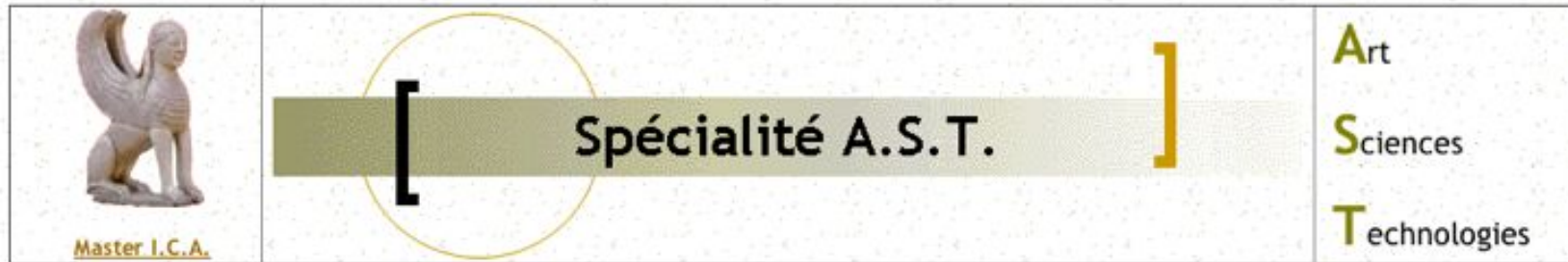


Master I.C.A.



Traitement interactif de l'image et du son

Méthodes mathématiques pour la création musicale :
aspects théoriques, informatiques et cognitifs

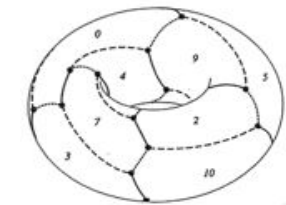
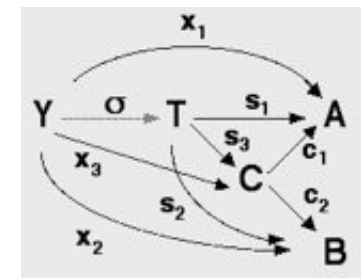
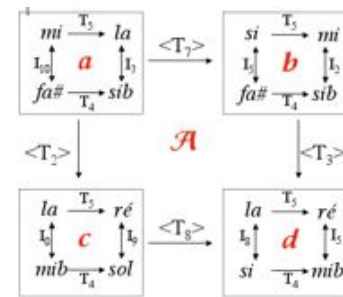
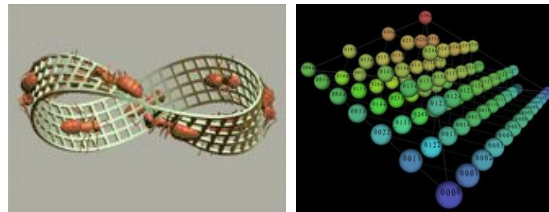
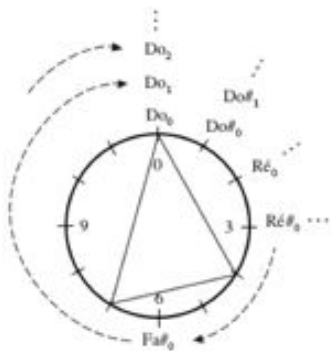
– Moreno Andreatta –

Equipe Représentations Musicales
IRCAM/CNRS/UPMC UMR 9912

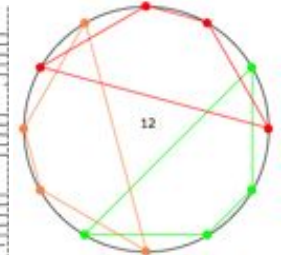
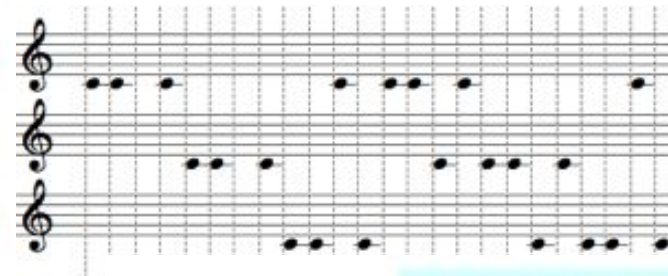
`Moreno.Andreatta@ircam.fr`

Première partie

- 1.) Rappel historique
- 2.) Représentation/formalisation et énumération/classification des structures musicales
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1 1	0 2	1 3	2 4	3 5	4 6	[4 8]
11 2	0 3	1 4	2 5	3 6	4 7	[9 3]
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9 3	10 4	11 5	6 0	7 1	8 2	[0 0]
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4 8	5 9	6 10	7 11	8 0	9 1	
5 8	6 9	7 10	8 11	9 0	10 1	
5 7	6 8	7 9	8 10	9 11	10 0	
6 7	7 8	8 9	9 10	10 11	11 0	
6 6	7 7	8 8	9 9	10 10	11 11	



$$Df(x) = f(x) - f(x-1)$$

$$\begin{aligned}
 f &= 7 \ 11 \ 10 \ 11 \ 7 \ 2 \ 7 \ 11 \ 10 \ 11 \ 7 \ 2 \ 7 \ 11 \dots \\
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 D^k f &= \dots
 \end{aligned}$$

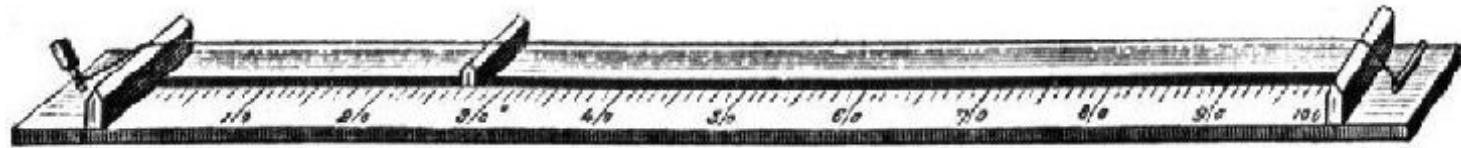


Mathématiques/Musique...une histoire récente!

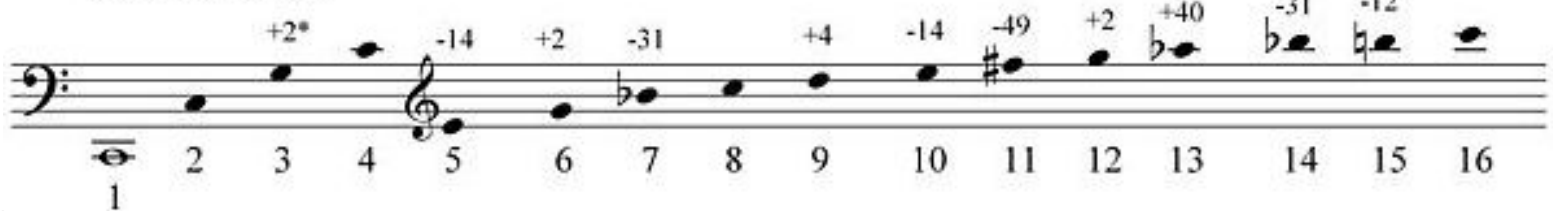
- 1999 : 4^e Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (G. Assayag, H.G. Feichtinger, J.F. Rodrigues, Springer, 2001)
- 2000-2001 : Séminaire *MaMuPhi*, *Penser la musique avec les mathématiques ?* (Assayag, Mazzola, Nicolas éd., Coll. « M/S », Ircam/Delatour, 2006)
- 2000-2003 : International Seminar on *MaMuTh* (*Perspectives in Mathematical and Computational Music Theory*) (Mazzola, Noll, Luis-Puebla eds, epOs, 2004)
- 2003 : *The Topos of Music* (G. Mazzola et al.)
- 2003: *Music and Mathematics. From Pythagoras to Fractals* (J. Fauvel et al.)
- 2001 - 2011 : Séminaire *MaMuX* de l'Ircam
- 2004 - 2011 : Séminaire *mamuphi* (Ens/Ircam)
- 2006 : *Mathematical Theory of Music* (F. Jędrzejewski), Coll. « M/S »
- 2007 : *La vérité du beau dans la musique* (G. Mazzola), Coll. « Musique/Sciences »
- 2007 : *Journal of Mathematics and Music* (Taylor & Francis) et MCM 2007
- 2007: *Music. A Mathematical Offering* (Dave Benson), CUP
- 2008: *Music Theory and Mathematics* (Jack Douthett et al.), URP
- 2009 : *Computational Music Science Series* (Springer)
- 2009 : MCM 2009 (Yale) et Proceedings chez Springer
- 2010 : Mathematics Subject Classification : 00A65 Mathematics and music
- 2011 : Conférence de la SMCM (Ircam, 15-17 juin 2011)



De Pythagore... à la théorie des groupes



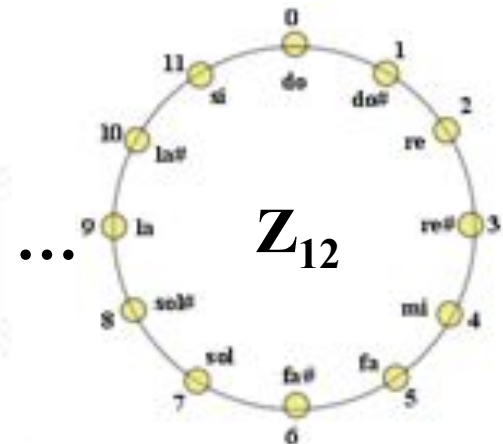
i suoni armonici



Physique

* in cents, confrontati con la scala temperata

Mathématiques

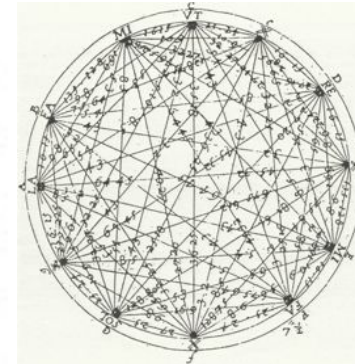


Musique et mathématiques : deux destinées parallèles

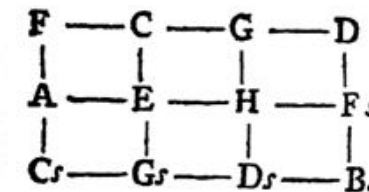
MUSIQUE	MATHS
500 av. J. C. Relation hauteur/longueur corde. La musique est source d'inspiration pour la théorie des nombres et la géométrie.	Nombres naturels et rationnels
300 a.J. Invention (théorique) de la gamme chromatique tempérée égale par Aristoxène de Tarente) et prémonition de la théorie des groupes . Isomorphismes entre les logarithmes (intervalles musicaux) et les exponentiels (longueur d'une corde)	Aucune relation.
1000 Invention de la représentation bidimensionnelle des hauteurs	Aucune correspondance
1500 Aucune reprise des concepts précédents	Nombres négatifs. Construction des rationnels
1600 Aucune relation	Nombres réels et les logarithmes
Martin Mersenne (1588-1648) : combinatoire musicale	Calcul des probabilités
1700 La fugue comme un automate abstrait. Manipulation inconsciente du groupe de Klein	Nombres complexes (Euler, Gauss), les quaternions (Hamilton), continuité (Cauchy), structure de groupe (Galois, Abel)
Leonhard Euler : Speculum Musicum (1773)	Théorie des graphes
1900 Libération de la prison de la tonalité (Loquin, Hauer, Schoenberg)	Nombres infinis et transfinis (Cantor). Axiomatique de Peano. Théorie de la mesure (Lebesgue, Borel)
1920 Formalisation radicale des macrostructures à travers le système sériel (Schoenberg)	Aucun développement de la théorie des nombres.
Ernst Krenek (1900-1991) : les axiomes dans le système dodécaphonique	David Hilbert, Les fondements de la géométrie (1899)



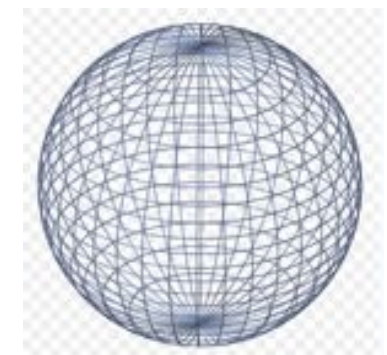
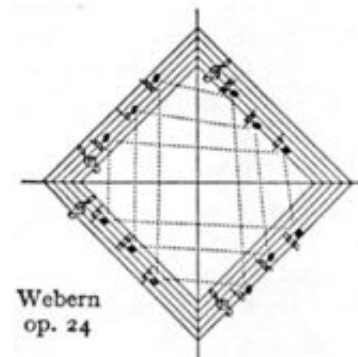
Pythagore et le monochorde, VI^e-V^e siècle av. J. C.



Mersenne, *Harmonicorum Libri XII*, 1648



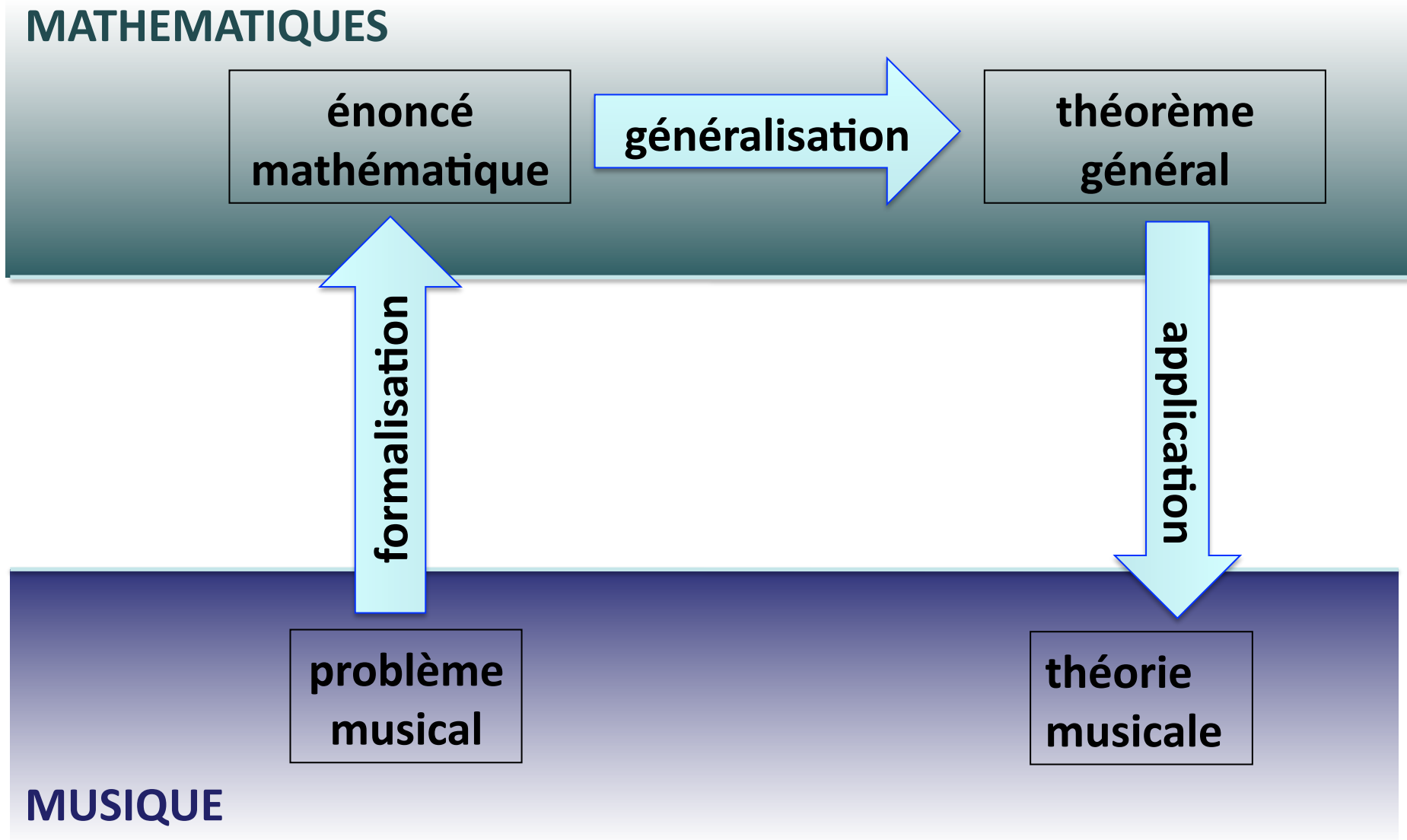
Euler : *Speculum musicum*, 1773



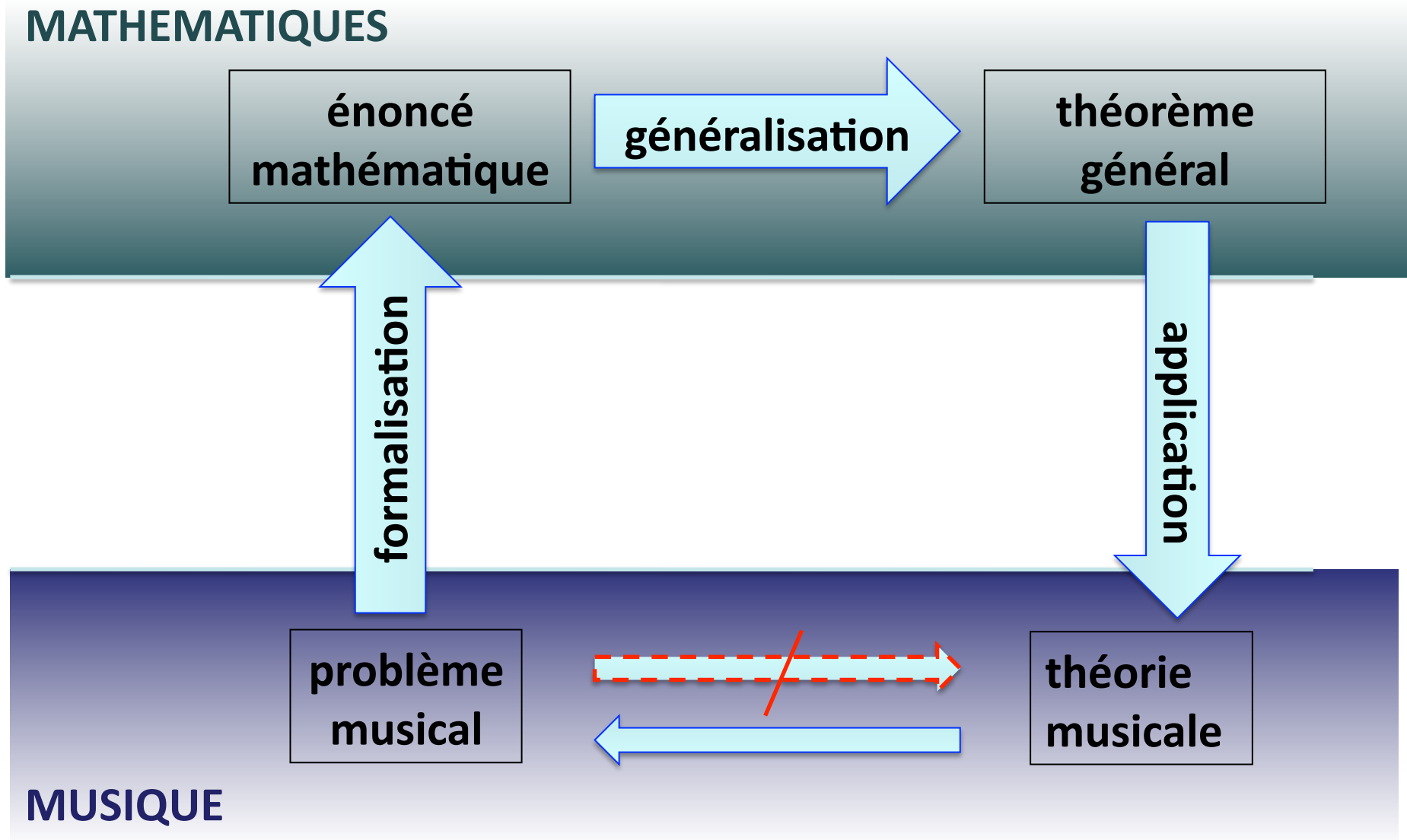
Iannis Xenakis, *Musique. Architecture*, Tournai, Casterman, 1971, (New, revised edition: Tournai, Casterman, 1976)



