Master I.C.A.

Master I.C.A.

Spécialité A.S.T.

Art
Sciences
Technologies

Traitement interactif de l'image et du son

Workshop OpenMusic

– Jean Bresson // Moreno Andreatta –
Equipe Représentations Musicales
IRCAM/CNRS/UPMC UMR 9912

"MathTools": an algebraic environment within OpenMusic visual programming language

OM=♬+∑

Computational Music Theory:

- Classification and Enumeration of musical structures
  - Chords/scales, motifs and rhythms:
    - ♬ Catalogues (Costère, Zalewski, Vieru, Forte, Carter, Morris, Mazzola, Estrada, …)
    - ∑ Combinatorial algebra, Polya Enumeration Theory, Burnside Lemma, Discrete Fourier Transform
  - Rhythmic Tiling Canons (by translation, inversion and augmentation)
    - ♬ Messiaen, Vieru, Levy, Johnson, Bloch, Wild, Lanza, …
    - ∑ Group factorization theory
    - ∑

Computational Music Analysis:

- *Set Theory,* Transformational Analysis and Sieve Theory
  - Pitch-class sets, interval vectors and IFUNC, Z-relations:
    - ♬ Carter, Vieru, Xenakis
    - ∑ Group Actions, Homometry, TFD
  - Transformational progression/network, *K-nets*
    - ♬ Generalized Interval Systems (David Lewin)
    - ∑ Group action and category theory

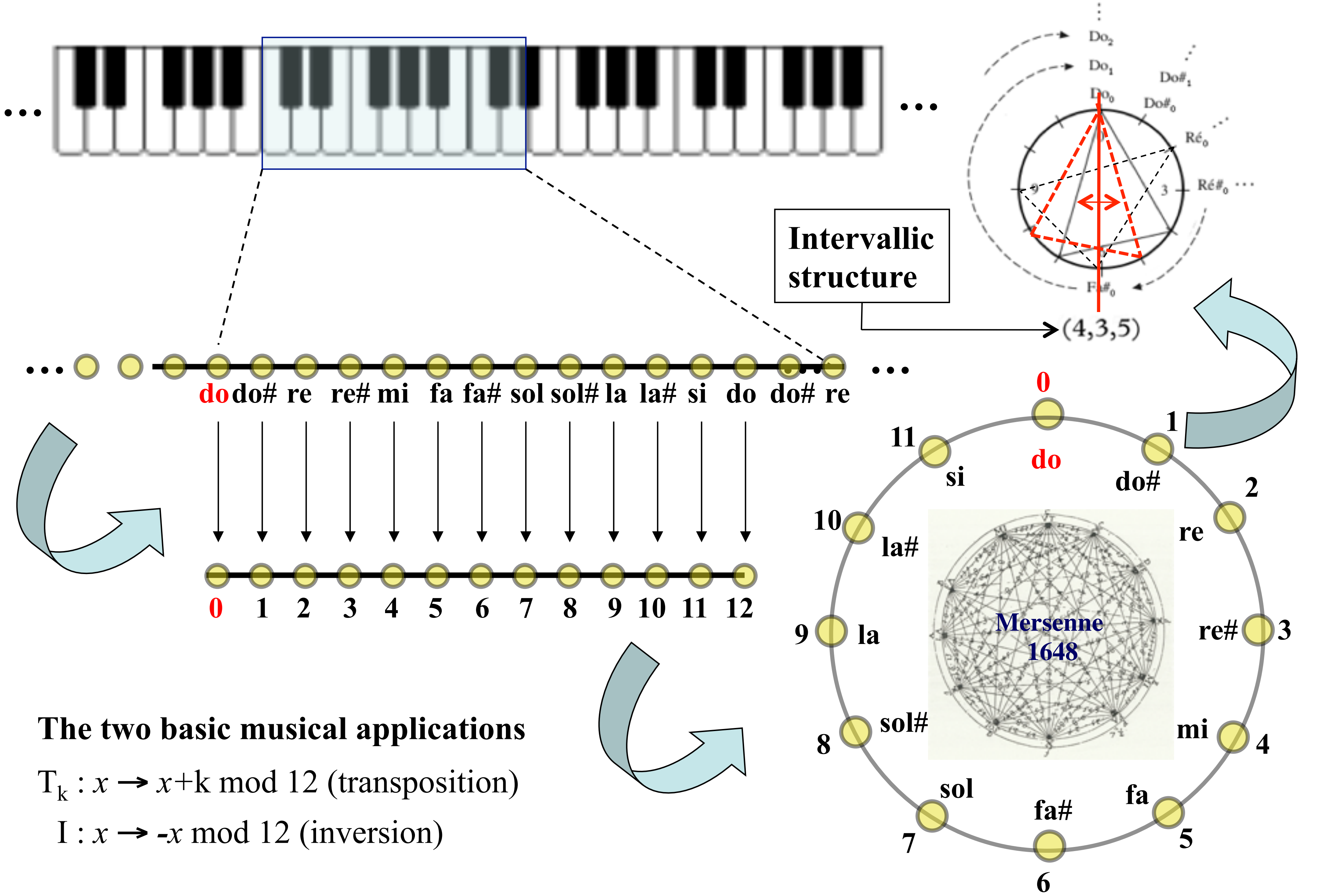
Computer Aided-Composition:

- Cf. *The OM Composer's Book (2 volumes).*
  Edited by C. Agon, G. Assayag and J. Bresson

http://recherche.ircam.fr/equipes/repmus/OpenMusic/

➔**`http://repmus.ircam.fr/openmusic/home`**

Octave equivalence and mod 12 congruence

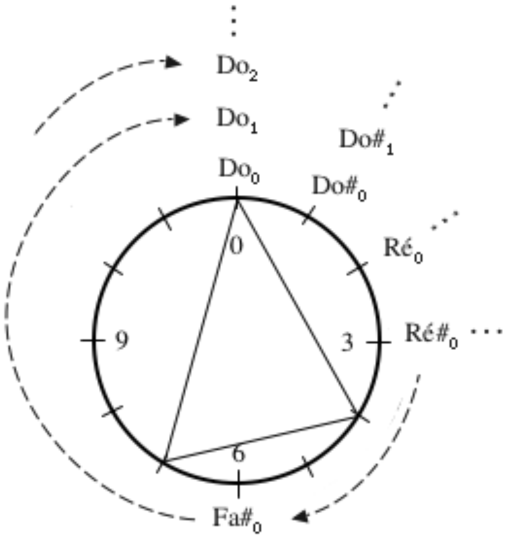


The two basic musical applications

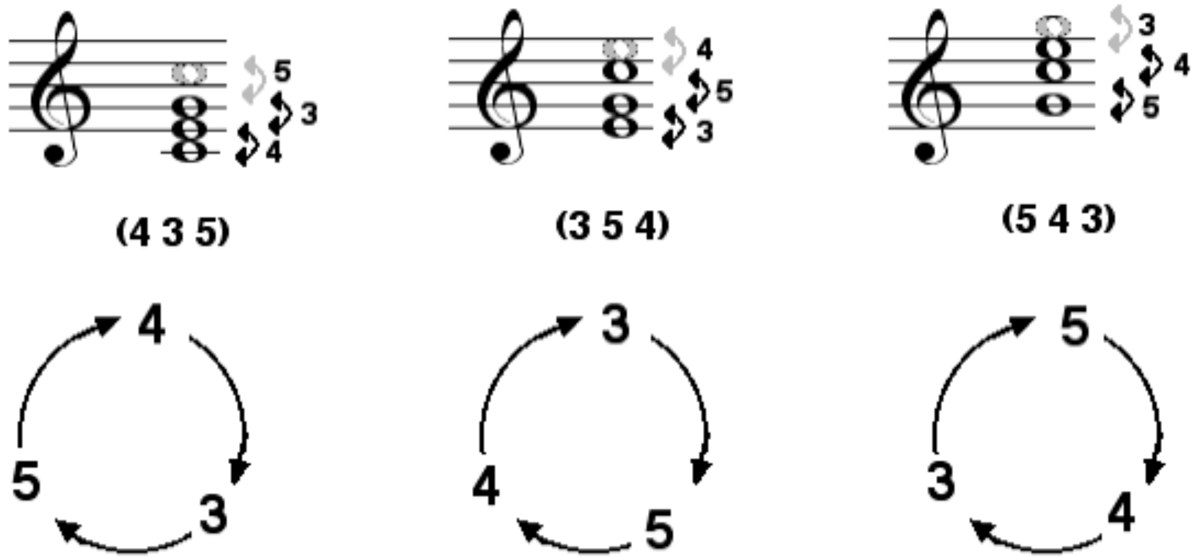
$T_k : x \rightarrow x+k \bmod 12$ (transposition)

$I : x \rightarrow -x \bmod 12$ (inversion)

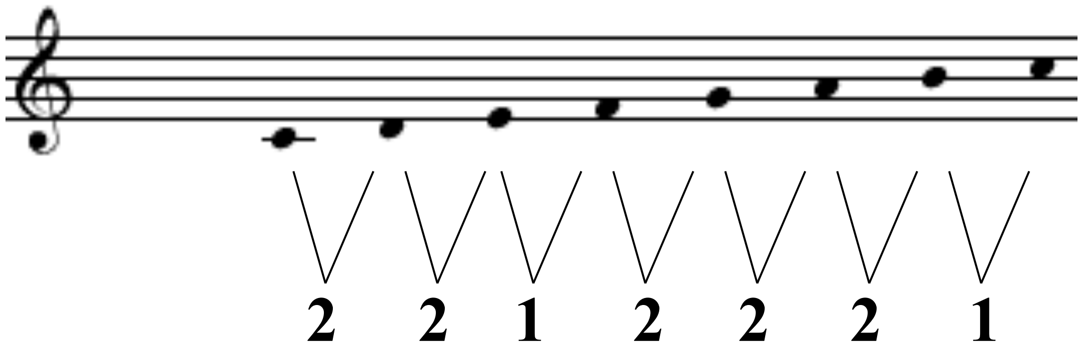
Circular representation and intervallic structure

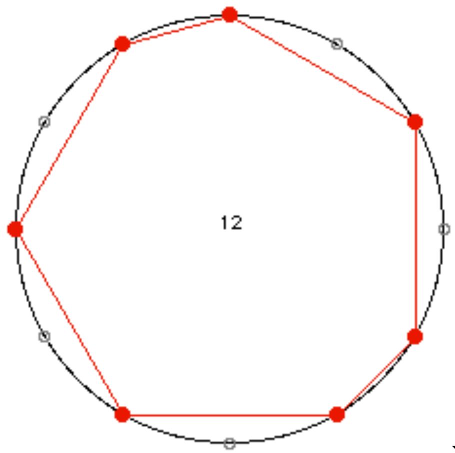


(4,3,5)



The « inversions » of a chord are all circular permutations on an intervallic structure

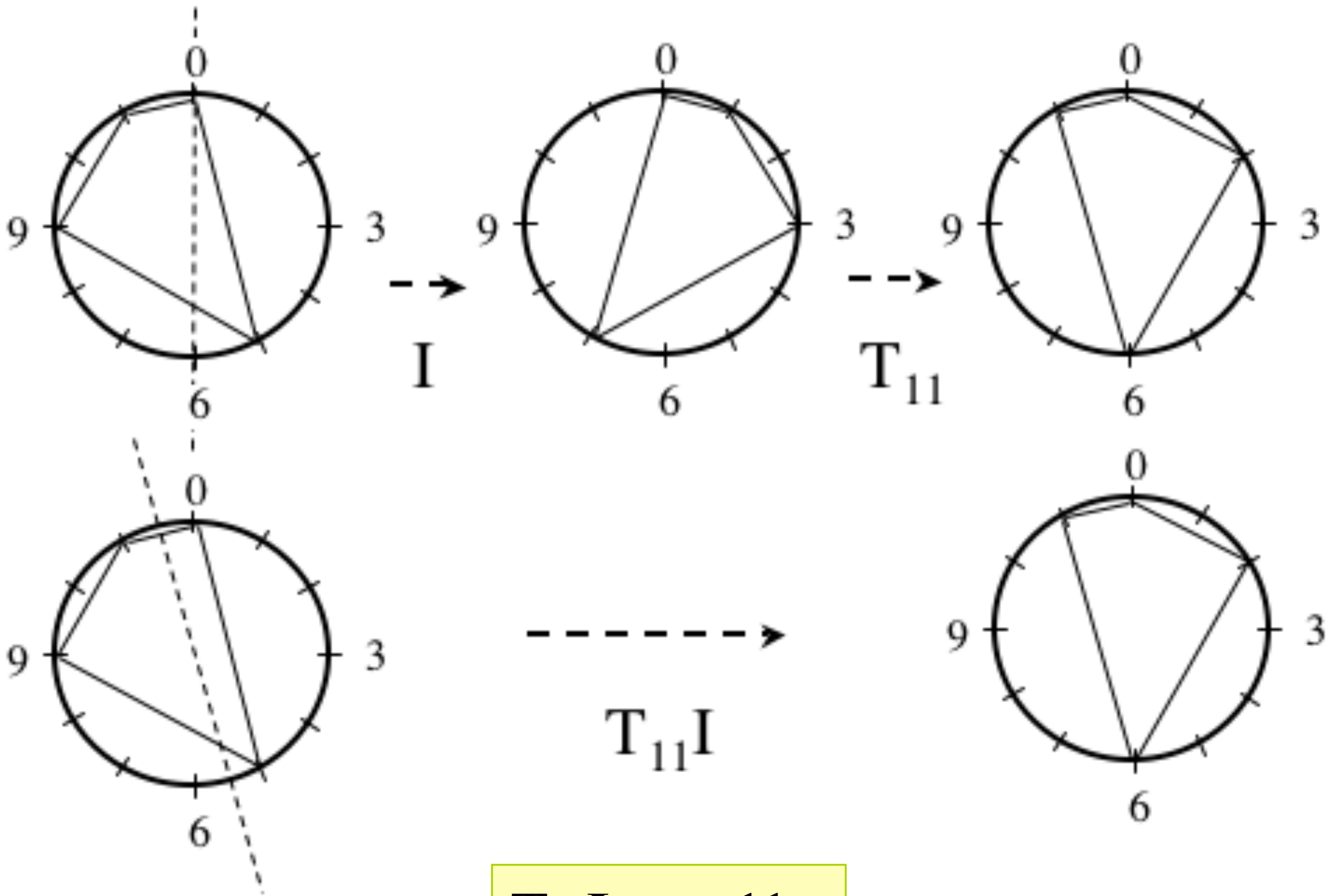




Transposition and Inversion

I: $x \to 12-x$

T_k: $x \to k+x$



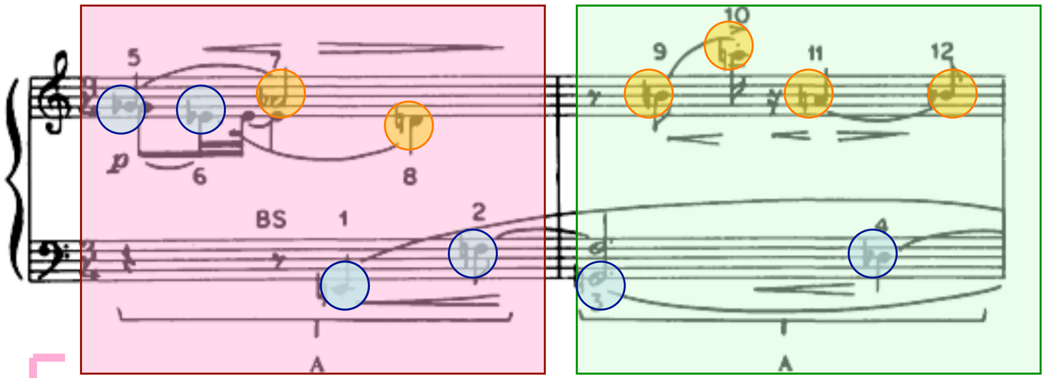
$T_{11}I$: $x \to 11-x$

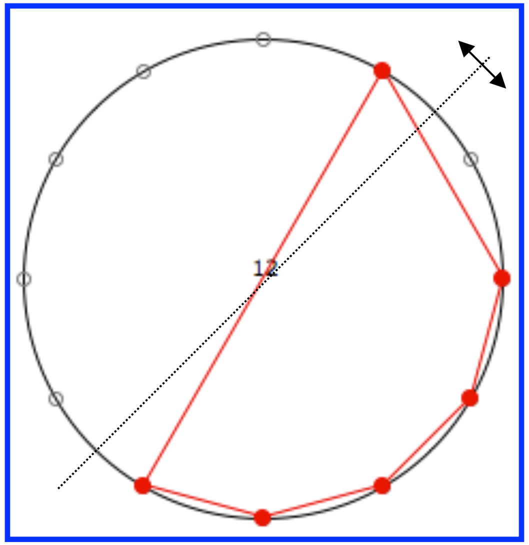
{0, 5, 9, 11} → {11, 6, 3, 0}

➔ 

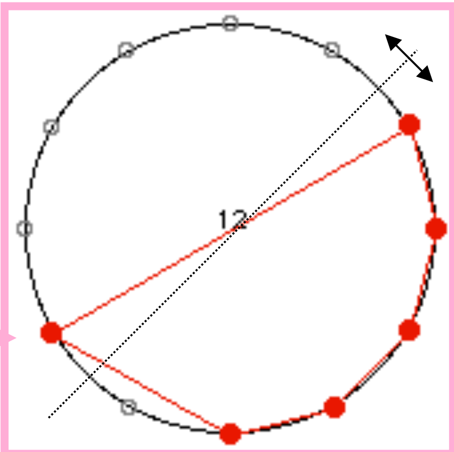
Serialism and hexachordal combinatoriality

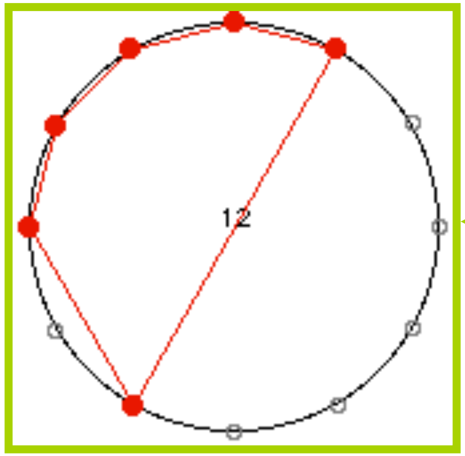
Schoenberg: Suite Op.25, Minuetto

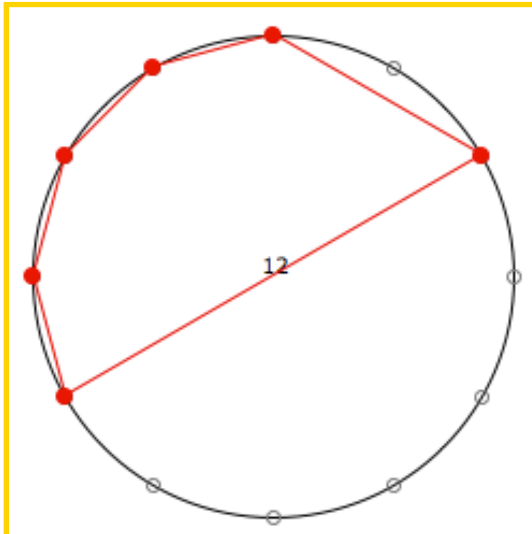




Double combinatoriality

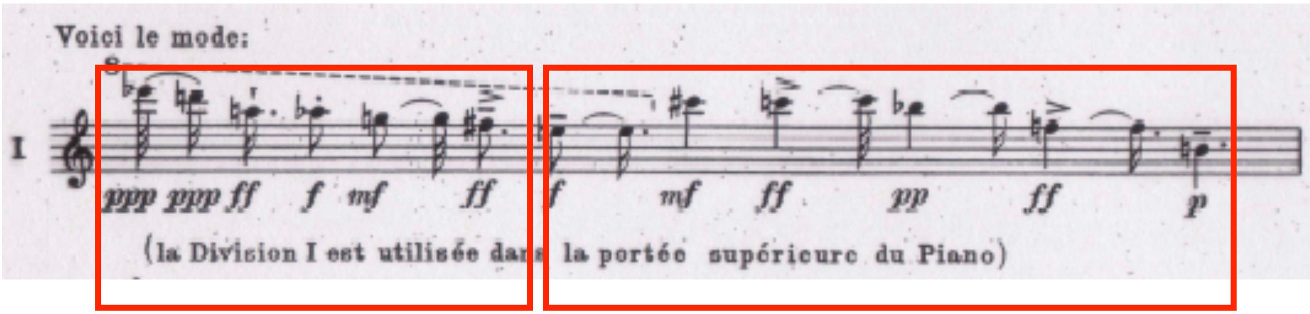


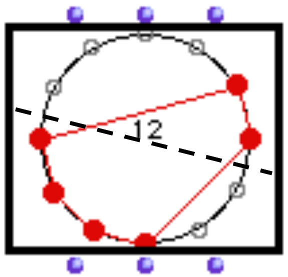




Hexachordal Combinatoriality in Messiaen

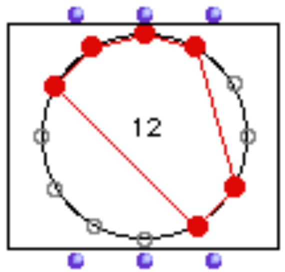
- Mode de valeurs et d'intensités (1950)





