

Une introduction à l'analyse musicale computationnelle

ATIAM, UE MSV

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Qu'est-ce que la musicologie computationnelle ?

• Overview

- Walter B. Hewlett & Eleanor Selfridge-Field: « Computing in Musicology, 1966-91 », *Computers and the Humanities*, 25, p. 381-392, 1991
- Marc Leman et Albrecht Schneider, « Origin and Nature of Cognitive and Systematic Musicology : An Introduction », dans Marc Leman (eds), 1997, p. 13-29
- Richard Parncutt, « Systematic Musicology and the History of Western Musical Scholarship », *Journal of Interdisciplinary Music Studies*, 2007, 1(1), p. 1-32

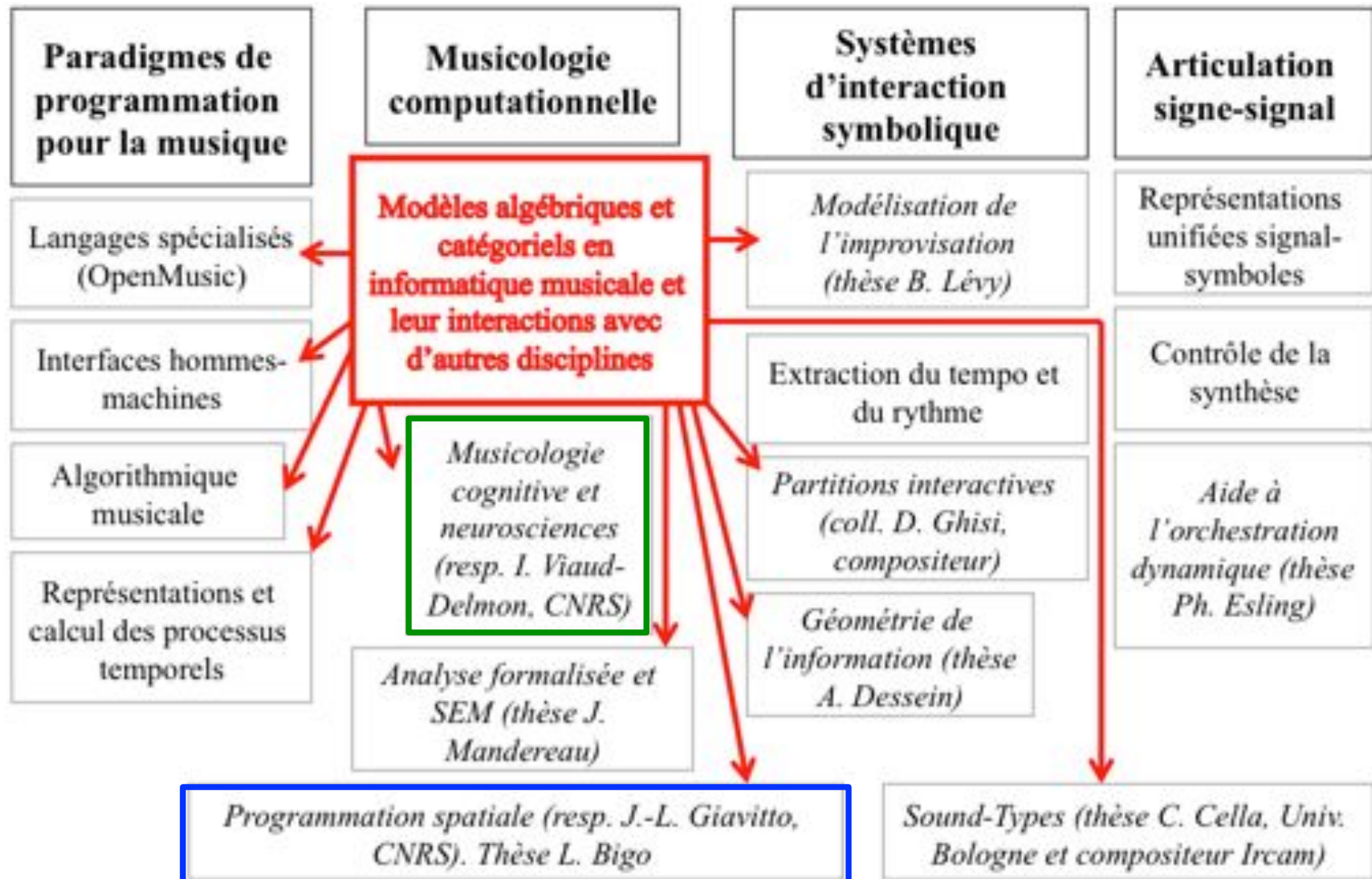
• Ouvrages de référence

- Marc Leman (ed.), *Music, Gestalt and Computing. Studies in Cognitive and Systematic Musicology*, Springer, 1997
- Eric Clarke et Nicholas Cook (eds), *Empirical Musicology. Aims, Methods, Prospects*, Oxford University Press, 2004
- André Riotte & Marcel Mesnage: *Formalismes et modèles musicaux* (2 volumes), Collection « Musique/Sciences », Ircam-Delatour France, 2006

• Quelques travaux universitaires (à l'Ircam)

- Benoit Mathieu, *Outils informatiques d'analyse musicale*, DEA, ENST Brest / Ircam, 2002
- Moreno Andreatta, *Méthodes algébriques en musique et musicologie du XXe siècle : aspects théoriques, analytiques et compositionnels*, thèse de doctorat, EHESS/Ircam, 2003
- Olivier Lartillot, *Un système d'analyse musicale computationnelle suivant une modélisation cognitive de l'écoute*, thèse, UPMC/Ircam, 2004
- Yun-Kang Ahn, *L'analyse musicale computationnelle*, thèse, UPMC/Ircam, 2009

La musicologie computationnelle au sein de RepMus : le projet MISA



La place des mathématiques dans la musicologie systématique

Guido Adler : « Umfang, Methode und Ziel der Musikwissenschaft » (1885)

| II. Systematisch. | | | |
|---|--|---|---|
| Aufstellung der in den einzelnen Zweigen der Tonkunst zuhöchst stehenden Gesetze. | | | |
| A. Erforschung und Begründung derselben in der | B. Aesthetik der Tonkunst. | C. Musikalische Pädagogik und Didaktik | D. Musikologie (Untersuchung und Vergleichung zu ethnographischen Zwecken). |
| 1. <i>Harmonik</i> (tonal od. tonlich). 2. <i>Rhythmik</i> (temporär oder zeitlich). 3. <i>Melik</i> (Cohärenz von tonal und temporär). | 1. Vergleichung und Werthschätzung der Gesetze und deren Relation mit den apperzipirenden Subjecten behufs Feststellung der <i>Kriterien des musikalisch Schönen</i> . 2. Complex unmittelbar und mittelbar damit zusammenhängender Fragen. | (Zusammenstellung der Gesetze mit Rücksicht auf den Lehrzweck) 1. Tonlehre, 2. Harmonielehre, 3. Kontrapunkt, 4. Compositionslehre, 5. Instrumentationslehre, 6. Methoden des Unterrichtes im Gesang und Instrumentalspiel. | |
| Hilfwissenschaften: Akustik und Mathematik. Physiologie (Tonempfindungen). Psychologie (Tonvorstellungen, Tonurtheile und Tongefühle). Logik (das musikalische Denken). Grammatik, Metrik und Poetik. Pädagogik Ästhetik etc. | | | |

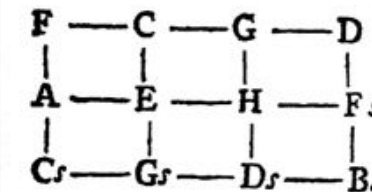
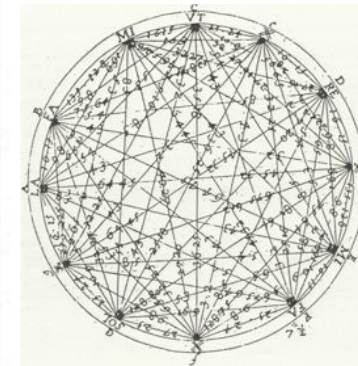
« La deuxième grande partie de la musicologie est la partie systématique; cette partie se base sur la partie historique. (...) L'accent de l'observation réside dans l'analogie de la méthode musicologique avec la méthode scientifique ».

Un court survol historique

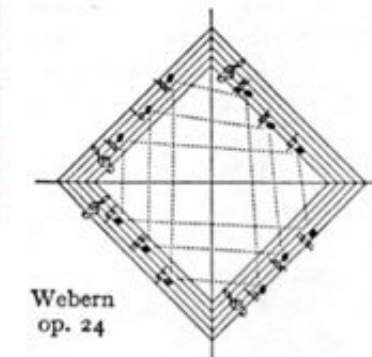


Mersenne,
Harmonicorum
Libri XII, 1648

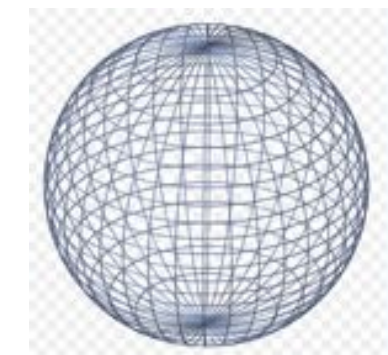
| MUSIQUE | MATHS |
|--|---|
| 500 av. J. C. Relation hauteur/longueur corde. La musique est source d'inspiration pour la théorie des nombres et la géométrie. | Nombres naturels et rationnels |
| 300 a.J. Invention (théorique) de la gamme chromatique tempérée égale par Aristoxène de Tarente) et prémonition de la théorie des groupes . Isomorphismes entre les logarithmes (intervalles musicaux) et les exponentiels (longueur d'une corde) | Aucune relation. |
| 1000 Invention de la représentation bidimensionnelle des hauteurs | Aucune correspondance |
| 1500 Aucune reprise des concepts précédents | Nombres négatifs. Construction des rationnels |
| 1600 Aucune relation | Nombres réels et les logarithmes |
| Marin Mersenne (1588-1648) : combinatoire musicale | Calcul des probabilités |
| 1700 La fugue comme un automate abstrait. Manipulation inconsciente du groupe de Klein | Nombres complexes (Euler, Gauss), les quaternions (Hamilton), continuité (Cauchy), structure de groupe (Galois, Abel) |
| Leonhard Euler : Speculum Musicum [1773] | Théorie des graphes |
| 1900 Libération de la prison de la tonalité (Loquin, Hauer, Schoenberg) | Nombres infinis et transfinis (Cantor). Axiomatique de Peano. Théorie de la mesure (Lebesgue, Borel) |
| 1920 Formalisation radicale des macrostructures à travers le système sériel (Schoenberg) | Aucun développement de la théorie des nombres. |
| Ernst Krenek (1900-1991) : les axiomes dans le système dodécaphonique | David Hilbert, Les fondements de la géométrie (1899) |



Euler : *Speculum musicum*, 1773



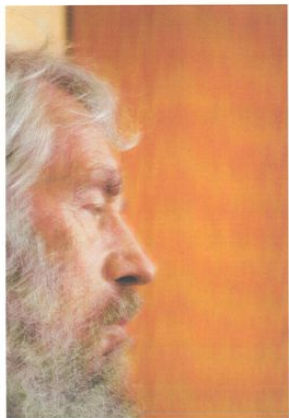
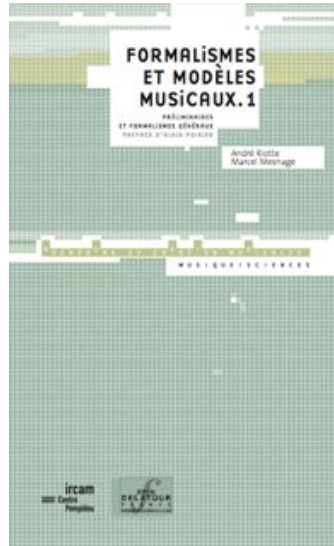
Webern
op. 24



Iannis Xenakis, *Musique. Architecture*, Tournai, Casterman, 1971, 176 p. (New, revised edition: Tournai, Casterman, 1976, 238 p.)

L'analyse formalisée ou les entités formelles en musique

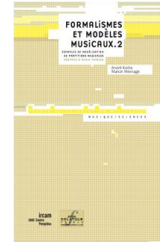
André Riotte e Marcel Mesnage



- « Anamorphoses » d'André Riotte
- « La terrasse des audiences du clair de lune » de Claude Debussy : esquisse d'analyse modélisée
- La mise en évidence de régularités locales : le « Mode de valeurs et d'intensités » de Messiaen
- Un exemple d'invention structurelle : le « Mikrokosmos » de Béla Bartok
- Un modèle informatique de la « Pièce pour quatuor à cordes » n°1 de Stravinsky
- Les « Variations pour piano », op. 27, d'Anton Werbern
- L'« Invention à deux voix » n°1 de J.-S. Bach
- Un modèle informatique du « Troisième Regard sur l'Enfant Jésus » d'Olivier Messiaen
- Un modèle de la « Valse sentimentale », Op. 50, n°13, de Franz Schubert
- Un automate musical construit à partir d'une courte pièce de Béla Bartok (Mikrokosmos n°39)



« Entités formelles pour l'analyse musicale » *Marcel Mesnage (1998)*



A. Schoenberg : *Klavierstück Op. 33a*, 1929

The image displays a musical score for A. Schoenberg's *Klavierstück Op. 33a*, 1929, in 4/4 time. The score is presented in two systems of staves. The first system consists of a treble clef staff and a bass clef staff. The second system consists of a bass clef staff and a treble clef staff. The score is annotated with several colored boxes and arrows:

- Blue rounded rectangles highlight specific chordal structures in both systems.
- Yellow diamonds highlight specific chordal structures in both systems.
- Green dotted boxes highlight specific chordal structures in both systems.
- Blue dashed arrows point from the blue rounded rectangles to the first, second, and fifth matrices below.
- Yellow dashed arrows point from the yellow diamonds to the third and fourth matrices below.
- Green dashed arrows point from the green dotted boxes to the first and sixth matrices below.

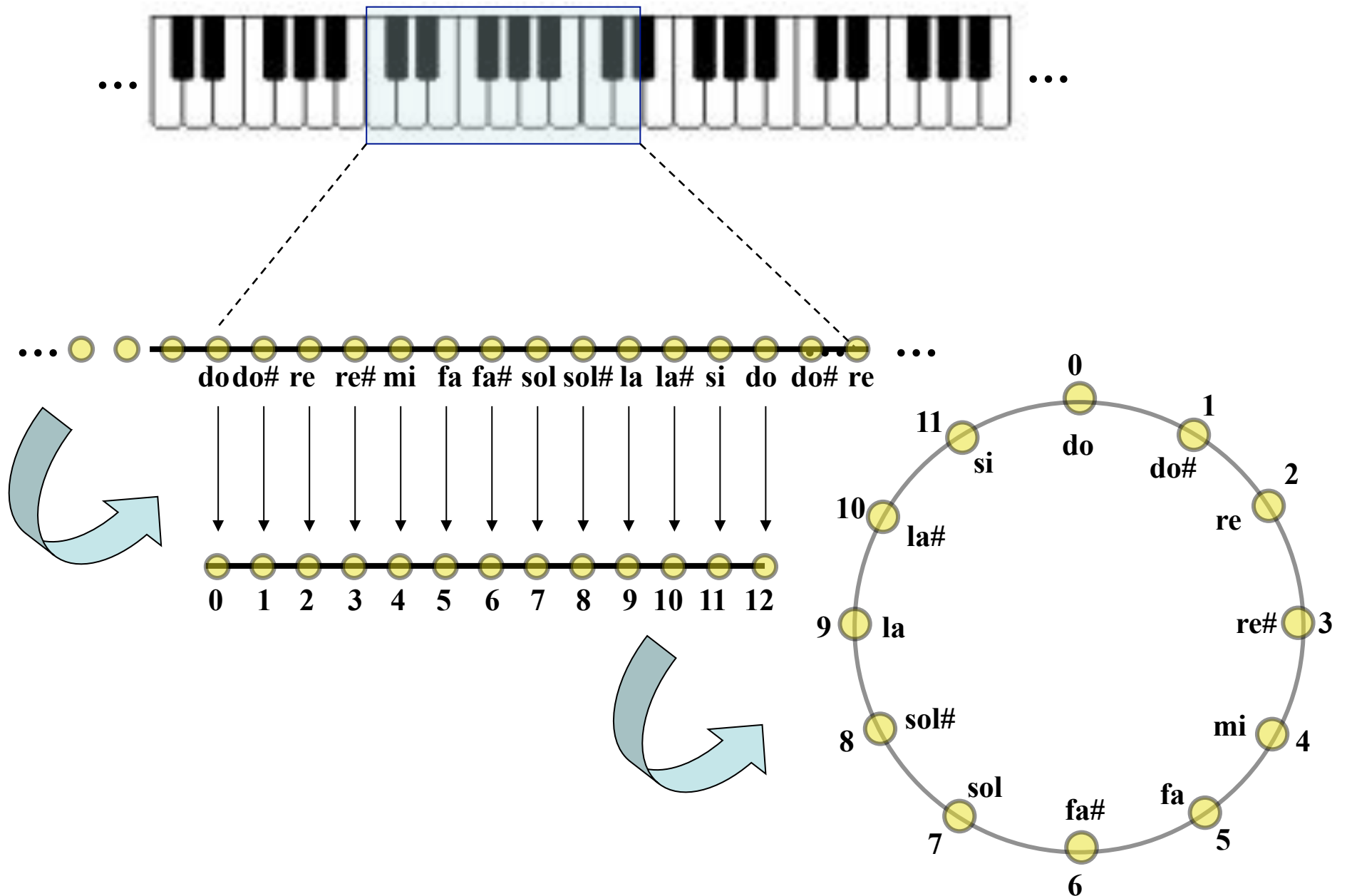
Below the score, six twelve-tone matrices are shown, each with a set of numbers (0-11) arranged in a circle and a set of numbers (1-6) arranged in a line below it. The matrices are:

| | | | | | |
|---------------------|-----------------------|-------------------------|--------------------------|-----------------------|---------------------|
| 0-5511 (1 2 5 6) | 9-4233 (2 3 4 5 6) | 8-6231 (1 2 3 4 5 6) | 11-6132 (1 2 3 4 5 6) | 0-4332 (2 3 4 5 6) | 3-5511 (1 2 5 6) |
|---------------------|-----------------------|-------------------------|--------------------------|-----------------------|---------------------|

Arrows at the bottom indicate transformations between the matrices:

- A large arrow labeled T_3 spans from the first matrix to the second.
- An arrow labeled T_1I spans from the second matrix to the third.
- An arrow labeled T_1I spans from the third matrix to the fourth.
- An arrow labeled T_1I spans from the fourth matrix to the fifth.
- An arrow labeled T_1I spans from the fifth matrix to the sixth.

Reduction à l'octave et congruence modulo 12



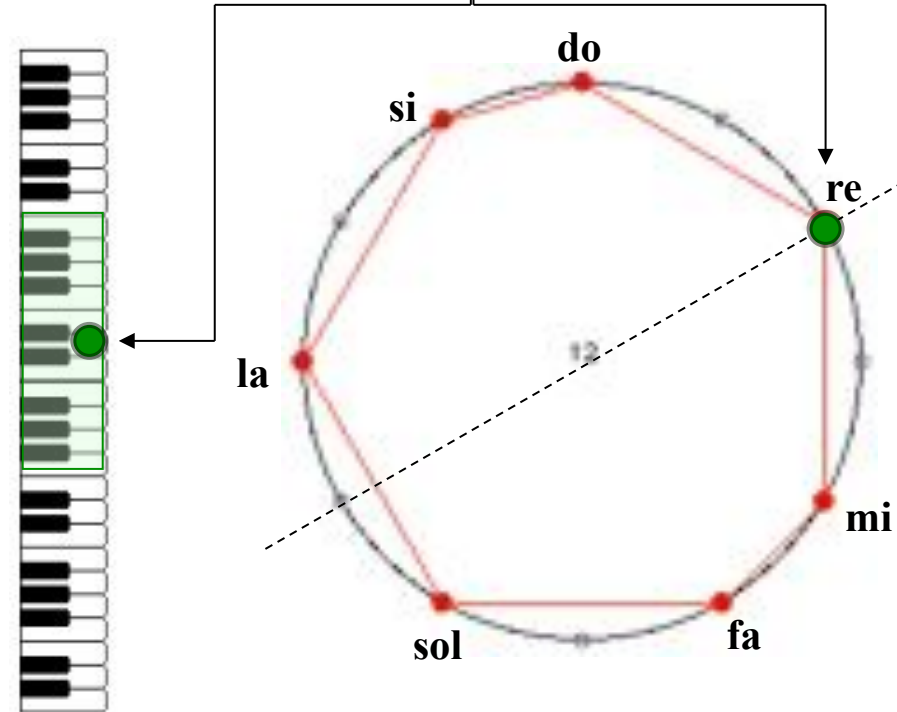
Un peu d'histoire...



