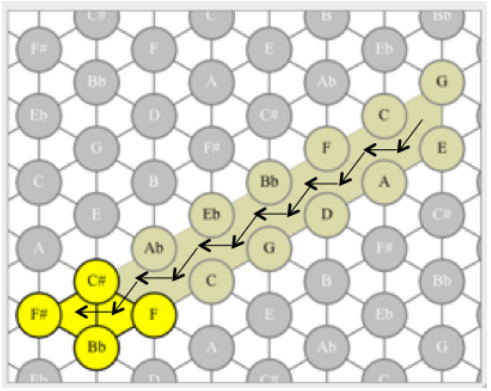
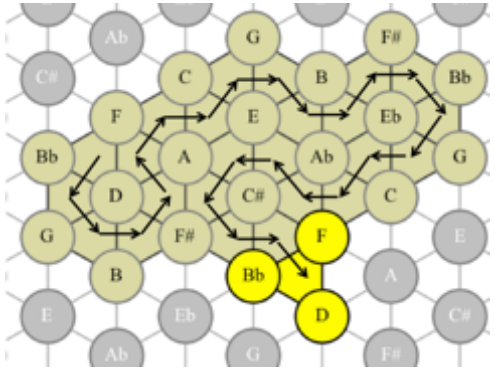
« Les maths…un exercice de musique ! Voyage d'initiation au cœur des recherches 'mathémusicales' »

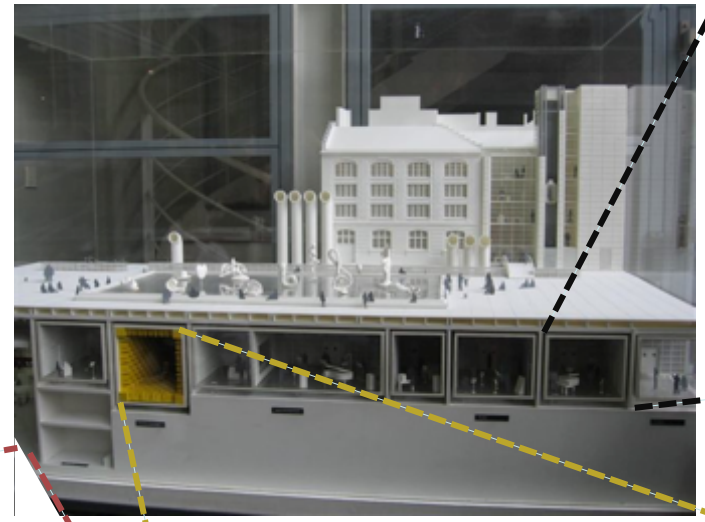


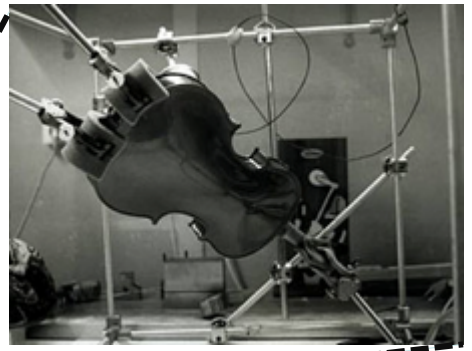
Auditorium du Lycée La Fontaine,
21 mars 2014



Moreno Andreatta
Equipe Représentations Musicales
IRCAM / CNRS UMR 9912 / UPMC
Moreno.Andreatta@ircam.fr

Être chercheur en musique à l'IRCAM

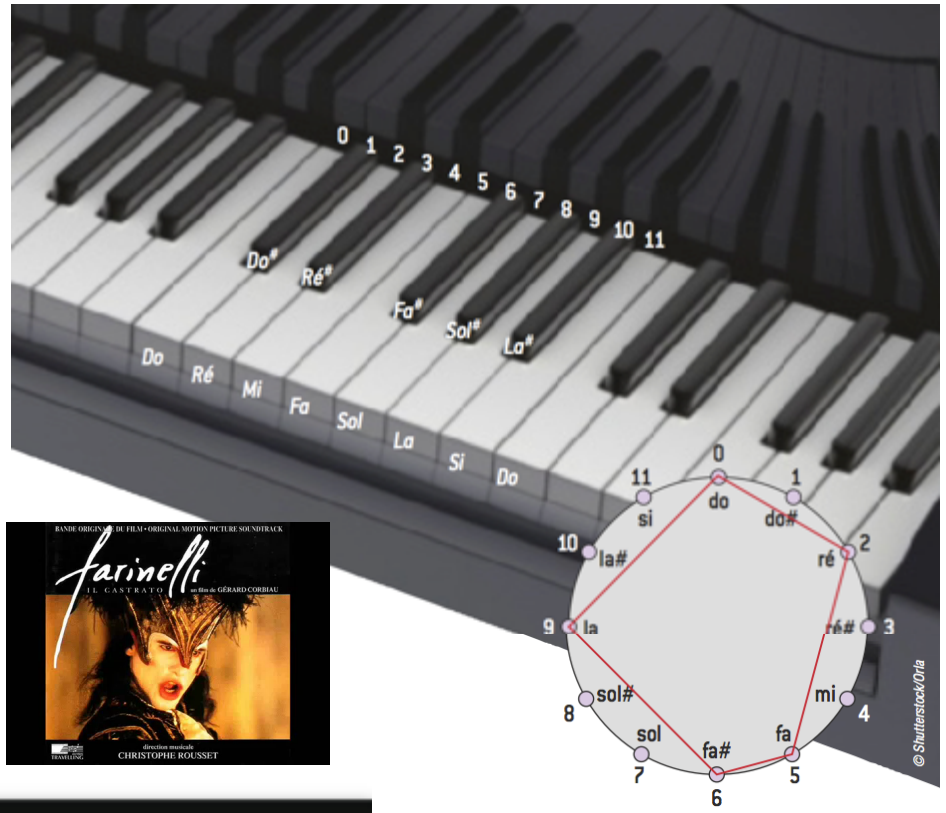




www.ircam.fr

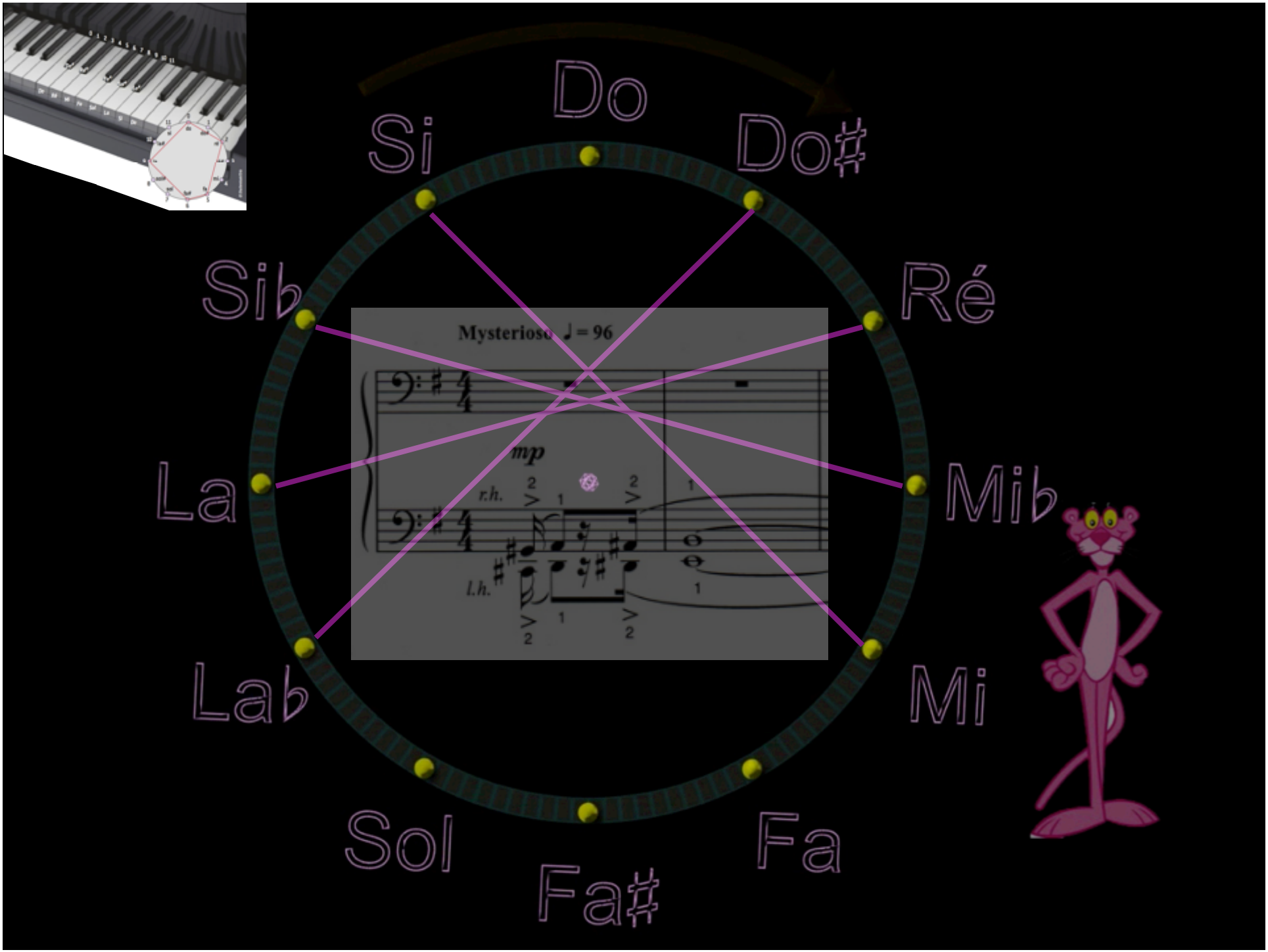
Être chercheur en musique à l'IRCAM

MusiqueLab 2



OMAX (logiciel d'improvisation)

www.ircam.fr


Do
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Mysterioso ♩ = 96
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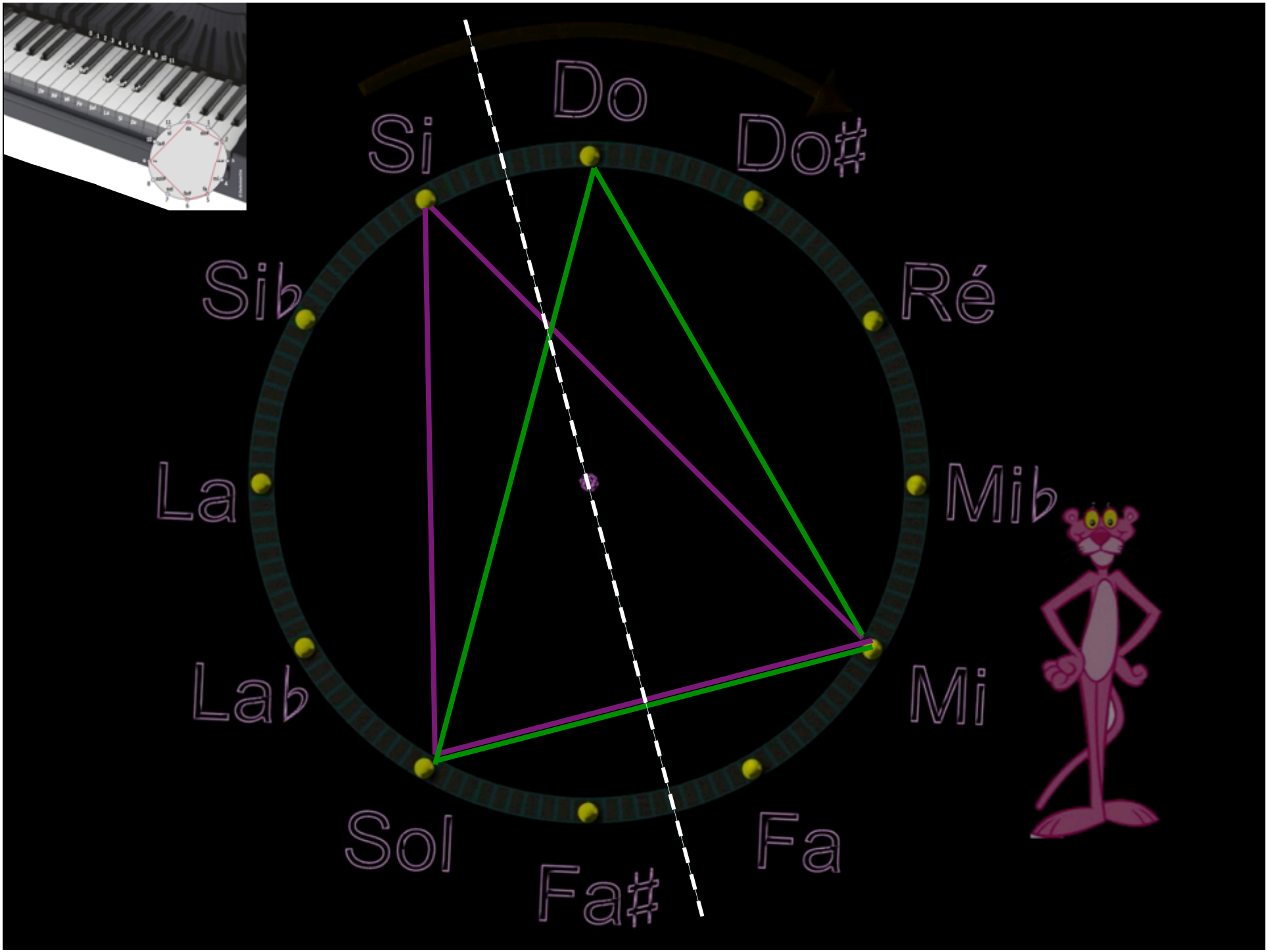
Si
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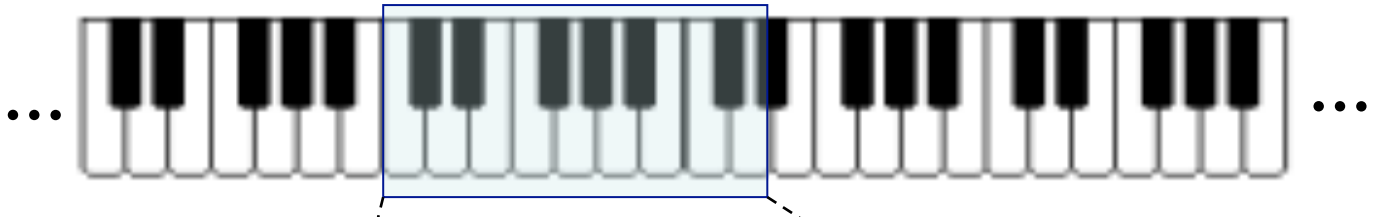
La réduction à l'octave et le cadran d'horloge



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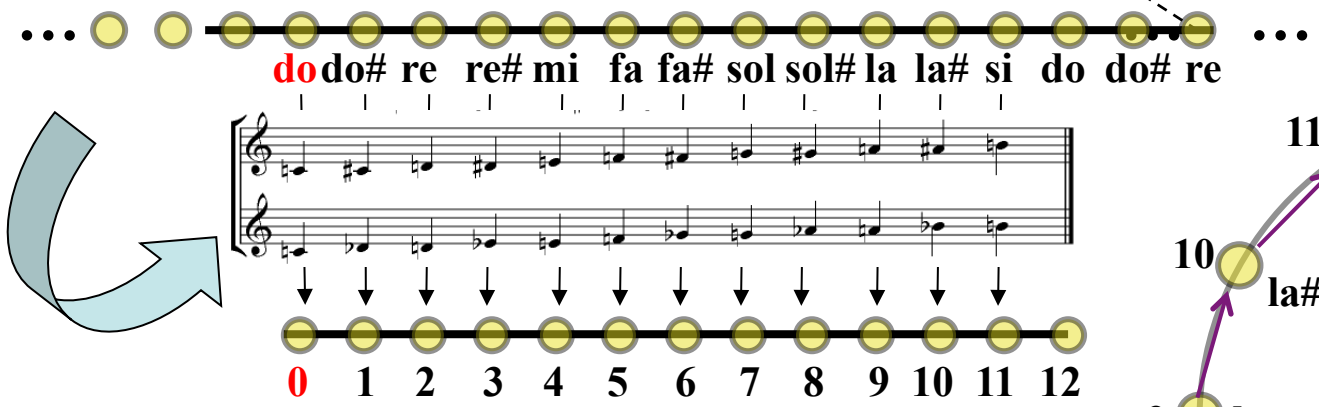
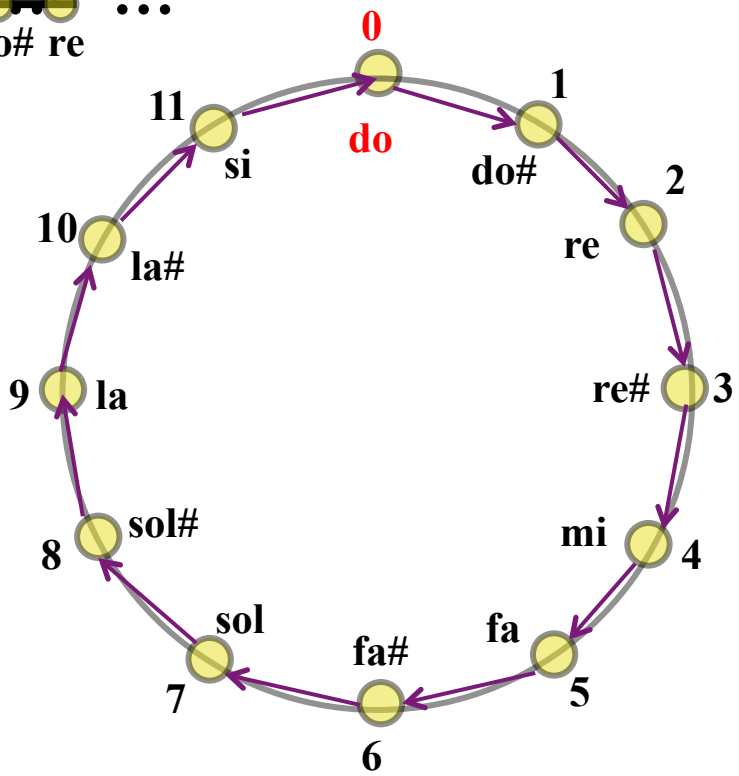
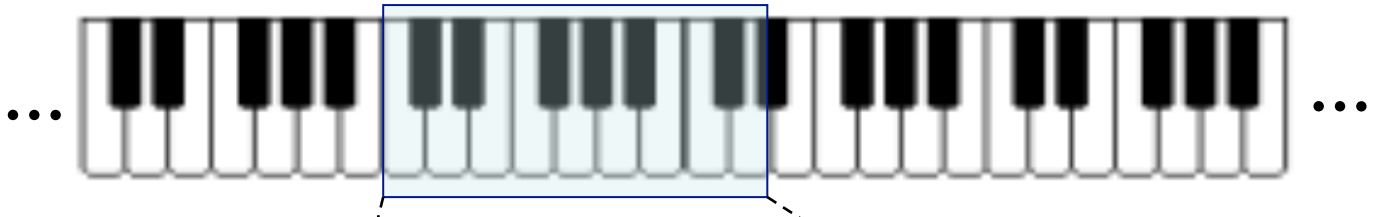
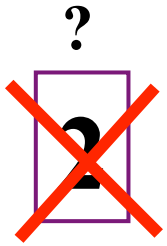


