PARCOURS DU MASTER SCIENCES ET TECHNOLOGIES
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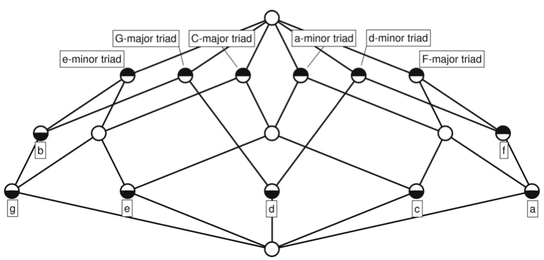
The interplay between algebra and geometry in computational musicology



Moreno Andreatta
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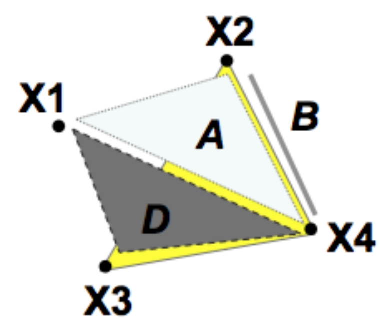
http://www.ircam.fr/repmus.html



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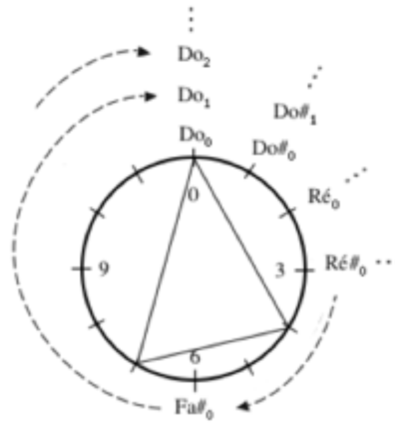
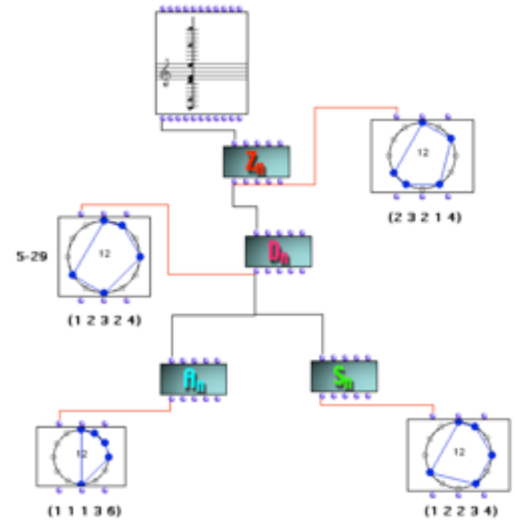
Algebraic structures and combinatorics

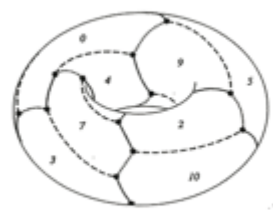
- Definition of group
  - Cyclic group **Z/*n*Z** of order *n*
  - Dihedral group $\mathbf{D}_{2n}$ of order $2n$
  - Affine group $\mathbf{Aff}_n$ of order $\varphi(n)\cdot n$
- Action of G on S
  - Action and equivalence relations
  - Stabiliser of an element and TL
  - Simply transitive action and GIS
  - Burnside Lemma and orbits
  - (Interval content and homometry)
- K-nets and isographies
  - Strong, positive and negative
- Tonnetz
  - Dual actions
  - Anisotropic Tonnetz

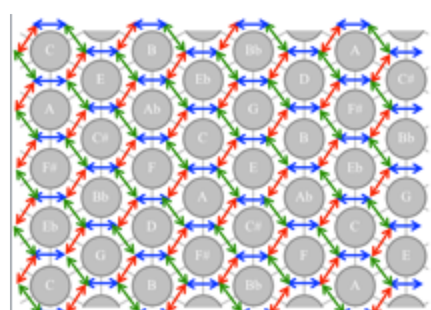
$$\begin{array}{rccl} \varphi & : & \mathbb{N}^* & \longrightarrow \mathbb{N}^* \\ & & n & \longmapsto \mathrm{card}(\{m \in \mathbb{N}^* \mid m \leq n \text{ et } m \text{ premier avec } n\}) \end{array}$$

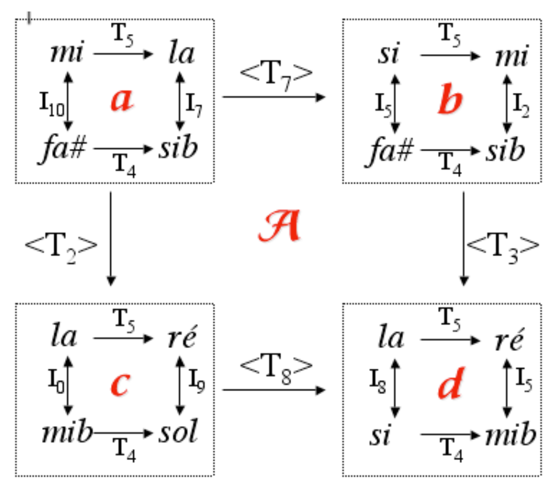
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\phi(n)$	0	1	2	2	4	2	6	4	6	4	10	4	12	6	8	8



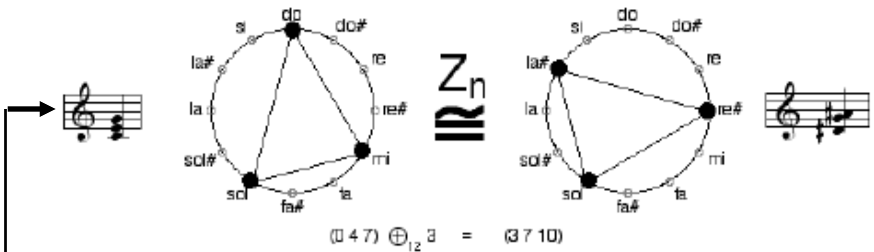








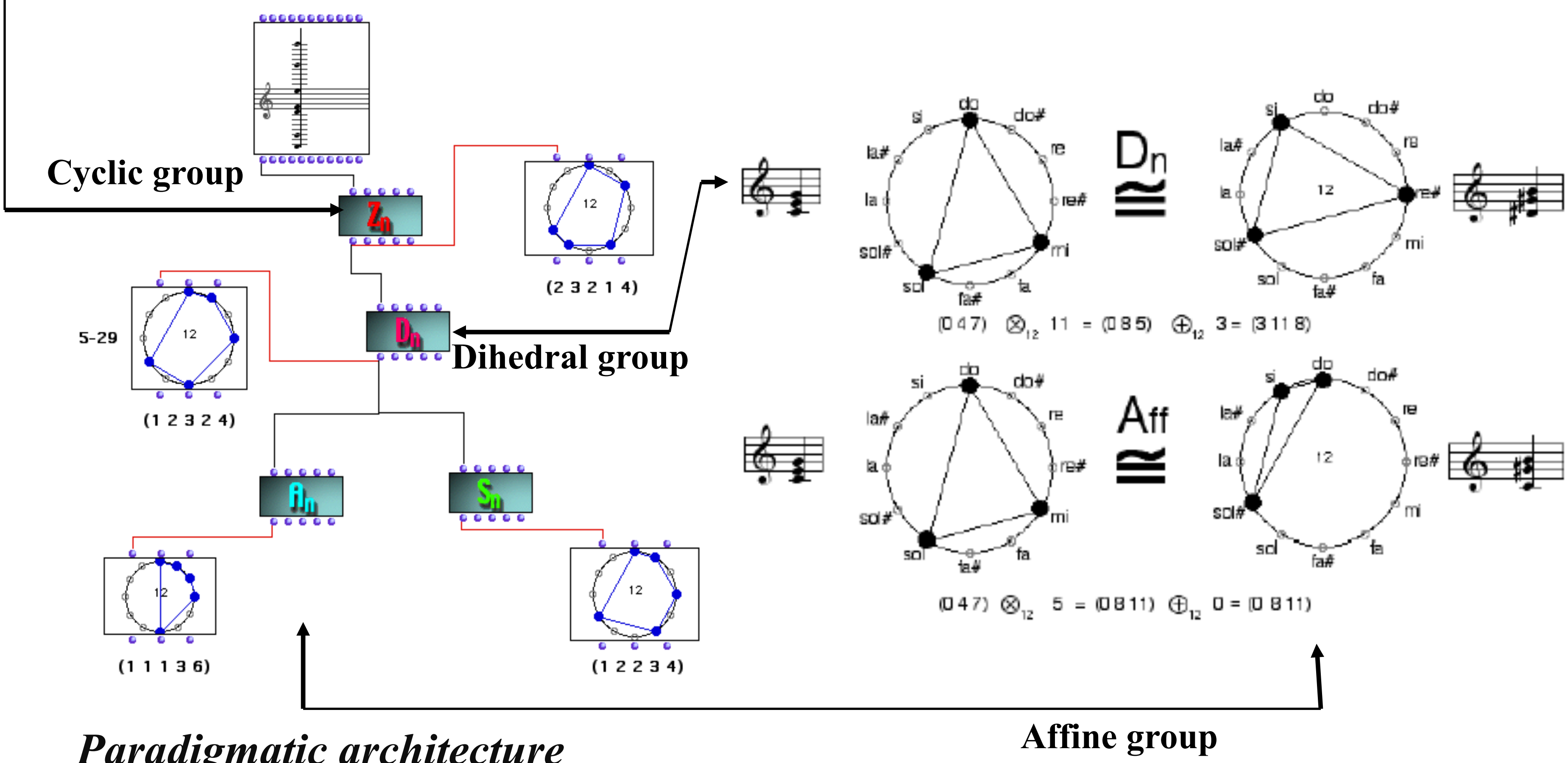
Equivalence classes of chords (up to a group action)



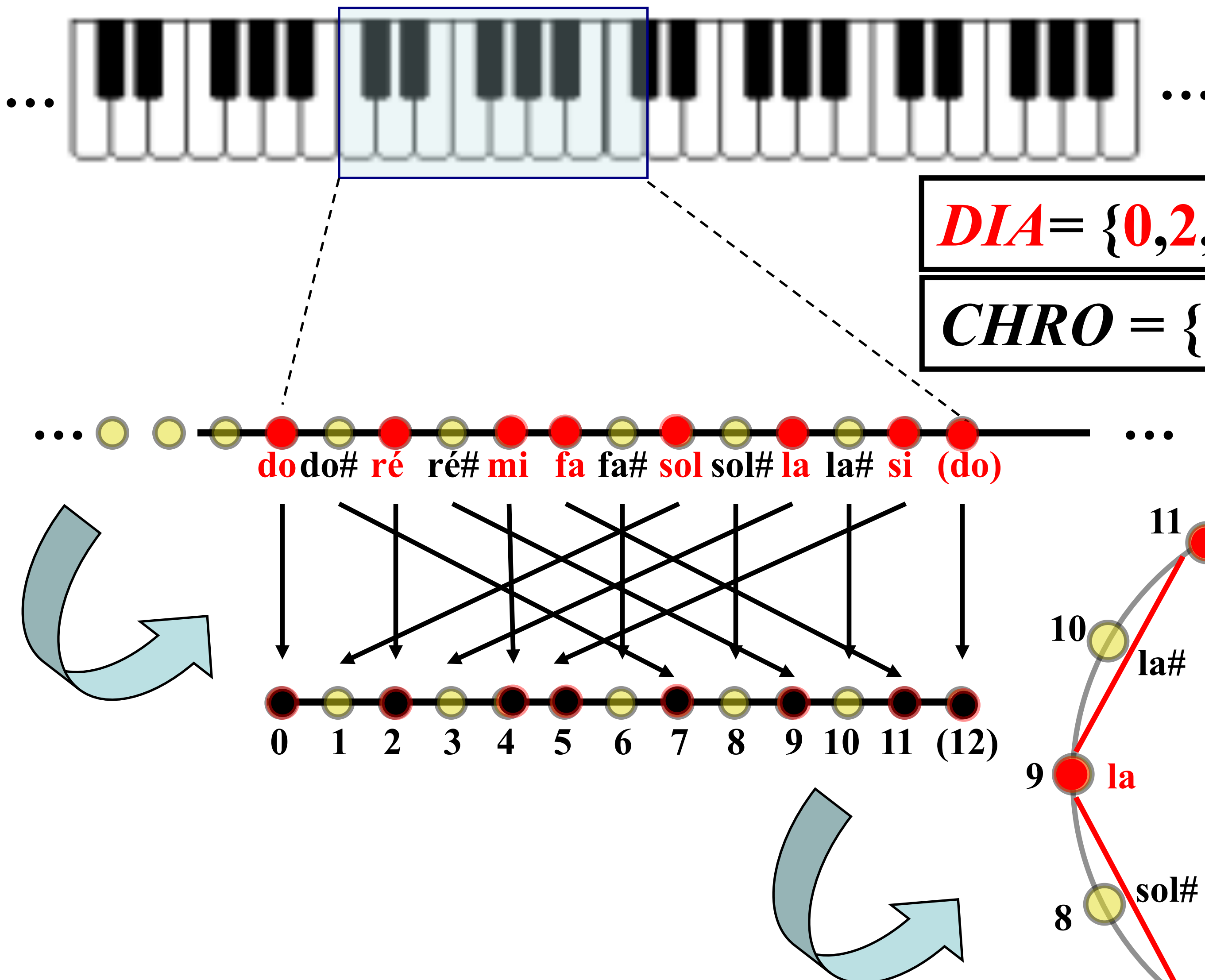
$$\mathbf{Z_{12}} = < T_k \mid (T_k)^{12} = T_0 >$$

$$\mathbf{D_{12}} = < T_k , I \mid (T_k)^{12} = I^2 = T_0 , ITI = I(IT)^{-1} >$$

$$\mathbf{Aff} = \{ f \mid f(x) = ax+b,\ a \in (\mathbf{Z_{12}})^*,\ b \in \mathbf{Z_{12}} \}$$

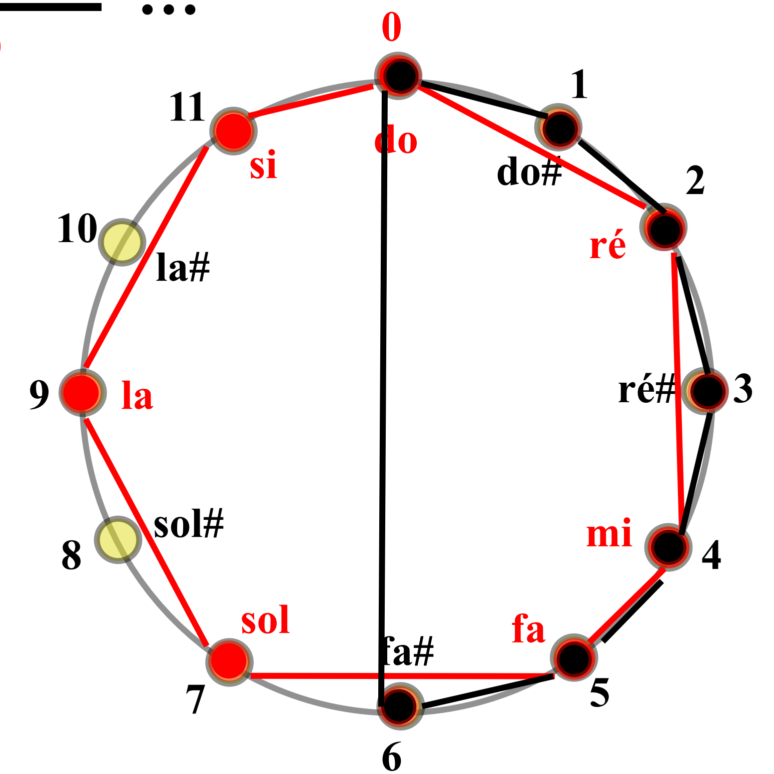


Augmentations are multiplications...



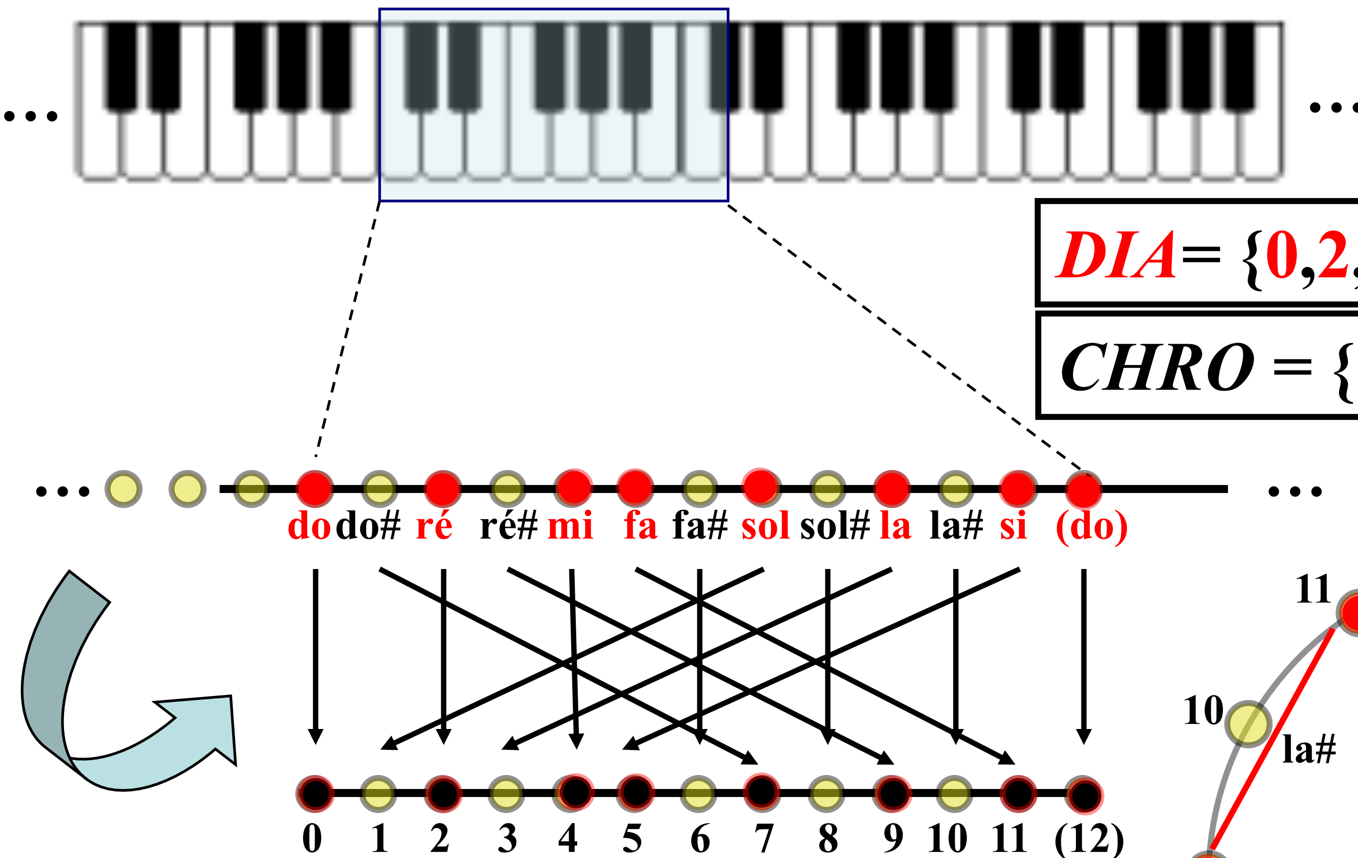
DIA= {0,2,4,5,7,9,11}

CHRO = {0,1,2,3,4,5,6}



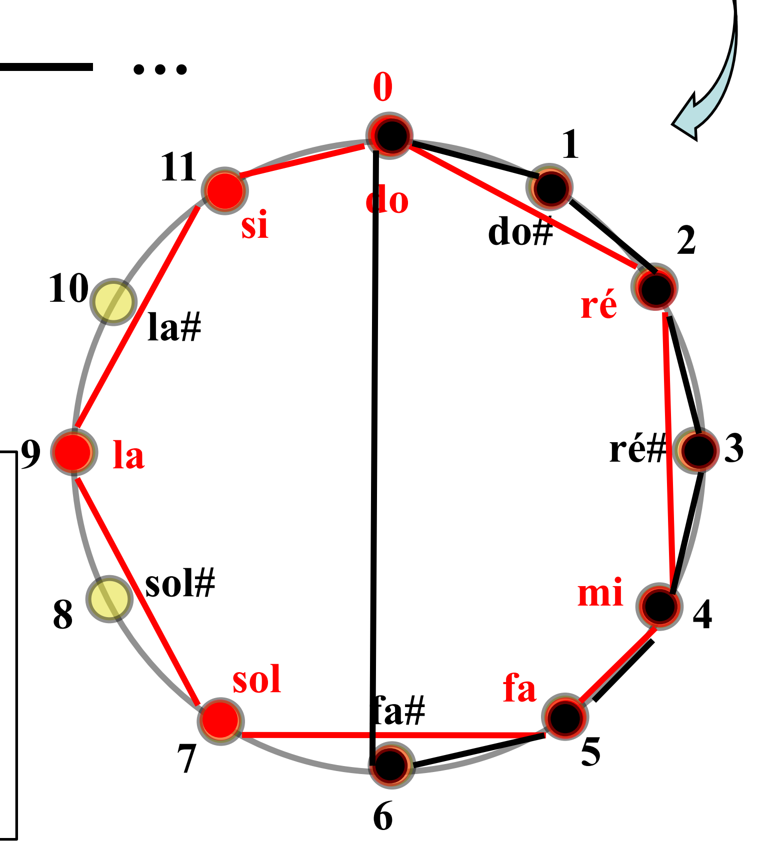
... or affine transformations!

