
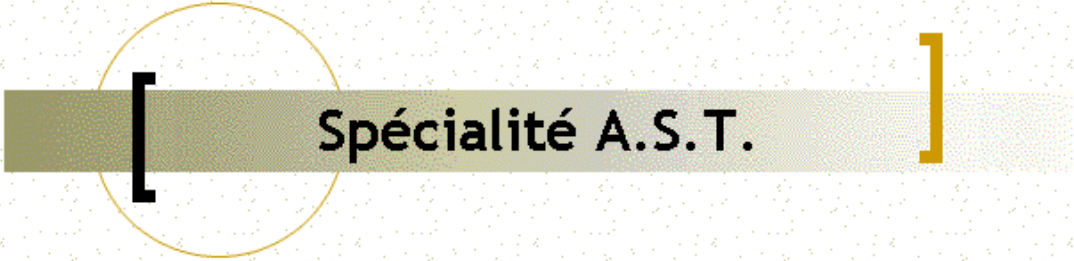


Master I.C.A.

 <u>Master I.C.A.</u>		A rt S ciences T echnologies
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Traitement interactif de l'image et du son

Workshop OpenMusic

– Jean Bresson // Moreno Andreatta –
Equipe Représentations Musicales
IRCAM/CNRS/UPMC UMR 9912

“MathTools”: an algebraic environment within OpenMusic visual programming language



OM= $\text{♩} + \Sigma$

Computational Music Theory:

- Classification and Enumeration of musical structures
 - Chords/scales, motifs and rhythms:
 - ♩ Catalogues (Costère, Zalewski, Vieru, Forte, Carter, Morris, Mazzola, Estrada, ...)
 - Σ Combinatorial algebra, Polya Enumeration Theory, Burnside Lemma, Discrete Fourier Transform
 - Rhythmic Tiling Canons (by translation, inversion and augmentation)
 - ♩ Messiaen, Vieru, Levy, Johnson, Bloch, Wild, Lanza, ...
 - Σ Group factorization theory
 - Σ

Computational Music Analysis:

- *Set Theory*, Transformational Analysis and Sieve Theory
 - Pitch-class sets, interval vectors and IFUNC, Z-relations:
 - ♩ Carter, Vieru, Xenakis
 - Σ Group Actions, Homometry, TFD
 - Transformational progression/network, *K-nets*
 - ♩ Generalized Interval Systems (David Lewin)
 - Σ Group action and category theory



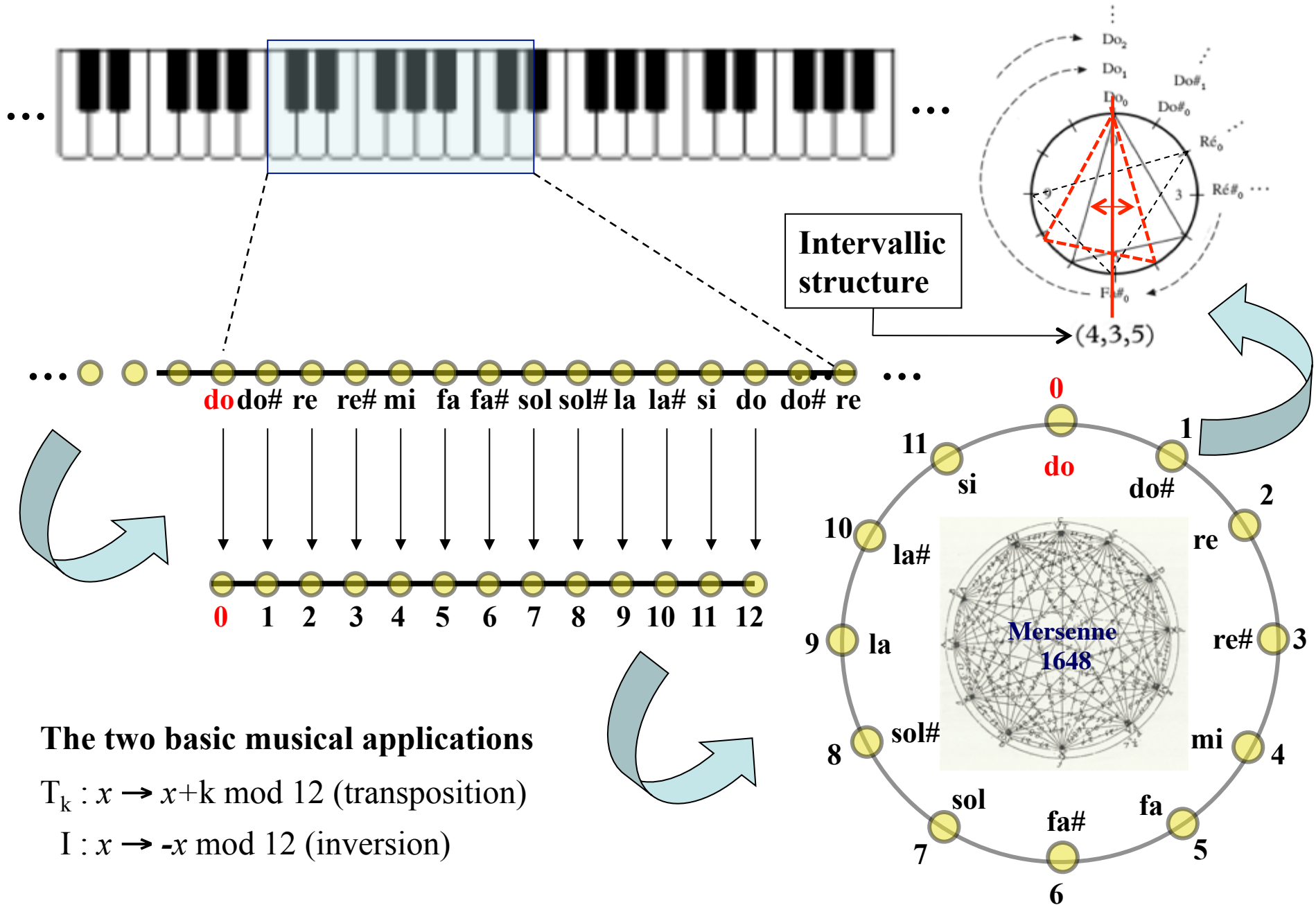
Computer Aided-Composition:

- Cf. *The OM Composer's Book (2 volumes)*.
Edited by C. Agon, G. Assayag and J. Bresson

<http://recherche.ircam.fr/equipes/repmus/OpenMusic/>

→ <http://repmus.ircam.fr/openmusic/home>

Octave equivalence and mod 12 congruence

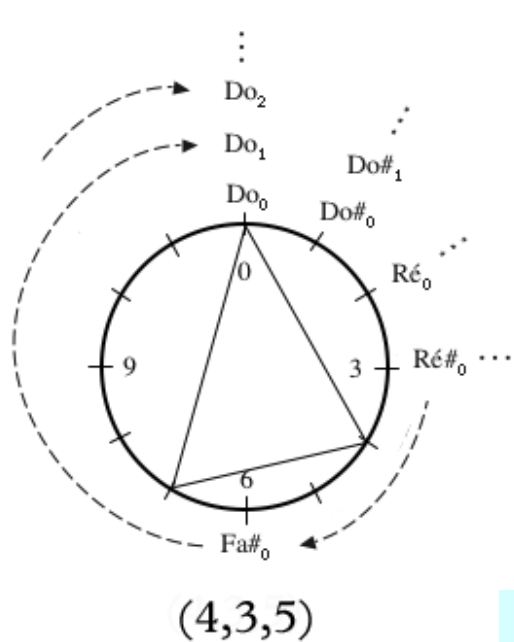


The two basic musical applications

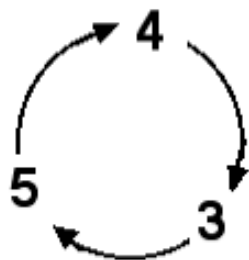
$$T_k : x \rightarrow x+k \pmod{12} \text{ (transposition)}$$

$$I : x \rightarrow -x \pmod{12} \text{ (inversion)}$$

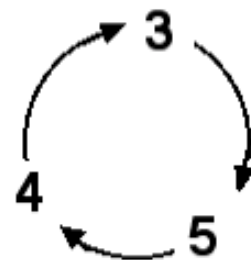
Circular representation and intervallic structure



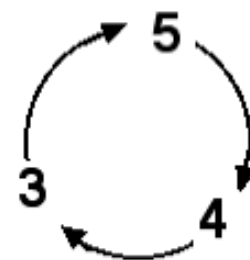
$(4\ 3\ 5)$



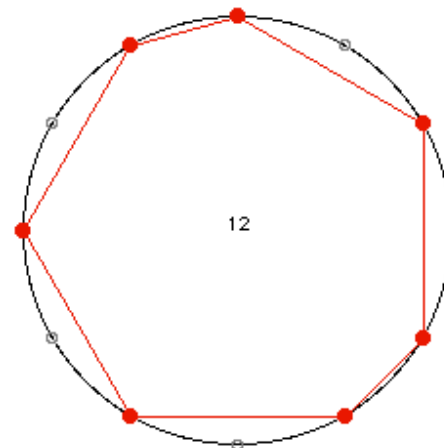
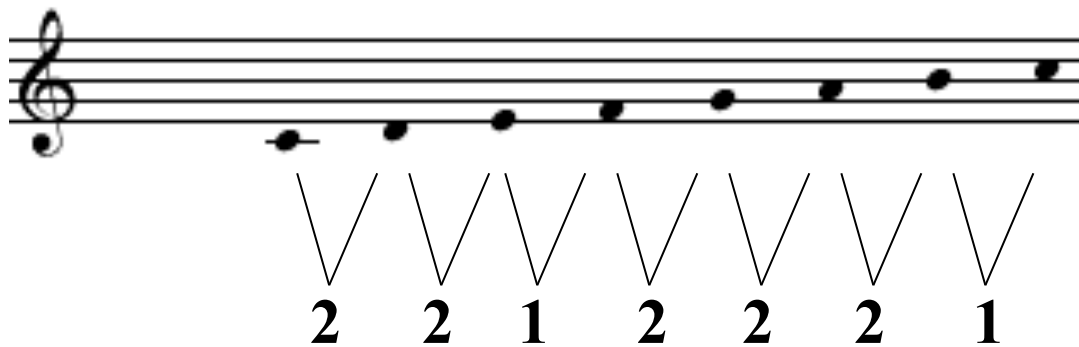
$(3\ 5\ 4)$



$(5\ 4\ 3)$



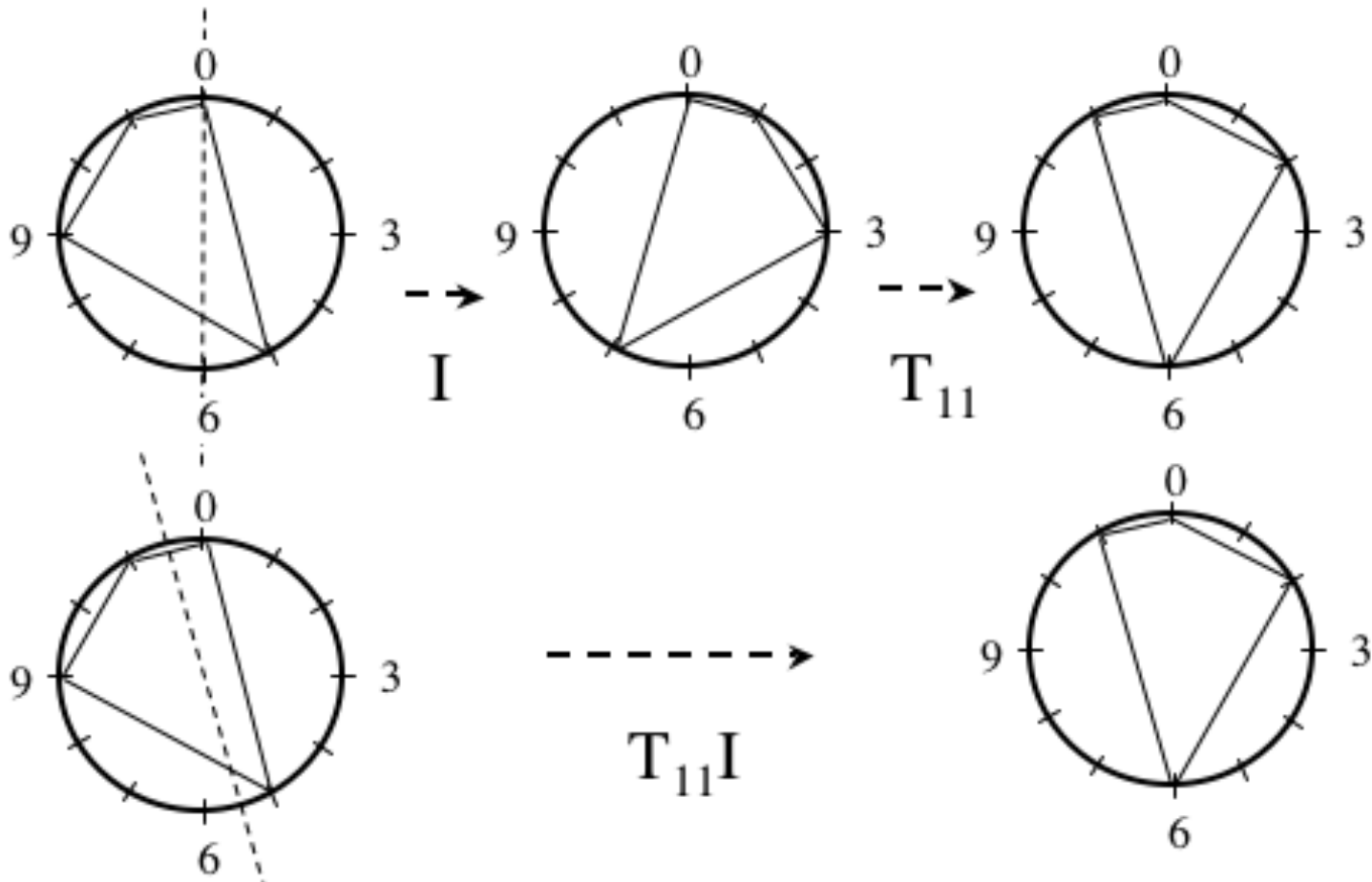
The « inversions » of a chord are all circular permutations on an intervallic structure



Transposition and Inversion

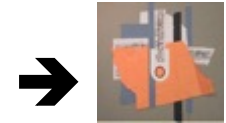
$$I: x \rightarrow 12-x$$

$$T_k: x \rightarrow k+x$$



$$T_{11}I: x \rightarrow 11-x$$

$$\{0, 5, 9, 11\} \longrightarrow \{11, 6, 3, 0\}$$



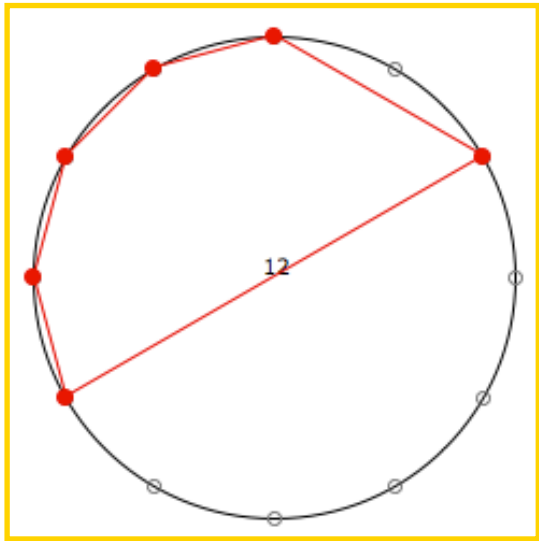
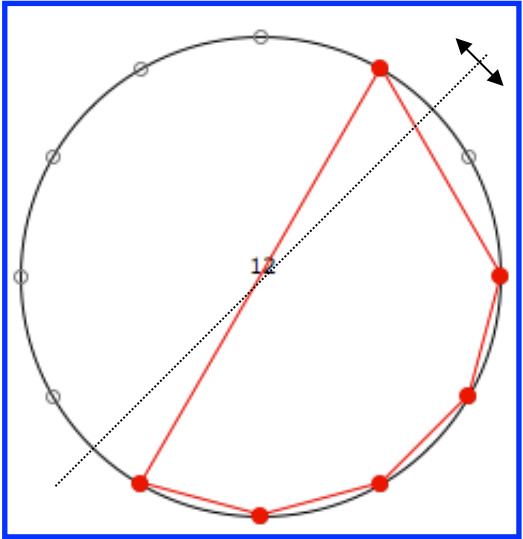
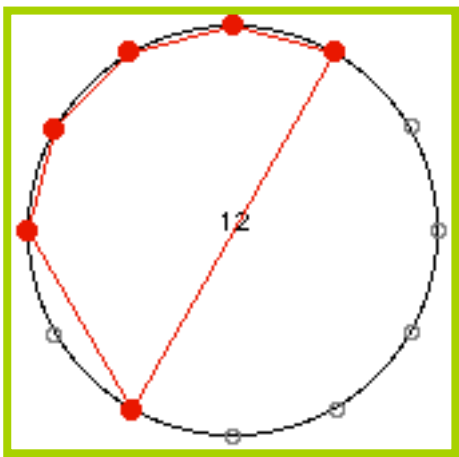
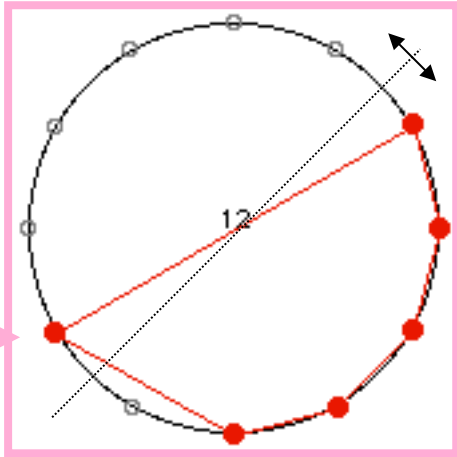
Serialism and hexachordal combinatoriality

