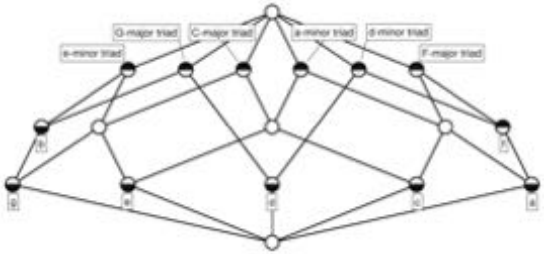
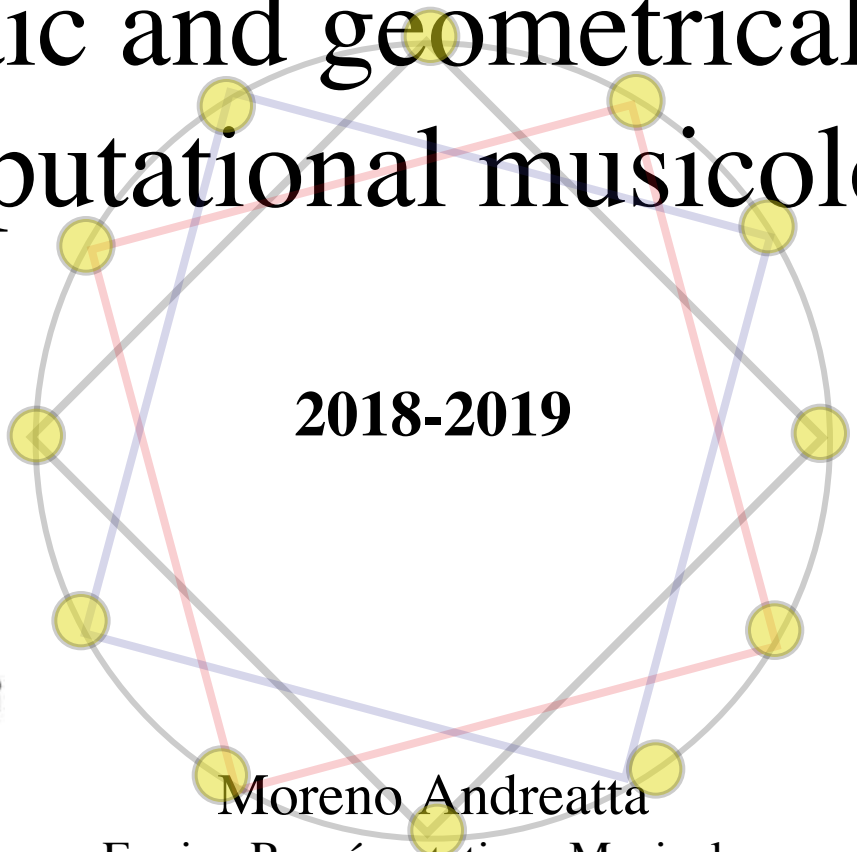
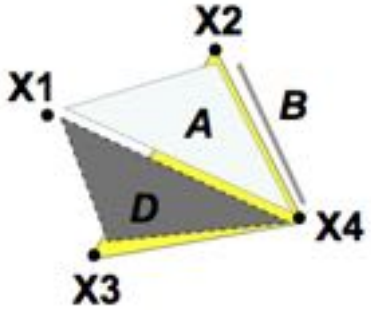
PARCOURS DU MASTER SCIENCES ET TECHNOLOGIES
UPMC • IRCAM • TELECOM PARISTECH

Algebraic and geometrical models in computational musicology (II)

2018-2019







Moreno Andreatta
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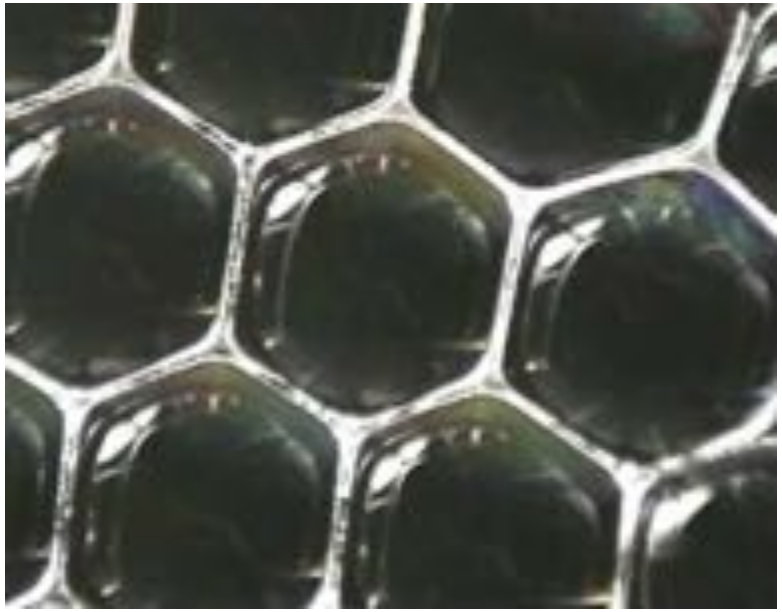
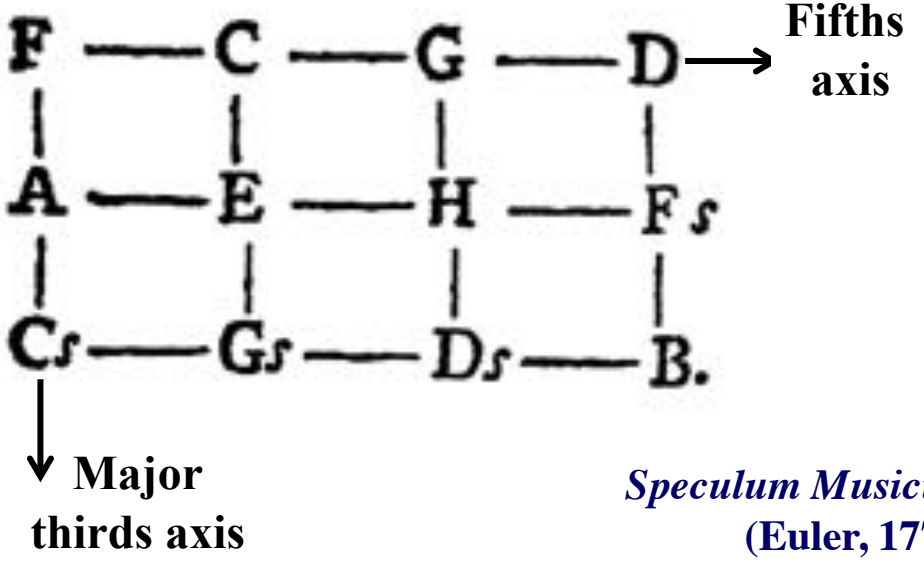
http://www.ircam.fr/repmus.html

ircam Centre Pompidou
cnrs
SORBONNE UNIVERSITÉ
IRMA Institut de Recherche Mathématique Avancée
USIAS Université de Strasbourg Institut d'Études Avancées

The *Tonnetz*
(or 'honeycomb' hexagonal tiling)





Speculum Musicum
(Euler, 1773)

The *Tonnetz*
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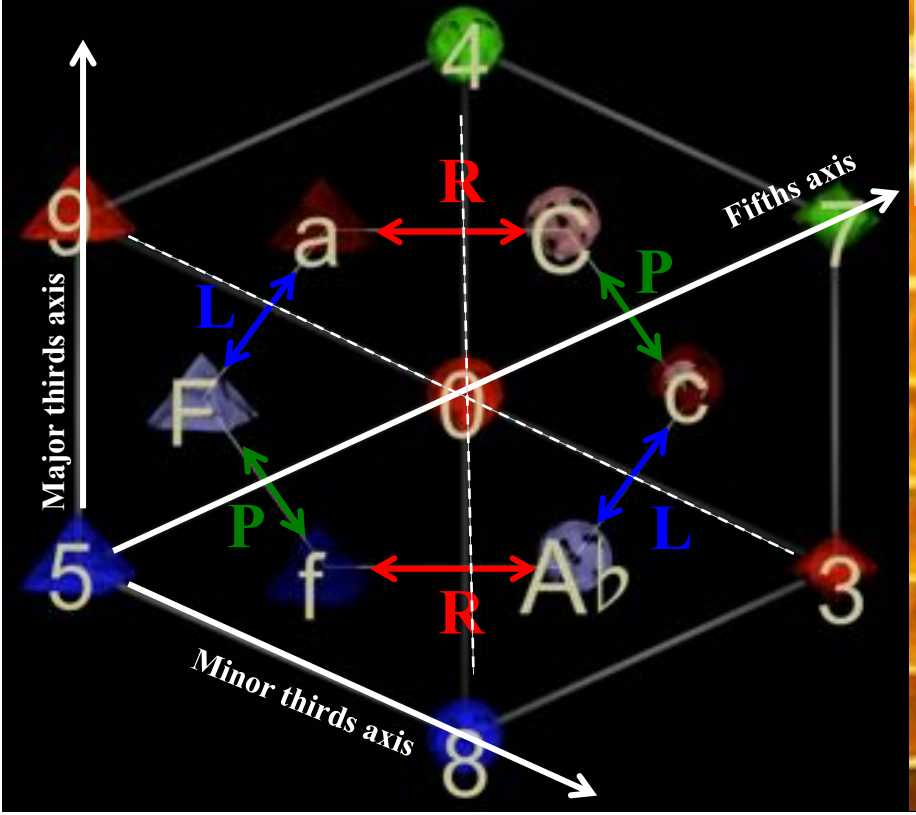

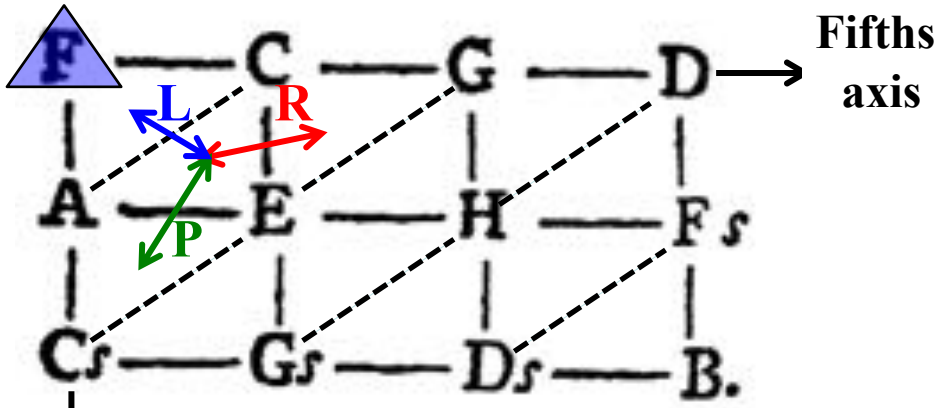

Speculum Musicum (Euler, 1773)

The *Tonnetz*
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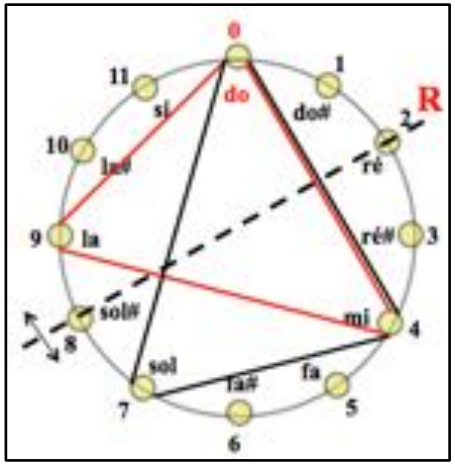
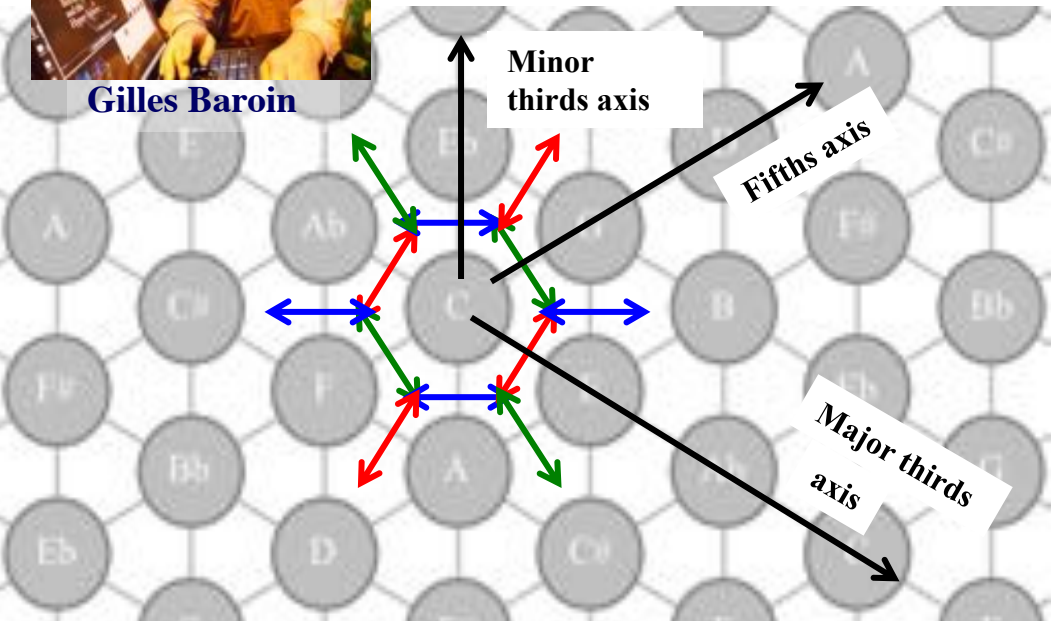
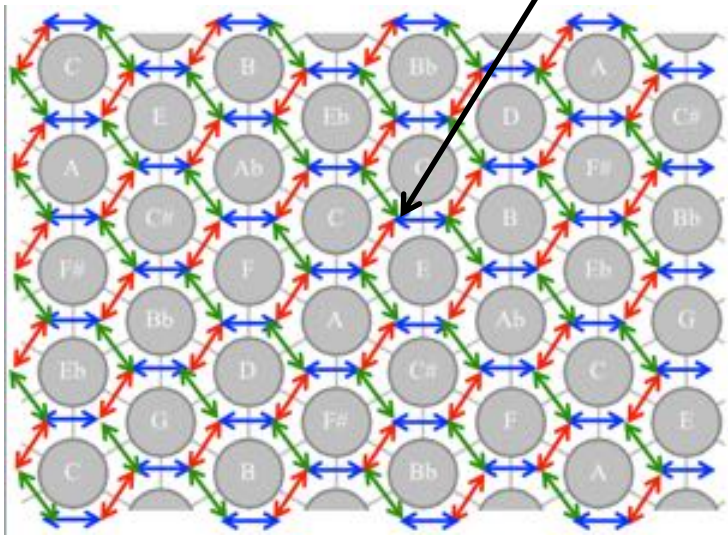
Gilles Baroin

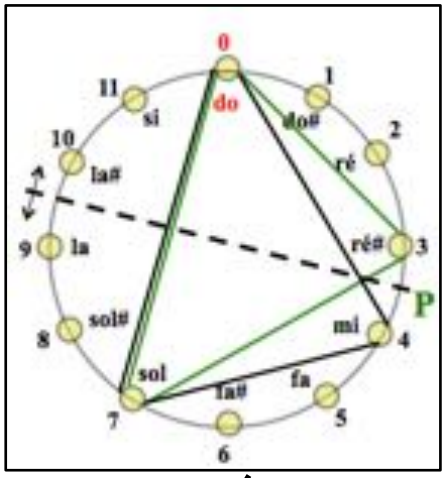


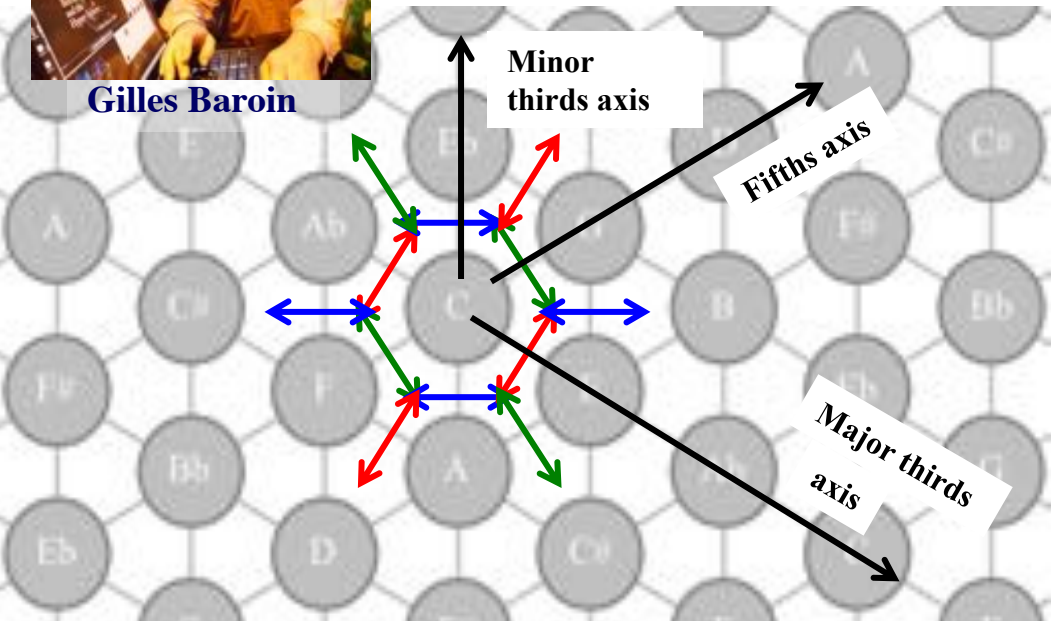


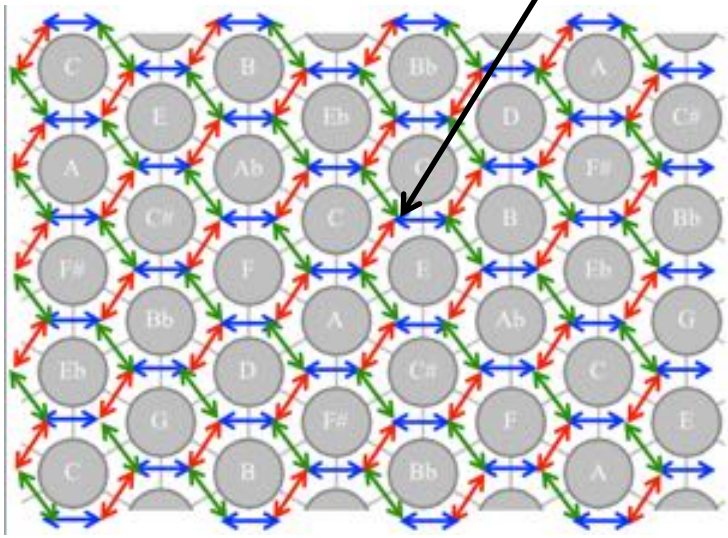


The *Tonnetz*
(or hexagonal tiling honeycomb)

Gilles Baroin







The *Tonnetz*
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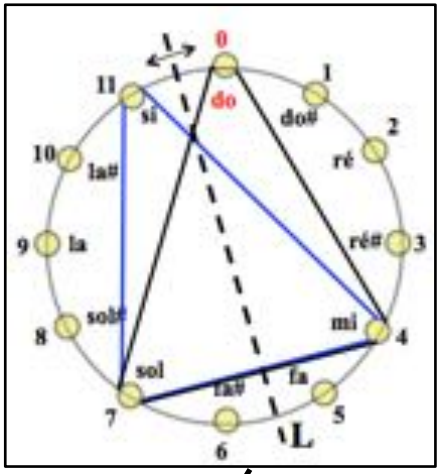
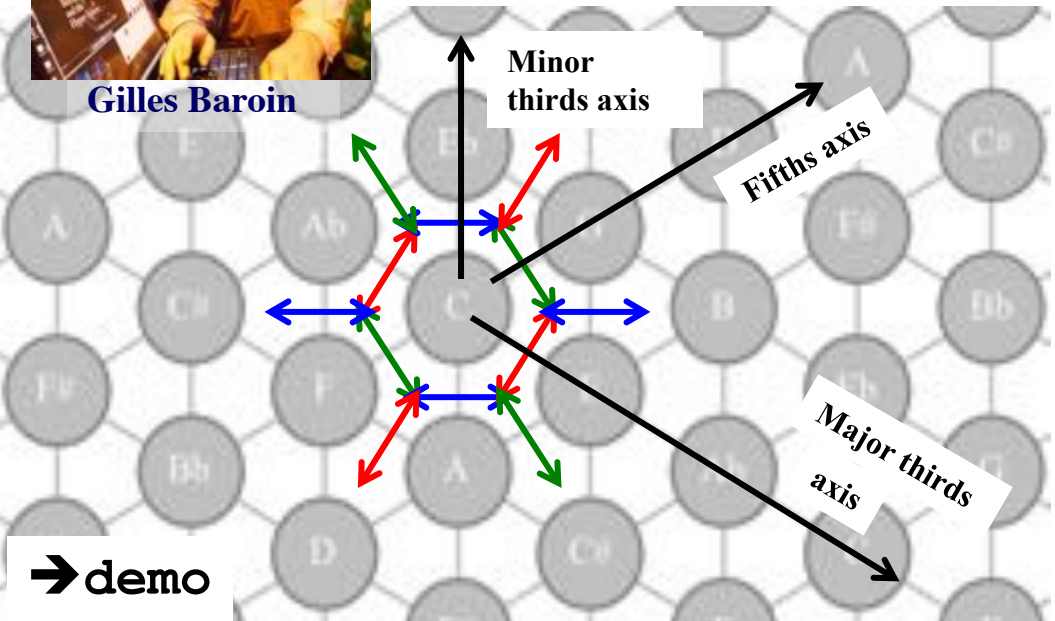
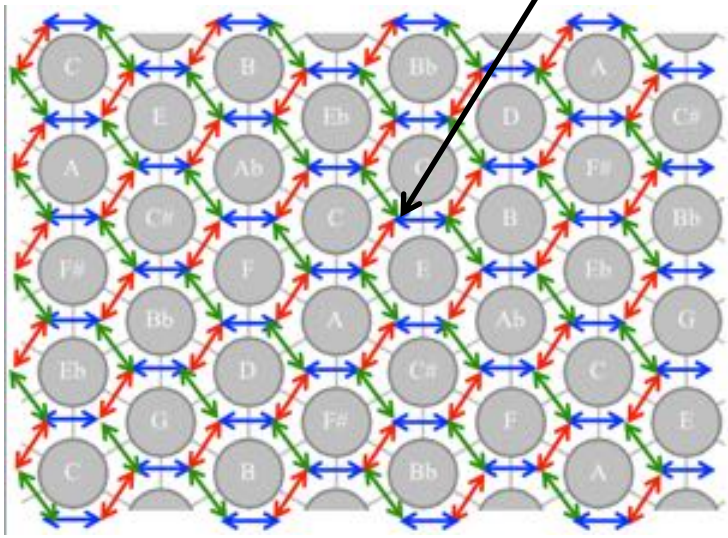
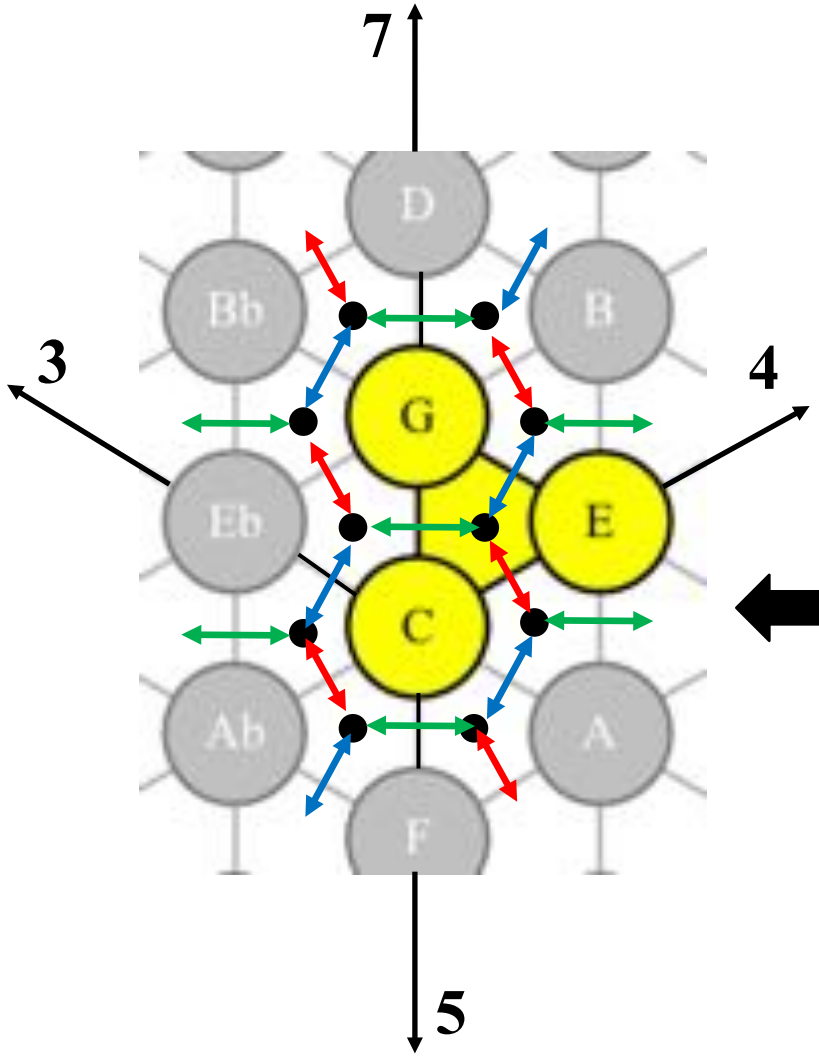
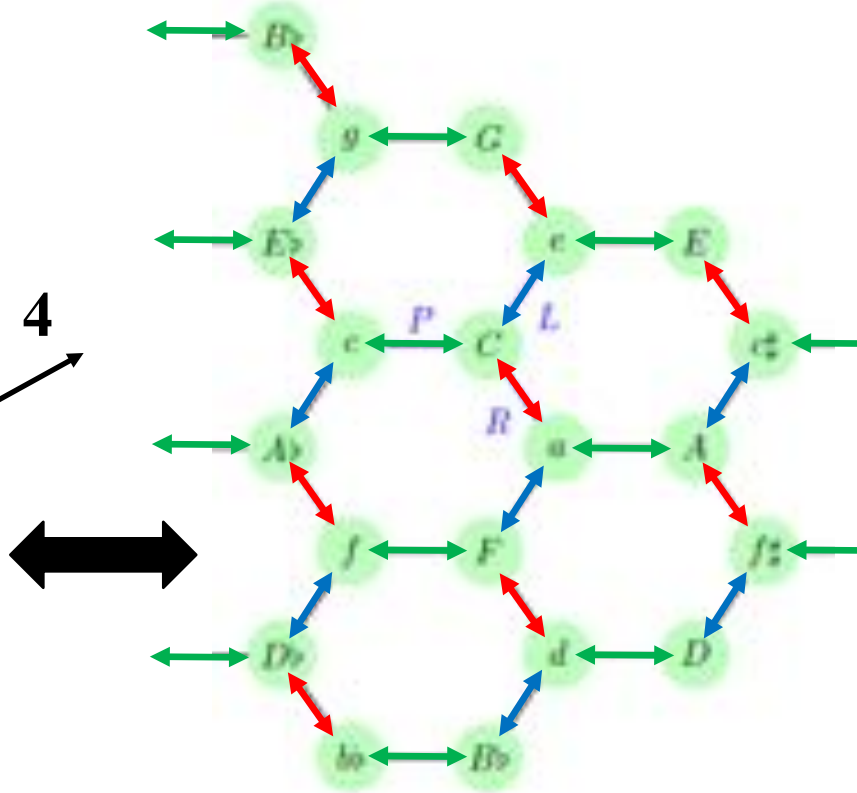






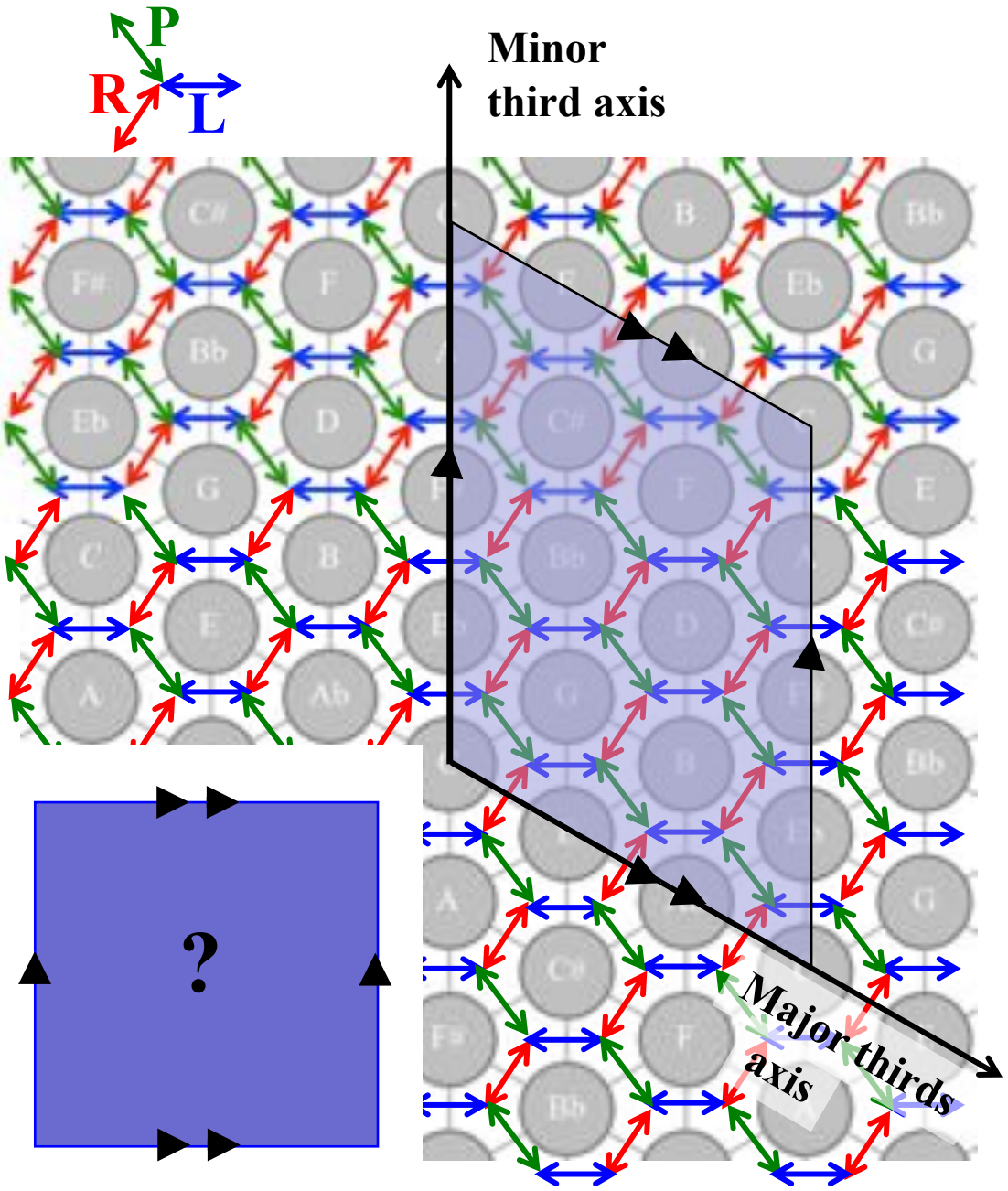
From the Tonnetz to the dual one (and vice-versa)

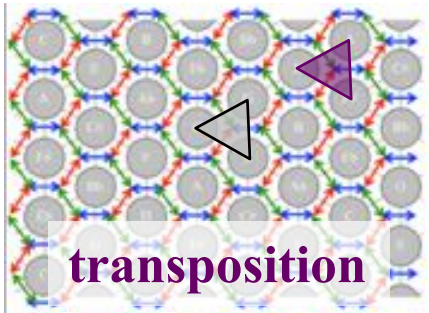


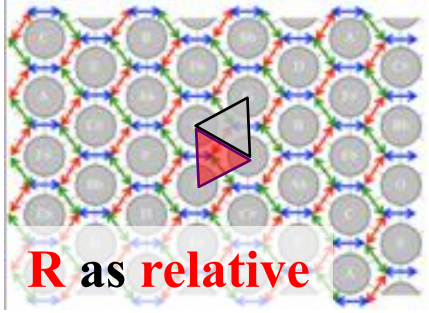


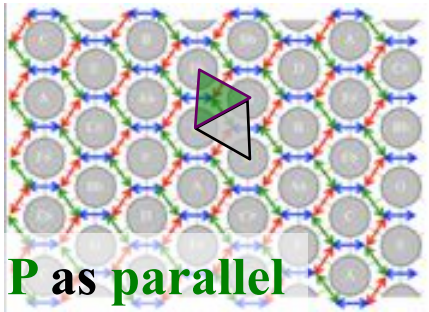
[Sonia Cannas, 2018]

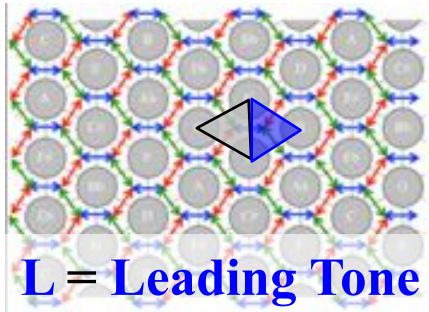
The *Tonnetz*, its symmetries and its topological structure