Camille Durutte:

- *Technie, ou lois générales du système harmonique* (1855)
- *Résumé élémentaire de la Technie harmonique, et complément de cette Technie* (1876)

| | | | | | | | | | | | | |
|------------------|-----------------|-------|----|----|-----|----|----|----|----|-------|-----------------|-----------------|
| Sol ^b | Re ^b | | Fa | Ut | Sol | Re | La | Mi | Si | | Re ^x | La ^x |
| -15 | -14 | | -3 | -2 | -1 | 0 | +1 | +2 | +3 | | +14 | +15 |



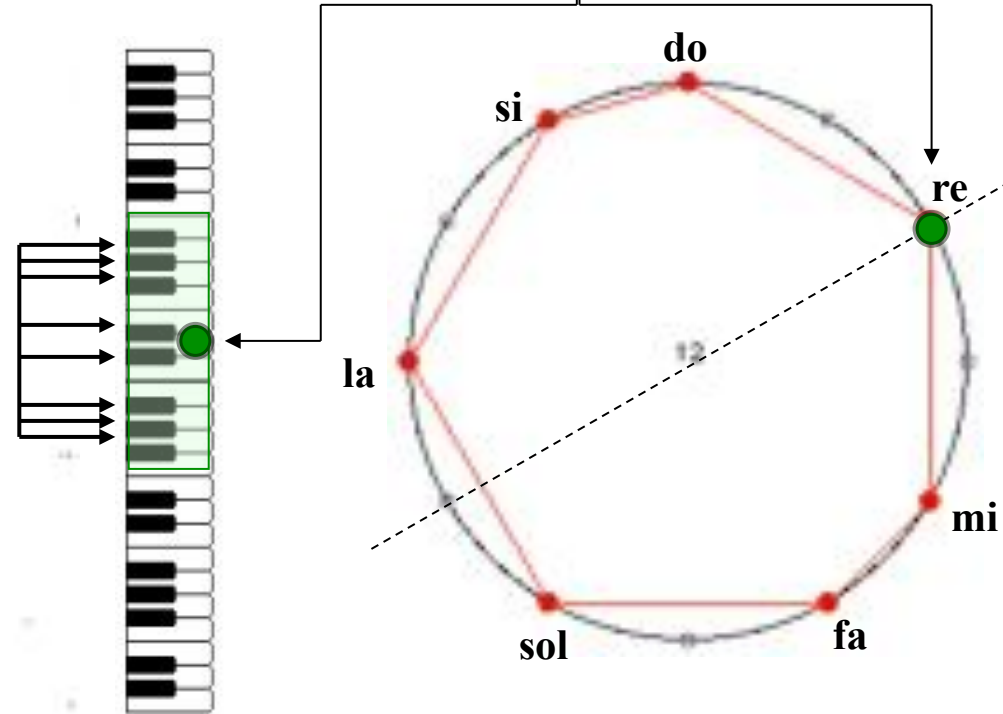
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| | | | | | | | | | | | | |
|------------------|-----------------|-------|----|----|-----|----|----|----|----|-------|-----------------|-----------------|
| Sol ^b | Re ^b | | Fa | Ut | Sol | Re | La | Mi | Si | | Re ^x | La ^x |
| -15 | -14 | | -3 | -2 | -1 | 0 | +1 | +2 | +3 | | +14 | +15 |



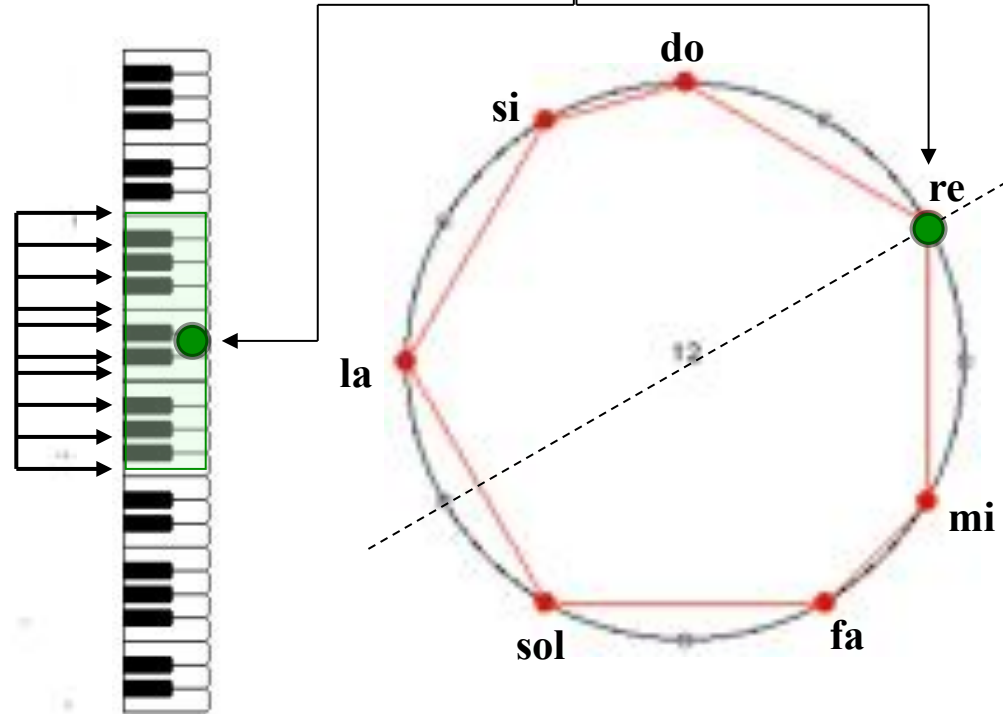
Un peu d'histoire...



Camille Durutte:

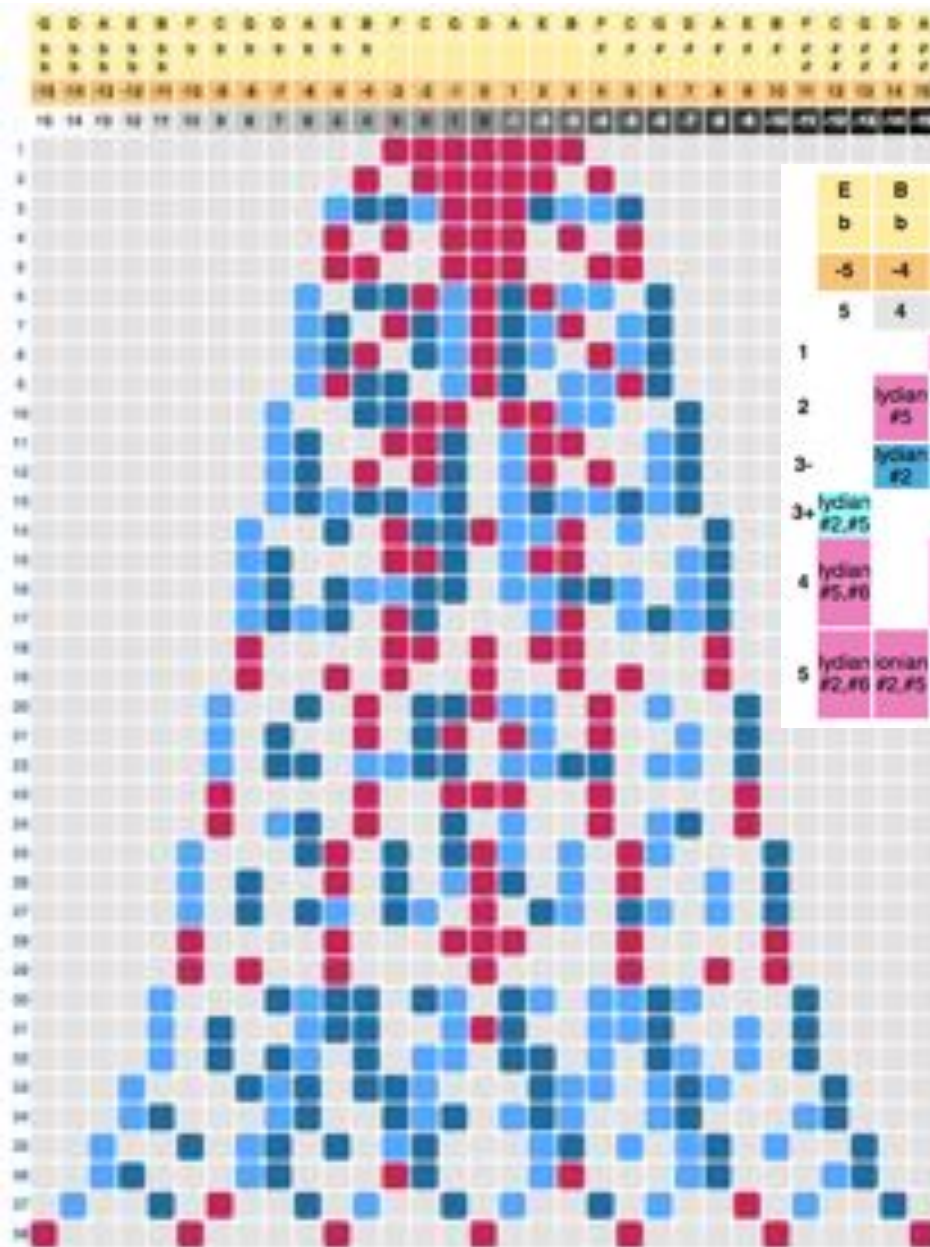
- *Technie, ou lois générales du système harmonique* (1855)
- *Résumé élémentaire de la Technie harmonique, et complément de cette Technie* (1876)

| | | | | | | | | | | | | |
|------------------|-----------------|-------|----|----|-----|----|----|----|----|-------|-----------------|-----------------|
| Sol ^b | Re ^b | | Fa | Ut | Sol | Re | La | Mi | Si | | Re ^x | La ^x |
| -15 | -14 | | -3 | -2 | -1 | 0 | +1 | +2 | +3 | | +14 | +15 |



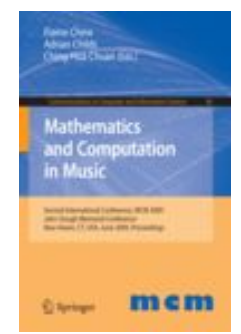
La cloche diatonique centrée autour du ré (P. Audétat & co.)

<http://www.cloche-diatonique.ch/>

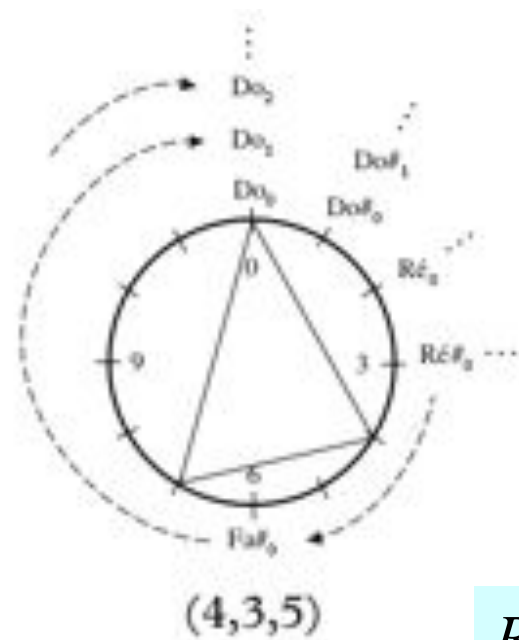


| | E | B | F | C | G | D | A | E | B | F | C | |
|----|--------------|--------------|------------------|-----------|--------------|------------------------------|------------------|-----------|--------------|-----------------|-----------------|-----------------|
| | b | b | | | | | | | | # | # | |
| | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | |
| | 5 | 4 | 3 | 2 | 1 | 0 | -1 | -2 | -3 | -4 | -5 | |
| 1 | | | lydian | ionian | mixolydian | dorian | aeolian | phrygian | locrian | | | diatonic |
| 2 | lydian #5 | | | lydian b7 | ionian b3 | mixolydian b5 or aeolian 3 | phrygian 6 | locrian 2 | | locrian b4 | | minor melodic |
| 3 | | lydian #2 | augmented ionian | | dorian #4 | aeolian 7 | phrygian 3 | locrian 6 | | | locrian b4,bb7 | minor harmonic |
| 3+ | lydian #2,#5 | | | lydian b3 | ionian b6 | mixolydian b2 | dorian b5 | | phrygian b4 | locrian b6,7 | | major harmonic |
| 4 | lydian #5,#6 | | lydian #5,b7 | | lydian b6,b7 | ionian b2,b3 or phrygian 6,7 | locrian 2,3 | | locrian 2,b4 | | locrian bb3,b4 | unisonic |
| 5 | lydian #2,#6 | ionian #2,#5 | | | aeolian #4,7 | ionian b2,b6 or phrygian 3,7 | mixolydian b2,b5 | | | phrygian b4,bb7 | locrian bb3,bb7 | double harmonic |

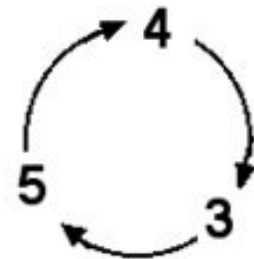
Junod, J., Audétat, P., Agon, C., Andreatta, M., « A Generalisation of Diatonicism and the Discrete Fourier Transform as a Mean for Classifying and Characterising Musical Scales », Second International Conference MCM 2009, vol. 38, New Haven, 2009, pp. 166-179



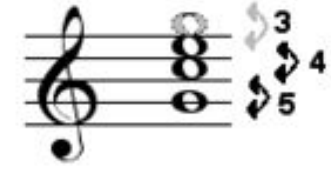
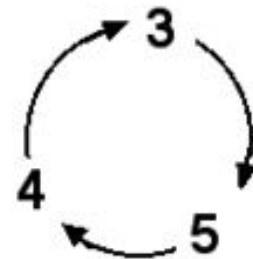
Représentation circulaire et structure intervallique



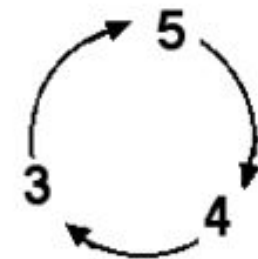
(4 3 5)



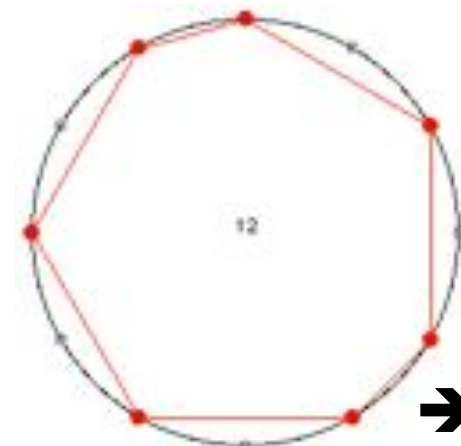
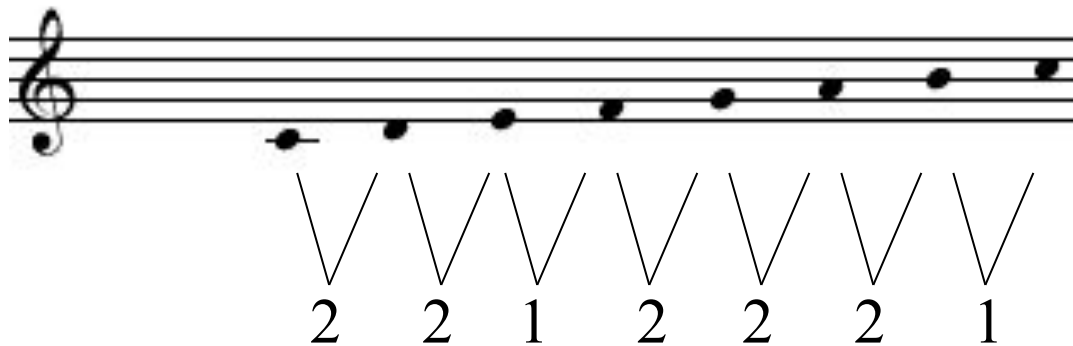
(3 5 4)



(5 4 3)

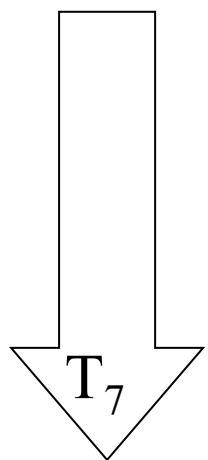
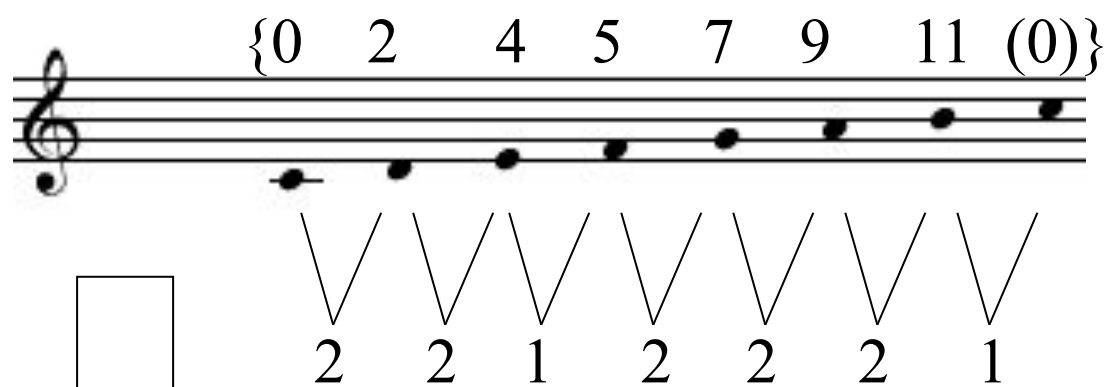


Renversements = permutations circulaires de la structure intervallique

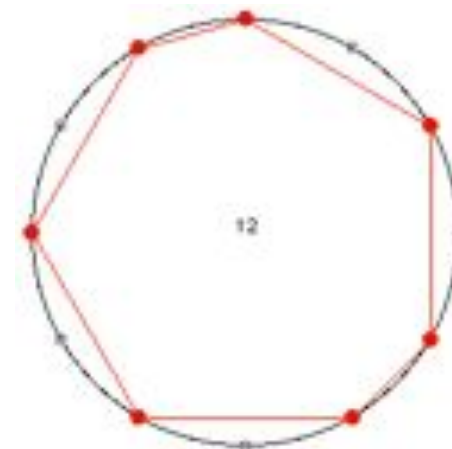
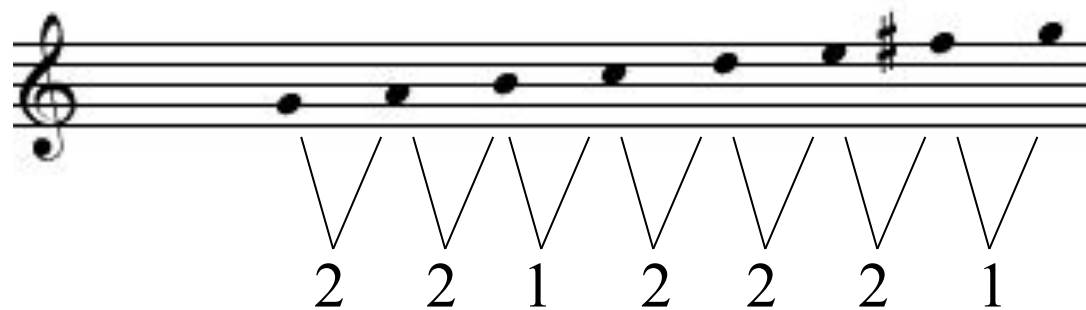


