PARCOURS DU MASTER SCIENCES ET TECHNOLOGIES
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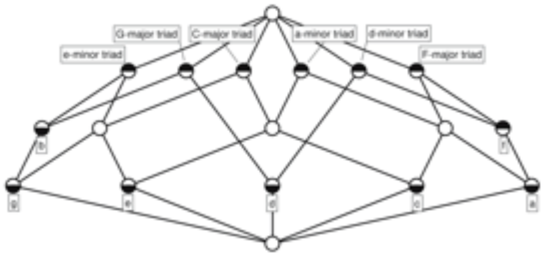
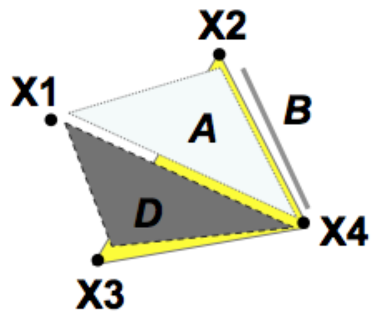
The interplay between algebra and geometry in computational musicology III

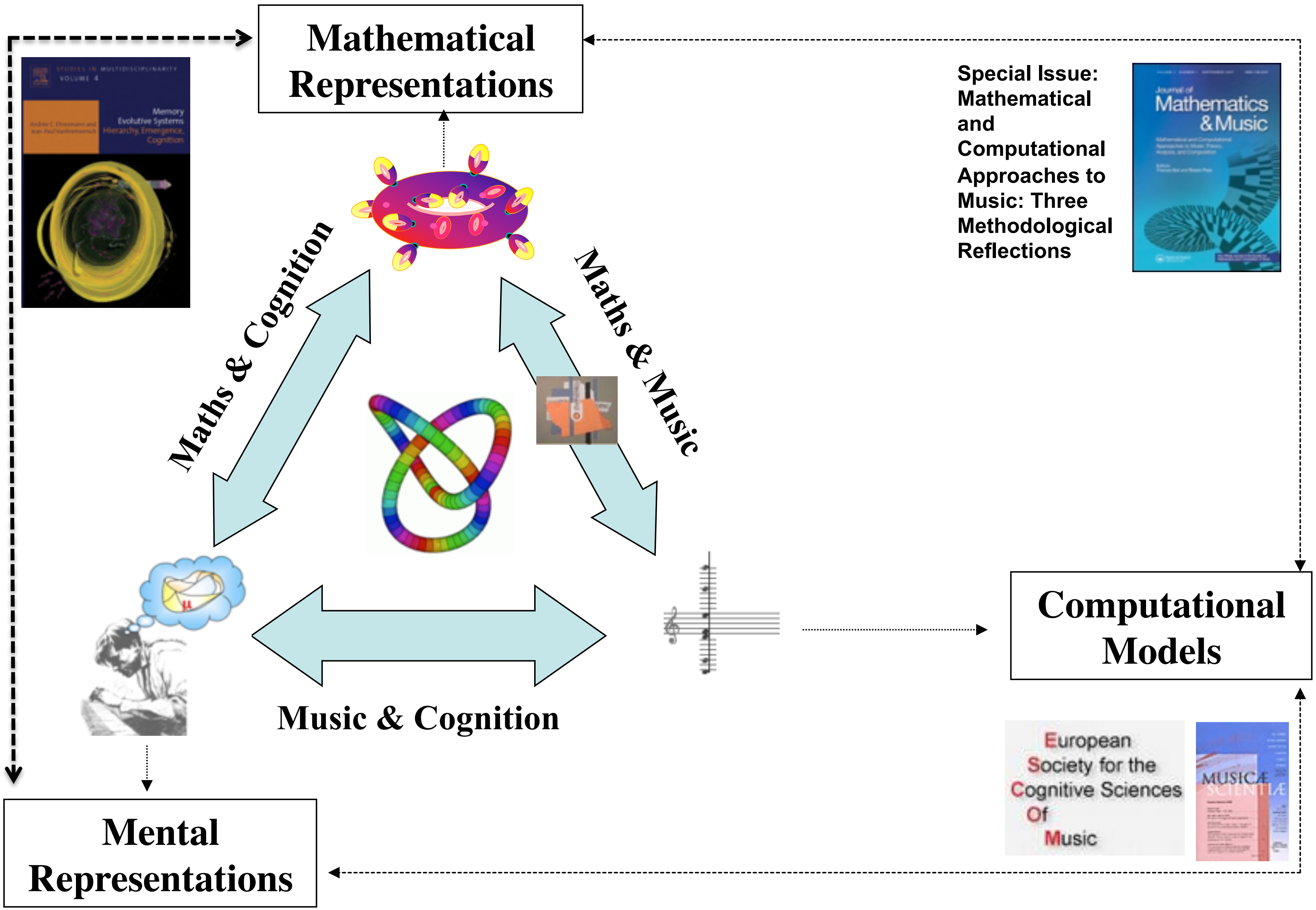




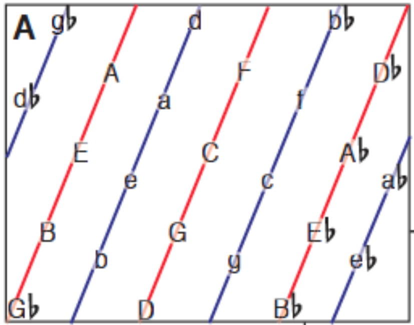
Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS/UPMC
http://www.ircam.fr/repmus.html

Bridging the gap: mathematical and cognitive approaches

http://recherche.ircam.fr/equipes/repmus/mamux/Cognition.html



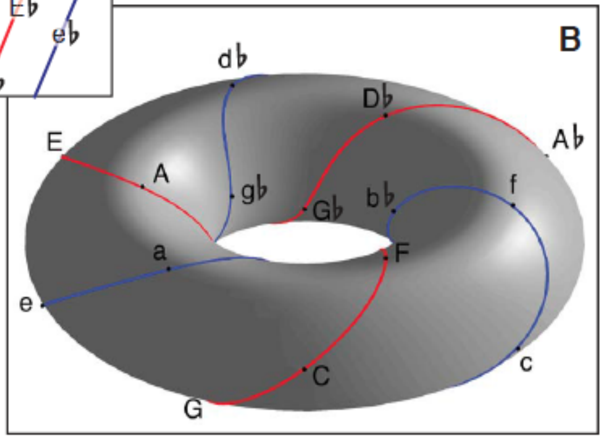
Neurosciences et maillage hexagonal des hauteurs



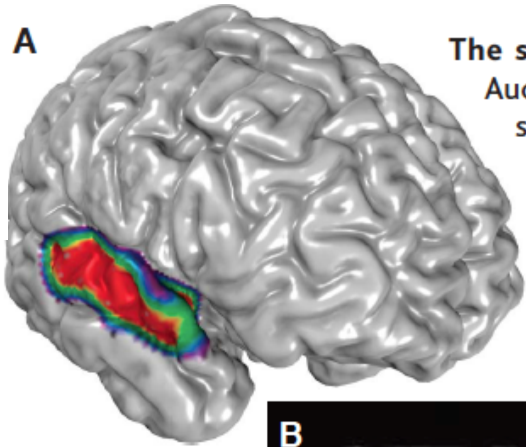
PERSPECTIVES: NEUROSCIENCE

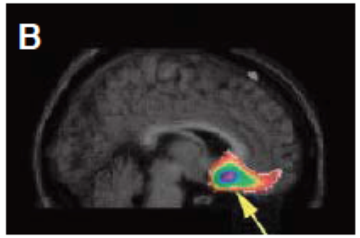
Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl

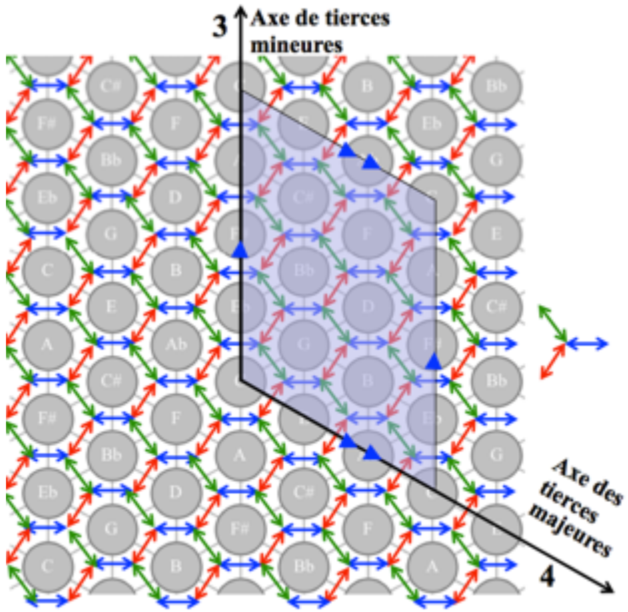


Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). **(B)** Musical keys as points on the surface of a torus.





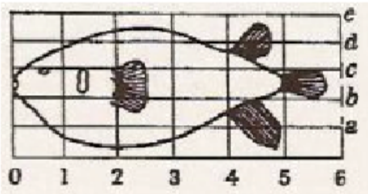
The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. **(B)** Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

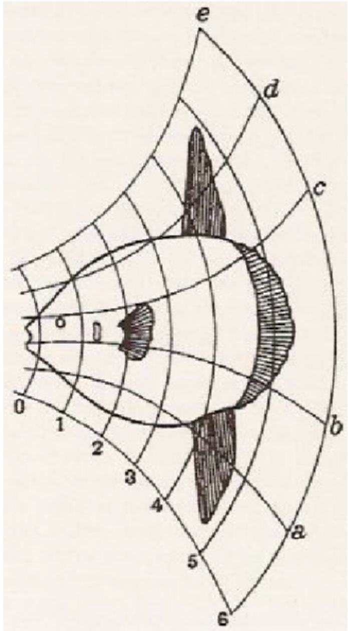


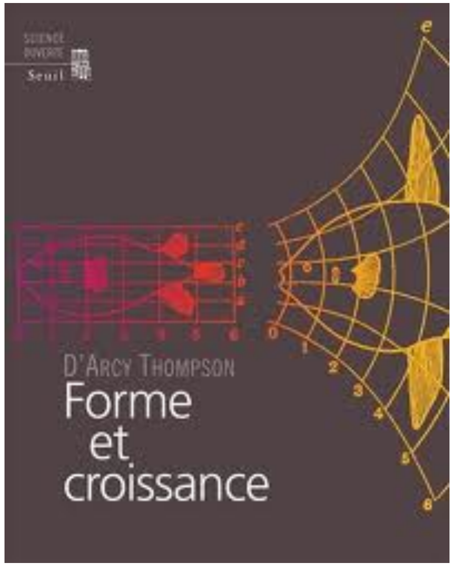
Acotto E. et M. Andreatta (2012), « Between Mind and Mathematics. Different Kinds of Computational Representations of Music », *Mathematics and Social Sciences*, n° 199, 2012(3), p. 9-26.

