

Introduction générale à la recherche 'mathémusicale'

RAIM 2013

Institut Henri Poincaré,
18 novembre 2013



Moreno Andreatta
Equipe Représentations Musicales /
IRCAM/CNRS UMR 9912



RAIM 2013
<http://raim2013.lip6.fr>
Séance Arithmétique et Musique
Institut Henri Poincaré, 18 Novembre 2013
(14h - 18h30)

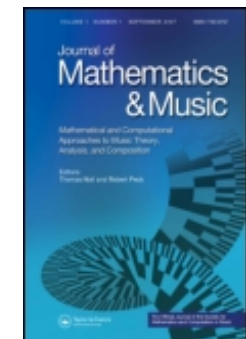
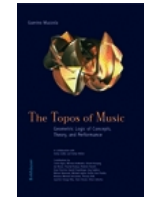
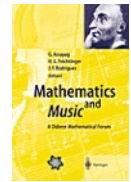
Programme :

- 14h00-14h30 – **Moreno Andreatta** – Introduction générale sur la recherche mathémusicale
- 14h30-14h45 – **Jean-Paul Allouche** – Quelques compléments sur la théorie des mots, les automates, les pavages et l'inconstance
- 14h45-15h15 – **Emmanuel Amiot** – Le mot Lambda
- 15h30-16h00 – **Thomas Noll** – Sturmian Involution, Lattice-Path-Transformations and their Application to the Investigation of Diatonicity

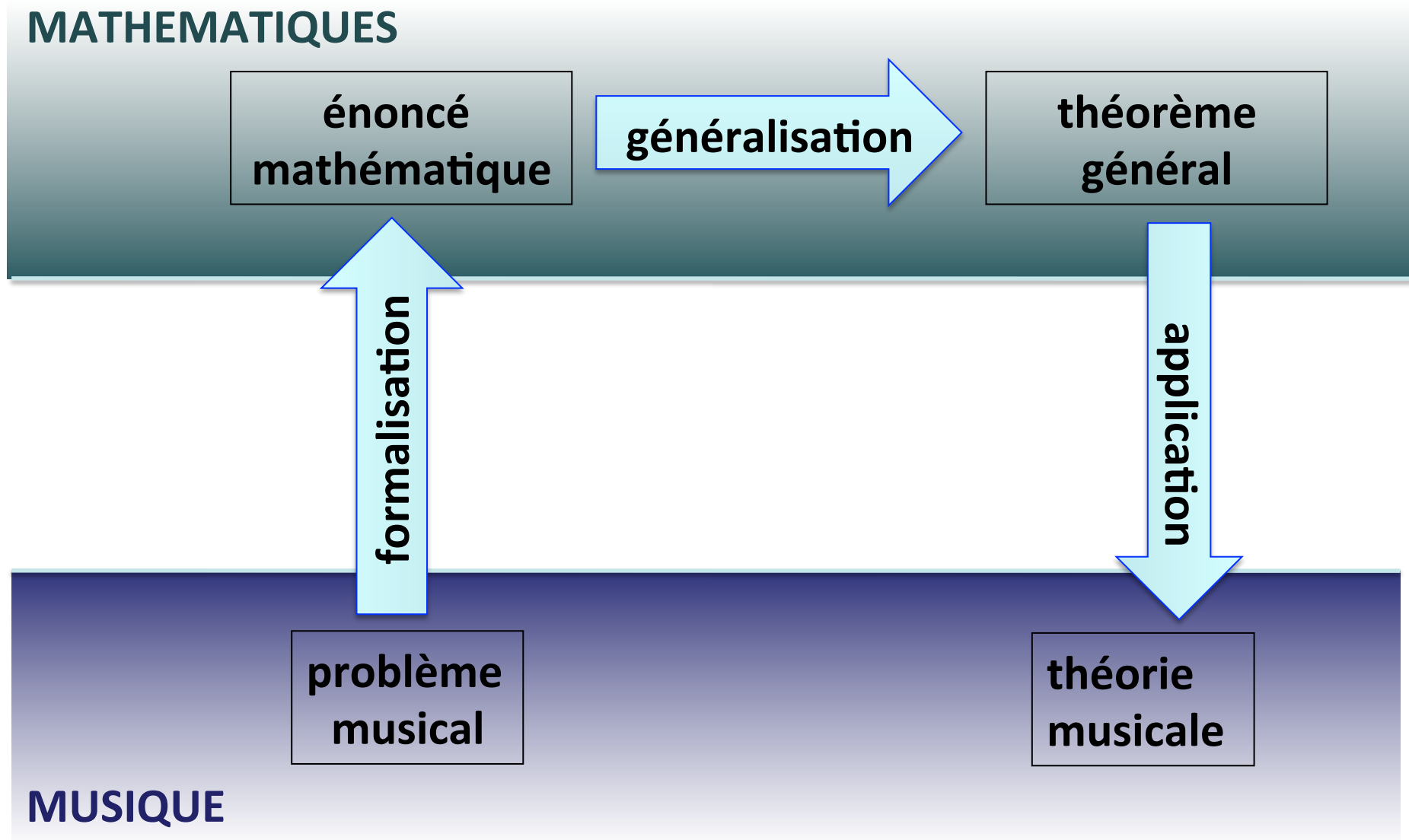
- 16h30-17h00 – **Hélianthe Caure** – Pavages rythmiques : aspects algébriques et algorithmiques
- 17h15-17h45 – **Philippe Esling** et **Carlos Agon**, Séries temporelles et orchestration
- 18h00-18h30 Discussion générale sur arithmétique, musique et informatique

Maths/Musique et Info : un historique

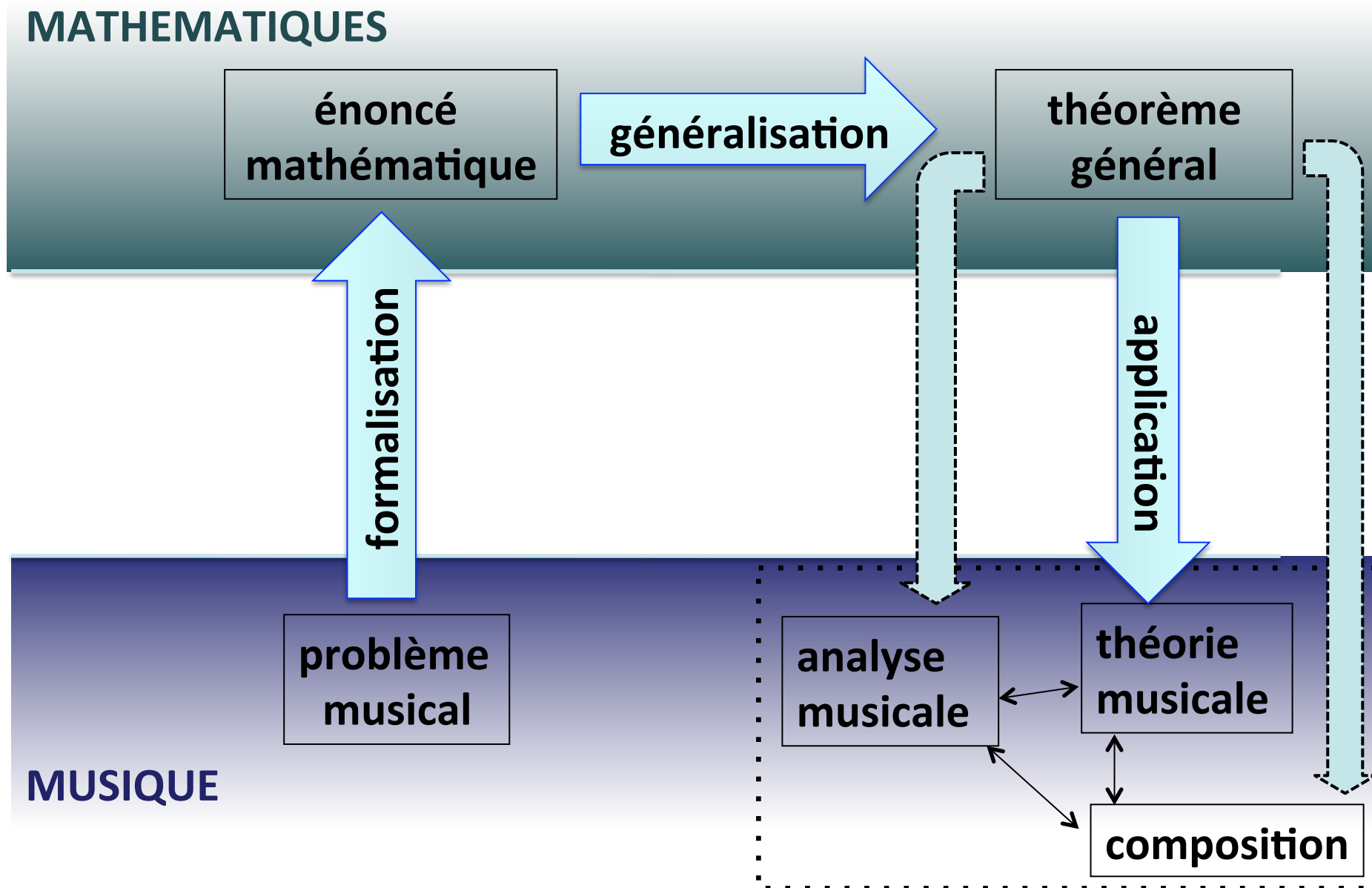
- 1999: 4^e Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (G. Assayag, H.G. Feichtinger, J.F. Rodrigues, Springer, 2001)
- 2000-2001: *MaMuPhi Seminar, Penser la musique avec les mathématiques ?* (Assayag, Mazzola, Nicolas éd., Coll. 'Musique/Sciences', Ircam/Delatour, 2006)
- 2000-2003: International Seminar on *MaMuTh (Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Luis-Puebla eds, epOs, 2004)
- 2003: *The Topos of Music* (G. Mazzola et al.)
- 2001-....: *MaMuX Seminar* at Ircam
- 2004-....: *mamuphi Seminar* (Ens/Ircam)
- 2006: Collection 'Musique/Sciences' (Ircam/Delatour France)
- 2007: *Journal of Mathematics and Music* (Taylor & Francis) and SMCMM
- 2007: First MCM 2007 (Berlin) and Proceedings by Springer
- 2007-....: AMS Special Session on Mathematical Techniques in Musical Analysis
- 2009: *Computational Music Science* (eds: G. Mazzola, M. Andreatta, Springer)
- 2009: MCM 2009 (Yale University) and Proceedings by Springer
- 2010: Mathematics Subject Classification : 00A65 Mathematics and music
- 2011: MCM 2011 (Ircam, 15-17 June 2011) and Proceedings LNCS Springer
- 2013: MCM 2013 (McGill University, Canada, 12-14 June 2013) - Springer



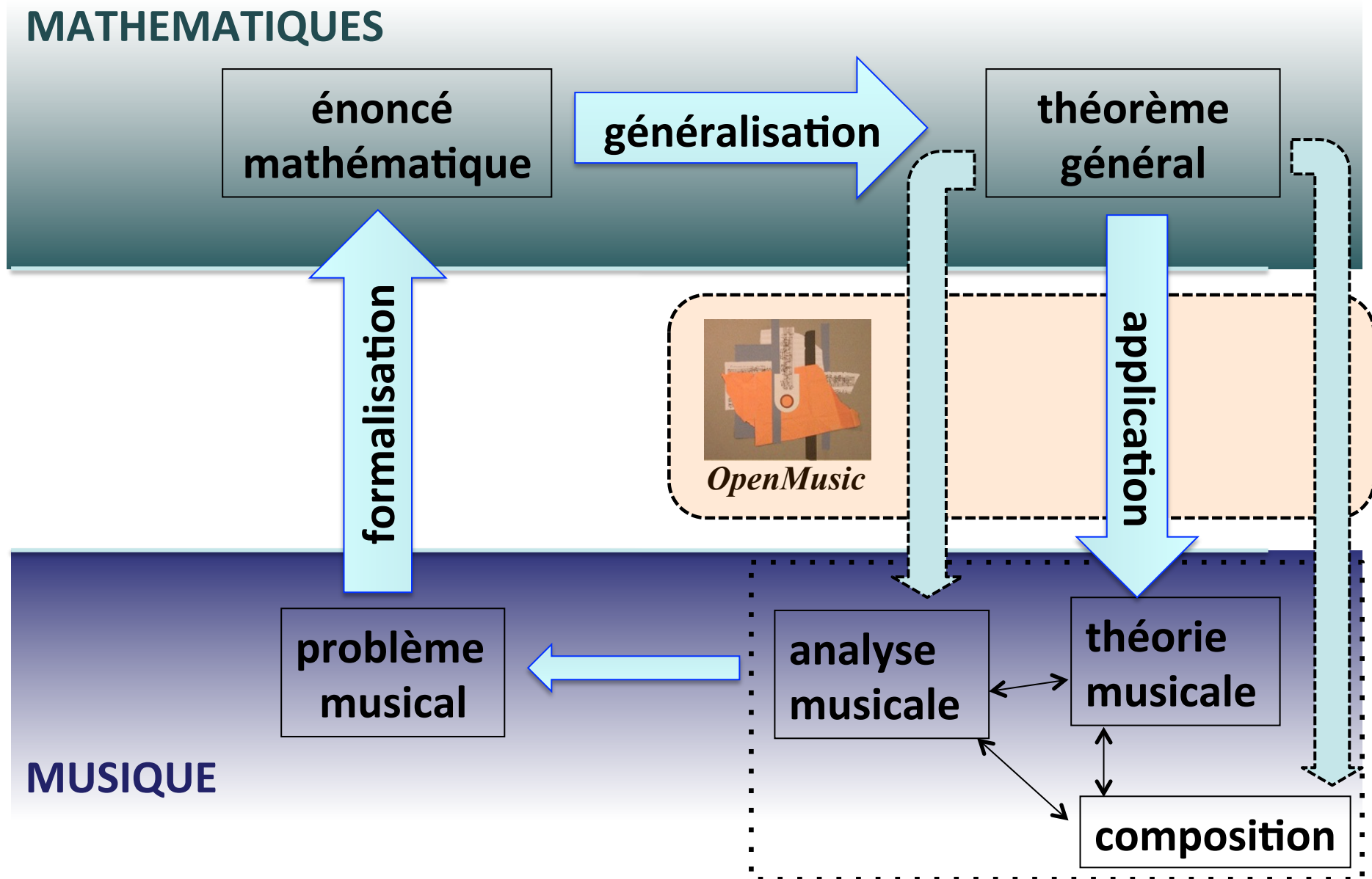
La place de l'info dans la dynamique mathémusicale