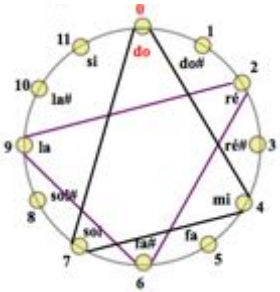


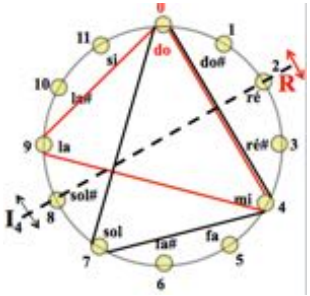


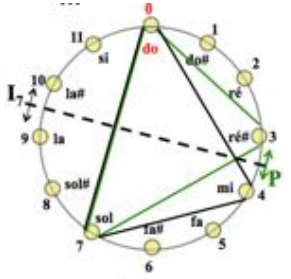


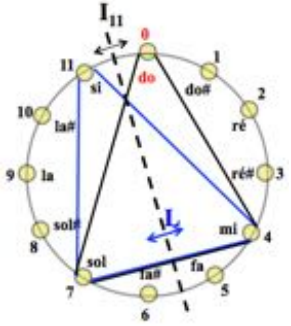












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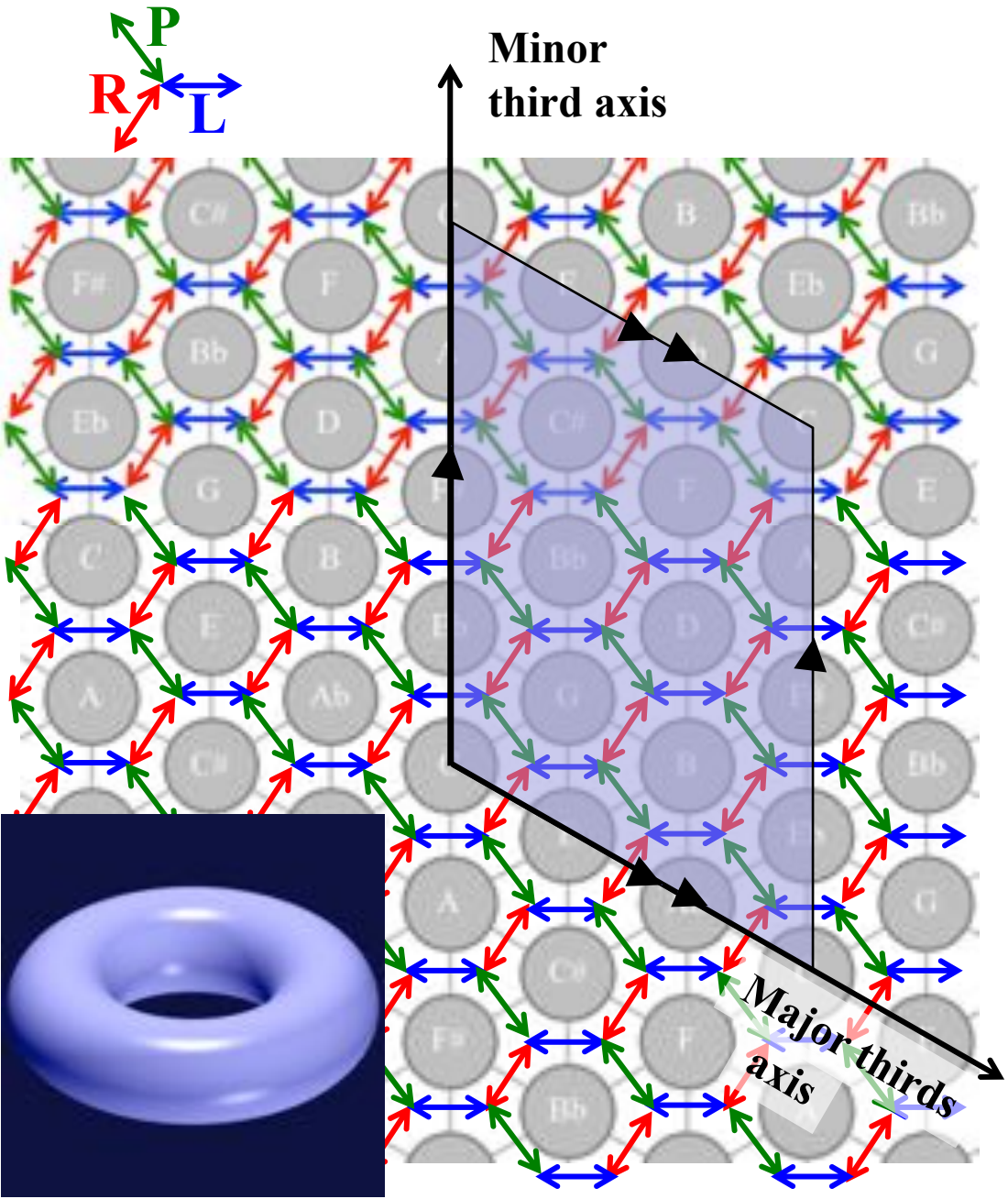

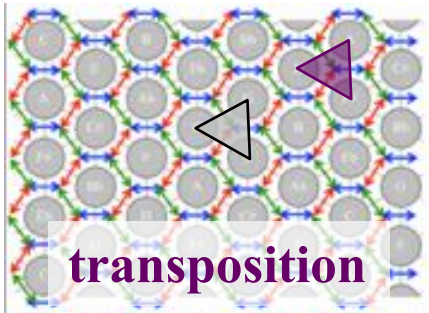

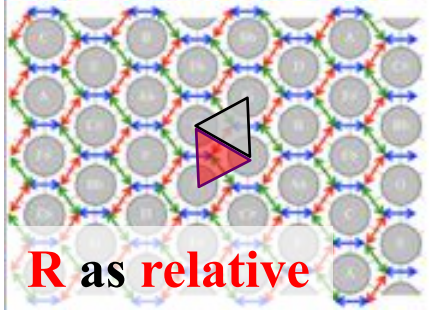

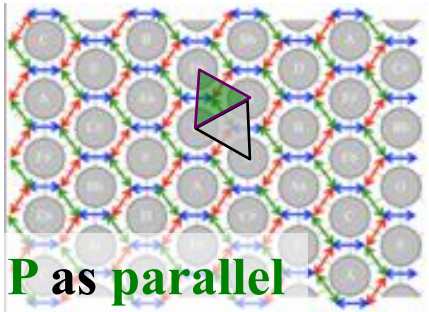

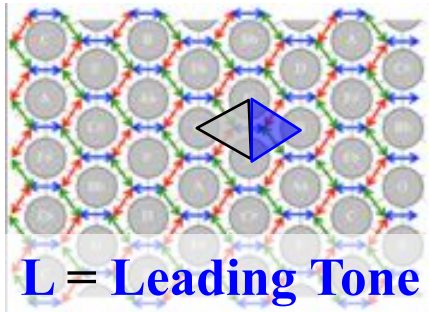

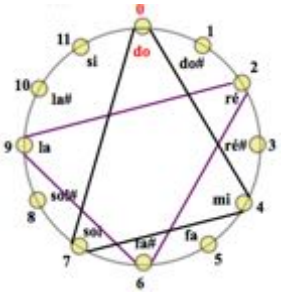

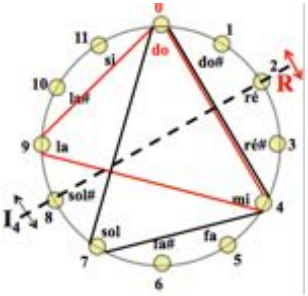

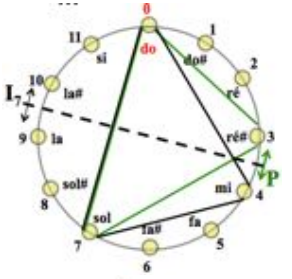

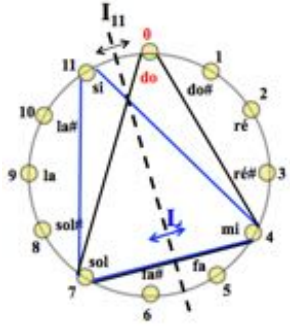

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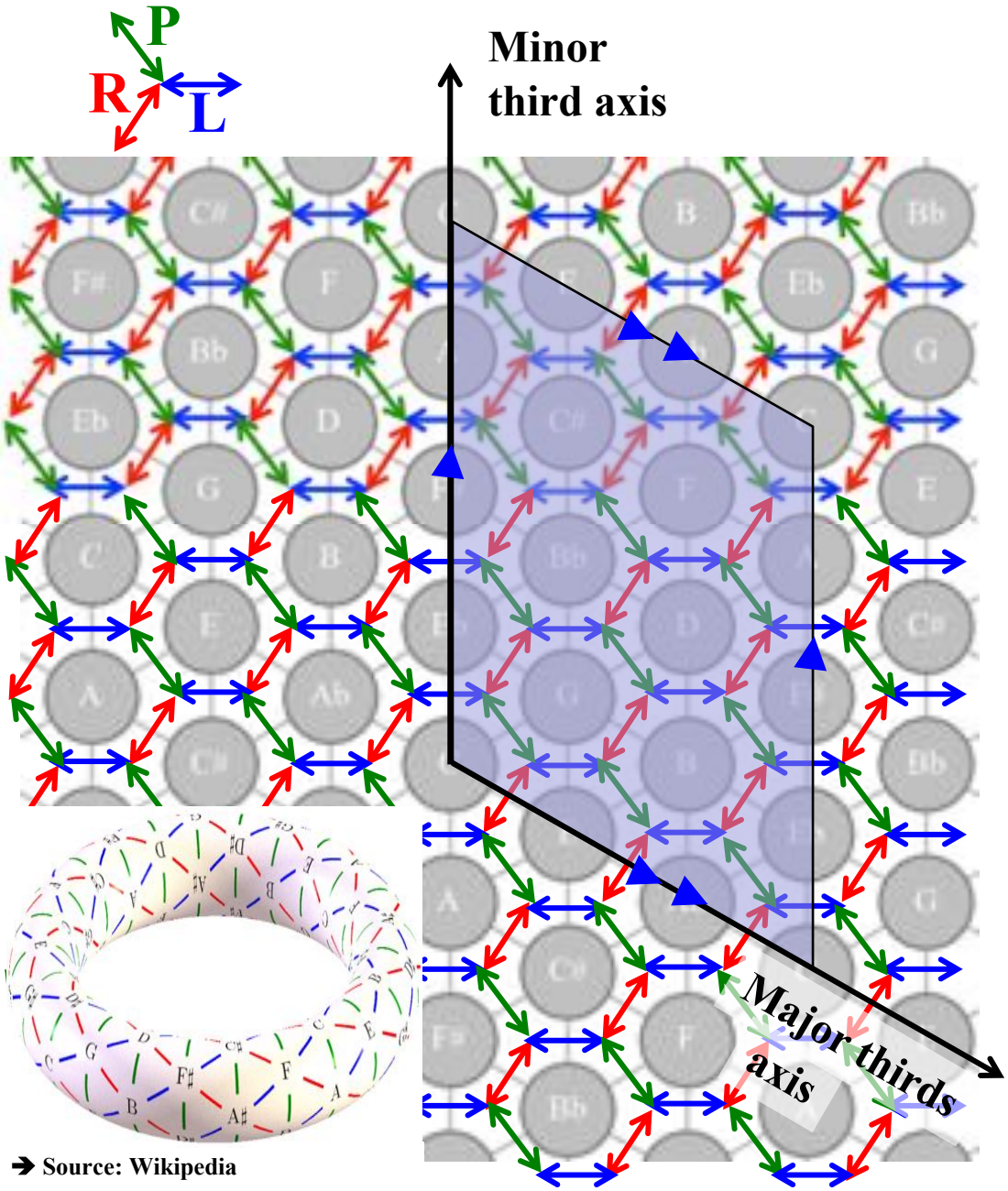
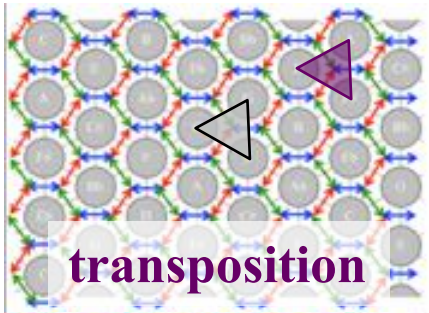
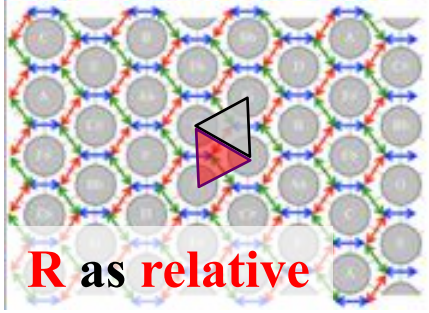
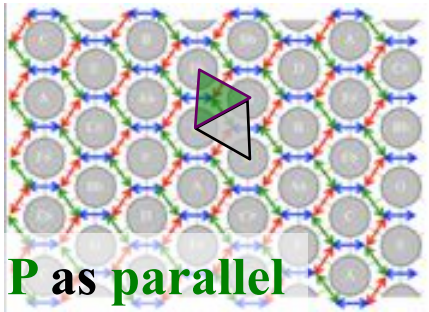
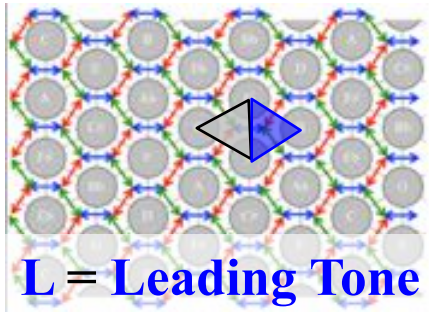


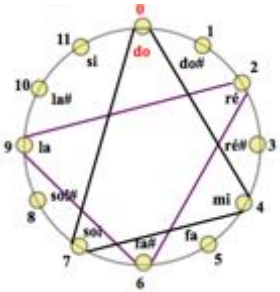
➔ Source: Wikipedia

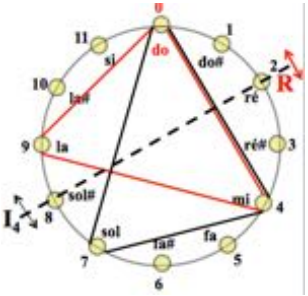


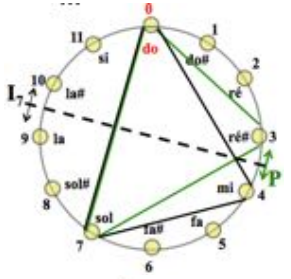


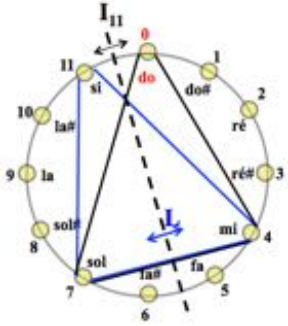










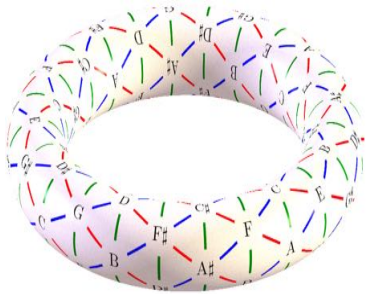


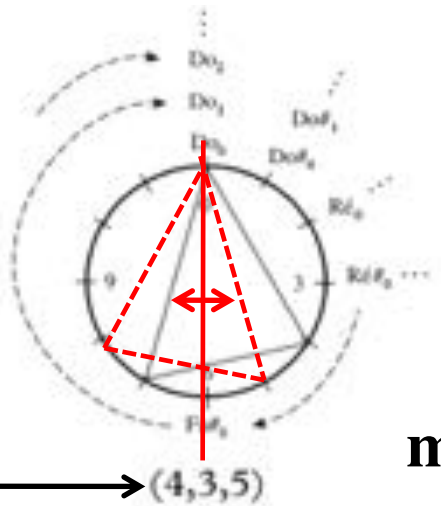
Louis Bigo

The *Tonnetz* as a simplicial complex

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

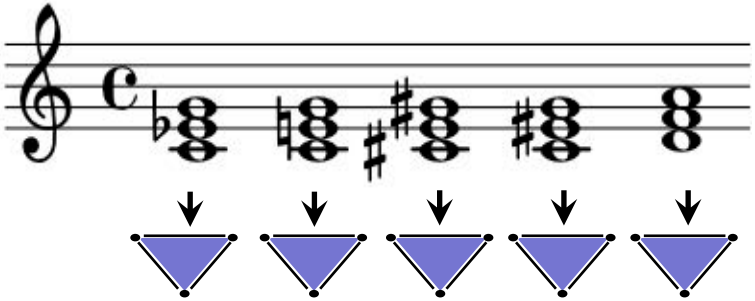
- Assembling chords related by some equivalence relation
  - Equivalence up to transposition/inversion:



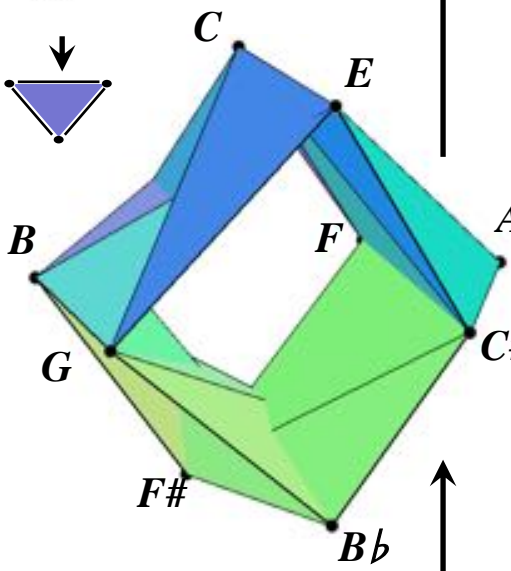


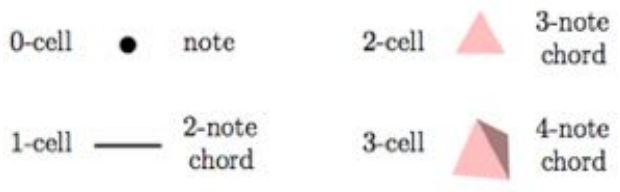
Intervallic structure

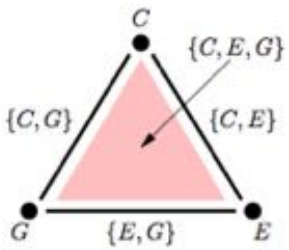
(4,3,5)

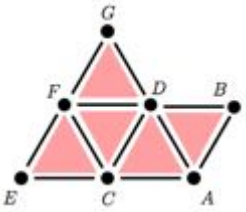


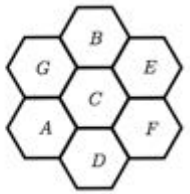
major/minor triads

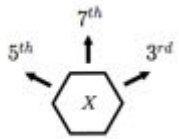










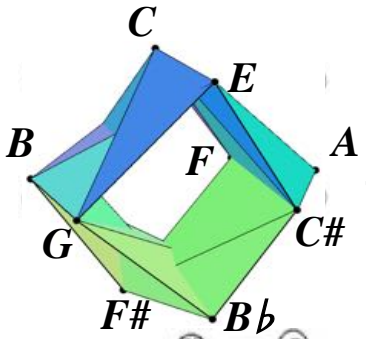


Classifying Chord Complexes

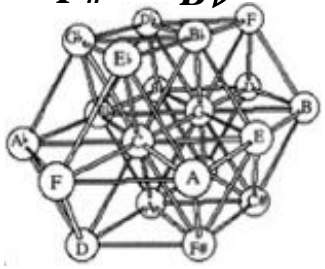
L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- Complexes enumeration in the chromatic system

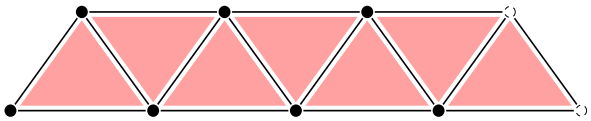
K_{TI}[3,4,5]
[Cohn – 1997]



K_{TI}[2,3,3,4]
[Gollin - 1998]



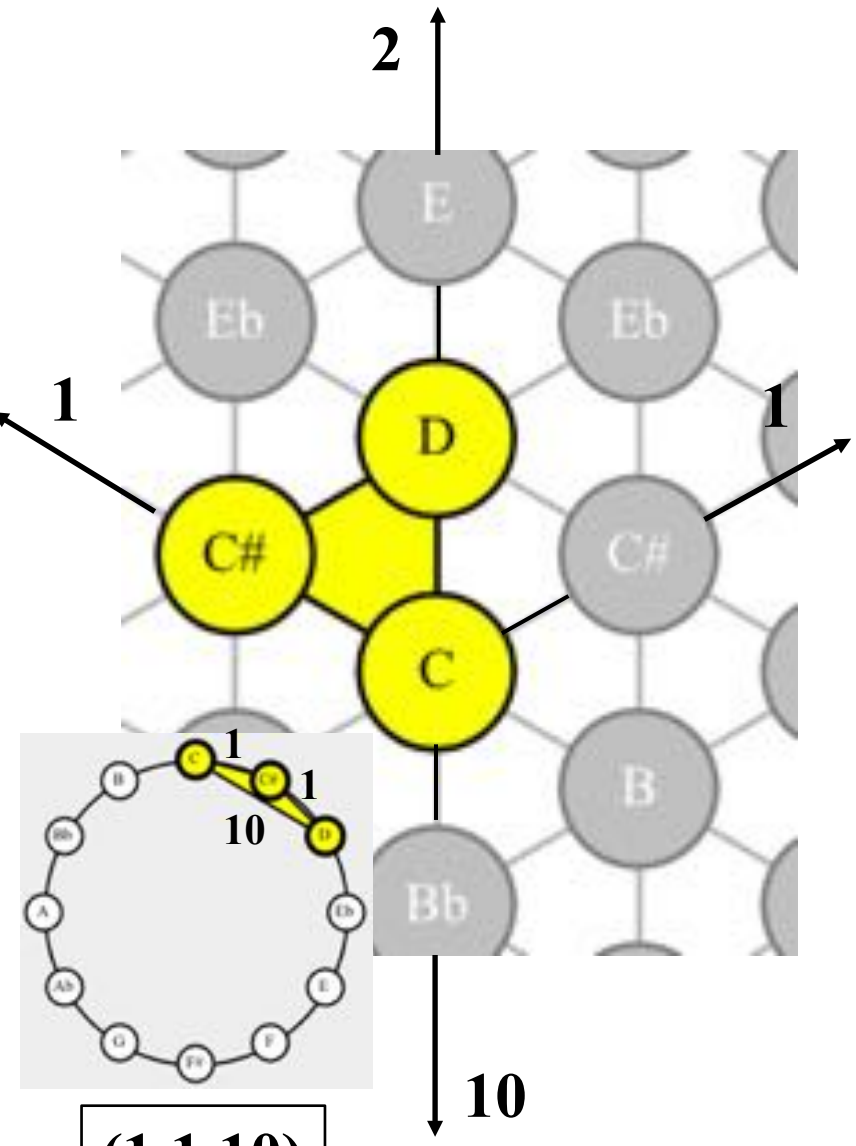
K_{T}[2,2,3]
[Mazzola – 2002]



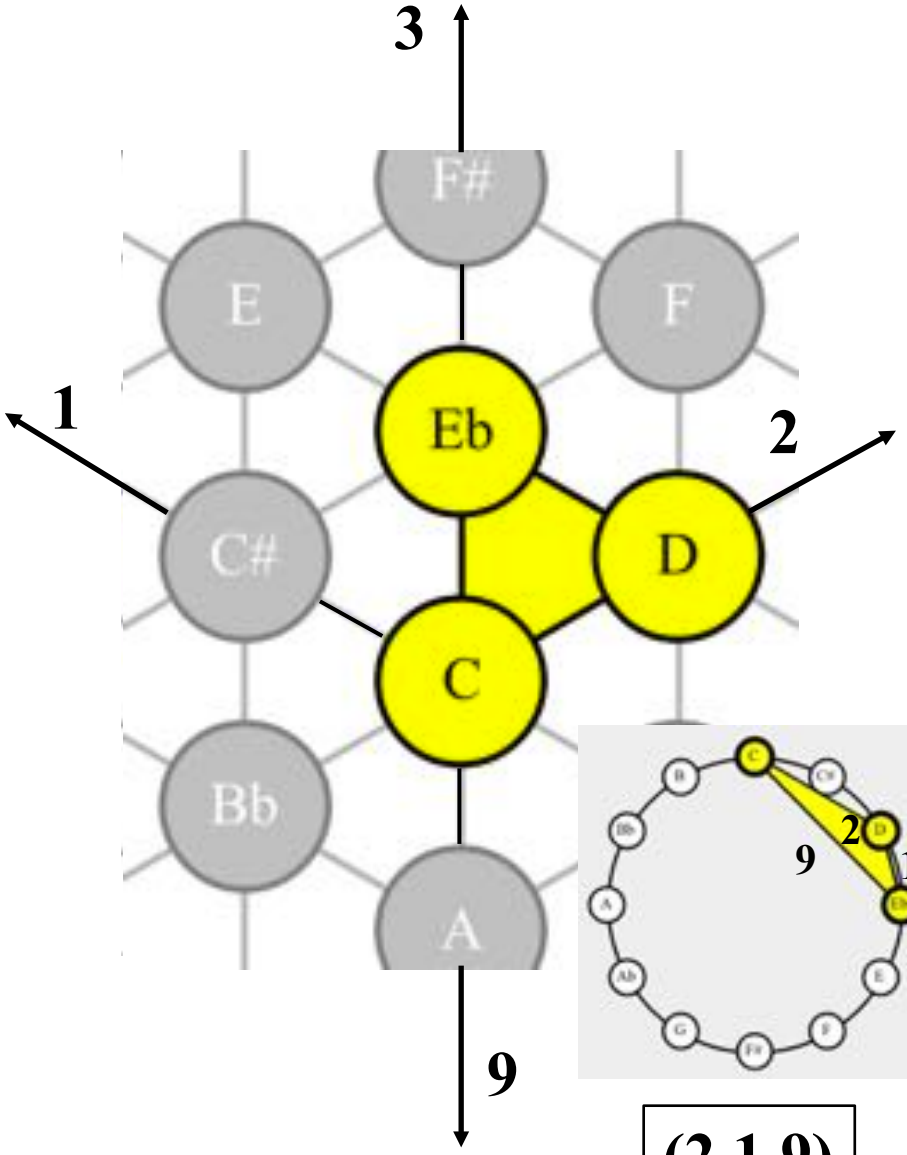
…

d	complexe	taille	b_n	p-v	χ
-	$\mathcal{K}_\emptyset$	0	0		0
0	$\mathcal{K}_{TI}[0]$	0	[0]		0
1	$\mathcal{K}_{TI}[1,11]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[2,10]$	12	[2, 2]		0
	$\mathcal{K}_{TI}[3,9]$	12	[3, 3]		0
	$\mathcal{K}_{TI}[4,8]$	12	[4, 4]		0
	$\mathcal{K}_{TI}[5,7]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[6,6]$	6	[6, 0]		6
2	$\mathcal{K}_{TI}[1,1,10]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[1,2,9]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,3,8]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,4,7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,5,6]$	24	[1, 1, 6]		6
	$\mathcal{K}_{TI}[2,2,8]$	12	[2, 2, 0]		0
	$\mathcal{K}_{TI}[2,3,7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[2,4,6]$	24	[2, 2, 6]		6
	$\mathcal{K}_{TI}[2,5,5]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[3,3,6]$	12	[3, 0, 3]		6
	$\mathcal{K}_{TI}[3,4,5]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[4,4,4]$	4	[4, 0, 0]		4
3	$\mathcal{K}_{TI}[1,1,1,9]$	12	[1, 1, 0, 0]	x	0
	$\mathcal{K}_{TI}[1,1,2,8]$	24	[1, 1, 12, 0]		12
	$\mathcal{K}_{TI}[1,1,3,7]$	24	[1, 2, 13, 0]		12
	$\mathcal{K}_{TI}[1,1,4,6]$	24	[1, 1, 18, 0]		18
	$\mathcal{K}_{TI}[1,1,5,5]$	12	[1, 1, 6, 0]		6

The panoply of *Tonnetze* at the service of the analyst

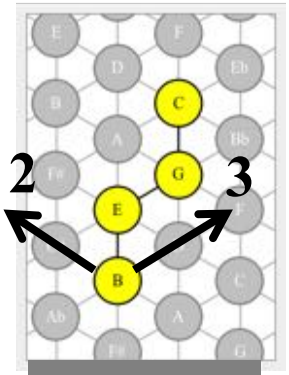


(1,1,10)

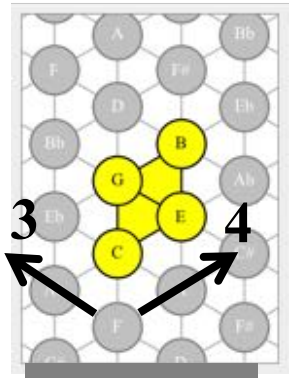


(2,1,9)

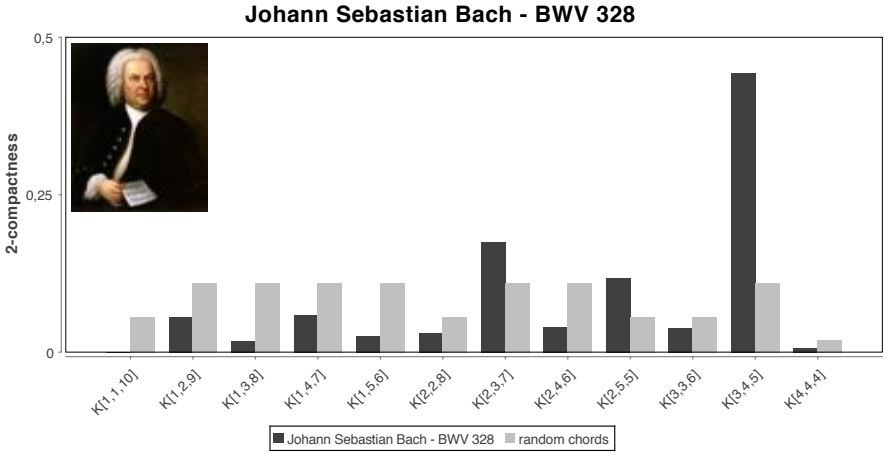
The geometric character of musical logic

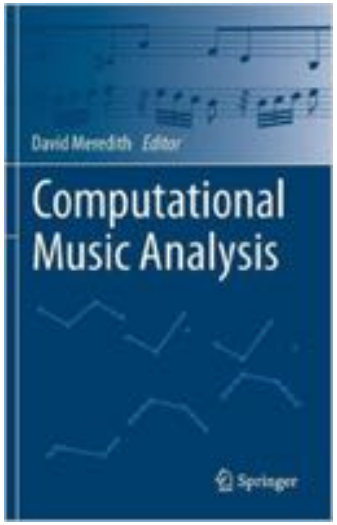


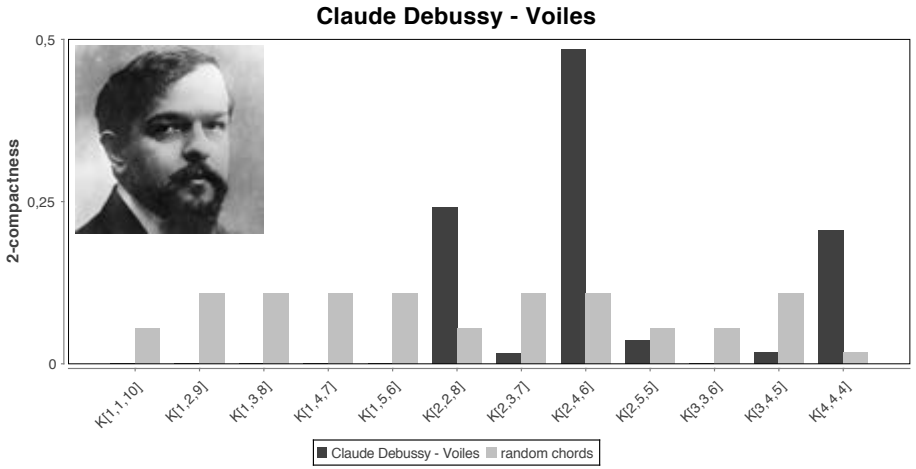
T[2,3,7]

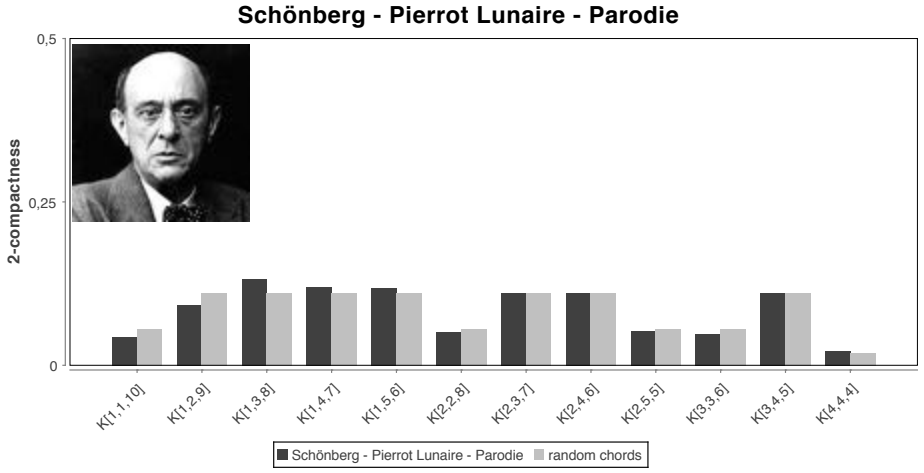


T[3,4,5]

