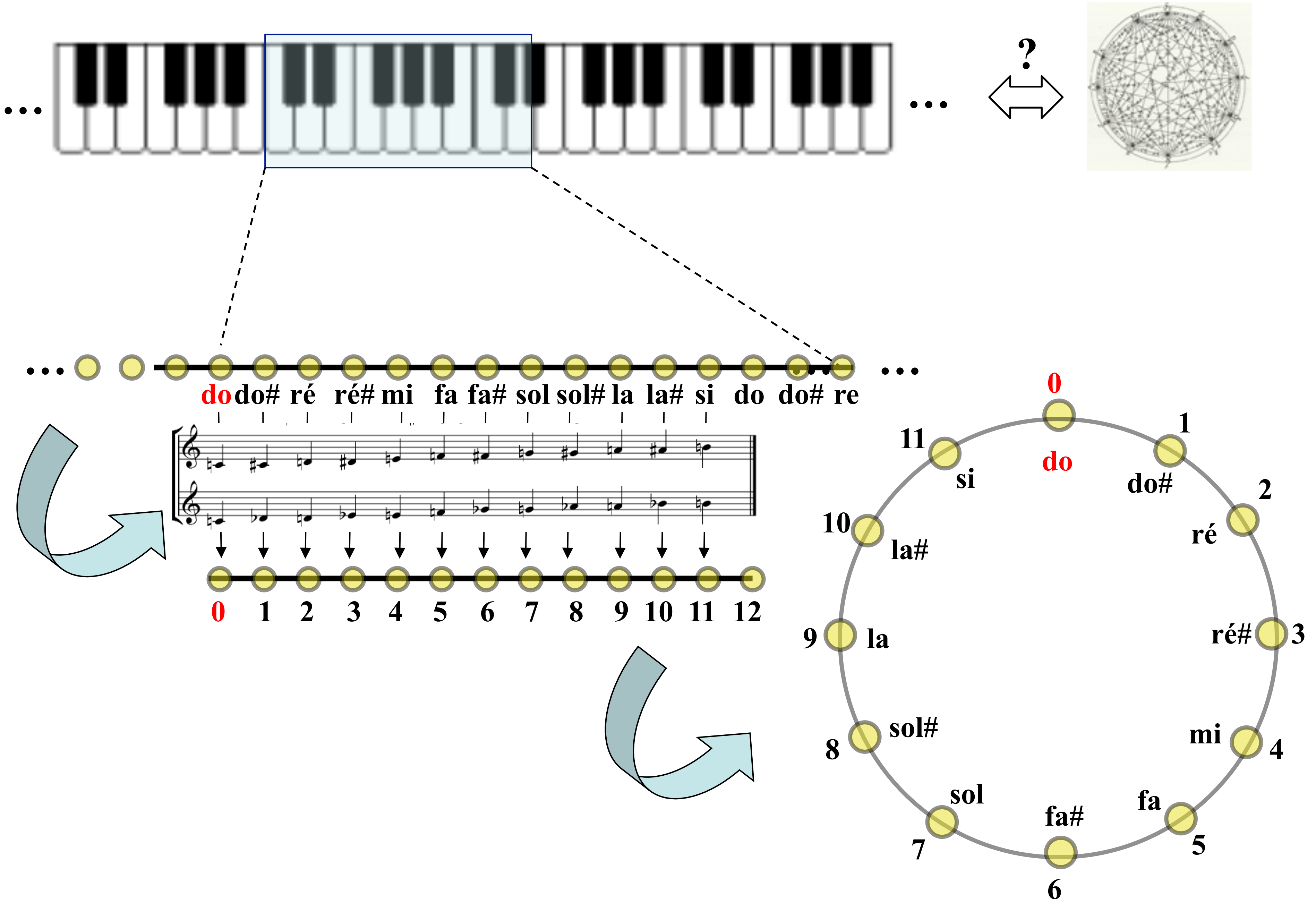
Thus the number of n-tone rows is

$$\tfrac{1}{2}[(n-1)! + (n-1)(n-3)\cdots(2)] \quad \text{if } n \text{ is odd};$$
$$\tfrac{1}{2}[(n-1)! + (n-2)(n-4)\cdots(2)(1+n/2)] \quad \text{if } n \text{ is even}.$$

For example, there are 9985920 twelve tone rows, a fact which does not seem to be in the literature.

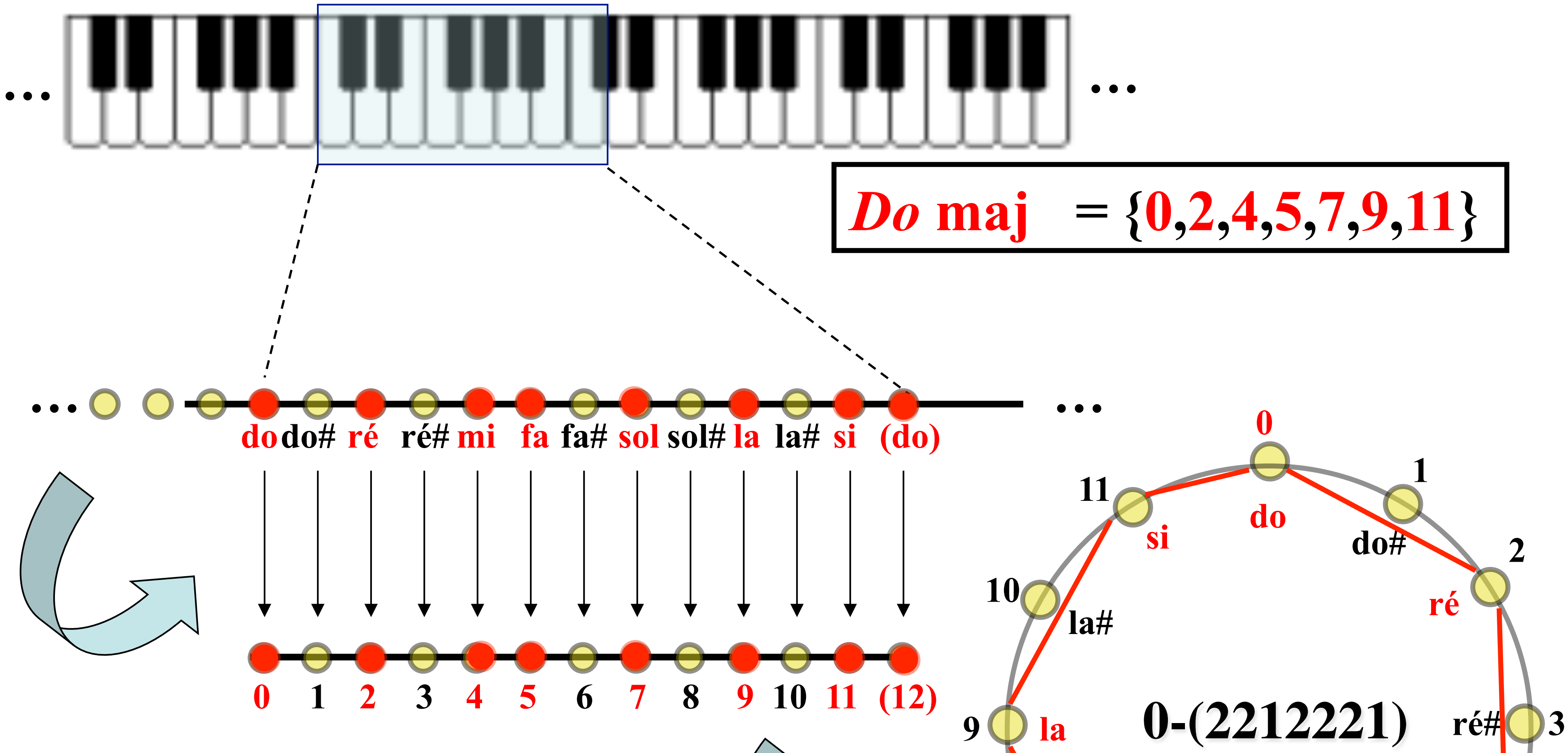
[D. Reiner, « Enumeration in Music Theory », *Amer. Math. Month.* 1985]

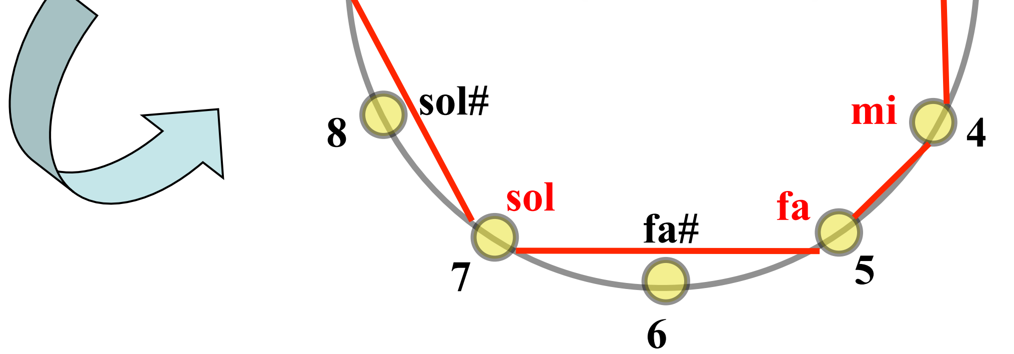
Octave reduction and mod 12 congruence



A musical scale as a polygon in a circle

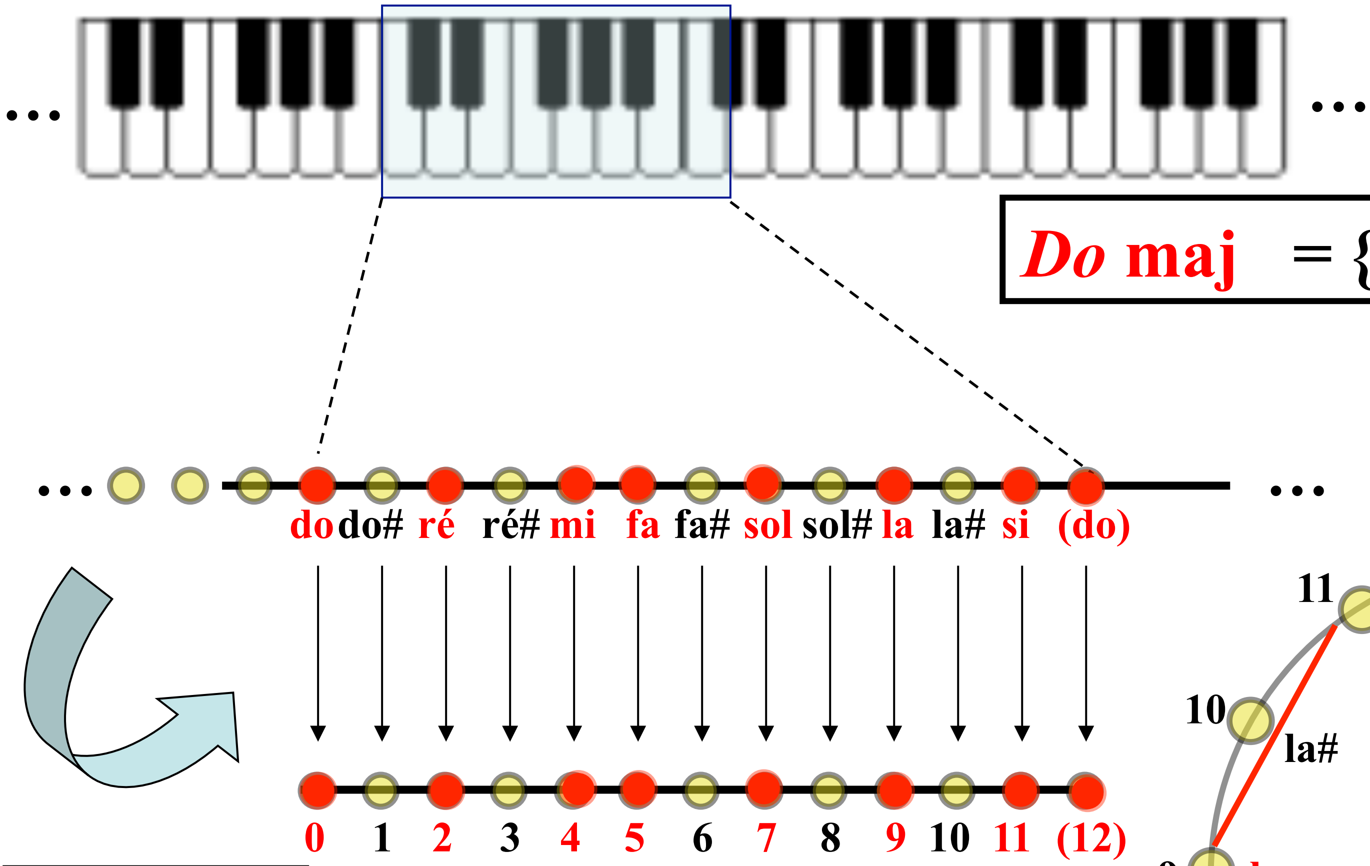
Do maj = {0,2,4,5,7,9,11}



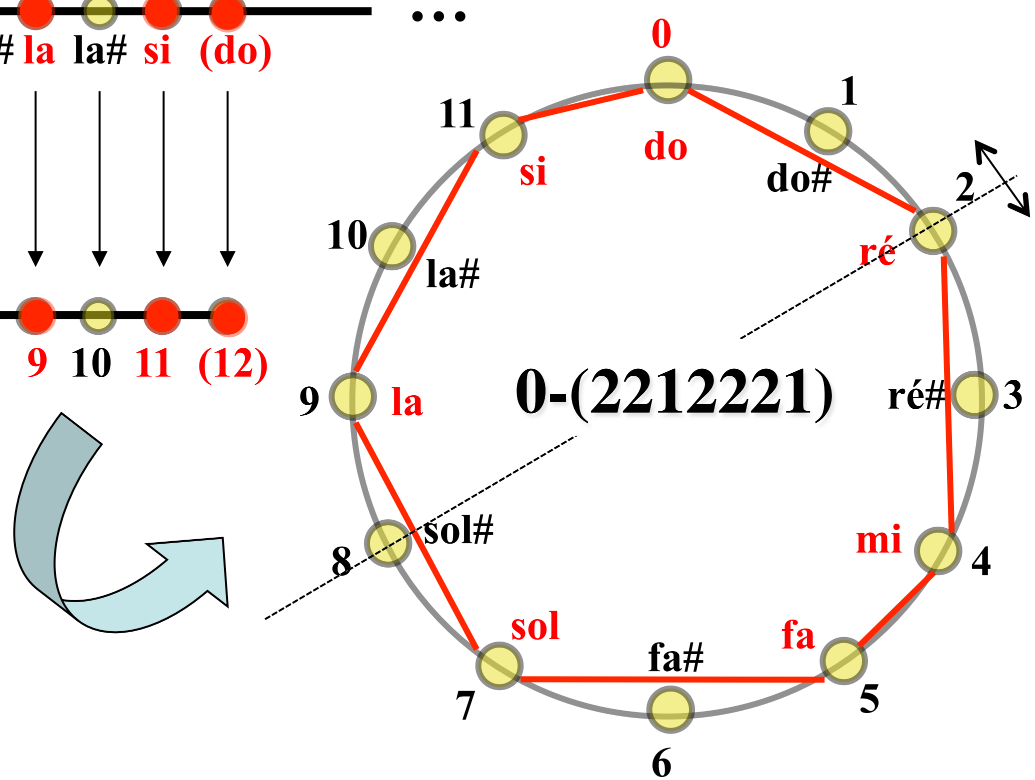


A. Riotte M. Mesnage

A musical scale as a polygon in a circle

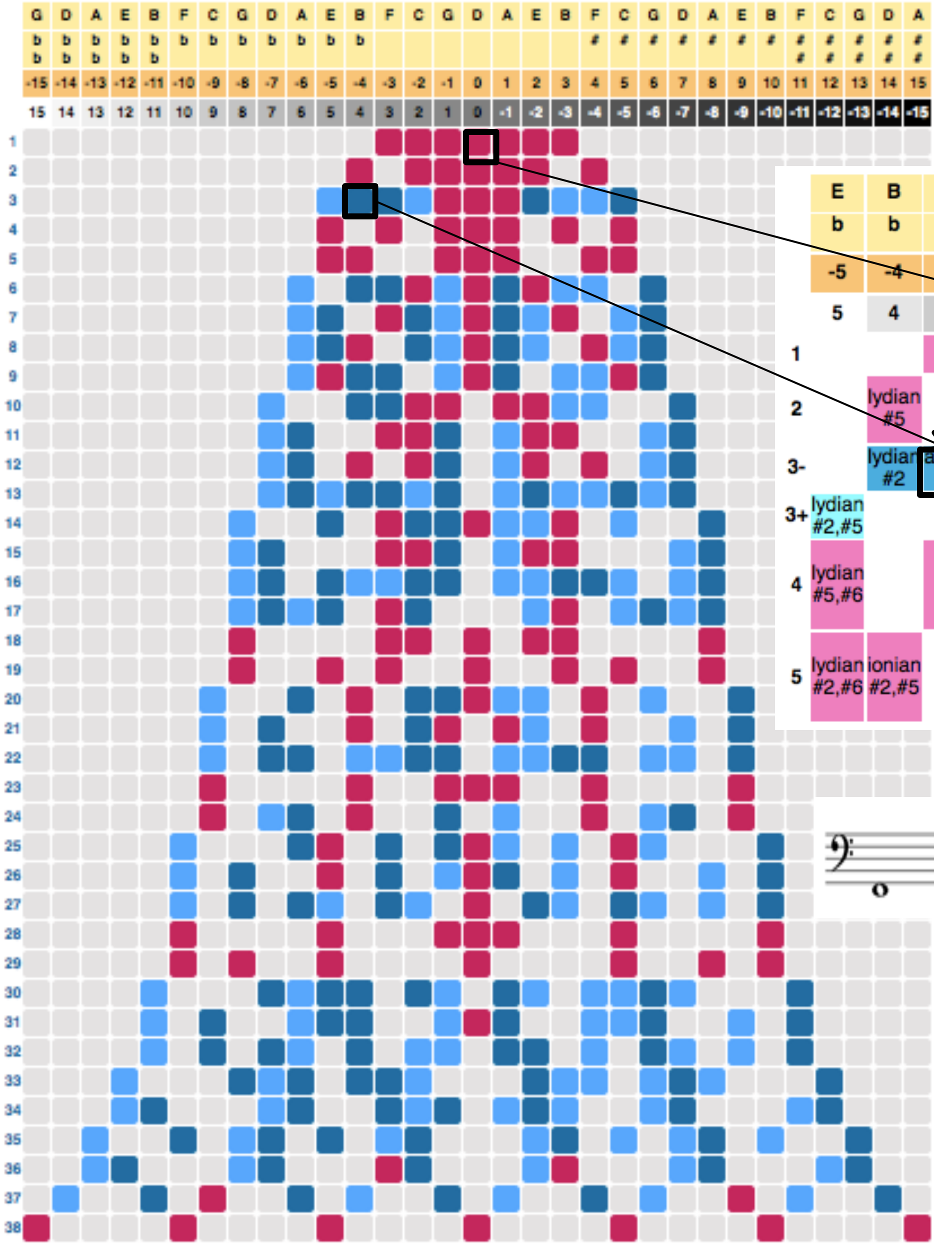


Do maj = {0,2,4,5,7,9,11}

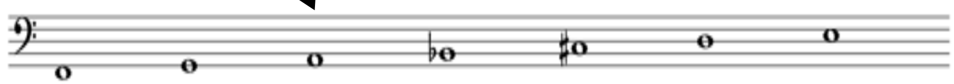


Camille Durutte

The diatonic bell (P. Audétat & co.)