Affine transformations and DIA/CHRO duality



DIA= {0,2,4,5,7,9,11}

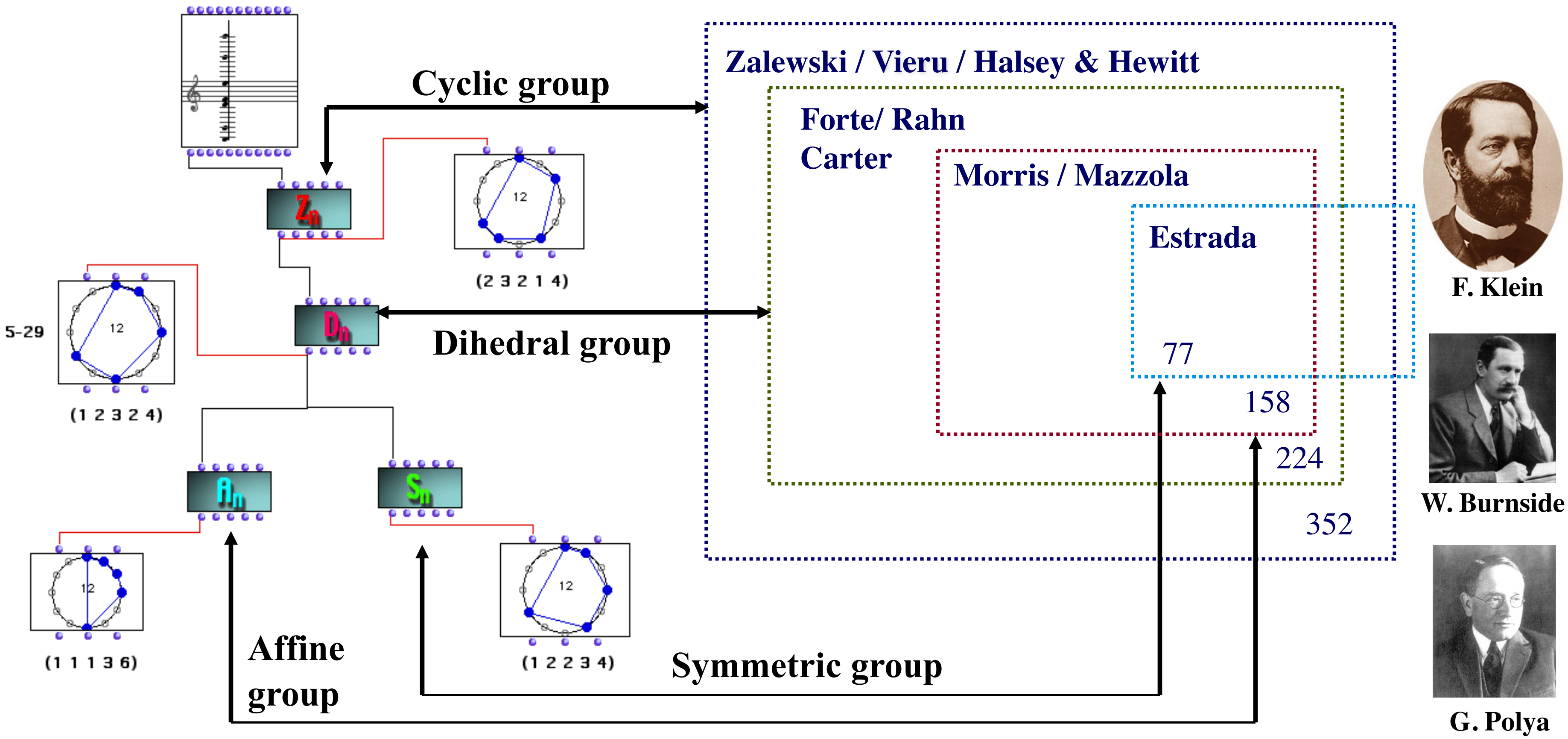
CHRO = {0,1,2,3,4,5,6}



« Le **diatonisme** et le **chromatisme** ne peuvent pas être envisagés en termes de simplicité ou de complexité, comme on le pensait jadis. Il s'agit plutôt d'une question **d'unité des contraires** dans le groupe **Z/12Z** »

(A. Vieru, « The Musical Signification of Multiplication by 7. Diatonicity and Chromaticity », *Muzica,* 1995)

Group actions and the classification of musical structures



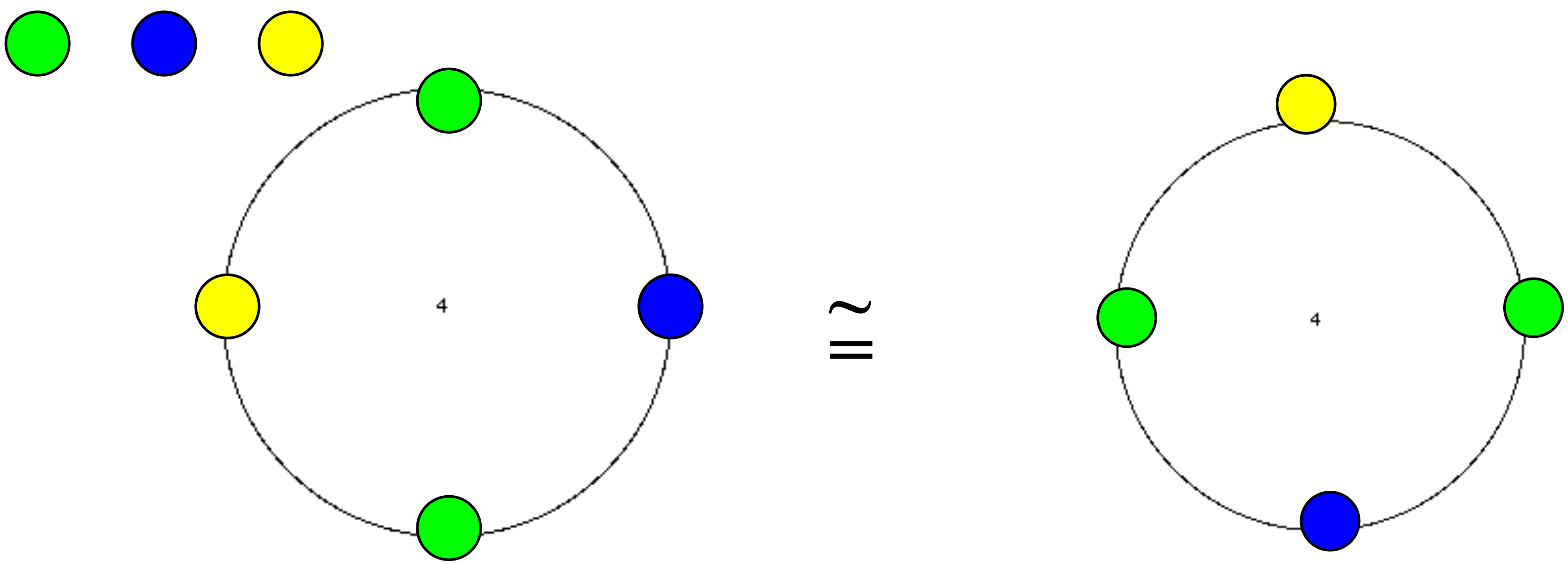
	1	2	3	4	5	6	7	8	9	10	11	12
Z_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1
S_n	***1***	***6***	***12***	***15***	***12***	***11***	***7***	***5***	***3***	***2***	***1***	***1***

Enumeration of chord classes (modulo a group action)

Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

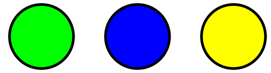
$$X^g = \{x \in X : gx = x\}$$

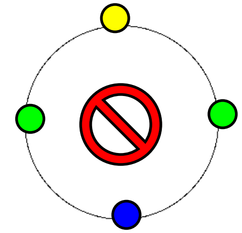


?

How many possible configurations could you find?

Enumeration of chord classes (modulo a group action)





Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

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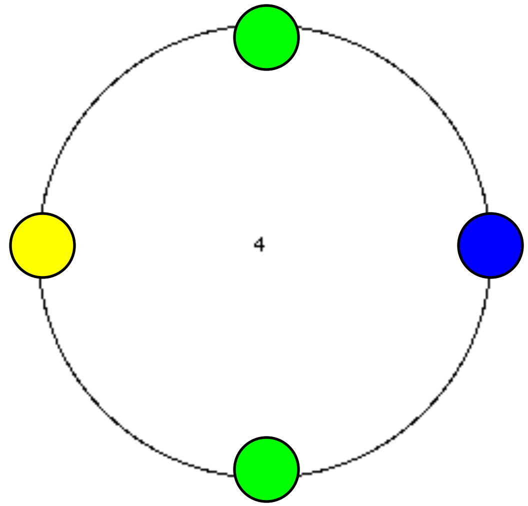


Action of **Z/4Z**

T_0 = identity
T_1 = rotation by 90°
T_2 = rotation by 180°
T_3 = rotation by 270°

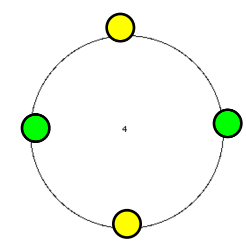
Possible configurations = 3^4 = 81

T_0 fixes all configurations => $|X^{T_0}|$=81

T_1 fixes all monochromatic configurations => $|X^{T_1}|$=3

T_3 idem

T_2 fixes all «double-diameter » configurations => $|X^{T_1}| = 3^2 = 9$



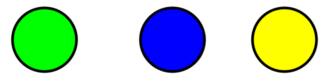
$n = 1/4\ (81+3+3+9) = 24$

Enumeration of chord classes (modulo a group action)

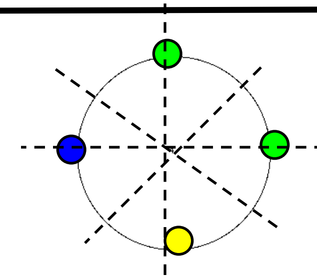
Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$



Action de $\mathbf{Z}_4$



Transformation	*Action*	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	$0\to 0,\ 1\to 1,\ 2\to 2,\ 3\to 3$	(0)(1)(2)(3)	4	$3^4 = 81$	1^4	$t_1^{\,4}$
T_1	$0\to 1\to 2\to 3\to 0$	(0 1 2 3)	1	$3^1 = 3$	4^1	$t_4^{\,1}$
T_2	$0\to 2\to 0,\ 1\to 3\to 1$	(0 2)(1 3)	2	$3^2 = 9$	2^2	$t_2^{\,2}$
T_3	$0\to 3\to 2\to 1\to 0$	(0 3 2 1)	1	$3^1 = 3$	4^1	$t_4^{\,1}$

Julian Hook, « Why are there 29 Tetrachords? A Tutorial on Combinatorics and Enumeration in Music Theory », MTO, 13(4), 2007

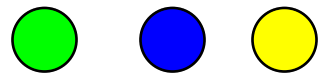
$$n = 1/4\ (81+3+3+9) = 24$$

Enumeration of chord classes (modulo a group action)

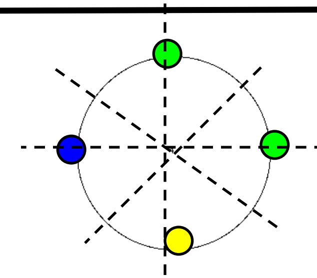
Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$



Action of $\mathbf{D_4}$



Transformation	*Action*	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	$0\to0,\ 1\to1,\ 2\to2,\ 3\to3$	(0)(1)(2)(3)	4	$3^4 = 81$	1^4	$t_1^{\,4}$
T_1	$0\to1\to2\to3\to0$	(0 1 2 3)	1	$3^1 = 3$	4^1	$t_4^{\,1}$
T_2	$0\to2\to0,\ 1\to3\to1$	(0 2)(1 3)	2	$3^2 = 9$	2^2	$t_2^{\,2}$
T_3	$0\to3\to2\to1\to0$	(0 3 2 1)	1	$3^1 = 3$	4^1	$t_4^{\,1}$
I	$0\to0,\ 1\to3\to1,\ 2\to2$	(0)(1 3)(2)	3	$3^3 = 27$	$1^2 2^1$	$t_1^{\,2}t_2^{\,1}$
T_1I	$0\to1\to0,\ 2\to3\to2$	(0 1)(2 3)	2	$3^2 = 9$	2^2	$t_2^{\,2}$
T_2I	$0\to2\to0,\ 1\to1,\ 3\to3$	(0 2)(1)(3)	3	$3^3 = 27$	$1^2 2^1$	$t_1^{\,2}t_2^{\,1}$
T_3I	$0\to3\to0,\ 1\to2\to1$	(0 3)(1 2)	2	$3^2 = 9$	2^2	$t_2^{\,2}$

Julian Hook, « Why are there 29 Tetrachords? A Tutorial on Combinatorics and Enumeration in Music Theory », MTO, 13(4), 2007

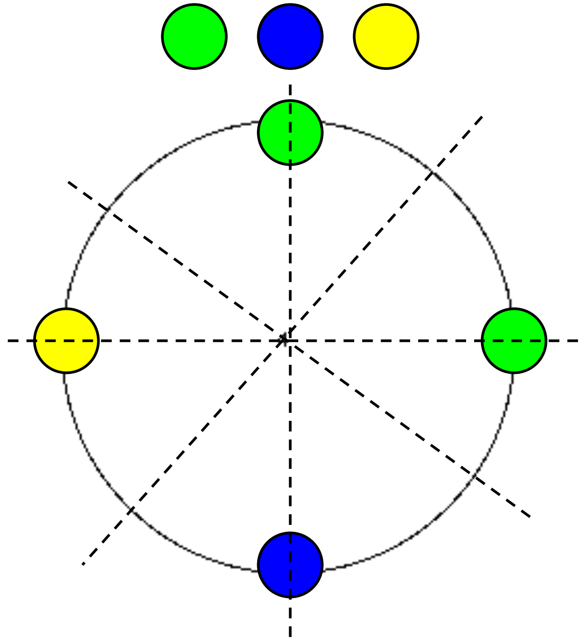
$$n = 1/8\ (81+3+3+9+27+9+27+9) = 168/8=21$$

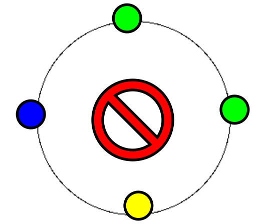
Enumeration of chord classes (modulo a group action)

Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$

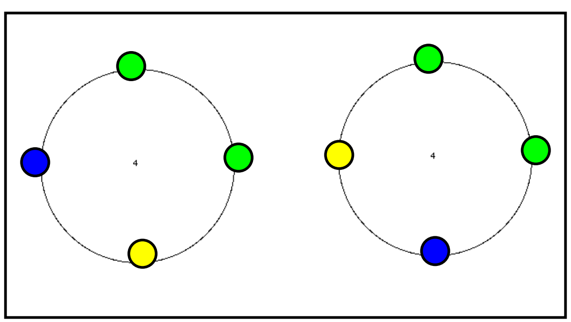


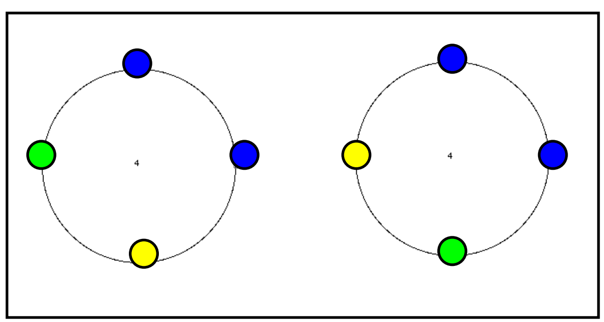


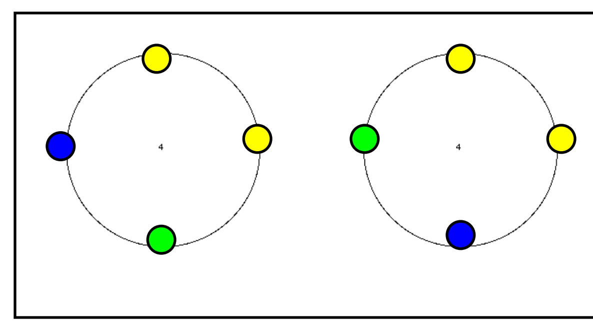
Action of $\mathbf{D_4}$

T_0 = id	T_0I = inversion
T_1 = rot 90°	T_1I = inv.
T_2 = rot 180°	T_2I = inv.
T_3 = rot 270°	T_3I = inv.