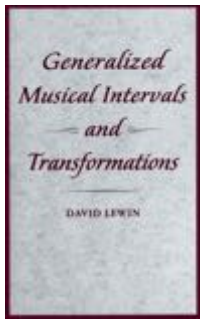




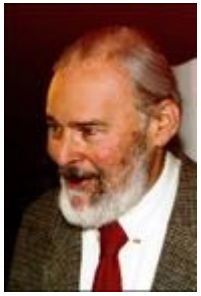




Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015



Système d'Intervalles Généralisés - Système Généralisé d'Intervalles
David Lewin's *Generalized Interval System* [GMIT, 1987]


D. Lewin

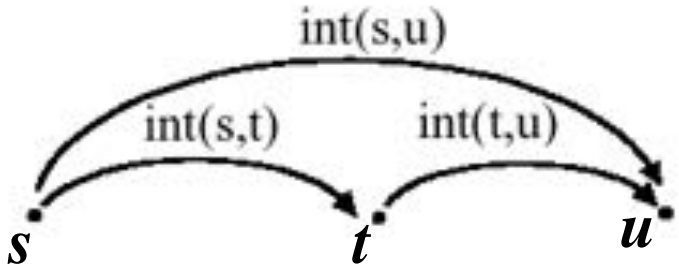
GIS = (*S*, *G*, int)

S **= set**

(*G*,•) = group of intervals

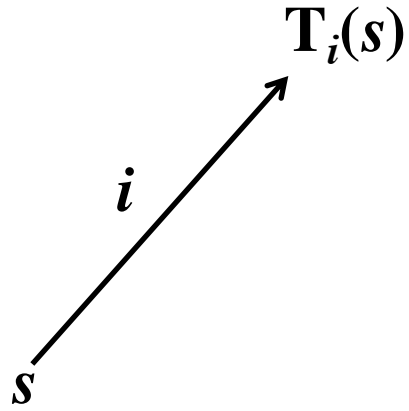
int = intervallic function

$$S \times S \xrightarrow{\text{int}} G$$



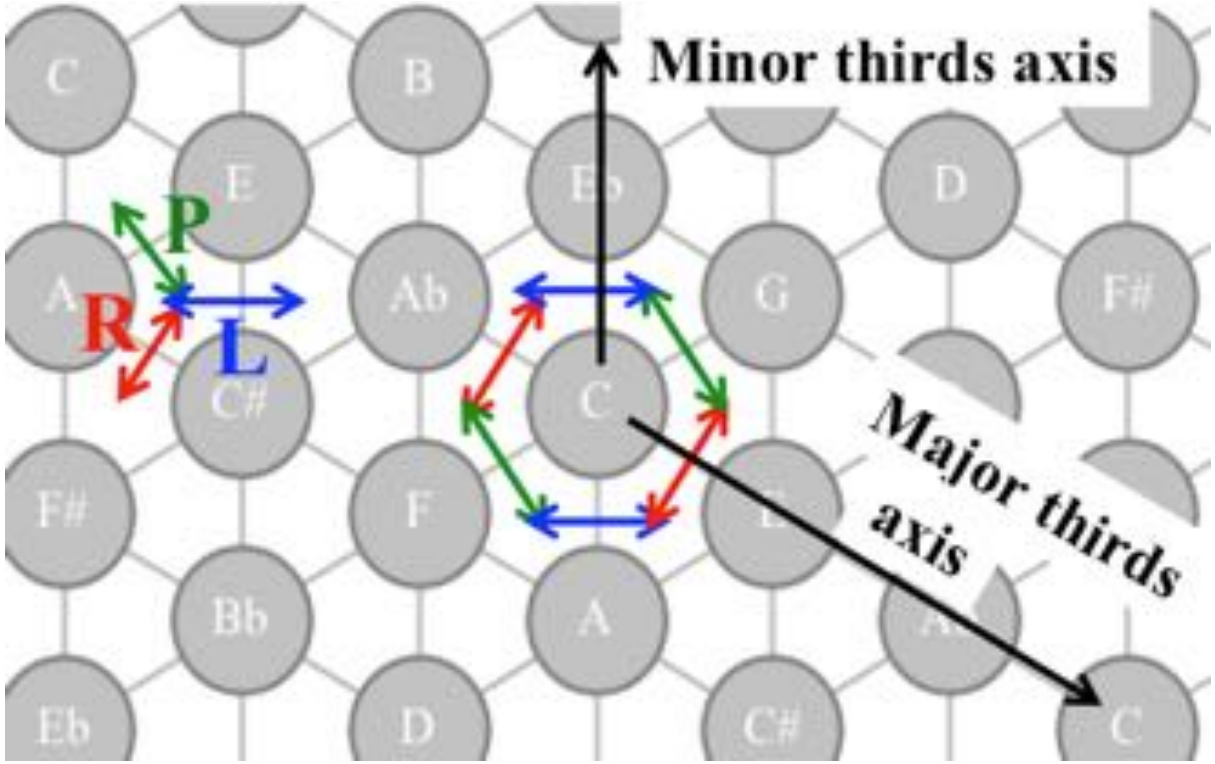
Simply transitive action

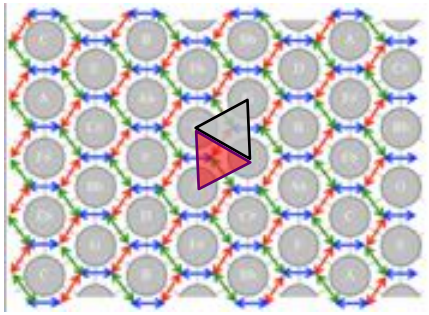
1. **For all objects *s*, *t*, *u* in *S* :**
   int(*s*, *t*)•int(*t*, *u*) = int(*s*, *u*)
2. **For all object *s* in *S* and for all interval *i* in *G* there exists a unique object *t* in *S* such that int(*s*, *t*) = *i***

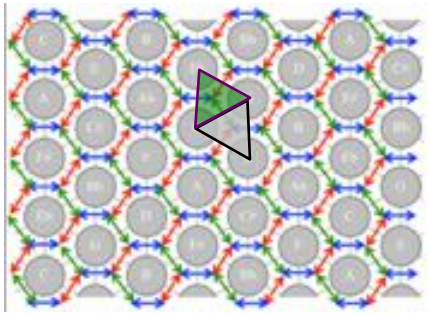


Let $\tau = \{T_i ; i \in G\}$ be the group of transpositions
GIS = (*S*, *G*, int) $\Leftrightarrow$ $\tau \times S \rightarrow S$ such that $(T_i , s) \rightarrow T_i(s)$ where $\text{int}(s, T_i(s)) = i$

The *Tonnetz* as "Generalized Interval System"

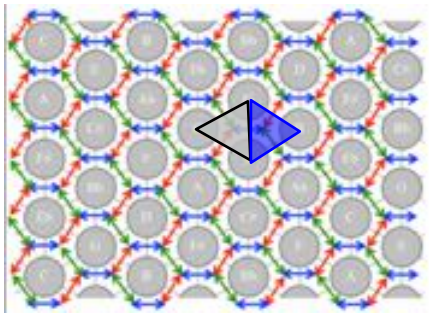




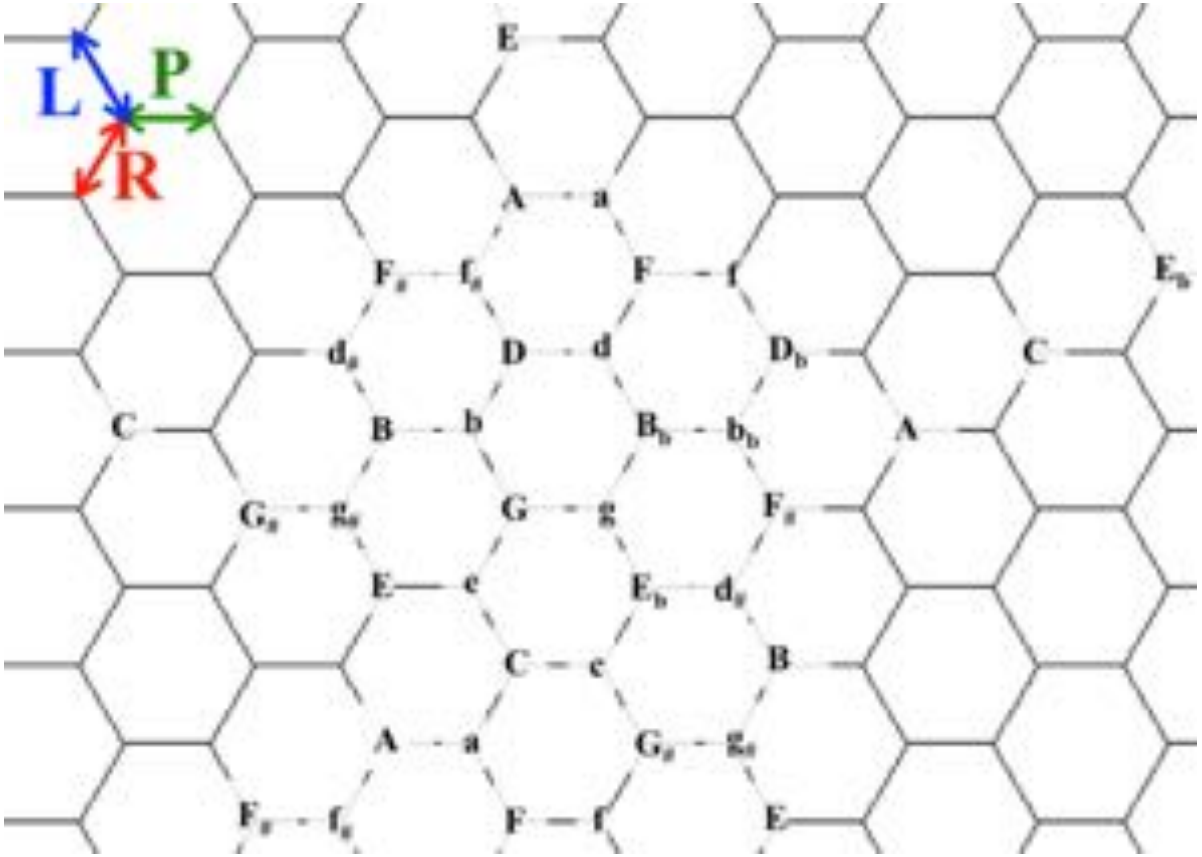


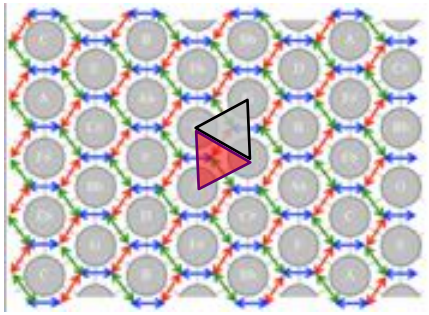
$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \; ; \; LRL = L(LR)^{-1} \rangle$

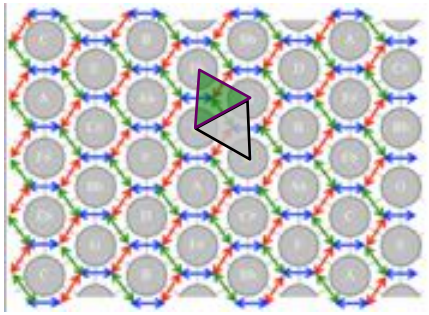
ρ acts in a simply transitive way on the set *S* of the 24 consonant triads



The *Tonnetz* as "Generalized Interval System"







$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \; ; \; LRL = L(LR)^{-1} \rangle$$

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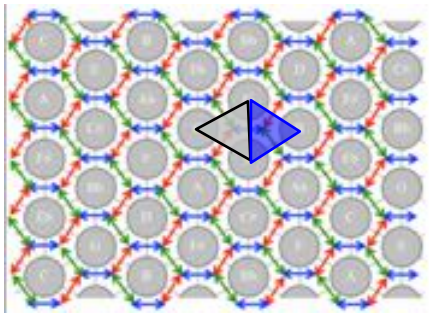
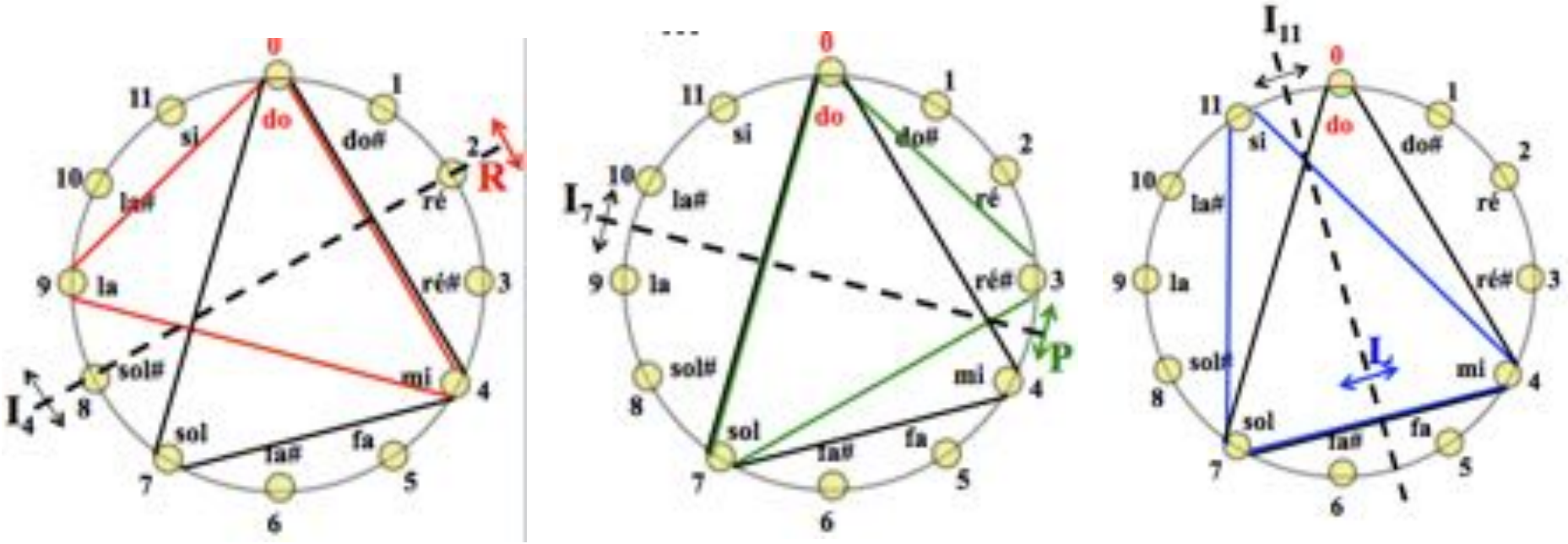


A different GIS structure on the same set *S*



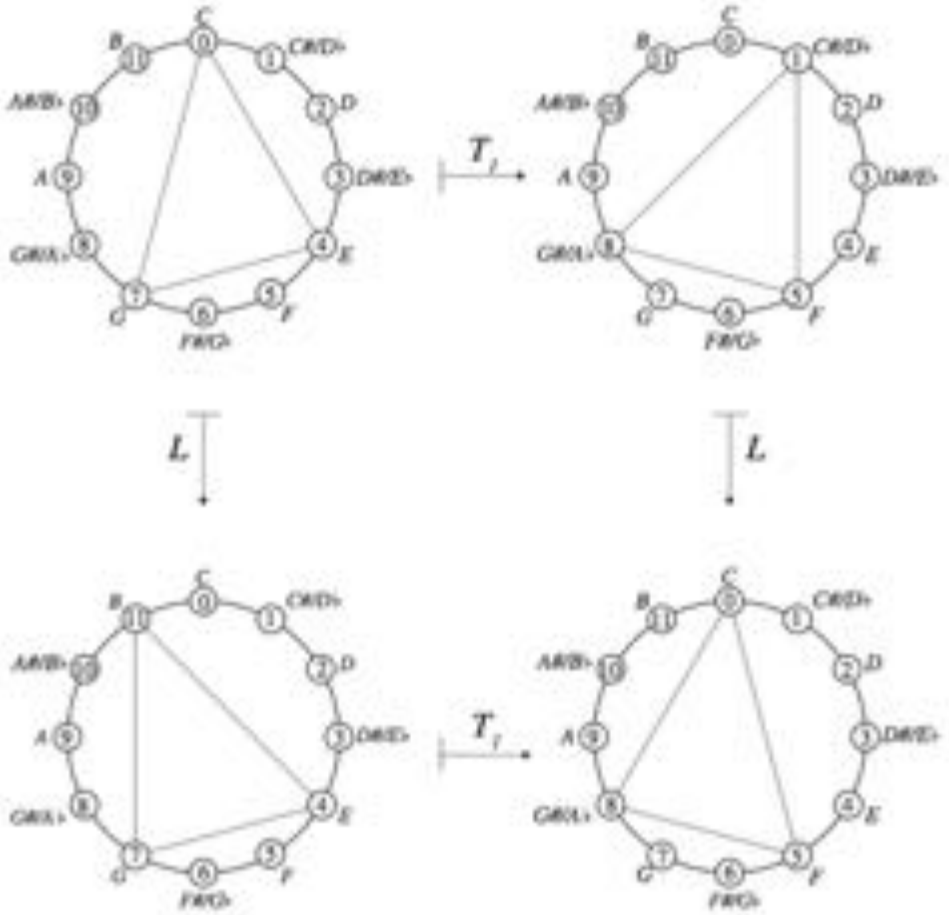
$I_4: x \rightarrow 4-x$ $I_7: x \rightarrow 7-x$ $I_{11}: x \rightarrow 11-x$

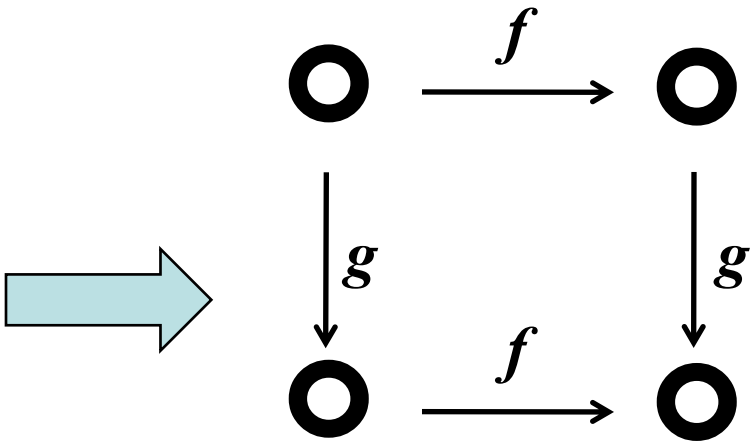
$D_{12} = \langle I, T \mid I^2 = T^{12} = 1 ; ITI = I(IT)^{-1} \rangle$

D_{12} acts in a simply transitive way on the set *S* of the 24 consonant triads

Two “dual” actions on the set of consonant triads

$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \; ; \; LRL = L(LR)^{-1} \rangle$ $\leftrightarrow$ $D_{12} = \langle I, T \mid I^2 = T^{12} = 1 \; ; \; ITI = I(IT)^{-1} \rangle$





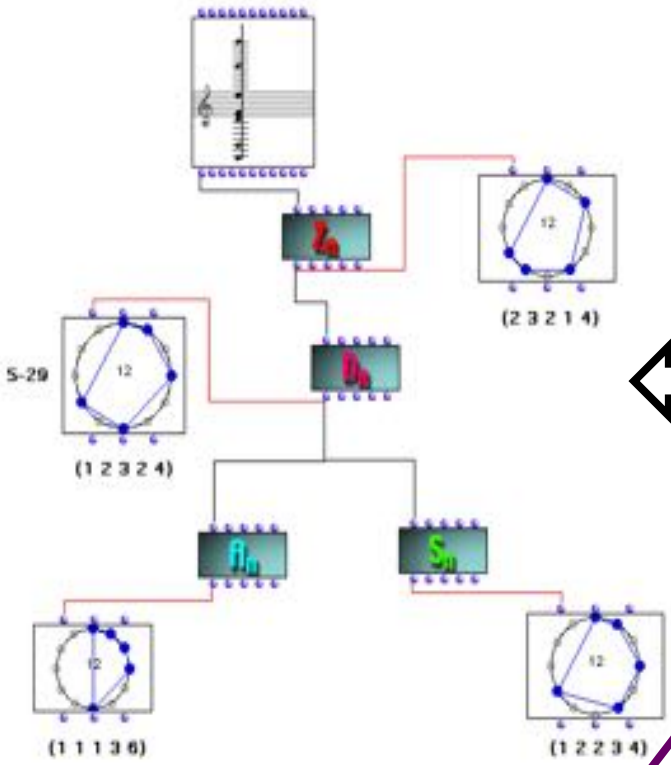
Every diagram commutes

$\forall f \in D_{12}$

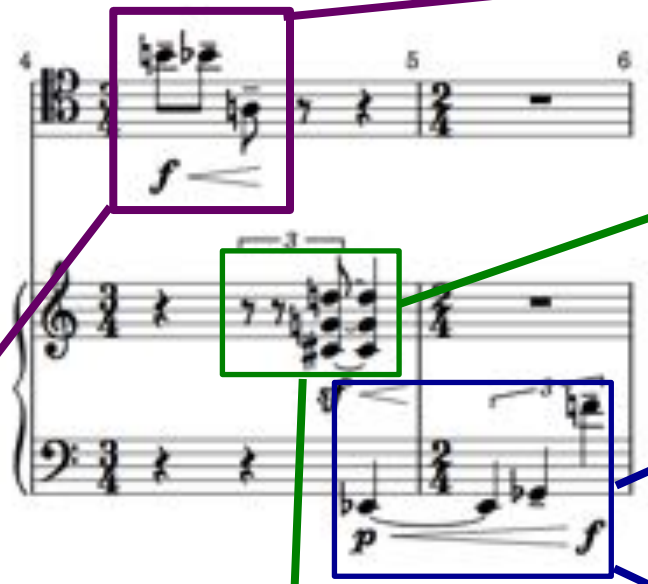
$\forall g \in \rho$

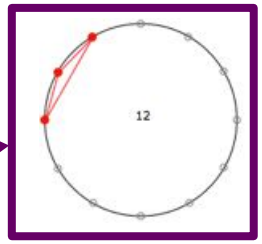
➔ **Alissa Crans, Tom Fiore and Ramon Satyendra, « Musical Actions of Dihedral Groups », *The American Mathematical Monthly*, Vol. 116 (2009), No. 6: 479 - 495**

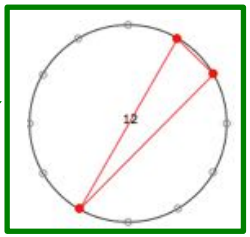
K-Nets and the paradigmatic approach

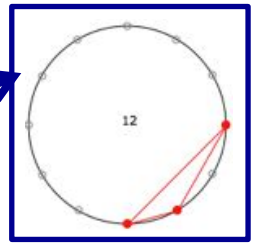


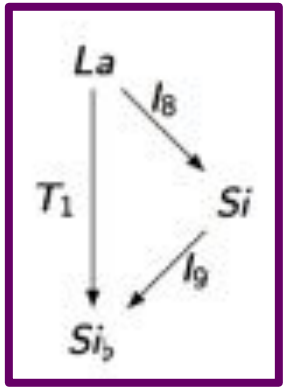
Anton Webern, *Drei Kleine Stücke*, Op. 11/2











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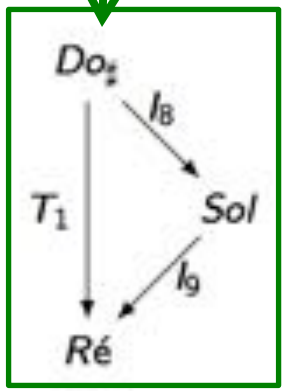

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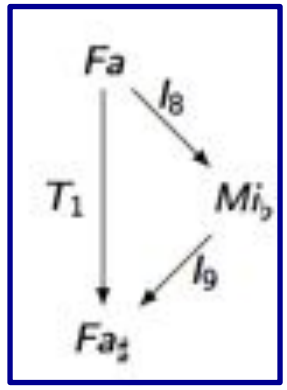
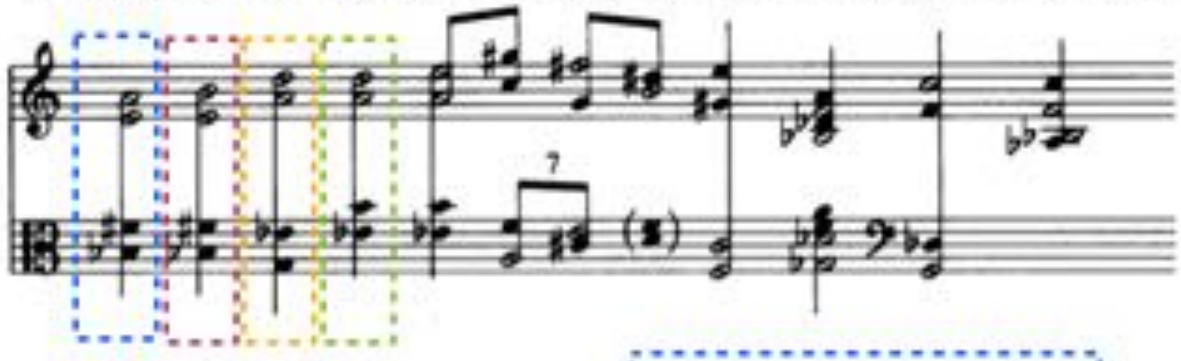
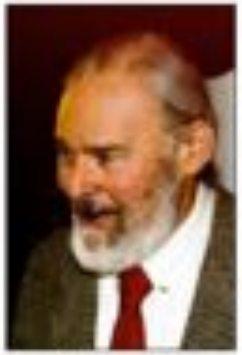


K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994

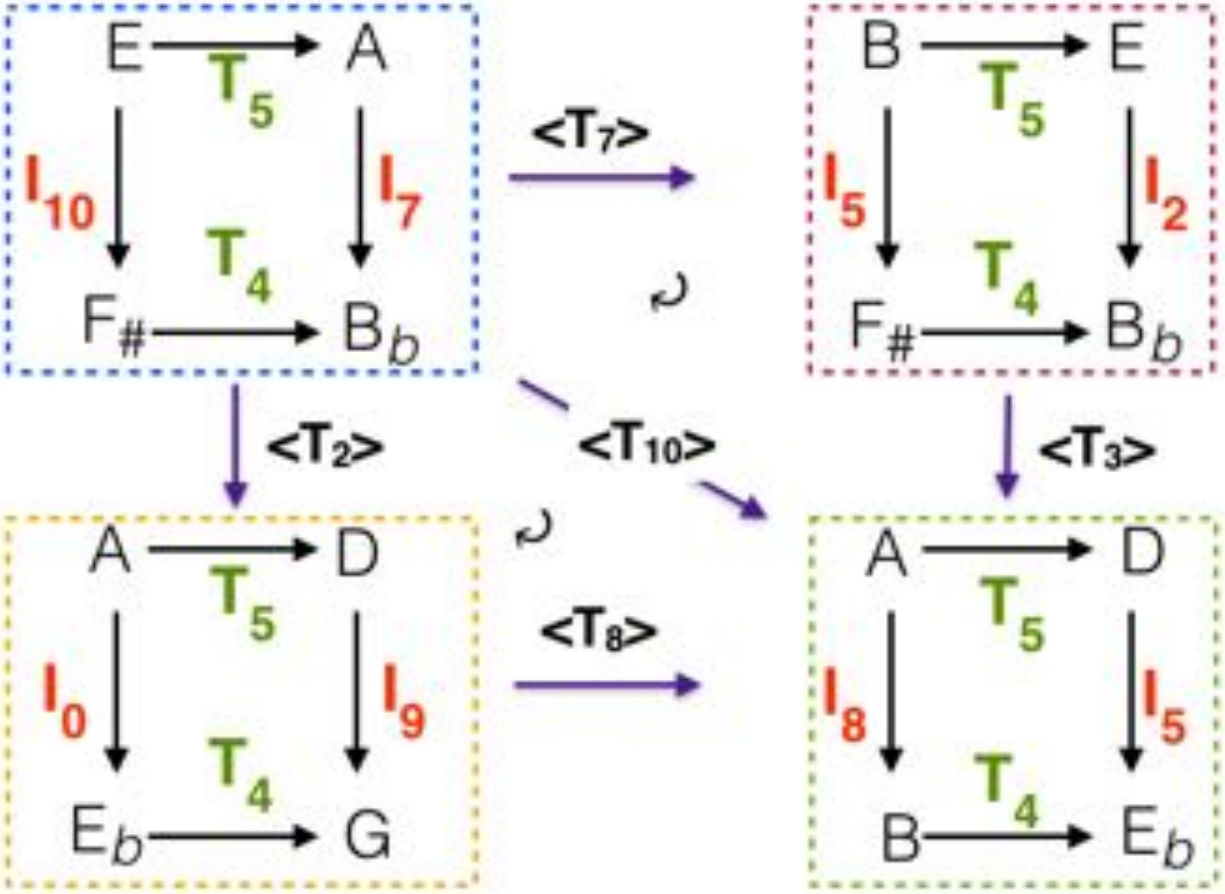




D. Lewin

H. Klumpenhouwer

$\langle T_k \rangle : T_m \rightarrow T_m$
$I_m \rightarrow I_{k+m}$



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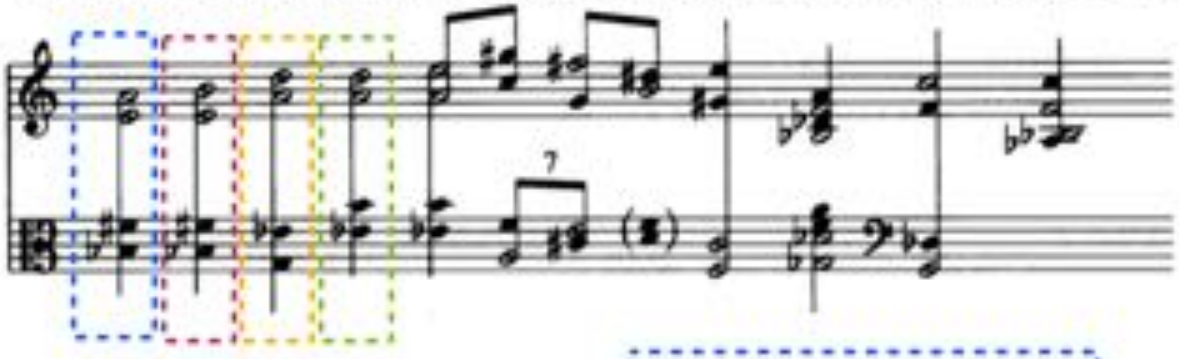
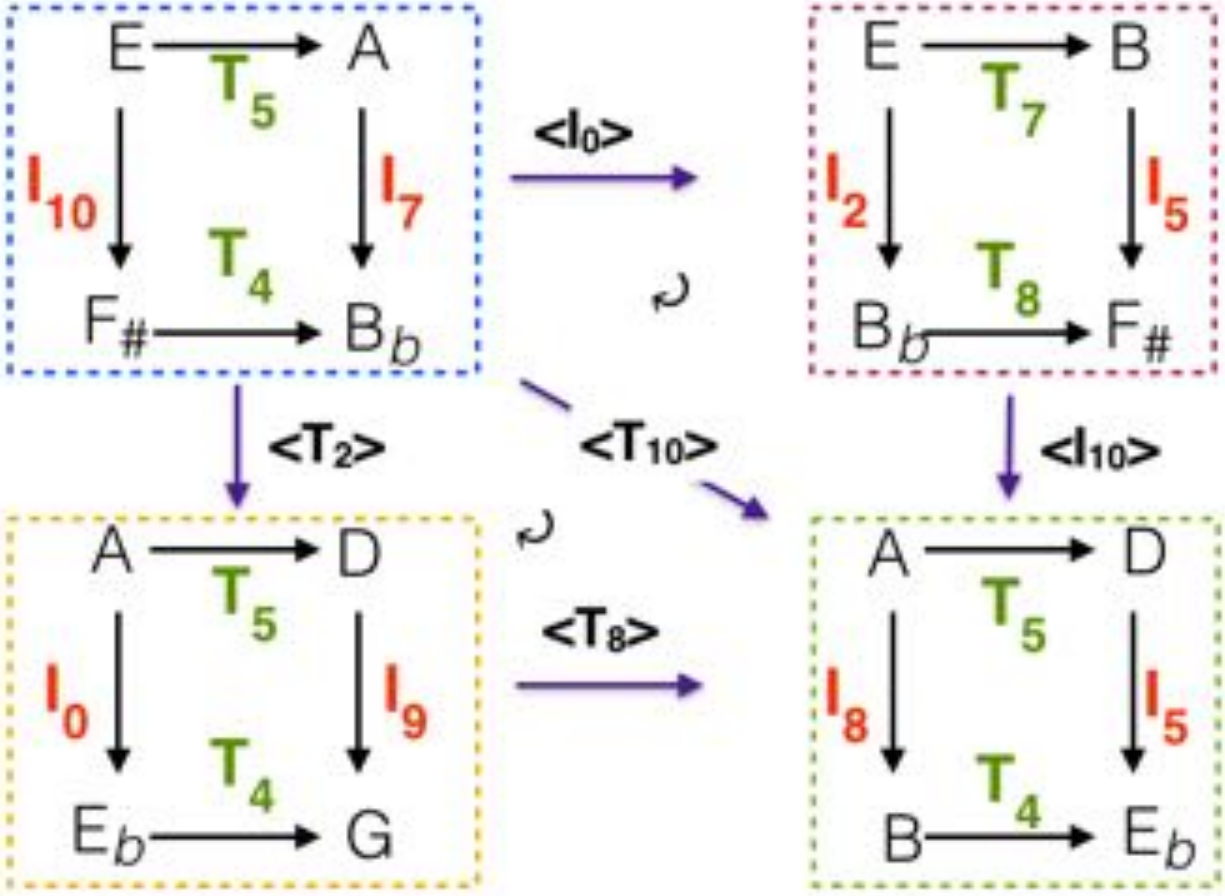