	E b	B b	F	C	G	D	A	E	B	F #	C #	
	-5	-4	-3	-2	-1	0	1	2	3	4	5	
	5	4	3	2	1	0	-1	-2	-3	-4	-5	
1			lydian	ionian	mixolydian	dorian	aeolian	phrygian	locrian			diatonic
2		lydian #5		lydian b7	ionian b3	mixolydian b6 or aeolian 3	phrygian 6	locrian 2		locrian b4		minor melodic
3-		lydian #2	augmented ionian		dorian #4	aeolian 7	phrygian 3	locrian 6			locrian b4,bb7	minor harmonic
3+	lydian #2,#5			lydian b3	ionian b6	mixolydian b2	dorian b5		phrygian b4	locrian bb7		major harmonic
4	lydian #5,#6		lydian #5,b7		lydian b6,b7	ionian b2,b3 or phrygian 6,7	locrian 2,3		locrian 2,b4		locrian bb3,b4	unitonic
5	lydian #2,#6	ionian #2,#5			aeolian #4,7	ionian b2,b6 or phrygian 3,7	mixolydian b2,b5			phrygian b4,bb7	locrian bb3,bb7	double harmonic



http://www.cloche-diatonique.ch/

Junod, J., Audétat, P., Agon, C., Andreatta, M., « A Generalisation of Diatonicism and the Discrete Fourier Transform as a Mean for Classifying and Characterising Musical Scales », Second International Conference MCM 2009, vol. 38, New Haven, 2009, pp. 166-179

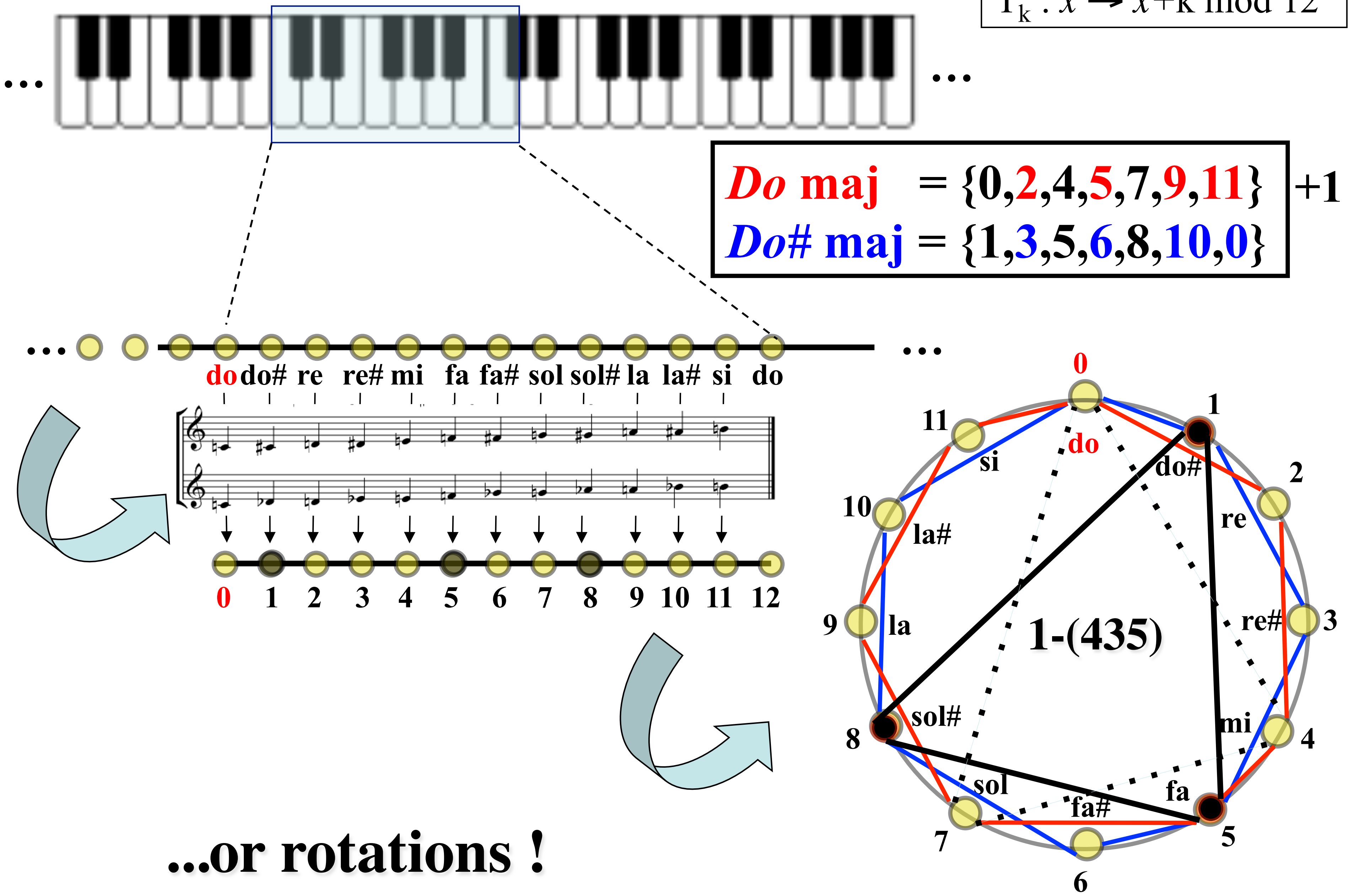
Musical transpositions are additions...

$T_k : x \rightarrow x + k \bmod 12$

Do maj = {0,2,4,5,7,9,11}

Do# maj = {1,3,5,6,8,10,0}

+1



...or rotations !

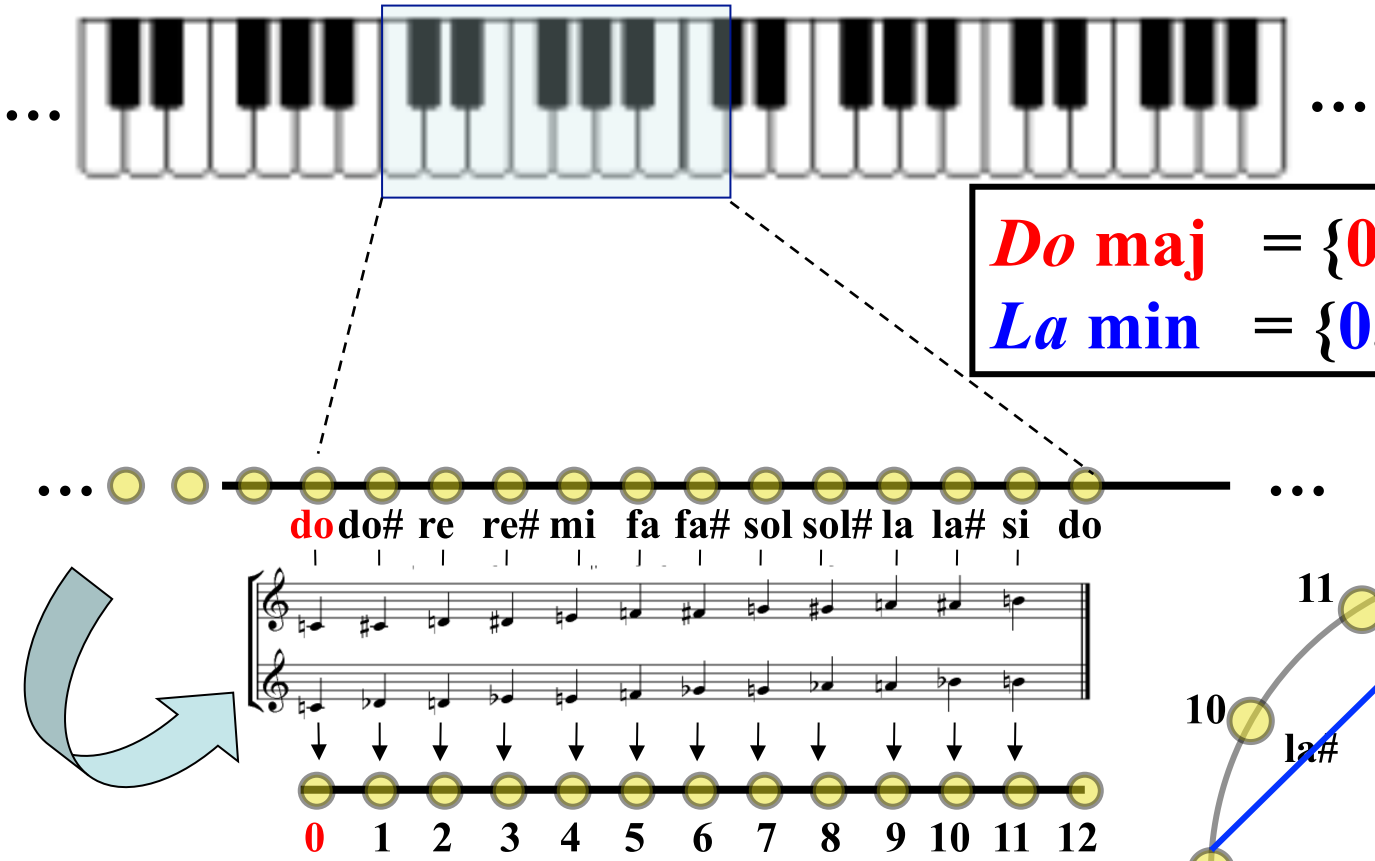
Musical inversions are differences...

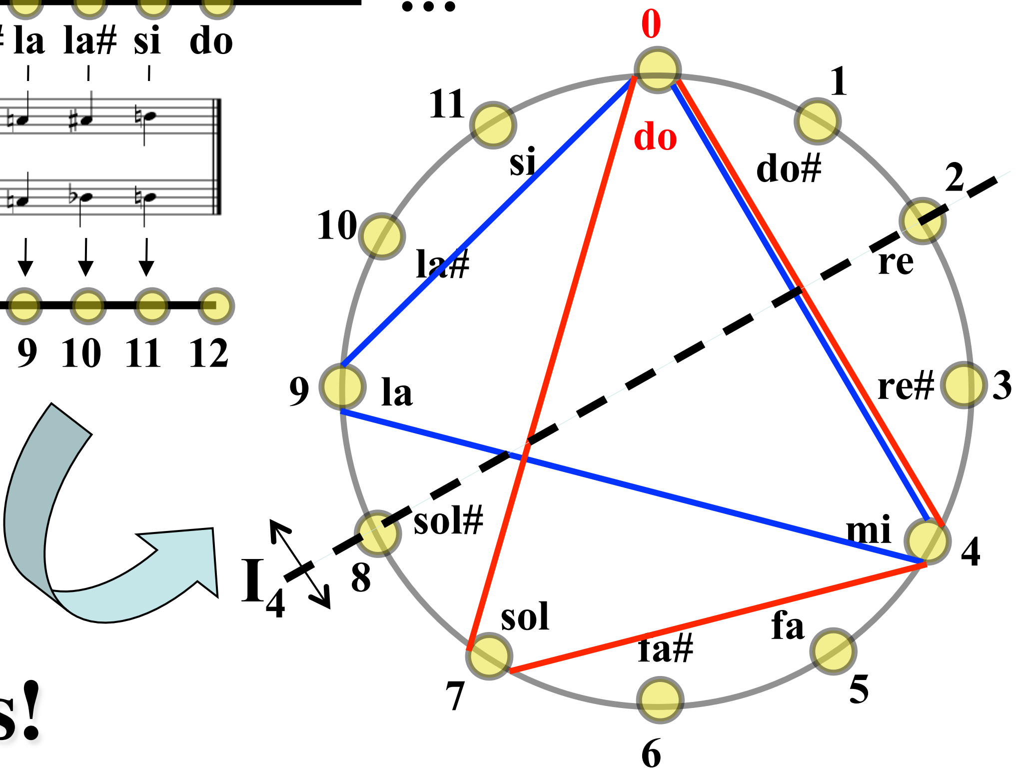
$\mathrm{I} : x \to -x \bmod 12$

***Do* maj** $= \{0,4,7\}$

***La* min** $= \{0,4,9\}$

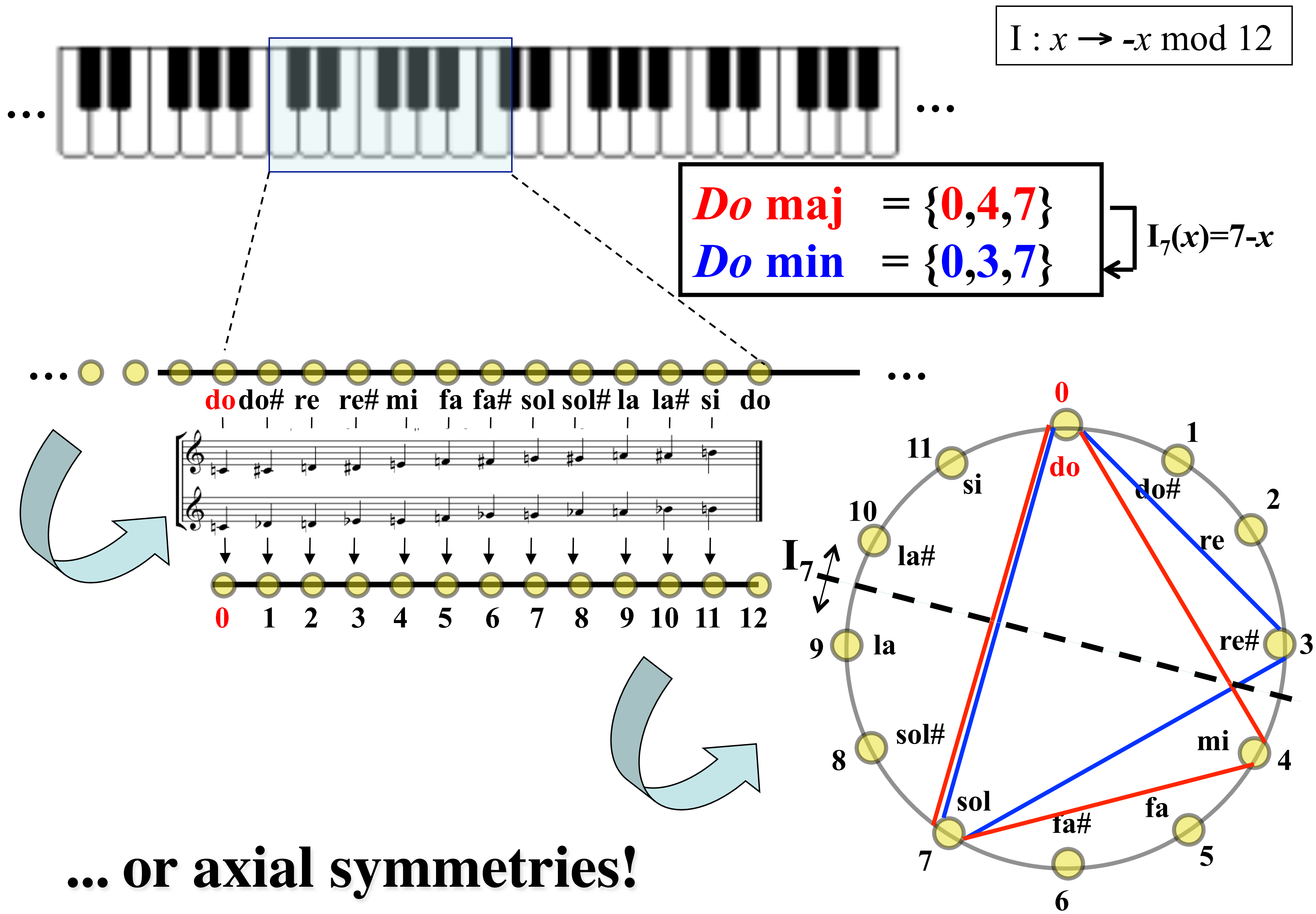
$\mathrm{I}_4(x) = 4 - x$





... or axial symmetries!

