The interplay between algebra and geometry in computational musicology

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Neurosciences et maillage hexagonal des hauteurs

“[The notion of transformation] comes from a work which played for me a very important role and which I have read during the war in the United States: *On Growth and Form*, in two volumes, by D'Arcy Wentworth Thompson, originally published in 1917. The author (...) proposes an interpretation of the visible transformations between the species (animals and vegetables) within a same gender. This was fascinating, in particular because I was quickly realizing that this perspective had a long tradition: behind Thompson, there was Goethe’s botany and behind Goethe, Albert Dürer with his *Treatise of human proportions*” (Lévi-Strauss, conversation with Eribon, 1988).
Creative processes and conceptual blending


Minimal network for the *conceptual blending*  
[Fauconnier & Turner, 2002]

[...] Conceptual Blending is as an elaboration of other works related to creativity, namely Bisociation, Metaphor and Conceptual Combination. As such, it attracts the attention of computational creativity modelers and, regardless of how Fauconnier and Turner describe its processes and principles, it is unquestionable that there is some kind of blending happening in the creative mind.

Category Theory and Cognition


Category theory offers a re-conceptualization for cognitive science, analogous to the one that Copernicus provided for astronomy, where representational states are no longer the center of the cognitive universe — replaced by the relationships between the maps that transform them [S. Phillips, W. H. Wilson, 2010].
Abstract
This article presents a first attempt at establishing a category-theoretical model of creative processes. The model, which is applied to musical creativity, discourse theory, and cognition, suggests the relevance of the notion of “colimit” as a unifying construction in the three domains as well as the central role played by the Yoneda Lemma in the categorical formalization of creative processes.

From conceptual to structural blending

- J. Goguen, « An Introduction to Algebraic Semiotics, with Applications to User Interface Design », 1999

The category of sign systems with semiotic morphisms has some additional structure over that of a category: it is an *ordered category*, because of the orderings by quality of representation that can be put on its morphisms. This extra structure gives a richer framework for considering blends; I believe this approach captures what Fauconnier and Turner have called « emergent » structure, without needing any other machinery. [Goguen, 1999, p. 32]
Towards a categorical explanation of music perception?

(a) Processus de « colimite » à la base des systèmes évolutifs à mémoire (Ehresmann et Vanbremeersch, 2007) ; (b) réseau minimal pour le « blending conceptuel » (Fauconnier & Turner, 2002) et exemple de Klumpenhouwer Network (ou K-net).

« La théorie des catégories est une théorie des constructions mathématiques, qui est macroscopique, et procède d’étage en étage. Elle est un bel exemple d’abstraction réfléchissante, cette dernière reprenant elle-même un principe constructeur présent dès le stade sensori-moteur. Le style catégoriel qui est ainsi à l’image d’un aspect important de la genèse des facultés cognitives, est un style adéquat à la description de cette genèse »

K-nets as a transformational construction


\[<T_k>: T_m \rightarrow T_m\]
\[I_m \rightarrow I_{k+m}\]

\[<T_{k} \cdot T_{m}> = <T_{k+m}>\]
K-nets as a transformational construction


\[
\langle T_k \rangle : T_m \rightarrow T_m
\]
\[
I_m \rightarrow I_{k+m}
\]

\[
\langle I_k \rangle : T_m \rightarrow T_{-m}
\]
\[
I_m \rightarrow I_{k-m}
\]

\[
\langle T_k \rangle \cdot \langle T_m \rangle = \langle T_{k+m} \rangle
\]
\[
\langle I_k \rangle \cdot \langle I_m \rangle = \langle T_{m-k} \rangle
\]
K-nets as a transformational construction


\[
\begin{align*}
\langle T_k \rangle & : T_m \rightarrow T_m \\
I_m & \rightarrow I_{k+m} \\
\langle I_k \rangle & : T_m \rightarrow T_{-m} \\
I_m & \rightarrow I_{k-m} \\
\langle T_k \rangle \cdot \langle T_m \rangle & = \langle T_{k+m} \rangle \\
\langle T_k \rangle \cdot \langle I_m \rangle & = \langle I_{m-k} \rangle \\
\langle I_m \rangle \cdot \langle T_k \rangle & = \langle I_{k+m} \rangle \\
\langle I_k \rangle \cdot \langle I_m \rangle & = \langle T_{m-k} \rangle
\end{align*}
\]
Transformational vs set-theoretical approaches

Some theoretical difficulties with the isographic relations

CONCLUSION
There are K-Nets which are not always isographic to a given one, i.e. the isographic relations are highly sensitive to the transformations used to label the arrows. Is it possible to overstep this theoretical limitation? Which new definition of K-nets allows one to do that?
Making and Using a Pcset Network for Stockhausen's Klavierstück III

Three interpretations:
- Henck
- Kontarsky
- Tudor
« Making and Using a Pcset Network for Stockhausen's Klavierstück III »

« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special group of transformations is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges all pentachord forms of the music into a spatial configuration that illustrates network structure, for this particular phenomenon, over the entire piece. »

David Lewin, Musical Form and Transformation, YUP 1993
«Making and Using a Pcset Network for Stockhausen's Klavierstück III»

Lewin 1993
« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their potentially functional interrelationships, in a very compactly organized little spatial configuration. »

« [...] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - it makes the abstract space of such a world accessible to our sensibilities. That is to say that the story projects what one would traditionally call form. »
Listening paths within the piece

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Pass 1 (mm. 1-5).

- Horizontal arrows within boxes = J0; between boxes = J3 or J9
- Vertical arrows within boxes = T6; between boxes = Te or T1
- Diagonal arrows within boxes = J6; between boxes = Je or J1

Pass 2 (mm. 5-8) goes back and elaborates the beginning area of pass 1.

- Horizontal arrows within boxes = J0; between boxes = J3 or J9
- Vertical arrows within boxes = T6; between boxes = Te or T1
- Diagonal arrows within boxes = J6; between boxes = Je or J1
Listening paths within the piece

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Pass 3 (mm. 8-10) picks up and elaborates the ending area of pass 1.

Pass 4 (mm. 9-16) expands the p8 + P8 area of pass 3 to activate P2 and p2 as well. P2 is the “essential” incipit of pass 4; p2 is the end of the pass, and of the piece.

Horizontal arrows within boxes = J0; between boxes = J3 or J9
Vertical arrows within boxes = T6; between boxes = T0 or T1
Diagonal arrows within boxes = J6; between boxes = Je or J1

Listening exercise: « do you hear it? » vs « can you hear it? »

Stockhausen: Klavierstück III (Analyse de D. Lewin)

« I take the question ‘Can you hear it’ » to mean something like this: After studying the analysis in examples 2.5 and 2.6, do you find it possible to focus your aural attention upon aspects of the acoustic signal that seem to engage the signifiers of that analysis? […] For me, the interesting questions involve the extent and ways in which I am satisfied and dissatisfied when focusing my aural attention in that manner. It is important to ask those questions about any systematic analysis of any musical composition ». 

Example 2.7. An ear-training aid for listening to P/p forms and their interrelations.
Can you hear it? Yes, we can!


« A cognitive model is derived to show that singleton-tetrachord interaction is salient in facilitating the mental formation of common-tone-preserving percepts, and it serves as perceptual information that determines the acquisition of implicit pitch pattern knowledge for pitch-detection tasks, but only for atonally well-trained musicians. »