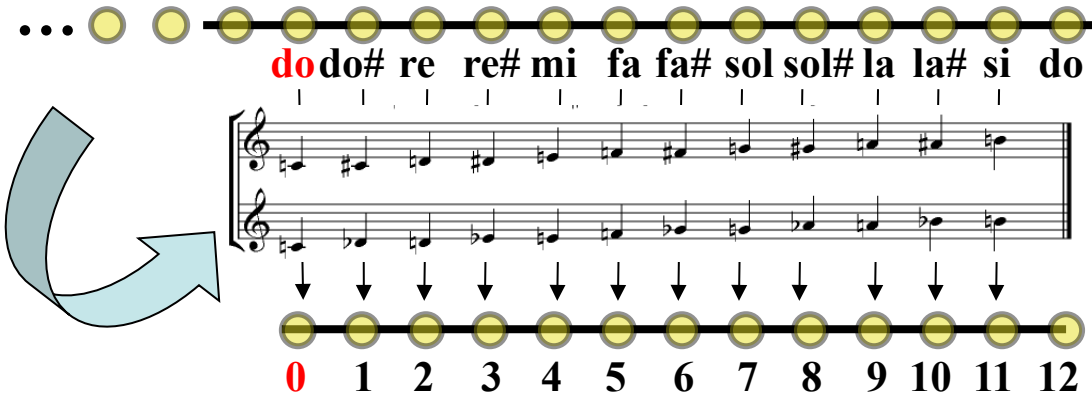


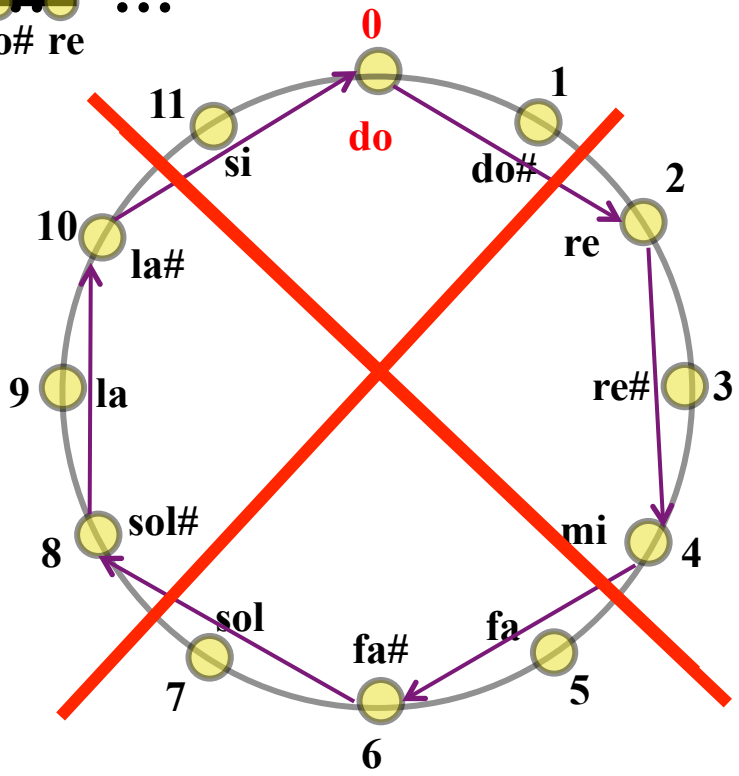
➔ **Quels sont les autres entiers qui engendrent le cercle ?**

La réduction à l'octave et le cadran d'horloge









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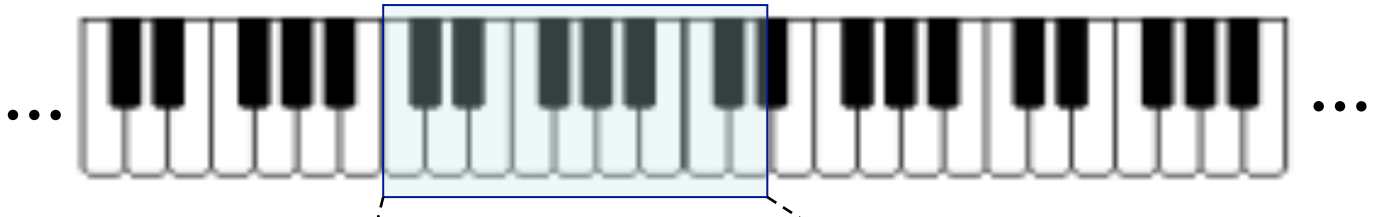

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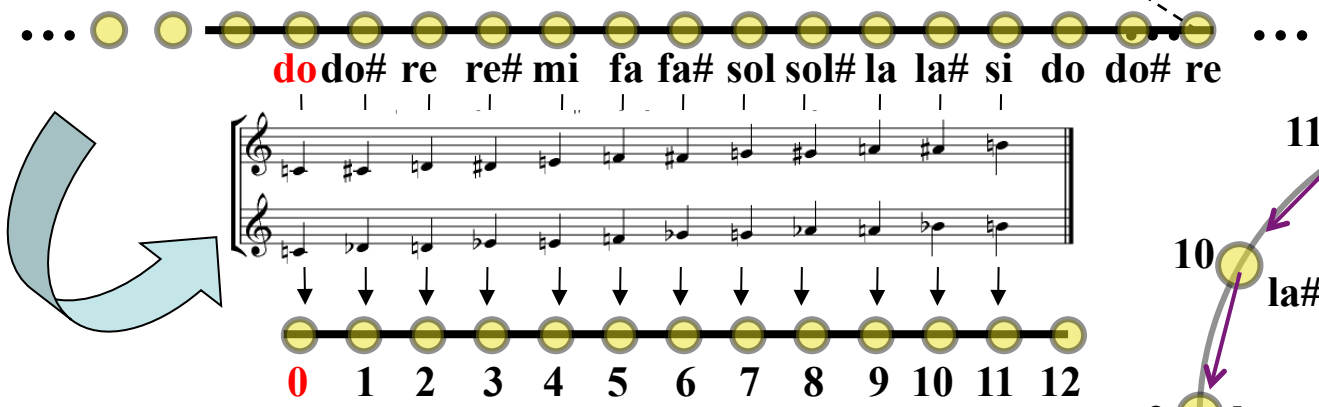

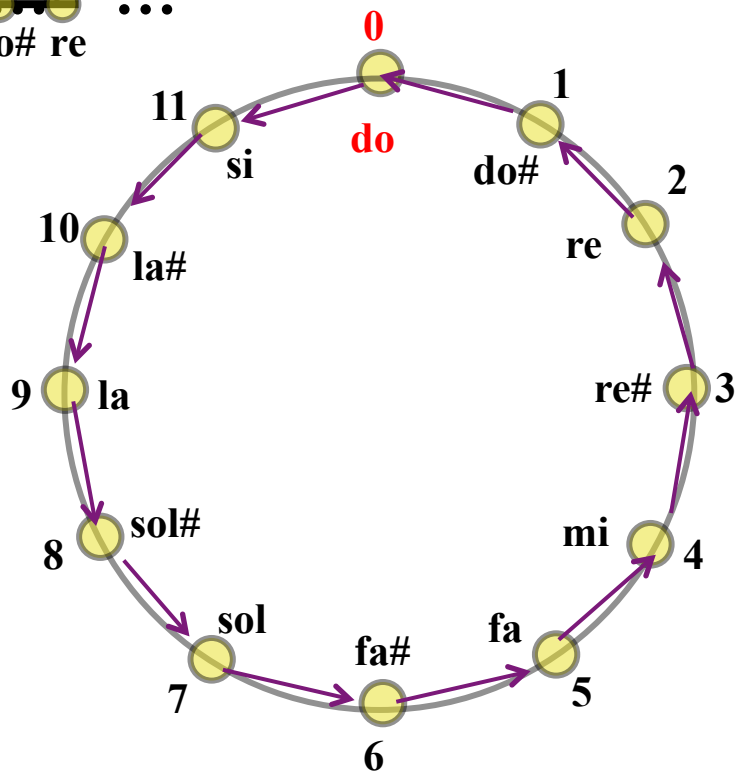

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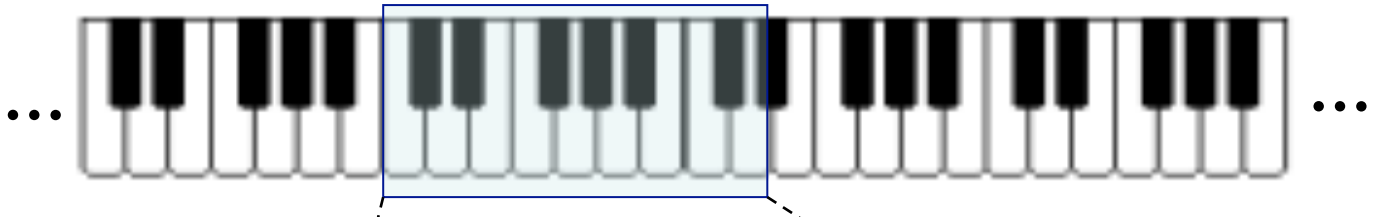

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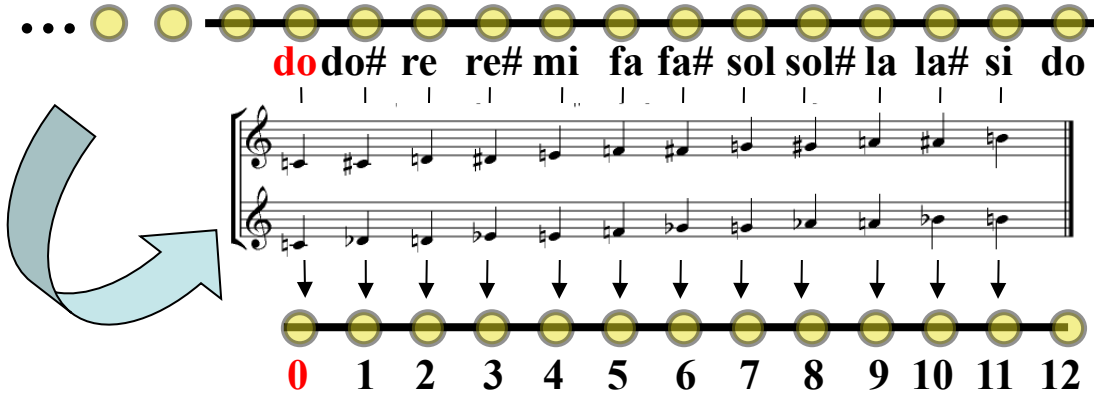
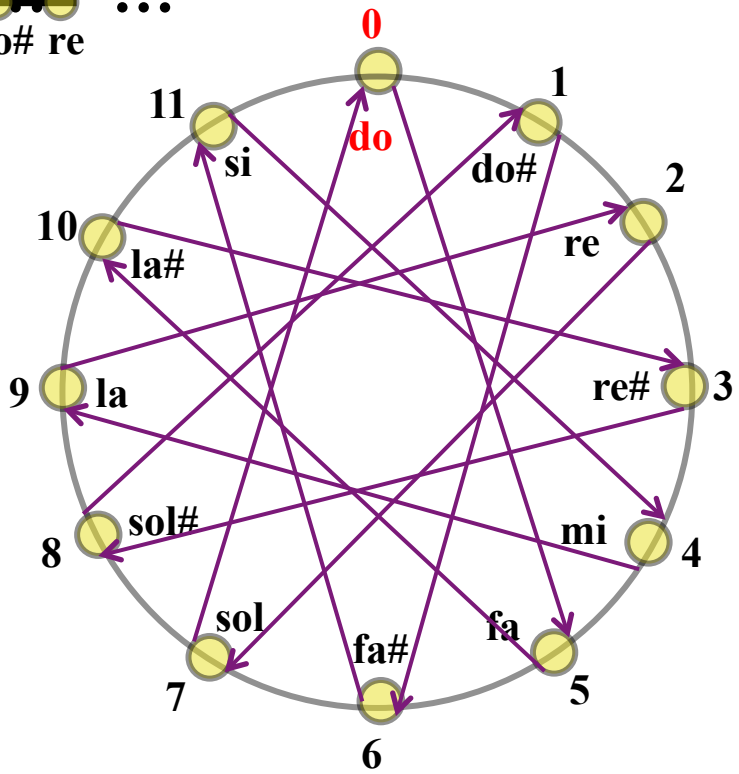
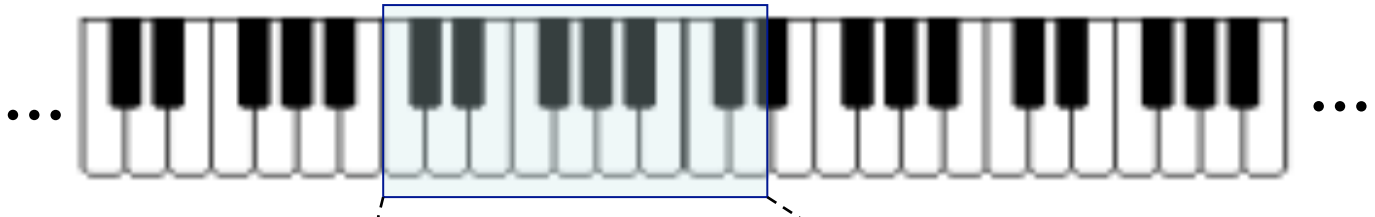




Cycle des quartes

La réduction à l'octave et le cadran d'horloge



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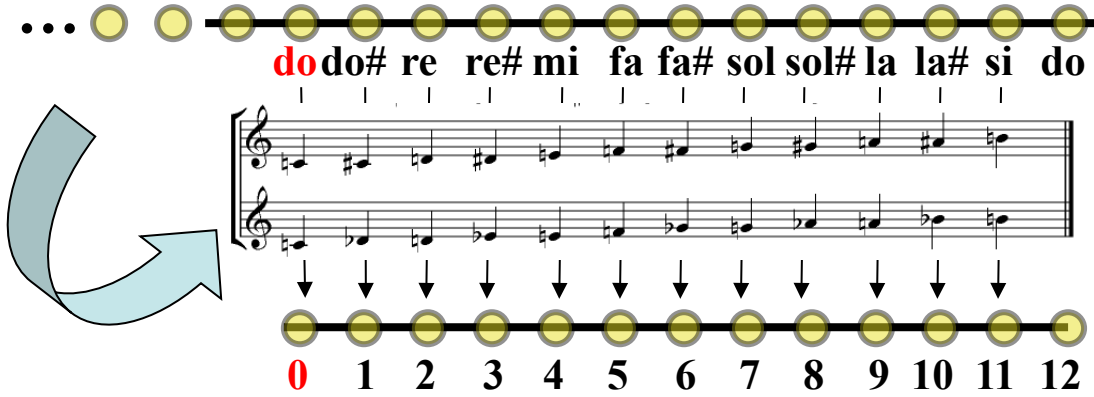
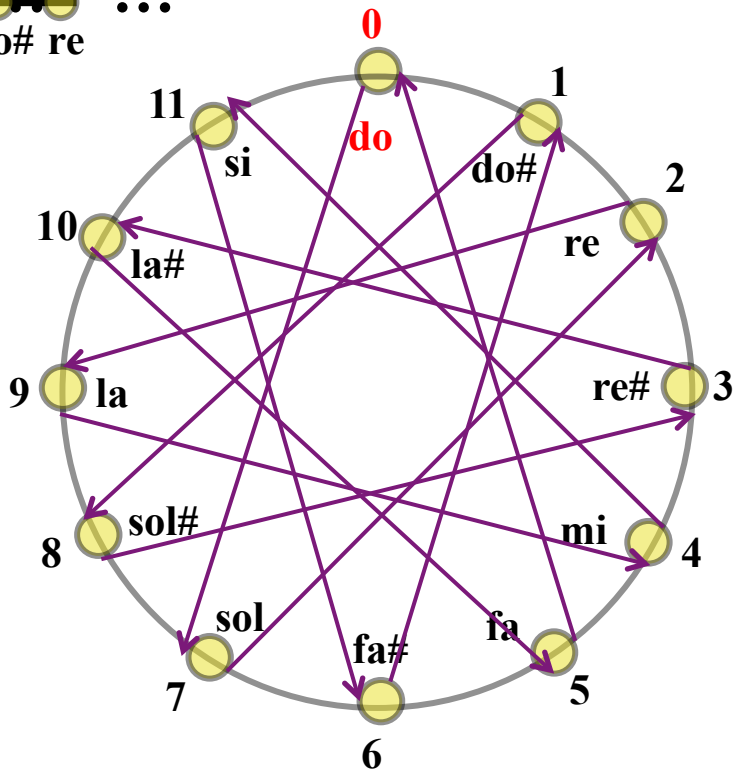
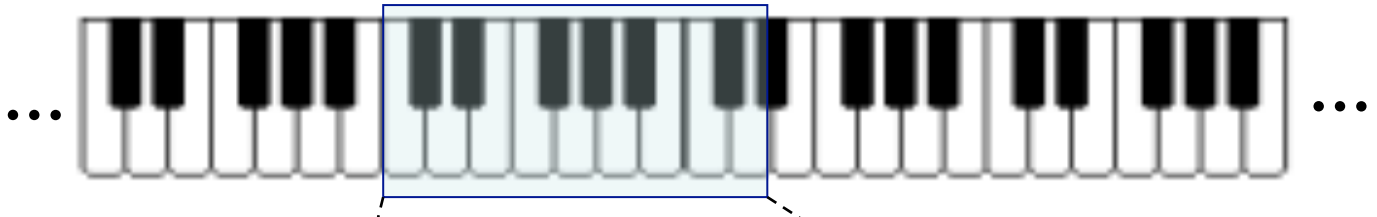
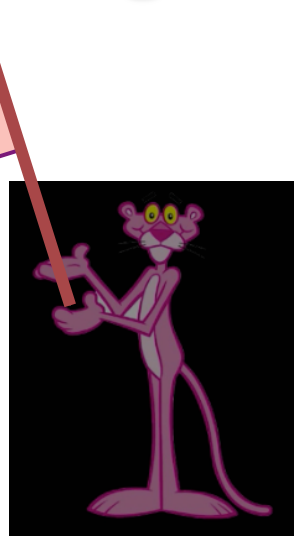