Musical inversions are differences...



... or axial symmetries!

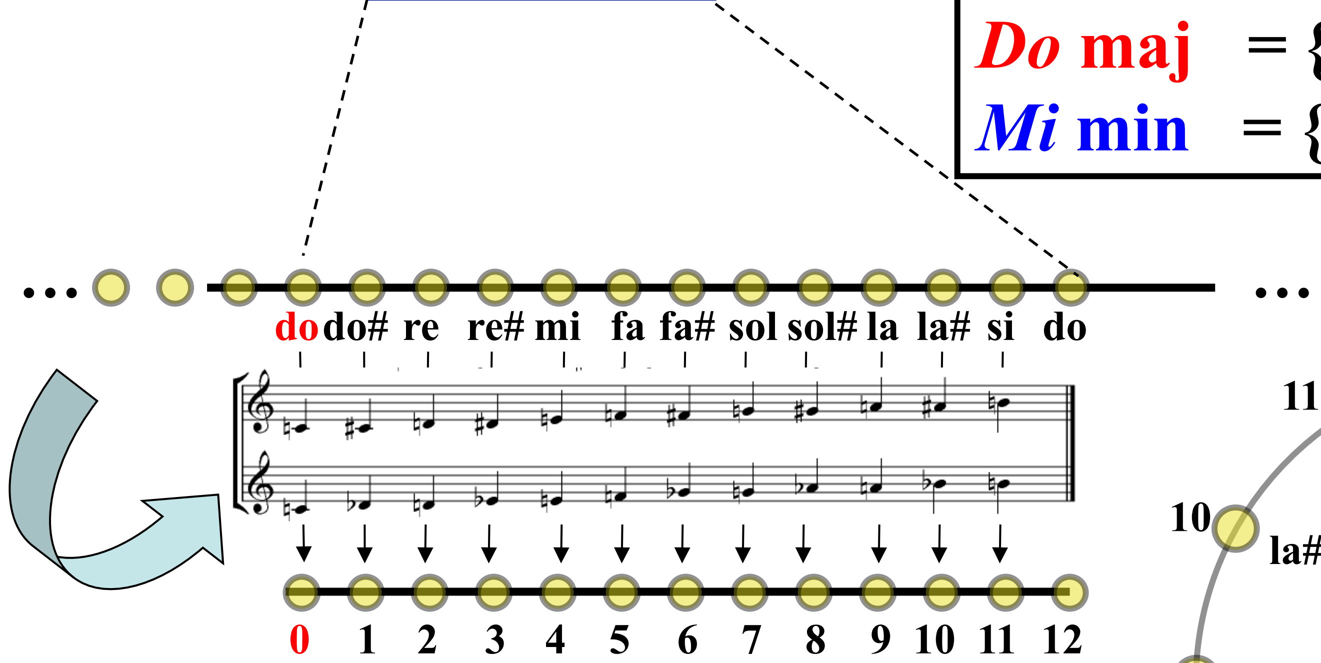
Musical inversions are differences...

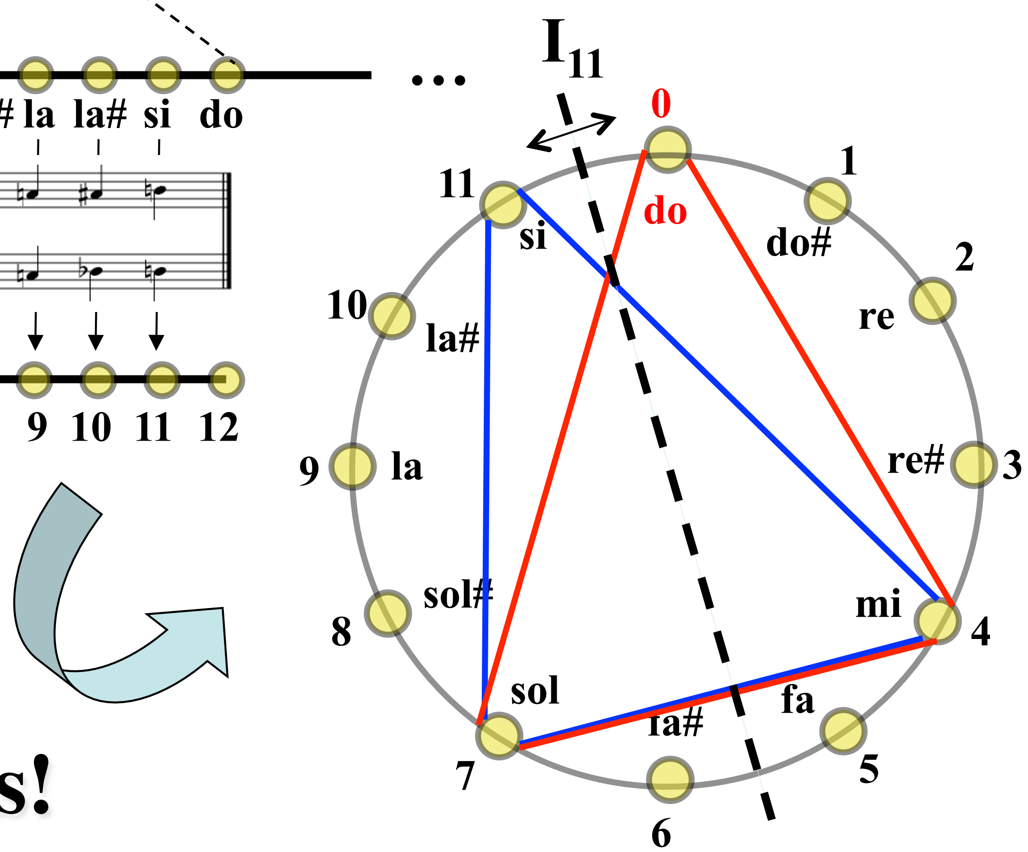
I : $x \rightarrow -x \bmod 12$

***Do* maj** = {0,4,7}

***Mi* min** = {4,7,11}

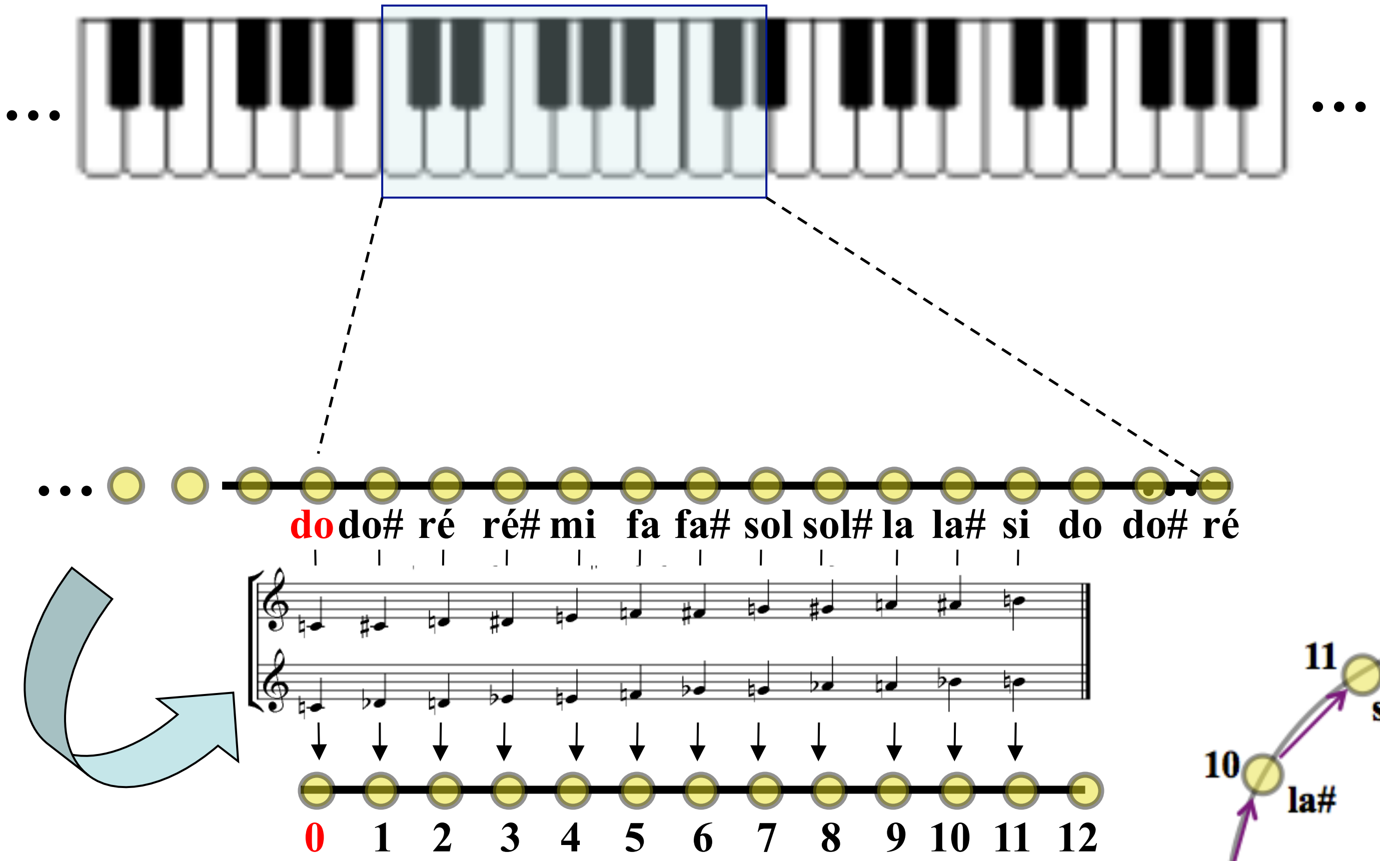
$I_{11}(x)=11-x$





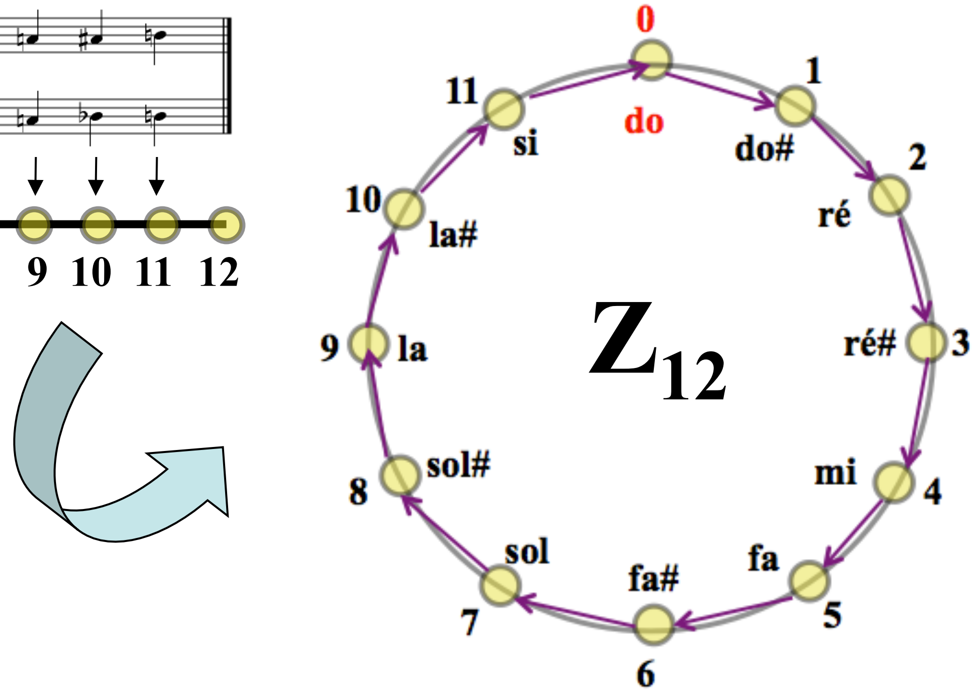
... or axial symmetries!

The equal tempered space is a cyclic group

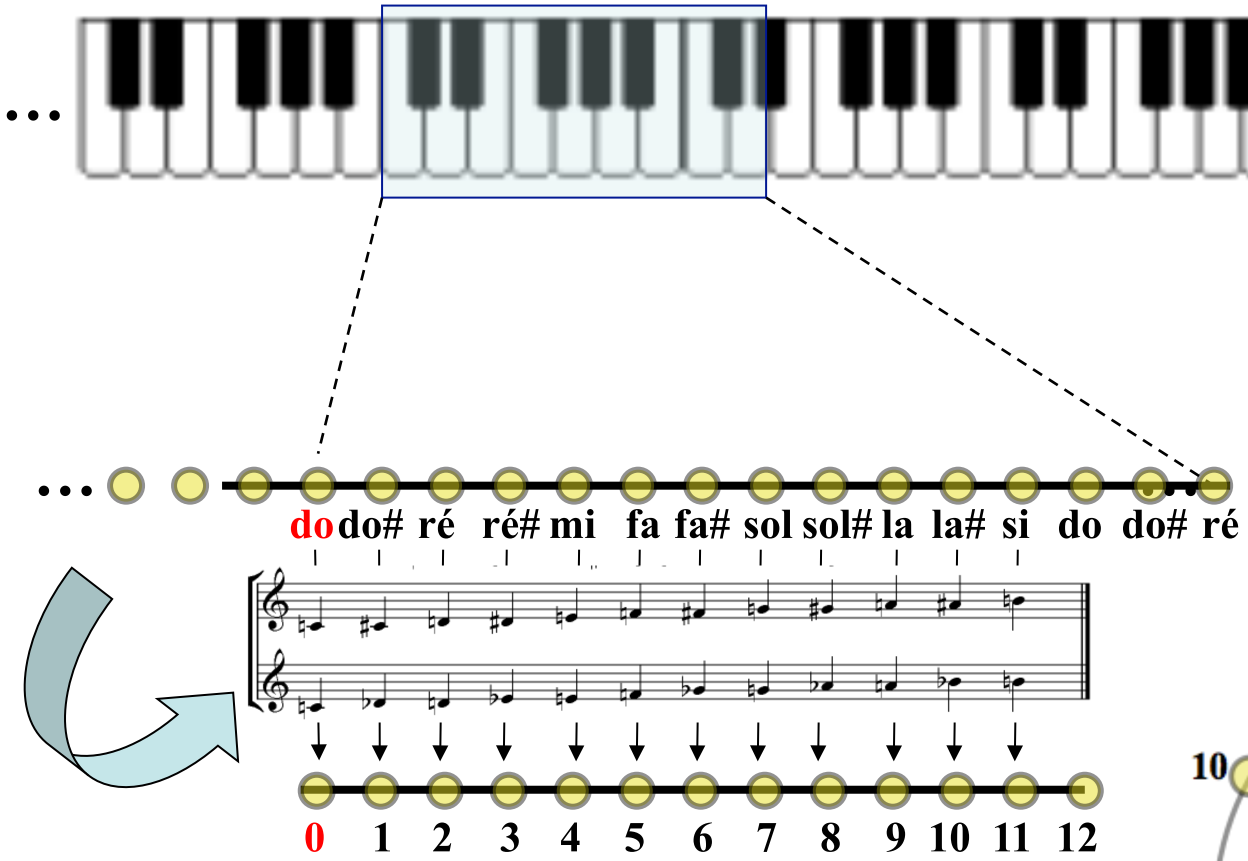


The generators of the cyclic group of order 12 are the transpositions T_1, T_5, T_7 et T_{11}