⇨ 21=24-3







Enumeration of transposition chord classes

Transformation	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	(0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(A)(B)	12	$2^{12} = 4096$	1^{12}	t_1^{12}
T_1	(0 1 2 3 4 5 6 7 8 9 A B)	1	$2^1 = 2$	12^1	t_{12}^1
T_2	(0 2 4 6 8 A)(1 3 5 7 9 B)	2	$2^2 = 4$	6^2	t_6^2
T_3	(0 3 6 9)(1 4 7 A)(2 5 8 B)	3	$2^3 = 8$	4^3	t_4^3
T_4	(0 4 8)(1 5 9)(2 6 A)(3 7 B)	4	$2^4 = 16$	3^4	t_3^4
T_5	(0 5 A 3 8 1 6 B 4 9 2 7)	1	$2^1 = 2$	12^1	t_{12}^1
T_6	(0 6)(1 7)(2 8)(3 9)(4 A)(5 B)	6	$2^6 = 64$	2^6	t_2^6
T_7	(0 7 2 9 4 B 6 1 8 3 A 5)	1	$2^1 = 2$	12^1	t_{12}^1
T_8	(0 8 4)(1 9 5)(2 A 6)(3 B 7)	4	$2^4 = 16$	3^4	t_3^4
T_9	(0 9 6 3)(1 A 7 4)(2 B 8 5)	3	$2^3 = 8$	4^3	t_4^3
T_{10}	(0 A 8 6 4 2)(1 B 9 7 5 3)	2	$2^2 = 4$	6^2	t_6^2
T_{11}	(0 B A 9 8 7 6 5 4 3 2 1)	1	$2^1 = 2$	12^1	t_{12}^1

Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$X^g = \{x \in X : gx = x\}$

Action of $\mathbf{Z}_{12}$

(Hook, MTO)

chords = 1/12[4096+2+4+8+16+2+64+2+16+8+4+2]=4224/12=352

Enumeration of pitch-class sets

Transformation	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	(0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(A)(B)	12	$2^{12} = 4096$	1^{12}	t_1^{12}
T_1	(0 1 2 3 4 5 6 7 8 9 A B)	1	$2^1 = 2$	12^1	t_{12}^1
T_2	(0 2 4 6 8 A)(1 3 5 7 9 B)	2	$2^2 = 4$	6^2	t_6^2
T_3	(0 3 6 9)(1 4 7 A)(2 5 8 B)	3	$2^3 = 8$	4^3	t_4^3
T_4	(0 4 8)(1 5 9)(2 6 A)(3 7 B)	4	$2^4 = 16$	3^4	t_3^4
T_5	(0 5 A 3 8 1 6 B 4 9 2 7)	1	$2^1 = 2$	12^1	t_{12}^1
T_6	(0 6)(1 7)(2 8)(3 9)(4 A)(5 B)	6	$2^6 = 64$	2^6	t_2^6
T_7	(0 7 2 9 4 B 6 1 8 3 A 5)	1	$2^1 = 2$	12^1	t_{12}^1
T_8	(0 8 4)(1 9 5)(2 A 6)(3 B 7)	4	$2^4 = 16$	3^4	t_3^4
T_9	(0 9 6 3)(1 A 7 4)(2 B 8 5)	3	$2^3 = 8$	4^3	t_4^3
T_{10}	(0 A 8 6 4 2)(1 B 9 7 5 3)	2	$2^2 = 4$	6^2	t_6^2
T_{11}	(0 B A 9 8 7 6 5 4 3 2 1)	1	$2^1 = 2$	12^1	t_{12}^1
I	(0)(1 B)(2 A)(3 9)(4 8)(5 7)(6)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_1I	(0 1)(2 B)(3 A)(4 9)(5 8)(6 7)	6	$2^6 = 64$	2^6	t_2^6
T_2I	(0 2)(1)(3 B)(4 A)(5 9)(6 8)(7)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_3I	(0 3)(1 2)(4 B)(5 A)(6 9)(7 8)	6	$2^6 = 64$	2^6	t_2^6
T_4I	(0 4)(1 3)(2)(5 B)(6 A)(7 9)(8)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_5I	(0 5)(1 4)(2 3)(6 B)(7 A)(8 9)	6	$2^6 = 64$	2^6	t_2^6
T_6I	(0 6)(1 5)(2 4)(3)(7 B)(8 A)(9)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_7I	(0 7)(1 6)(2 5)(3 4)(8 B)(9 A)	6	$2^6 = 64$	2^6	t_2^6
T_8I	(0 8)(1 7)(2 6)(3 5)(4)(9 B)(A)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_9I	(0 9)(1 8)(2 7)(3 6)(4 5)(A B)	6	$2^6 = 64$	2^6	t_2^6
$T_{10}I$	(0 A)(1 9)(2 8)(3 7)(4 6)(5)(B)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
$T_{11}I$	(0 B)(1 A)(2 9)(3 8)(4 7)(5 6)	6	$2^6 = 64$	2^6	t_2^6

Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$X^g = \{x \in X : gx = x\}$

Action of $\mathbf{D_{12}}$

(Hook, MTO)

chords = 1/12[4096+2+4+8+16+2+64+2+16+8+4+2]=4224/12=352

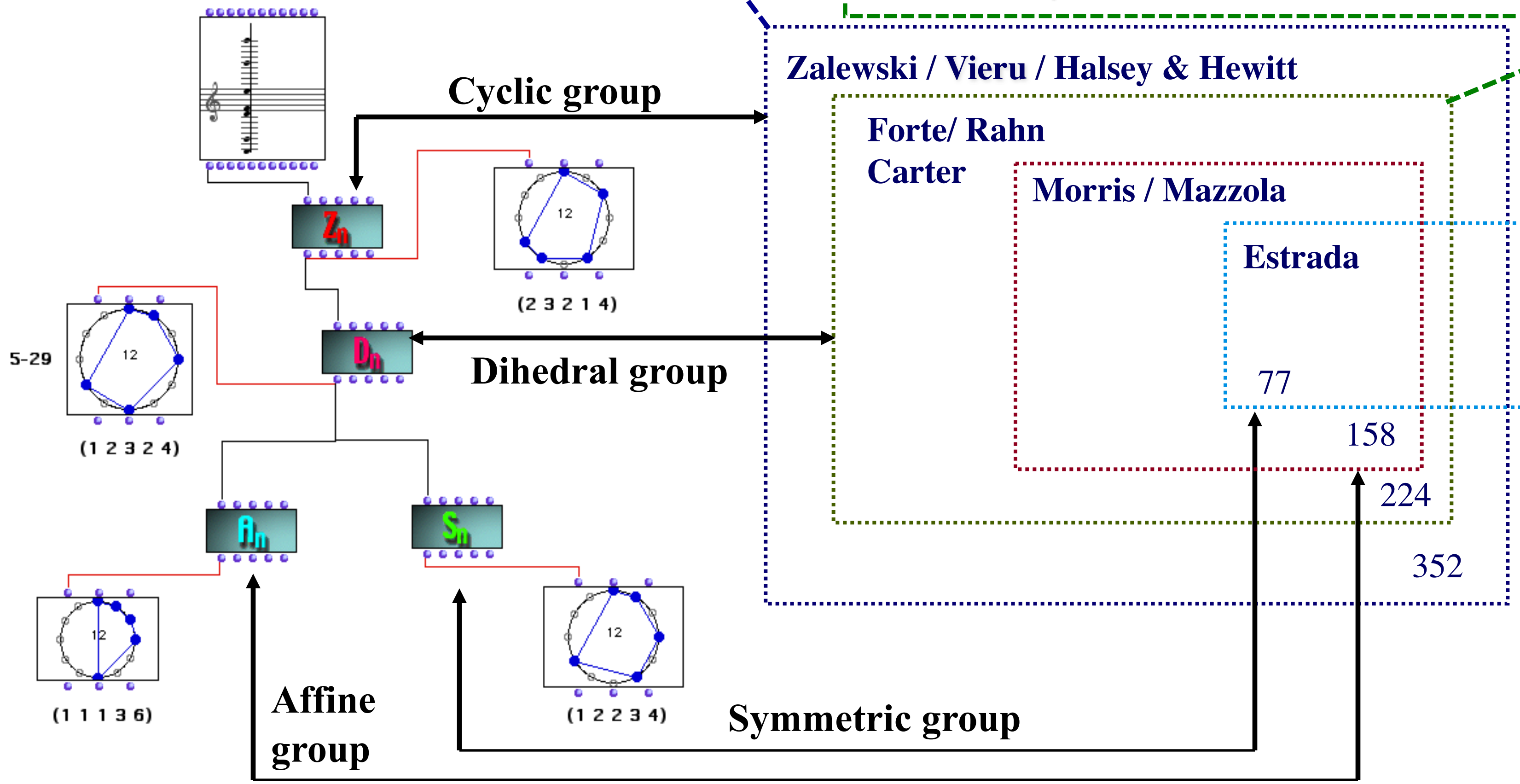
chords = 1/24[4224+1152] = 224

A group action based classification of musical structures

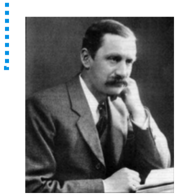
$$\#\text{ of }k\text{-chords} = \frac{1}{n}\sum_{j|(n,k)} \phi(j)\binom{n/j}{k/j} = \frac{1}{n}\Phi_n(k)$$

$$\#\text{ of }k\text{-chords} = \begin{cases} \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n-1)/2}{[k/2]}\right], & \text{if } n \text{ is odd,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{n/2}{k/2}\right], & \text{if } n \text{ is even and } k \text{ is even,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n/2)-1}{[k/2]}\right], & \text{if } n \text{ is even and } k \text{ is odd.} \end{cases}$$

Paradigmatic architecture



F. Klein



W. Burnside

G. Polya

- D. Halsey & E. Hewitt, “Eine gruppentheoretische Methode in der Musik-theorie”, *Jahr. der Dt. Math.-Vereinigung*, 80, 1978
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- H. Fripertinger, “Enumeration of non-isomorphic canons”, *Tatra Mt. Math. Publ.*, 2001

From permutations to algebraic combinatorics

114 Marin Mersenne, *Harmonicorum Libri XII*, 1648

LIBER SEPTIMVS.

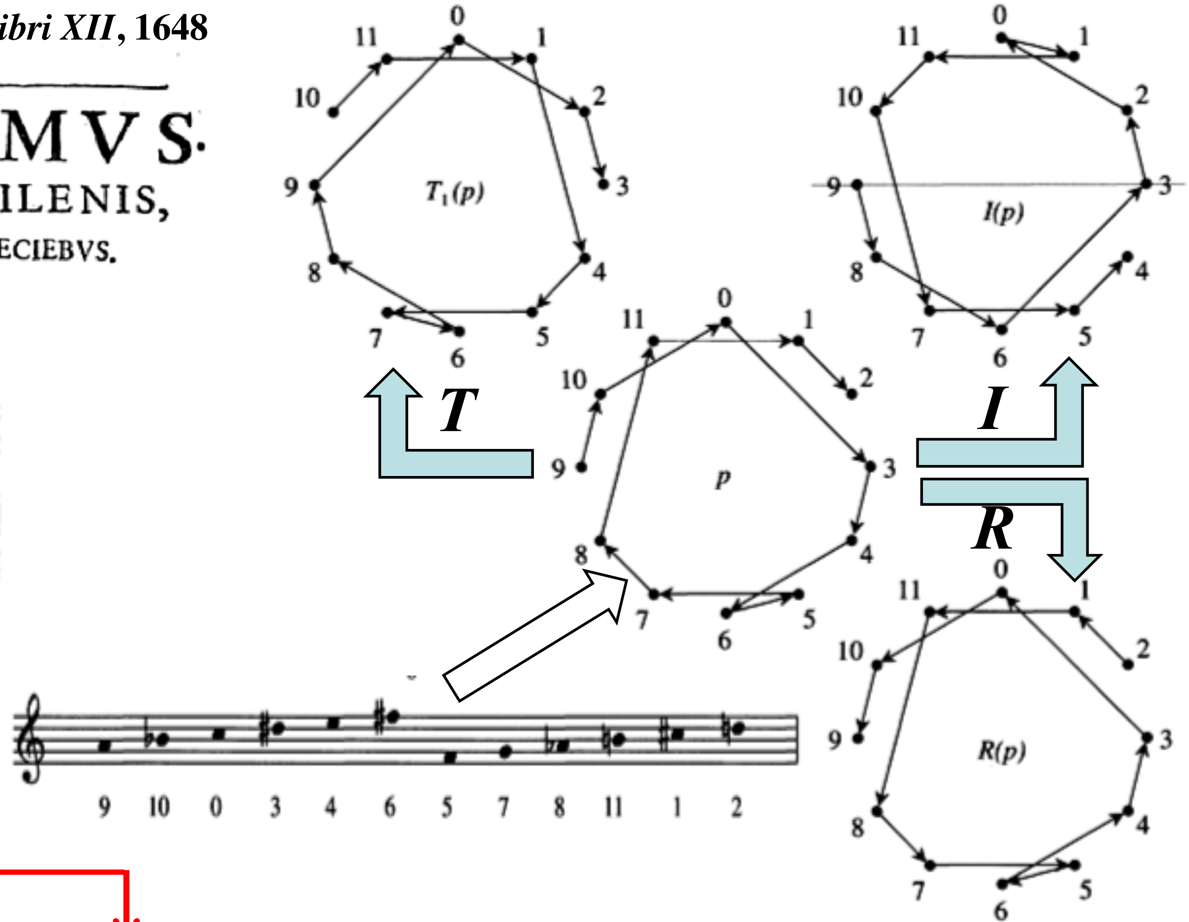
DE CANTIBVS, SEV CANTILENIS,

EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 22.

I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	**479001600**
XIII	6127020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000

?



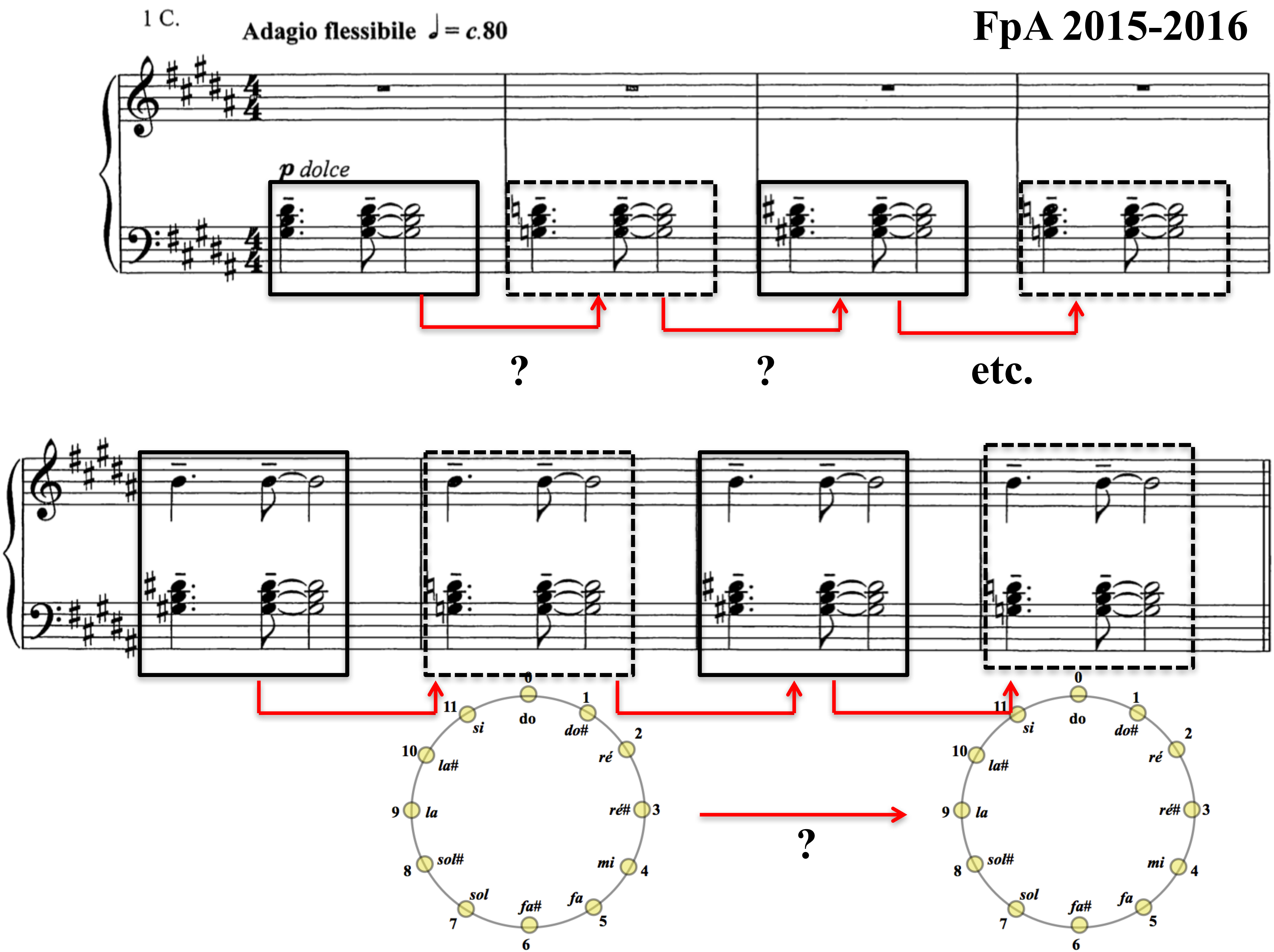
Thus the number of n-tone rows is

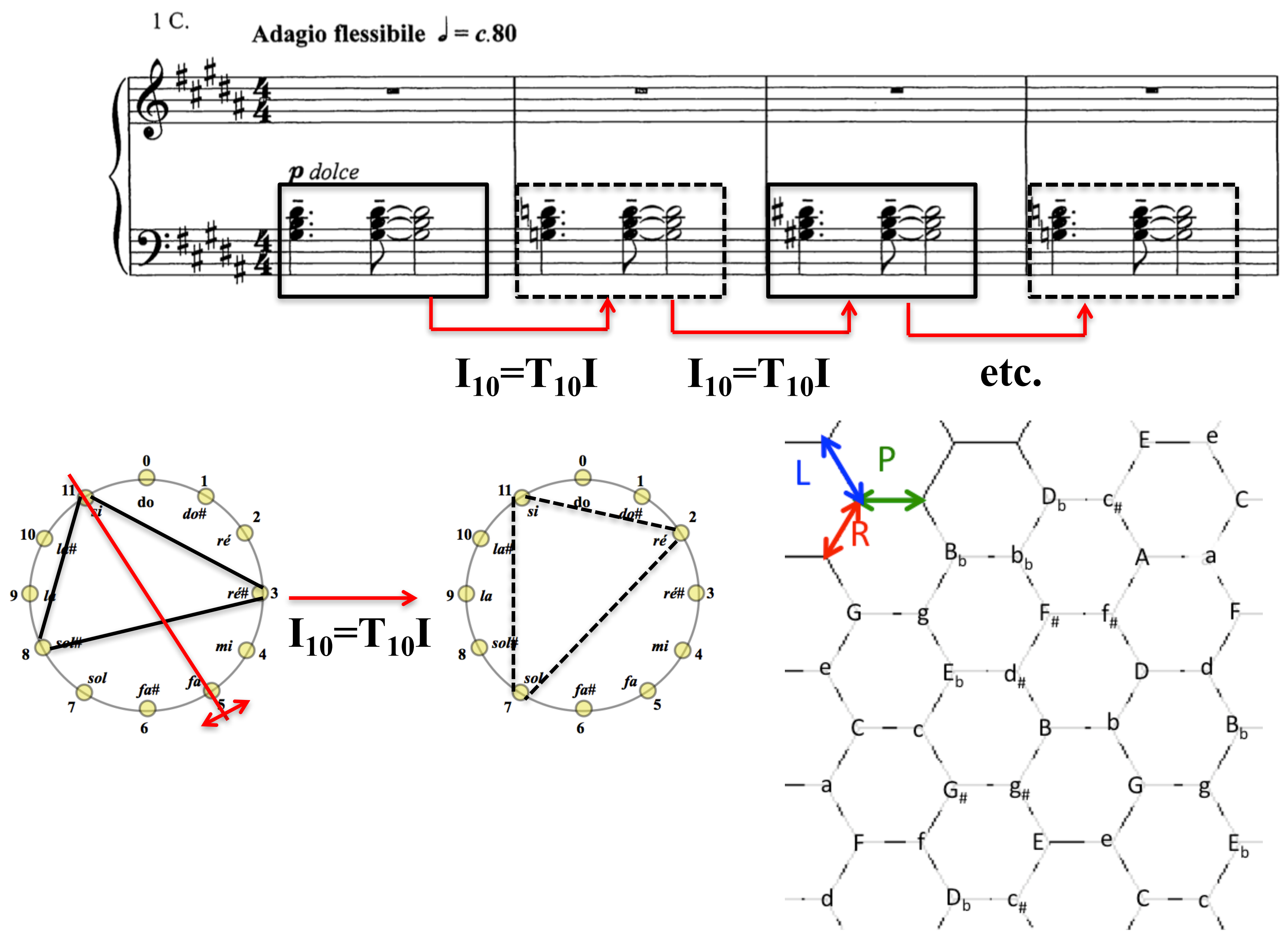
$$\tfrac{1}{4}[(n-1)! + (n-1)(n-3)\cdots(2)] \quad \text{if } n \text{ is odd;}$$

$$\tfrac{1}{4}[(n-1)! + (n-2)(n-4)\cdots(2)(1+n/2)] \quad \text{if } n \text{ is even.}$$