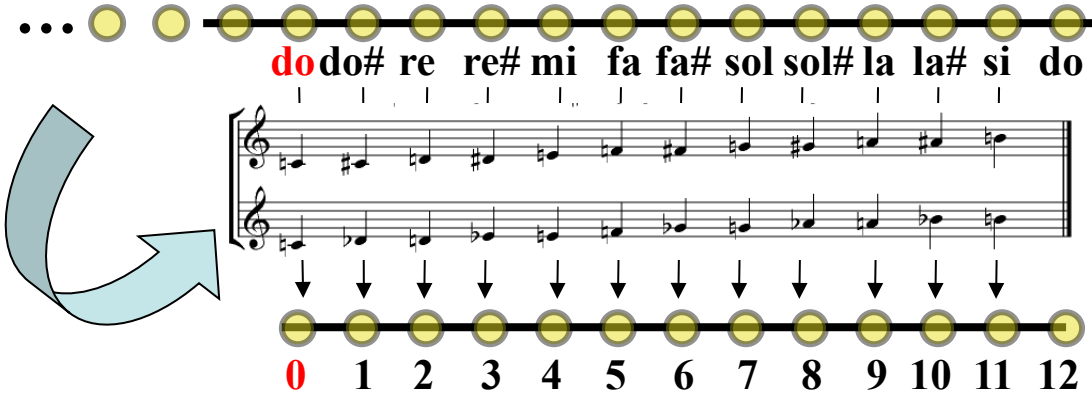


Cycle des quintes

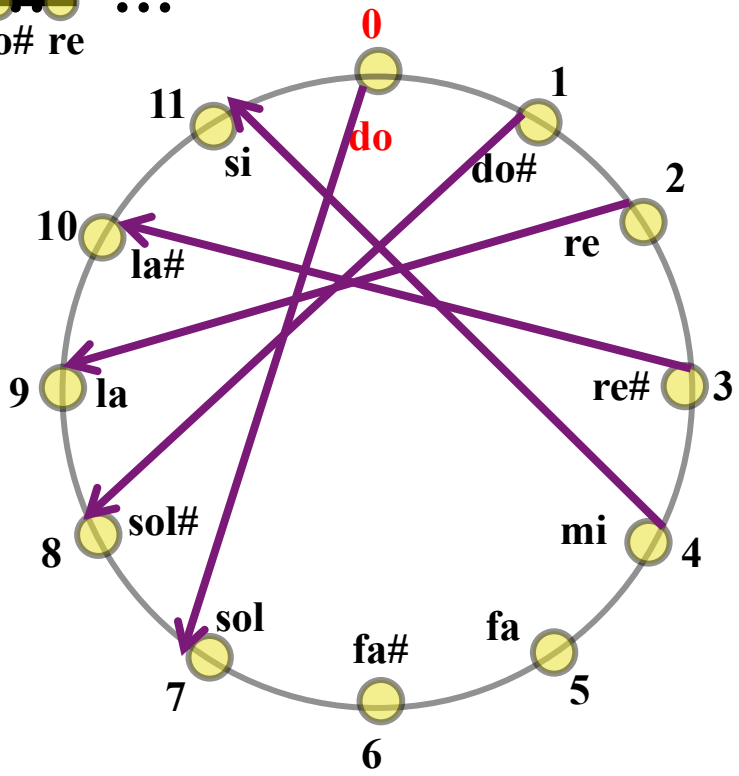
La réduction à l'octave et le cadran d'horloge



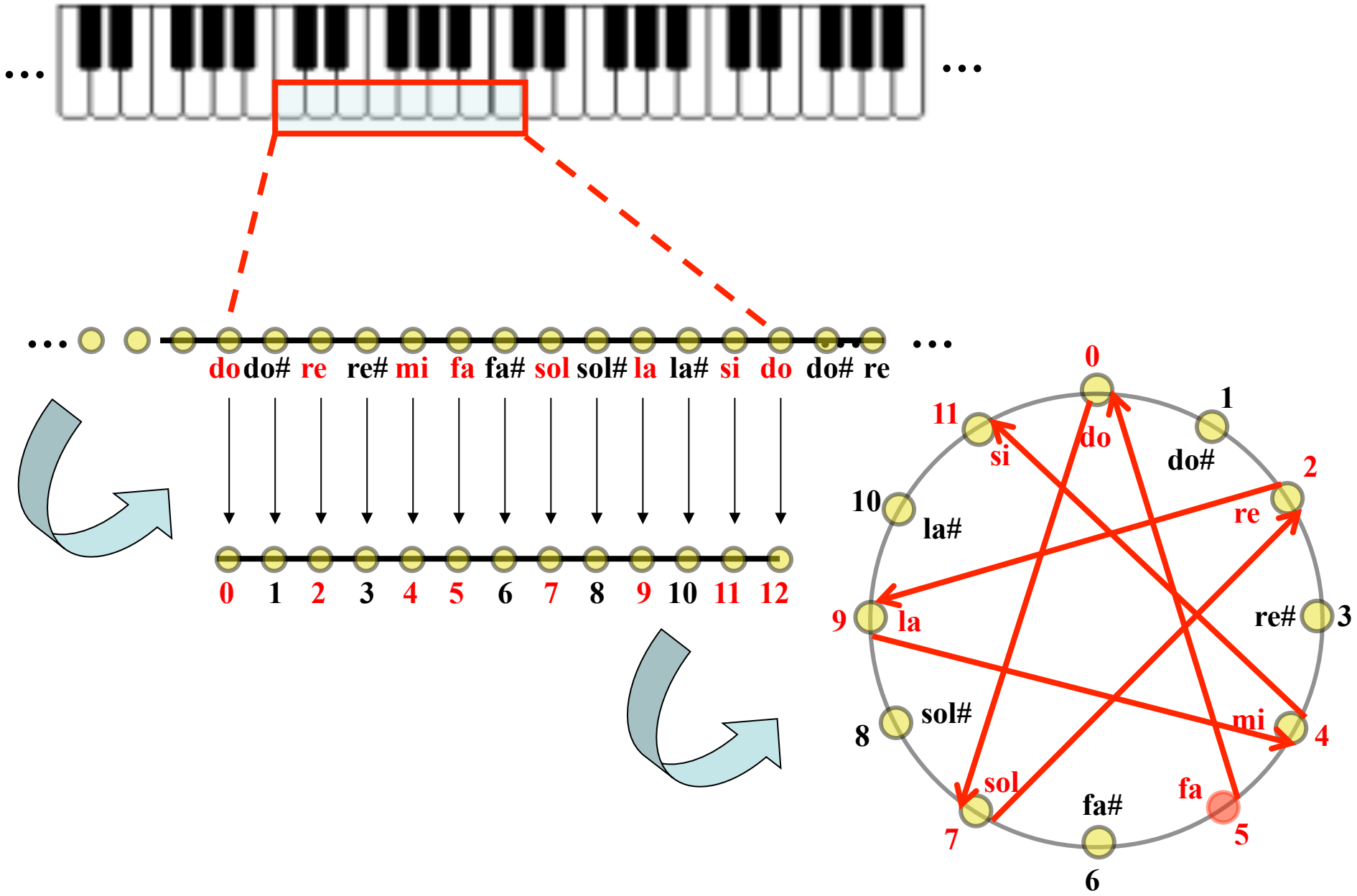




Cycle des quintes

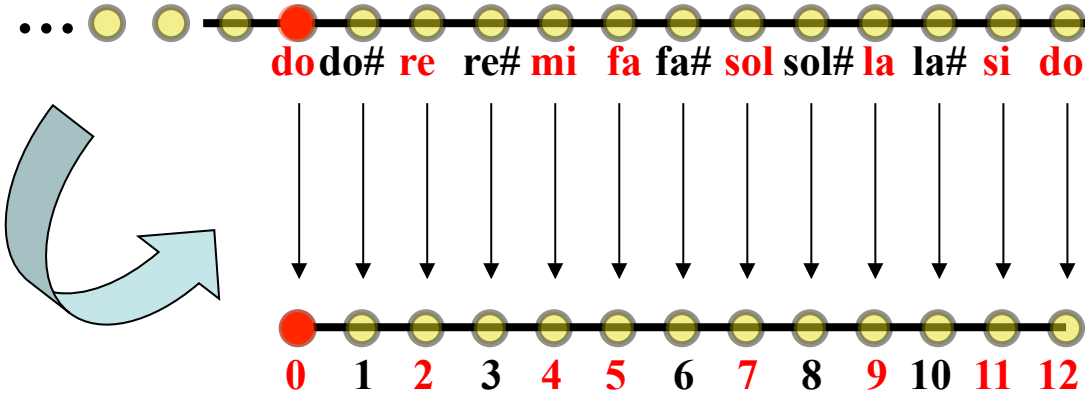


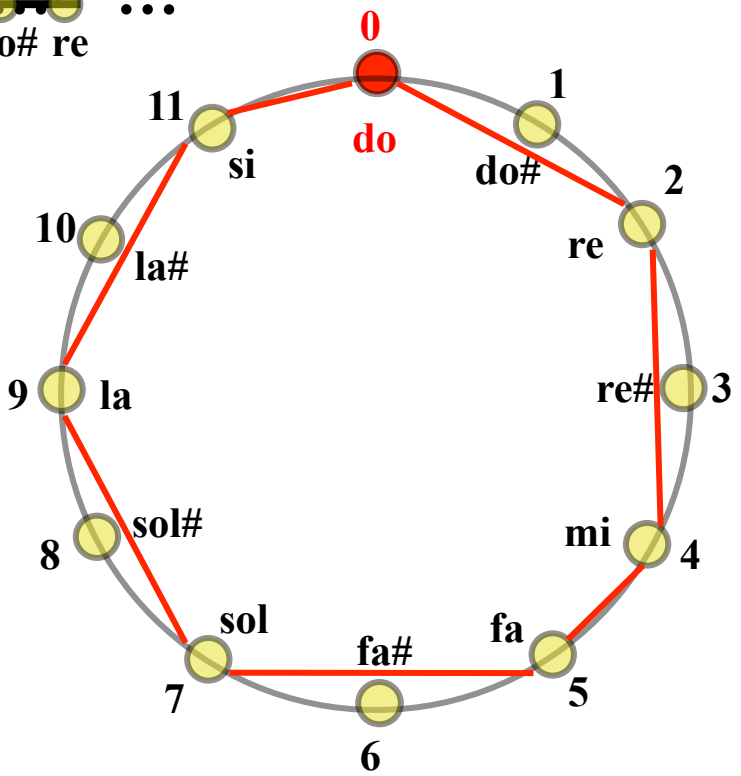
Le cycle des quintes et la gamme diatonique



Le cycle des quintes et la gamme diatonique

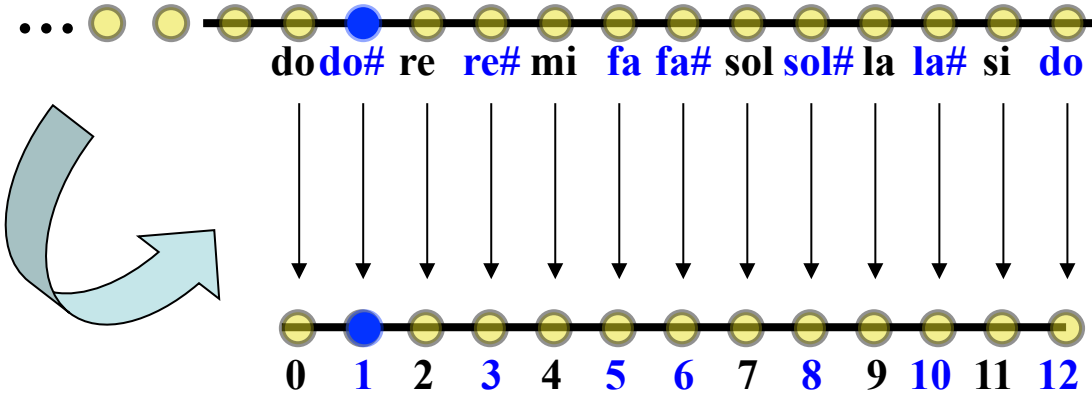
***Do* maj = gamme de *do* majeur**

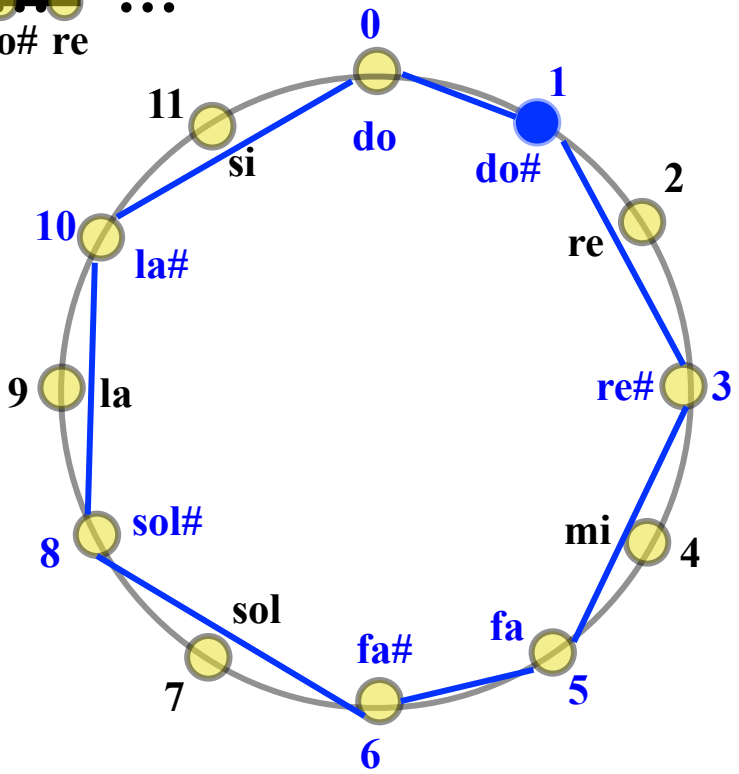




Les rotations d'un polygone sont les transpositions

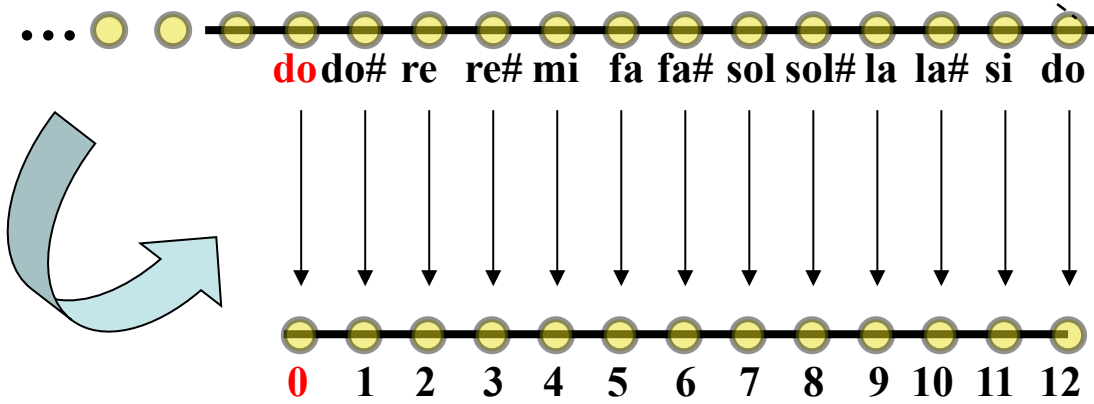
***Do#* maj = gamme de *do#* majeur**

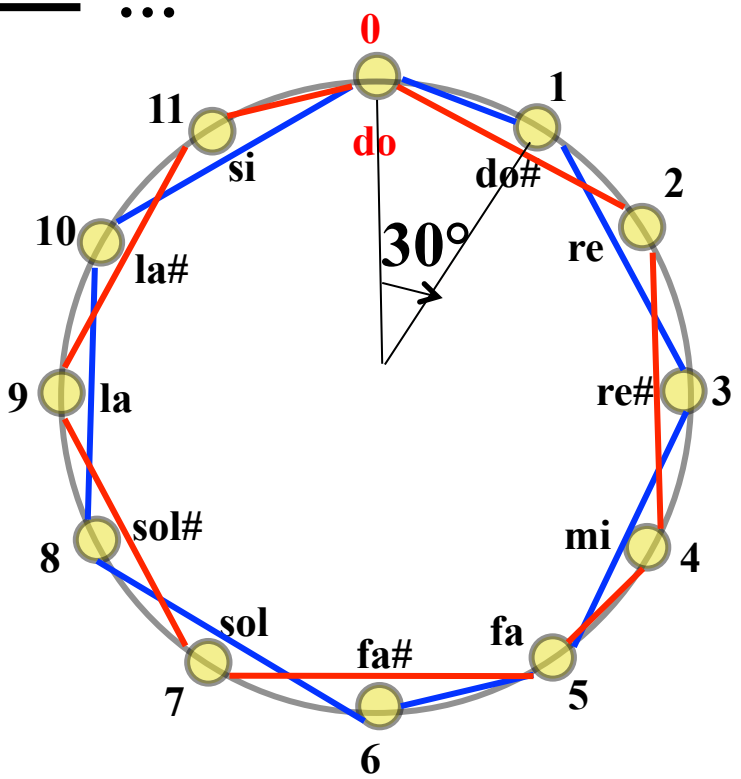




Les transpositions sont des additions...

