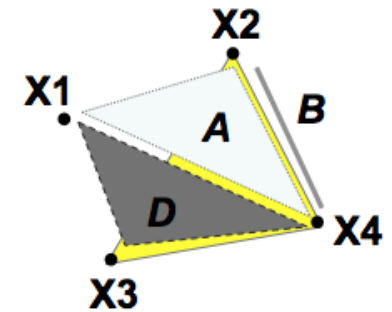
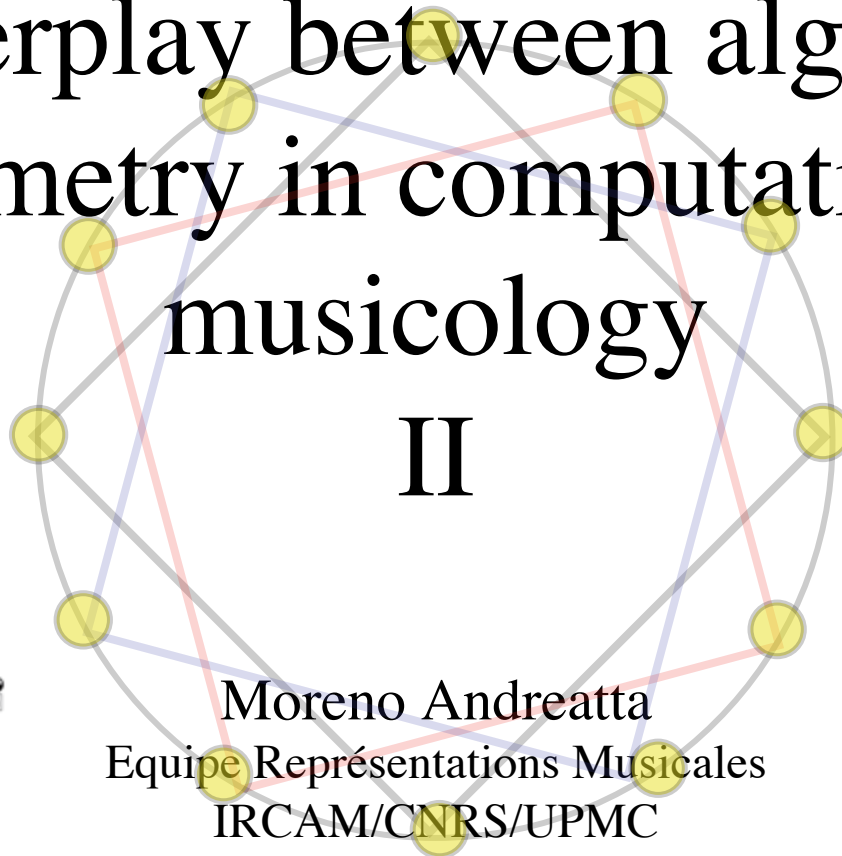
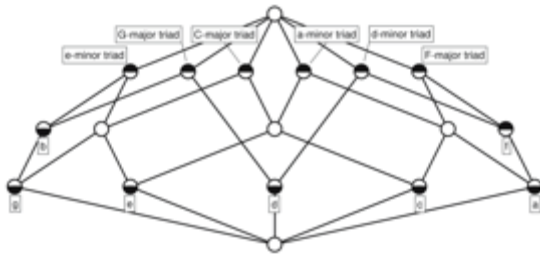




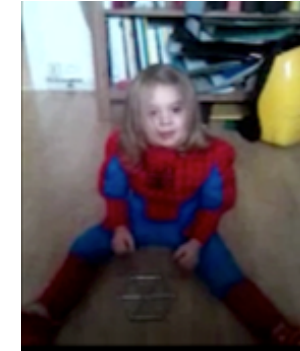
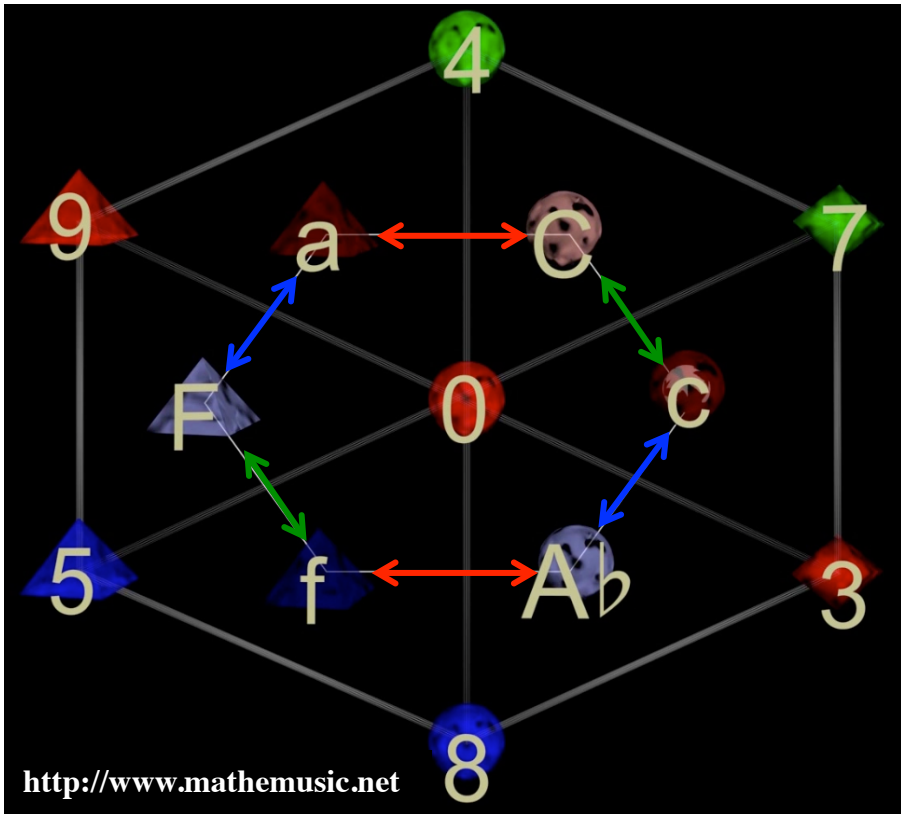
# The interplay between algebra and geometry in computational musicology

## II

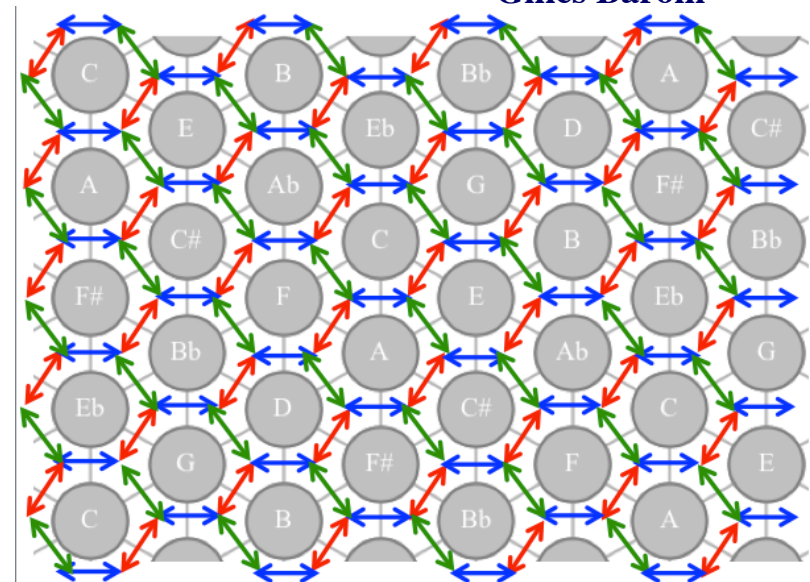
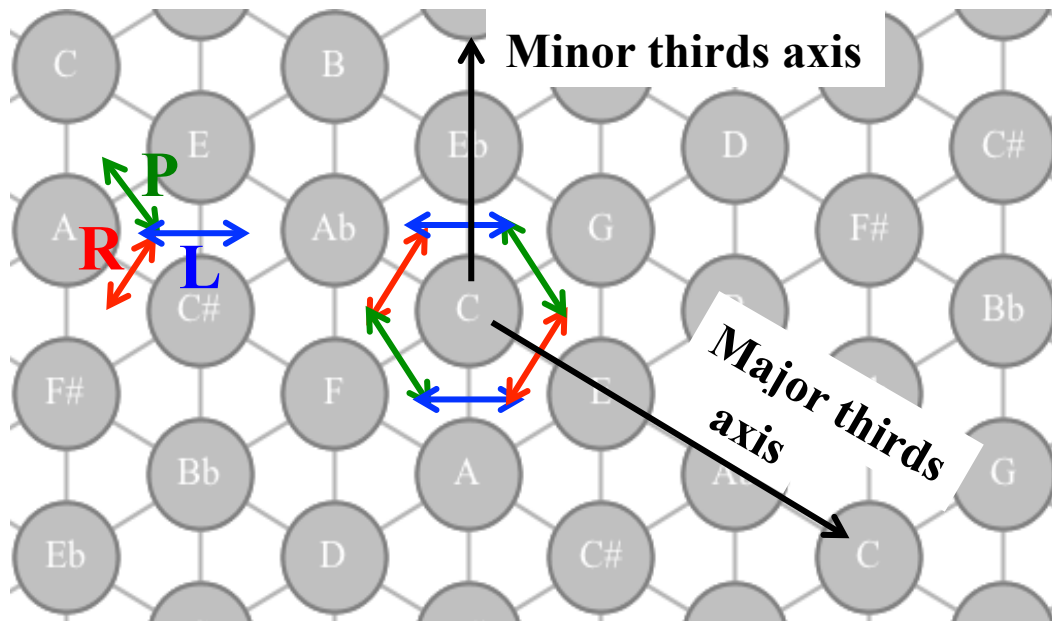


Moreno Andreatta  
Equipe Représentations Musicales  
IRCAM/CNRS/UPMC  
<http://www.ircam.fr/repmus.html>

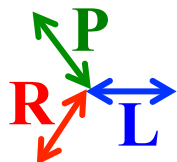
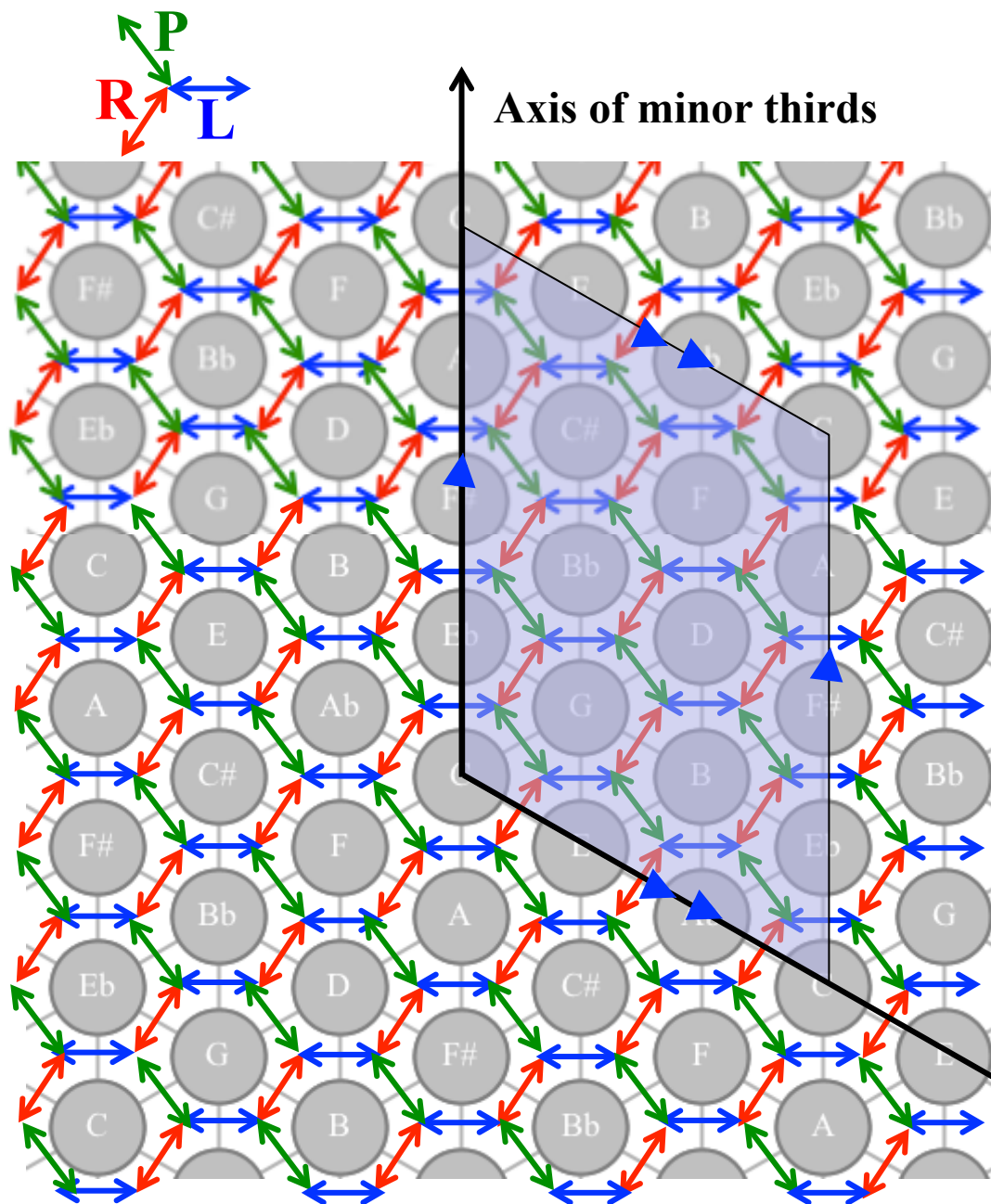
# The Tonnetz (or hexagonal tiling honeycomb)



Gilles Baroin



# The Tonnetz as a torus



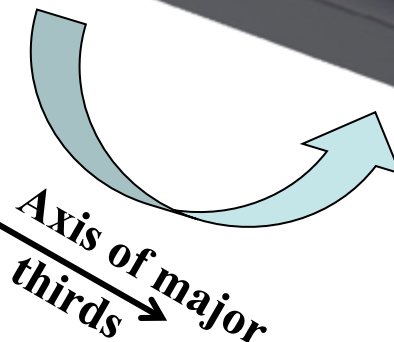
<b>F</b>	—	<b>C</b>	—	<b>G</b>	—	<b>D</b>
<b>A</b>	—	<b>E</b>	—	<b>H</b>	—	<b>F<sub>s</sub></b>
<b>C<sub>s</sub></b>	—	<b>G<sub>s</sub></b>	—	<b>D<sub>s</sub></b>	—	<b>B</b>



*Speculum Musicum* (Euler, 1773)

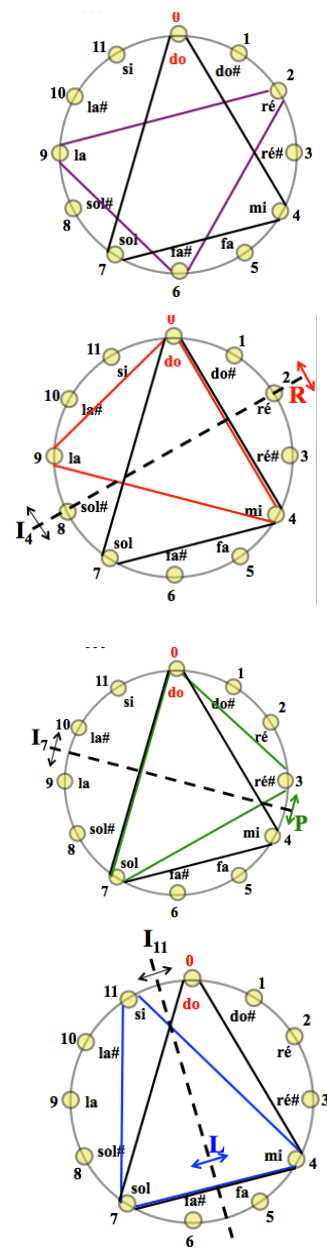
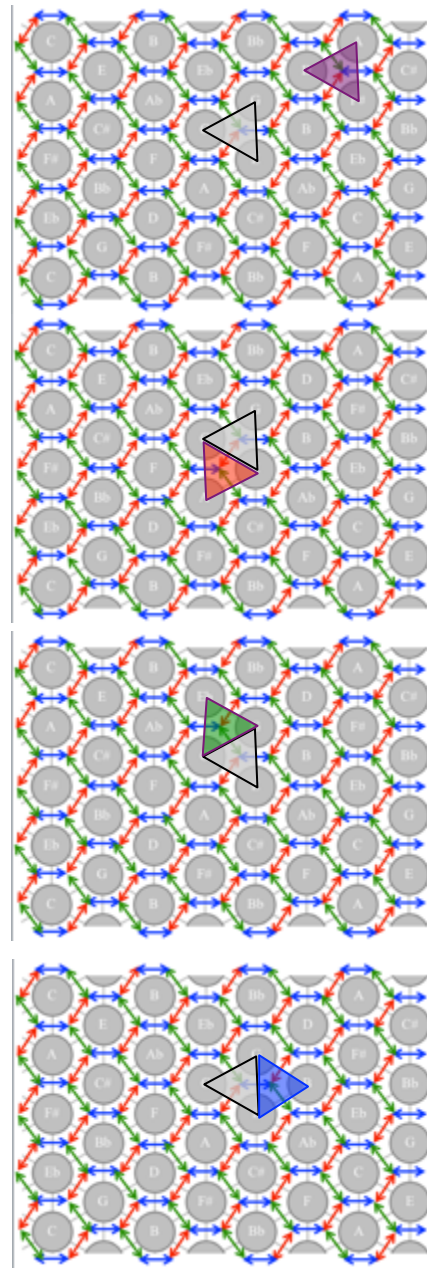
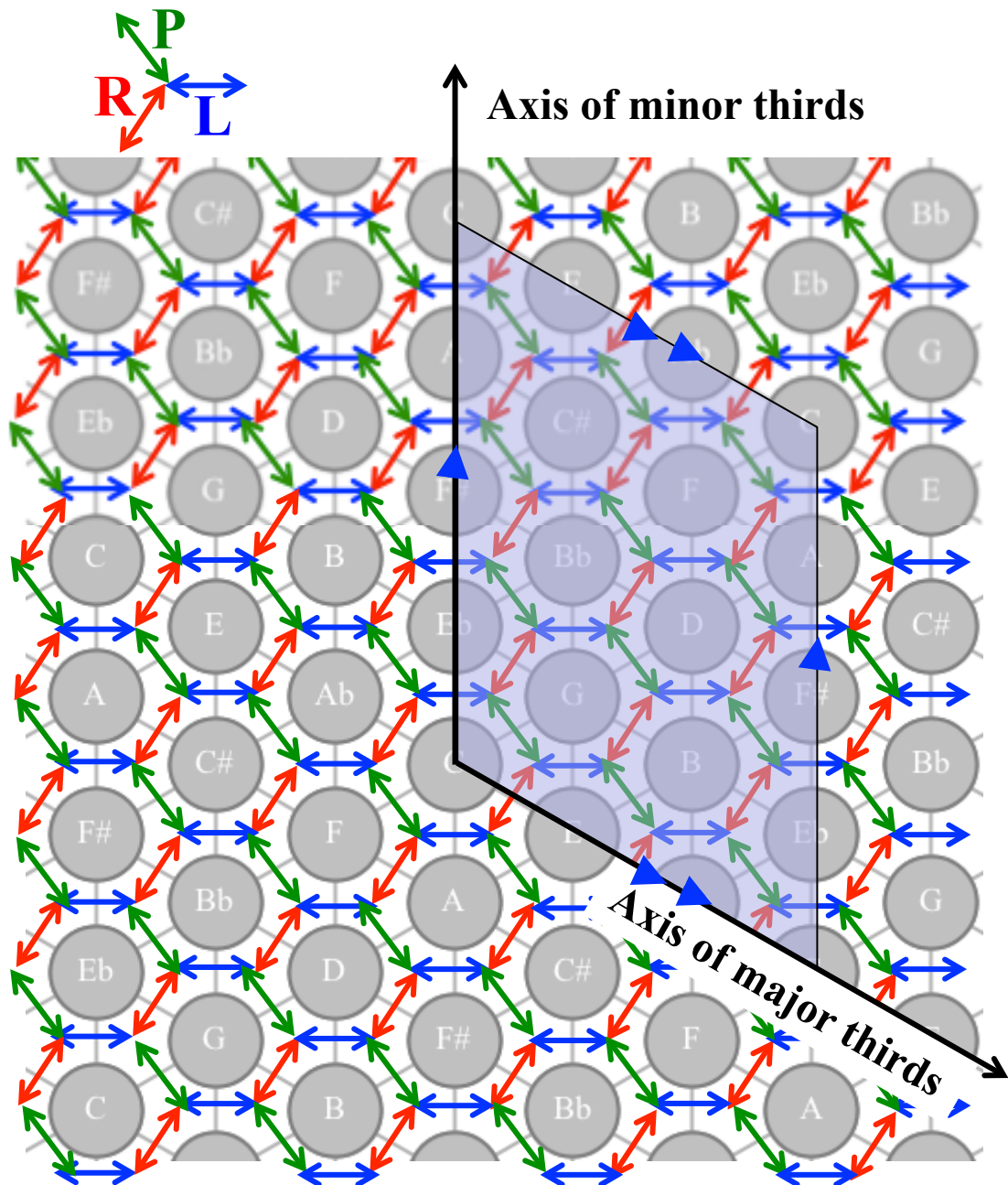