$$Z_{12} = < T_1 \mid (T_1)^{12} = T_0 >$$

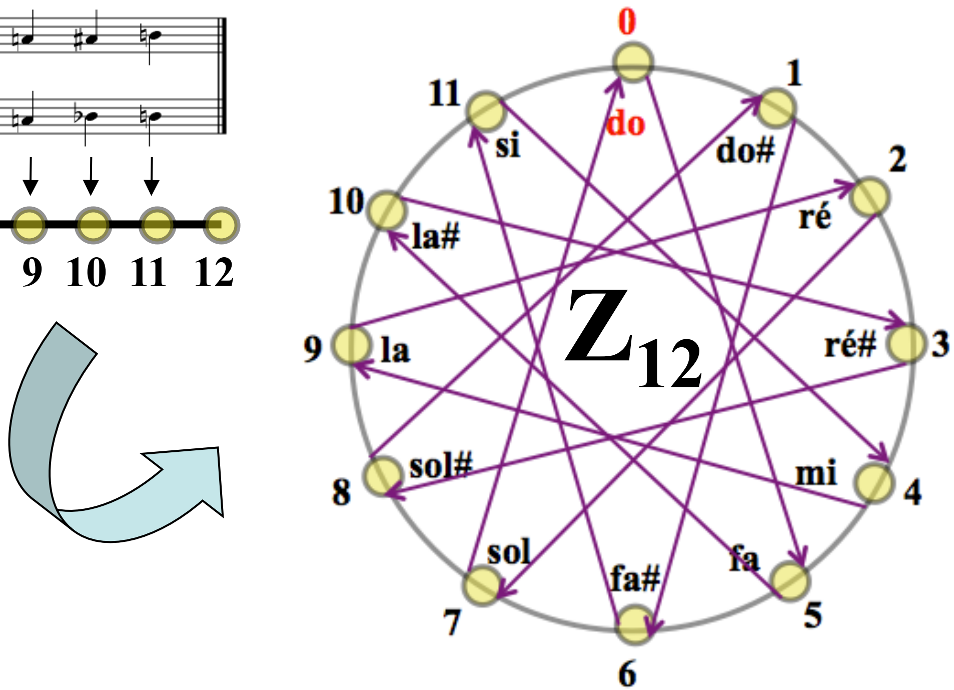


The equal tempered space is a cyclic group

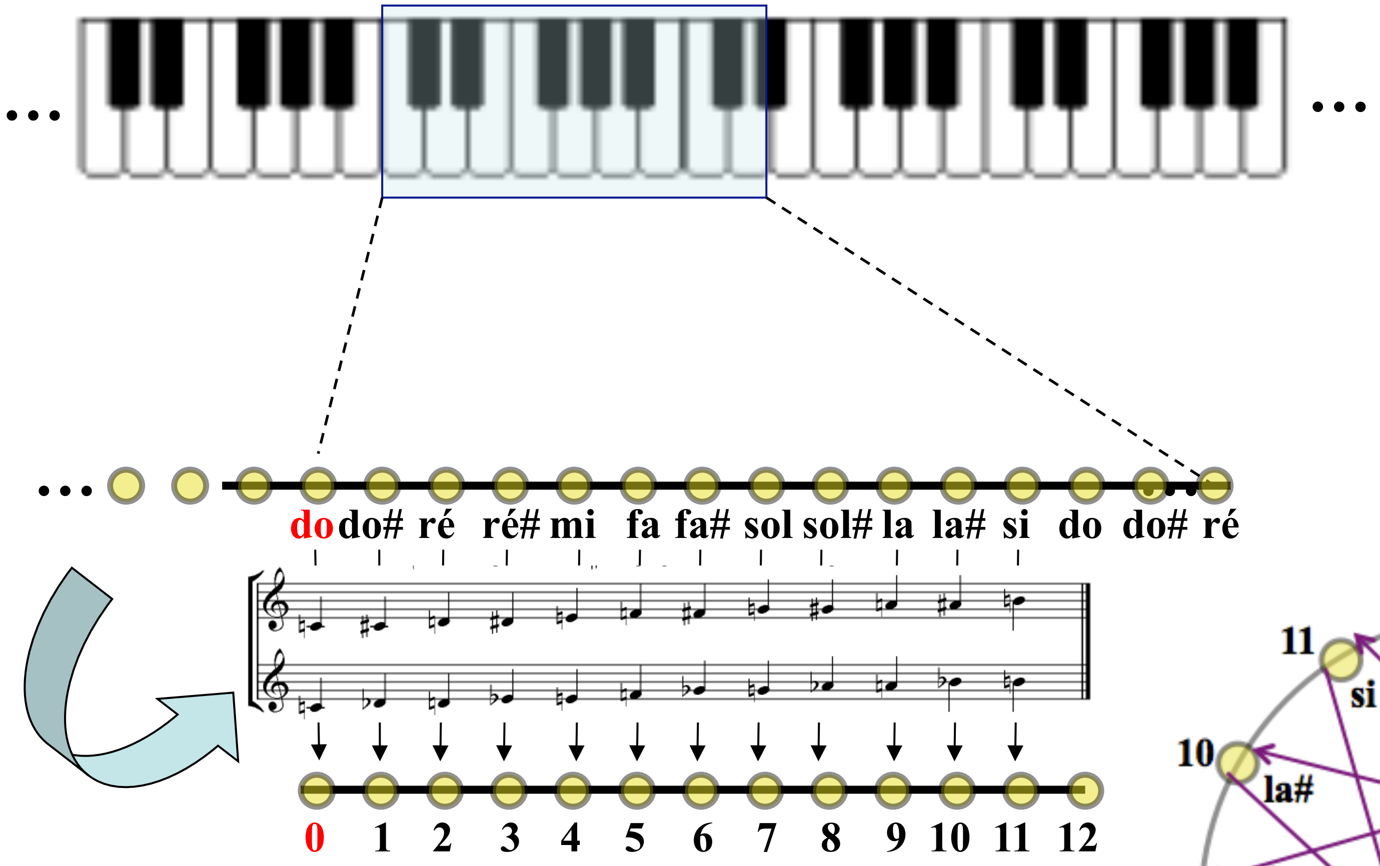


The generators of the cyclic group of order 12 are the transpositions T_1, T_5, T_7 et T_{11}

$$\mathbf{Z_{12}} = \langle T_1 \mid (T_1)^{12} = T_0 \rangle =$$
$$= \langle \mathbf{T_5} \mid (\mathbf{T_5})^{12} = \mathbf{T_0} \rangle$$

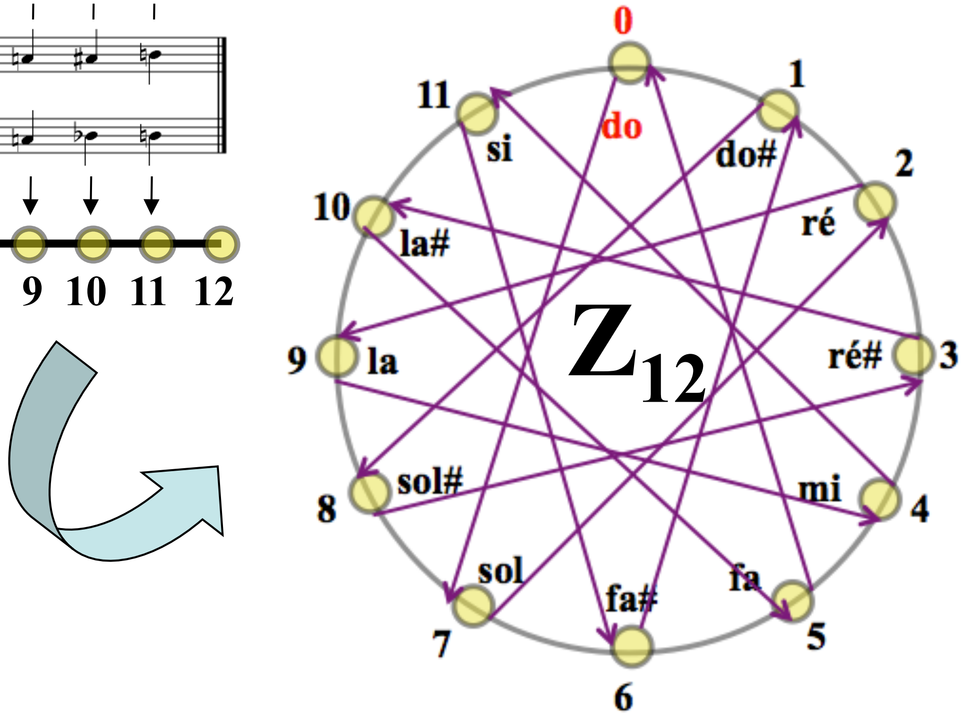


The equal tempered space is a cyclic group

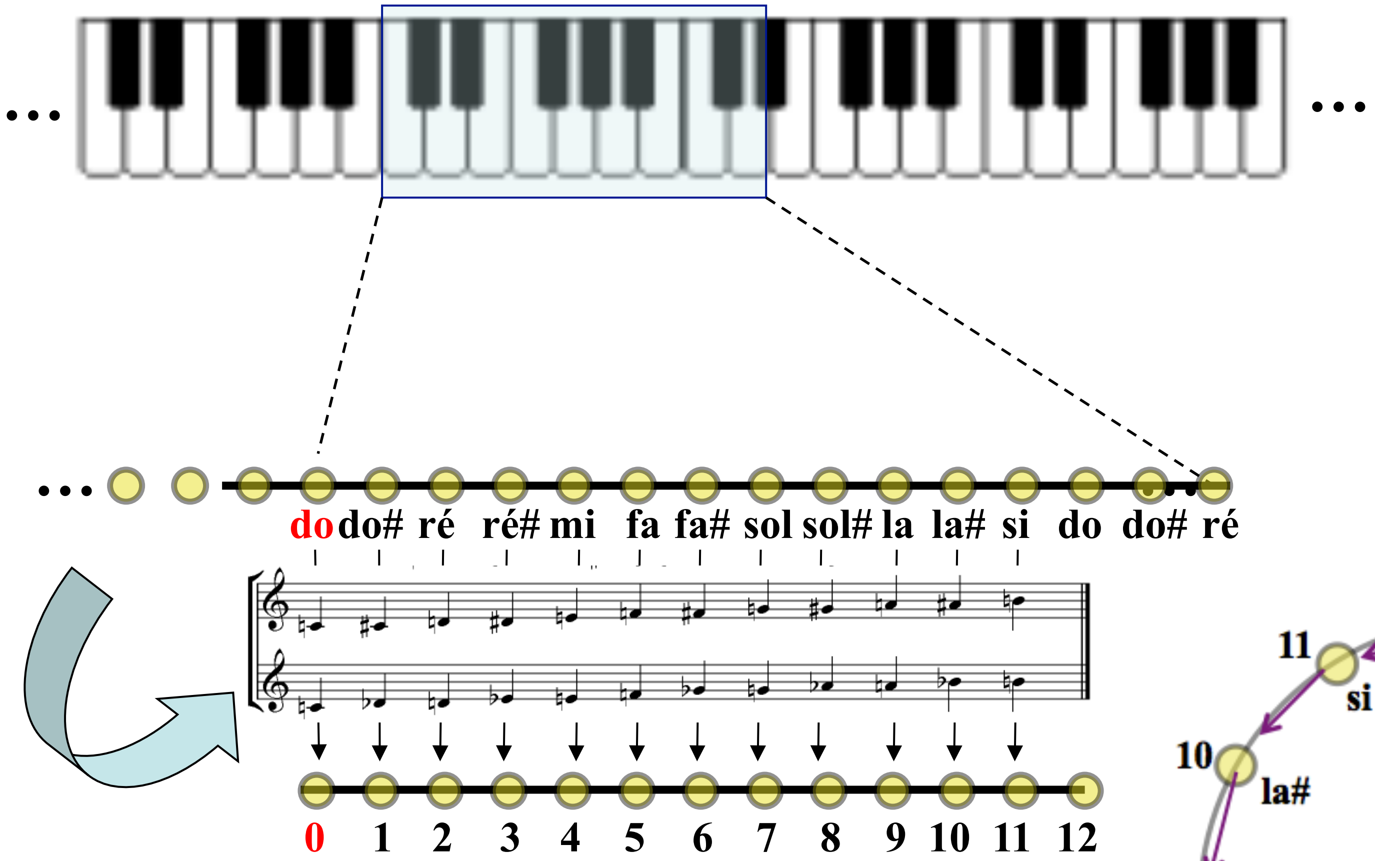


The generators of the cyclic group of order 12 are the transpositions T_1 , T_5 , T_7 et T_{11}

$$\mathbf{Z_{12}} = \langle T_1 \mid (T_1)^{12} = T_0 \rangle =$$
$$= \langle T_5 \mid (T_5)^{12} = T_0 \rangle =$$
$$\mathbf{= \langle T_7 \mid (T_7)^{12} = T_0 \rangle}$$

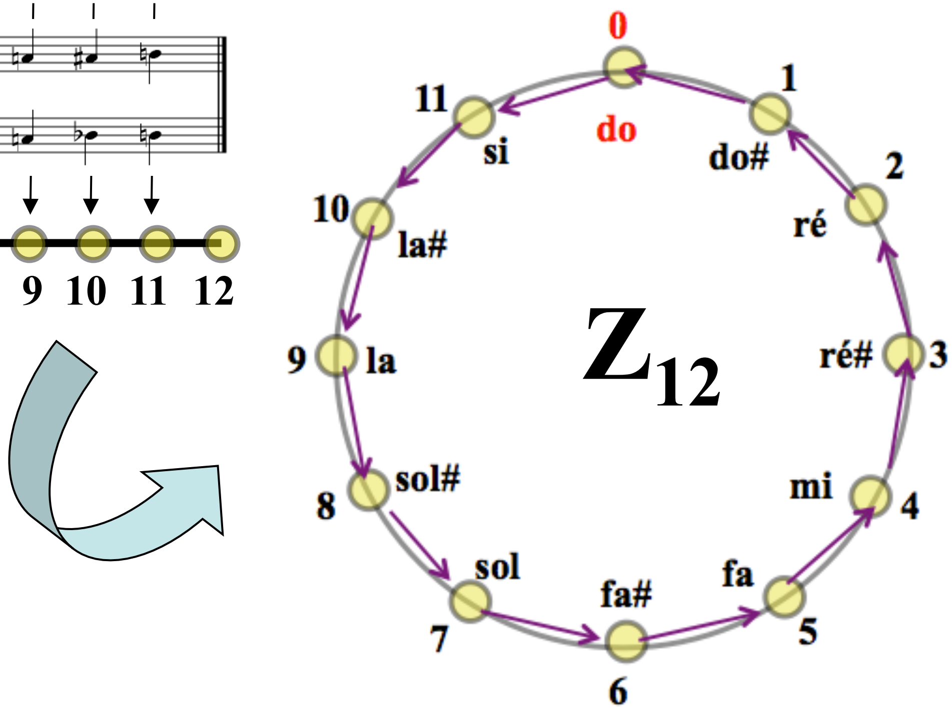


The equal tempered space is a cyclic group



The generators of the cyclic group of order 12 are the transpositions T_1, T_5, T_7 et T_{11}

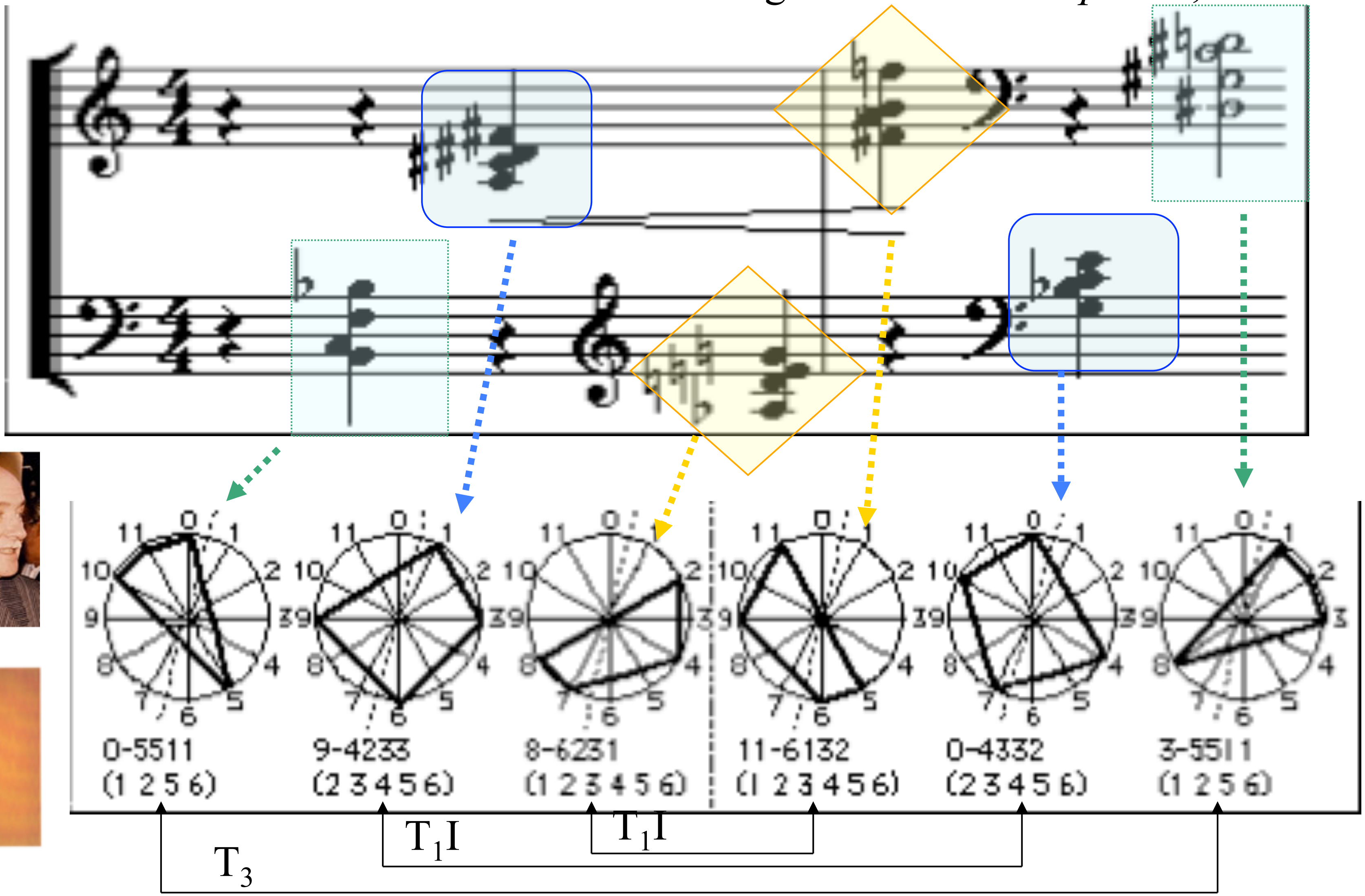
$$\mathbf{Z_{12}} = < T_1 \mid (T_1)^{12} = T_0 > =$$
$$= < T_5 \mid (T_5)^{12} = T_0 > =$$
$$= < T_7 \mid (T_7)^{12} = T_0 > =$$
$$\mathbf{= < T_{11} \mid (T_{11})^{12} = T_0 >}$$



« Entités formelles pour l'analyse musicale »

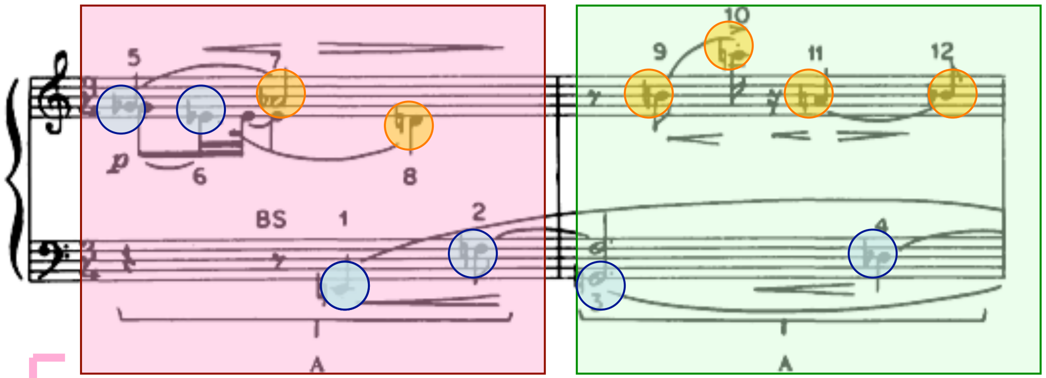
Marcel Mesnage (1998)