01-circular-representation

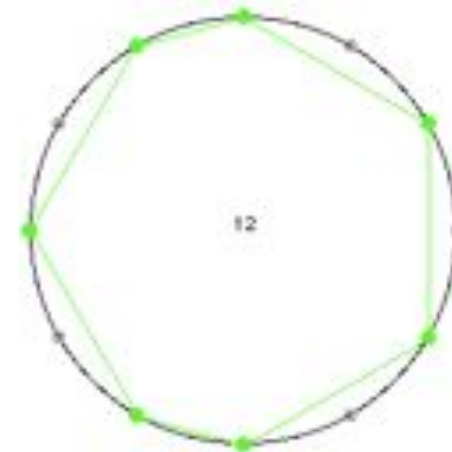
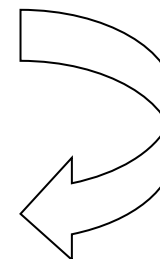
Transformations géométriques : la transposition



$T_7(x) = 7 + x \pmod{12}$




$\alpha = 210^\circ$



Equivalence modulo la transposition


Transformations géométriques : l'inversion

{0 2 4 5 7 9 11 (0)}



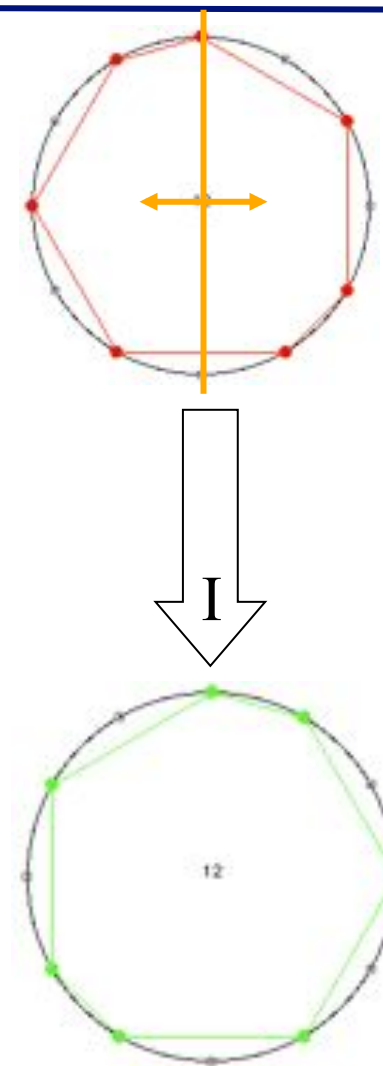
2 2 1 2 2 2 1

$I(x) = -x \pmod{12}$



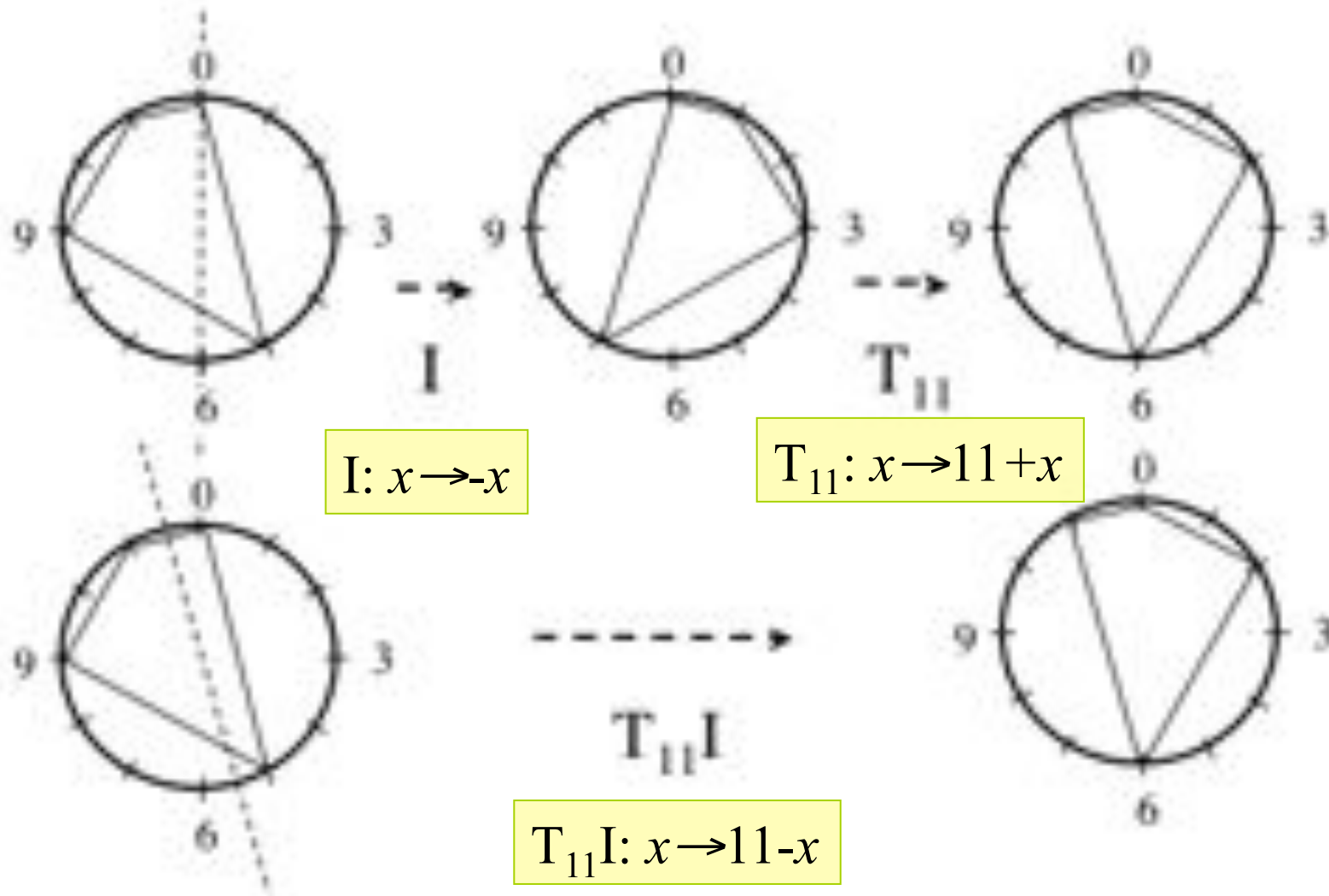
2 1 2 2 1 2 2

The diagram illustrates the inversion of a musical scale. The top staff shows a scale with notes corresponding to pitch classes {0, 2, 4, 5, 7, 9, 11} and intervals {2, 2, 1, 2, 2, 2, 1}. A large arrow labeled 'I' points down to the inverted scale. The bottom staff shows the inverted scale with notes corresponding to pitch classes {1, 10, 8, 7, 5, 4, 2} and intervals {2, 1, 2, 2, 1, 2, 2}. A light blue box contains the formula $I(x) = -x \pmod{12}$.



*Equivalence modulo
l'inversion*

La Set Theory: équivalence modulo transposition/inversion

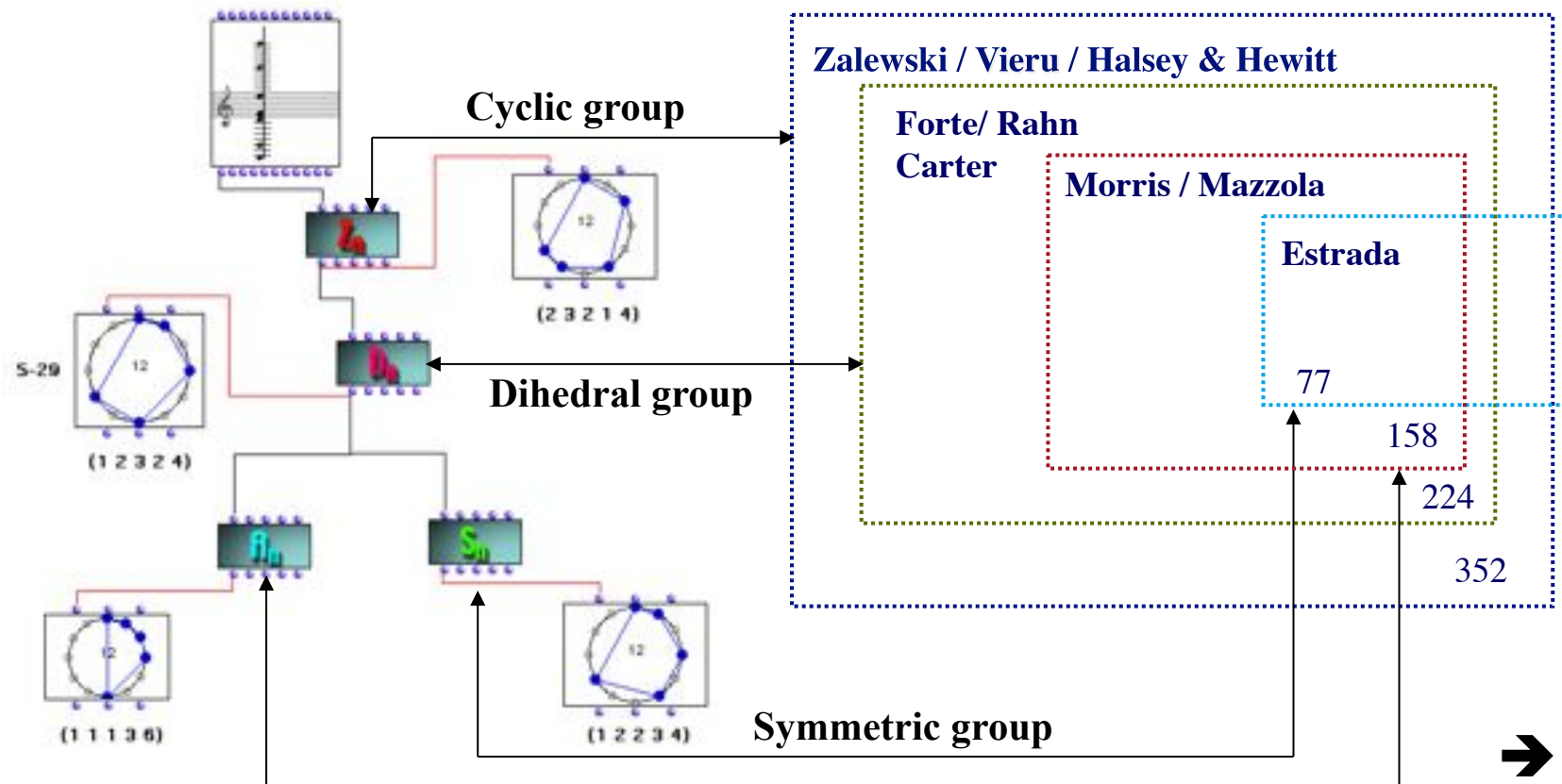


$$\{0, 5, 9, 11\} \longrightarrow \{11, 6, 3, 0\}$$

Enumeration of musical structures

| $G \setminus k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-----------------|---|---|----|----|----|----|----|----|----|----|----|----|
| C_{12} | 1 | 6 | 19 | 43 | 66 | 80 | 66 | 43 | 19 | 6 | 1 | 1 |
| D_{12} | 1 | 6 | 12 | 29 | 38 | 50 | 38 | 29 | 12 | 6 | 1 | 1 |
| $Aff_1(Z_{12})$ | 1 | 5 | 9 | 21 | 25 | 34 | 25 | 21 | 9 | 5 | 1 | 1 |

Set Theory



Paradigmatic architecture

Affine group



07-paradigms

Exercice : retrouver les symétries dans une série (I)

Schoenberg: Serenade Op.24, Mouvement 5

Diagram showing a musical staff with four groups of notes highlighted by red boxes. Below each group are three downward-pointing arrows and a set of three ellipses in curly braces, representing a sequence of notes.

Diagram showing four circles, each with a '12' in the center. An arrow points from each of the four groups of notes in the musical staff above to one of the circles, indicating a mapping or transformation.

(... , ... , ...)

Exercice : retrouver les symétries dans une série (I)

Schoenberg: Serenade Op.24, Mouvement 5

The image displays a musical staff with four groups of notes, each enclosed in a red box. Below each group are three downward-pointing arrows leading to a set of three numbers in curly braces. Below these are four chromatic circles, each with a '12' in the center and a polygonal shape drawn on its circumference. Arrows point from the first two sets of numbers to the first two circles, and from the last two sets to the last two circles. The first two circles have red polygons, and the last two have green polygons.

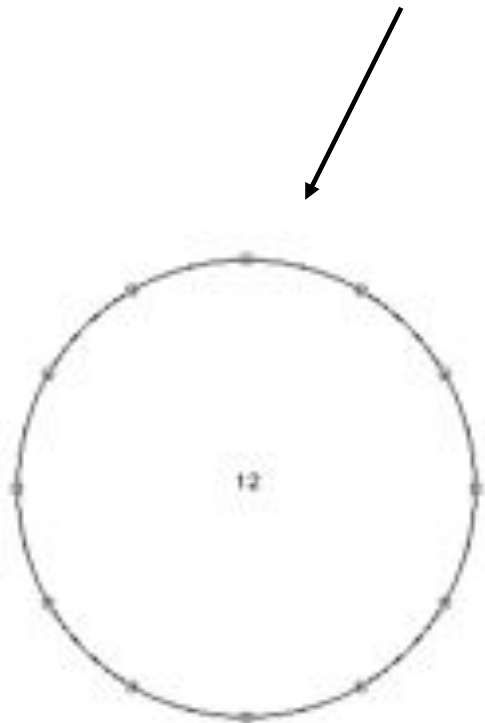
$\{9, 10, 0\}$ $\{3, 4, 6\}$ $\{5, 7, 8\}$ $\{11, 1, 2\}$

$(1, 2, 9)$ $(1, 2, 9)$ $(2, 1, 9)$ $(2, 1, 9)$

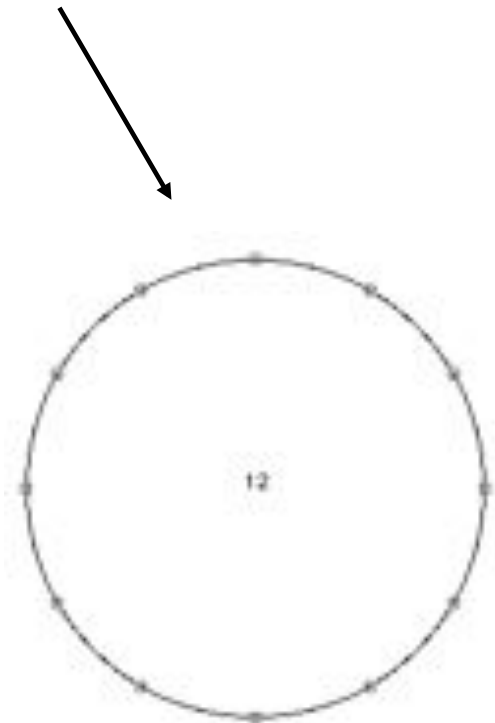
Exercice : retrouver les symétries dans une série

Schoenberg: Serenade Op.24, Mouvement 5

The image shows a musical staff with a treble clef. Two segments of the series are highlighted with red boxes. The first box covers the first six notes, and the second box covers the last six notes. Below each box, a vertical arrow points to a set of six dots enclosed in curly braces, representing the pitch classes of that segment.



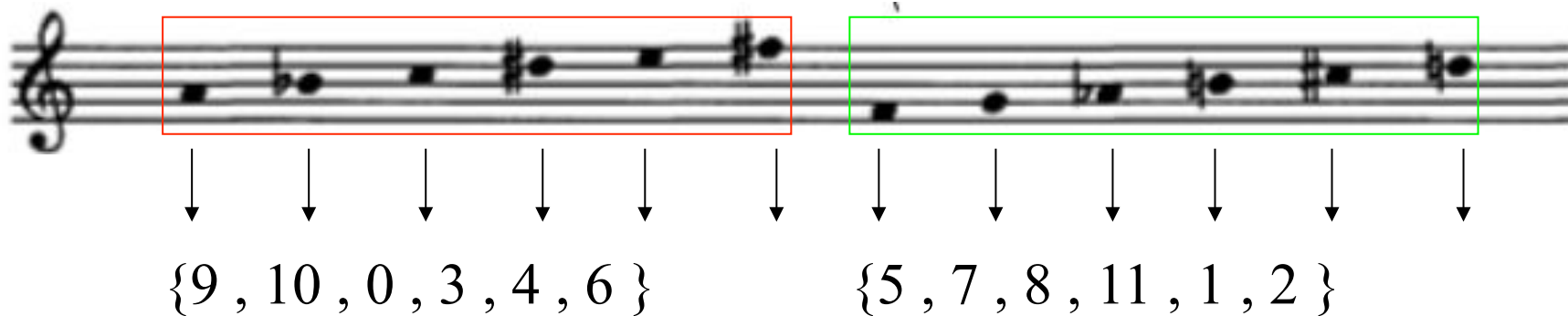
(... , ... , ... , ... , ... , ...)



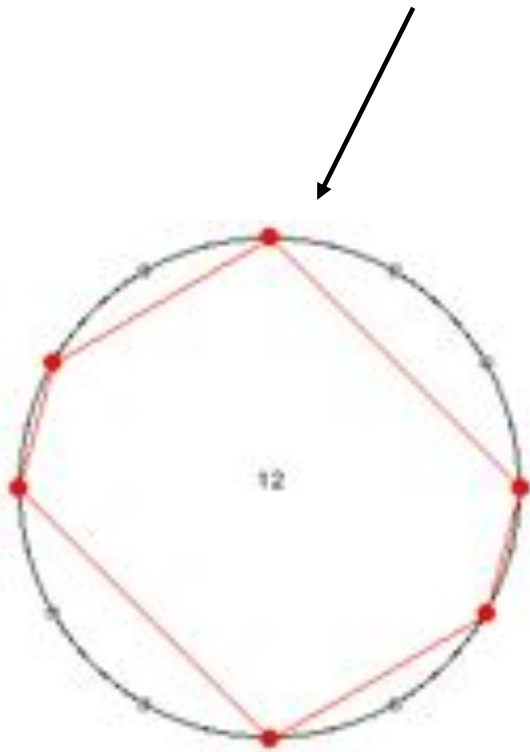
(... , ... , ... , ... , ... , ...)