Schoenberg: Suite Op.25, Minuetto

5 6 7 8 9 10 11 12 1 2 3 4

BS A

Double combinatoriality



Hexachordal Combinatorality in Messiaen →



- Mode de valeurs et d'intensités (1950)

Modéré

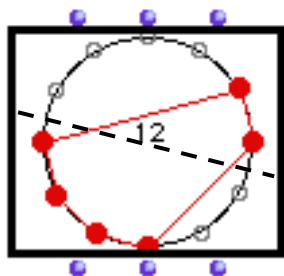
PIANO



Voici le mode:

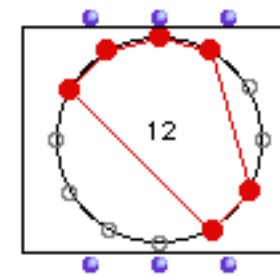
I

(la Division I est utilisée dans la portée supérieure du Piano)



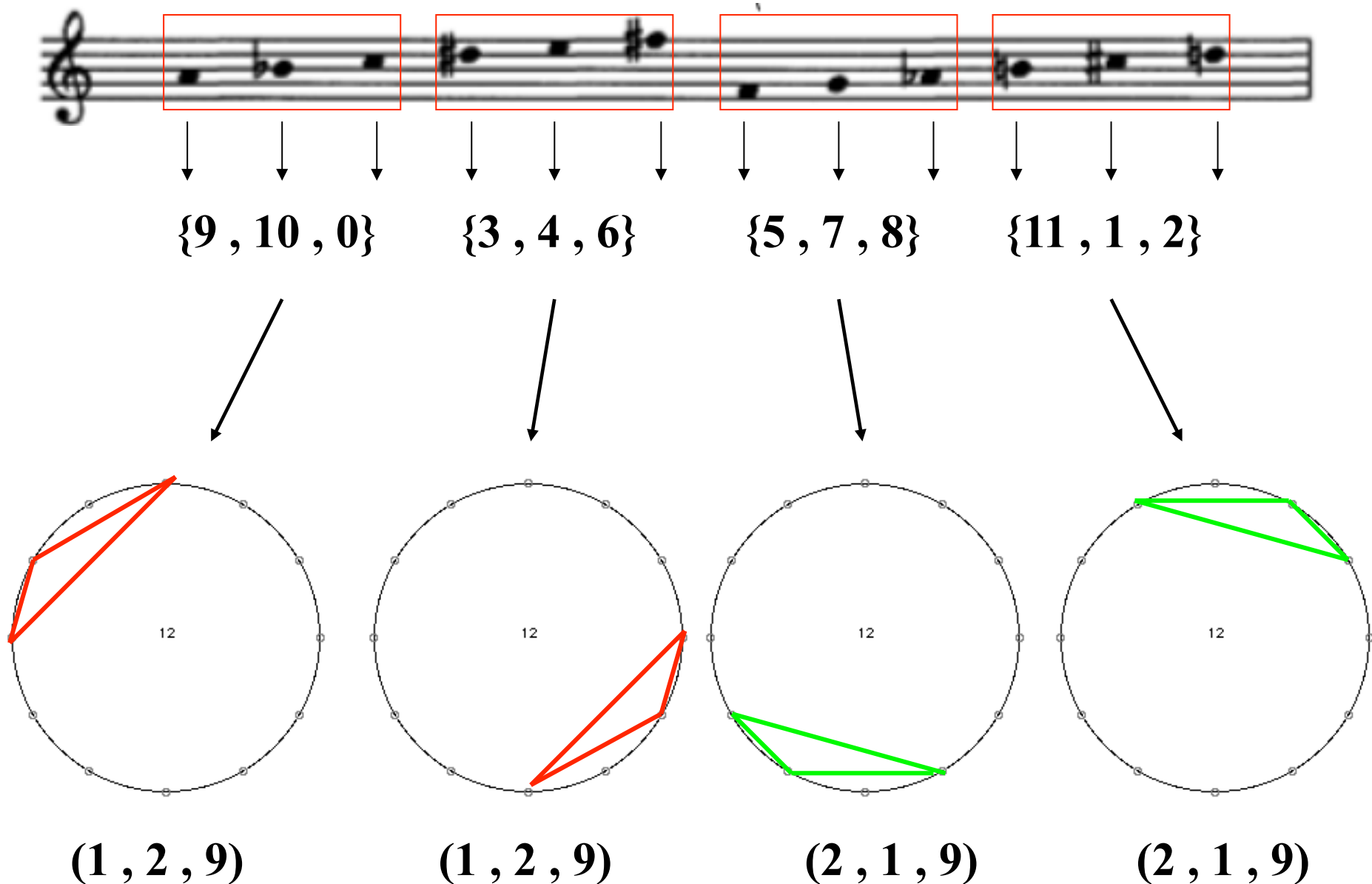
$$\{3,2,9,8,7,6\} \longrightarrow \{4,5,10,11,0,1\}$$

$$T_7I : x \rightarrow 7-x$$



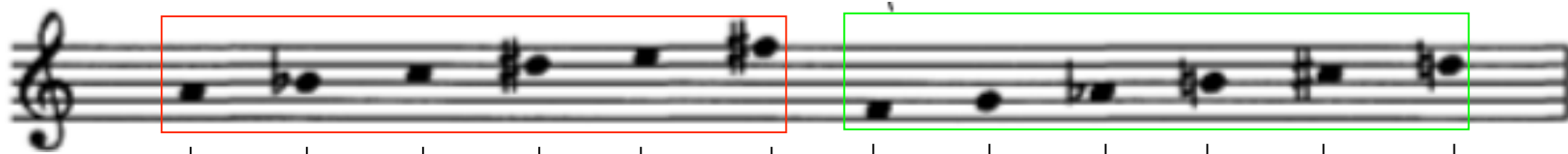
Symmetries in a Twelve-Tone Row : partitioning trichords

Schoenberg: Serenade Op.24, Mouvement 5

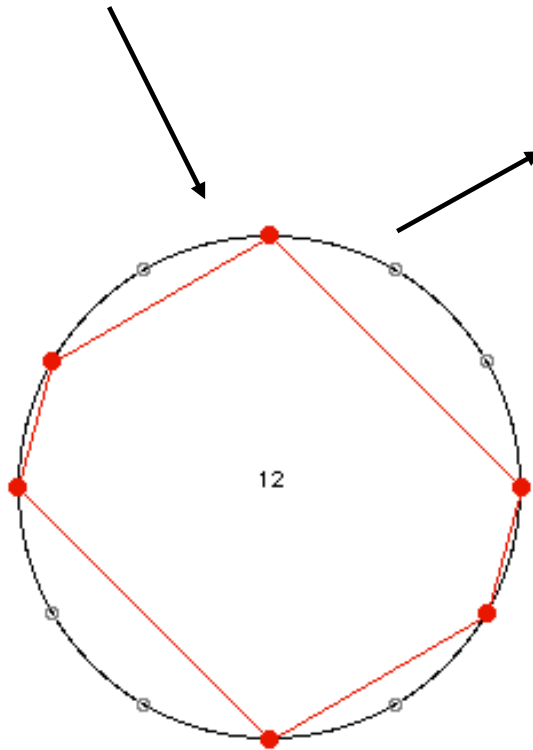


Hexachordal Combinatorality and Transpositional Symmetry

Schoenberg: Serenade Op.24, Mouvement 5



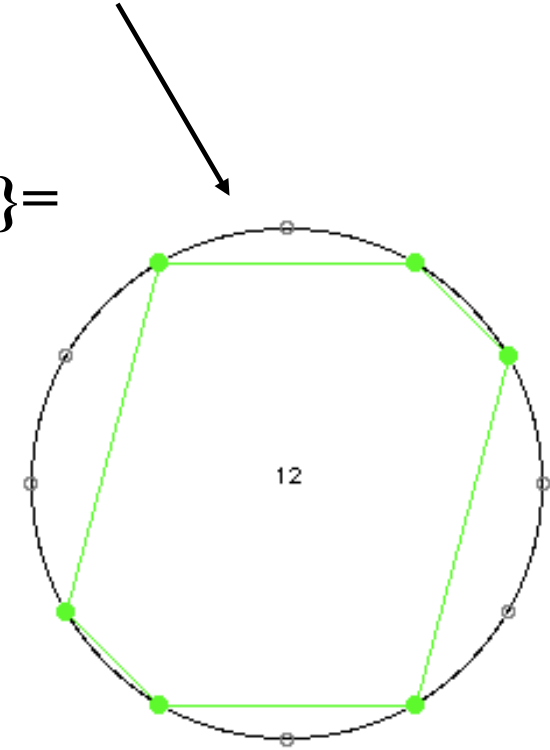
$$A = \{9, 10, 0, 3, 4, 6\} \quad \{5, 7, 8, 11, 1, 2\}$$



(3, 1, 2, 3, 1, 2)

$$\begin{aligned} T_6\{9,10,0,3,4,6\} &= \\ &= \{6+9, 6+10, 6, 6+3, 6+4, 6+6\} = \\ &= \{3, 4, 6, 9, 10, 0\} \end{aligned}$$

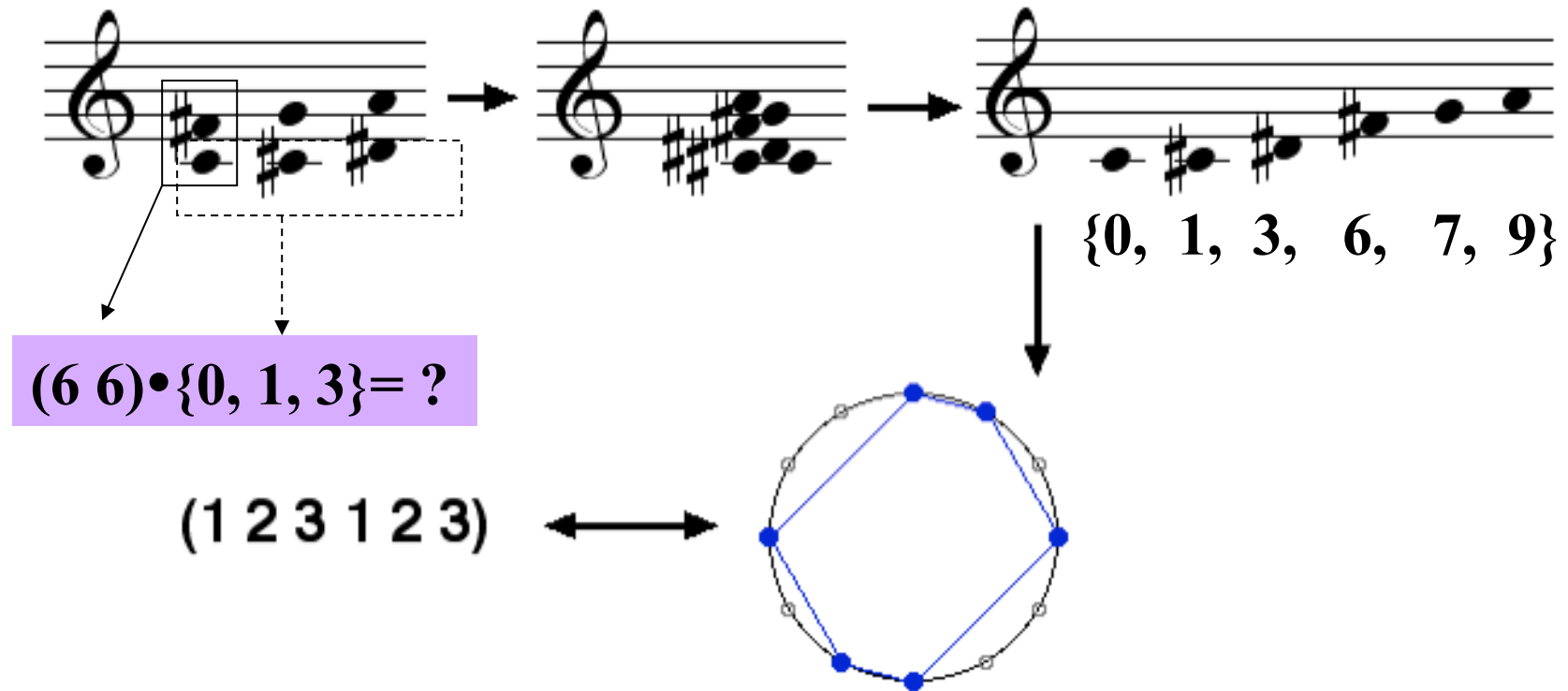
$$T_6(A) = A$$



(2, 1, 3, 2, 1, 3)



Chord multiplication (Boulez) & TC (Cohn)



$$(6\ 6) \bullet \{0, 1, 3\} =$$

$$= ((6\ 6) \bullet \{0\}) \cup ((6\ 6) \bullet \{1\}) \cup ((6\ 6) \bullet \{3\}) =$$

$$= \{0, 6\} \cup \{1, 7\} \cup \{3, 9\} =$$

$$= \{0, 1, 3, 6, 7, 9\}.$$



Equivalence classes of chords

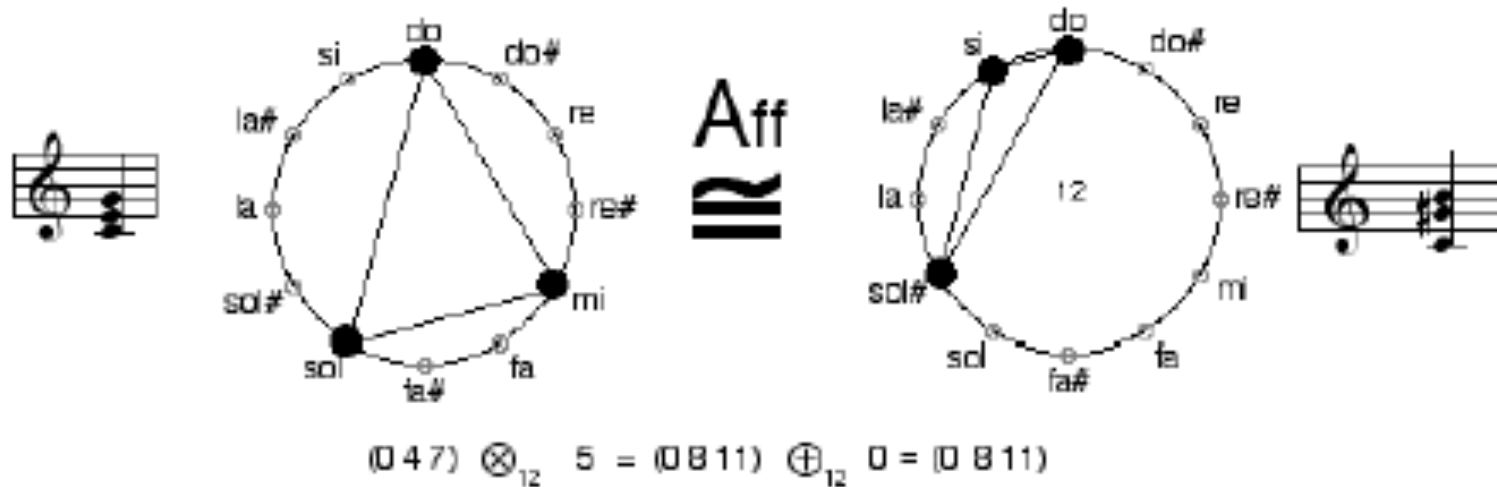


Transposition

$$T_3\{0, 4, 7\} = 3 + \{0, 4, 7\} = \{3, 7, 10\}$$

Transposition and/or inversion

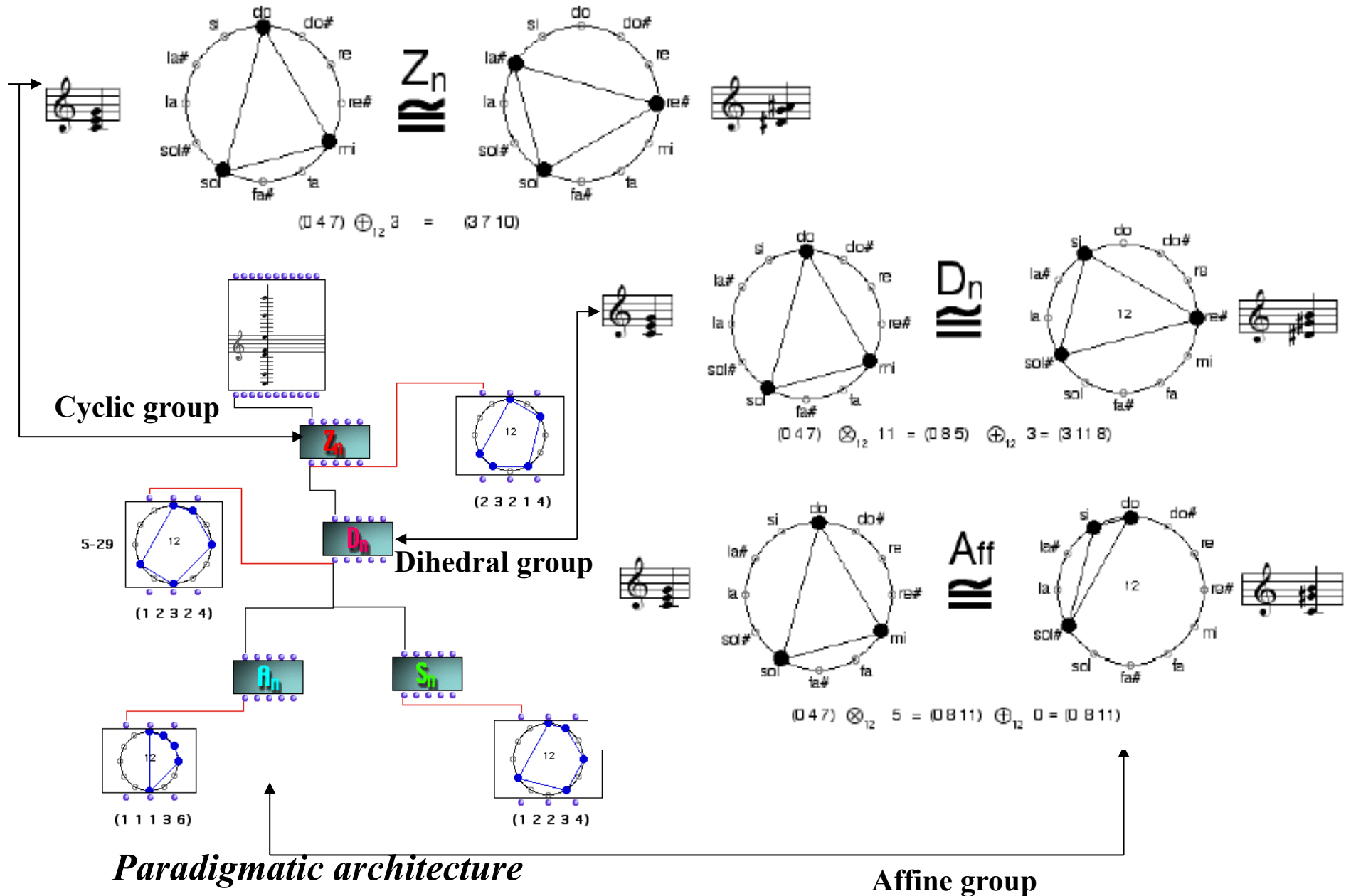
$$T_3I\{0, 4, 7\} = 3 + \{0, -4, -7\} = \{3, 11, 8\}$$