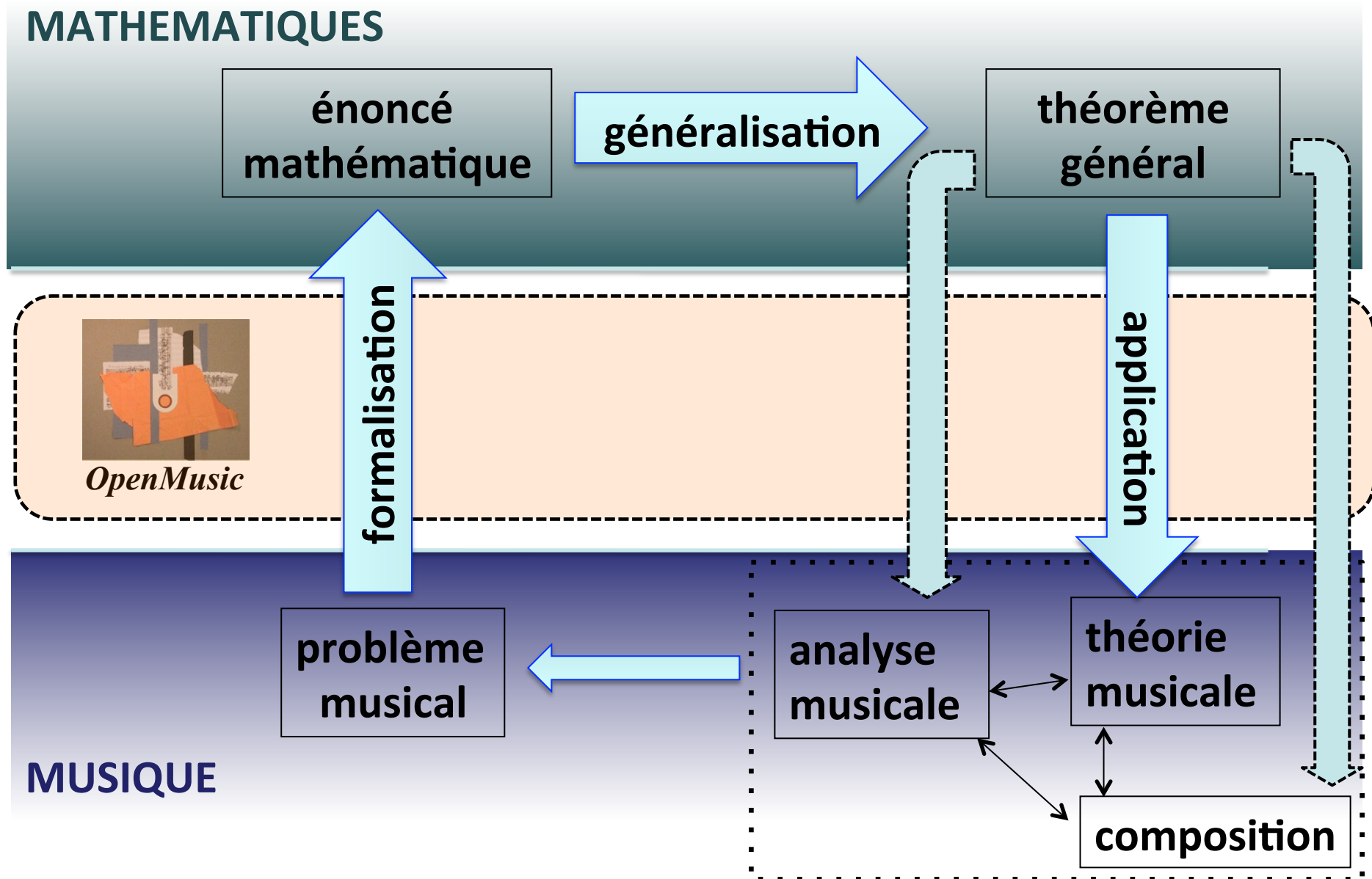
La place de l'info dans la dynamique mathémusicale



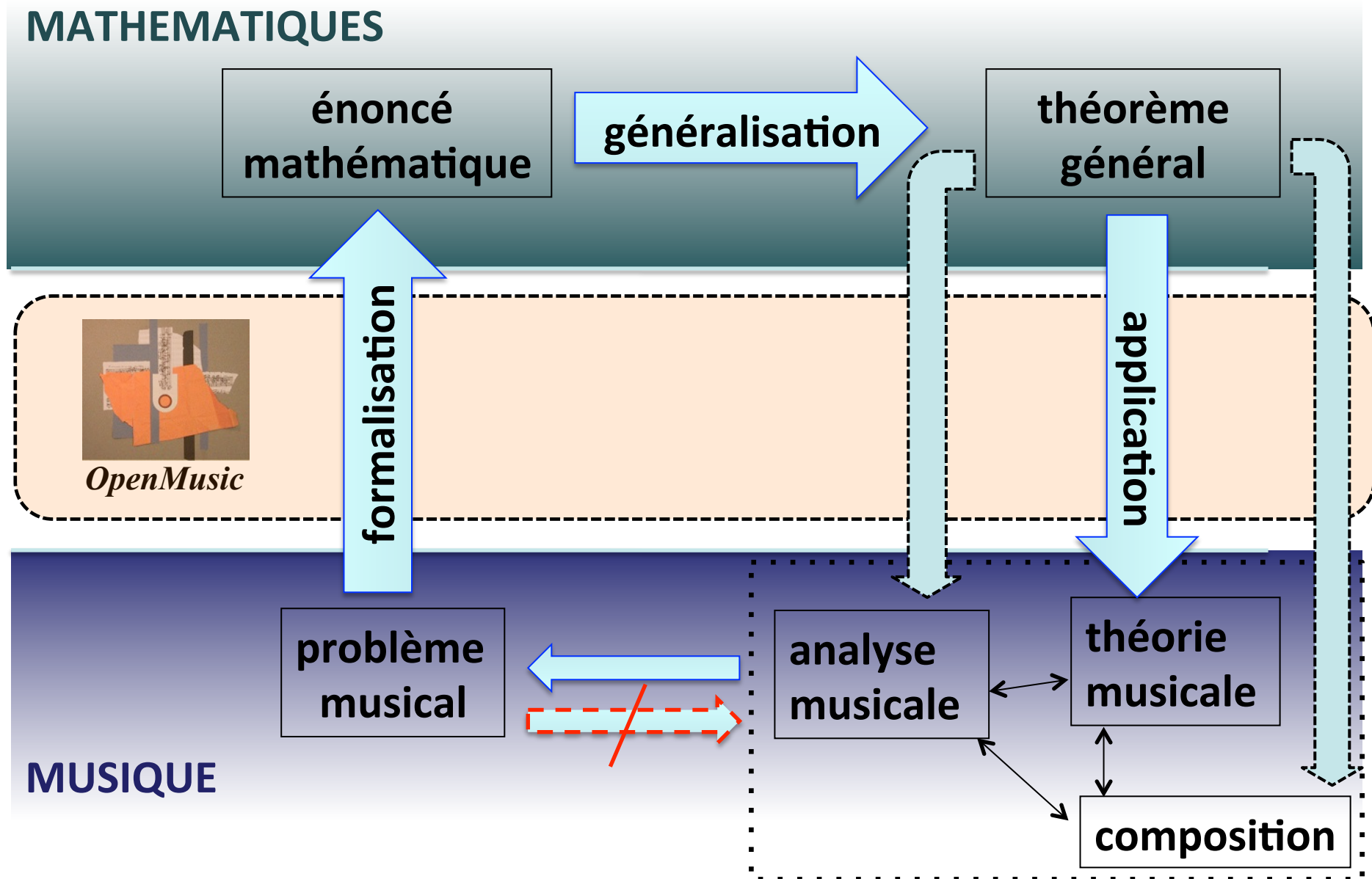
La place de l'info dans la dynamique mathémusicale



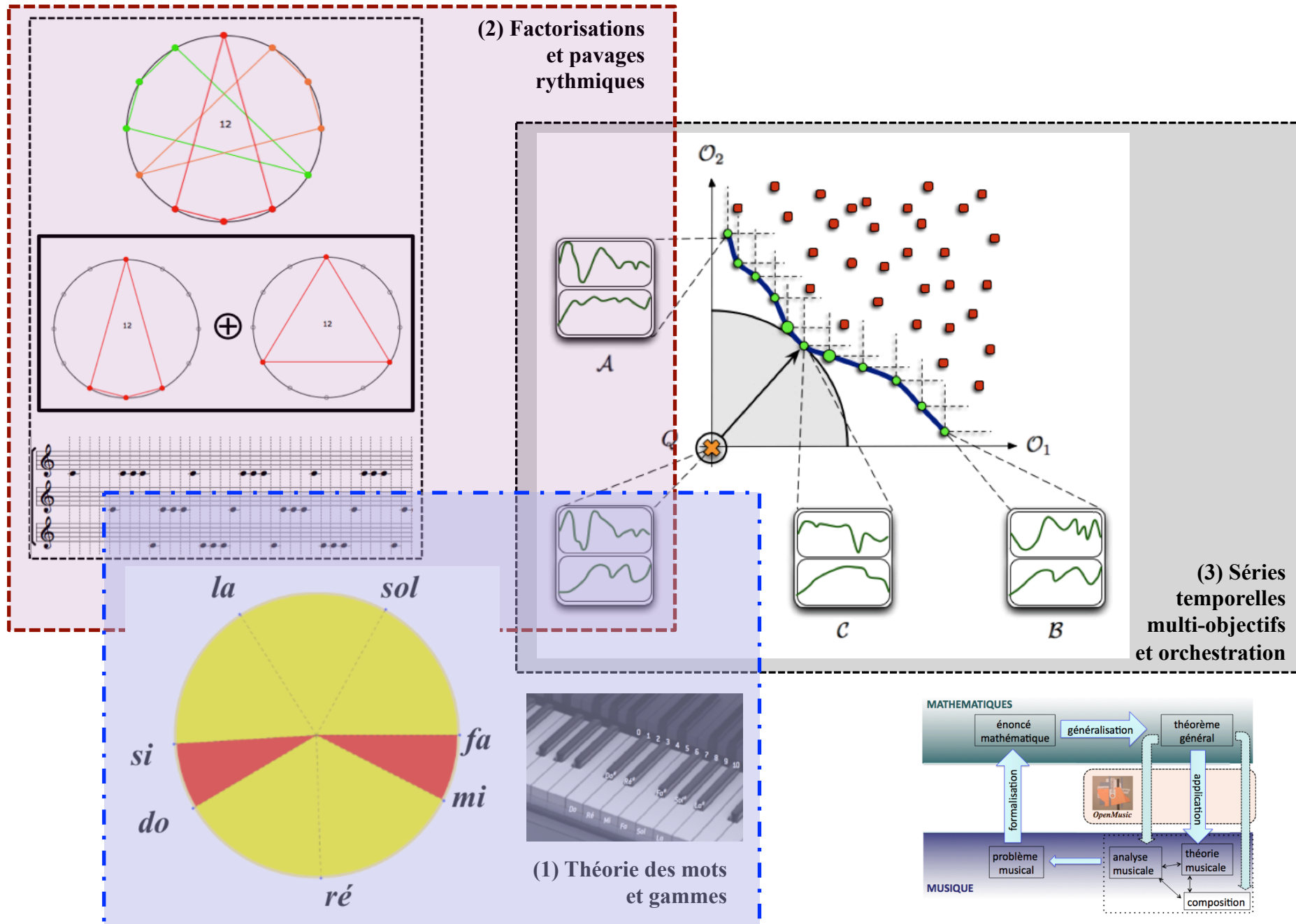
La place de l'info dans la dynamique mathémusicale



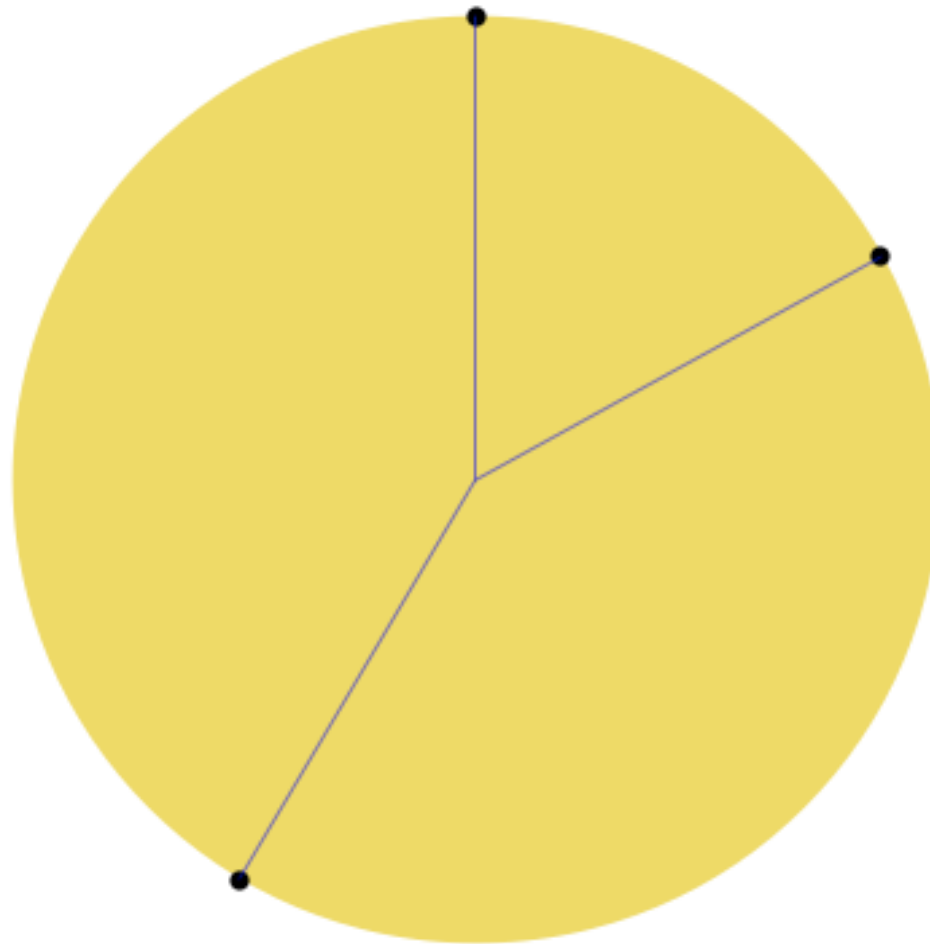
La place de l'info dans la dynamique mathémusicale



Trois sujets de recherche pour trois paramètres musicaux

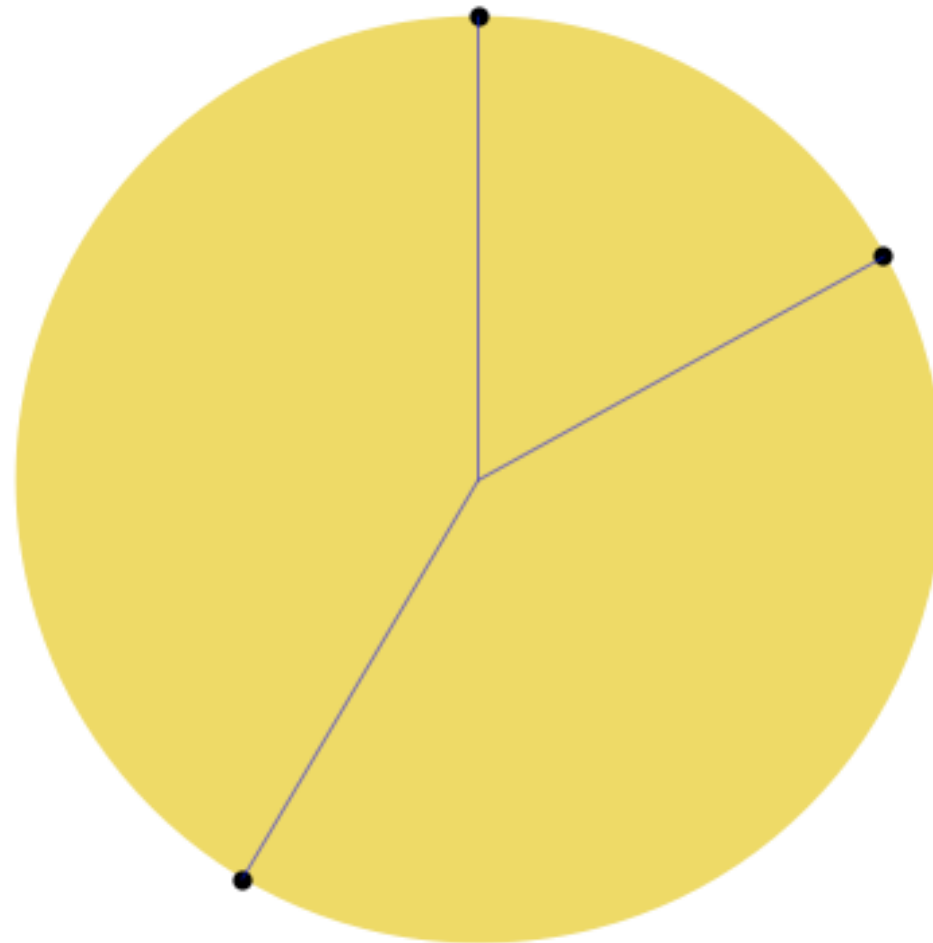


La théorie arithmétique du tempérament musical



k points engendrés par la rotation d'un angle α partitionnent le cercle en au plus trois longueurs différentes d'intervalle

Le théorème des trois intervalles (Steinhaus, ?/1958)