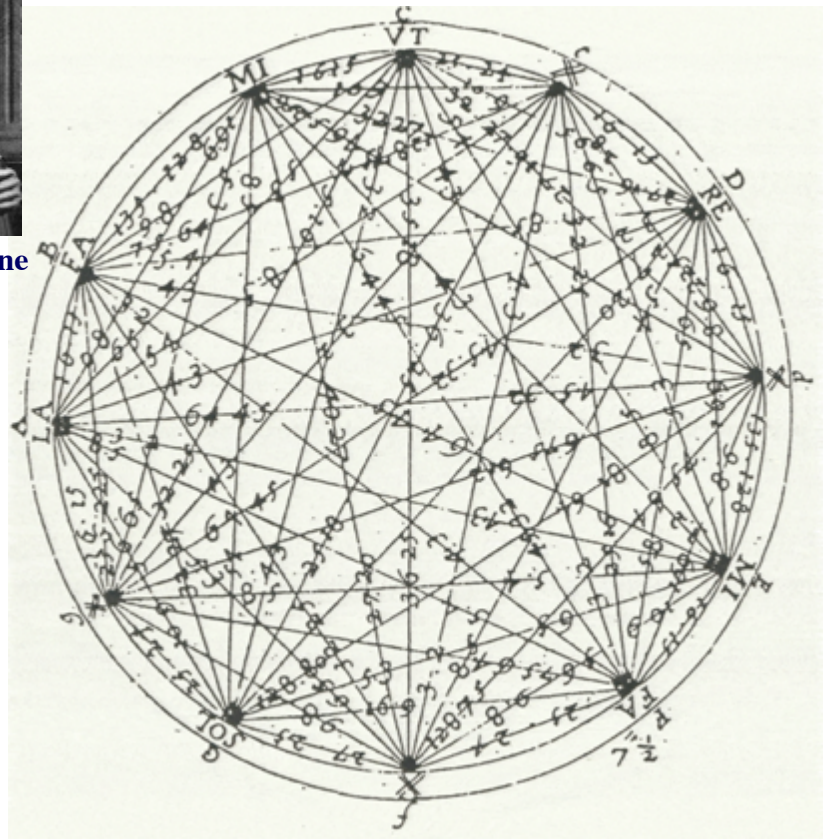
Do maj = {0,2,4,5,7,9,11} +1
Do# maj = {1,3,5,6,8,10,0}



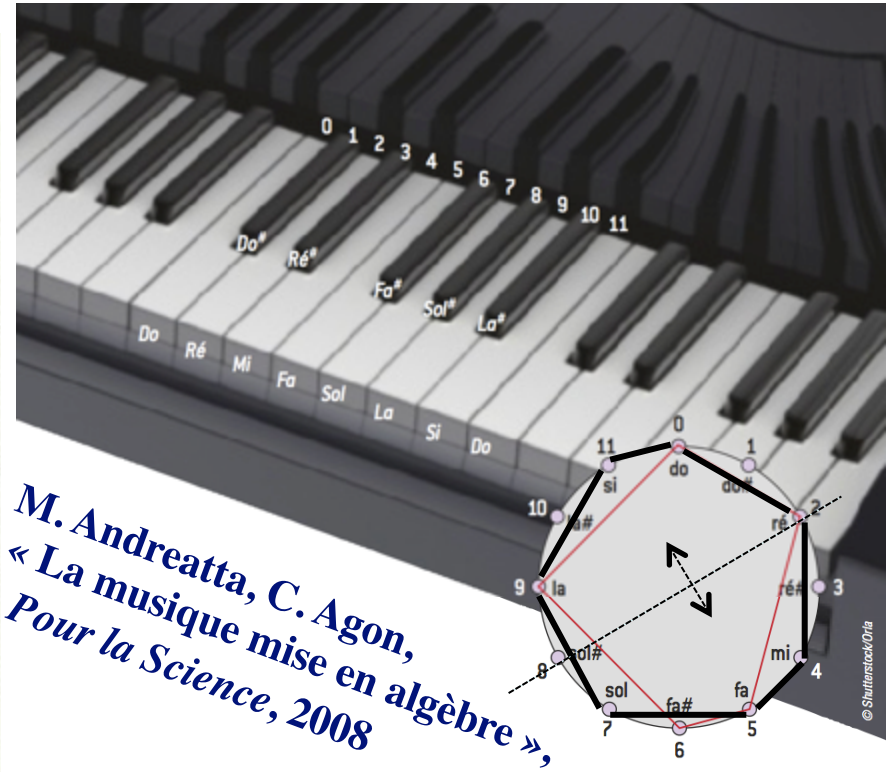


... ou des rotations !

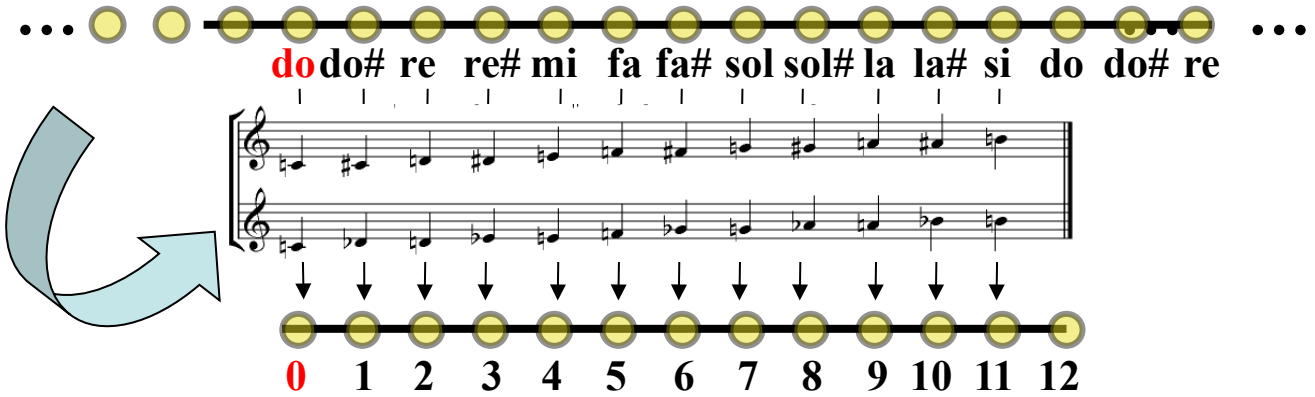
Marin Mersenne



Harmonicorum Libri XII, 1648



M. Andreatta, C. Agon, « La musique mise en algèbre », *Pour la Science*, 2008



Mersenne et la naissance de la combinatoire musicale

114 **Marin Mersenne,** ***Harmonicorum Libri XII*****, 1648**

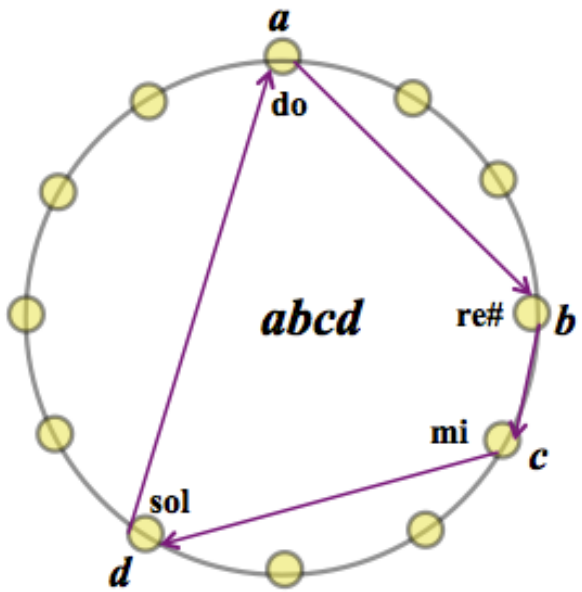
LIBER SEPTIMVS.

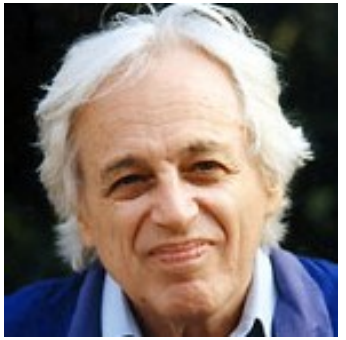
DE CANTIBVS, SEV CANTILENIS,

EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 22.

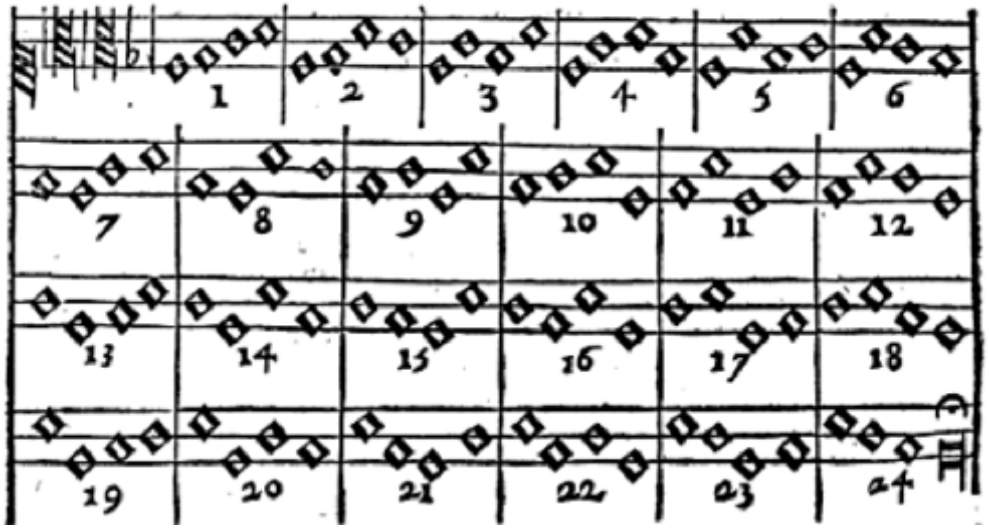
I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	361880
X	3618800
XI	39916800
XII	479001600
XIII	6127010800
XIV	87178191200
XV	1307674368000
XVI	20922789888000
XVII	355687418096000
XVIII	6402373705718000
XIX	121645100408832000
XX	2432901008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000



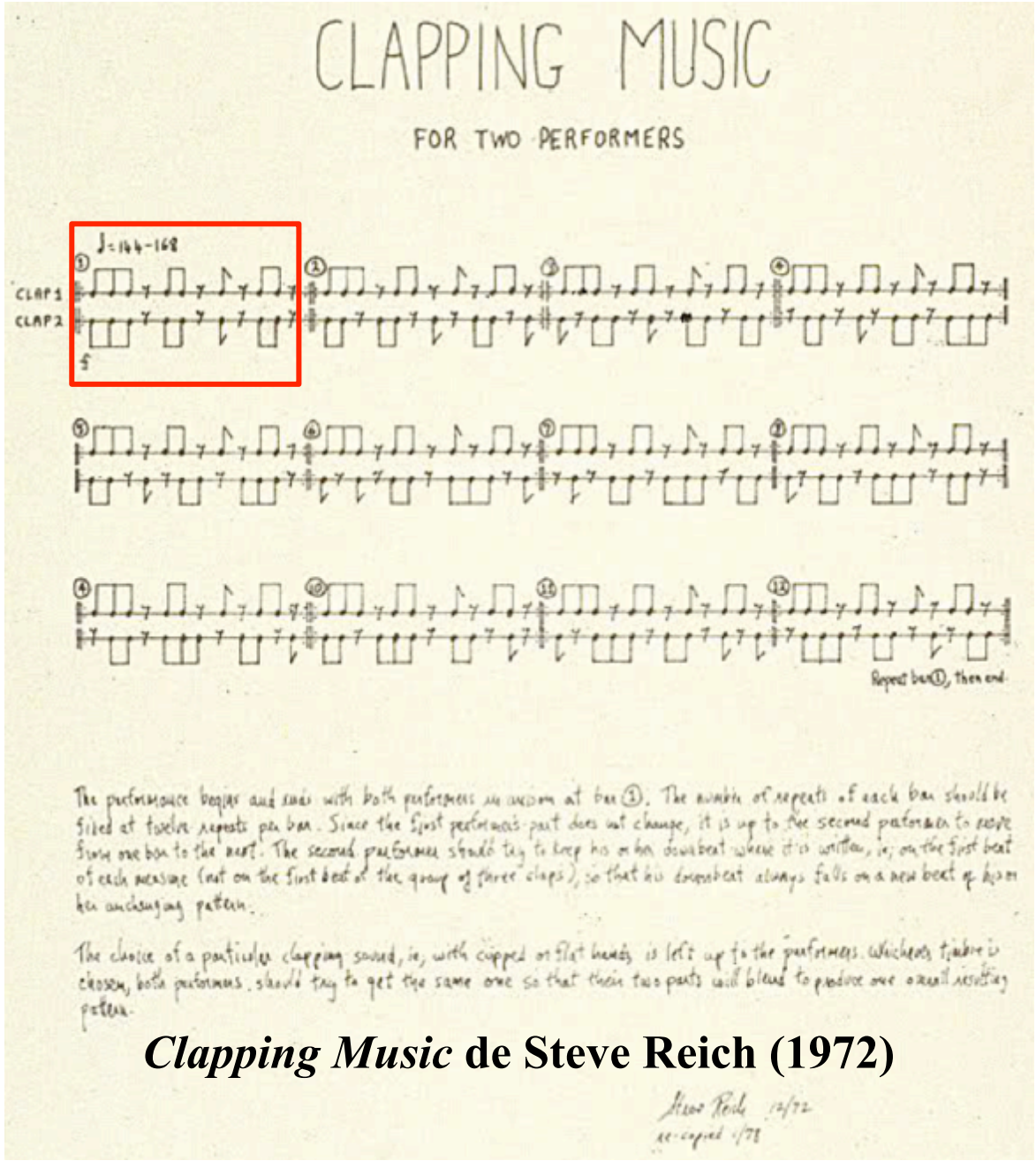


Six Bagatelles
(G. Ligeti, 1953)

Varietas, seu Combinatio quatuor notarum.



Le « cercle rythmique » et ses rotations


CLAPPING MUSIC

FOR TWO PERFORMERS

♩=144-168

CLAP 1

CLAP 2

Repeat bar ①, then end

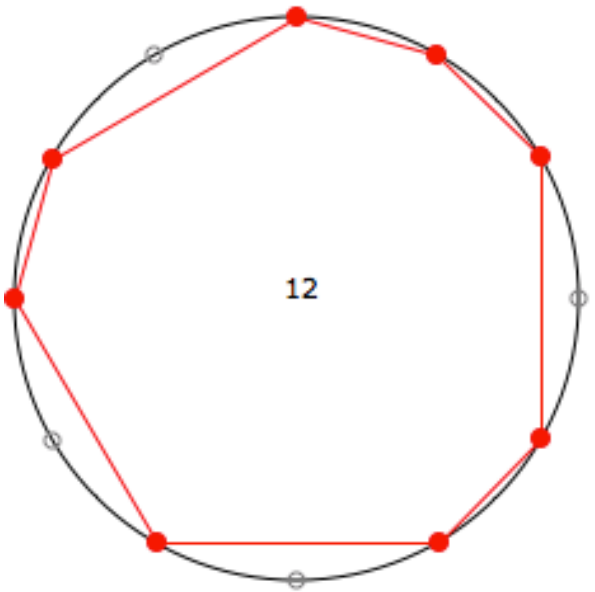
The performance begins and ends with both performers in unison at bar ③. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer's part does not change, it is up to the second performer to move from one bar to the next. The second performer should try to keep his or her downbeat where it is written, i.e. on the first beat of each measure (not on the first beat of the group of three claps), so that his downbeat always falls on a new beat of his or her unchanging pattern.

The choice of a particular clapping sound, i.e. with cupped or flat hands, is left up to the performers. Whichever timbre is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall resulting pattern.

Steve Reich 12/72

re-copied 1/73

***Clapping Music* de Steve Reich (1972)**



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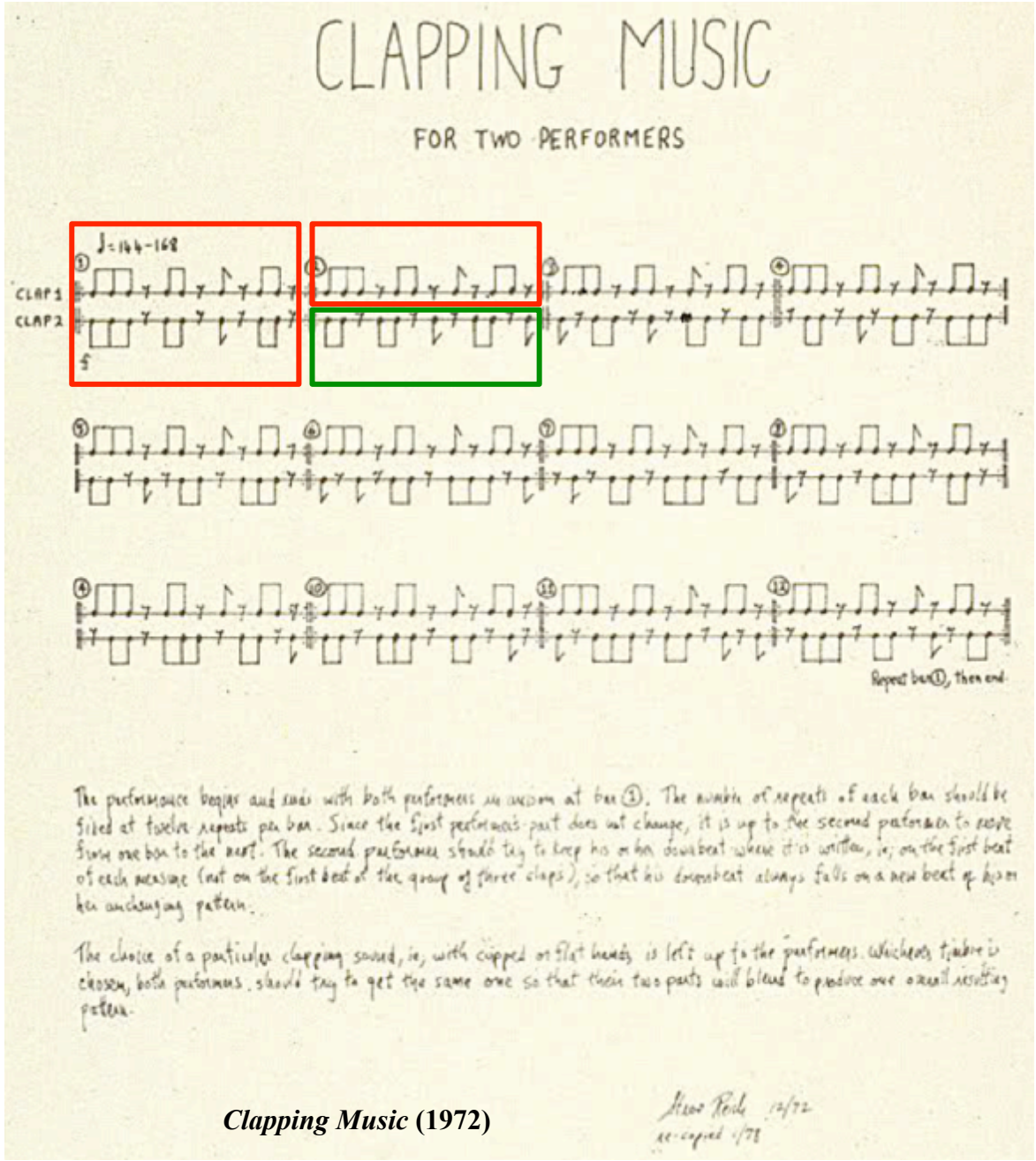