D. Lewin

H. Klumpenhouwer

$\langle T_k \rangle : T_m \rightarrow T_m$
$I_m \rightarrow I_{k+m}$

$\langle I_k \rangle : T_m \rightarrow T_{-m}$
$I_m \rightarrow I_{k-m}$



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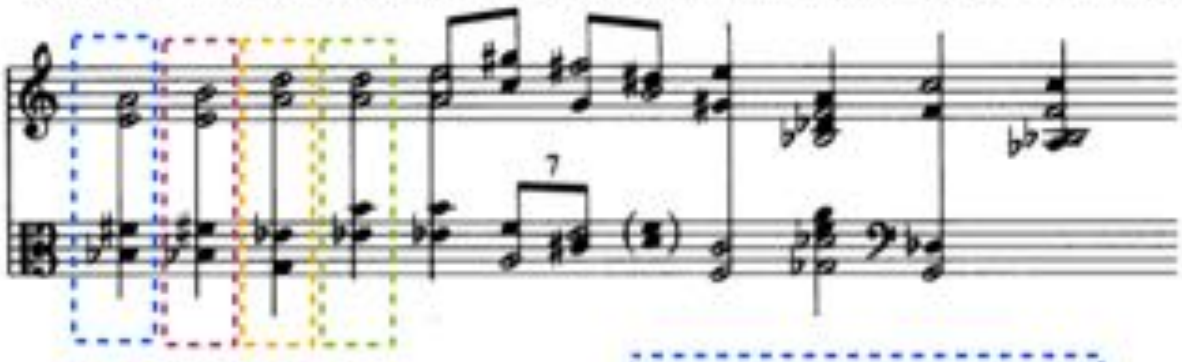

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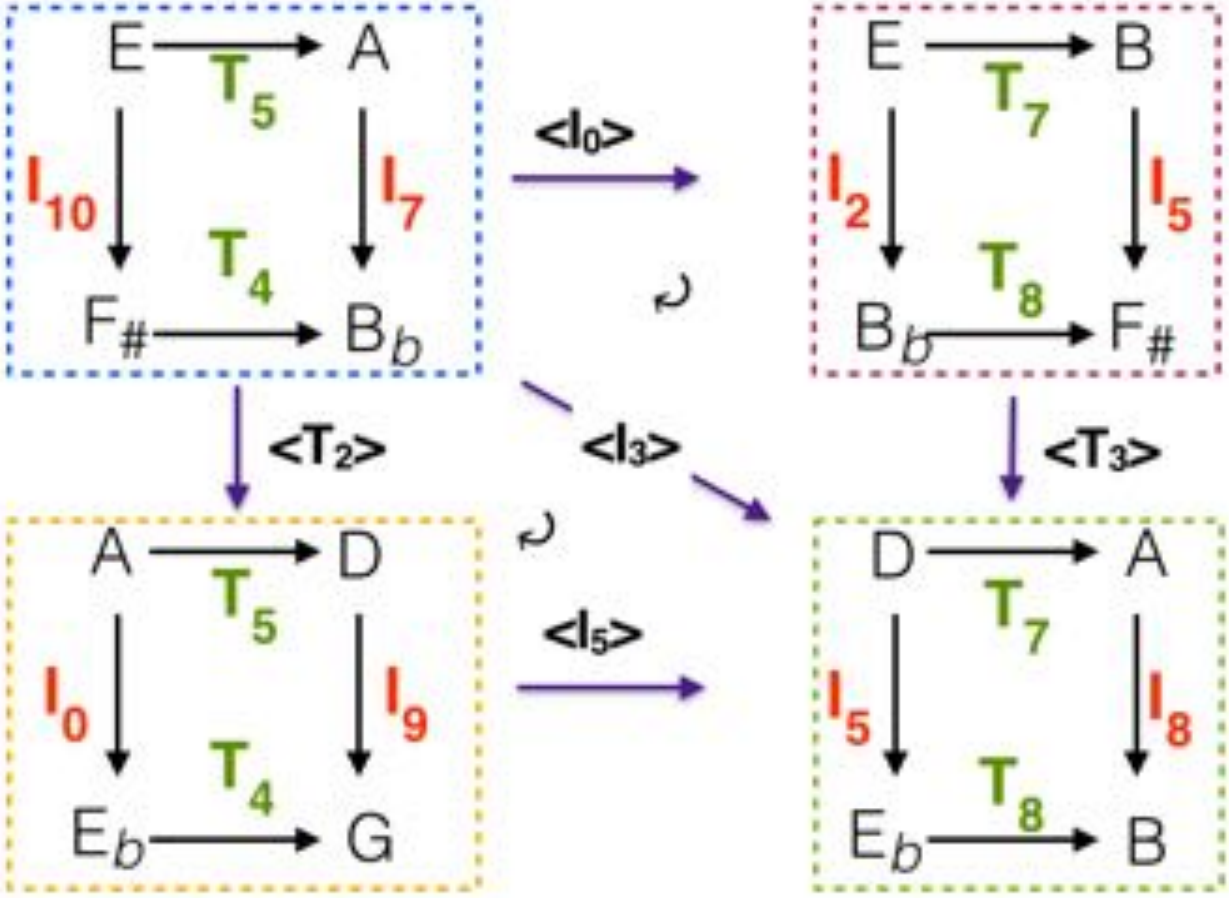
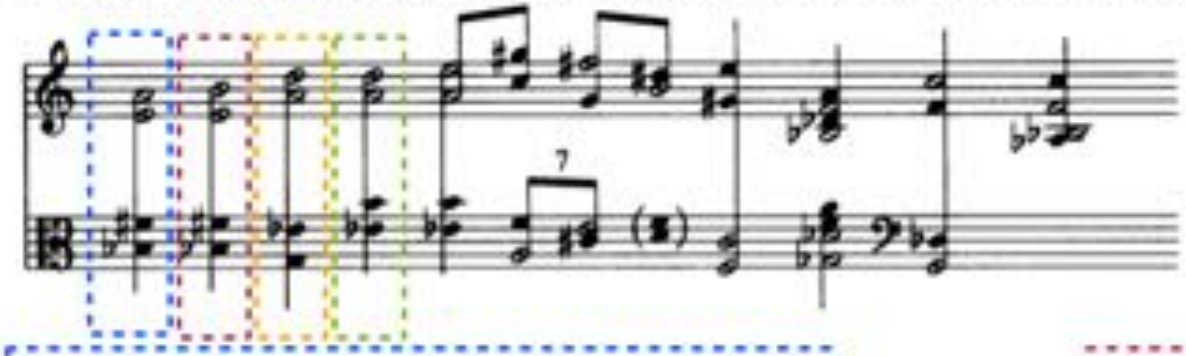
$I_m \rightarrow I_{k+m}$

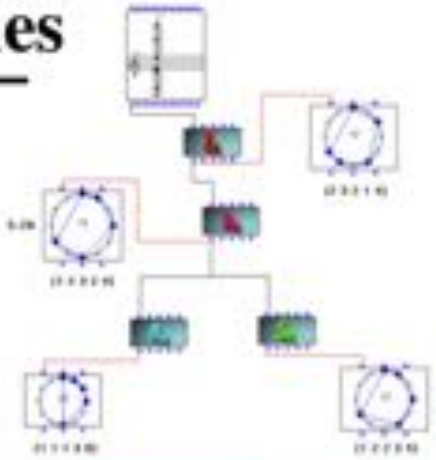
$\langle I_k \rangle : T_m \rightarrow T_{-m}$
$I_m \rightarrow I_{k-m}$

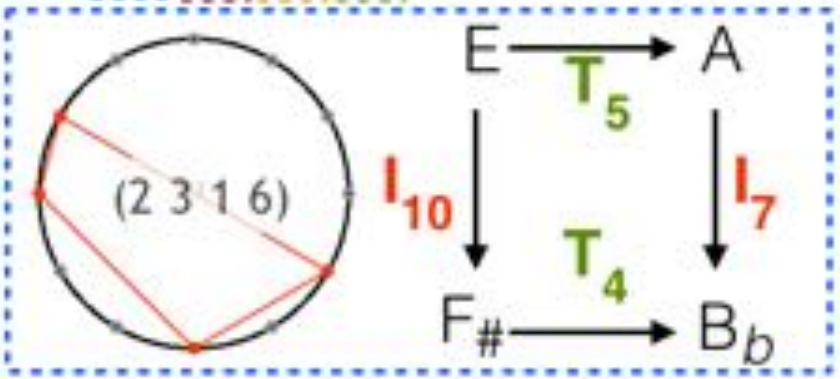


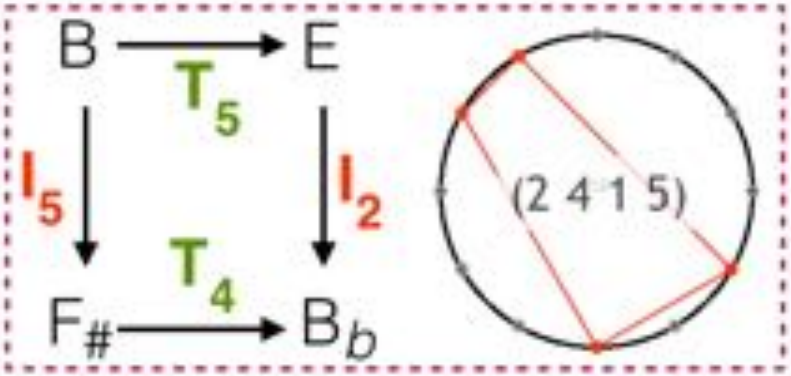
Transformational *vs* set-theoretical approaches

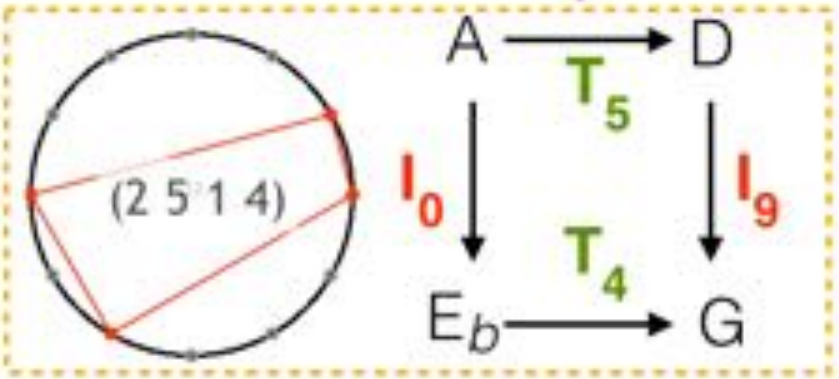
D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994

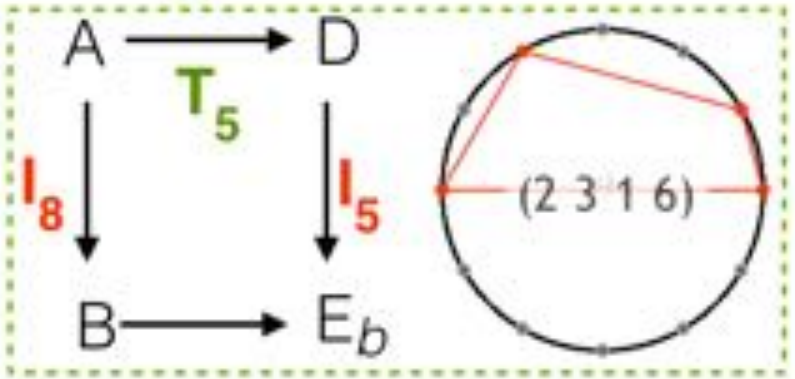




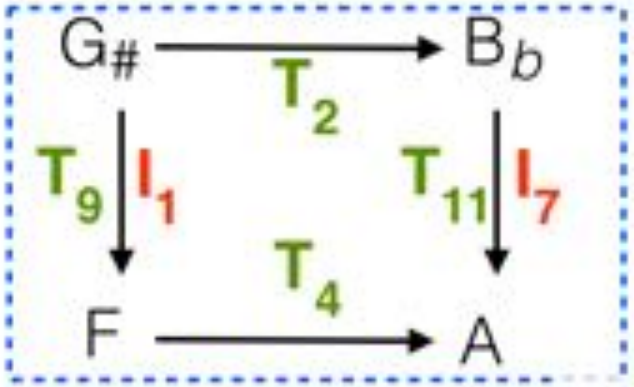


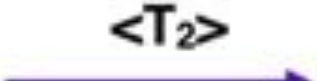


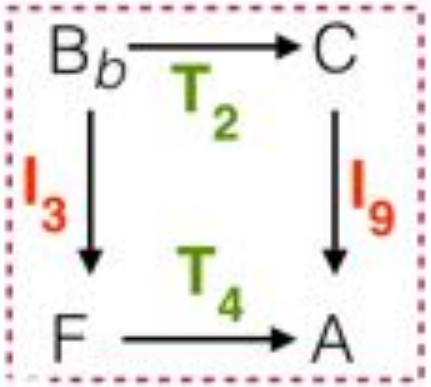


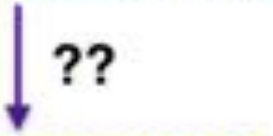


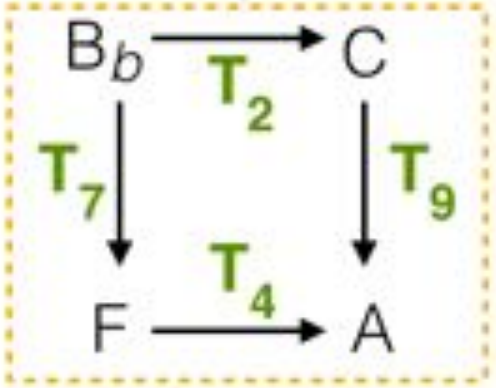
Some theoretical difficulties with the isographic relations

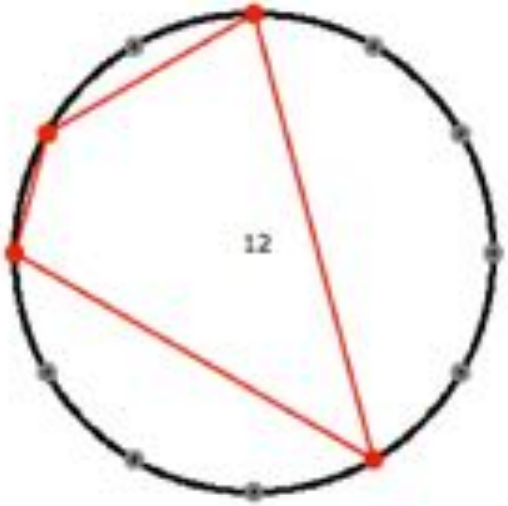








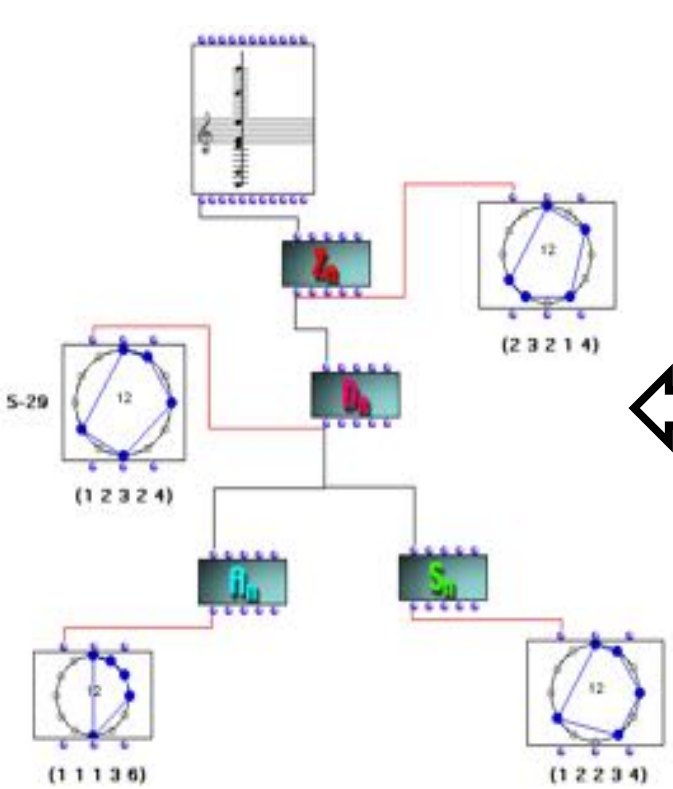




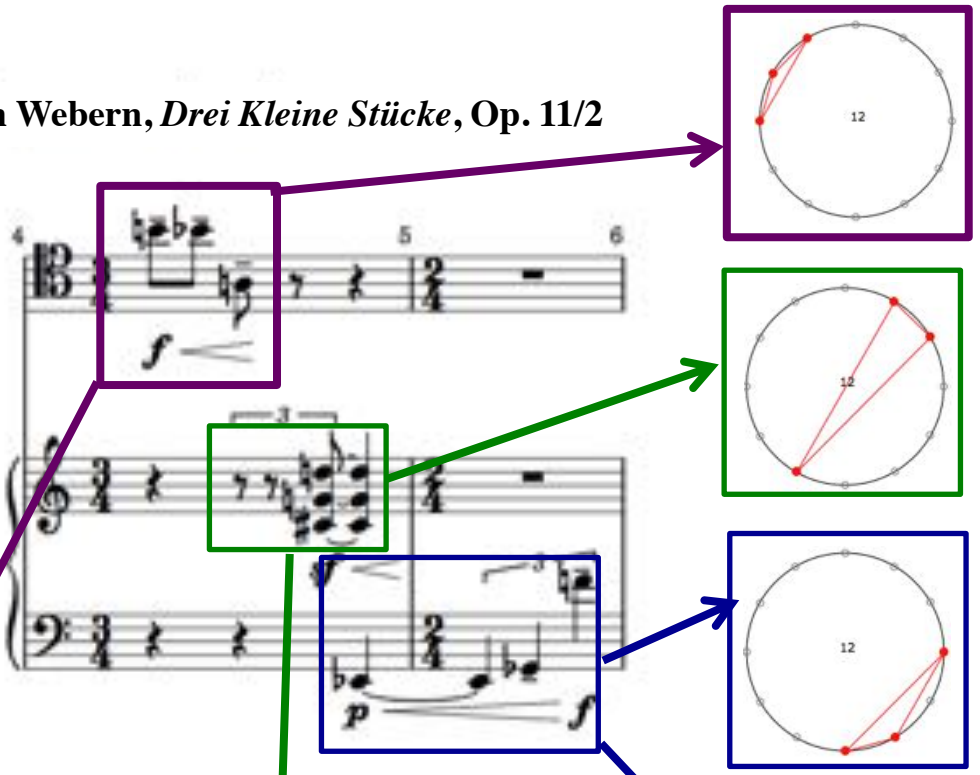
CONCLUSION

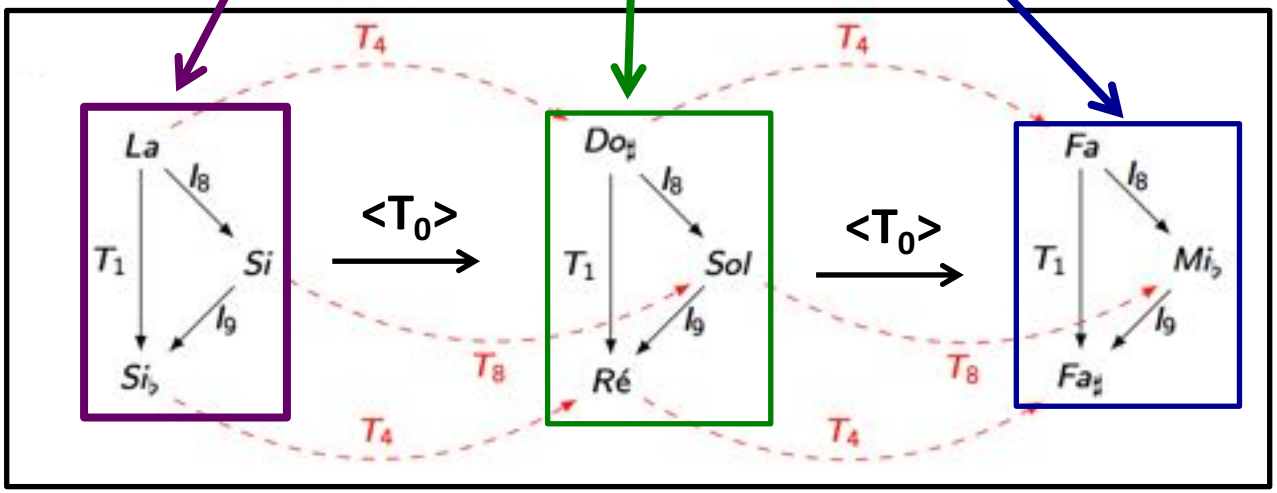
There are K-Nets which are not always isographic to a given one, i.e. the isographic relations are highly sensitives to the transformations used to label the arrows.
Is it possible to overstep this theoretical limitation? Which new definition of K-nets allows one to do that?

The categorical vs paradigmatic action-based approach

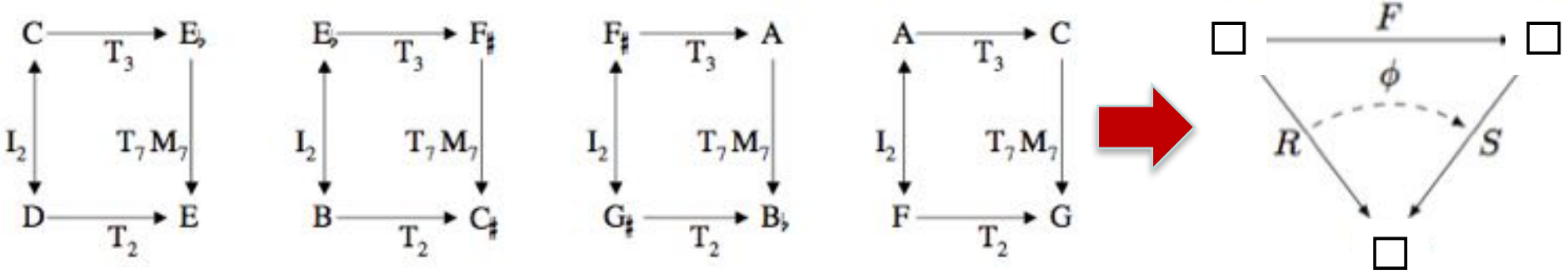


Anton Webern, *Drei Kleine Stücke*, Op. 11/2





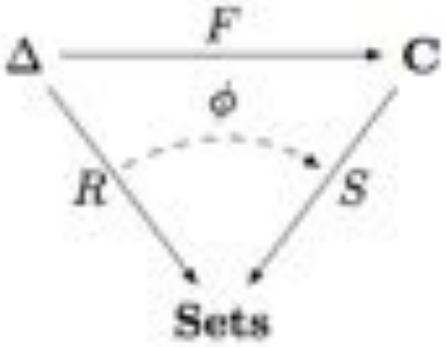
From K-Nets to category-based PK-Nets



Definition 1 *Let* **C** *be a category, and* S *a functor from* **C** *to the category* **Sets**. *Let* Δ *be a small category and* R *a functor from* Δ *to* **Sets**. *A PK-net of form* R *and of support* S *is a 4-tuple* (R, S, F, ϕ), *in which*

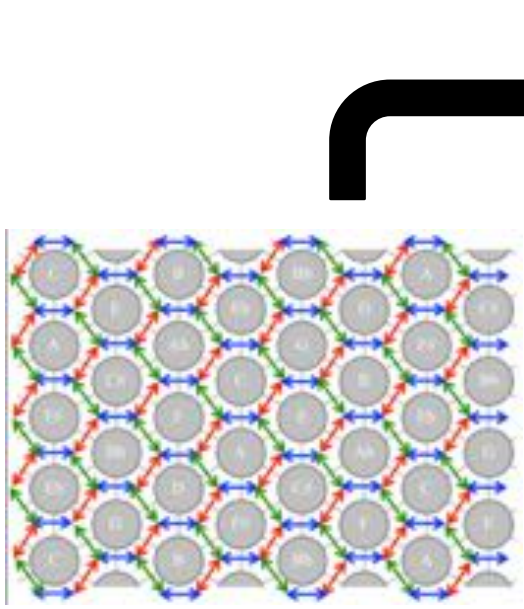
- F *is a functor from* Δ *to* **C**,
- *and* ϕ *is a natural transformation from* R *to* SF.

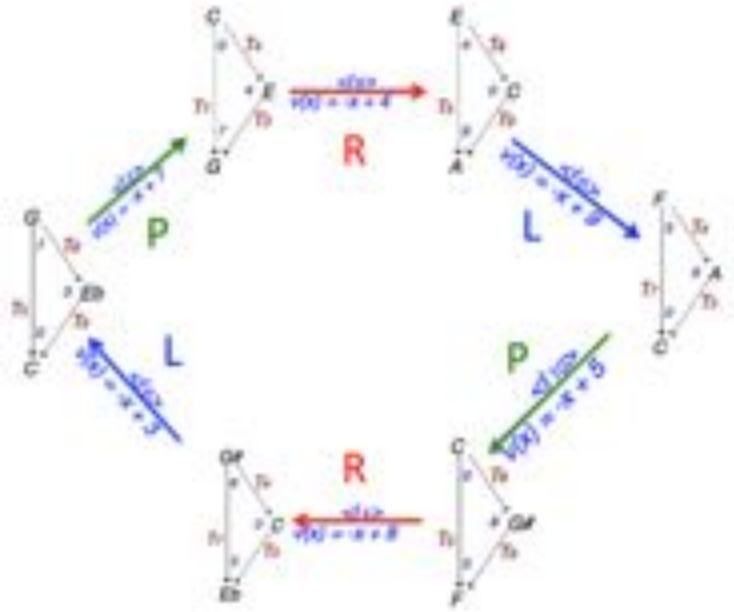
The definition of a PK-net is summed up by the following diagram:

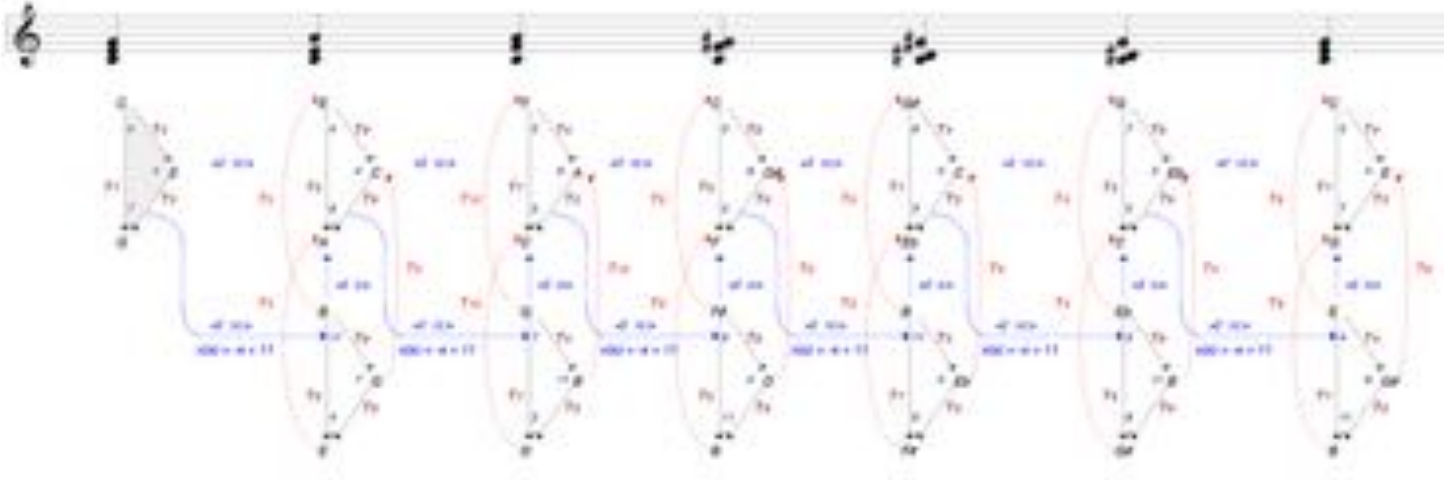


Popoff A., M. Andreatta, A. Ehresmann, « A Categorical Generalization of Klumpenhouwer Networks », MCM 2015, Queen Mary University, Springer, p. 303-314

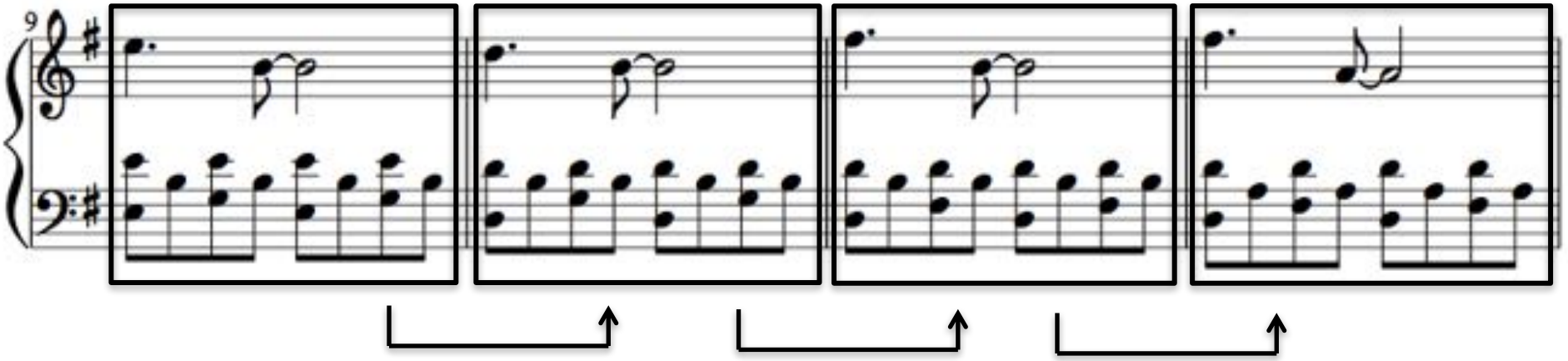
Topological vs categorical construction of the Tonnetz







- **Popoff A., C. Agon, M. Andreatta, A. Ehresmann (2016), « From K-Nets to PK-Nets: A Categorical Approach », PNM, 54(1)**
- **Popoff A., M. Andreatta, A. Ehresmann, « Relational PK-Nets for Transformational Music Analysis » (forthcoming in the JMM)**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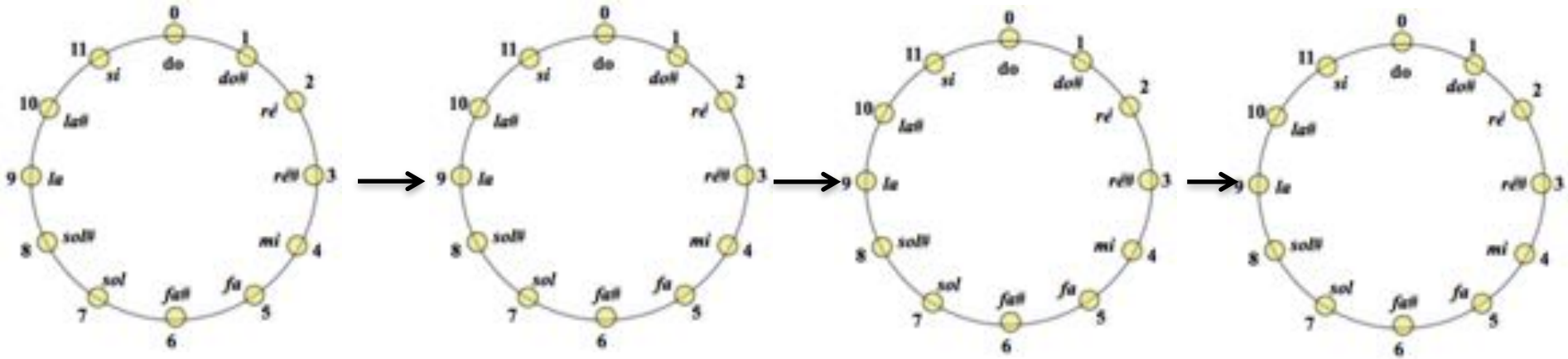
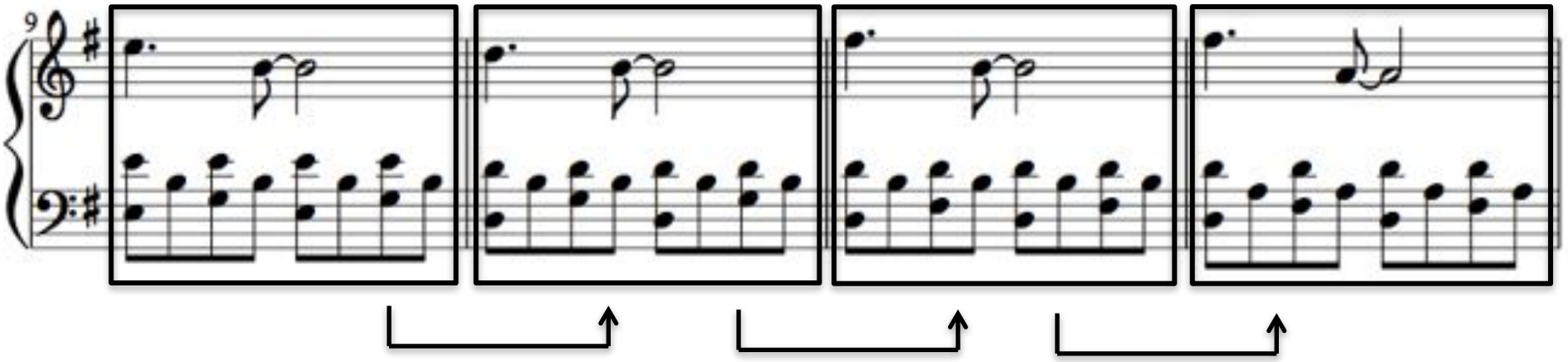