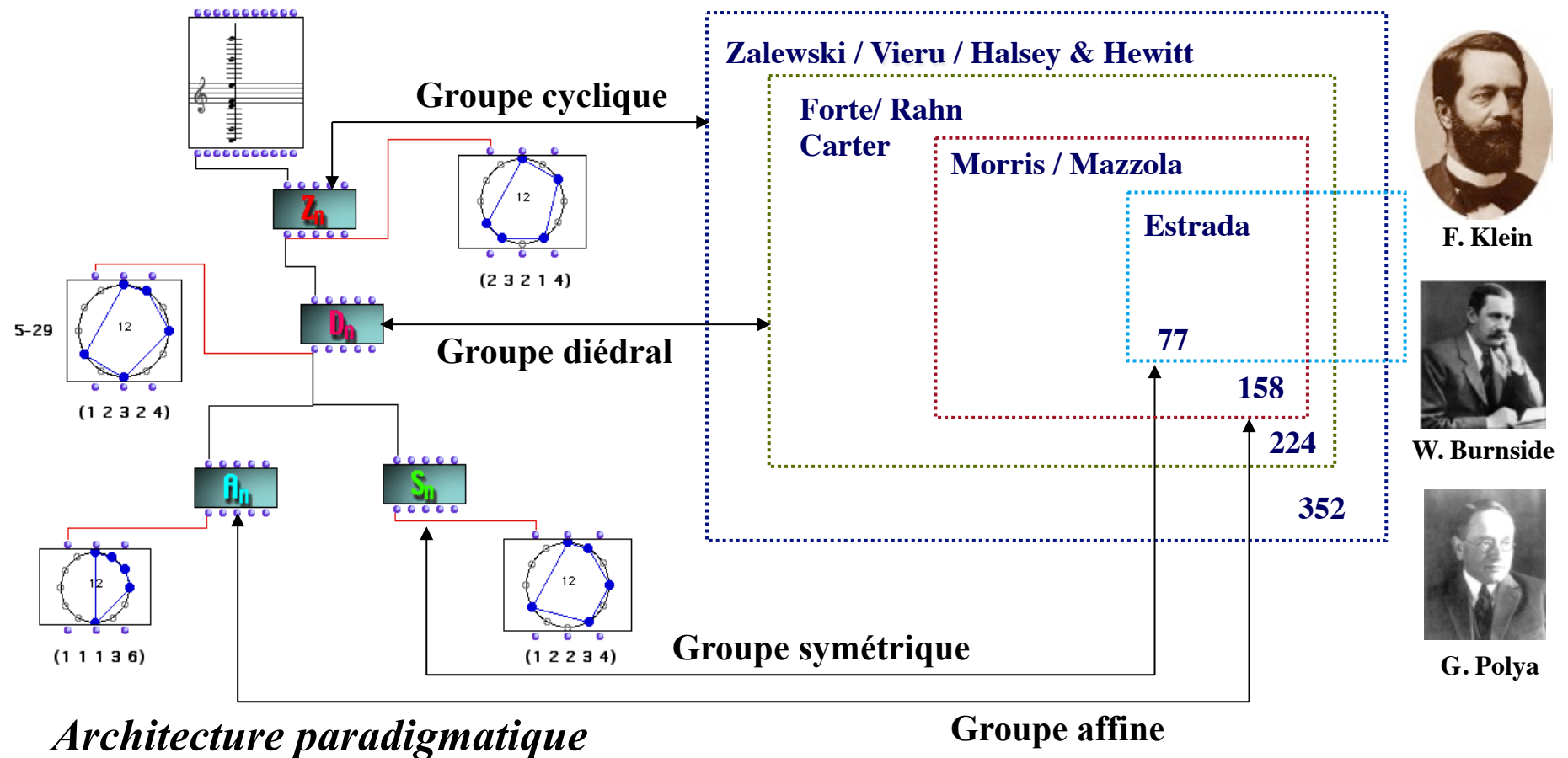
Multiplication (or affine transformation)

$$M_5\{0, 4, 7\} = 5 \times \{0, 4, 7\} = \{0, 8, 11\}$$

Equivalence relation between musical structures



Classification paradigmatique des structures musicales



« [C'est la notion de groupe qui] donne un sens précis à l'idée de structure d'un ensemble [et] permet de déterminer les éléments efficaces des transformations en réduisant en quelque sorte à son schéma opératoire le domaine envisagé. [...] L'objet véritable de la science est le système des relations et non pas les termes supposés qu'il relie. [...] Intégrer les résultats - symbolisés - d'une expérience nouvelle revient [...] à créer un canevas nouveau, un groupe de transformations plus complexe et plus compréhensif »

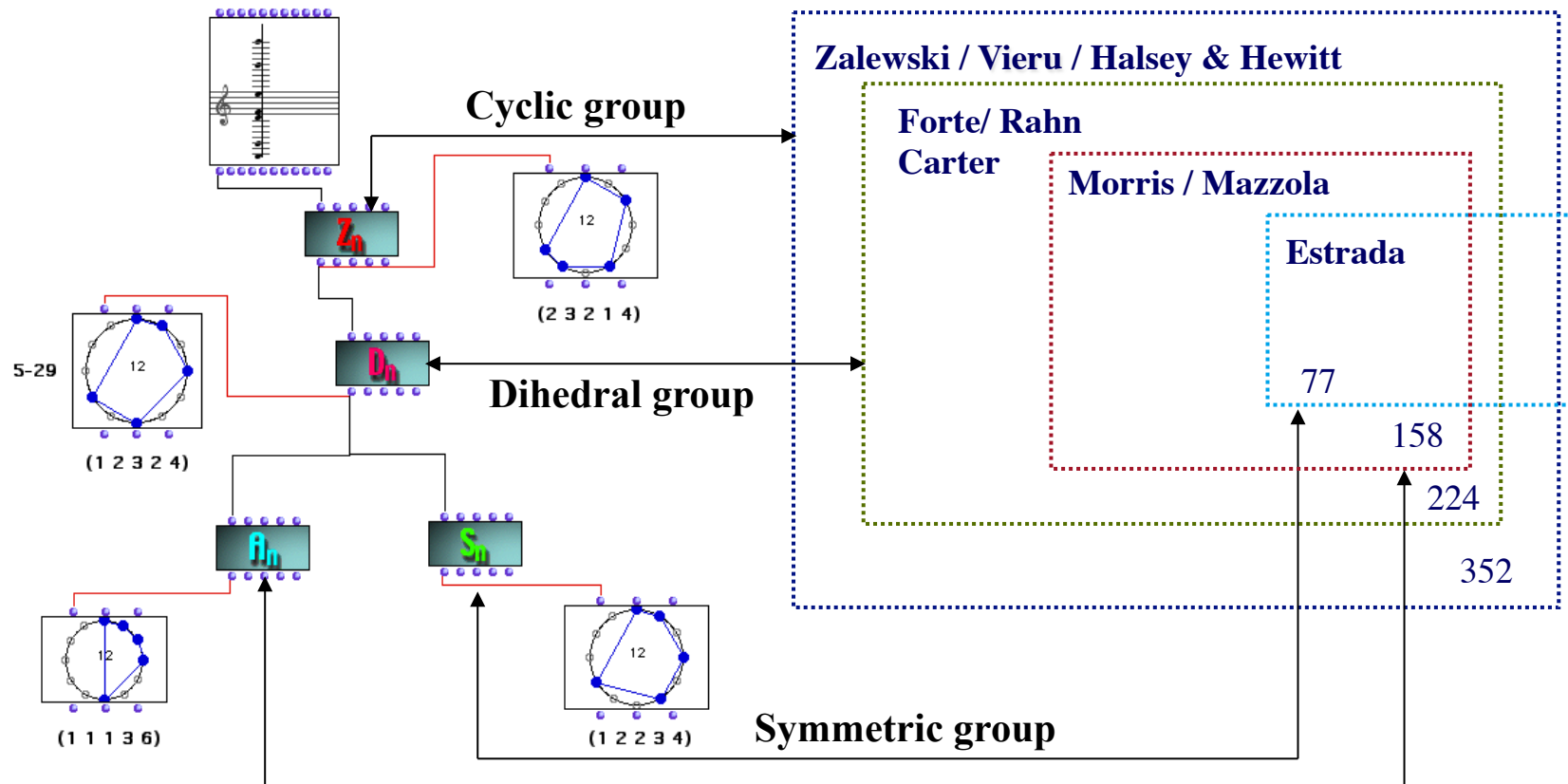


G.-G. Granger

G.-G. Granger : « Pygmalion. Réflexions sur la pensée formelle », 1947

Enumeration of musical structures

$G \setminus k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\text{Aff}_1(\mathbb{Z}_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

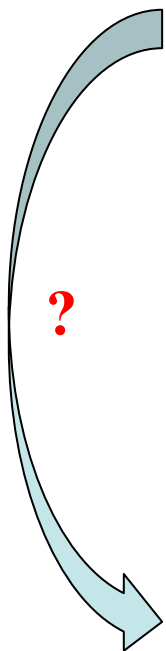


Paradigmatic architecture

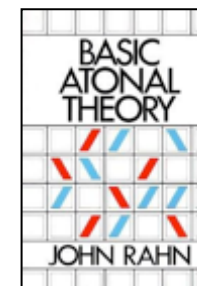
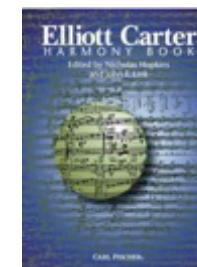
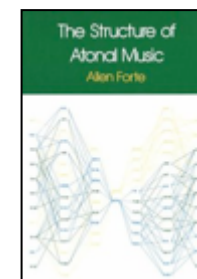
Affine group



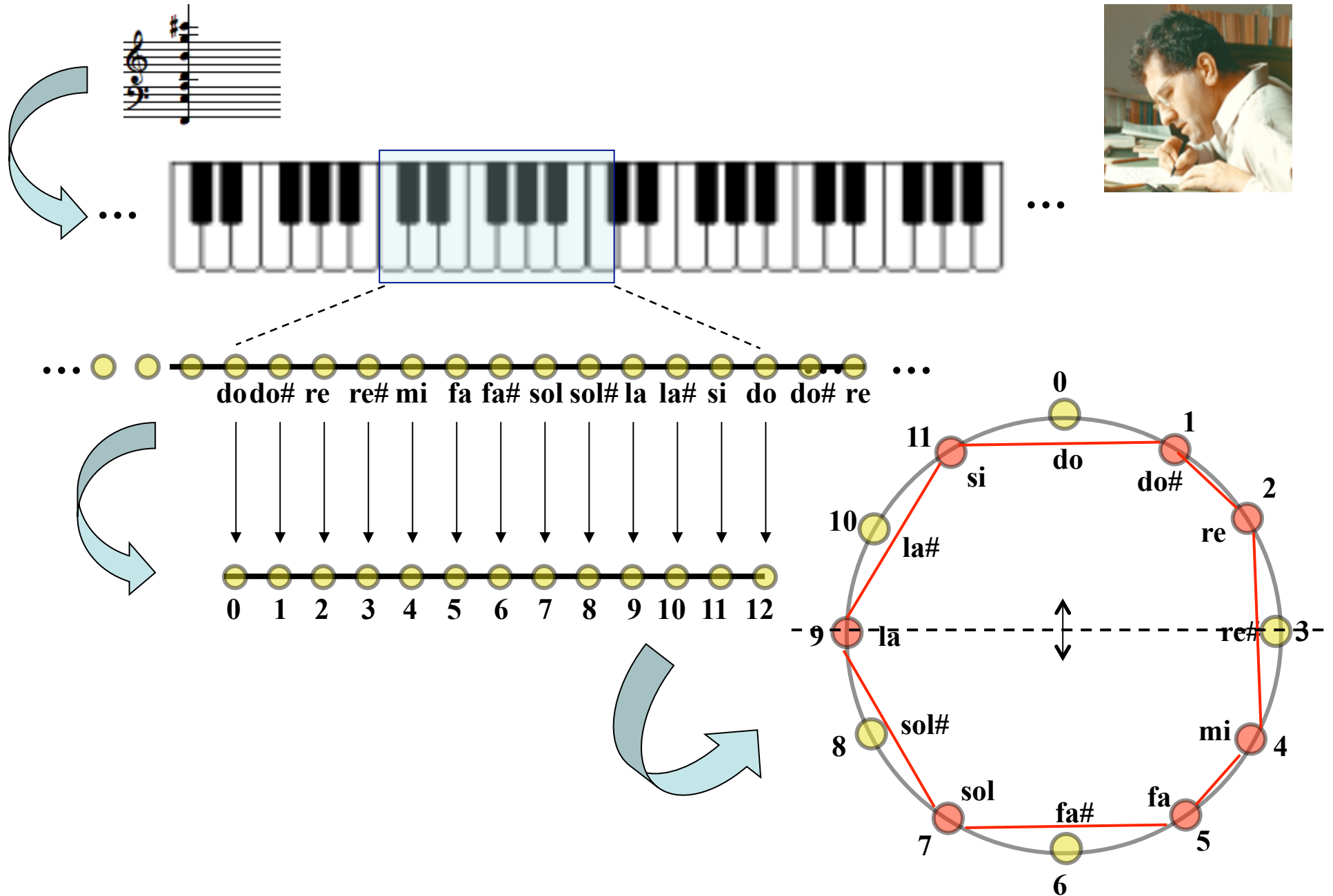
A set-theoretical exercise by Célestin Deliège



5-30	0,1,4,6,8	121321	7-30	0,1,2,4,6,8,9	343542
5-31	0,1,3,6,9	114112	7-31	0,1,3,4,6,7,9	336333
5-32	0,1,4,6,9	113221	7-32	0,1,3,4,6,8,9	335442
5-33(12)	0,2,4,6,8	040402	7-33	0,1,2,4,6,8,10	262623
5-34(12)	0,2,4,6,9	032221	7-34	0,1,3,4,6,8,10	254442
5-35(12)	0,2,4,7,9	032140	7-35	0,1,3,5,6,8,10	254361
5-Z36	0,1,2,4,7	222121	7-Z36	0,1,2,3,5,6,8	444342
5-Z37(12)	0,3,4,5,8	212320	7-Z37	0,1,3,4,5,7,8	434541
5-Z38	0,1,2,5,8	212221	7-Z38	0,1,2,4,5,7,8	434442
6-1(12)	0,1,2,3,4,5	543210			
6-2	0,1,2,3,4,6	443211			
6-Z3	0,1,2,3,5,6	433221	6-Z36	0,1,2,3,4,7	
6-Z4(12)	0,1,2,4,5,6	432321	6-Z37(12)	0,1,2,3,4,8	
6-5	0,1,2,3,6,7	422232			
6-Z6(12)	0,1,2,5,6,7	421242	6-Z38(12)	0,1,2,3,7,8	
6-7(6)	0,1,2,6,7,8	420243			
6-8(12)	0,2,3,4,5,7	343230			
6-9	0,1,2,3,5,7	342231			
6-Z10	0,1,3,4,5,7	333321	6-Z39	0,2,3,4,5,8	
6-Z11	0,1,2,4,5,7	333231	6-Z40	0,1,2,3,5,8	
6-Z12	0,1,2,4,6,7	332232	6-Z41	0,1,2,3,6,8	
6-Z13(12)	0,1,3,4,6,7	324222	6-Z42(12)	0,1,2,3,6,9	