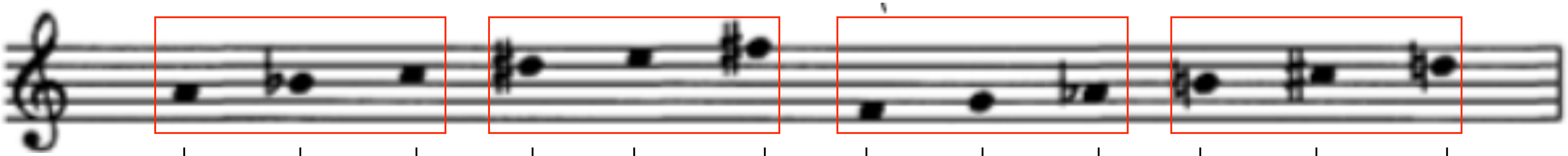
$\{3,2,9,8,7,6\} \longrightarrow \{4,5,10,11,0,1\}$

$T_7I : x \rightarrow 7\text{-}x$

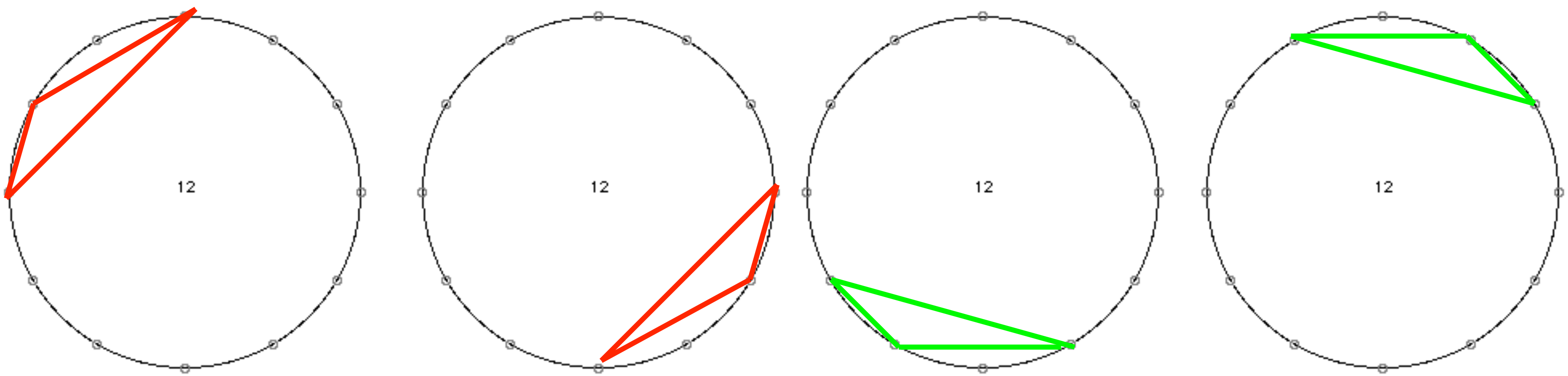


Symmetries in a Twelve-Tone Row : partitioning trichords

Schoenberg: Serenade Op.24, Mouvement 5



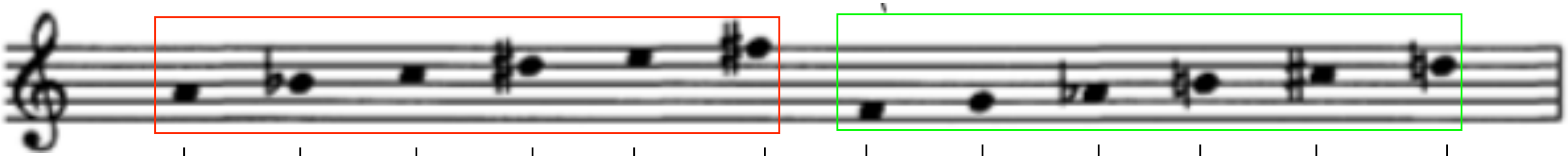
{9 , 10 , 0}   {3 , 4 , 6}   {5 , 7 , 8}   {11 , 1 , 2}



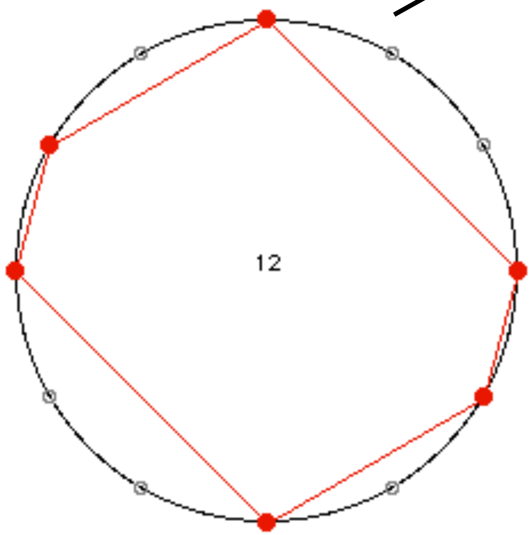
(1 , 2 , 9)   (1 , 2 , 9)   (2 , 1 , 9)   (2 , 1 , 9)

Hexachordal Combinatoriality and Transpositional Symmetry

Schoenberg: Serenade Op.24, Mouvement 5



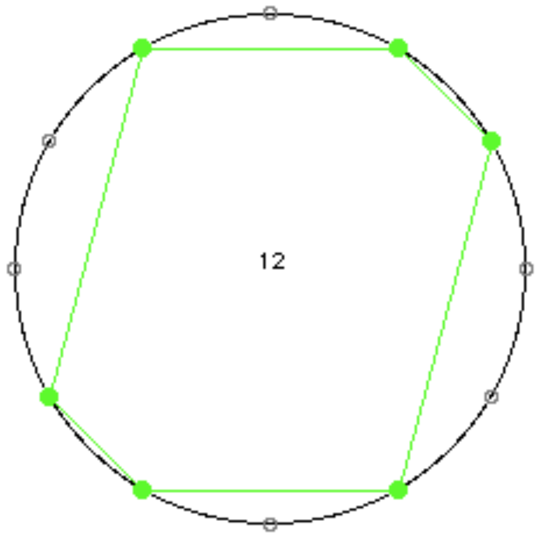
A = {9 , 10 , 0 , 3, 4 , 6} {5 , 7, 8, 11, 1, 2}



(3, 1, 2, 3, 1, 2)

T6{9,10,0,3,4,6}=
={6+9,6+10,6,6+3,6+4,6+6}=
={3,4,6,9,10,0}

T6(A)=A



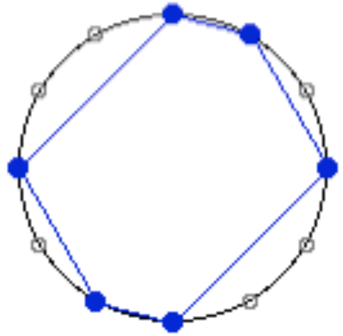
(2, 1, 3, 2, 1, 3)

Chord multiplication (Boulez) & TC (Cohn)

{0, 1, 3, 6, 7, 9}

(6 6)•{0, 1, 3}= ?

(1 2 3 1 2 3) ⟷



(6 6)•{0, 1, 3}=

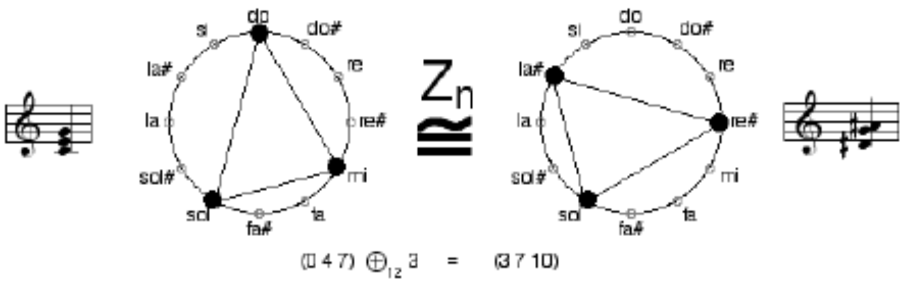
=((6 6)•{0})∪((6 6)•{1})∪((6 6)•{3})=

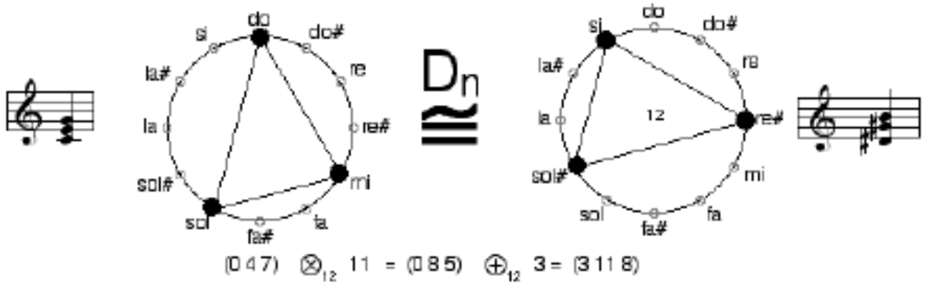
={0,6}∪{1,7}∪{3,9}=

={0,1,3,6,7,9}.

➔ 

Equivalence classes of chords



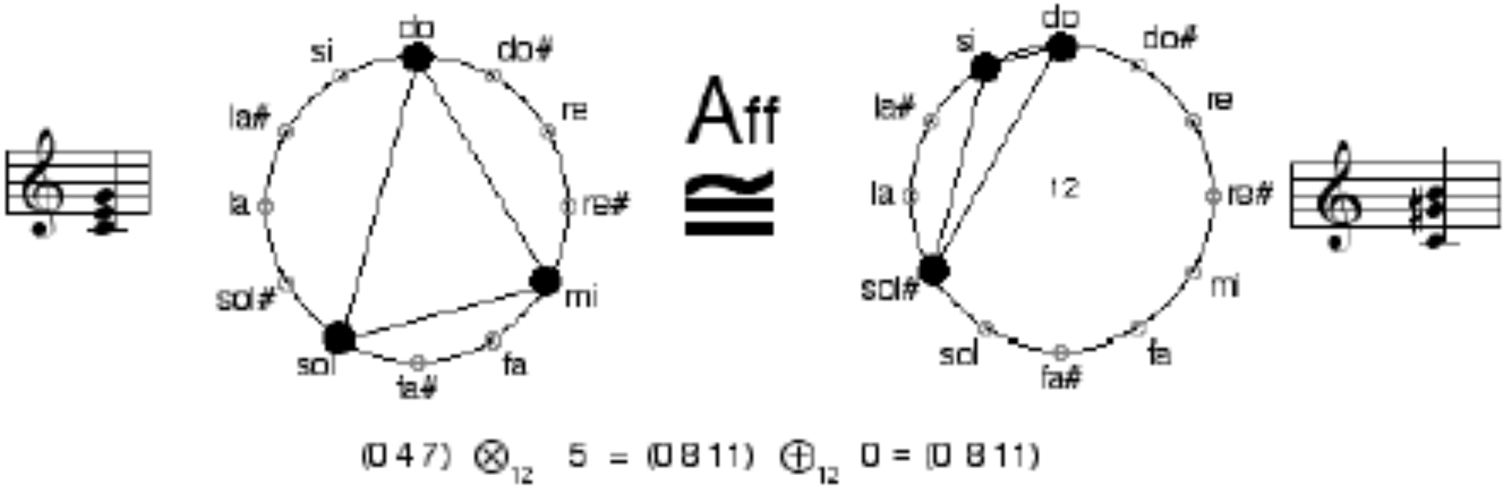


Transposition

$T_3\{0, 4, 7\} = 3+\{0, 4, 7\} = \{3, 7, 10\}$

Transposition and/or inversion

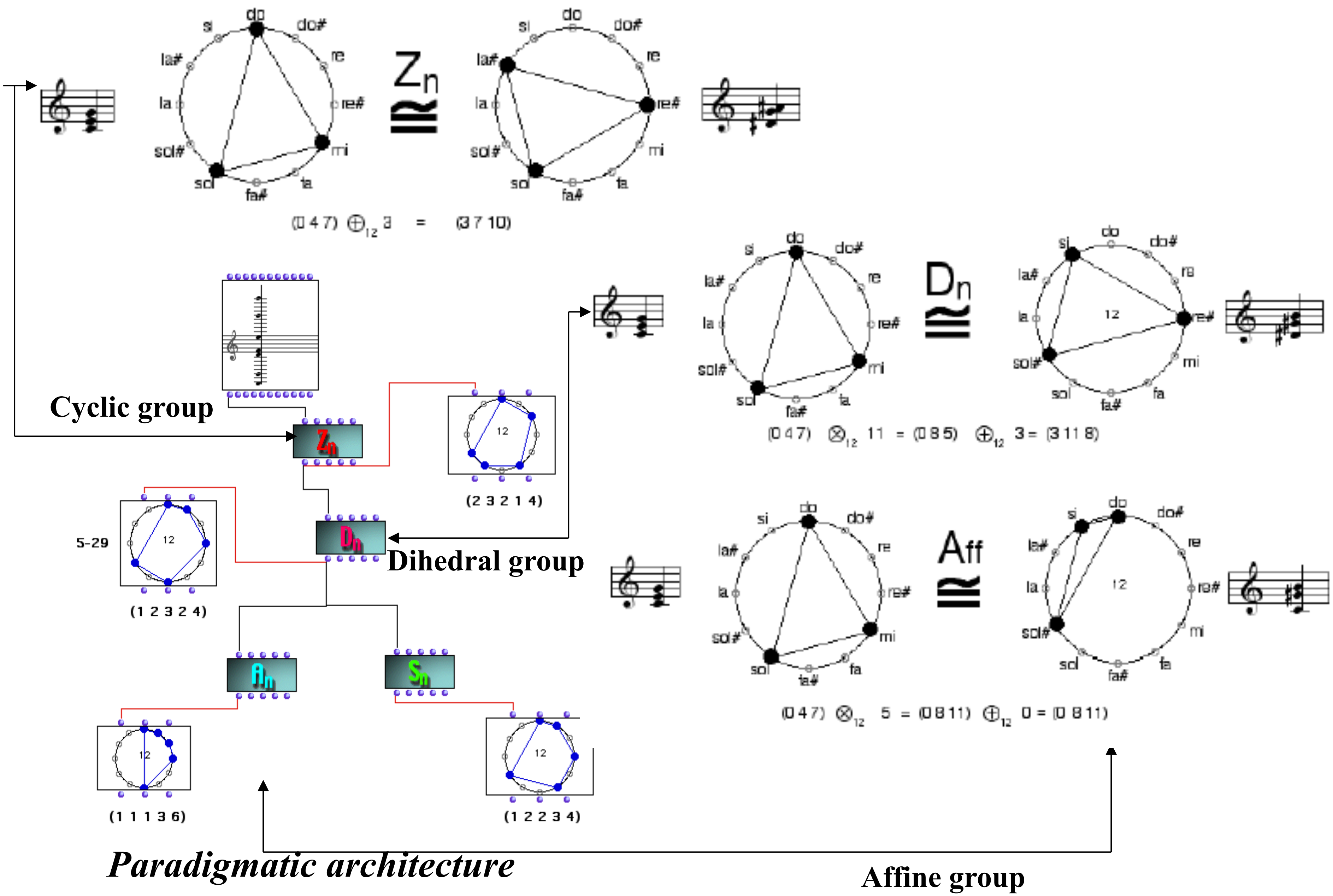
$T_3I\{0, 4, 7\} = 3+\{0, -4, -7\} = \{3, 11, 8\}$



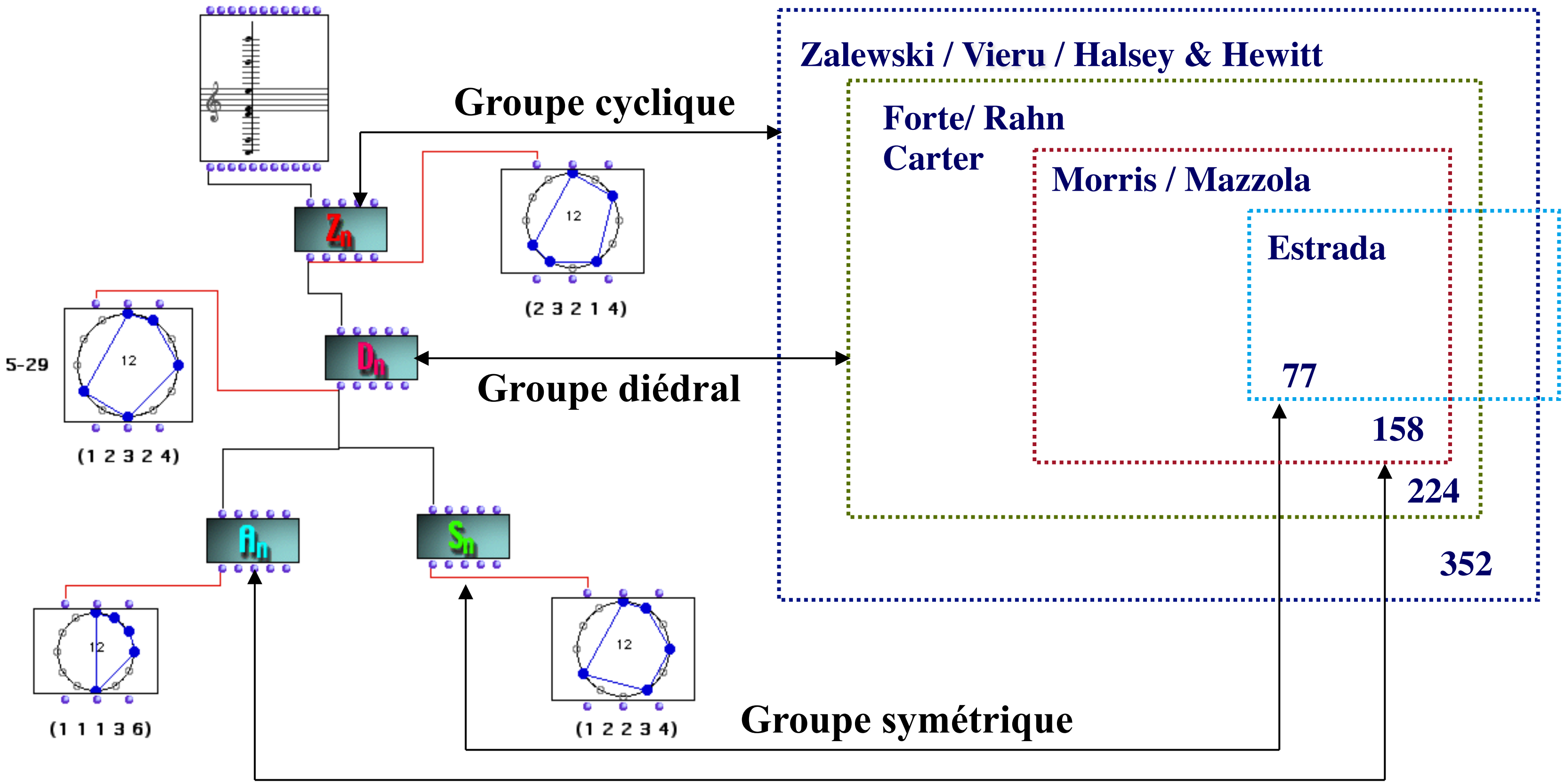
Multiplication (or affine transformation)