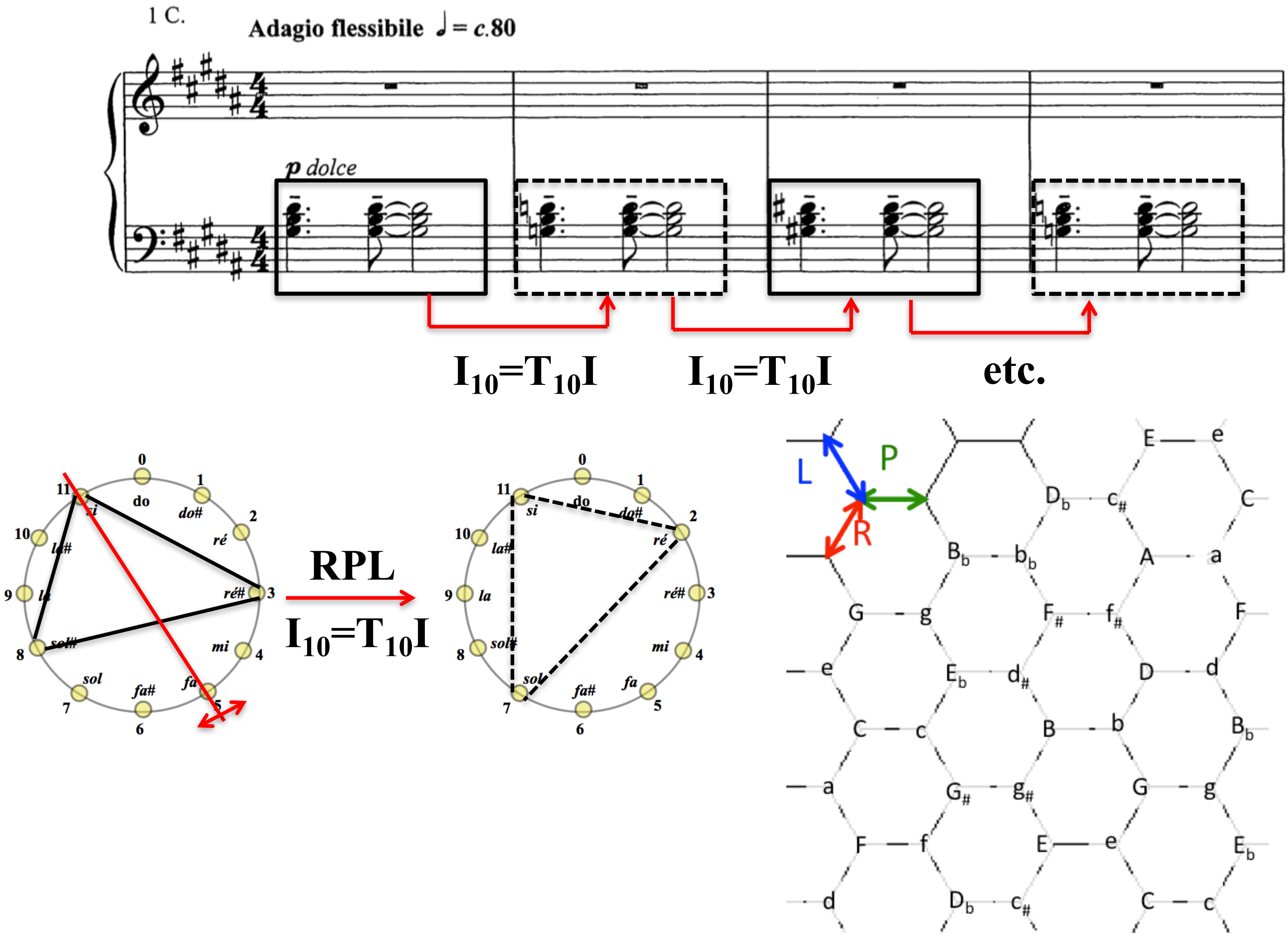
For example, there are 9985920 twelve tone rows, a fact which does not seem to be in the literature.

[D. Reiner, « Enumeration in Music Theory », *Amer. Math. Month.* 1985]

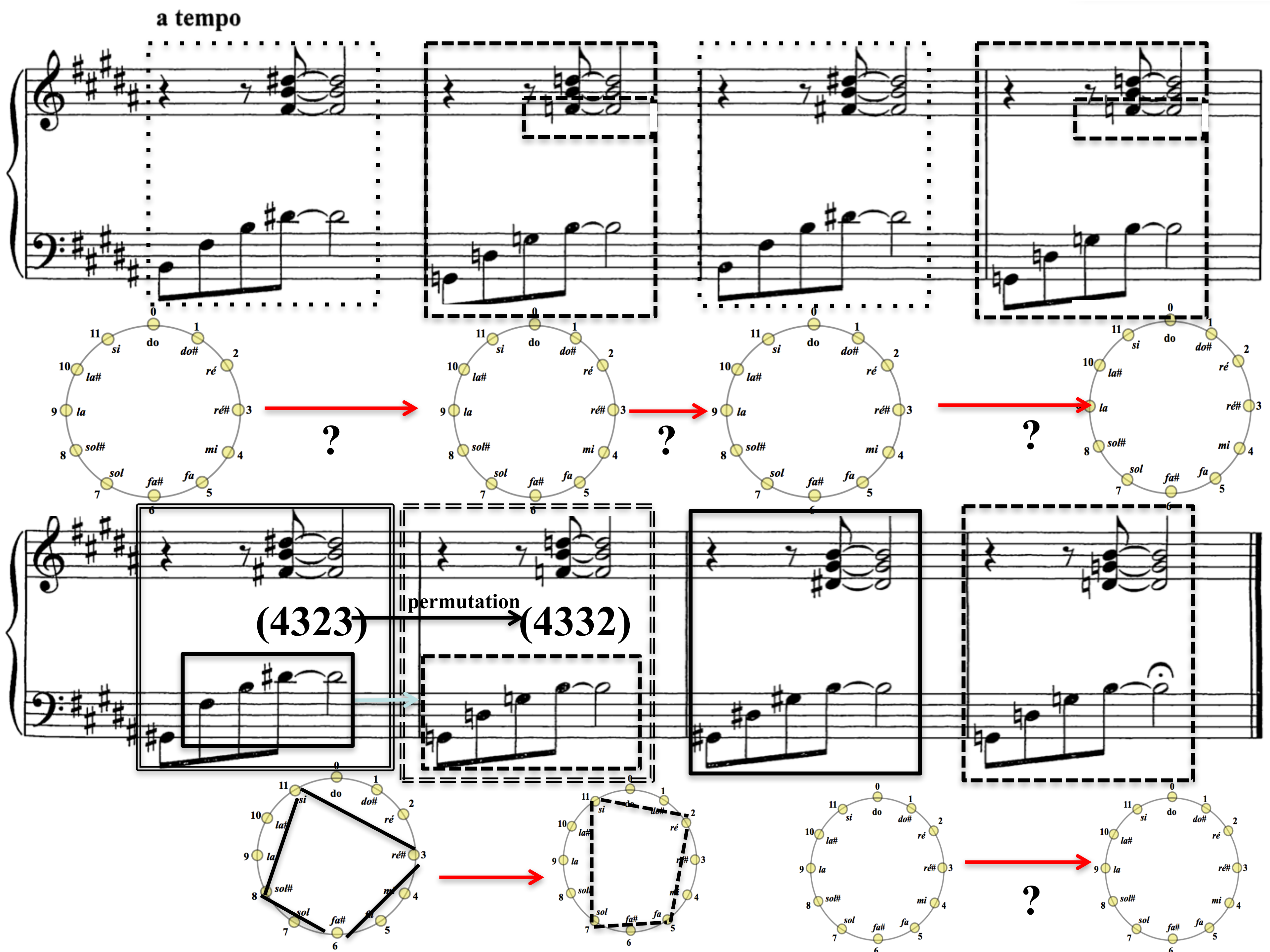

FpA 2015-2016
1 C.
Adagio flessibile ♩ = c.80
p dolce
?
?
etc.
?
0 do
1 do#
2 ré
3 ré#
4 mi
5 fa
6 fa#
7 sol
8 sol#
9 la
10 la#
11 si



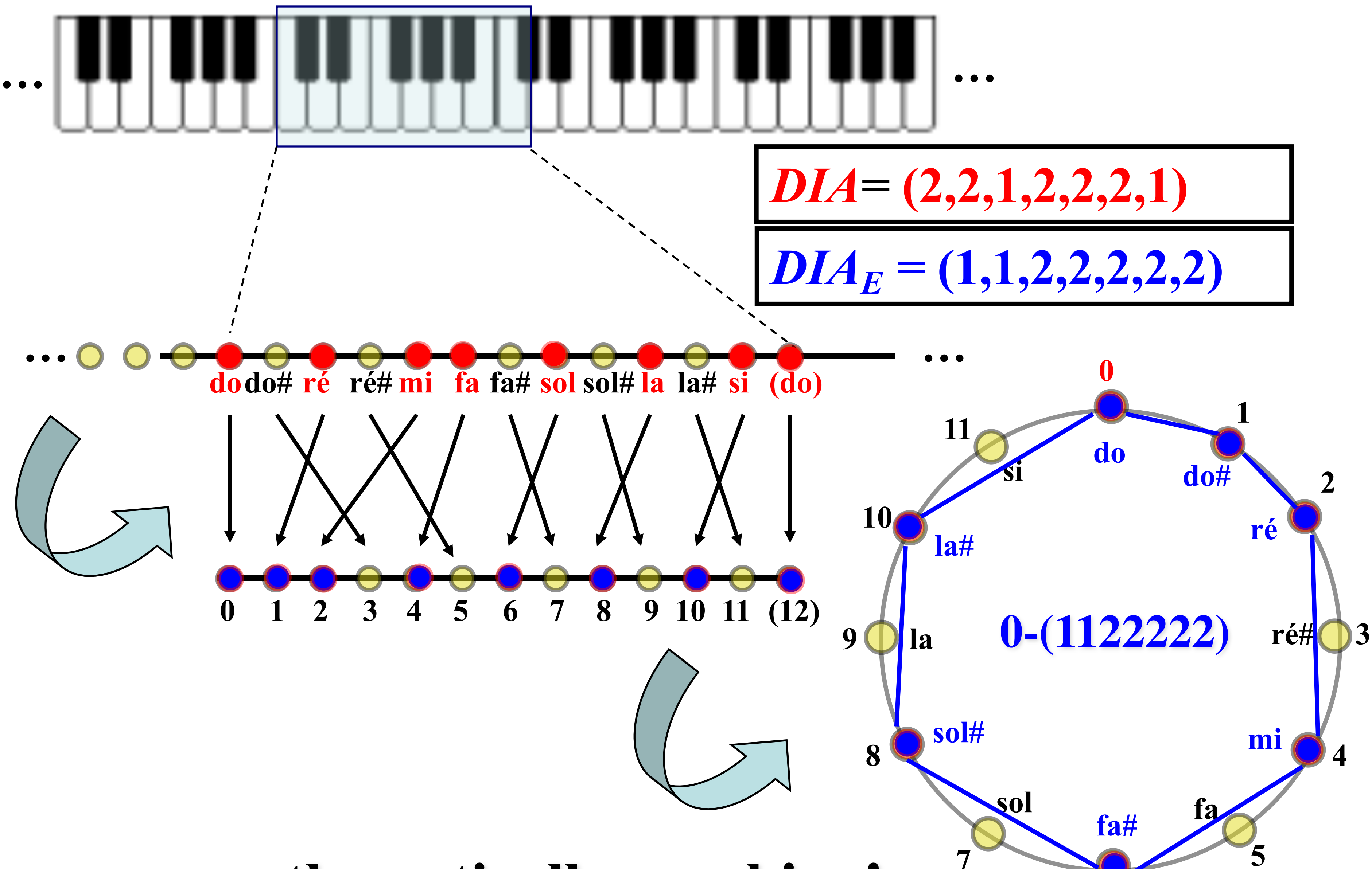
1 C.
Adagio flessibile ♩ = c.80
p dolce
I10=T10I
I10=T10I
etc.
0 do
1 do#
2 ré
3 ré#
4 mi
5 fa
6 fa#
7 sol
8 sol#
9 la
10 la#
11 si
I10=T10I
L
P
R
E e
Db c#
C
Bb bb
A a
G g
F# f#
F
e
Eb d#
D d
C c
B b
Bb
a
G# g#
G g
F f
E e
Eb
d
Db c#
C c



1 C.
Adagio flessibile ♩ = c.80
p dolce
I10=T10I
I10=T10I
etc.
0 do 1 do# 2 ré 3 ré# 4 mi 5 fa 6 fa# 7 sol 8 sol# 9 la 10 la# 11 si
RPL
I10=T10I
0 do 1 do# 2 ré 3 ré# 4 mi 5 fa 6 fa# 7 sol 8 sol# 9 la 10 la# 11 si
L
P
R
E e
Db c#
C
Bb bb
A a
G g
F# f#
F
e
Eb d#
D d
C c
B b
Bb
a
G# g#
G g
F f
E e
Eb
d
Db c#
C c


a tempo
?
?
?
(4323)
permutation
(4332)
?
0 do
1 do#
2 ré
3 ré#
4 mi
5 fa
6 fa#
7 sol
8 sol#
9 la
10 la#
11 si

Permutations are 'partitions'...



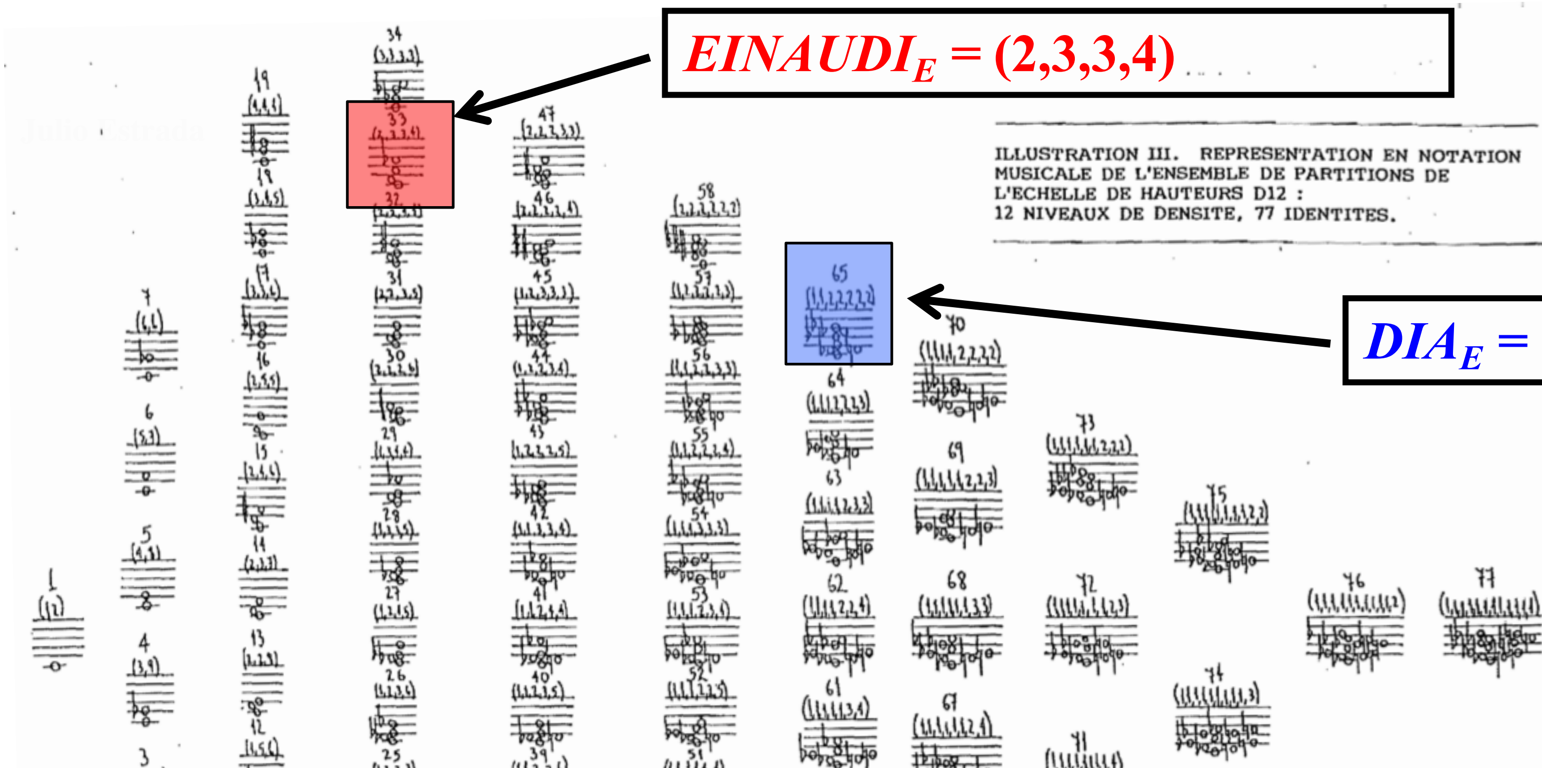
$DIA = (2,2,1,2,2,2,1)$

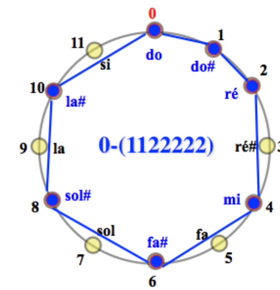
$DIA_E = (1,1,2,2,2,2,2)$

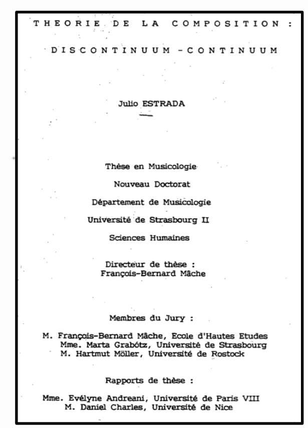
... mathematically speaking!