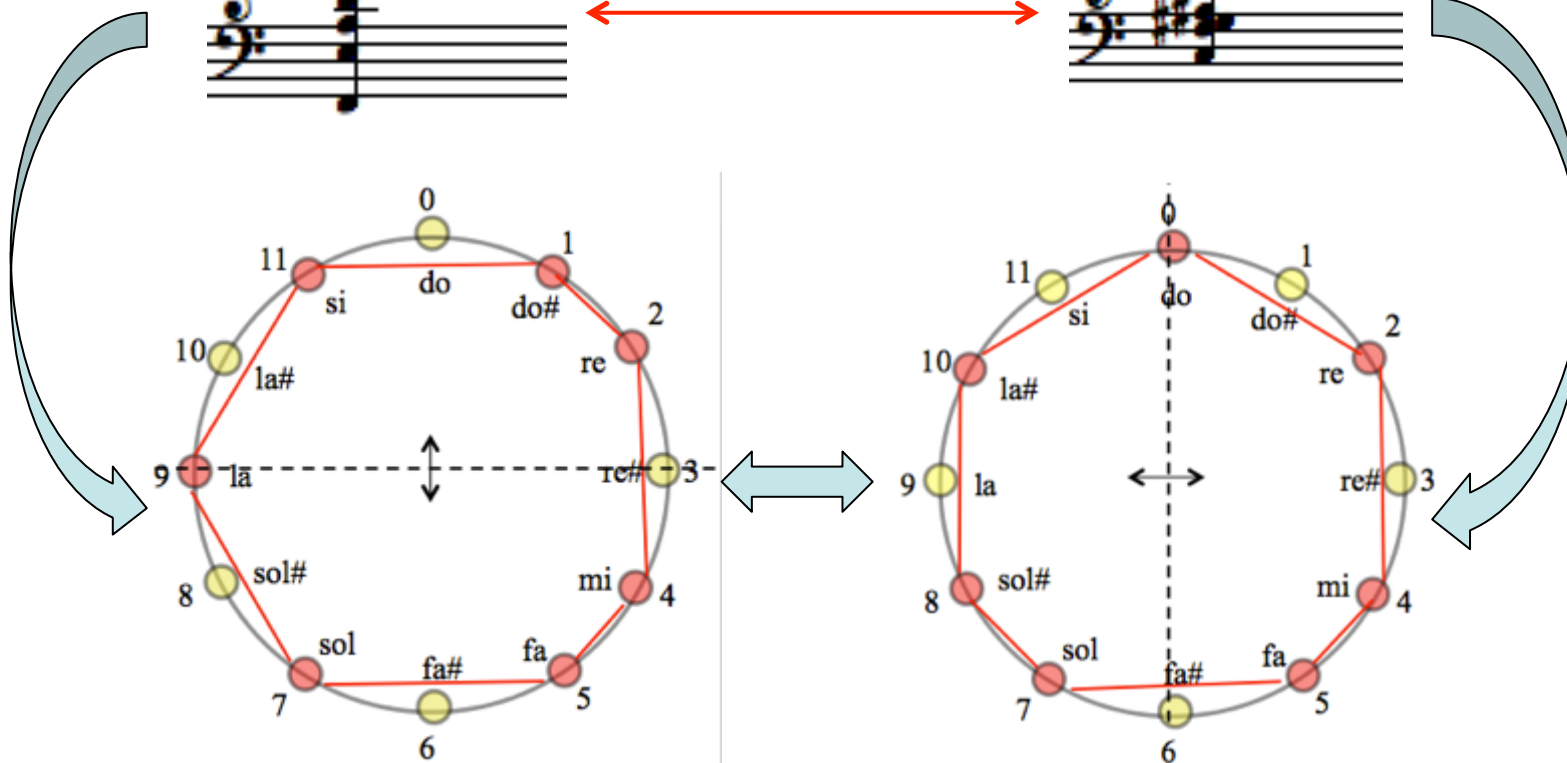
A set-theoretical exercise by Célestin Deliège



A set-theoretical exercise by Célestin Deliège



transposition



pcset

Interval Content

name

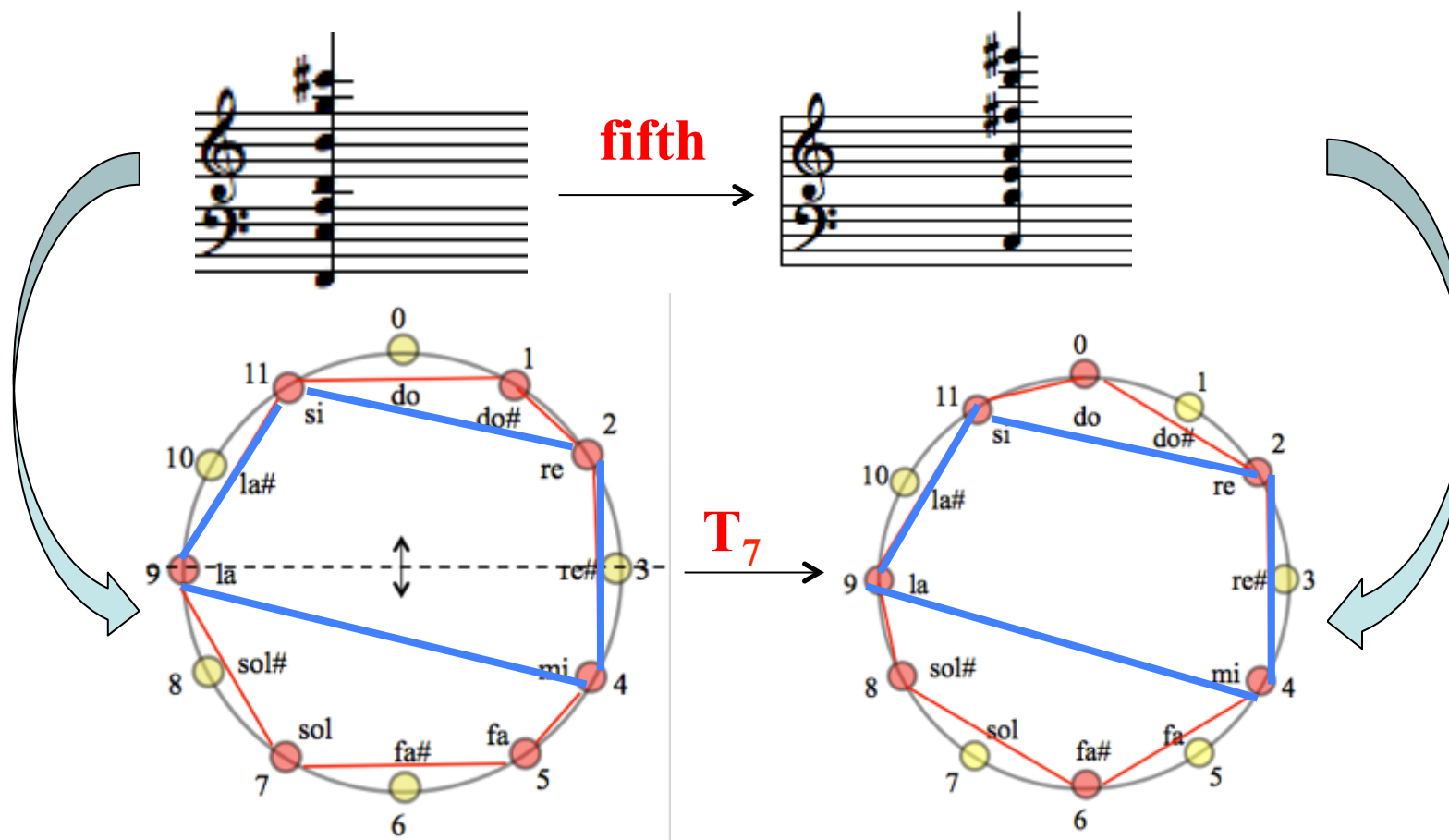
(0 1 3 4 6 8 10)

[7 2 5 4 4 4 4 4 4 4 5 2]

7-34



A set-theoretical exercise by Célestin Deliège



pcset

Interval Content

name

(0 1 3 4 6 8 10)

[7 2 5 4 4 4 4 4 4 4 5 2]

7-34

T_0

... T_7



Computational Music Analysis: the French tradition

Formalismes et modèles musicaux (André Riotte & Marcel Mesnage)

- « Anamorphoses » d'André Riotte
- « La terrasse des audiences du clair de lune » de Claude Debussy : esquisse d'analyse modélisée
- La mise en évidence de régularités locales : le « Mode de valeurs et d'intensités » de Messiaen
- Un exemple d'invention structurelle : le « Mikrokosmos » de Béla Bartok
- Un modèle informatique de la « Pièce pour quatuor à cordes » n°1 de Stravinsky
- Les « Variations pour piano », op. 27, d'Anton Werbern
- L'« Invention à deux voix » n°1 de J.-S. Bach
- Un modèle informatique du « Troisième Regard sur l'Enfant Jésus » d'Olivier Messiaen
- Un modèle de la « Valse sentimentale », Op. 50, n°13, de Franz Schubert
- Un automate musical construit à partir d'une courte pièce de Béla Bartok (Mikrokosmos n°39)





« Entités formelles pour l'analyse musicale » Marcel Mesnage (1998)



A. Schoenberg : *Klavierstück Op. 33a*, 1929

The image displays a musical score for A. Schoenberg's *Klavierstück Op. 33a*, 1929, with several formal entities highlighted. The score is in 4/4 time and features a complex harmonic structure. The highlighted entities are:

- Entity 1 (blue box): 0-5511 (1 2 5 6)
- Entity 2 (yellow box): 9-4233 (2 3 4 5 6)
- Entity 3 (yellow box): 8-6231 (1 2 3 4 5 6)
- Entity 4 (yellow box): 11-6132 (1 2 3 4 5 6)
- Entity 5 (blue box): 0-4332 (2 3 4 5 6)
- Entity 6 (blue box): 3-5511 (1 2 5 6)

Below the score, six circular diagrams represent these formal entities. Each diagram is a circle with a grid of numbers (0-11) and a set of numbers (1-6) below it. The diagrams are labeled with transformation types: T_3 for the first entity, and T_{1I} for the others. The diagrams are arranged in a row, with a vertical dashed line separating the first three from the last three.

« Making and Using a Pcset Network for Stockhausen's Klavierstück III »

Measures 1-4 of the musical score. The top staff is in treble clef and the bottom in bass clef. Measure 1 has a 4/8 time signature, measure 2 has a 5/8 time signature, and measure 3 has a 3/8 time signature. Dynamics include *p*, *mf*, *f*, and *ff*. Fingerings are indicated with numbers 1-5 and slurs.

Three interpretations:



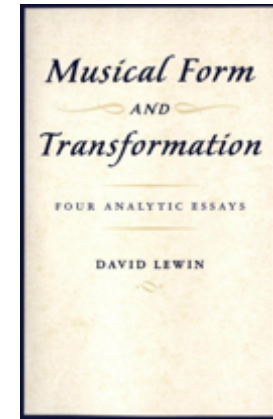
Henck



Kontarsky



Tudor



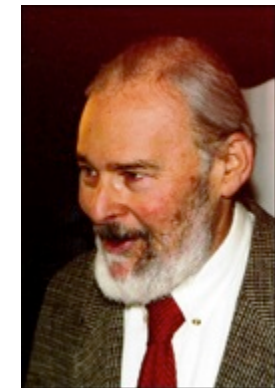
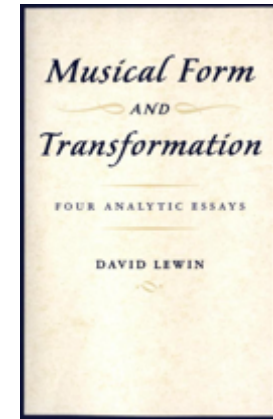
Measures 5-8 of the musical score. Measure 5 has a 7/8 time signature, measure 6 has a 4/8 time signature, and measure 7 has a 3/8 time signature. Dynamics include *f*, *p*, *mf*, and *ff*. Fingerings and slurs are present.



Measures 11-14 of the musical score. Measure 11 has a 7/8 time signature, measure 12 has a 5/8 time signature, and measure 13 has a 3/8 time signature. Dynamics include *mf*, *f*, *ff*, and *ff*. Fingerings and slurs are present.

« Making and Using a Pcset Network for Stockhausen's Klavierstück III »

The image shows a musical score for Stockhausen's Klavierstück III. The score is in 4/8, 5/8, and 3/8 time signatures. It features various dynamics such as *p*, *mf*, and *f*. Three colored boxes highlight specific passages: a red box around the first measure, a green box around the second measure, and a blue box around the third measure. Below the score are three circular diagrams, each labeled '12', representing pentachord forms. Arrows point from the red, green, and blue boxes to the first, second, and third diagrams respectively, with question marks above each arrow.



« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

« *Making and Using a Pcset Network for Stockhausen's Klavierstück III* »

Lewin 1993

The image shows a musical score for Klavierstück III in 4/8 time. It consists of two staves: a treble clef staff and a bass clef staff. The music is divided into three measures. The first measure is in 4/8 time, the second in 5/8, and the third in 3/8. Dynamics include piano (p), mezzo-forte (mf), and forte (f). There are various musical notations such as slurs, accents, and articulation marks. Three overlapping boxes are drawn around the first two measures, with arrows pointing from them to the pcset data below.

SI: (1, 1, 1, 3, 6)

(6, 3, 1, 1, 1)

(6, 3, 1, 1, 1)

IFUNC: [5 3 2 2 1 1 1 1 2 2 3]

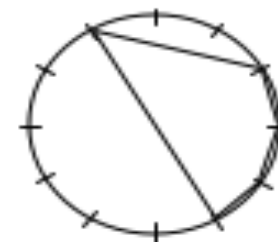
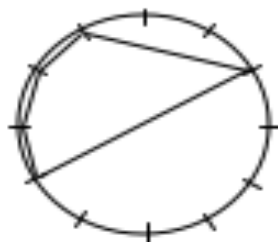
[5 3 2 2 1 1 1 1 2 2 3]

[5 3 2 2 1 1 1 1 2 2 3]

VI: [3 2 2 1 1 1]

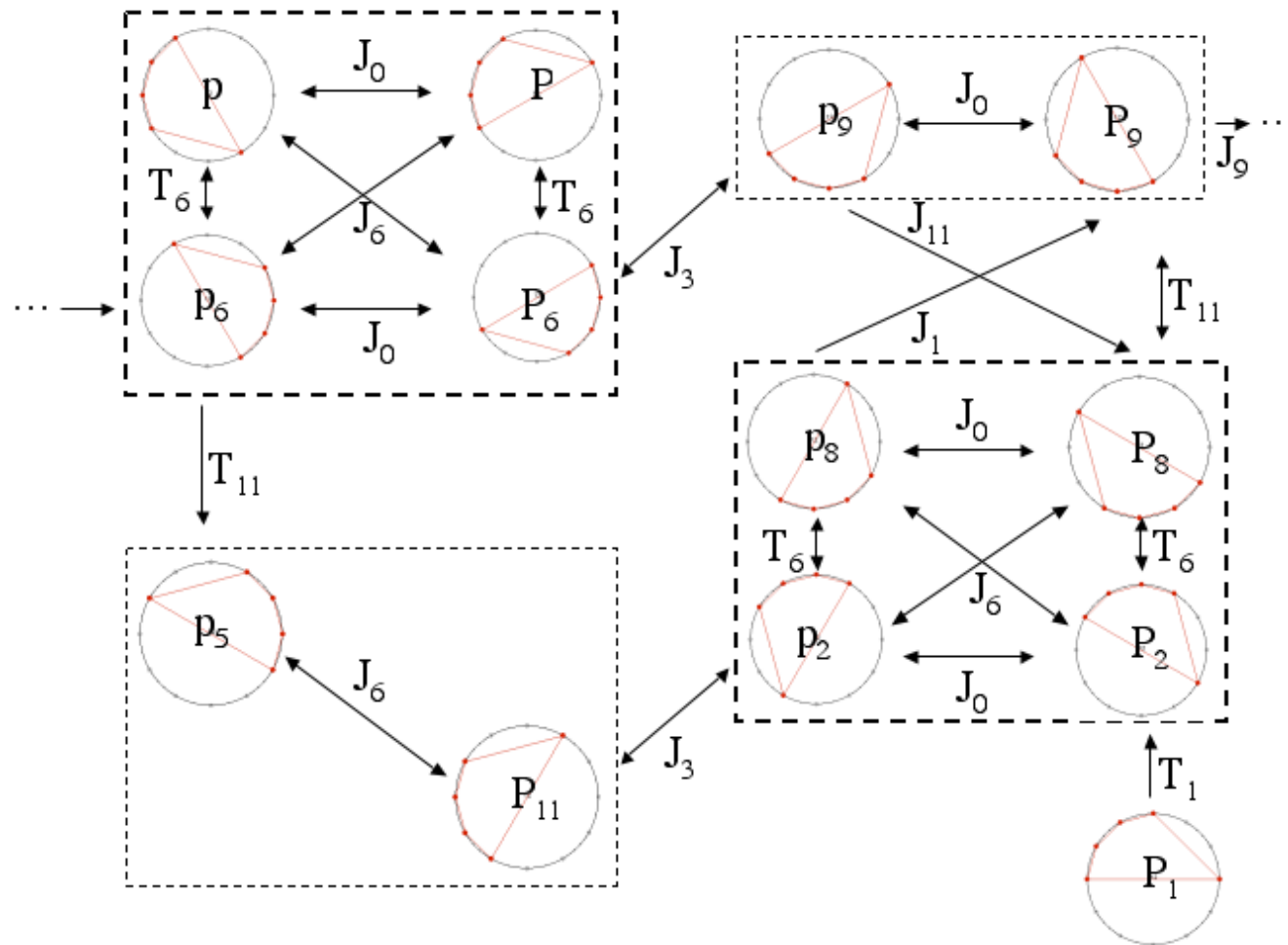
[3 2 2 1 1 1]

[3 2 2 1 1 1]



Transformational Network

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)



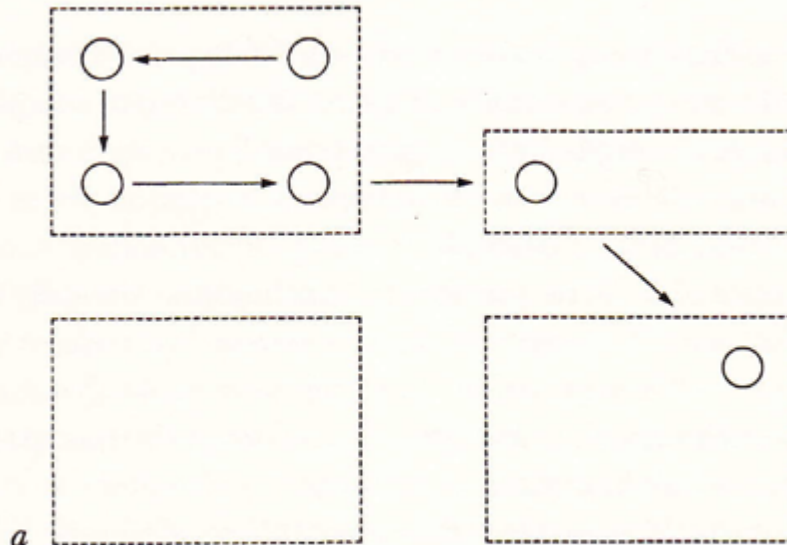
« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their **potentially functional interrelationships**, in a very compactly organized little **spatial configuration**. »

« [...] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - **it makes the abstract space of such a world accessible to our sensibilities**. That is to say that the story projects what one would traditionally call *form*. »

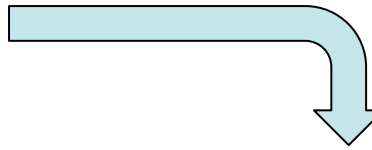
Listening paths within the piece

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

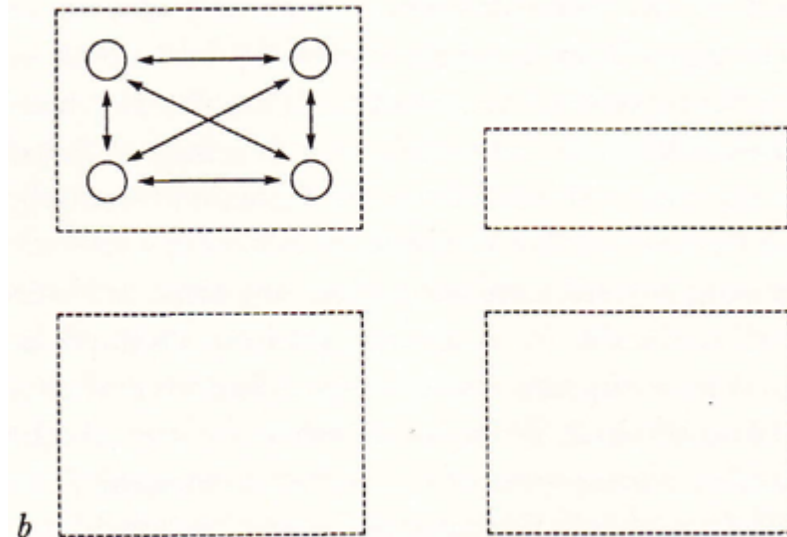
Pass 1 (mm. 1-5).