0 do
1 doti
2 ré
3 réti
4 mi
5 fa
6 fati
7 sol
8 solti
9 la
10 lati
11 si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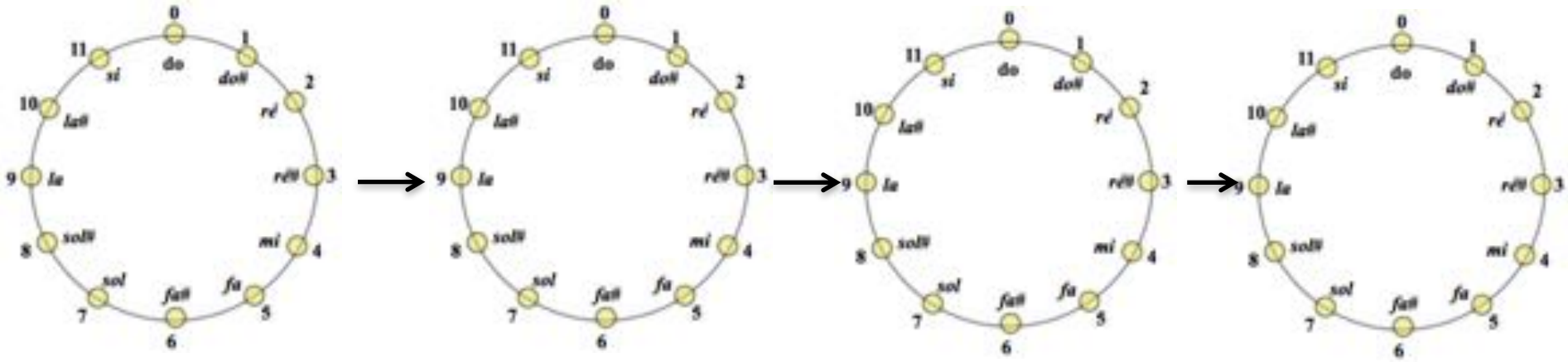
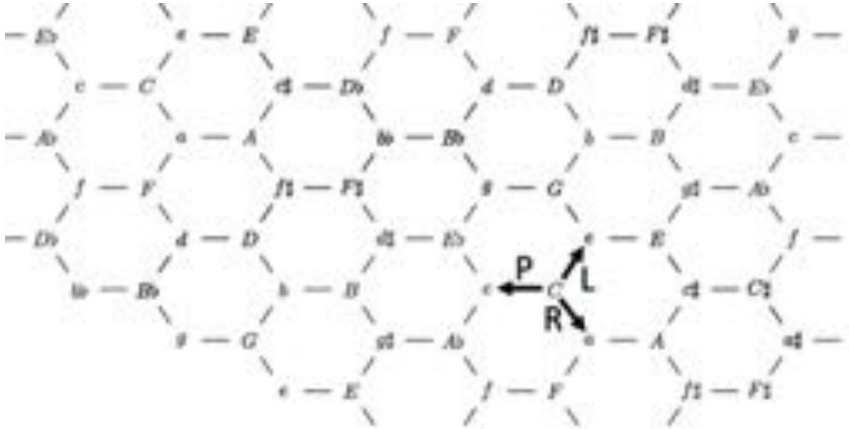
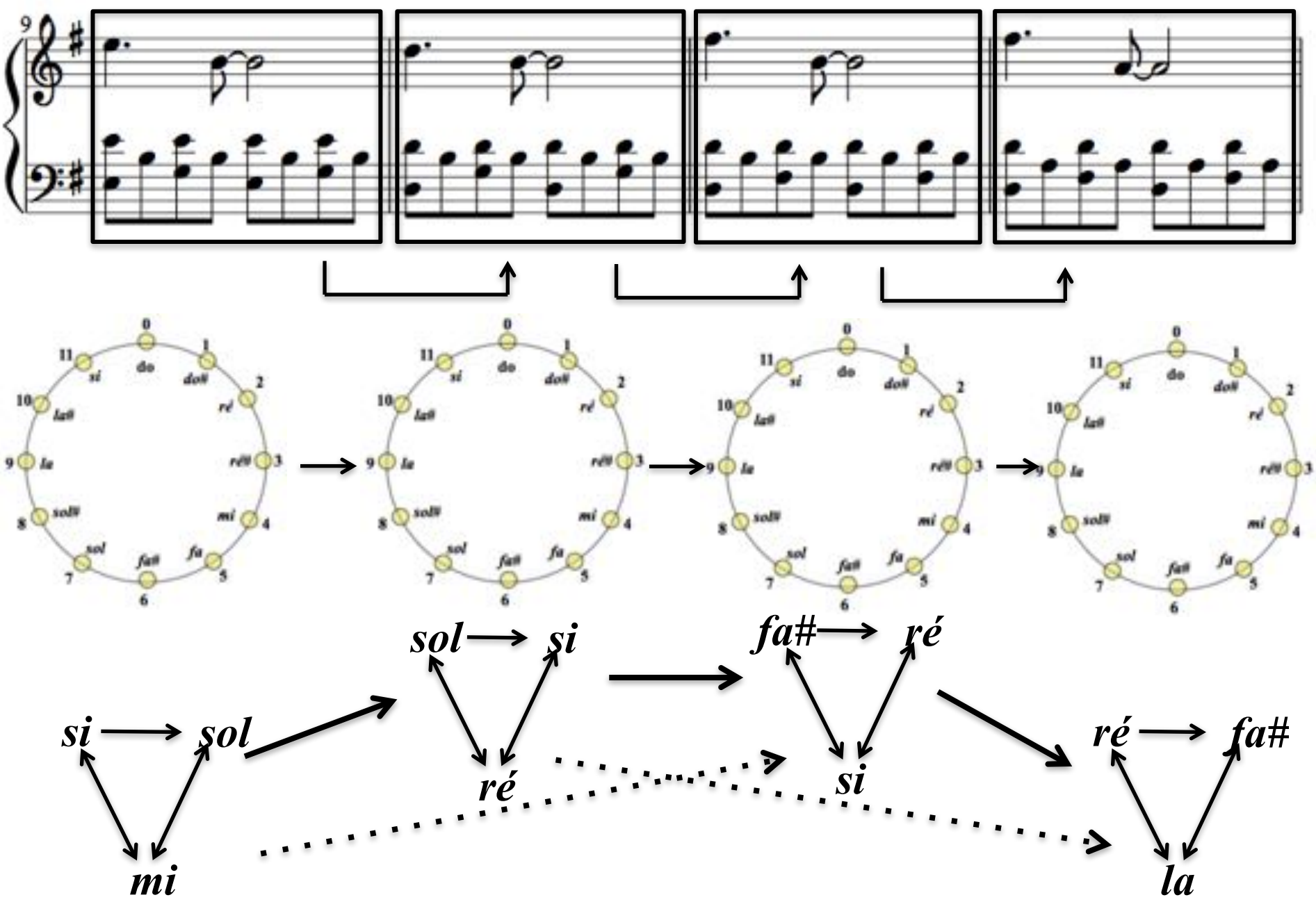
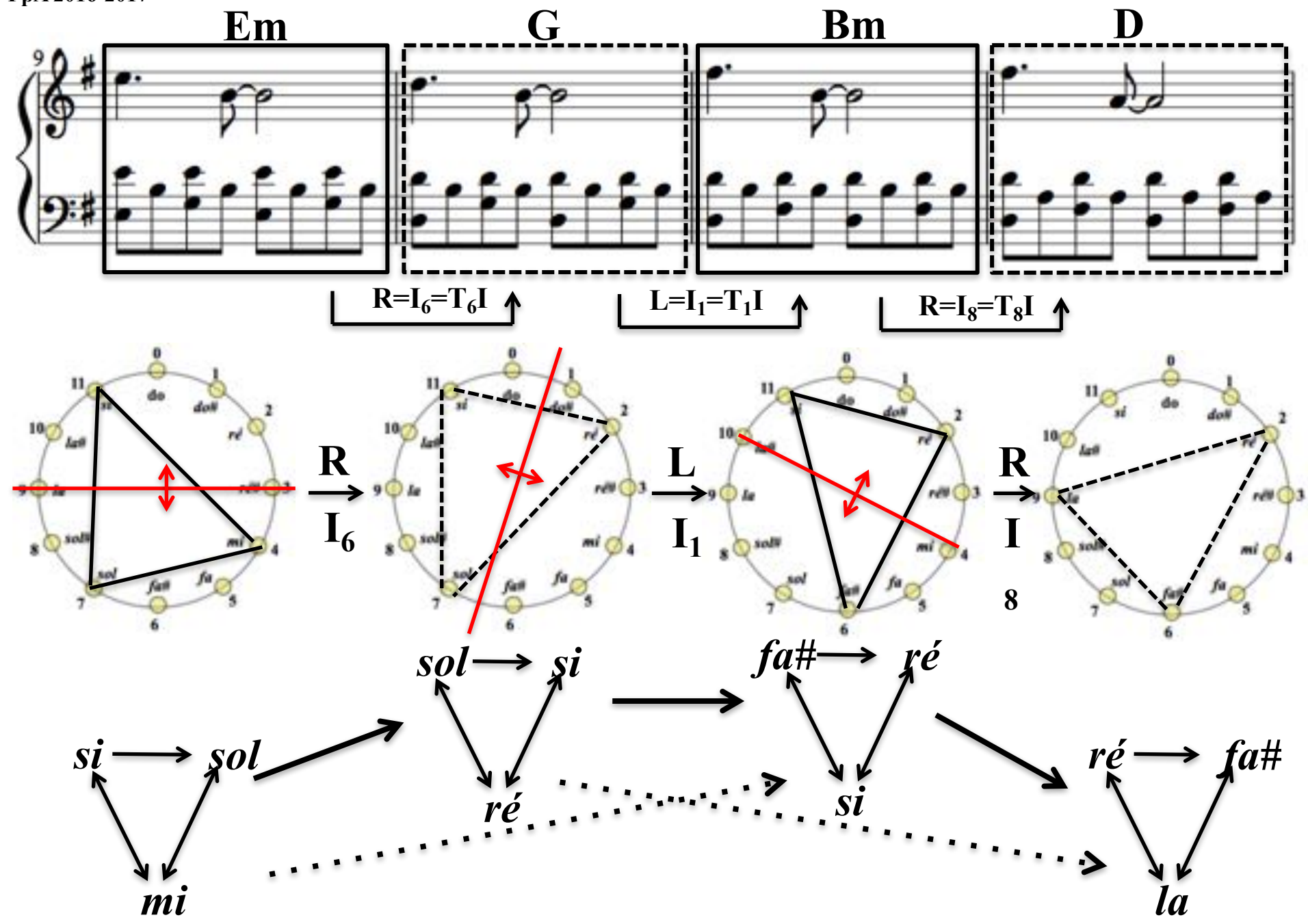
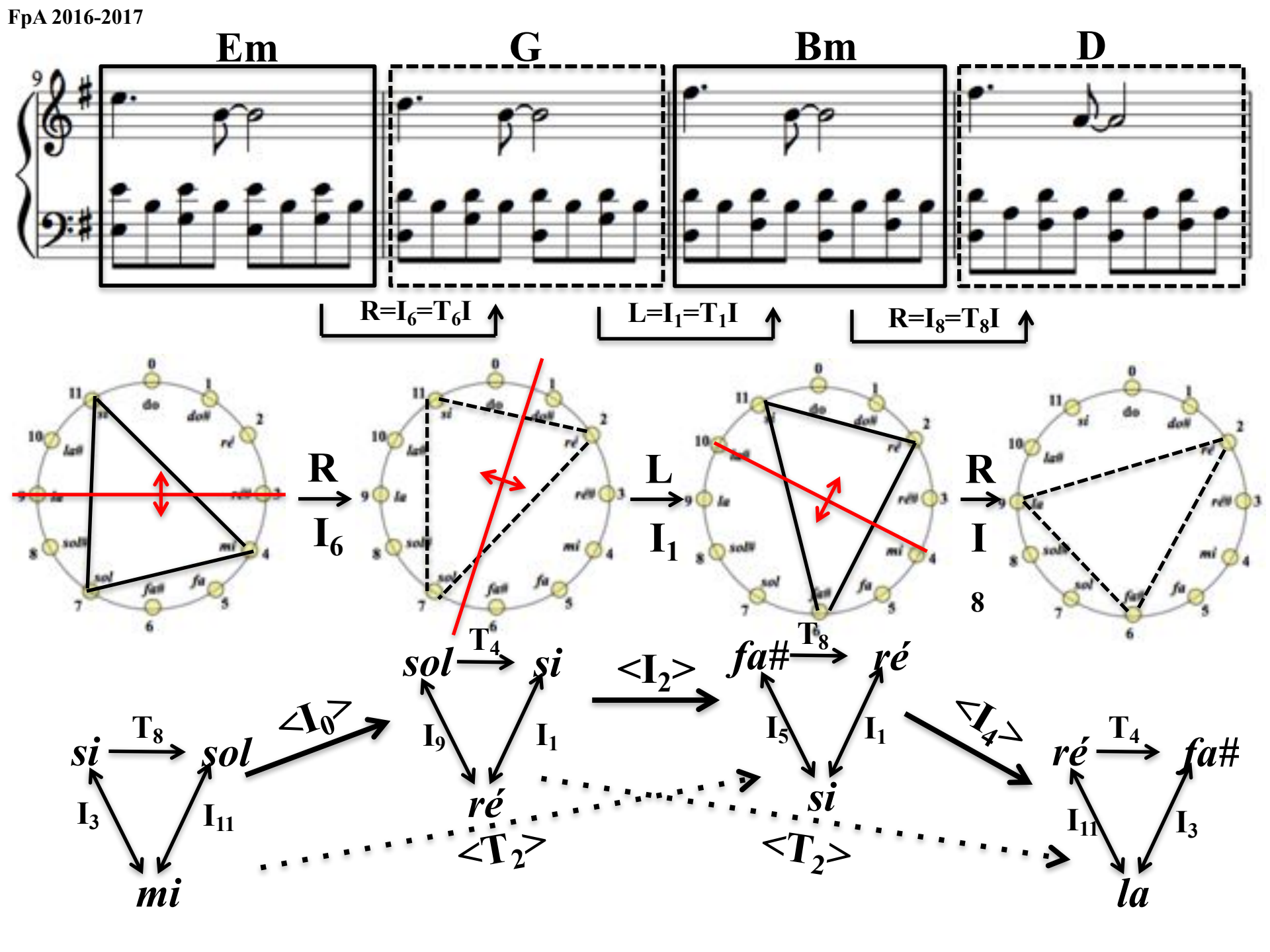

0 do
1 doti
2 ré
3 réti
4 mi
5 fa
6 fati
7 sol
8 solti
9 la
10 lati
11 si


P
L
R


9
0 do
1 doll
2 ré
3 réll
4 mi
5 fa
6 fa#
7 sol
8 solll
9 la
10 lall
11 si
sol → si
fa# → ré
si → sol
ré → fa#
ré
si
mi
la


Em
G
Bm
D
R=I6=T6I
L=I1=T1I
R=I8=T8I
R
I6
L
I1
R
I
8
sol → si
fa# → ré
si → sol
ré → fa#
ré
si
mi
la


Em
G
Bm
D
R=I6=T6I
L=I1=T1I
R=I8=T8I
R
I6
L
I1
R
I
8
T4
T8
sol
si
fa#
ré
<I2>
<I0>
<I4>
I9
I1
I5
I1
si
T8
sol
I3
I11
mi
ré
T4
fa#
I11
I3
la
<T2>
<T2>

From Circles to Hypersheres Chapter 6

Two Dimensions

Traditional Chordal Space

Gilles Baroin

➔ www.mathemusic.net

Harmonic Progressions

In Paolo Conte

Sotto le Stelle del Jazz

Supervision Moreno Andreatta
Modelisation Gilles Baroin 2016

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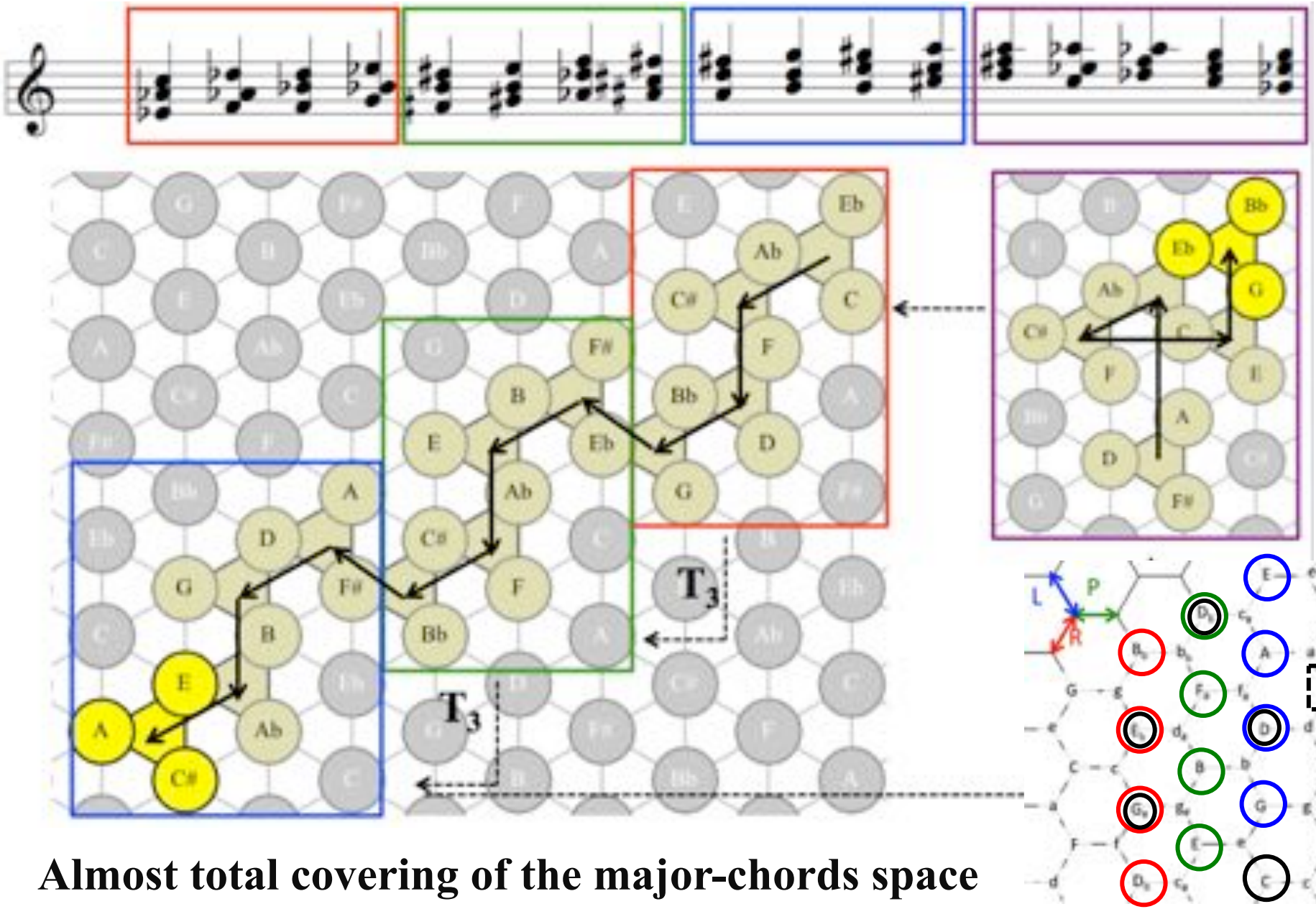
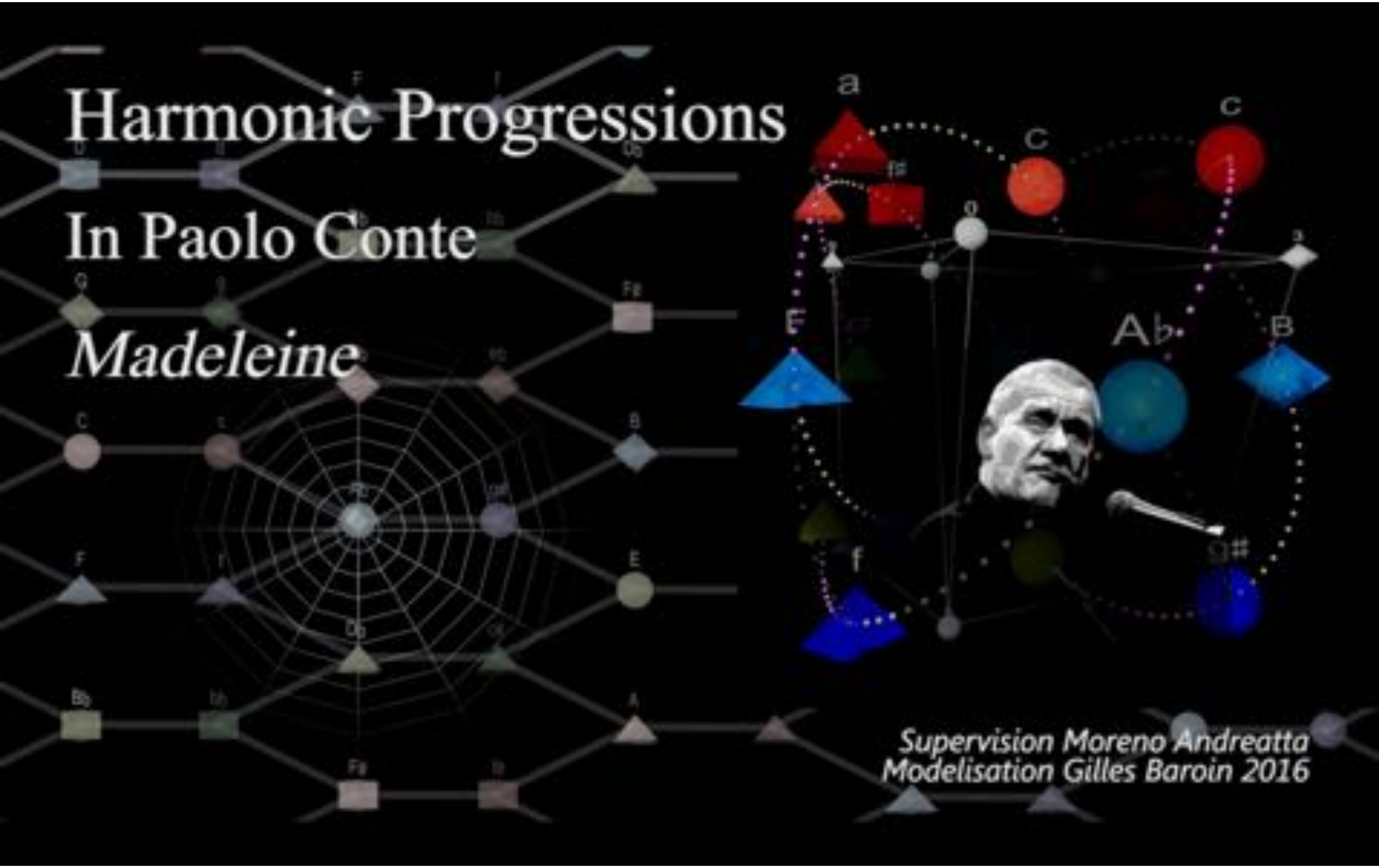
Symmetries in Paolo Conte's *Madeleine*

La_b Re_b Si_b Mi_b Si Mi Re_b $Fa_\#$ Re Sol Mi La Re La_b Re_b Do Mi_b



Almost total covering of the major-chords space



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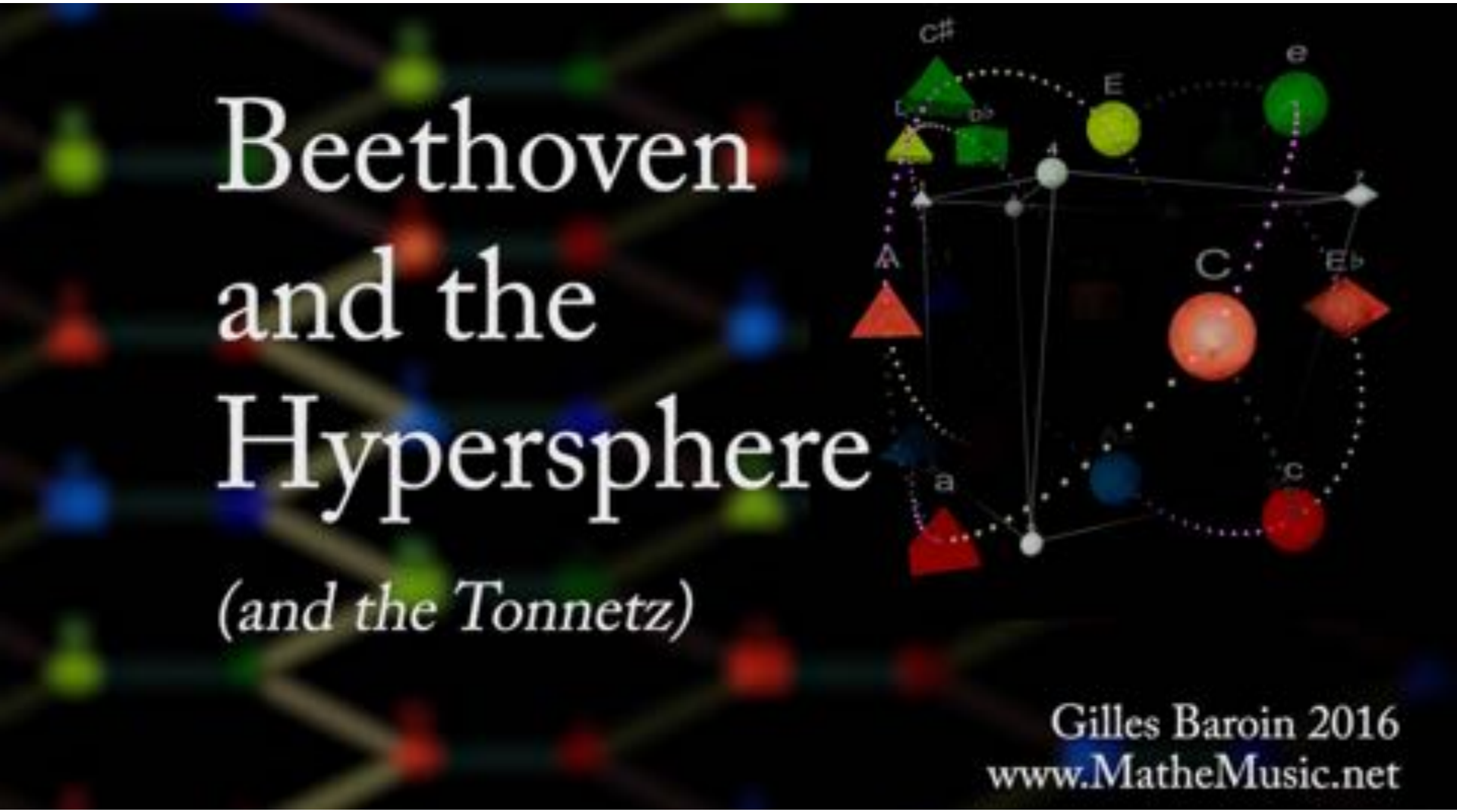

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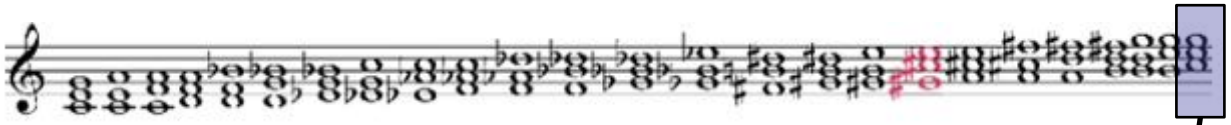
Reading Beethoven backwards



time

Le Blé en Herbe

(Polo/Moreno/Dieu)

Plonger comme un enfant, cheveux au vent
Sous l'océan du blé en herbe
Marée d'épis couleur d'amande
Qui tendent à caresser le ciel

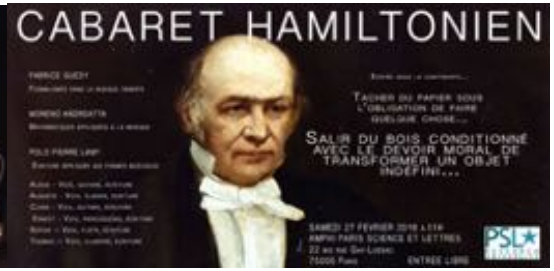
Algues tendres de mille plages
Frôlant le ventre des nuages
Cheveux de pluie, dos de poissons
Qui frissonnent à l'unisson

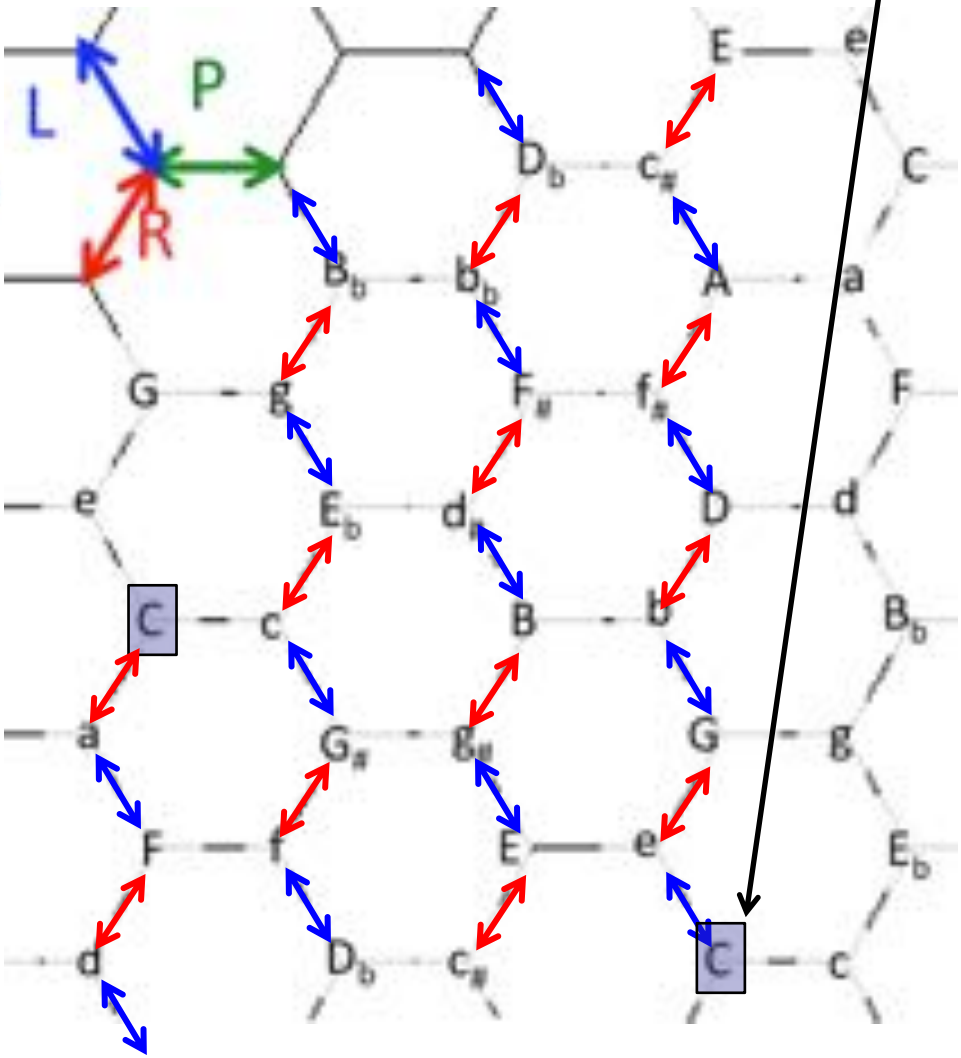
Suivre le bord des continents
Dans l'océan du blé en herbe
Pécher le corail du pavot
Dans le sang des coquelicots

Croiser matin dans l'herbe folle
Deux tourterelles qui s'envolent
Suivre les jeux des hirondelles
Sur le paysage éternel
Nager comme un enfant, cheveux au vent
Sous l'océan
Du blé en herbe

Marée de fruits au goût amer
Acide et salée comme la mer

Vers l'îlôt d'un petit village
Vers un château d'eau sur la plage
Quand tout s'éteint avant l'orage
Quand se lève le vent du large
Sur le blé vert





Aprile, a Hamiltonian « decadent » song

Do←do_m←Sol#←fa_m←Fa←la_m←La←$fa\#_m$←Fa#←sib_m←Do#←$do\#_m$

↘ La ↗ ↗

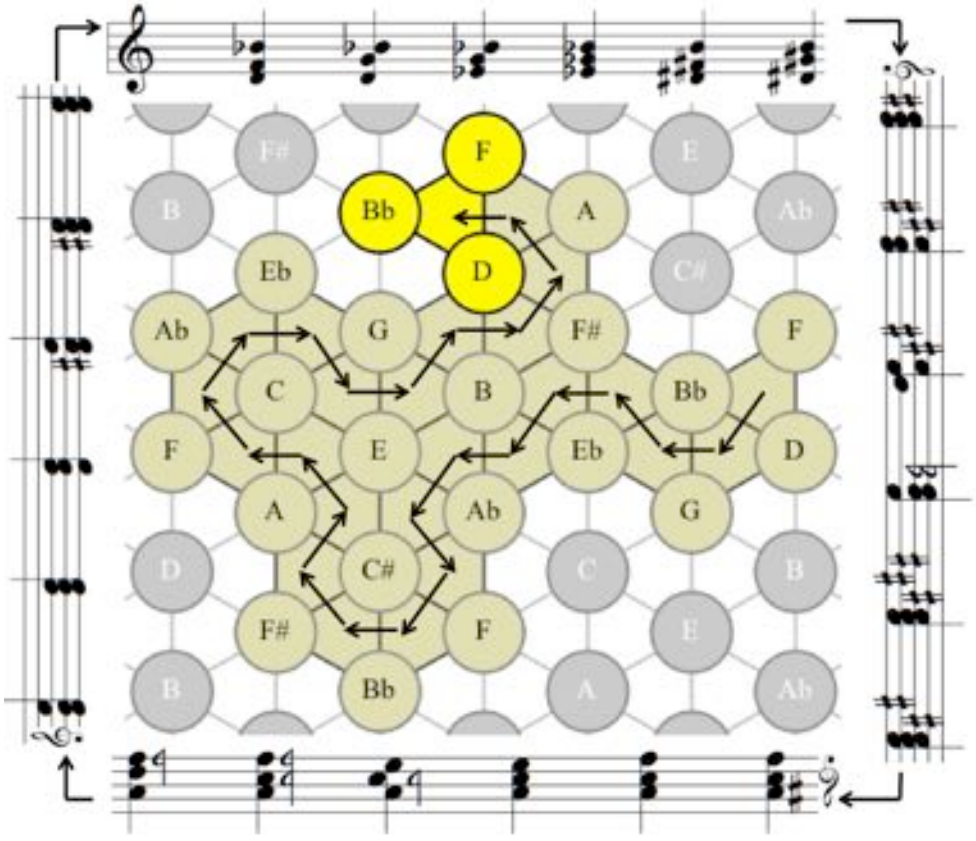
mi_m→Sol→si_m→Ré→$ré_m$→Sib→sol_m→Mib→mib_m→Si→$sol\#_m$→Mi

Socchiusa è la finestra, sul giardino.
Un'ora passa lenta, sonnolenta.
Ed ella, ch'era attenta, s'addormenta
A quella voce che già si lamenta,
Che si lamenta in fondo a quel giardino.