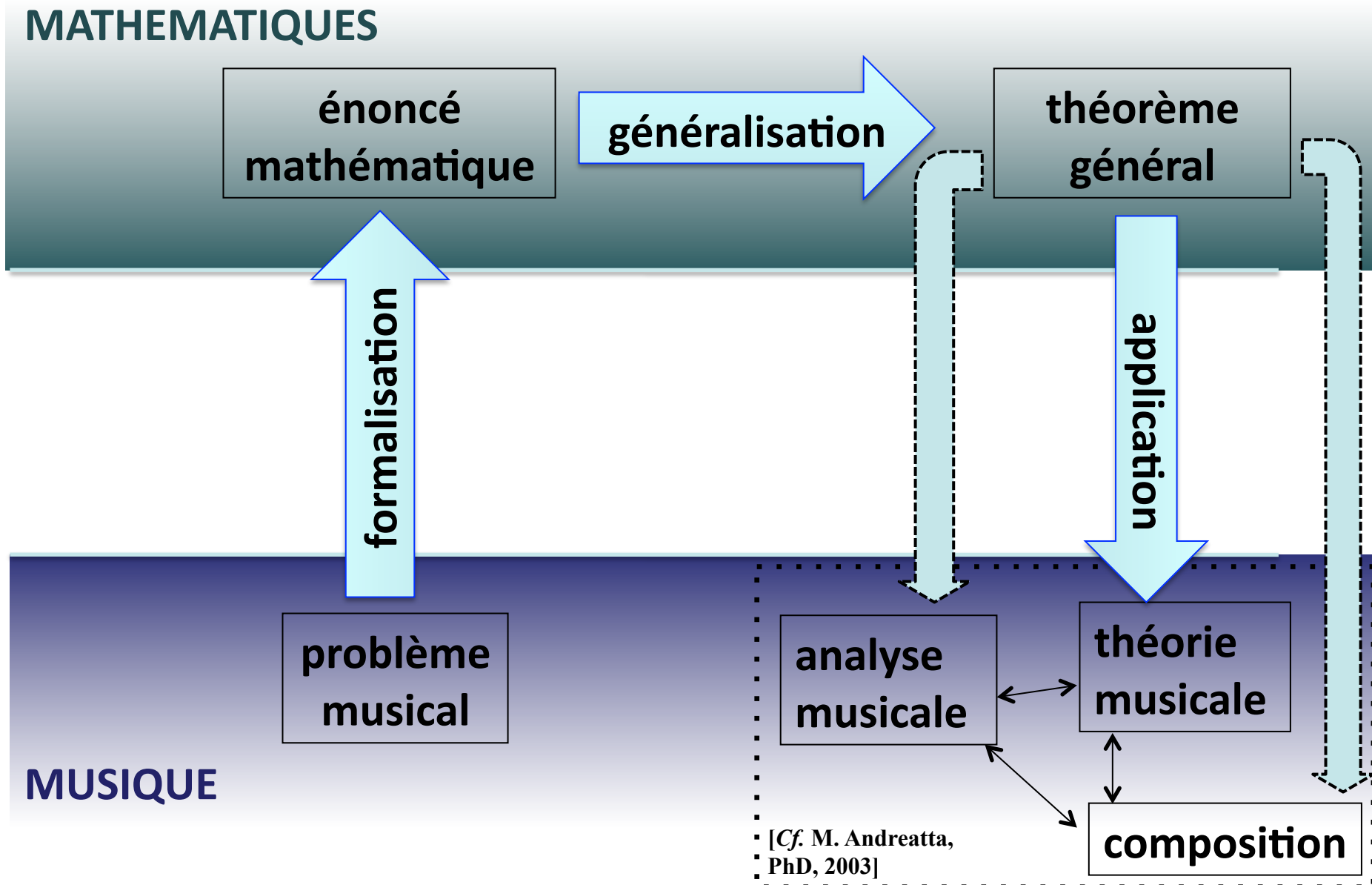
Double mouvement d'une dynamique mathémusicale



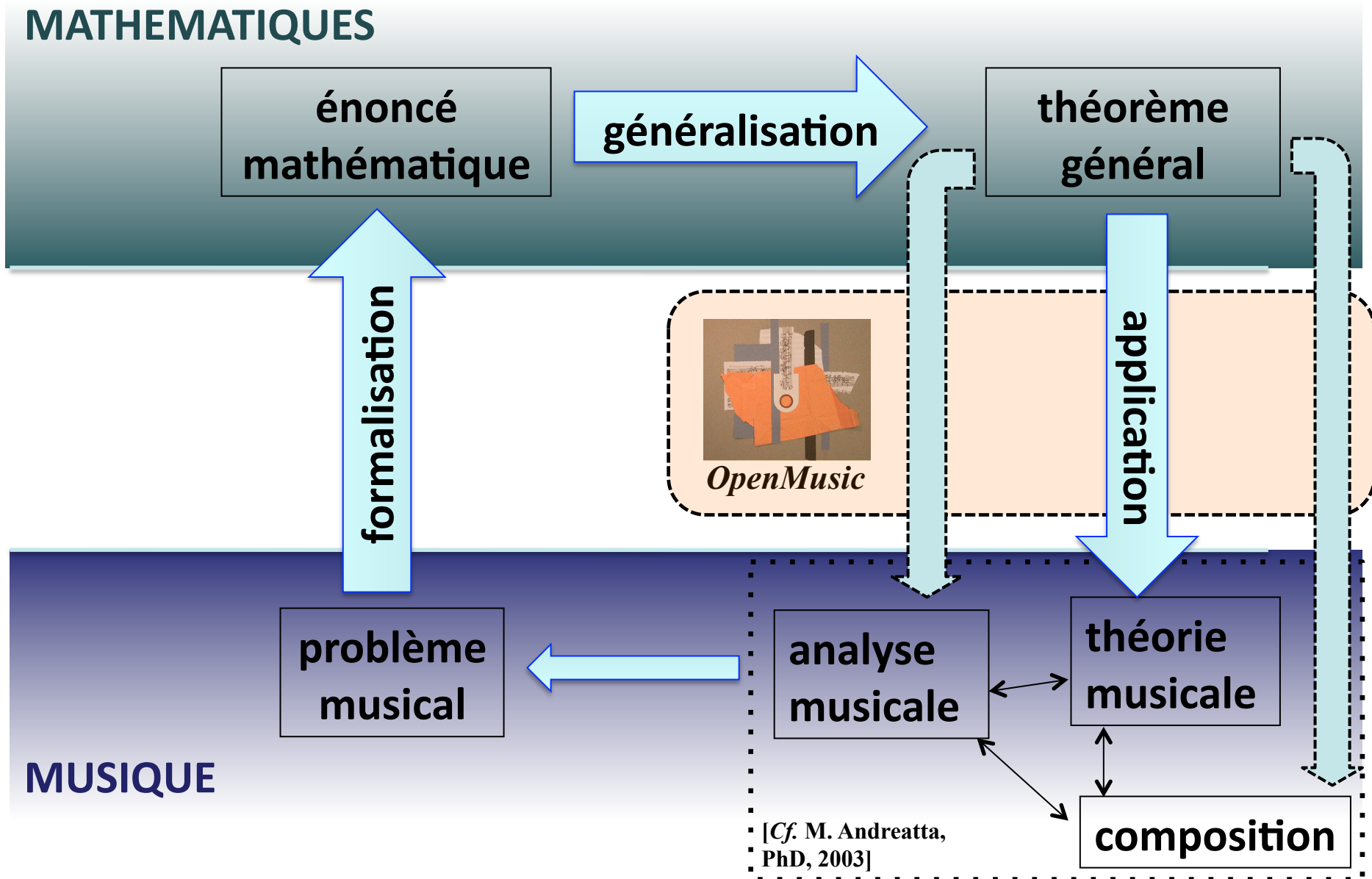
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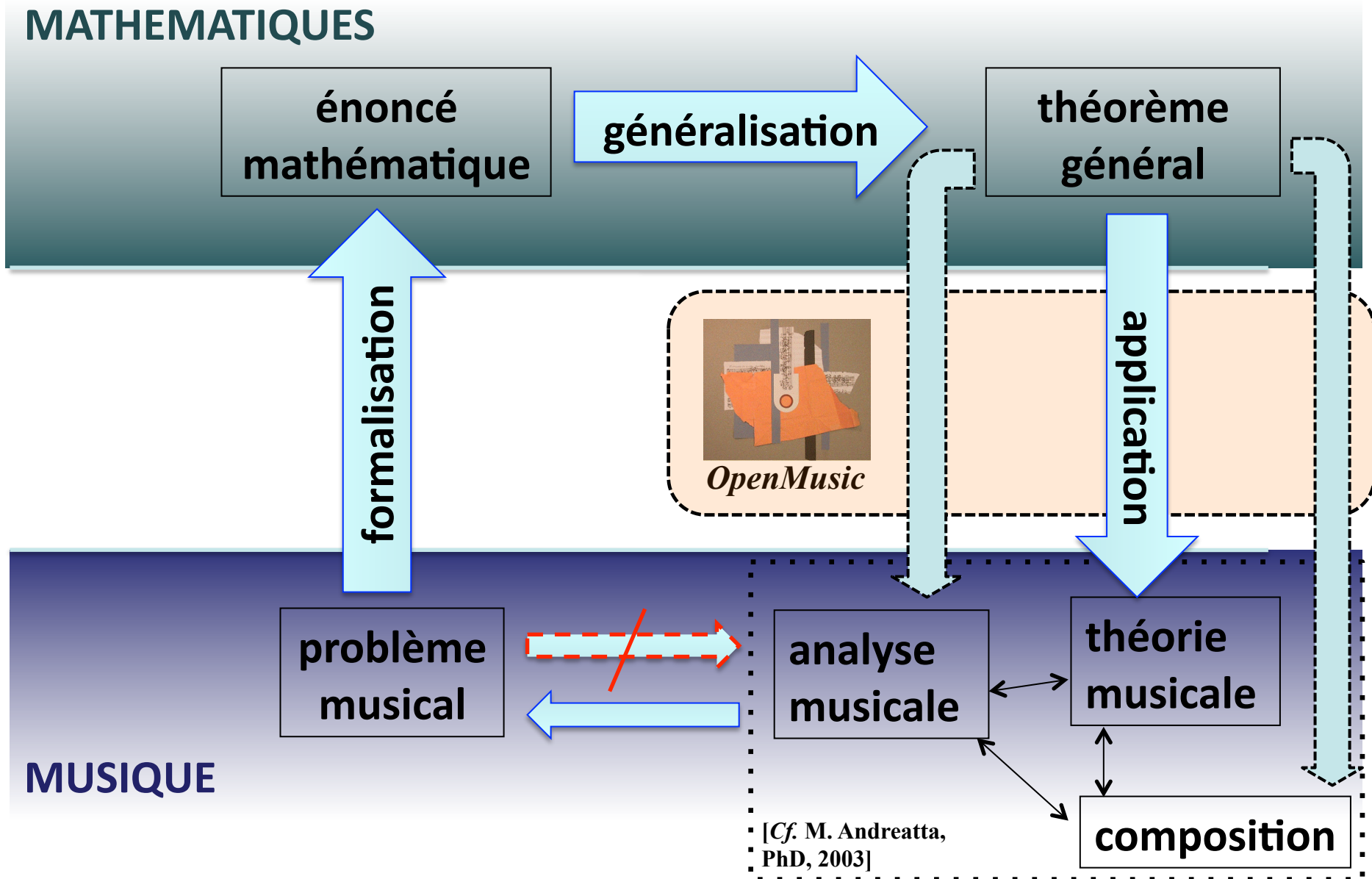
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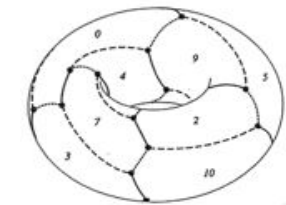
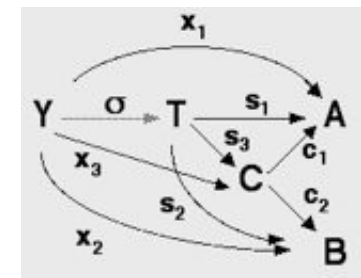
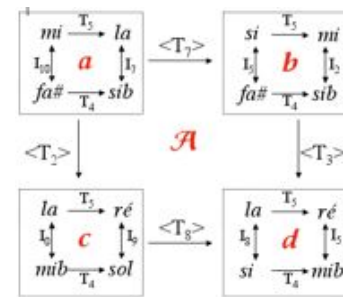
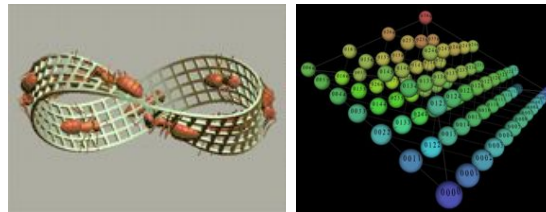
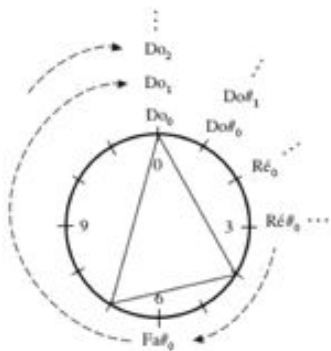


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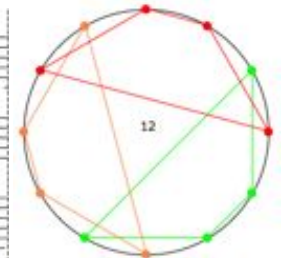
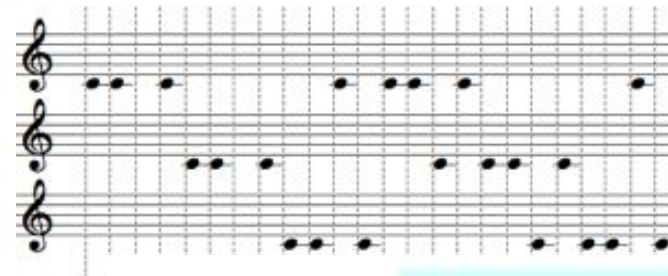


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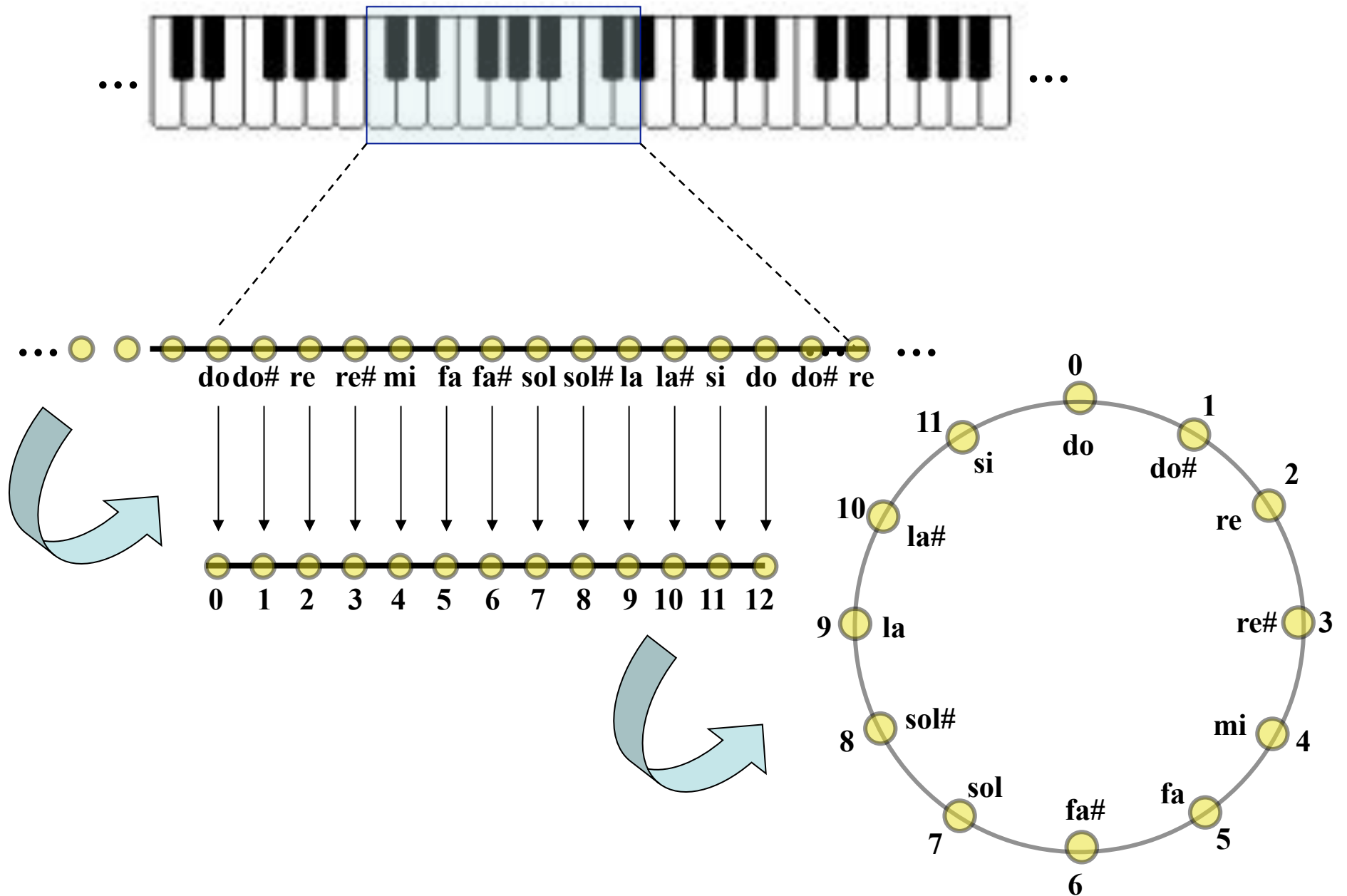


Quelques notions algébriques de base

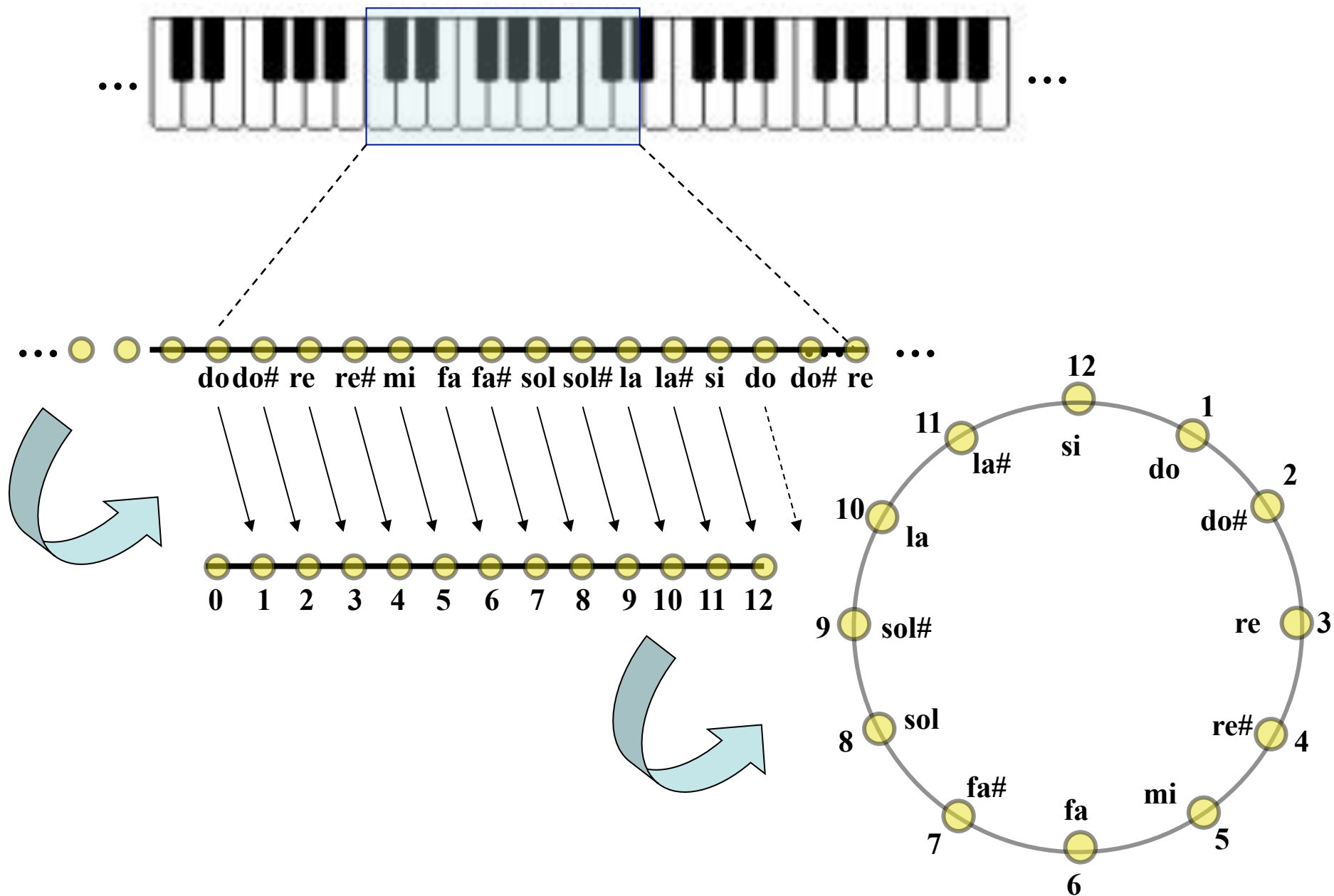
1.) Représentation/formalisation et énumération/classification des structures musicales