EMPG European Mathematical Psychology Group

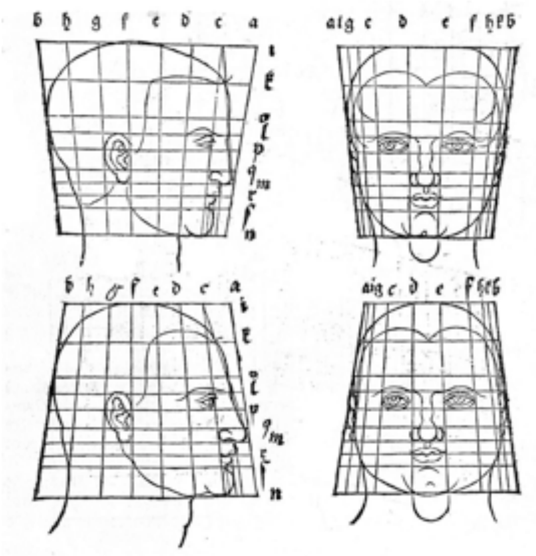
The morphological *vs* the mathematical genealogy of the structuralism





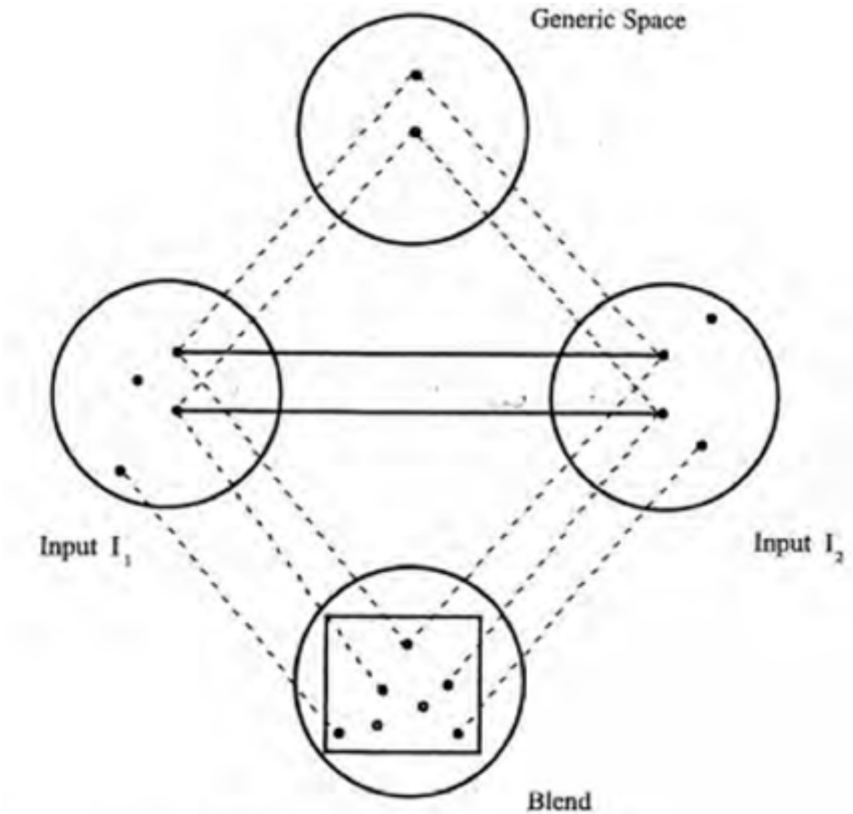


"[The notion of **transformation**] comes from a work which played for me a very important role and which I have read during the war in the United States : *On Growth and Form*, in two volumes, by **D'Arcy Wentworth Thompson**, originally published in 1917. The author (...) proposes an interpretation of the visible transformations between the species (animals and vegetables) within a same gender. This was fascinating, in particular because I was quickly realizing that this perspective had a long tradition: behind Thompson, there was **Goethe's** botany and behind Goethe, **Albert Dürer** with his *Treatise of human proportions*" (Lévi-Strauss, conversation with Eribon, 1988).

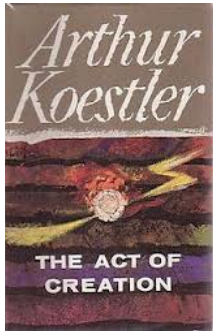


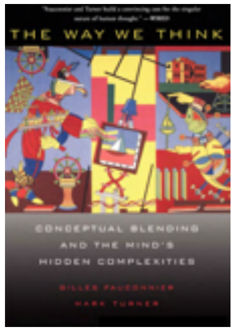
Creative processes and conceptual blending

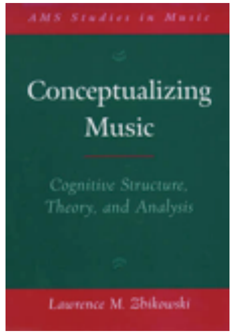
- A. Koestler, *The act of creation,* 1964
- **L. Zbikowski, « Seeger's Unitary Field Theory Reconsidered ». In: Yung, Bell & Helen Rees (eds). *Understanding Charles Seeger, Pioneer in American Musicology*. Illinois: University of Illinois Press. 1999: 130-149.**
- G. Fauconnier & M. Turner, *The Way We Think*, 2002
- L. Zbikowski, *Conceptualizing Music: Cognitive Structure, Theory, and Analysis*, 2002
- F. C. Pereira, *Creativity and Artificial Intelligence - A Conceptual Blending Approach*, 2007

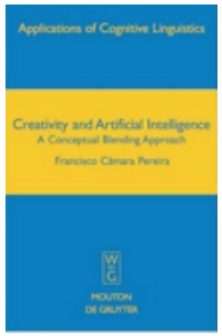


Minimal network for the *conceptual blending* [Fauconnier & Turner, 2002]







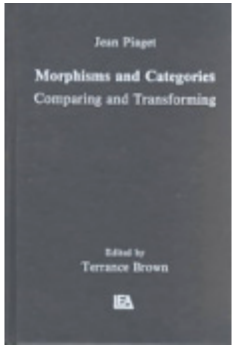


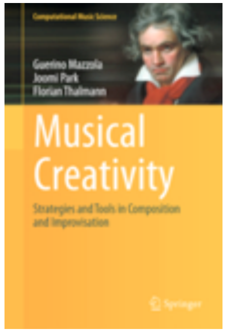
[…] Conceptual Blending is as an elaboration of other works related to creativity, namely Bisociation, Metaphor and Conceptual Combination. As such, it attracts the attention of computational creativity modelers and, regardless of how Fauconnier and Turner describe its processes and principles, it is unquestionable that there is some kind of blending happening in the creative mind.

F. C. Pereira, *Creativity and Artificial Intelligence - A Conceptual Blending Approach*, 2007

Category Theory and Cognition

- G. S. Halford & W. H. Wilson, “A Category Theory Approach to Cognitive Development”, *Cognitive Psychology*, 12, 1980
- **J. Piaget, Gil Henriques et Edgar Ascher, *Morphisms and Categories: Comparing and Transforming* (orig. French, 1990)**
- J. Macnamara & G. E. Reyes, *The Logical Foundation of Cognition*, OUP, 1994
- A. Ehresmann, J.-P Vanbremeerch, *Memory Evolutive Systems, Hierarchy, Emergence, Cognition*, 2007
- **A. Ehresmann, J.-P. Vanbremeerch, “MENS, a mathematical model for cognitive systems”, *Journal of Mind Theory*, 2009**
- S. Phillips, W. H. Wilson, “Categorial Compositionality: A Category Theory Explanation for the Systematicity of Human Cognition”, PLoS Comp. Biology, 6(7), July 2010
- S. Phillips, W. H. Wilson, “Categorial Compositionality II: Universal Constructions and a General Theory of (Quasi-)Systematicity in Human Cognition, PLoS Comp. Biology, 7(8), August 2011
- A. Ehresmann, “MENS, an Info-Computational Model for (Neuro-)cognitive Systems Capable of Creativity”, *Entropy*, 2012
- **G. Mazzola, *Musical Creativity*, Springer, 2012**
- M. Andreatta, Andreatta M., A. Ehresmann, R. Guitart, G. Mazzola, “Towards a categorical theory of creativity”, Fourth International Conference, MCM 2013, McGill University, Montreal, June 12-14, 2013, Springer, 2013.





Category theory offers a re-conceptualization for cognitive science, analogous to the one that Copernicus provided for astronomy, where representational states are no longer the center of the cognitive universe —replaced by the relationships between the maps that transform them [S. Phillips, W. H. Wilson, 2010].

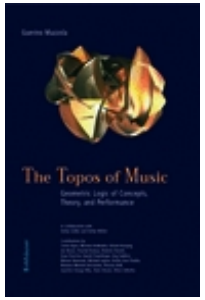
Towards a categorical theory of creativity (in music, cognition and discourse)

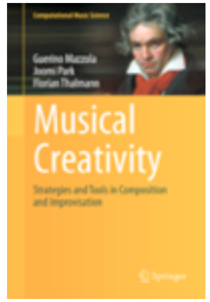
Abstract

This article presents a first attempt at establishing a **category-theoretical model of creative processes**. The model, which is applied to musical creativity, discourse theory, and cognition, suggests the relevance of the notion of "colimit" as a unifying construction in the three domains as well as the central role played by the Yoneda Lemma in the categorical formalization of creative processes.