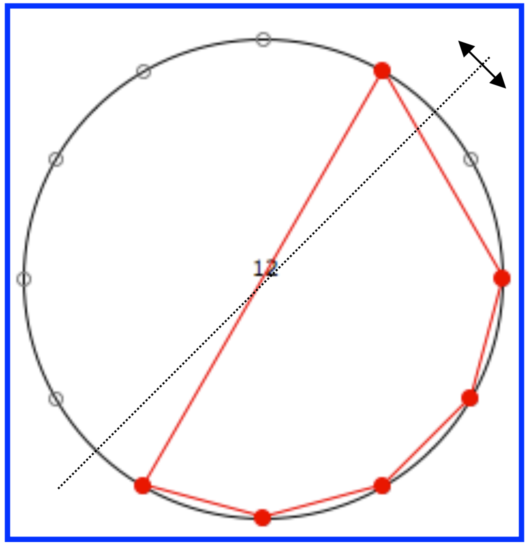
A. Schoenberg : *Klavierstück Op. 33a*, 1929



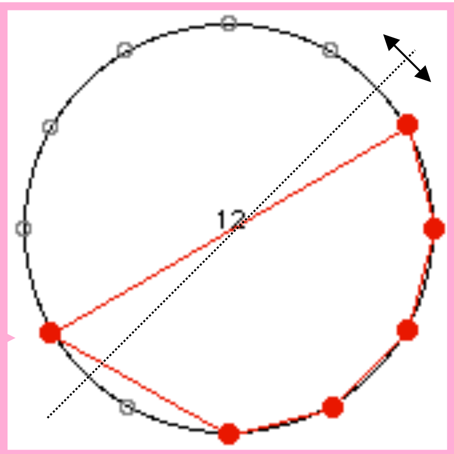
Serialism and hexachordal combinatoriality

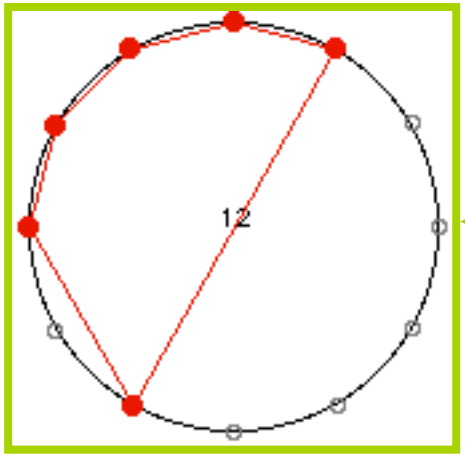
Schoenberg: Suite Op.25, Minuetto

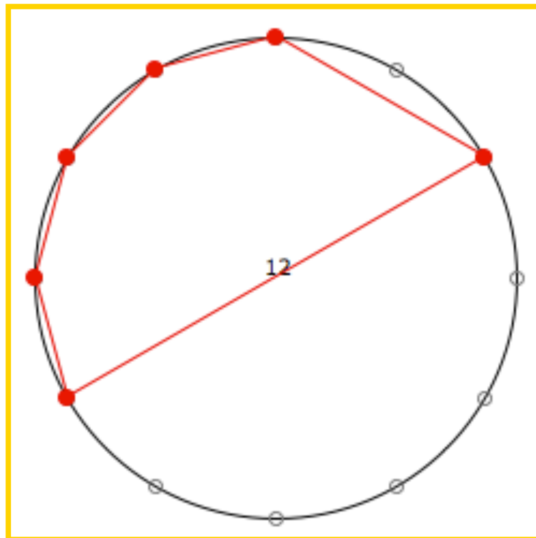




Double combinatoriality

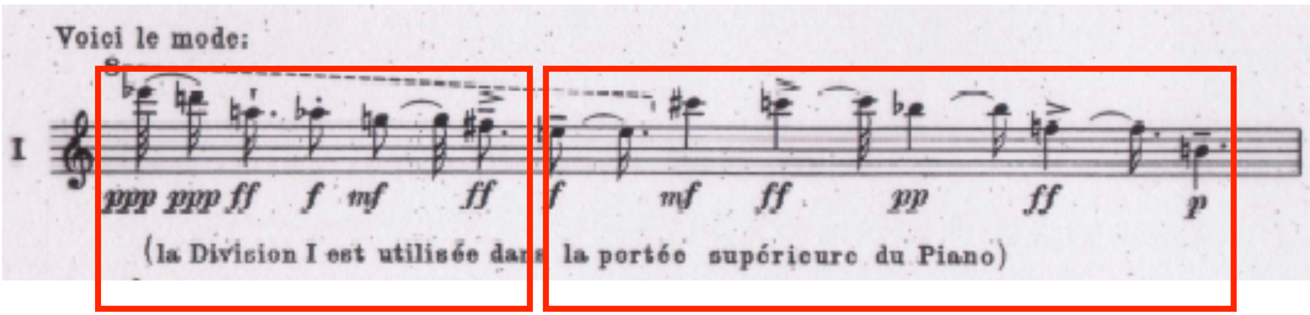


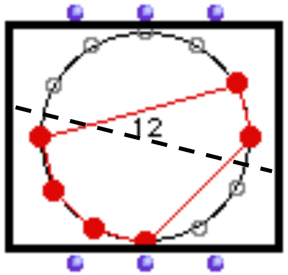




Hexachordal Combinatoriality in Messiaen

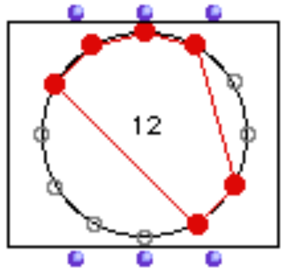
- Mode de valeurs et d'intensités (1950)





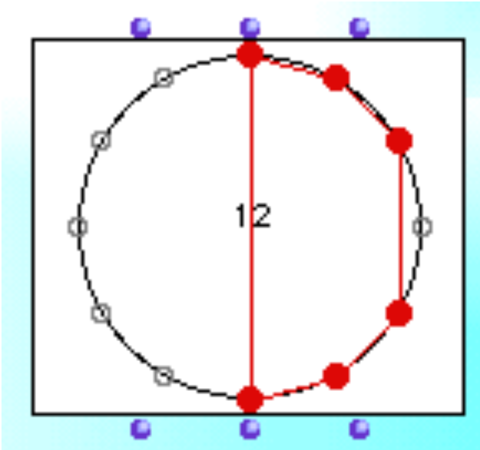
$$\{3,2,9,8,7,6\} \longrightarrow \{4,5,10,11,0,1\}$$

$T_7I : x \rightarrow 7\text{-}x$



A ‘Mathemusical’ Theorem

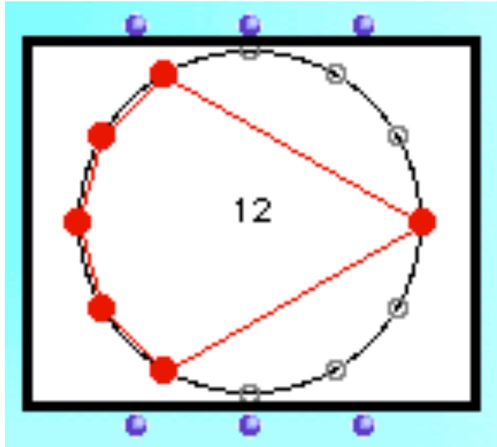
A



Z-relation

$$IC_A = IC_{A'}$$

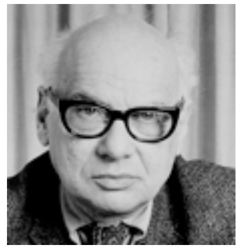
homometry



A’

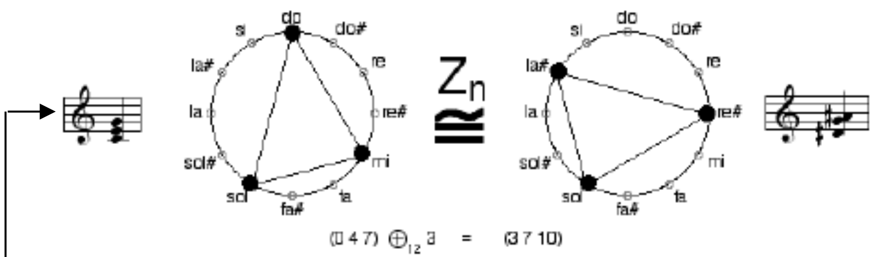
$$IC_A = [4, 3, 2, 3, 2, 1\ ] \quad = \quad [4, 3, 2, 3, 2, 1\ ] = IC_{A'}$$

Babbitt’s Hexachord Theorem:
A hexacord and its complement have the same interval content



(Proofs by Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, …, Amiot, …)

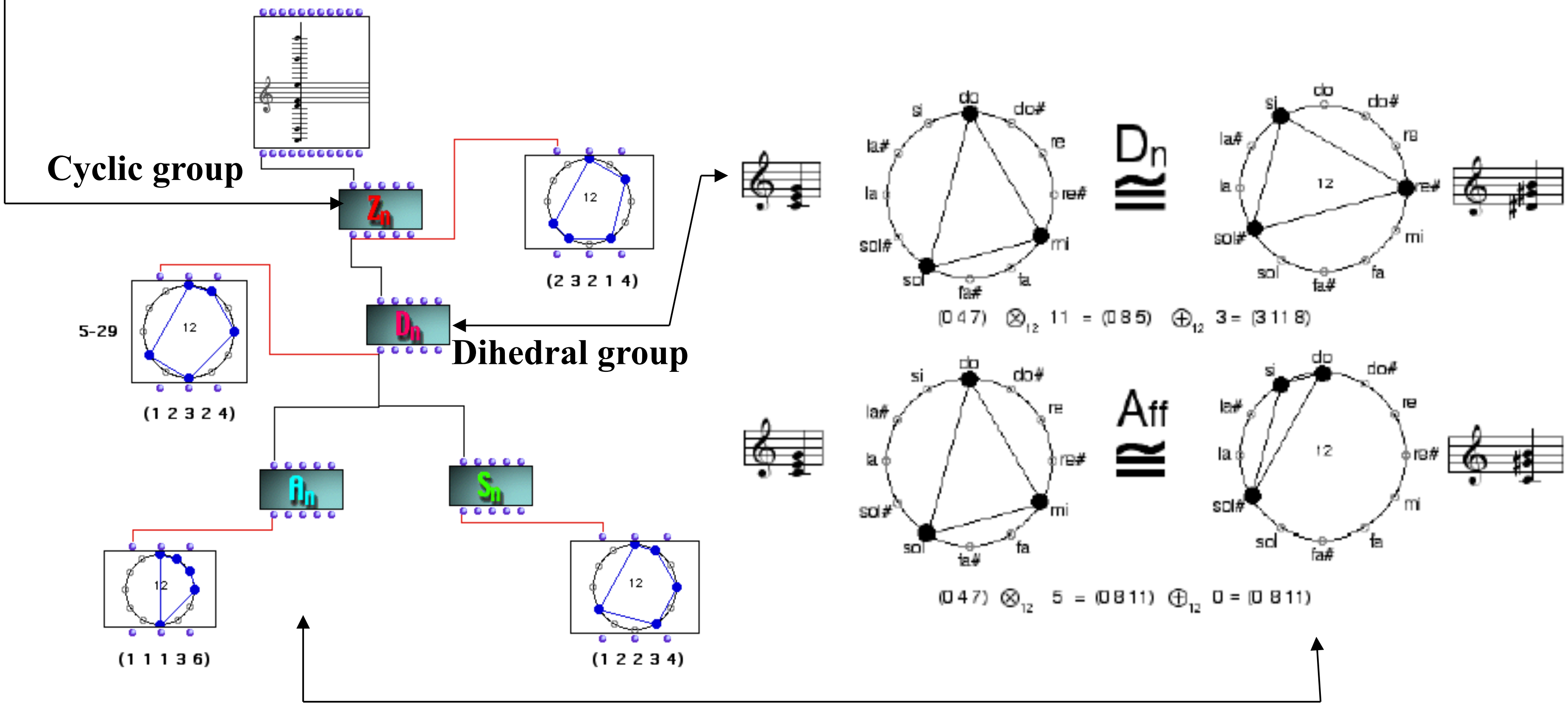
Equivalence classes of chords (up to a group action)



$\mathbf{Z_{12}} = < T_k \mid (T_k)^{12} = T_0 >$

$\mathbf{D_{12}} = < T_k\, ,\, I \mid (T_k)^{12} = I^2 = T_0\, ,\, ITI = I(IT)^{-1} >$

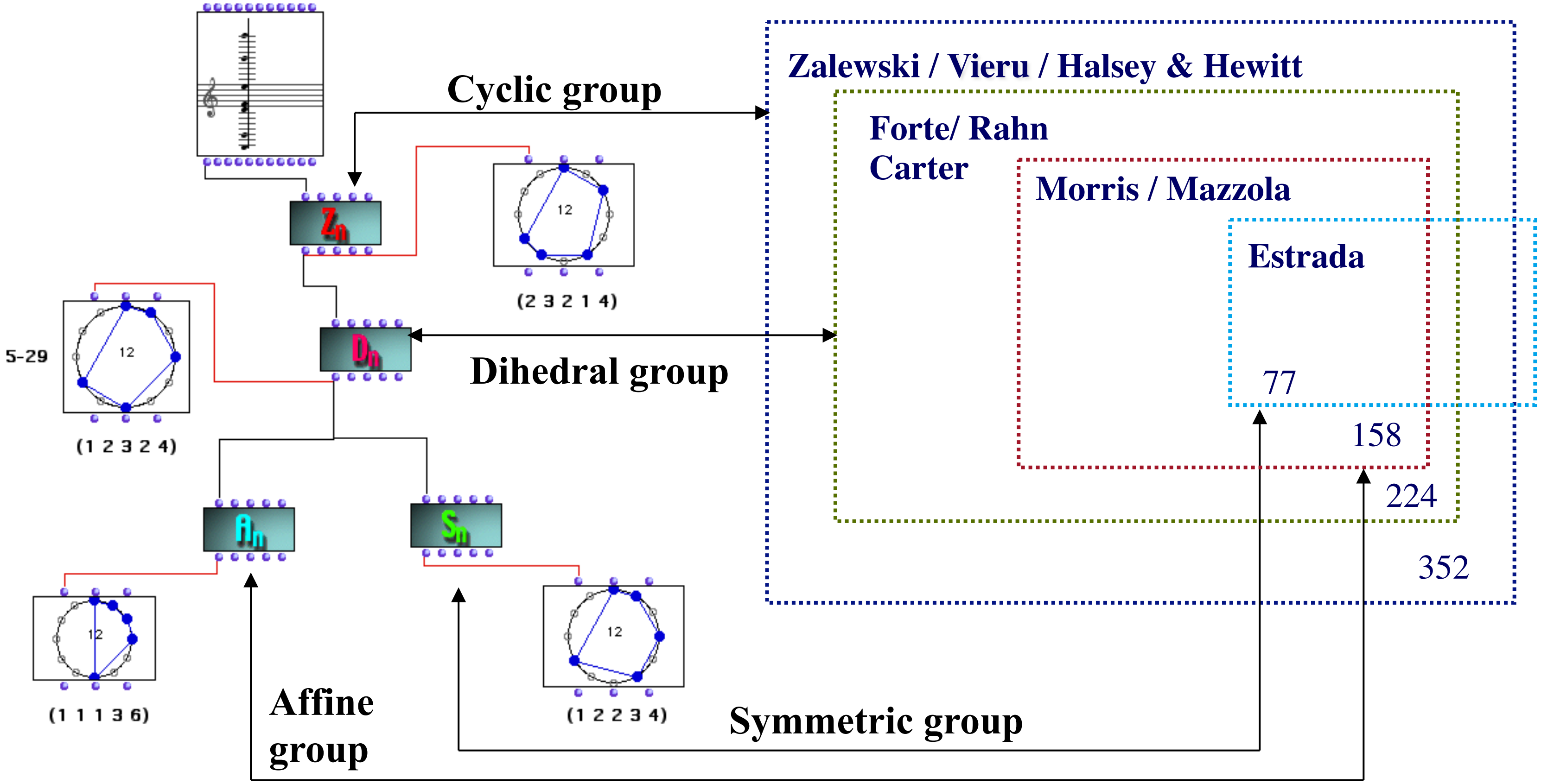
$\mathbf{Aff} = \{ f \mid f(x) = ax + b,\ a \in (\mathbf{Z_{12}})^*,\ b \in \mathbf{Z_{12}} \}$