Non è che voce d'acque su la pietra:
E quante volte, quante volte udita!
Quell'amore e quell'ora in quella vita
S'affondan come ne l'onda infinita
Stretti insieme il cadavere e la pietra.

Ella stende l'angoscia sua nel sonno.
L'angoscia è forte, e il sonno è così lieve!
(Par la luce d'april quasi una neve
che sia tiepida.)
Ed ella certo deve soffrire,
Vagamente, anche nel sonno.

G. D'Annunzio (1863-1938)



ACTIONS Math'n'pop

Aprile (d'après Gabriele D'Annunzio)

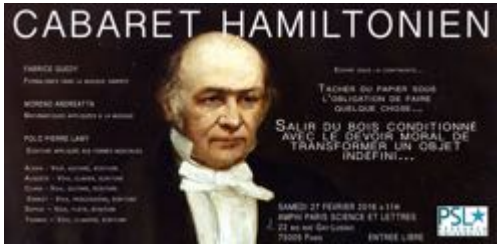
Tangente pour mieux comprendre le monde L'aventure mathématique

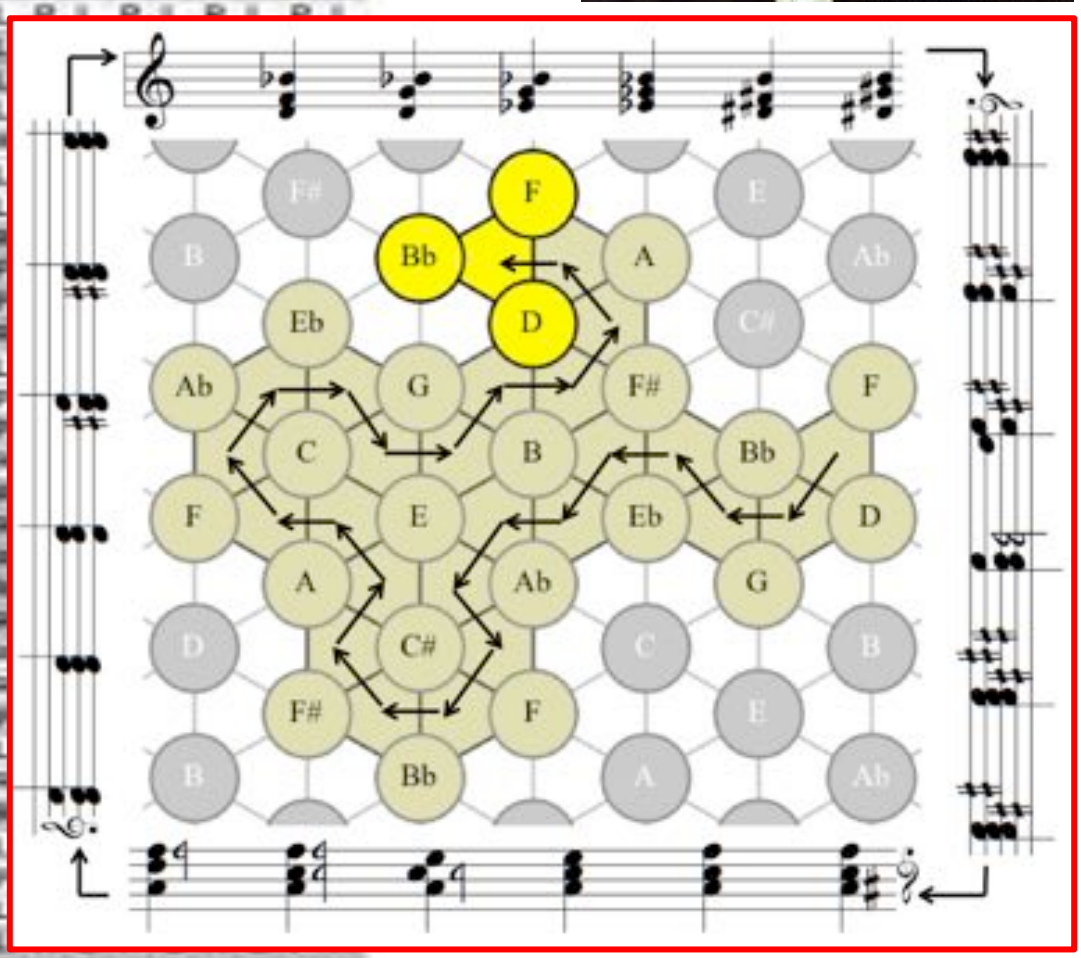
The collection of 124 Hamiltonian Cycles

ACTIONS Math'n'pop

Aprile (d'après Gabriele D'Annunzio)

Tangente pour mieux comprendre le monde — L'aventure mathématique





G. Albini & S. Antonini, University of Pavia, 2008

Aprile (d'après Gabriele D'Annunzio)

Tangente pour mieux comprendre le monde
L'aventure mathématique

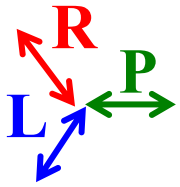
M. Andreatta, « Math'n pop : symétries et cycles hamiltoniens en chanson », *Tangente*

Hamiltonian Cycles with inner periodicities

8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR

L P L P L R …
P L P L R L …
L P L R L P …
P L R L P L …
L R L P L P …
R L P L P L …

min. 1'02"



Music: M. Andreatta
Arrangement and mix: M. Bergomi & S. Geravini (*Perfect Music Production*)
Mastering: A. Cutolo (Massive Arts Studio, Milan)

La sera non è più la tua canzone
(Mario Luzi, 1945, in *Poesie sparse*)

La sera non è più la tua canzone,
è questa roccia d'ombra traforata
dai lumi e dalle voci senza fine,
la quiete d'una cosa già pensata.

Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d'una cosa conosciuta,
per nome hai una parola ch'è passata
nell'intimo del cuore e s'è perduta.

Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell'essere, (compiuta).

L'aria tace ed il tempo dietro a te
si leva come un'arida montagna
dove vaga il tuo spirito e si perde,
un vento raro scivola e ristagna.

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From poetry to song writing: hamiltonian compositional strategies

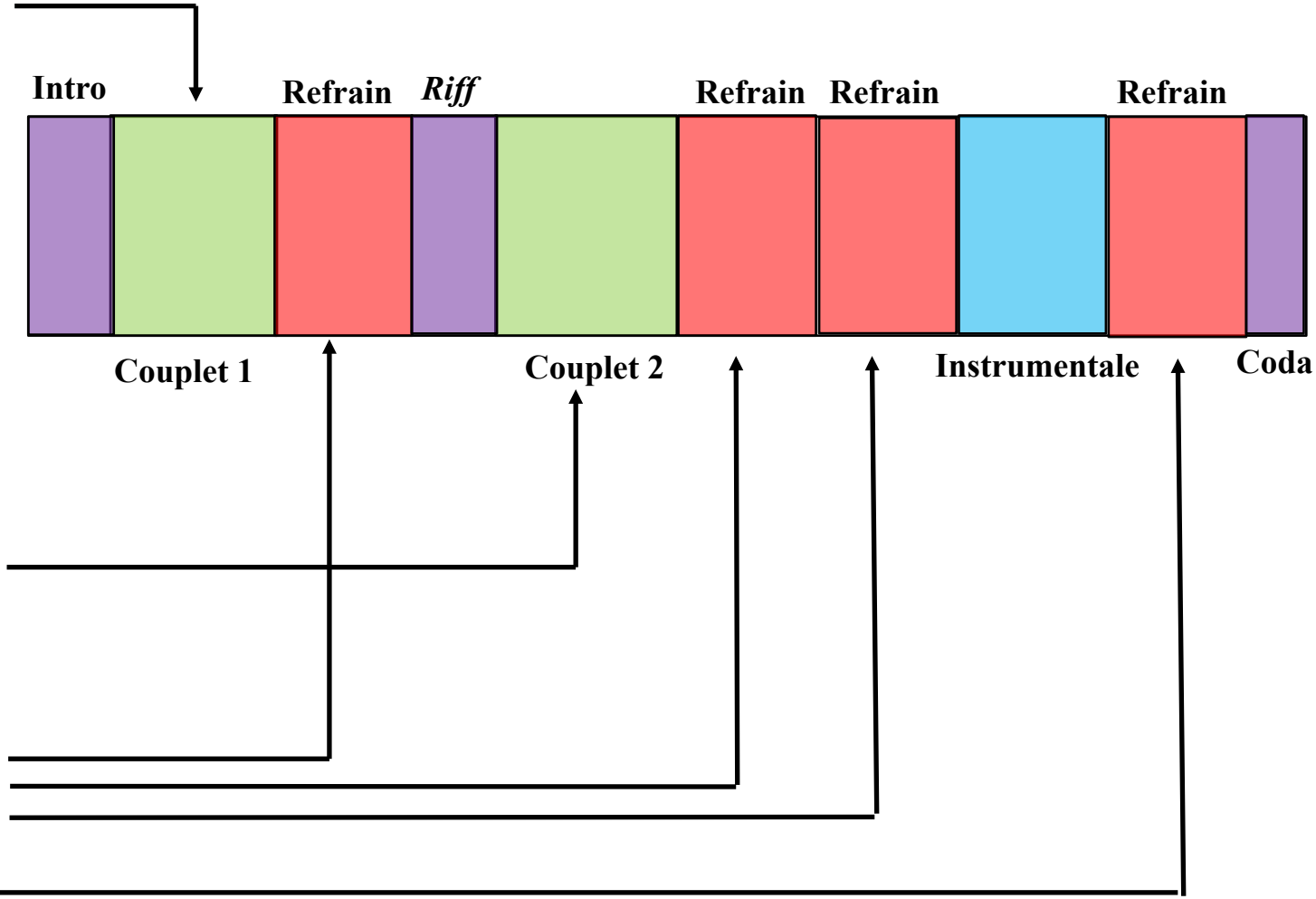
A part (Andrée Chedid, poème tiré du recueil *Rhymes* Collection Poésie/Gallimard (n. 527), Gallimard, 2018)

À part le temps
Et ses rouages
À part la terre
En éruptions
À part le ciel
Pétrisseur de nuages
À part l'ennemi
Qui génère l'ennemi

À part le désamour
Qui ronge l'illusion
À part la durée
Qui moisit nos visages

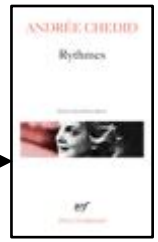
À part les fléaux
À part la tyrannie
À part l'ombre et le crime
Nos batailles nos outrages

Je te célèbre ô Vie
Entre cavités et songes
Intervalle convoité
Entre le vide et le rien

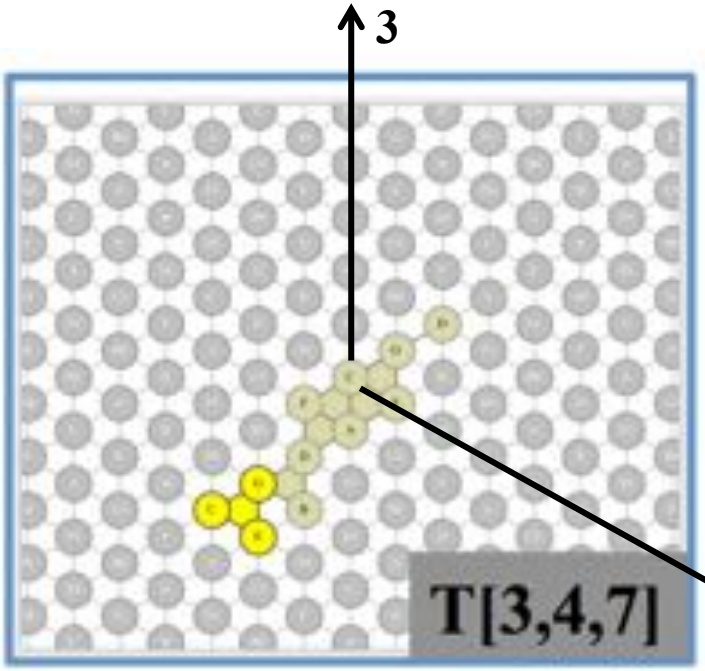


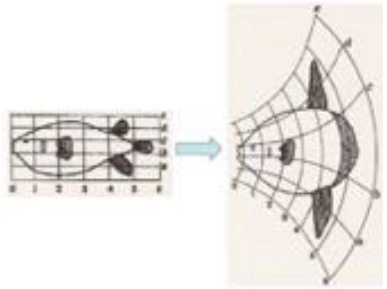
The catalogue of 28 hamiltonian cycles (with inner symmetry)

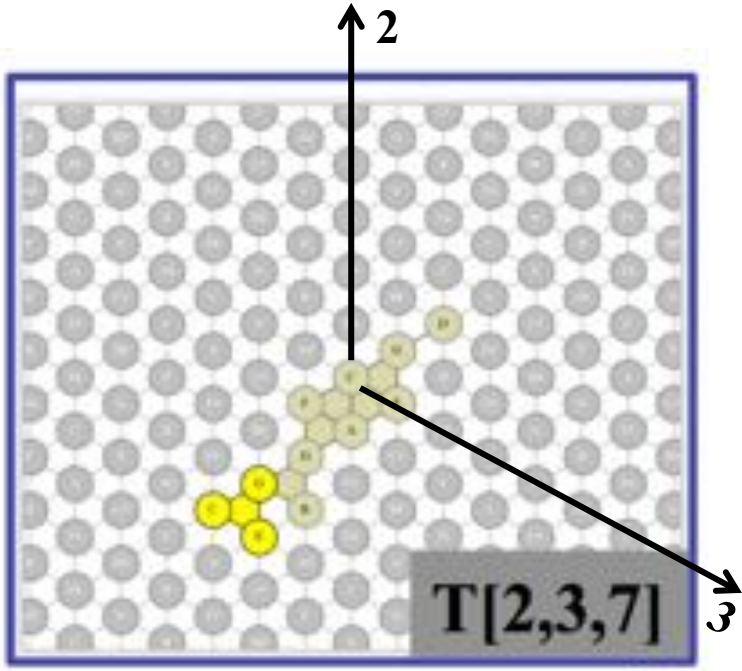
1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPRPR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPRLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRLRPRP
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPRL
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRLRLRP
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm--RLRPRPRP
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em--RLRPRPRL
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLRP
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm--RLRLRPRP
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL

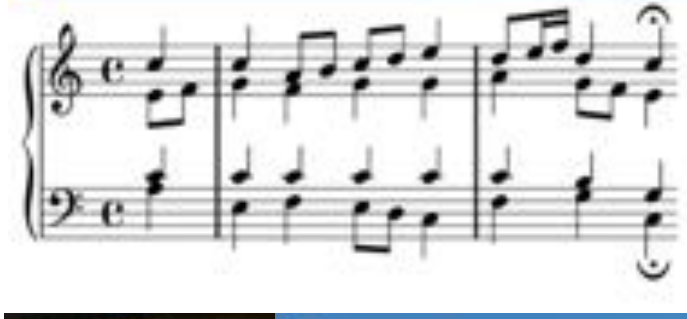


The musical style...is the space!

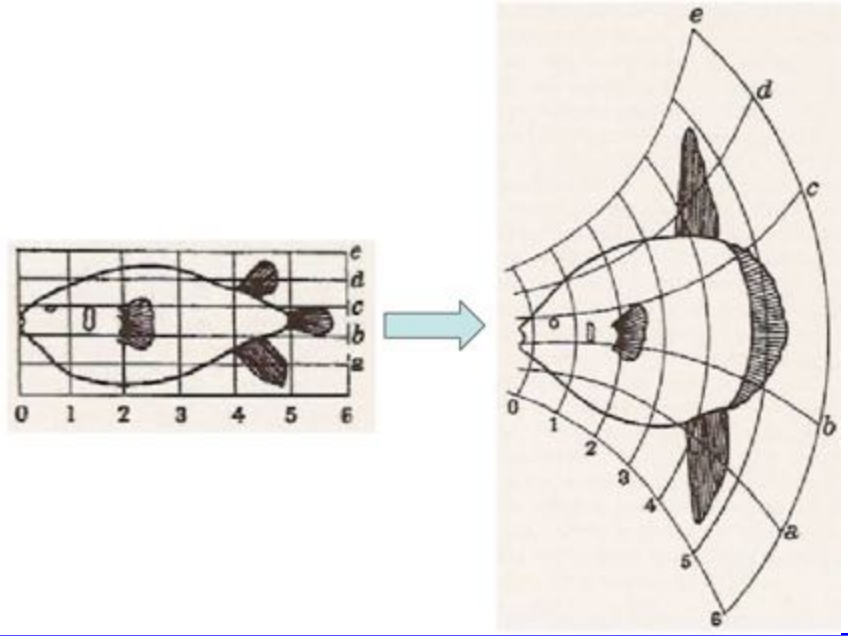




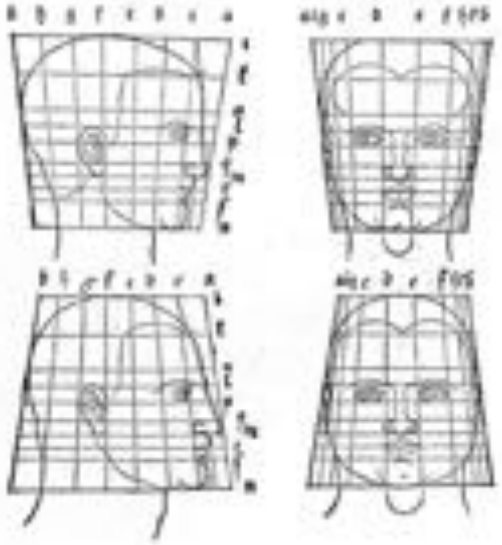




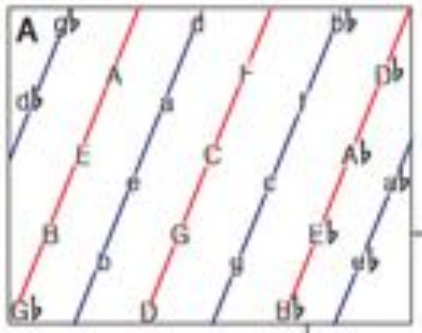
The morphological *vs* the mathematical genealogy of the structuralism



"[The notion of **transformation**] comes from a work which played for me a very important role and which I have read during the war in the United States : *On Growth and Form*, in two volumes, by **D'Arcy Wentworth Thompson**, originally published in 1917. The author (...) proposes an interpretation of the visible transformations between the species (animals and vegetables) within a same gender. This was fascinating, in particular because I was quickly realizing that this perspective had a long tradition: behind Thompson, there was **Goethe's** botany and behind Goethe, **Albert Dürer** with his *Treatise of human proportions*" (Lévi-Strauss, conversation with Eribon, 1988).



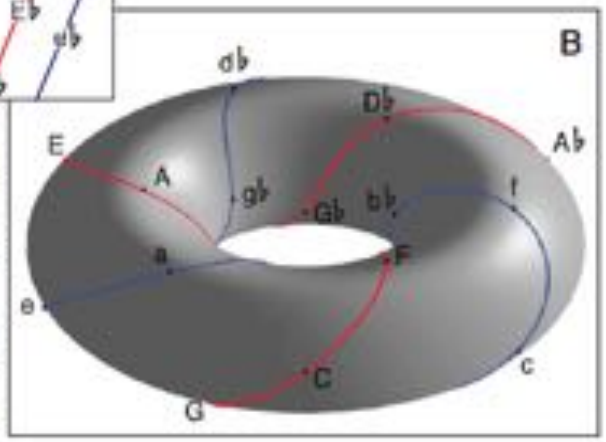
Some cognitive implications of mathemusical research



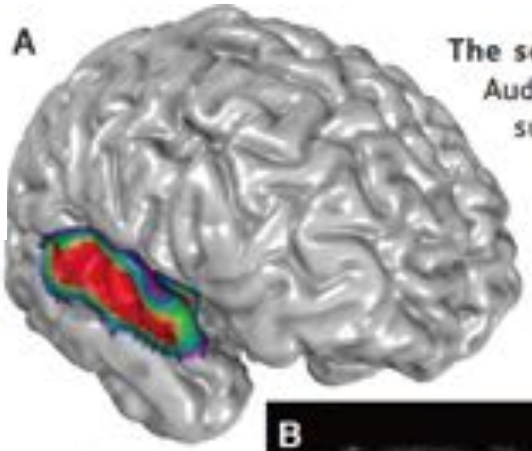
PERSPECTIVES: NEUROSCIENCE

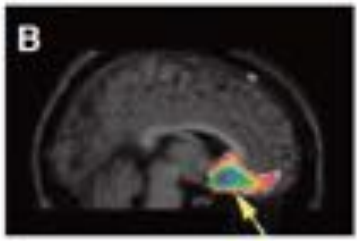
Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl

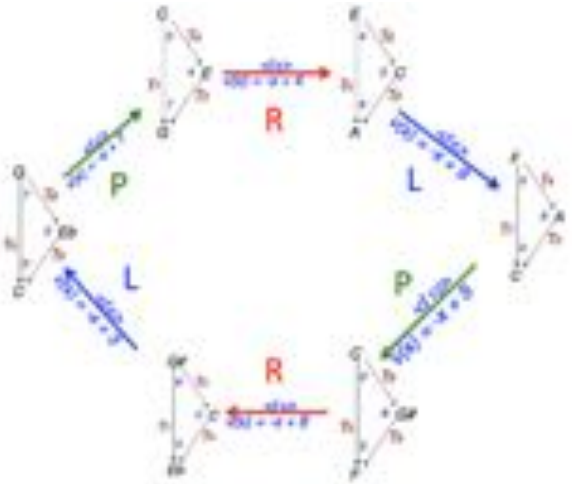


Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (B) Musical keys as points on the surface of a torus.





The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

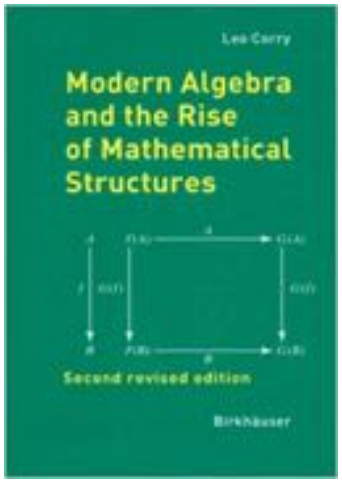


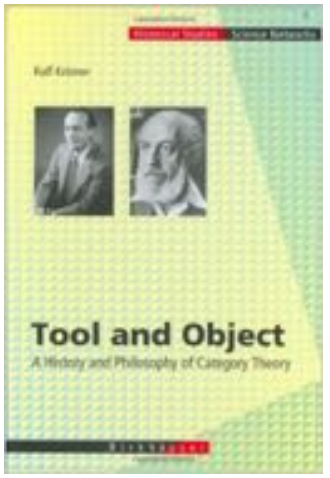
« La **théorie des catégories** est une théorie des constructions mathématiques, qui est macroscopique, et procède d'étage en étage. Elle est un bel exemple d'**abstraction réfléchissante**, cette dernière reprenant elle-même un principe constructeur présent dès le stade sensori-moteur. Le **style catégoriel** qui est ainsi à l'image d'un aspect important de la genèse des facultés cognitives, est un style adéquat à la description de cette genèse »

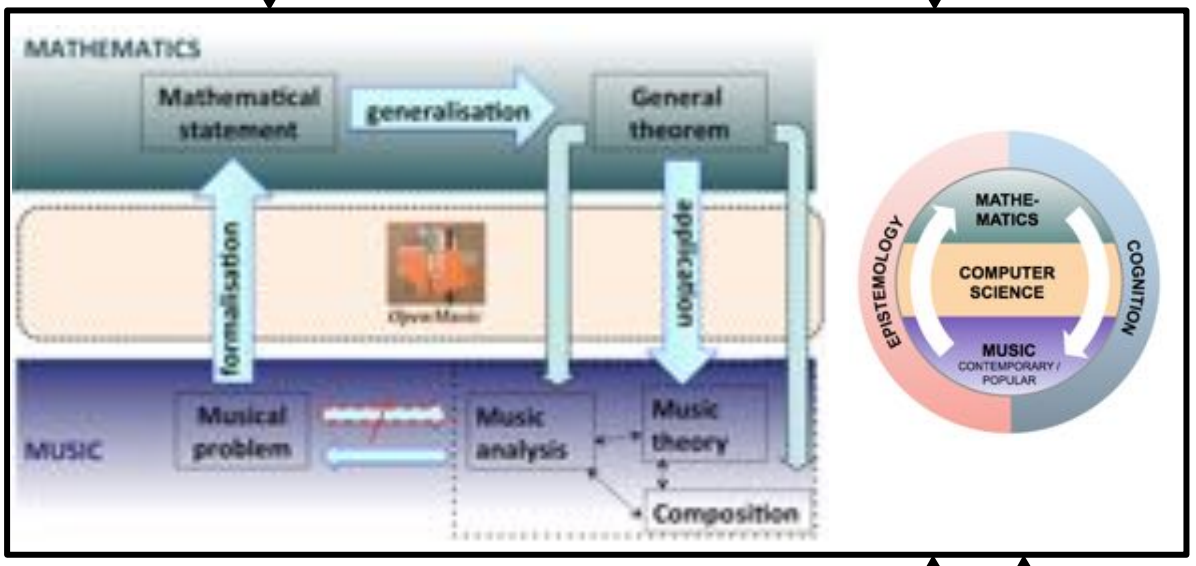
J. Piaget

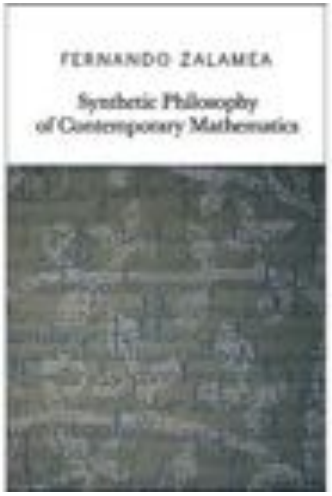
Jean Piaget, Gil Henriques et Edgar Ascher, ***Morphismes et Catégories. Comparer et transformer*****, 1990**

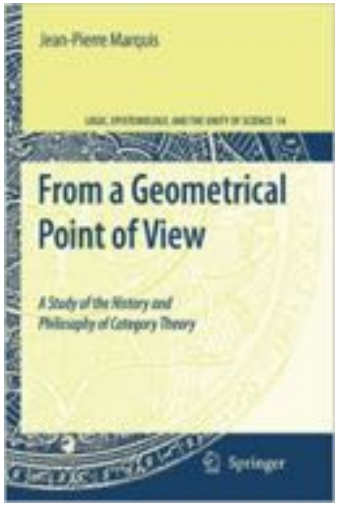
Which type of philosophy for the *mathemusical* research?