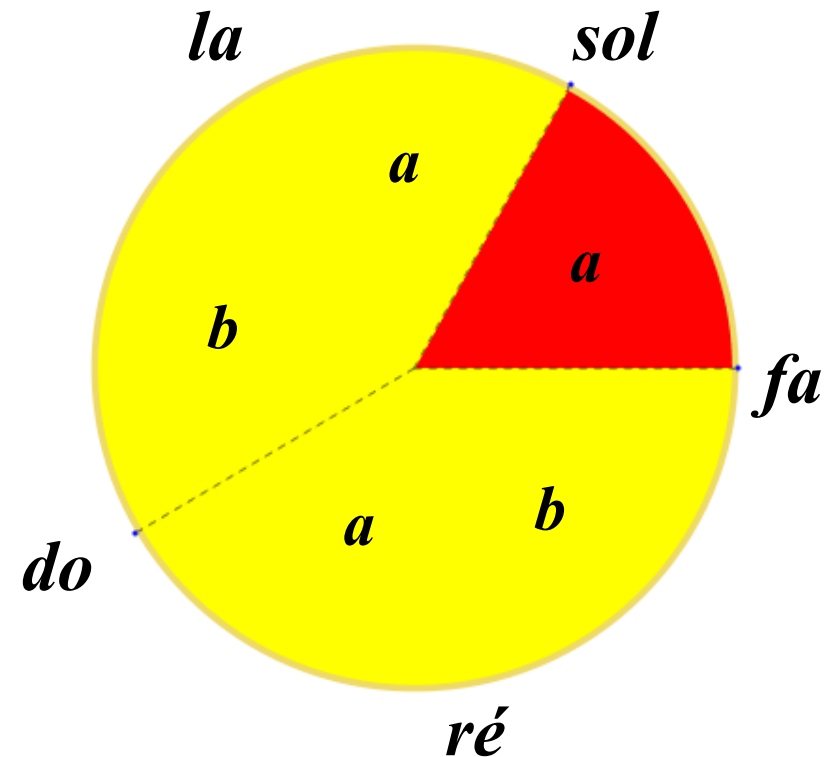


k points engendrés par la rotation d'un angle α partitionnent le cercle en au plus trois longueurs différentes d'intervalle

Deux gammes bien connues : la pentatonique

k=5 points engendrés par la rotation d'un angle $\alpha = \log_2 3/2$ partitionnent le cercle en exactement deux différentes d'intervalles avec une distribution bien répartie (maximally even) de ces intervalles

➔ Well-formed scales
(Carey & Clampitt, 1989)



(a a b a b) [Christoffel word]



Deux gammes bien connues : la diatonique

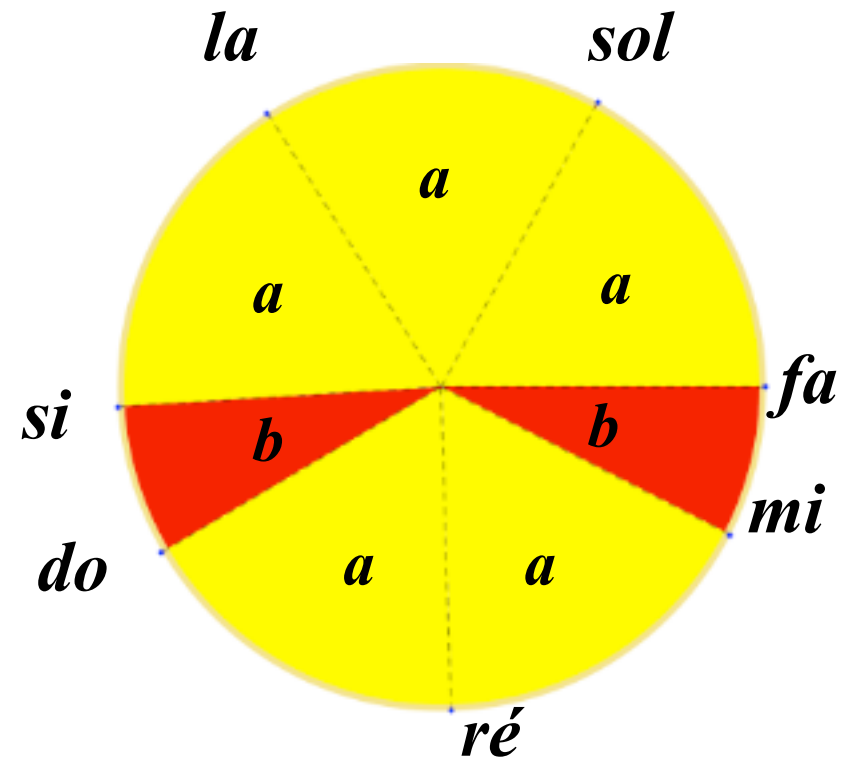
k=7 points engendrés par la rotation d'un angle $\alpha = \log_2 3/2$ partitionnent le cercle en exactement deux longueurs différentes d'intervalle avec une distribution bien répartie (maximally even) de ces intervalles

→ Well-formed scales (Carey & Clampitt, 1989)

→ *Cela touche aux suites Sturmiennes et aux mots de Christoffel (version finie).*

$b \rightarrow ab$

$(a \text{ } a \text{ } b \text{ } a \text{ } b) \rightarrow (a \text{ } a \text{ } a \text{ } b \text{ } a \text{ } a \text{ } b)$ [Christoffel word]

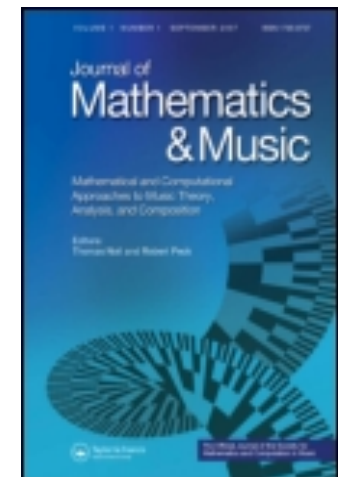
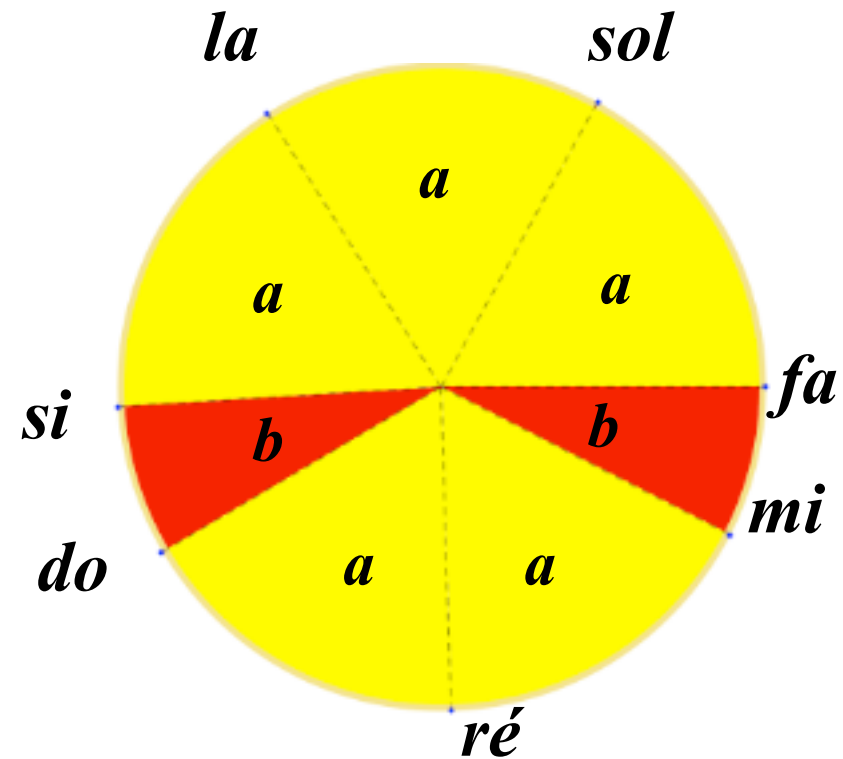


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k=7 points engendrés par la rotation d'un angle $\alpha = \log_2 3/2$ partitionnent le cercle en exactement deux longueurs différentes d'intervalle avec une distribution bien répartie (maximally even) de ces intervalles

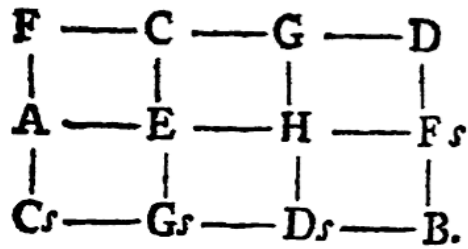
→ Well-formed scales (Carey & Clampitt, 1989)

→ *Cela touche aux suites Sturmiennes et aux mots de Christoffel (version finie).*

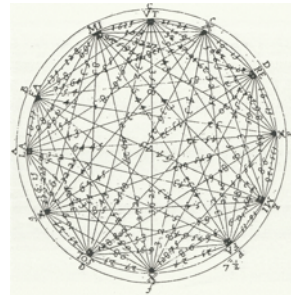


- P. Alessandri, V. Berthé, « Three distance theorems and combinatorics on words », *Enseign. Math.* 2, 44 1998, 103–132.
- D. Clampitt (ed.), special issue *Journal of Mathematics and Music*, Special Issue, Vol. 1, No. 2, July 2007, 73 78

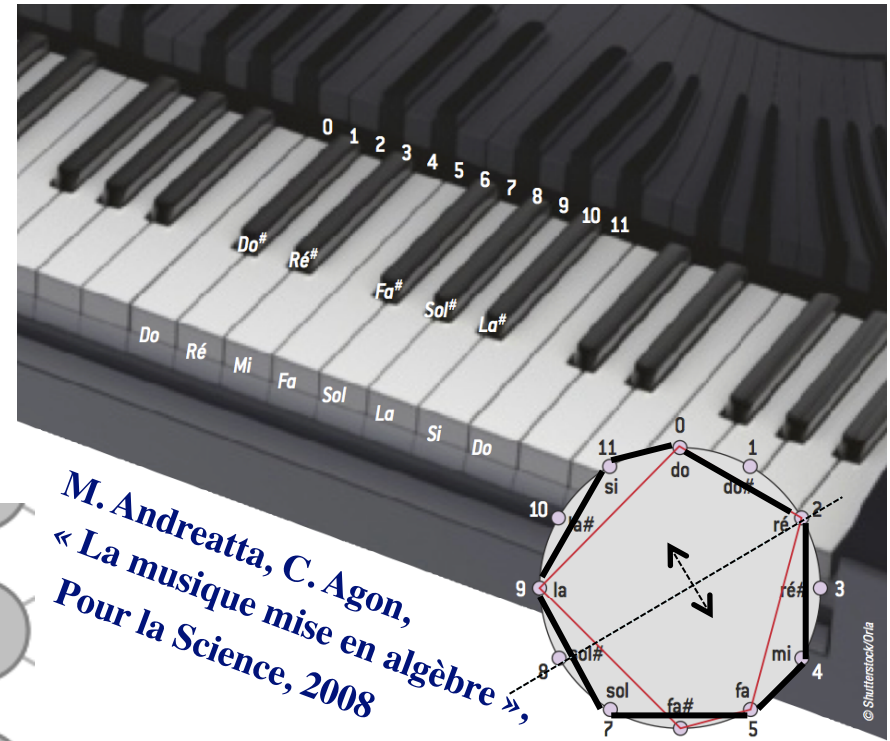
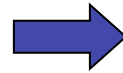
Représentations géométriques des structures musicales



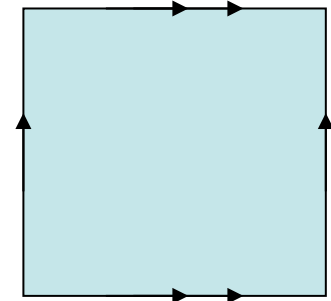
Euler (1773)



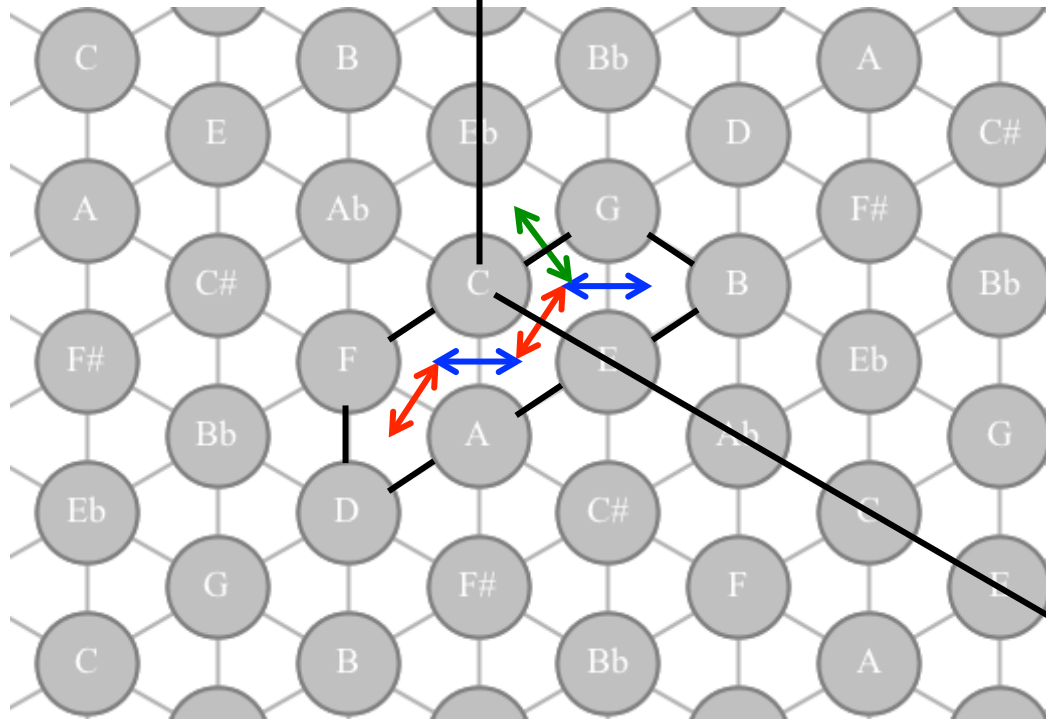
Mersenne (1648)



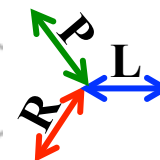
M. Andreatta, C. Agon,
« La musique mise en algèbre »,
Pour la Science, 2008



Axe de tierces mineures



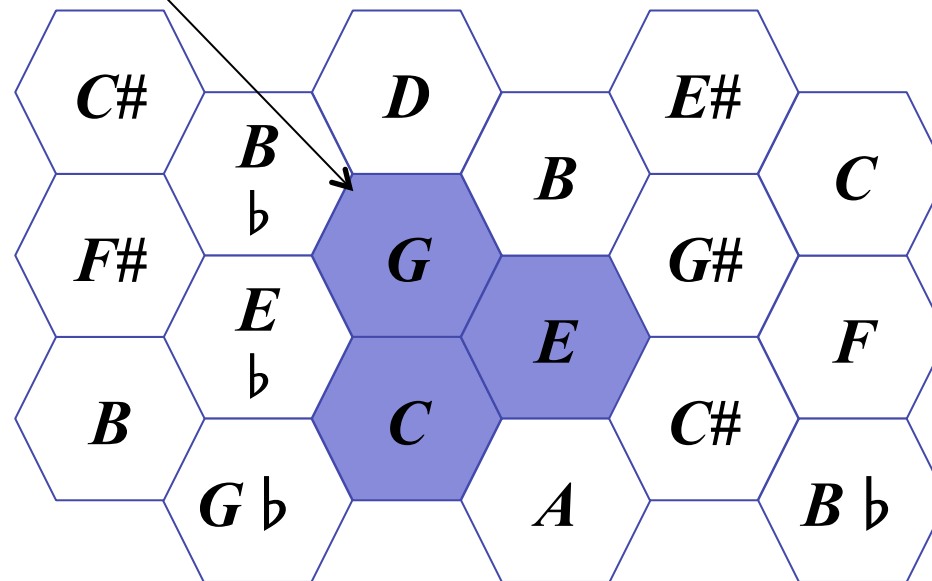
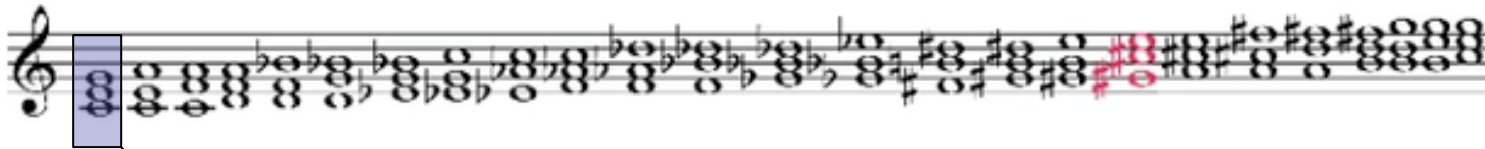
Hexachord (Louis Bigo, 2013)