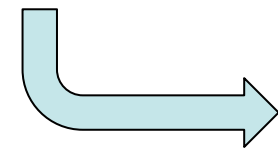
a
horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1



Pass 2 (mm. 5-8) goes back and elaborates the beginning area of pass 1.



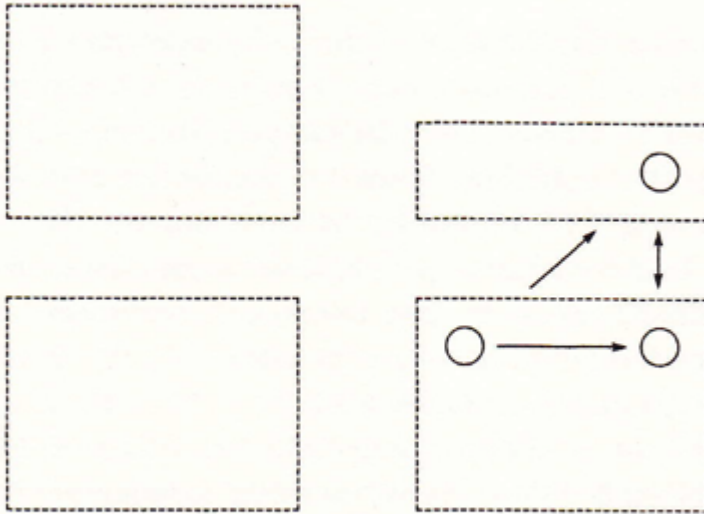
b
horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1



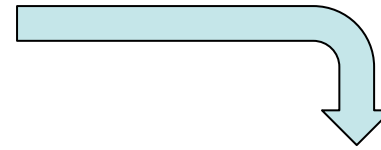
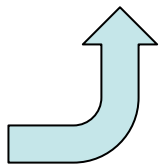
Listening paths within the piece

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

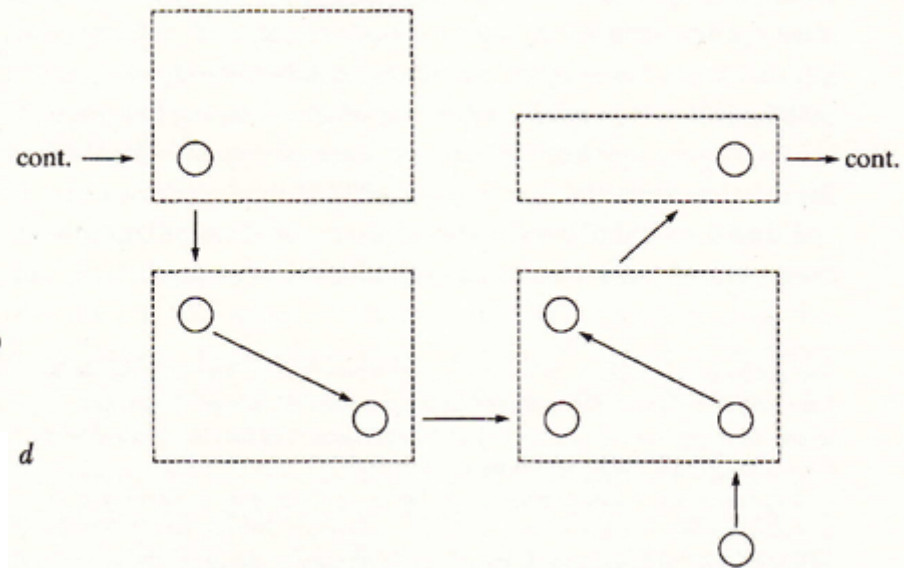
Pass 3 (mm. 8-10) picks up and elaborates the ending area of pass 1.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1



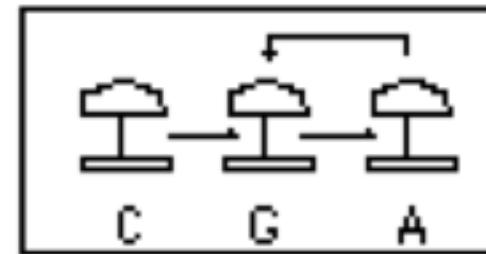
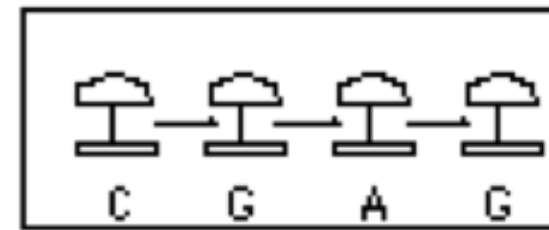
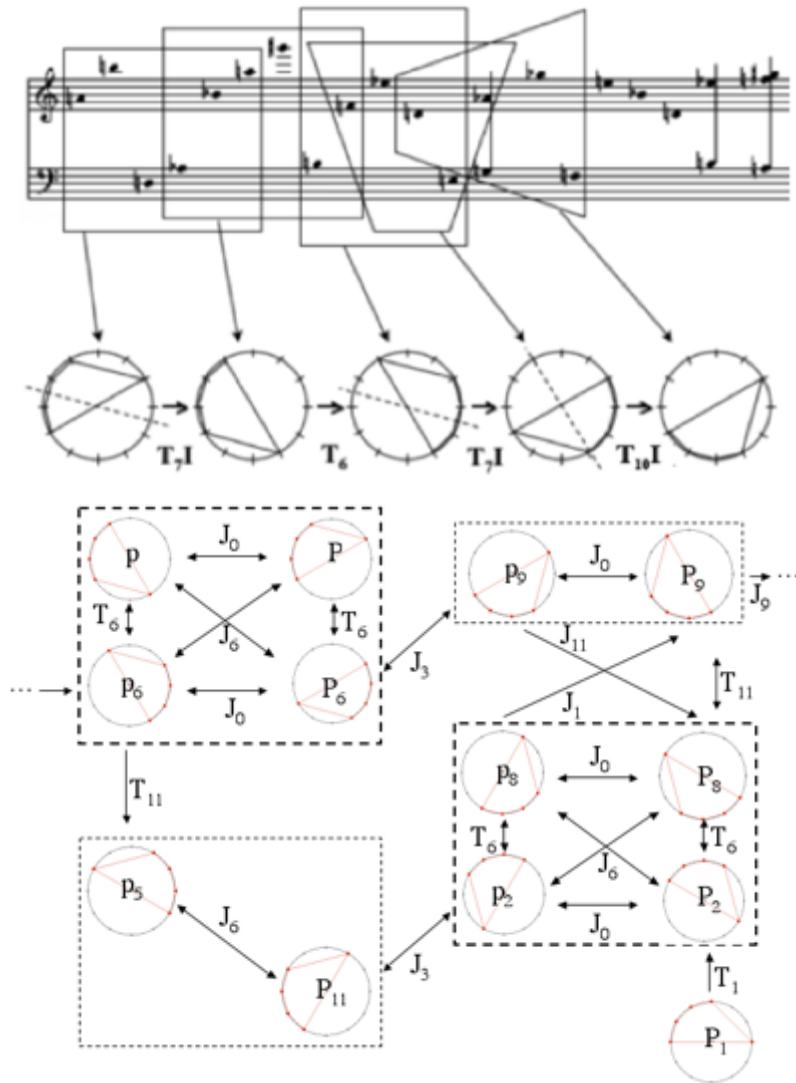
Pass 4 (mm. 9-16) expands the p8 + P8 area of pass 3 to activate P2 and p2 as well. P2 is the "essential" incipit of pass 4; p2 is the end of the pass, and of the piece.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Transformational Networks and Music Cognition

Bamberger, J. (1986). Cognitive issues in the development of musically gifted children. In *Conceptions of giftedness* (eds., R. J. Sternberg, & J. E. Davidson), pp. 388-413. Cambridge University Press, Cambridge



Bamberger, J. (2006). "What develops in musical development?" In G. MacPherson (ed.) *The child as musician: Musical development from conception to adolescence*. Oxford, U.K. Oxford University Press.

Listening exercise: « do you hear it? » vs « can you hear it? »

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

The image displays three systems of musical notation for Stockhausen's *Klavierstück III*. Each system consists of two staves (treble and bass clef) with notes and rests. Above the first staff of each system are interval labels: m. 1 (P0), m. 1-2 (p0), m. 2 (p6), m. 2-3 (P6), m. 2-5 (p9), m. 2-5 (P8). The second system has labels: m. 5-7 (P6), m. 5-7 (p6), m. 5-7 (P0), m. 5-7 (p0), m. 8-10 (p8), m. 8-10 (P8), m. 8-10 (P9). The third system has labels: m. 9-11 (P1), m. 10-11 (P2), m. 11-12 (p8), m. 11-12 (P9), m. 11-13 (p6), m. 12-13 (p5), m. 13-14 (Pe), m. 13-15 (p2). The notation includes various accidentals and note heads.

Example 2.7. An ear-training aid for listening to P/p forms and their interrelations.

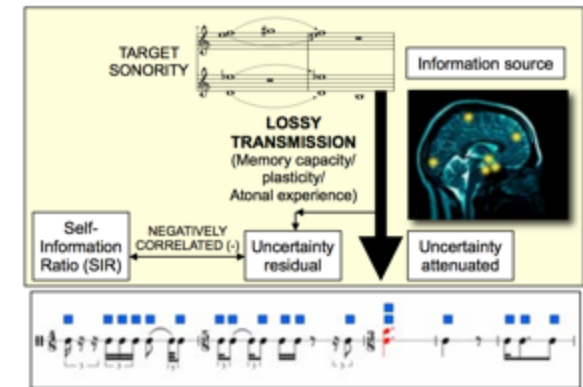
« I take the question ‘Can you hear it » to mean something like this: After studying the analysis in examples 2.5 and 2.6, do you find it possible to focus your **aural attention** upon aspects of the acoustic signal that seem to engage the signifiers of that analysis? [...] For me, the interesting questions involve the extent and ways in which I am satisfied and dissatisfied when **focusing my aural attention** in that manner. It is important to ask those questions about any systematic analysis of any musical composition ».



Can you hear it? Yes, we can!

Figure 5 displays musical notation for Phase I pitch-detection tasks. It is divided into two parts, I and V. Each part shows target sonorities (circled in dashed-line boxes) and corresponding melodic excerpts. Part I includes target sonorities P₀, P₆, and P₈. Part V includes target sonorities P₉, P₅, and P₂. Fingerings and dynamics are indicated throughout the melodic lines.

FIGURE 5. Six target sonorities used for Phase I pitch-detection tasks (circled in dashed-line boxes): Single Pentachords appeared in form of either 'st' or 'ts' according to Lewin's ear-training aid (*MFT*, Example 2.7, p. 42). Their corresponding melodies are either Excerpt I or V.



« A cognitive model is derived to show that singleton-tetrachord interaction is salient in facilitating the mental formation of common-tone-preserving percepts, and it serves as perceptual information that determines the acquisition of implicit pitch pattern knowledge for pitch-detection tasks, but only for atonally well-trained musicians. »

Y. Cao, J. Wild, B. Smith, S. McAdams, « The Perception and Learning of Contextually-defined Inversion Operators in Transformational Pitch Patterns », 5th International Conference of Students of Systematic Musicology (SysMus12), Montreal, 2012.

Elliott Carter : 90+ (1994)



• Chord combinatorics

- Hexacords
- Tetrachords
- Triads
- Z-relation

• all-interval series

- *Link-chords*



(piano: John Snijders)

mille e novanta auguri a caro Goffredo

90+

Elliott Carter
(1994)

♩ = 96

Piano

(senza pedale)*

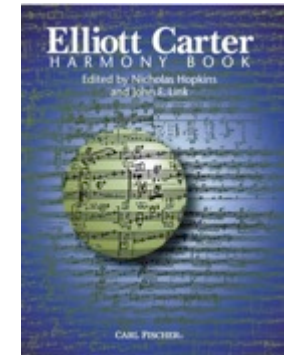
* Use pedal only to join one chord to another *legato*, as in mm. 1-13, 16-21, 36-43, and 45-48.

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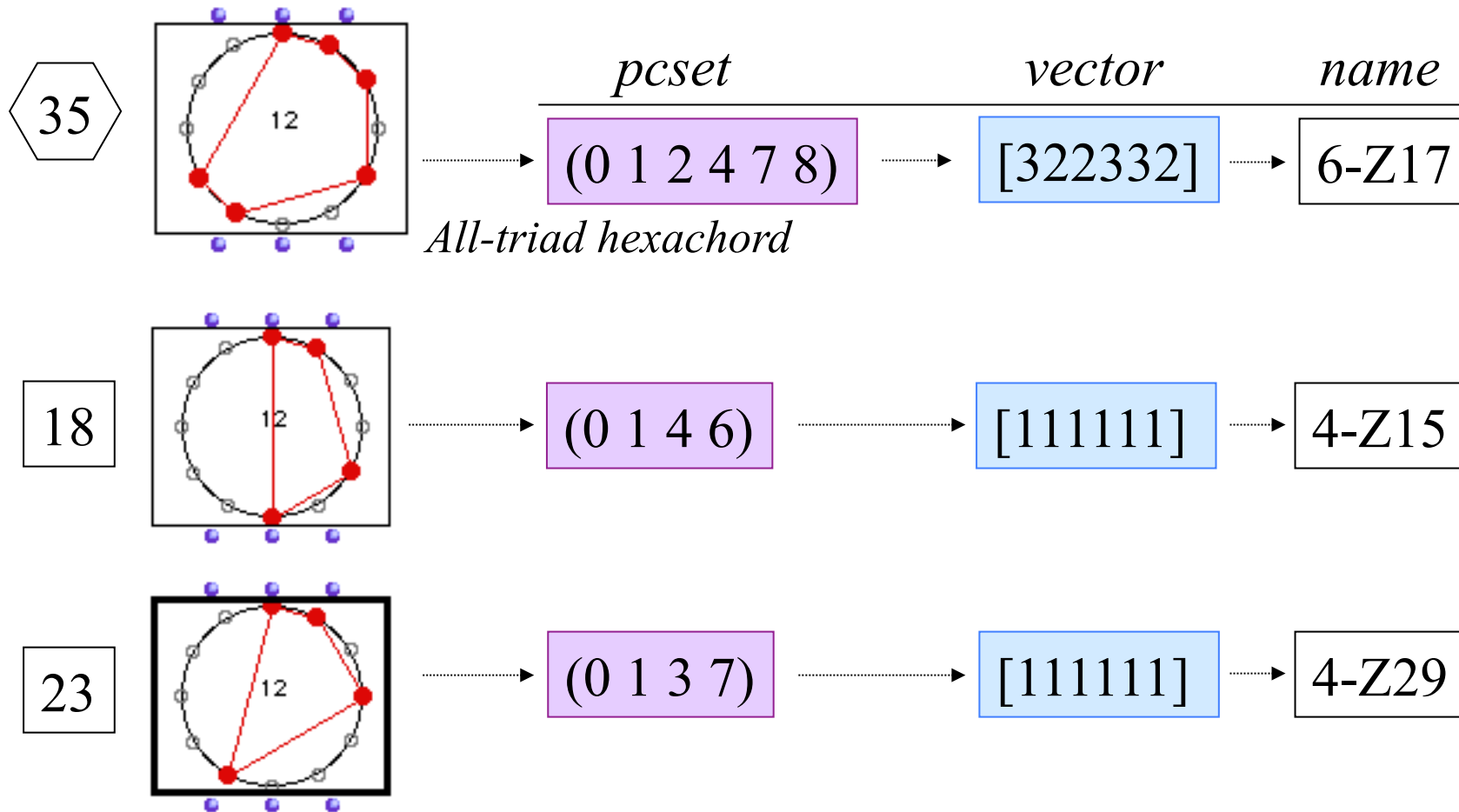
PIB 503

Printed in U.S.A.