MCM 2013

MATHEMATICS AND COMPUTATION IN MUSIC





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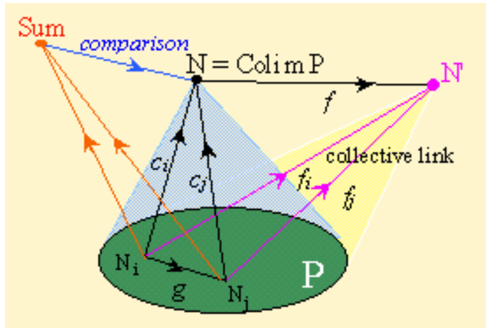


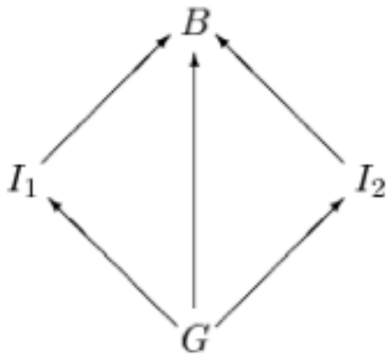
Andreatta M., A. Ehresmann, R. Guitart, G. Mazzola, « Towards a categorical theory of creativity », Fourth International Conference, MCM 2013, McGill University, Montreal, June 12-14, 2013, Springer, 2013.

From conceptual to structural blending

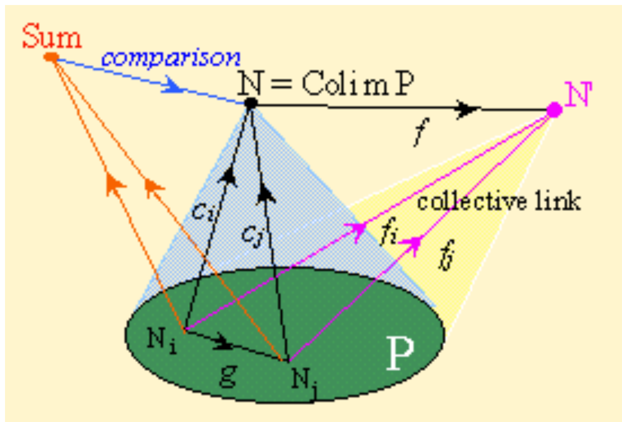
- J. Goguen, « A Categorical Manifesto », *Math. Structures in Computer Science*, 1991.
- J. Goguen, « An Introduction to Algebraic Semiotics, with Applications to User Interface Design », 1999
- **J. Goguen, « Musical Qualia, Context, Time, and Emotion », in *Journal of Consciousness Studies* 11, 3/4, 117-147, 2004**
- J. Goguen, « What is a Concept? », *International Conference on Comp. Science*, 2005
- A. Ehresmann, J.-P Vanbremeerch, Memory Evolutive Systems, Hierarchy, Emergence, Cognition, 2007

The **category of sign systems** with **semiotic morphisms** has some additional structure over that of a category: it is an *ordered category*, because of the orderings by quality of representation that can be put on its morphisms. This extra structure gives a richer framework for considering blends; I believe this approach captures what Fauconnier and Turner have called « emergent » structure, without needing any other machinery. [Goguen, 1999, p. 32]

Algebraic/ structural semiotics

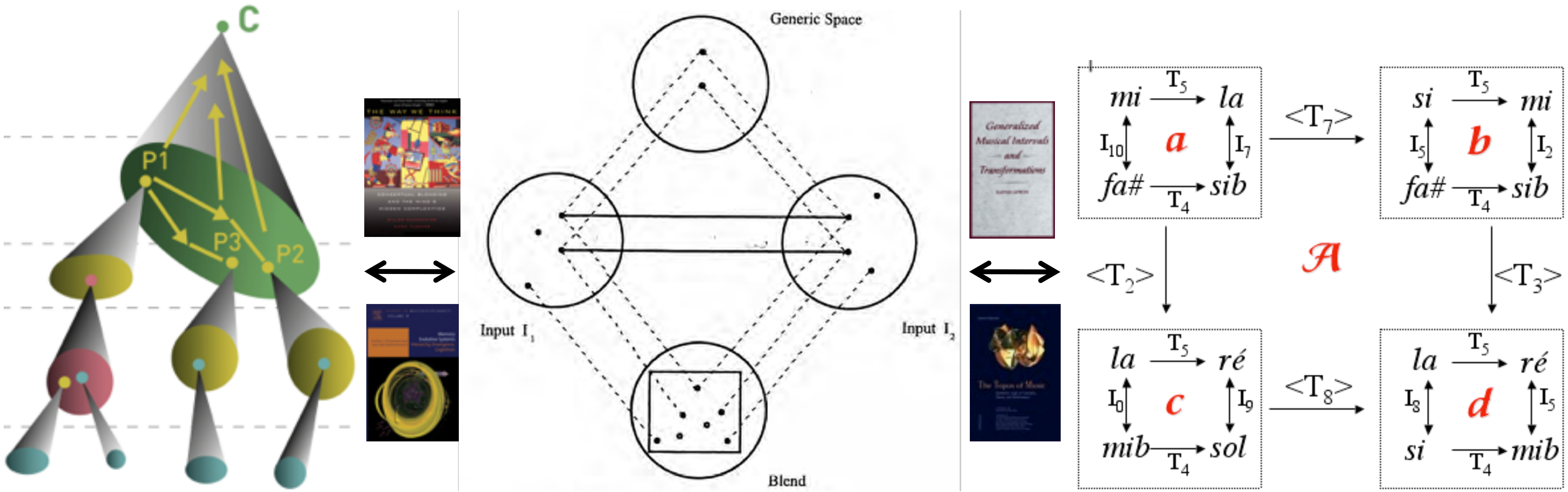


structural blending



Colimit of a diagram

Towards a categorical explanation of music perception?



(a) **(b)** **(c)**

(a) Processus de « colimite » à la base des systèmes évolutifs à mémoire (Ehresmann et Vanbremeersch, 2007) ; (b) réseau minimal pour le « blending conceptuel » (Fauconnier & Turner, 2002) et exemple de Klumpenhouwer Network (ou *K-net*).

« La **théorie des catégories** est une théorie des constructions mathématiques, qui est macroscopique, et procède d'étage en étage. Elle est un bel exemple d'**abstraction réfléchissante**, cette dernière reprenant elle-même un principe constructeur présent dès le stade sensori-moteur. Le **style catégoriel** qui est ainsi à l'image d'un aspect important de la **genèse des facultés cognitives**, est un style adéquat à la description de cette genèse »

J. Piaget

Jean Piaget, Gil Henriques et Edgar Ascher, *Morphismes et Catégories. Comparer et transformer*, 1990

K-nets as a transformational construction

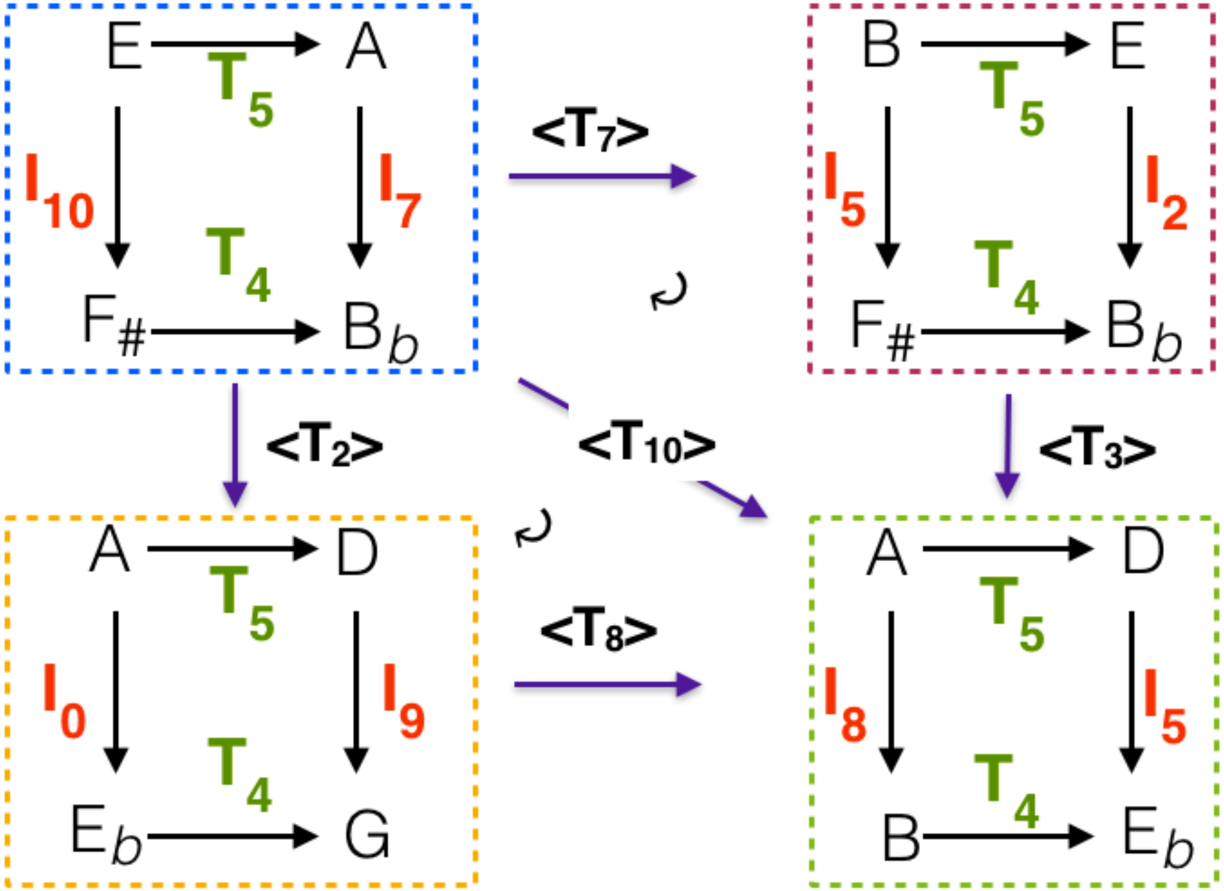
D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994