The permutohedron as a combinatorial space

Julio Estrada, *Théorie de la composition : discontinuum – continuum*, université de Strasbourg II, 1994







THEORIE DE LA COMPOSITION :

DISCONTINUUM - CONTINUUM

Julio ESTRADA

Thèse en Musicologie

Nouveau Doctorat

Département de Musicologie

Université de Strasbourg II

Sciences Humaines

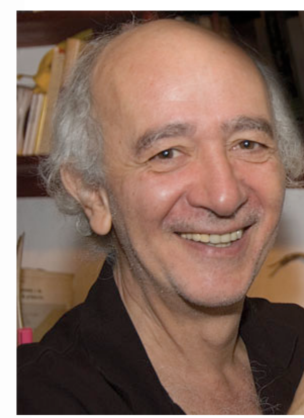
Directeur de thèse :
François-Bernard Mâche

Membres du Jury :

M. François-Bernard Mâche, Ecole d'Hautes Etudes
Mme. Marta Grabótz, Université de Strasbourg
M. Hartmut Möller, Université de Rostock

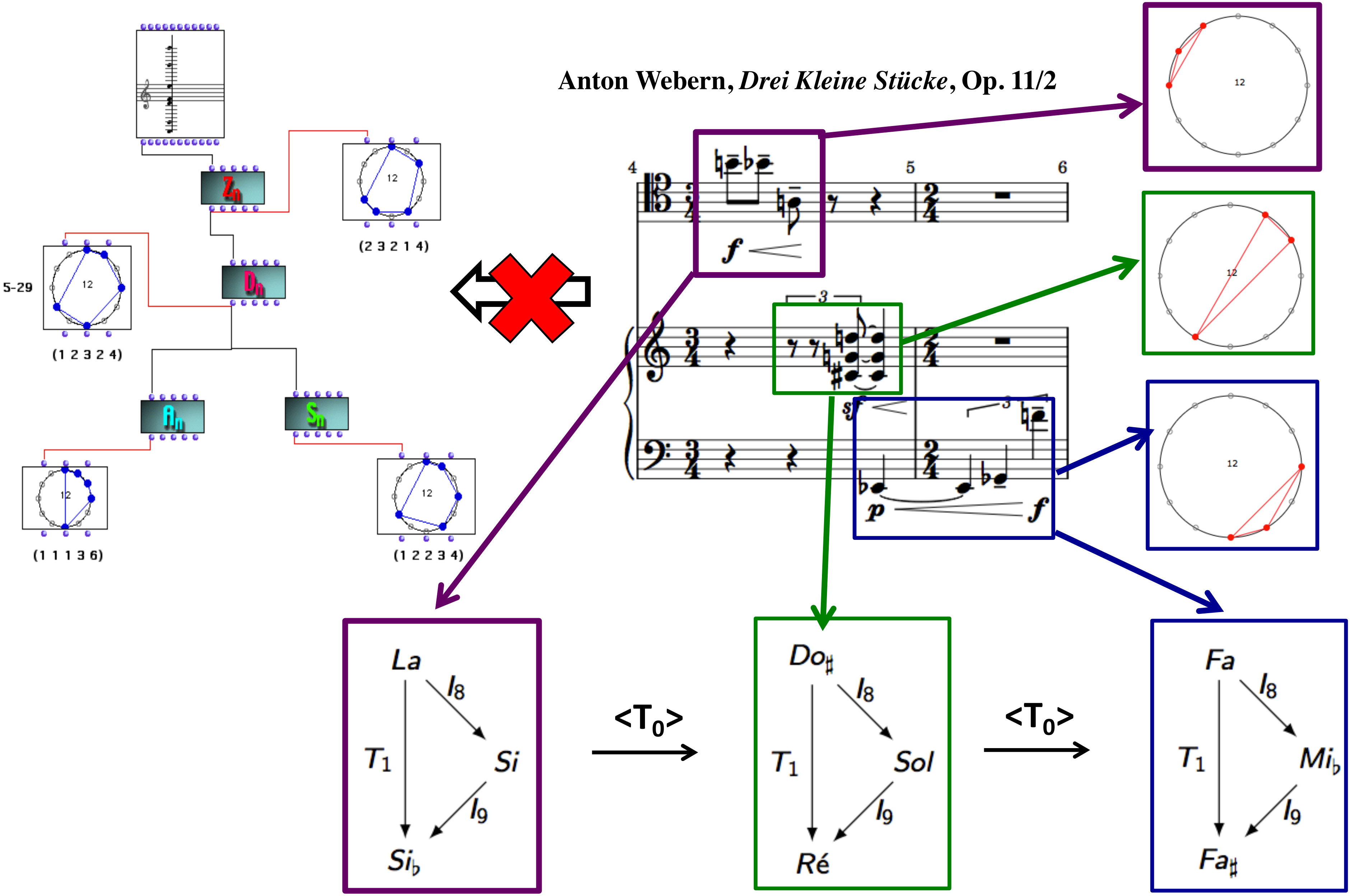
Rapports de thèse :

Mme. Evélyne Andreani, Université de Paris VIII
M. Daniel Charles, Université de Nice



J. Estrada

K-Nets and the paradigmatic approach



K-nets as a transformational construction

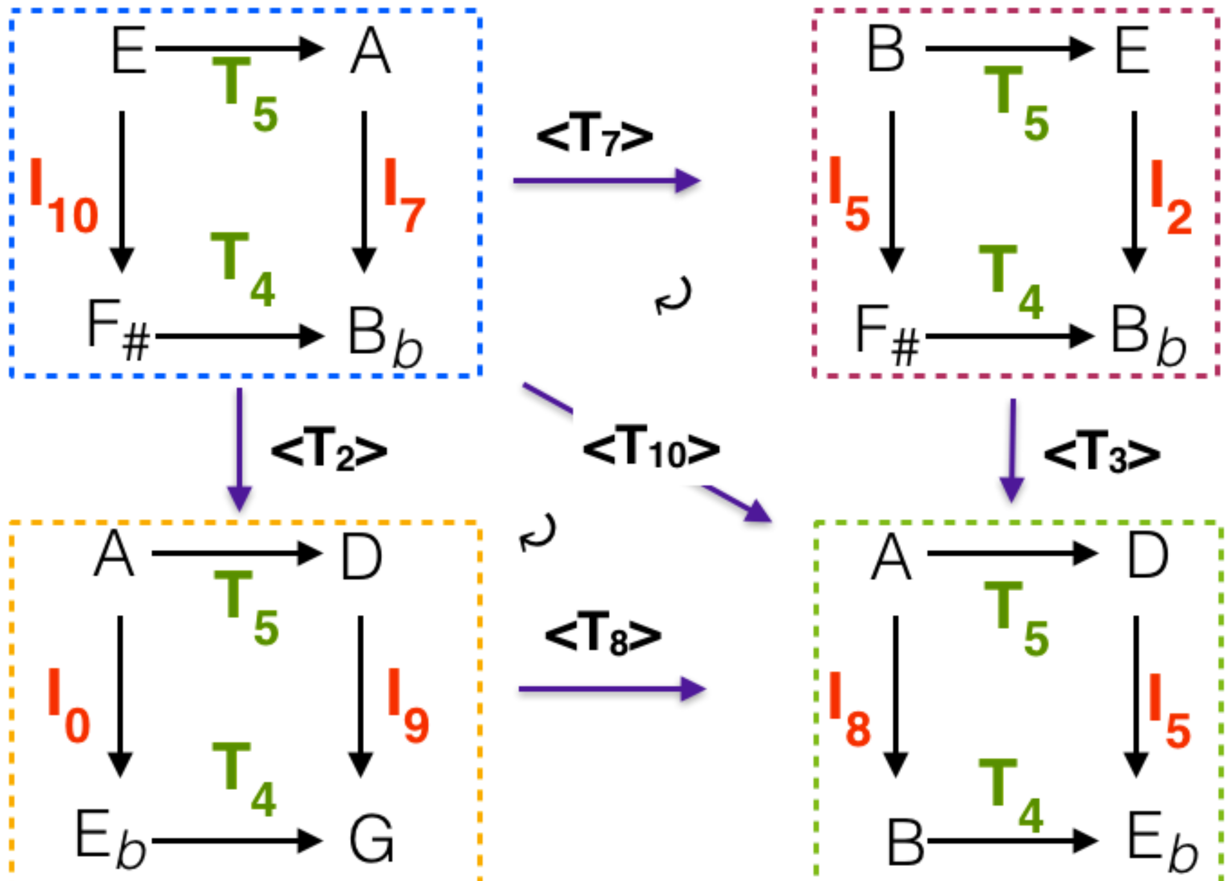
D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994

D. Lewin

H. Klumpenhouwer

$\langle T_k \rangle : T_m \to T_m$
$I_m \to I_{k+m}$

$\langle T_k \rangle \cdot \langle T_m \rangle = \langle T_{k+m} \rangle$



K-nets as a transformational construction

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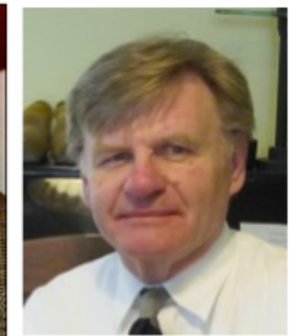
D. Lewin

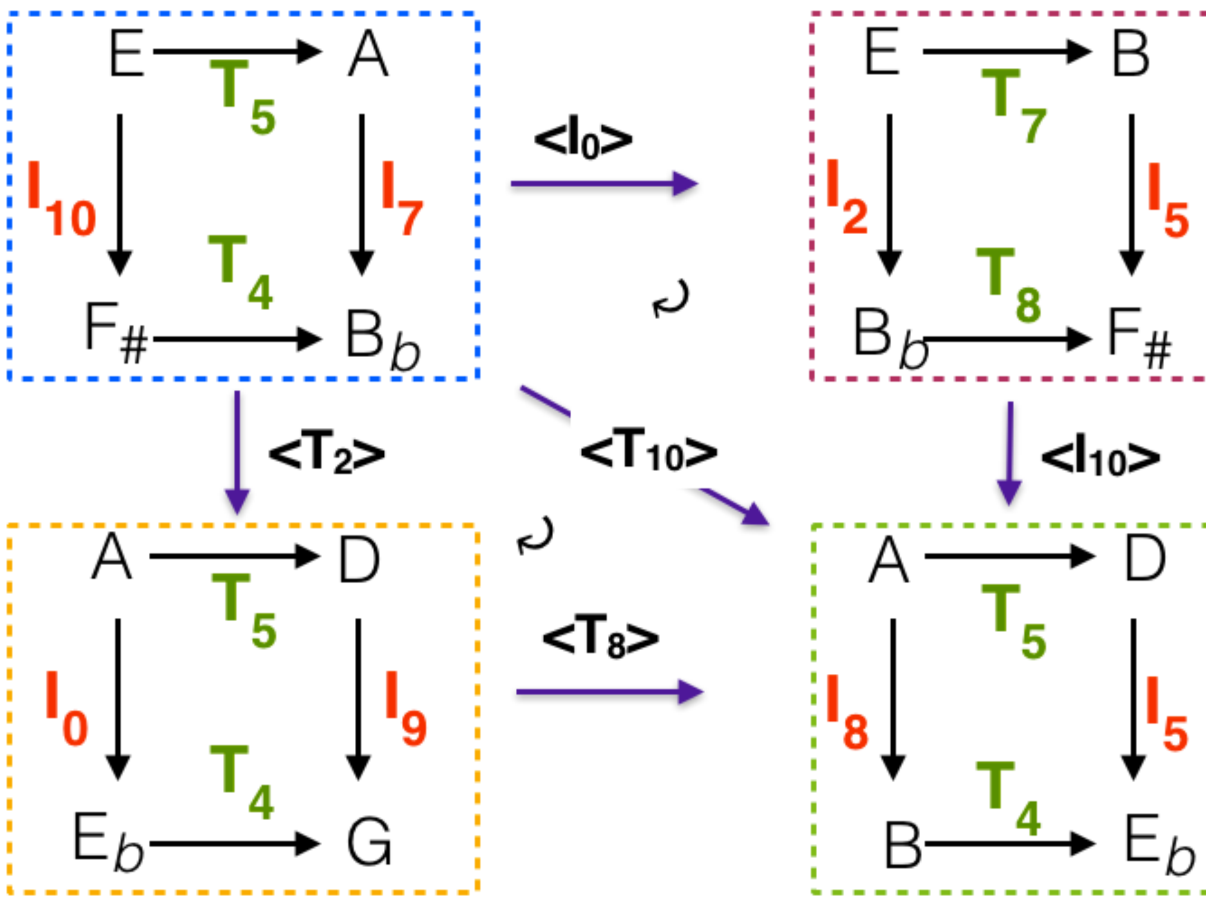


H. Klumpenhouwer

$\langle T_k \rangle : T_m \rightarrow T_m$
$\qquad I_m \rightarrow I_{k+m}$

$\langle I_k \rangle : T_m \rightarrow T_{-m}$
$\qquad I_m \rightarrow I_{k-m}$

$\langle T_k \rangle \bullet \langle T_m \rangle = \langle T_{k+m} \rangle$
$\langle I_k \rangle \bullet \langle I_m \rangle = \langle T_{m-k} \rangle$



K-nets as a transformational construction

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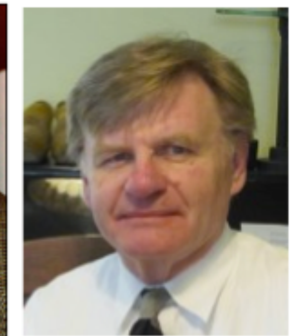
D. Lewin



H. Klumpenhouwer

$$\langle T_k \rangle : T_m \to T_m \qquad I_m \to I_{k+m}$$

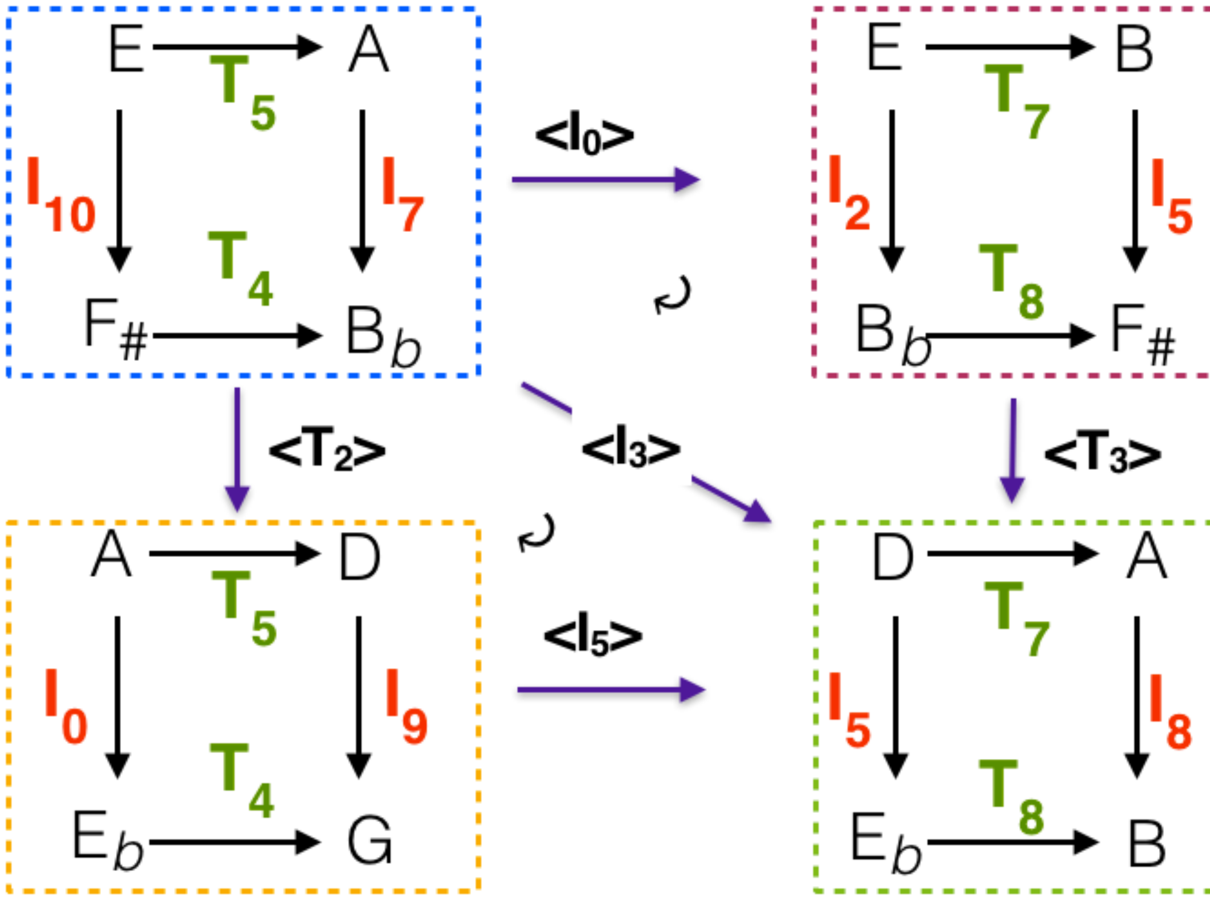
$$\langle I_k \rangle : T_m \to T_{-m} \qquad I_m \to I_{k-m}$$

$$\langle T_k \rangle \bullet \langle T_m \rangle = \langle T_{k+m} \rangle$$

$$\langle T_k \rangle \bullet \langle I_m \rangle = \langle I_{m-k} \rangle$$

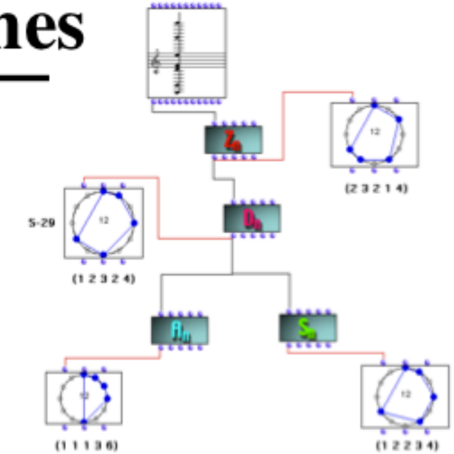
$$\langle I_m \rangle \bullet \langle T_k \rangle = \langle I_{k+m} \rangle$$

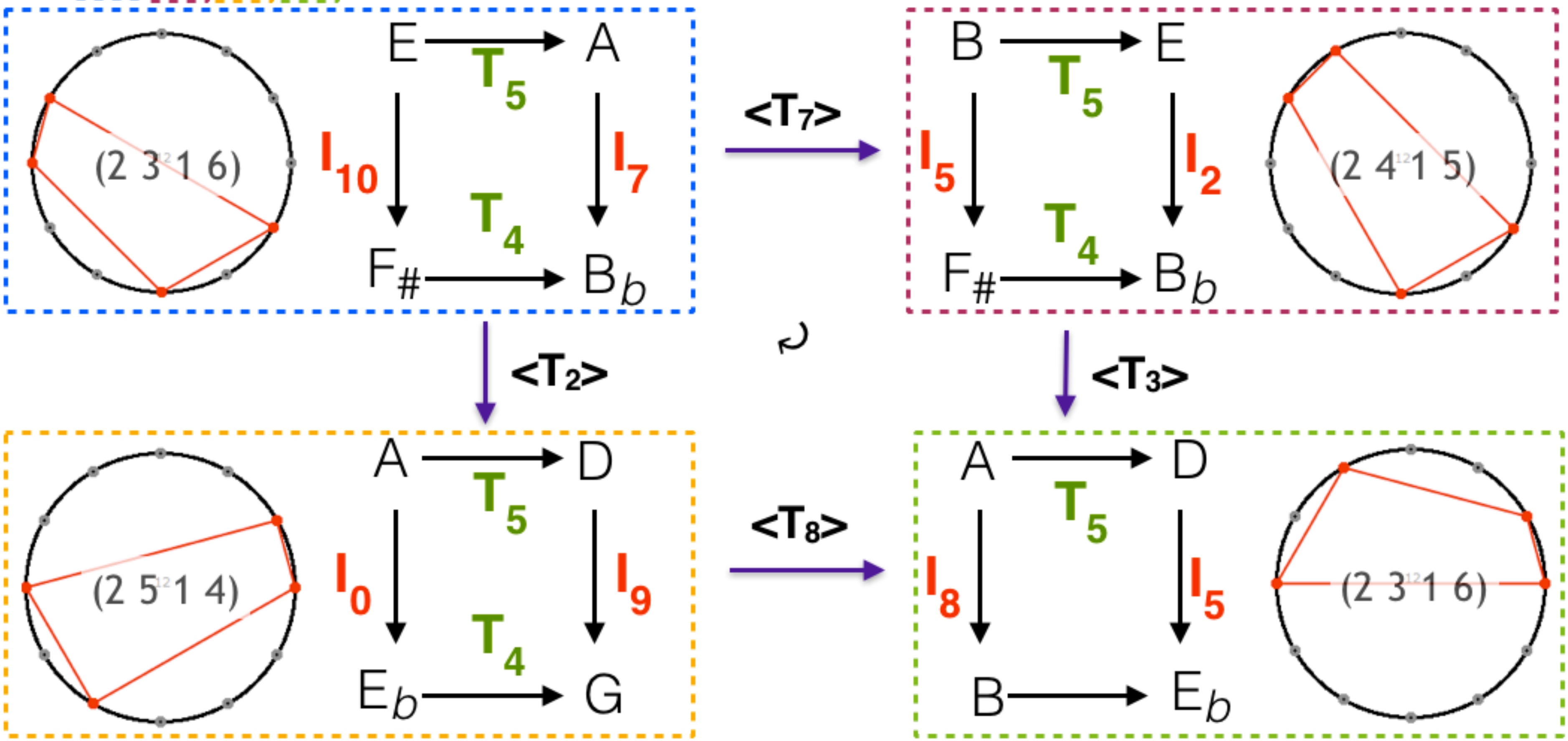
$$\langle I_k \rangle \bullet \langle I_m \rangle = \langle T_{m-k} \rangle$$