Exercice : retrouver les symétries dans une série

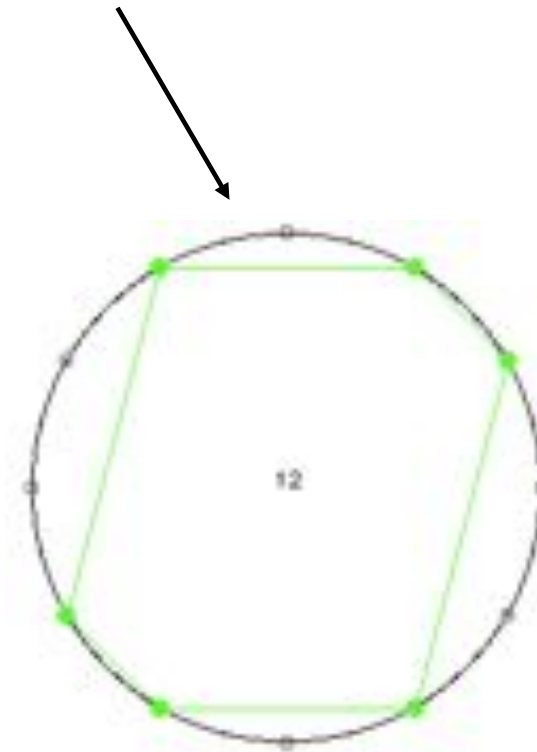
Schoenberg: Serenade Op.24, Mouvement 5



$\{9, 10, 0, 3, 4, 6\}$ $\{5, 7, 8, 11, 1, 2\}$



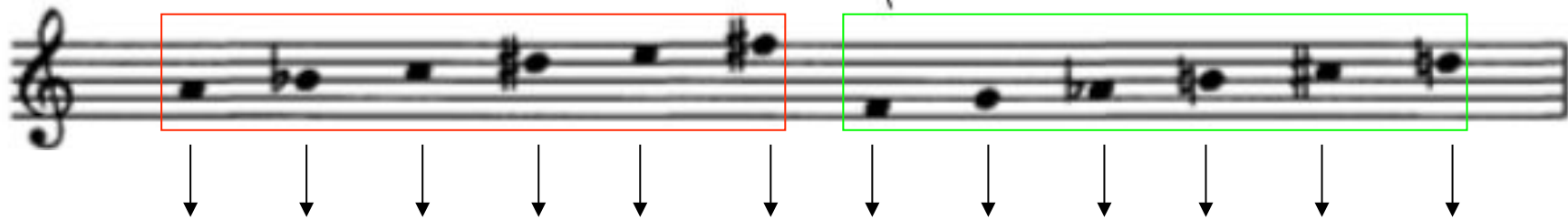
(3, 1, 2, 3, 1, 2)



(2, 1, 3, 2, 1, 3)

“Combinatorialité” et symétrie par transposition

Schoenberg: Serenade Op.24, Mouvement 5

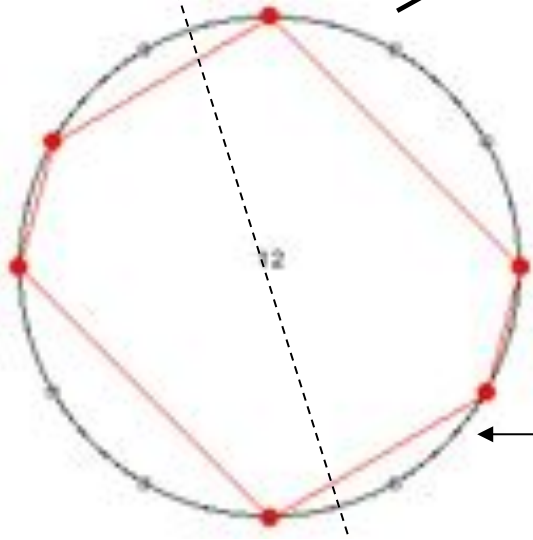


$$A = \{9, 10, 0, 3, 4, 6\} \quad \{5, 7, 8, 11, 1, 2\} = A'$$

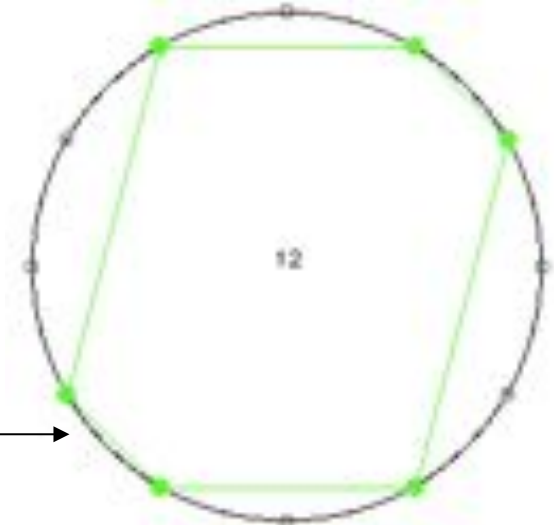
$$\begin{aligned} T_6\{9,10,0,3,4,6\} &= \\ &= \{6+9, 6+10, 6, 6+3, 6+4, 6+6\} = \\ &= \{3, 4, 6, 9, 10, 0\} \end{aligned}$$

$$T_6(A) = A$$

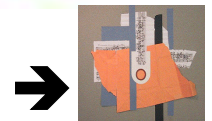
$$I_{11} = T_{11} I$$



(3, 1, 2, 3, 1, 2)

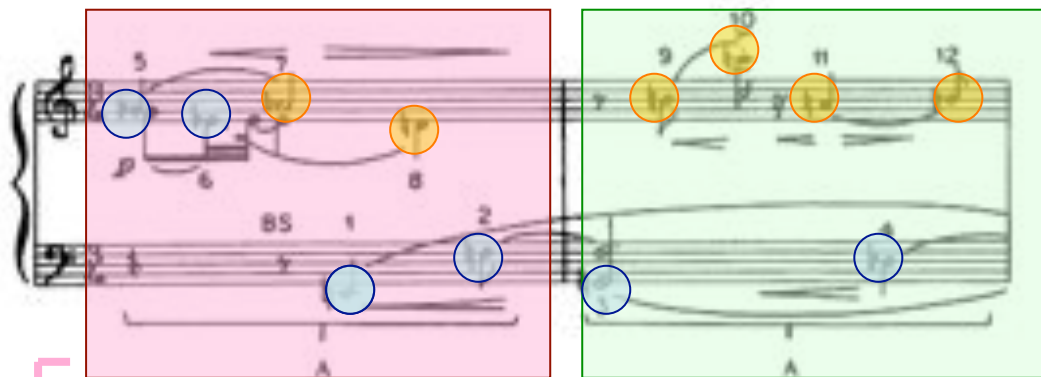


(2, 1, 3, 2, 1, 3)



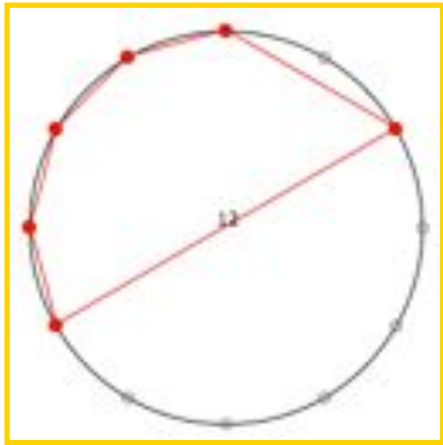
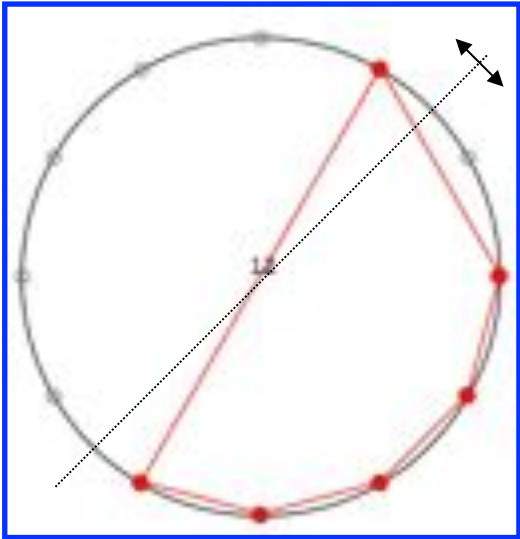
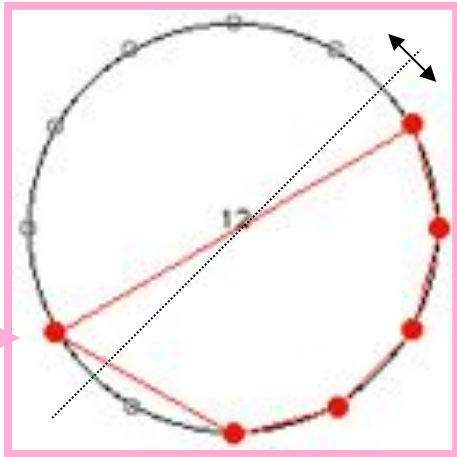
Serialisme and hexachordal combinatoriality

Schoenberg: Suite Op.25, Minuetto



A musical score for piano, showing two staves. The upper staff has notes 5, 6, 7, 8, 9, 10, 11, 12. The lower staff has notes 1, 2, 3, 4. Some notes are circled in blue and orange. The score is divided into two sections: a pink-shaded section on the left and a green-shaded section on the right.


Double combinatoriality

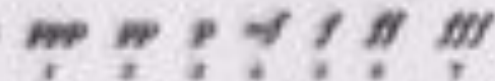


« Sérialisme intégrale » chez Messiaen...

- Mode de valeurs et d'intensités (1950)

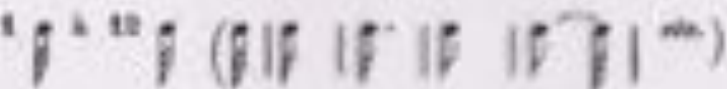
Ce morceau utilise un mode de hauteurs (16 sons), de valeurs (24 durées), d'attaques (32 attaques), et d'intensités (7 nuances). Il est entièrement écrit dans le mode.

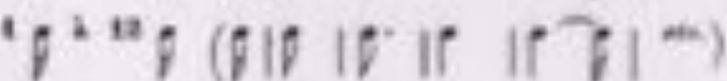
Attaches: 
(avec l'attaque normale, sans signe, cela fait 32.)

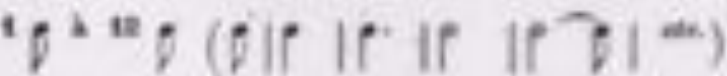
Intensités: 

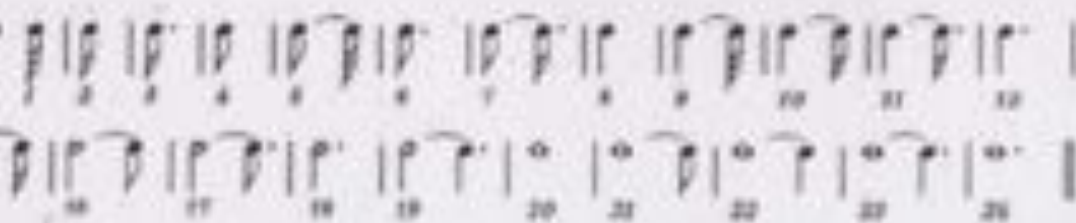
Sons: Le mode se partage en 3 Divisions ou ensembles mélodiques de 12 sons, s'étendant chacun sur plusieurs octaves, et croisés entre eux. Tous les sons de même son sont différents comme hauteur, comme valeur, et comme intensité.

Valeurs:

Division I: durées chromatiques de 1 

Division II: durées chromatiques de 1 


Division III: durées chromatiques de 1 

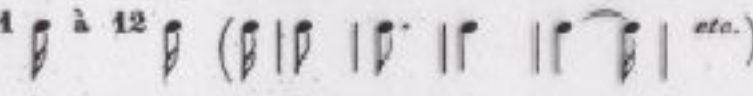
Au total 24 durées: 

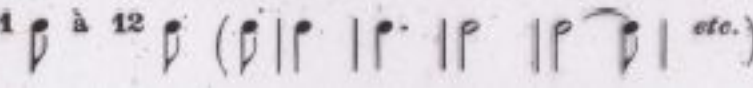


« Sérialisme intégrale » chez Messiaen...

- Mode de valeurs et d'intensités (1950)

Division I: durées chromatiques de 1 à 12 ( etc.)

Division II: durées chromatiques de 1 à 12 ( etc.)

Division III: durées chromatiques de 1 à 12 ( etc.)



Voici le mode:

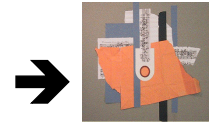
I 
(la Division I est utilisée dans la portée supérieure du Piano)

II 
(la Division II est utilisée dans la portée médiane du Piano)

III 
(la Division III est utilisée dans la portée inférieure du Piano)

...et « combinatorialité » d'hexacordes

- Mode de valeurs et d'intensités (1950)



03-messiaen

Modéré

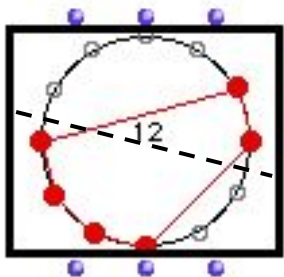
PIANO



Voici le mode:

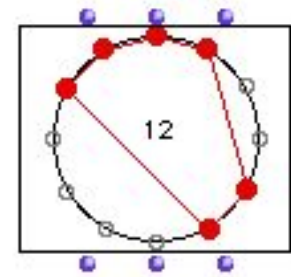
I

(la Division I est utilisée dans la portée supérieure du Piano)



$$\{3,2,9,8,7,6\} \longrightarrow \{4,5,10,11,0,1\}$$

$$T_7I : x \rightarrow 7-x$$



Procédés algorithmiques dans le sérialisme intégrale

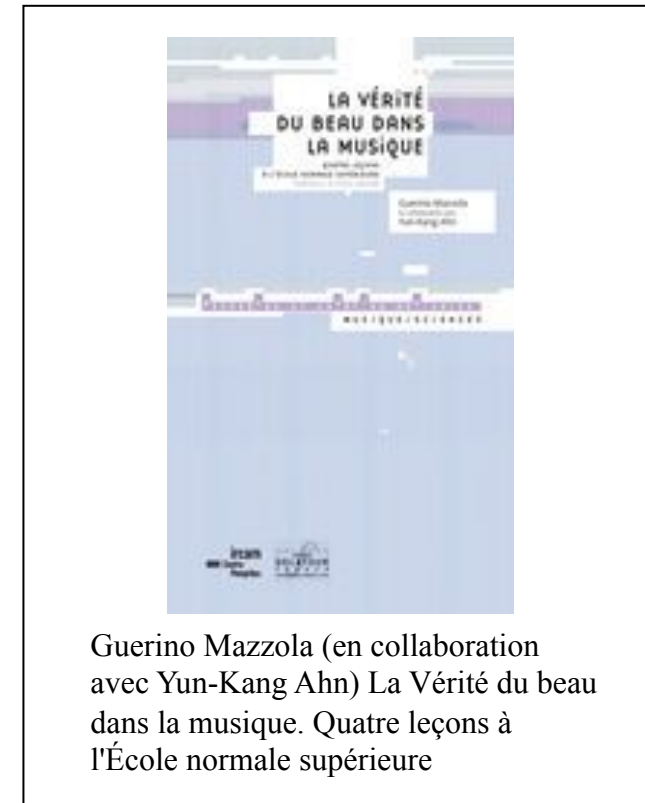
- *Structures 1a* pour deux piano de Boulez (1952/1961)

| S | | | | | | | | | | | | I | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 1 | 7 | 3 | 10 | 12 | 9 | 2 | 11 | 6 | 4 | 8 | 5 |
| 3 | 8 | 4 | 5 | 6 | 11 | 1 | 9 | 12 | 3 | 7 | 10 | 7 | 11 | 10 | 12 | 9 | 8 | 1 | 6 | 5 | 3 | 2 | 4 |
| 3 | 4 | 1 | 2 | 8 | 9 | 10 | 5 | 6 | 7 | 12 | 11 | 3 | 10 | 1 | 7 | 11 | 6 | 4 | 12 | 9 | 2 | 5 | 8 |
| 4 | 5 | 2 | 8 | 9 | 12 | 3 | 6 | 11 | 1 | 10 | 7 | 10 | 12 | 7 | 11 | 6 | 5 | 3 | 9 | 8 | 1 | 4 | 2 |
| 5 | 8 | 6 | 9 | 12 | 10 | 4 | 11 | 7 | 2 | 3 | 1 | 12 | 9 | 11 | 6 | 5 | 4 | 10 | 8 | 2 | 7 | 3 | 1 |
| 6 | 11 | 9 | 12 | 10 | 3 | 5 | 7 | 1 | 8 | 4 | 2 | 6 | 8 | 6 | 5 | 4 | 3 | 12 | 2 | 1 | 11 | 10 | 7 |
| 7 | 1 | 10 | 3 | 4 | 5 | 11 | 2 | 9 | 12 | 6 | 9 | 2 | 1 | 4 | 3 | 10 | 12 | 8 | 7 | 11 | 5 | 9 | 6 |
| 8 | 9 | 5 | 6 | 11 | 7 | 2 | 12 | 10 | 4 | 1 | 3 | 11 | 6 | 12 | 9 | 8 | 2 | 7 | 5 | 4 | 10 | 1 | 3 |
| 9 | 12 | 6 | 11 | 7 | 1 | 8 | 10 | 3 | 5 | 2 | 4 | 6 | 5 | 9 | 8 | 2 | 1 | 11 | 4 | 3 | 12 | 7 | 10 |
| 10 | 3 | 7 | 1 | 2 | 8 | 12 | 4 | 5 | 11 | 9 | 6 | 4 | 3 | 2 | 1 | 7 | 11 | 5 | 10 | 12 | 8 | 6 | 9 |
| 11 | 7 | 12 | 10 | 3 | 4 | 6 | 1 | 2 | 9 | 5 | 8 | 8 | 2 | 5 | 4 | 3 | 10 | 9 | 1 | 7 | 6 | 12 | 11 |
| 12 | 10 | 11 | 7 | 1 | 2 | 9 | 3 | 4 | 6 | 8 | 5 | 5 | 4 | 8 | 2 | 1 | 7 | 6 | 3 | 10 | 9 | 11 | 12 |

FIGURE 4.3 – Les matrices S et I servant de base à la composition de Boulez.



FIGURE 4.4 – Les trois premières lignes de la matrice S détaillées.



Guerino Mazzola (en collaboration avec Yun-Kang Ahn) *La Vérité du beau dans la musique. Quatre leçons à l'École normale supérieure*

- G. Ligeti, « Pierre Boulez. Decision and automatism in Structure 1a », *Die Reihe*, vol. IV, pp. 36-62, 1959
- Y.- Kang Ahn, « L'analyse musicale computationnelle », thèse, Paris 6/Ircam, 2009

Procédés algorithmiques dans le sérialisme intégrale

- *Structures 1a* pour deux piano de Boulez (1952/1961)

| S | | | | | | | | | | | | I | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 1 | 7 | 3 | 10 | 12 | 9 | 2 | 11 | 6 | 4 | 8 | 5 |
| 3 | 8 | 4 | 5 | 6 | 11 | 1 | 9 | 12 | 3 | 7 | 10 | 7 | 11 | 10 | 12 | 9 | 8 | 1 | 6 | 5 | 3 | 2 | 4 |
| 3 | 4 | 1 | 2 | 8 | 9 | 10 | 5 | 6 | 7 | 12 | 11 | 3 | 10 | 1 | 7 | 11 | 6 | 4 | 12 | 8 | 2 | 5 | 8 |
| 4 | 5 | 2 | 8 | 9 | 12 | 3 | 6 | 11 | 1 | 10 | 7 | 10 | 12 | 7 | 11 | 6 | 5 | 3 | 9 | 8 | 1 | 4 | 2 |
| 5 | 8 | 6 | 9 | 12 | 10 | 4 | 11 | 7 | 2 | 3 | 1 | 12 | 9 | 11 | 6 | 5 | 4 | 10 | 8 | 2 | 7 | 3 | 1 |
| 6 | 11 | 9 | 12 | 10 | 3 | 5 | 7 | 1 | 8 | 4 | 2 | 9 | 8 | 6 | 5 | 4 | 3 | 12 | 2 | 1 | 11 | 10 | 7 |
| 7 | 1 | 10 | 3 | 4 | 5 | 11 | 2 | 9 | 12 | 8 | 9 | 2 | 1 | 4 | 3 | 10 | 12 | 8 | 7 | 11 | 5 | 8 | 6 |
| 8 | 9 | 5 | 6 | 11 | 7 | 2 | 12 | 10 | 4 | 1 | 3 | 11 | 8 | 12 | 9 | 8 | 2 | 7 | 5 | 4 | 10 | 1 | 3 |
| 9 | 12 | 6 | 11 | 7 | 1 | 8 | 10 | 3 | 5 | 2 | 4 | 6 | 5 | 9 | 8 | 2 | 1 | 11 | 4 | 3 | 12 | 7 | 10 |
| 10 | 3 | 7 | 1 | 2 | 8 | 12 | 4 | 5 | 11 | 9 | 6 | 4 | 3 | 2 | 1 | 7 | 11 | 5 | 10 | 12 | 8 | 6 | 9 |
| 11 | 7 | 12 | 10 | 3 | 4 | 6 | 1 | 2 | 9 | 5 | 8 | 8 | 2 | 5 | 4 | 3 | 10 | 9 | 1 | 7 | 6 | 12 | 11 |
| 12 | 10 | 11 | 7 | 1 | 2 | 9 | 3 | 4 | 6 | 8 | 5 | 5 | 4 | 8 | 2 | 1 | 7 | 6 | 3 | 10 | 9 | 11 | 12 |

FIGURE 4.3 – Les matrices S et I servant de base à la composition de Boulez.