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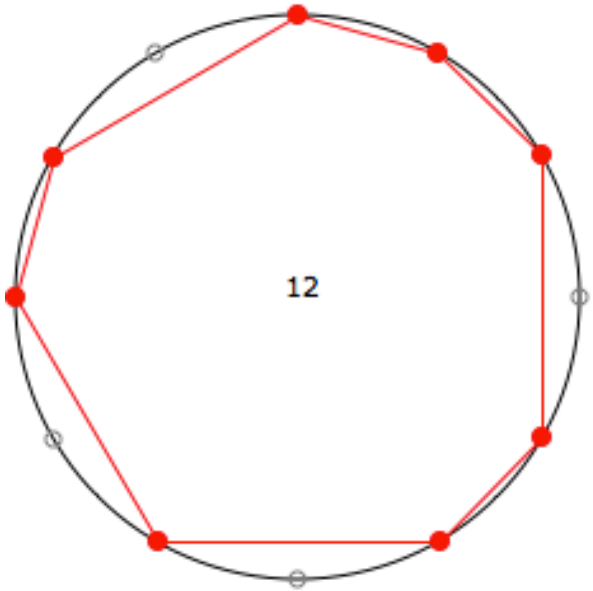
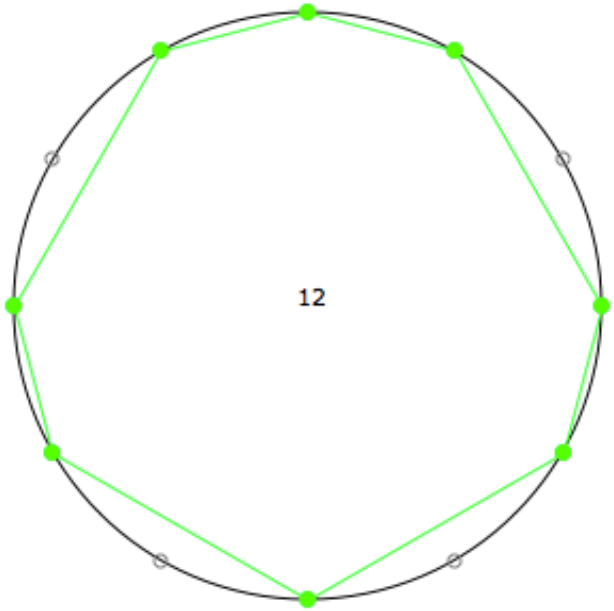
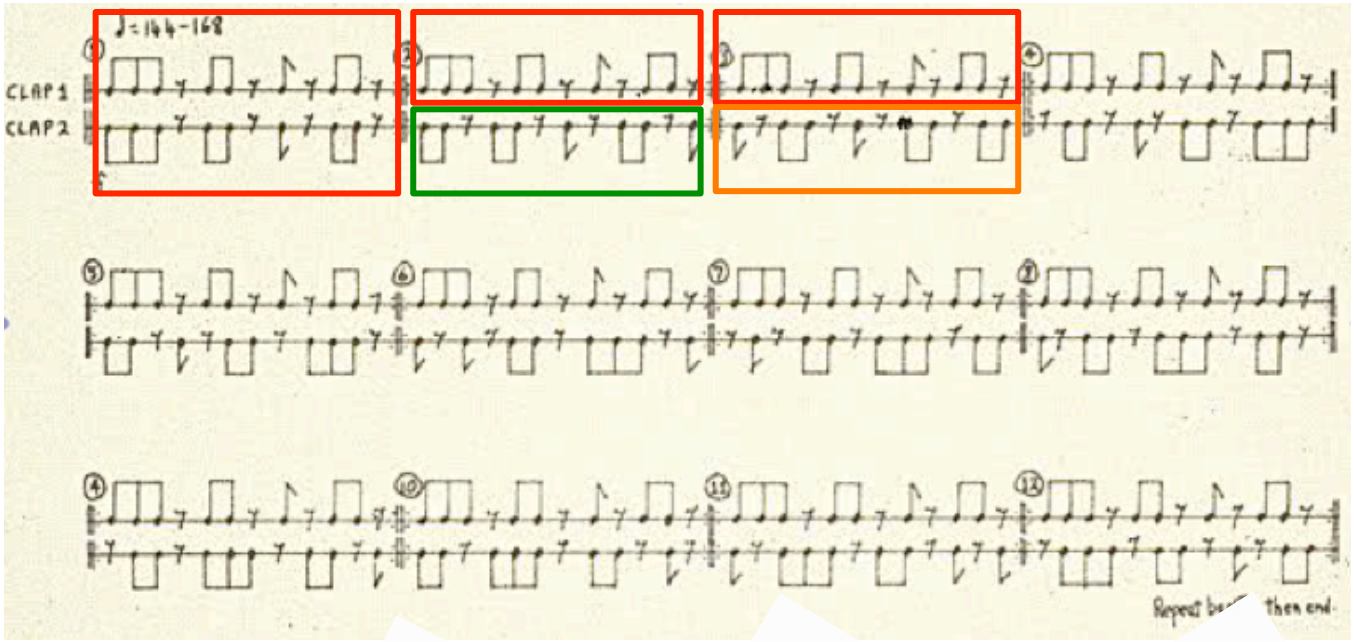




Le « cercle rythmique » et ses rotations



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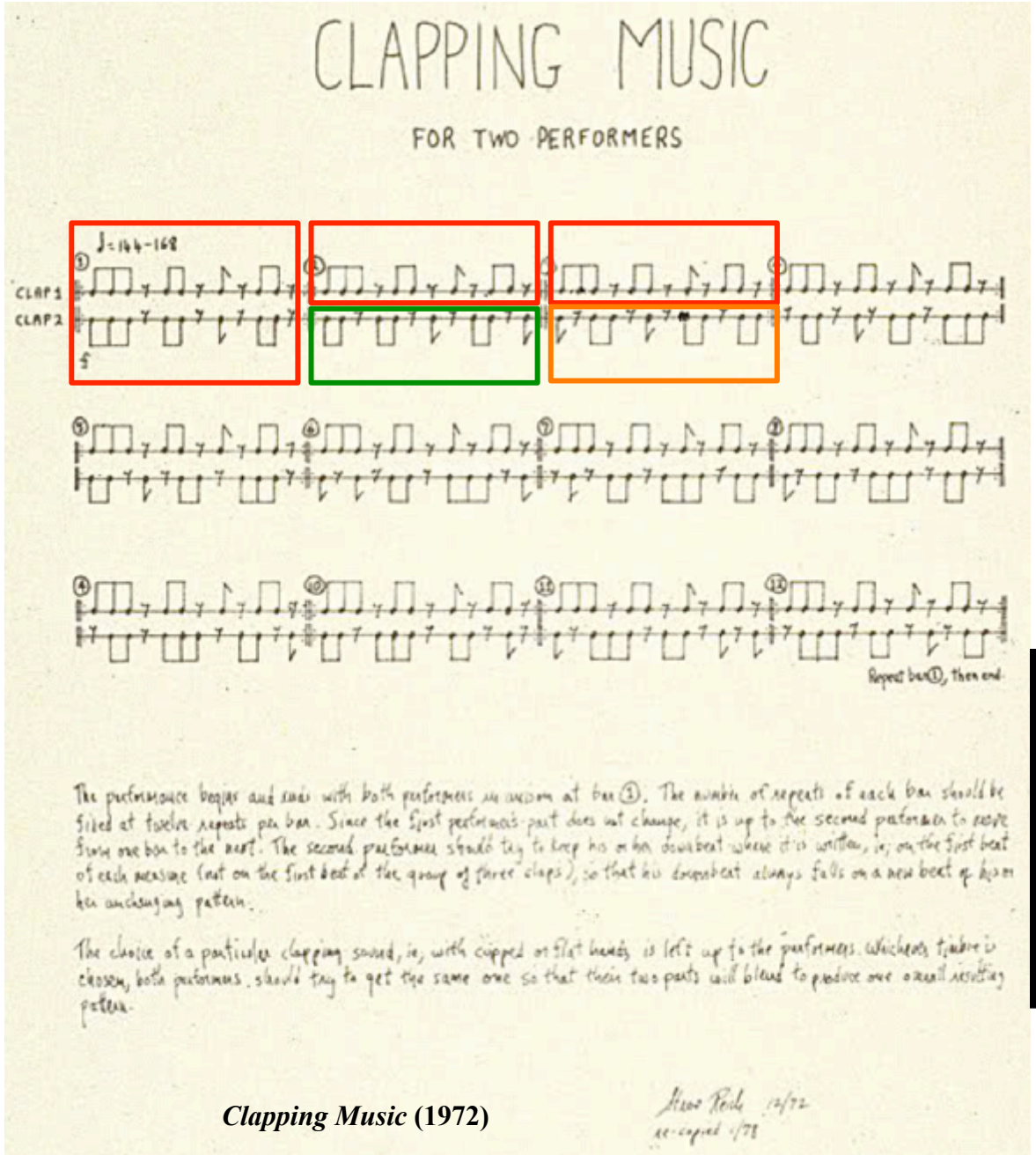
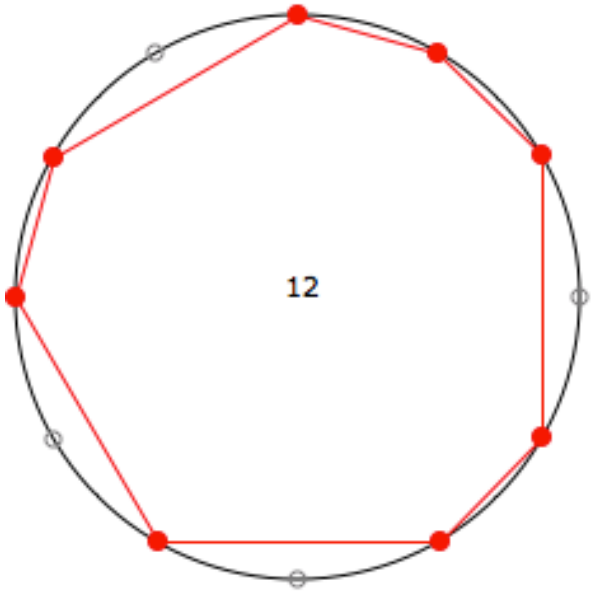
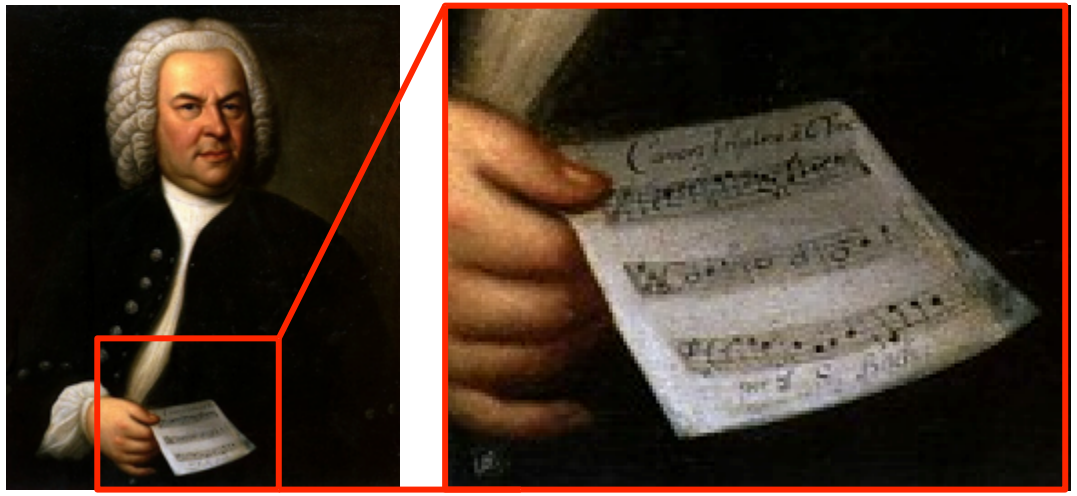


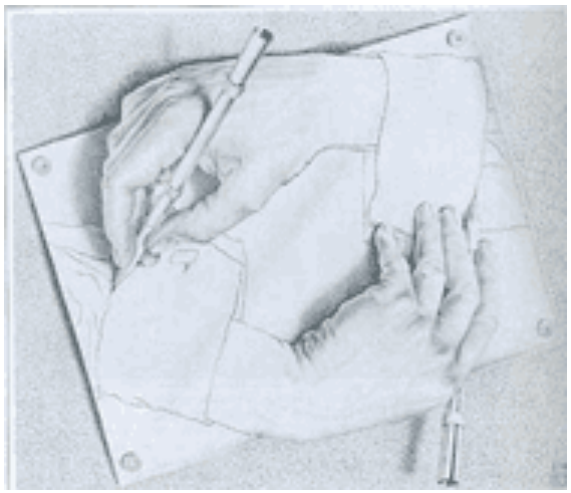
Clapping Music (1972)

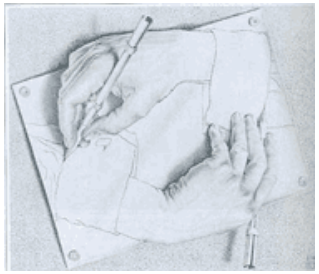


Gerubach's Scrolling Score Project
`http://www.gerubach.com`

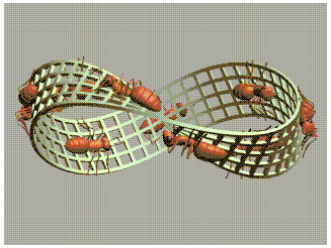
Canons énigmatiques chez Bach et géométrie

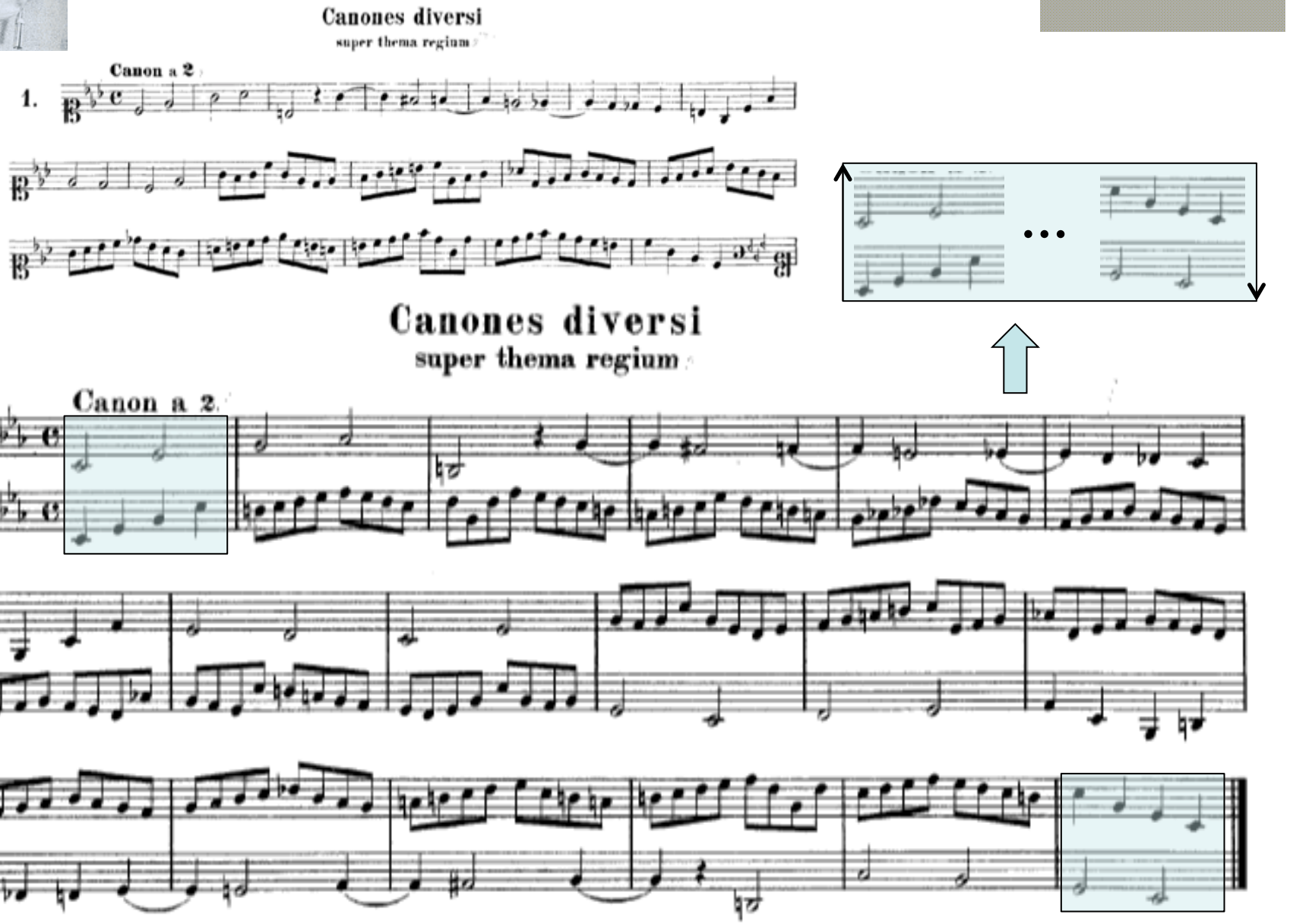






Ma fin est mon début (mais renversé !)