Axe des tierces
majeures



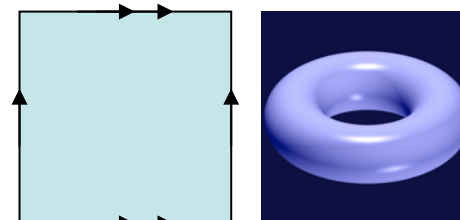
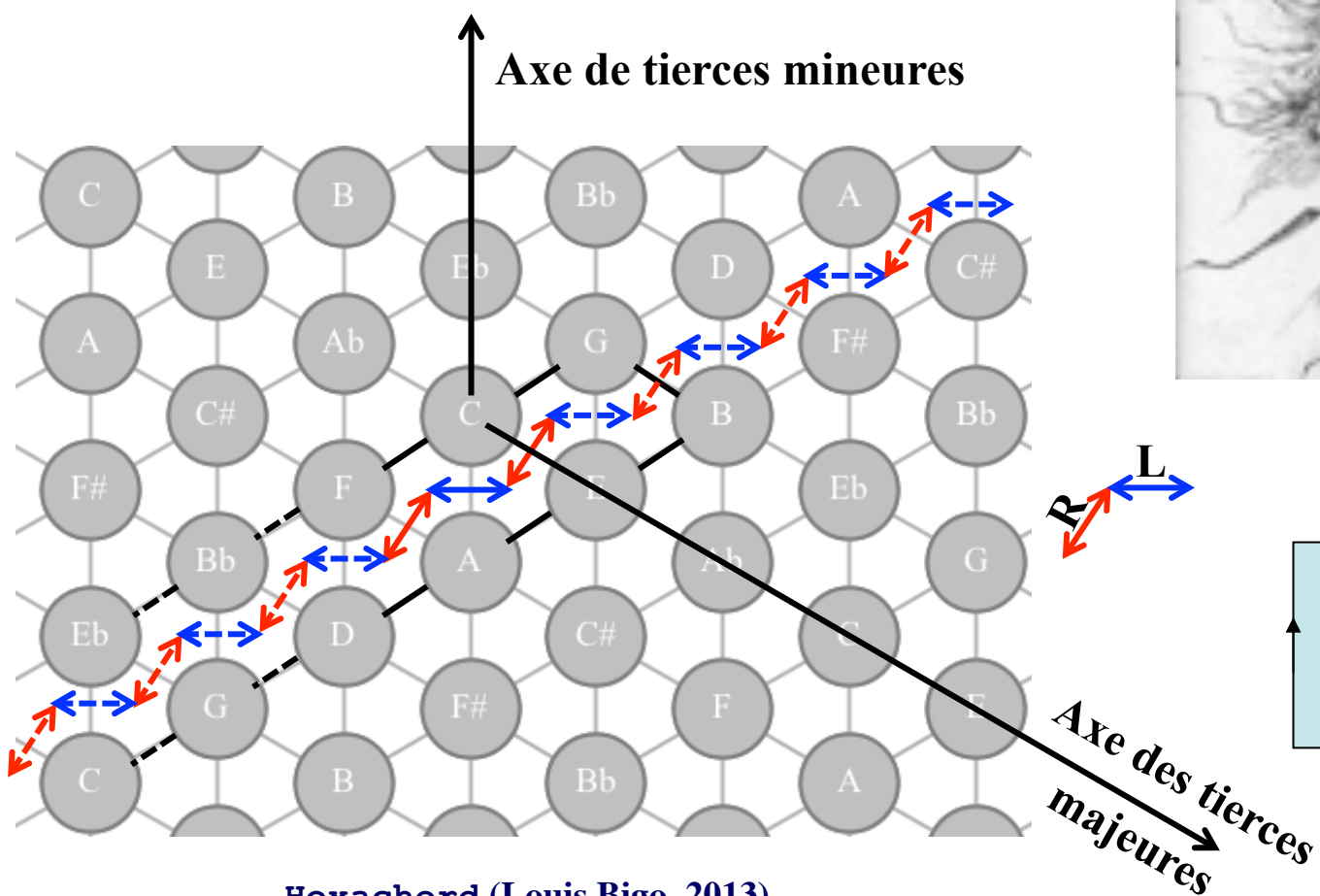
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



b



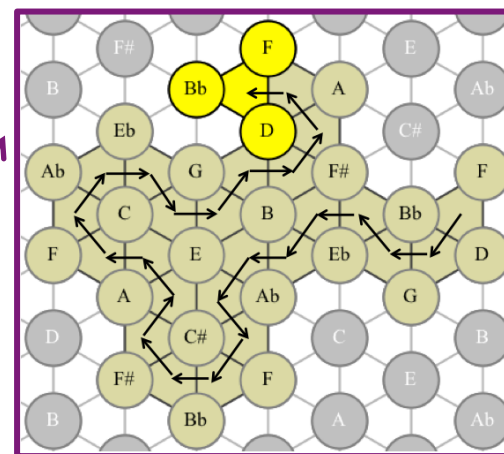
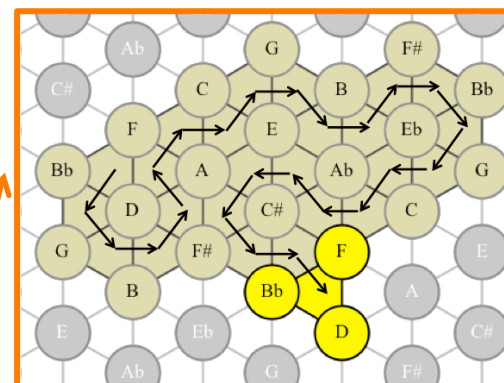
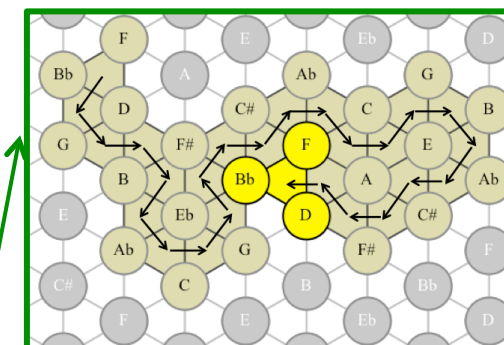
Trajectoires hamiltoniennes dans le *Tonnetz*



Hexachord (Louis Bigo, 2013)

Enumération des cycles hamiltoniens dans le *Tonnetz*

#41	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R
#62	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P
#13	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L
#4	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L
#58	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L
#19	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	L
#7	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L
#27	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L
#21	P	R	P	L	P	L	P	R	L	P	L	P	L	P	R	P	L	P	L	P
#8	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	P	R	P	L	P
#30	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	P	R	P
#22	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	P
#10	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P
#31	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	L	P	R
#9	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P
#6	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P
#33	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R
#44	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	P	R	P	R
#40	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	P	R
#38	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	L	R
#34	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	L	R
#42	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R
#43	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	P	R
#39	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	L	R	L	R
#50	L	R	L	P	L	R	L	P	R	L	P	L	R	P	L	R	P	L	R	P
#37	P	R	L	R	L	P	L	R	L	P	R	L	P	L	R	P	L	R	P	L
#25	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R
#16	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	P	L	R
#23	P	L	P	R	P	L	R	P	R	L	P	L	R	L	P	R	L	P	L	R
#1	P	L	R	P	L	P	R	P	L	R	P	R	L	P	L	R	L	P	R	L
#59	L	P	L	R	P	L	P	R	P	L	R	P	R	L	P	L	R	L	P	R
#48	L	R	L	P	L	R	P	L	P	R	P	L	R	P	L	R	L	P	L	R
#18	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	L	R	L	P	L
#60	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	L	R	L	P
#2	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	L	R	L	P