Affine group

Paradigmatic architecture

Group actions and the classification of musical structures



F. Klein

W. Burnside

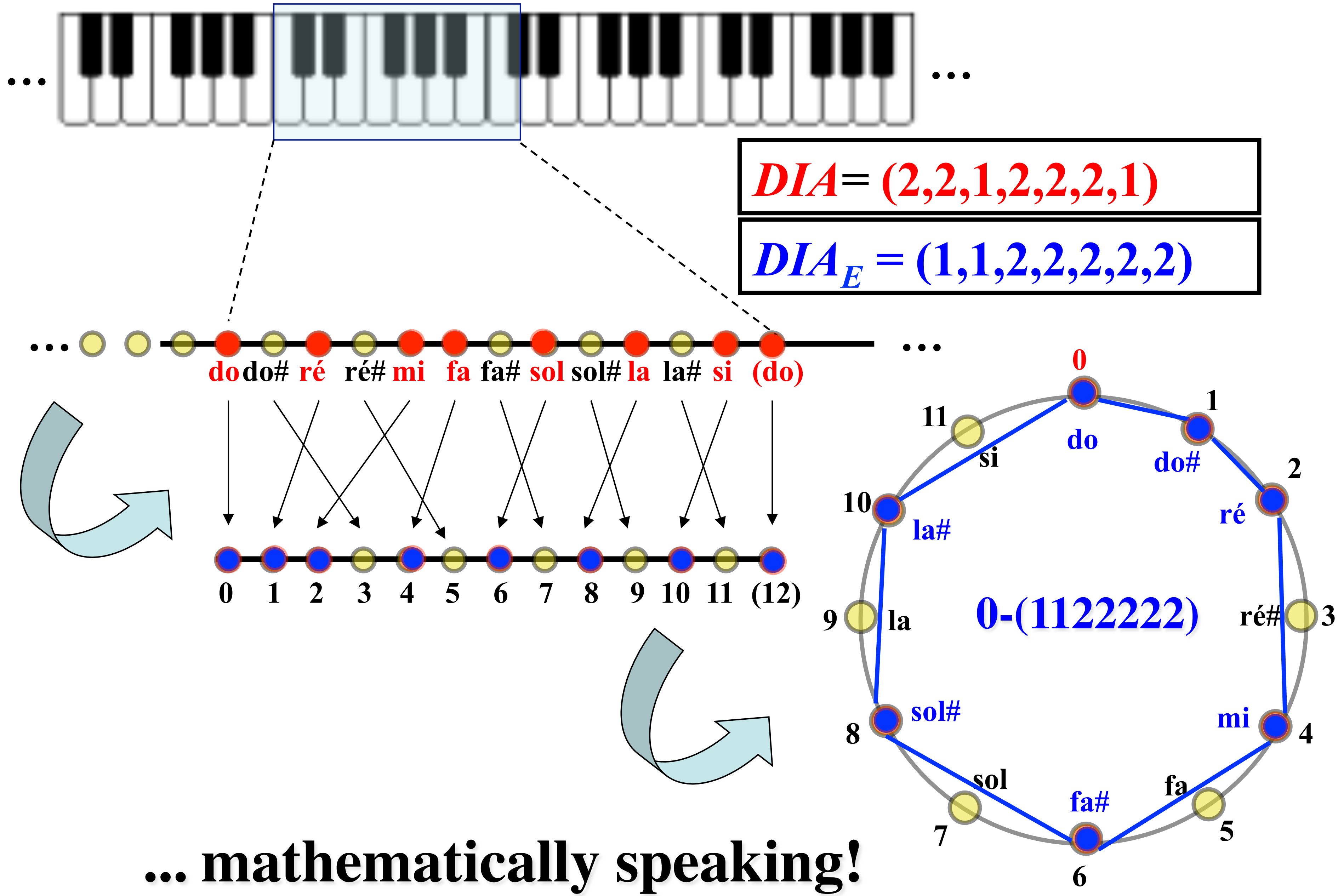
G. Polya

	1	2	3	4	5	6	7	8	9	10	11	12
Z_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1
S_n	*1*	*6*	*12*	*15*	*12*	*11*	*7*	*5*	*3*	*2*	*1*	*1*

Permutations are ‘partitions’...

$DIA = (2,2,1,2,2,2,1)$

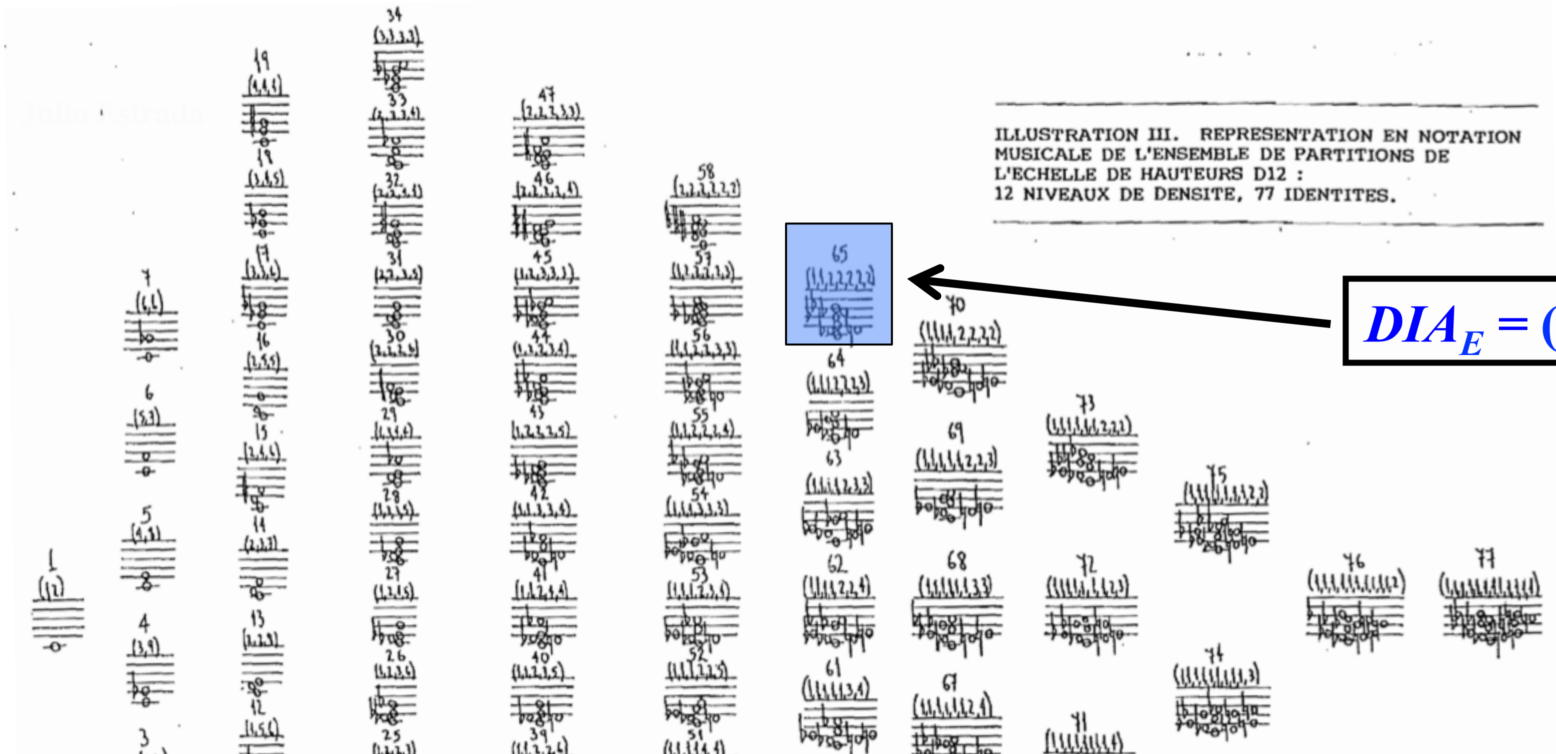
$DIA_E = (1,1,2,2,2,2,2)$



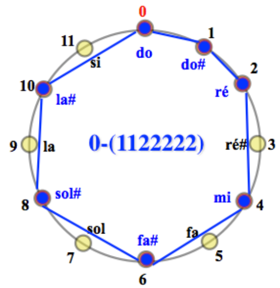
... mathematically speaking!

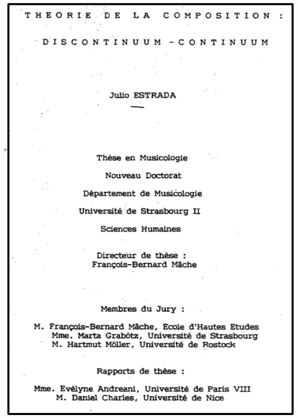
The permutohedron as a combinatorial space

Julio Estrada, *Théorie de la composition : discontinuum – continuum,* université de Strasbourg II, 1994



$DIA_E = (1,1,2,2,2,2,2)$




THEORIE DE LA COMPOSITION :

DISCONTINUUM - CONTINUUM

Julio ESTRADA

Thèse en Musicologie

Nouveau Doctorat

Département de Musicologie

Université de Strasbourg II

Sciences Humaines

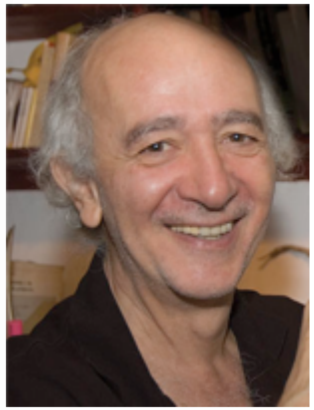
Directeur de thèse :
François-Bernard Mâche

Membres du Jury :

M. François-Bernard Mâche, Ecole d'Hautes Etudes
Mme. Marta Grabócz, Université de Strasbourg
M. Hartmut Möller, Université de Rostock

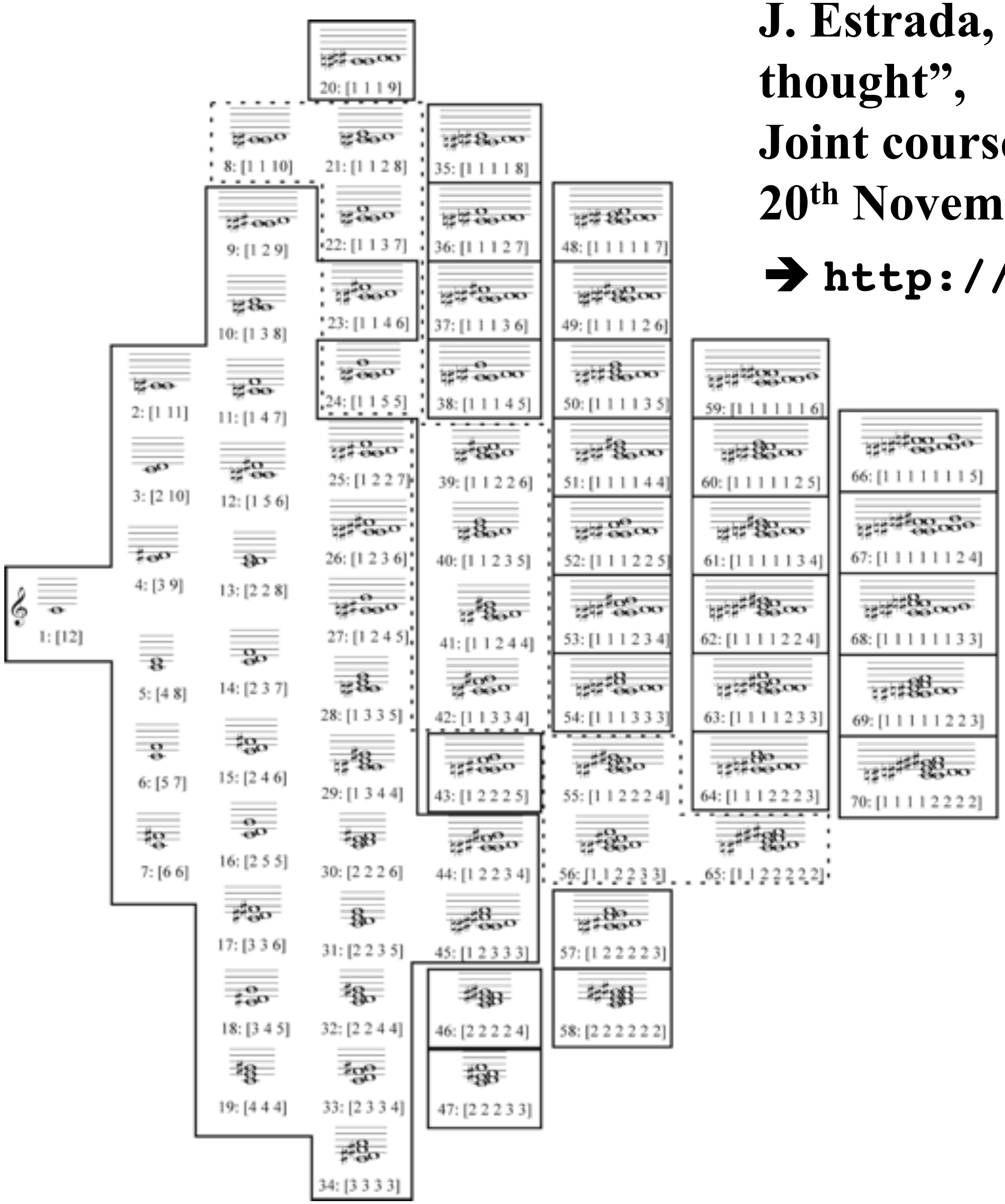
Rapports de thèse :

Mme. Evélyne Andreani, Université de Paris VIII
M. Daniel Charles, Université de Nice