- Relation d'équivalence
- Définition de groupe
 - Groupe cyclique $\mathbf{Z}/n\mathbf{Z}$ d'ordre n
 - Groupe diédral \mathbf{D}_{2n} d'ordre $2n$
 - Groupe affine \mathbf{Aff}_n d'ordre $\varphi(n) \cdot n$
 - Groupe symétrique \mathbf{S}_n d'ordre $n!$
 - Groupe de Klein
- Action d'un groupe sur un ensemble
 - Action transitive et simplement transitive
 - Stabilisateur d'un élément (d'un ensemble)
- Factorisation d'un groupe

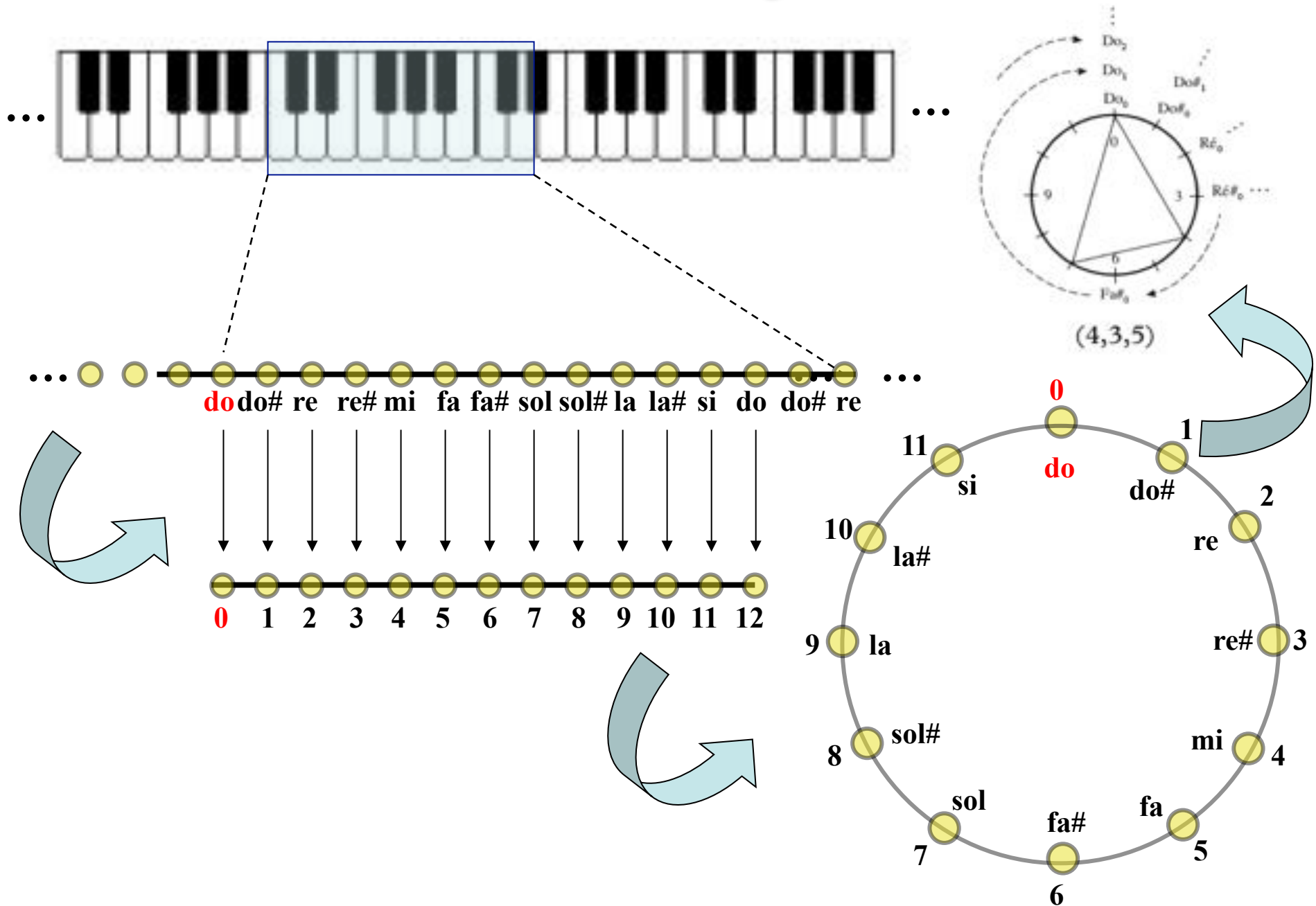
Réduction à l'octave et congruence modulo 12



Réduction à l'octave et congruence modulo 12



Réduction à l'octave et congruence modulo 12



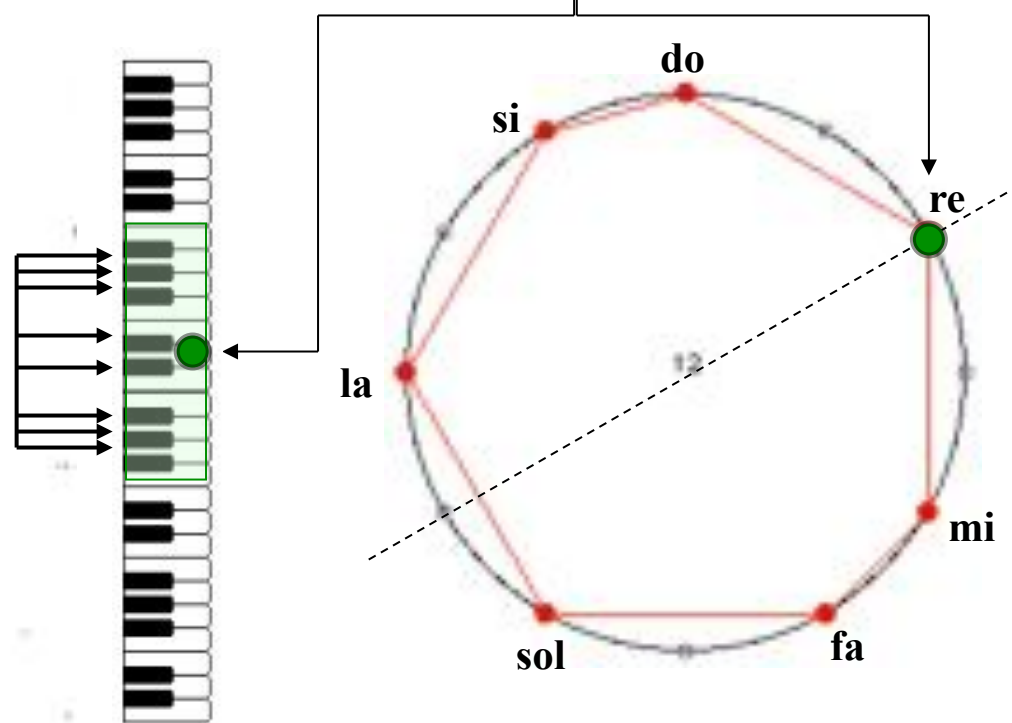
Un peu d'histoire...



Camille Durutte:

- *Technie, ou lois générales du système harmonique* (1855)
- *Résumé élémentaire de la Technie harmonique, et complément de cette Technie* (1876)

So	bb	Re	bb	Fa	Ut	Sol	Re	La	Mi	Si	Re	La
-15	-14	-3	-2	-1	0	+1	+2	+3	+14	+15		



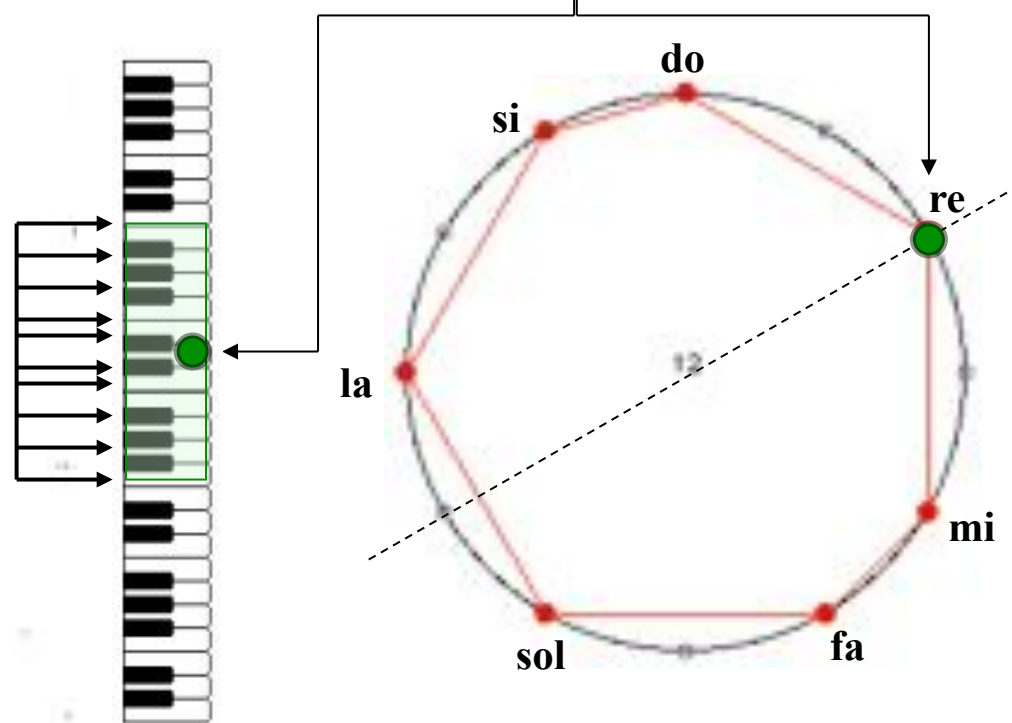
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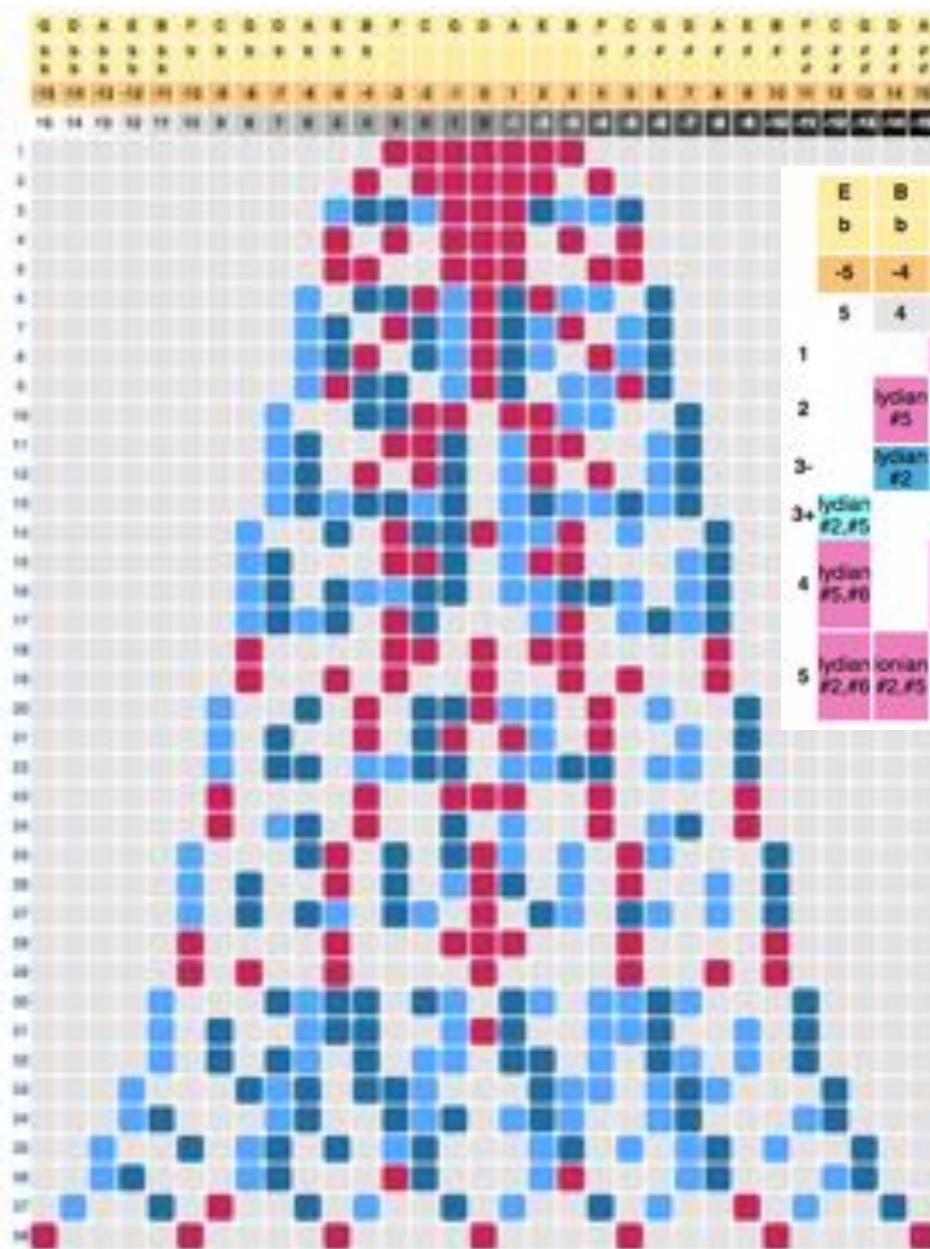
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-15	-14	-3	-2	-1	0	+1	+2	+3	+14	+15		



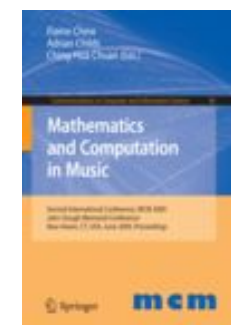
La cloche diatonique centrée autour du ré (P. Audétat & co.)

<http://www.cloche-diatonique.ch/>

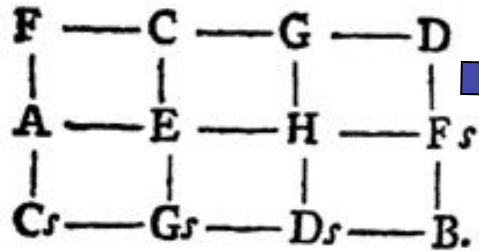


	E	B	F	C	G	D	A	E	B	F	C	
	b	b								#	#	
	-5	-4	-3	-2	-1	0	1	2	3	4	5	
	5	4	3	2	1	0	-1	-2	-3	-4	-5	
1			lydian	ionian	mixolydian	dorian	aeolian	phrygian	locrian			diatonic
2	lydian #5			lydian b7	ionian b3	mixolydian b5 or aeolian 3	phrygian 6	locrian 2		locrian b4		minor melodic
3		lydian #2	augmented ionian		dorian #4	aeolian 7	phrygian 3	locrian 6			locrian b4,bb7	minor harmonic
3+	lydian #2,#5			lydian b3	ionian b6	mixolydian b2	dorian b5		phrygian b4	locrian b6,7		major harmonic
4	lydian #5,#6		lydian #5,b7		lydian b6,b7	ionian b2,b3 or phrygian 6,7	locrian 2,3		locrian 2,b4		locrian bb3,b4	unisonic
5	lydian #2,#6	ionian #2,#5			aeolian #4,7	ionian b2,b6 or phrygian 3,7	mixolydian b2,b5			phrygian b4,bb7	locrian bb3,bb7	double harmonic

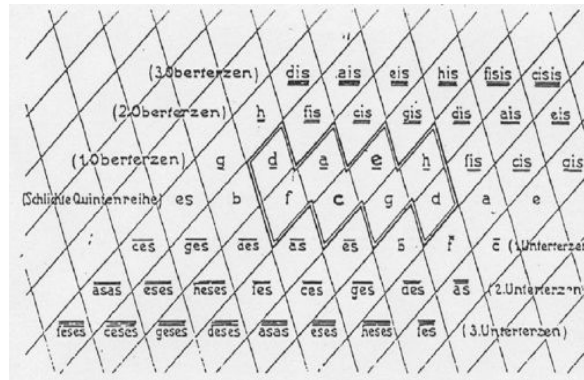
Junod, J., Audétat, P., Agon, C.,
 Andreatta, M., « A Generalisation of
 Diatonicism and the Discrete Fourier
 Transform as a Mean for Classifying and
 Characterising Musical Scales », Second
 International Conference MCM 2009,
 vol. 38, New Haven, 2009, pp. 166-179



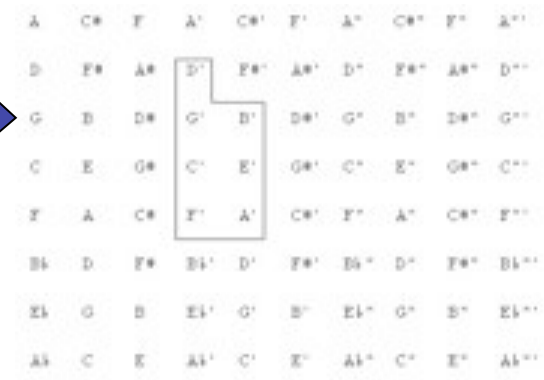
Approches géométriques : d'Euler au *Tonnetz*



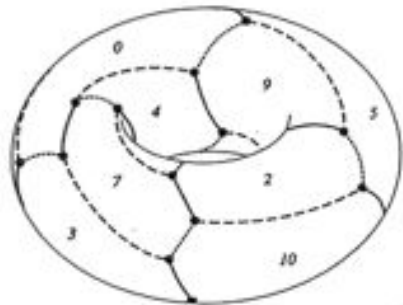
Euler : *Speculum musicum*, 1773



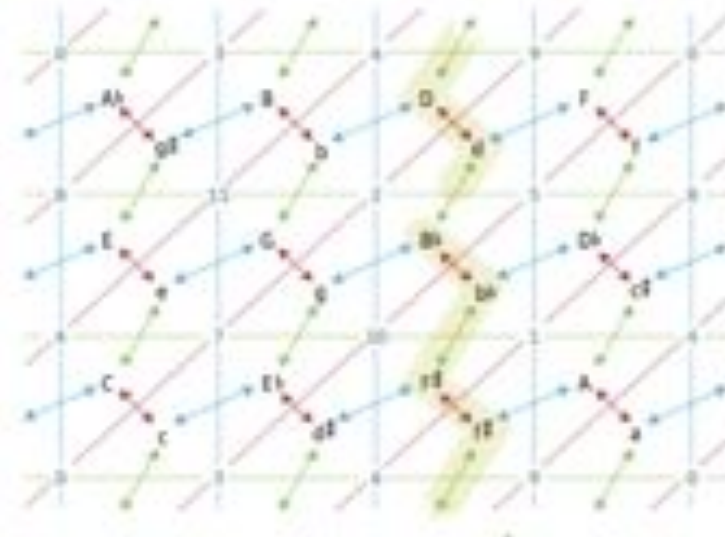
Hugo Riemann : « Ideen zu einer Lehre von den Tonvorstellung », 1914



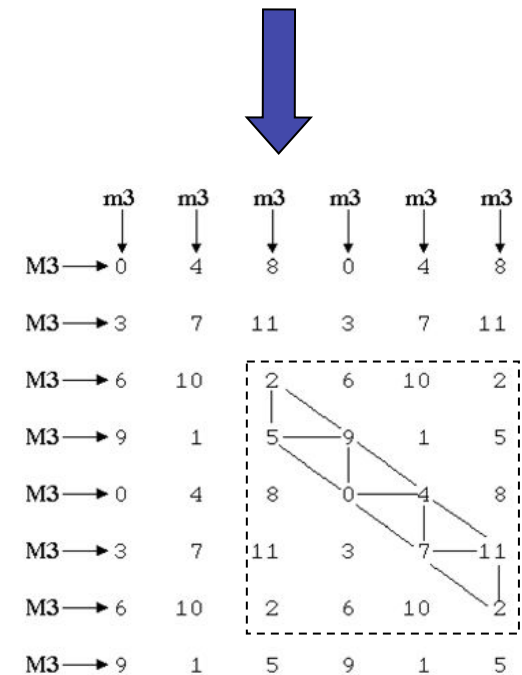
Longuet-Higgins (1962)



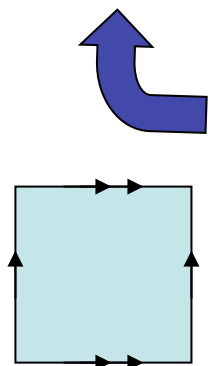
Douthett & Steinbach, *JMT*, 1998



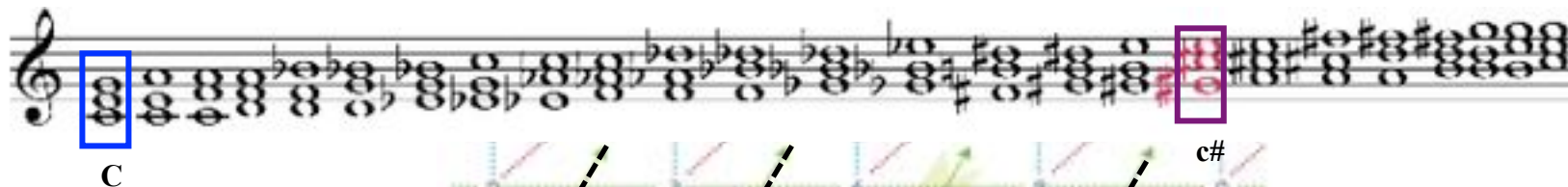
J. Hook, « Exploring Musical Space », *Science*, 2006