$M_5\{0, 4, 7\} = 5\times\{0, 4, 7\} = \{0, 8, 11\}$

Equivalence relation between musical structures



Classification paradigmatique des structures musicales



Architecture paradigmatique

F. Klein

W. Burnside

G. Polya

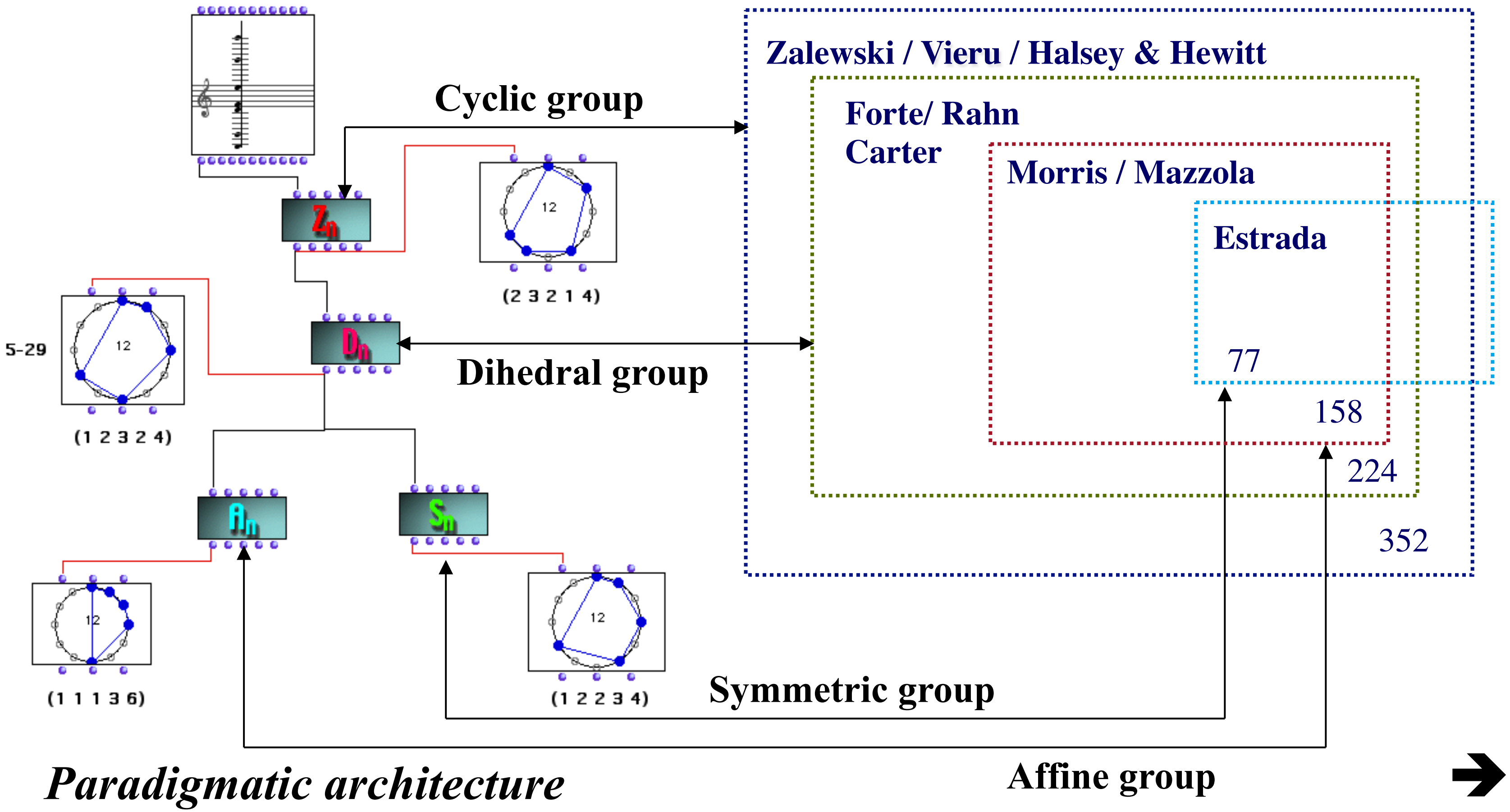
« [C'est la notion de groupe qui] donne un sens précis à l'idée de structure d'un ensemble [et] permet de déterminer les éléments efficaces des transformations en réduisant en quelque sorte à son schéma opératoire le domaine envisagé. […]L'objet véritable de la science est le système des relations et non pas les termes supposés qu'il relie. […] Intégrer les résultats - symbolisés - d'une expérience nouvelle revient […] à créer un canevas nouveau, un groupe de transformations plus complexe et plus compréhensif »

G.-G. Granger : « Pygmalion. Réflexions sur la pensée formelle », 1947

G.-G. Granger

Enumeration of musical structures

$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12	
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1	
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1	**Set Theory**
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1	



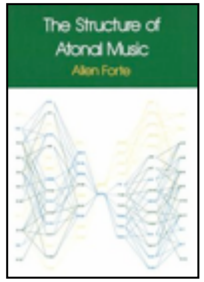
Paradigmatic architecture

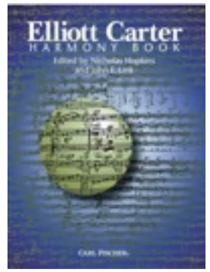
A set-theoretical exercice by Célestin Deliège

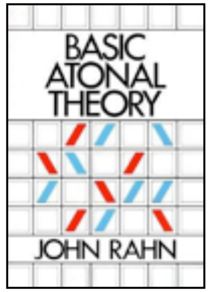
?

?

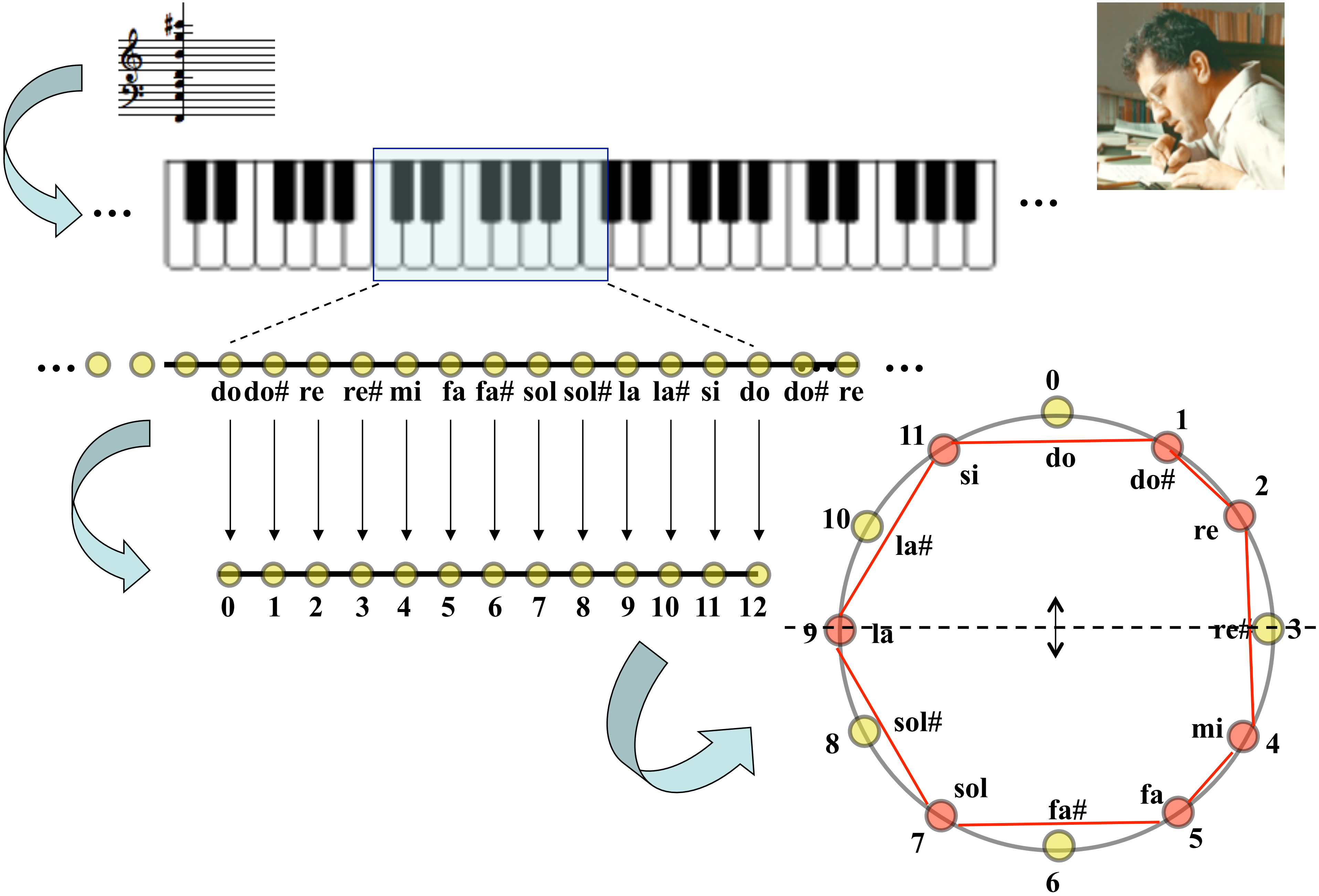
5-30	0,1,4,6,8	121321	7-30	0,1,2,4,6,8,9	343542
5-31	0,1,3,6,9	114112	7-31	0,1,3,4,6,7,9	336333
5-32	0,1,4,6,9	113221	7-32	0,1,3,4,6,8,9	335442
5-33(12)	0,2,4,6,8	040402	7-33	0,1,2,4,6,8,10	262623
5-34(12)	0,2,4,6,9	032221	7-34	0,1,3,4,6,8,10	254442
5-35(12)	0,2,4,7,9	032140	7-35	0,1,3,5,6,8,10	254361
5-Z36	0,1,2,4,7	222121	7-Z36	0,1,2,3,5,6,8	444342
5-Z37(12)	0,3,4,5,8	212320	7-Z37	0,1,3,4,5,7,8	434541
5-Z38	0,1,2,5,8	212221	7-Z38	0,1,2,4,5,7,8	434442
6-1(12)	0,1,2,3,4,5	543210			
6-2	0,1,2,3,4,6	443211			
6-Z3	0,1,2,3,5,6	433221	6-Z36	0,1,2,3,4,7	
6-Z4(12)	0,1,2,4,5,6	432321	6-Z37(12)	0,1,2,3,4,8	
6-5	0,1,2,3,6,7	422232			
6-Z6(12)	0,1,2,5,6,7	421242	6-Z38(12)	0,1,2,3,7,8	
6-7(6)	0,1,2,6,7,8	420243			
6-8(12)	0,2,3,4,5,7	343230			
6-9	0,1,2,3,5,7	342231			
6-Z10	0,1,3,4,5,7	333321	6-Z39	0,2,3,4,5,8	
6-Z11	0,1,2,4,5,7	333231	6-Z40	0,1,2,3,5,8	
6-Z12	0,1,2,4,6,7	332232	6-Z41	0,1,2,3,6,8	
6-Z13(12)	0,1,3,4,6,7	324222	6-Z42(12)	0,1,2,3,6,9	



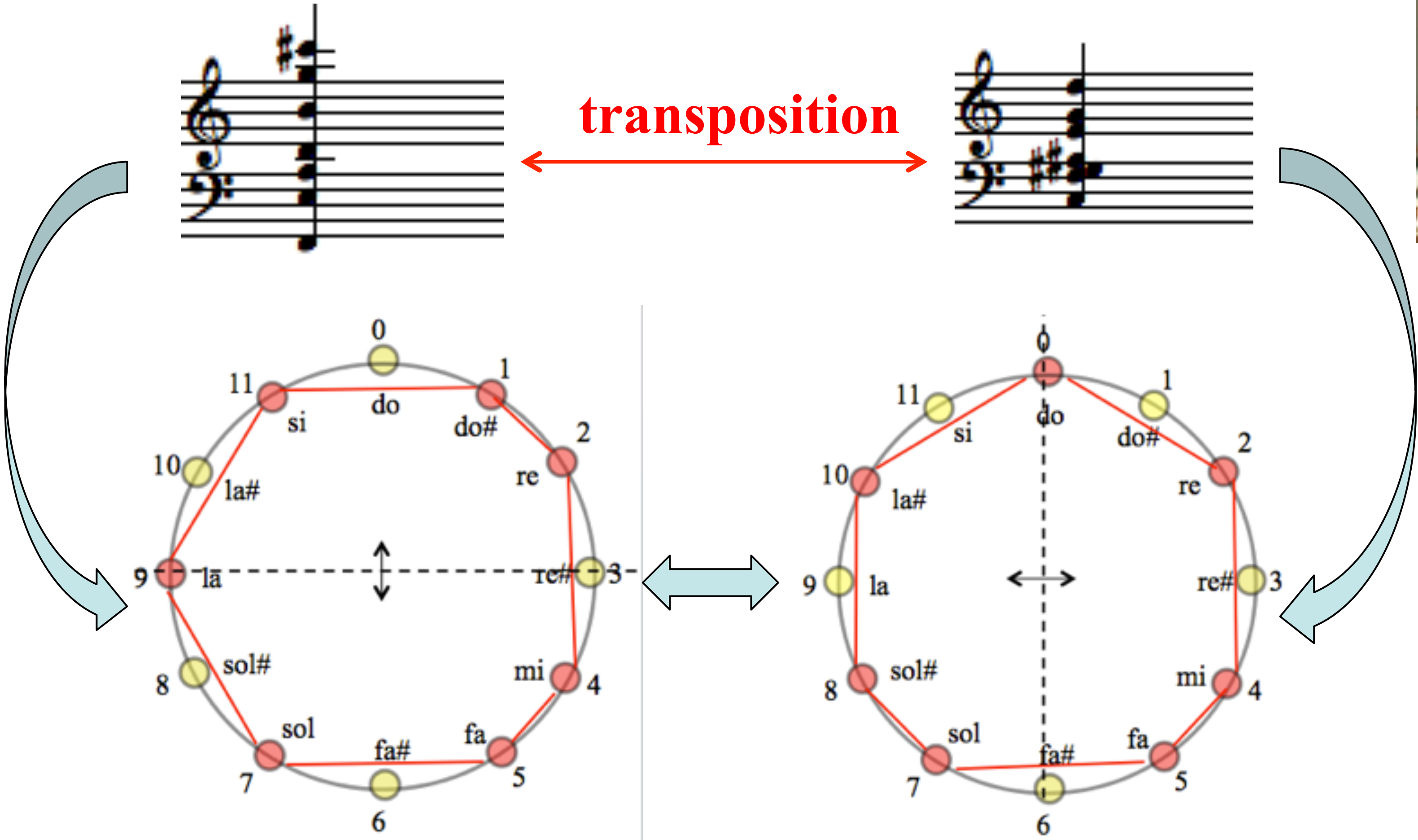




A set-theoretical exercice by Célestin Deliège

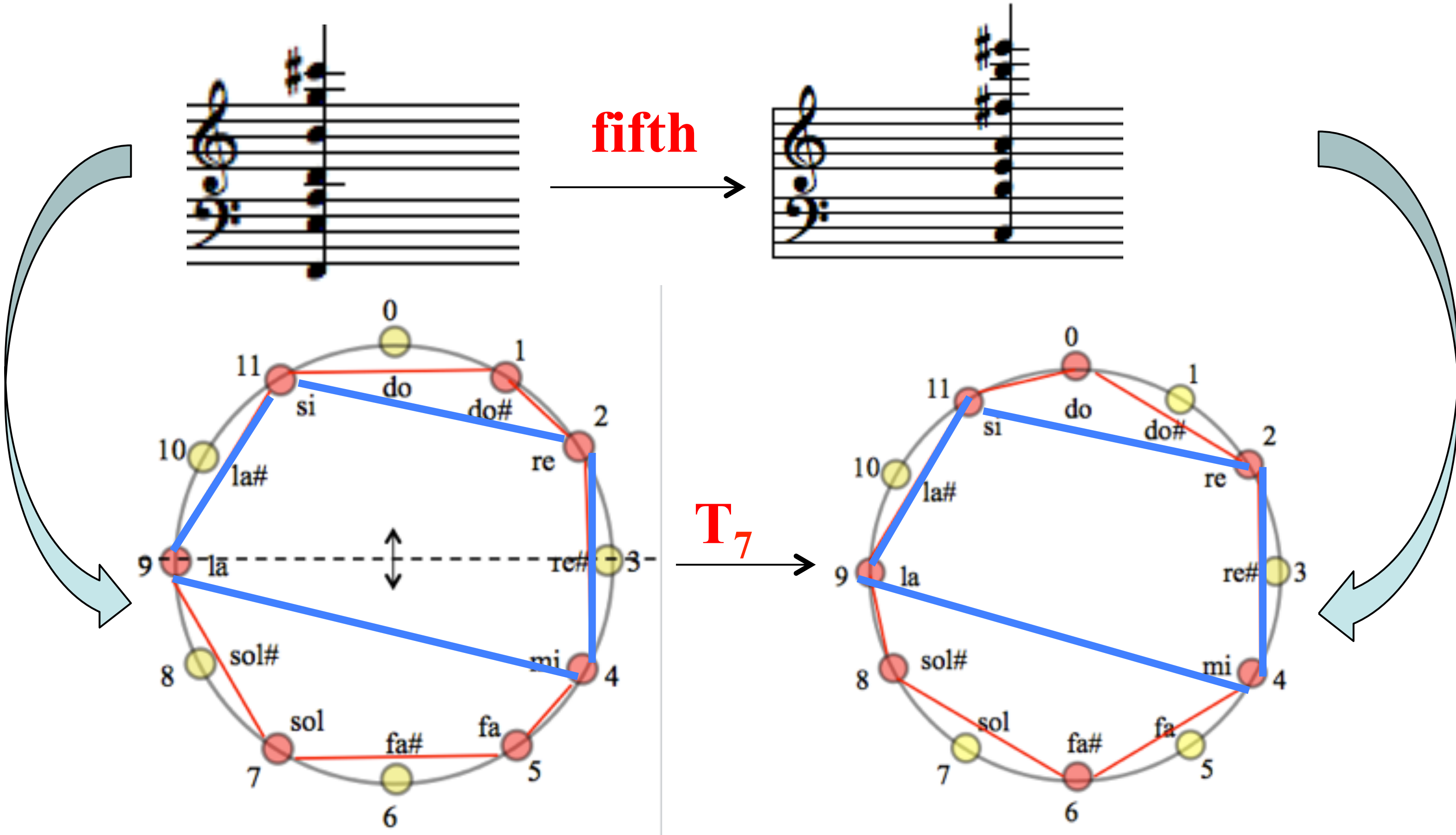


A set-theoretical exercice by Célestin Deliège



pcset	*Interval Content*	*name*
(0 1 3 4 6 8 10) →	[7 2 5 4 4 4 4 4 4 4 5 2] →	7-34