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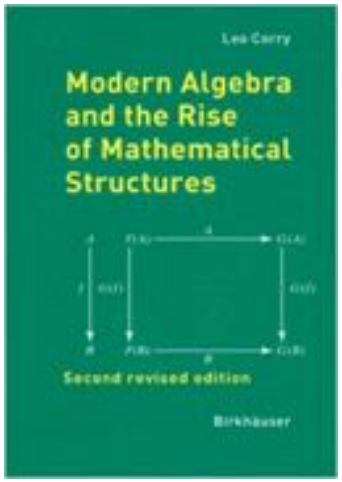
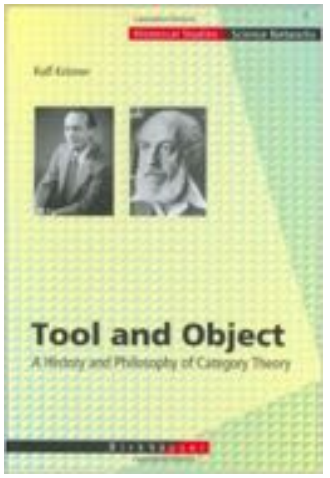
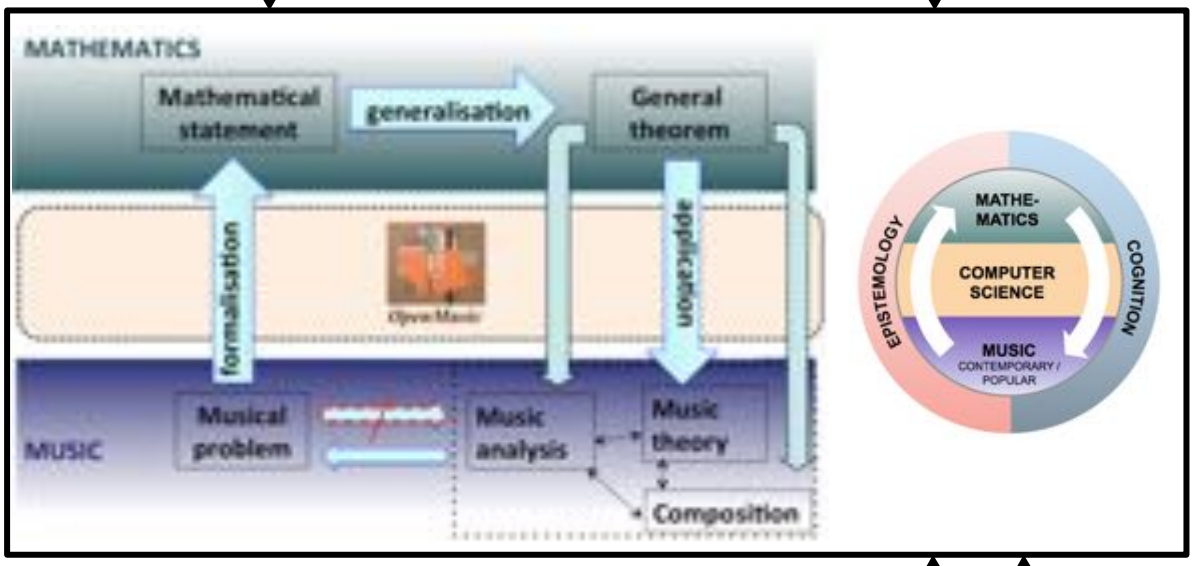
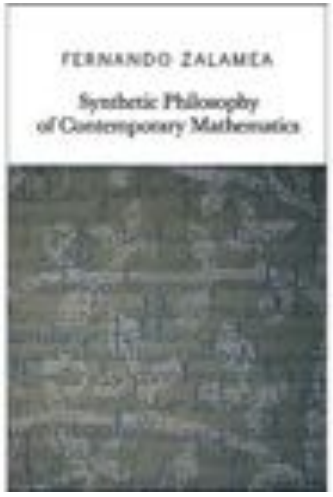
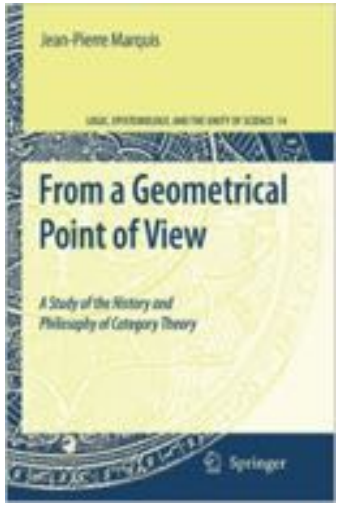




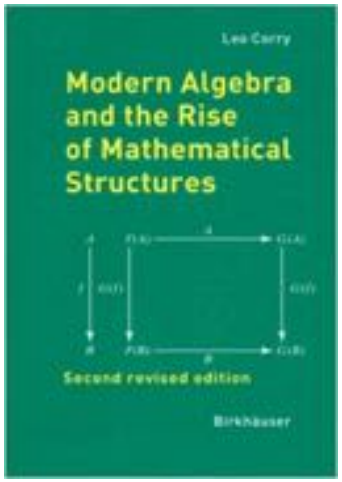


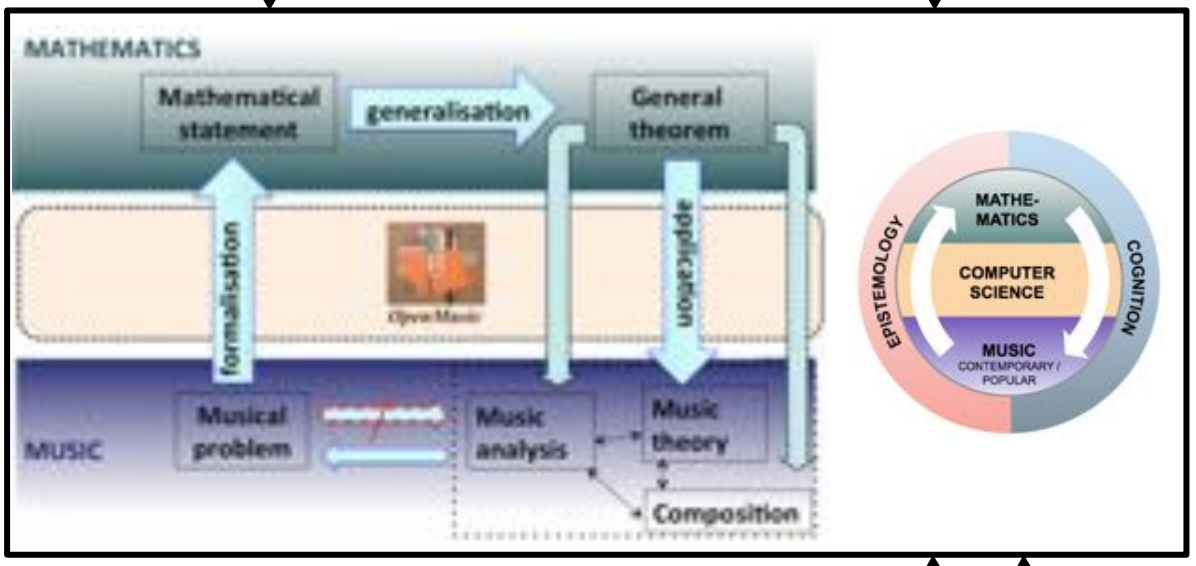


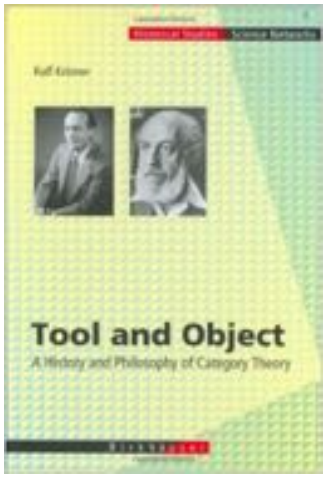
*A **synthetic** vision allows us to link together apparently distant strata of mathematics and culture, helping us to break down many artificial barriers. Not only can today's mathematics be appreciated through epistemic, ontic, phenomenological and aesthetic modes, but in turn, it should help to transform **philosophy.***

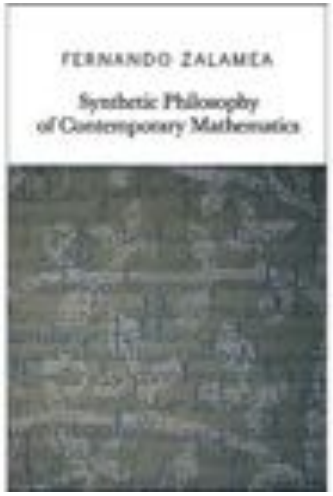


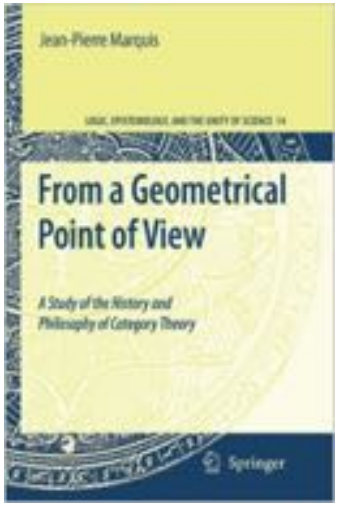
Which type of philosophy for the *mathemusical* research?











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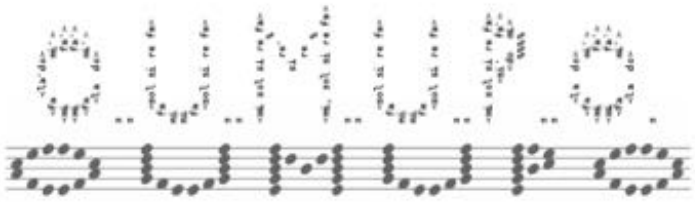
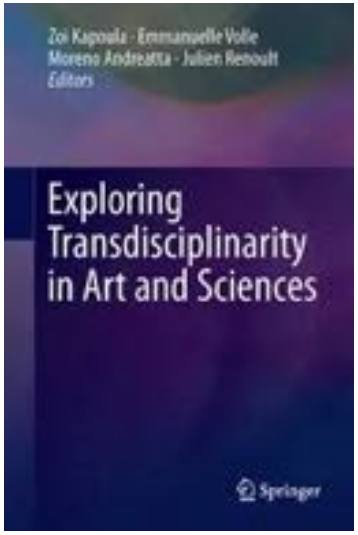
From the OuLiPo to the OuMuPo (ouvroir de musique potentielle)



http://oumupo.org/



M. Andreatta et al., « Music, mathematics and language: chronicles from the Oumupo sandbox », in Kapoula, Z., Volle, E., Renoult, J., Andreatta, M. (Eds.), *Exploring Transdisciplinarity in Art and Sciences*, Springer, 2018

Valentin Villenave

Mike Solomon

Jean-François Piette

Martin Granger

Joseph Boisseau

Moreno Andreatta

Tom Johnson

Symmetries and algorithmic processes in *Muse*

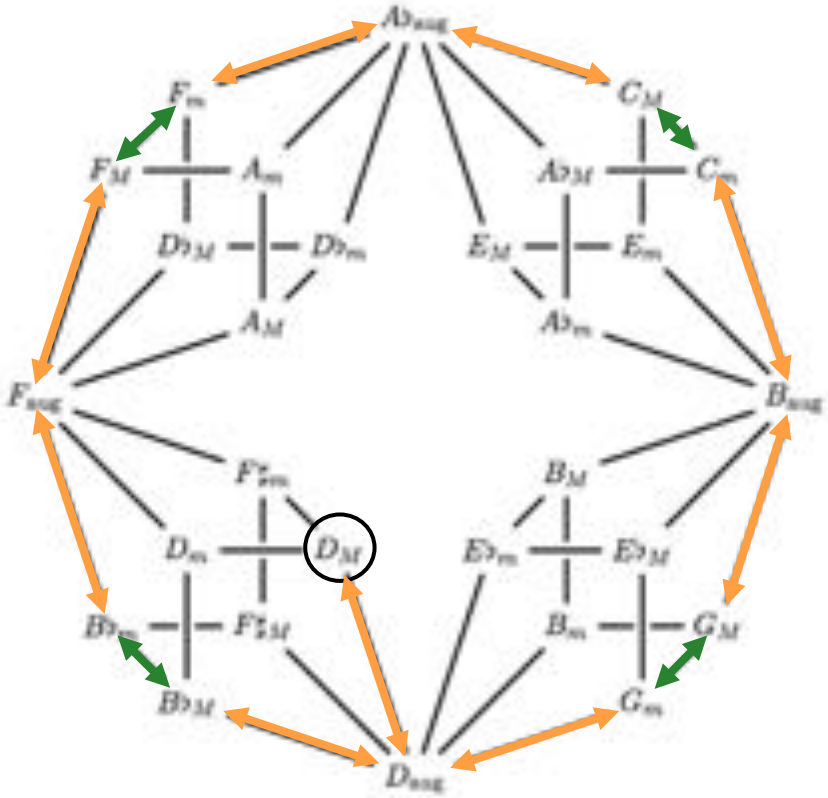
"Take a bow" (*Black Holes and Revelations*, 2006)

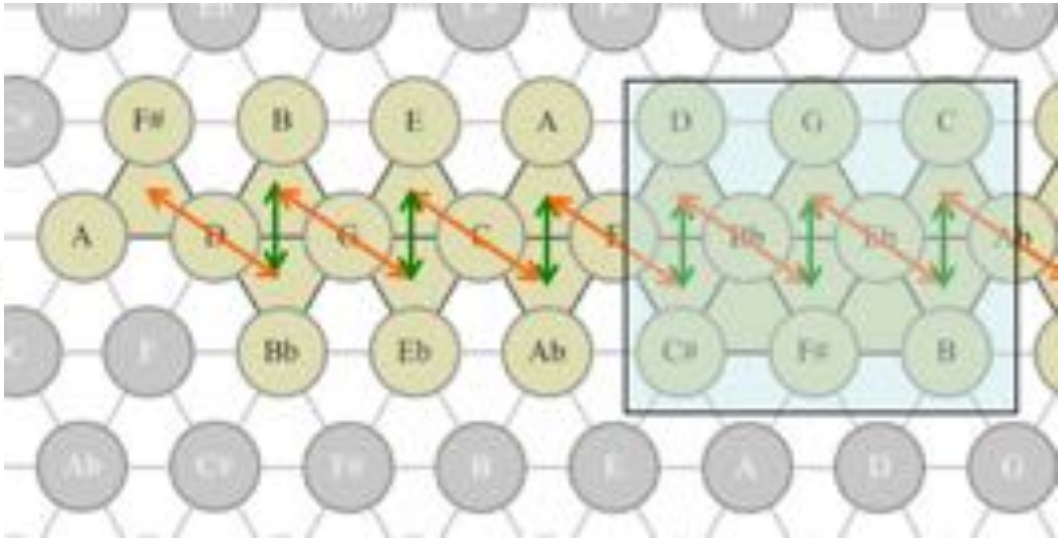
Louis Bigo

Hexachord (Louis Bigo, 2013)

Temporal axis

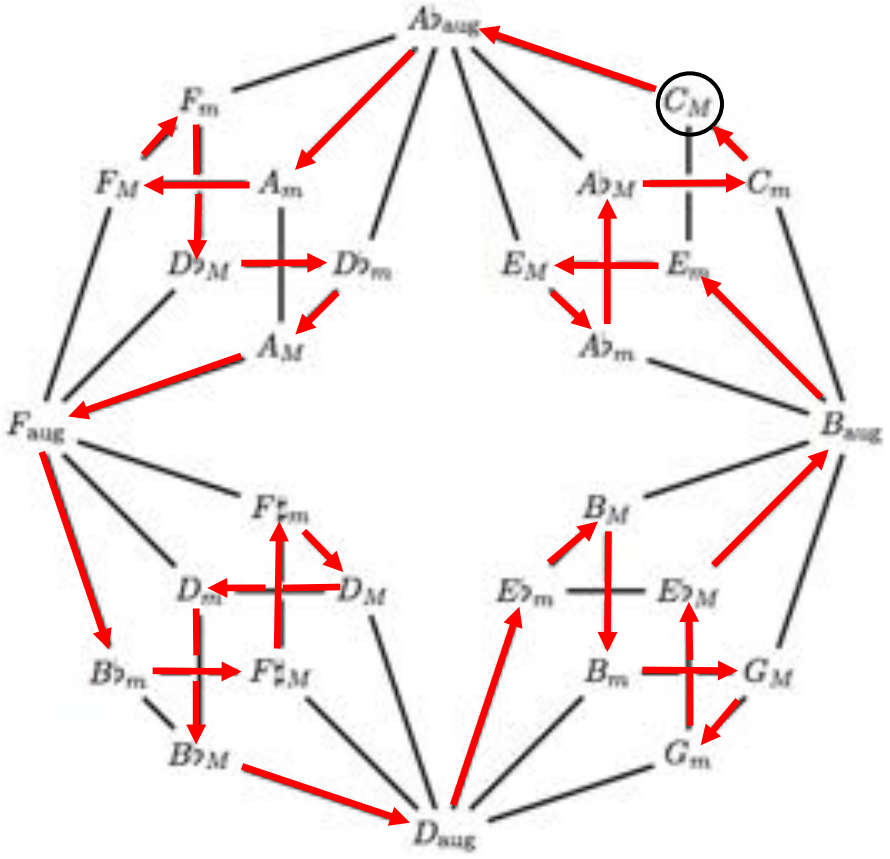
Representing *Muse's* progression in the Cube Dance





J. Douthett, P. Steinbach, Parsimonious Graphs: A Study in Parsimony, Contextual Transformation, and Modes of Limited Transposition, *Journal of Music Theory*, 42/2, 1998.

The Gunner's Hamiltonian Dream (an *oumoupian* experiment on a song by Pink-Floyd)



The Gunner's dream (R. Waters, 1983 / M. Andreatta, 2018)

C C+
Floating down through the clouds
Am F
Memories come rushing up to meet me now.
Fm
In the space between the heavens
C# C#m
and in the corner of some foreign field
A F+ Bbm
I had a dream.
F# F#m D Dm
I had a dream.
Bb
Good-bye Max.
D+
Good-bye Ma.
Ebm B
After the service when you're walking slowly to the car
Bm G
And the silver in her hair shines in the cold November air

Gm
You hear the tolling bell
Eb
And touch the silk in your lapel
G+ Em E G#m
And as the tear drops rise to meet the comfort of the band
G# Cm
You take her frail hand
C
And hold on to the dream.

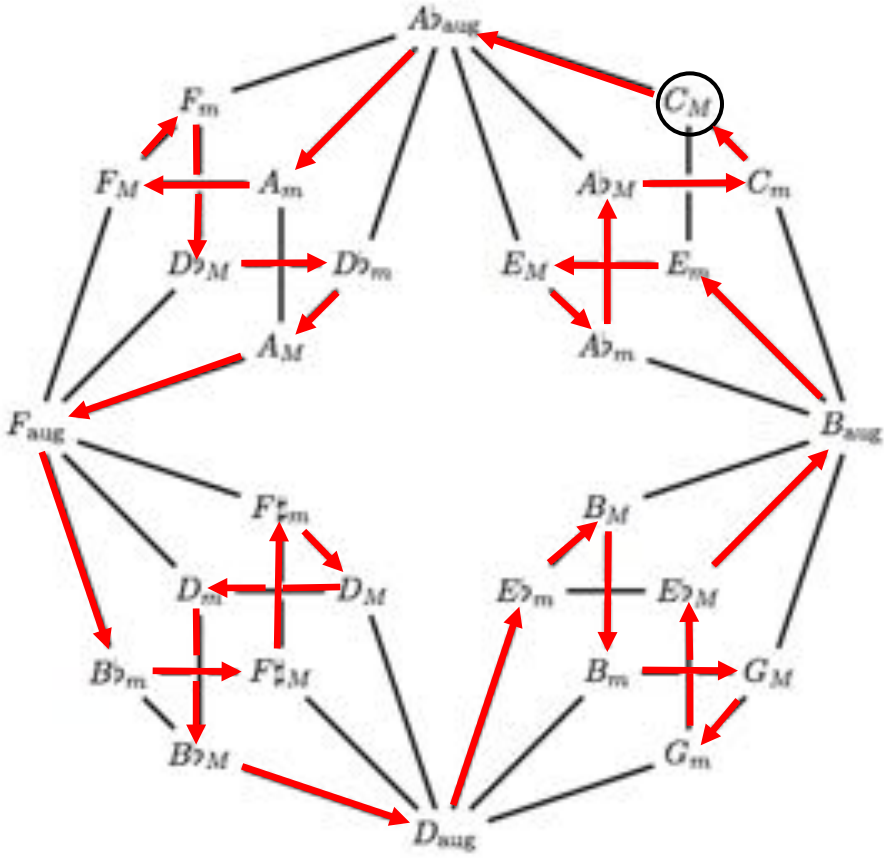
The three main hamiltonian cycles (C_M = C, C_m = Cm, C_{aug} = C+)

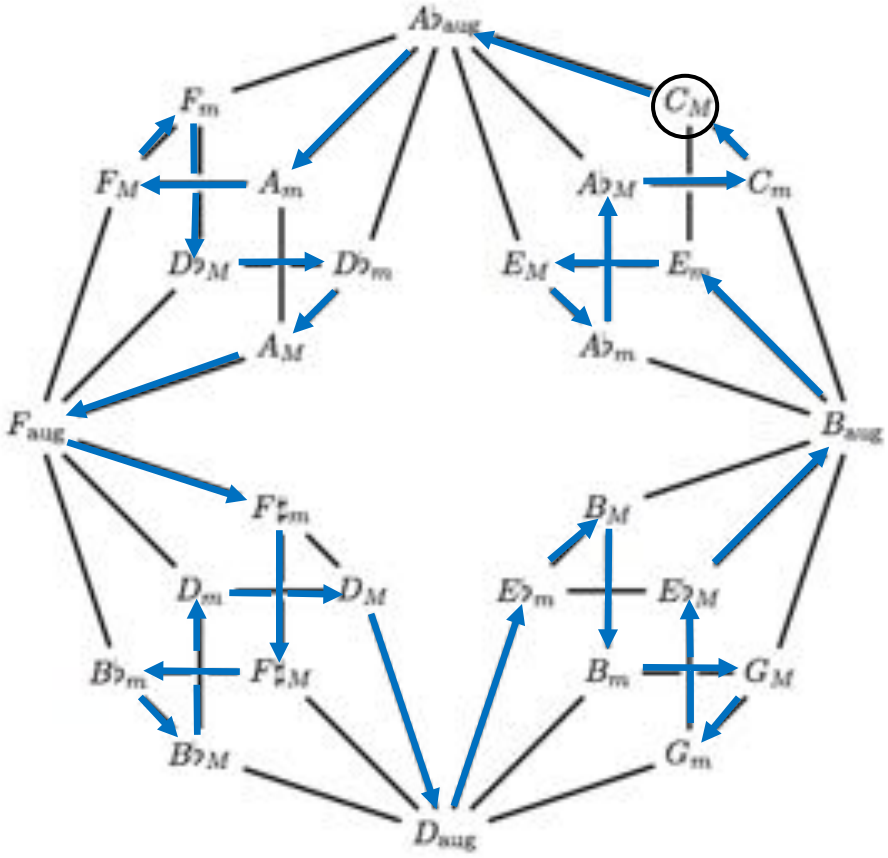
C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->Bbm-->F#-->F#m-->D-->Dm-->Bb-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->F#-->Bbm-->Bb-->Dm-->D-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->D-->Dm-->Bb-->Bbm-->F#-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Cm-->G#-->G#m-->E-->Em-->C

The Gunner's Hamiltonian Dream (an *oumoupian* experiment on a song by Pink-Floyd)





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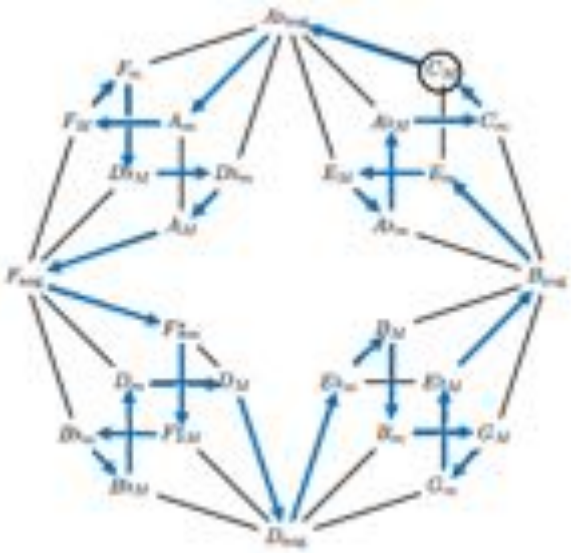
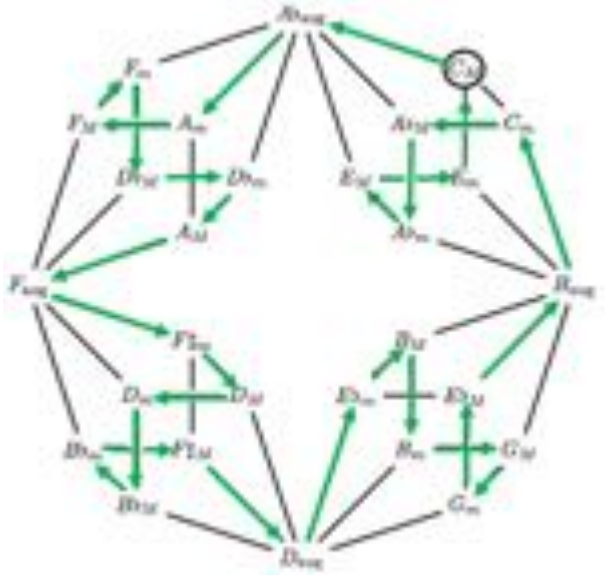
C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->Bbm-->F#-->F#m-->D-->Dm-->Bb-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->F#-->Bbm-->Bb-->Dm-->D-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->D-->Dm-->Bb-->Bbm-->F#-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Cm-->G#-->G#m-->E-->Em-->C

The Gunner's Hamiltonian Dream (an *oumoupian* experiment on a song by Pink-Floyd)





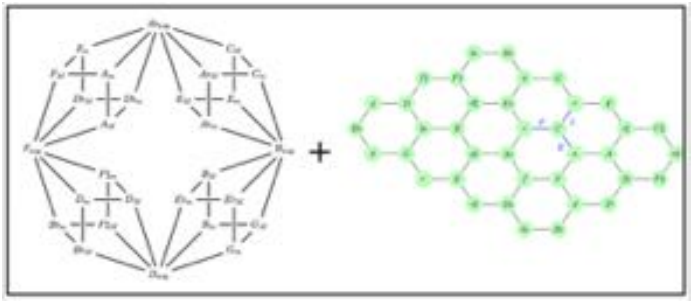
The three main hamiltonian cycles (C_M = C, C_m = Cm, C_{aug} = C+)

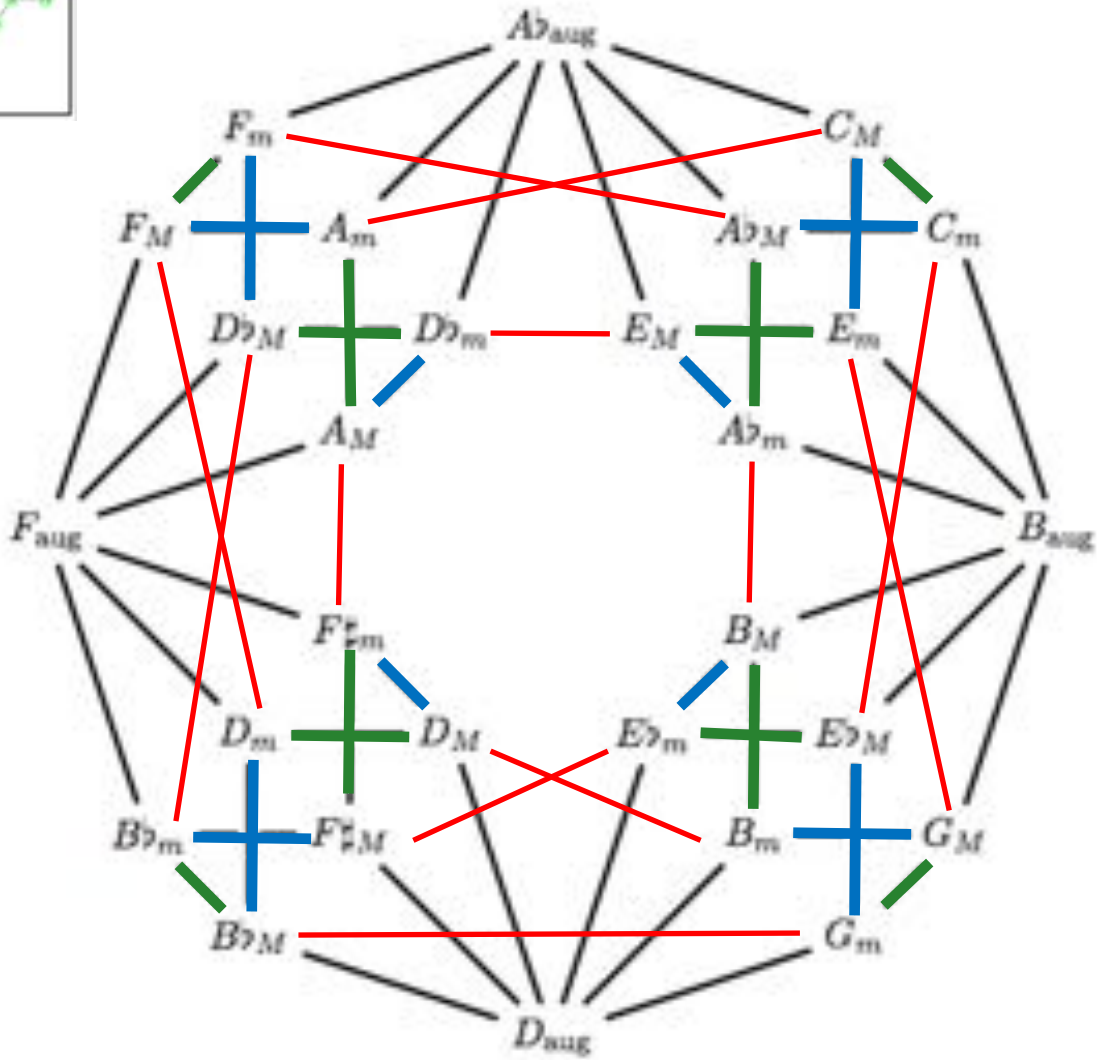
C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->Bbm-->F#-->F#m-->D-->Dm-->Bb-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->F#-->Bbm-->Bb-->Dm-->D-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

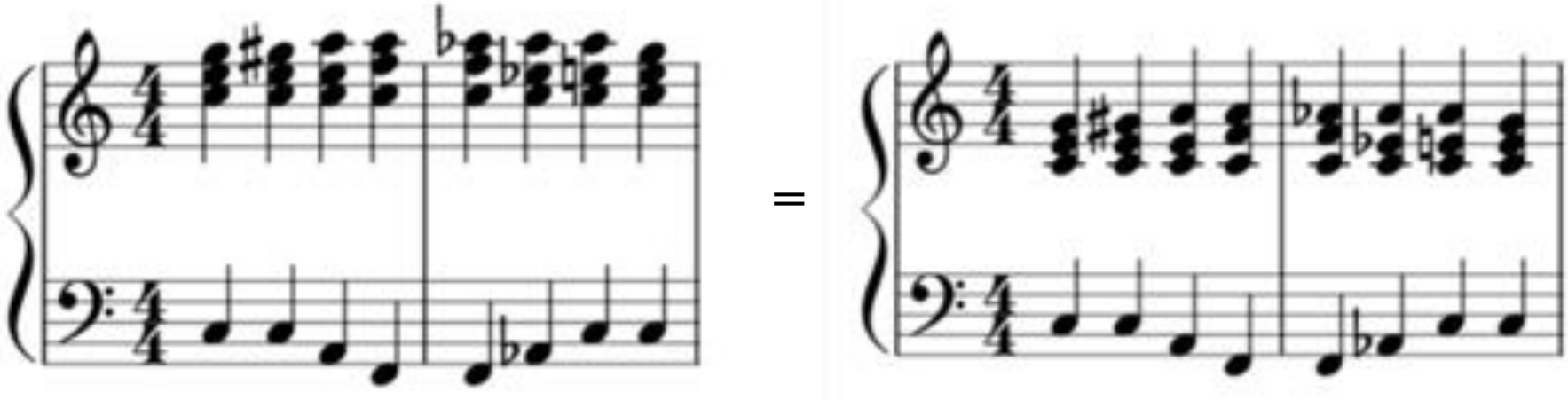
C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->D-->Dm-->Bb-->Bbm-->F#-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Cm-->G#-->G#m-->E-->Em-->C

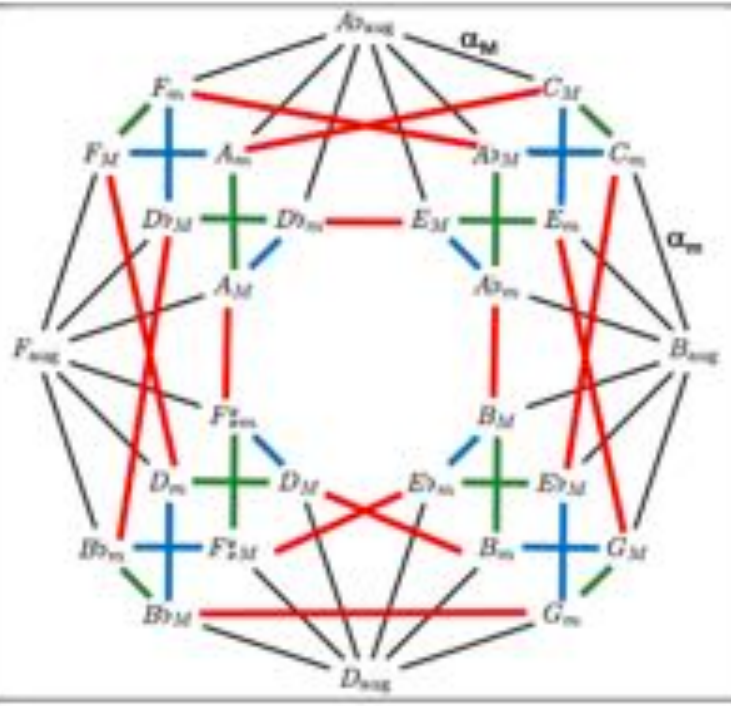
Embedding the Cube Dance into the Tonnetz





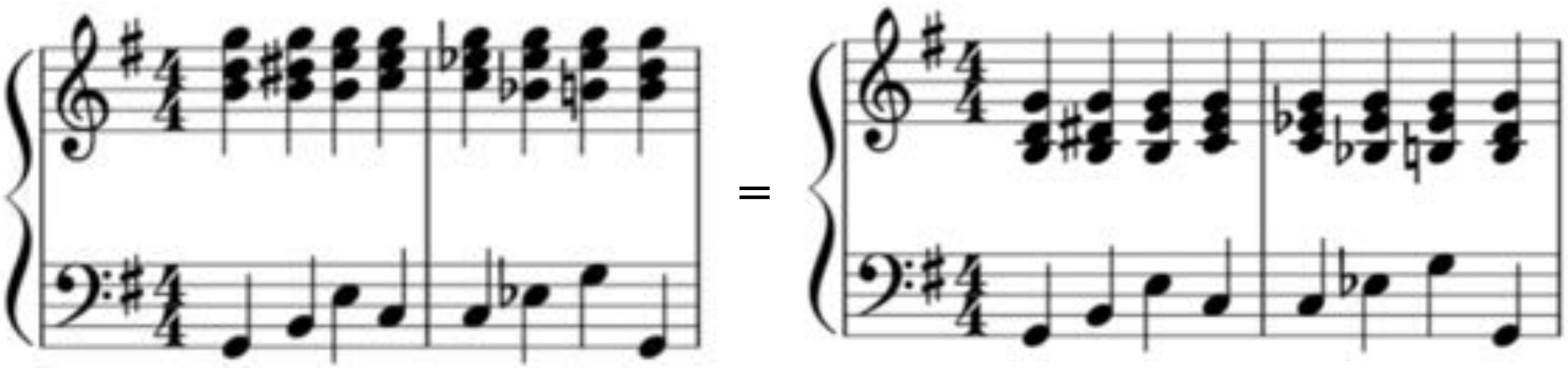
➔ **http://repmus.ircam.fr/moreno/chanson**

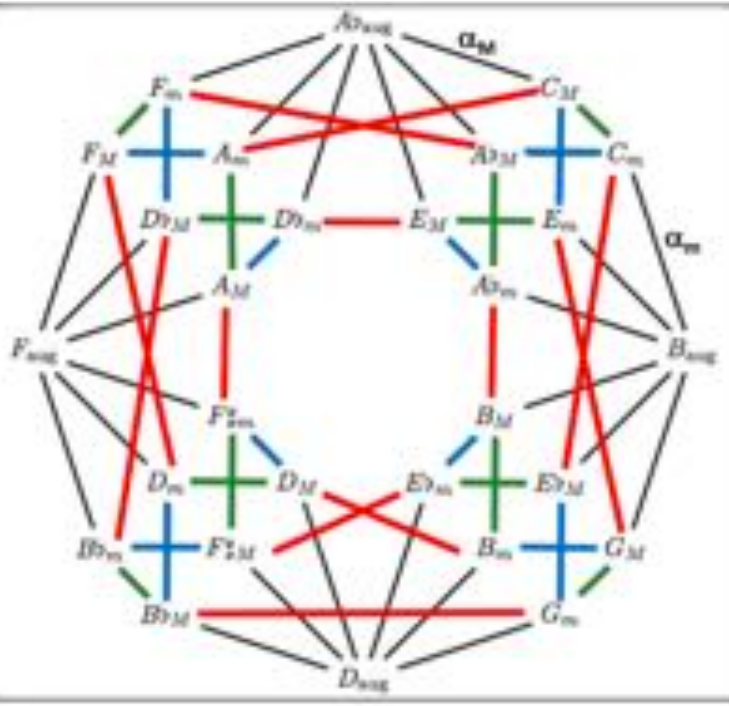




Suite d'accords et d'opérateurs P, L, R, α_M ou α_m, sur le modèle :