Balzano (1980)

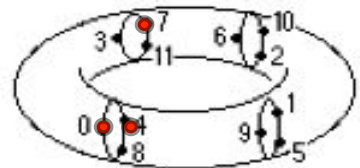
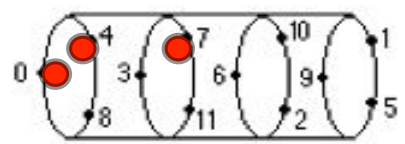
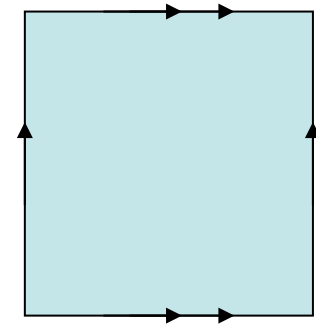
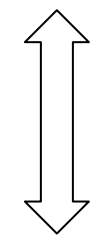
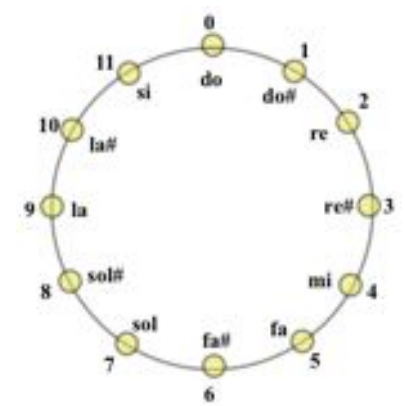
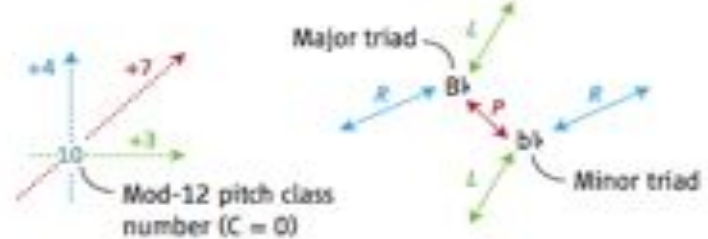
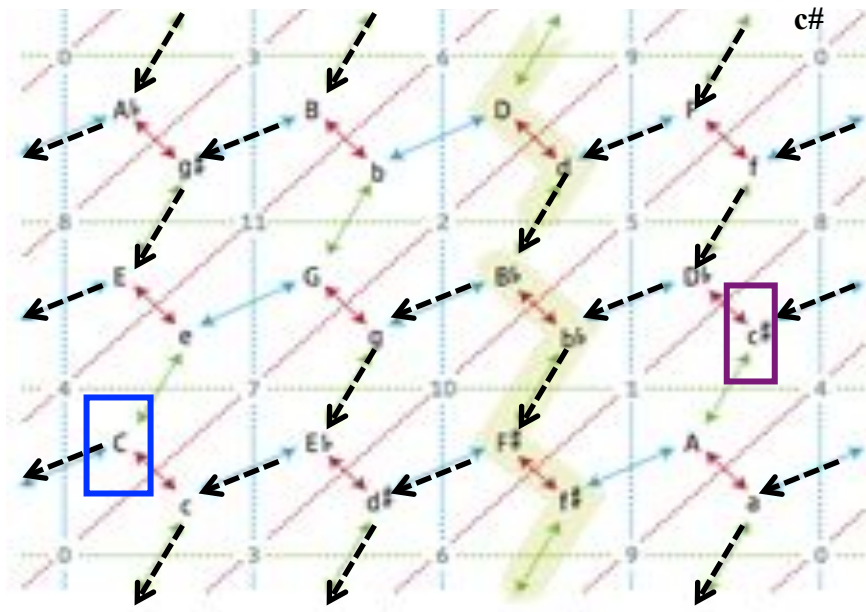
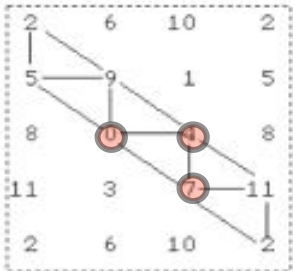
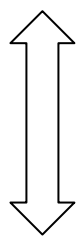


Procédés algorithmiques en musique tonale

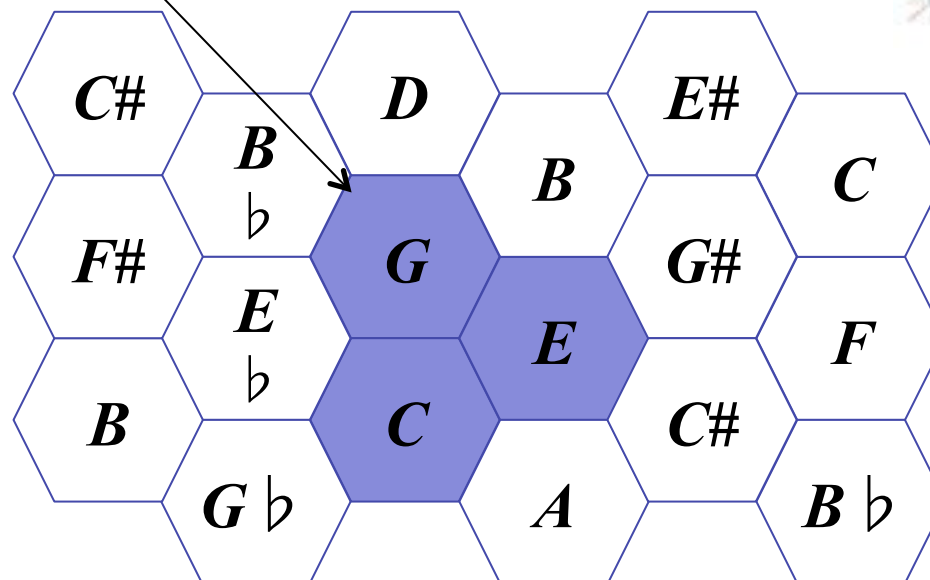
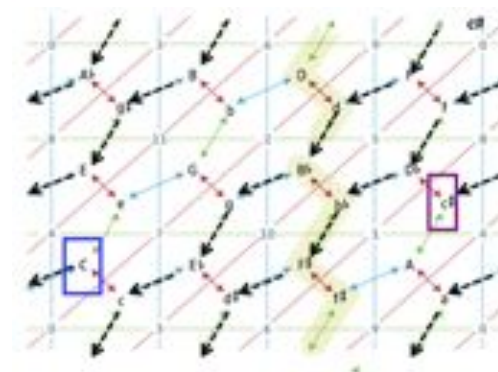
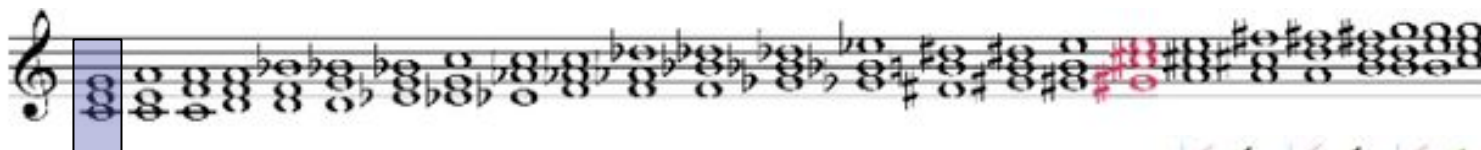


F	C	G	D
A	E	H	F _s
C _s	G _s	D _s	B.

Euler : *Speculum musicum*, 1773



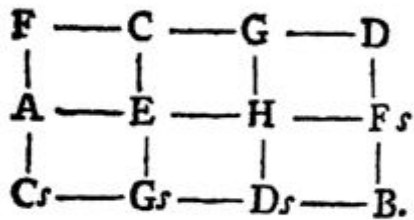
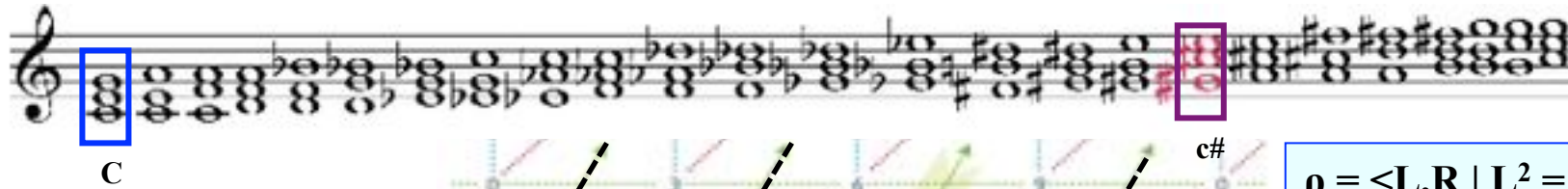
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



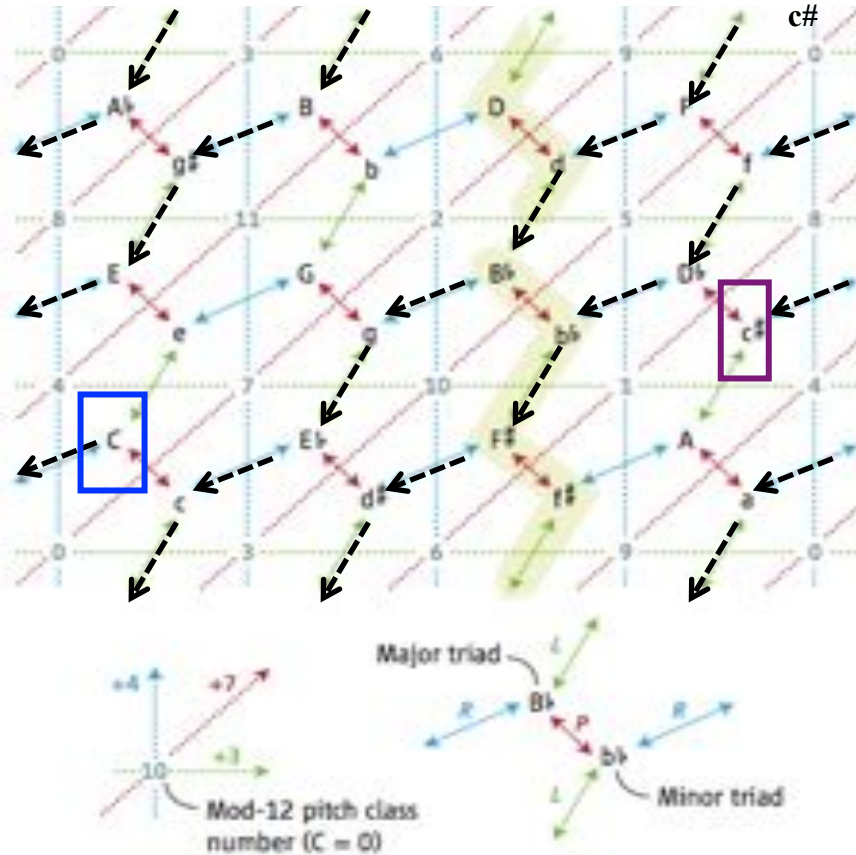
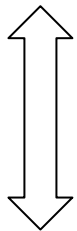
b



Analyse neo-riemannienne et programmation spatiale



Euler : *Speculum musicum*, 1773



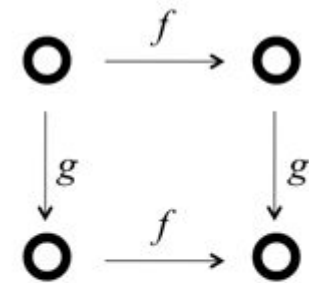
$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \rangle$$

$$LRL = L(LR)^{-1}$$

↕ dualité

$$D_{12} = \langle I, T \mid I^2 = T^{12} = 1 \rangle$$

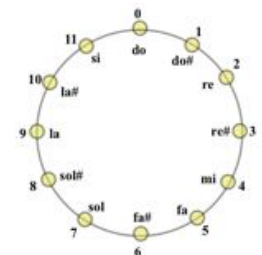
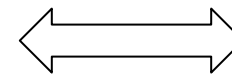
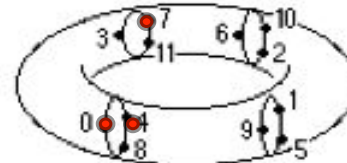
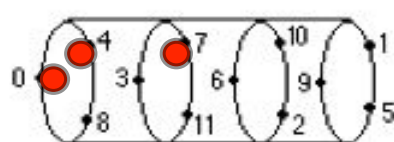
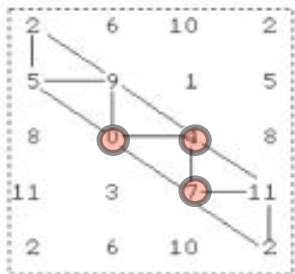
$$ITI = I(IT)^{-1}$$



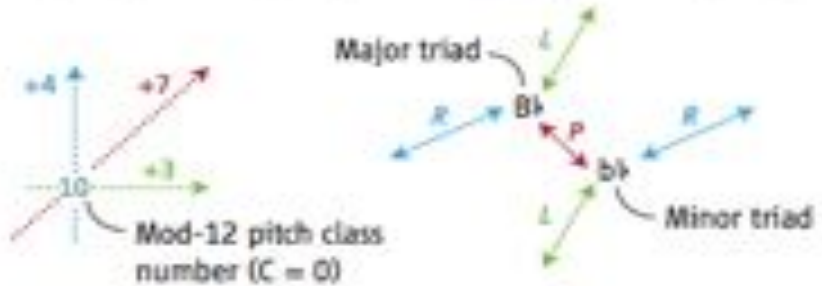
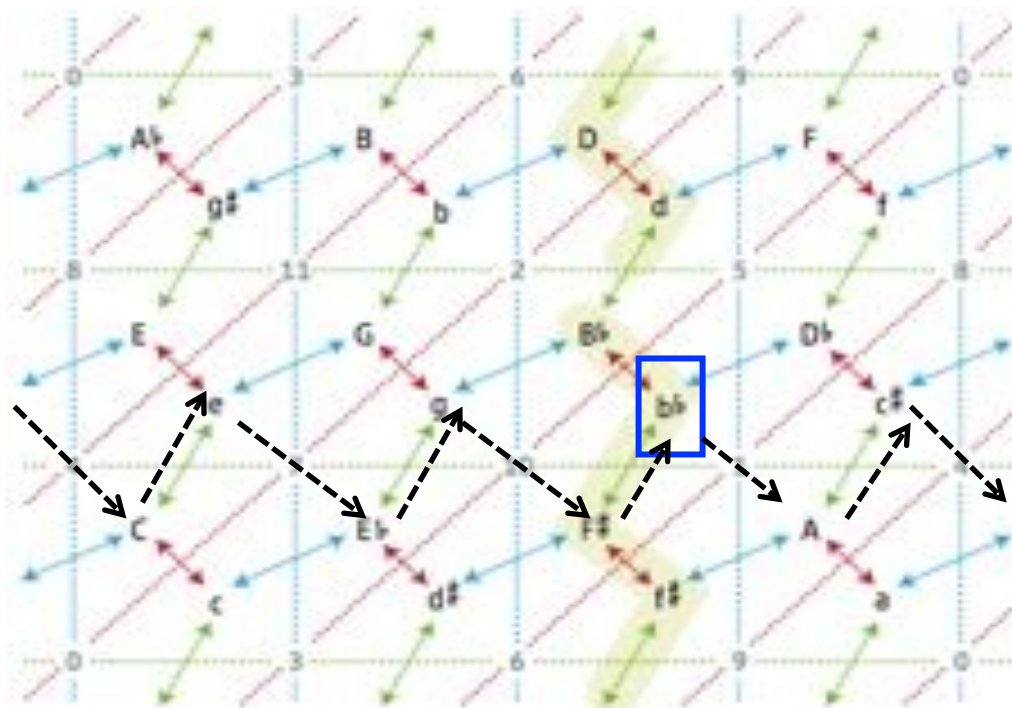
Tout diagramme commute

$$\forall f \in D_{12}$$

$$\forall g \in \rho$$

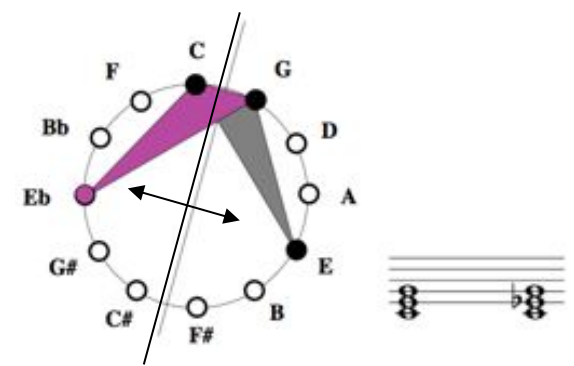


Cycles dans le réseau hexagonal

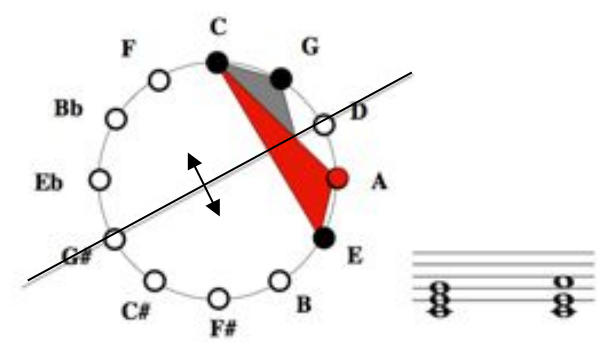


[J. Hook 06]

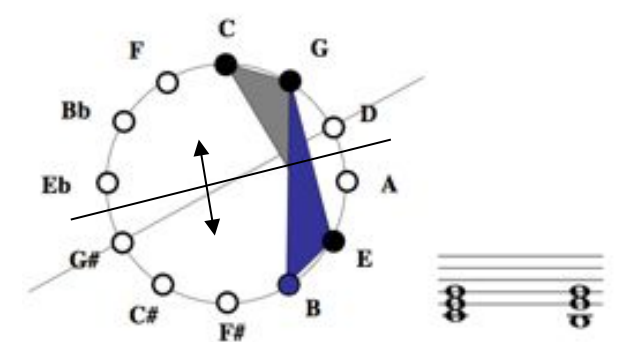
(Neo-)Riemannian Operation P = „Parallel“ [Noll04]



(Neo-)Riemannian Operation R = „Relative“



(Neo-)Riemannian Operation L = „Leading-Tone“



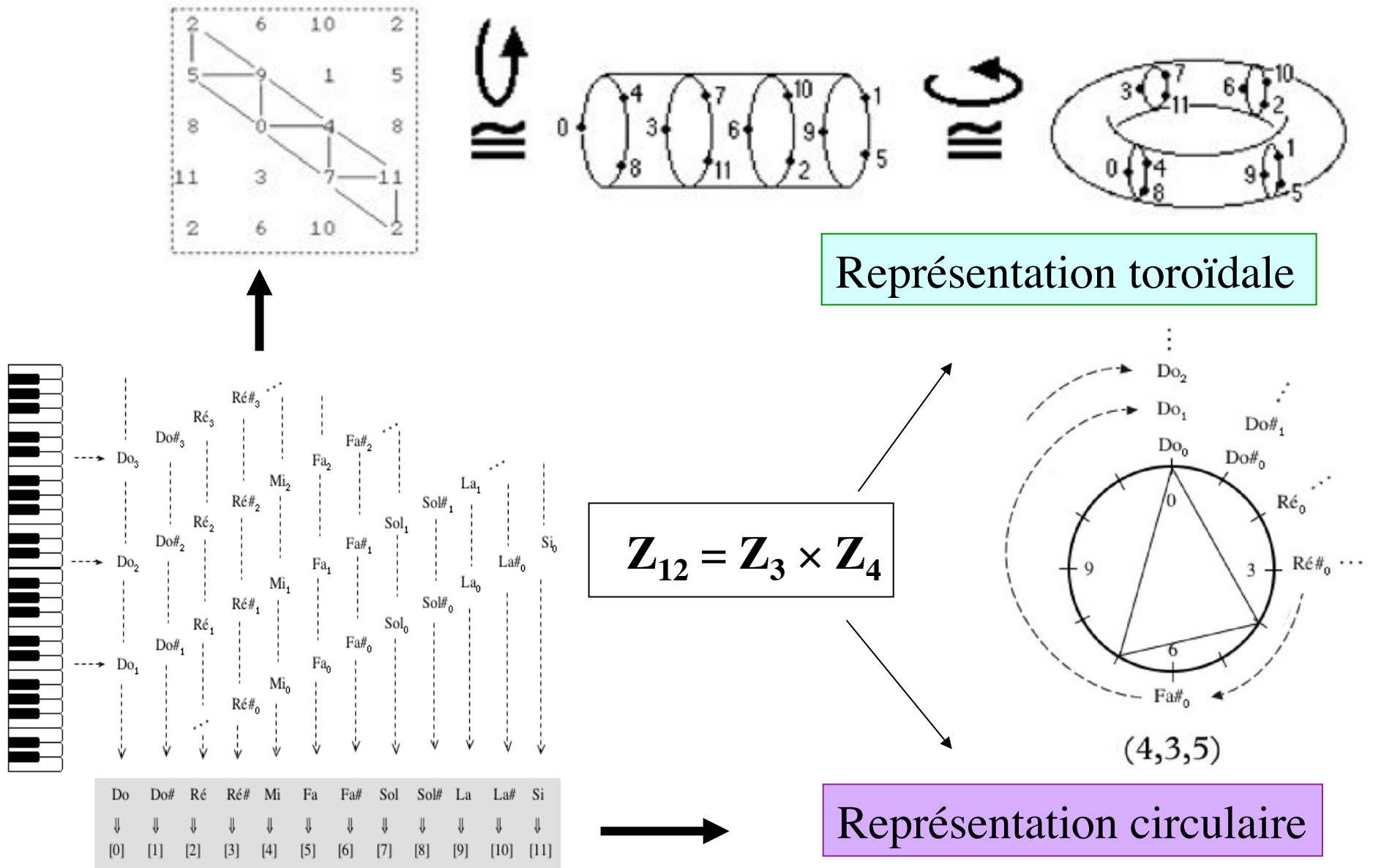
Recherche des cycles hamiltoniens dans LPR