A set-theoretical exercice by Célestin Deliège



pcset		*Interval Content*		*name*
(0 1 3 4 6 8 10)	→	[7 2 5 4 4 4 4 4 4 4 5 2]	→	7-34

↑ T_0 … ↑ T_7

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Computational Music Analysis: the French tradition

Formalismes et modèles musicaux (André Riotte & Marcel Mesnage)

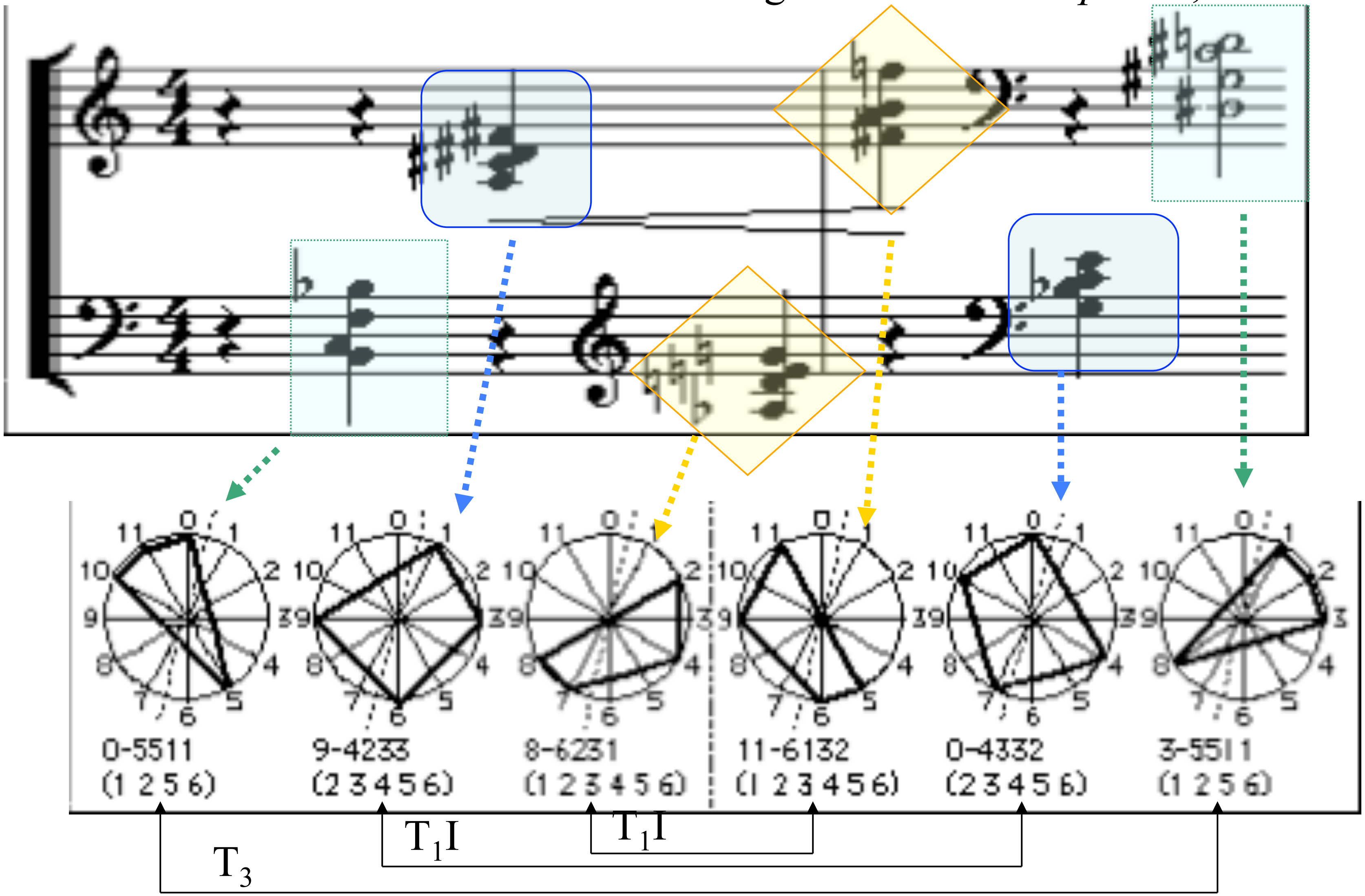
- « Anamorphoses » d'André Riotte
- « La terrasse des audiences du clair de lune » de Claude Debussy : esquisse d'analyse modélisée
- La mise en évidence de régularités locales : le « Mode de valeurs et d'intensités » de Messiaen
- Un exemple d'invention structurelle : le « Mikrokosmos » de Béla Bartok
- Un modèle informatique de la « Pièce pour quatuor à cordes » n°1 de Stravinsky
- Les « Variations pour piano », op. 27, d'Anton Werbern
- L'« Invention à deux voix » n°1 de J.-S. Bach
- Un modèle informatique du « Troisième Regard sur l'Enfant Jésus » d'Olivier Messiaen
- Un modèle de la « Valse sentimentale », Op. 50, n°13, de Franz Schubert
- Un automate musical construit à partir d'une courte pièce de Béla Bartok (Mikrokosmos n°39)



« Entités formelles pour l'analyse musicale »

Marcel Mesnage (1998)

A. Schoenberg : *Klavierstück Op. 33a*, 1929



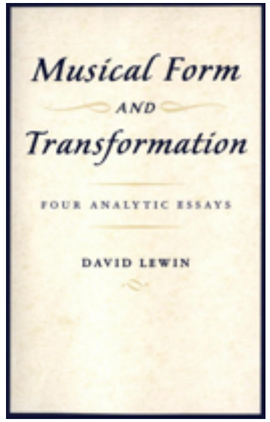
« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »

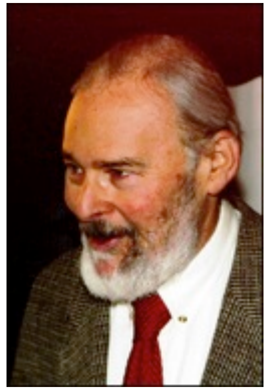
Three interpretations:

Henck

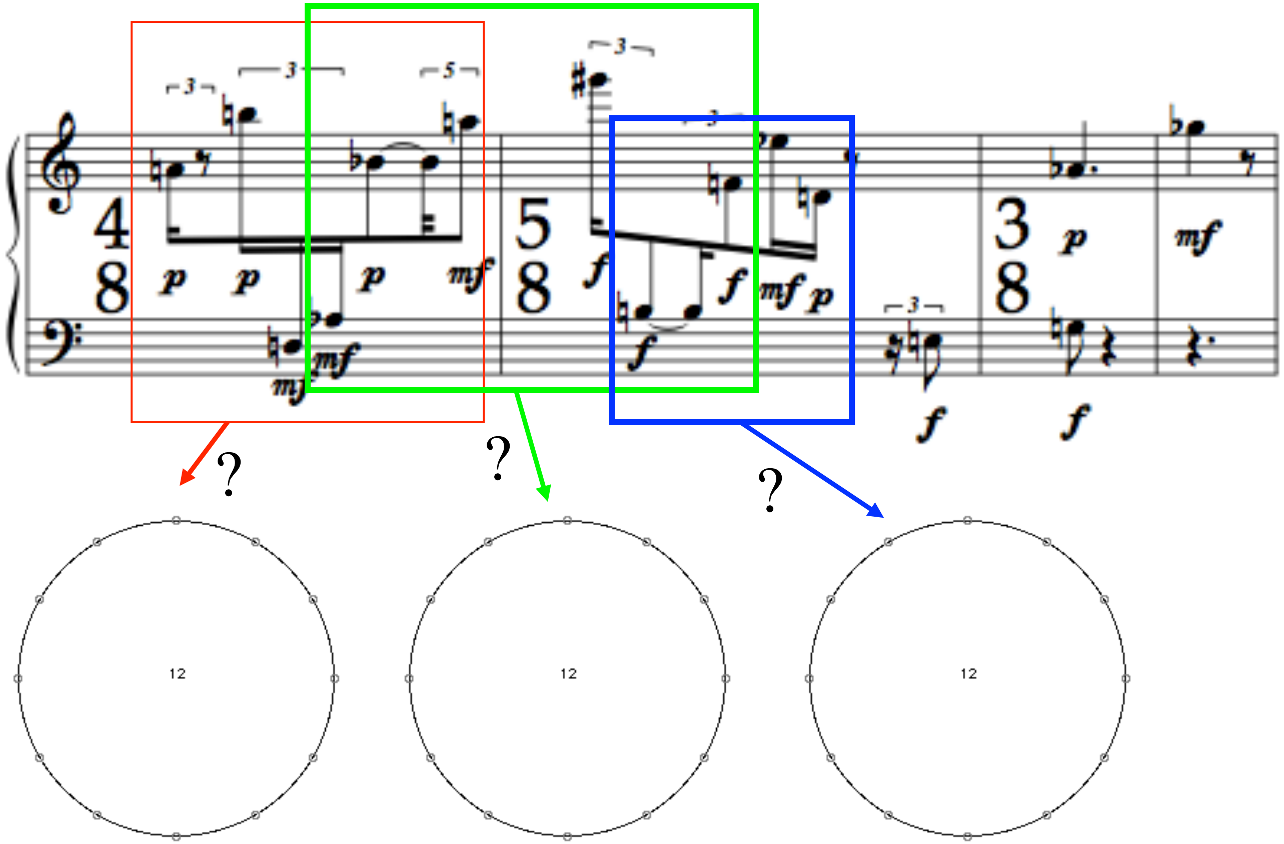
Kontarsky

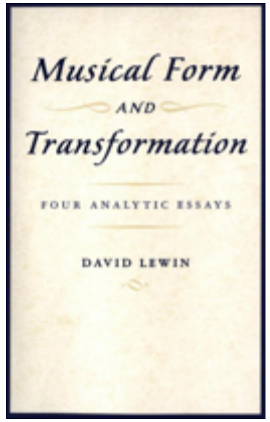
Tudor

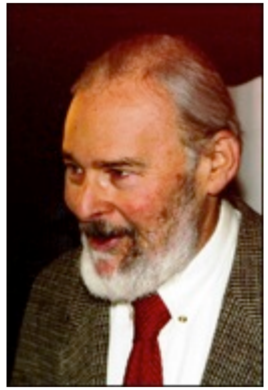




« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »





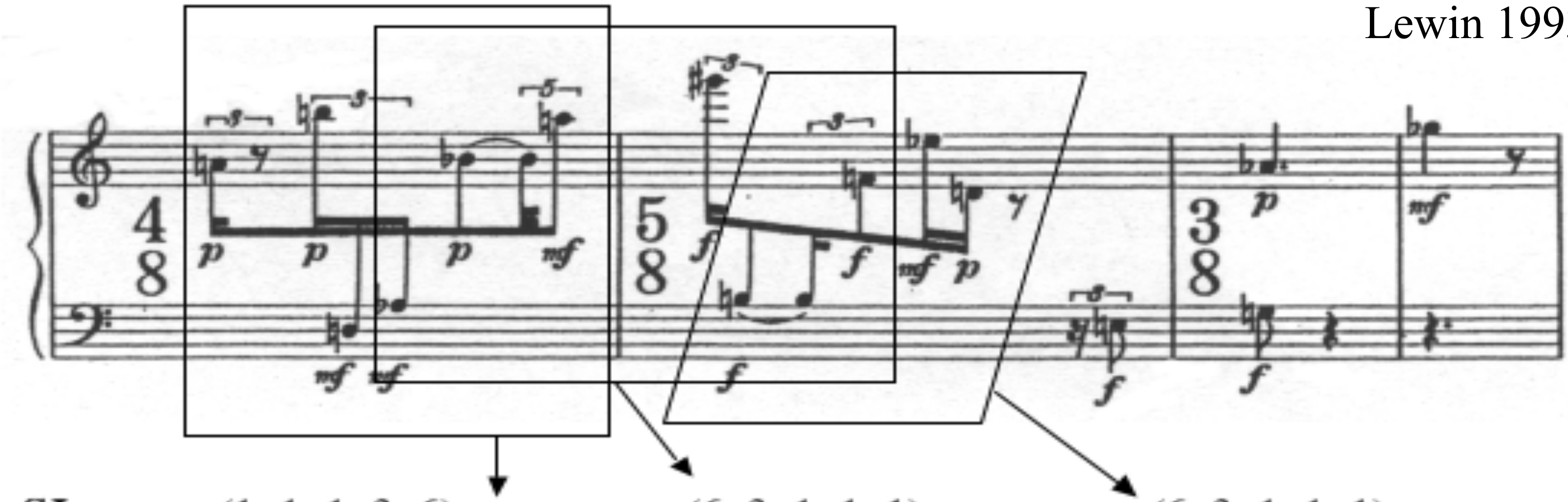


« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

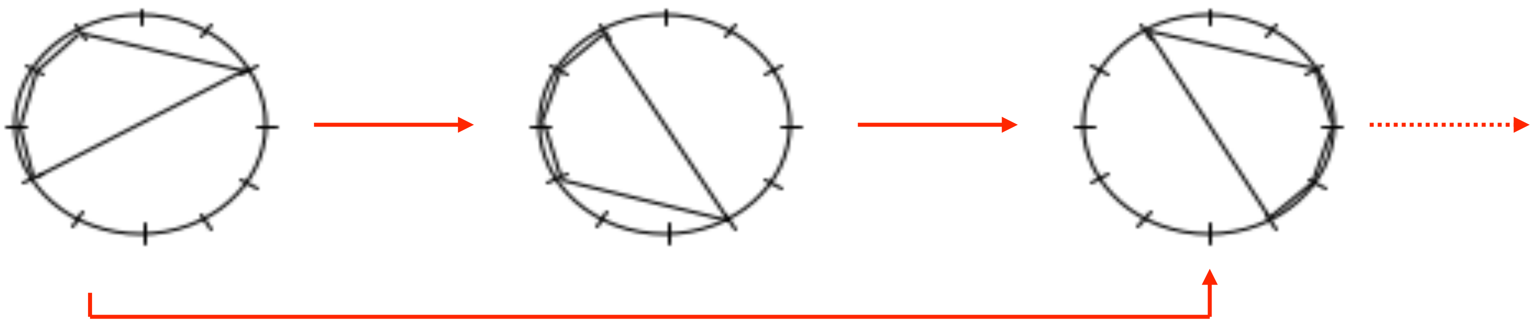
David Lewin, *Musical Form and Transformation,* YUP 1993

« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »

Lewin 1993

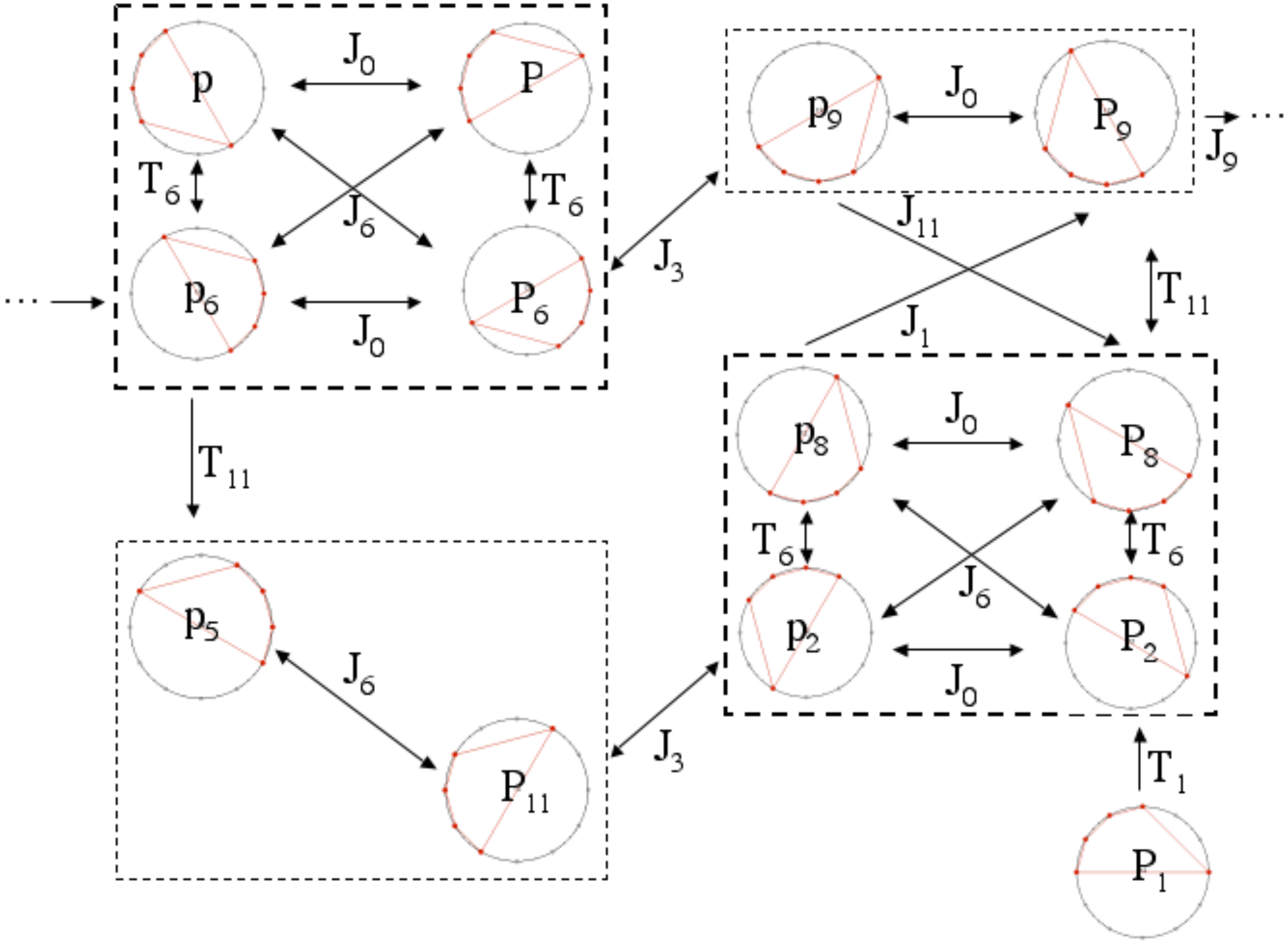


SI:	(1, 1, 1, 3, 6)	(6, 3, 1, 1, 1)	(6, 3, 1, 1, 1)
IFUNC:	[5 3 2 2 1 1 1 1 1 2 2 3]	[5 3 2 2 1 1 1 1 1 2 2 3]	[5 3 2 2 1 1 1 1 1 2 2 3]
VI:	[3 2 2 1 1 1]	[3 2 2 1 1 1]	[3 2 2 1 1 1]



Transformational Network

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)



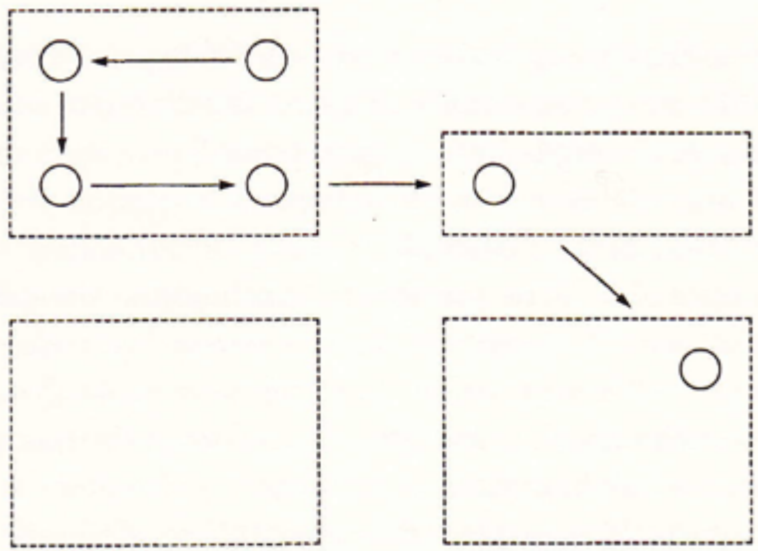
« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their **potentially functional interrelationships**, in a very compactly organized little **spatial configuration**. »

« […] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - **it makes the abstract space of such a world accessible to our sensibilities**. That is to say that the story projects what one would traditionally call *form*. »

Listening paths within the piece

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

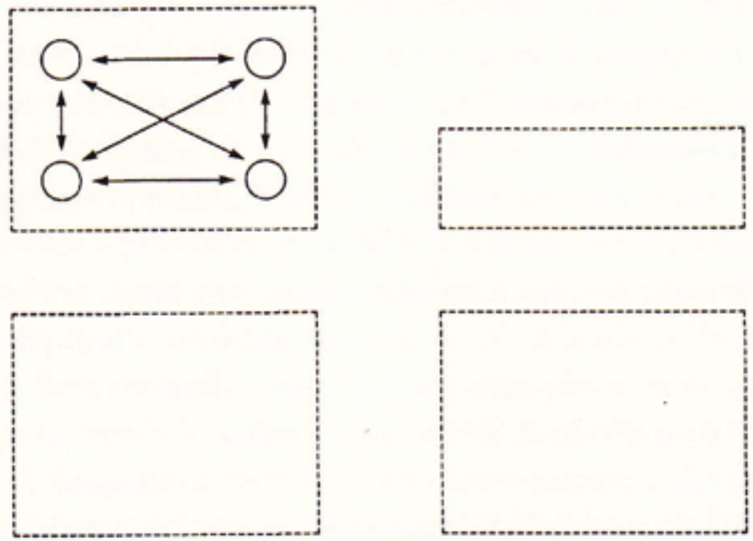
Pass 1 (mm. 1-5).



a

horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Pass 2 (mm. 5-8) goes back and elaborates the beginning area of pass 1.