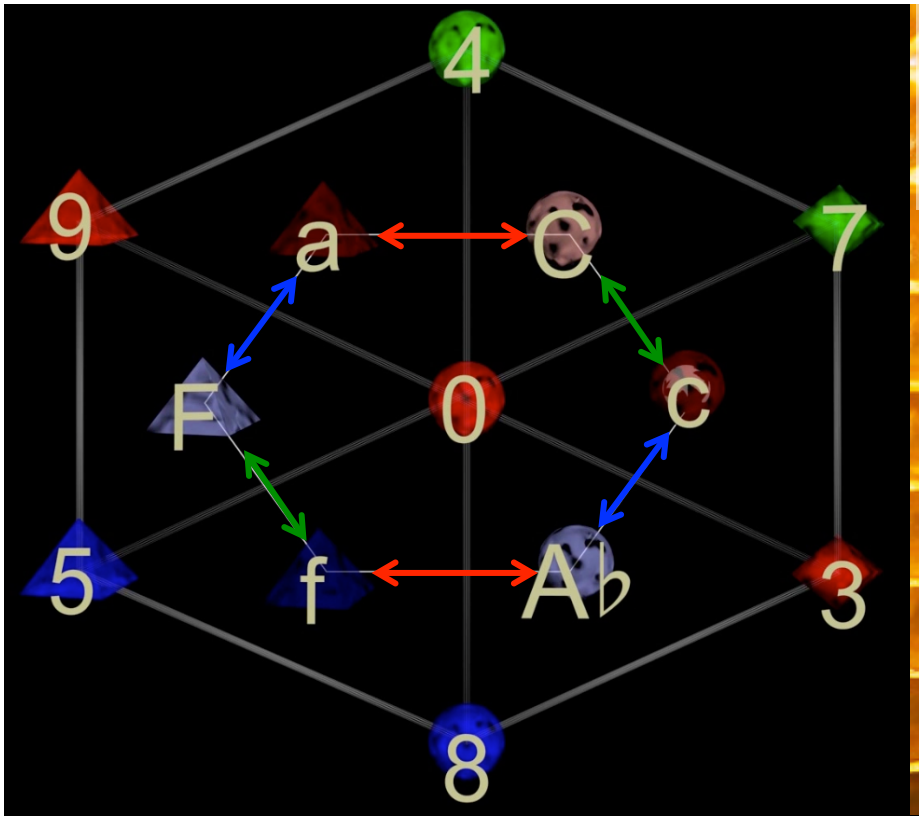




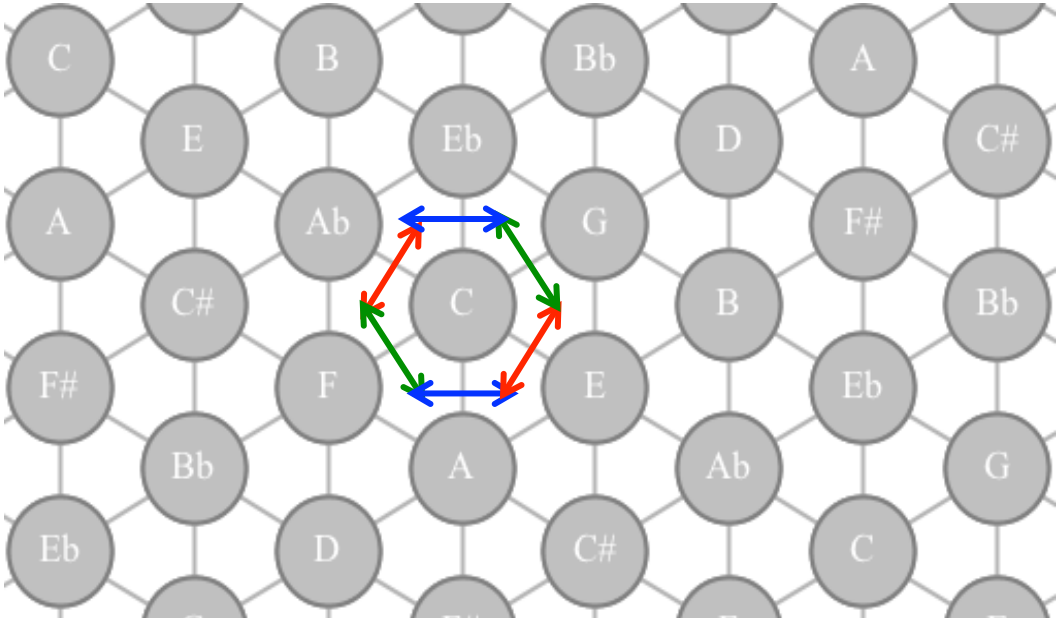
http://www.josleys.com/Canon/Canon.html

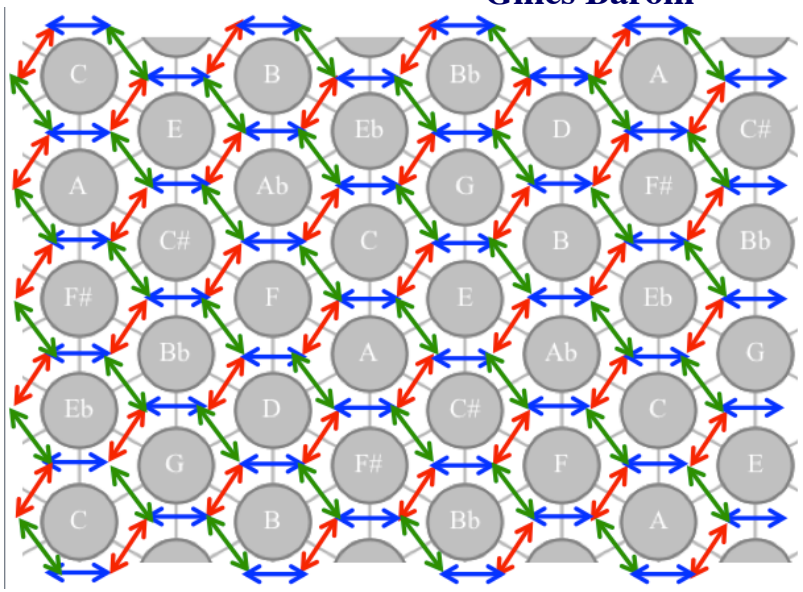
[min. 1’14”]

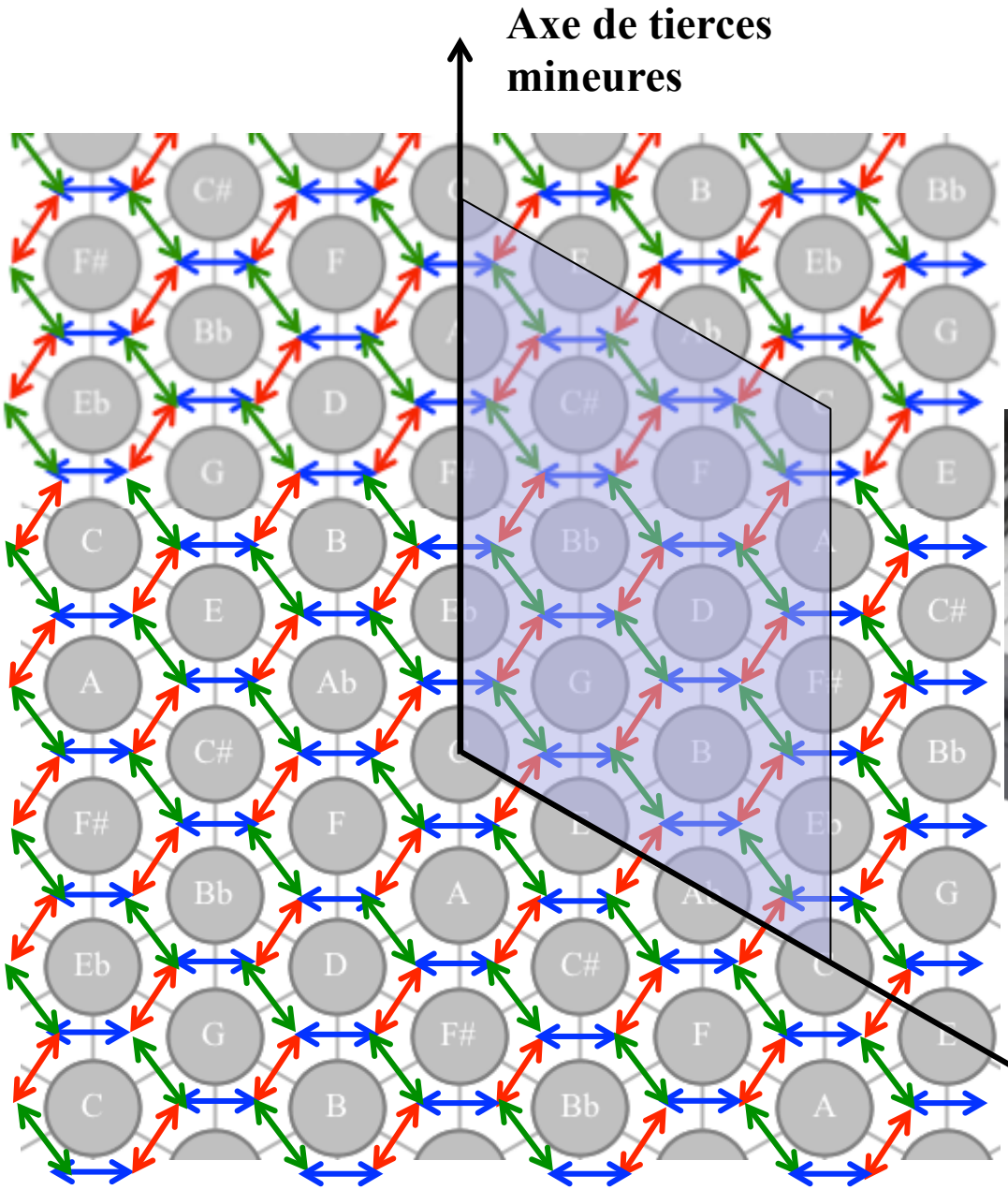
Le *Tonnetz*
(ou nid musical d'abeilles)

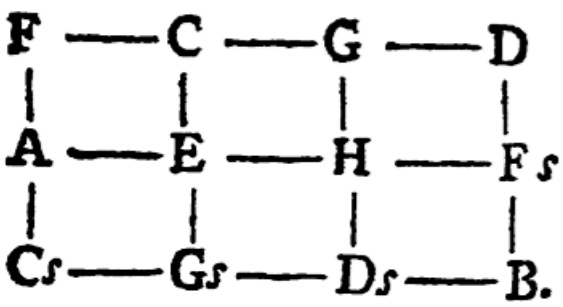


Gilles Baroin

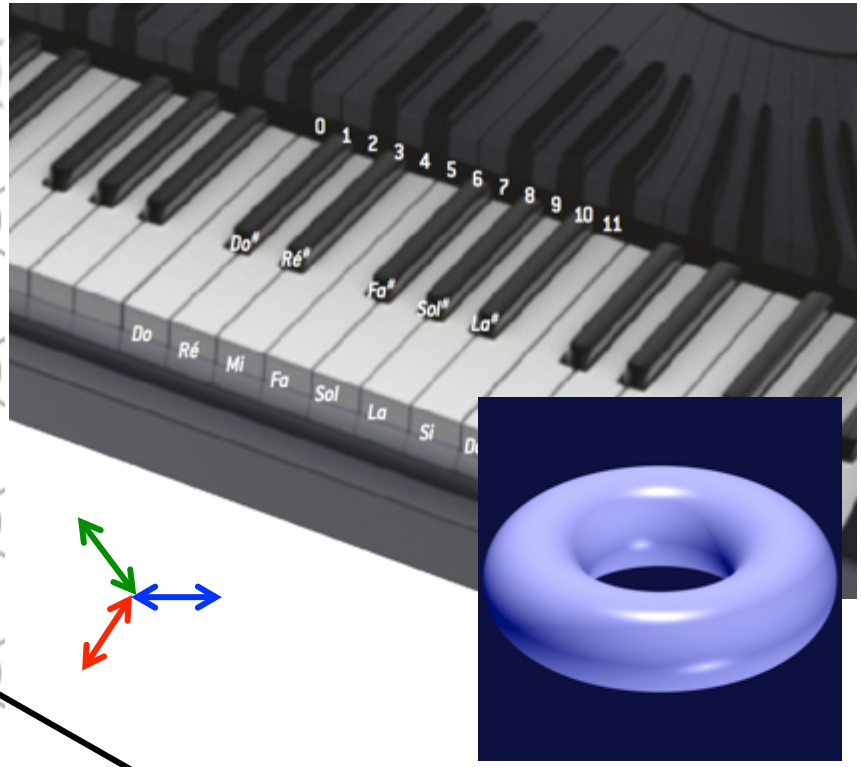






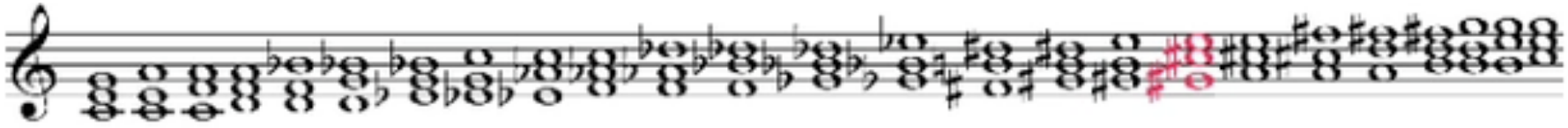


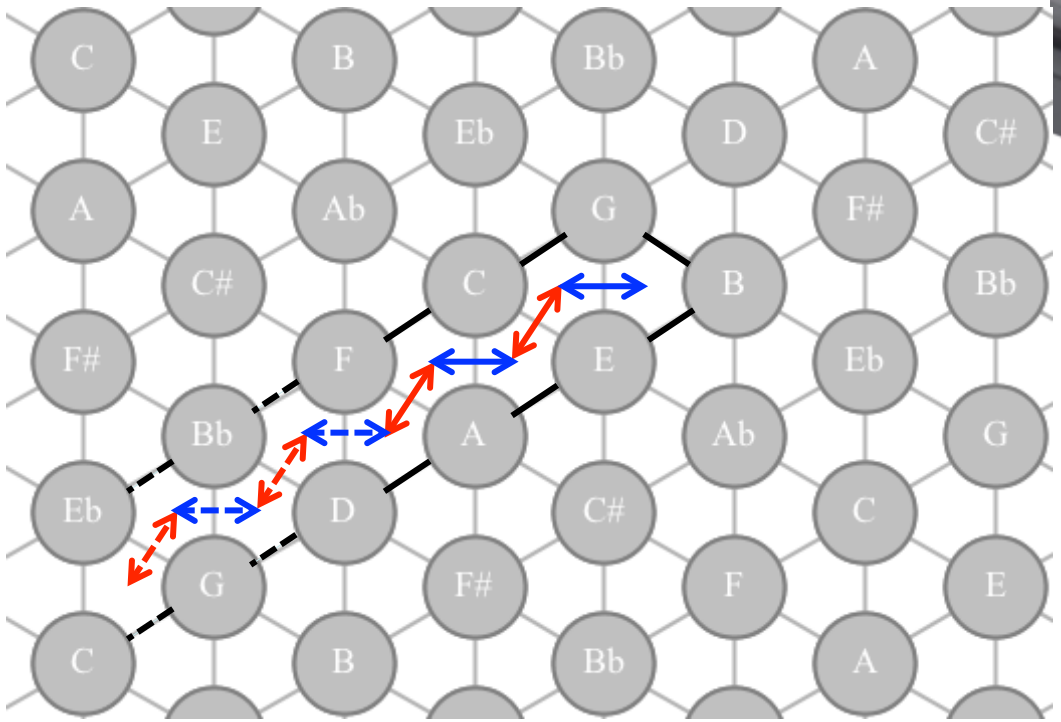
Speculum Musicum (Euler, 1773)



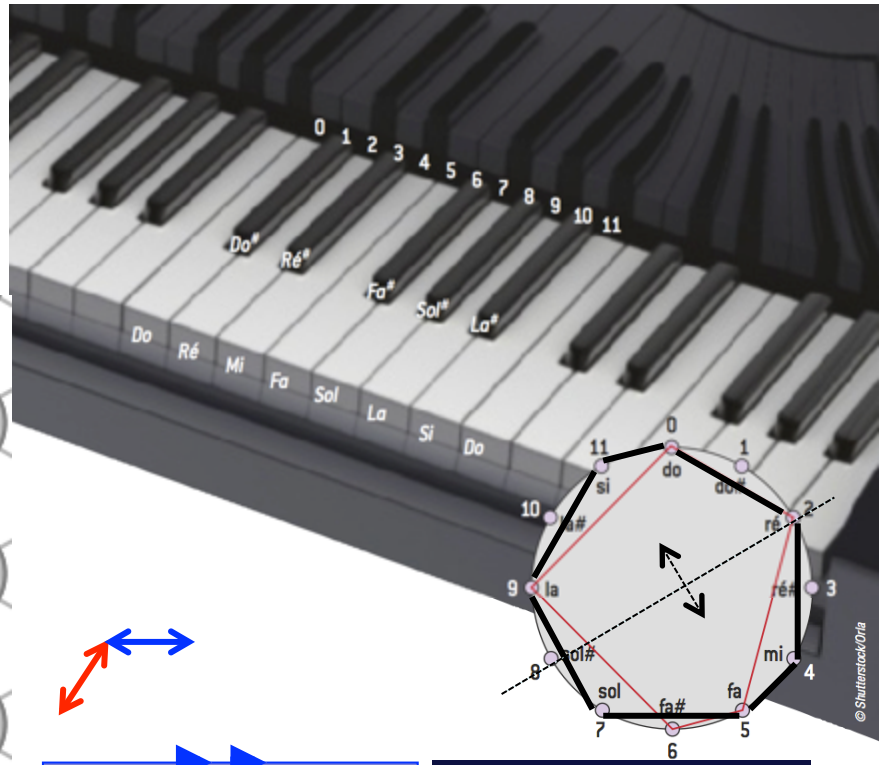
Tore

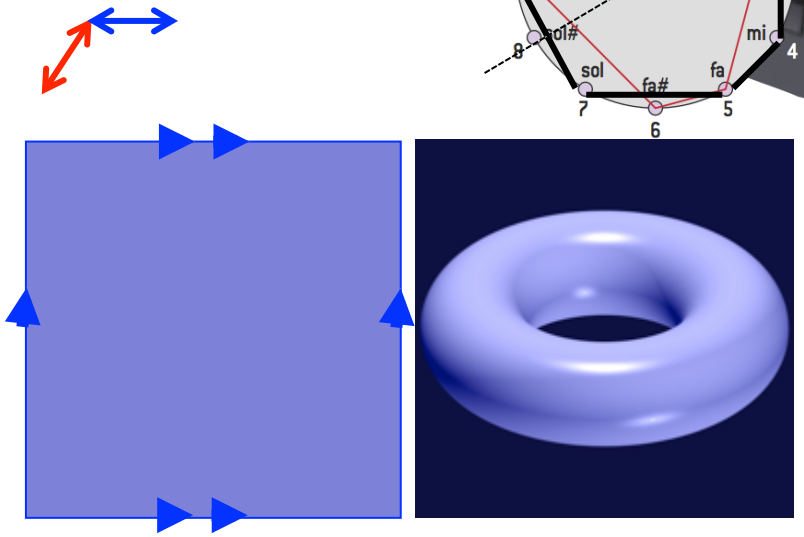
Progressions harmoniques comme trajectoires spatiales