D. Lewin

H. Klumpenhouwer

$$\langle T_k \rangle : T_m \to T_m$$
$$I_m \to I_{k+m}$$

$$\langle T_k \rangle \bullet \langle T_m \rangle = \langle T_{k+m} \rangle$$



K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994

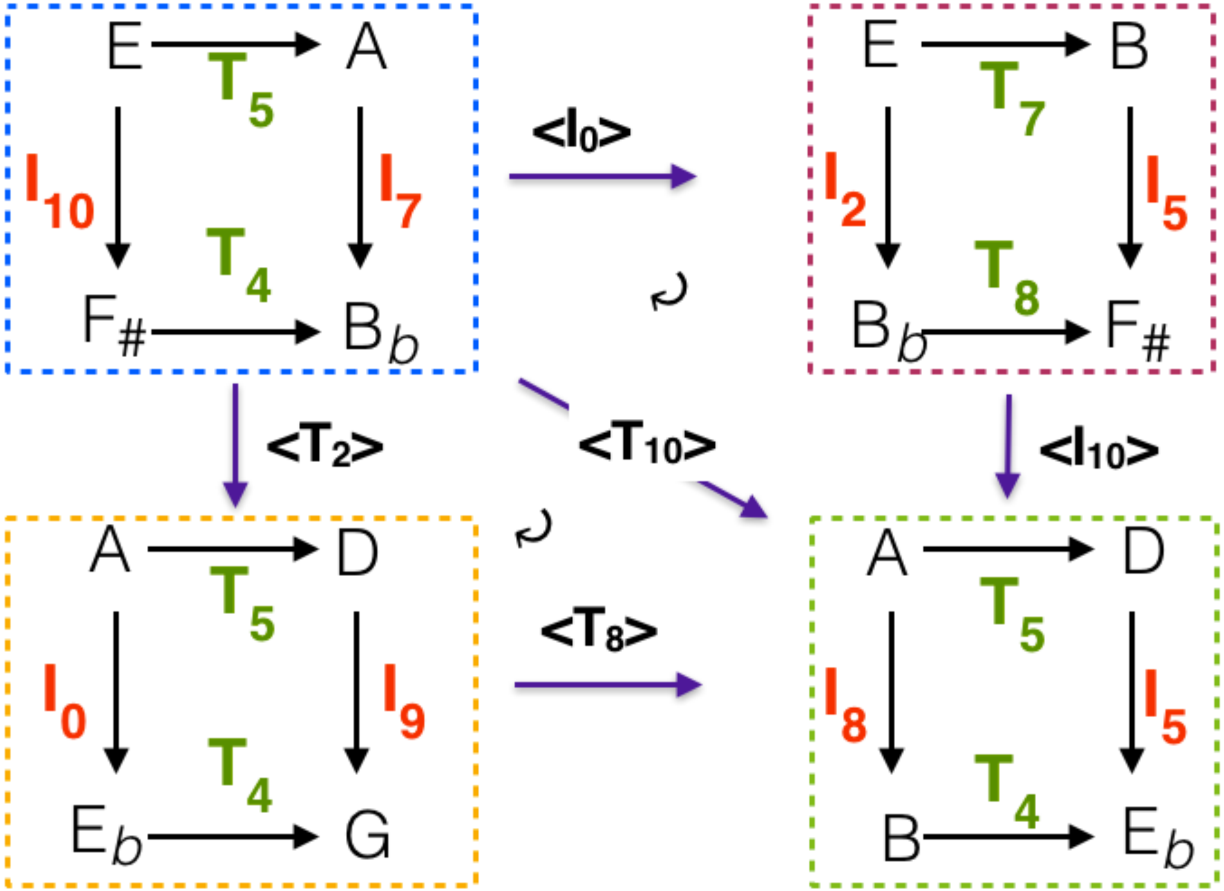
D. Lewin

H. Klumpenhouwer

$\langle T_k \rangle : T_m \rightarrow T_m$
$\qquad\quad I_m \rightarrow I_{k+m}$

$\langle I_k \rangle : T_m \rightarrow T_{-m}$
$\qquad\quad I_m \rightarrow I_{k-m}$

$\langle T_k \rangle \bullet \langle T_m \rangle = \langle T_{k+m} \rangle$
$\langle I_k \rangle \bullet \langle I_m \rangle = \langle T_{m-k} \rangle$



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D. Lewin

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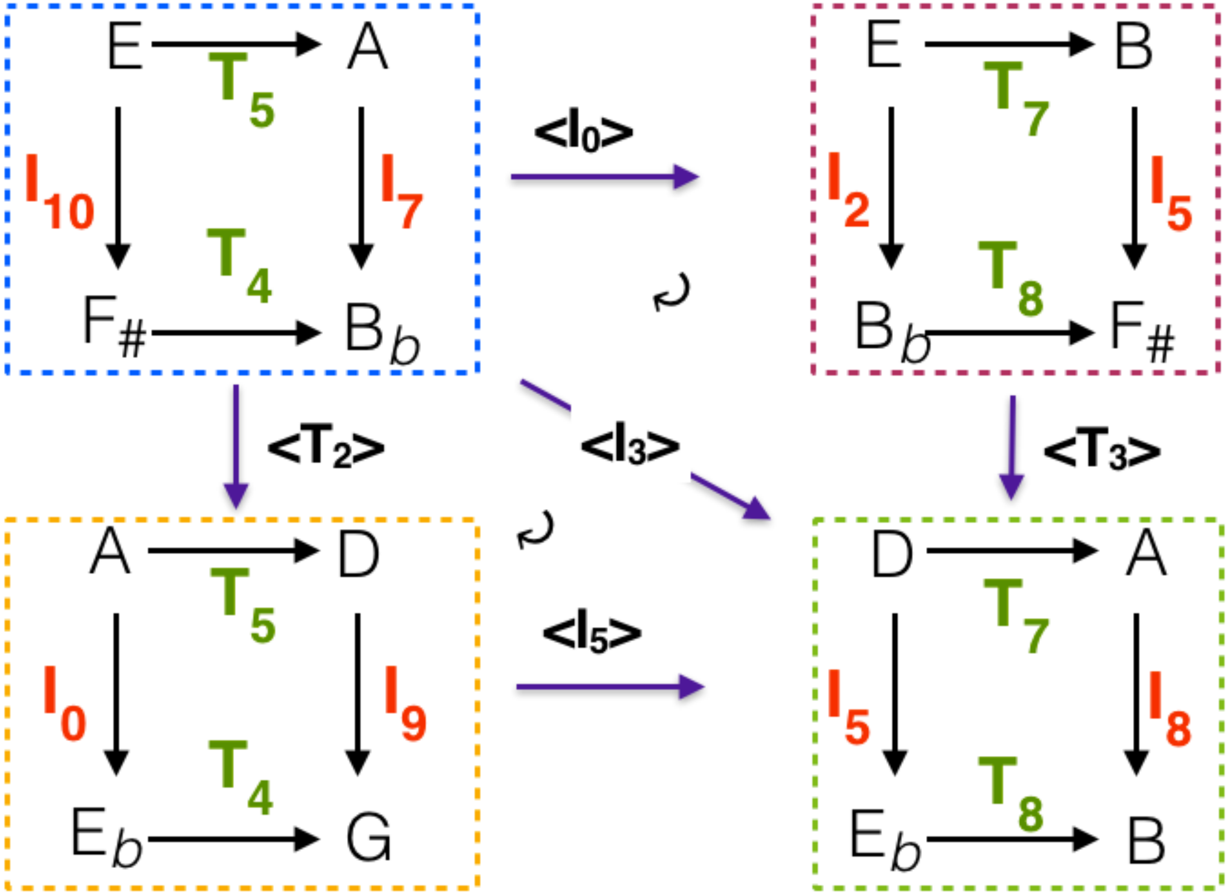
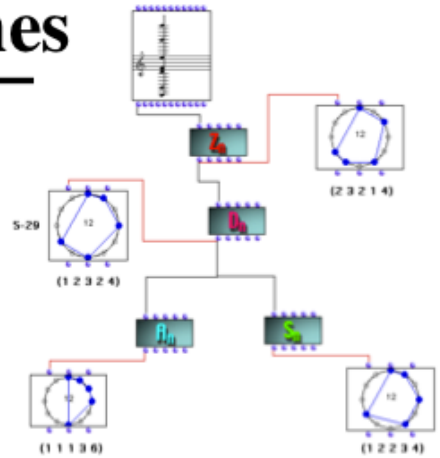
$\langle I_k \rangle : T_m \rightarrow T_{-m}$
$\qquad\qquad I_m \rightarrow I_{k-m}$