FIGURE 4.4 – Les trois premières lignes de la matrice S détaillées.

« Rétrospectivement, je ne considère plus cette pièce comme représentative de l'œuvre de Boulez, ni même pour la conception sérielle, mais plutôt comme un paradigme de musique presque algorithmique, se rapprochant de la pensée informatique. La voie que Boulez a poursuivie après la composition du premier volume des *Structures* l'a conduit à se détourner dans ses nouvelles œuvres de tout automatisme, tout en restant attaché à l'idée de constructivisme » (Ligeti 1959)

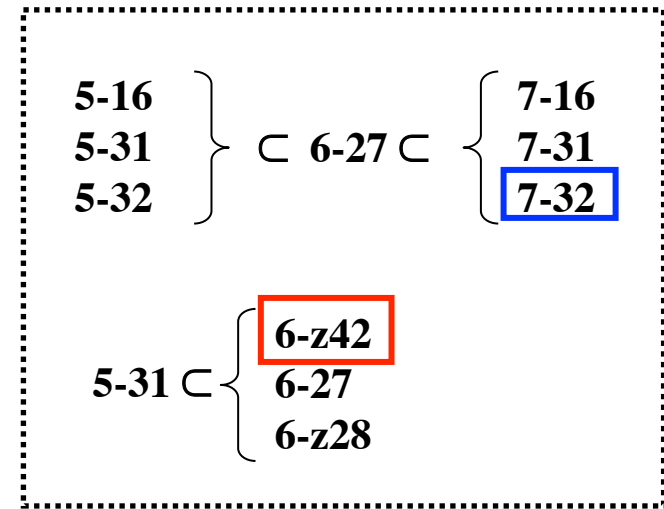
- G. Ligeti, « Pierre Boulez. Decision and automatism in Structure 1a », *Die Reihe*, vol. IV, pp. 36-62, 1959
- Y.- Kang Ahn, « L'analyse musicale computationnelle », thèse, Paris 6/Ircam, 2009

Set Theory Analysis (Allen Forte)

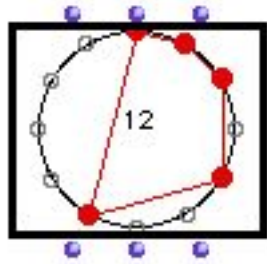
Ex. 89. *Sacrificial Dance*: R142+3

Ex. 90. *Sacrificial Dance*: R144

| name | pcs | vector | name | pcs | vector |
|-----------|-------------|--------|-----------|----------------|--------|
| 5-30 | 0,1,4,6,8 | 121321 | 7-30 | 0,1,2,4,6,8,9 | 343542 |
| 5-31 | 0,1,3,6,9 | 114112 | 7-31 | 0,1,3,4,6,7,9 | 336333 |
| 5-32 | 0,1,4,6,9 | 113221 | 7-32 | 0,1,3,4,6,8,9 | 335442 |
| 5-33(12) | 0,2,4,6,8 | 040402 | 7-33 | 0,1,2,4,6,8,10 | 262623 |
| 5-34(12) | 0,2,4,6,9 | 032221 | 7-34 | 0,1,3,4,6,8,10 | 254442 |
| 5-35(12) | 0,2,4,7,9 | 032140 | 7-35 | 0,1,3,5,6,8,10 | 254361 |
| 5-Z36 | 0,1,2,4,7 | 222121 | 7-Z36 | 0,1,2,3,5,6,8 | 444342 |
| 5-Z37(12) | 0,3,4,5,8 | 212320 | 7-Z37 | 0,1,3,4,5,7,8 | 434541 |
| 5-Z38 | 0,1,2,5,8 | 212221 | 7-Z38 | 0,1,2,4,5,7,8 | 434442 |
| 6-1(12) | 0,1,2,3,4,5 | 543210 | | | |
| 6-2 | 0,1,2,3,4,6 | 443211 | 6-Z36 | 0,1,2,3,4,7 | |
| 6-23 | 0,1,2,3,5,6 | 433221 | 6-Z37(12) | 0,1,2,3,4,8 | |
| 6-24(12) | 0,1,2,4,5,6 | 432321 | 6-Z38(12) | 0,1,2,3,7,8 | |
| 6-5 | 0,1,2,3,6,7 | 422232 | | | |
| 6-Z6(12) | 0,1,2,5,6,7 | 421242 | | | |
| 6-7(6) | 0,1,2,6,7,8 | 420243 | | | |
| 6-8(12) | 0,2,3,4,5,7 | 343230 | | | |
| 6-9 | 0,1,2,3,5,7 | 342231 | | | |
| 6-Z10 | 0,1,3,4,5,7 | 333321 | 6-Z39 | 0,2,3,4,5,8 | |
| 6-Z11 | 0,1,2,4,5,7 | 333231 | 6-Z40 | 0,1,2,3,5,8 | |
| 6-Z12 | 0,1,2,4,6,7 | 332232 | 6-Z41 | 0,1,2,3,6,8 | |
| 6-Z13(12) | 0,1,3,4,6,7 | 324222 | 6-Z42(12) | 0,1,2,3,6,9 | |



PCS Theory: interval vector and Z-relation



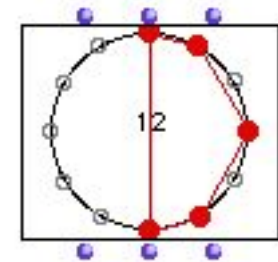
| | | |
|-----------|-------------|--------|
| 5-30 | 0,1,4,6,8 | 121321 |
| 5-31 | 0,1,3,6,9 | 114112 |
| 5-32 | 0,1,4,6,9 | 113221 |
| 5-33(12) | 0,2,4,6,8 | 040402 |
| 5-34(12) | 0,2,4,6,9 | 032221 |
| 5-35(12) | 0,2,4,7,9 | 032140 |
| 5-Z36 | 0,1,2,4,7 | 222121 |
| 5-Z37(12) | 0,3,4,5,8 | 212320 |
| 5-Z38 | 0,1,2,5,8 | 212221 |
| 6-1(12) | 0,1,2,3,4,5 | 543210 |
| 6-2 | 0,1,2,3,4,6 | 443211 |

| | | |
|-----------|-------------|--------|
| 5-Z36 | 0,1,2,4,7 | 222121 |
| 6-Z4(12) | 0,1,2,4,5,6 | 432321 |
| 6-5 | 0,1,2,3,6,7 | 422232 |
| 6-Z6(12) | 0,1,2,5,6,7 | 421242 |
| 6-7(6) | 0,1,2,6,7,8 | 420243 |
| 6-8(12) | 0,2,3,4,5,7 | 343230 |
| 6-9 | 0,1,2,3,5,7 | 342231 |
| 6-Z10 | 0,1,3,4,5,7 | 333321 |
| 6-Z11 | 0,1,2,4,5,7 | 333231 |
| 6-Z12 | 0,1,2,4,6,7 | 332232 |
| 6-Z13(12) | 0,1,3,4,6,7 | 324222 |

| | | |
|-------|----------------|--------|
| 7-30 | 0,1,2,4,6,8,9 | 343542 |
| 7-31 | 0,1,3,4,6,7,9 | 336333 |
| 7-32 | 0,1,3,4,6,8,9 | 335442 |
| 7-33 | 0,1,2,4,6,8,10 | 262623 |
| 7-34 | 0,1,3,4,6,8,10 | 254442 |
| 7-35 | 0,1,3,5,6,8,10 | 254361 |
| 7-Z36 | 0,1,2,3,5,6,8 | 444342 |
| 7-Z37 | 0,1,3,4,5,7,8 | 434541 |
| 7-Z38 | 0,1,2,4,5,7,8 | 434442 |

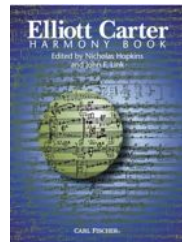
| | |
|-----------|-------------|
| 6-Z36 | 0,1,2,3,4,7 |
| 6-Z37(12) | 0,1,2,3,4,8 |
| 6-Z38(12) | 0,1,2,3,7,8 |

| | |
|-----------|-------------|
| 6-Z39 | 0,2,3,4,5,8 |
| 6-Z40 | 0,1,2,3,5,8 |
| 6-Z41 | 0,1,2,3,6,8 |
| 6-Z42(12) | 0,1,2,3,6,9 |



5-Z12

Elliott Carter : 90+ (1994)



- **Combinatoire d'accords**
 - Hexacordes
 - Tétracordes
 - Triades
 - Relation Z
- **Séries tous-intervalles**
 - *Link-chords*

 (piano: John Snijders)

90+ Elliott Carter (1994)

♩ = 96

Piano

milie e novanta auguri a caro Geffredo

(senza pedale)*

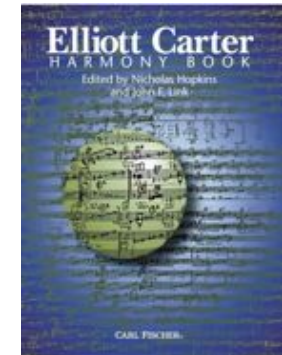
* Use pedal only to join one chord to another legato, as in mm. 1-13, 16-21, 36-43, and 45-48.

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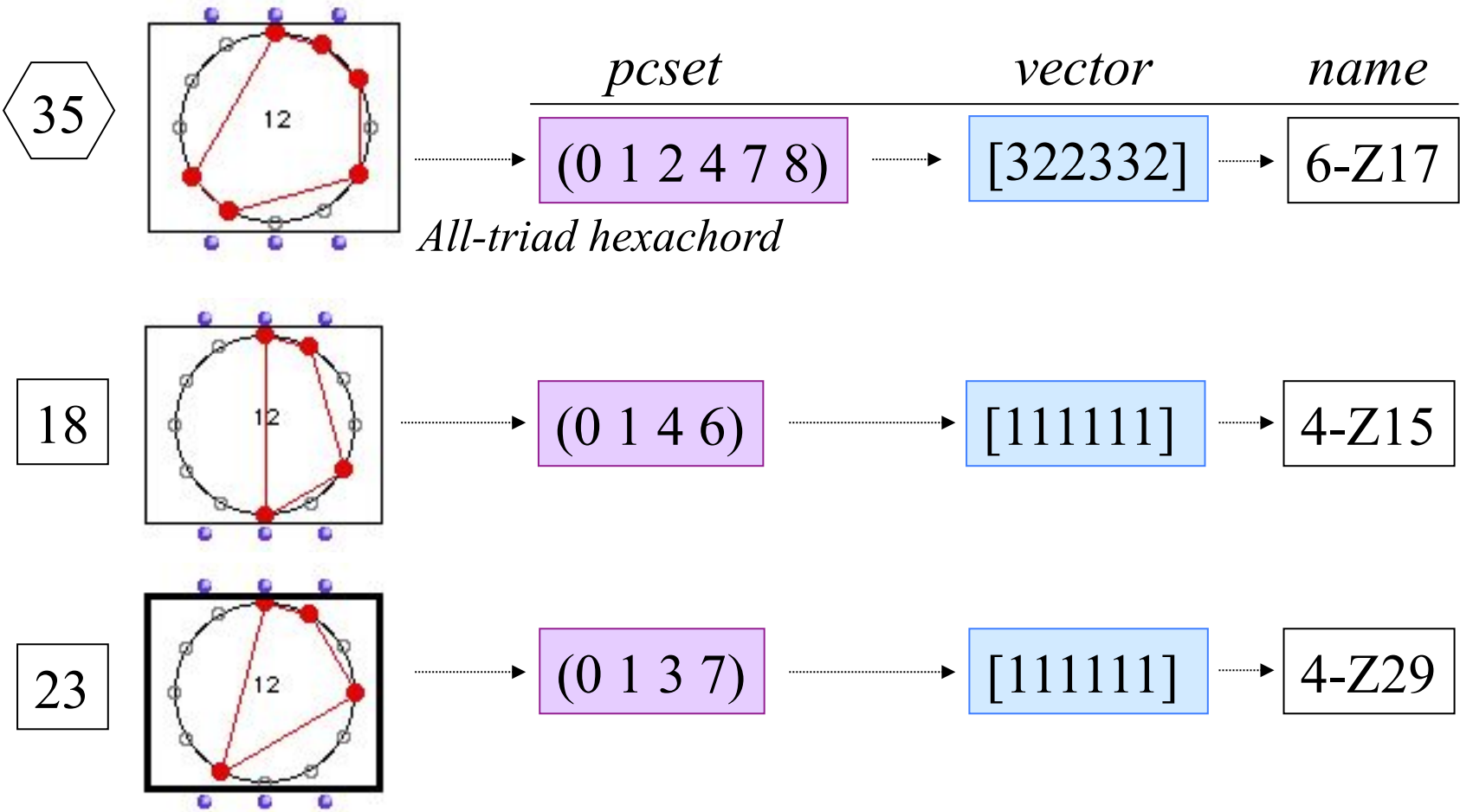
PSB 503

Printed in U.S.A.

Elliott Carter: 90+ (1994)

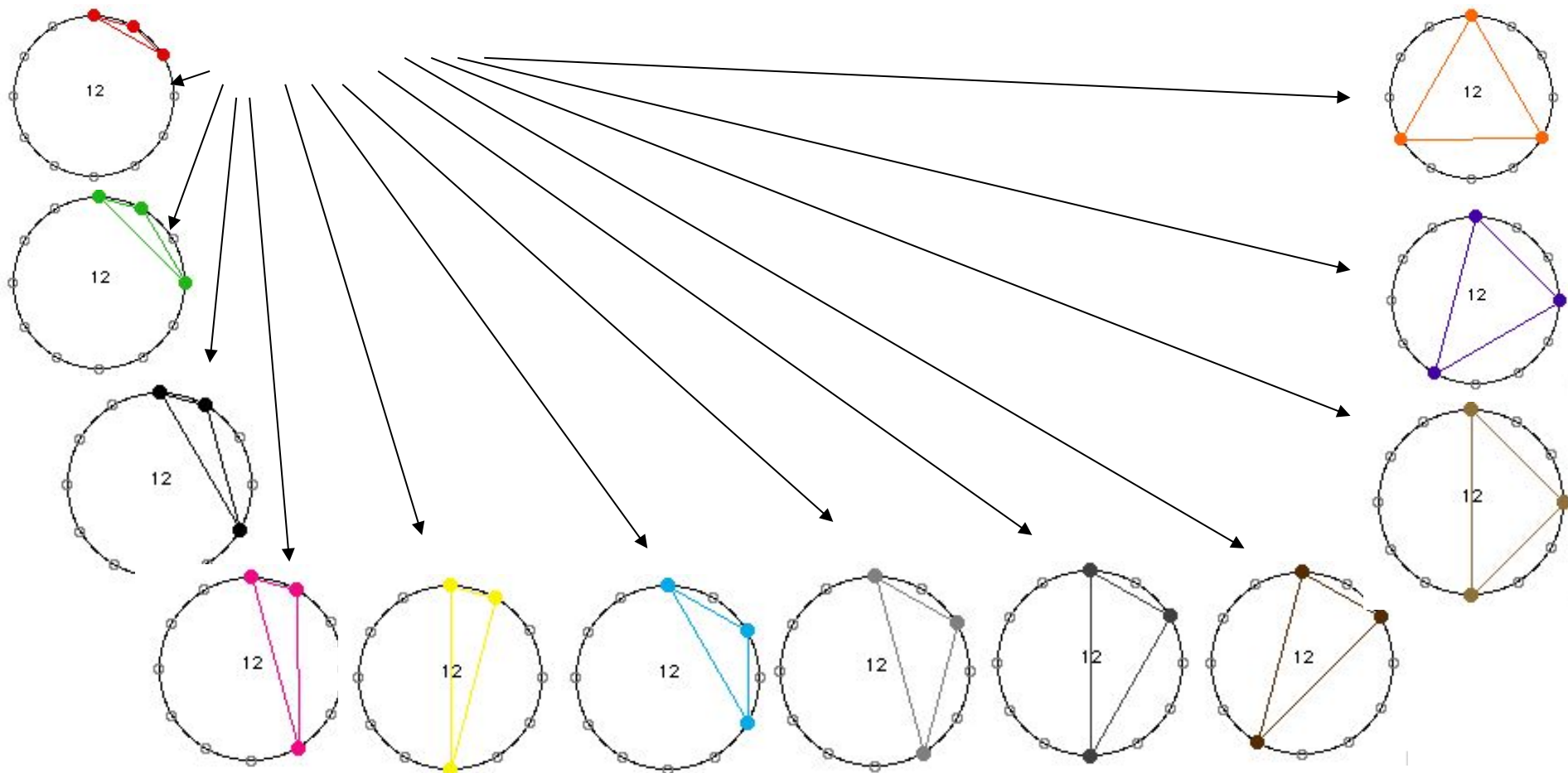
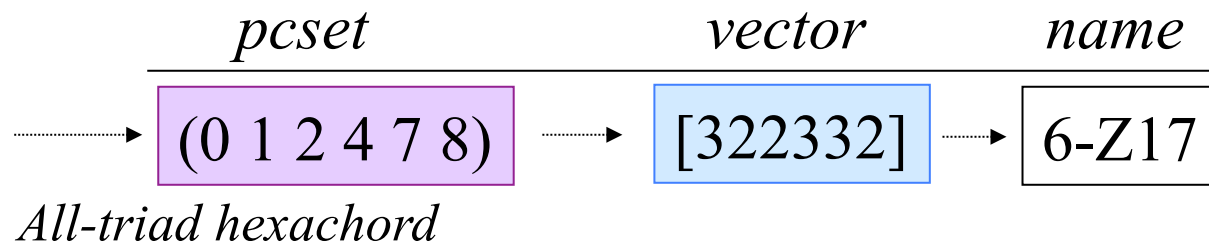
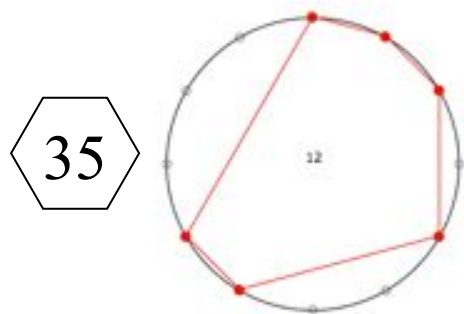


« From about 1990, I have reduced my vocabulary of chords more and more to the six note chord n° 35 and the four note chords n° 18 and 23, which encompass all the intervals » (Harmony Book, 2002, p. ix)



Elliott Carter: 90+ (1994)

| $G \setminus k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---------------------------------|---|---|----|----|----|----|----|----|----|----|----|----|
| C_{12} | 1 | 6 | 19 | 43 | 66 | 80 | 66 | 43 | 19 | 6 | 1 | 1 |
| D_{12} | 1 | 6 | 12 | 29 | 38 | 50 | 38 | 29 | 12 | 6 | 1 | 1 |
| $\text{Aff}_1(\mathbb{Z}_{12})$ | 1 | 5 | 9 | 21 | 25 | 34 | 25 | 21 | 9 | 5 | 1 | 1 |





« Entités formelles pour l'analyse musicale » *Marcel Mesnage (1998)*



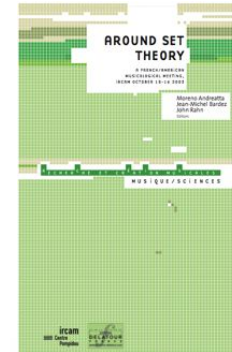
A. Schoenberg : *Klavierstück Op. 33a*, 1929

The image displays a musical score for A. Schoenberg's *Klavierstück Op. 33a*, 1929, with formal entities highlighted by colored boxes and arrows pointing to corresponding tone circle diagrams below. The score is in 4/4 time and features a complex harmonic structure. The formal entities are represented by tone circle diagrams, each with a unique configuration of notes and a corresponding set of numbers below it. The diagrams are arranged in a sequence, with arrows indicating the relationships between them. The first diagram is labeled T_3 and the others are labeled T_1I . The diagrams are arranged in a sequence, with arrows indicating the relationships between them. The first diagram is labeled T_3 and the others are labeled T_1I .