J. Estrada

The permutohedron as a musical conceptual space

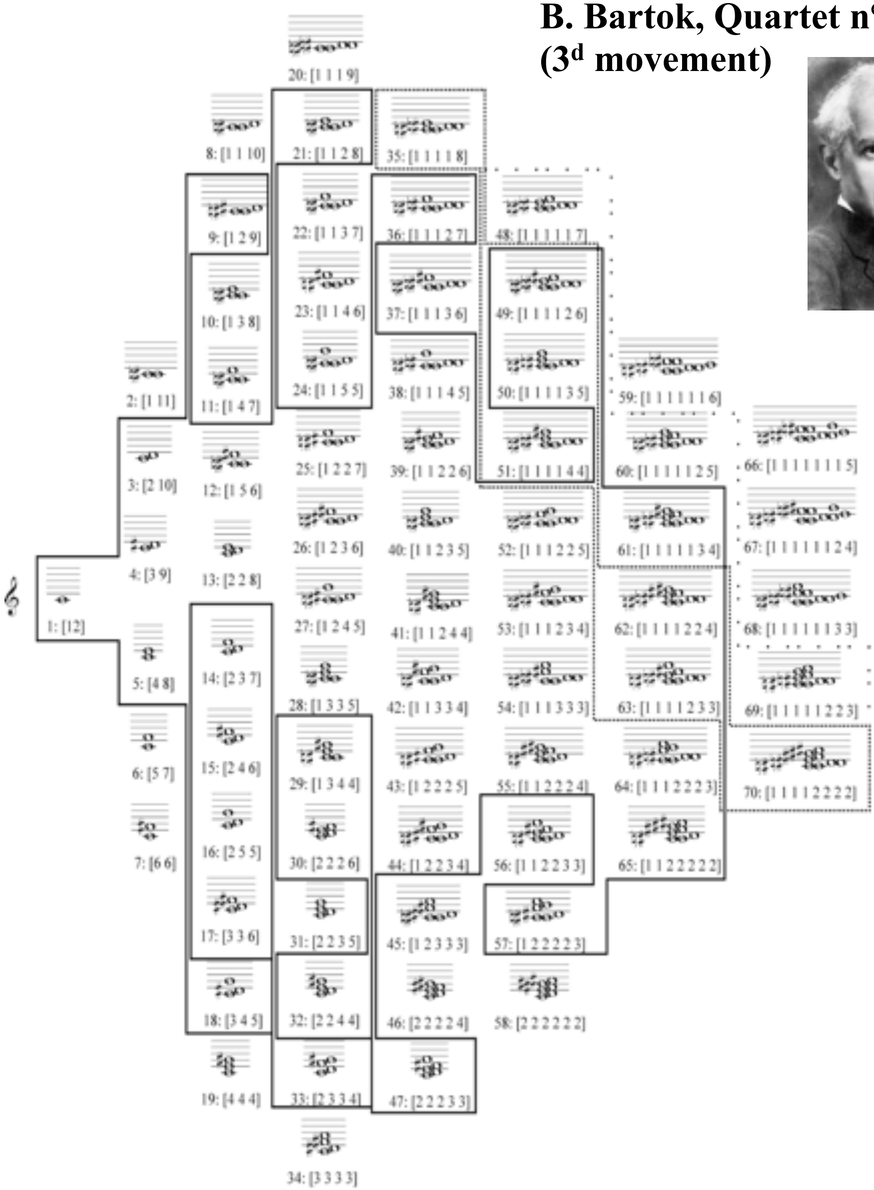



J. Estrada, "The intervallic thought",
Joint course ATIAM/Cursus ,
20th November 2012

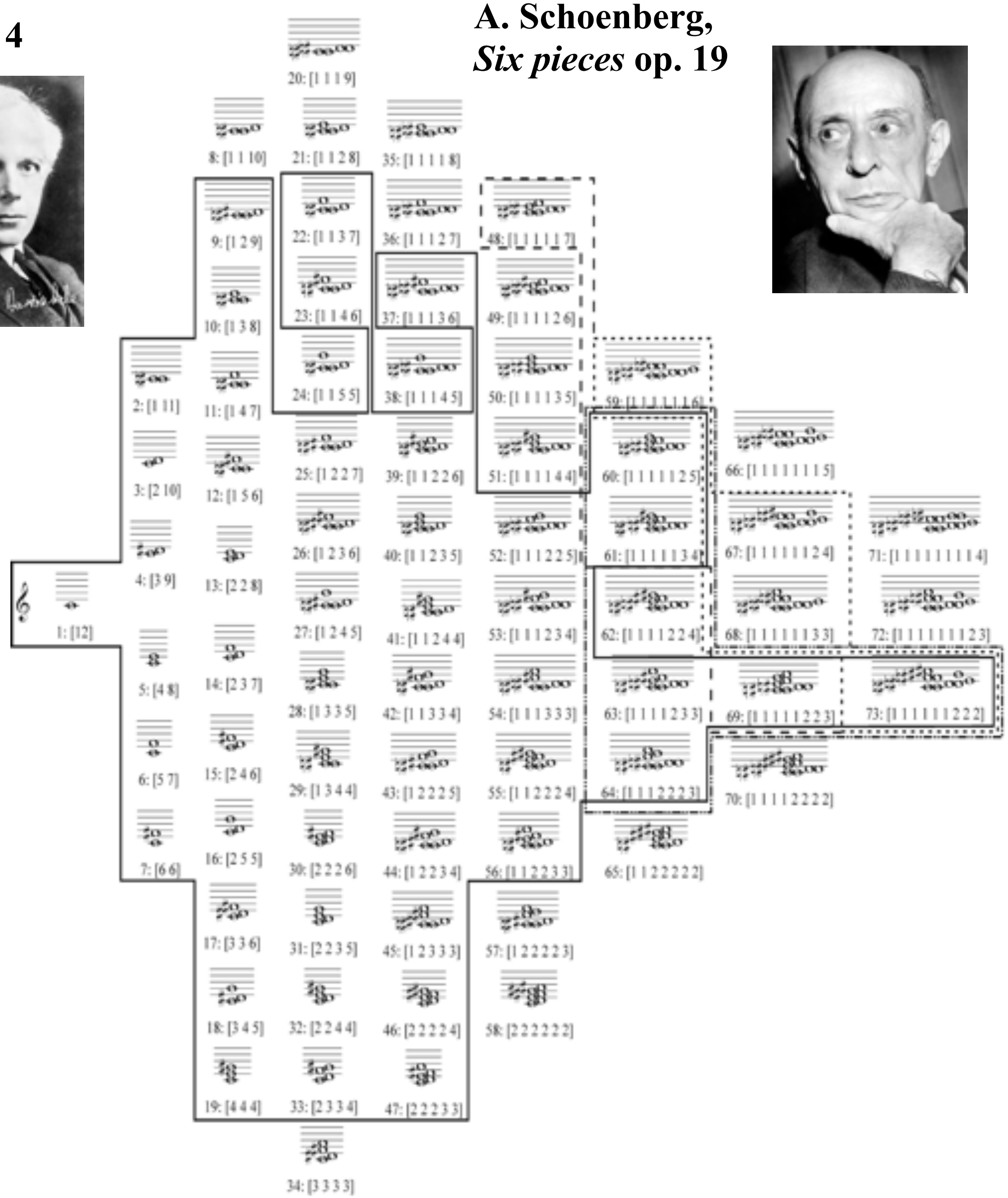
➔ **`http://ressources.ircam.fr/`**

L. Van Beethoven,
Quatuor n° 17

The permutohedron as a musical conceptual space

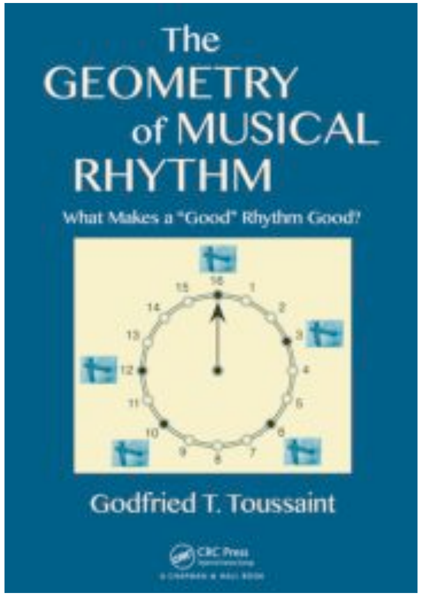


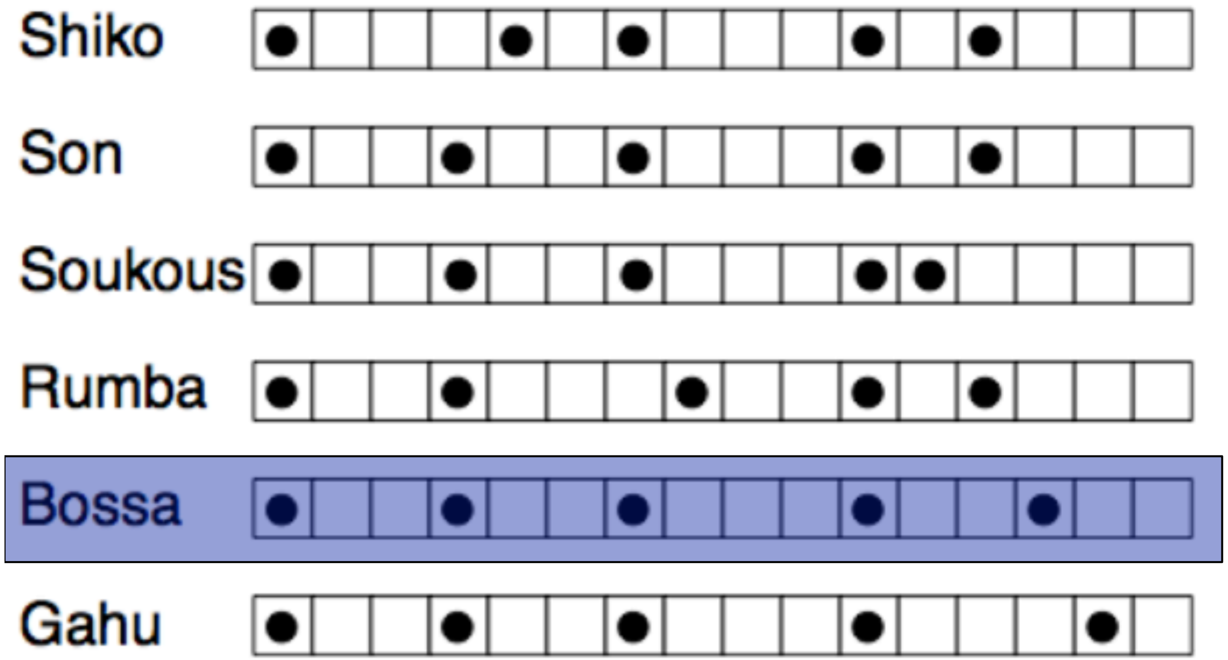
B. Bartok, Quartet n° 4
(3^{d} movement)

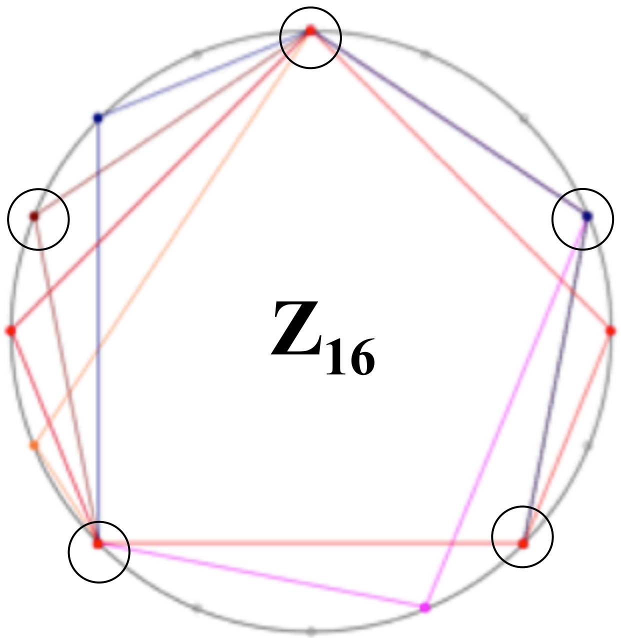


A. Schoenberg,
***Six pieces* op. 19**

Circular representation of rhythmic structures

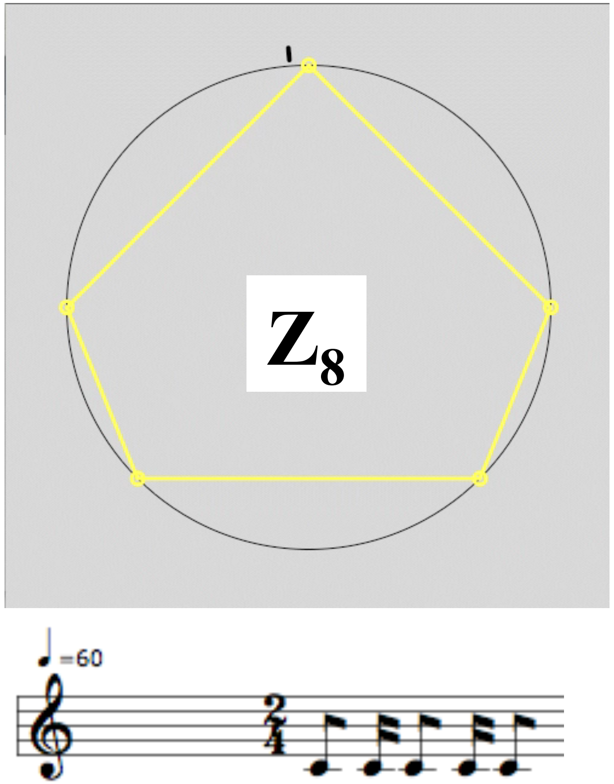






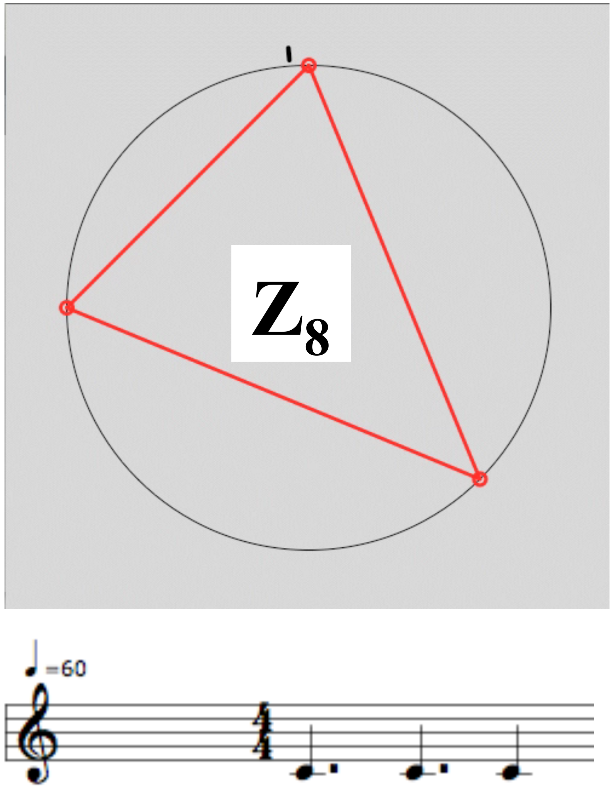
The geometry of African-Cuban rhythms

Cinquillo



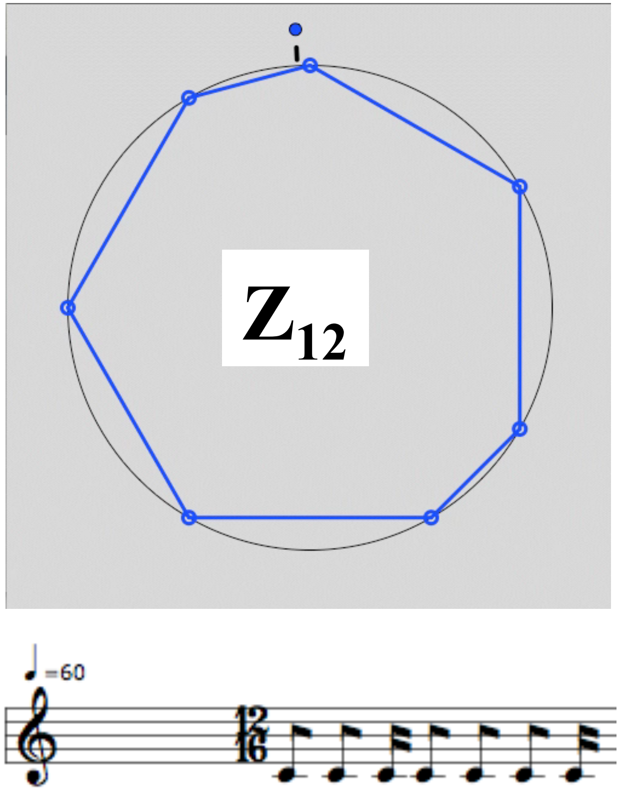
0-(2 1 2 1 2)

Trecillo



0-(3 3 2)

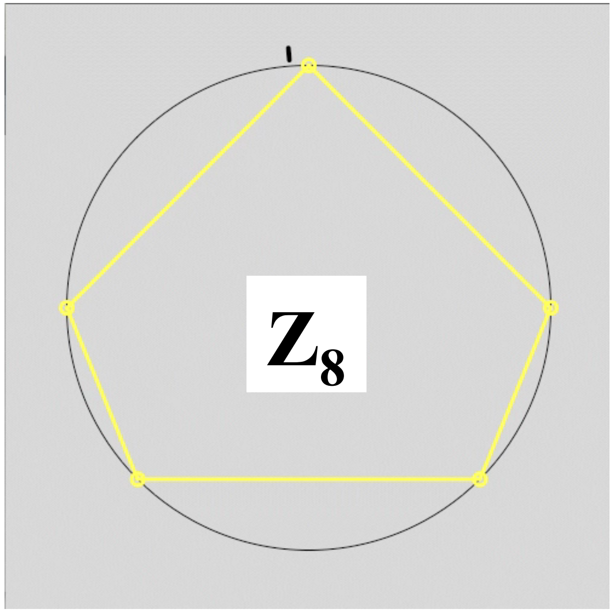
Bembé (Abadja)



0-(2 2 1 2 2 2 1)

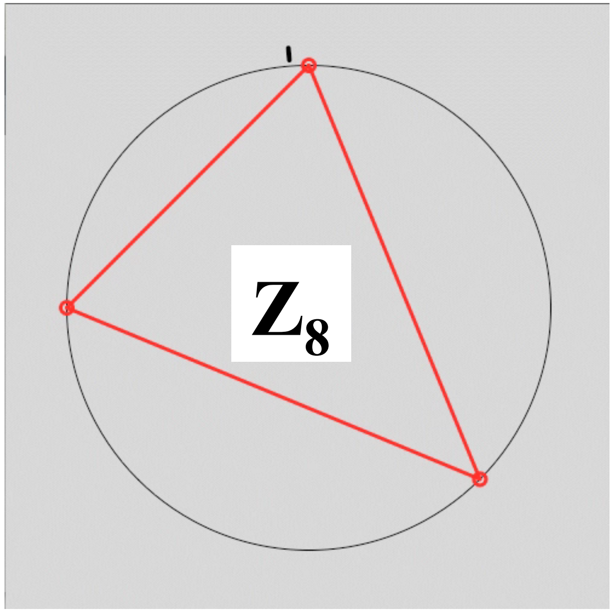
The geometry of African-Cuban rhythms

Cinquillo



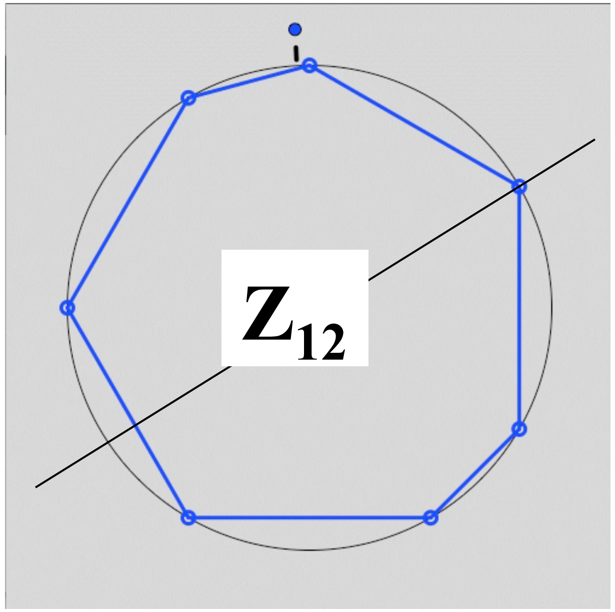
0-(2 1 2 1 2)

Trecillo



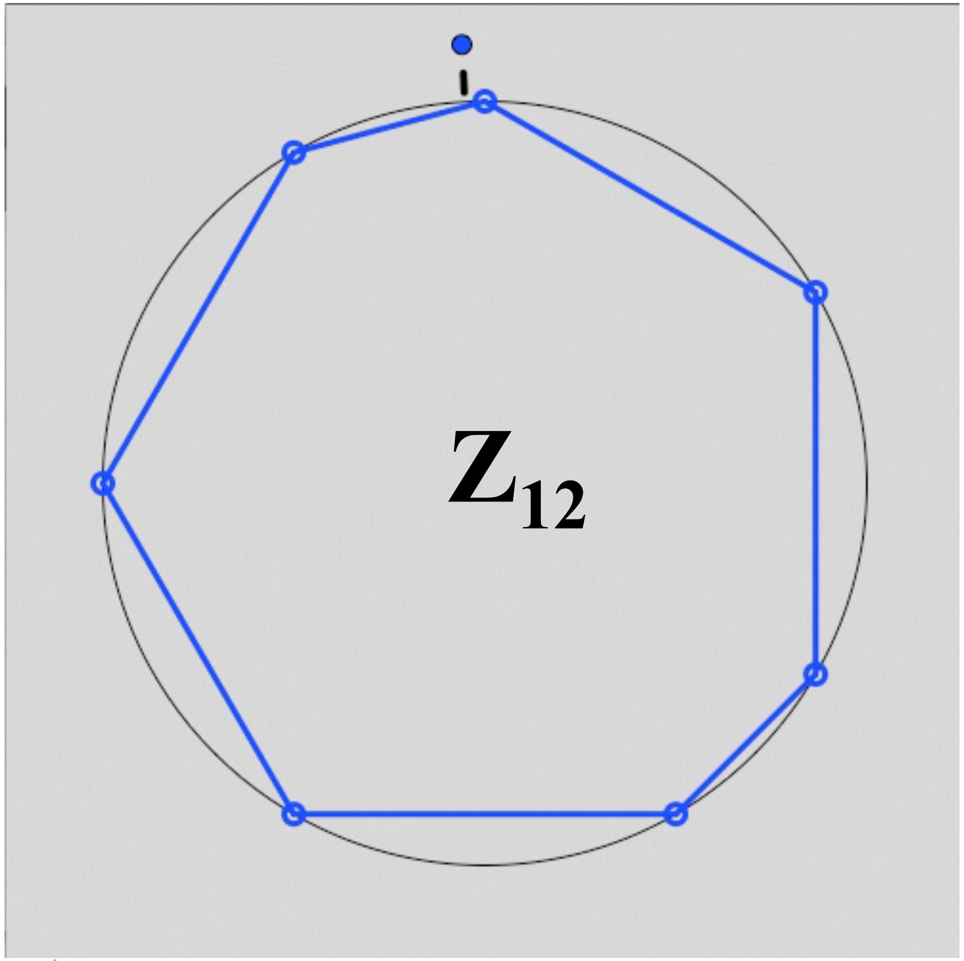
0-(3 3 2)