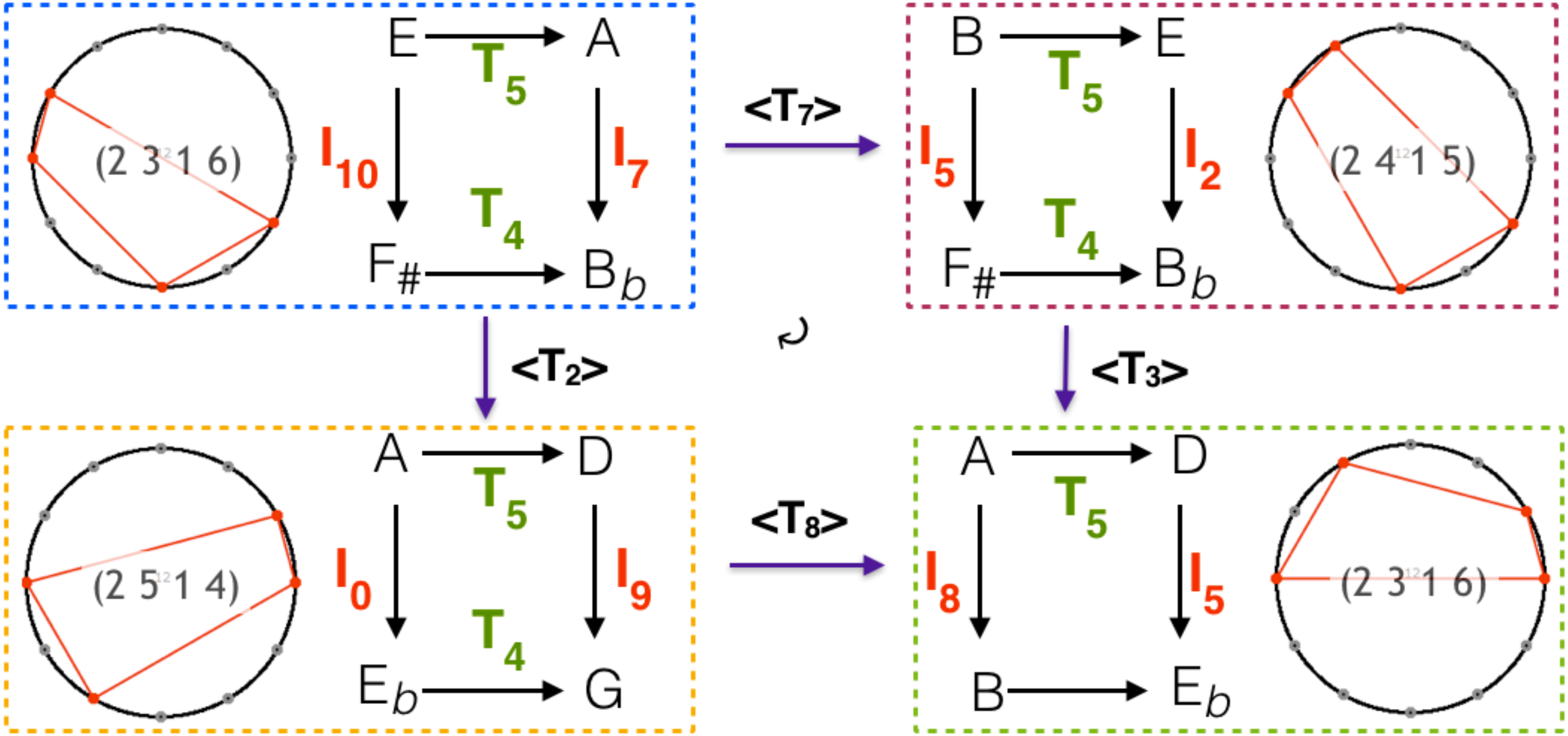
$\langle T_k \rangle \bullet \langle T_m \rangle = \langle T_{k+m} \rangle$
$\langle T_k \rangle \bullet \langle I_m \rangle = \langle I_{m-k} \rangle$
$\langle I_m \rangle \bullet \langle T_k \rangle = \langle I_{k+m} \rangle$
$\langle I_k \rangle \bullet \langle I_m \rangle = \langle T_{m-k} \rangle$



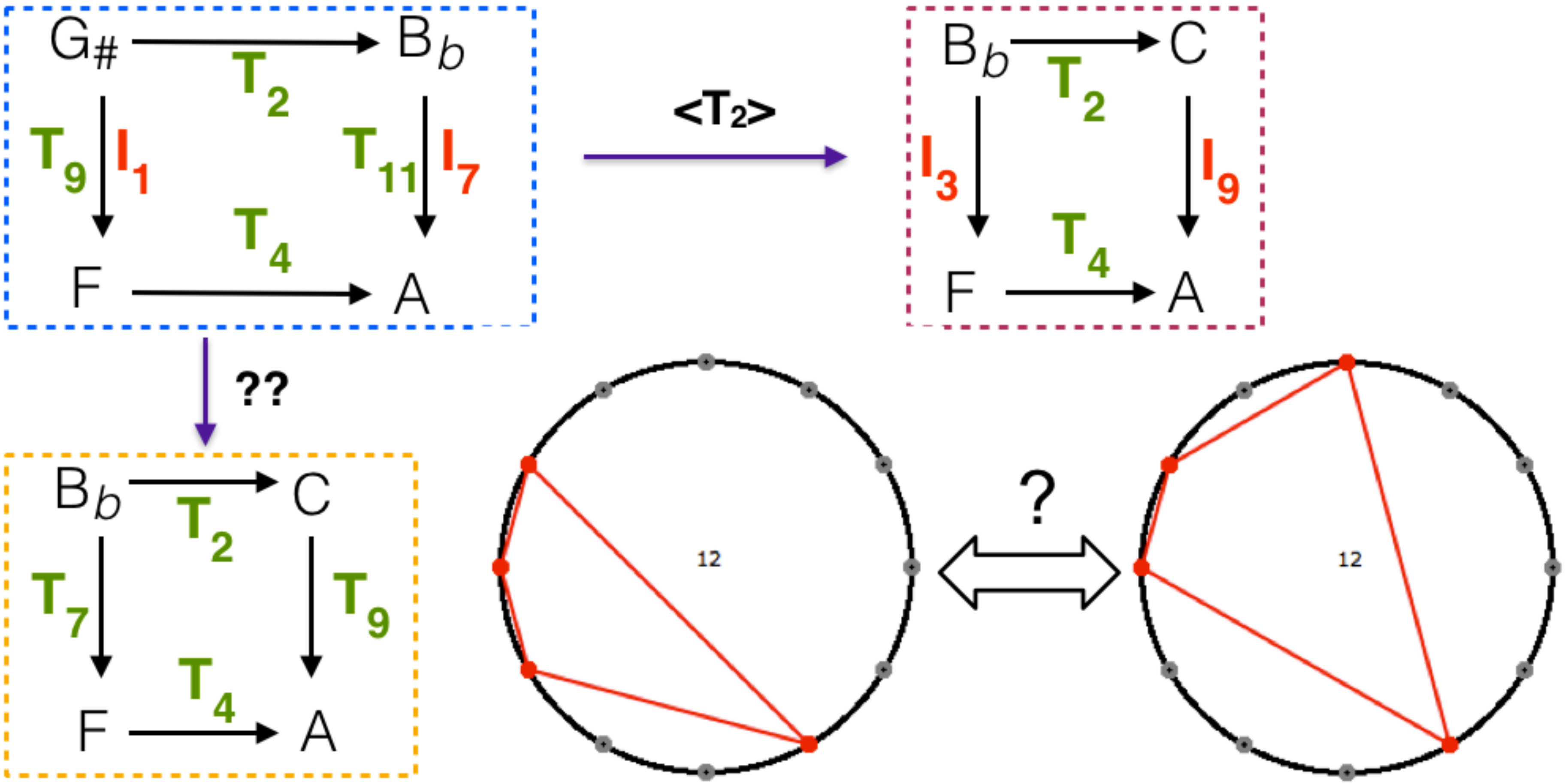
Transformational *vs* set-theoretical approaches

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994





Some theoretical difficulties with the isographic relations



CONCLUSION
There are K-Nets which are not always isographic to a given one, i.e. the isographic relations are highly sensitives to the transformations used to label the arrows.
Is it possible to overstep this theoretical limitation? Which new definition of K-nets allows one to do that?

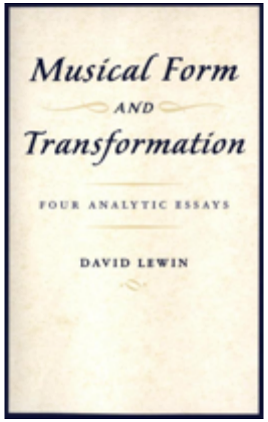
« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »

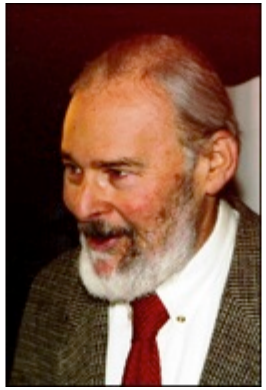
Three interpretations:

Henck

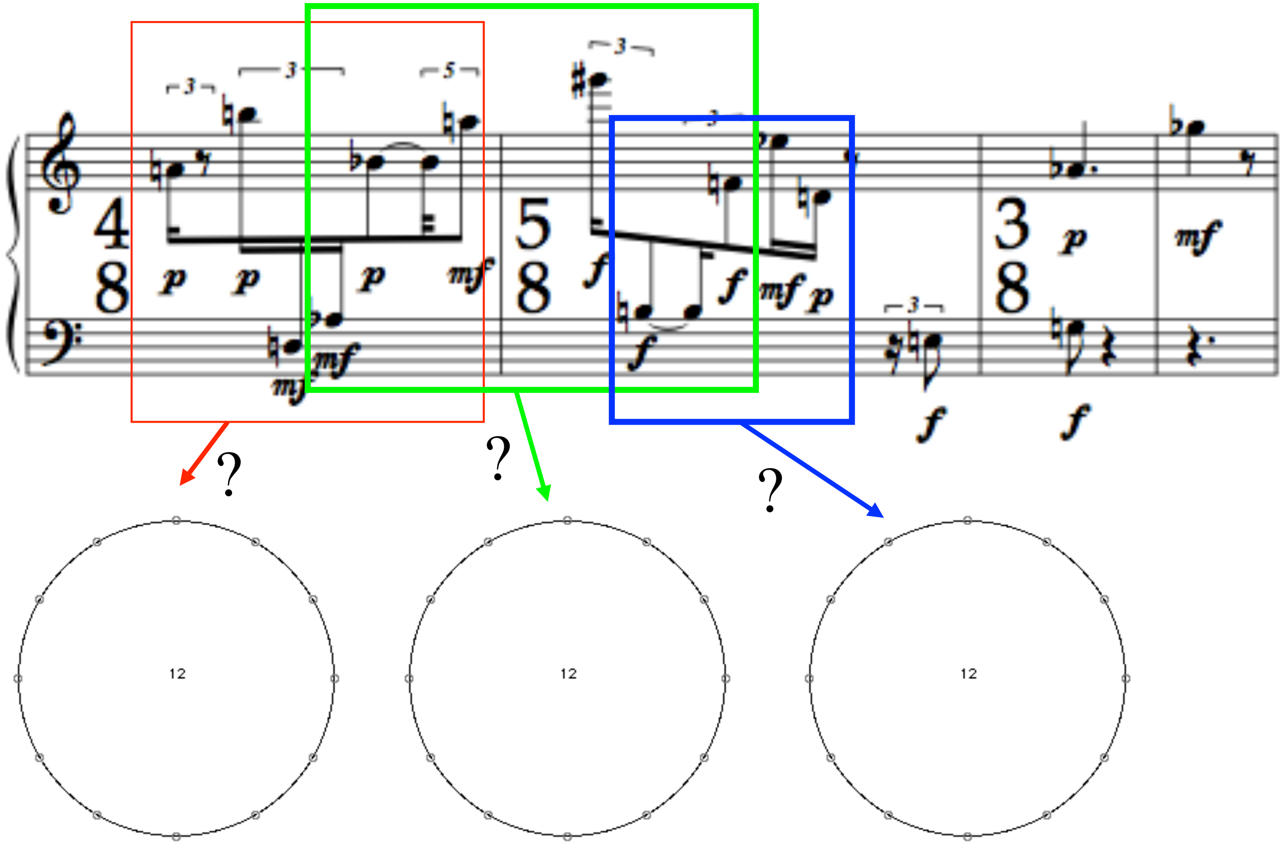
Kontarsky

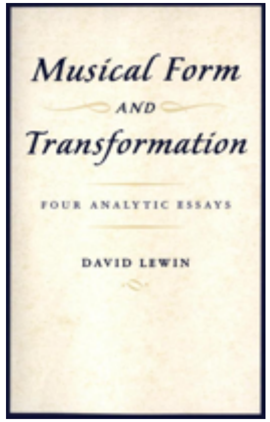
Tudor

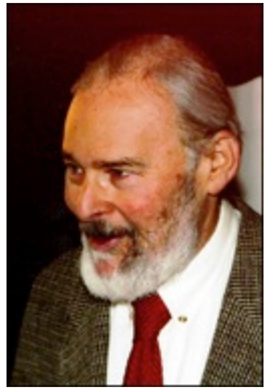




« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »





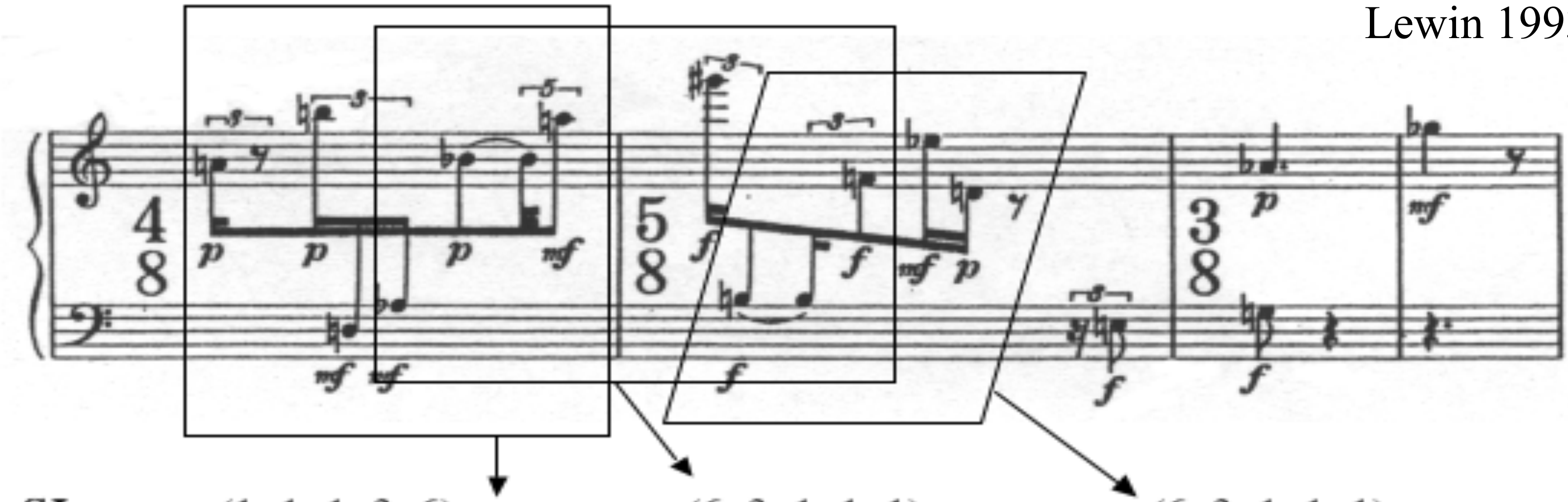


« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

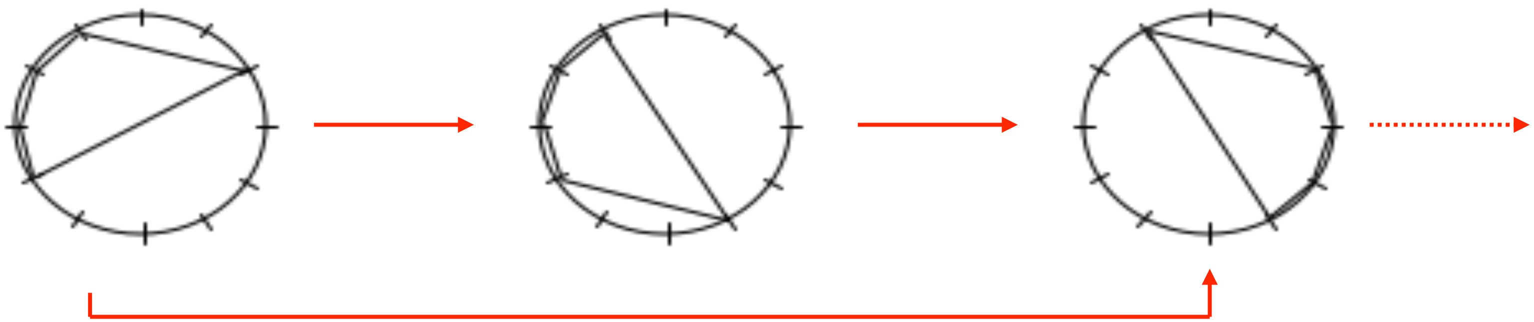
David Lewin, *Musical Form and Transformation,* YUP 1993

« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »

Lewin 1993

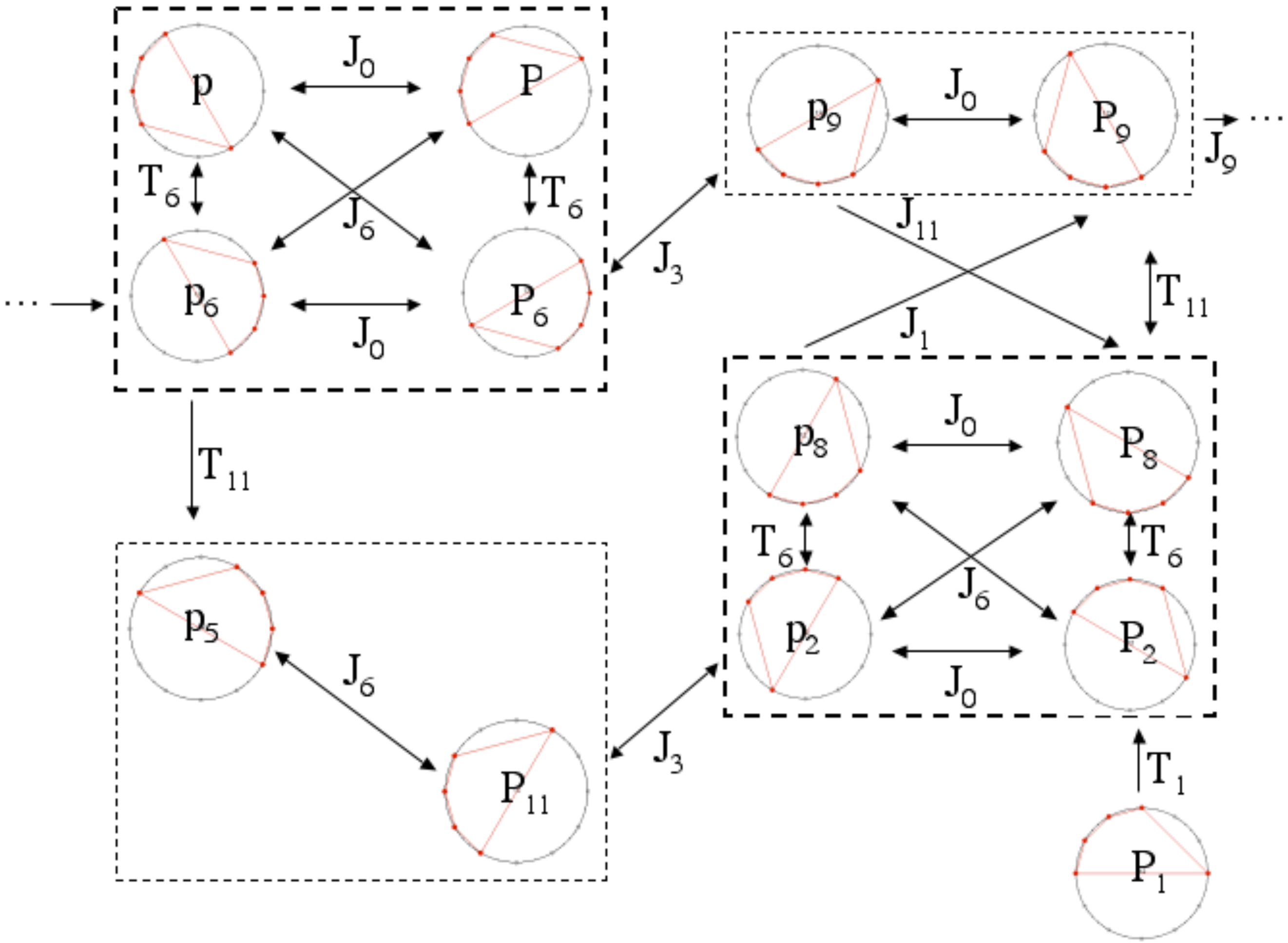


SI:	(1, 1, 1, 3, 6)	(6, 3, 1, 1, 1)	(6, 3, 1, 1, 1)
IFUNC:	[5 3 2 2 1 1 1 1 1 2 2 3]	[5 3 2 2 1 1 1 1 1 2 2 3]	[5 3 2 2 1 1 1 1 1 2 2 3]
VI:	[3 2 2 1 1 1]	[3 2 2 1 1 1]	[3 2 2 1 1 1]