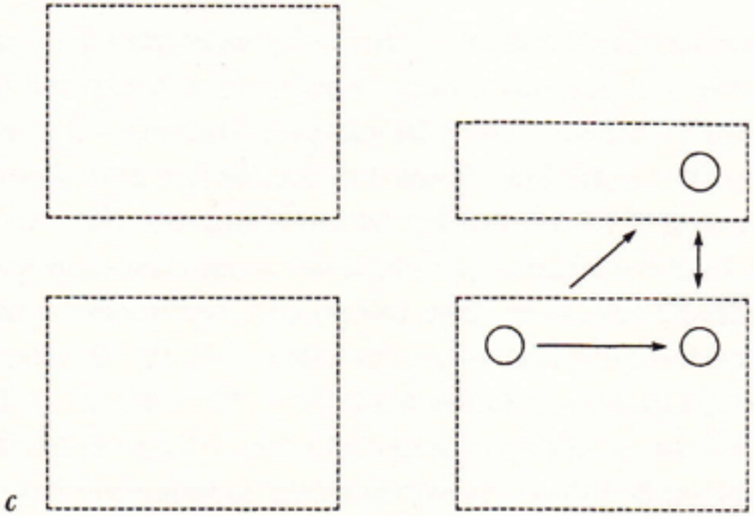
b

horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Listening paths within the piece

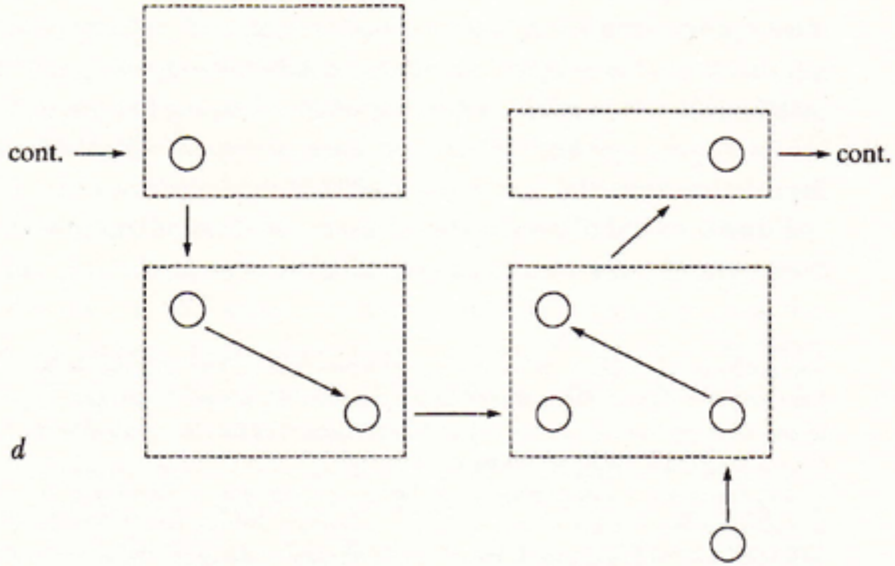
Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Pass 3 (mm. 8-10) picks up and elaborates
the ending area of pass 1.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

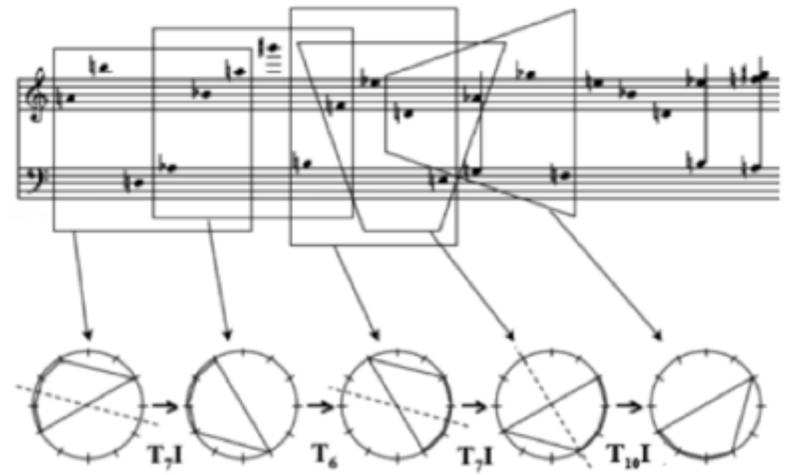
Pass 4 (mm. 9-16) expands the p8 + P8 area of pass 3
to activate P2 and p2 as well. P2 is the "essential" incipit
of pass 4; p2 is the end of the pass, and of the piece.

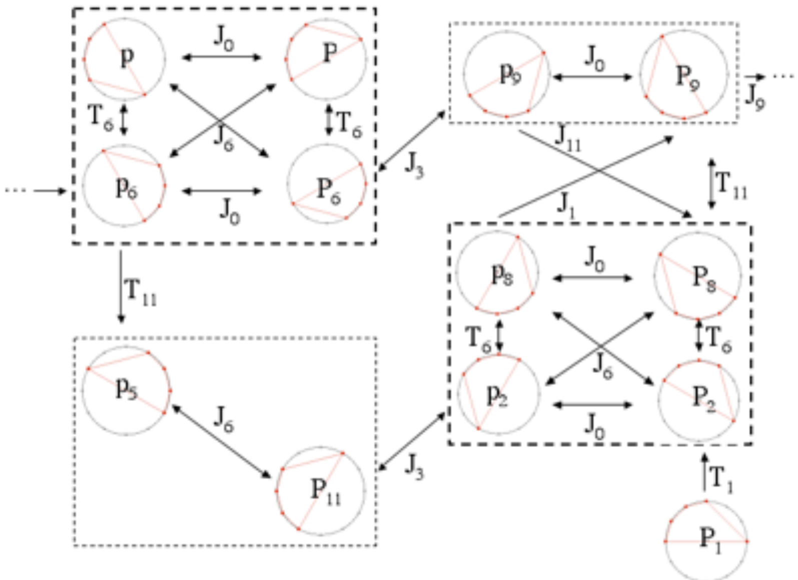


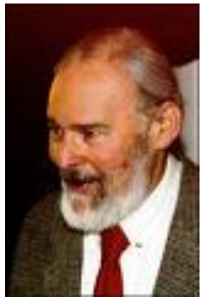
horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

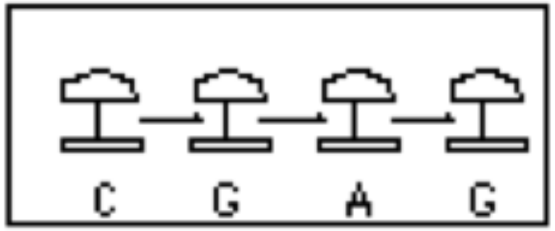
Transformational Networks and Music Cognition

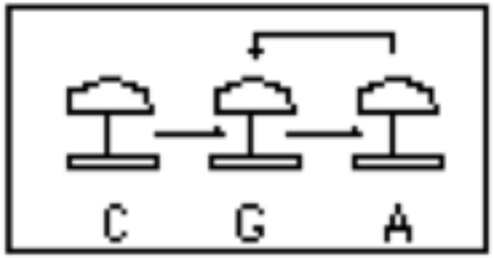
Bamberger, J. (1986). Cognitive issues in the development of musically gifted children. In *Conceptions of giftedness (eds., R. J. Sternberg, & J. E. Davidson), pp. 388-413. Cambridge University Press, Cambridge*











Bamberger, J. (2006). "What develops in musical development?" In G. MacPherson (ed.) *The child as musician: Musical development from conception to adolescence.* Oxford, U.K. Oxford University Press.

Listening exercise: « do you hear it? » *vs* « can you hear it? »

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Example 2.7. An ear-training aid for listening to P/p forms and their inter-relations.

« I take the question ‘Can you hear it » to mean something like this: After studying the analysis in examples 2.5 and 2.6, do you find it possible to focus your **aural attention** upon aspects of the acoustic signal that seem to engage the signifiers of that analysis? […] For me, the interesting questions involve the extent and ways in which I am satisfied and dissatisfied when **focusing my aural attention** in that manner. It is important to ask those questions about any systematic analysis of any musical composition ».

Can you hear it? Yes, we can!

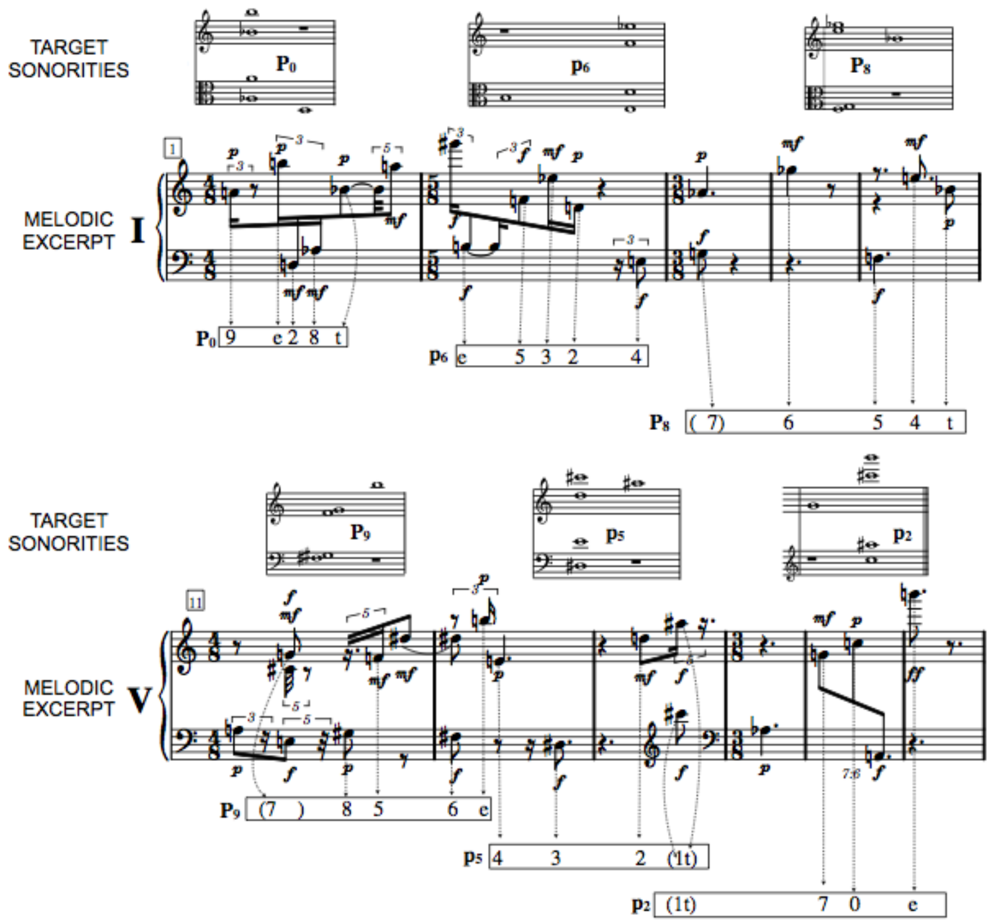
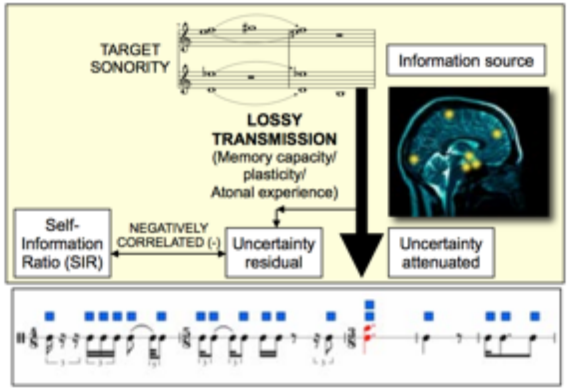


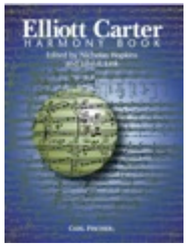
FIGURE 5. Six target sonorities used for Phase I pitch-detection tasks (circled in dashed-line boxes): Single Pentachords appeared in form of either 'st' or 'ts' according to Lewin's ear-training aid (*MFT*, Example 2.7, p. 42). Their corresponding melodies are either Excerpt I or V.



« A cognitive model is derived to show that singleton-tetrachord interaction is salient in facilitating the mental formation of common-tone-preserving percepts, and it serves as perceptual information that determines the acquisition of implicit pitch pattern knowledge for pitch-detection tasks, but only for atonally well-trained musicians. »

Y. Cao, J. Wild, B. Smith, S. McAdams, « The Perception and Learning of Contextually-defined Inversion Operators in Transformational Pitch Patterns », 5th International Conference of Students of Systematic Musicology (SysMus12), Montreal, 2012.

Elliott Carter : 90+ (1994)