C e E gB B dE c cD f Ck cA A W Fa aA g G b D d F a	#53
C c Dg g Aa aCk f cD gB dE Fa W A cE E e G b D d F a	#52
C e G g Aa d F f cD c Dd dE Fa aCk cE gB B b D W A a	#45
C c Dk f Ck cA A W D b B gE E e G g Dd dE Fa aA d F a	#29
C e G g Ck c cD gE E cA A W Fa dE B b D d Aa aCk f F a	#49
C c Ck f Ck aA d F a A cE E gB B b D W Fa dE Dg g G e	#11
C e E gB B dE Fa W D b G g Dc c Ck f F d Aa aCk cA a	#54
C c Dk g G b D W A a F d Aa aFa dE B gE c Dd f Ck cE e	#14
C c Ck gB B dE Dg g Aa d F f Ck aFa W D b G e E cA a	#24
C c Dk dE B b D W Fa aCk f cD gE E cA a F d Aa g G e	#20
C e E cE Ck f F d Aa aFa dE B gE c Dd g G b D W A a	#55
C c Dk g G e E gE cA f Ck cA A W D b B dE Fa aA d F a	#38
C e G g Aa d D b B dE Dc c Ck gE E cA A W Fa aCk f F a	#47
C c Ck f Ck aA d D dE dE Fa W D d F a A cE E gB B b G e	#5
C e E gB B dE Fa W A cE Ck aA d F a G g Dd c cA f F a	#57
C c Ck f F a A W D d Aa g Dd dE Fa aCk cE gB B b G e	#3
C e E gB B b G g Aa d D W A cE Ck aA Fa dE c cD f F a	#61
C e E cE A W Fa dE B gE c Dd g G b D d Aa aCk f F a	#56
C c Ck g Aa aFa dE B b G e E gE cA f Ck cA A W D d F a	#35
C c Dk g G b D W Fa dE B gE f Ck aA d F a A cE E e	#15
C c Ck f Ck cE E gB B b D W A a F d Aa aFa dE Dg g G e	#12
C c Ck gB B dE Dg g Aa d D b G e E cE A W Fa aCk f F a	#28
C c Dk g Aa aCk f cD gE E cA a F d D W Fa dE B b G e	#17
C e G g Dk c cD gE E cE cD f F d Aa aFa dE B b D W A a	#51
C e E gB B dE c cD f F d Aa g G b D W Fa aCk cA a	#52
C e G g Aa d F f Ck aA Fa W D b B dE Dc c Ck gE E cA a	#46
C c Ck f Ck cA A W Fa aA g Dd dE B gE E e G b D d F a	#28

#53 L P L R L P R L R L P L R P L P R P L R P R L R
#52 P R L R P R L R P R L R P R L R P R L R P R L R
#45 L R P R L R P R L R P R L R P R L R P R L R P R
#29 P L R L P L R L R P R L P R P L P R L P L R L R
#49 L R P L R L P L R L R P R L P R P L P R L P L R
#11 P L R L R P L R L P L R L R P R L P R P L P R L
#54 L P L R L R P L R L P L R L R P R L P R P L P R
#14 P R L P L R L R P L R L P L R L R P R L P R P L
#24 P L P R L P L R L R P L R L P L R L R P R L P R
#20 P R P L P R L P L R L R P L R L P L R L R P R L
#55 L P R P L P R L P L R L R P L R L P L R L R P R
#36 P R L P R P L P R L P L R L R P L R L P L R L R
#47 L R P R L P R P L P R L P L R L R P L R L P L R
#5 P L R L R P R L P R P L P R L P L R L R P L R L
#57 L P L R L R P R L P R P L P R L P L R L R P L R
#3 P L R P L P R L P L R L P R L R P R L R P L R L
#61 L P L R P L P R L P L R L P R L R P R L R P L R
#56 L P R L R P R L R P L R L P L R P L P R L P L R
#35 P R L R P L R L P L R P L P R L P L R L P R L R
#15 P R L P L R L P R L R P R L R P L R L P L R P L
#12 P L R L P R L R P R L R P L R L P L R P L P R L
#26 P L P R L P L R L R P R L R P R L R P L R L P L R
#17 P R L R P R L R P L R L P L R P L P R L P R L L
#51 L R P L R L P L R P L P R L P L R L P R L R P R
#52 L P L R L P R L R P R L R P L R L P L R P L R P R
#46 L R P R L R P L R L P L R P L P R L P L R L P R
#28 P L R L P L R P L P R L P L R L P R L R P R L R

62 cycles hamiltoniens classés en 8 types

Equivalence algébrique entre représentations géométriques

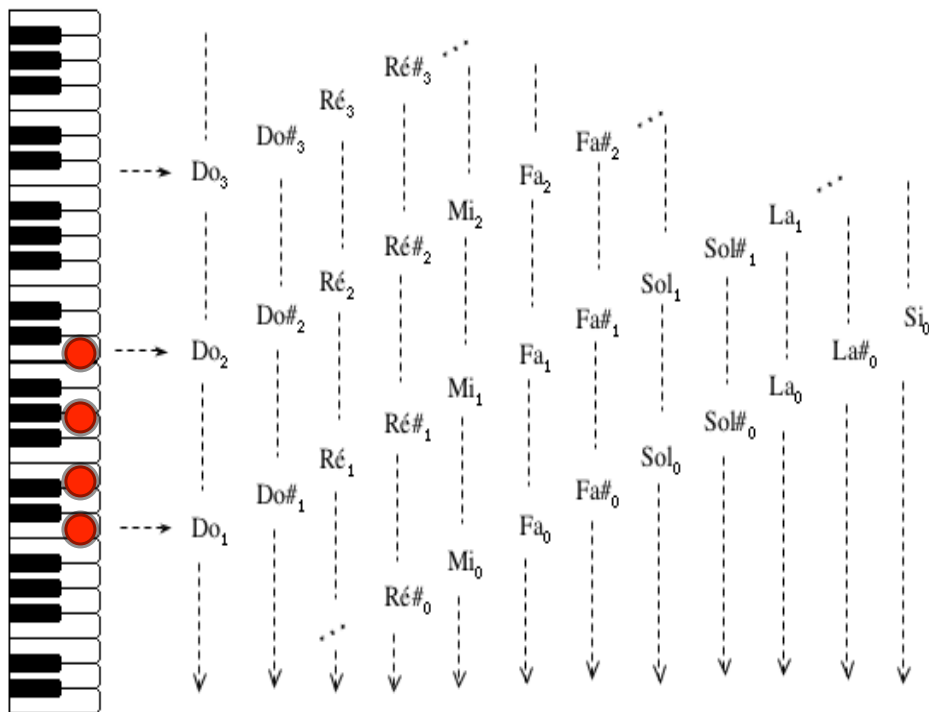


Représentation toroïdale

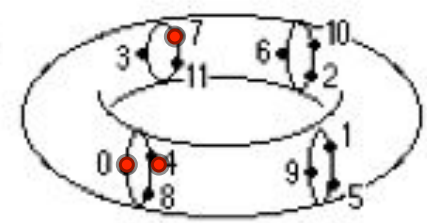
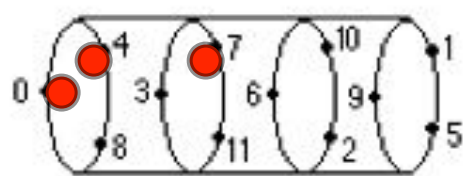
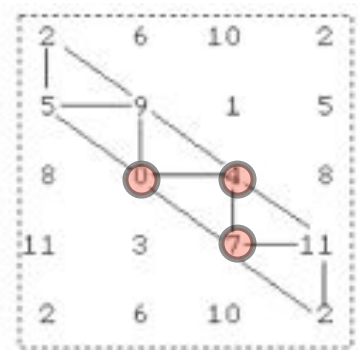
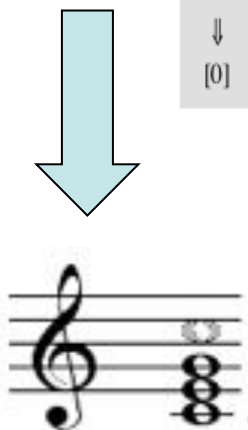
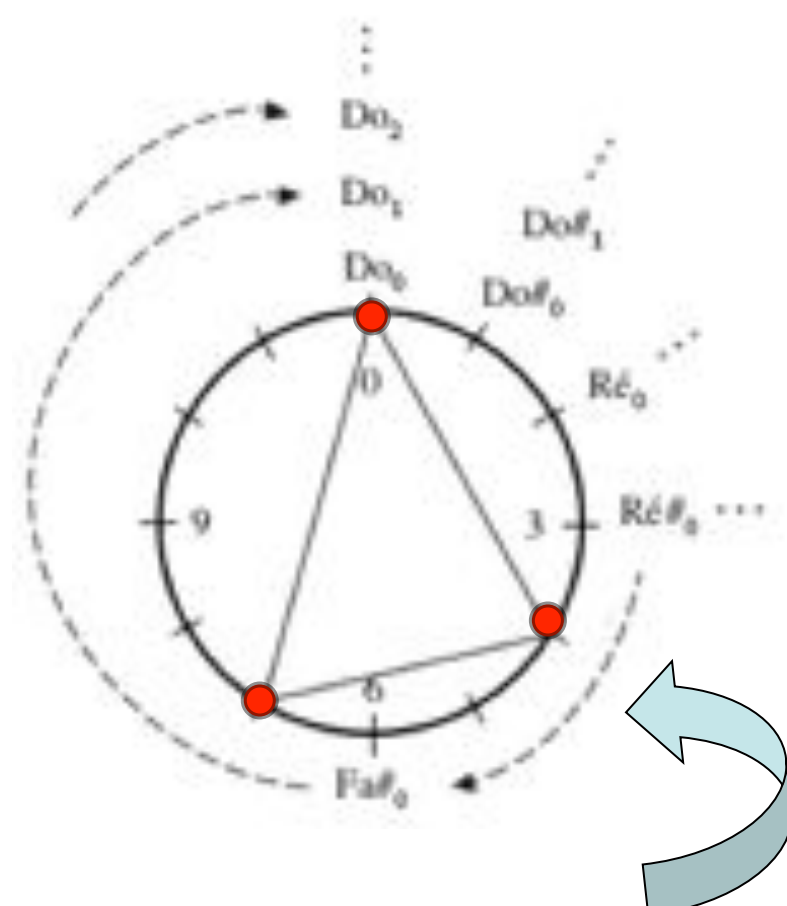
$$Z_{12} = Z_3 \times Z_4$$

Représentation circulaire

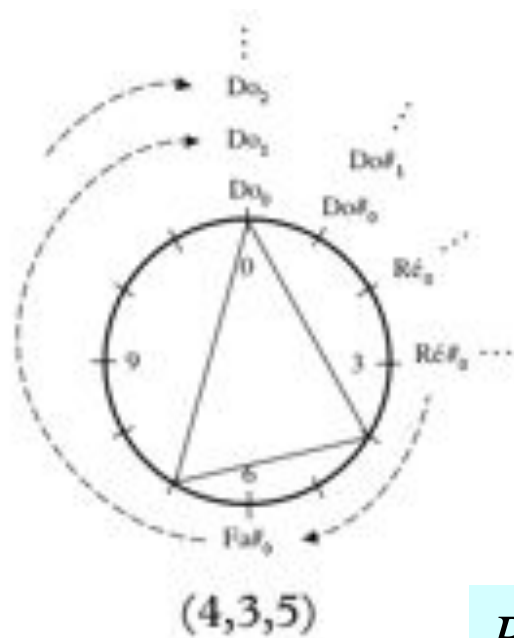
Reduction à l'octave et congruence modulo 12



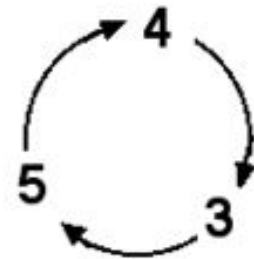
Do	Do#	Ré	Ré#	Mi	Fa	Fa#	Sol	Sol#	La	La#	Si
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]



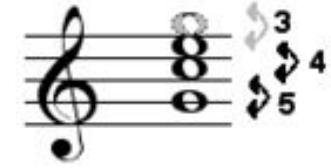
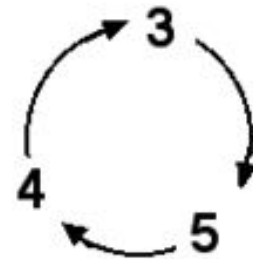
Représentation circulaire et structure intervallique



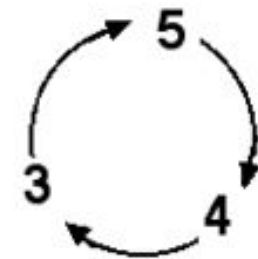
(4 3 5)



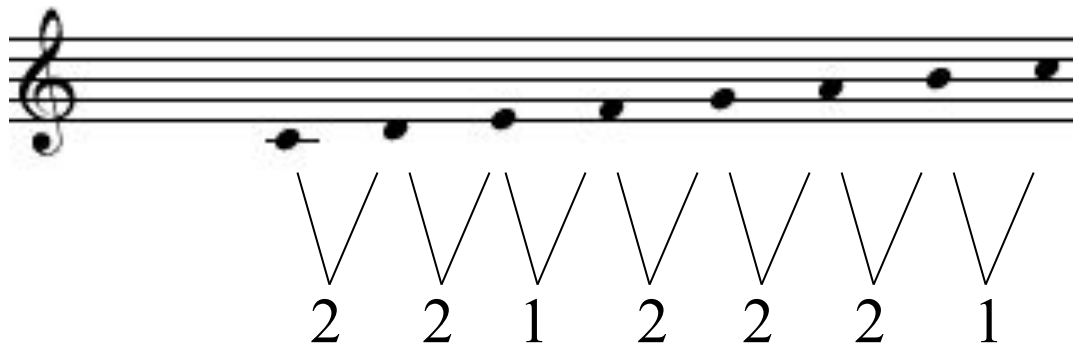
(3 5 4)



(5 4 3)

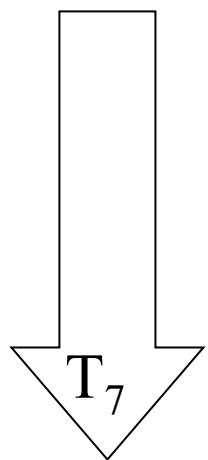
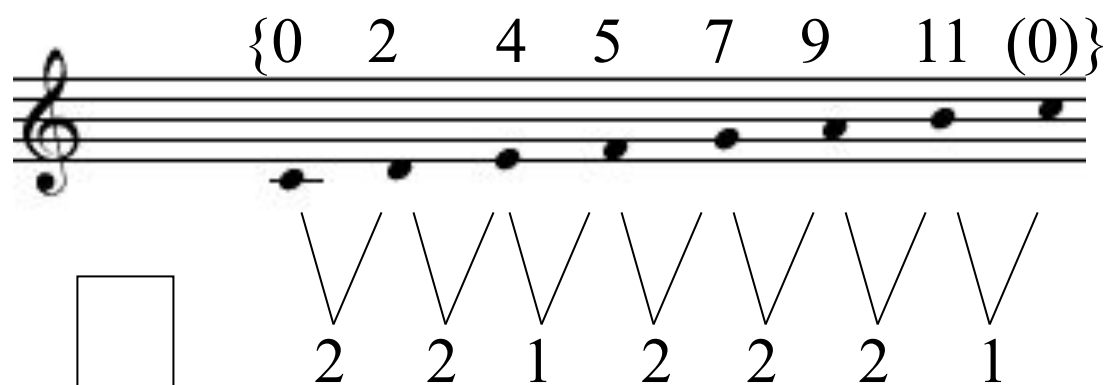


Renversements = permutations circulaires de la structure intervallique

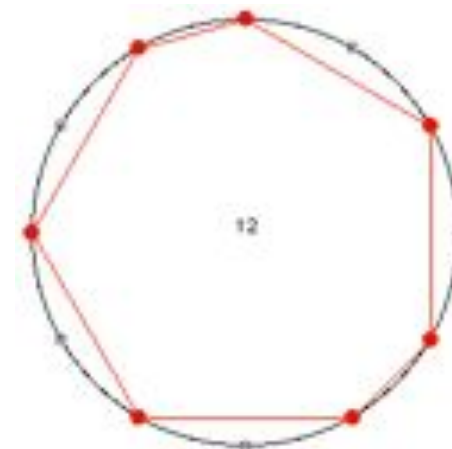
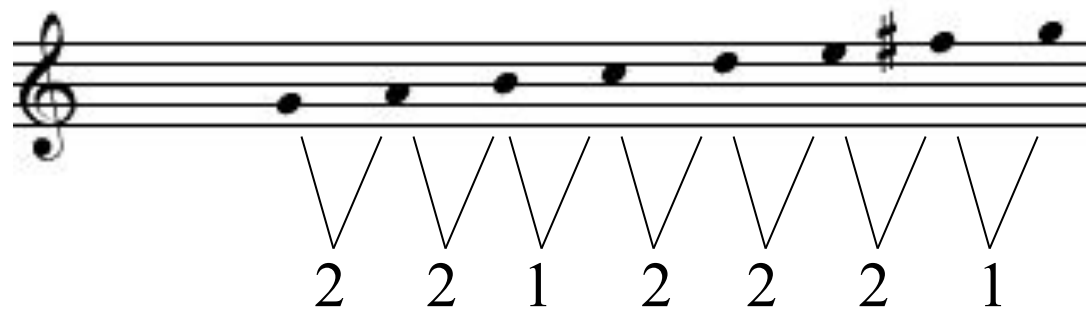


12

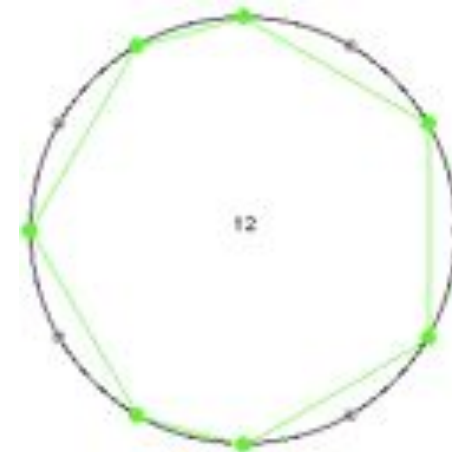
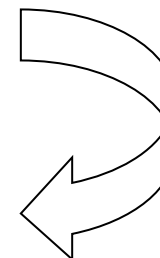
Transformations géométriques : la transposition



$T_7(x) = 7 + x \pmod{12}$

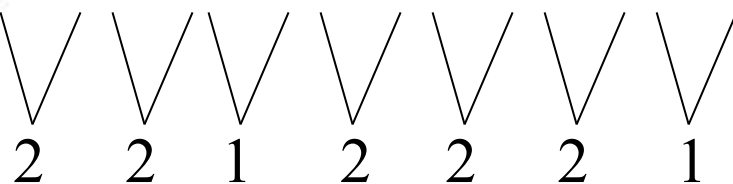


$\alpha = 210^\circ$

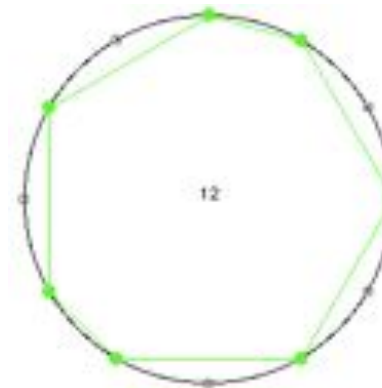
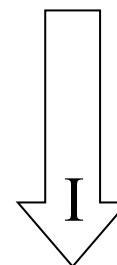
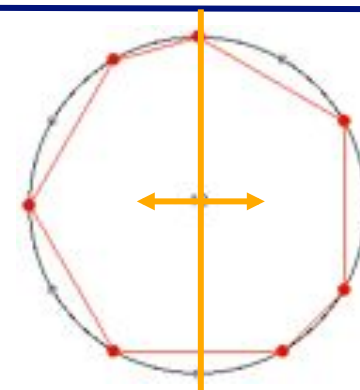
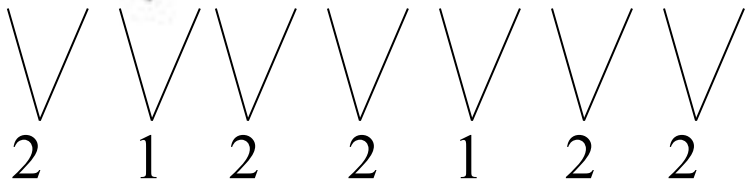
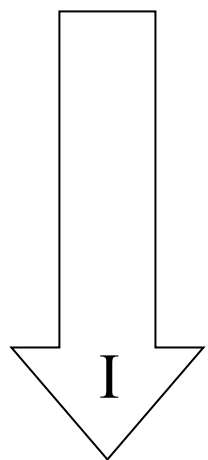