Transformational Network

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)



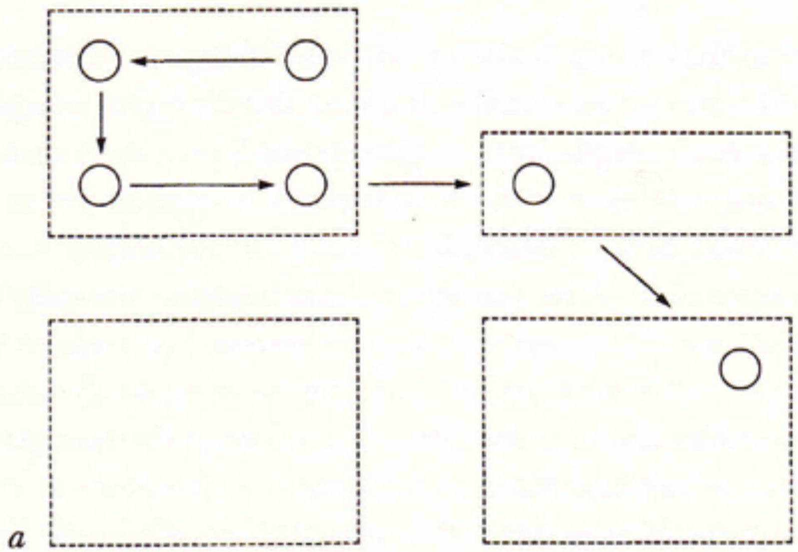
« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their **potentially functional interrelationships**, in a very compactly organized little **spatial configuration**. »

« […] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - **it makes the abstract space of such a world accessible to our sensibilities**. That is to say that the story projects what one would traditionally call *form*. »

Listening paths within the piece

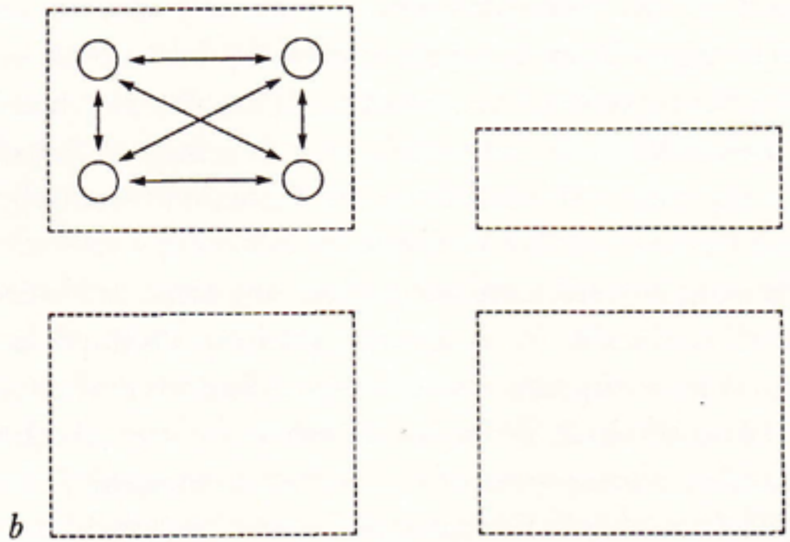
Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Pass 1 (mm. 1-5).



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Pass 2 (mm. 5-8) goes back and elaborates
the beginning area of pass 1.

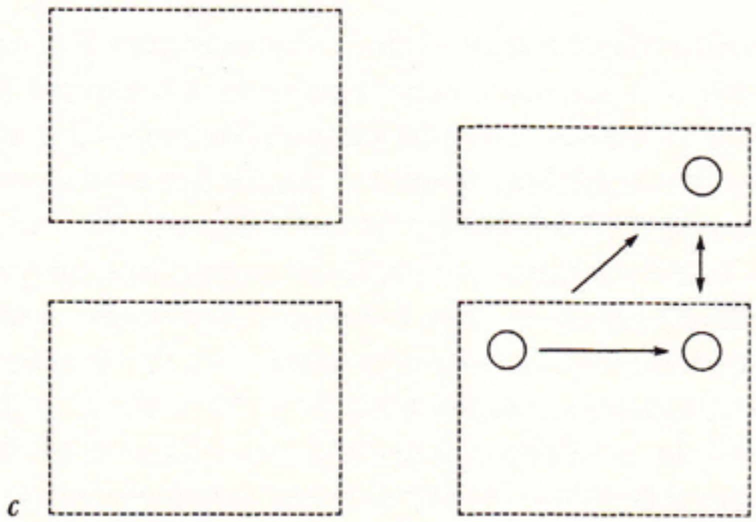


horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Listening paths within the piece

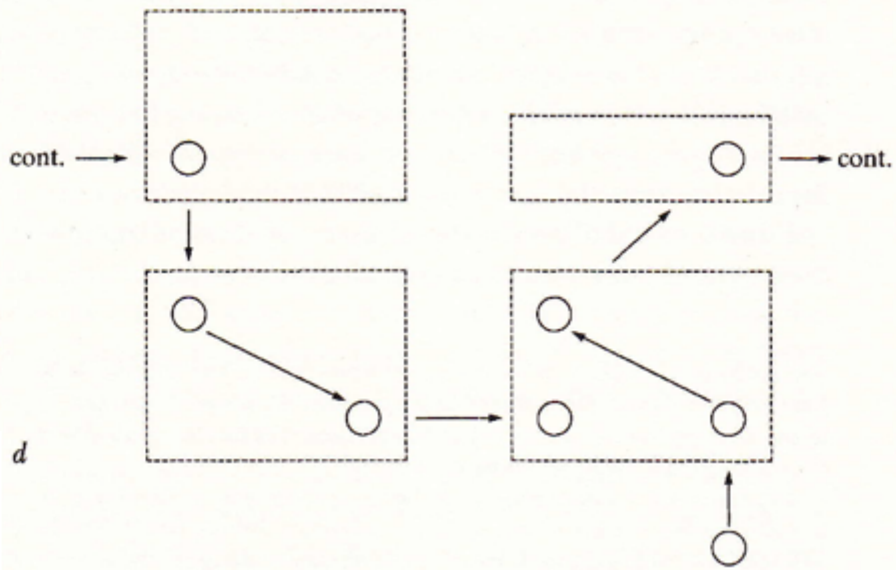
Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Pass 3 (mm. 8-10) picks up and elaborates
the ending area of pass 1.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

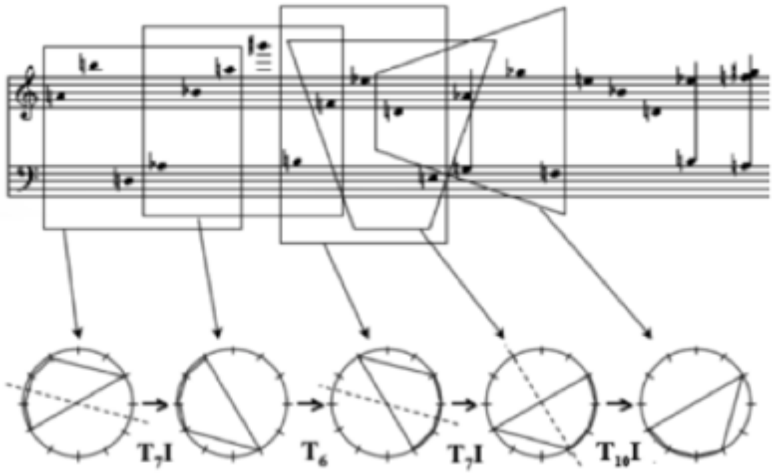
Pass 4 (mm. 9-16) expands the p8 + P8 area of pass 3
to activate P2 and p2 as well. P2 is the "essential" incipit
of pass 4; p2 is the end of the pass, and of the piece.