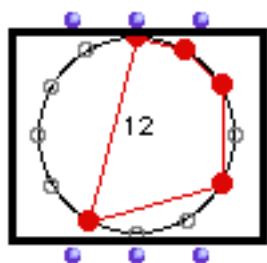
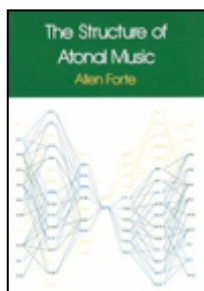
Elliott Carter: 90+ (1994)



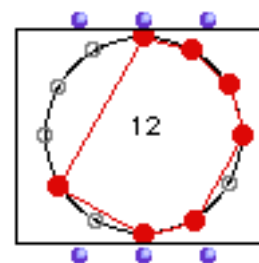
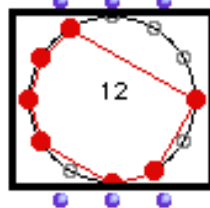
« From about 1990, I have reduced my vocabulary of chords more and more to the six note chord n° 35 and the four note chords n° 18 and 23, which encompass all the intervals » (Harmony Book, 2002, p. ix)



Allen Forte's Catalogue (1973) and the Z relation



complémentation



A. Forte (1926-)

5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140
5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211

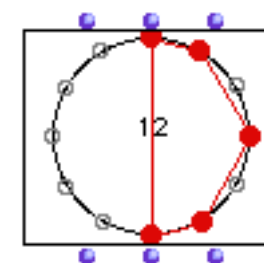
7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442

5-Z36 0,1,2,4,7 222121

7-Z36 0,1,2,3,5,6,8 444342

6-Z4(12)	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

6-Z39	0,2,3,4,5,8	
6-Z40	0,1,2,3,5,8	
6-Z41	0,1,2,3,6,8	
6-Z42(12)	0,1,2,3,6,9	

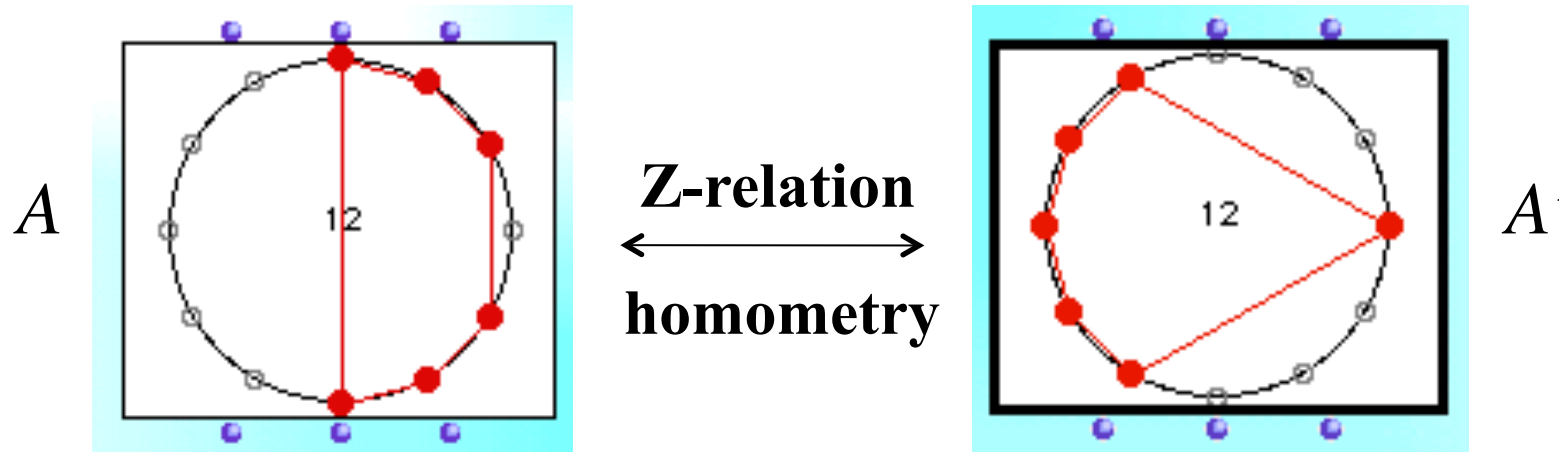
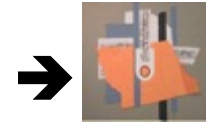


5-Z12



A 'Mathemusical' Theorem

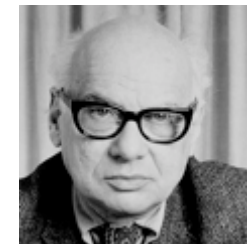
$$IC_A(k) = \text{Card}\{(x, y) \in A \times A \mid x + k = y\}$$



$$IC_A = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = IC_{A'}$$

Babbitt's Hexachord Theorem:

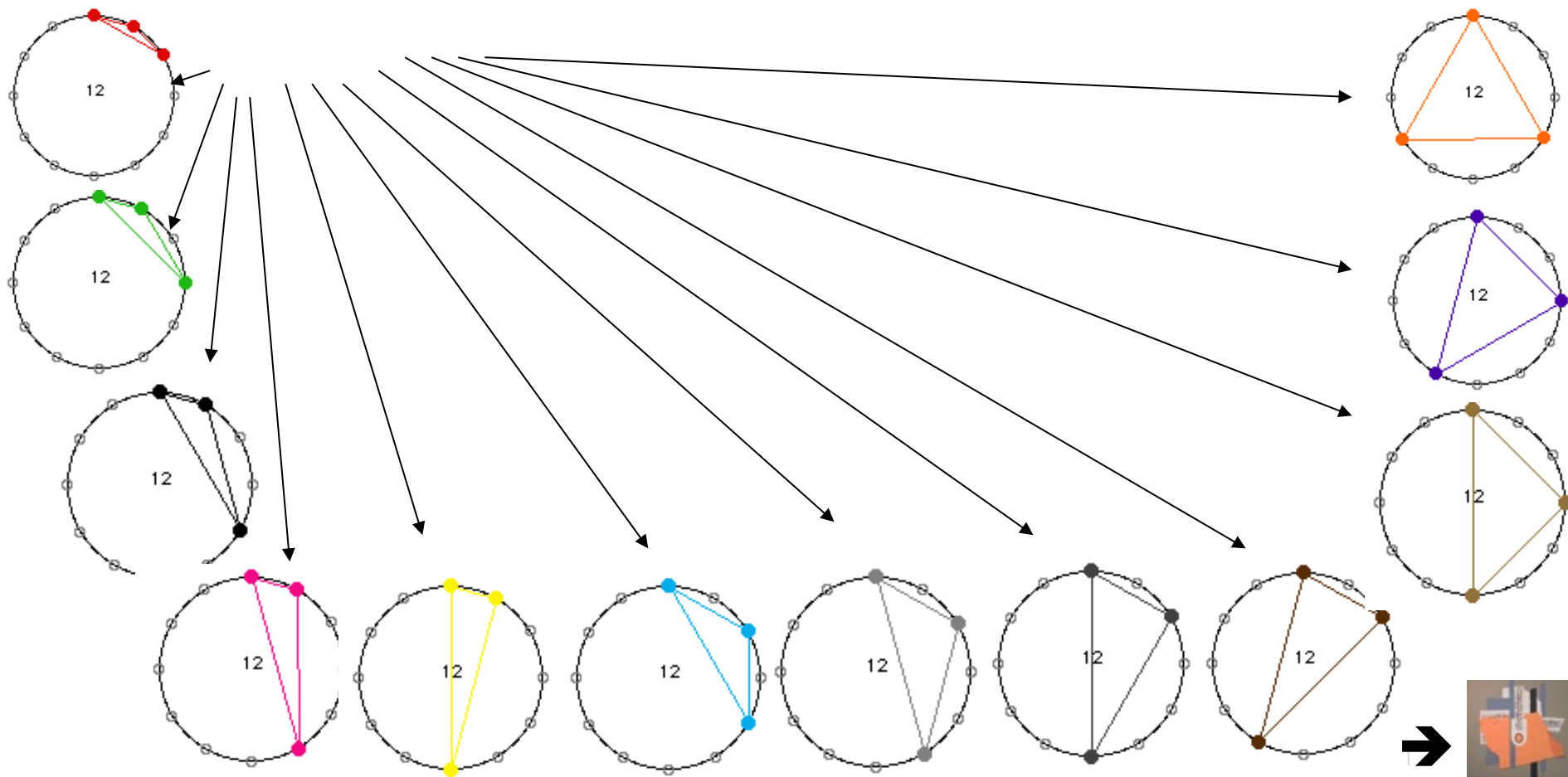
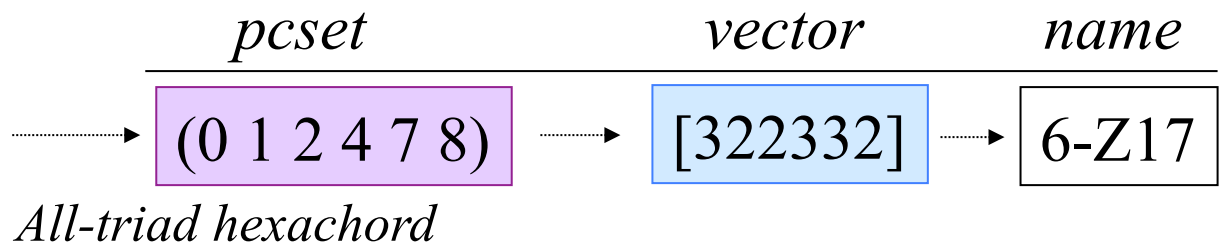
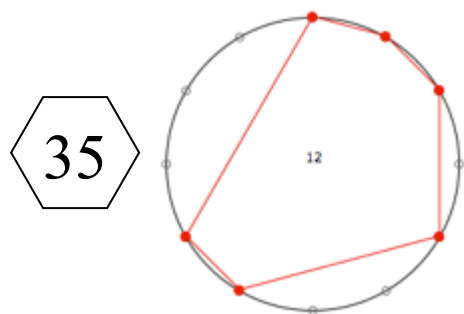
A hexachord and its complement have the same interval content



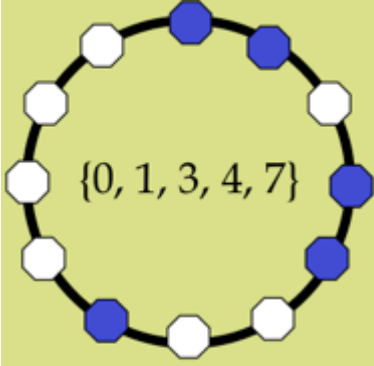
(Proofs by Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, ..., Amiot, ...)

Elliott Carter: 90+ (1994)


$G \setminus k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\text{Aff}_1(\mathbb{Z}_{12})$	1	5	9	21	25	34	25	21	9	5	1	1



High-order 'interval' content: Lewin's mv^k vector









$\{0, 1, 3, 4, 7\}$



3-set (*prime forms*):

- $\{0, 1, 2\} \rightarrow 0$ copies
- $\{0, 1, 3\} \rightarrow 2$ copies
- $\{0, 1, 4\} \rightarrow 3$ copies
- $\{0, 1, 5\} \rightarrow 0$ copies
- $\{0, 1, 6\} \rightarrow 1$ copie
- $\{0, 2, 4\} \rightarrow 0$ copies
- $\{0, 2, 5\} \rightarrow 0$ copies
- $\{0, 2, 6\} \rightarrow 1$ copie
- $\{0, 2, 7\} \rightarrow 0$ copies
- $\{0, 3, 6\} \rightarrow 1$ copie
- $\{0, 3, 7\} \rightarrow 2$ copies
- $\{0, 4, 8\} \rightarrow 0$ copies

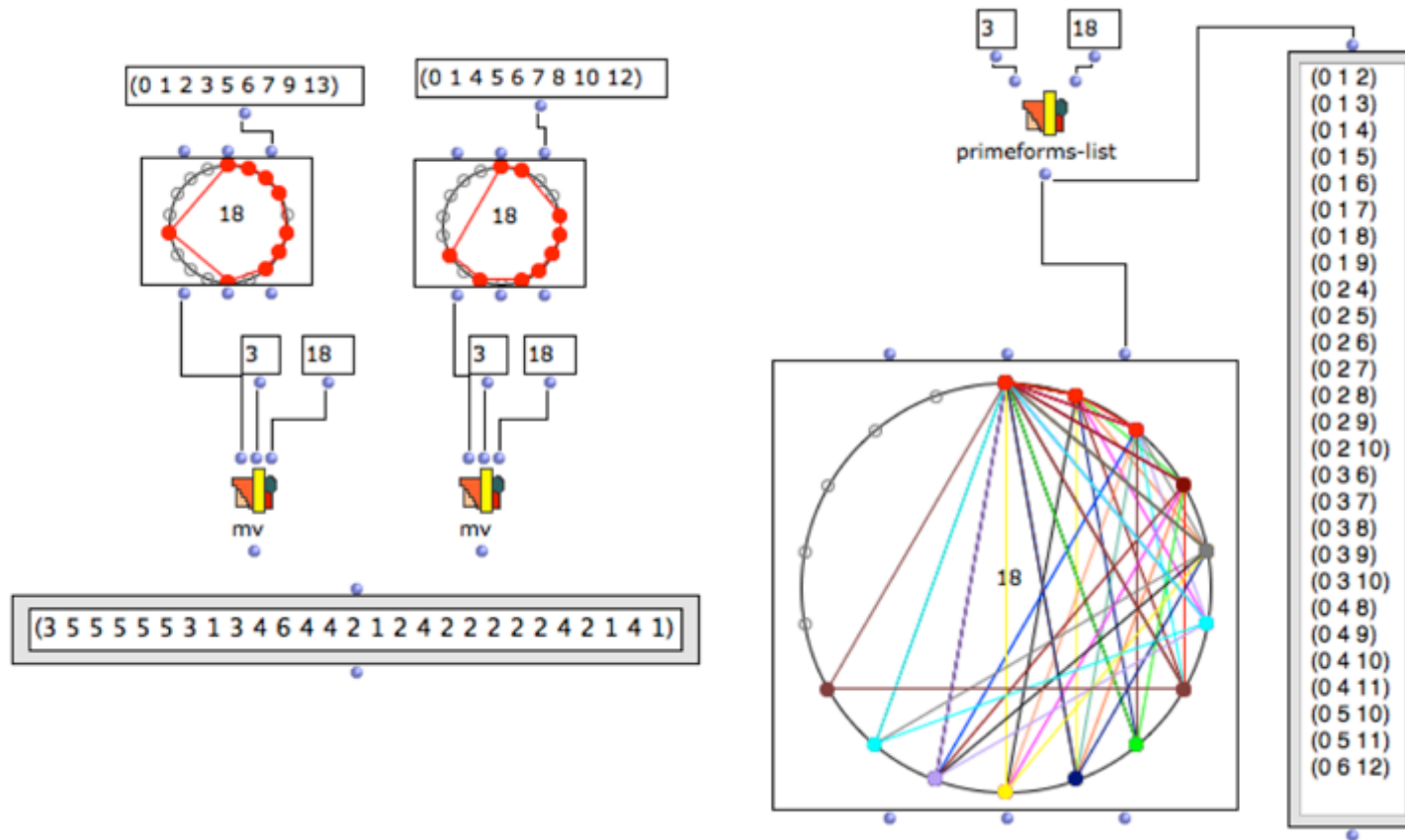
3-Subsets:

- 2 / $\{0, 1, 3\}$ / 
- 3 / $\{0, 1, 4\}$ / 
- 1 / $\{0, 1, 7\}$ / 
- 1 / $\{0, 2, 6\}$ / 
- 1 / $\{0, 3, 6\}$ / 
- 2 / $\{0, 3, 7\}$ / 

$mv^3(\{0,1,3,4,7\})=[0, 2, 3, 0, 1, 0, 0, 1, 0, 1, 2, 0]$

• Daniele Ghisi, "From Z-relation to homometry: an introduction and some musical examples", Séminaire MaMuX, 10 décembre 2010 [<http://repmus.ircam.fr/mamux/saisons/saison10-2010-2011/2010-12-10>]

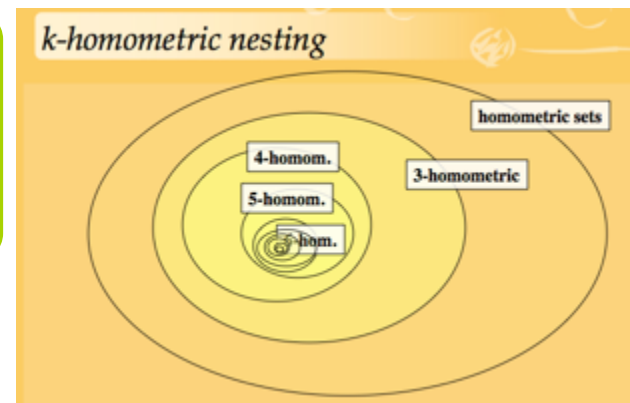
High-order Z-relation and k -homometric nesting