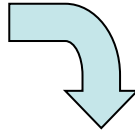
| Diagram | Notes | Labels |
|---------|--------------------------|--------|
| 1 | 0-5511 (1 2 5 6) | T_3 |
| 2 | 9-4233 (2 3 4 5 6) | T_1I |
| 3 | 8-6231 (1 2 3 4 5 6) | T_1I |
| 4 | 11-6132 (1 2 3 4 5 6) | T_1I |
| 5 | 0-4332 (2 3 4 5 6) | T_1I |
| 6 | 3-5511 (1 2 5 6) | T_1I |

Analysis by Allen Forte: visualisation

Arnold Schoenberg: *Klavierstück* op. 19 n° 4

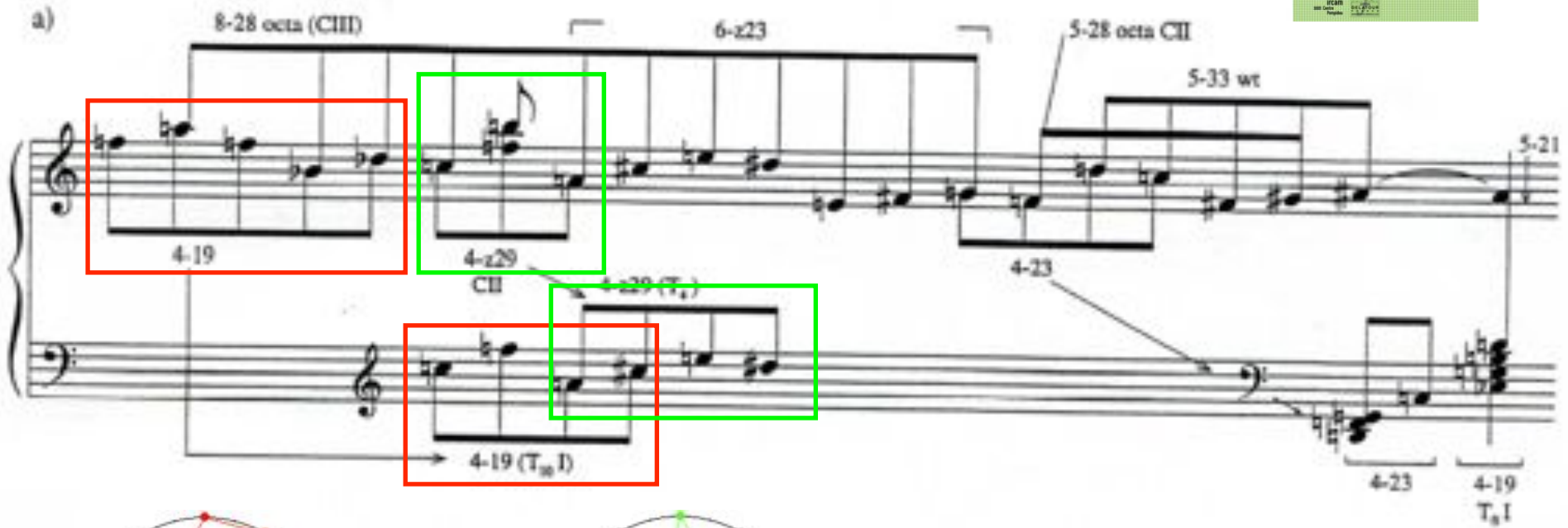


reduction



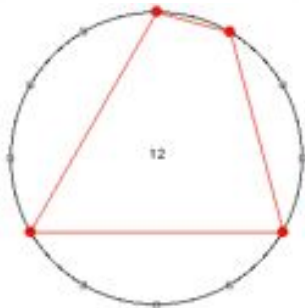
Rasch, aber leicht (♩)
poco rit. ... leicht

a)




8-28 octa (CII) 6-z23 5-28 octa CII 5-33 wt 5-21

4-19 4-z29 4-23 4-19 (T₉ I) 4-z29 (T₉ z) 4-23 4-19 T₉ I



Towards an assisted analysis of the *Klavierstück* op. 19 n° 4

score



reduction





Diagram illustrating the reduction of the score, showing the original score and the reduced score with highlighted sections (red, green, blue, and red boxes).



Three circular diagrams illustrating chord structures on a 12-tone scale (labeled 12):

- Red triangle (1, 4, 7)
- Green triangle (1, 3, 6)
- Black pentagon (1, 2, 4, 7, 11)

« *Making and Using a Pcset Network for Stockhausen's Klavierstück III* »



Trois interprétations :



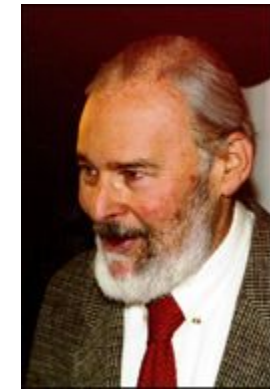
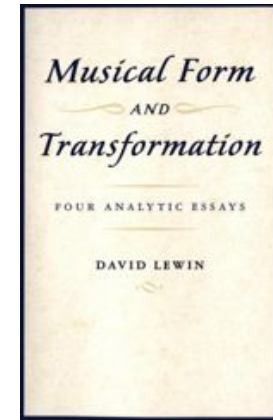
Henck



Kontarsky

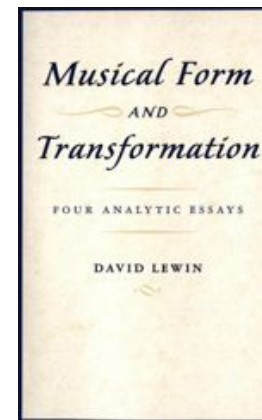


Tudor



« Making and Using a Pcset Network for Stockhausen's Klavierstück III »

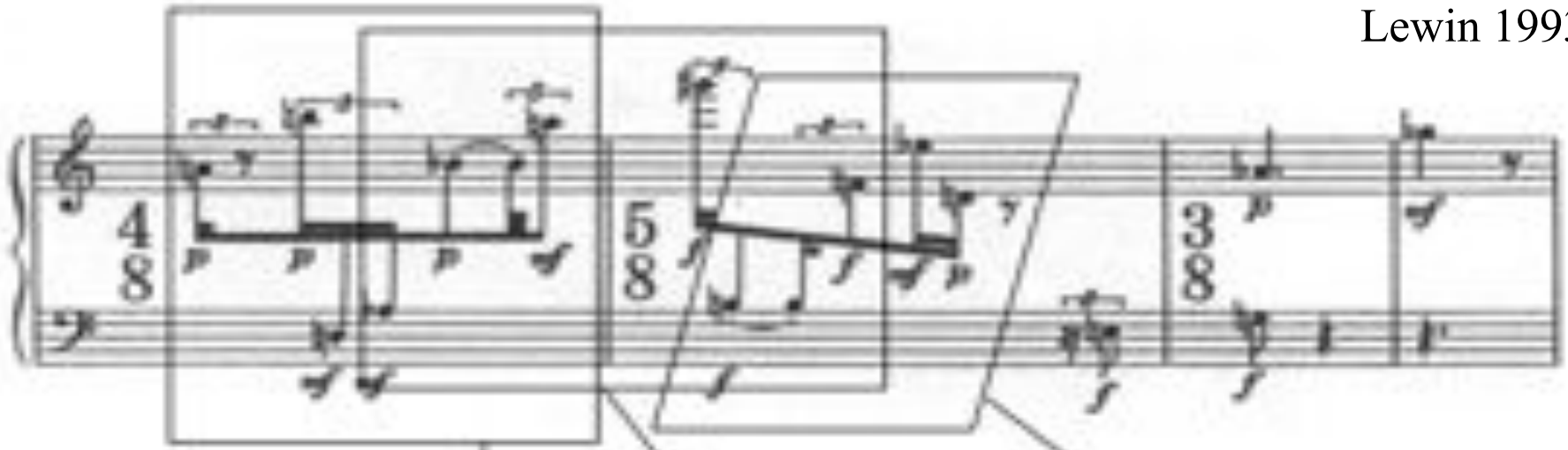
The image shows a musical score for Stockhausen's Klavierstück III, consisting of two staves (treble and bass clef). The score is divided into three measures with time signatures 4/8, 5/8, and 3/8. The first measure has dynamics *p p p mf*. The second measure has dynamics *f mf p*. The third measure has dynamics *f f*. Three sections of the score are highlighted with colored boxes: a red box around the first measure, a green box around the first two measures, and a blue box around the second measure. Three arrows with question marks point from these boxes to three identical pentachord diagrams below. Each diagram is a circle with 12 points on its circumference, representing a 12-tone scale.



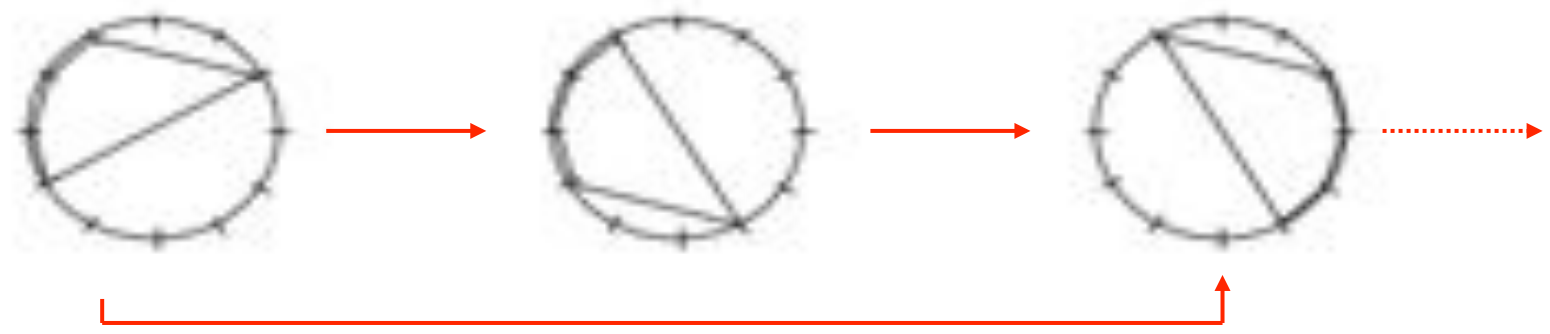
« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

« *Making and Using a Pcset Network for Stockhausen's Klavierstück III* »

Lewin 1993

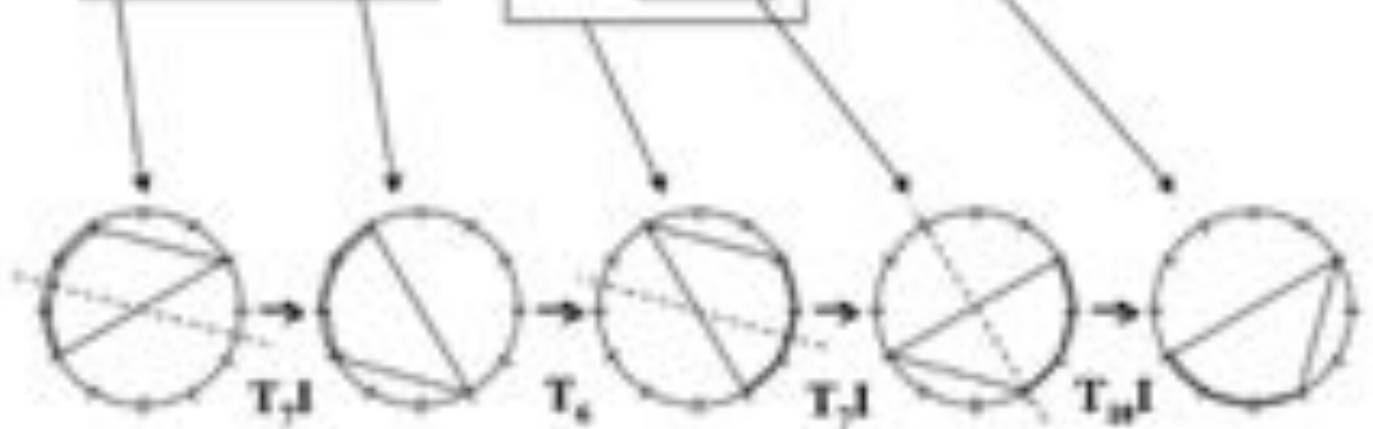


SI: (1, 1, 1, 3, 6) (6, 3, 1, 1, 1) (6, 3, 1, 1, 1)
 IFUNC: [5 3 2 2 1 1 1 1 2 2 3] [5 3 2 2 1 1 1 1 2 2 3] [5 3 2 2 1 1 1 1 2 2 3]
 VI: [3 2 2 1 1 1] [3 2 2 1 1 1] [3 2 2 1 1 1]



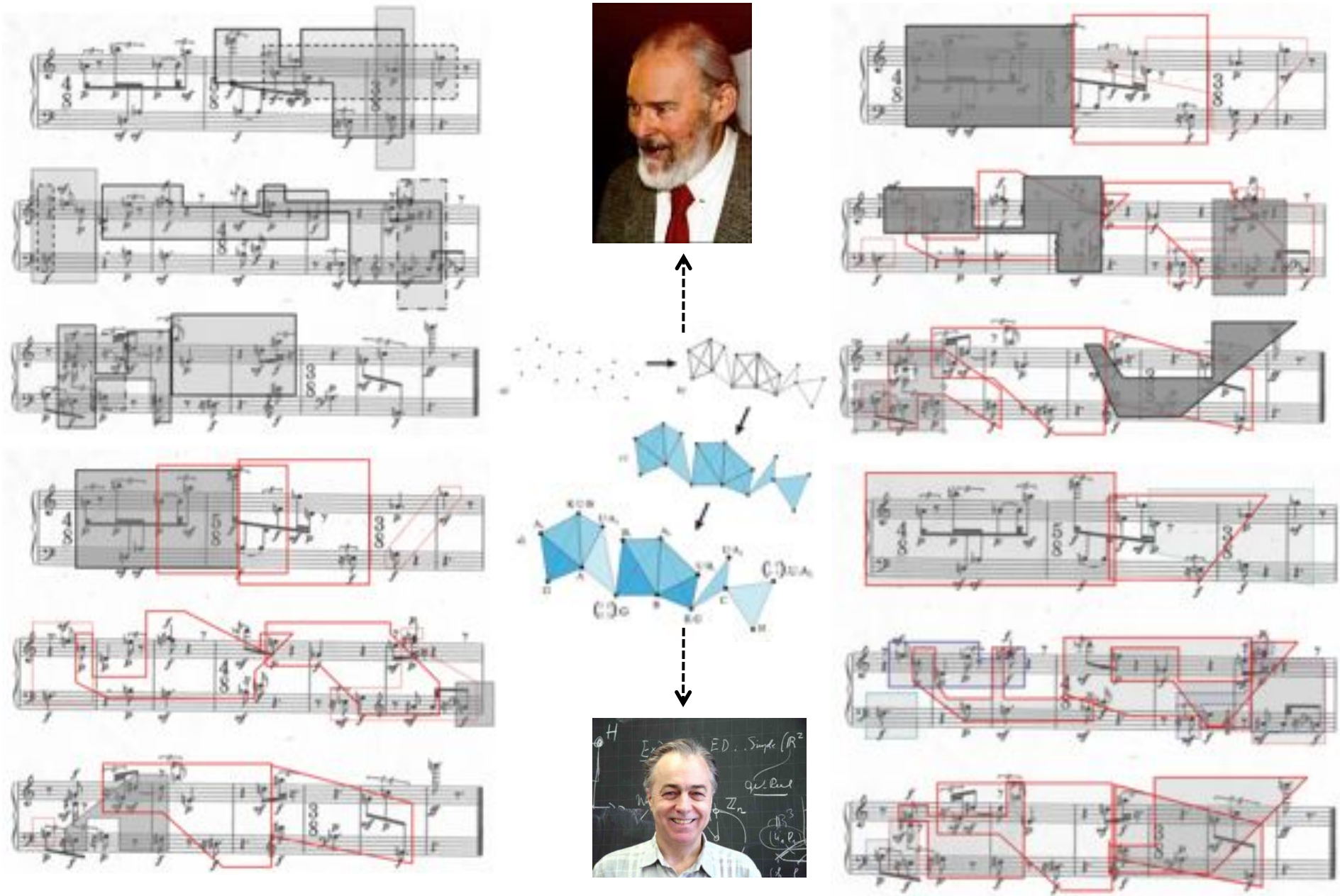
Segmentation par « imbrication »: progression transformationnelle

Stockhausen: *Klavierstück III* (Analisi di D. Lewin)



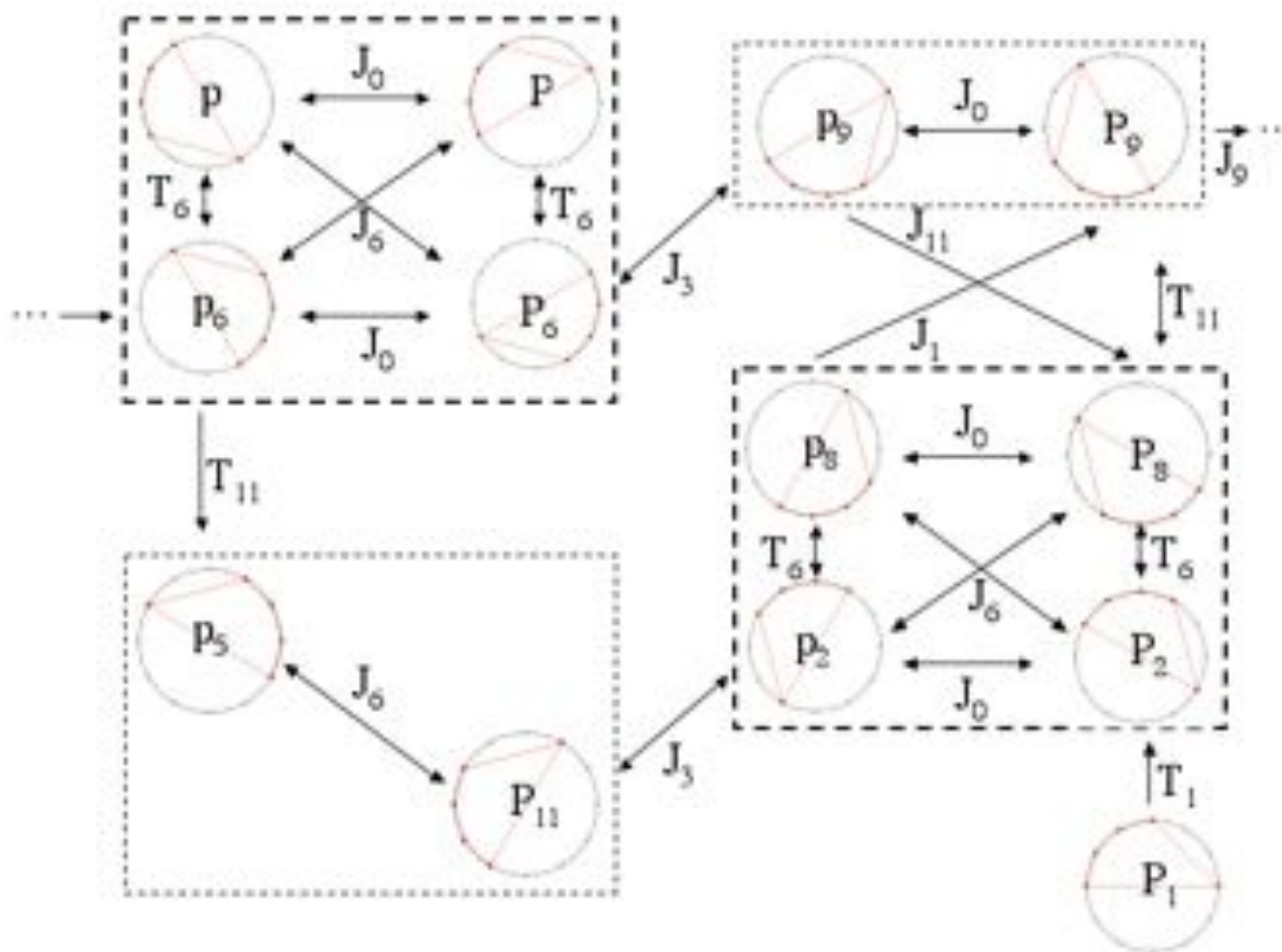
Vers une modélisation informatique de l'analyse transformationnelle