**Torus**





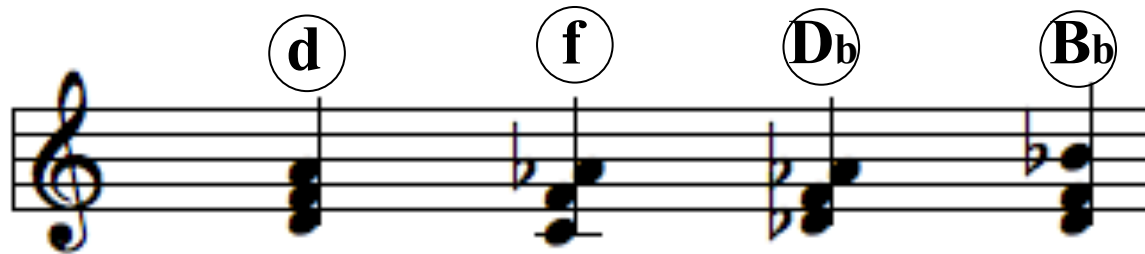
# The *Tonnetz* and its symmetries



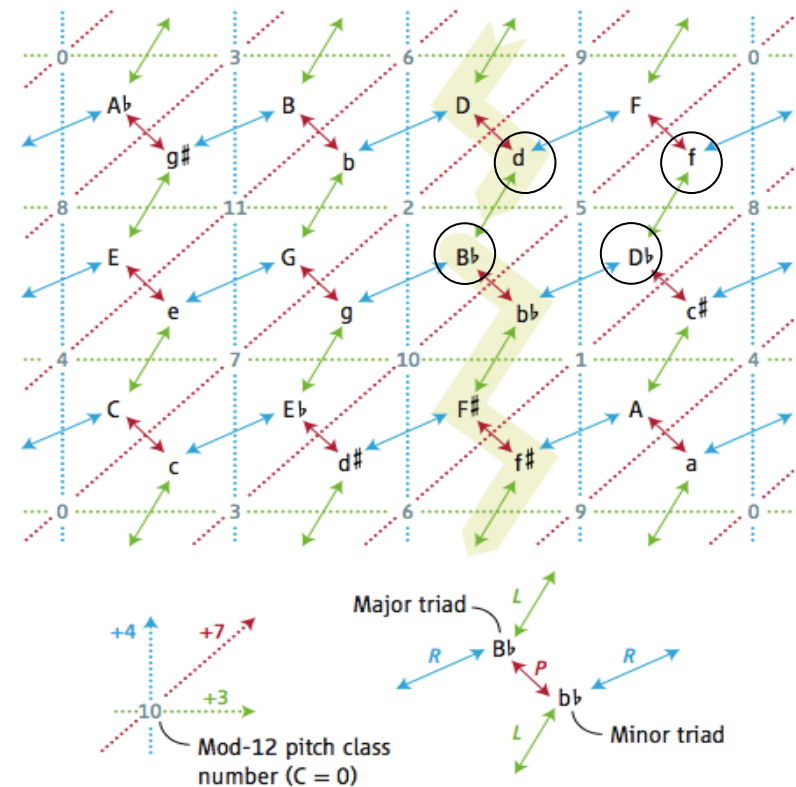
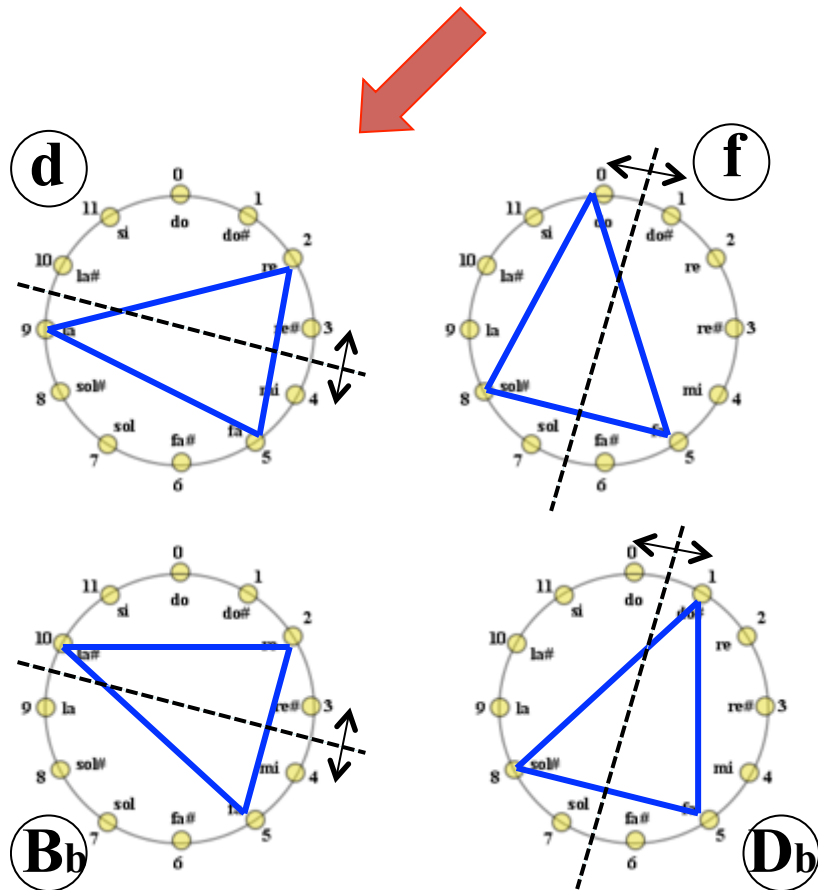


# Spatial symmetries in pop music

Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), 177-199, 2004



*Shake the disease* - 1985  
(Depeche Mode) – min. 2'17"



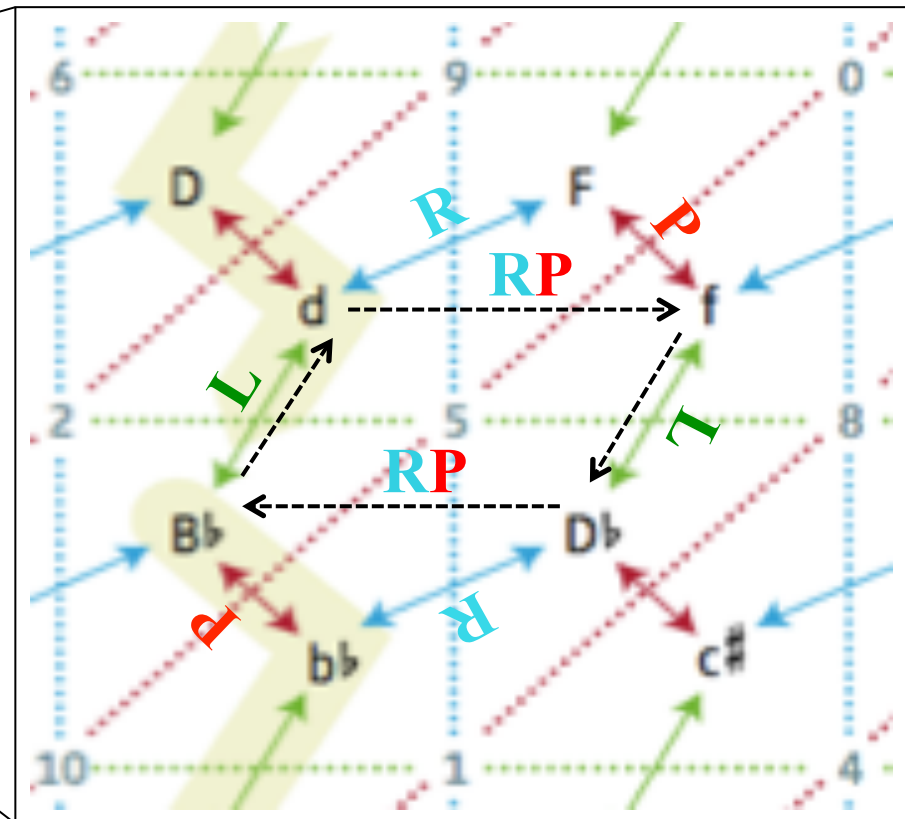
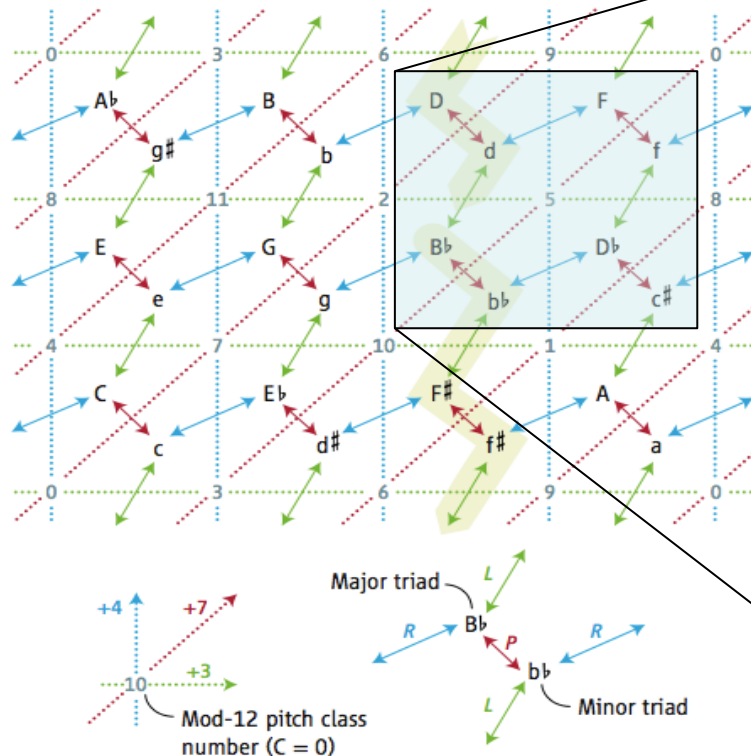
# Trajectories and harmonic progressions in the *Tonnetz*

Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), 177-199, 2004

*Shake the disease* - 1985  
(Depeche Mode)



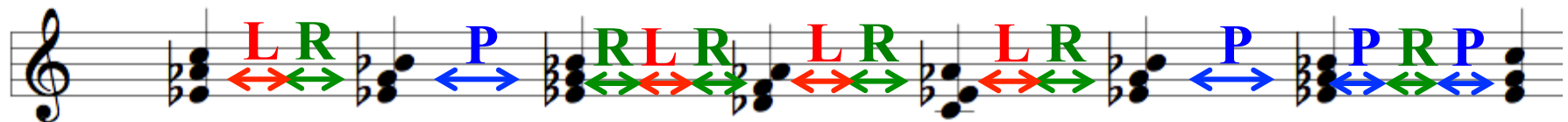
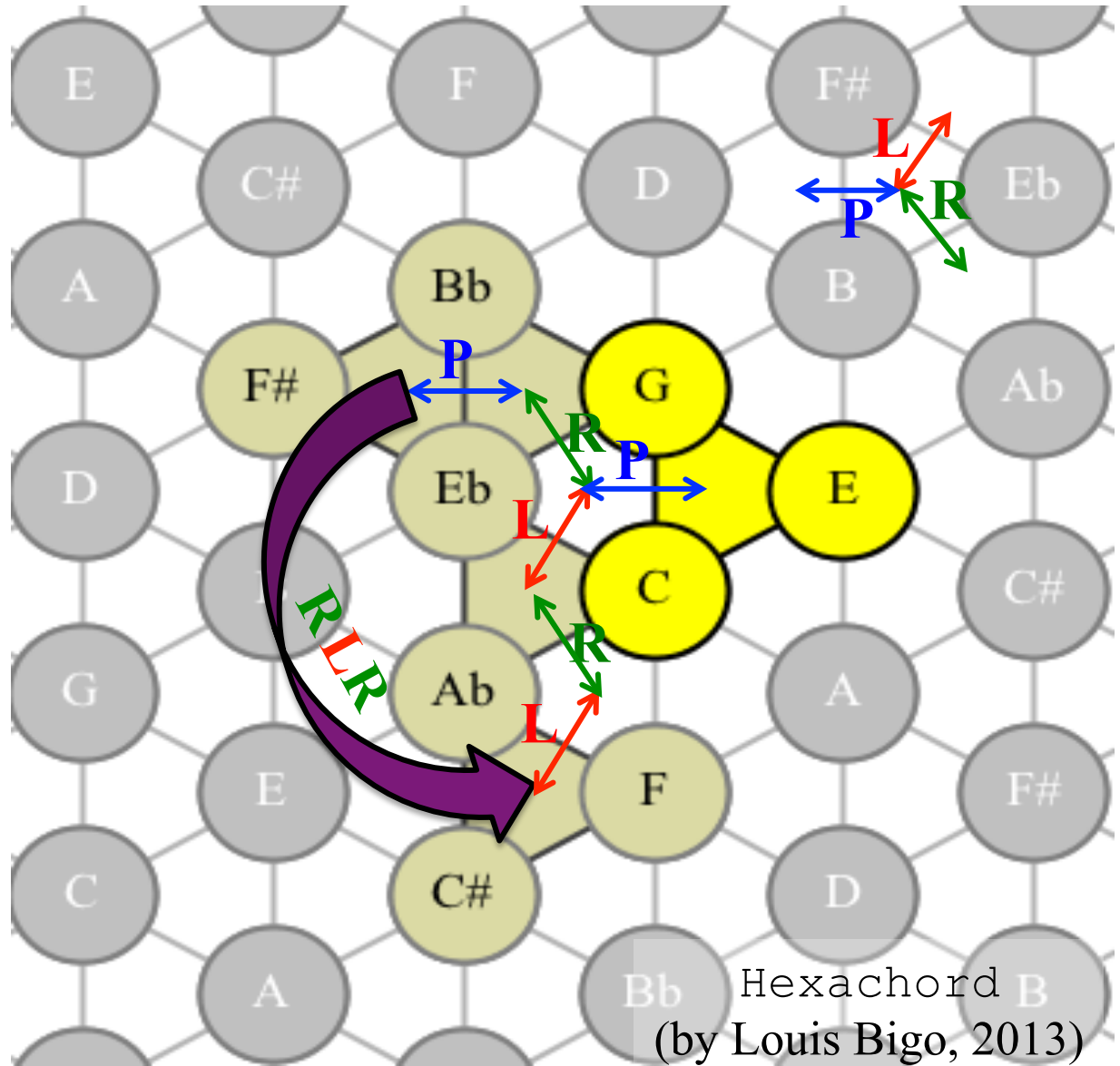
Sequence: **RPLRPL**







min. 0'33''

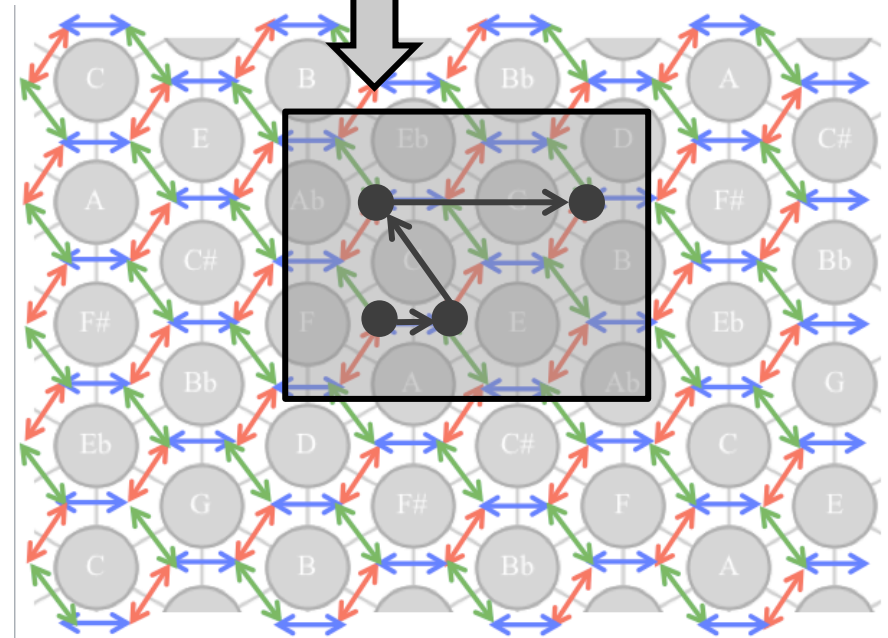


# Symmetries in Frank Zappa's music

Synthesizer

*Fine* *D.C. al Fine*

[Guy Capuzzo, *Music Theory Spectrum*, 2004]



« Easy Meat » - 1981 (Frank Zappa)  
min. 1'44" – 2'39"

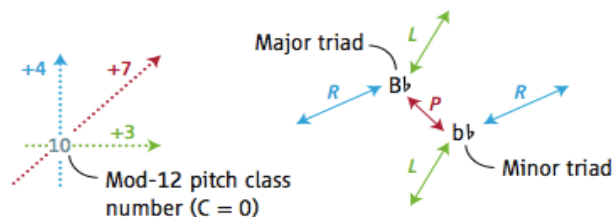
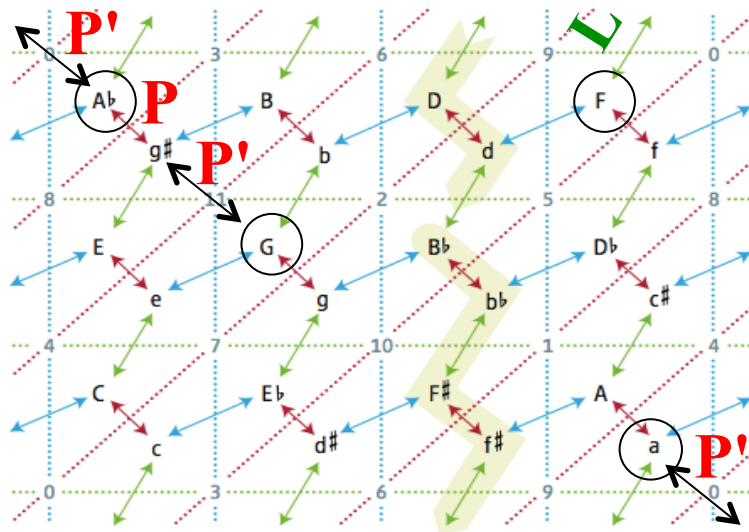


# Symmetries in Frank Zappa's music: the generating cell

Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), 177-199, 2004

« Easy Meat » - 1981 (Frank Zappa)

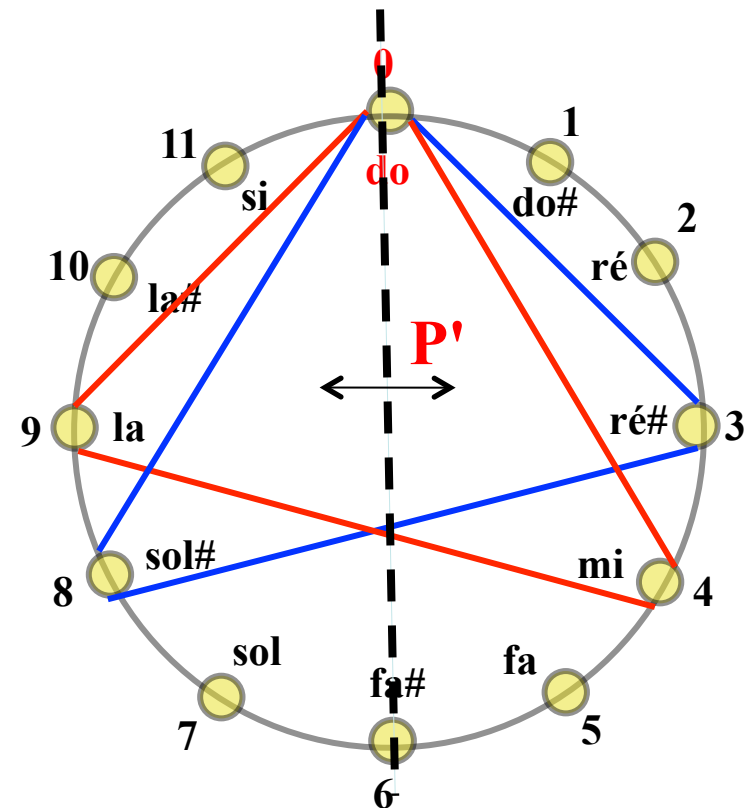
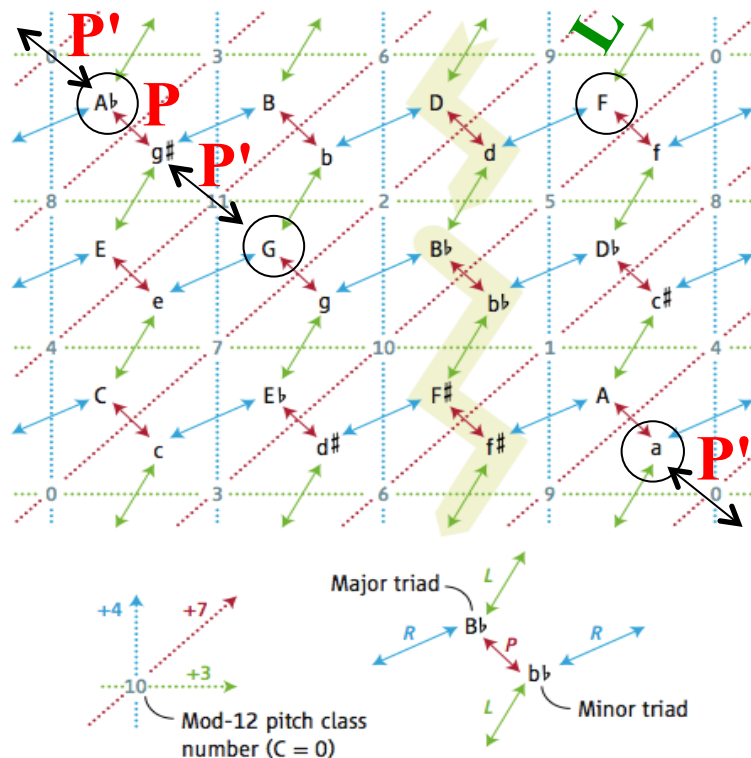
$$F+ \xrightarrow{L} A- \xrightarrow{P'} Ab+ \xrightarrow{PP'} G+$$



# Symmetries in Zappa's music: the P' transformation

- Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), p. 177-199, 2004

« Easy Meat » - 1981 (Frank Zappa)