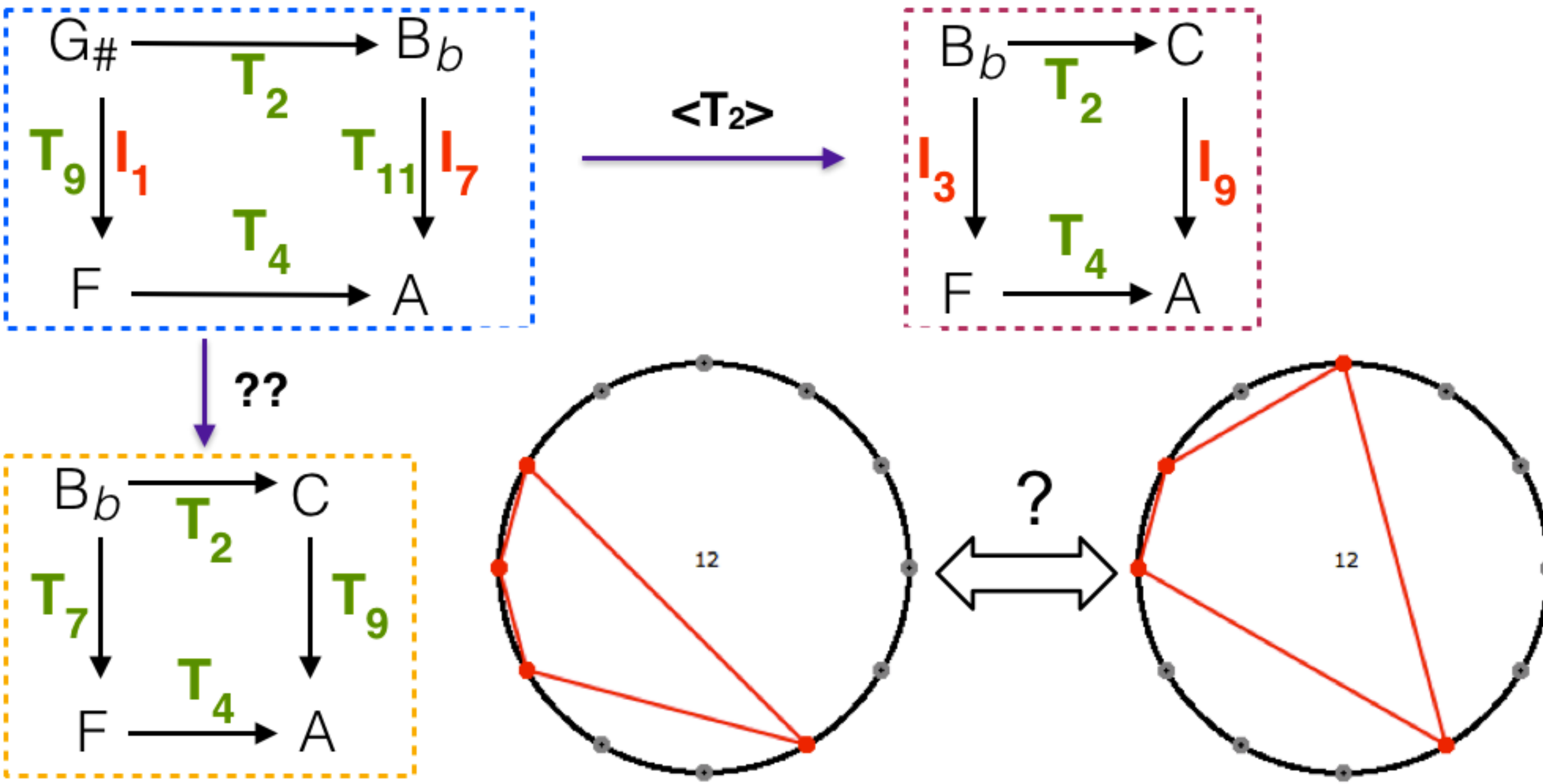
Transformational *vs* set-theoretical approaches

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994

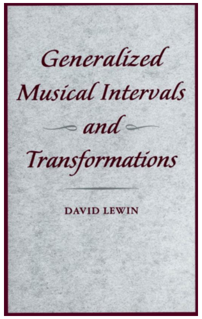




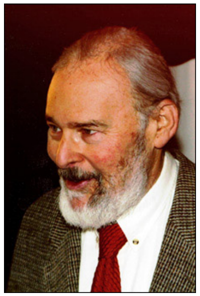
Some theoretical difficulties with the isographic relations



CONCLUSION
There are K-Nets which are not always isographic to a given one, i.e. the isographic relations are highly sensitives to the transformations used to label the arrows.
Is it possible to overstep this theoretical limitation? Which new definition of K-nets allows one to do that?



Système d'Intervalles Généralisés - Système Généralisé d'Intervalles
David Lewin's *Generalized Interval System* [GMIT, 1987]



D. Lewin

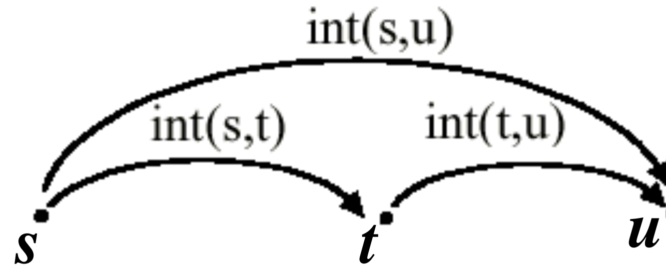
GIS = (S, G, int)

S = ensemble

(G,•) = groupe d'intervalles

int = fonction intervallique

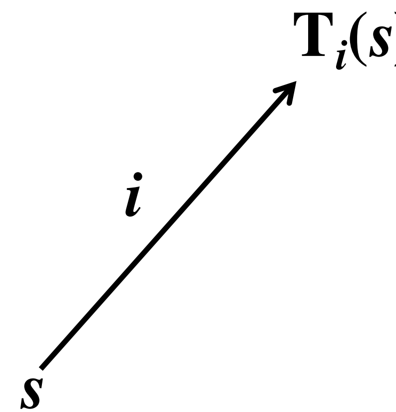
$S \times S \xrightarrow{\text{int}} G$



Action simplément transitive

1. Pour tous objets s, t, u dans S :
   int(s, t)•int(t, u) = int(s, u)

2. Pour tout objet s dans S et tout intervalle i dans G il y a un seul objet t dans S tel que int(s, t) = i



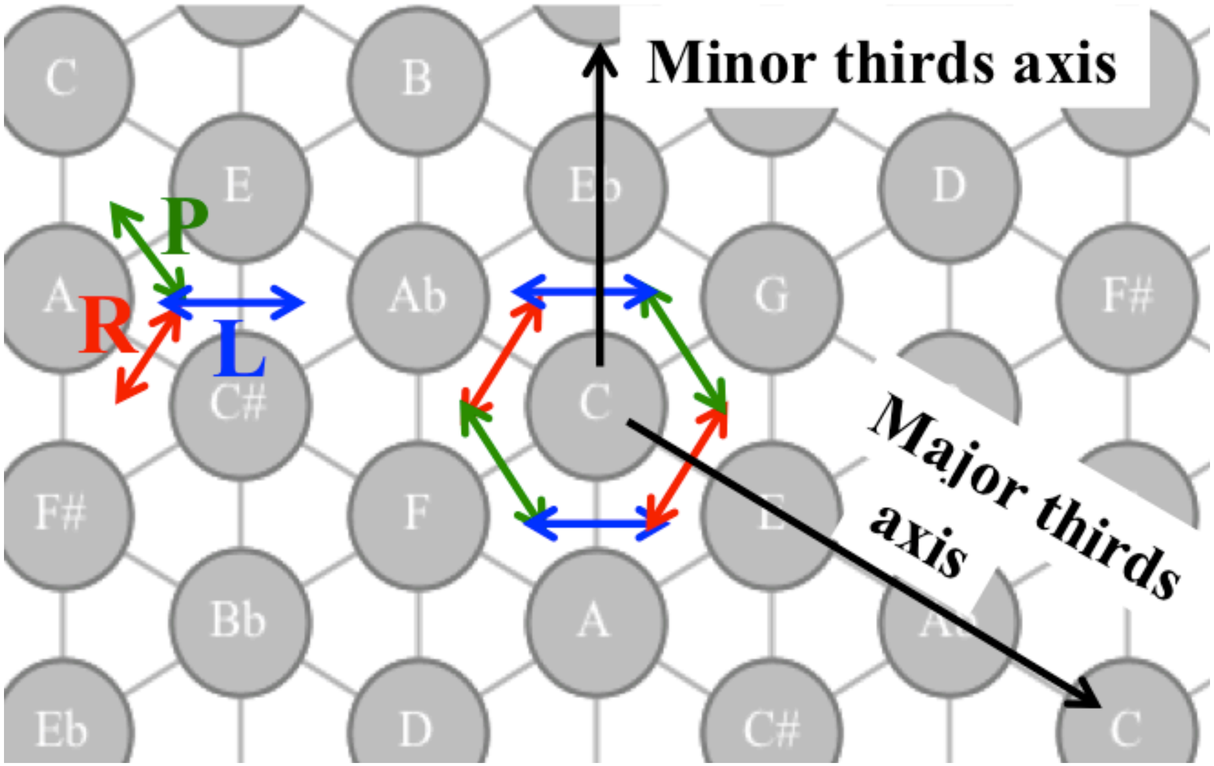
Soit $\tau = \{T_i \, ; \, i \in G\}$ le groupe des transpositions

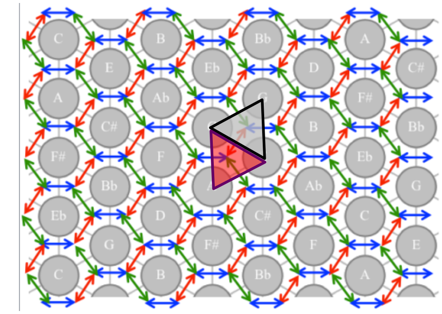
GIS = (S, G, int) $\Leftrightarrow$ $\tau \times S \rightarrow S$ telle que $(T_i, s) \rightarrow T_i(s)$ où int(s, $T_i(s)$) = i

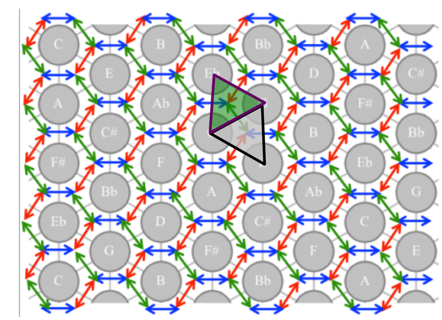
➔ Extension de la théorie transformationnelle aux groupoïdes et aux actions générales de groupoïdes (thèse J. Mandereau, 2011-2013)

➔ Liens avec les Systèmes Evolutifs à Mémoire (thèse G. Genuys, 2014-2017)

The *Tonnetz* as "Generalized Interval System"

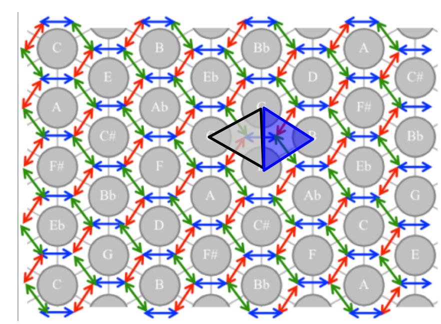




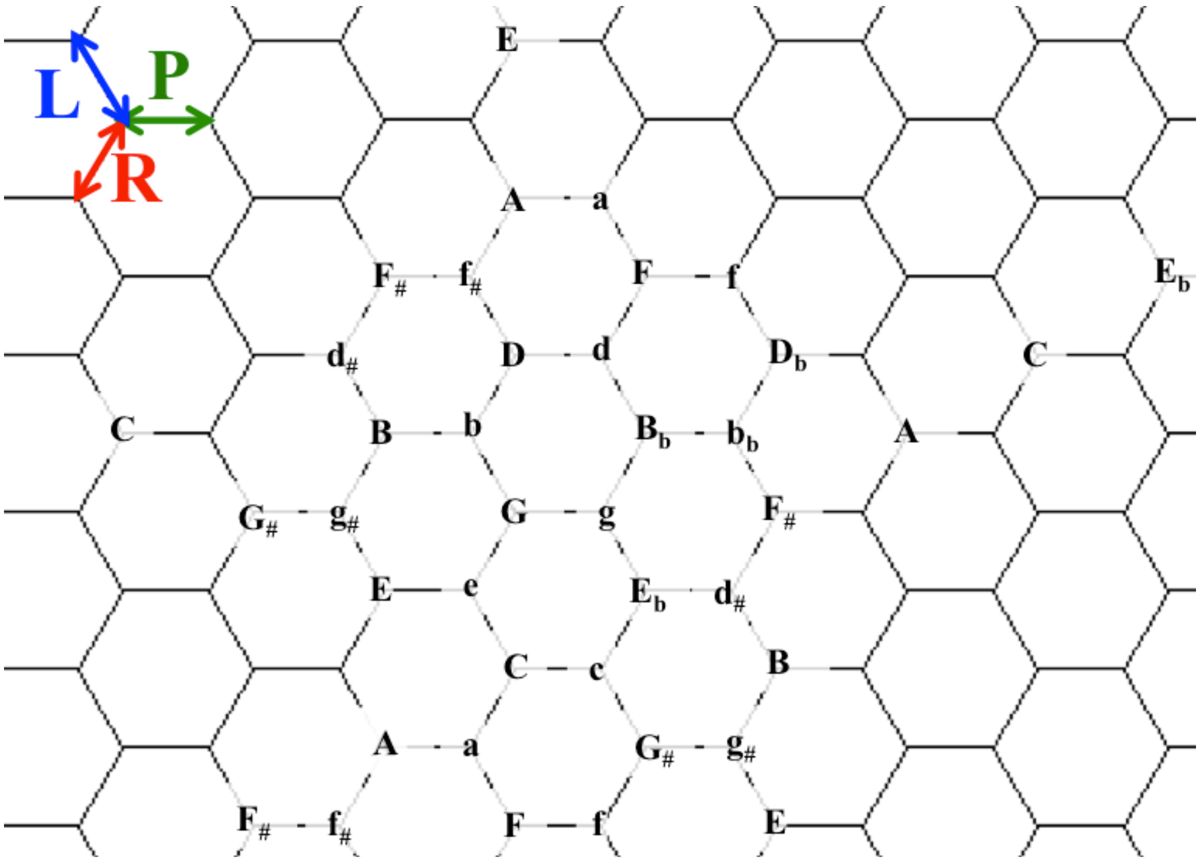


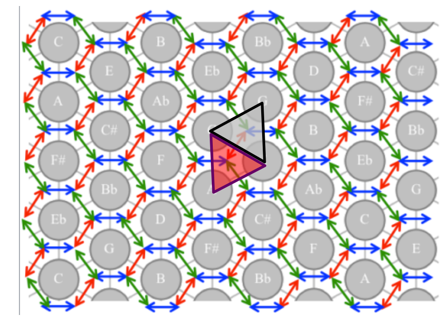
$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \; ; \; LRL = L(LR)^{-1} \rangle$

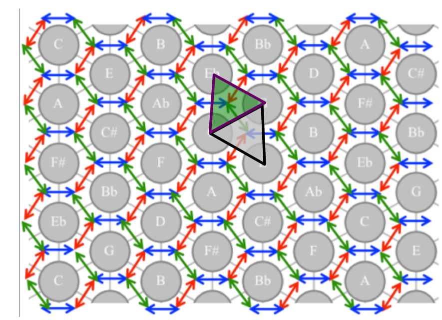
ρ acts in a simply transitive way on the set *S* of the 24 consonant triads



The *Tonnetz* as "Generalized Interval System"







$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \; ; \; LRL = L(LR)^{-1} \rangle$$

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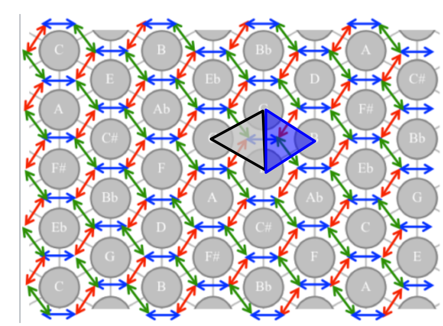
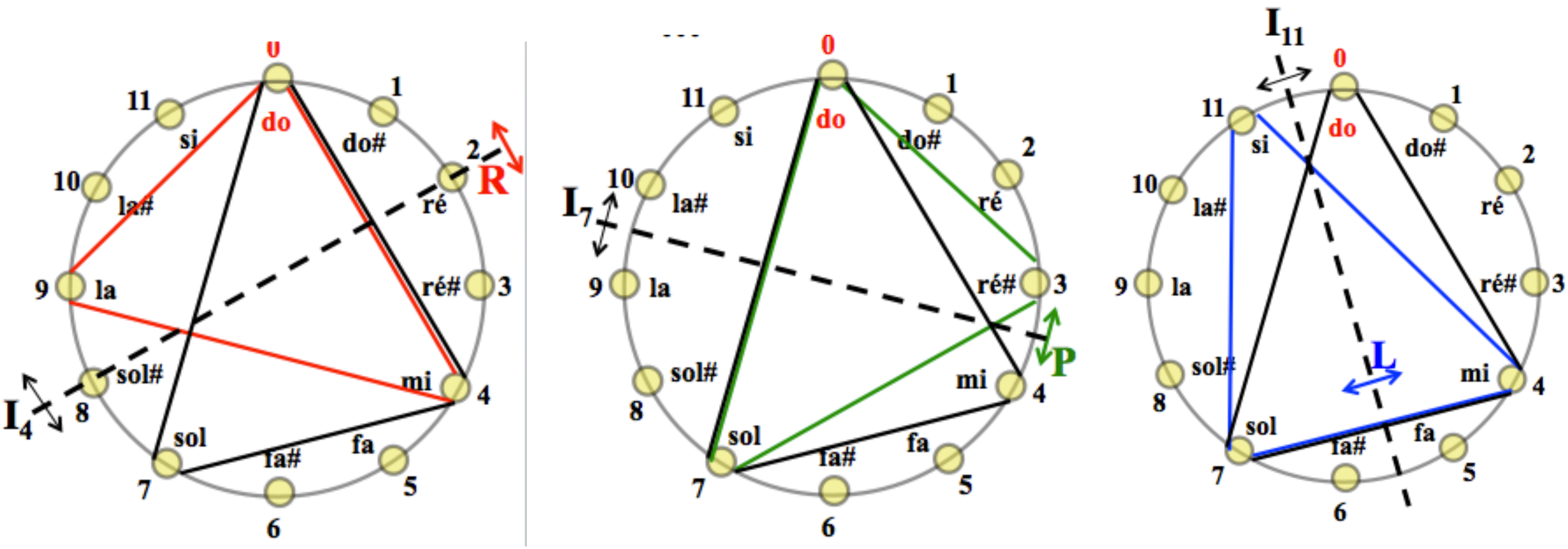