G. Albini et S. Antonini, « Hamiltonian Cycles in the Topological Dual of the Tonnetz », MCM 2009, LNCS Springer

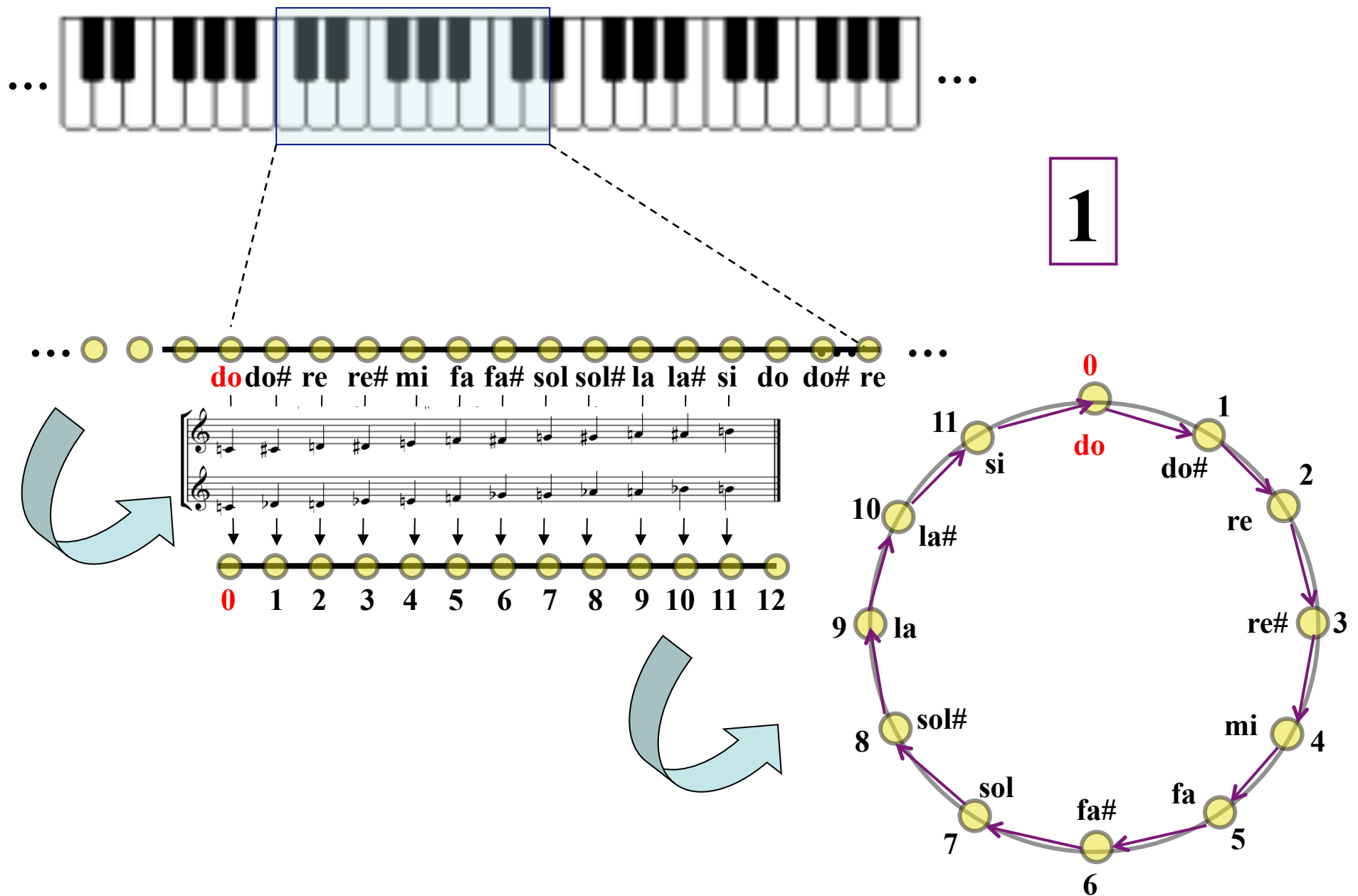


Tangente
L'aventure mathématique

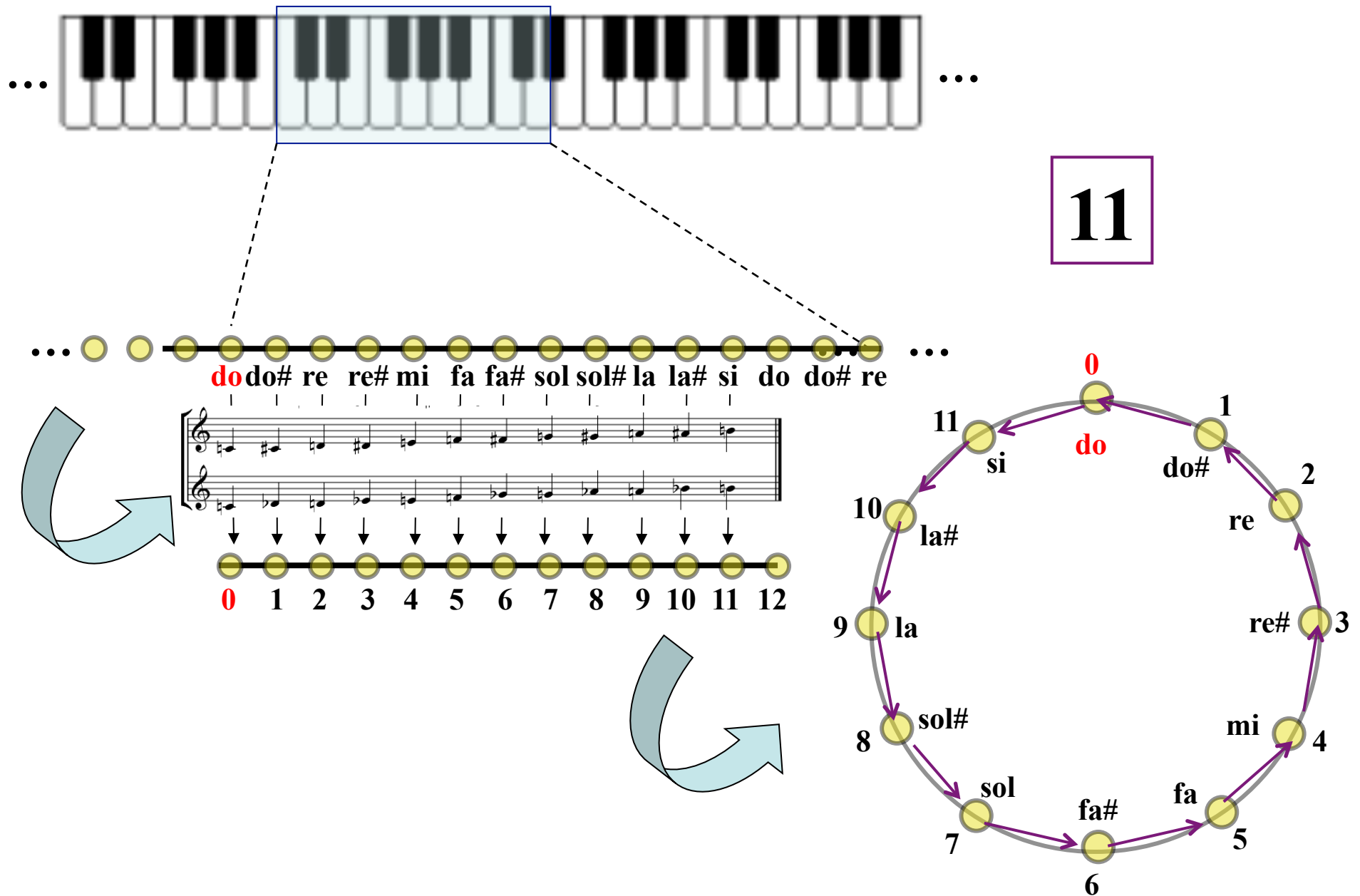
pour mieux comprendre
le monde



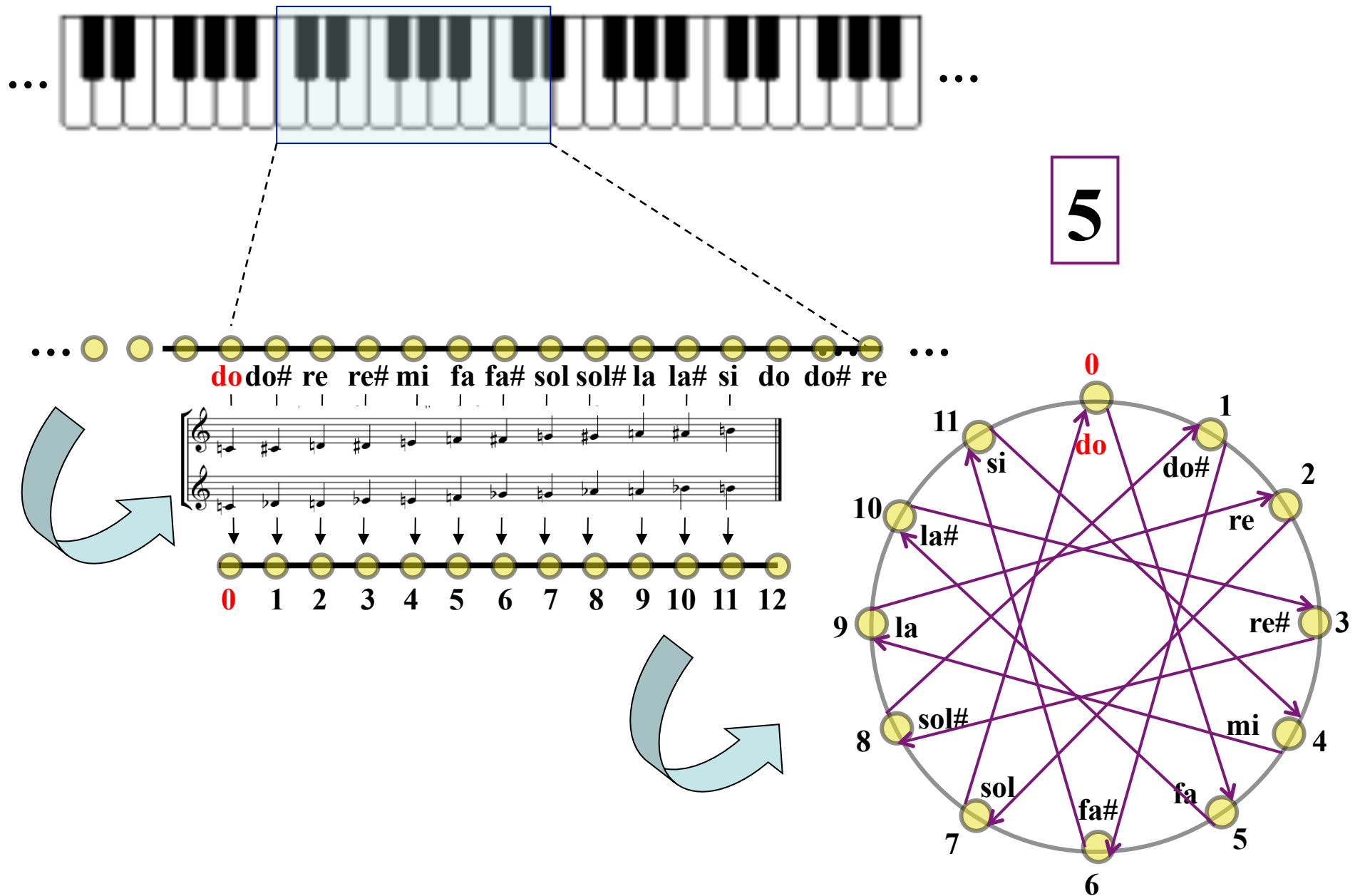
Réduction à l'octave et congruence modulo 12



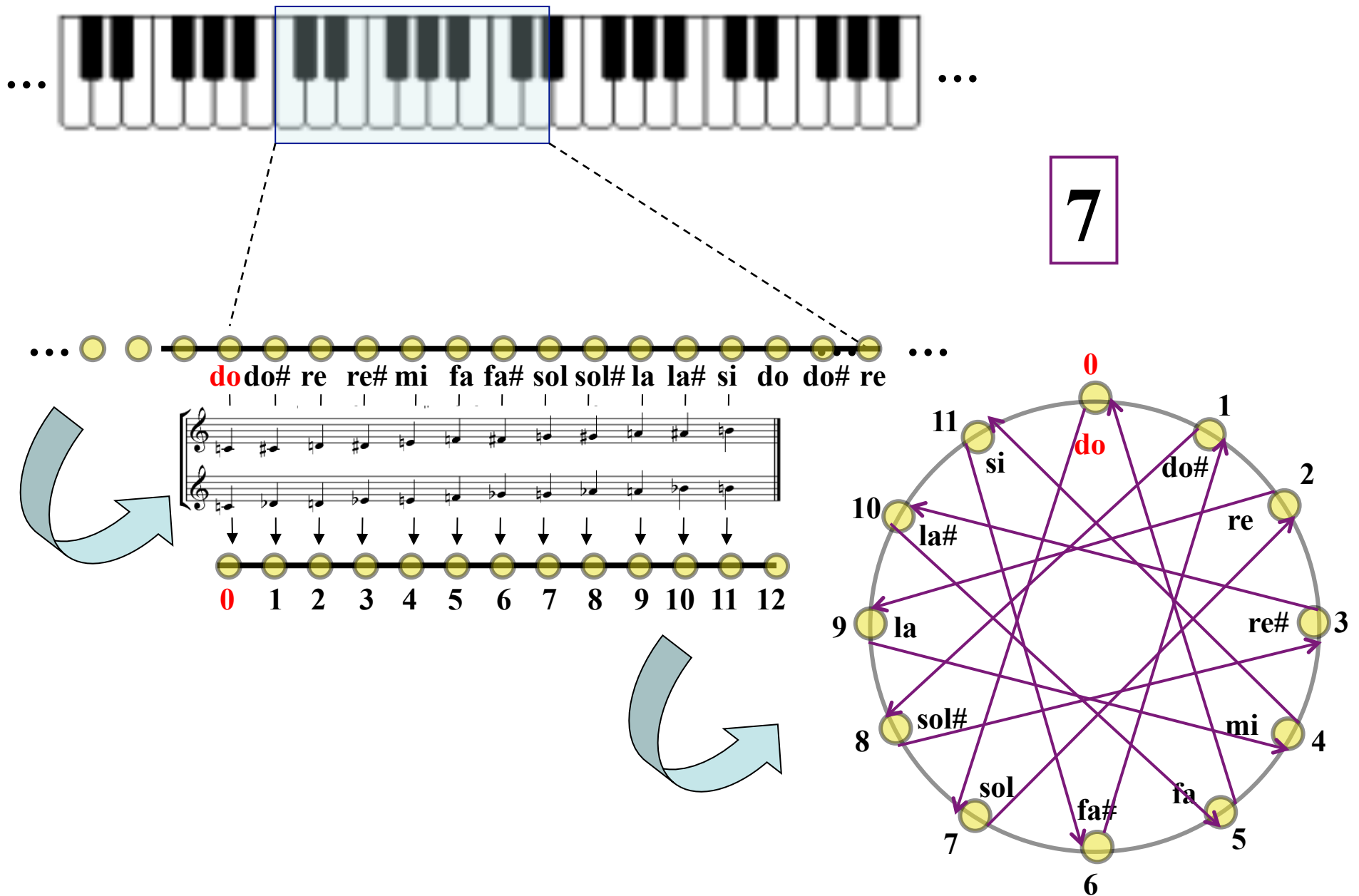
Réduction à l'octave et congruence modulo 12



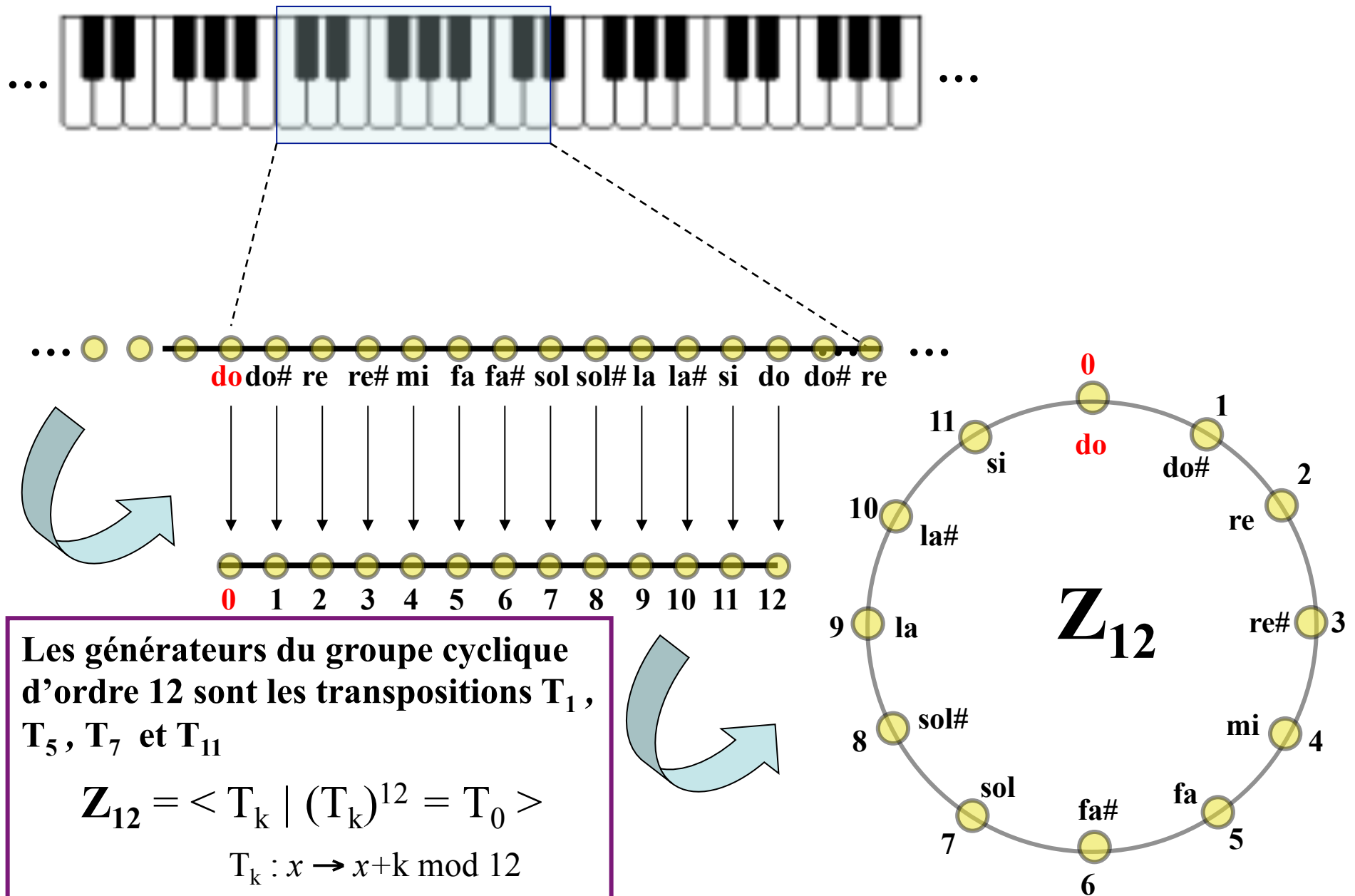
Réduction à l'octave et congruence modulo 12



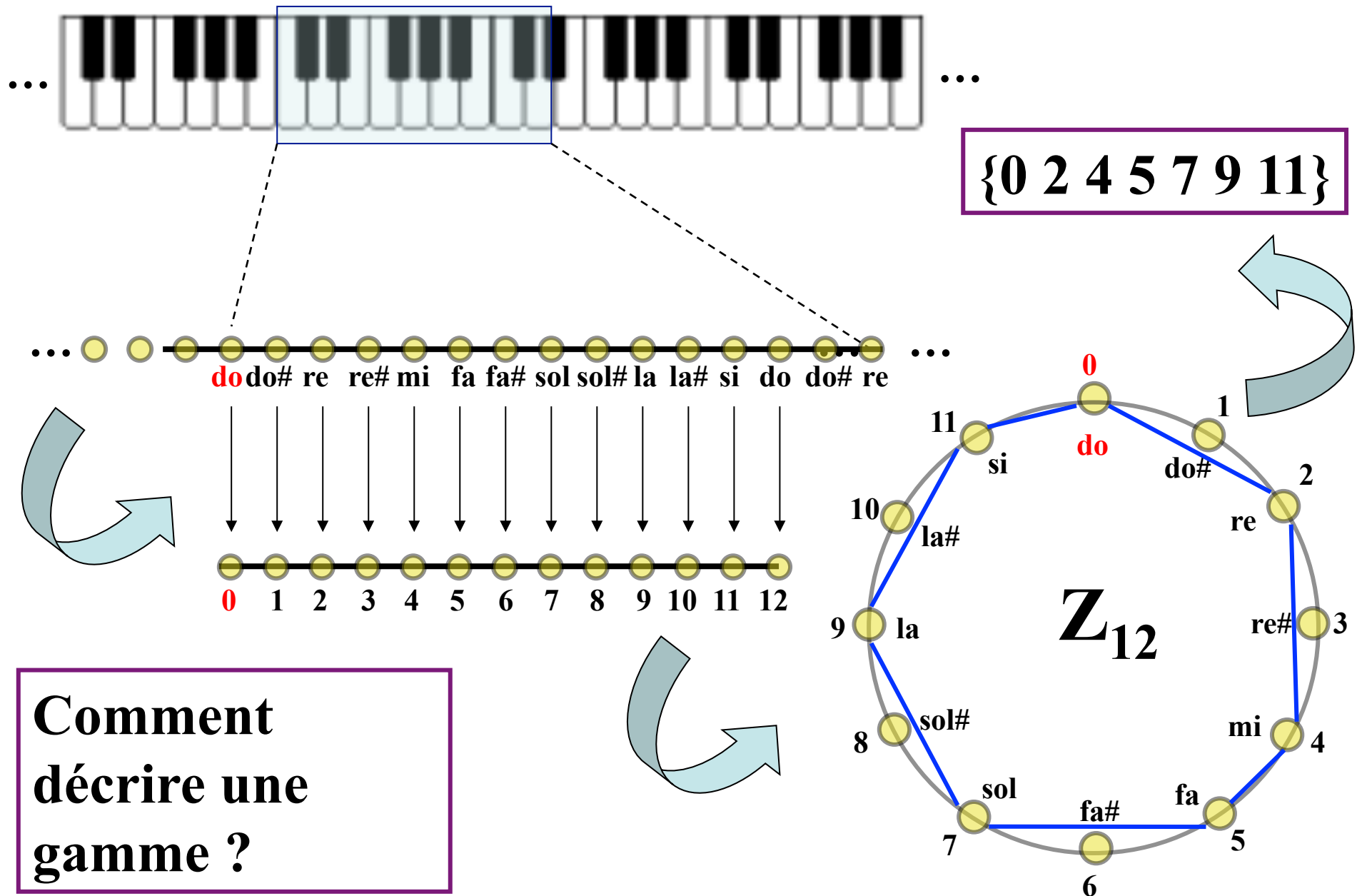
Réduction à l'octave et congruence modulo 12