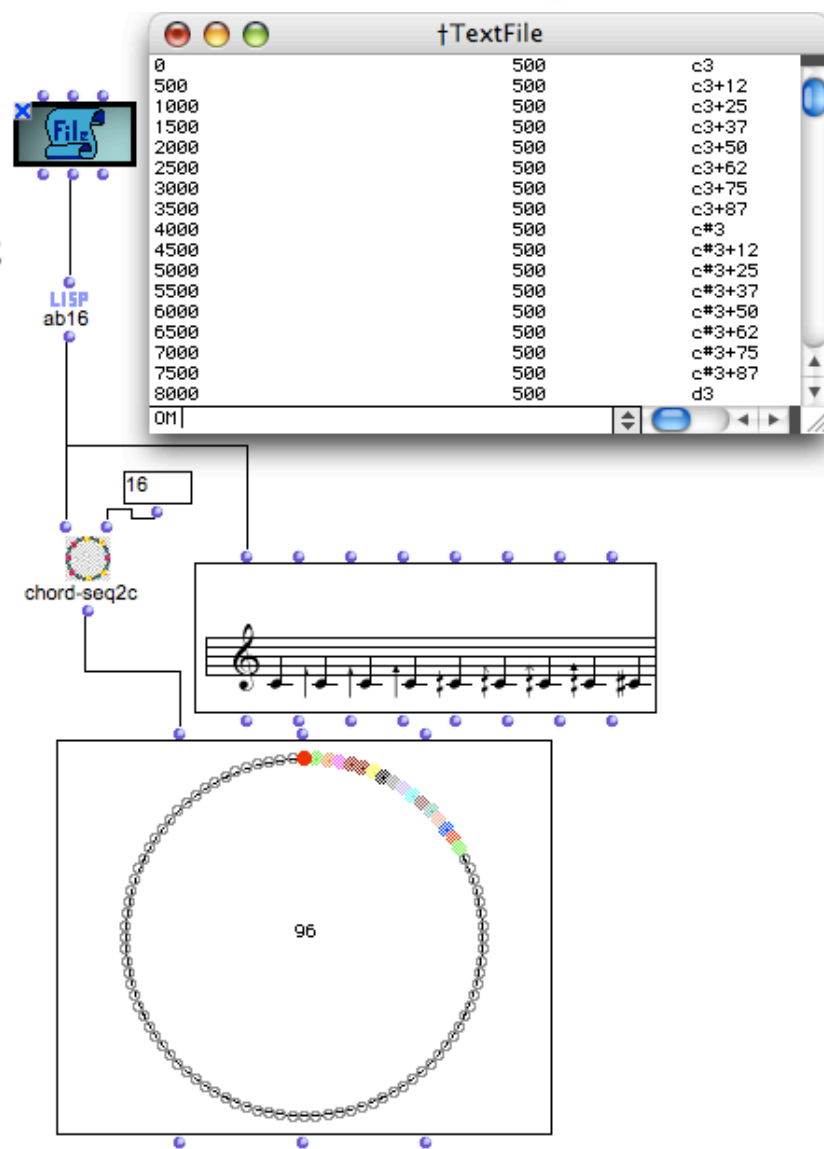
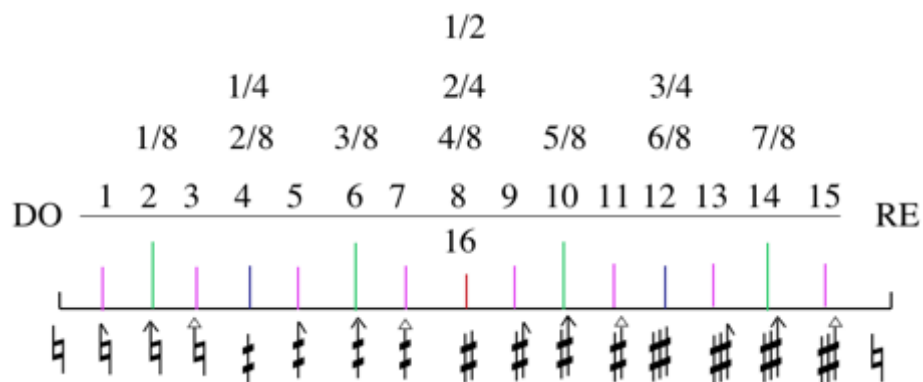
$$\mathbf{Z}_{18} \quad A = \{0, 1, 2, 3, 5, 6, 7, 9, 13\}$$

$$\quad \quad B = \{0, 1, 4, 5, 6, 7, 8, 10, 12\}$$

$mv^3(A) = mv^3(B) \rightarrow A$ and B are Z-related

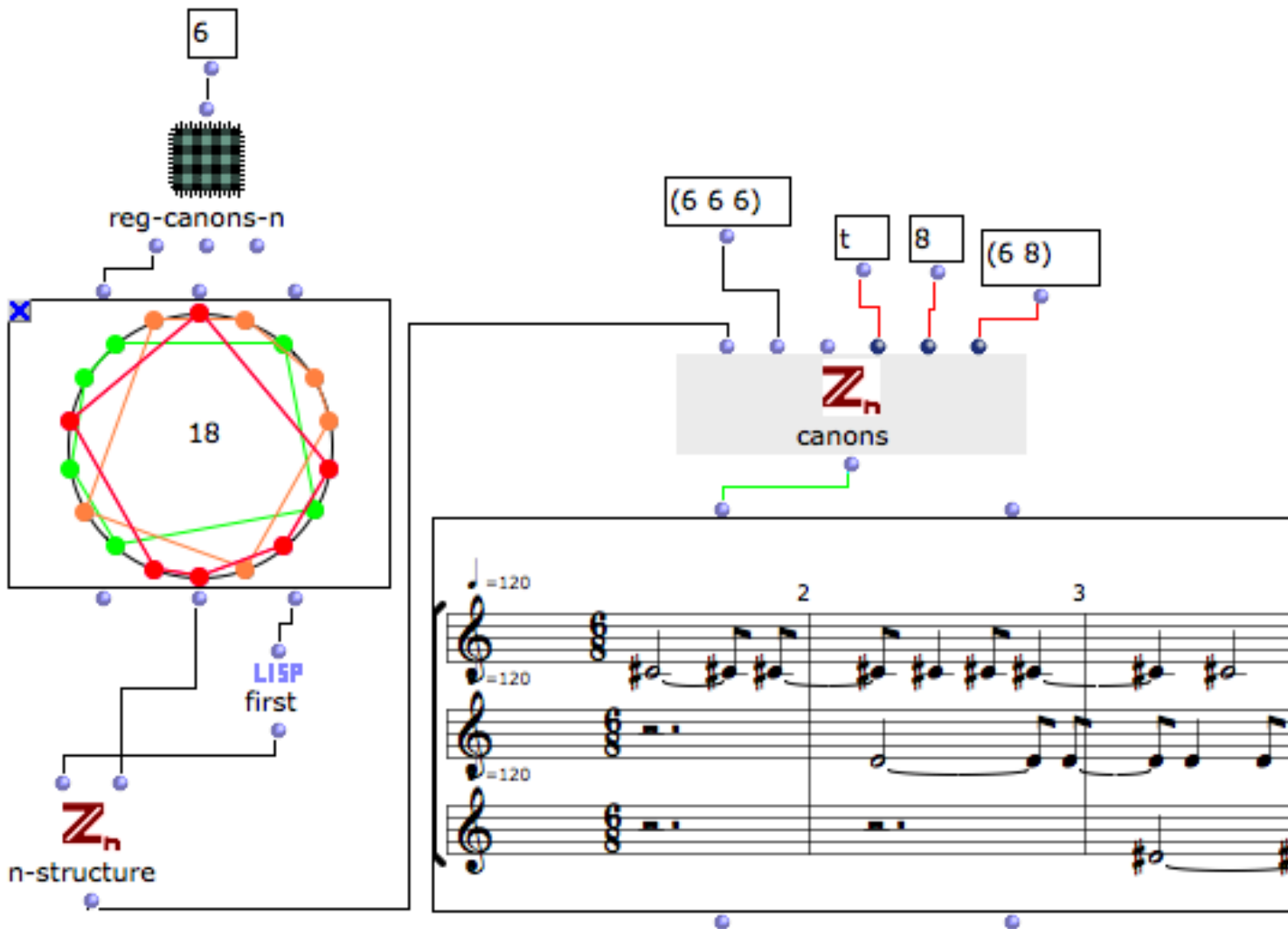


Microtonal composition (Alain Bancquart)



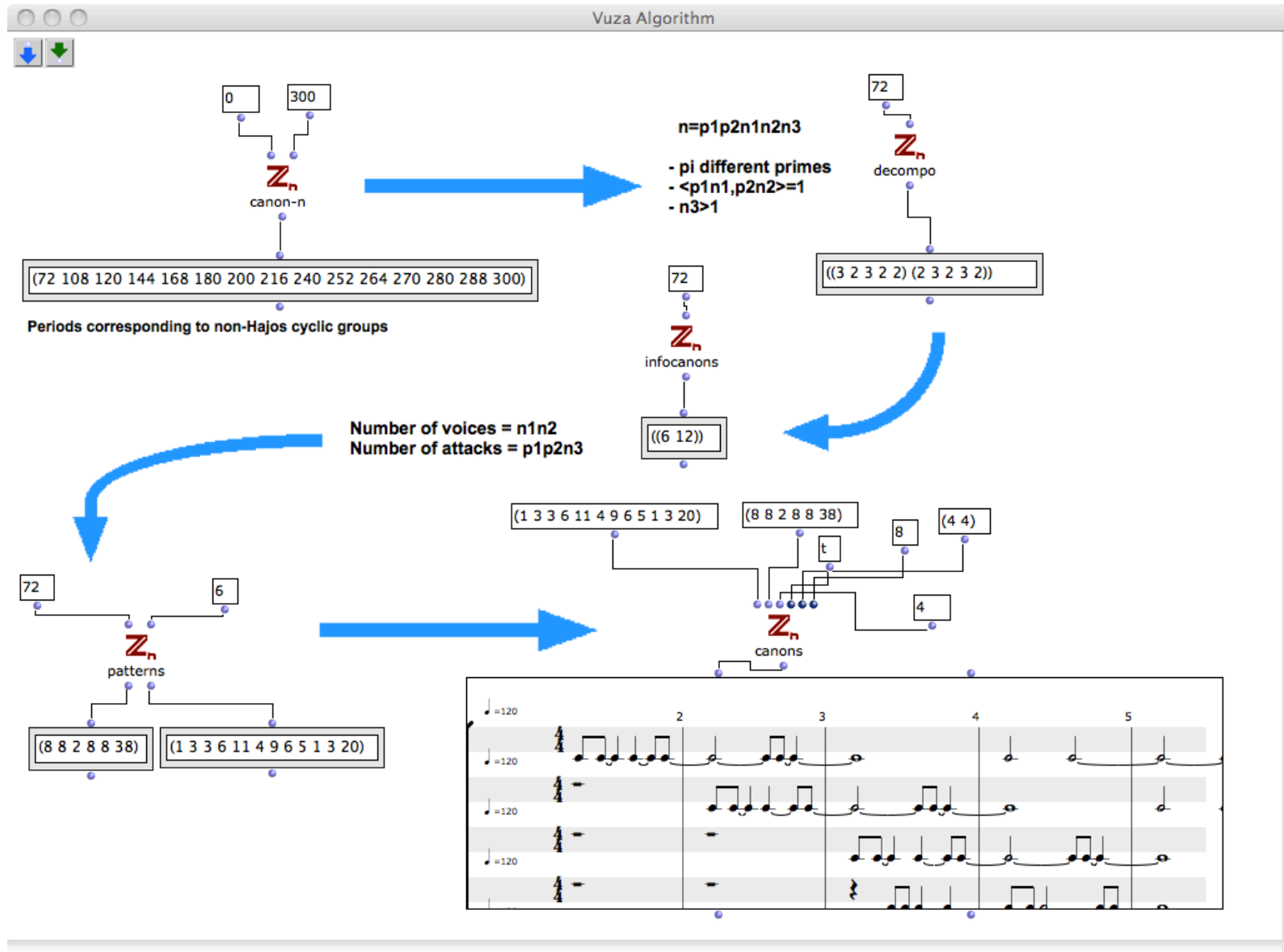
A. Bancquart, M. Andreatta, et C. Agon, « Microtonal Composition », The OM Composer's Book 2, éd. Jean Bresson, Carlos Agon, Gérard Assayag (Ircam/Delatour France, Sampzon), 2008, p. 279-302.

Construction explicite d'un rythme k -asymétrique



16-k-asymmetry-canons

Algorithme de Vuza et implémentation en OpenMusic



17-vuza algorithm

Classification « paradigmatique » des canons mosaïques de Vuza

Résultat : uniquement deux « types » de canons différents (à une transformation affine près, i.e. $f: \mathbb{Z}_{72} \rightarrow \mathbb{Z}_{72}$ t.q. $f(x) = ax + b$ avec $a \in (\mathbb{Z}_{72})^*$ et $b \in \mathbb{Z}_{72}$)



• R. Tijdeman: “Decomposition of the Integers as a direct sum of two subsets”, *Number Theory*, Cambridge University Press, 1995. The fundamental Lemma: R pave $\mathbb{Z}_n \Rightarrow aR$ pave \mathbb{Z}_n $\langle a, n \rangle = 1$

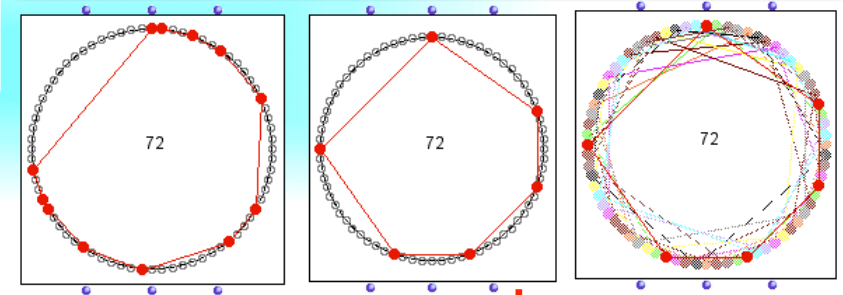
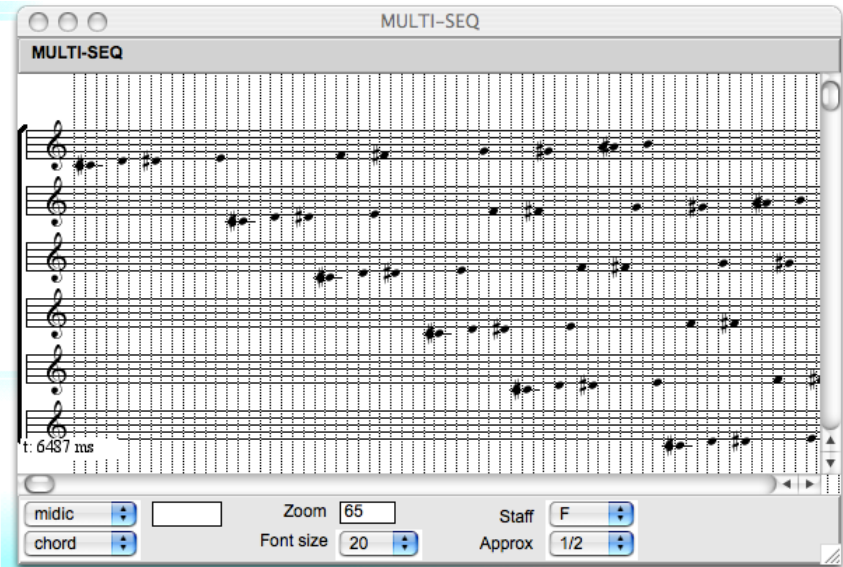
$\{Z_n\}$
 R (1 3 3 6 11 4 9 6 5 1 3 20)
 (20 3 1 5 6 9 4 11 6 3 3 1)
 (1 4 1 19 4 1 6 6 7 4 13 6)
 (6 13 4 7 6 6 1 4 19 1 4 1)
 (1 5 15 4 5 6 6 3 4 17 3 3)
 (3 3 17 4 3 6 6 5 4 15 5 1)

S (8 8 2 8 8 38)
 (16 2 14 2 16 22)
 (14 8 10 8 14 18)

$\{O_n\}$
 R (1 3 3 6 11 4 9 6 5 1 3 20)
 (1 4 1 19 4 1 6 6 7 4 13 6)
 (1 5 15 4 5 6 6 3 4 17 3 3)

S (8 8 2 8 8 38)
 (16 2 14 2 16 22)
 (14 8 10 8 14 18)

$\{Af_n\}$
 R (1 3 3 6 11 4 9 6 5 1 3 20)
 (1 4 1 19 4 1 6 6 7 4 13 6)
 S (14 8 10 8 14 18)



$\mathbb{Z}/72\mathbb{Z} = R \oplus S$

Collection « Musique/ Sciences » (dir. J.-M. Bardez & M. Andreatta)



F. Lévy



G. Bloch



M. Lanza



T. Johnson

1999



18-Catalogue-Z72

Mauro Lanza

Canons de Vuza et périodicités locales



- *La descrizione del diluvio* (Ricordi, 2007-2008)

[...] Le choix des notes et des durées est fait en cherchant à souligner certaines quasi-périodicités du canon de Vuza, et cela donne à chaque voix un caractère beaucoup plus “redondant”.