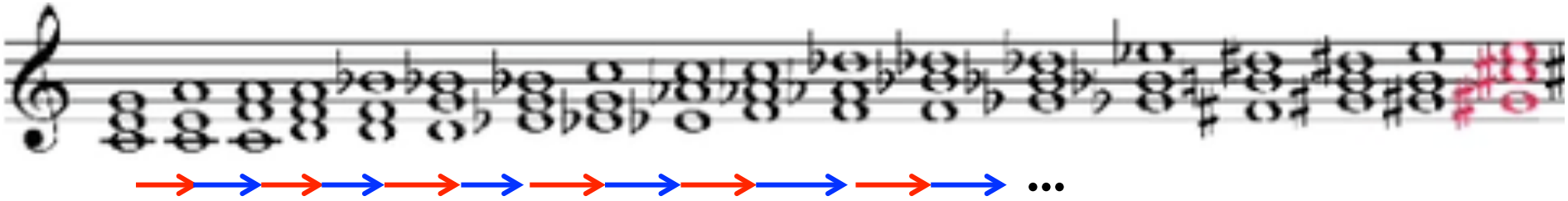


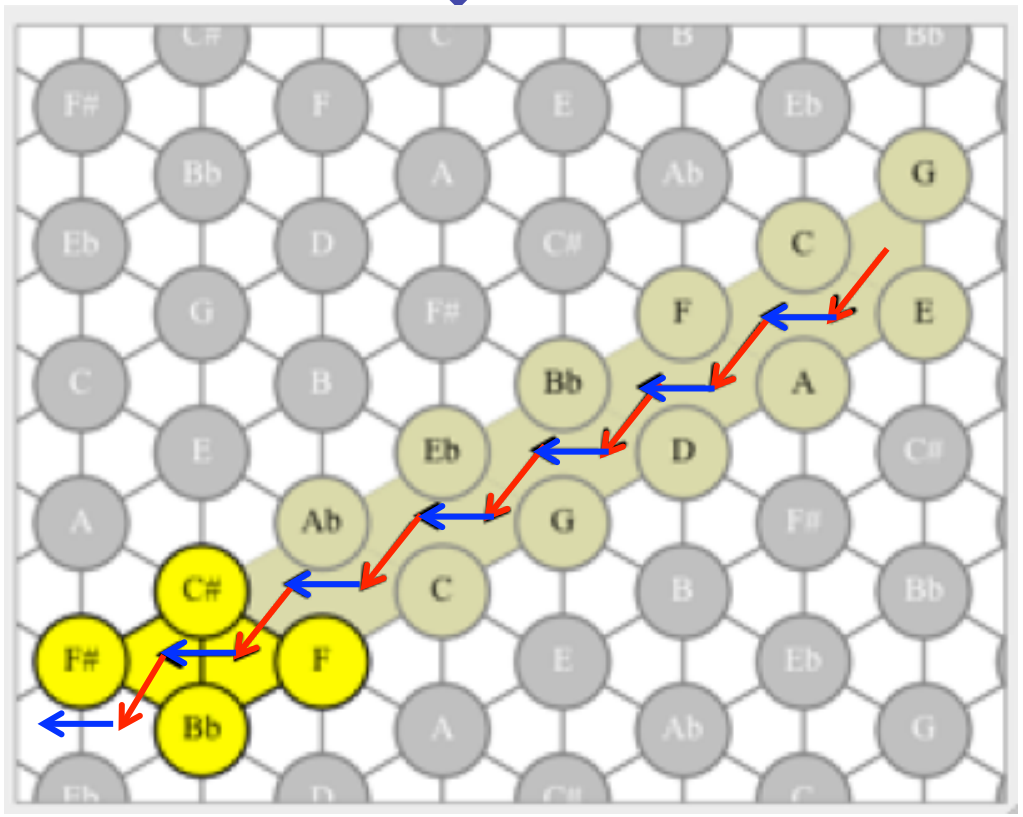
Hexachord (Louis Bigo, 2013)





Extrait du 2e mouvement de la Symphonie No. 9 (L. van Beethoven)

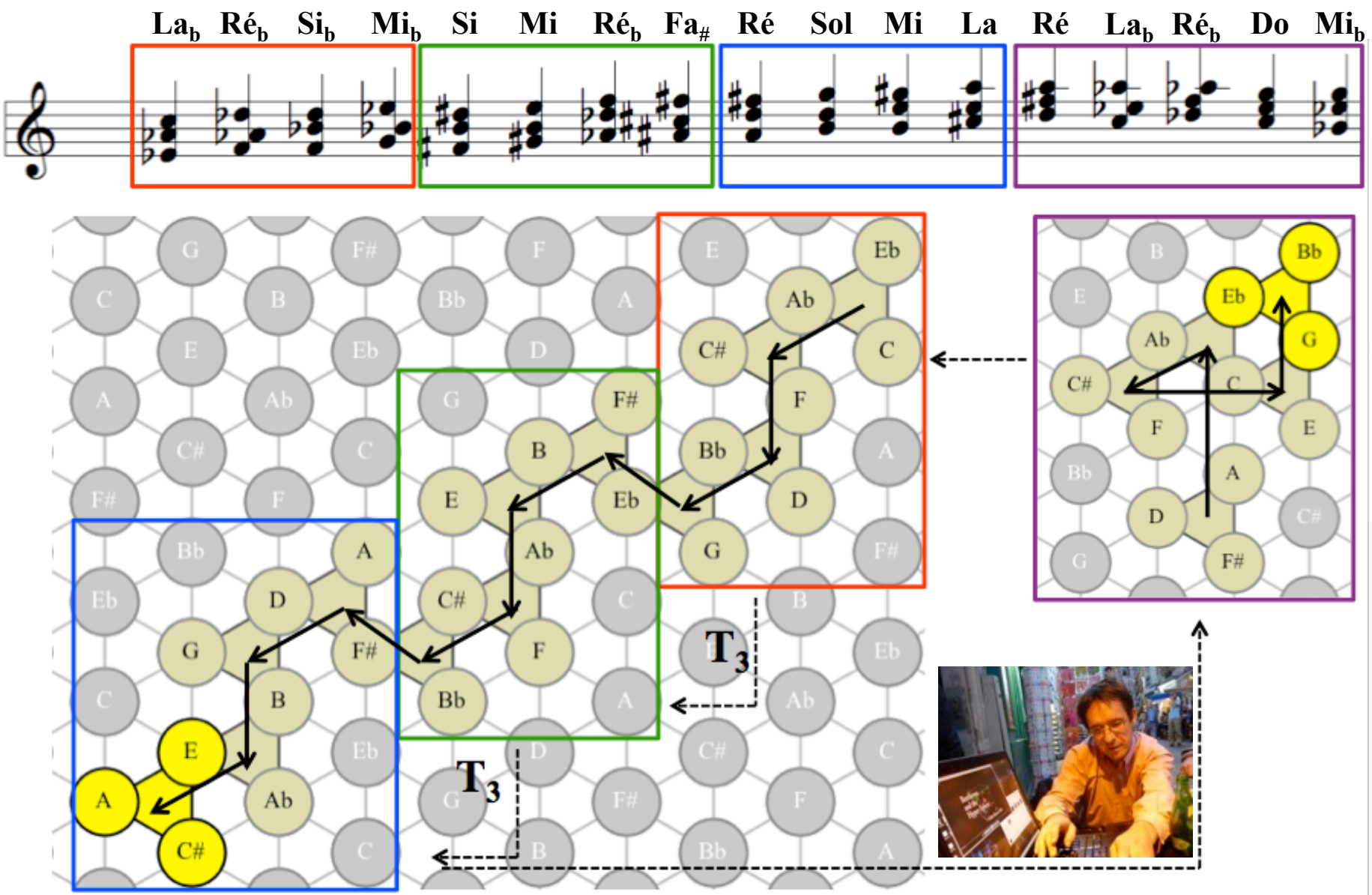




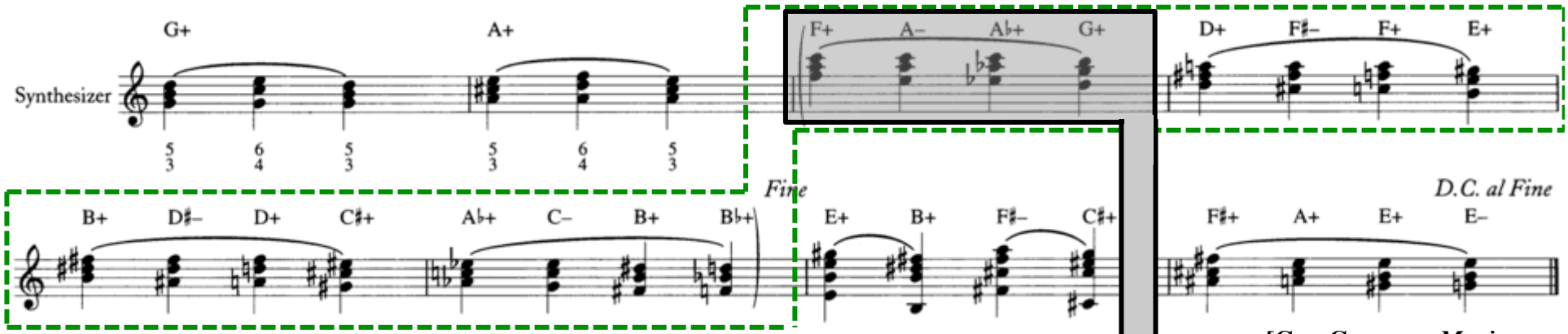
L

R

Progressions harmoniques dans « Madeleine » (Paolo Conte)

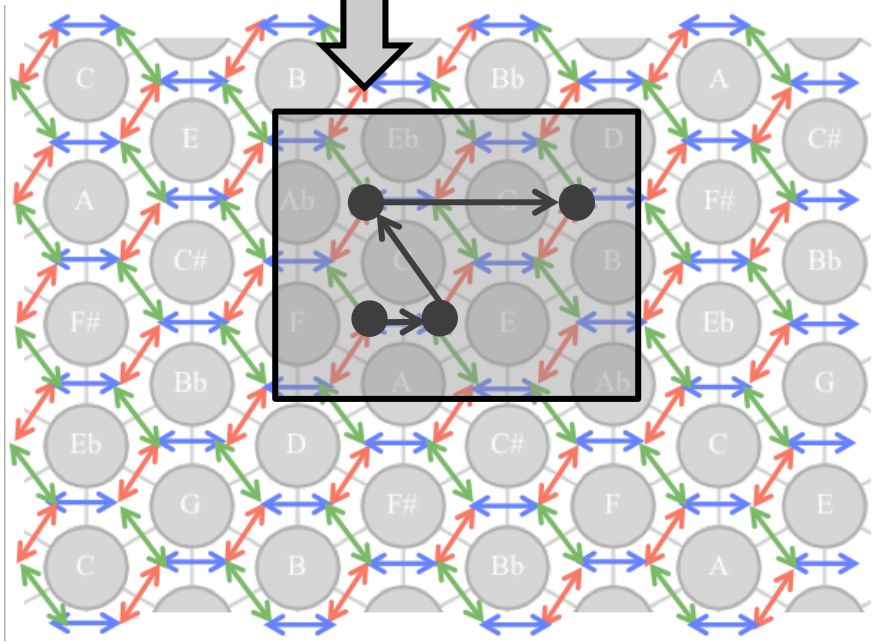


Progressions harmoniques chez Zappa: où sont les symétries ?

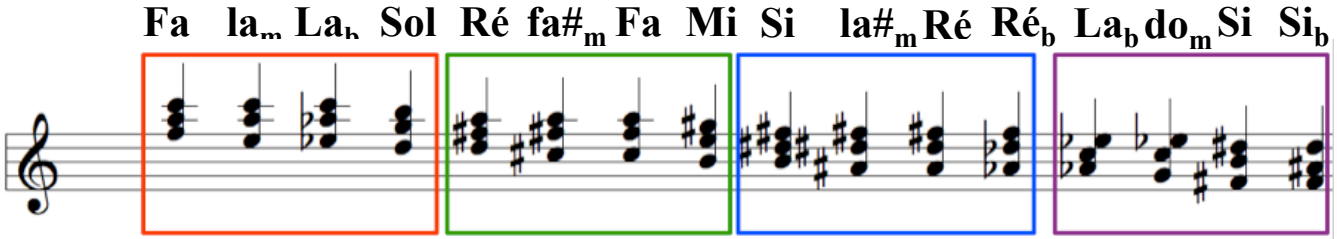


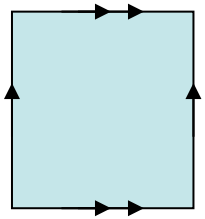
[Guy Capuzzo, *Music Theory* Spectrum, 2004]

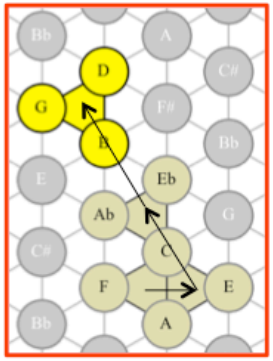
« Easy Meat » - 1981 (Frank Zappa)
http://www.youtube.com/watch?v=-MyVgK3osVk
min. 1'44" – 2'39"

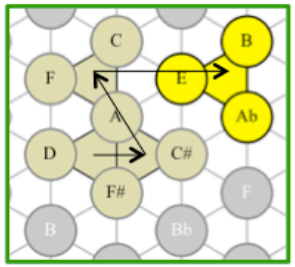


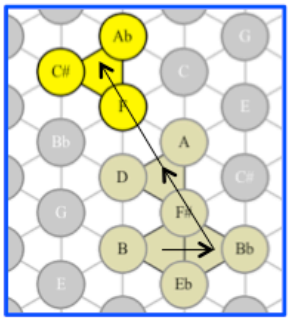
La trajectoire de la progression harmonique

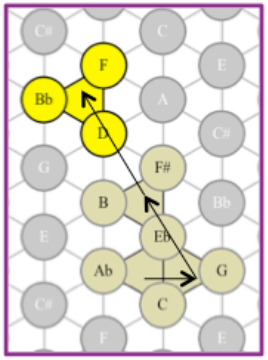


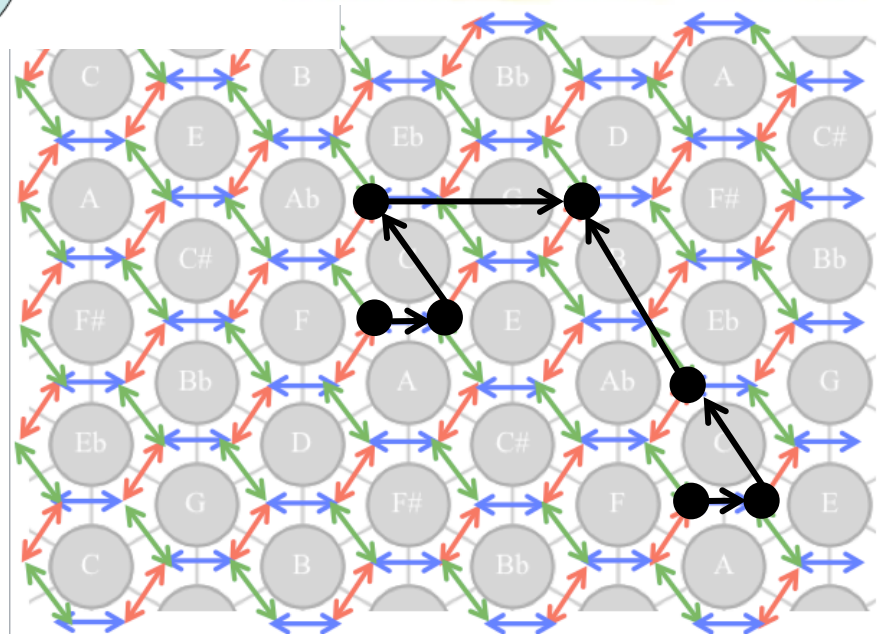












Aprile, chanson hamiltonienne « décadente »

Do←do$_m$←Sol#←fa$_m$←Fa←la$_m$←La←fa#$_m$←Fa#←sib$_m$←Do#←do#$_m$

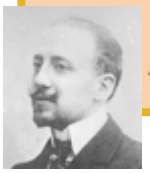
mi$_m$→Sol→si$_m$→Ré→ré$_m$→Sib→sol$_m$→Mib→mib$_m$→Si→sol#$_m$→Mi

La

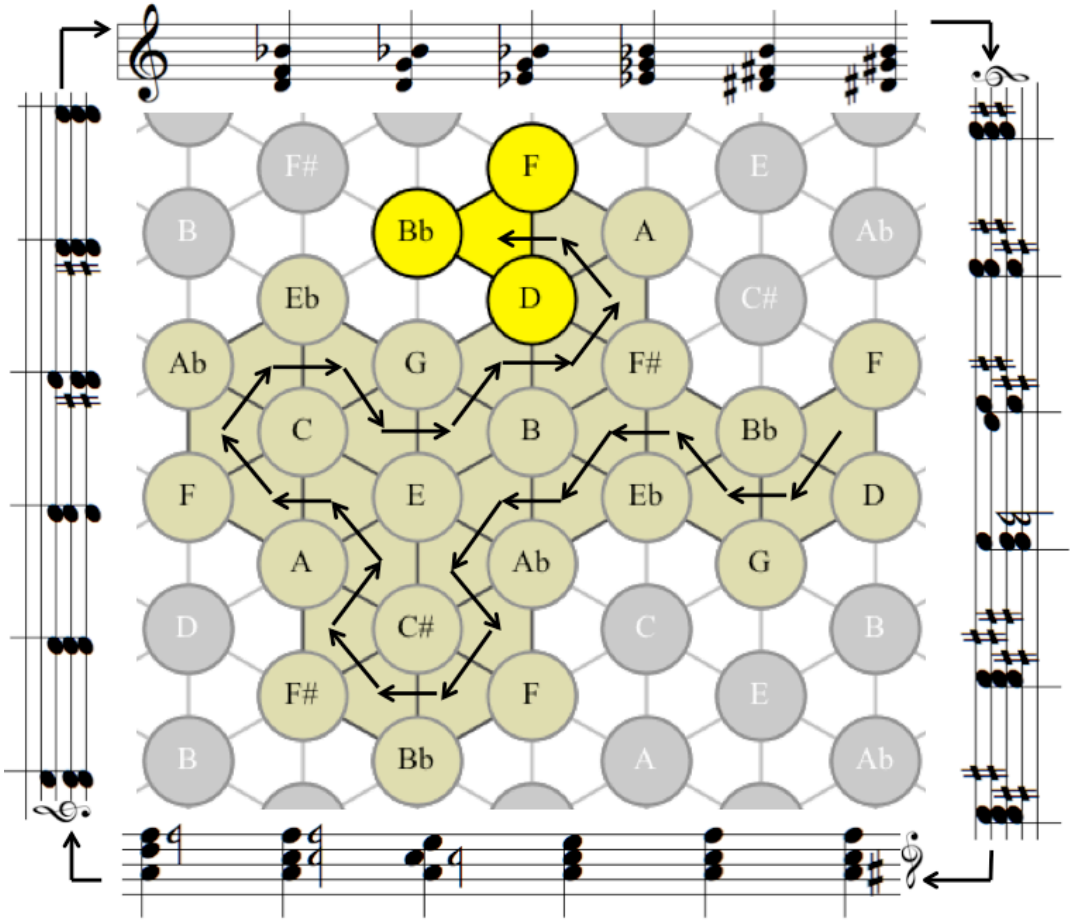
La fenêtre est entr'ouverte, sur le jardin.
Une heure passe, lente, somnolente.
Et elle, d'abord attentive, finit par s'endormir.
À cette voix qui là-bas se lamente,
Qui se lamente au fond de ce jardin.