A different GIS structure on the same set *S*



$I_4 : x \to 4\text{-}x$ $I_7 : x \to 7\text{-}x$ $I_{11} : x \to 11\text{-}x$

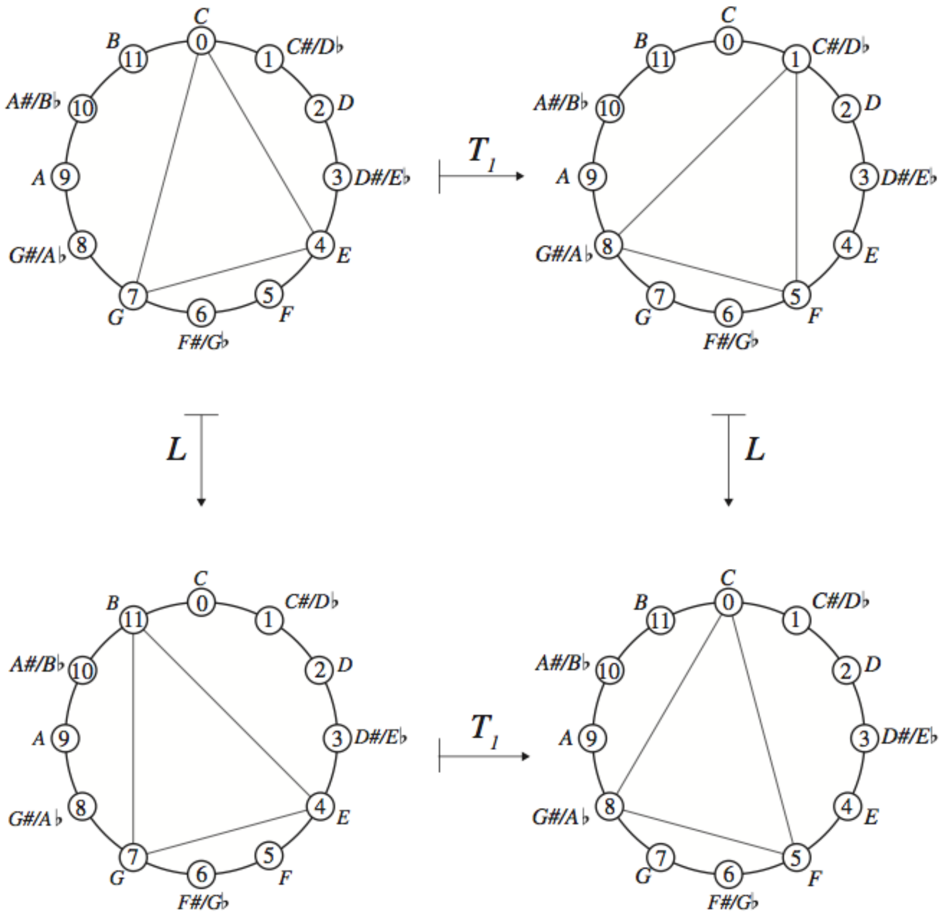
$$D_{12} = \langle I, T \mid I^2 = T^{12} = 1 \ ; \ ITI = I(IT)^{-1} \rangle$$

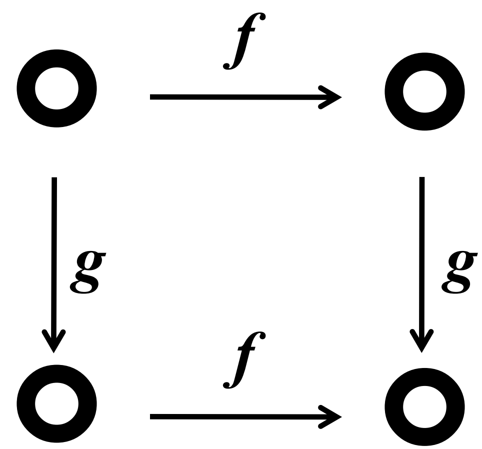
D_{12} acts in a simply transitive way on the set *S* of the 24 consonant triads

Two "dual" actions on the set of consonant triads

$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1\ ;\ LRL = L(LR)^{-1} \rangle$ $\leftrightarrow$ $D_{12} = \langle I, T \mid I^2 = T^{12} = 1\ ;\ ITI = I(IT)^{-1} \rangle$

$\Rightarrow$ **ρ is the centralizer of D_{12} (and conversely)**





Every diagram commutes

$\forall f \in D_{12}$

$\forall g \in \rho$

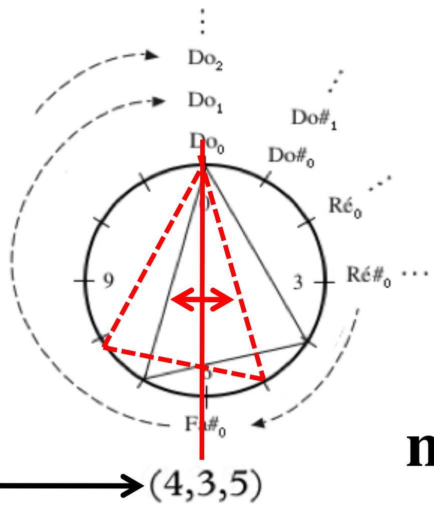
➔ **Alissa Crans, Tom Fiore and Ramon Satyendra, « Musical Actions of Dihedral Groups », *The American Mathematical Monthly*, Vol. 116 (2009), No. 6: 479 - 495**

The *Tonnetz* as a simplicial complex

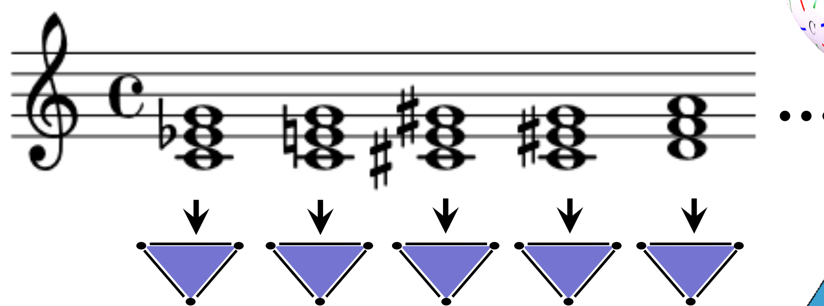
L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

Louis Bigo

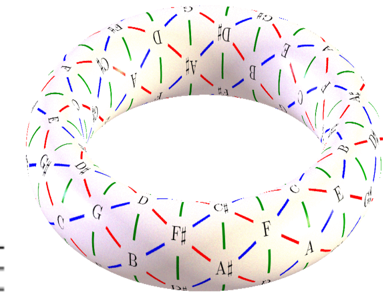
- Assembling chords related by some equivalence relation
  - Equivalence up to transposition/inversion:

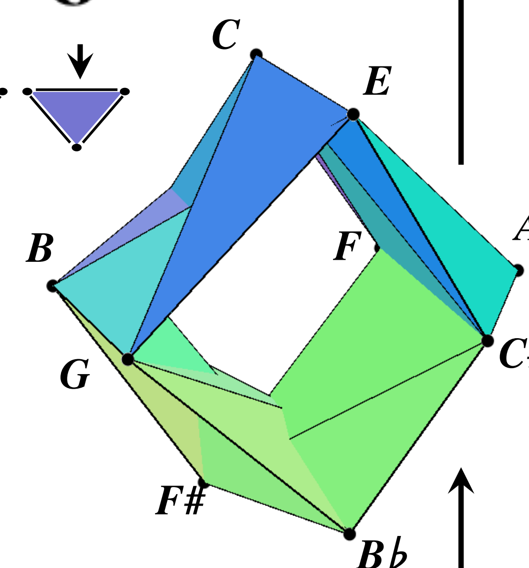


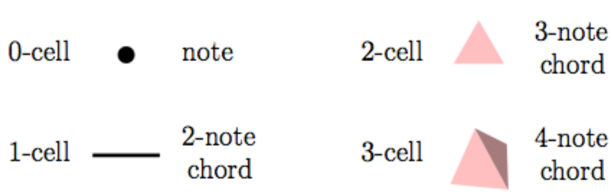
Intervallic structure

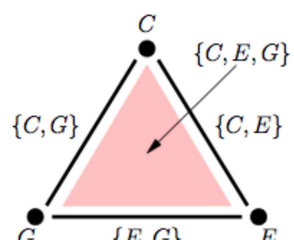


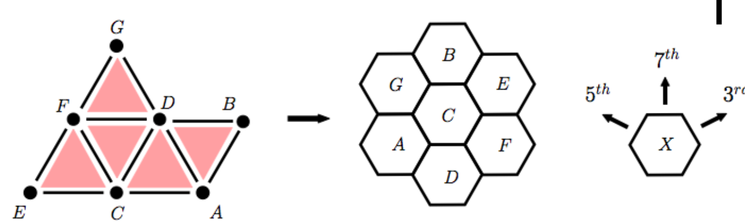
major/minor triads









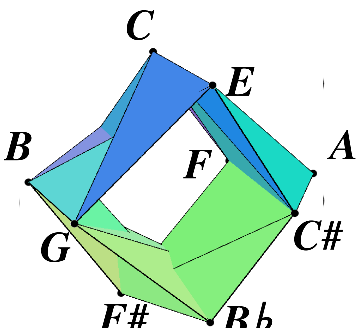


Classifying Chord Complexes

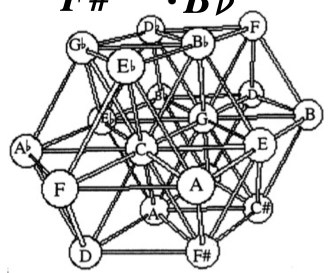
L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- Complexes enumeration in the chromatic system

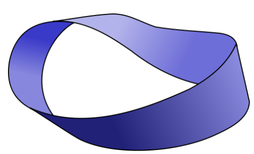
K_{TI}[3,4,5]
[Cohn – 1997]

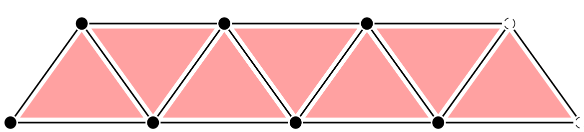


K_{TI}[2,3,3,4]
[Gollin - 1998]



K_{T}[2,2,3]
[Mazzola – 2002]



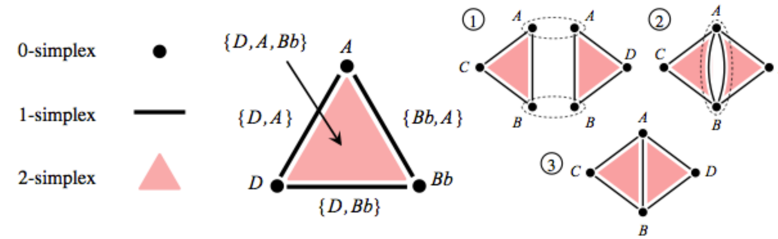


…

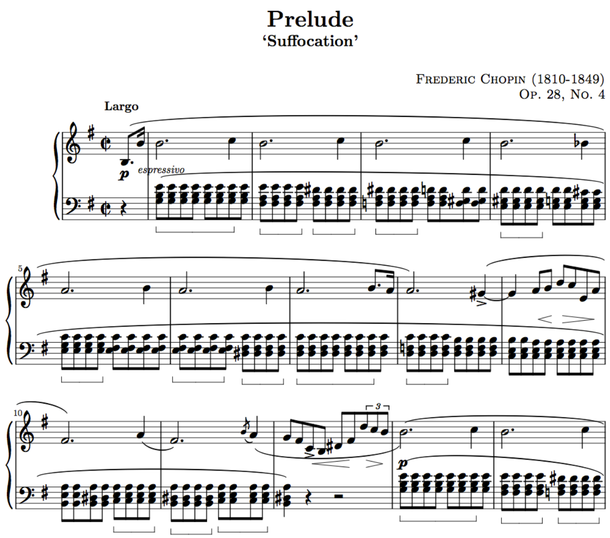
d	complexe	taille	b_n	p-v	χ
-	$\mathcal{K}_\emptyset$	0	0		0
0	$\mathcal{K}_{TI}[0]$	0	[0]		0
1	$\mathcal{K}_{TI}[1, 11]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[2, 10]$	12	[2, 2]		0
	$\mathcal{K}_{TI}[3, 9]$	12	[3, 3]		0
	$\mathcal{K}_{TI}[4, 8]$	12	[4, 4]		0
	$\mathcal{K}_{TI}[5, 7]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[6, 6]$	6	[6, 0]		6
2	$\mathcal{K}_{TI}[1, 1, 10]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[1, 2, 9]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1, 3, 8]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1, 4, 7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1, 5, 6]$	24	[1, 1, 6]		6
	$\mathcal{K}_{TI}[2, 2, 8]$	12	[2, 2, 0]		0
	$\mathcal{K}_{TI}[2, 3, 7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[2, 4, 6]$	24	[2, 2, 6]		6
	$\mathcal{K}_{TI}[2, 5, 5]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[3, 3, 6]$	12	[3, 0, 3]		6
	$\mathcal{K}_{TI}[3, 4, 5]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[4, 4, 4]$	4	[4, 0, 0]		4
	$\mathcal{K}_{TI}[1, 1, 1, 9]$	12	[1, 1, 0, 0]	x	0
	$\mathcal{K}_{TI}[1, 1, 2, 8]$	24	[1, 1, 12, 0]		12
	$\mathcal{K}_{TI}[1, 1, 3, 7]$	24	[1, 2, 13, 0]		12
	$\mathcal{K}_{TI}[1, 1, 4, 6]$	24	[1, 1, 18, 0]		18
	$\mathcal{K}_{TI}[1, 1, 5, 5]$	12	[1, 1, 6, 0]		6

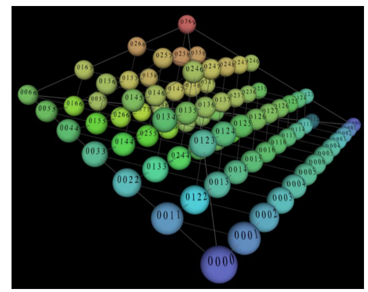
Towards a topological signature of a musical piece
A structural approach in Music Information Retrieval

The simplices and their self-assembly



The score



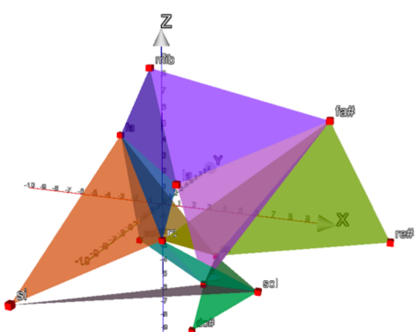


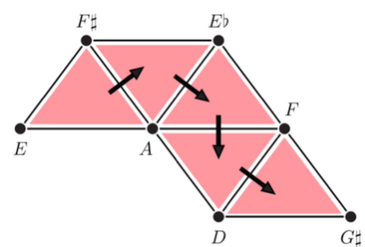
Orbifold-based Voice Leading (D. Tymoczko)

Topological signature?

Score reduction

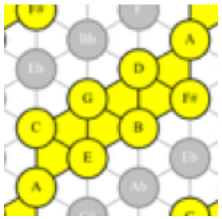
The simplicial complex generated by the piece



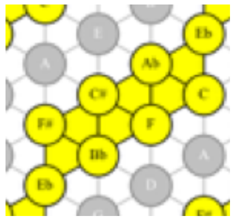


A specific trajectory in the complex

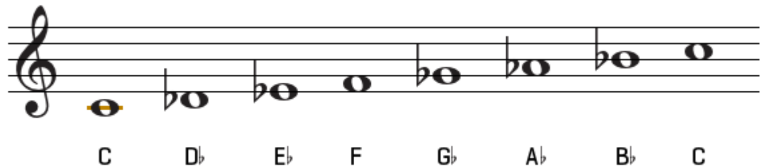
Towards a physically-based anisotropic *Tonnetz*