Equivalence modulo la transposition

Transformations géométriques : l'inversion

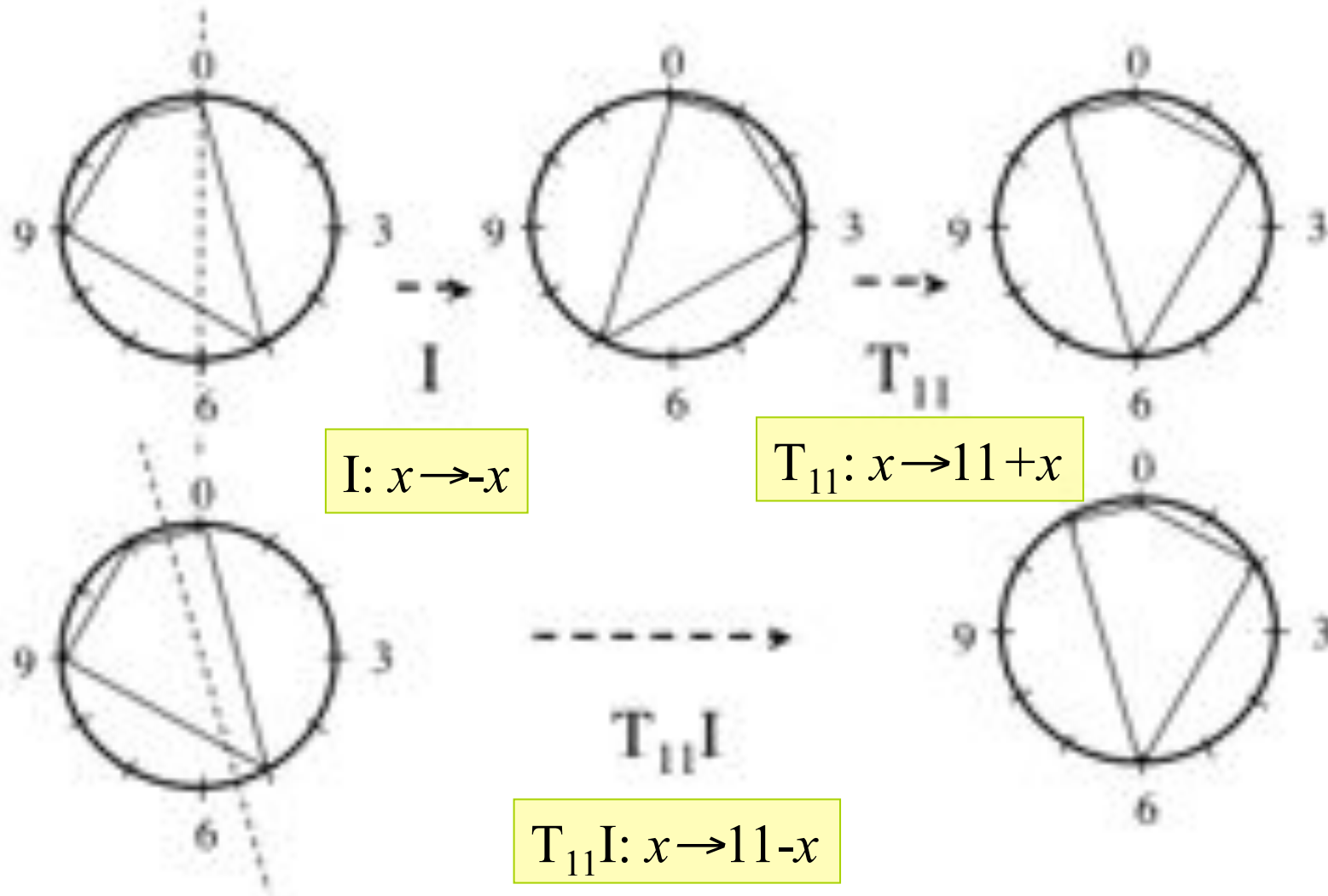


$$I(x) = -x \pmod{12}$$



*Equivalence modulo
l'inversion*

La Set Theory: équivalence modulo transposition/inversion

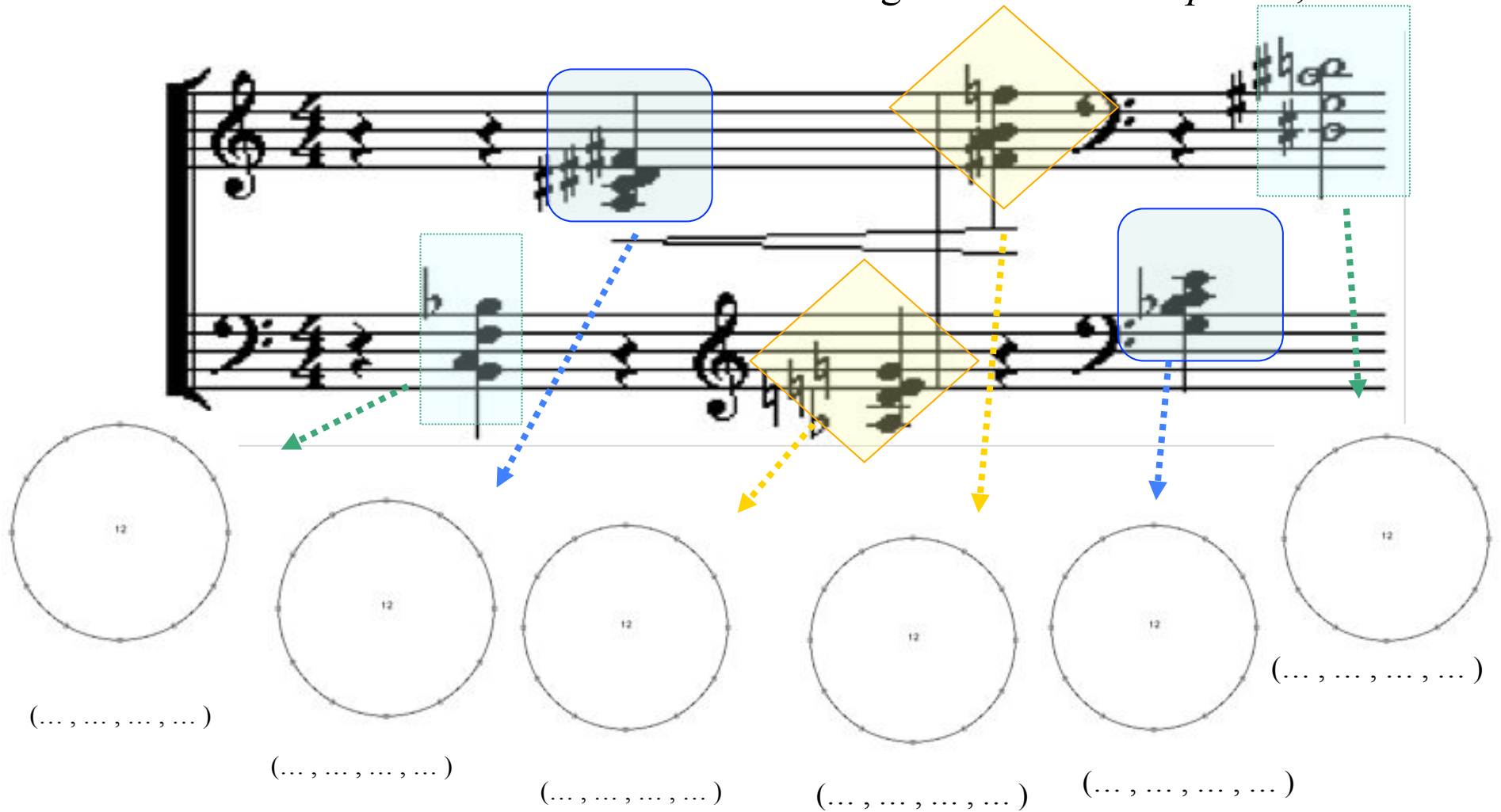


$$\{0, 5, 9, 11\} \longrightarrow \{11, 6, 3, 0\}$$

L'analyse formalisée ou les entités formelles en musique

André Riotte e Marcel Mesnage

A. Schoenberg : *Klavierstück Op. 33a*, 1929





L'analyse formalisée ou les entités formelles en musique

André Riotte & Marcel Mesnage

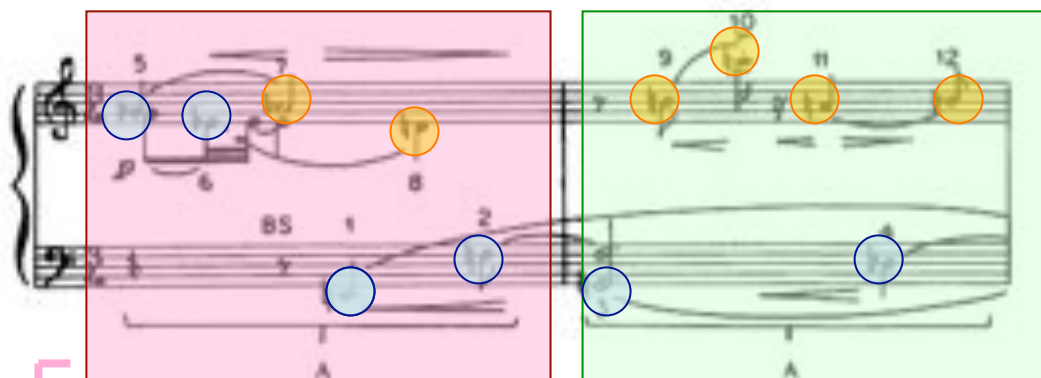


A. Schoenberg : *Klavierstück Op. 33a*, 1929

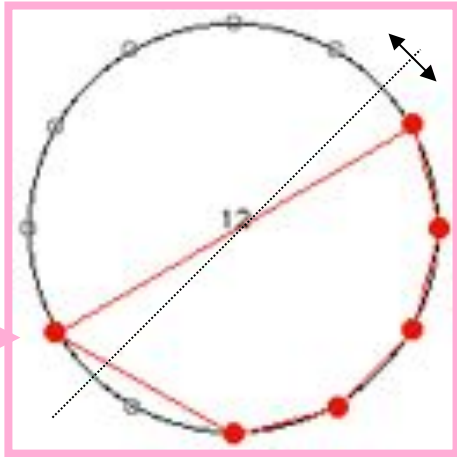
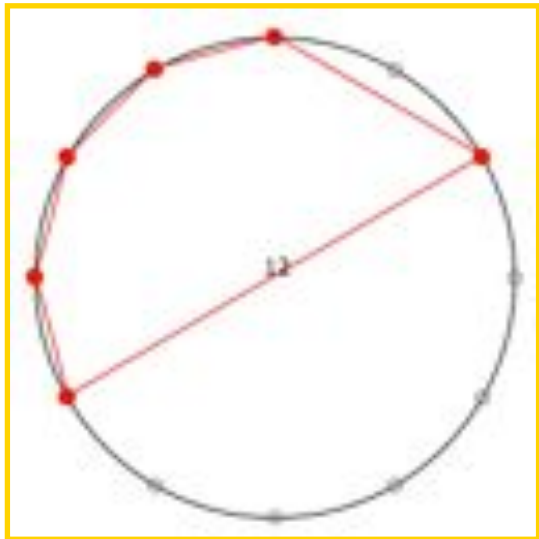
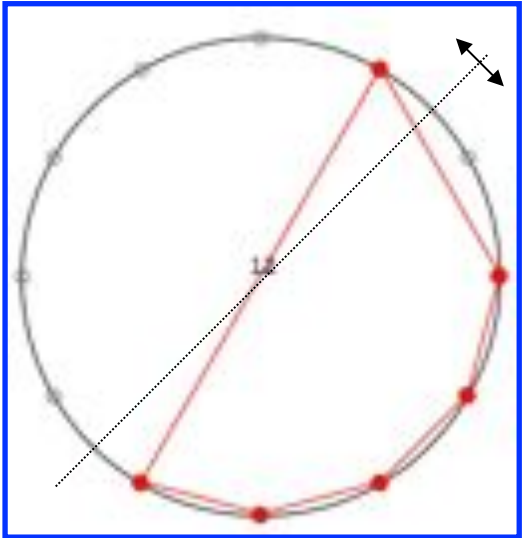
The image shows a musical score for A. Schoenberg's *Klavierstück Op. 33a* in 4/4 time. The score is annotated with several colored boxes and arrows indicating formal analysis. Below the score are six circular diagrams, each representing a formal entity. Each diagram is a circle divided into 12 segments, with numbers 1-12 around the perimeter. A central point is connected to various segments by lines, forming a specific geometric pattern. Below each diagram is a set of numbers: 0-5511 (1 2 5 6), 9-4233 (2 3 4 5 6), 8-6231 (1 2 3 4 5 6), 11-6132 (1 2 3 4 5 6), 0-4332 (2 3 4 5 6), and 3-5511 (1 2 5 6). Arrows labeled T_3 and T_1I indicate transformations between these entities. On the left, there are two book covers titled 'FORMALISMES ET MODÈLES MUSICRUX.1' and 'FORMALISMES ET MODÈLES MUSICRUX.2'.

Sérialisme et combinatorialité des hexacordes

Schoenberg: Suite Op.25, Minuetto



Double “combinatorialité”



Exercice : retrouver les symétries dans une série (I)

Schoenberg: Serenade Op.24, Mouvement 5

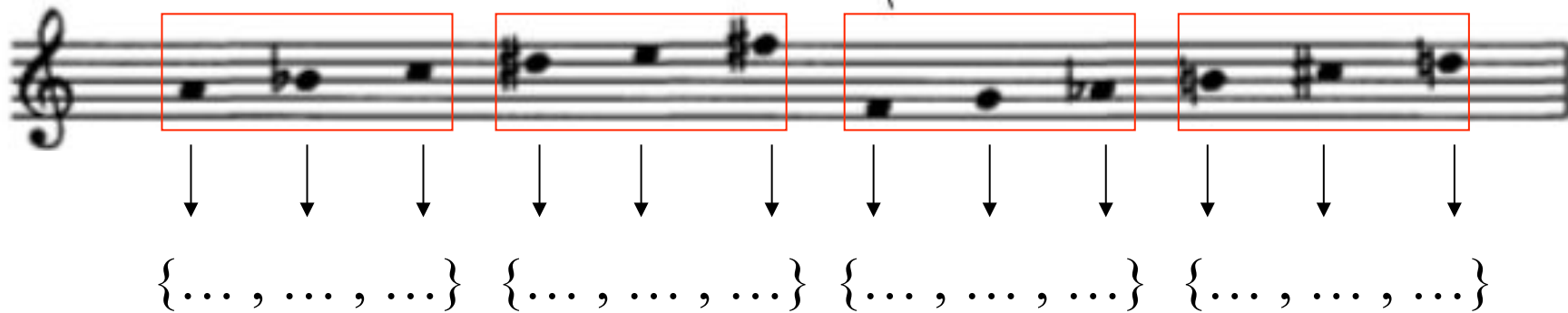


Diagram illustrating the first step of the exercise: identifying symmetries in a musical series. The notation shows a sequence of notes on a staff, with four groups of notes highlighted by red boxes. Below each group, three arrows point to a set of three dots in curly braces, representing the initial identification of potential symmetries.

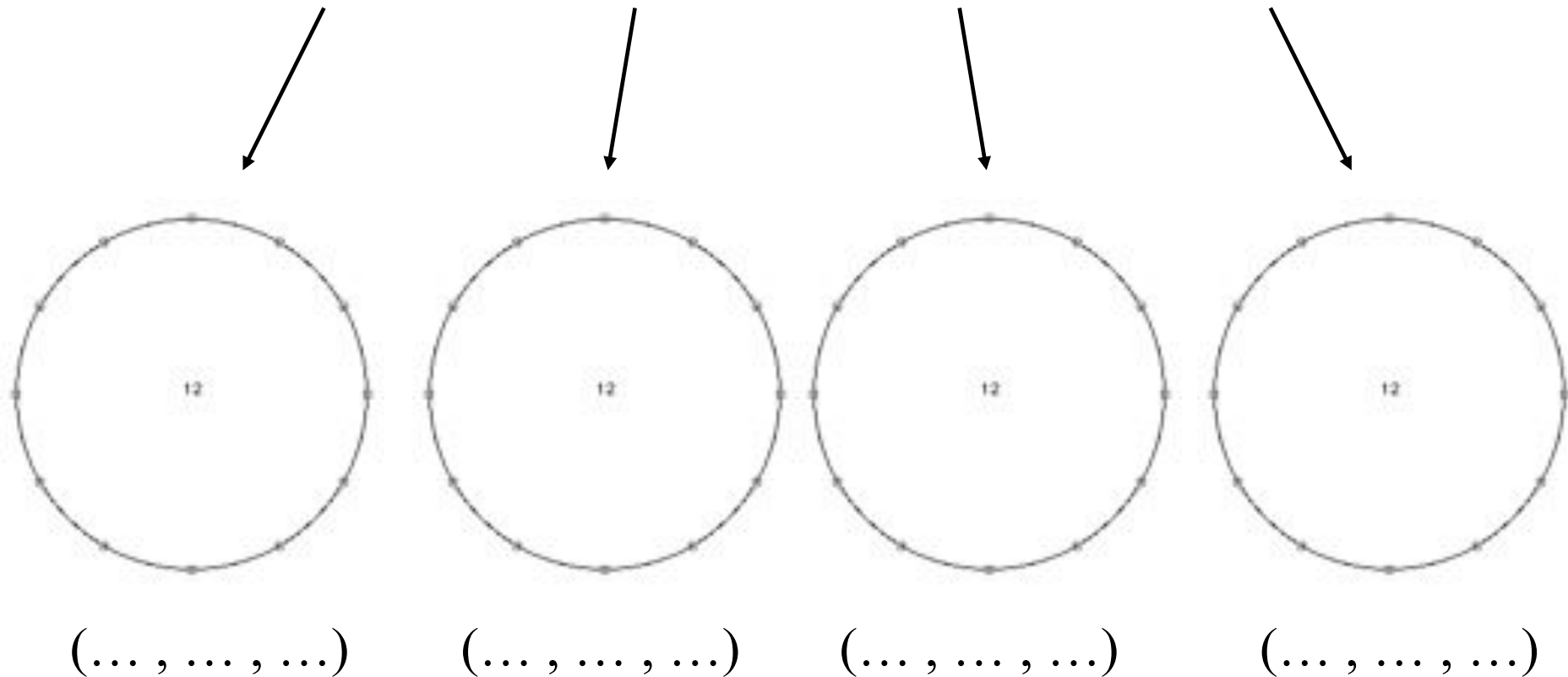


Diagram illustrating the second step of the exercise: mapping the identified symmetries onto a 12-tone circle. Four circles, each labeled '12', are shown. Arrows point from the boxes above to the circles, indicating the mapping of the identified symmetries. Below each circle, a set of three dots in parentheses represents the resulting symmetry.

Exercice : retrouver les symétries dans une série (I)

Schoenberg: Serenade Op.24, Mouvement 5

The image illustrates the process of identifying symmetries in a musical series. It begins with a musical staff showing four groups of notes, each enclosed in a red box. Below each group are three downward-pointing arrows leading to a set of three numbers in curly braces: $\{9, 10, 0\}$, $\{3, 4, 6\}$, $\{5, 7, 8\}$, and $\{11, 1, 2\}$. Below these sets are four circular diagrams, each representing a 12-tone circle with a '12' in the center. The first and second circles have red lines connecting three points on the circle, representing the set $(1, 2, 9)$. The third and fourth circles have green lines connecting three points on the circle, representing the set $(2, 1, 9)$. Arrows point from the musical sets to their corresponding circular diagrams.