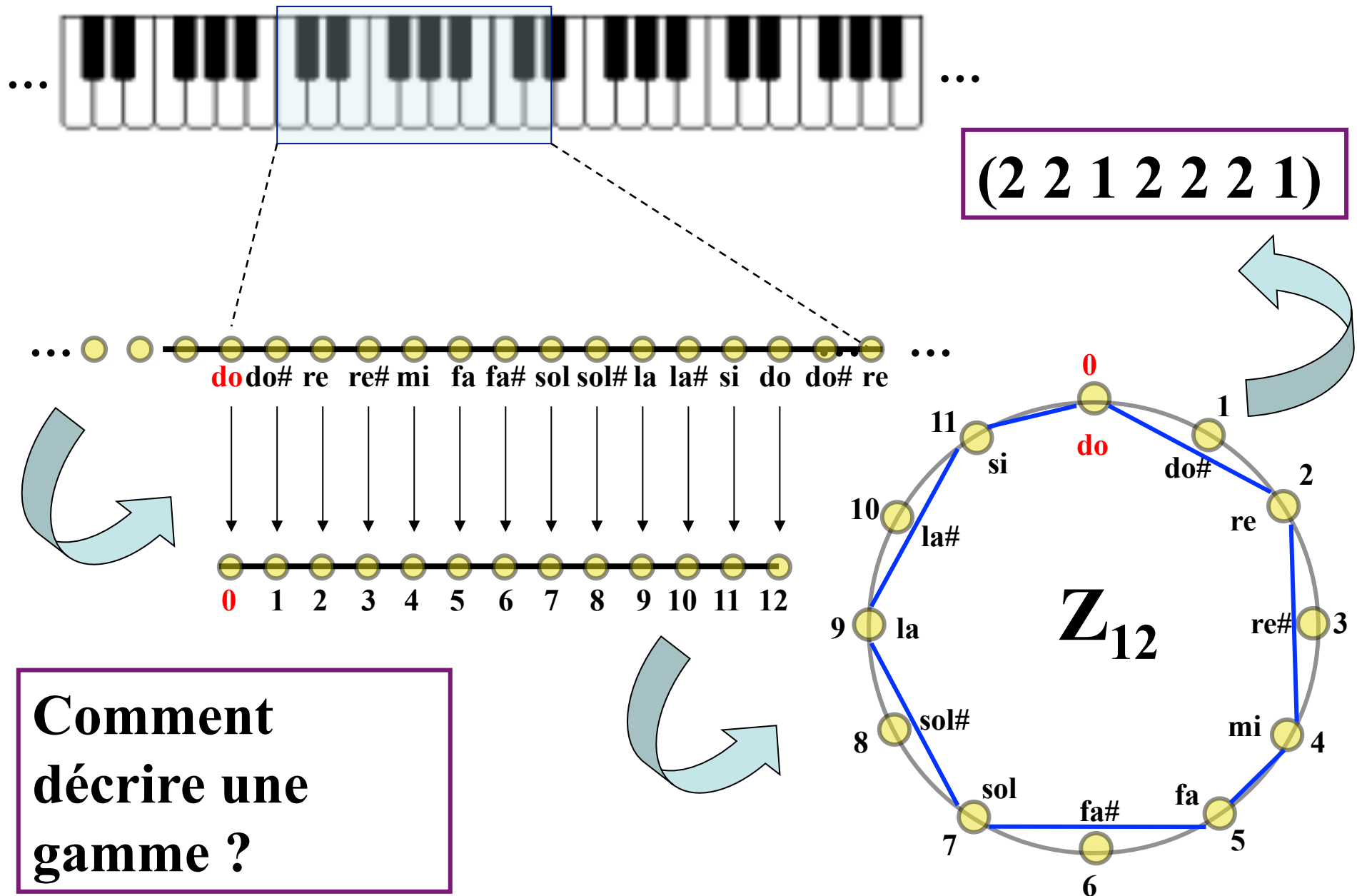
Réduction à l'octave et congruence modulo 12



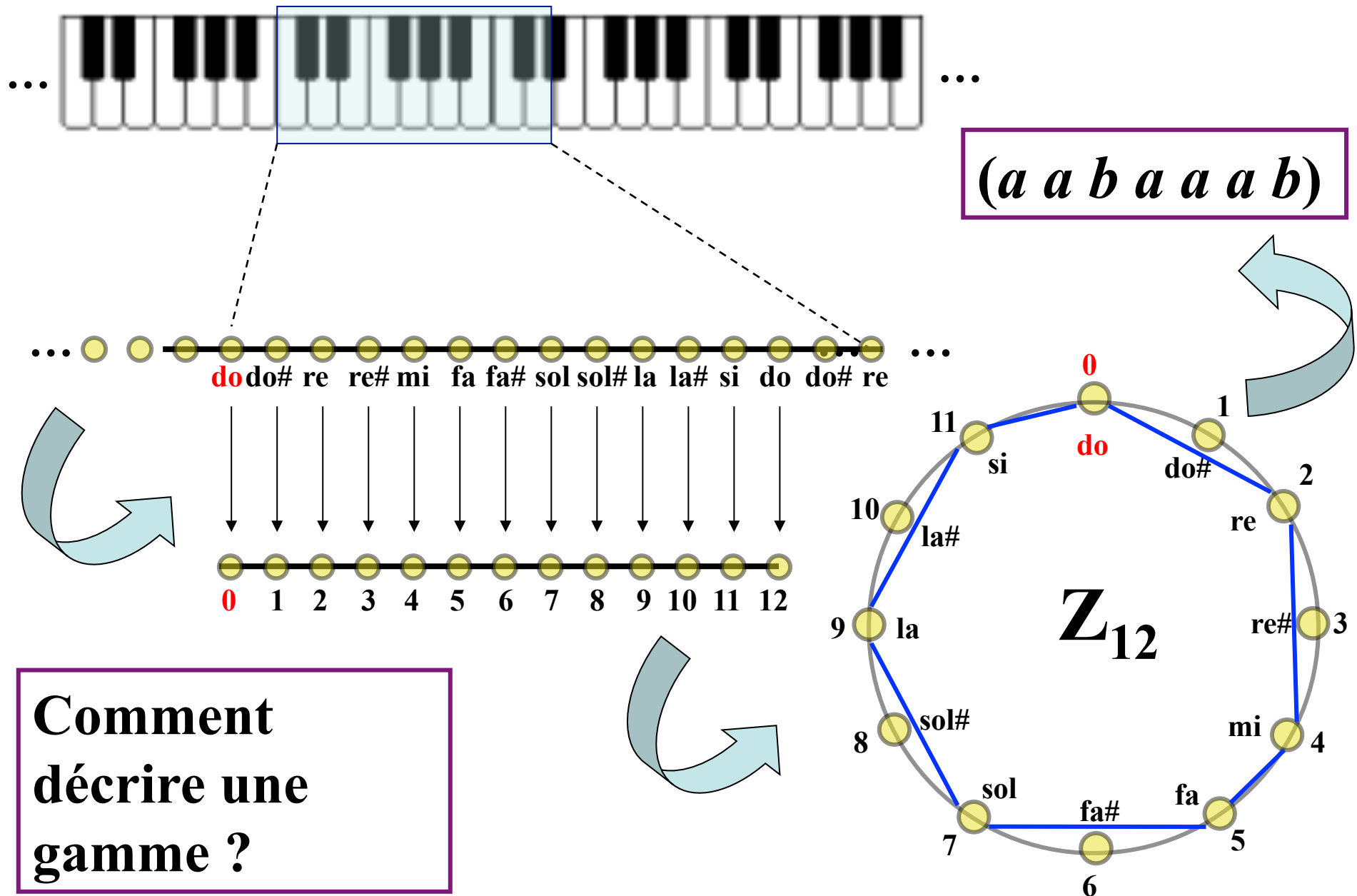
Réduction à l'octave et congruence modulo 12



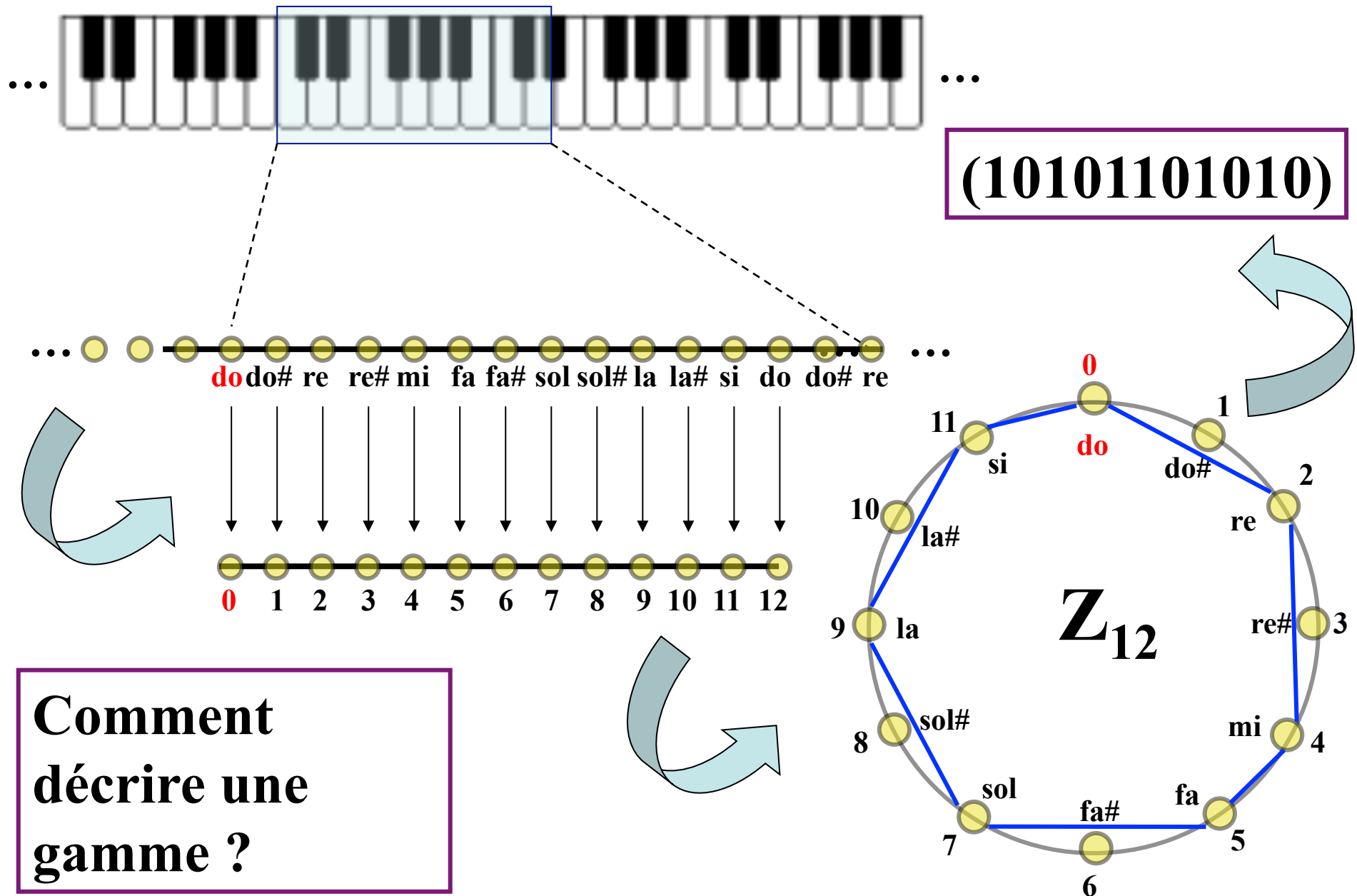
Réduction à l'octave et congruence modulo 12



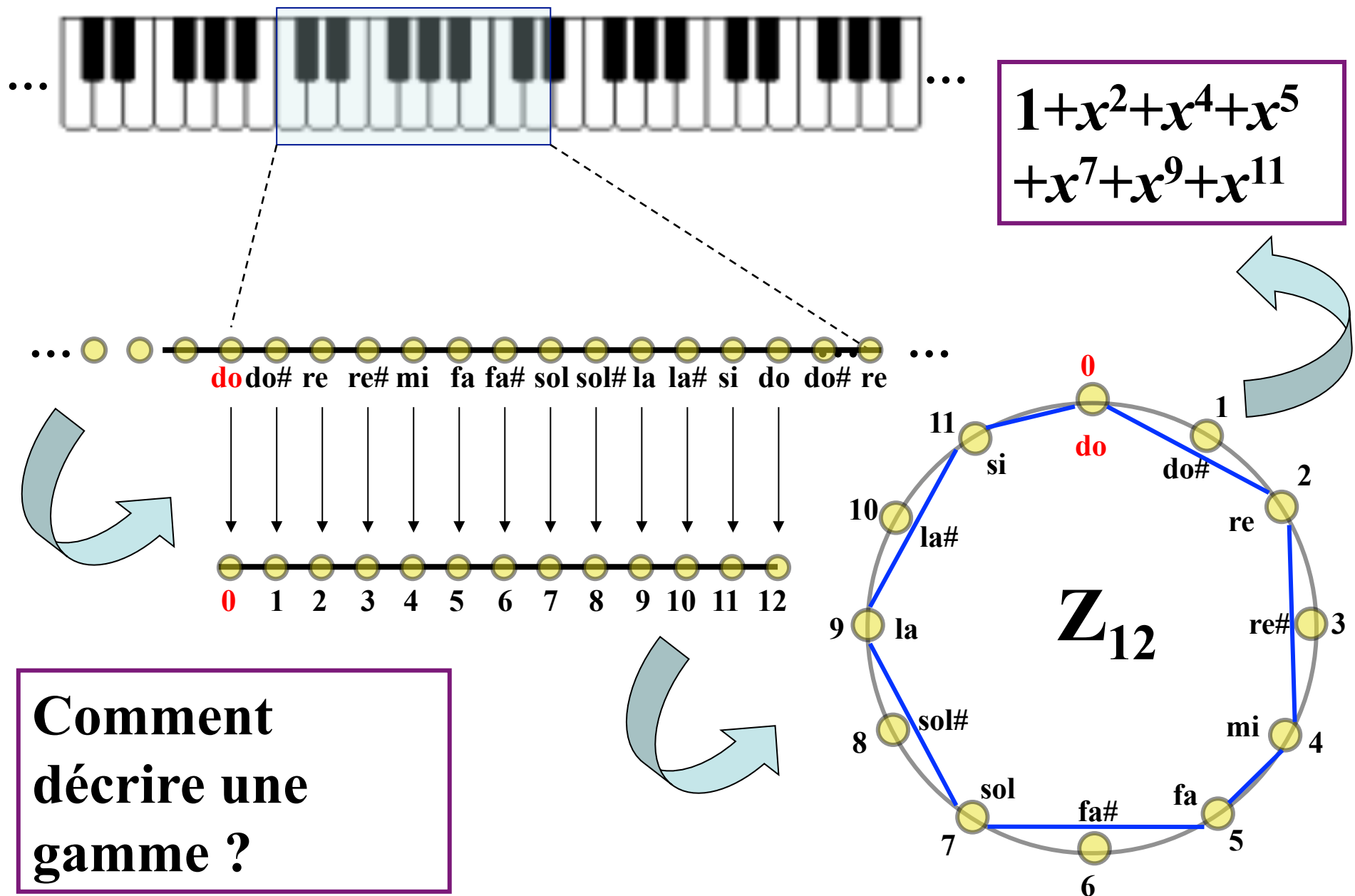
Réduction à l'octave et congruence modulo 12



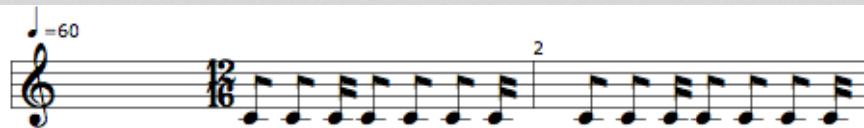
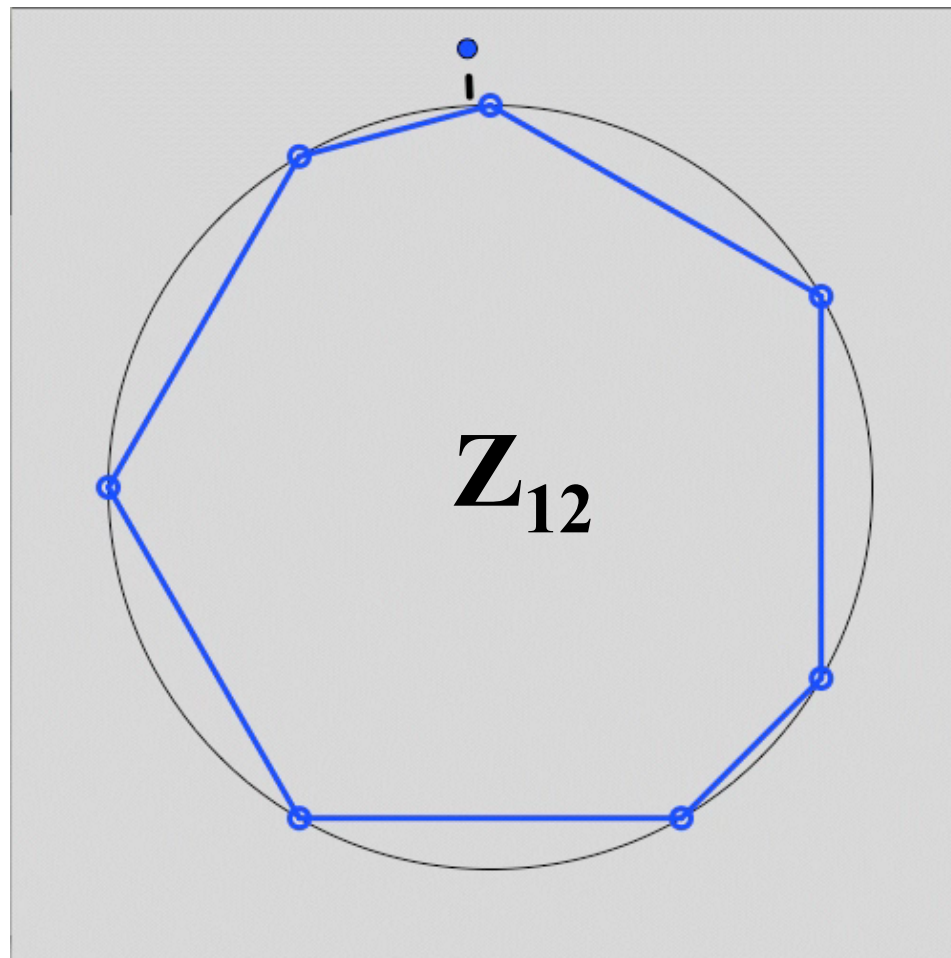
Réduction à l'octave et congruence modulo 12