YunKang Ahn, L'analyse musicale computationnelle, thèse, Université de Paris VI / Ircam, déc 2009



Reseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)



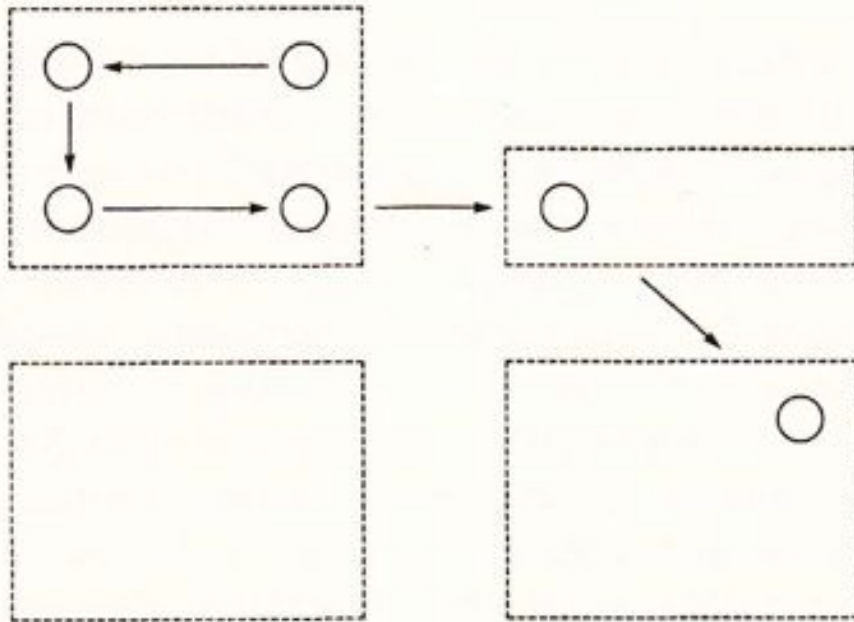
« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their **potentially functional interrelationships**, in a very compactly organized little **spatial configuration**. »

« [...] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - **it makes the abstract space of such a world accessible to our sensibilities**. That is to say that the story projects what one would traditionally call *form*. »

Parcours multiples d'écoute dans un réseau transformationnel

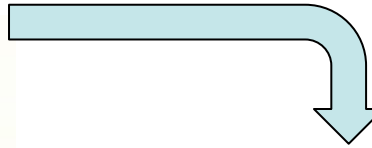
Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Pass 1 (mm. 1-5).

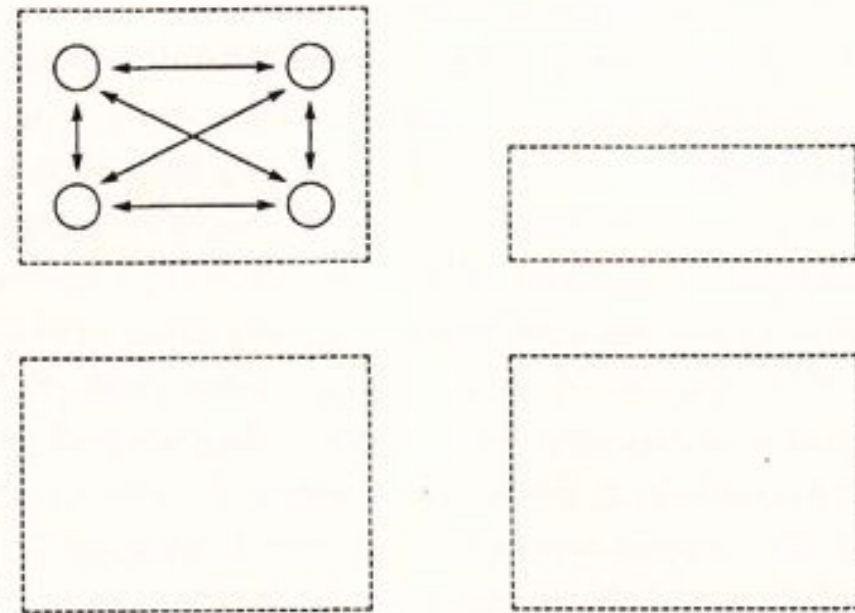


a

horizontal arrows within boxes = J0; between boxes = J3 or J9
 vertical arrows within boxes = T6; between boxes = Te or T1
 diagonal arrows within boxes = J6; between boxes = Je or J1

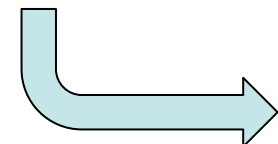


Pass 2 (mm. 5-8) goes back and elaborates the beginning area of pass 1.



b

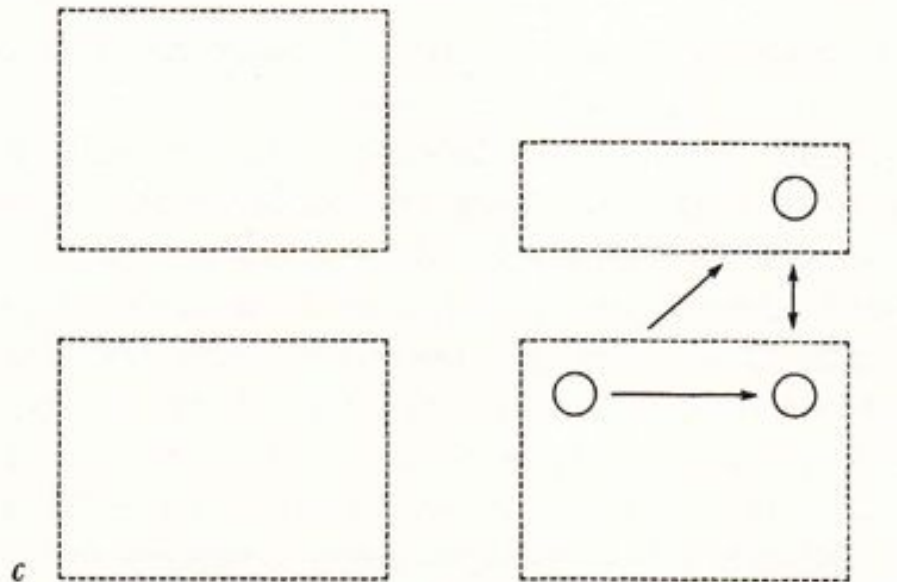
horizontal arrows within boxes = J0; between boxes = J3 or J9
 vertical arrows within boxes = T6; between boxes = Te or T1
 diagonal arrows within boxes = J6; between boxes = Je or J1



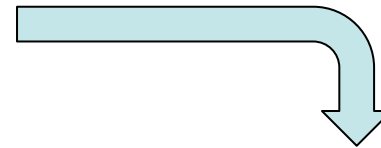
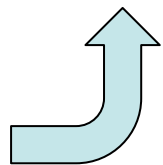
Parcours multiples d'écoute dans un réseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

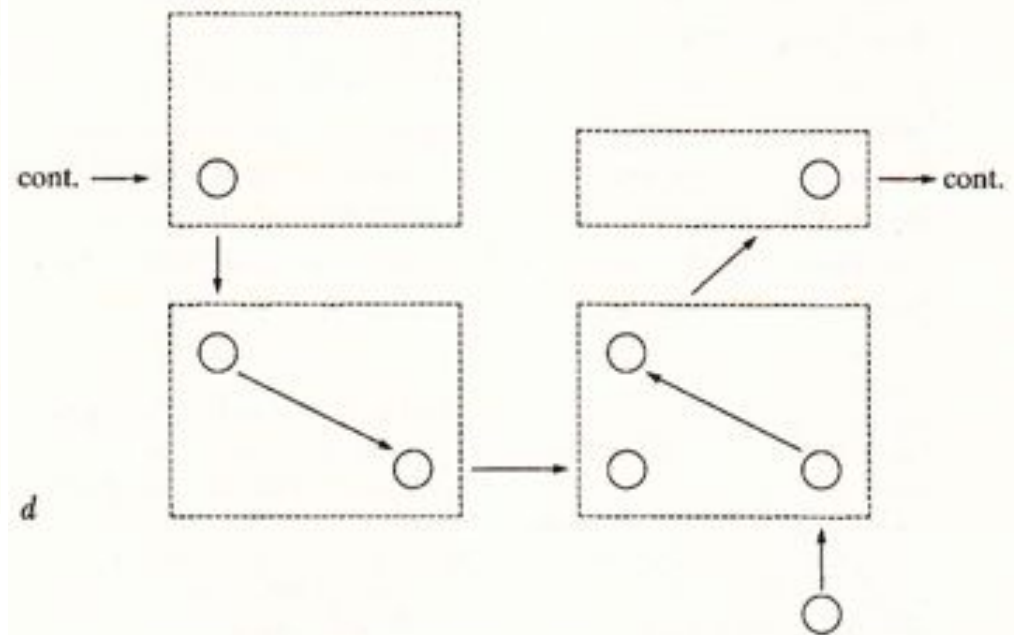
Pass 3 (mm. 8-10) picks up and elaborates the ending area of pass 1.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1



Pass 4 (mm. 9-16) expands the p8 + P8 area of pass 3 to activate P2 and p2 as well. P2 is the "essential" incipit of pass 4; p2 is the end of the pass, and of the piece.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

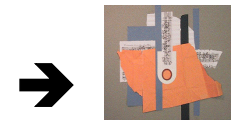
Exercices d'écoute : « do you hear it? » vs « can you hear it? »

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

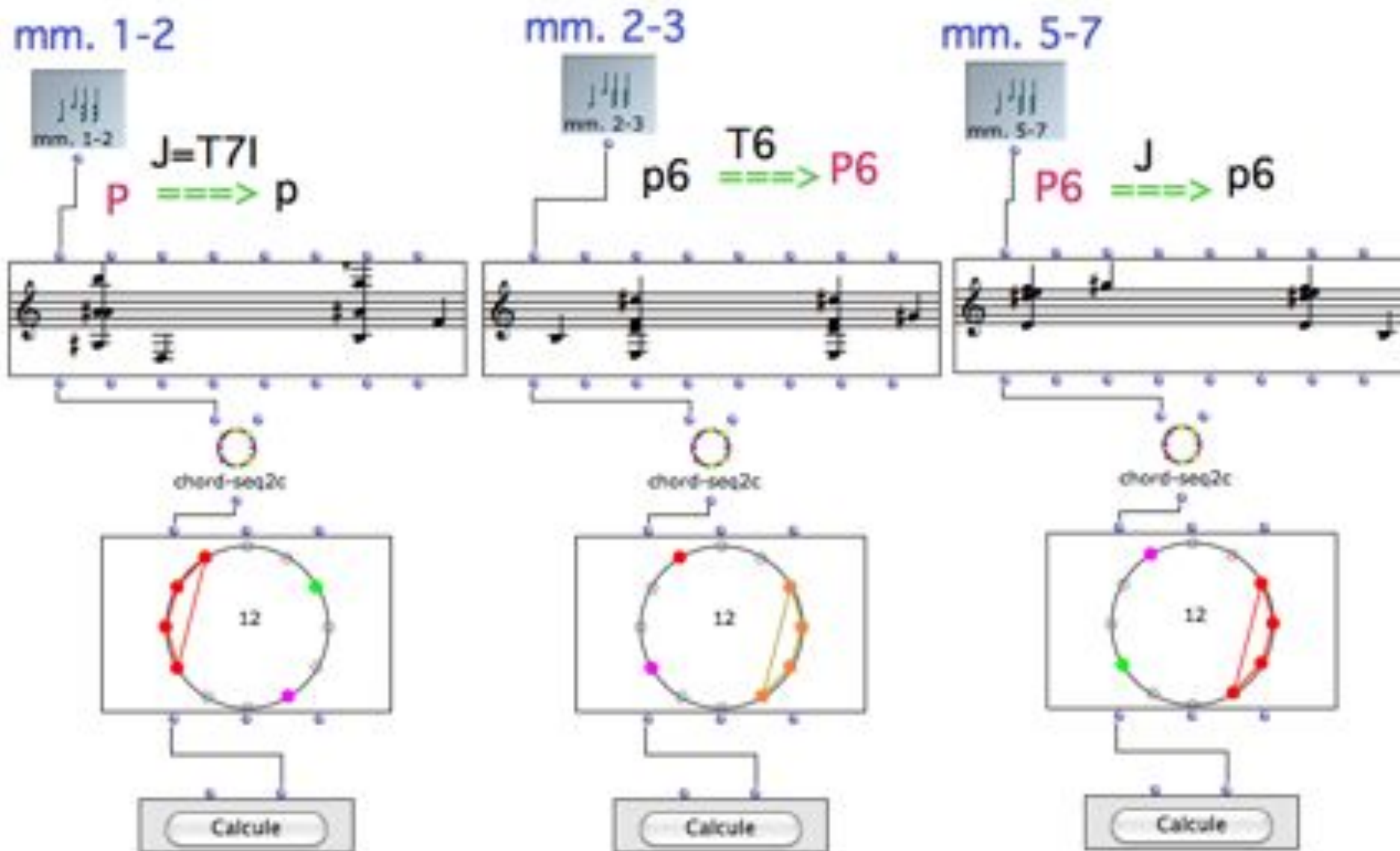
The image displays three systems of musical notation for Stockhausen's *Klavierstück III*. Each system consists of a treble and bass clef staff. The notes are represented by small circles with stems. Below each staff, pitch class intervals are labeled. The first system (measures 1-6) has intervals: P0, p0, p6, P6, p9, P8. The second system (measures 7-10) has intervals: P6, p6, P0, p0, p8, P8, P9. The third system (measures 11-15) has intervals: P1, P2, p8, P9, p6, p5, P6, p2. Above the first two systems, measure numbers and groupings are indicated: m. 1, 1-2, 2, 2-3, 2-5, 2-5 for the first system; and m. 5-7, 5-7, 5-7, 5-7, 8-10, 8-10, 8-10 for the second system.

Example 2.7. An ear-training aid for listening to P/p forms and their inter-relations.

« I take the question ‘Can you hear it » to mean something like this: After studying the analysis in examples 2.5 and 2.6, do you find it possible to focus your **aural attention** upon aspects of the acoustic signal that seem to engage the signifiers of that analysis? [...] For me, the interesting questions involve the extent and ways in which I am satisfied and dissatisfied when **focusing my aural attention** in that manner. It is important to ask those questions about any systematic analysis of any musical composition ».



Computer-Aided Transformational Analysis in OpenMusic



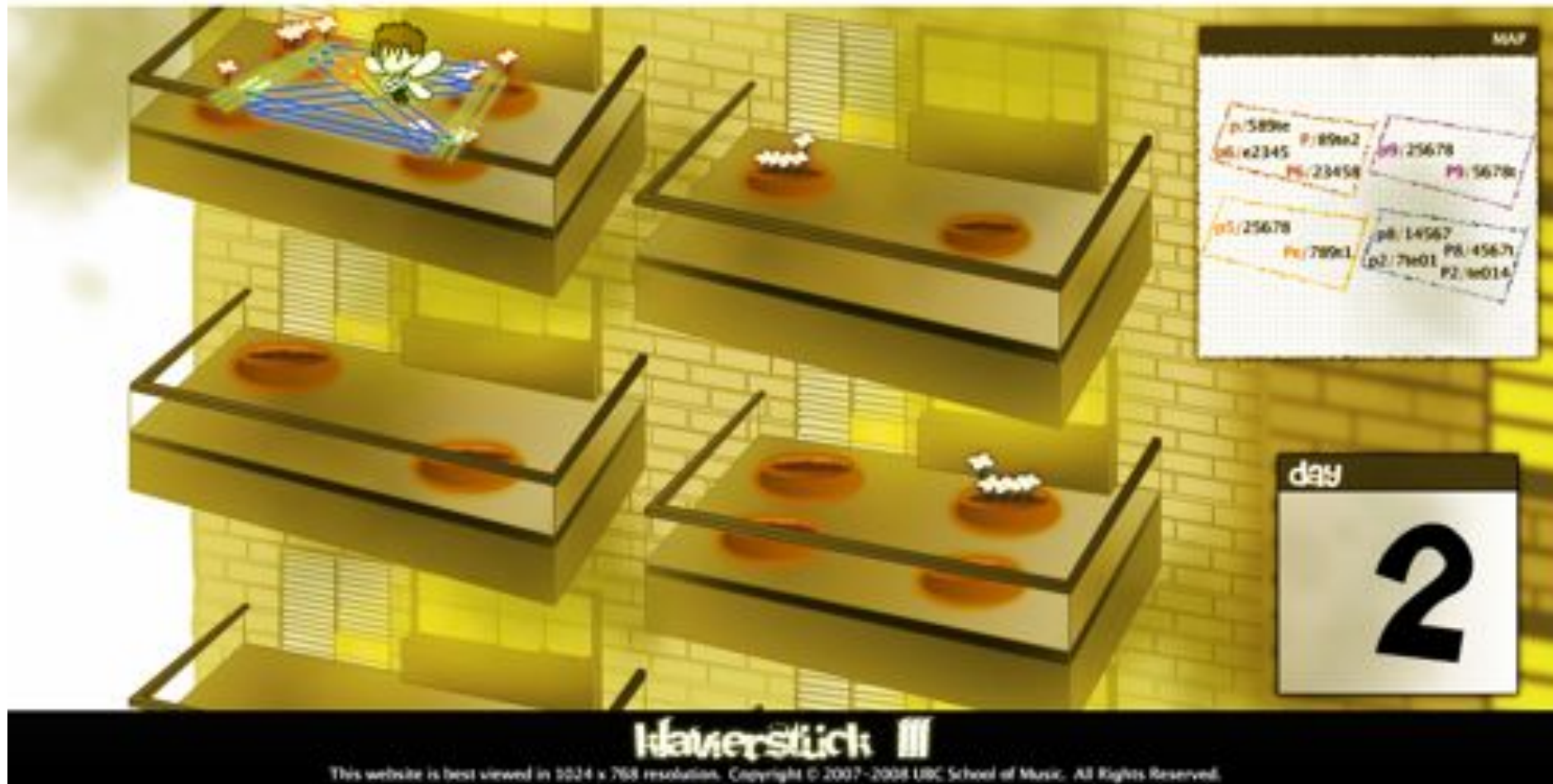
→ <http://recherche.ircam.fr/equipes/repmus/OpenMusic/>

→ OpenMusic

Visualisations multimédia de l'analyse transformationnelle

R. Attas : Metaphors in Motion: Agents and Representation in Transformational Analysis, *MTO*, 15(1), 2009
<http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html>

Animation 1. Klavierstück III



Visualisations multimédia de l'analyse transformationnelle

R. Attas : Metaphors in Motion: Agents and Representation in Transformational Analysis, *MTO*, 15(1), 2009
<http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html>