Accord #1 —(operateur #1)→ **Accord #2** —(operateur #2)→ **…** —(operateur #7)→ **Accord #8**

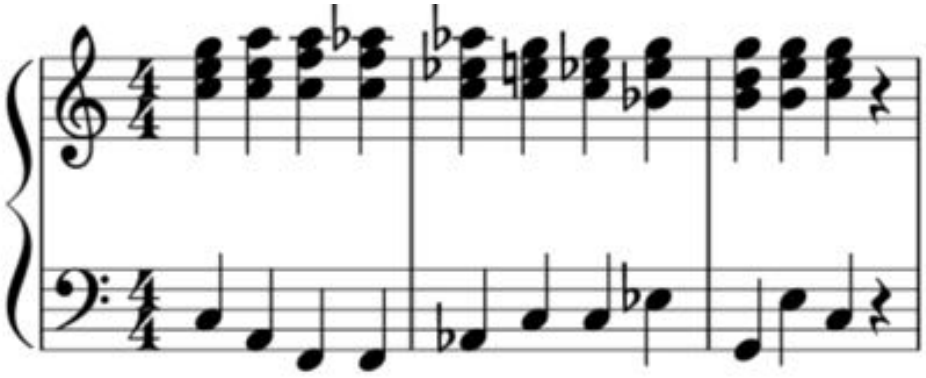


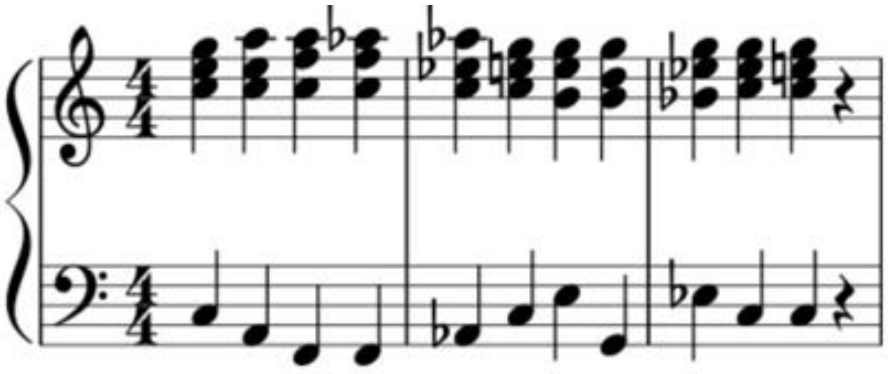


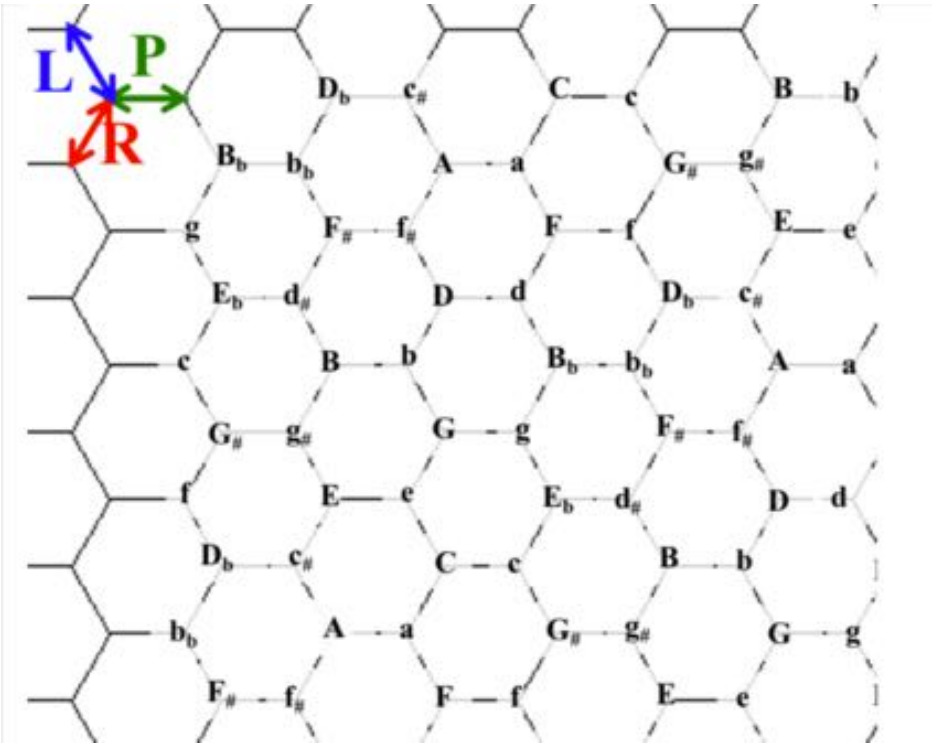
Suite d'accords et d'opérateurs P, L, R, α_M ou α_m, sur le modèle :

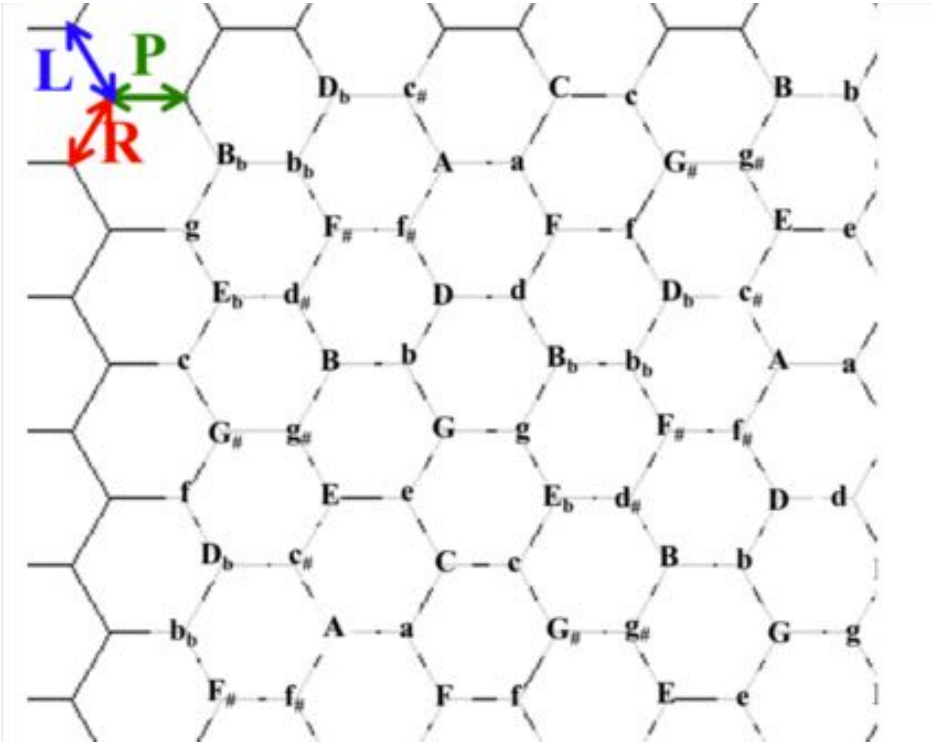
operateur #1 operateur #2 operateur #7

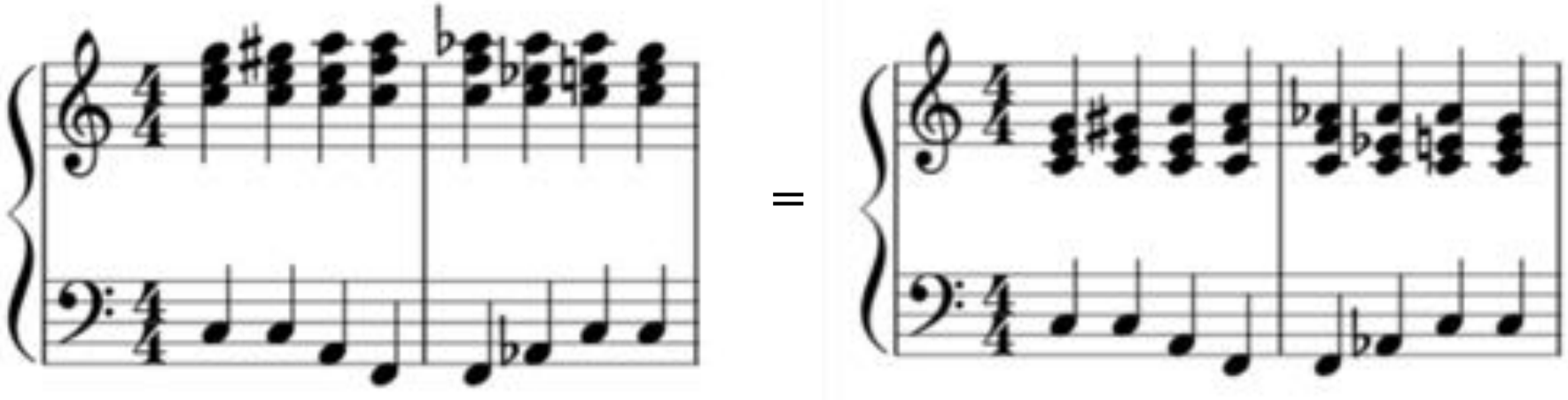
Accord #1 → Accord #2 → … → Accord #8

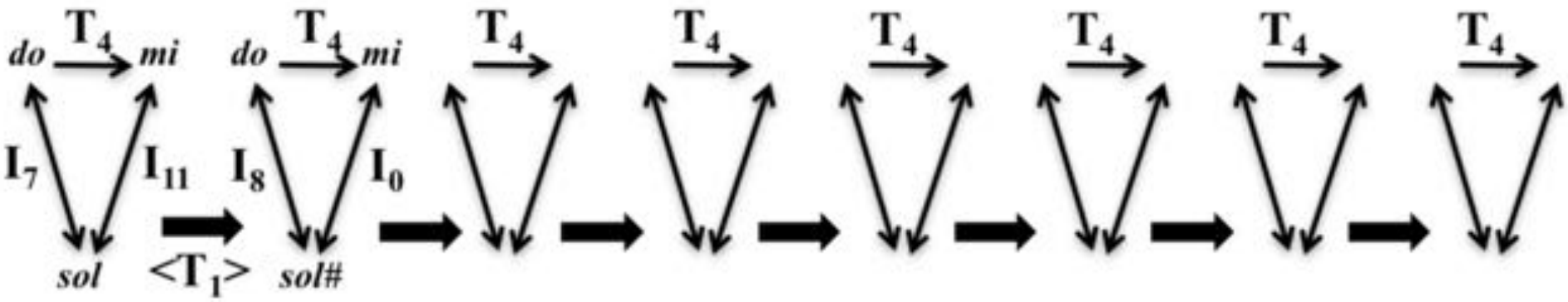





L
P
R


L
P
R

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mi
do
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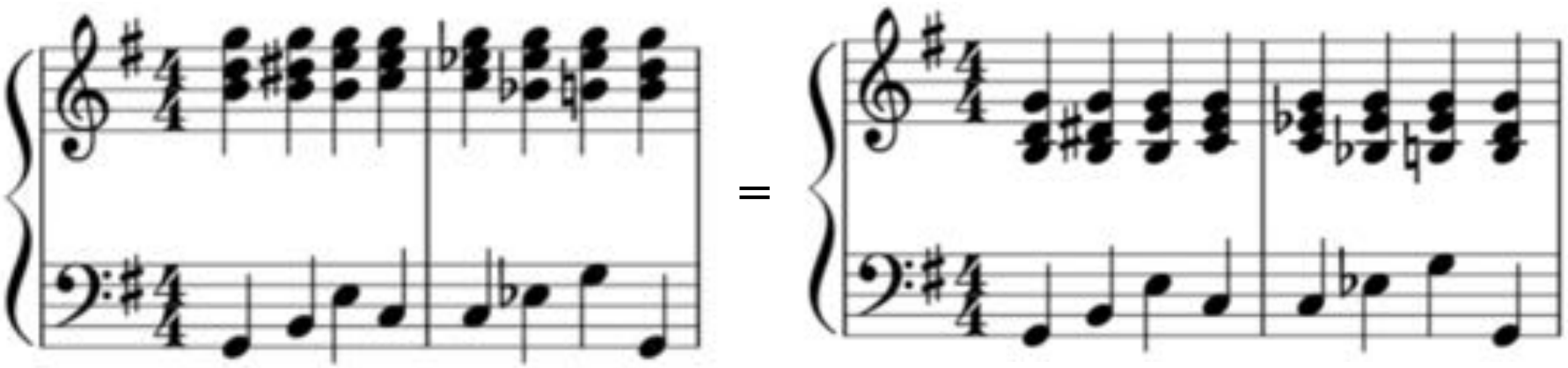
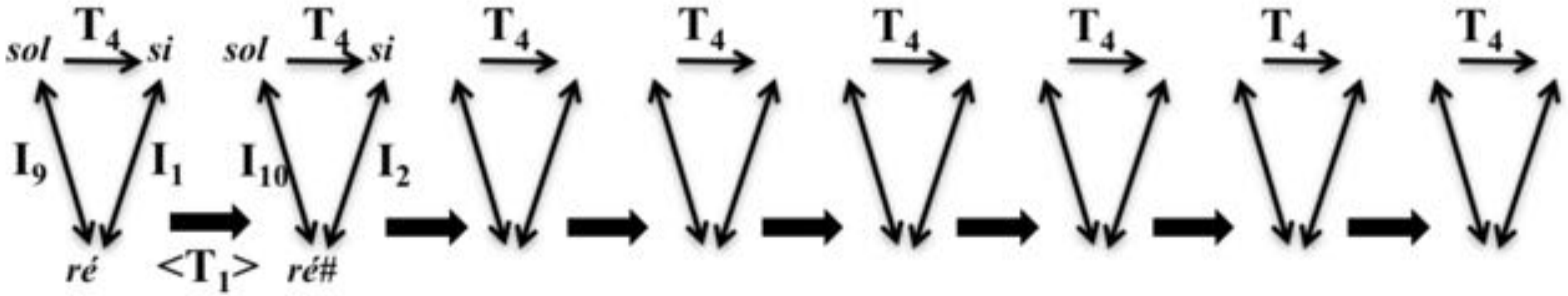
T4
T4
I7
I11
I8
I0
sol
<T1>
sol#




sol
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I9
I1
I10
I2
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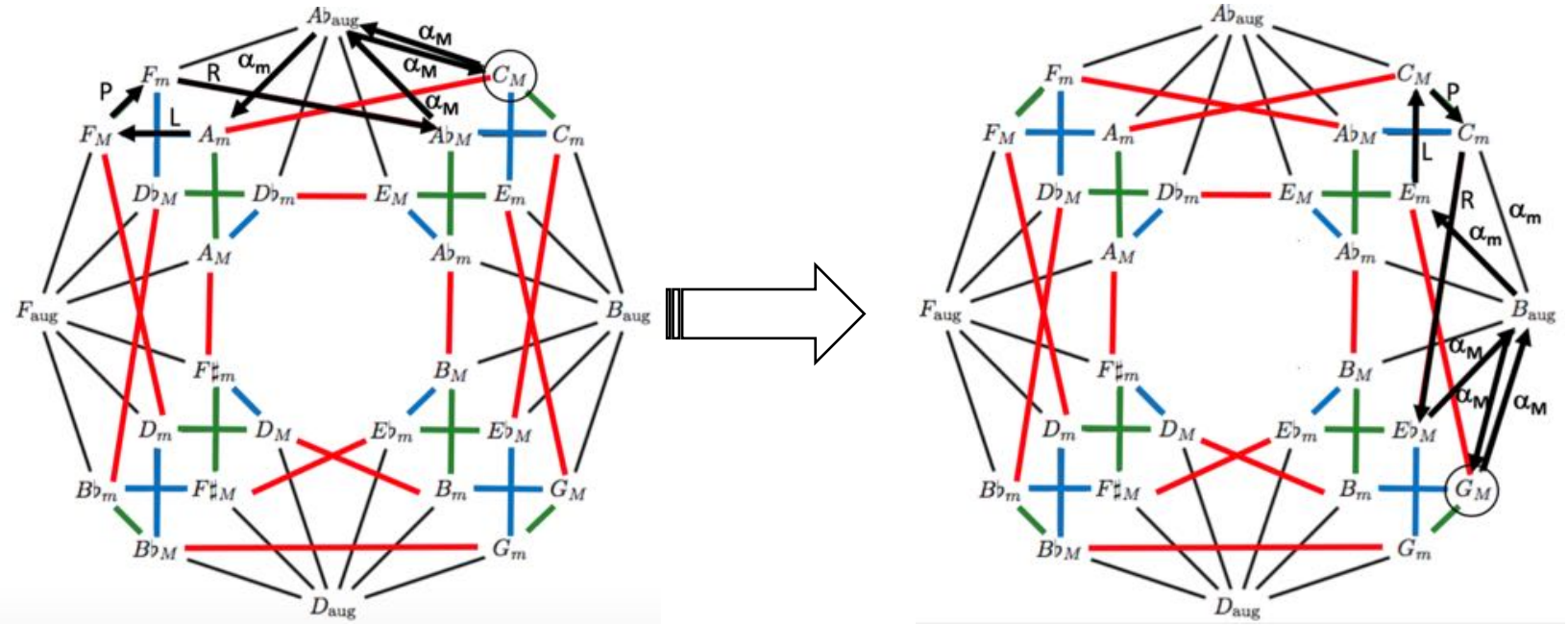
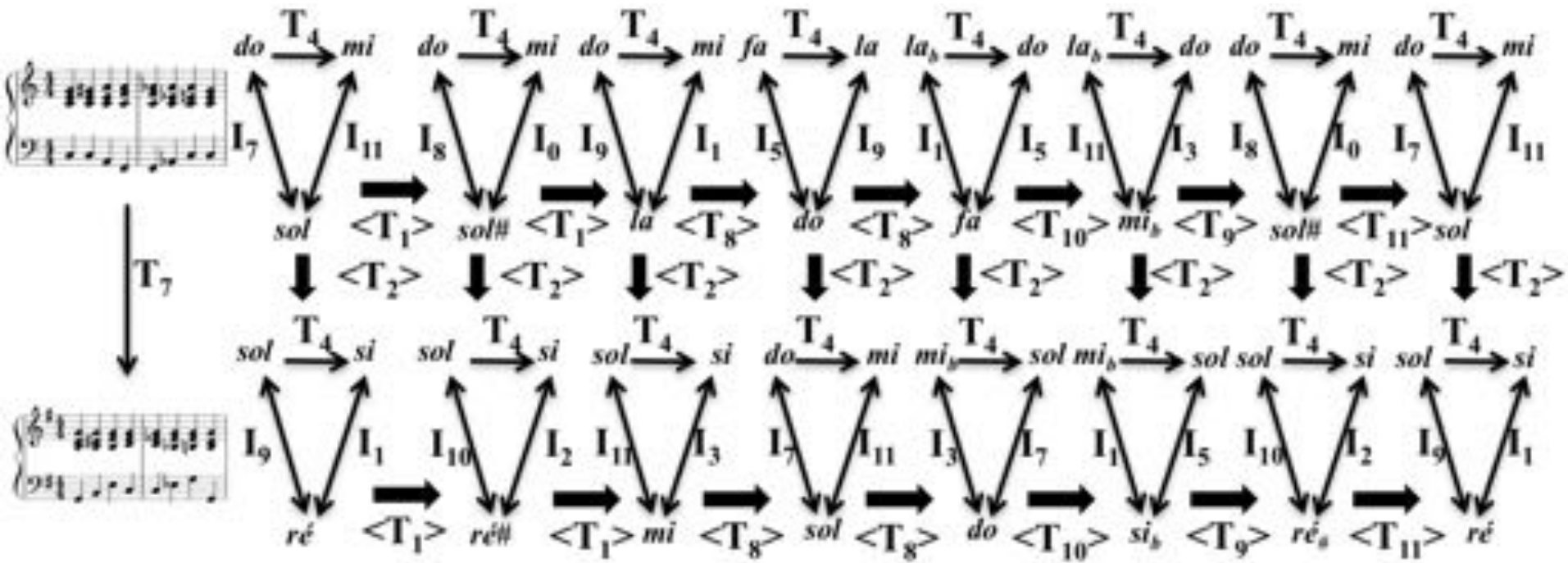
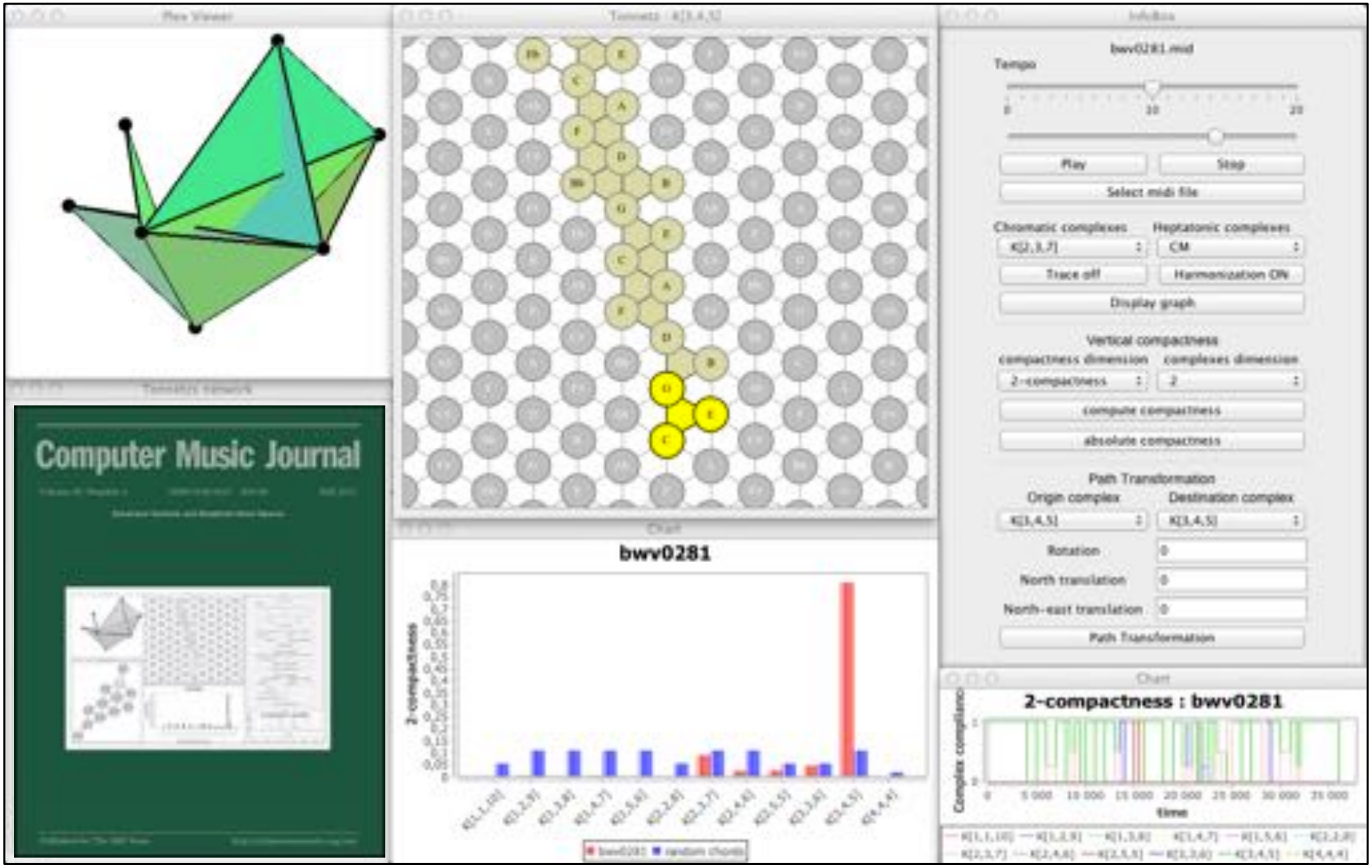
ré#

FpA 2018-2019 - Solution



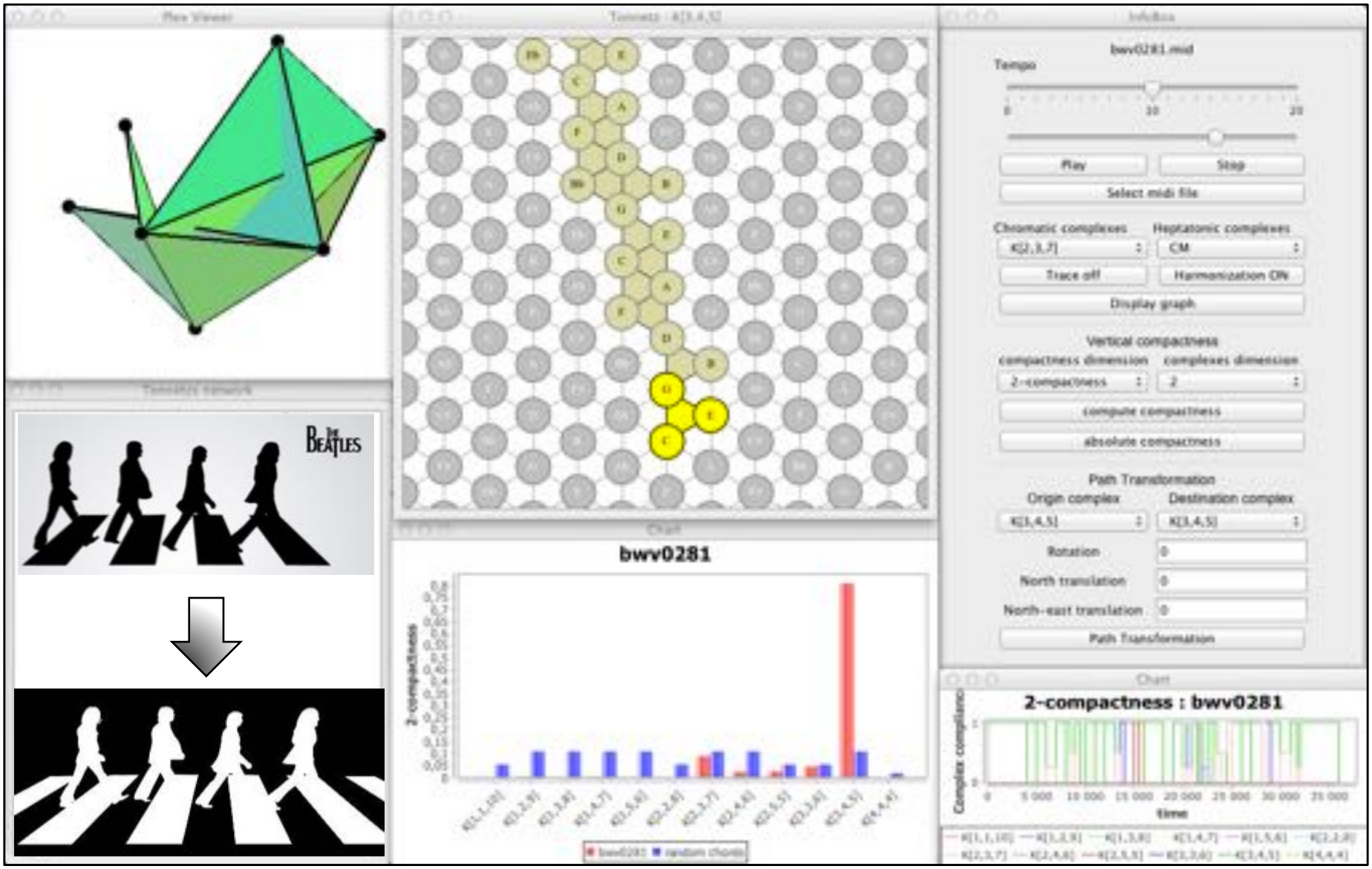


Spatial music analysis via Hexachord



➔**http://www.lacl.fr/~lbigo/hexachord**

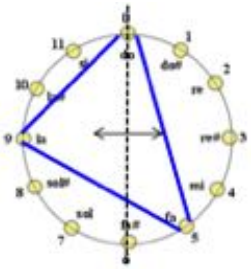
Keeping the space...but changing the trajectory!



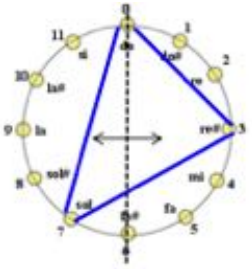
➔http://www.lacl.fr/~lbigo/hexachord

Keeping the space...but changing the trajectory!

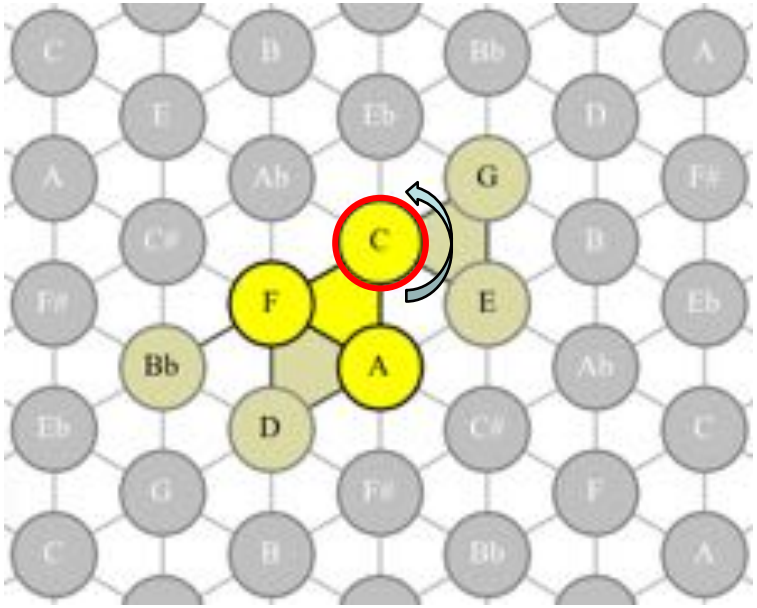
F major chord



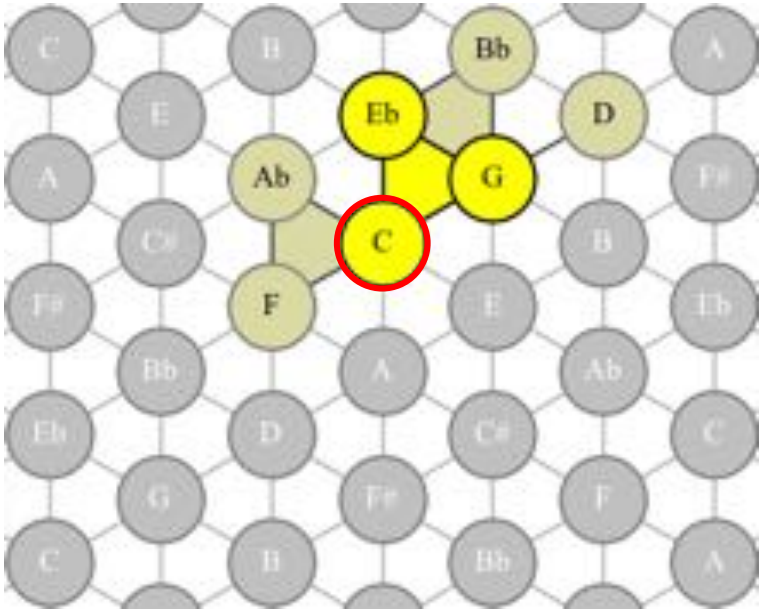
inversion



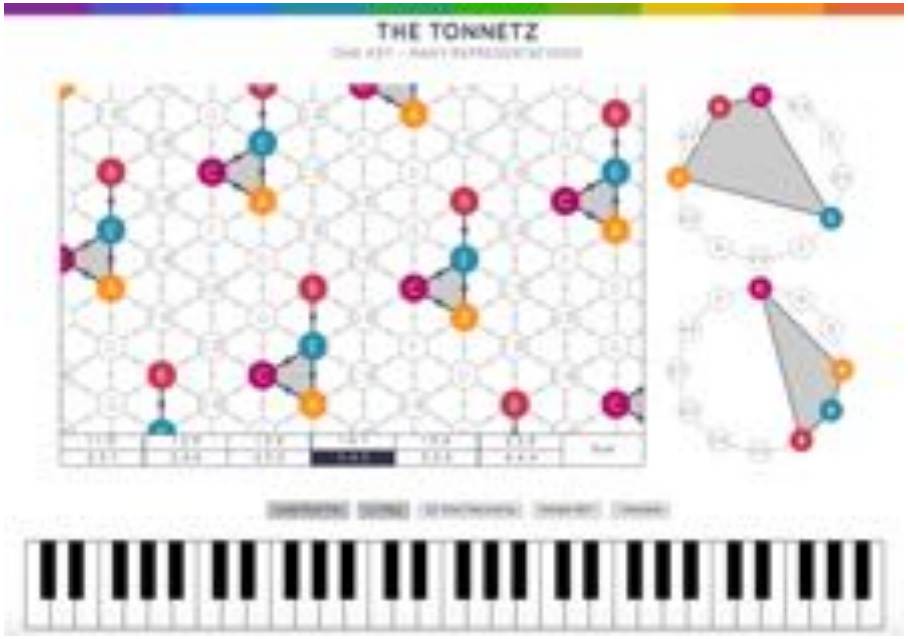
C minor chord



Rotation
(autour du *do*)



Tonnetz: a web application for ‘mathemusical’ outreach



➔ **https://guichaoua.gitlab.io/web-hexachord/**

Depuis 80 ans, nos connaissances bâtissent de nouveaux mondes

2019 2020 année des mathématiques

Math'n pop

Conférence Concert

Amphi Richelieu
3 October: 7.45 pm