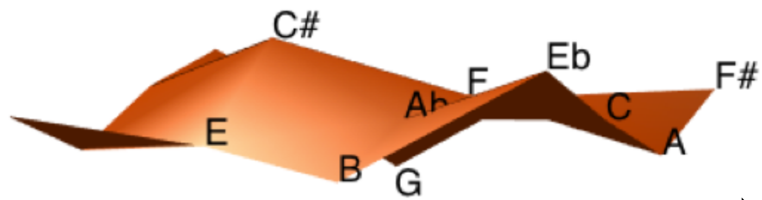


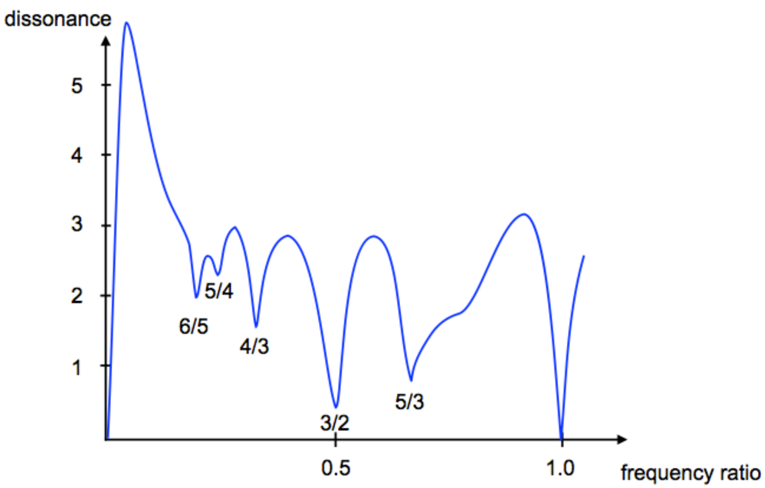
Ionian mode

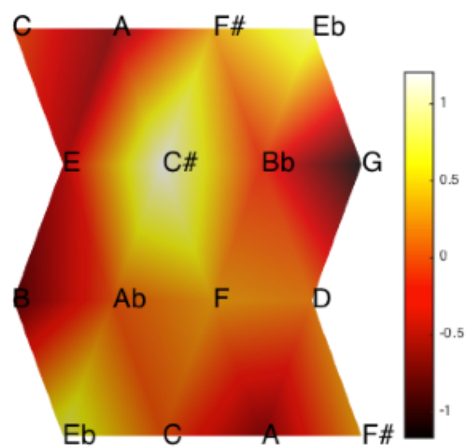


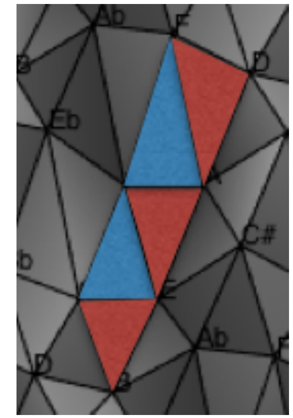
Locrian mode





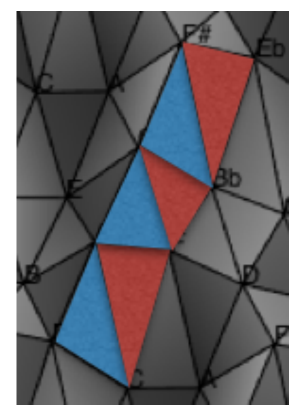






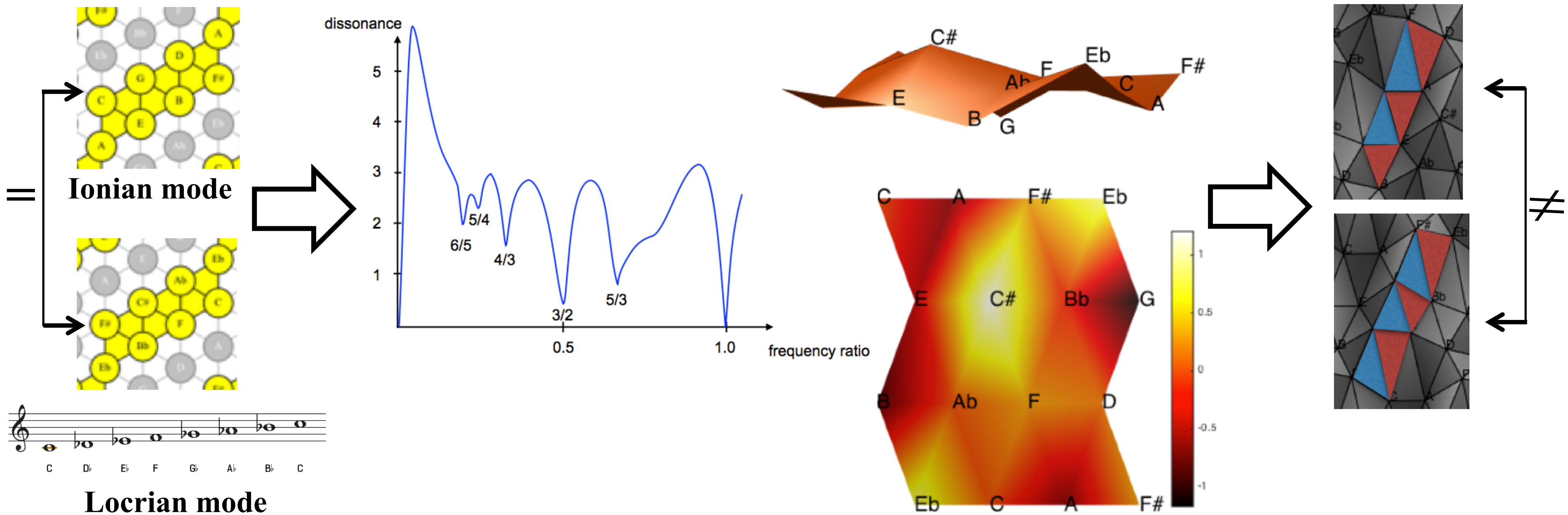
Ionian mode

≠



Locrian mode

Signal/Symbolic articulation in MIR



M. Bergomi, *Dynamics and Algebraic Topology Tools for Music in the Symbolic/Signal interaction domain*, ongoing PhD

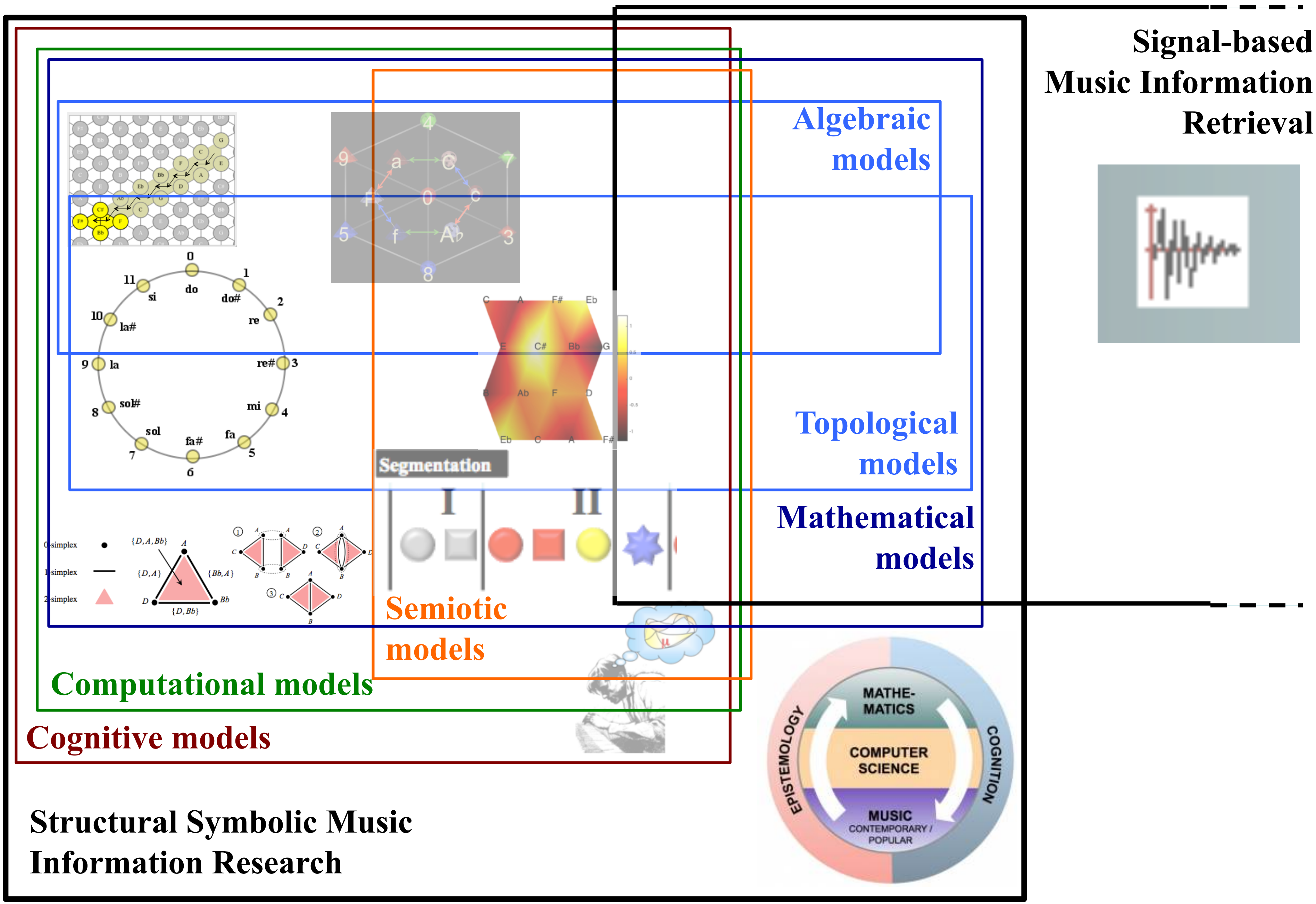
➔ **Towards a geometric dynamic modeling of a musical piece ?**

SPACE ⇄ **MUSIC**

?

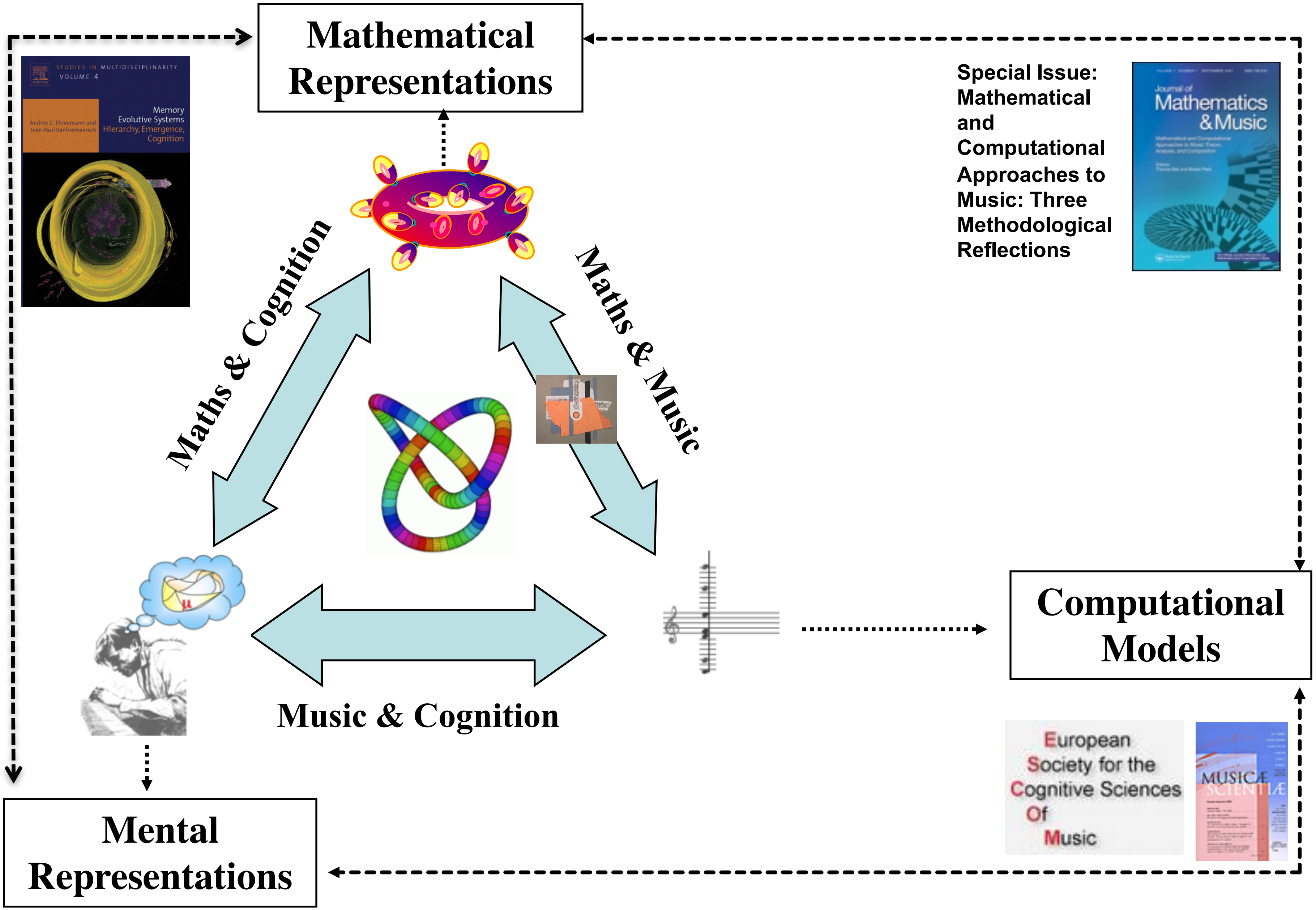
➔ **Towards a topological signature of a musical piece?**

The SMIR Project: Structural Music Information Research

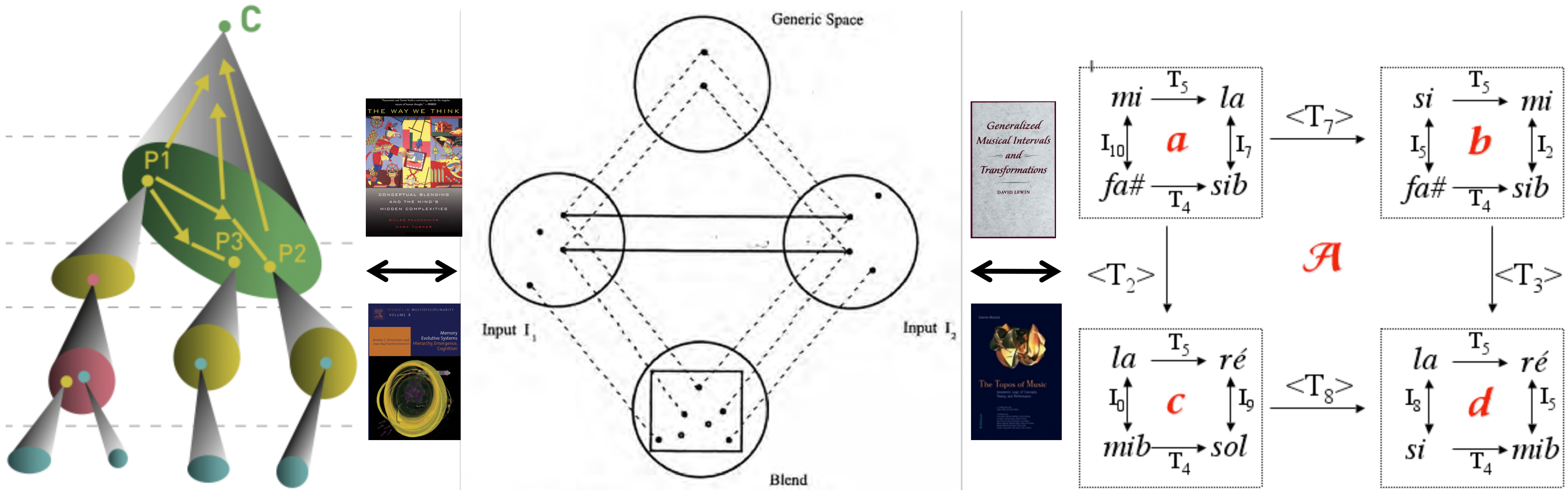


Bridging the gap: mathematical and cognitive approaches

http://recherche.ircam.fr/equipes/repmus/mamux/Cognition.html



Towards a categorical explanation of music perception?



(a) **(b)** **(c)**

(a) Processus de « colimite » à la base des systèmes évolutifs à mémoire (Ehresmann et Vanbremeersch, 2007) ; (b) réseau minimal pour le « blending conceptuel » (Fauconnier & Turner, 2002) et exemple de Klumpenhouwer Network (ou *K-net*).

> « La **théorie des catégories** est une théorie des constructions mathématiques, qui est macroscopique, et procède d'étage en étage. Elle est un bel exemple d'**abstraction réfléchissante**, cette dernière reprenant elle-même un principe constructeur présent dès le stade sensori-moteur. Le **style catégoriel** qui est ainsi à l'image d'un aspect important de la **genèse des facultés cognitives**, est un style adéquat à la description de cette genèse »

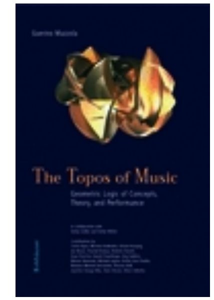
J. Piaget

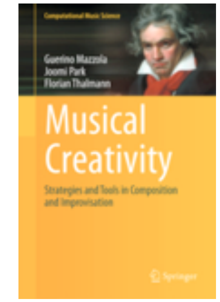
Jean Piaget, Gil Henriques et Edgar Ascher, *Morphismes et Catégories. Comparer et transformer*, 1990

Towards a categorical theory of creativity (in music, cognition and discourse)

Abstract

This article presents a first attempt at establishing a **category-theoretical model of creative processes**. The model, which is applied to musical creativity, discourse theory, and cognition, suggests the relevance of the notion of "colimit" as a unifying construction in the three domains as well as the central role played by the Yoneda Lemma in the categorical formalization of creative processes.





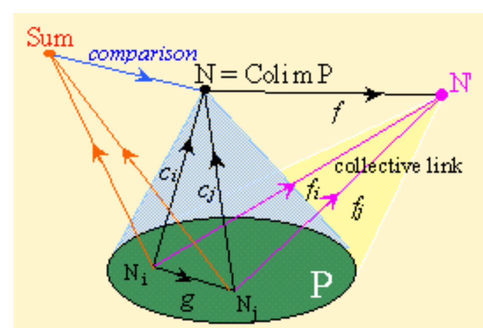
MAMUPHI

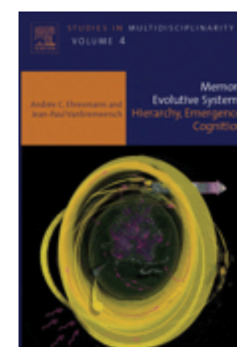
Séminaire MaMux

MCM 2013

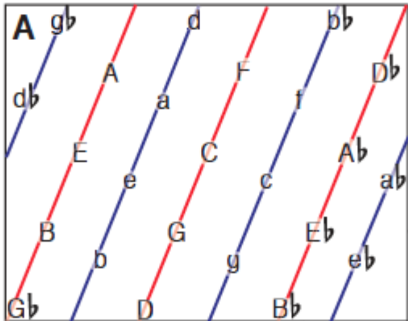
MATHEMATICS AND COMPUTATION IN MUSIC

Andreatta M., A. Ehresmann, R. Guitart, G. Mazzola, « Towards a categorical theory of creativity », Fourth International Conference, MCM 2013, McGill University, Montreal, June 12-14, 2013, Springer, 2013.





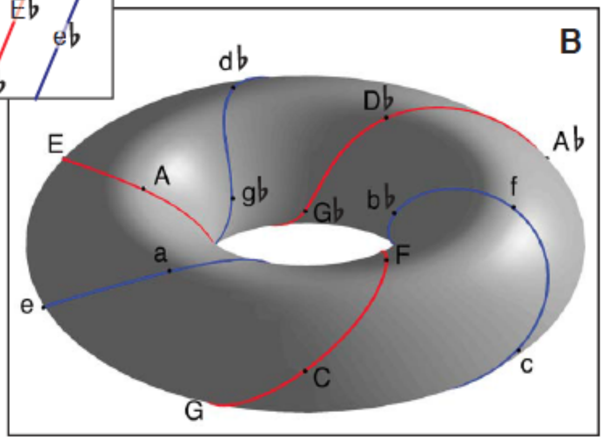
The neuronal foundation of the *Tonnetz*



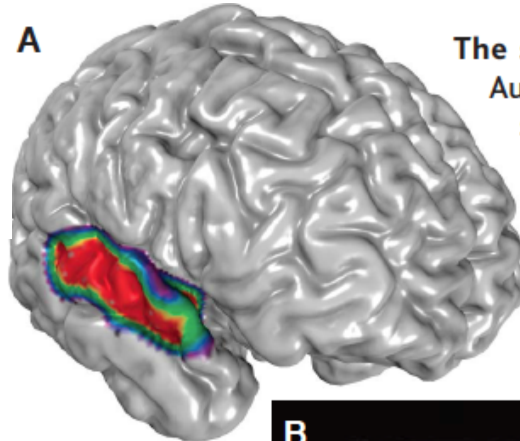
PERSPECTIVES: NEUROSCIENCE

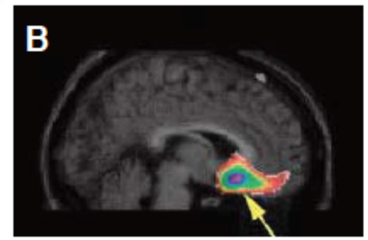
Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl

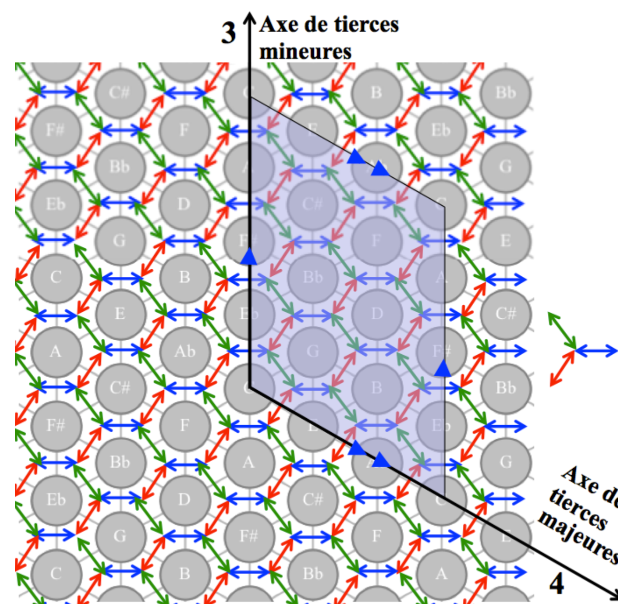


Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). **(B)** Musical keys as points on the surface of a torus.





The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. **(B)** Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

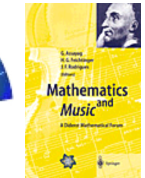


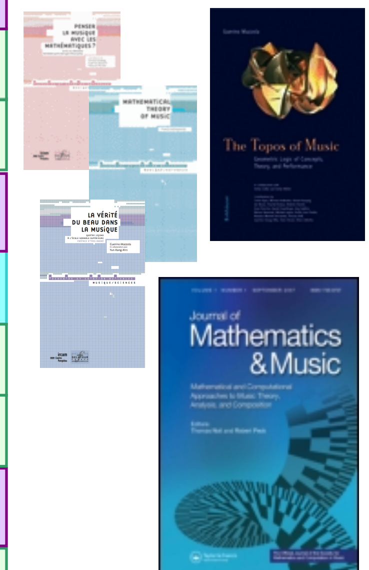
Acotto E. et M. Andreatta (2012), « Between Mind and Mathematics. Different Kinds of Computational Representations of Music », *Mathematics and Social Sciences*, n° 199, 2012(3), p. 9-26.

EUROPEAN MATHEMATICAL PSYCHOLOGY GROUP

Towards an institutionalisation of mathemusical research

- 1999: 4^{e} Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (G. Assayag, H.G. Feichtinger, J.F. Rodrigues, Springer, 2001)
- 2000-2001: *MaMuPhi Seminar, Penser la musique avec les mathématiques ?* (Assayag, Mazzola, Nicolas éds., Coll. ‘Musique/Sciences’, Ircam/Delatour, 2006)
- 2003: *The Topos of Music* (G. Mazzola et al.)
- 2001-…: *MaMuX* Seminar at Ircam
- 2004-…: *mamuphi* Seminar (Ens/Ircam)
- 2006: Collection ‘Musique/Sciences’ (Ircam/Delatour France)
- 2007: Journal of Mathematics and Music (Taylor & Francis) and SMCM
- 2007: First MCM 2007 (Berlin) and Proceedings by Springer
- 2007-…: AMS Special Session on Mathematical Techniques in Musical Analysis
- 2009: *Computational Music Science* (eds: G. Mazzola, M. Andreatta, Springer)
- 2009: MCM 2009 (Yale University) and Proceedings by Springer
- 2010: Mathematics Subject Classification : 00A65 Mathematics and music
- 2011: MCM 2011 (Ircam, 15-17 June 2011) and Proceedings LNCS Springer
- 2013: MCM 2013 (McGill University, Canada, 12-14 June 2013) - Springer
- 2015: MCM 2015 (Queen Mary University, Londres, 22-25 June 2013) - Springer
- 2017: MCM 2017 (UNAM, Mexico City, 26-29 June 2017)





Société Mathématique de France