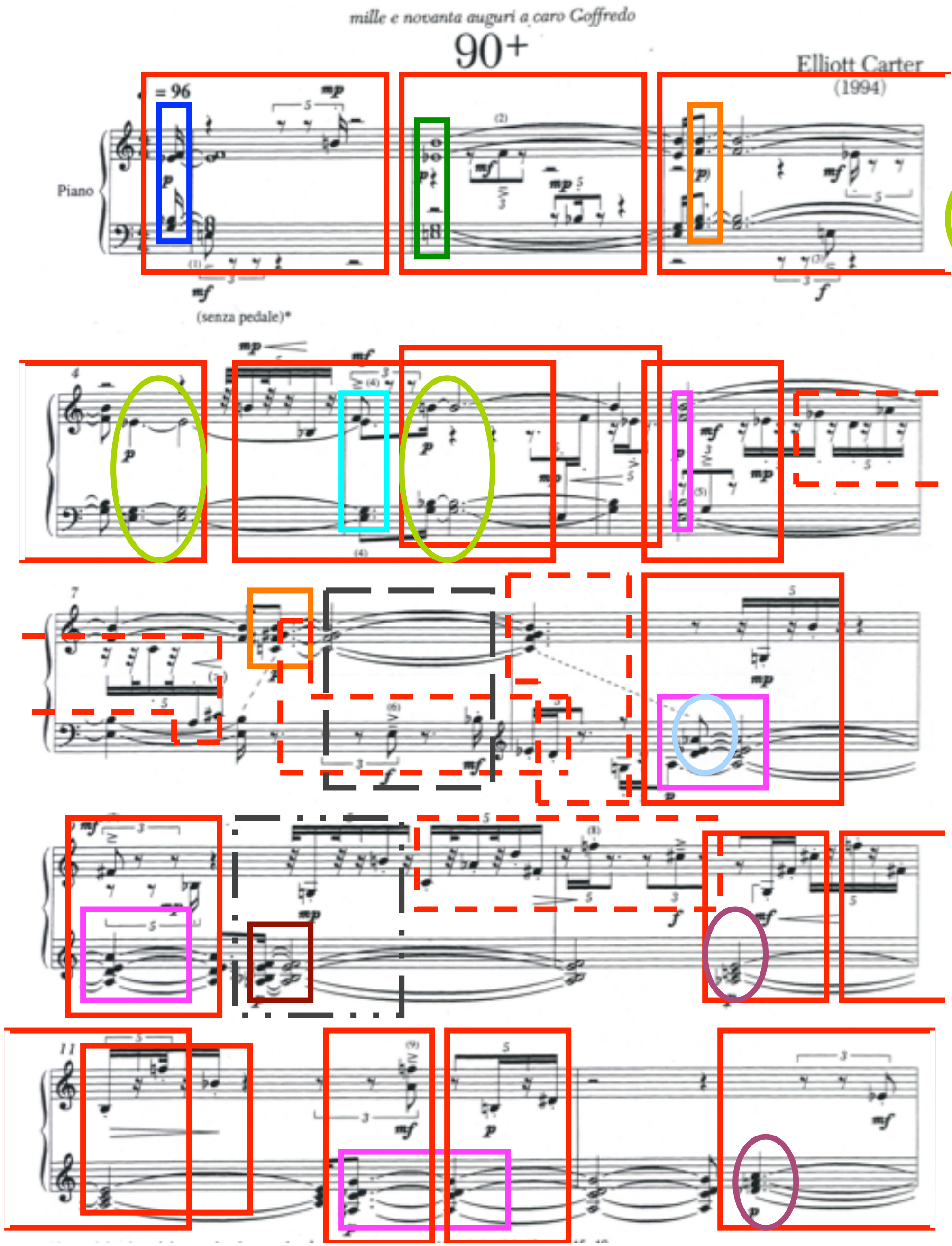


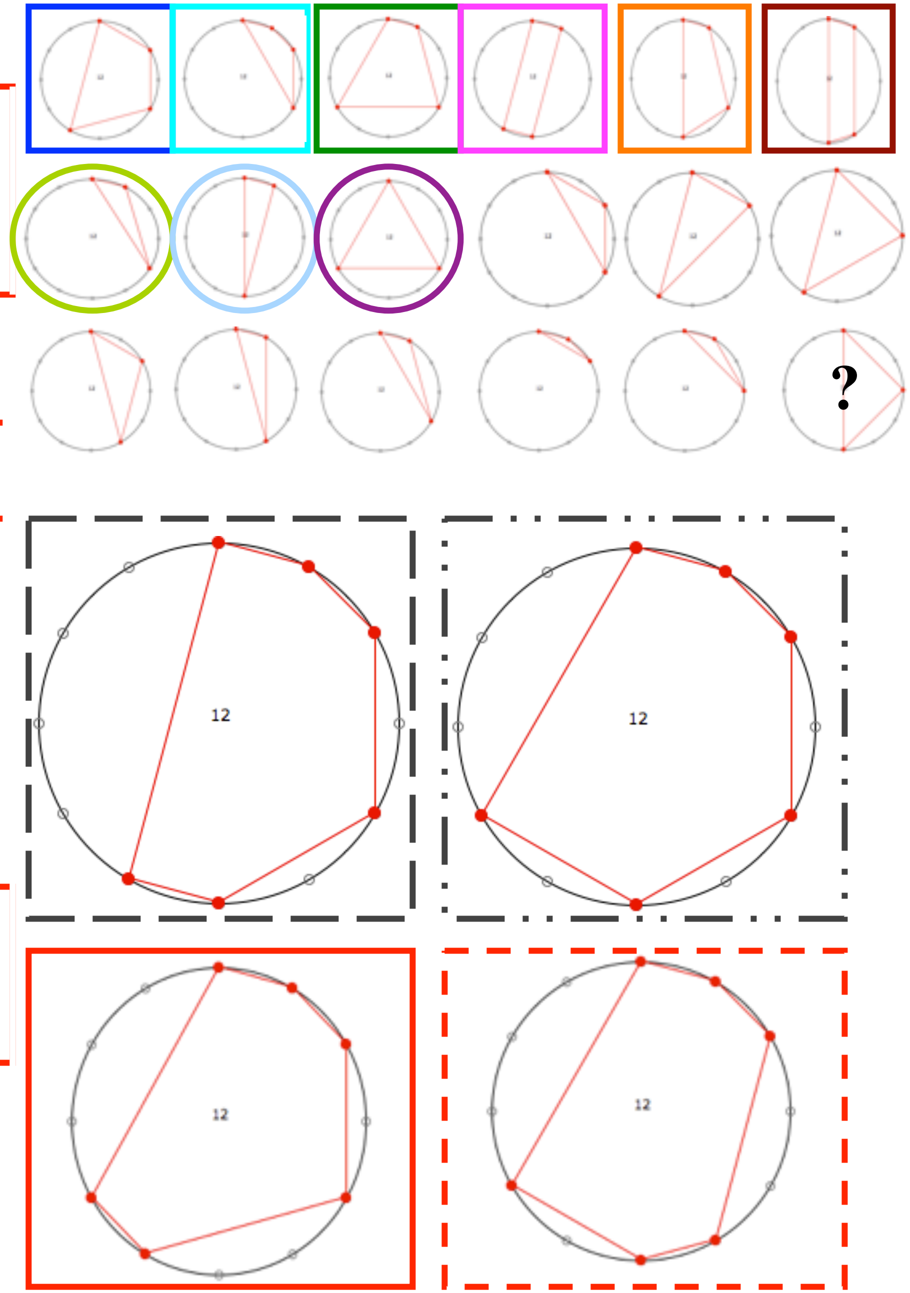
- **Chord combinatorics**
  - **Hexacords**
  - **Tetrachords**
  - **Triads**
  - **Z-relation**

- **all-interval series**
  - ***Link–chords***

(piano: John Snijders)

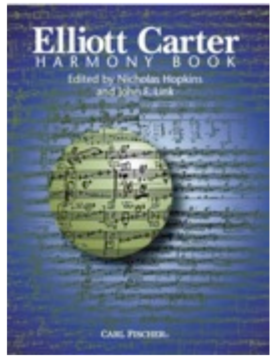
Elliott Carter : 90+ (1994) : tetra/trichordal combinatorics



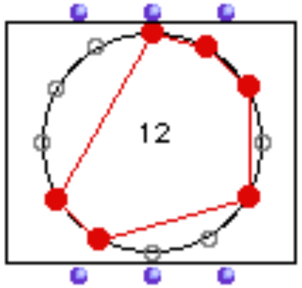


Elliott Carter: 90+ (1994)

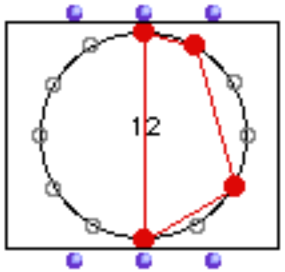
> « *From about 1990, I have reduced my vocabulary of chords more and more to the six note chord n° 35 and the four note chords n° 18 and 23, which encompass all the intervals* » (*Harmony Book*, 2002, p. ix)



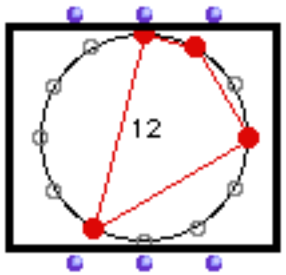
35



18



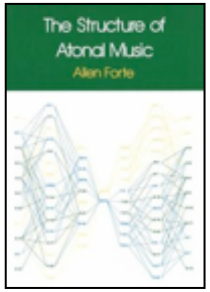
23

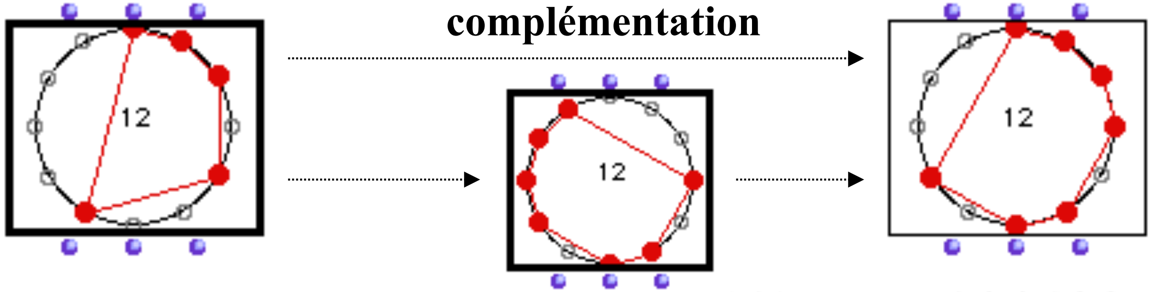


Chord	*pcset*	*vector*	*name*
35	(0 1 2 4 7 8)	[322332]	6-Z17
18	(0 1 4 6)	[111111]	4-Z15
23	(0 1 3 7)	[111111]	4-Z29

All-triad hexachord

Allen Forte's Catalogue (1973) and the Z relation





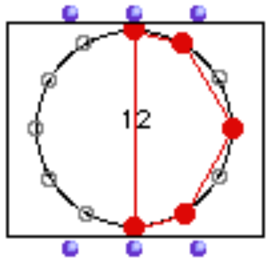

A. Forte (1926-)

5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140
5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211
6-Z4(12)	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

5-Z36 0,1,2,4,7 222121

7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442
6-Z37(12)	0,1,2,3,4,8	
6-Z38(12)	0,1,2,3,7,8	
6-Z39	0,2,3,4,5,8	
6-Z40	0,1,2,3,5,8	
6-Z41	0,1,2,3,6,8	
6-Z42(12)	0,1,2,3,6,9	

7-Z36 0,1,2,3,5,6,8 444342

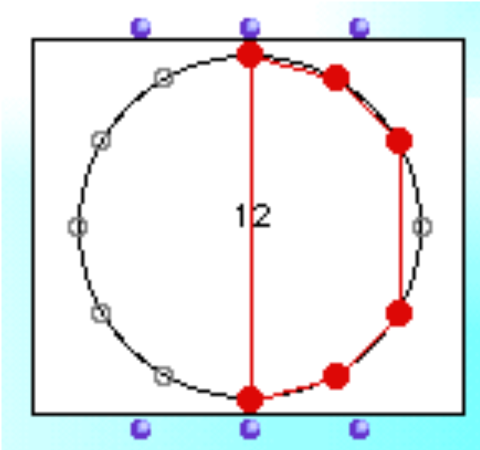


5-Z12

A 'Mathemusical' Theorem

$$IC_A(k) = \mathrm{Card}\{(x,y) \in A \times A \mid x + k = y\}$$

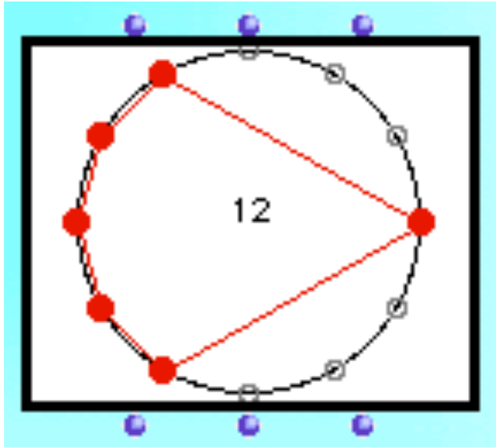
→ 

A 

Z-relation

↔

homometry

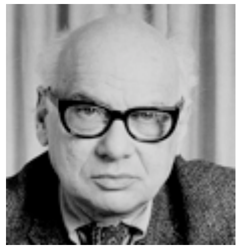


A'

$$IC_A = [4, 3, 2, 3, 2, 1] \;=\; [4, 3, 2, 3, 2, 1] = IC_{A'}$$

Babbitt's Hexachord Theorem:

A hexacord and its complement have the same interval content

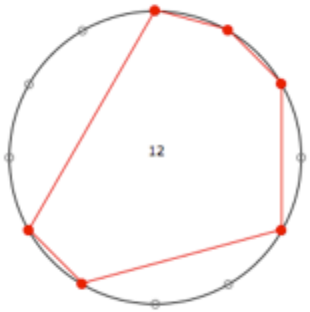


(Proofs by Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, …, Amiot, …)

Elliott Carter: 90+ (1994)

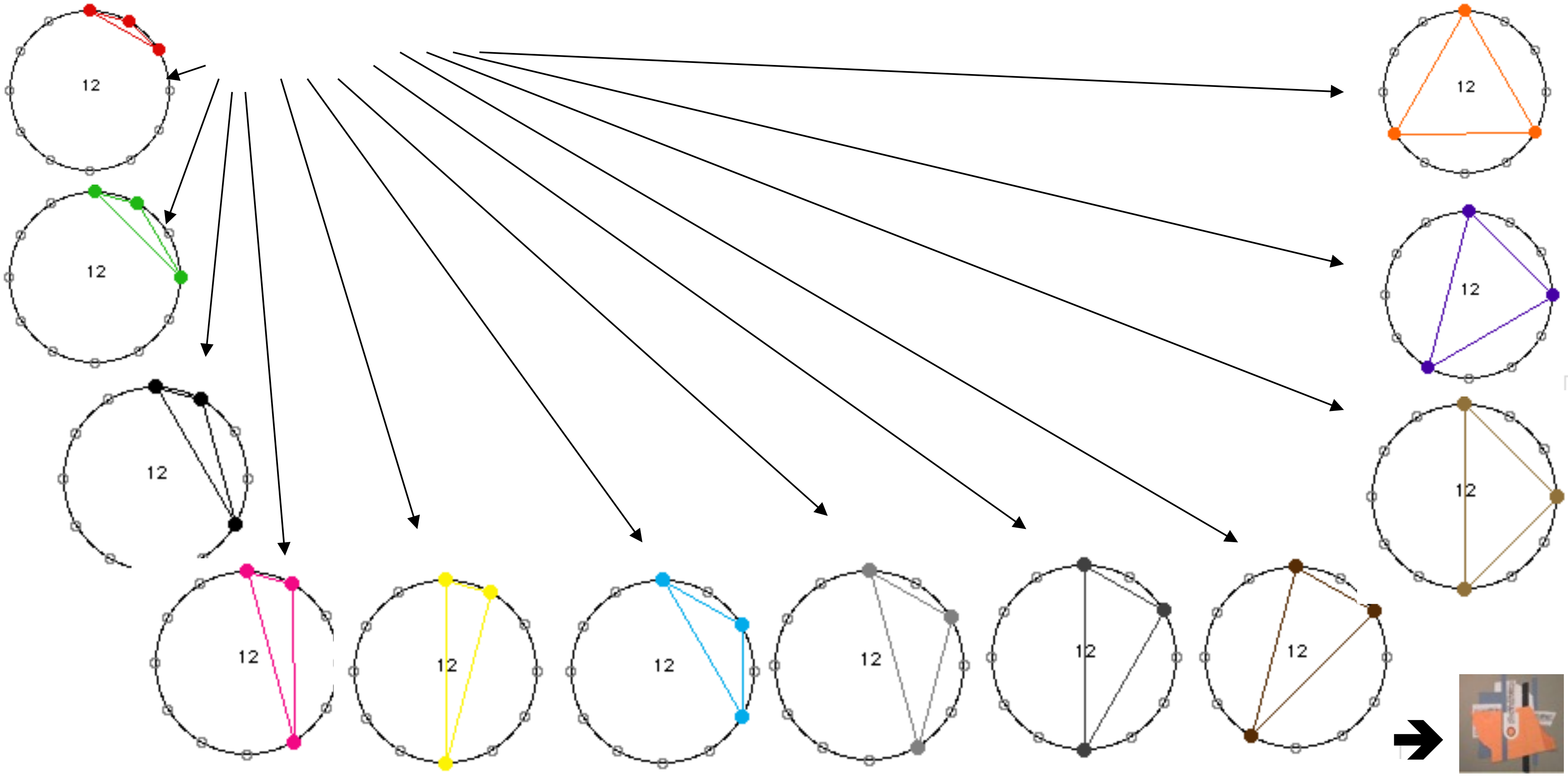
$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

35

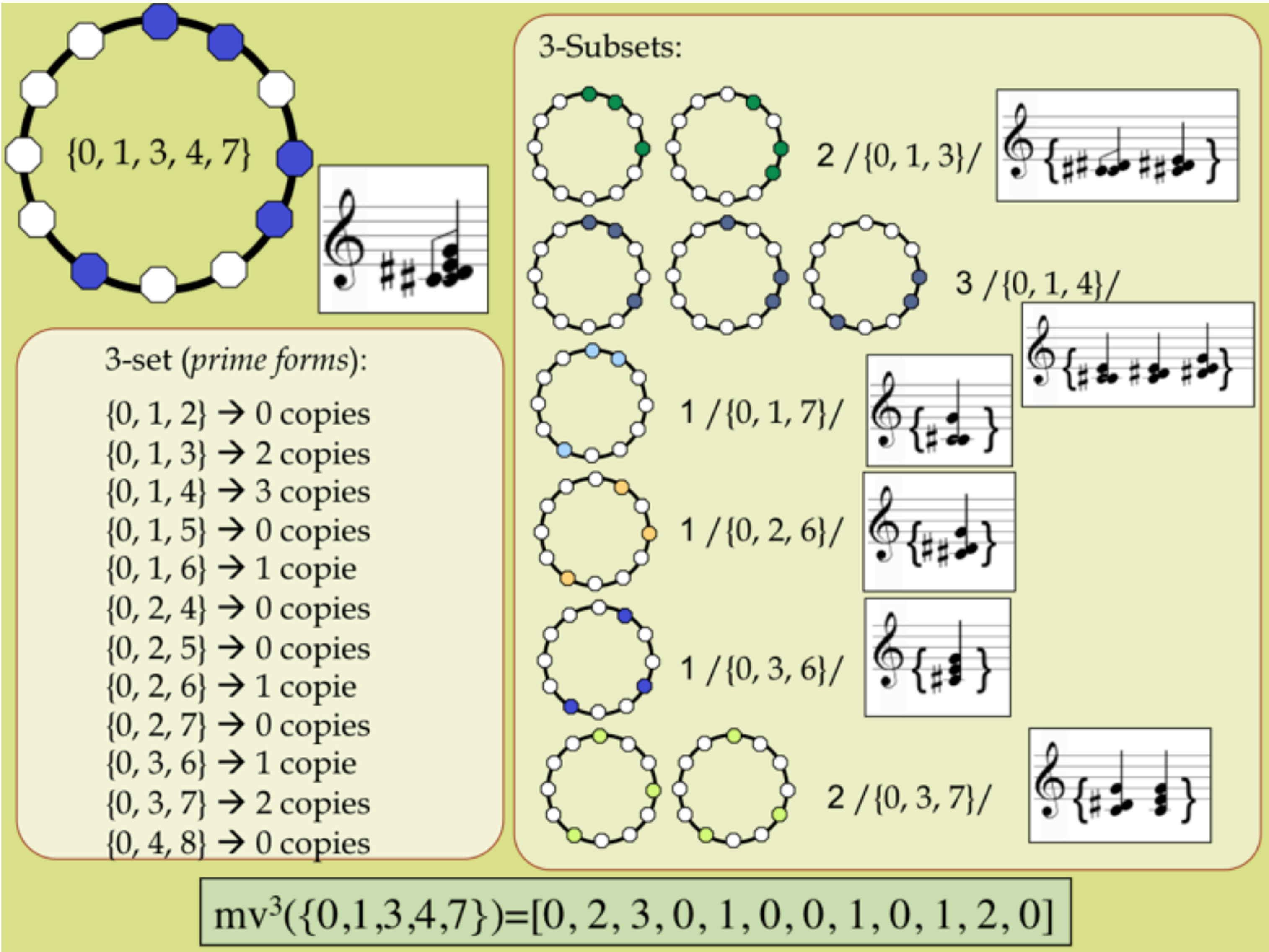


pcset	*vector*	*name*
(0 1 2 4 7 8)	[322332]	6-Z17

All-triad hexachord

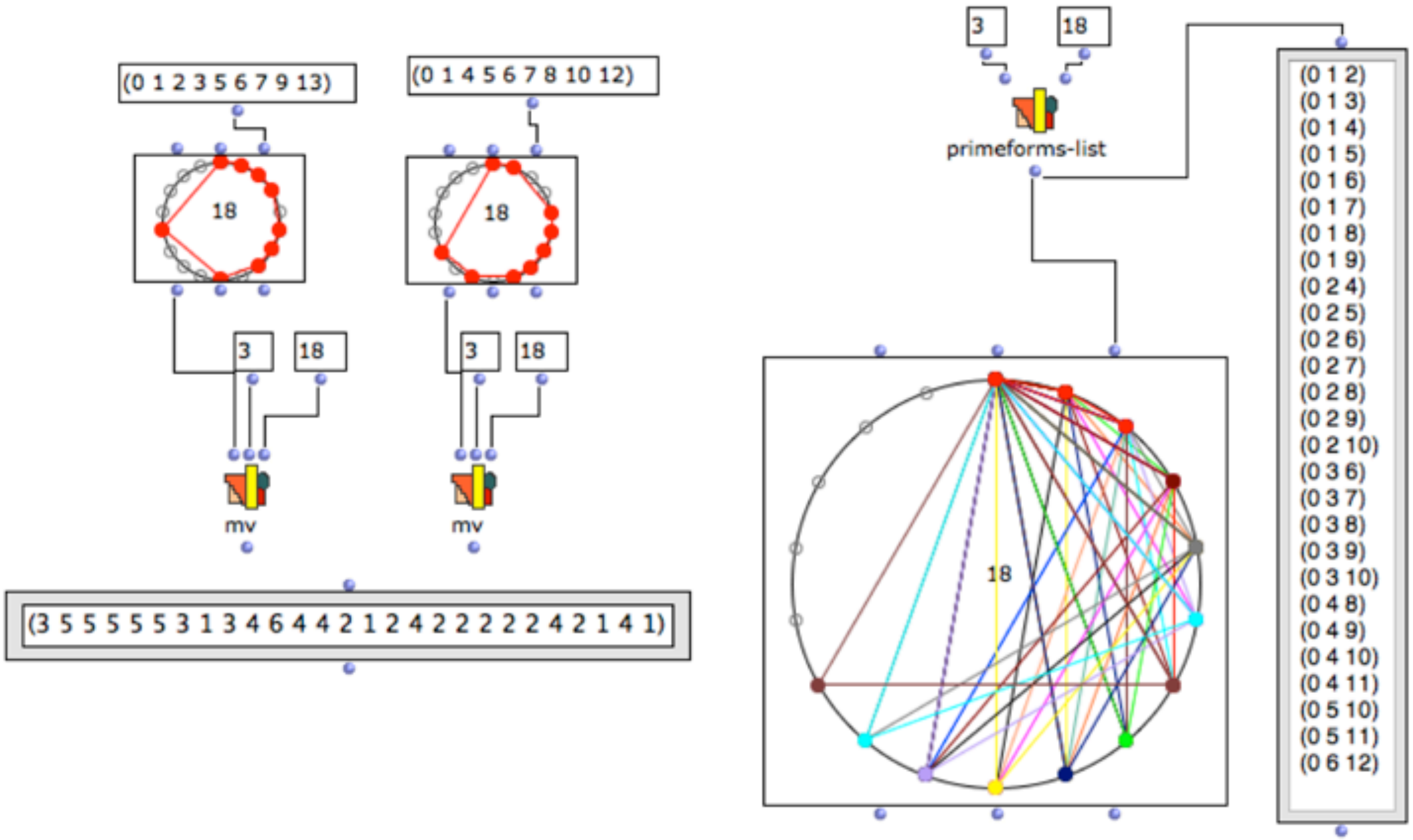


High-order ‘interval’ content: Lewin’s mv^k vector



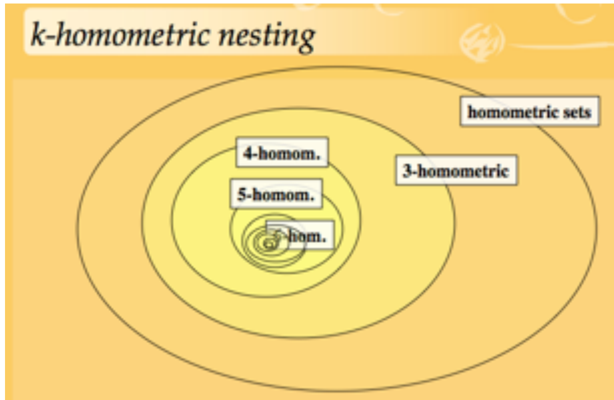
• Daniele Ghisi, "From Z-relation to homometry: an introduction and some musical examples", Séminaire MaMuX, 10 décembre 2010 [http://repmus.ircam.fr/mamux/saisons/saison10-2010-2011/2010-12-10]