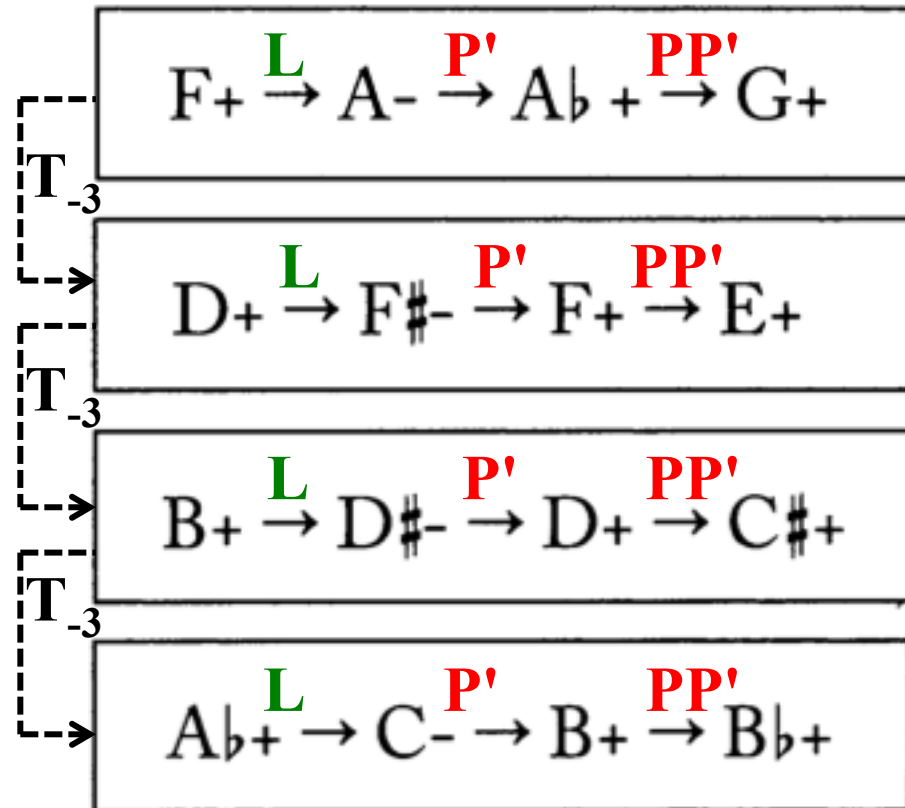
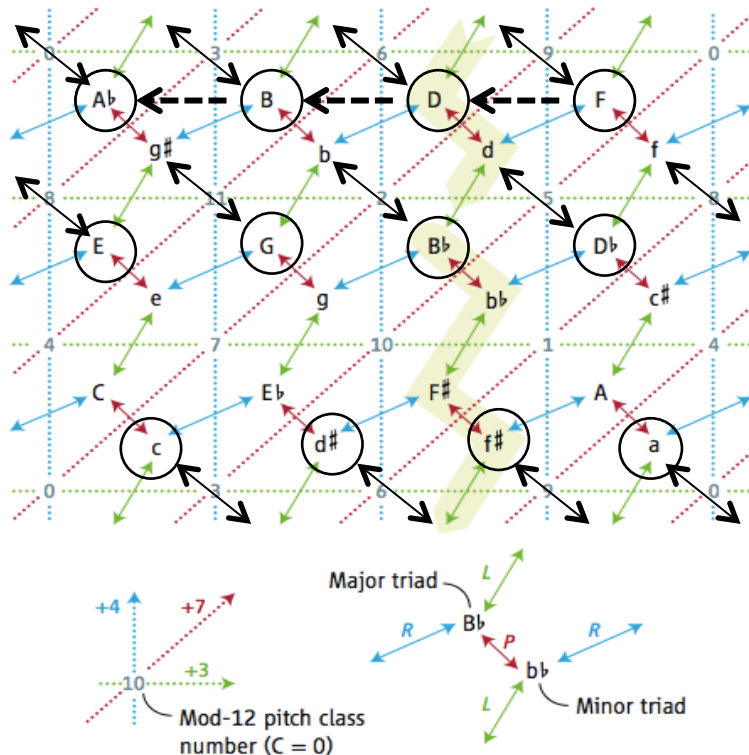


# The generating cell and its spatial transformations

Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), 177-199, 2004

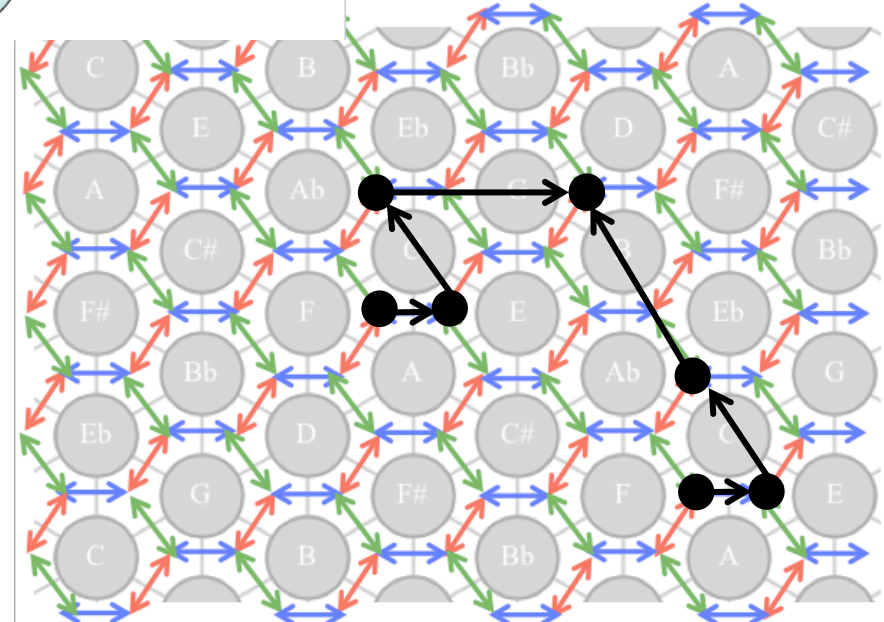
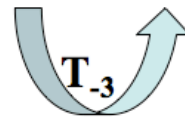
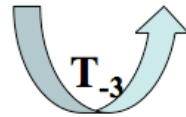
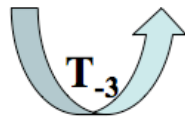
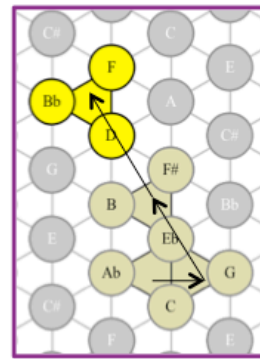
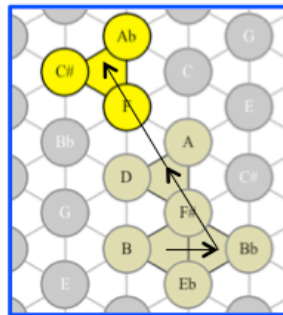
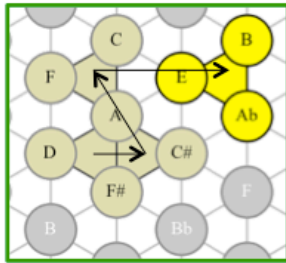
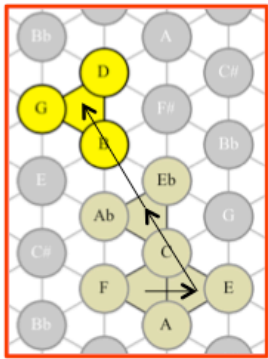
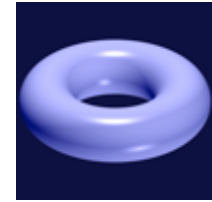
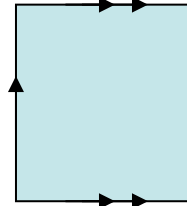
« Easy Meat » - 1981 (Frank Zappa)

The score shows two staves of music. The top staff is labeled 'Synthesizer' and features chords G+, A+, F+, A-, Ab+, G+, D-, F#-, F+, and E-. The bottom staff features chords B-, D-, D+, C#+, Ab-, C-, B+, Bb+, E+, B+, F#-, C#+, F#+, A+, E+, and E-. Transformations are indicated by arrows and labels: L (green), P' (red), and PP' (red). The score is divided into sections marked 'Fine' and 'D.C. al Fine'.



# The trajectory of the harmonic progression

Fa la<sub>m</sub> La<sub>r</sub> Sol Ré fa<sub>m</sub> Fa Mi Si la<sub>m</sub> Ré Ré<sub>b</sub> La<sub>b</sub> do<sub>m</sub> Si Si<sub>b</sub>



# Symmetries in Paolo Conte's *Madeleine*

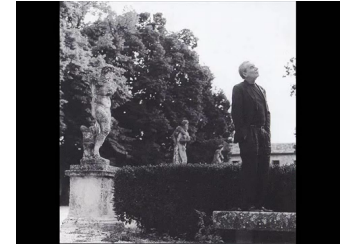
**Preludio** *Moderato*

**Chorus**

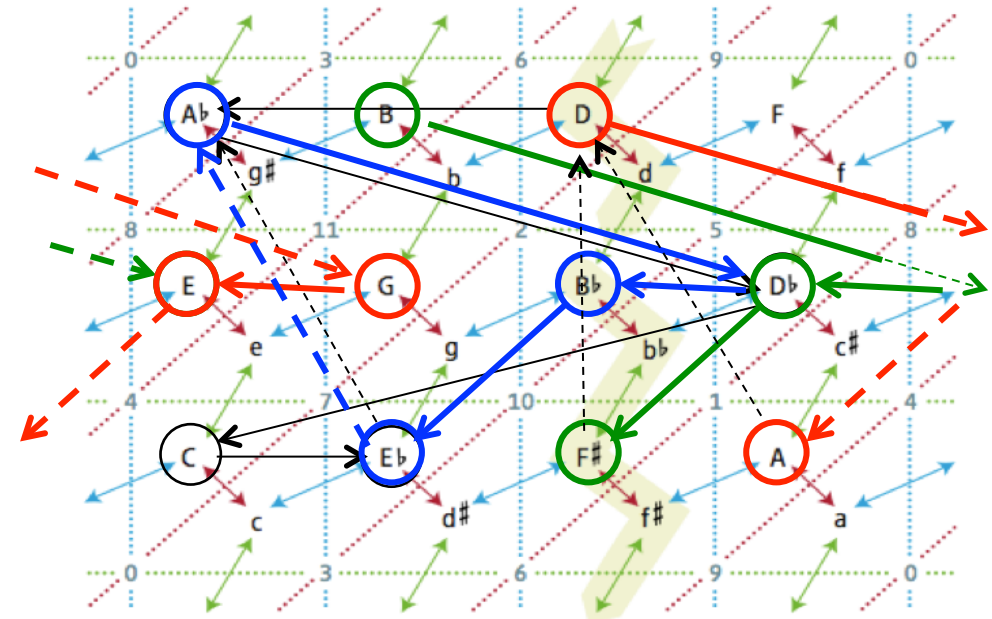
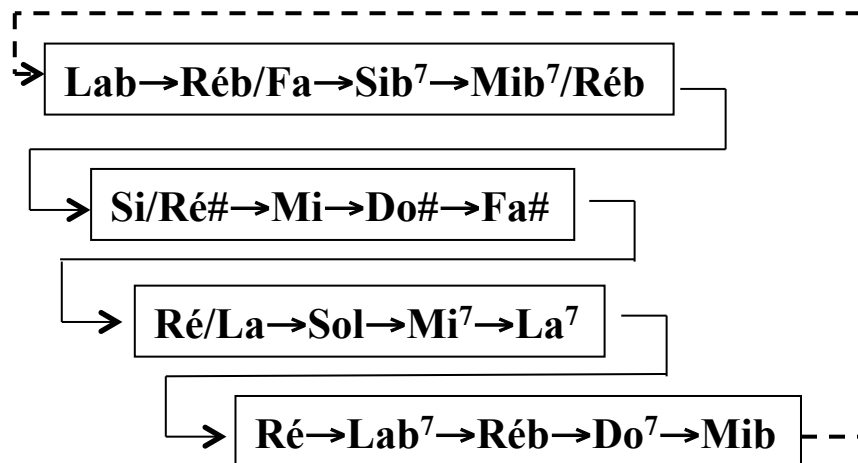
Lab → Réb/Fa → Sib<sup>7</sup> → Mib<sup>7</sup>/Réb

Qui, tut - to, il meglio è già qui, non ci so - no pa  
 Tan - to io ca - pi - sco sol - tan to il tut - to del - le tre  
 [Ma] qual - che vol - ta, è co - che qual - cu - no, è tor

ro - le per spie ga re ed in - tu - re e ca - pi - re, Ma de - leine, e se mai ri - cor - da - re...  
 ma - ni e la can - zo - ne per ca - da - ta e ni - tro - va - ta, come un' al - tra un' al - tra vi - ta...  
 na - to nel - to cer - te rez - ze...



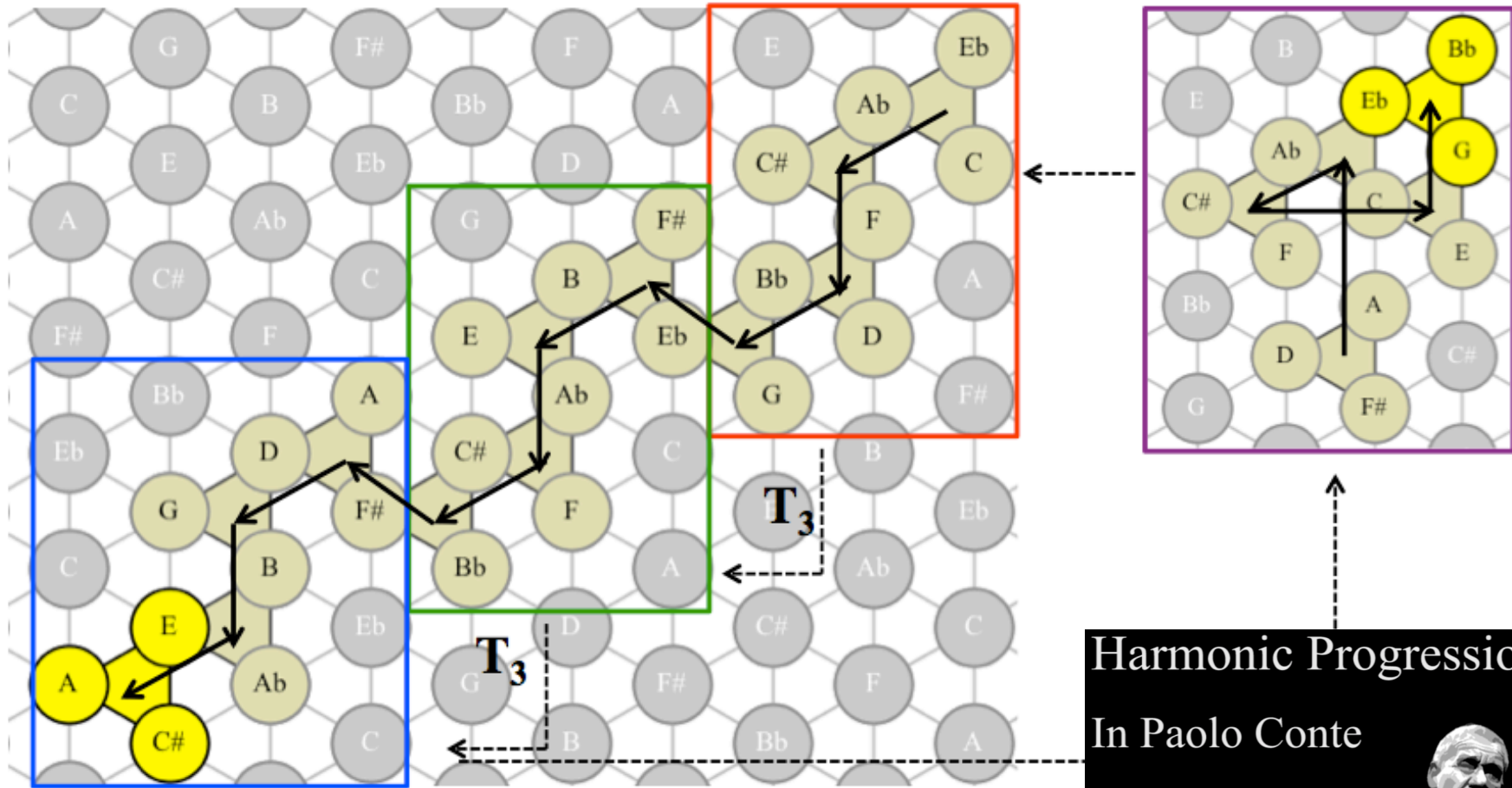
S. La Via, *Poesia per musica e musica per poesia*.  
 Dai trovatori a Paolo Conte, Carocci, 2006





# Madeleine's spatial trajectory

La<sub>b</sub> Ré<sub>b</sub> Si<sub>b</sub> Mi<sub>b</sub> Si Mi Ré<sub>b</sub> Fa<sub>##</sub> Ré Sol Mi La Ré La<sub>b</sub> Ré<sub>b</sub> Do Mi<sub>b</sub>



Harmonic Progressions  
In Paolo Conte  
*Madeleine*



<http://www.mathemusic.net>

# Partial covering of the *Tonnetz*



**Preludio** *Moderato*

Lab → Réb/Fa → Sib<sup>7</sup> → Mib<sup>7</sup>/Réb

**Chorus**

Lab Réb/Fa Sib<sup>7</sup> Mib<sup>7</sup>/Réb Si/Ré# Mi Do# Fa#

Qui, tut - to, il meglio è già qui, non ci so - no pa  
 Tan - to io ca - pi - sco sol - tan to il tut - to del - le tre  
 [Ma] qual - che vol - ta, è co - che qual - cu - no è tor

Re/La Sol Mi<sup>7</sup> La<sup>7</sup> Re Lab<sup>7</sup> Réb Do<sup>7</sup> Mib<sup>7</sup>

ro - le per spie - ga - re ed in - tu - re e ca - pi - re, Ma de - leine, e se mai ri - cor - da - re...  
 ma - ni e la can - zo - ne per ca - da - ta e ri - tro - va - ta, come un' al - tra un' al - tra vi - ta...  
 na - to sot - to cer - te rez - ze...

Missing major chord

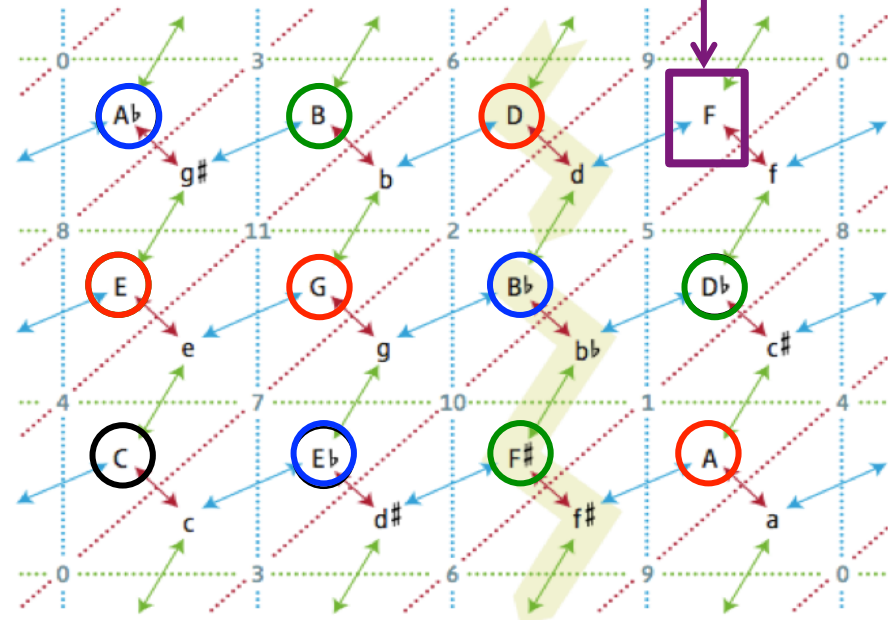
S. La Via, *Poesia per musica e musica per poesia*.  
 Dai trovatori a Paolo Conte, Carocci, 2006

→ Lab → Réb/Fa → Sib<sup>7</sup> → Mib<sup>7</sup>/Réb

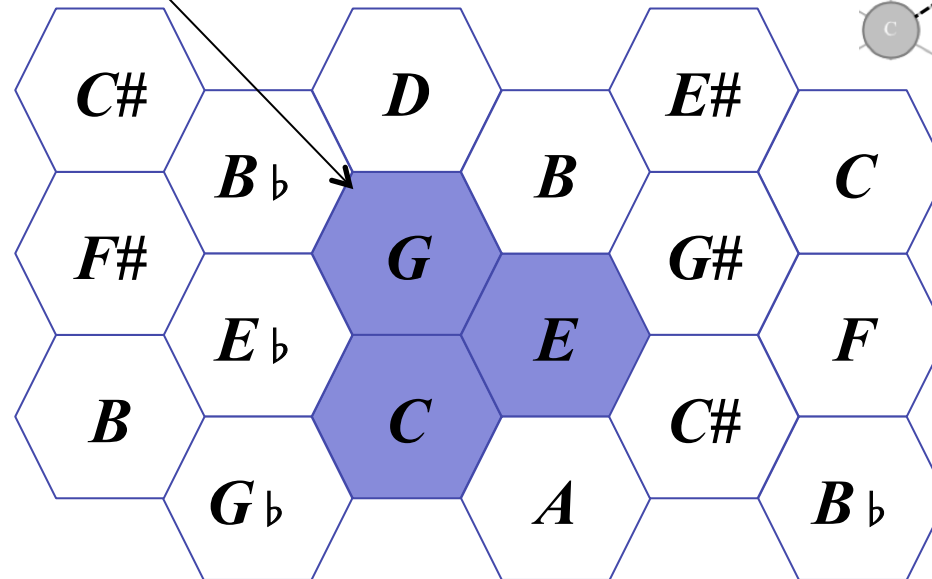
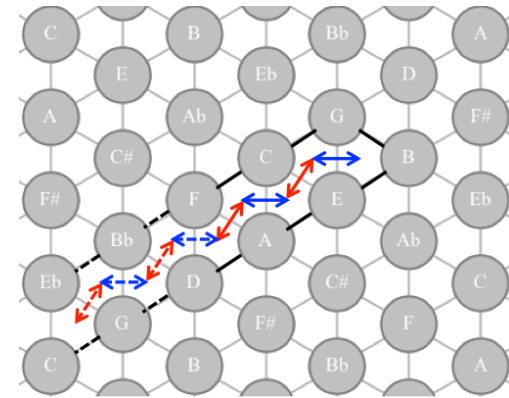
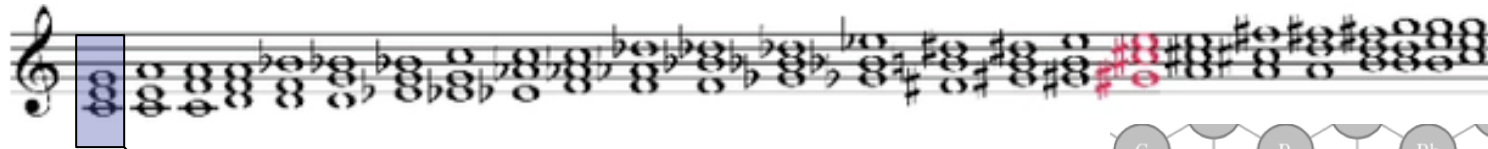
→ Si/Ré# → Mi → Do# → Fa#