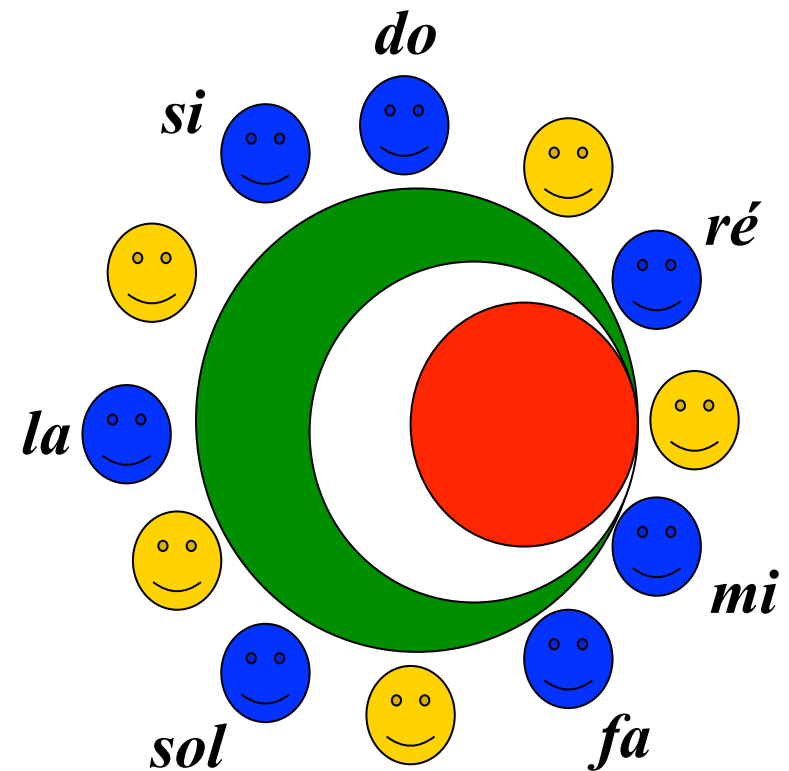
Réduction à l'octave et congruence modulo 12



Un pattern rythmique « bien reparti »



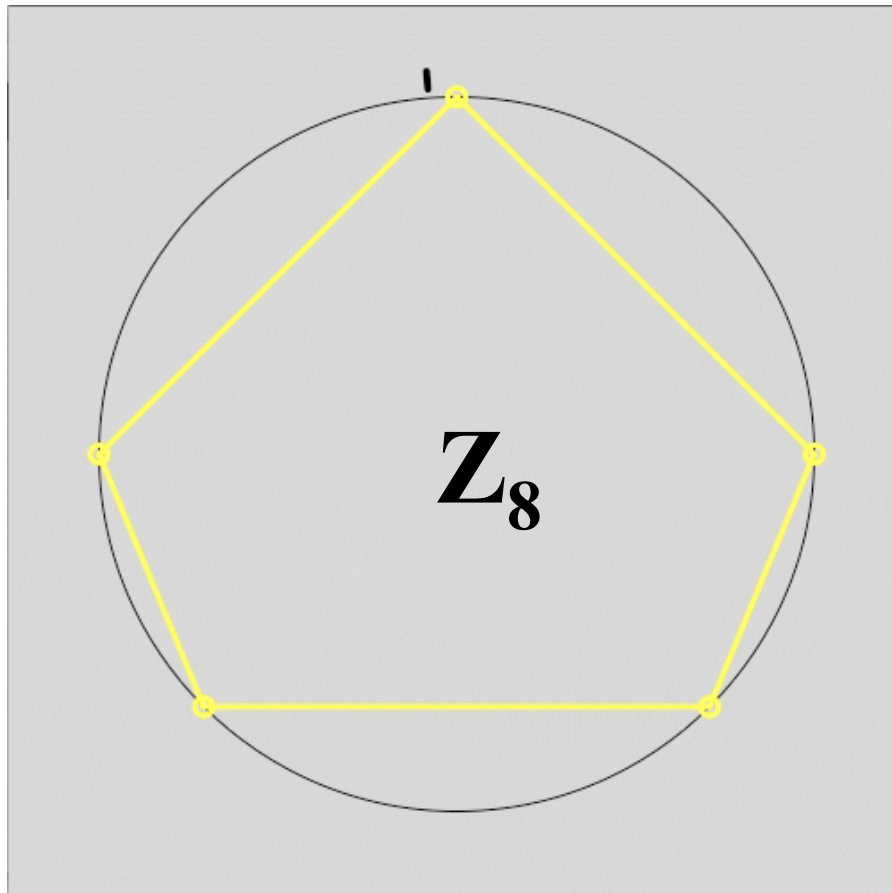
[maximally even]



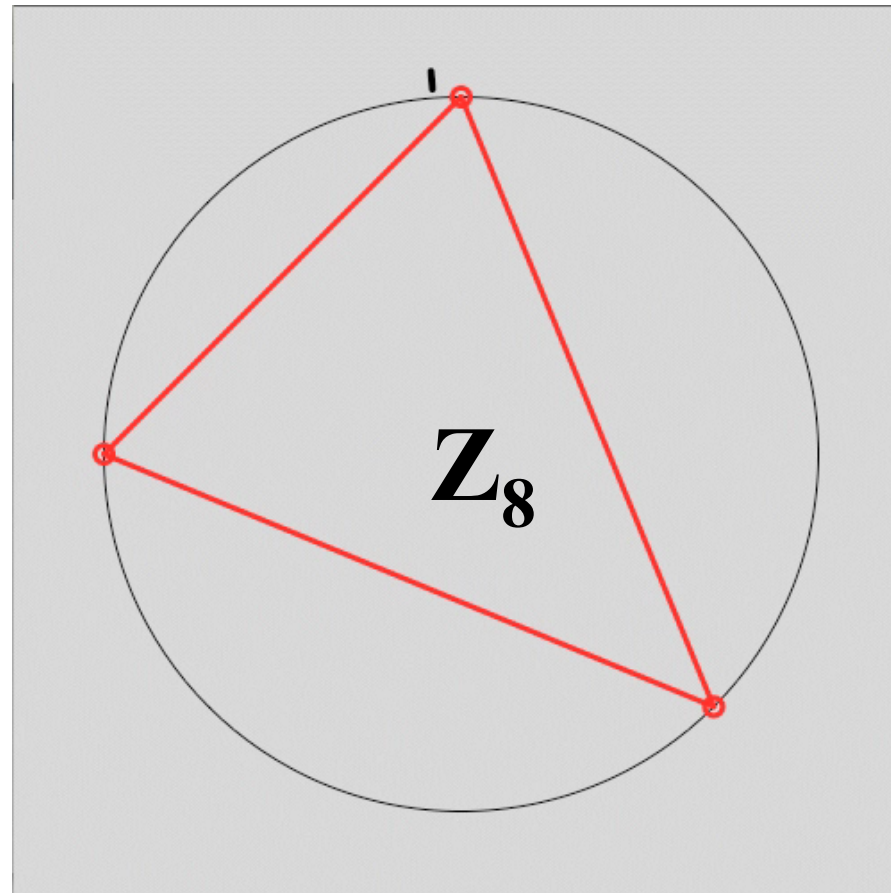
Jack Douthett & Richard Krantz, "Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction", *J. Math. Phys.* 37 (7), July 1996

Rythmes traditionnels « bien repartis »

cinquillo

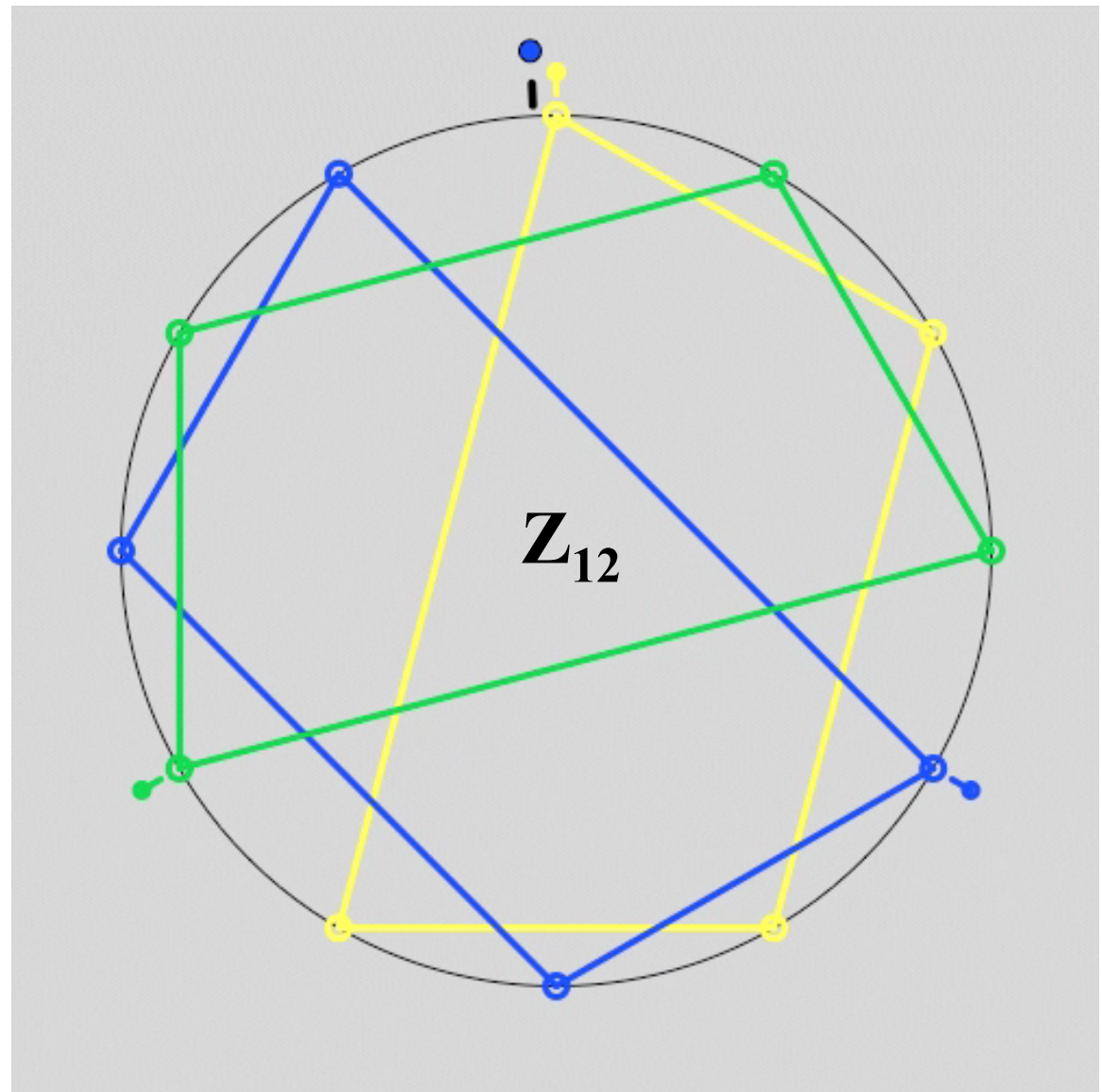
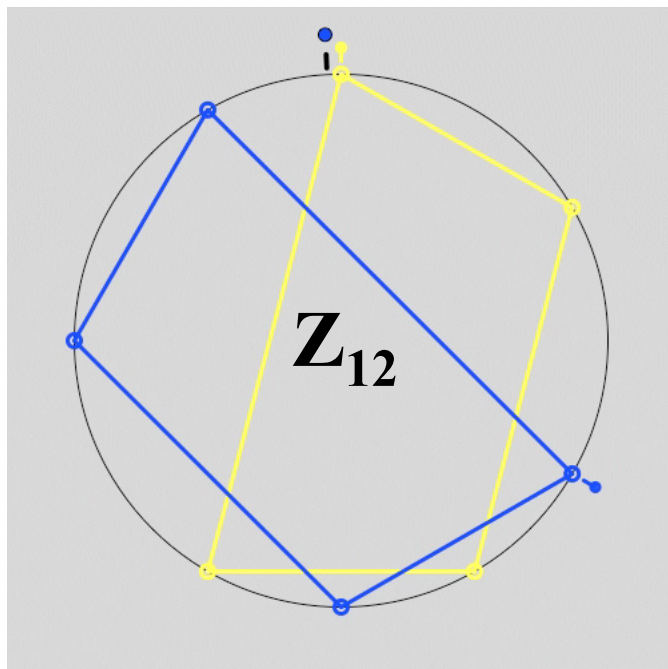
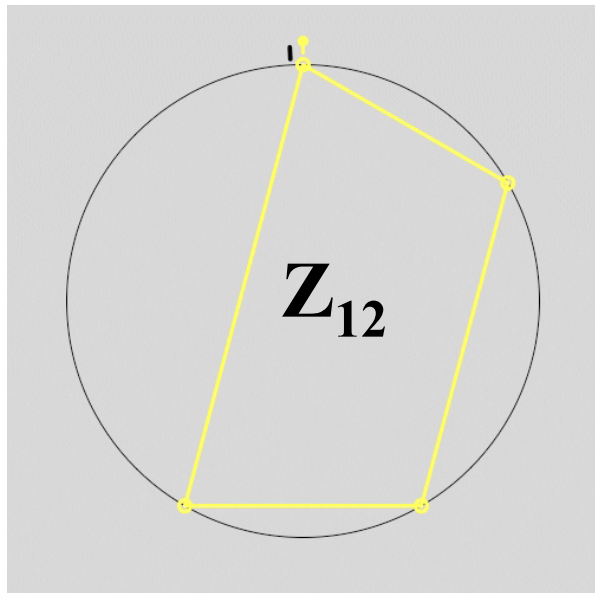


trecillo



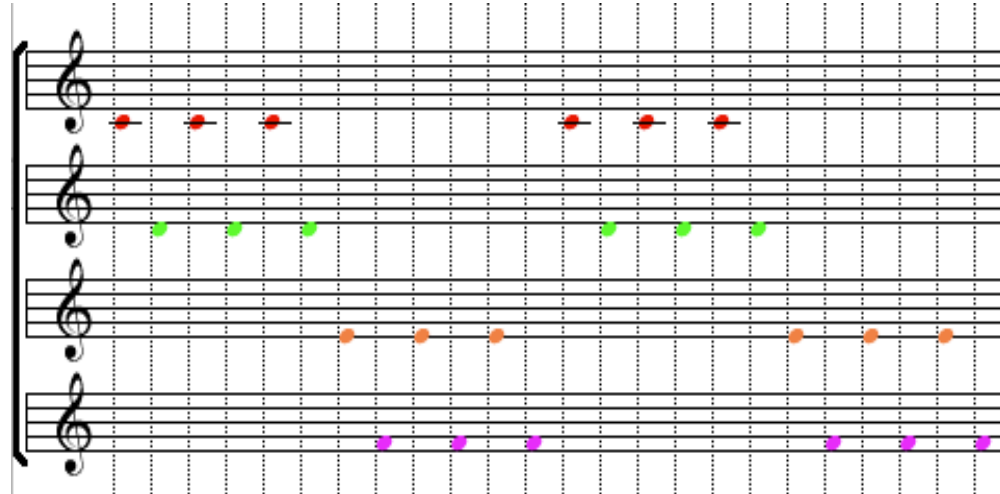
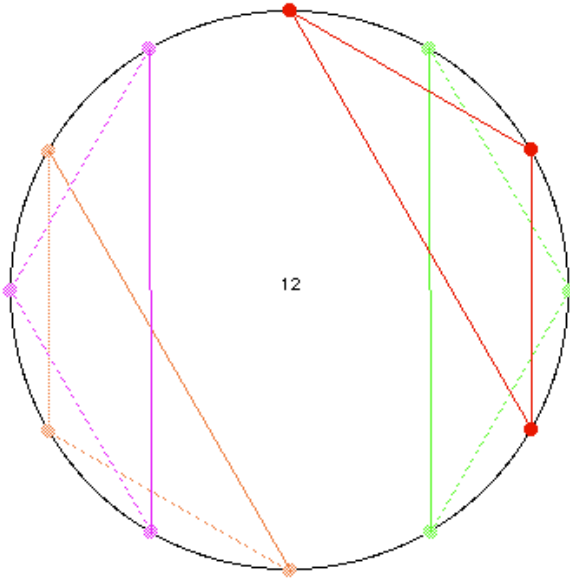
The image consists of three diagrams illustrating the construction of a 12-pointed star polygon. The top-left diagram shows a circle with a yellow pentagon inscribed. The top-right diagram shows the same circle with a blue pentagon inscribed. The bottom diagram shows the circle with both the yellow and blue pentagons inscribed, forming a 12-pointed star polygon.