High-order Z-relation and *k*-homometric nesting

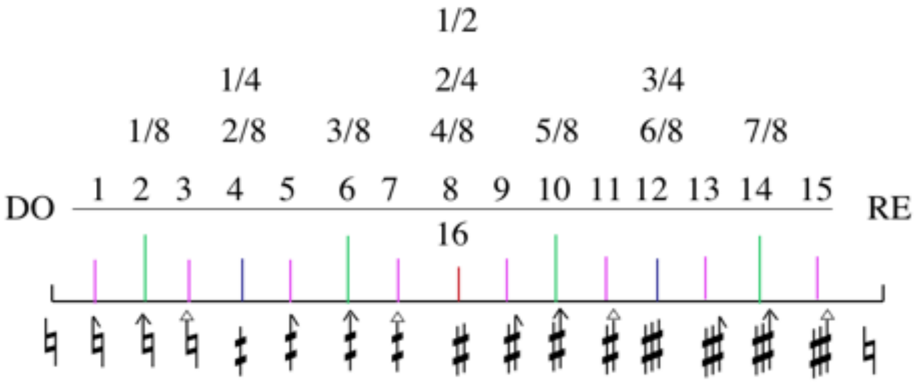


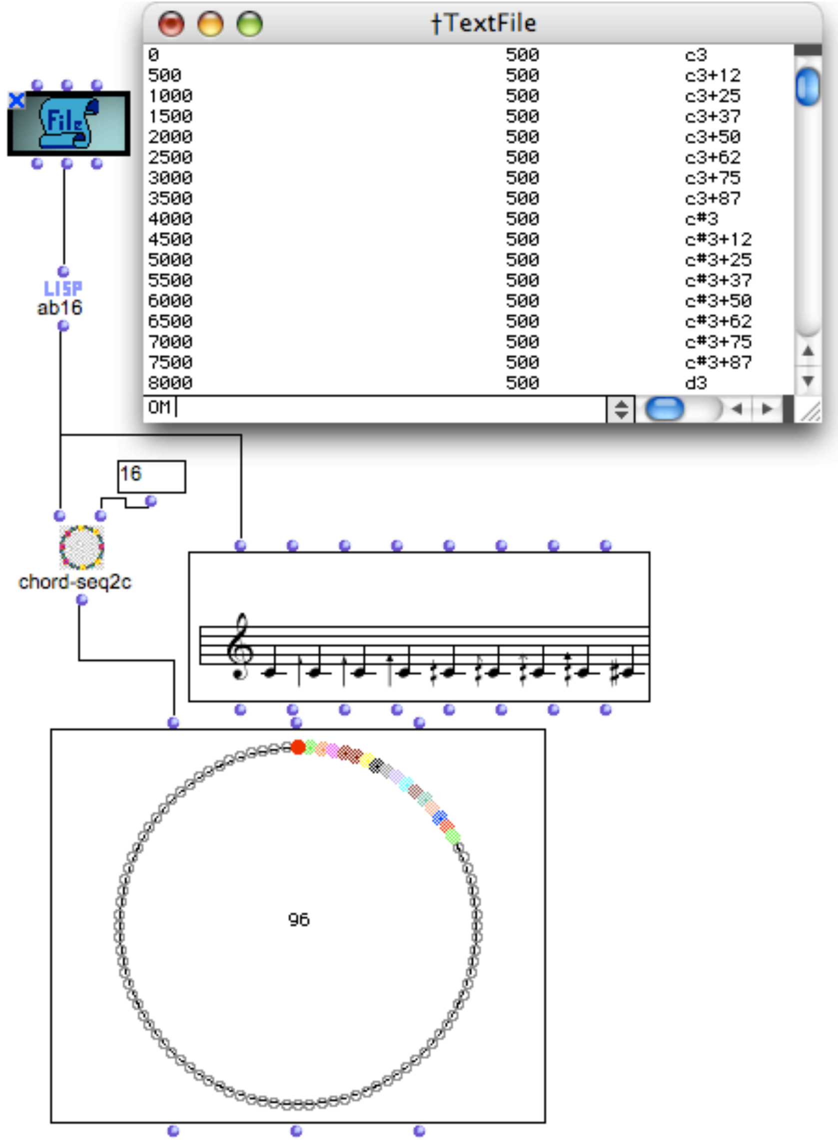
Z_{18} A = {0, 1, 2, 3, 5, 6, 7, 9, 13}
B = {0, 1, 4, 5, 6, 7, 8, 10, 12}

$mv^3(A) = mv^3(B)$ ➔ A and B are Z-related



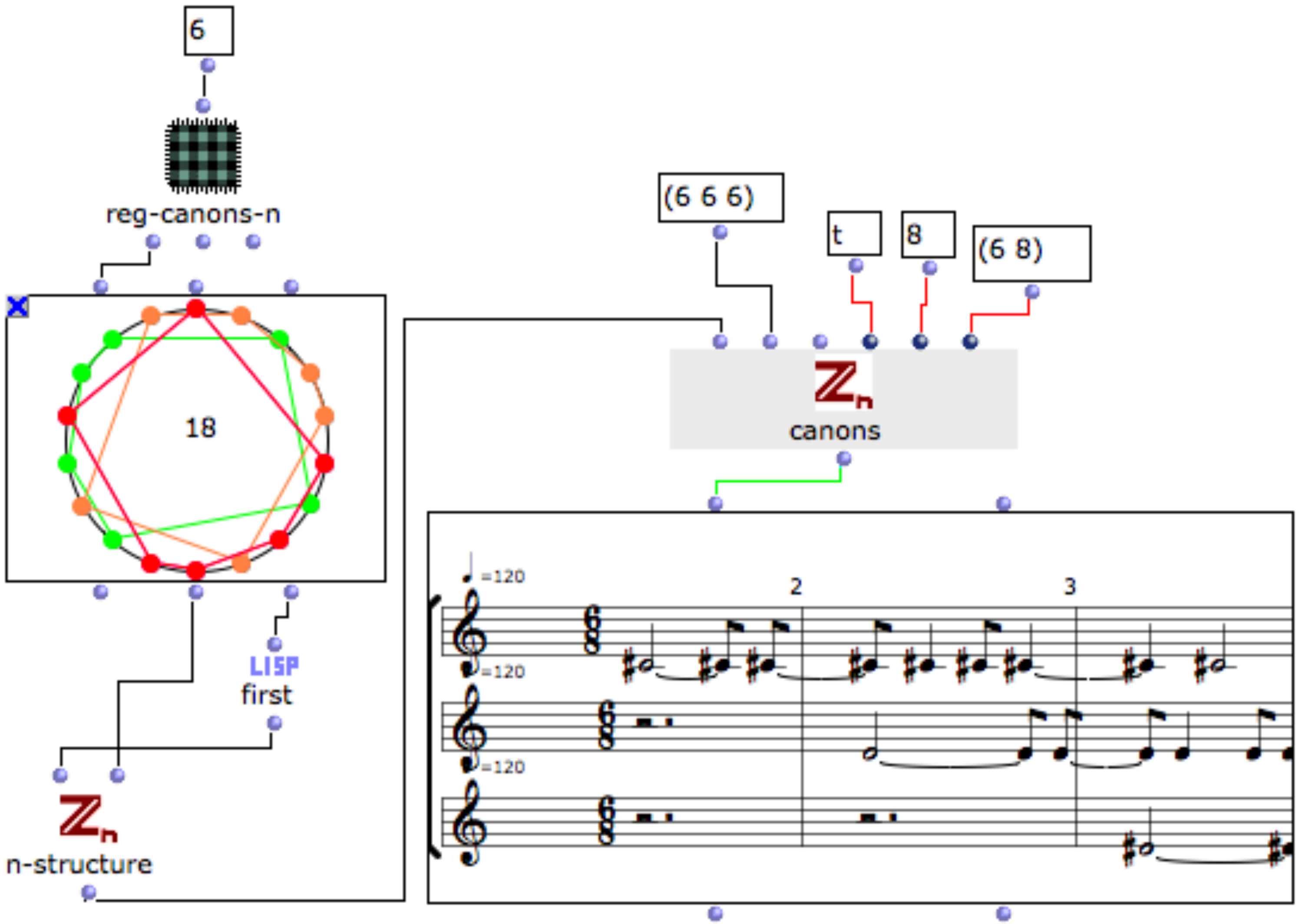
Microtonal composition (Alain Bancquart)





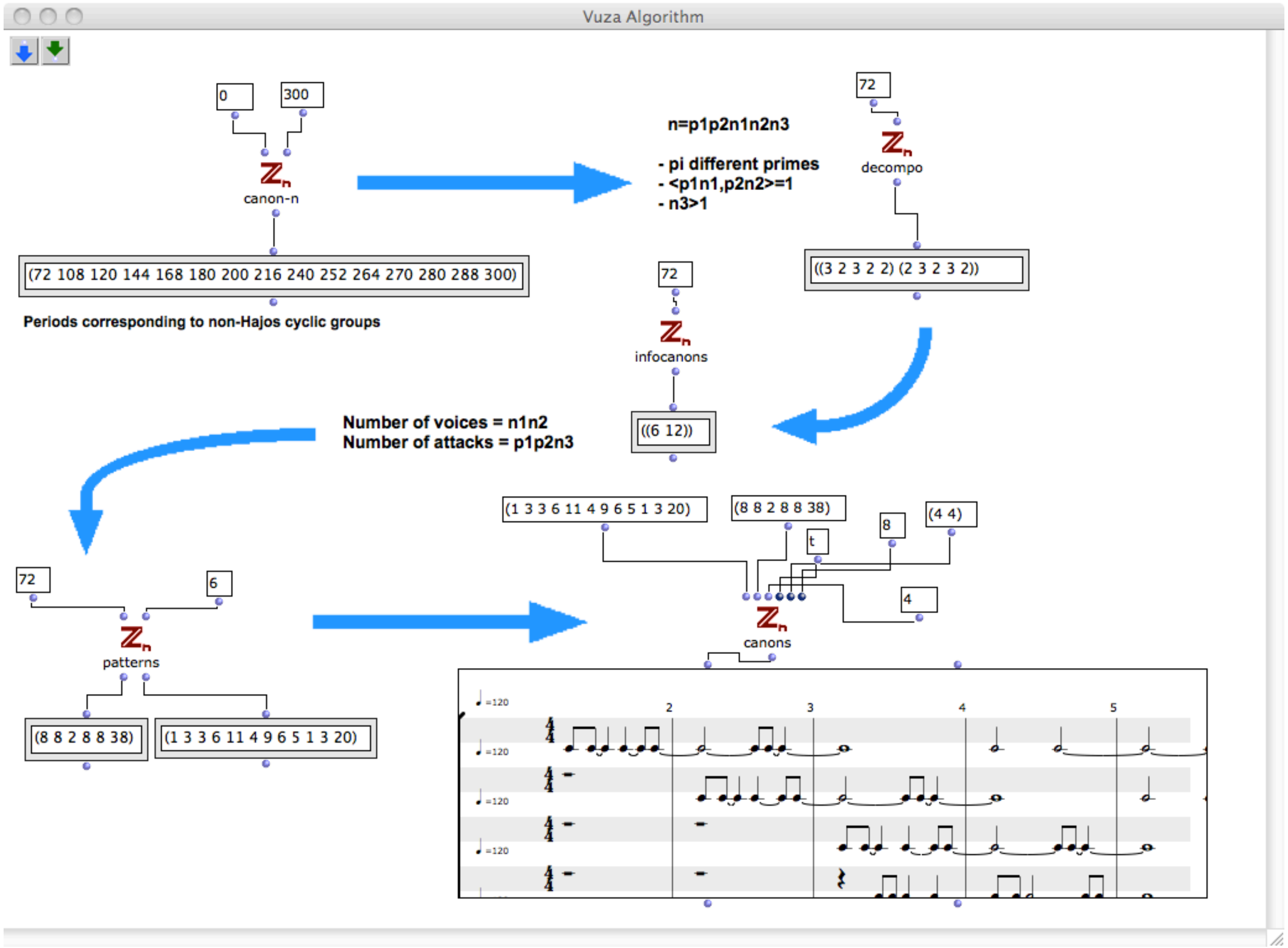
A. Bancquart, M. Andreatta, et C. Agon, « Microtonal Composition », The OM Composer's Book 2, éd. Jean Bresson, Carlos Agon, Gérard Assayag (Ircam/Delatour France, Sampzon), 2008, p. 279-302.

Construction explicite d'un rythme *k*-asymétrique



16-k-asymmetry-canons

Algorithme de Vuza et implémentation en OpenMusic



➔ 

17-vuza algorithm

Classification « paradigmatique » des canons mosaïques de Vuza

Résultat : uniquement deux « types » de canons différents (à une *transformation affine* près, i.e.
$f: Z_{72} \rightarrow Z_{72}$ t.q.
$f(x)=ax+b$ avec $a \in (Z_{72})^*$ et $b \in Z_{72}$

- **R. Tijdeman: "Decomposition of the Integers as a direct sum of two subsets", *Number Theory*, Cambridge University Press, 1995.**
The fundamental Lemma: *R* pave Z_n => *aR* pave Z_n
$\langle a,n \rangle=1$

{ Zn }

R
(1 3 3 6 11 4 9 6 5 1 3 20)
(20 3 1 5 6 9 4 11 6 3 3 1)
(1 4 1 19 4 1 6 6 7 4 13 6)
(6 13 4 7 6 6 1 4 19 1 4 1)
(1 5 15 4 5 6 6 3 4 17 3 3)
(3 3 17 4 3 6 6 5 4 15 5 1)

S
(8 8 2 8 8 38)
(16 2 14 2 16 22)
(14 8 10 8 14 18)

{ Dn }

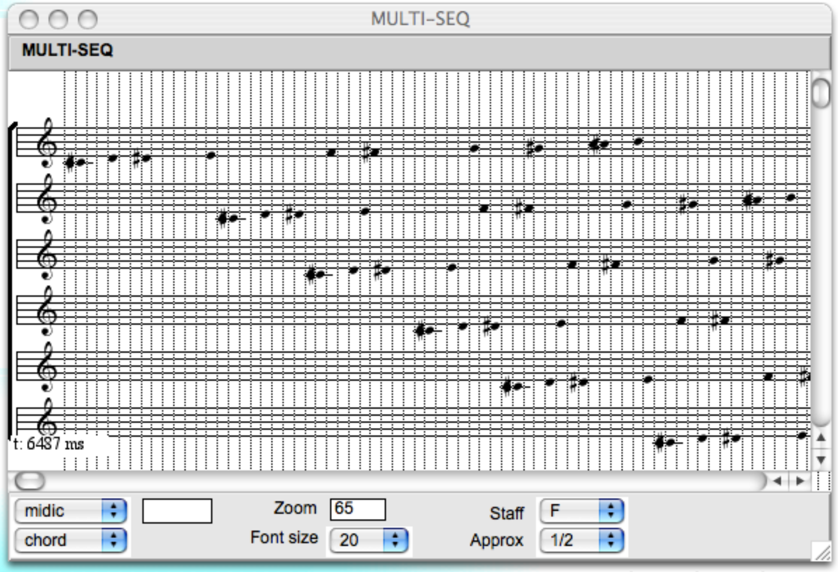
R
(1 3 3 6 11 4 9 6 5 1 3 20)
(1 4 1 19 4 1 6 6 7 4 13 6)
(1 5 15 4 5 6 6 3 4 17 3 3)

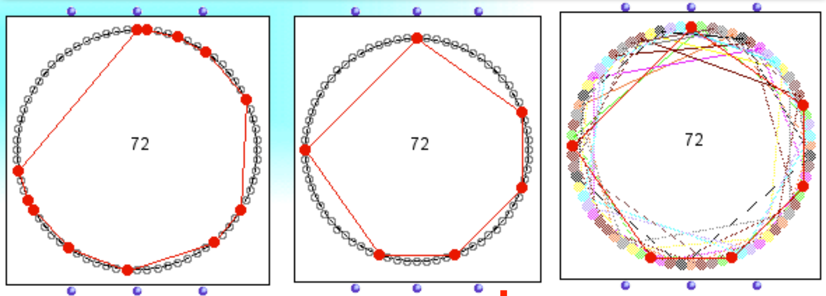
S
(8 8 2 8 8 38)
(16 2 14 2 16 22)
(14 8 10 8 14 18)

{ Afn }

R
(1 3 3 6 11 4 9 6 5 1 3 20)
(1 4 1 19 4 1 6 6 7 4 13 6)

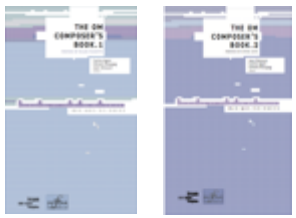
S
(14 8 10 8 14 18)





Z/72Z =R⊕S

Collection « Musique/ Sciences » (dir. J.-M. Bardez & M. Andreatta)



F. Lévy G. Bloch M. Lanza T. Johnson

1999

18-Catalogue-Z72

Mauro Lanza

Canons de Vuza et périodicités locales

• *La descrizione del diluvio* (Ricordi, 2007-2008)

[…] Le choix des notes et des durées est fait en cherchant à souligner certaines quasi-périodicités du canon de Vuza, et cela donne à chaque voix un caractère beaucoup plus "redondant".