$\{9, 10, 0\}$ $\{3, 4, 6\}$ $\{5, 7, 8\}$ $\{11, 1, 2\}$

$(1, 2, 9)$ $(1, 2, 9)$ $(2, 1, 9)$ $(2, 1, 9)$

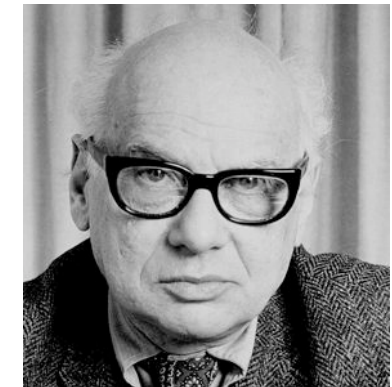
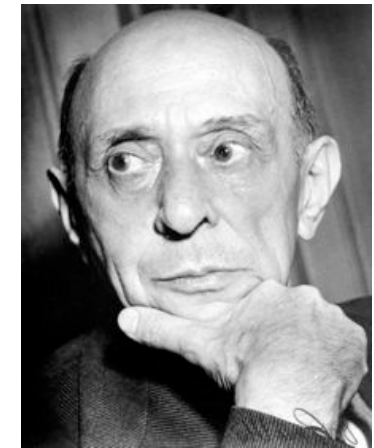
Vers une formalisation algébrique du système dodécaphonique

S
Série originelle

I
Inversion

R
Rétrogradation

IR
Rétrogradation inverse



Le système dodécaphonique est « *un ensemble d'éléments, **relations** entre les éléments et **operations** sur les éléments. [...] Une vraie mathématisation aurait besoin d'une formulation et d'une présentation dictées par le fait que le système dodécaphonique est un **groupe de permutations** qui est façonné [shaped] par la structure de ce modèle mathématique* »

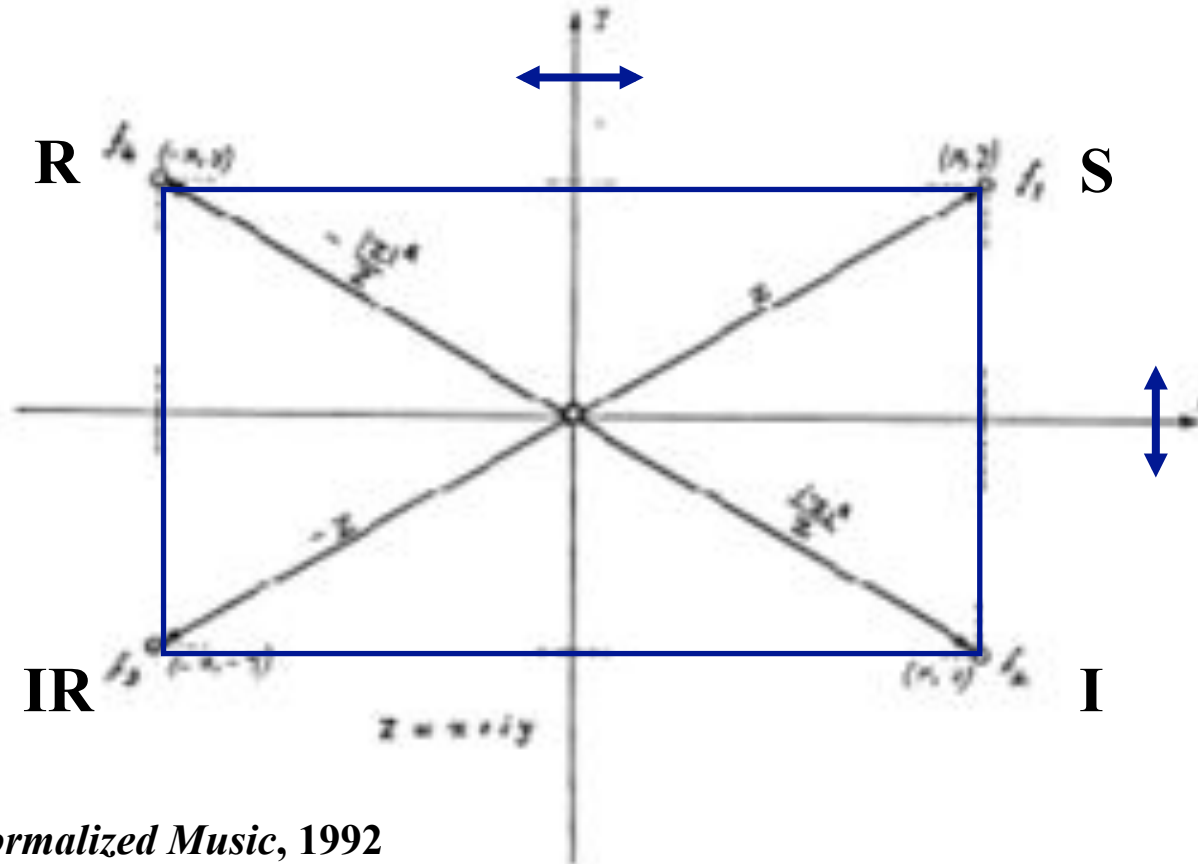
Opérations dodécaphoniques et structures algébriques

Série d'origine
Inversion
Rétrogradation
Rétrogradation inverse

	S	I	R	RI
S	S	I	R	RI
I	I	S	RI	R
R	R	RI	S	I
RI	RI	R	I	S

Structure de *groupe*

- Clôture
- Existence de l'élément neutre
- Existence de l'inverse
- Associativité



Felix Klein

Iannis Xenakis, *Formalized Music*, 1992

Vers l'émergence de la notion de groupe en musique

Ernst Krenek et l'approche axiomatique en musique

- *The Relativity of Scientific Systems*
- *The Significance of Axioms*
- *Axioms in music*
- *Musical Theory and Musical Practice*

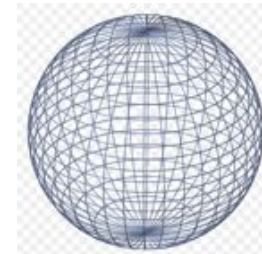
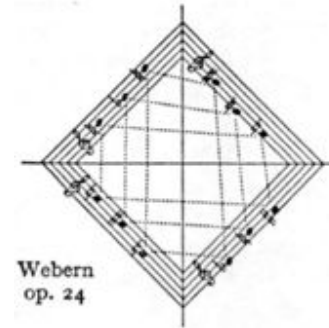
Ernst Krenek : *Über Neue Musik*, 1937
(Engl. Transl. *Music here and now*, 1939)



Physicists and mathematicians are far in advance of musicians in realizing that their respective sciences do not serve to establish a concept of the universe conforming to an objectively existent nature



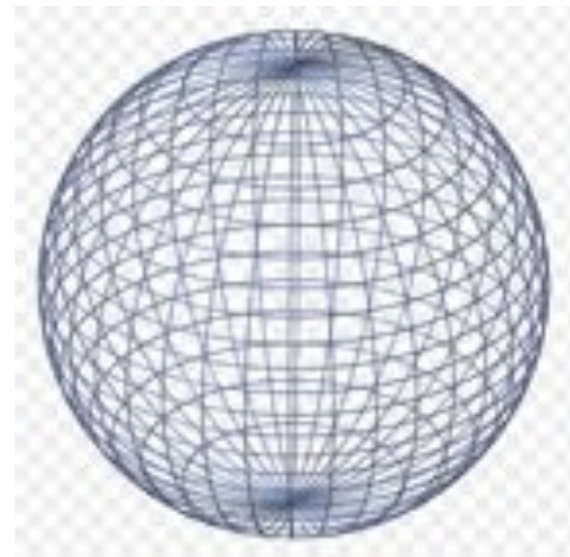
As the study of axioms eliminates the idea that axioms are something absolute, conceiving them instead as free propositions of the human mind, just so would this musical theory free us from the concept of major/minor tonality [...] as an irrevocable law of nature.



L'approche axiomatique en mathématique

David Hilbert: les fondements axiomatiques de la géométrie et le rôle de l'intuition

*In order to be constructed in a right way, geometry [...] only needs few simple principles. These principles are called the **axioms** of the geometry. [...] This study (of the axioms) goes back to the **logical analysis of our spatial intuition** (Grundlage der Geometrie, 1899).*

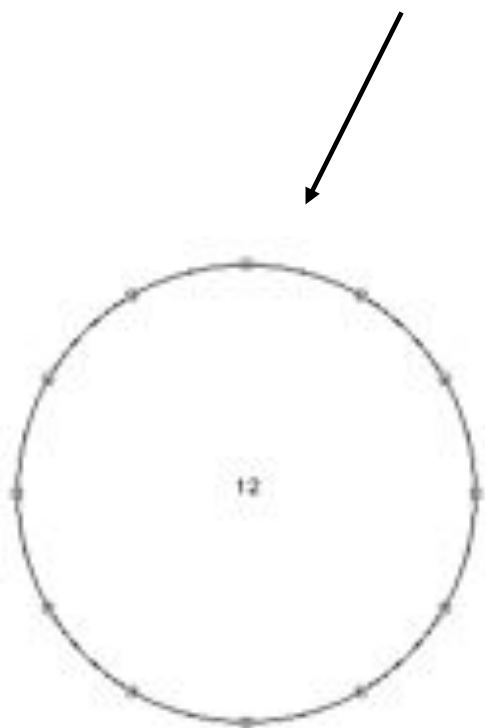


*At the moment there are two tendencies in mathematics. From one side, the tendency toward abstraction aims at 'crystallizing' the logical relations inside of a study object and at organizing this material in a systematic way. But there is also a tendency towards the **intuitive understanding** which aims at understanding the **concret meaning of their relations** (Anschauliche Geometrie, 1932)*

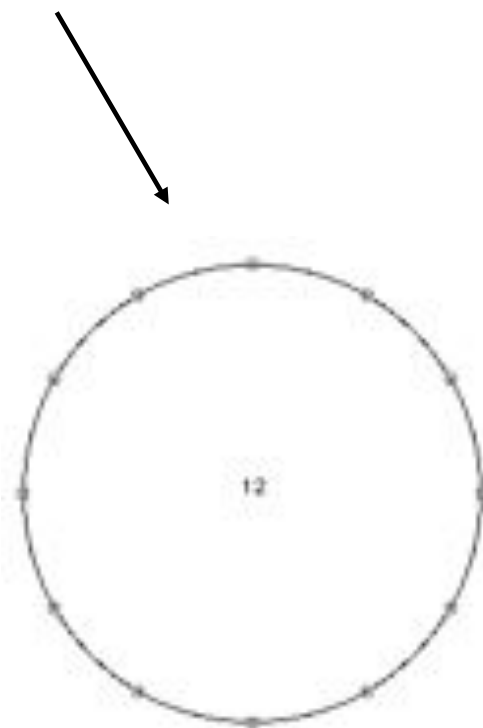
Exercice : retrouver les symétries dans une série

Schoenberg: Serenade Op.24, Mouvement 5

The image shows a musical staff with a treble clef. Two segments of the series are highlighted with red boxes. The first box covers the first six notes, and the second box covers the last six notes. Below each box, a vertical arrow points to a set of six dots enclosed in curly braces, representing the pitch classes of that segment.



(... , ... , ... , ... , ... , ...)

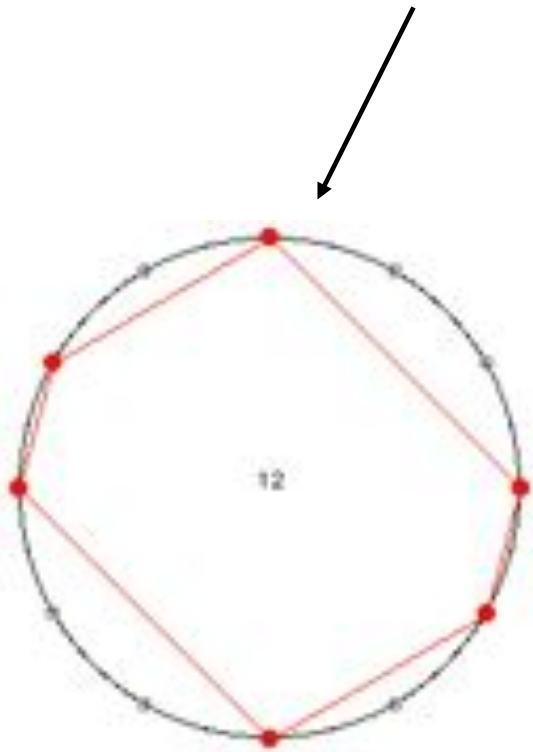


(... , ... , ... , ... , ... , ...)

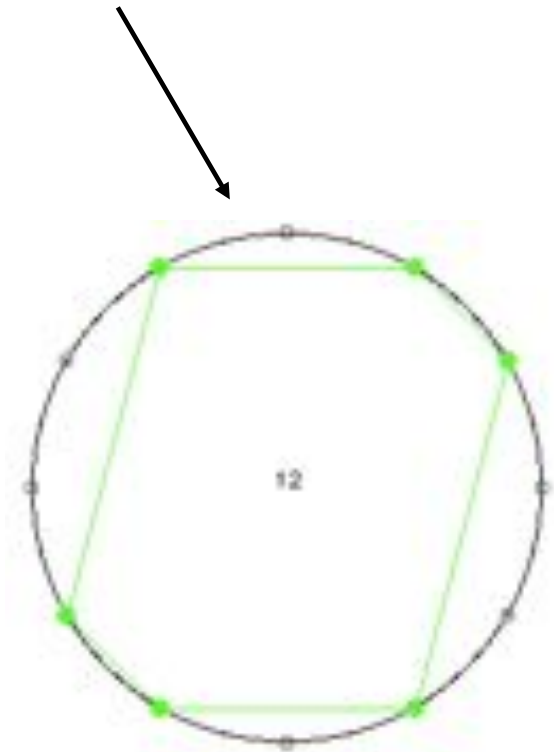
Exercice : retrouver les symétries dans une série

Schoenberg: Serenade Op.24, Mouvement 5

$\{9, 10, 0, 3, 4, 6\}$ $\{5, 7, 8, 11, 1, 2\}$



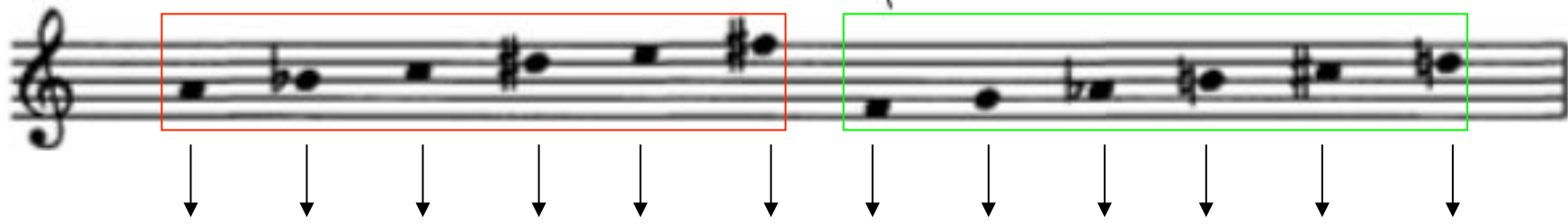
(3, 1, 2, 3, 1, 2)



(2, 1, 3, 2, 1, 3)

“Combinatorialité” et symétrie par transposition

Schoenberg: Serenade Op.24, Mouvement 5

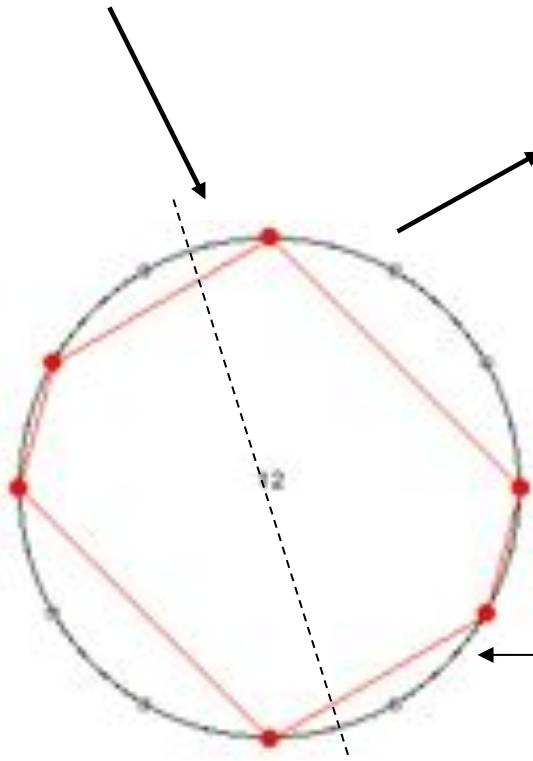


$$A = \{9, 10, 0, 3, 4, 6\} \quad \{5, 7, 8, 11, 1, 2\} = A'$$

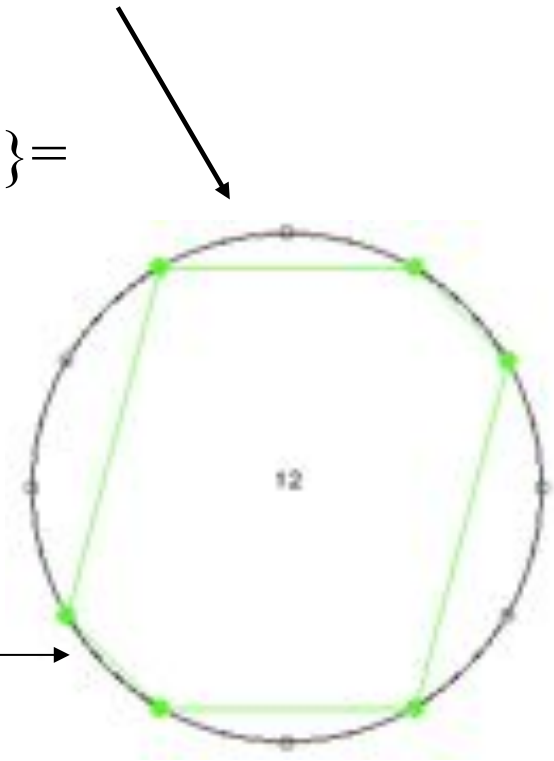
$$\begin{aligned} T_6\{9,10,0,3,4,6\} &= \\ &= \{6+9, 6+10, 6, 6+3, 6+4, 6+6\} = \\ &= \{3,4,6,9,10,0\} \end{aligned}$$

$$T_6(A) = A$$

$$I_{11} = T_{11} I$$

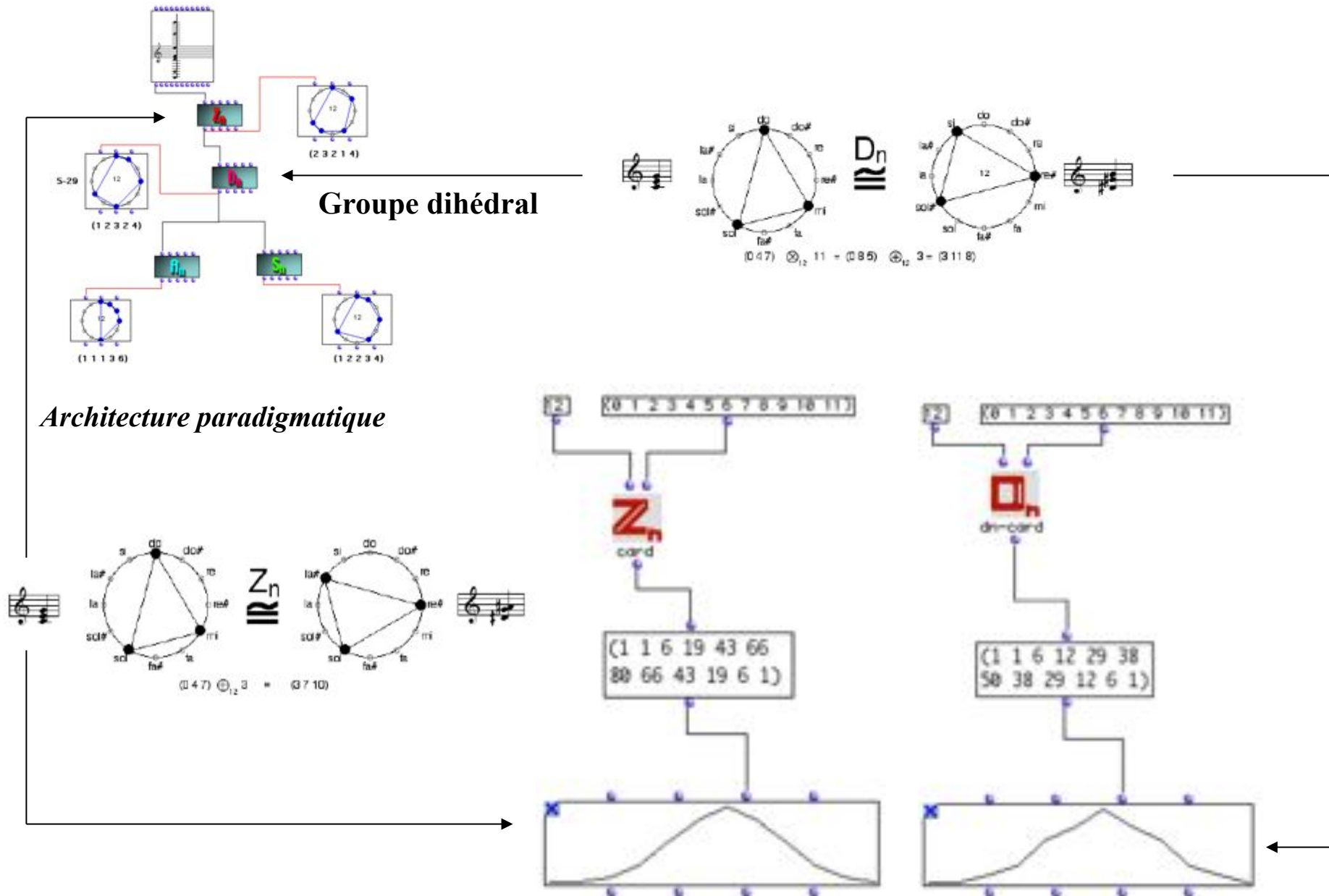


(3, 1, 2, 3, 1, 2)