→ Ré/La → Sol → Mi<sup>7</sup> → La<sup>7</sup>

→ Ré → Lab<sup>7</sup> → Réb → Do<sup>7</sup> → Mib



# Extract of the 2<sup>nd</sup> movement of the Symphony No. 9 (L. van Beethoven)

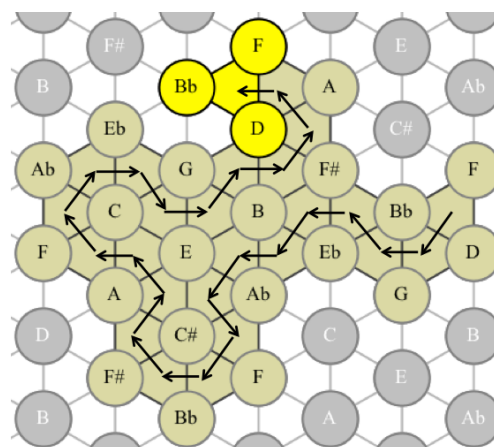
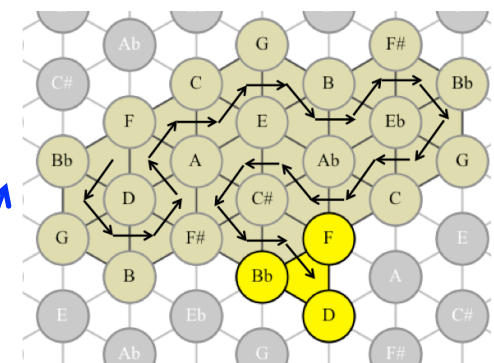
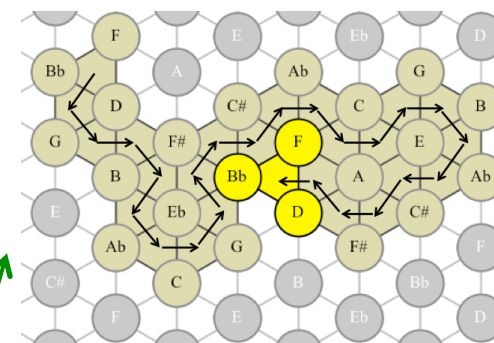
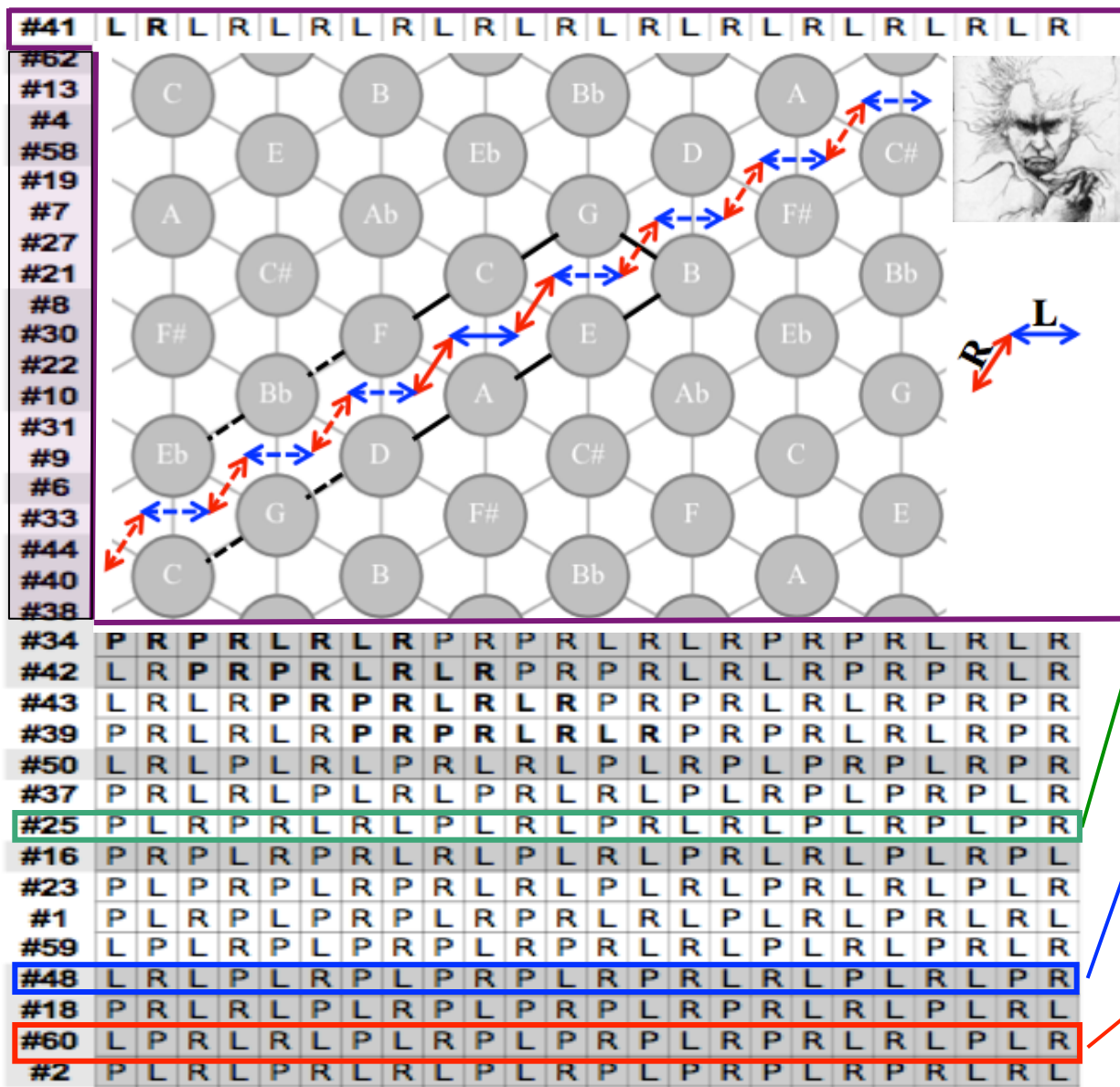


**b**

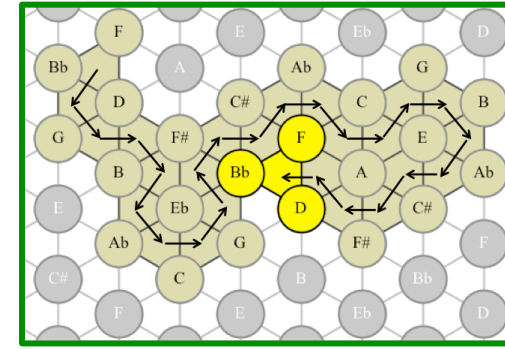
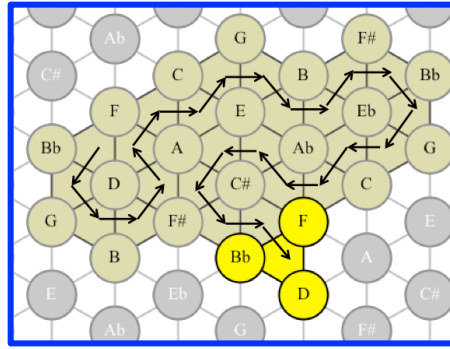
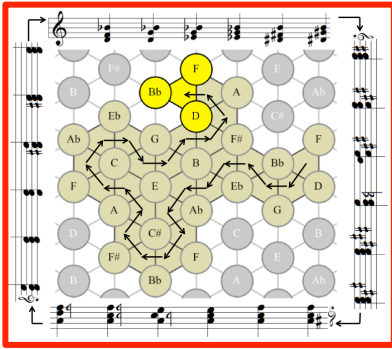




# Enumeration of Hamiltonian Cycles in the *Tonnetz*







**Do** ← do<sub>m</sub> ← Sol# ← fa<sub>m</sub> ← Fa ← la<sub>m</sub> ← La ← fa#<sub>m</sub> ← Fa# ← sib<sub>m</sub> ← Do# ← do#<sub>m</sub>

↙ **mi<sub>m</sub>** → Sol → si<sub>m</sub> → Ré → ré<sub>m</sub> → Sib → sol<sub>m</sub> → Mib → mib<sub>m</sub> → Si → sol#<sub>m</sub> → Mi ↘

La ↗ ↘

**Do** → mi<sub>m</sub> → Mi → sol#<sub>m</sub> → Si → ré#<sub>m</sub> → Re# → do<sub>m</sub> → Lab → fa<sub>m</sub> → Do# → do#<sub>m</sub>

↙ la<sub>m</sub> ← Fa ← ré<sub>m</sub> ← Ré ← si<sub>m</sub> ← Sol ← sol<sub>m</sub> → Sib ← sib<sub>m</sub> ← Fa# ← fa#<sub>m</sub> ← La ↘

**Mi** ← mi<sub>m</sub> ← Do ← la<sub>m</sub> ← Fa ← fa<sub>m</sub> ← Reb ← sib<sub>m</sub> ← Fa# ← mib<sub>m</sub> ← Mib ← do<sub>m</sub>

↙ do#<sub>m</sub> → La → fa#<sub>m</sub> → Ré → ré<sub>m</sub> → Sib → sol<sub>m</sub> → Sol → si<sub>m</sub> → Si → sol#<sub>m</sub> → Sol# ↘

La ↗ ↘



**M. Andreatta, « Math'n pop : symétries et cycles hamiltoniens en chanson », *Tangente***

*Aprile*

4D & 2D Visualizations  
Hamiltonian Cycles  
M.Andreatta, G.Baroin 2013

Lyrics: Gabriele d'Annunzio  
Music and Vocals: Moreno Andreatta  
Hypersphere and Ideogramms: Gilles Baroin  
Original "Chicken Wire" graph: J.Douthett, P.Steinbach

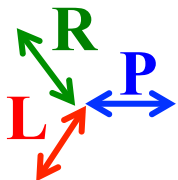
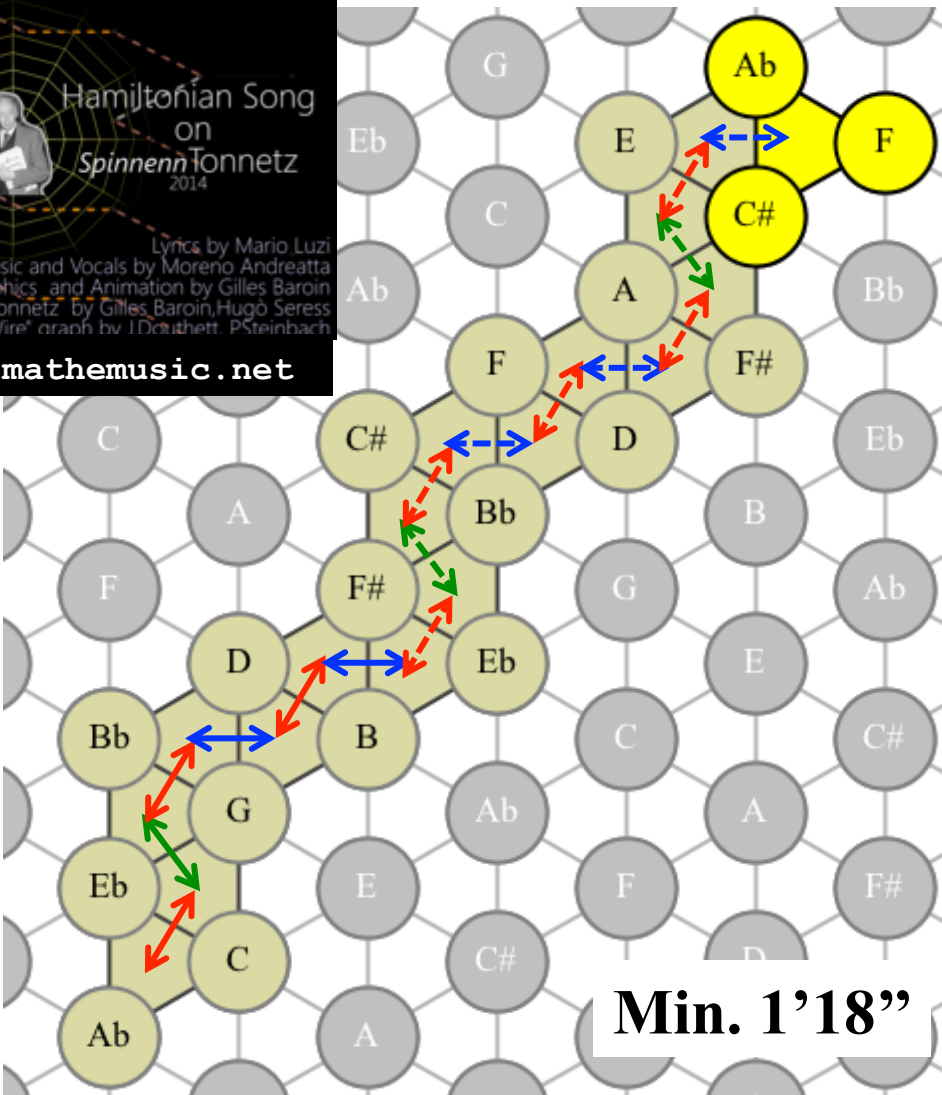
<http://www.mathemusic.net>



# Hamiltonian Cycles with inner periodicities

#41	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R			
#62	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	R	
#13	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P
#4	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R
#58	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P

L P L P L R L P L P L R ...  
 P L P L R L ...  
 L P L R L P ...  
 P L R L P L ...  
**L R L P L P ...**  
 R L P L P L ...



La sera non è più la tua canzone  
 (Mario Luzi, 1945, tratto da *Poesie sparse*)

La sera non è più la tua canzone,  
 è questa roccia d'ombra traforata  
 dai lumi e dalle voci senza fine,  
 la quiete d'una cosa già pensata.

Ah questa luce viva e chiara viene  
 solo da te, sei tu così vicina  
 al vero d'una cosa conosciuta,  
 per nome hai una parola ch'è passata  
 nell'intimo del cuore e s'è perduta.

Caduto è più che un segno della vita,  
 riposi, dal viaggio sei tornata  
 dentro di te, sei scesa in questa pura  
 sostanza così tua, così romita  
 nel silenzio dell'essere, (computa).

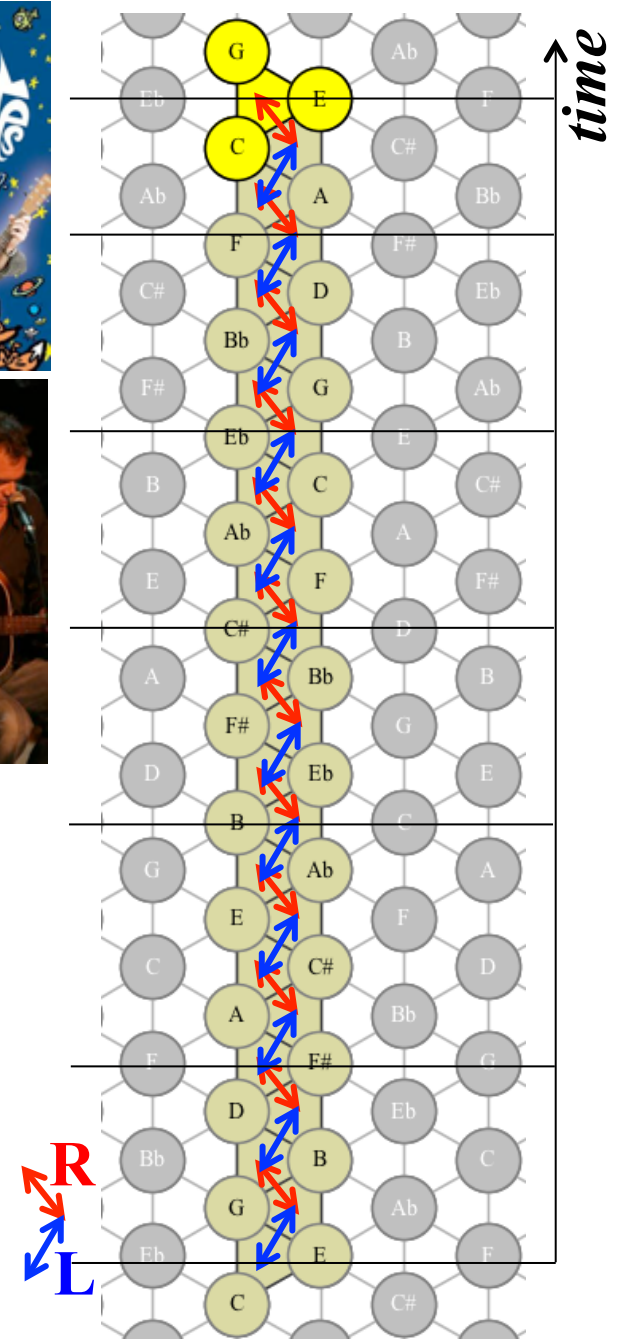
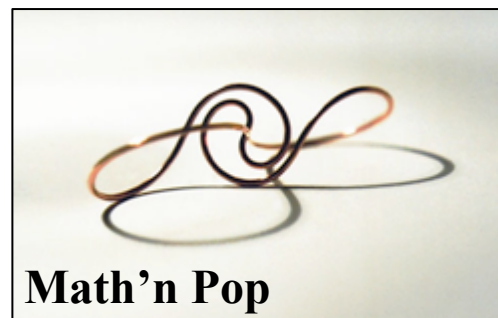
L'aria tace ed il tempo dietro a te  
 si leva come un'arida montagna  
 dove vaga il tuo spirito e si perde,  
 un vento raro scivola e ristagna.



# Le blé en herbe

(Polo/Moreno/Dieu)

Plonger comme un enfant, cheveux au vent	Croiser matin dans l'herbe folle
Sous l'océan du blé en herbe	Deux tourterelles qui s'envolent
Marée d'épis couleur d'amande	Suivre les jeux des hirondelles
Qui tendent à caresser le ciel	Sur le paysage éternel
	Nager comme un enfant, cheveux au vent
Algues tendres de mille plages	Sous l'océan
Frôlant le ventre des nuages	Du blé en herbe
Cheveux de pluie, dos de poissons	
Qui frissonnent à l'unisson	Marée de fruits au goût amer
	Acide et salée comme la mer
Suivre le bord des continents	
Dans l'océan du blé en herbe	Vers l'ilôt d'un petit village
Pêcher le corail du pavot	Vers un château d'eau sur la plage
Dans le sang des coquelicots	Quand tout s'éteint avant l'orage
	Quand se lève le vent du large
	Sur le blé vert





# The use of constraints in arts



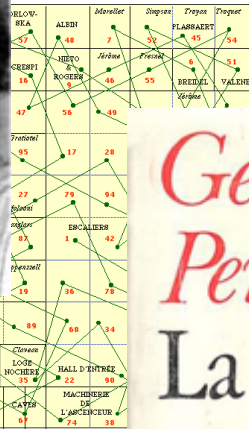
*Cent mille milliards de poèmes, 1961*



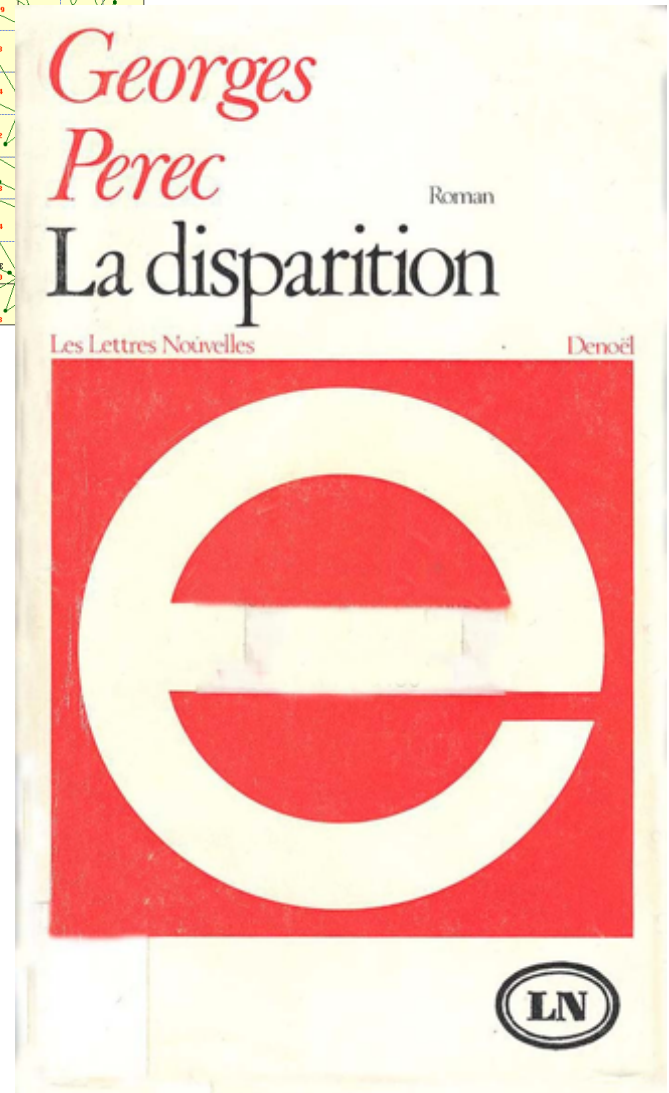
*La vie mode d'emploi,*



**Georges Perec**



**OuLiPo (Ouvroir de Littérature Potentielle)**



**Raymond Queneau**

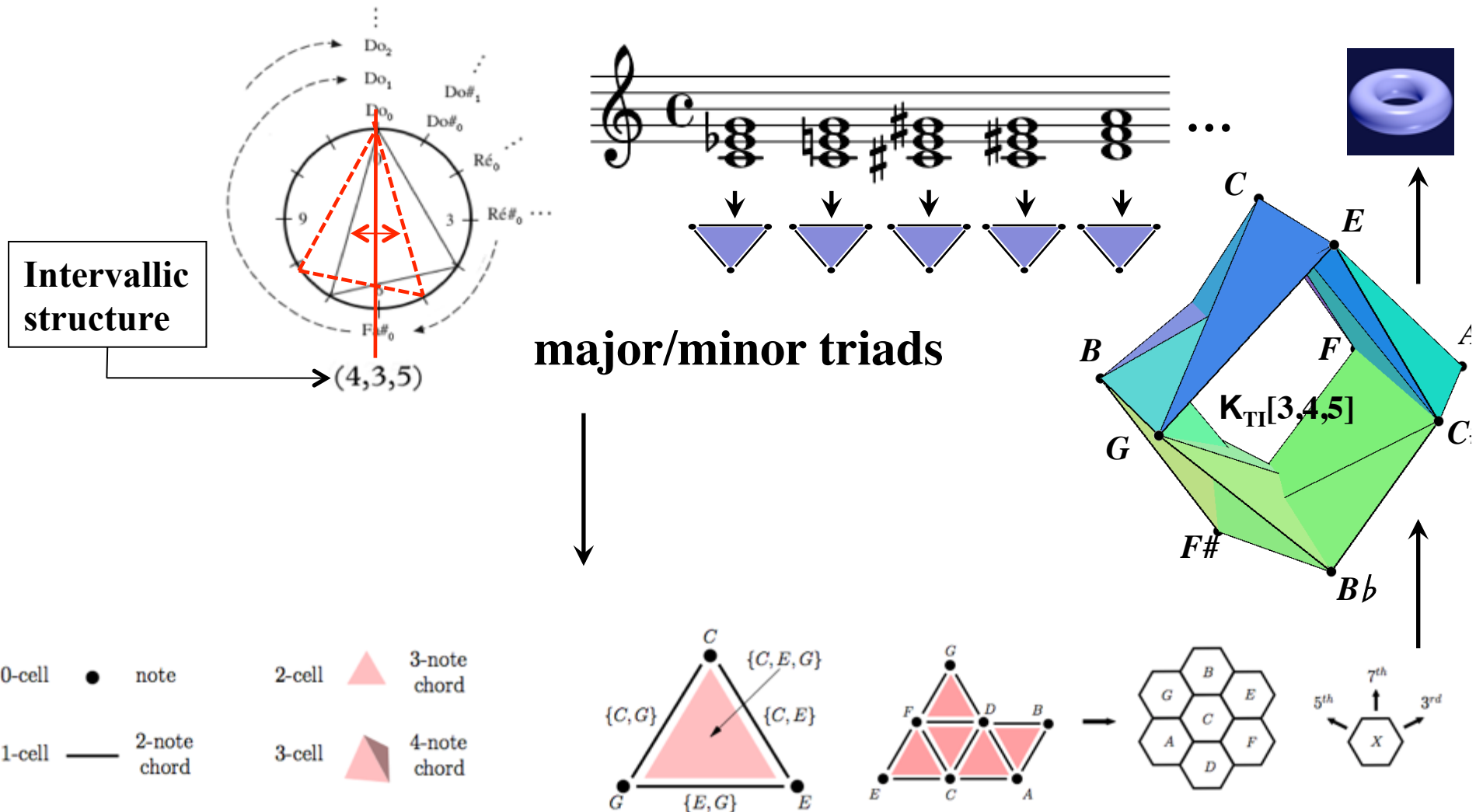


**Italo Calvino**  
*Il castello dei destini incrociati, 1969*

# Building Chord Complexes

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

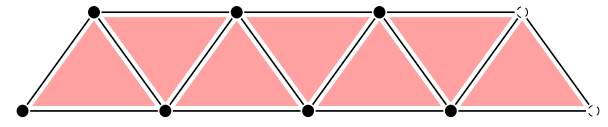
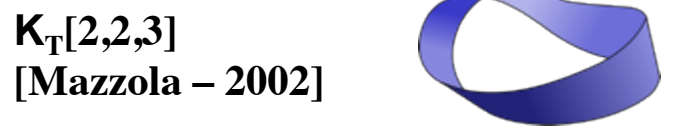
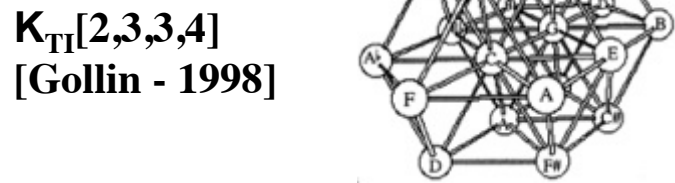
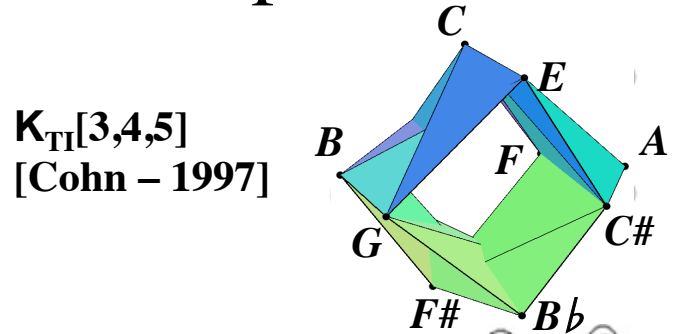
- Assembling chords related by some equivalence relation
  - Transposition/inversion: Dihedral group action on  $P(\mathbb{Z}_n)$