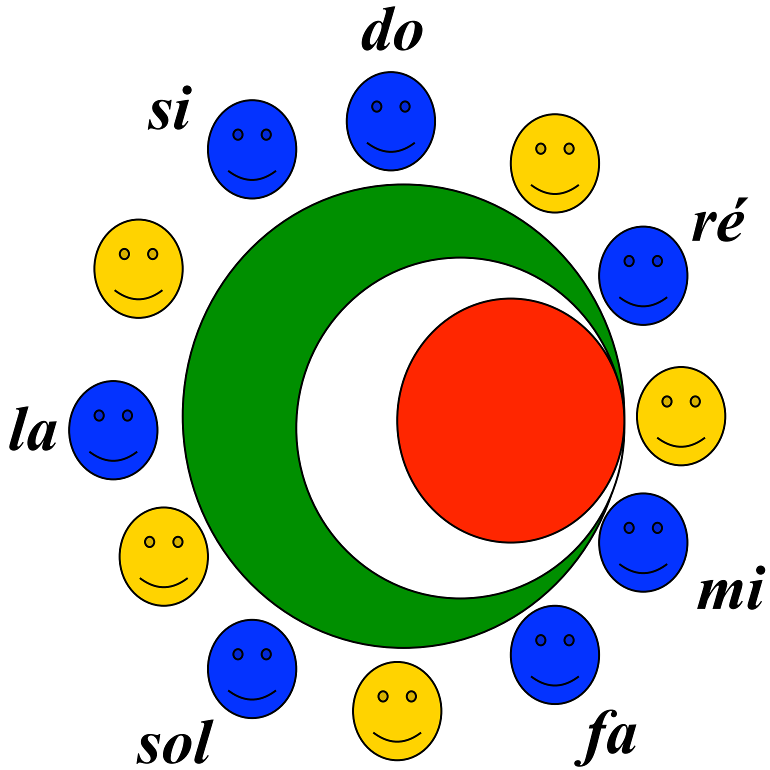
Bembé (Abadja)



0-(2 2 1 2 2 2 1)

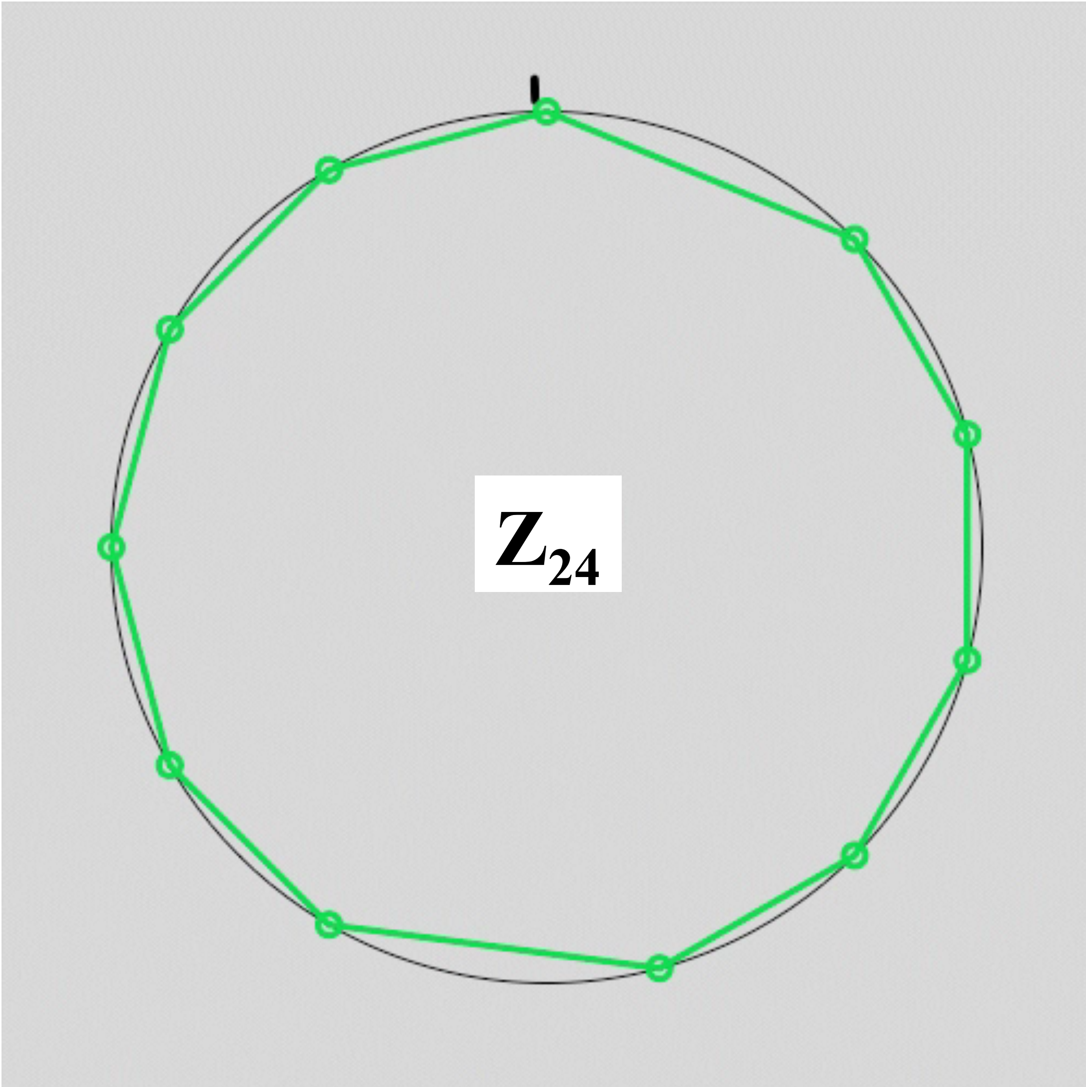
A maximally even rhythmic structure





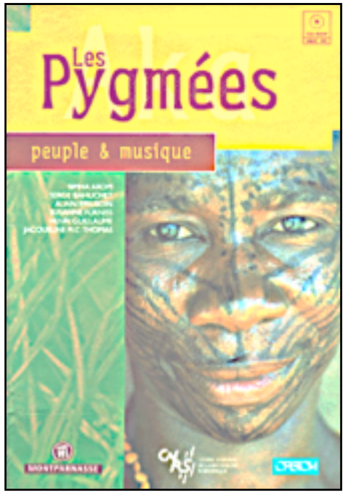
Jack Douthett & Richard Krantz, “Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction”, *J. Math. Phys.* 37 (7), July 1996

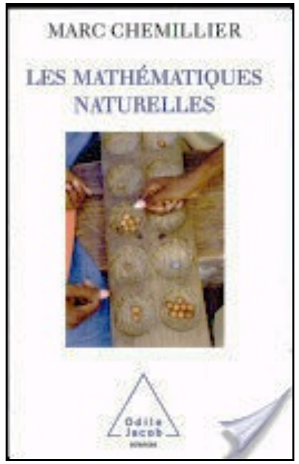
Odditive property of orally-trasmitted practices



Simha Arom

Marc Chemillier

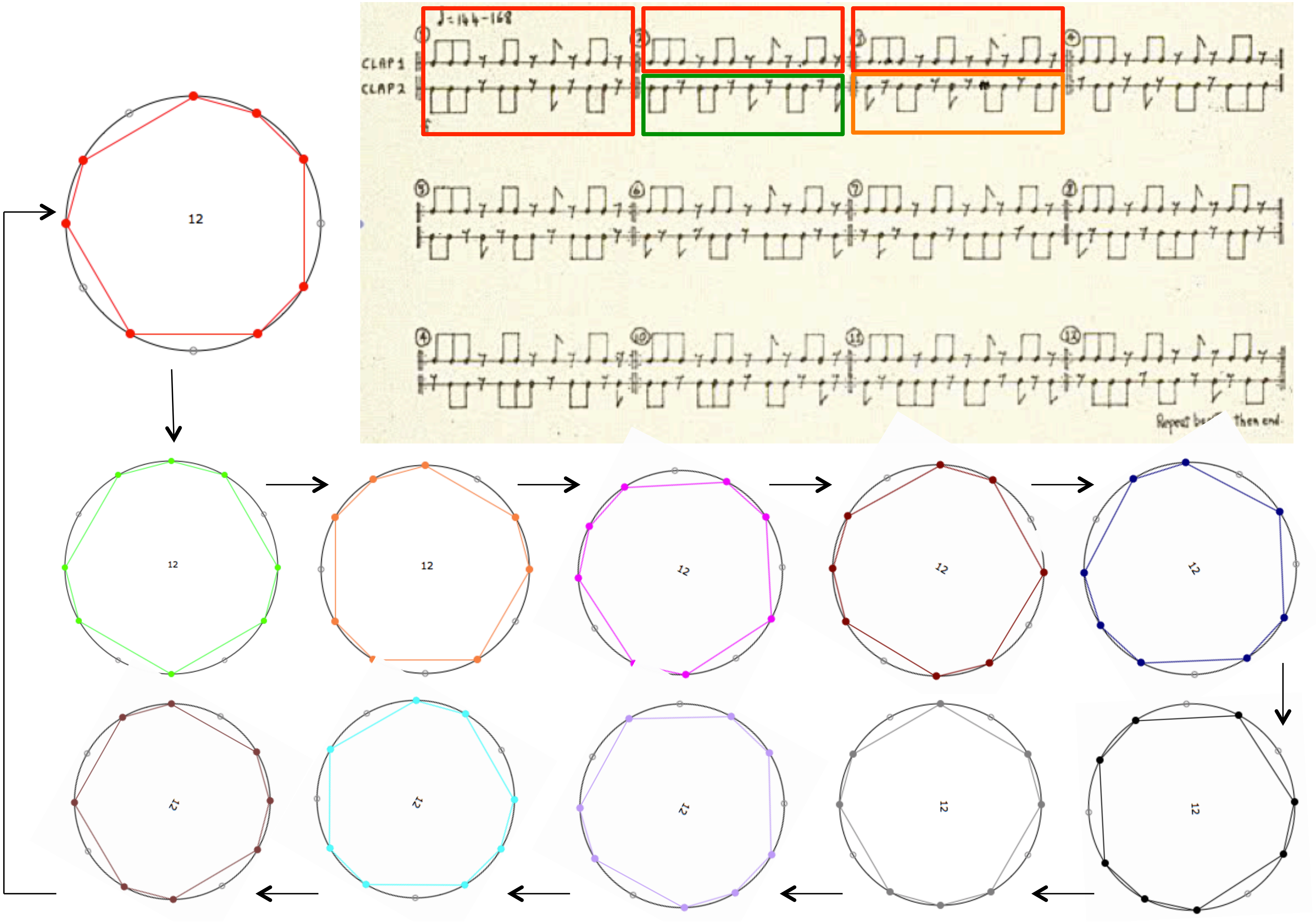




0-(3 2 2 2 2 3 2 2 2 2 2)

➔ 

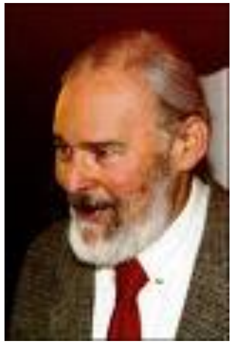
Cyclic permutations in minimalistic music



Husserl's bidimensional model of time perception

• **David Lewin, "Some Investigations into Foreground Rhythmic and Metric Patterning," Music Theory: Special Topics, ed. Richmond Browne (New York: Academic Press, 1981), 101–136.**

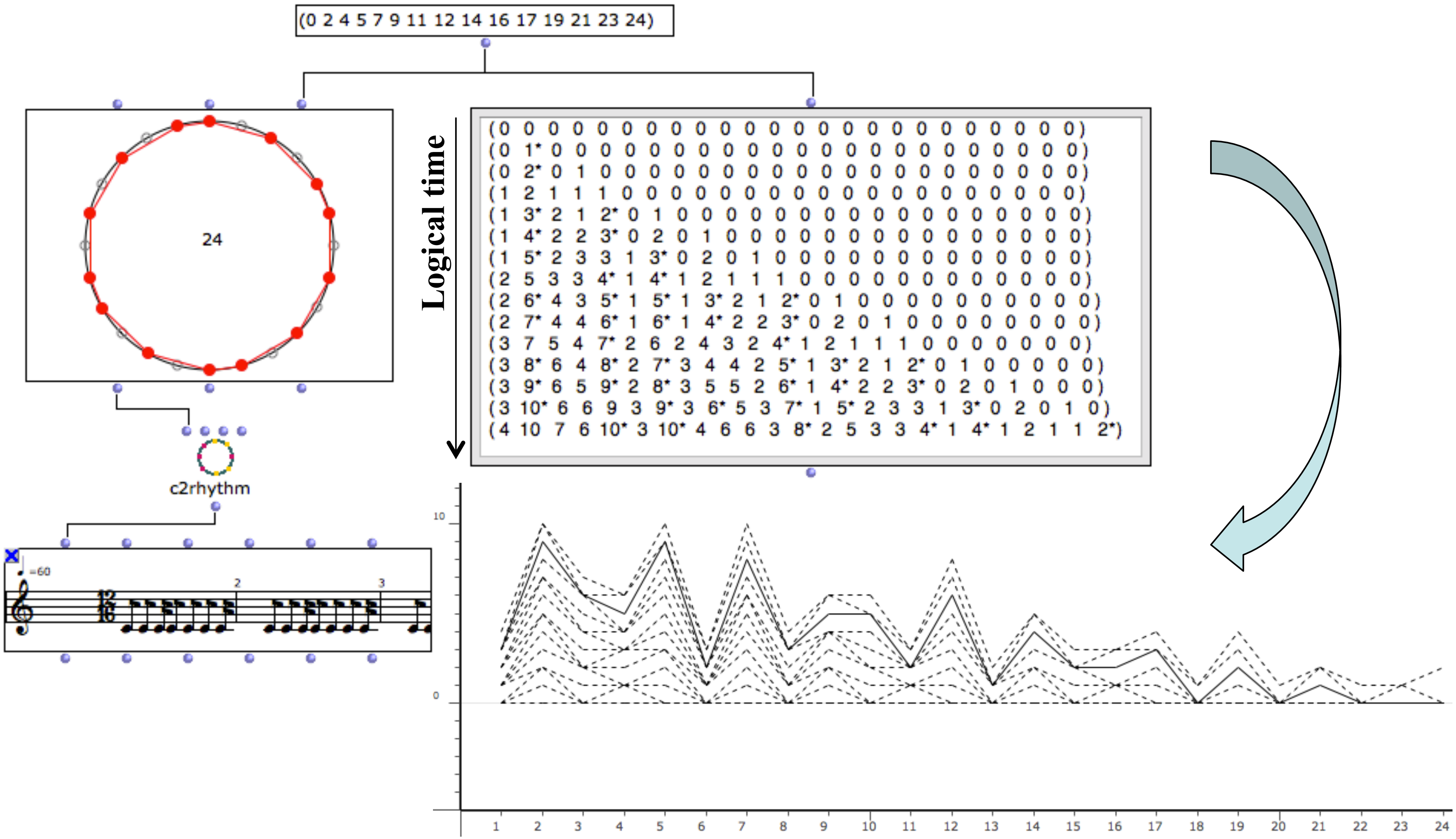
• **David Lewin (1986), « Music Theory, Phenomenology, and Modes of Perception », *Music Perception*, 3, 327-382.**



« L'article [Lewin 1981] développe un modèle numérique qui compte, à chaque position-comme-maintenant t [*"now"-time t*] le nombre de laps de temps [*time-spans*] que je retiens d'un passé récent pertinent ayant (eu) durée égale à d. On construit ainsi une fonction W(d,t) qui donne un vecteur progressif d'intervalles de durées [*"unfolding durational-interval vector"*] au fur et à mesure que le curseur-présent t avance. Le concept à la base de cette construction utilise un modèle husserlien bidimensionnel de la perception du temps [*Husserlian two-dimensional model of perceptual time*], un modèle qui exprime aussi bien les « impressions primaires » chez Husserl, impressions qui suivent les curseur-présent t, mais aussi les « retentions » chez Husserl, projections d'instants temporels passés [*projections of remembered past times*] (ainsi que durées passées) dans ma conscience présente [*into my present consciousness*]. Ensuite, dans le même article, j'envisage en quelque sorte les « protentions » chez Husserl, des projections d'attentes futures dans la conscience du présent » (Lewin, 1986/2006) »

➔ **Rythme « Abadja » (Afrique) = Rythme Bembé (Cuba)**

Grasping the unfolding durational content of a rhythmic pattern



Unfolding Rhythmic Interval Vector