Animation 2. Grow Your Own Pentachord

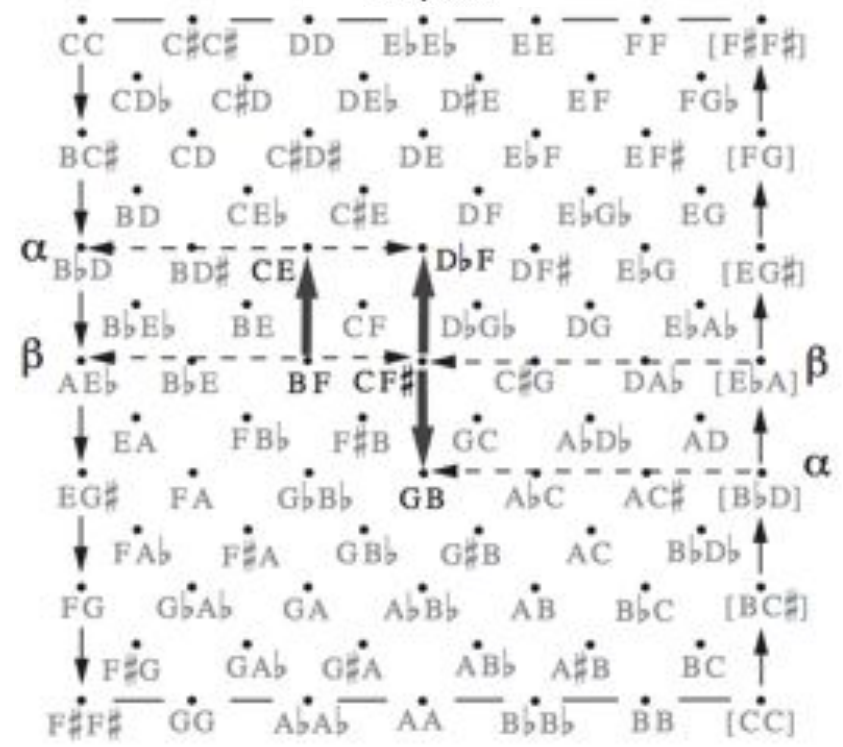
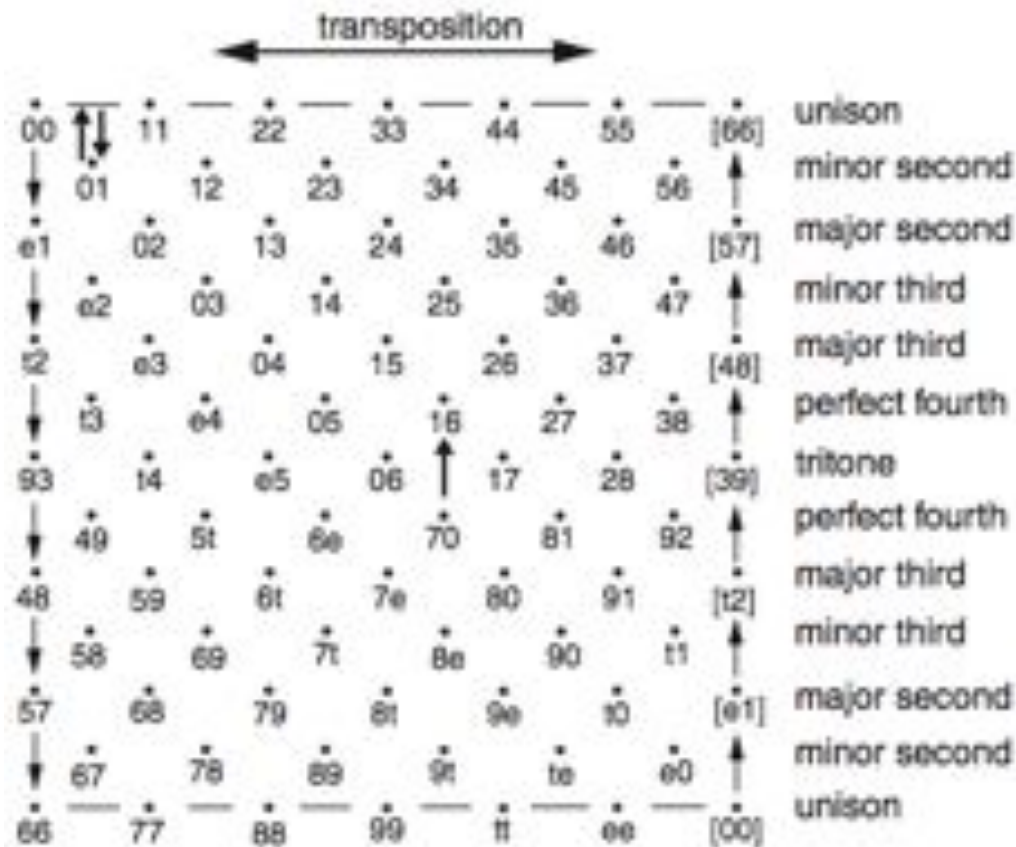
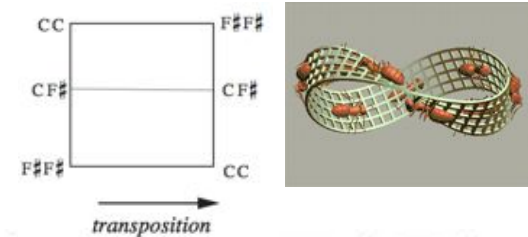
The screenshot displays the 'Grow Your Own Pentachord' interface. At the top left, there is a 'PLAYBACK SPEED' control with a slider. The central focus is a 3D illustration of a brown pot containing soil and several white flowers with yellow centers. To the left of the pot are two rows of glowing musical notes: the top row, labeled 'T WANDS', contains notes T1, T5, and Te; the bottom row, labeled 'J WANDS', contains notes J0, J1, J3, J6, J9, and J#.

In the top right corner, a 'PENTACHORD MAP' shows a network of nodes (circles) connected by arrows, representing the relationships between different pentachords. A red dot on the map is labeled '= You are here'. Below the map is a large black letter 'P' with '89te2' written next to it, and the text 'CURRENT PENTACHORD' below that.

At the bottom right, there are three buttons: 'INSTRUCTIONS', 'POT LAYOUT', and 'GUIDE'. The bottom of the interface features the text 'grow your own pentachord!' in a stylized font, followed by a footer: 'This website is best viewed in 1024 x 768 resolution. GROW YOUR OWN PENTACHORD! Copyright © 2007-2008 UBC School of Music. All Rights Reserved.'

Nouvelles représentations géométriques pour l'analyse

$$T^2 = \mathbf{R}/12\mathbf{Z} \times \mathbf{R}/12\mathbf{Z} \longrightarrow T^2 / S_2$$



Dmitri Tymoczko :
 « The Geometry of Musical Chords »,
Science, 313, 2006

[Tymoczko 2010]

$$T^2 = R/12Z \times R/12Z \longrightarrow T^2 / S_2$$

Largo

← transposition →

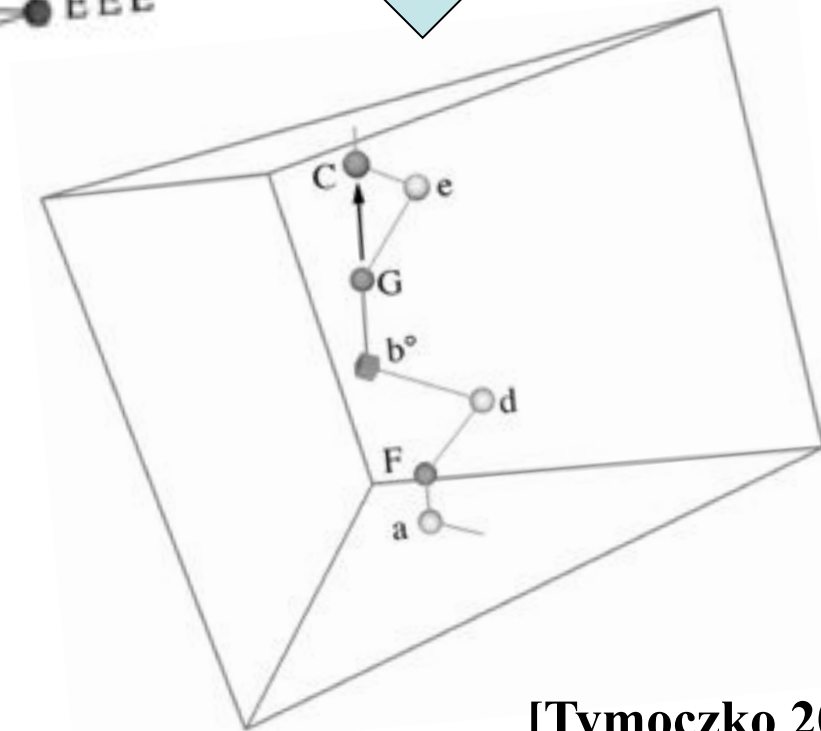
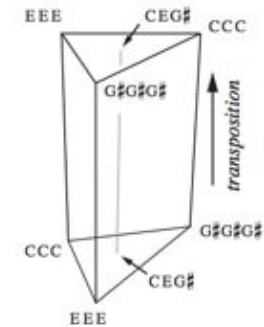
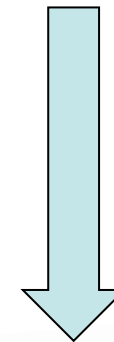
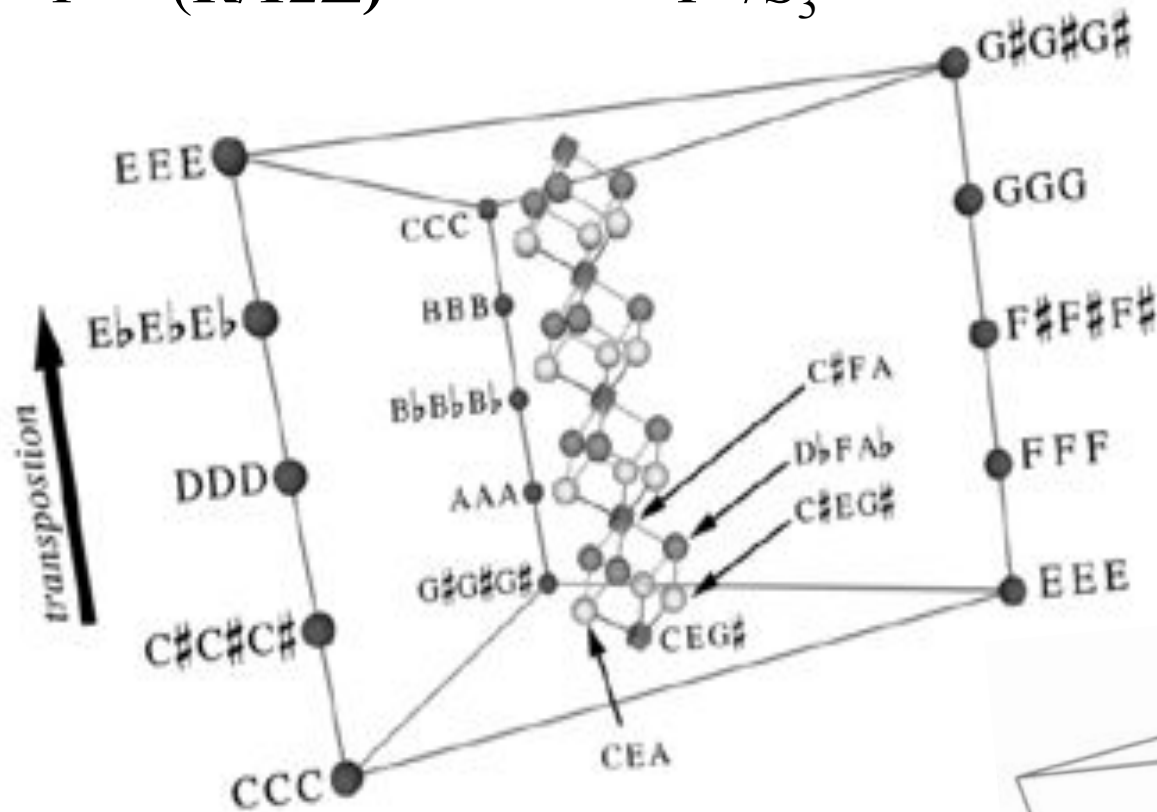
| | | | | | | |
|----|----|----|----|----|----|------|
| 00 | 11 | 22 | 33 | 44 | 55 | [66] |
| 01 | 12 | 23 | 34 | 45 | 56 | [57] |
| e1 | 02 | 13 | 24 | 35 | 46 | [48] |
| e2 | 03 | 14 | 25 | 36 | 47 | [39] |
| 52 | e3 | 04 | 15 | 26 | 37 | [22] |
| i3 | e4 | 05 | 16 | 27 | 38 | [13] |
| 93 | 14 | e5 | 06 | 17 | 28 | [04] |
| 49 | 5t | 6e | 70 | 81 | 92 | [95] |
| 48 | 59 | 6t | 7e | 80 | 91 | [86] |
| 58 | 69 | 7t | 8e | 90 | t1 | [77] |
| 57 | 68 | 79 | 8t | 9e | 0 | [68] |
| 67 | 78 | 89 | 9t | te | e0 | [59] |
| 66 | 77 | 88 | 99 | tt | ee | [00] |

unison
minor second
major second
minor third
major third
perfect fourth
tritone
perfect fourth
major third
minor third
major second
minor second
unison

| | | | | | | |
|-----|-----|-----|-----|------|------|-------|
| 00 | 11 | 22 | 33 | 44 | 55 | [66] |
| 01 | 12 | 23 | 34 | 45 | 56 | [57] |
| 111 | 02 | 13 | 24 | 35 | 46 | [48] |
| 112 | 03 | 14 | 25 | 36 | 47 | [39] |
| 102 | 113 | 04 | 15 | 26 | 37 | [22] |
| 103 | 114 | 05 | 16 | 27 | 38 | [13] |
| 93 | 10 | 115 | 60 | 71 | 82 | [93] |
| 49 | 510 | 611 | 70 | 81 | 92 | [84] |
| 48 | 59 | 610 | 711 | 80 | 91 | [102] |
| 58 | 69 | 710 | 811 | 90 | 101 | [93] |
| 57 | 68 | 79 | 810 | 911 | 100 | [111] |
| 67 | 78 | 89 | 910 | 1011 | 110 | [02] |
| 66 | 77 | 88 | 99 | 1010 | 1111 | [00] |

Dmitri Tymoczko :
« The Geometry of Musical Chords »,
Science, 313, 2006

$$T^3 = (\mathbb{R}/12\mathbb{Z})^3 \longrightarrow T^3 / S_3$$

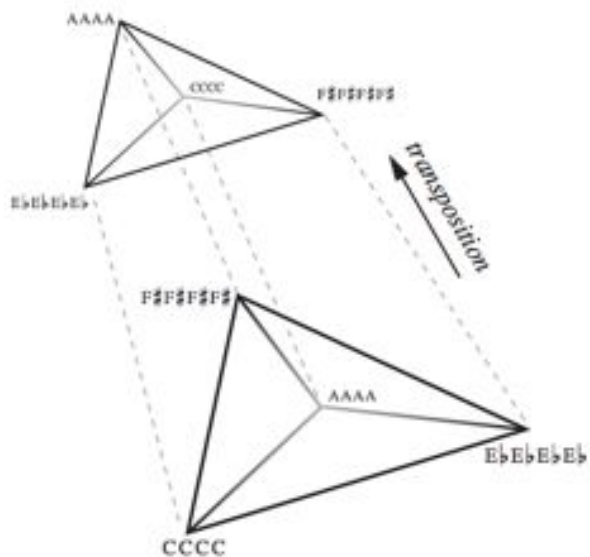


Dmitri Tymoczko :
 « The Geometry of Musical Chords »,
Science, 313, 2006

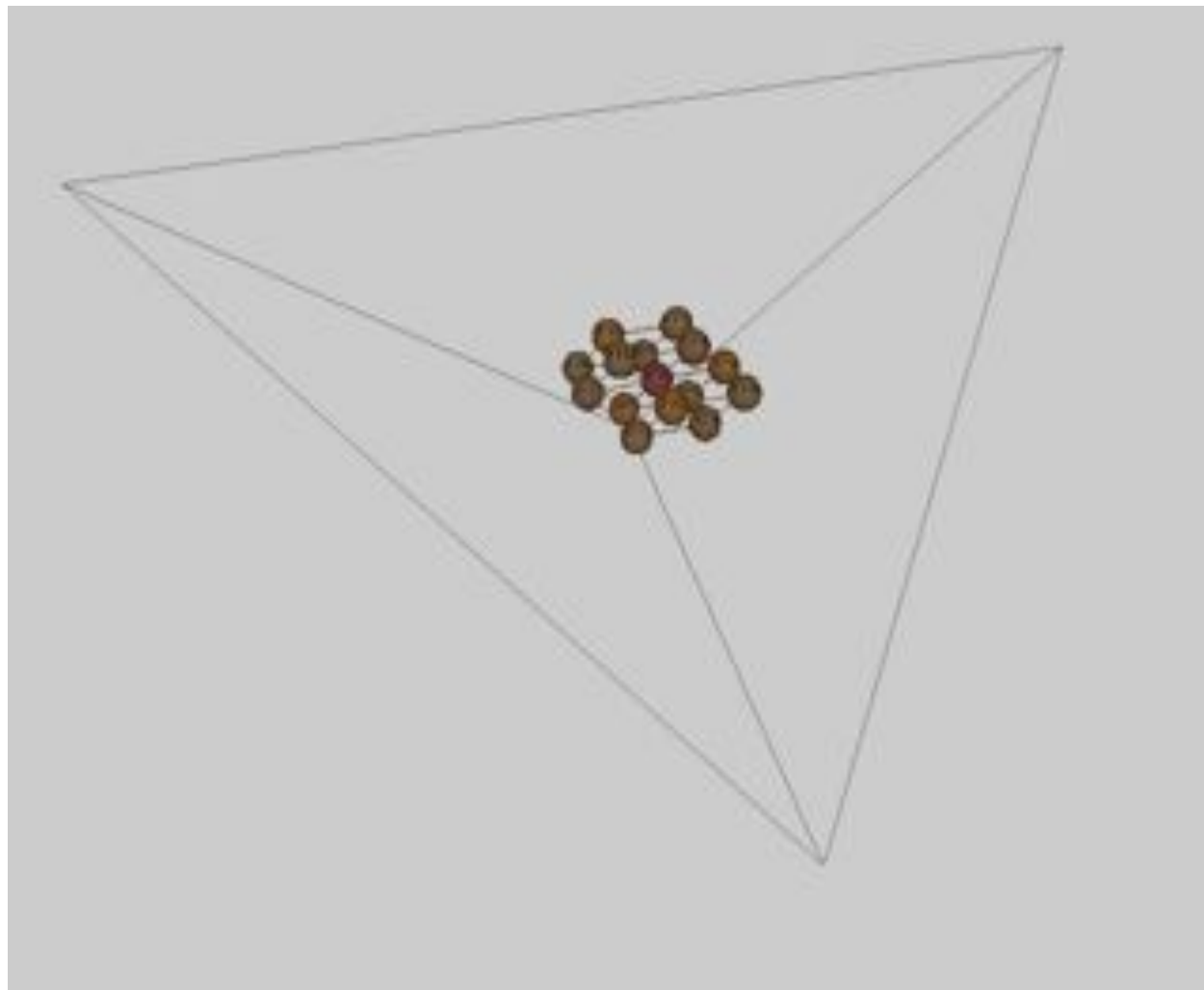
[Tymoczko 2010]

Se (dé)placer dans un espace de dim. 4

$$T^4 = (\mathbf{R}/12\mathbf{Z})^4 \longrightarrow T^4 / S_4$$



Largo



Dmitri Tymoczko, « The Geometry of Musical Chords », *Science*, 313, 2006