# Classifying Chord Complexes

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

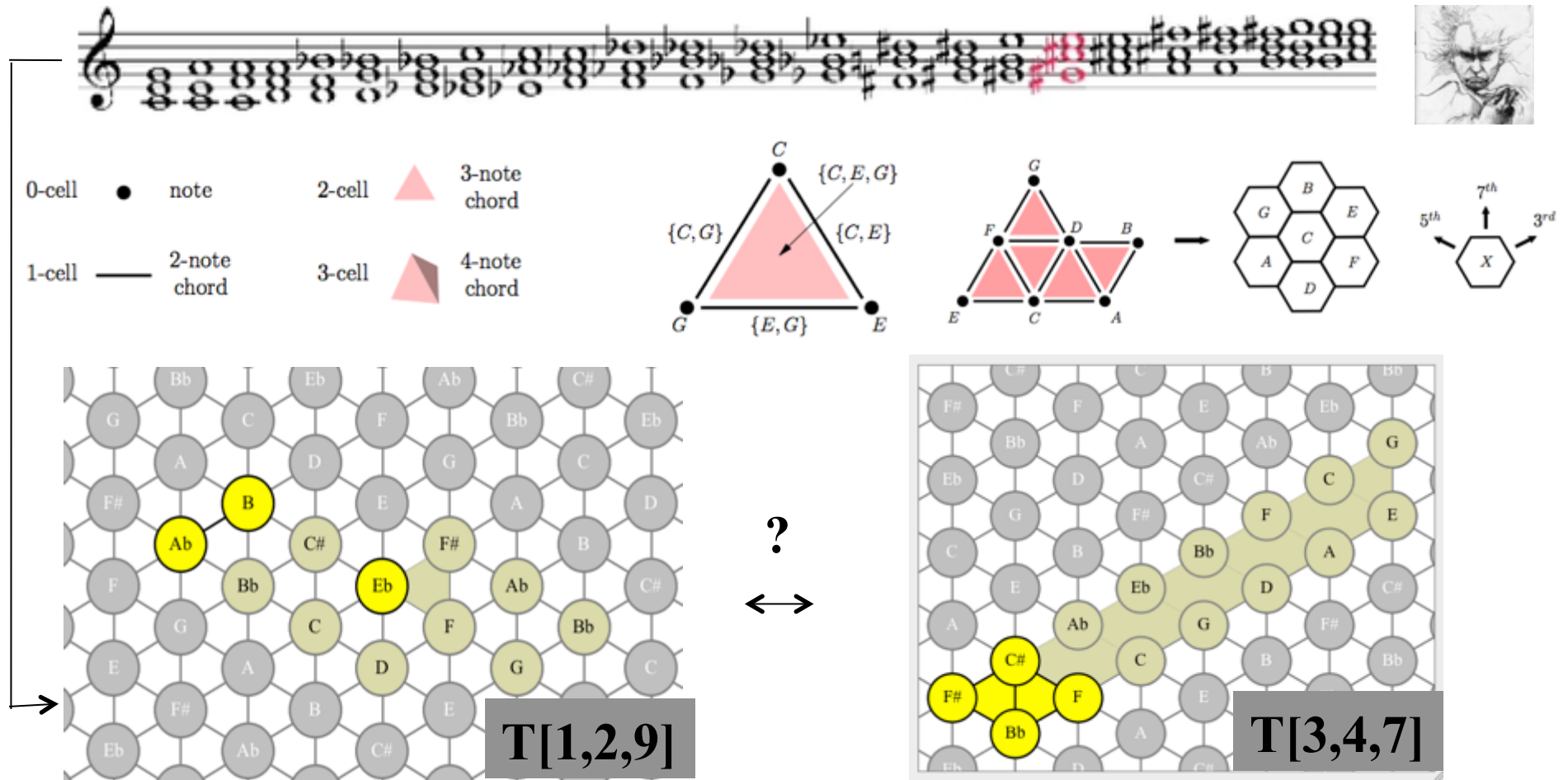
- Complexes enumeration in the chromatic system



...

$d$	complexe	taille	$b_n$	p-v	$\chi$
-	$\mathcal{K}_\emptyset$	0	0		0
0	$\mathcal{K}_{TI}[0]$	0	[0]		0
1	$\mathcal{K}_{TI}[1, 11]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[2, 10]$	12	[2, 2]		0
	$\mathcal{K}_{TI}[3, 9]$	12	[3, 3]		0
	$\mathcal{K}_{TI}[4, 8]$	12	[4, 4]		0
	$\mathcal{K}_{TI}[5, 7]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[6, 6]$	6	[6, 0]		6
2	$\mathcal{K}_{TI}[1, 1, 10]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[1, 2, 9]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1, 3, 8]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1, 4, 7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1, 5, 6]$	24	[1, 1, 6]		6
	$\mathcal{K}_{TI}[2, 2, 8]$	12	[2, 2, 0]		0
	$\mathcal{K}_{TI}[2, 3, 7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[2, 4, 6]$	24	[2, 2, 6]		6
	$\mathcal{K}_{TI}[2, 5, 5]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[3, 3, 6]$	12	[3, 0, 3]		6
	$\mathcal{K}_{TI}[3, 4, 5]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[4, 4, 4]$	4	[4, 0, 0]		4
	$\mathcal{K}_{TI}[1, 1, 1, 9]$	12	[1, 1, 0, 0]	x	0
	$\mathcal{K}_{TI}[1, 1, 2, 8]$	24	[1, 1, 12, 0]		12
	$\mathcal{K}_{TI}[1, 1, 3, 7]$	24	[1, 2, 13, 0]		12
	$\mathcal{K}_{TI}[1, 1, 4, 6]$	24	[1, 1, 18, 0]		18
	$\mathcal{K}_{TI}[1, 1, 5, 5]$	12	[1, 1, 6, 0]		6

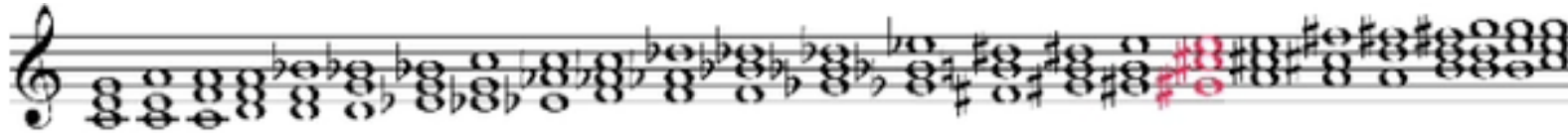
# Analyzing harmonic progressions as paths in a generic *Tonnetz*



- L. Bigo, M. Andreatta, J.-L. Giavitto, O. Michel, A. Spicher, « Computation and Visualization of Musical Structures in Chord-based Simplicial Complexes », MCM 2013, McGill University, Springer, LNCS.
- Bigo L., D. Ghisi, A. Spicher, M. Andreatta (2014), Proceedings ICMC|SMC|2014, 14-20 Sept. 2014, Athens (revised and enlarged version forthcoming in *Computer Music Journal*, 39(3), 2015)
- Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer (in press)



# Analyzing harmonic progressions as paths in *Hexachord*



**Plex Viewer**

**Tonnetz : K[3,4,5]**

**InfoBox**

Tempo: bwv0281.mid

0 10 20

Play Stop

Select midi file

Chromatic complexes: K[2,3,7]

Heptatonic complexes: CM

Trace off Harmonization ON

Display graph

Vertical compactness

compactness dimension: 2-compactness

complexes dimension: 2

compute compactness

absolute compactness

Path Transformation

Origin complex: K[3,4,5]

Destination complex: K[3,4,5]

Rotation: 0

North translation: 0

North-east translation: 0

Path Transformation

**Tonnetzs network**

**Chart**

**bwv0281**

Complex	2-compactness (approx.)
K[1,1,10]	0.05
K[1,2,9]	0.15
K[1,3,8]	0.15
K[1,4,7]	0.15
K[1,5,6]	0.15
K[2,2,8]	0.15
K[2,3,7]	0.15
K[2,4,6]	0.15
K[2,5,5]	0.15
K[3,3,6]	0.15
K[3,4,5]	0.80
K[4,4,4]	0.15

**Chart**

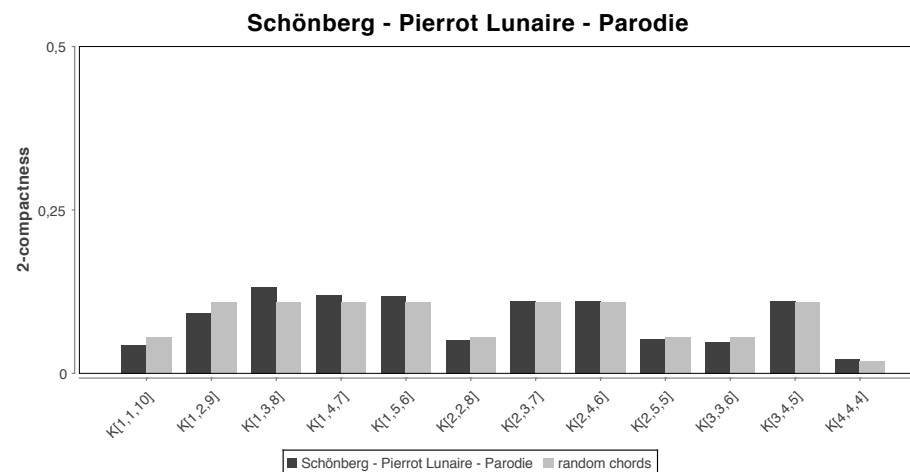
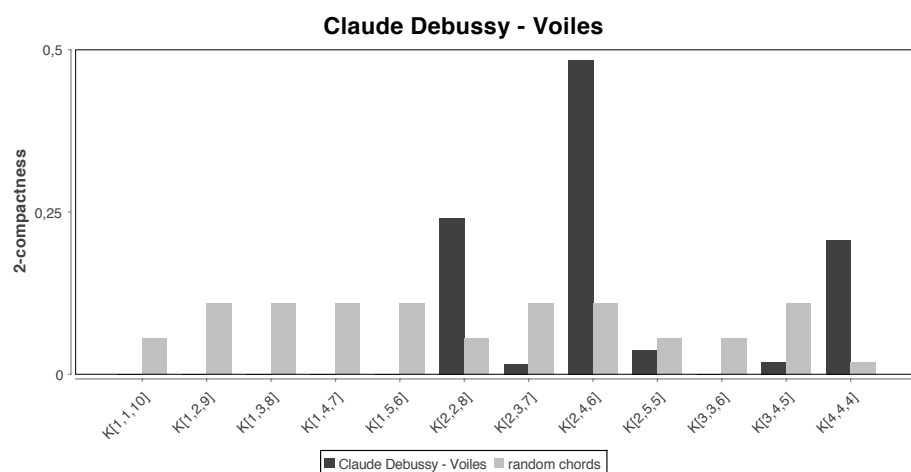
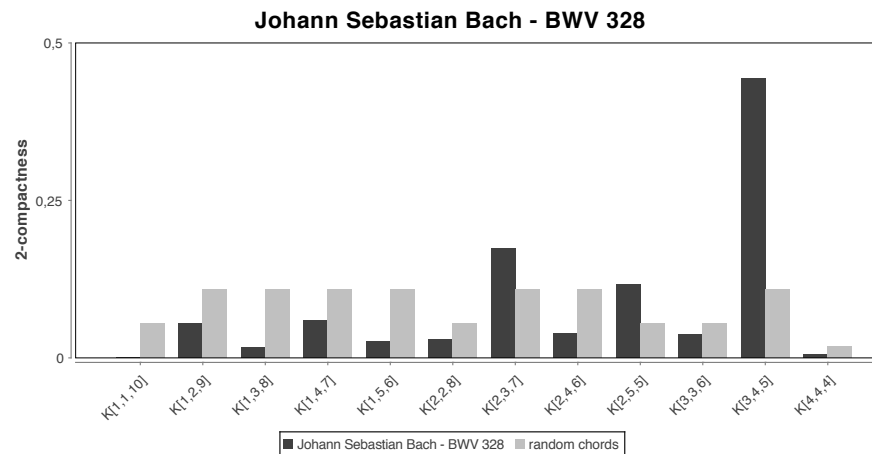
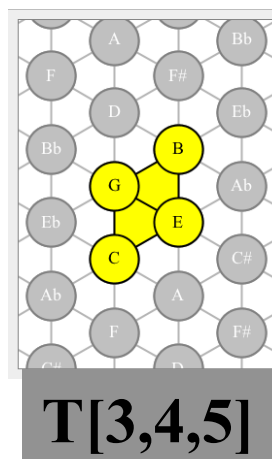
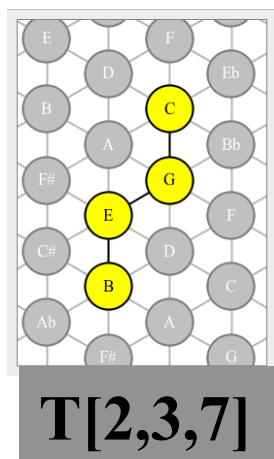
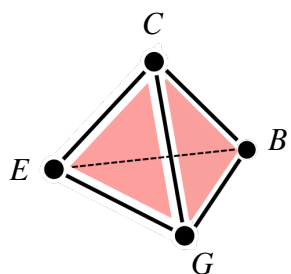
**2-compactness : bwv0281**

<http://www.lacl.fr/~lbiggo/hexachord>

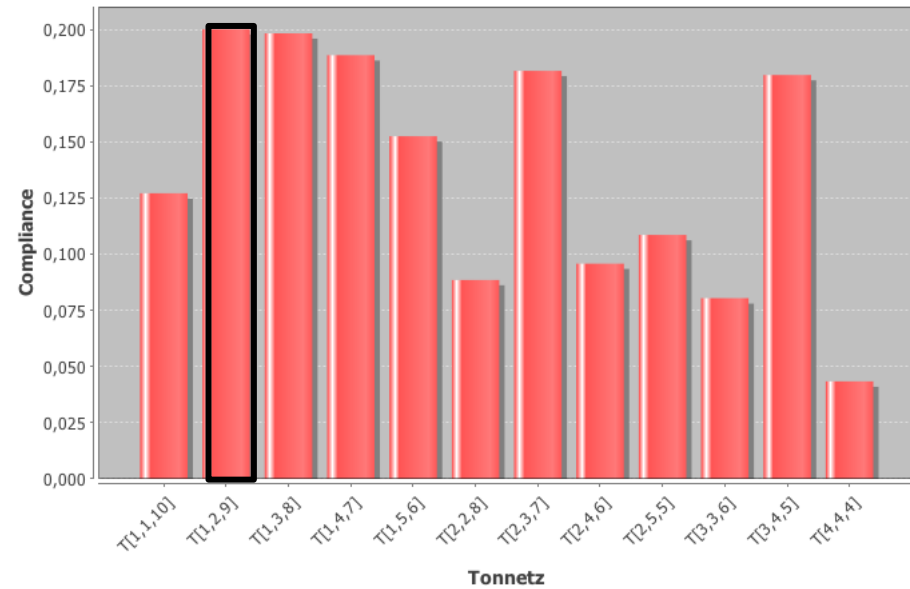
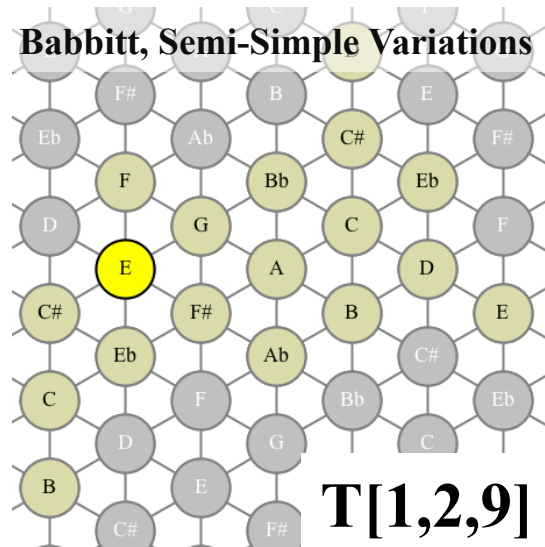
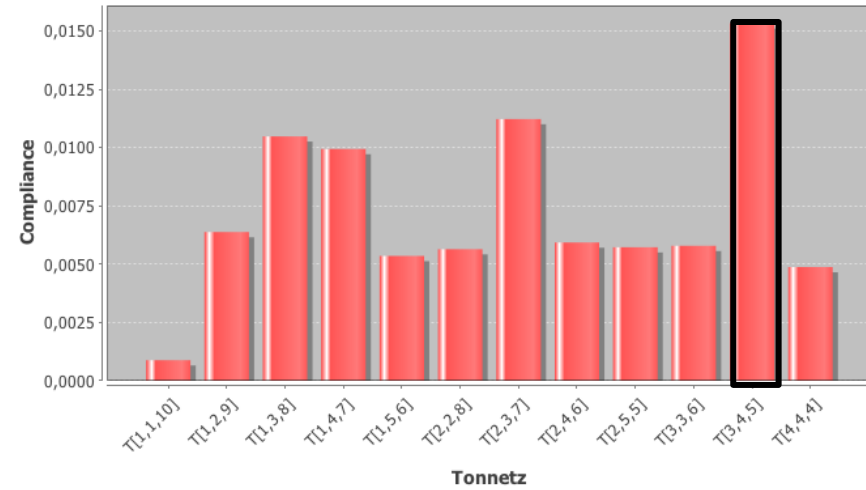
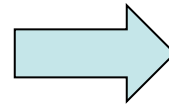
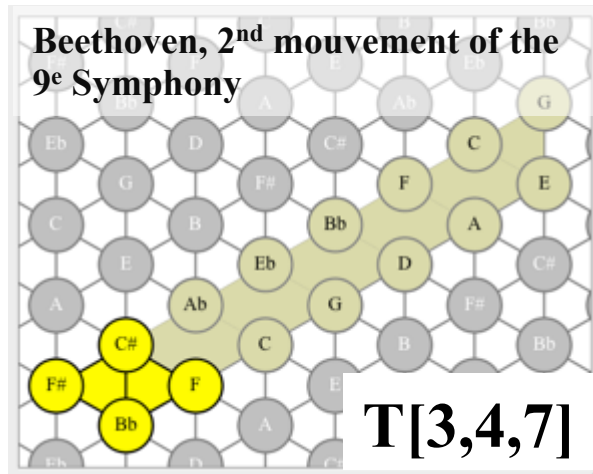
➔ demo

# The spatial character of the « musical style »

Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer (in press)



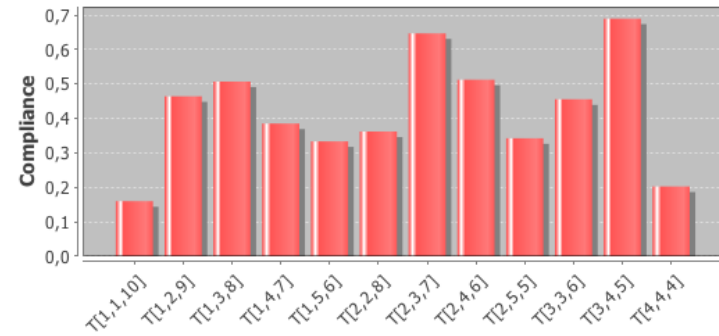
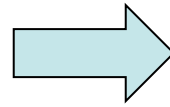
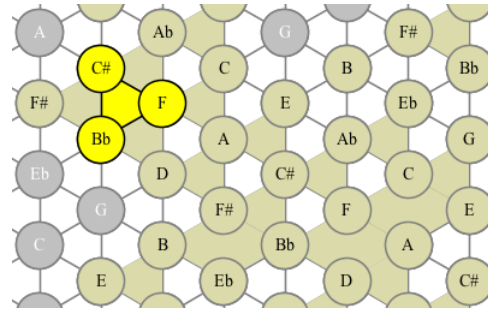
# The spatial character of the « musical style »