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)

(*1 3 25 27 1 3* 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 *1 3 25 27 1 3* 52)

(*1 3 25 27* 1 3 11 14 27 *1 3 25 27* 4 25 27 1 3 25 14 13 *1 3 25 27* 1 3 52)

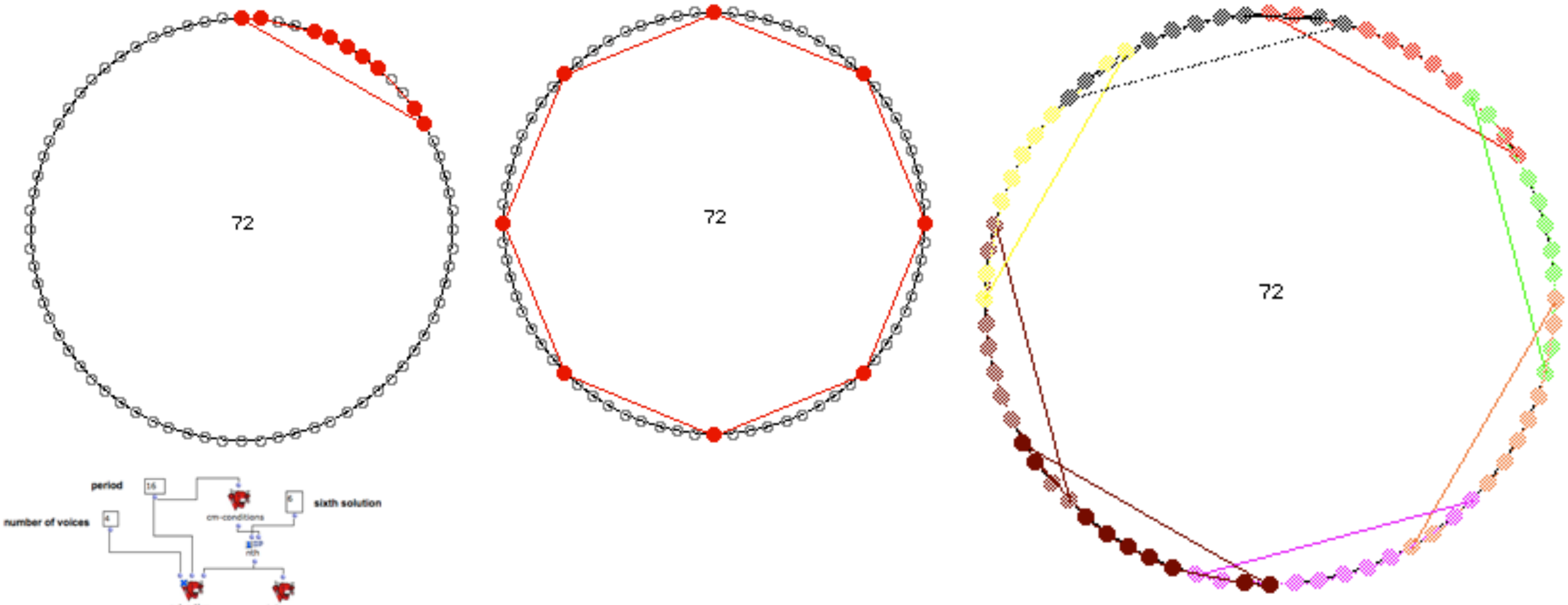
(*1 3 25* 27 1 3 11 14 27 *1 3 25* 27 4 25 27 *1 3 25* 14 13 *1 3 25* 27 1 3 52)

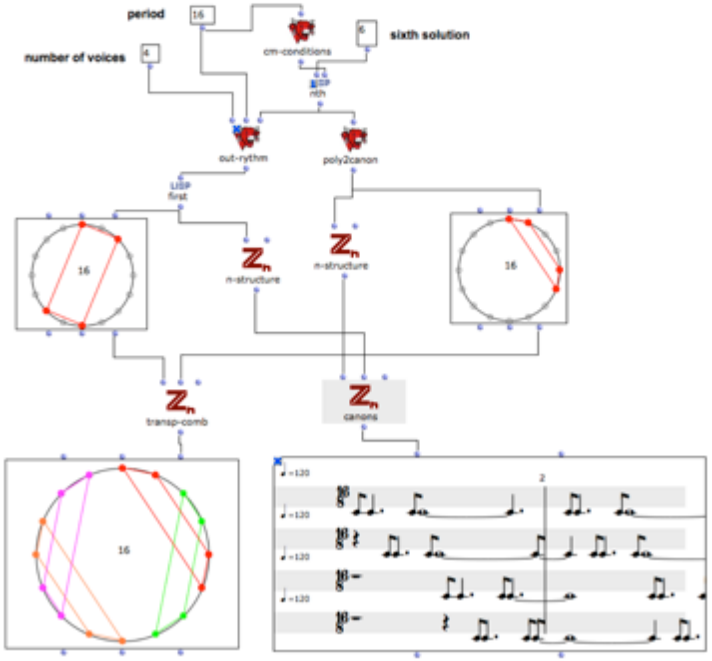
(*1 3* 25 27 *1 3* 11 14 27 *1 3* 25 27 4 25 27 *1 3* 25 14 13 *1 3* 25 27 *1 3* 52)

➔ 

19-mauro-diluvio

La famille des « canons cyclotomiques »



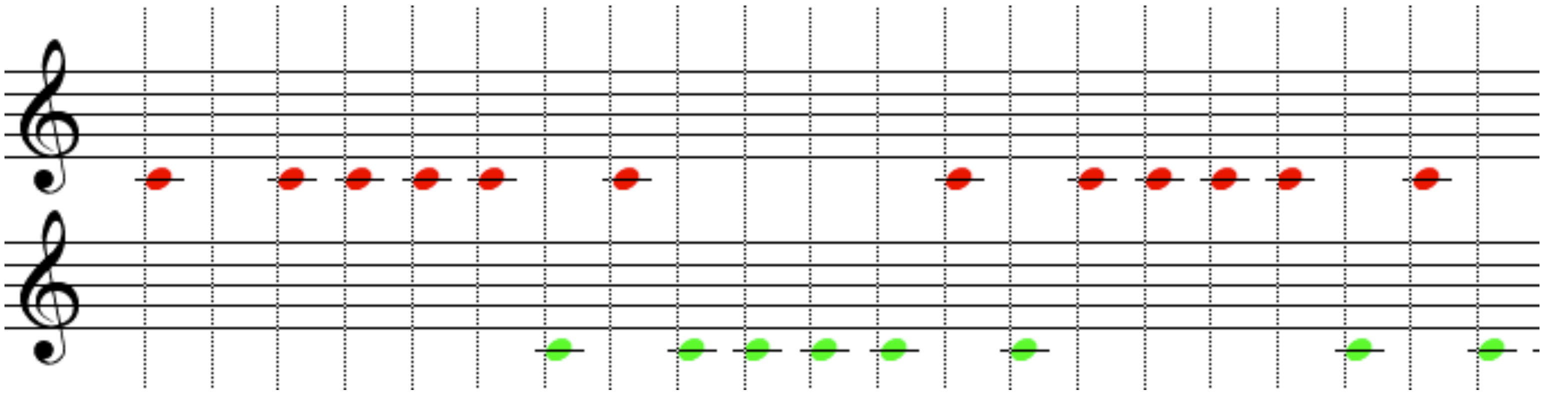


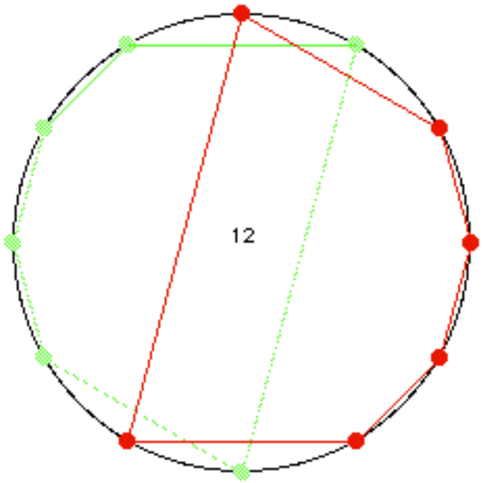
• E. Amiot, M. Andreatta, C. Agon, « Tiling the (musical) line with polynomial: some theoretical and implementational aspects », *ICMC*, Barcelona, 2005, 227-230
• C. Agon, M. Andreatta, «Modelling and Implementing Tiling Rhythmic Canons in OpenMusic Visual Programming Language », Perspectives of New Music, Special Issue, vol. 1-2, n° 49.

➔ 

20-Canons-Cyclo

Canons mosaïques par translation et augmentation





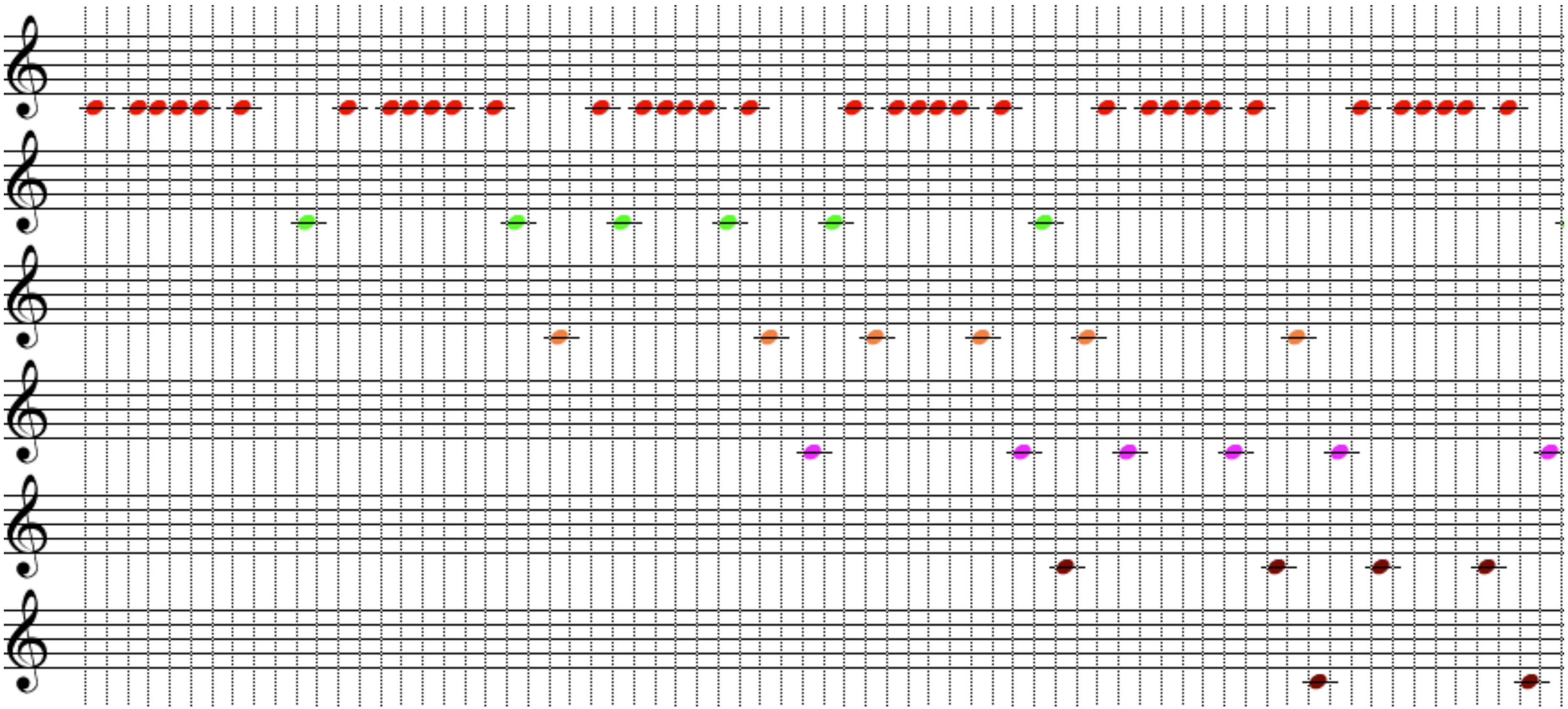

```
(((0 1 2 3 4 6) ((1 11)))
((0 1 2 3 4 5) ((1 11) (1 1)))
((0 1 2 3 5 7) ((1 11) (1 7)))
((0 1 3 4 7 8) ((1 5)))
((0 1 2 3 6 7) ((1 11)))
((0 1 3 4 6 9) ((1 11) (1 5)))
((0 1 3 6 7 9) ((1 11) (1 5)))
((0 1 2 6 7 8) ((1 11) (1 7) (1 5) (1 1)))
((0 1 4 5 8 9) ((1 11) (1 7) (1 5) (1 1)))
((0 1 2 5 6 7) ((1 7) (1 5)))
((0 2 3 4 5 7) ((1 11) (1 7) (1 5) (1 1)))
((0 1 4 5 6 8) ((1 11) (1 7)))
((0 1 2 4 5 7) ((1 5)))
((0 1 3 4 5 8) ((1 5) (1 1)))
((0 1 2 4 5 8) ((1 11)))
((0 1 2 4 6 8) ((1 11) (1 7)))
((0 2 3 4 6 8) ((1 11)))
((0 2 4 6 8 10) ((1 11) (1 7) (1 5) (1 1))))
```

➔ 

21-Augmented-Canons

Augmented Tiling Canons ou l'action du groupe affine

(en collaboration avec Thomas Noll)



➔ 

22-Augmented-Canons

Periodic sequences and finite difference calculus

$$Df(x)=f(x)-f(x-1).$$

f = **7 11 10 11 7 2** 7 11 10 11 7 2 7 11…

Df = **4 11 1 8 7 5** 4 11 1 8 7 5 4 11…

$D^2 f$ = **7 2 7 11 10** 11 7 2 7 11 10 11…

$D^3 f$ = **7 5 4 11 1 8** 7 5 4 11 1 8…

$D^k f$ = ……

V	0	3	8	7	11	0	11	10	6	9	0	9	1	2	9	8	4	3	6
VIII	0	0	0	0	3	3	7	2	0	0	0	6	3	3	3	4	8	0	0
IV	3	3	4	4	1	11	11	8	3	3	9	4	1	7	11	8	11	3	9
IX	0	0	0	0	0	3	6	[1]	3	3	3	3	9	0	3	6	[10]	6	6
IV	0	10	3	9	10	0	9	7	0	6	7	9	6	4	9	3	4	6	3

Anatol Vieru: *Zone d'oubli* pour alto (1973)

Periodic sequences and finite difference calculus

f = 11 6 7 2 3 10 11 6 …

Df = 7 1 7 1 7 1 7 1…

D^2f = 6 6 6 6 6…

D^4f = 0 0 0

Reducible sequences:
∃ $k \geq 1$ such that $D^k f = 0$

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f = 7 11 10 11 7 2 7 11 …

Df = 4 11 1 8 7 5 4 11 1 …

D^2f = 7 2 7 11 10 11 7 2 7 …

D^3f = 7 5 4 11 1 8 7 5 4 11 1 8…

D^4f = 10 11 7 2 7 11 10 11 …

D^5f = 1 8 7 5 4 11 1 8 …

D^6f = 7 11 10 11 7 2 7 11 …

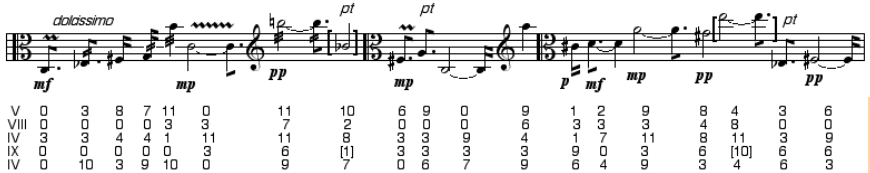
Reproducible sequences:
∃ $k \geq 1$ such that
$D^k f = f$

Periodic sequences and finite difference calculus

23-Decompo

$Df(x)=f(x)-f(x-1)$.

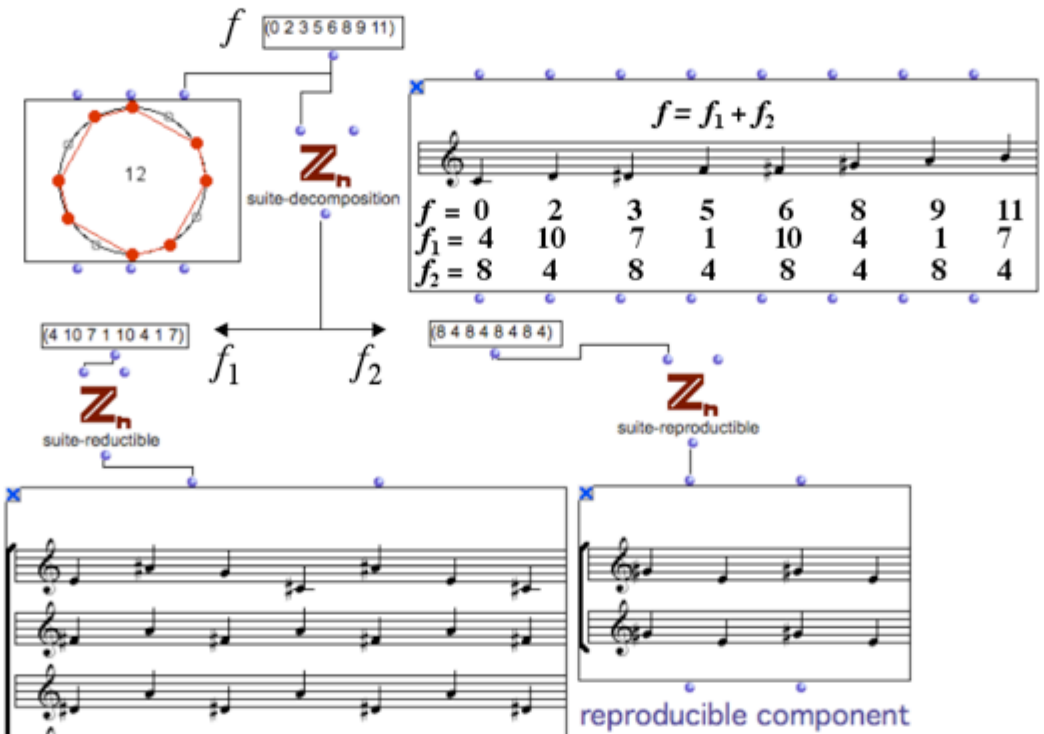
7 11 10 11 7 2 7 11 10 11 7 2 7 11…
4 11 1 8 7 5 4 11 1 8 7 5 4 11…
7 2 7 11 10 11 7 2 7 11 10 11…
7 5 4 11 1 8 7 5 4 11 1 8…
……



Anatol Vieru: *Zone d'oubli* for viola (1973)

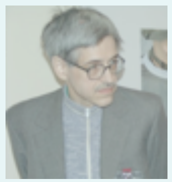
Reducible sequences:
∃ $k \geq 1$ such that $D^k f = 0$

Reproducible sequences:
∃ $k \geq 1$ such that $D^k f = f$



- **Decomposition theorem**

(Vuza & Andreatta, *Tatra M.*, 2001)
Every periodic sequence f can be decomposed in a unique way as a sum $f_1 + f_2$ of a reducible sequence f_1 and a reproducible sequence f_2



D. Vuza, M. Andreatta (2001), « On some properties of periodic sequences in Anatol Vieru's modal theory », Tatra Mountains Mathematical Publications, Vol. 23, p. 1-15