Les canons rythmiques mosaïques : type 1



$$\{0,2,5,7\} \oplus \{0,4,8\} \Leftrightarrow (1+x^2+x^5+x^7) \times (1+x^4+x^8)$$

Les canons rythmiques mosaïques : type 2

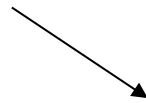
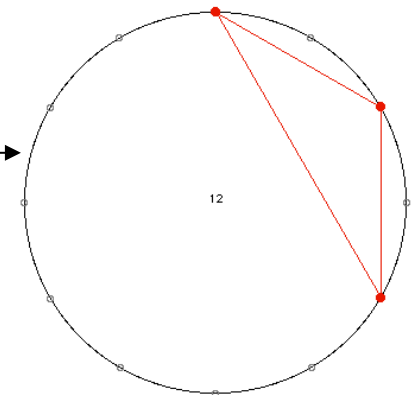
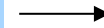


$$\mathbf{Z}_{12} = \mathbf{A} \oplus \mathbf{B}$$

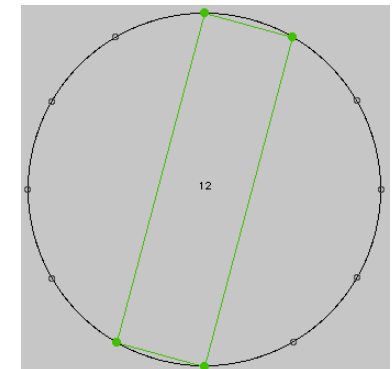
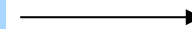
$$\mathbf{A} = \{0, 2, 4\}$$

$$\mathbf{B} = \{0, 1, 6, 7\}$$

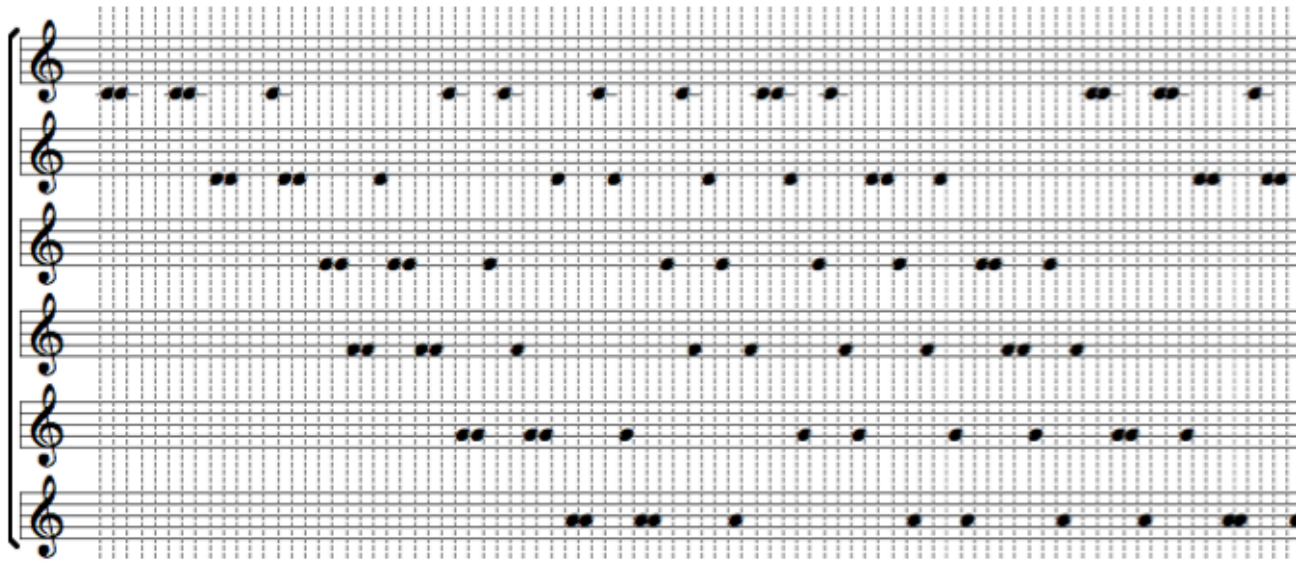
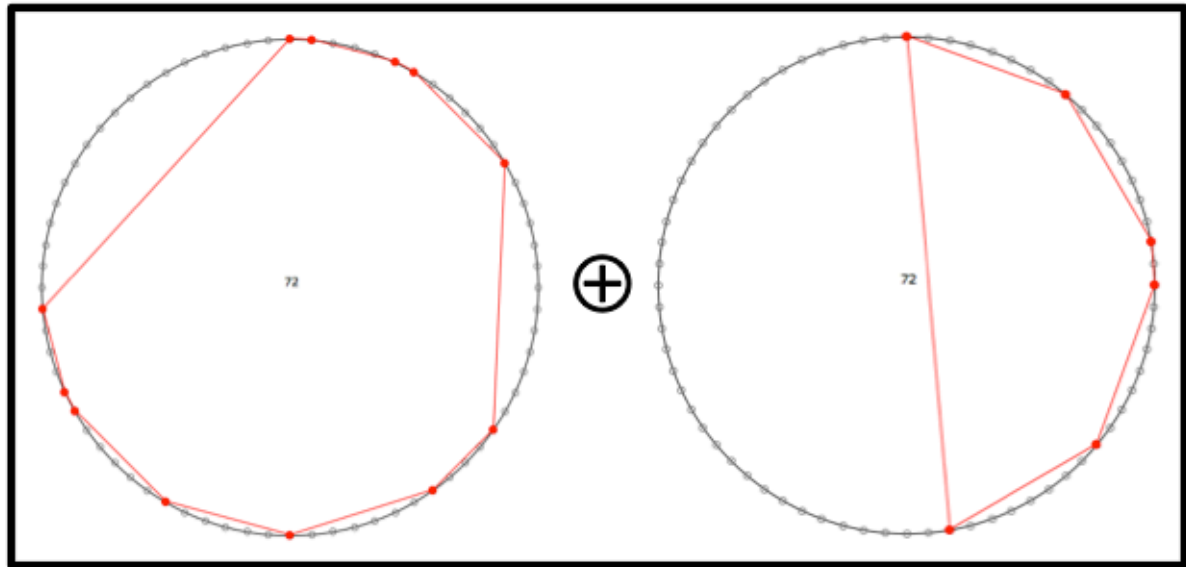
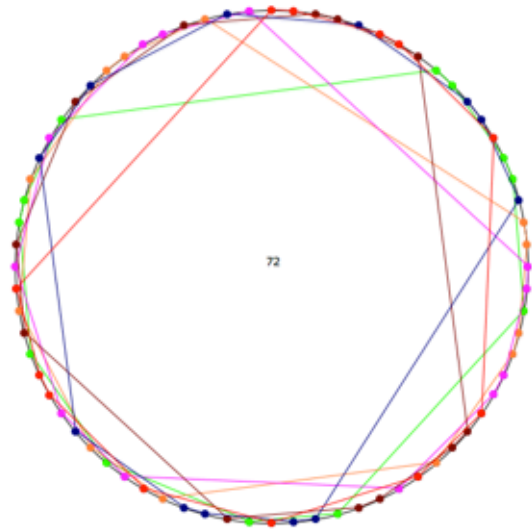
$$(2, 2, 8)$$



$$(1, 5, 1, 5)$$



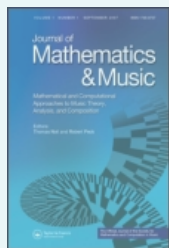
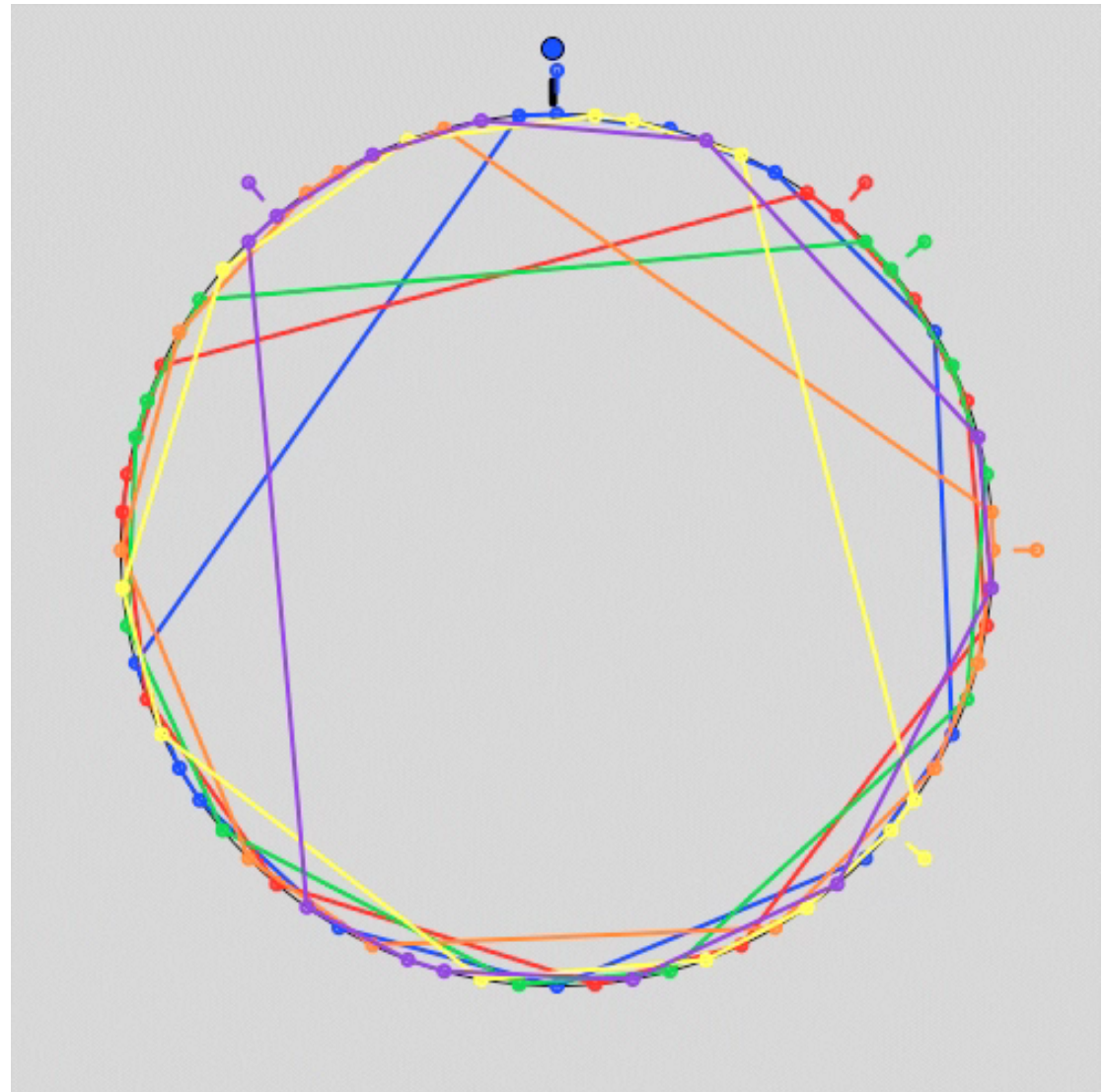
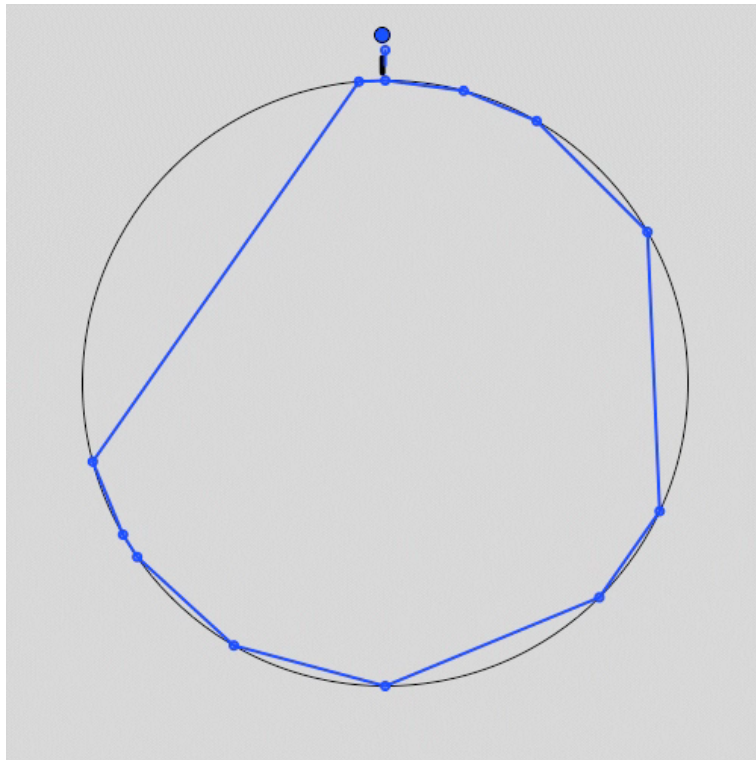
Les canons rythmiques mosaïques de Vuza : type 3



Dan Vuza



Les canons rythmiques mosaïques de Vuza : type 3



M. Andreatta & C. Agon
eds. Special Issue « Tiling
Problems in Music »,
*Journal of Mathematics and
Music*, juill., 3(2), 2009

$$(3,3,6,11,4,9,6,5,1,3,20,1) \bullet (8,2,8,8,38)$$

L'orchestration « par imitation stylistique »

The diagram illustrates the process of orchestration by stylistic imitation. It features a central sequence of musical elements connected by arrows:

- A single bass line in 2/4 time, marked *p*, with a speaker icon above it.
- A full orchestral score for *Segmentation des séries temporelles* by Maria Mannone, with a speaker icon above it.
- A comparison between two composers: a portrait of Olivier Messiaen on the left and a portrait of Igor Stravinsky on the right, separated by a dot and an equals sign.

The orchestral score includes parts for:

- Comi in Fa 1, 2 - 3, 4
- Comi in Fa 5, 6 - 7, 8
- Violini II
- Viola
- Violoncelli
- Contrabbassi

The score is marked *Tempo giusto, $\text{♩} = 50$* and includes various performance instructions such as *sempre*, *sempre simile*, *sempre stacc.*, and *non div.*

Maria Mannone, *Segmentation des séries temporelles pour l'orchestration automatique*, mémoire Master ATIAM, septembre 2013