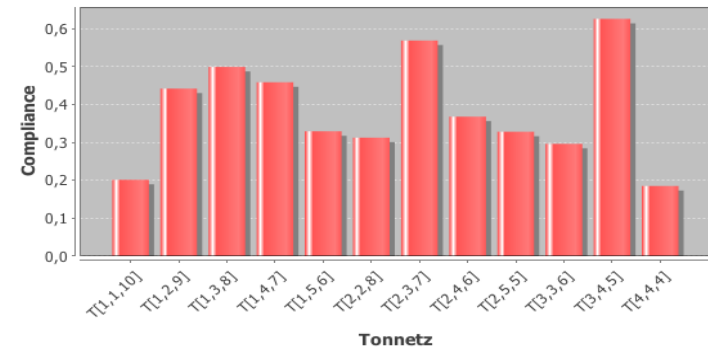
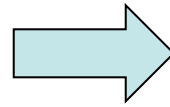
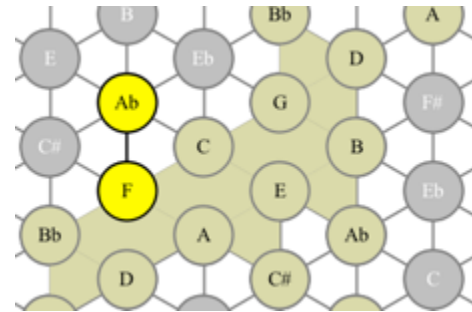
# The « shape » of space distributions in jazz standards



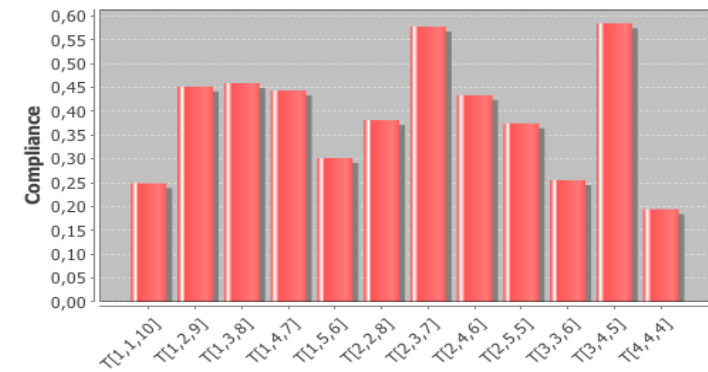
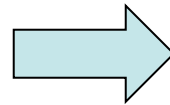
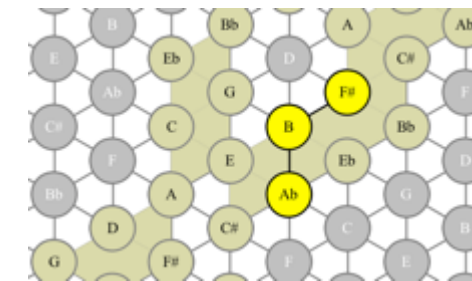
Thelonious Monk, *Brilliant Corners*



Chick Corea, *Eternal Child*



Bill Evans, *Turn Out the Stars*

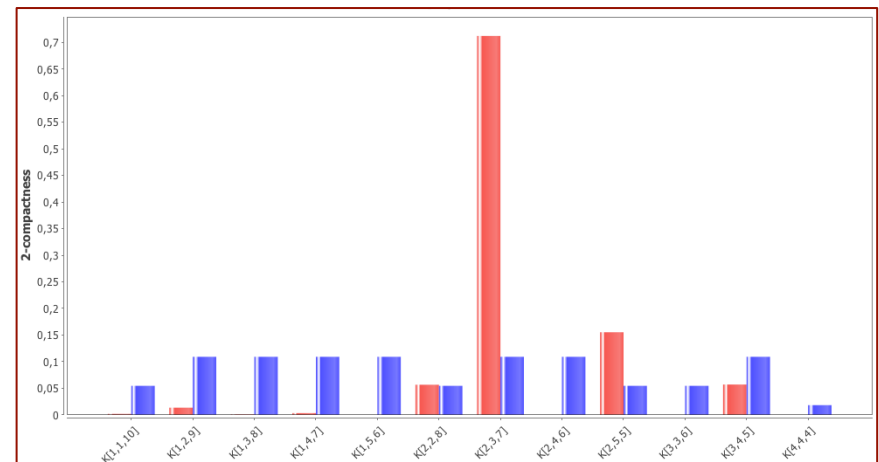
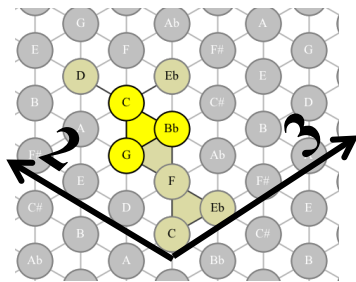
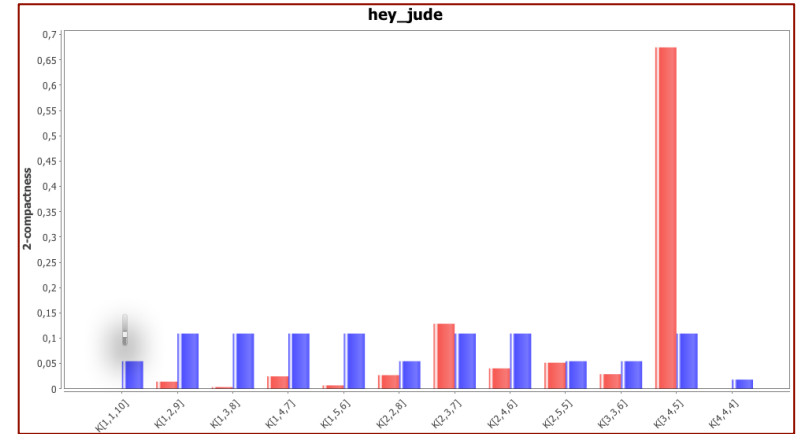
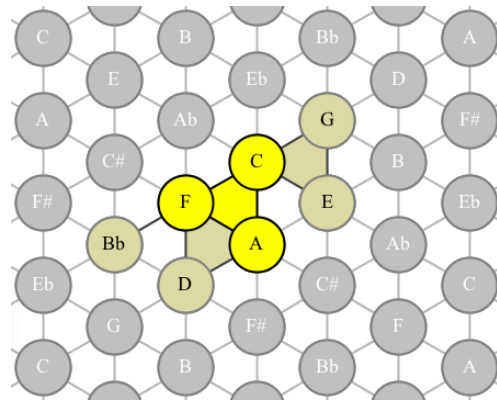




# Beatles' 'natural' Space and stylistic embeddings

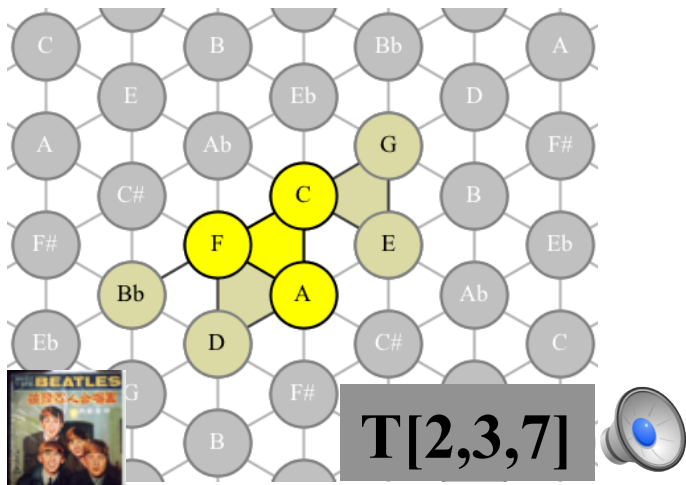
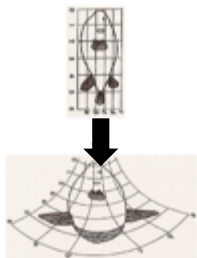
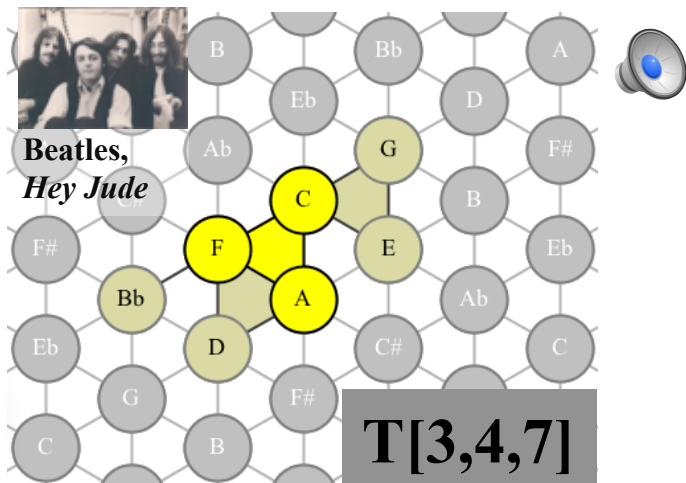


The Beatles,  
*Hey Jude*



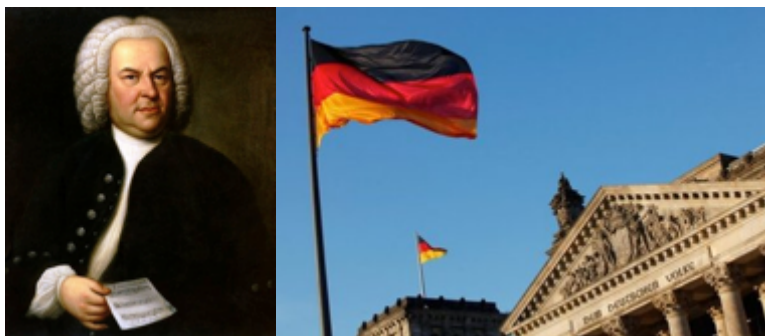
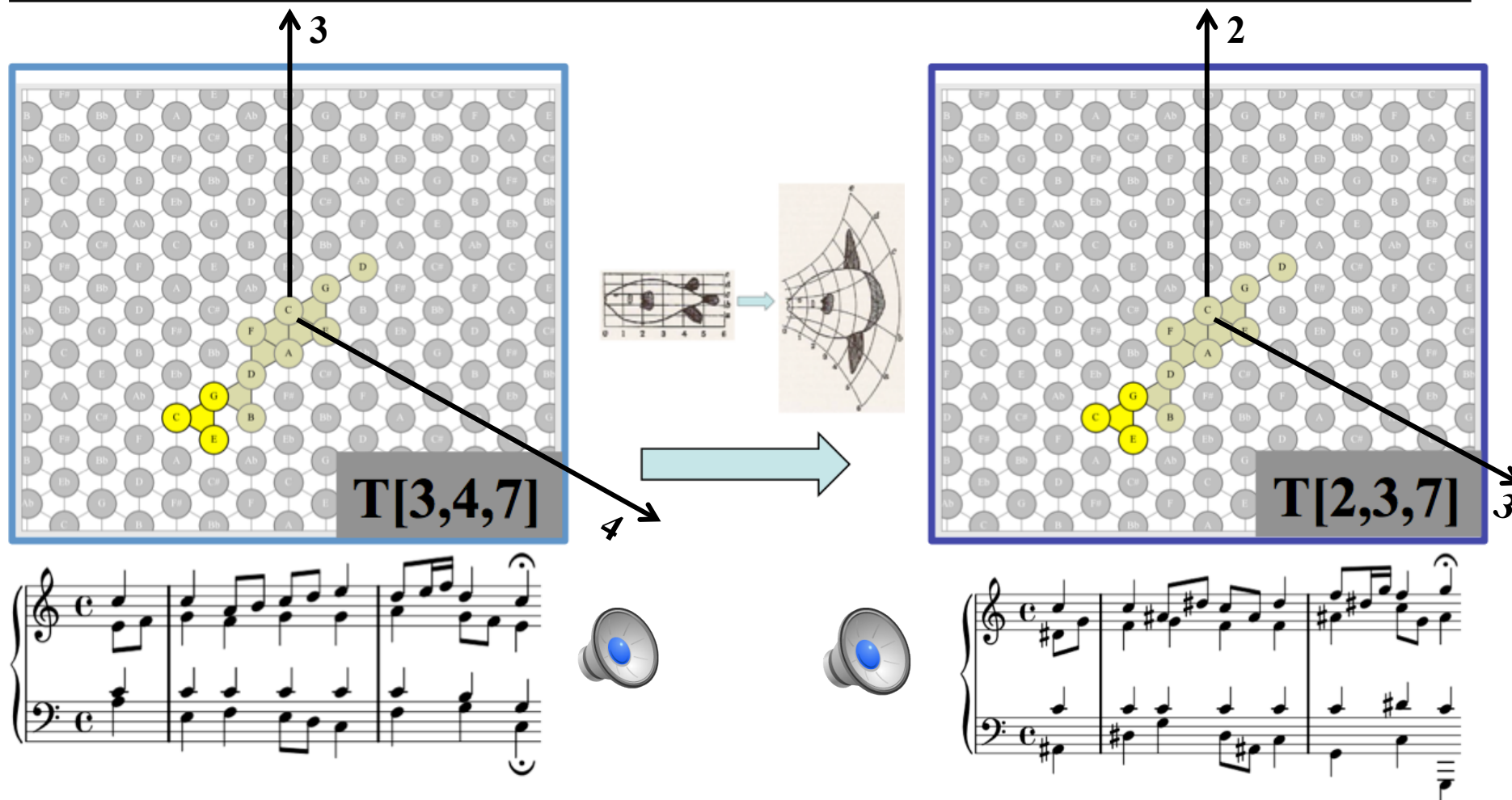
# Musically interesting Trajectory Transformations

Isomorphism from a support space to a different one



Transformation sur l'espace	Transformation sur la trajectoire	Transformation musicale
$\mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X_{\text{chro}}] \rightarrow \mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X_{\text{chro}}]$	Translation	Transposition chromatique
	Symétrie centrale	Inversion chromatique
	Rotation d'angle $\neq \pi$ Symétrie axiale	?
	Homothétie ( $\Leftrightarrow \mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X_{\text{chro}}] \rightarrow \mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X'_{\text{chro}}]$ )	?
$\mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X_{\text{hep}}]_{\mathbb{T}} \rightarrow \mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X_{\text{hep}}]_{\mathbb{T}}$	Translation	Transposition modale
	Symétrie centrale	Inversion modale
	Rotation d'angle $\neq \pi$ Symétrie axiale	?
	Homothétie ( $\Leftrightarrow \mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X_{\text{hep}}]_{\mathbb{T}} \rightarrow \mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X'_{\text{hep}}]_{\mathbb{T}}$ )	?
$\mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X_{\text{chro}}] \rightarrow \mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X'_{\text{chro}}]$	Plongement	?
$\mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X_{\text{hep}}]_{\mathbb{T}} \rightarrow \mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X'_{\text{hep}}]_{\mathbb{T}}$	Plongement	?
$\mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X_{\text{hep}}]_{\mathbb{T}} \rightarrow \mathcal{K}^u_{\mathbb{Z}\mathbb{Z}}[X_{\text{hep}}]_{\mathbb{T}'}$	Plongement	Transposition chromatique (+ transposition modale)
Trace $\rightarrow$ Trace	Isométrie	Permutation dans le temps des ensembles de notes
$\mathcal{K} \rightarrow \mathcal{K}$	Isométrie	?
$\mathcal{K} \rightarrow \mathcal{K}'$	Plongement	?

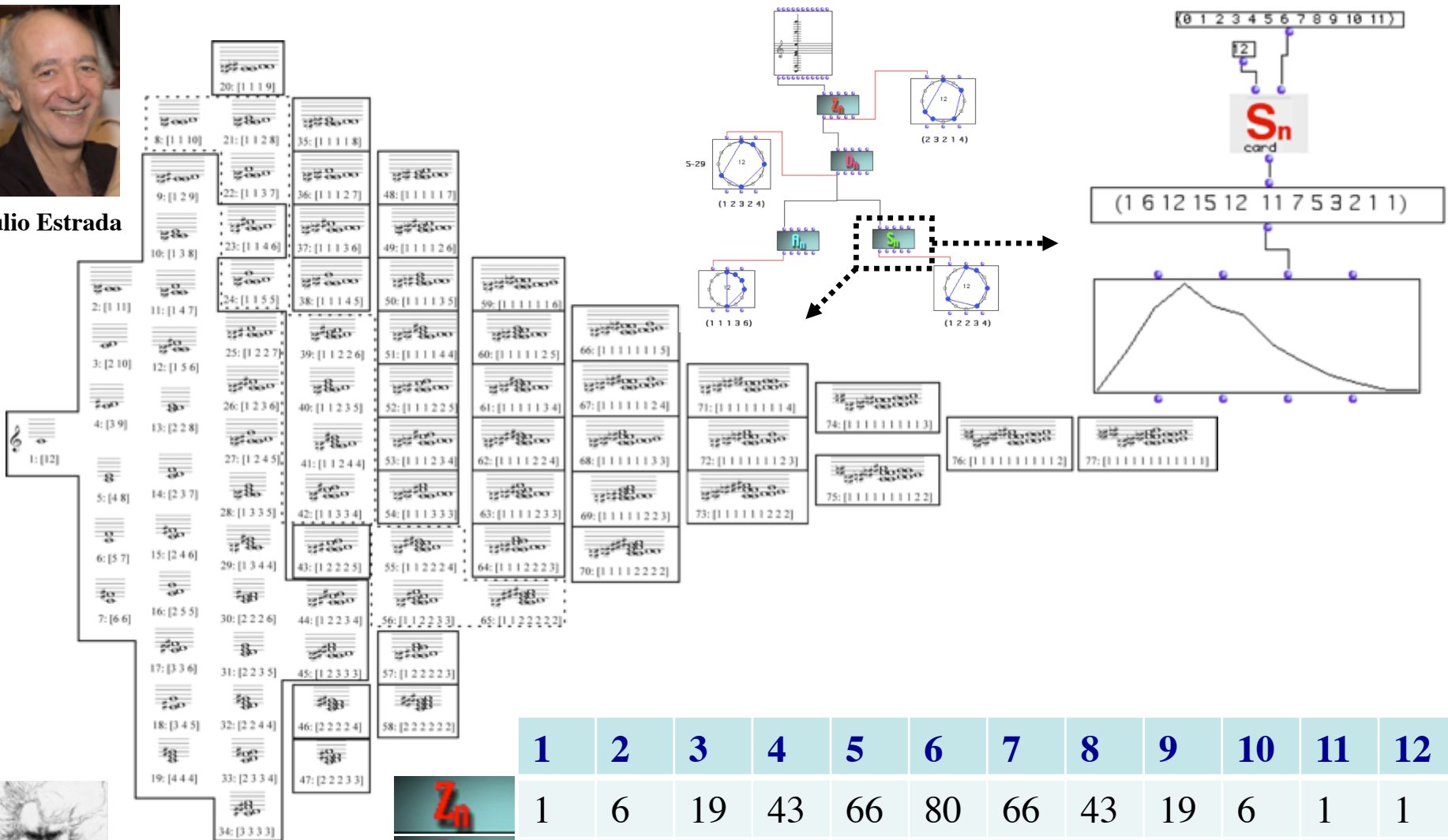
# A trajectory realized in different support spaces



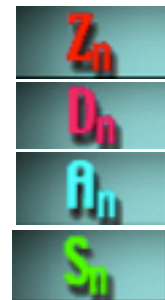
# The permutohedron as a parameter of style



Julio Estrada



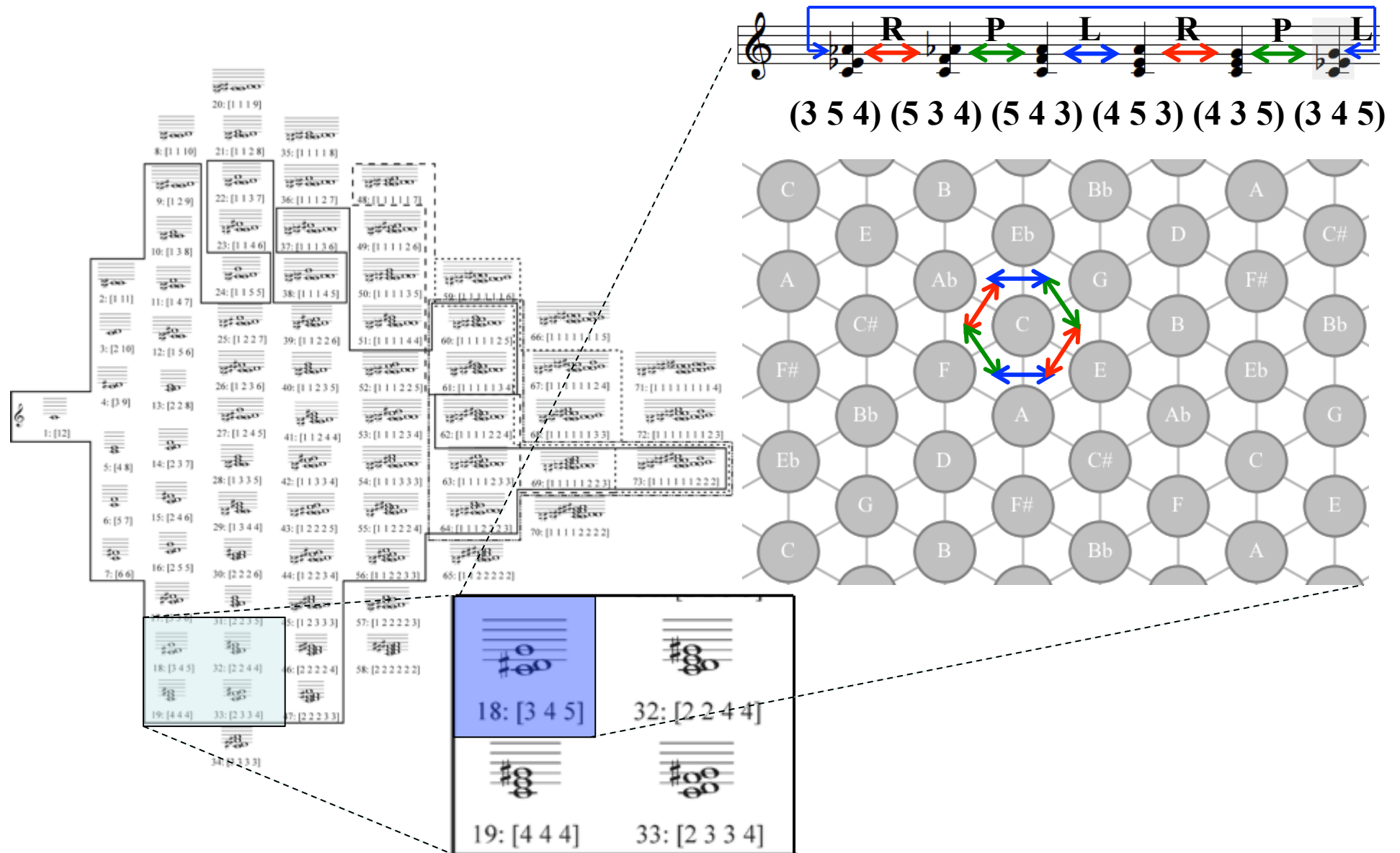
L. Van Beethoven,  
Quatuor n° 17



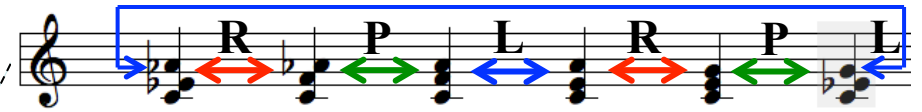
	1	2	3	4	5	6	7	8	9	10	11	12
$Z_n$	1	6	19	43	66	80	66	43	19	6	1	1
$D_n$	1	6	12	29	38	50	38	29	12	6	1	1
$A_n$	1	5	9	21	25	34	25	21	9	5	1	1
$S_n$	<b>1</b>	<b>6</b>	<b>12</b>	<b>15</b>	<b>12</b>	<b>11</b>	<b>7</b>	<b>5</b>	<b>3</b>	<b>2</b>	<b>1</b>	<b>1</b>