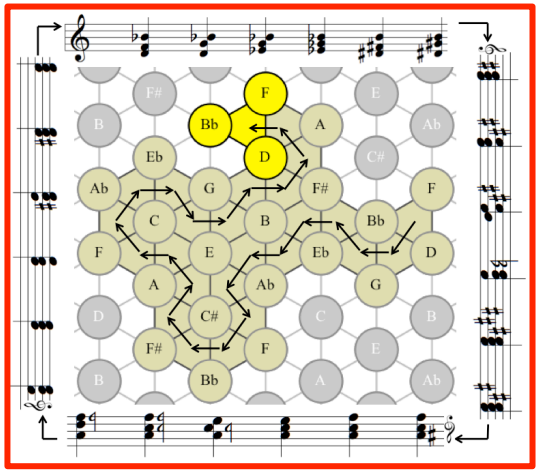
Ce n'est qu'une voix d'eau sur la pierre,
Et combien de fois, combien de fois entendue !
Cet amour et cette heure s'abîment dans cette vie
Comme s'abîment dans l'onde sans fin
Le cadavre et la pierre liés ensemble.

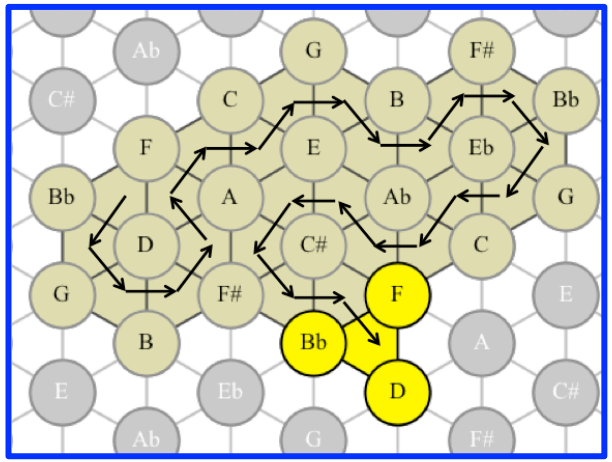
Elle détend son angoisse dans le sommeil.
Mais l'angoisse est forte, et le sommeil est si léger !
(La lumière d'avril ressemble presque à une neige
qui serait tiède.)
Et certes elle doit souffrir,
Vaguement souffrir, aussi dans le sommeil.

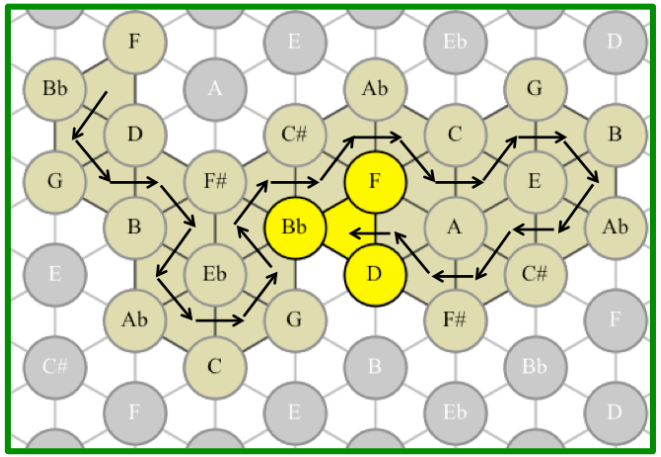


G. D'Annunzio (1863-1938)









Do←do_m←Sol#←fa_m←Fa←la_m←La←$fa\#_m$←Fa#←sib_m←Do#←$do\#_m$

(Premier cycle hamiltonien)

La

mi_m→Sol→si_m→Ré→$ré_m$→Sib→sol_m→Mib→mib_m→Si→$sol\#_m$→Mi

Do→mi_m→Mi→$sol\#_m$→Si→$ré\#_m$→Re#→do_m→Lab→fa_m→Do#→$do\#_m$

(Deuxième cycle hamiltonien)

la_m←Fa←$ré_m$←Ré←si_m←Sol←sol_m←Sib←sib_m←Fa#←$fa\#_m$←La

Mi←mi_m←Do←la_m←Fa←fa_m←Reb←sib_m←Fa#←mib_m←Mib←do_m

(Troisième cycle hamiltonien)

La

$do\#_m$→La→$fa\#_m$→Ré→$ré_m$→Sib→sol_m→Sol→si_m→Si→$sol\#_m$→Sol#

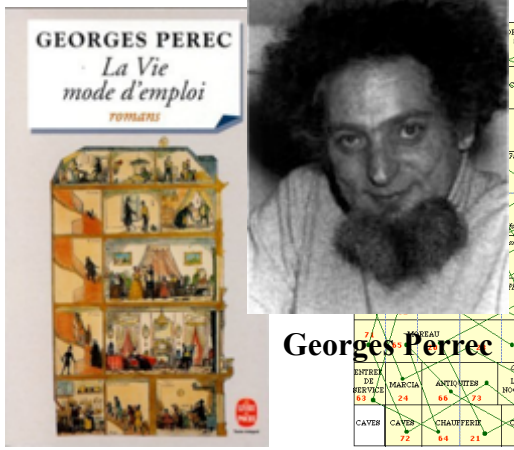
Aprile (d'après Gabriele D'Annunzio)

Tangente pour mieux comprendre le monde
L'aventure mathématique

➔ **M. Andreatta, « Math'n pop : symétries et cycles hamiltoniens en chanson », *Tangente***

Les contraintes dans l'art : de l'OuLiPo (Ouvroir de Littérature Potentielle)

Cent mille milliards de poèmes, 1961



Georges Perrec

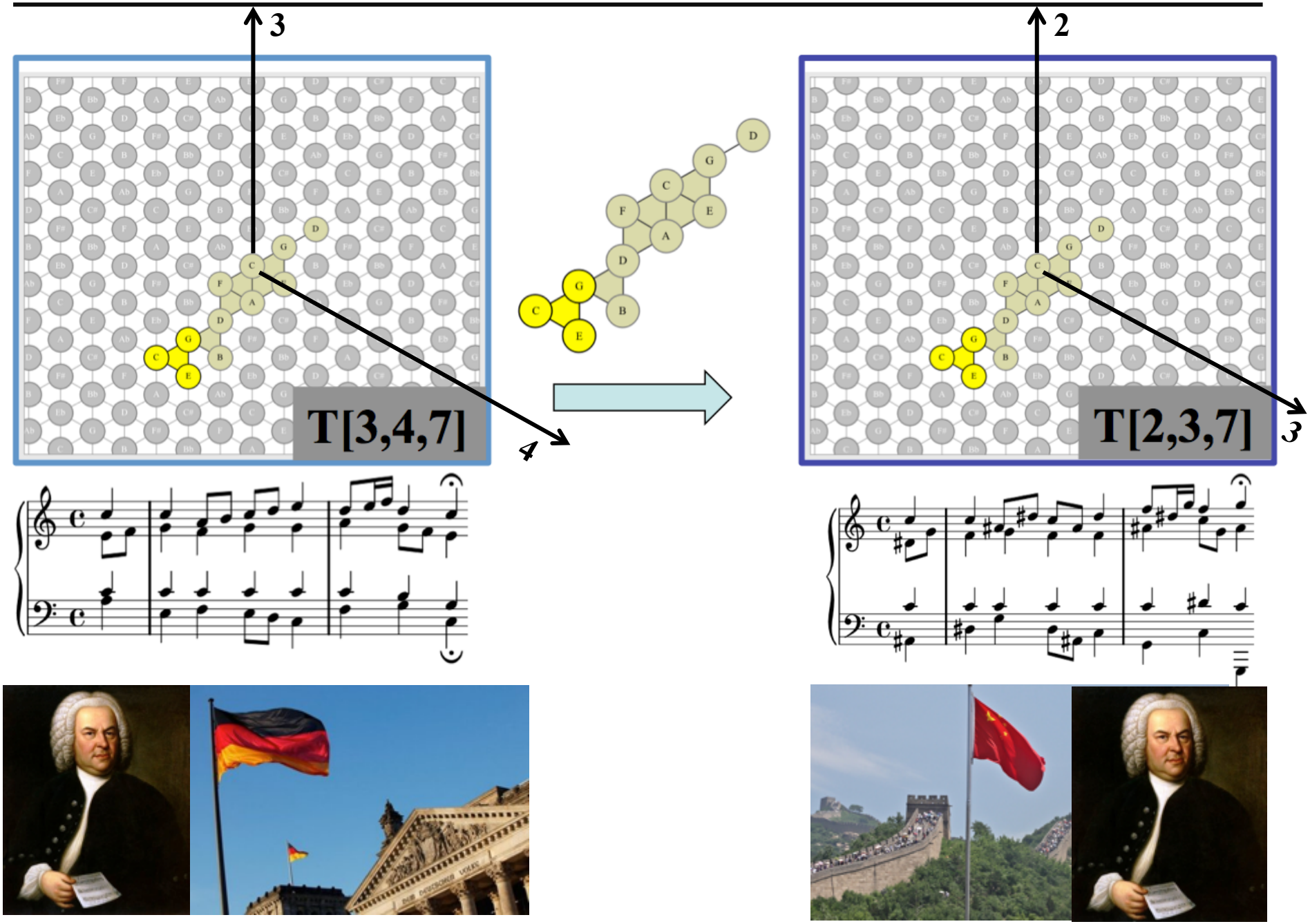
La vie mode d'emploi,

Raymond Queneau

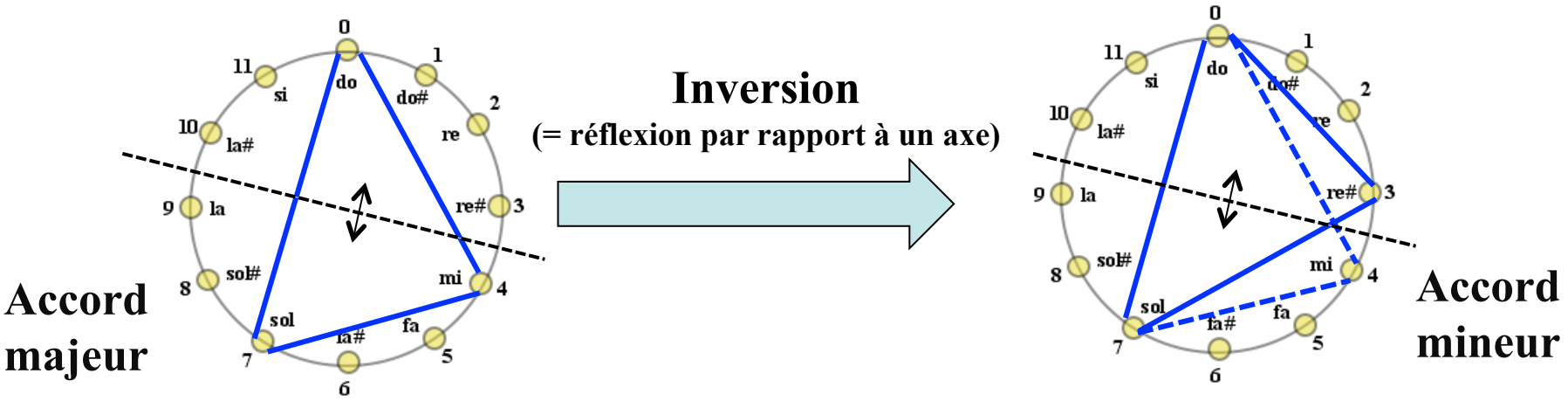
Italo Calvino

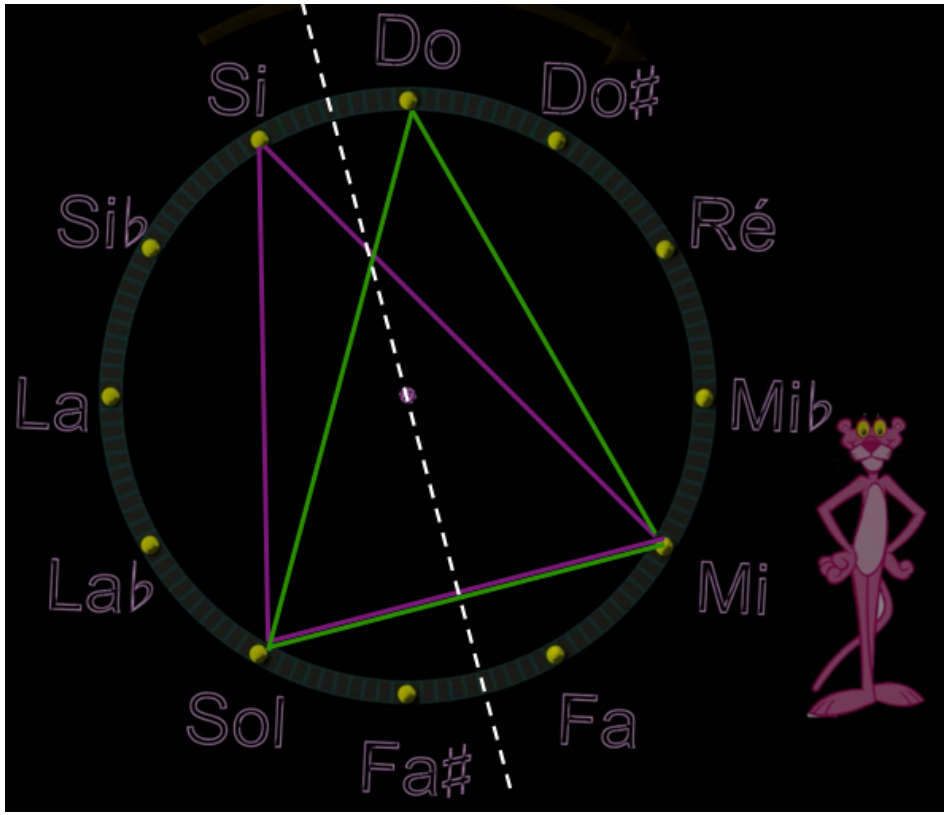
Le Château des destins croisés, 1969

...à l'OuMuPo : ouvroir de musique potentielle



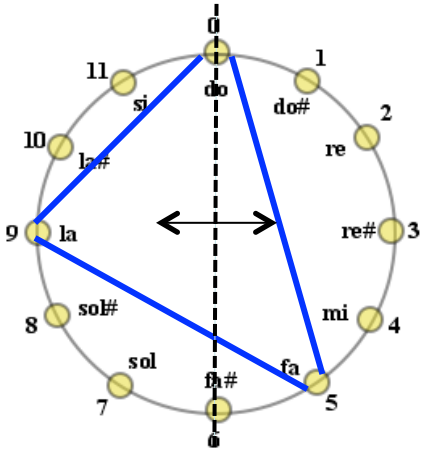
Premières expériences oumu(po)piennes sur les Beatles





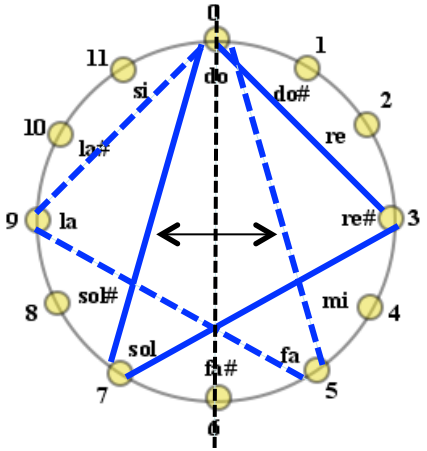
Premières expériences oumu(po)piennes sur les Beatles

Accord de *Fa* majeur



Beatles, Hey Jude
(version orig.)

inversion



Beatles, Hey Jude
(version transformée)

Accord de *do* mineur

T[3,4,7]

Rotation
(autour du *do*)

T[3,4,7]

Pour plus d'exemples musicaux voir la thèse de Louis Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013 ➔ `http://www.lacl.fr/~lbigo/scw13`

MERCI DE VOTRE ATTENTION !

Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS UMR 9912