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)

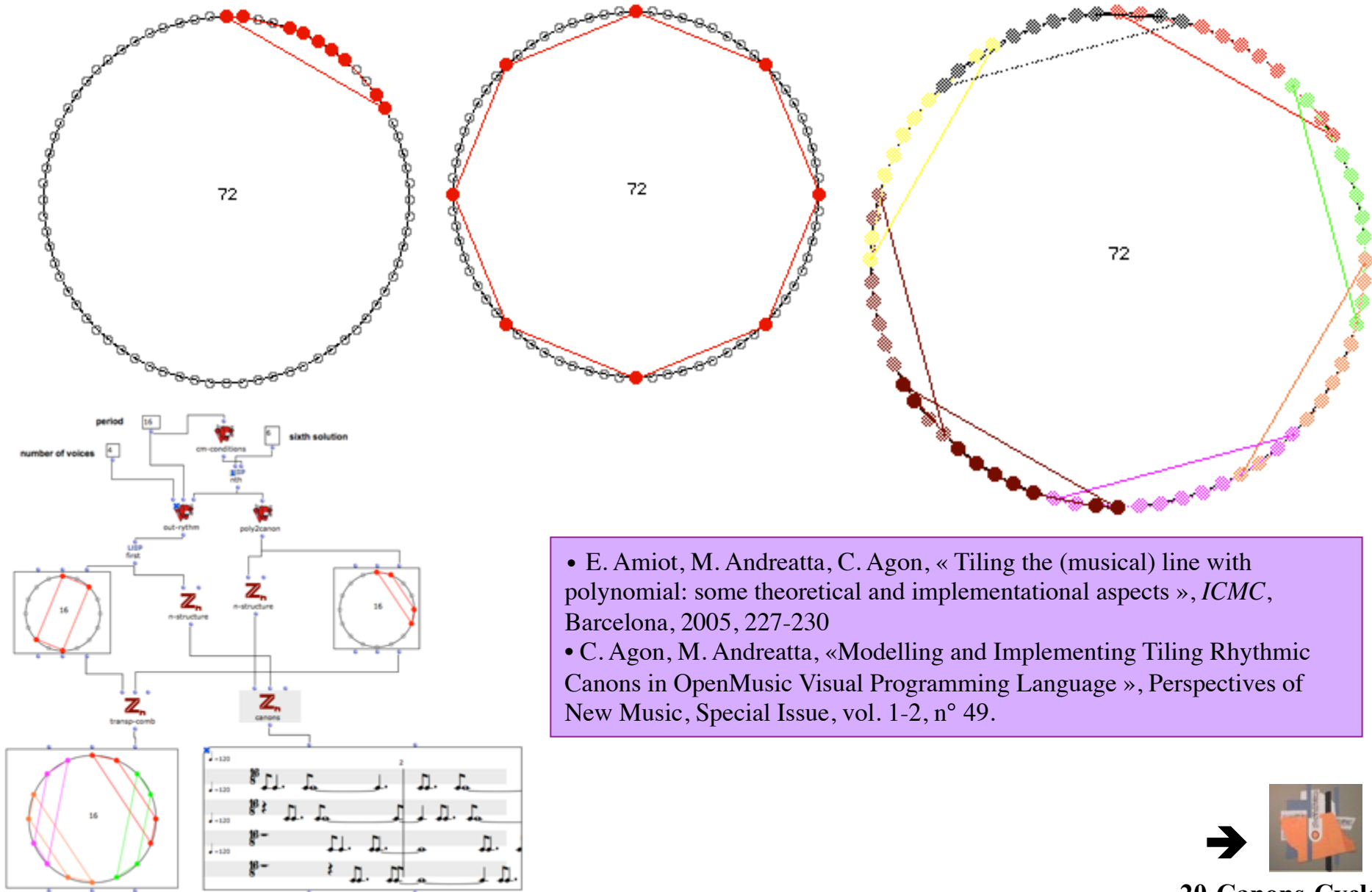
(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)



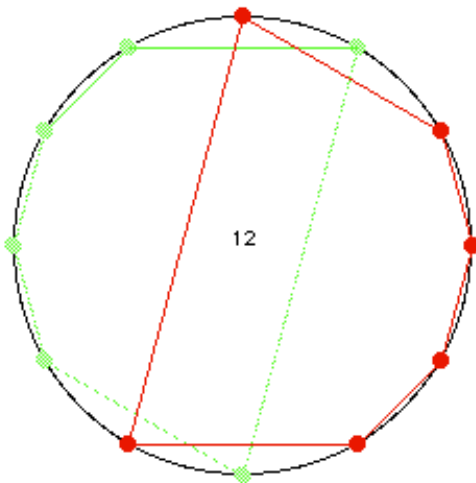
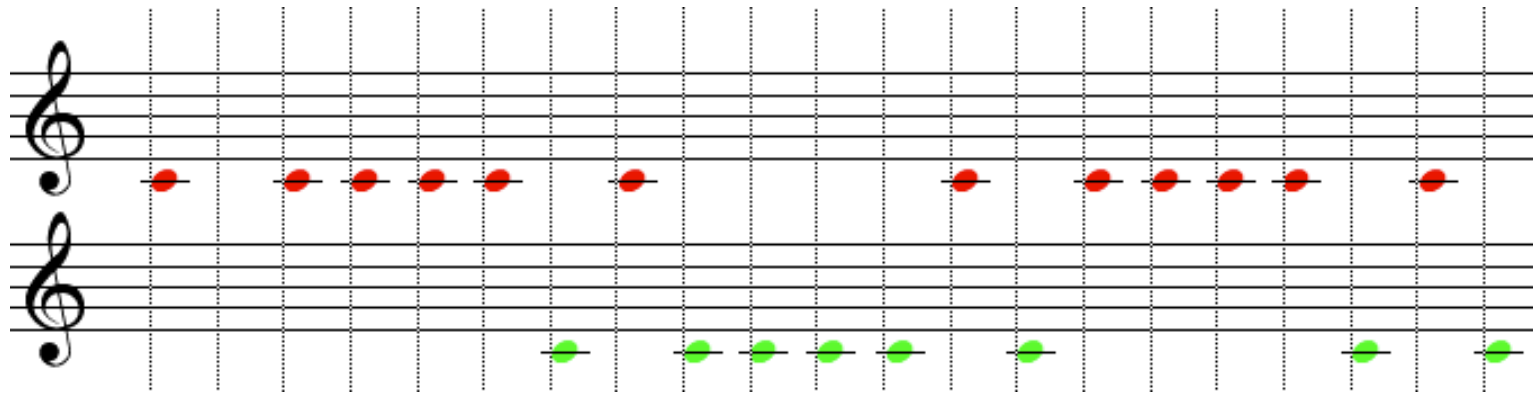
La famille des « canons cyclotomiques »



- E. Amiot, M. Andreatta, C. Agon, « Tiling the (musical) line with polynomial: some theoretical and implementational aspects », *ICMC*, Barcelona, 2005, 227-230
- C. Agon, M. Andreatta, «Modelling and Implementing Tiling Rhythmic Canons in OpenMusic Visual Programming Language », *Perspectives of New Music*, Special Issue, vol. 1-2, n° 49.



Canons mosaïques par translation et augmentation

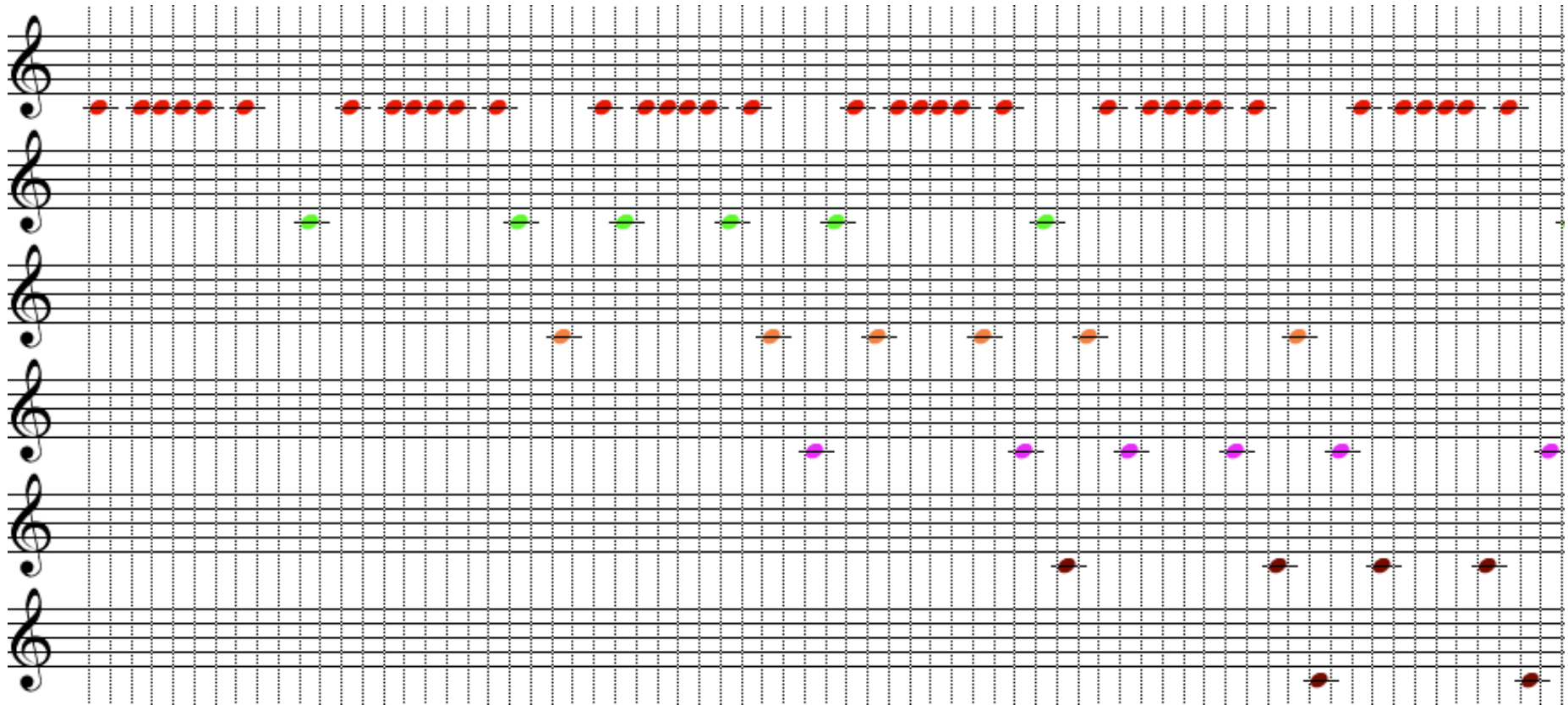


- ((0 1 2 3 4 6) ((1 11)))
- ((0 1 2 3 4 5) ((1 11) (1 1)))
- ((0 1 2 3 5 7) ((1 11) (1 7)))
- ((0 1 3 4 7 8) ((1 5)))
- ((0 1 2 3 6 7) ((1 11)))
- ((0 1 3 4 6 9) ((1 11) (1 5)))
- ((0 1 3 6 7 9) ((1 11) (1 5)))
- ((0 1 2 6 7 8) ((1 11) (1 7) (1 5) (1 1)))
- ((0 1 4 5 8 9) ((1 11) (1 7) (1 5) (1 1)))
- ((0 1 2 5 6 7) ((1 7) (1 5)))
- ((0 2 3 4 5 7) ((1 11) (1 7) (1 5) (1 1))) ←
- ((0 1 4 5 6 8) ((1 11) (1 7)))
- ((0 1 2 4 5 7) ((1 5)))
- ((0 1 3 4 5 8) ((1 5) (1 1)))
- ((0 1 2 4 5 8) ((1 11)))
- ((0 1 2 4 6 8) ((1 11) (1 7)))
- ((0 2 3 4 6 8) ((1 11)))
- ((0 2 4 6 8 10) ((1 11) (1 7) (1 5) (1 1)))



Augmented Tiling Canons ou l'action du groupe affine

(en collaboration avec Thomas Noll)



Periodic sequences and finite difference calculus

$$Df(x) = f(x) - f(x-1).$$

$$\begin{aligned}
 f &= 7 \ 11 \ 10 \ 11 \ 7 \ 2 \ 7 \ 11 \ 10 \ 11 \ 7 \ 2 \ 7 \ 11 \dots \\
 Df &= 4 \ 11 \ 1 \ 8 \ 7 \ 5 \ 4 \ 11 \ 1 \ 8 \ 7 \ 5 \ 4 \ 11 \dots \\
 D^2 f &= 7 \ 2 \ 7 \ 11 \ 10 \ 11 \ 7 \ 2 \ 7 \ 11 \ 10 \ 11 \dots \\
 D^3 f &= 7 \ 5 \ 4 \ 11 \ 1 \ 8 \ 7 \ 5 \ 4 \ 11 \ 1 \ 8 \dots \\
 D^k f &= \dots\dots
 \end{aligned}$$

V	0	3	8	7	11	0	11	10	6	9	0	9	1	2	9	8	4	3	6
VIII	0	0	0	0	3	3	7	2	0	0	0	6	3	3	3	4	8	0	0
IV	3	3	4	4	1	11	11	8	3	3	9	4	1	7	11	8	11	3	9
IX	0	0	0	0	0	3	6	[1]	3	3	3	3	9	0	3	6	[10]	6	6
IV	0	10	3	9	10	0	9	7	0	6	7	9	6	4	9	3	4	6	3

Anatol Vieru: *Zone d'oubli* pour alto (1973)

Periodic sequences and finite difference calculus

$$\begin{array}{rcl}
 f & = & 11 \ 6 \ 7 \ 2 \ 3 \ 10 \ 11 \ 6 \ \dots \\
 Df & = & \begin{array}{cccccc} & \backslash & / & \backslash & / & \backslash & / \\ & 7 & 1 & 7 & 1 & 7 & 1 \\ & \backslash & / & \backslash & / & \backslash & / \\ & 6 & 6 & 6 & 6 & 6 & \dots \end{array} \\
 D^2f & = & 6 \ 6 \ 6 \ 6 \ 6 \ \dots \\
 D^4f & = & 0 \ 0 \ 0
 \end{array}$$

Reducible sequences:
 $\exists k \geq 1$ such that $D^k f = 0$

$$\begin{array}{rcl}
 f & = & 7 \ 11 \ 10 \ 11 \ 7 \ 2 \ 7 \ 11 \ \dots \\
 Df & = & \begin{array}{cccccc} & \backslash & / & \backslash & / & \backslash & / \\ & 4 & 11 & 1 & 8 & 7 & 5 \\ & \backslash & / & \backslash & / & \backslash & / \\ & 7 & 2 & 7 & 11 & 10 & 11 \\ & \backslash & / & \backslash & / & \backslash & / \\ & 7 & 5 & 4 & 11 & 1 & 8 \end{array} \\
 D^2f & = & 7 \ 2 \ 7 \ 11 \ 10 \ 11 \ 7 \ 2 \ 7 \ \dots \\
 D^3f & = & 7 \ 5 \ 4 \ 11 \ 1 \ 8 \ 7 \ 5 \ 4 \ 11 \ 1 \ 8 \ \dots \\
 D^4f & = & 10 \ 11 \ 7 \ 2 \ 7 \ 11 \ 10 \ 11 \ \dots \\
 D^5f & = & 1 \ 8 \ 7 \ 5 \ 4 \ 11 \ 1 \ 8 \ \dots \\
 D^6f & = & 7 \ 11 \ 10 \ 11 \ 7 \ 2 \ 7 \ 11 \ \dots
 \end{array}$$

Reproducible sequences:
 $\exists k \geq 1$ such that
 $D^k f = f$

Periodic sequences and finite difference calculus



23-Decompo



$$Df(x) = f(x) - f(x-1).$$

7 11 10 11 7 2 7 11 10 11 7 2 7 11...
 4 11 1 8 7 5 4 11 1 8 7 5 4 11...
 7 2 7 11 10 11 7 2 7 11 10 11...
 7 5 4 11 1 8 7 5 4 11 1 8...

V	0	3	8	7	11	0	11	10	6	9	0	9	1	2	9	8	4	3	6
VIII	0	0	0	0	3	3	7	2	0	0	0	6	3	3	3	4	8	0	0
IV	0	3	0	4	4	1	11	8	3	3	3	4	1	7	11	8	11	10	9
IX	0	0	0	0	0	0	6	1	0	0	0	9	9	0	0	6	3	0	0
IV	0	10	9	9	10	0	9	7	0	6	7	9	6	4	9	9	4	6	9

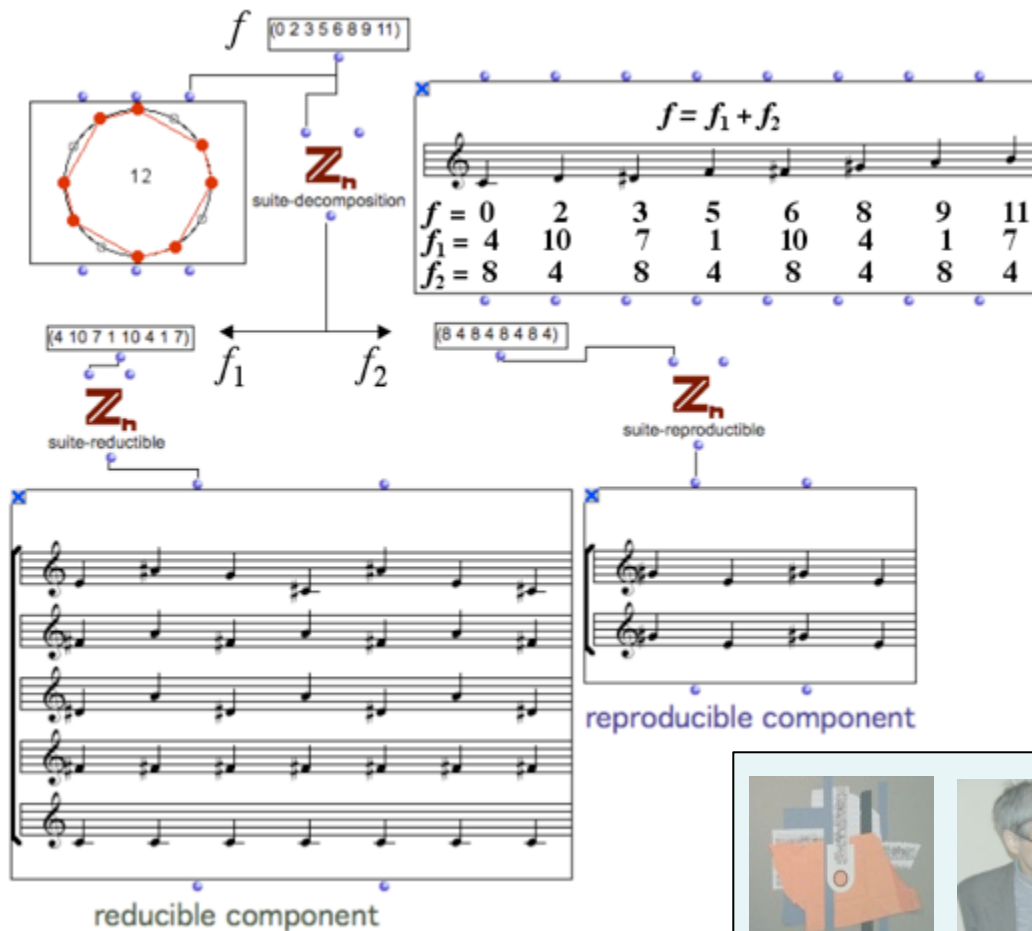
Anatol Vieru: *Zone d'oubli* for viola (1973)

Reducible sequences:

$$\exists k \geq 1 \text{ such that } D^k f = 0$$

Reproducible sequences:

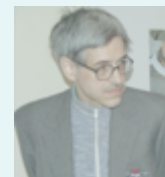
$$\exists k \geq 1 \text{ such that } D^k f = f$$



• Decomposition theorem

(Vuza & Andreatta, *Tatra M.*, 2001)

Every periodic sequence f can be decomposed in a unique way as a sum $f_1 + f_2$ of a reducible sequence f_1 and a reproducible sequence f_2



D. Vuza, M. Andreatta (2001), « On some properties of periodic sequences in Anatol Vieru's modal theory », *Tatra Mountains Mathematical Publications*, Vol. 23, p. 1-15