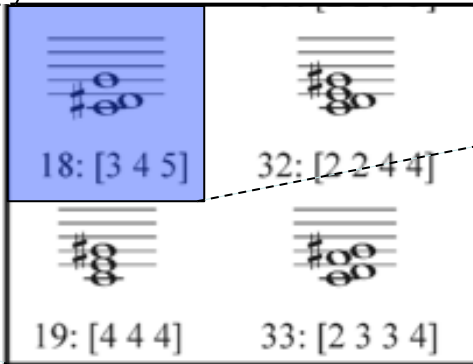
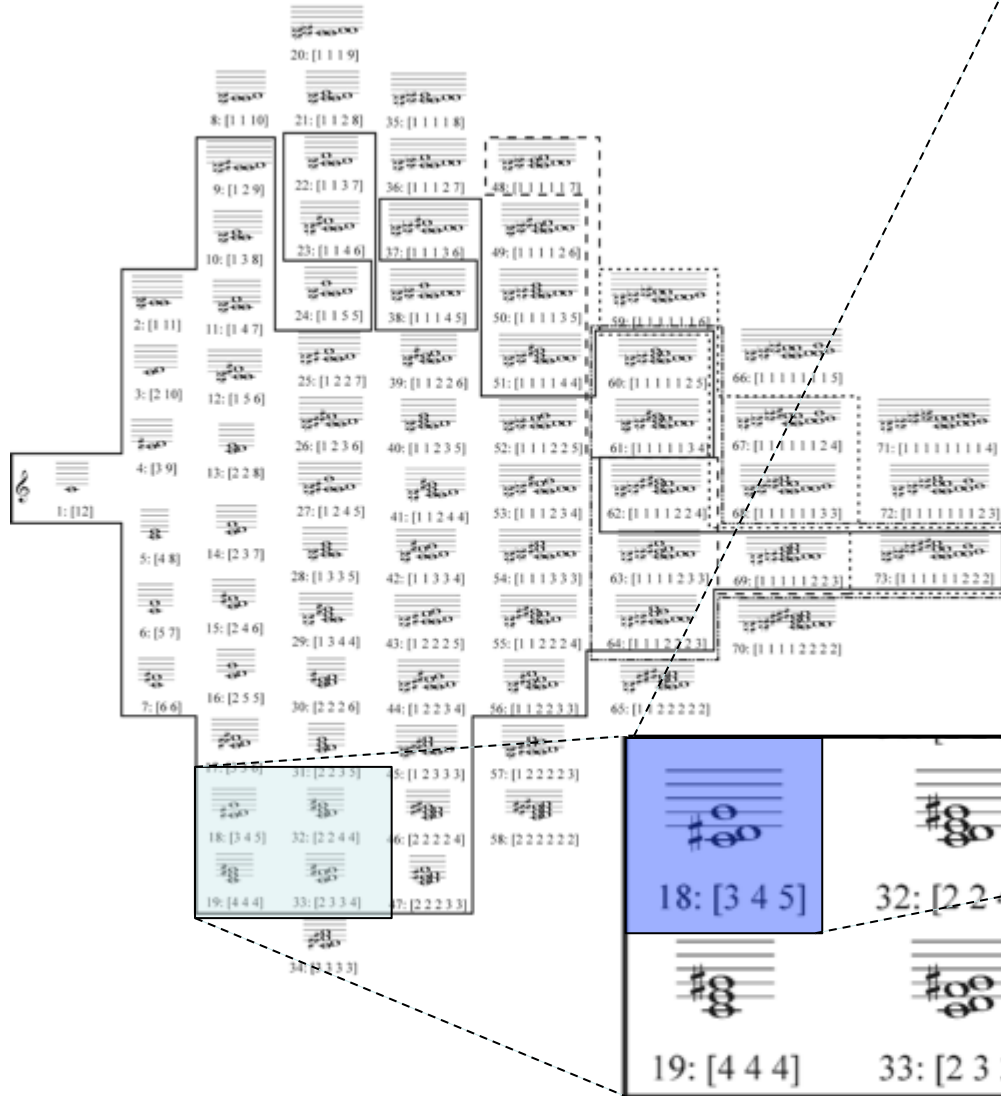
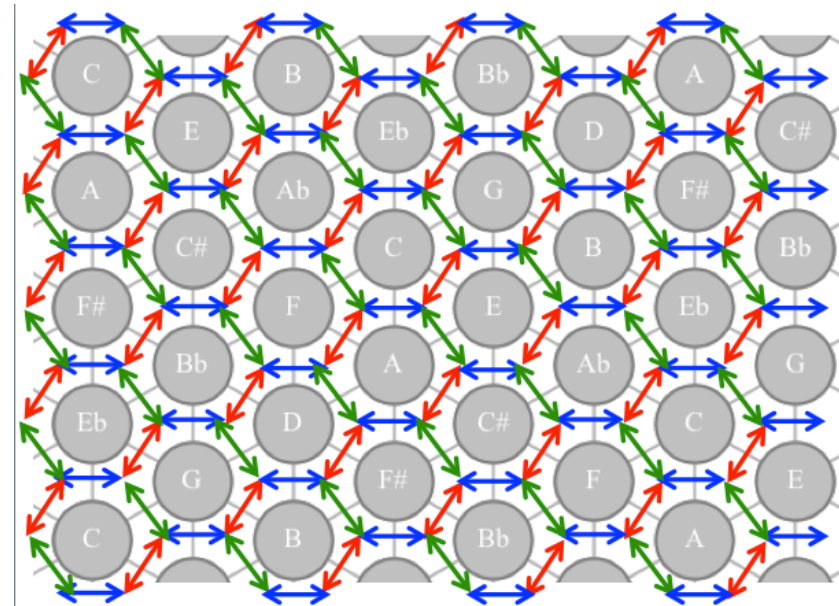
# Permutohedron and *Tonnetz*: a structural inclusion



# Permutohedron and *Tonnetz*: a structural inclusion



(3 5 4) (5 3 4) (5 4 3) (4 5 3) (4 3 5) (3 4 5)



# The permutohedron of 77 possible partitions of 12

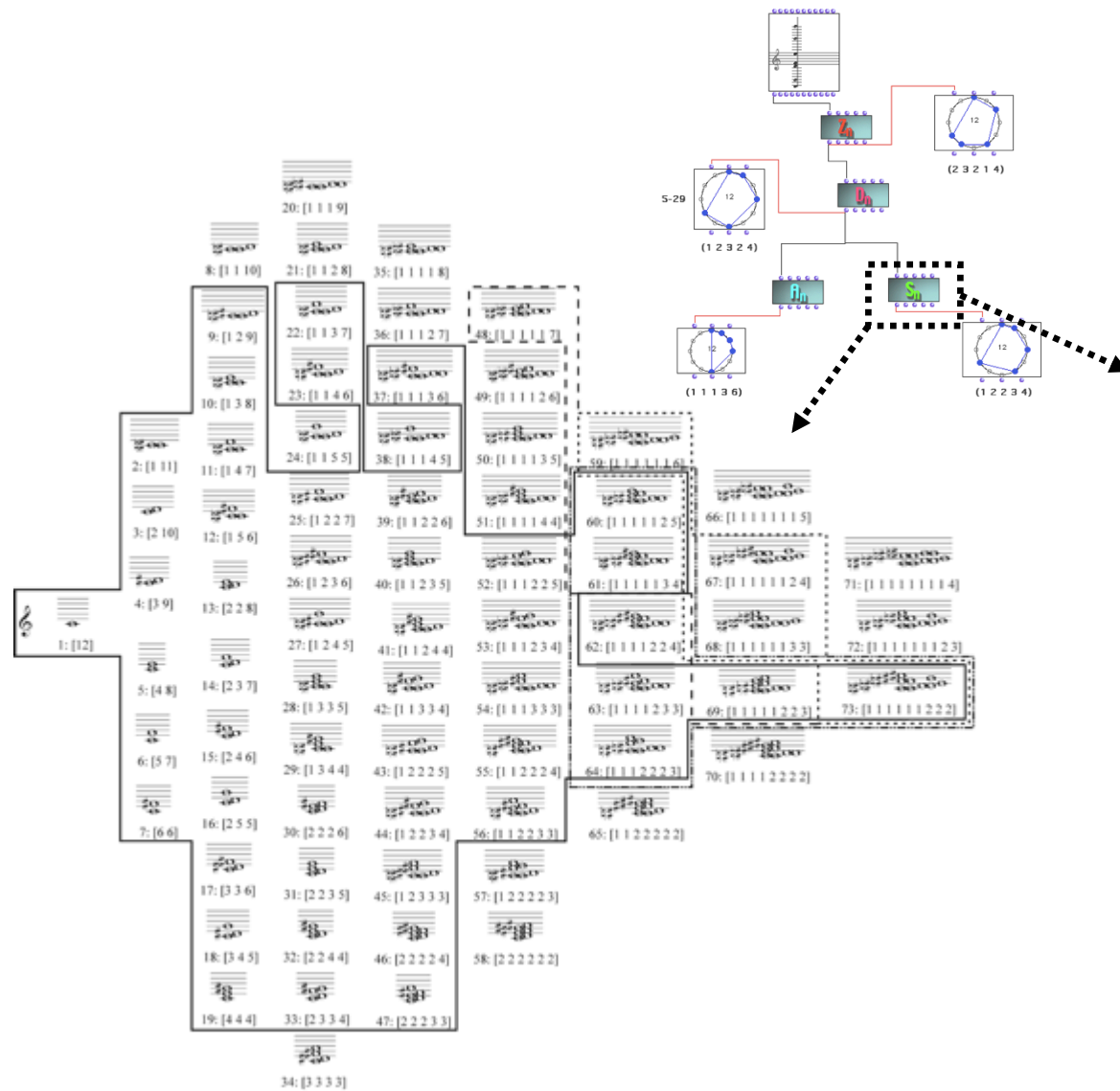


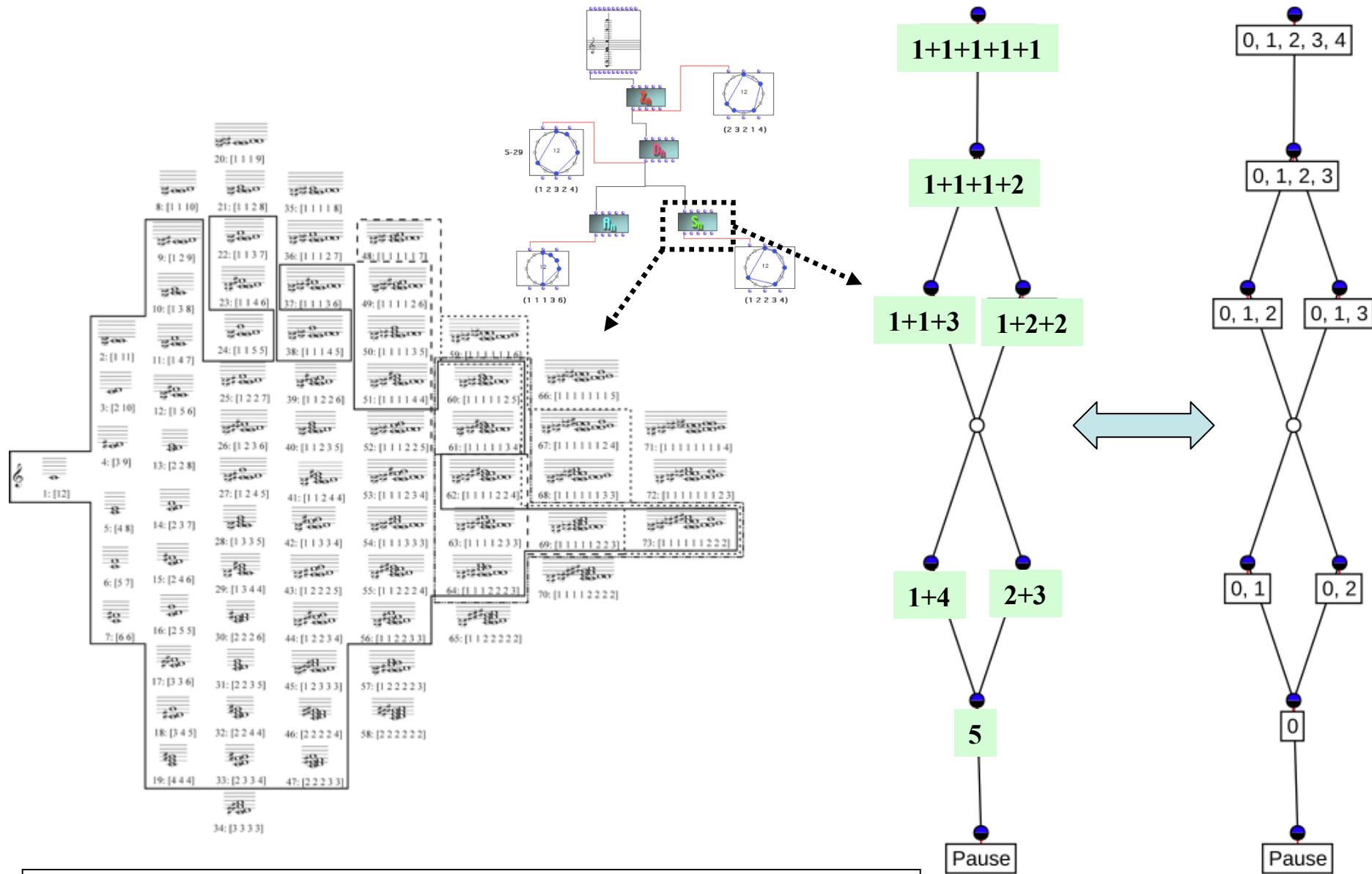
Tabelle 1  
Tabelle aller möglichen Intervallstrukturen

Nr.	Intervalle
0	—
1	12
2	6 + 6
3	5 + 7
4	4 + 8
5	4 + 4 + 4
6	3 + 9
7	3 + 4 + 5
8	3 + 3 + 6
9	3 + 3 + 3 + 3
10	2 + 10
11	2 + 5 + 5
12	2 + 4 + 6
13	2 + 3 + 7
14	2 + 3 + 3 + 4
15	2 + 2 + 8
16	2 + 2 + 4 + 4
17	2 + 2 + 3 + 5
18	2 + 2 + 2 + 6
19	2 + 2 + 2 + 3 + 3
20	2 + 2 + 2 + 2 + 4
21	2 + 2 + 2 + 2 + 2 + 2
22	1 + 11
23	1 + 5 + 6
24	1 + 4 + 7
25	1 + 3 + 8
26	1 + 2 + 9
27	1 + 3 + 4 + 4
28	1 + 3 + 3 + 5
29	1 + 2 + 4 + 5
30	1 + 2 + 3 + 6
31	1 + 2 + 2 + 7
32	1 + 2 + 3 + 3 + 3
33	1 + 2 + 2 + 3 + 4

*Studia Musicologica Academiae Scientiarum Hungaricae 9, 1967*

W. Reckziegel, "Musikanalyse und Wissenschaft", *Studia Musicologica* 9(1-2), 1967, 163-186

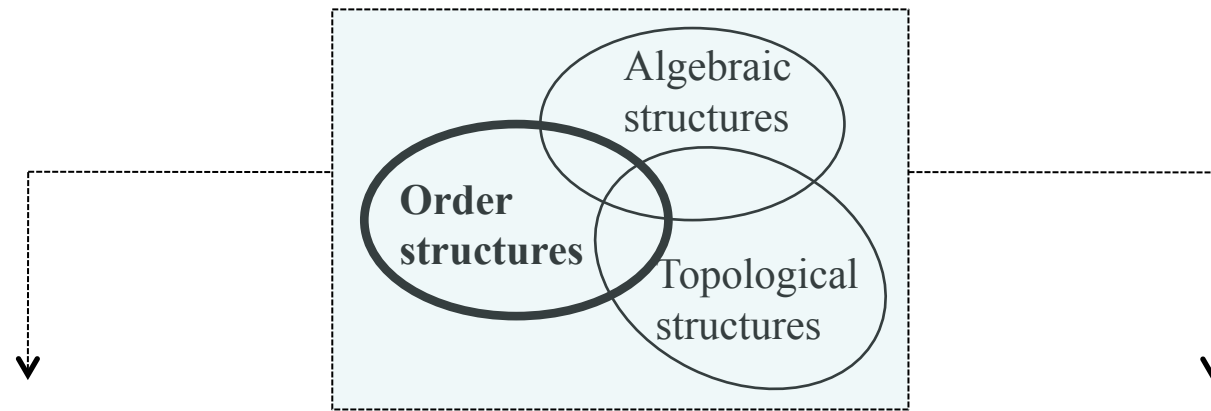
# The permutohedron as a lattice of formal concepts



• T. Schlemmer, M. Andreatta, « Using Formal Concept Analysis to represent Chroma Systems », MCM 2013, McGill Univ., Springer, LNCS.



# Formal Concept Analysis: the double history



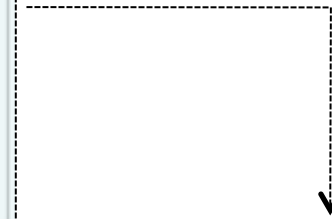
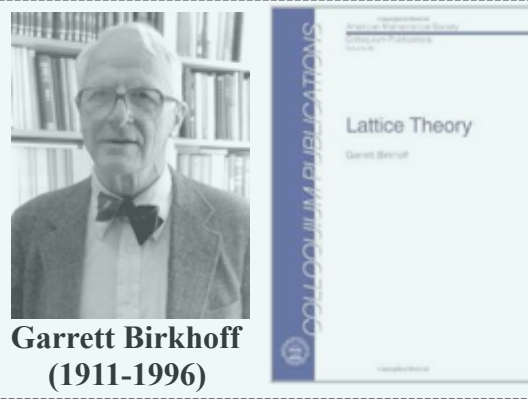
- M. Barbut, « Note sur l'algèbre des techniques d'analyse hiérarchique », in B. Matalon (éd.), *L'analyse hiérarchique*, Paris, Gauthier-Villars, 1965.
- M. Barbut, B. Monjardet, *Ordre et Classification. Algèbre et Combinatoire*, en deux tomes, 1970
- M. Barbut, L. Frey, « Techniques ordinales en analyse des données », Tome I, *Algèbre et Combinatoire des Méthodes Mathématiques en Sciences de l'Homme*, Paris, Hachette, 1971.
- B. Leclerc, B. Monjardet, « Structures d'ordres et sciences sociales », *Mathématiques et sciences humaines*, 193, 2011, 77-97



- R. Wille, « Mathematische Sprache in der Musiktheorie », in B. Fuchssteiner, U. Kulisch, D. Laugwitz, R. Liedl (Hrsg.): *Jahrbuch Überblicke Mathematik. B.I.-Wissenschaftsverlag*, Mannheim, 1980, p. 167-184.
- R. Wille, « Restructuring Lattice Theory: An approach based on Hierarchies of Concepts », I. Rival (ed.), *Ordered Sets*, 1982
- R. Wille, « Sur la fusion des contextes individuels », *Mathématiques et sciences humaines*, tome 85, 1984.
- B. Ganter & R. Wille, *Formal Concept Analysis: Mathematical Foundations*, Springer, Berlin, 1998



# Formal Concept Analysis: the common root



- M. Barbut, « Note sur l'algèbre des techniques d'analyse hiérarchique », in B. Matalon (éd.), *L'analyse hiérarchique*, Paris, Gauthier-Villars, 1965.
- M. Barbut, B. Monjardet, *Ordre et Classification. Algèbre et Combinatoire*, en deux tomes, 1970
- M. Barbut, L. Frey, « Techniques ordinales en analyse des données », Tome I, *Algèbre et Combinatoire des Méthodes Mathématiques en Sciences de l'Homme*, Paris, Hachette, 1971.
- B. Leclerc, B. Monjardet, « Structures d'ordres et sciences sociales », *Mathématiques et sciences humaines*, 193, 2011, 77-97

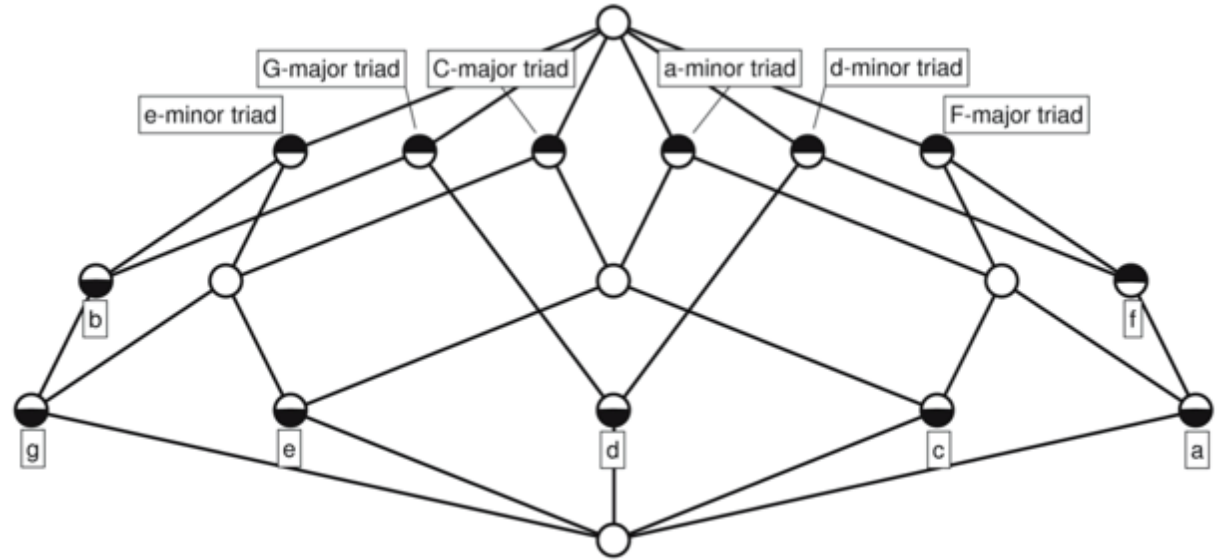


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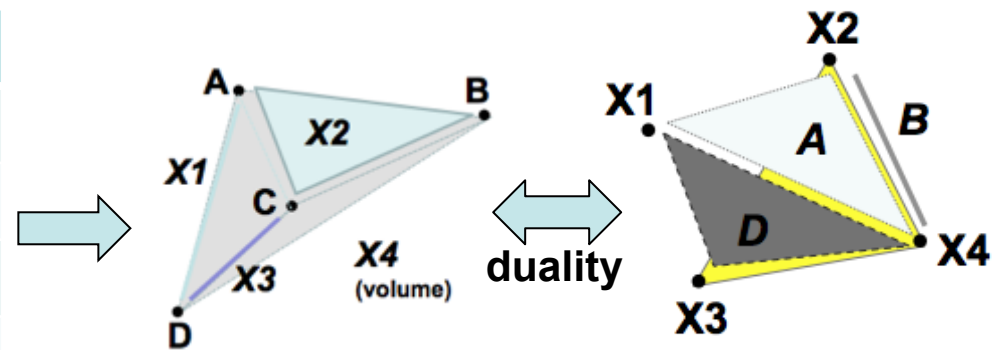