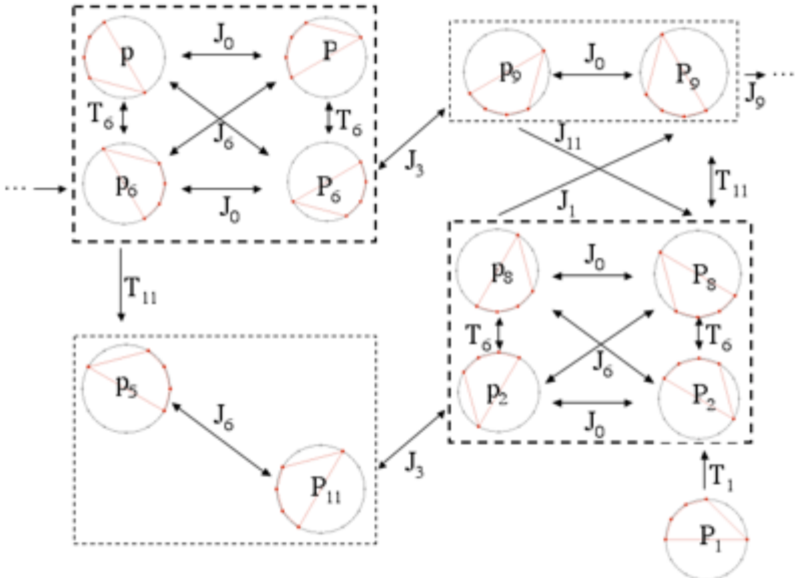


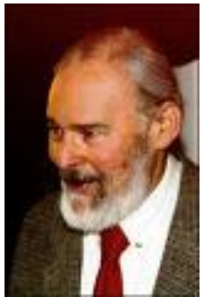
horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

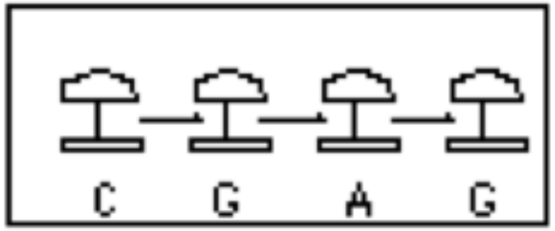
Transformational Networks and Music Cognition

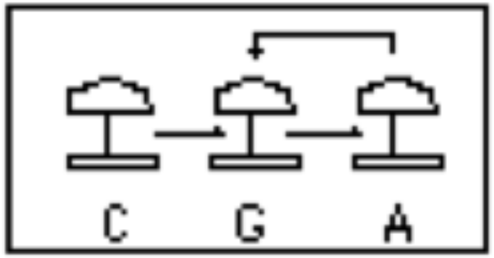
Bamberger, J. (1986). Cognitive issues in the development of musically gifted children. In *Conceptions of giftedness (eds., R. J. Sternberg, & J. E. Davidson), pp. 388-413. Cambridge University Press, Cambridge*







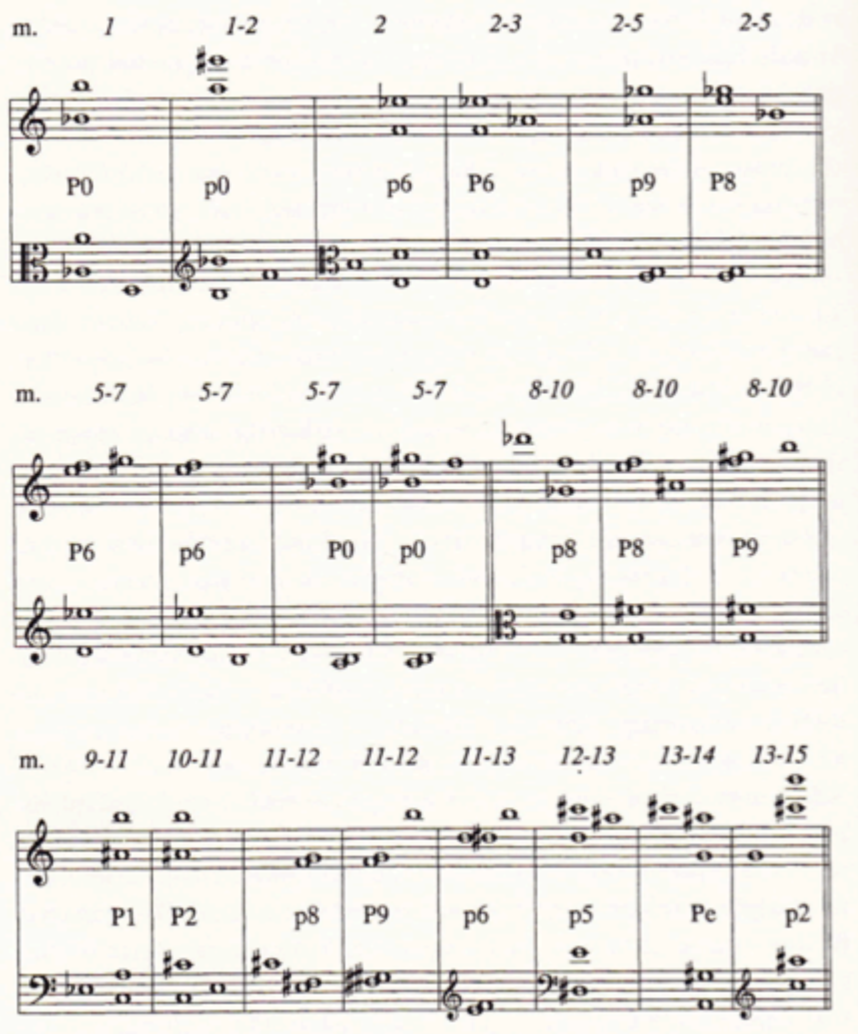




Bamberger, J. (2006). "What develops in musical development?" In G. MacPherson (ed.) *The child as musician: Musical development from conception to adolescence.* Oxford, U.K. Oxford University Press.

Listening exercise: « do you hear it? » *vs* « can you hear it? »

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)



Example 2.7. An ear-training aid for listening to P/p forms and their interrelations.

« I take the question 'Can you hear it » to mean something like this: After studying the analysis in examples 2.5 and 2.6, do you find it possible to focus your **aural attention** upon aspects of the acoustic signal that seem to engage the signifiers of that analysis? […] For me, the interesting questions involve the extent and ways in which I am satisfied and dissatisfied when **focusing my aural attention** in that manner. It is important to ask those questions about any systematic analysis of any musical composition ».

Can you hear it? Yes, we can!

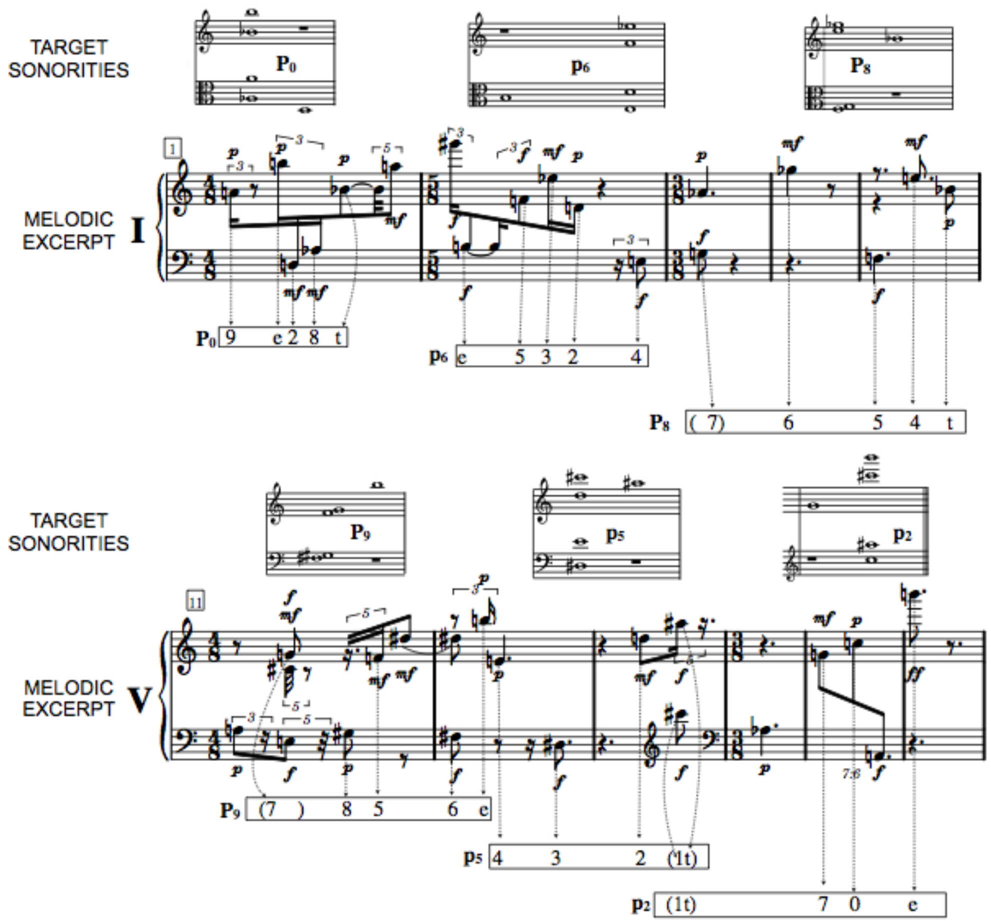
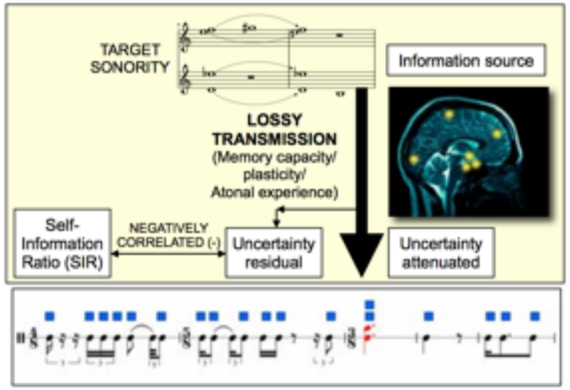


FIGURE 5. Six target sonorities used for Phase I pitch-detection tasks (circled in dashed-line boxes): Single Pentachords appeared in form of either 'st' or 'ts' according to Lewin's ear-training aid (*MFT*, Example 2.7, p. 42). Their corresponding melodies are either Excerpt I or V.



« A cognitive model is derived to show that singleton-tetrachord interaction is salient in facilitating the mental formation of common-tone-preserving percepts, and it serves as perceptual information that determines the acquisition of implicit pitch pattern knowledge for pitch-detection tasks, but only for atonally well-trained musicians. »

Y. Cao, J. Wild, B. Smith, S. McAdams, « The Perception and Learning of Contextually-defined Inversion Operators in Transformational Pitch Patterns », 5th International Conference of Students of Systematic Musicology (SysMus12), Montreal, 2012.