# Formal Concept Analysis and topology: the Q-analysis

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	×			×		×
d		×			×	
e	×		×			×
f		×		×		
g	×		×		×	
a		×		×		×
b			×		×	

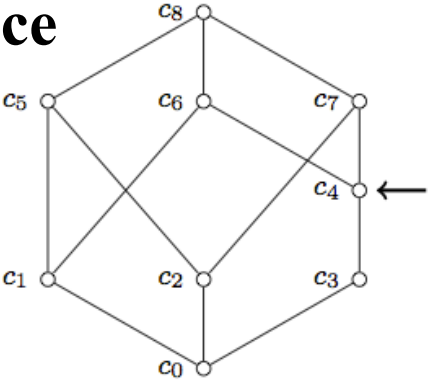


	A	B	C	D
X1	1	0	0	1
X2	1	1	1	0
X3	0	0	1	1
X4	1	1	1	1



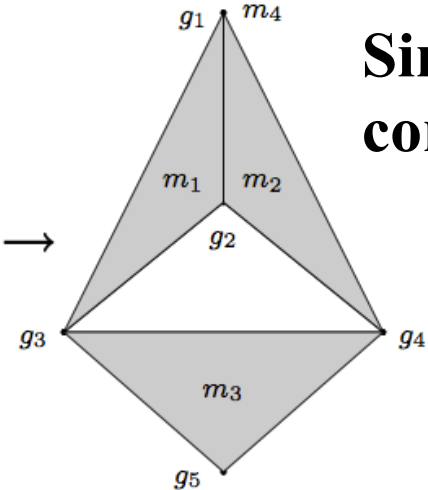
# Concept lattice vs simplicial complex

**Lattice**

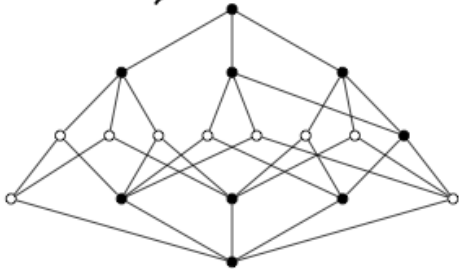
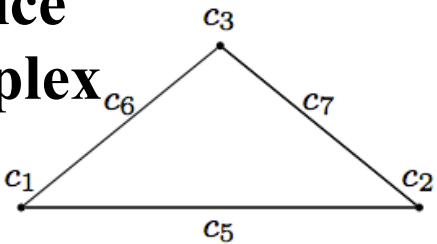


	$m_1$	$m_2$	$m_3$	$m_4$
$g_1$	×	×		×
$g_2$	×	×		
$g_3$	×		×	
$g_4$		×	×	
$g_5$			×	

**Simplicial complex**



**Lattice complex**



$\Delta$

$\zeta$

$\Gamma$

**Lattice-based and Topological Representations of Binary Relations with an Application to Music**

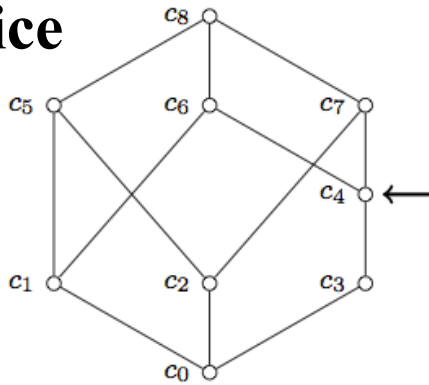
Anton Freund · Moreno Andreatta · Jean-Louis Giavitto

**Under reviewing**  
*(Annals of Mathematics and Artificial Intelligence)*



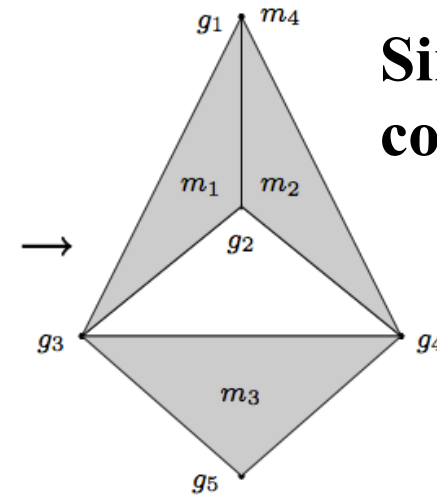
# Concept lattice vs simplicial complex

**Lattice**

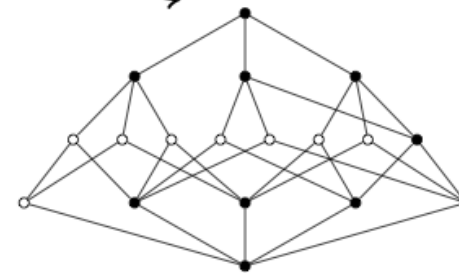
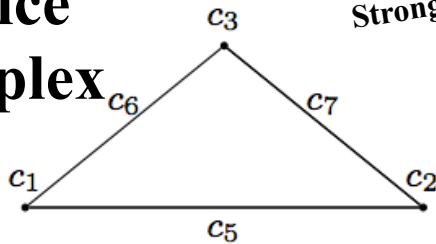


	$m_1$	$m_2$	$m_3$	$m_4$
$g_1$	×	×		×
$g_2$	×	×		
$g_3$	×		×	
$g_4$		×	×	
$g_5$			×	

**Simplicial complex**



**Lattice complex**



$\Delta$

$\zeta$   
Strong deformation retraction

$\Gamma$

**Conclusions:**

- The concept lattice alone cannot be fully reconstructed from the simplicial complex
- The simplicial complex cannot be fully determined from the concept lattice alone
- The concept lattice alone allows to determine the homotopy type of the simplicial complex

# The simplicial complex of a Chopin's *Prelude*

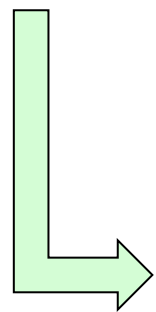
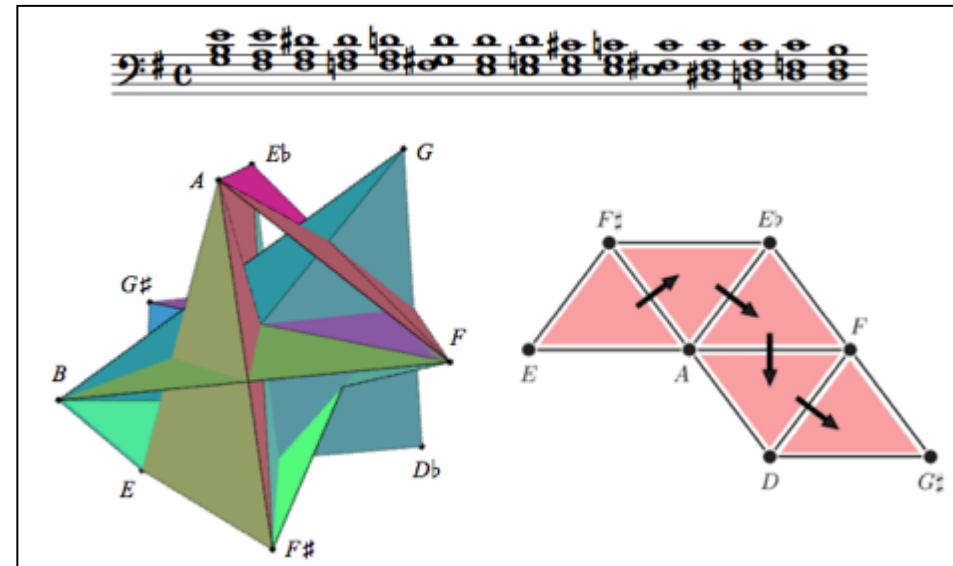
Prelude  
'Suffocation'

FREDERIC CHOPIN (1810-1849)  
Op. 28, No. 4

→ Hexachord  
(by Louis Bigo, 2013)

Largo

The musical score consists of three systems of piano notation. The first system shows the beginning of the piece with a *p* dynamic and a *capressivo* marking. The second system continues the piece with a *<* and *>* marking. The third system starts at measure 10 and includes a *p* dynamic marking.



0-simplex



{D, A, Bb}

1-simplex



{D, A}

{Bb, A}

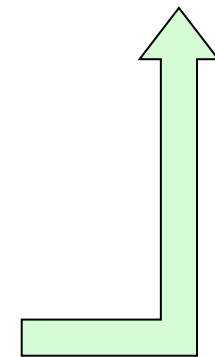
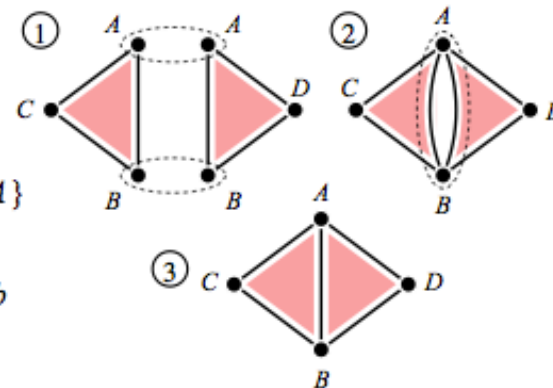
2-simplex



D

{D, Bb}

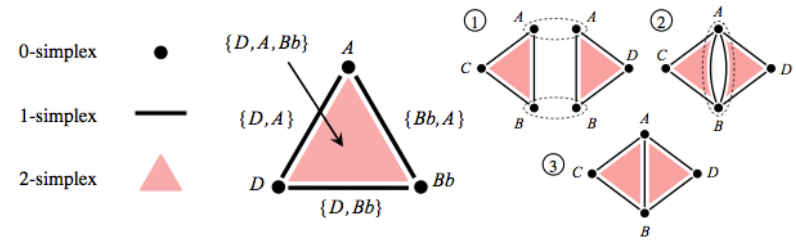
Bb



# Towards a topological signature of a musical piece

## A structural approach in Music Information Retrieval

The simplices and their self-assembly



The score

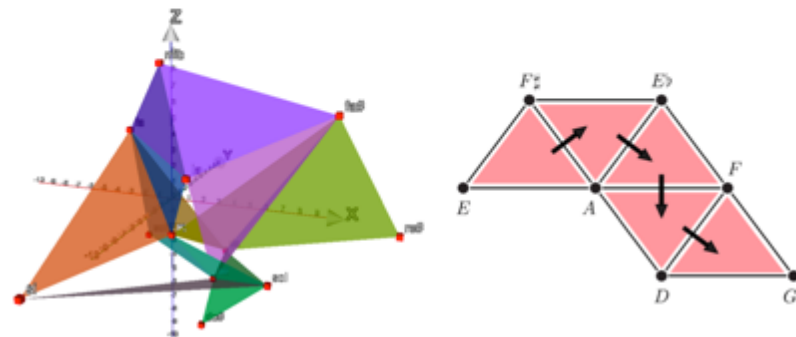
Prelude  
'Suffocation'

FREDERIC CHOPIN (1810-1849)  
Op. 28, No. 4

Largo

Score reduction

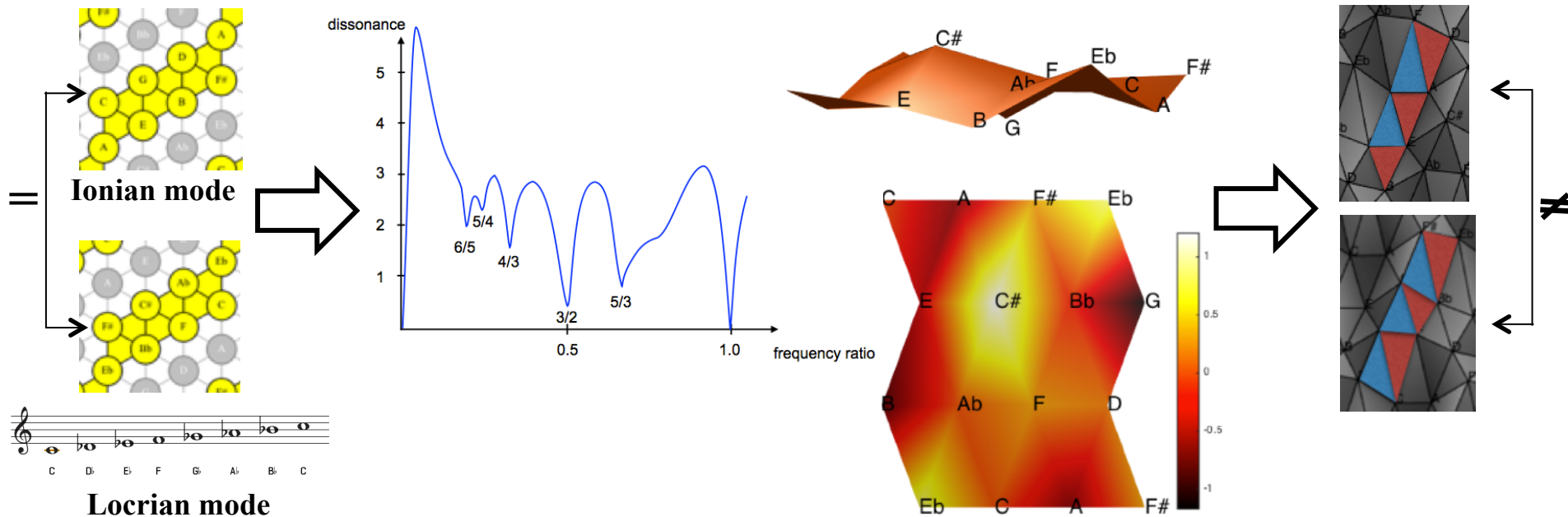
The simplicial complex generated by the piece



Topological signature?

A specific trajectory in the complex

# Signal/Symbolic articulation in MIR



M. Bergomi, *Dynamics and Algebraic Topology Tools for Music in the Symbolic/Signal interaction domain*, ongoing PhD

**→ Towards a geometric dynamic modeling of a musical piece ?**

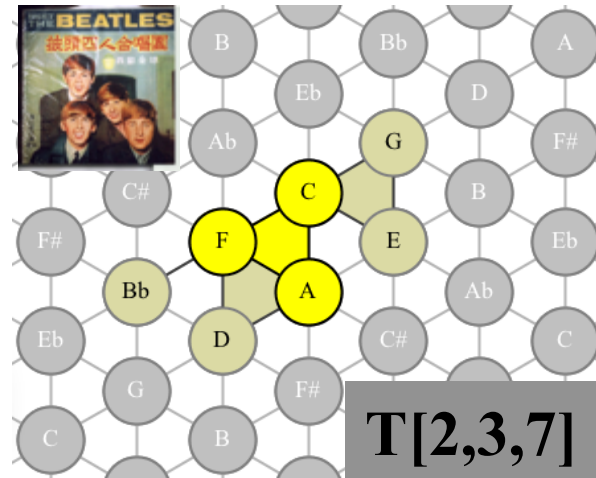
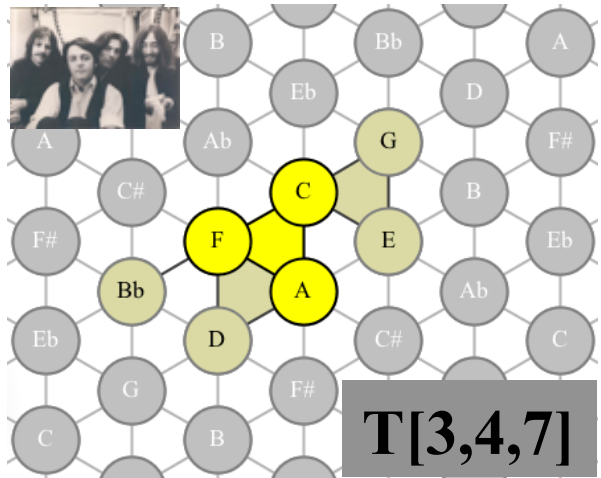
**SPACE**  $\rightleftarrows$  **MUSIC**  
 ?

**→ Towards a topological signature of a musical piece?**

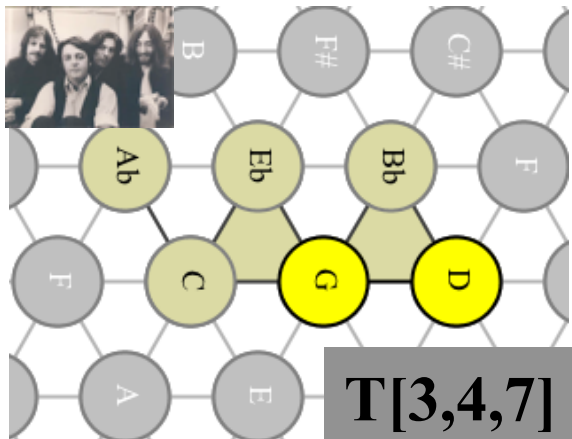


# Musically interesting Trajectory Transformations

## The "M" transformation



**M**



**M**

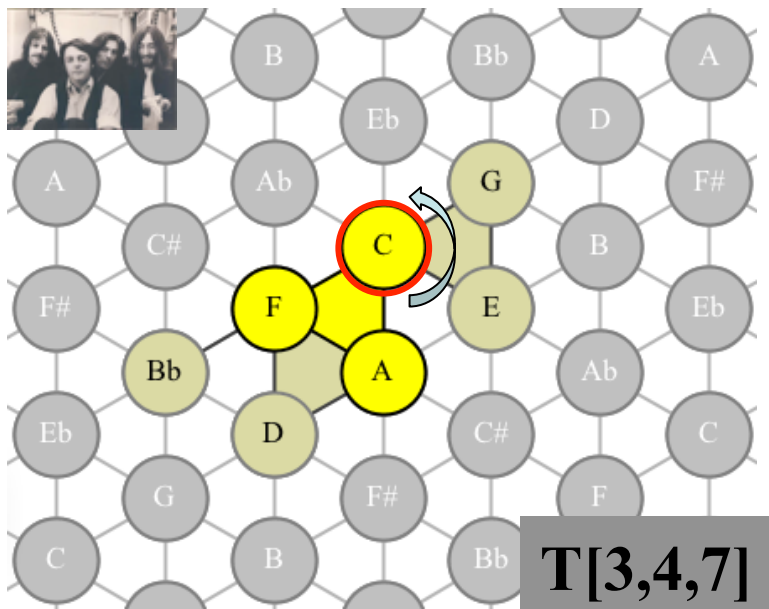
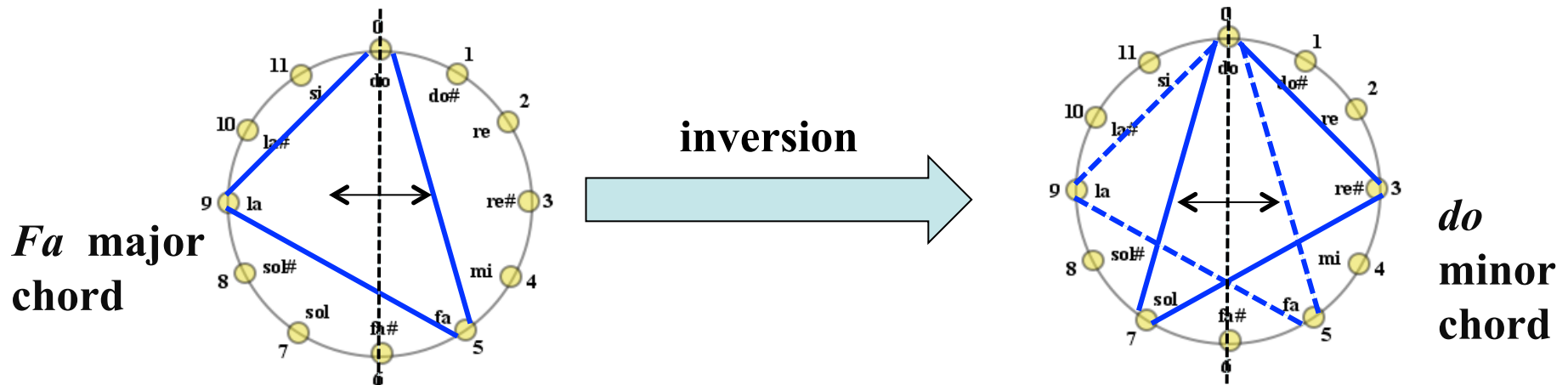


**M**



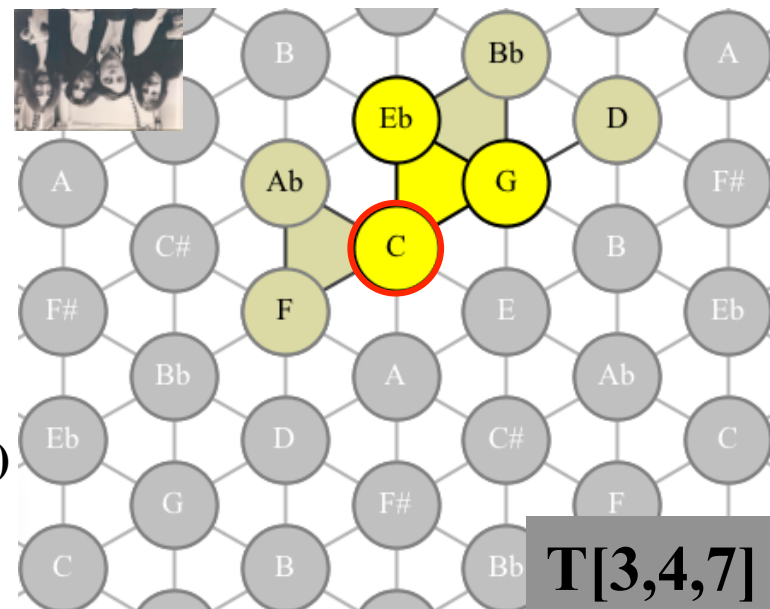
# Musically interesting Trajectory Transformations

Automorphism of the support space



Beatles, Hey Jude  
(orig. version)

Rotation  
(autour du do)



Beatles, Hey Jude  
(transformed version)

→ Hexachord