(2, 1, 3, 2, 1, 3)

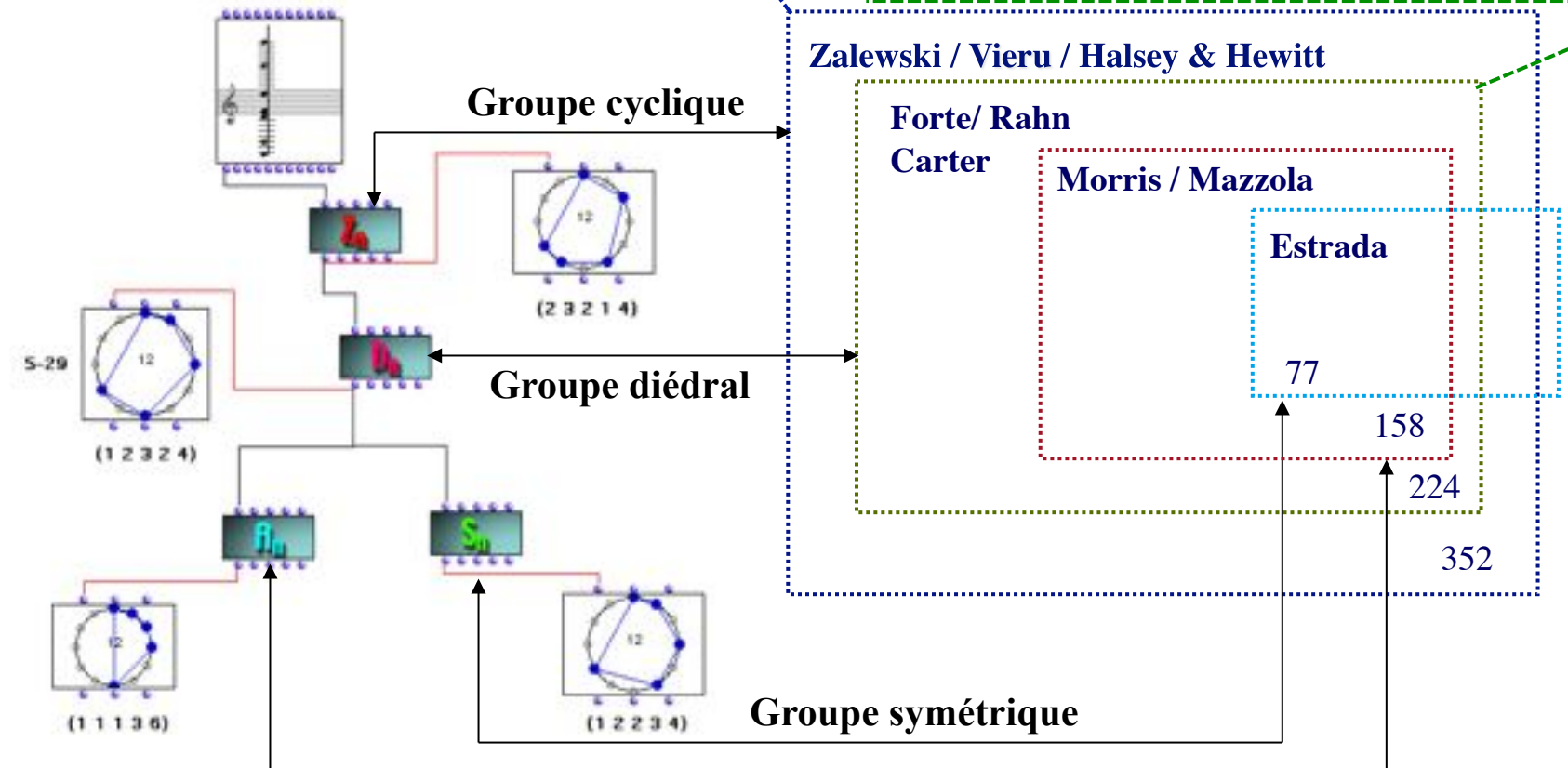
Classes d'équivalence d'accords et groupes de transformations



Classification paradigmatic des structures musicales

$$\# \text{ of } k\text{-chords} = \frac{1}{n} \sum_{j|(n,k)} \phi(j) \binom{n/j}{k/j} = \frac{1}{n} \Phi_n(k)$$

$$\# \text{ of } k\text{-chords} = \begin{cases} \frac{1}{2n} \left[\Phi_n(k) + n \binom{(n-1)/2}{[k/2]} \right], & \text{if } n \text{ is odd,} \\ \frac{1}{2n} \left[\Phi_n(k) + n \binom{n/2}{k/2} \right], & \text{if } n \text{ is even and } k \text{ is even,} \\ \frac{1}{2n} \left[\Phi_n(k) + n \binom{(n/2)-1}{[k/2]} \right], & \text{if } n \text{ is even and } k \text{ is odd.} \end{cases}$$



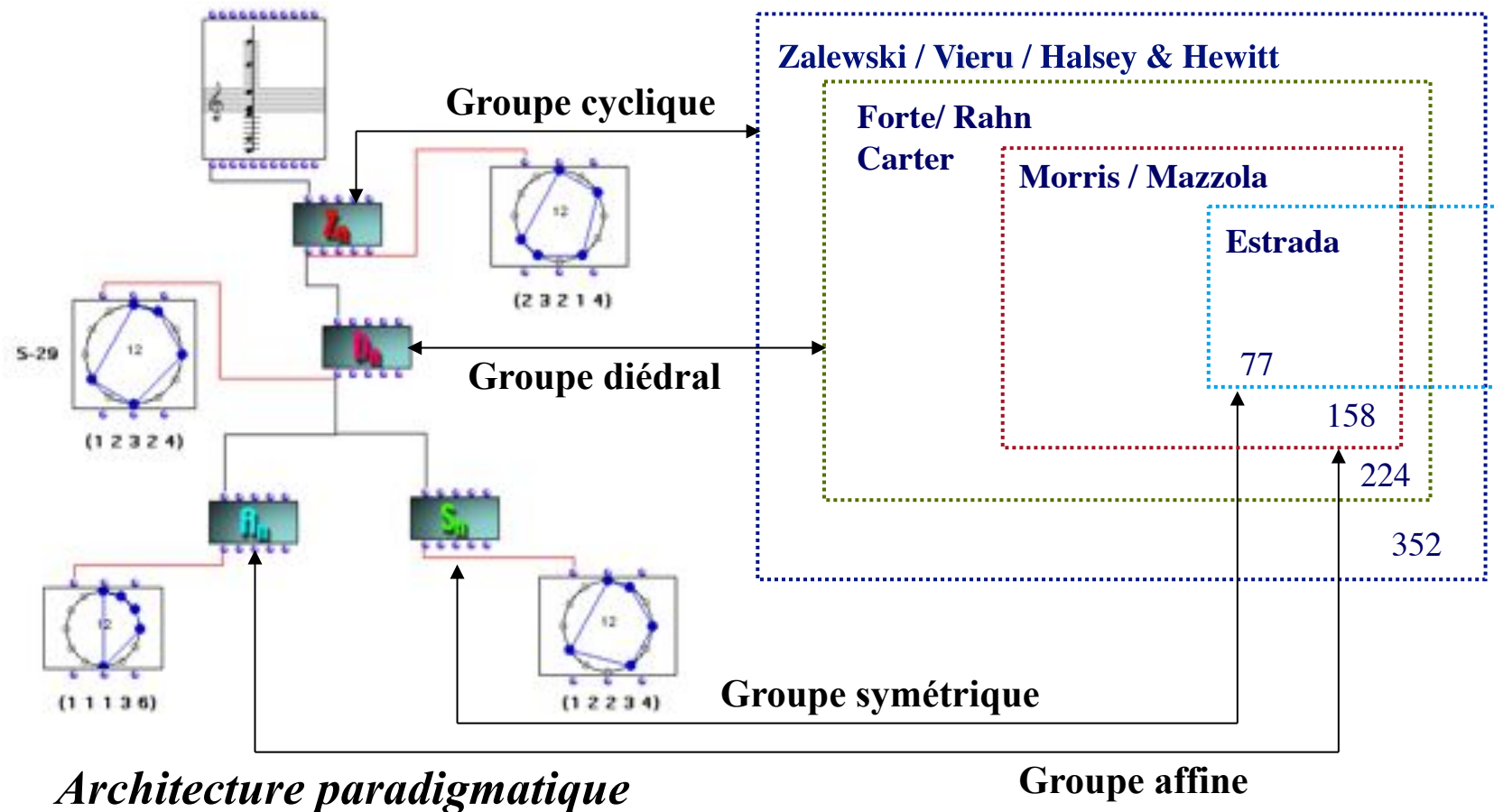
Architecture paradigmatic

Groupe affine

- D. Halsey & E. Hewitt: « Eine gruppentheoretische Methode in der Musik-theorie », *Jahresber. Der Dt. Math.-Vereinigung*, 80, 1978.
- D. Reiner: « Enumeration in Music Theory », *Amer. Math. Month.* 92:51-54, 1985
- R.C. Read: « Combinatorial problems in the theory of music », *Discrete Math.*, 1997
- H. Friepertinger: « Enumeration of mosaics », *Discrete Math.*, 1999

Classification paradigmatic des structures musicales

$G \setminus k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\text{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

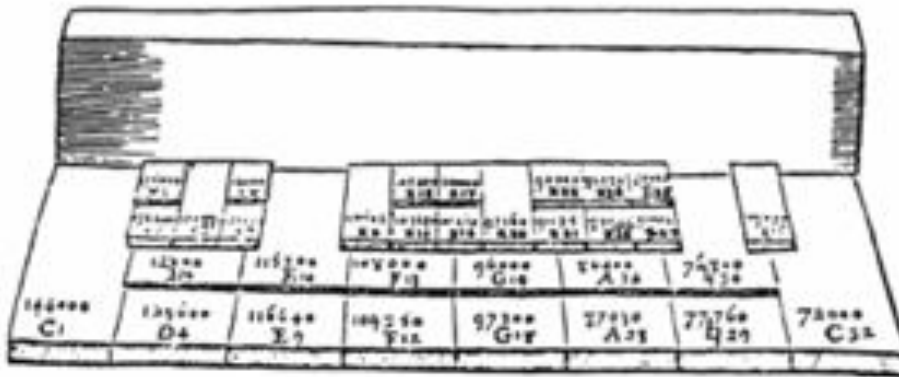


Autres tempéraments égaux

Z/31Z



Vitus Trasuntinis, 1606



Marin Mersenne, *Harmonie universelle*, 1636

Z/53Z



© Science and Society Picture Library

Robert Bosanquet, 1876

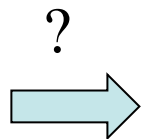
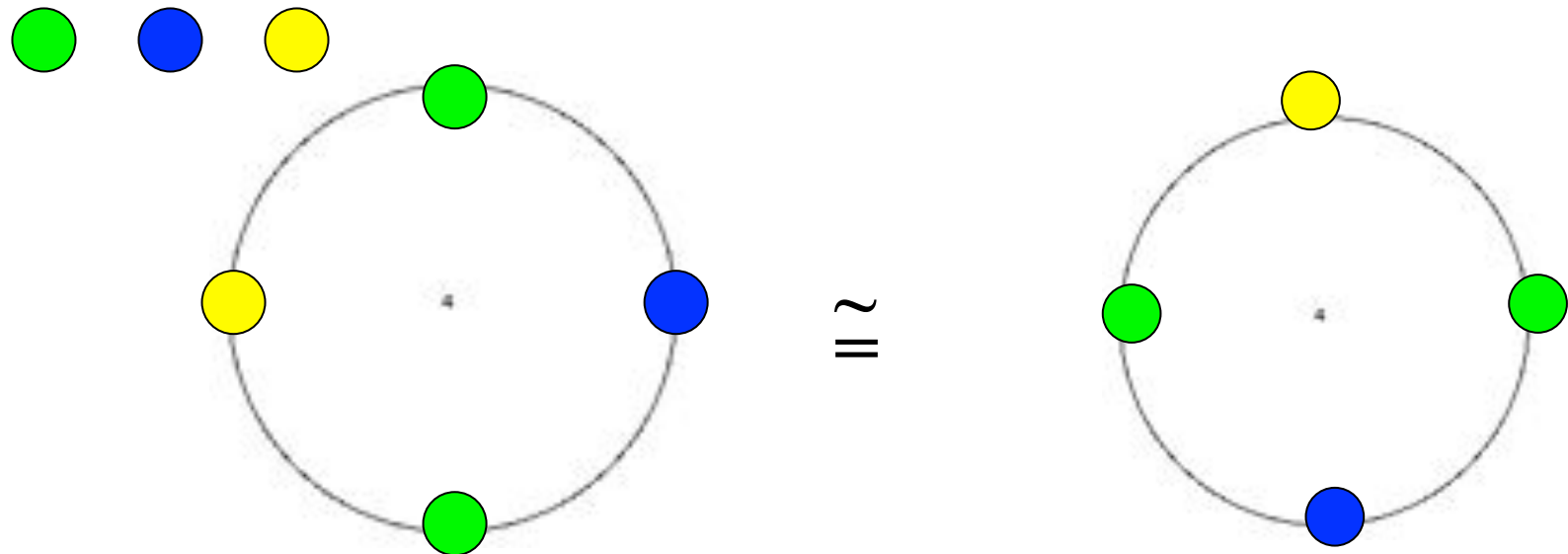
Enumeration des orbites par rapport à l'action d'un groupe



Lemme de Burnside

$$n = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$



Trouver le nombre de configurations possibles

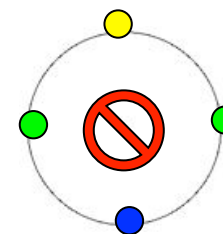
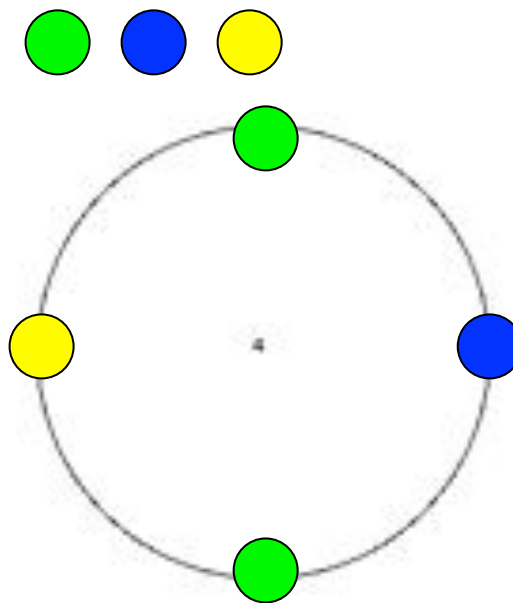
Énumération d'orbites par rapport à l'action d'un groupe



Lemme de Burnside

$$n = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$



Action de $\mathbf{Z}/4\mathbf{Z}$

T_0 = identité

T_1 = rotation de 90°

T_2 = rotation de 180°

T_3 = rotation de 270°

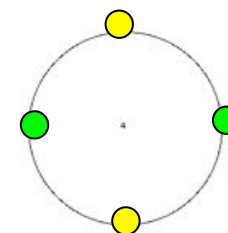
Configurations possibles = $3^4 = 81$

T_0 fixe toute configuration $\Rightarrow |X^{T_0}| = 81$

T_1 fixe toute configuration monochromes $\Rightarrow |X^{T_1}| = 3$

T_3 idem

T_2 fixe toute configuration «double-diamètre» $\Rightarrow |X^{T_2}| = 3^2 = 9$



→ $n = 1/4 (81+3+3+9) = 24$

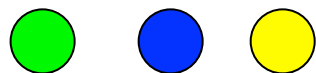
Enumération d'orbites par rapport à l'action d'un groupe



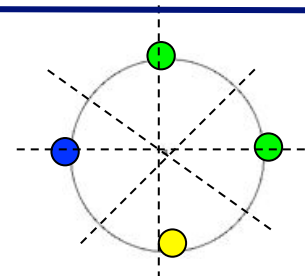
Lemme de Burnside

$$n = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$



Action de Z_4



<i>Transformation</i>	<i>Action</i>	<i>Cycle representation</i>	<i>No. of cycles</i>	<i>Fixed configs.</i>	<i>Cycle type</i>	<i>Cycle index</i>
T_0	$0 \rightarrow 0, 1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3$	$(0)(1)(2)(3)$	4	$3^4 = 81$	1^4	t_1^4
T_1	$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 0$	$(0\ 1\ 2\ 3)$	1	$3^1 = 3$	4^1	t_4^1
T_2	$0 \rightarrow 2 \rightarrow 0, 1 \rightarrow 3 \rightarrow 1$	$(0\ 2)(1\ 3)$	2	$3^2 = 9$	2^2	t_2^2
T_3	$0 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 0$	$(0\ 3\ 2\ 1)$	1	$3^1 = 3$	4^1	t_4^1

Julian Hook, « Why are there 29 Tetrachords? A Tutorial on Combinatorics and Enumeration in Music Theory », MTO, 13(4), 2007

$$n = 1/4 (81+3+3+9) = 24$$

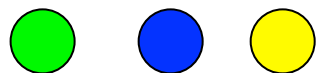
Enumération d'orbites par rapport à l'action d'un groupe



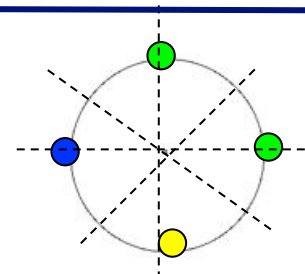
Lemme de Burnside

$$n = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$



Action de D_4



<i>Transformation</i>	<i>Action</i>	<i>Cycle representation</i>	<i>No. of cycles</i>	<i>Fixed configs.</i>	<i>Cycle type</i>	<i>Cycle index</i>
T_0	$0 \rightarrow 0, 1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3$	$(0)(1)(2)(3)$	4	$3^4 = 81$	1^4	t_1^4
T_1	$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 0$	$(0\ 1\ 2\ 3)$	1	$3^1 = 3$	4^1	t_4^1
T_2	$0 \rightarrow 2 \rightarrow 0, 1 \rightarrow 3 \rightarrow 1$	$(0\ 2)(1\ 3)$	2	$3^2 = 9$	2^2	t_2^2
T_3	$0 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 0$	$(0\ 3\ 2\ 1)$	1	$3^1 = 3$	4^1	t_4^1
I	$0 \rightarrow 0, 1 \rightarrow 3 \rightarrow 1, 2 \rightarrow 2$	$(0)(1\ 3)(2)$	3	$3^3 = 27$	$1^2 2^1$	$t_1^2 t_2^1$
$T_1 I$	$0 \rightarrow 1 \rightarrow 0, 2 \rightarrow 3 \rightarrow 2$	$(0\ 1)(2\ 3)$	2	$3^2 = 9$	2^2	t_2^2
$T_2 I$	$0 \rightarrow 2 \rightarrow 0, 1 \rightarrow 1, 3 \rightarrow 3$	$(0\ 2)(1)(3)$	3	$3^3 = 27$	$1^2 2^1$	$t_1^2 t_2^1$
$T_3 I$	$0 \rightarrow 3 \rightarrow 0, 1 \rightarrow 2 \rightarrow 1$	$(0\ 3)(1\ 2)$	2	$3^2 = 9$	2^2	t_2^2

Julian Hook, « Why are there 29 Tetrachords? A Tutorial on Combinatorics and Enumeration in Music Theory », MTO, 13(4), 2007

$$n = 1/8 (81+3+3+9+27+9+27+9) = 168/8=21$$

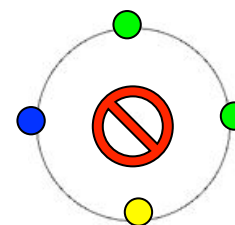
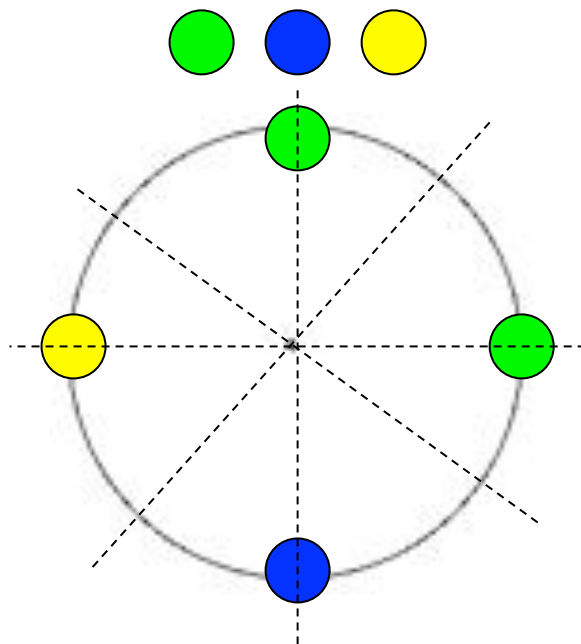
Enumération d'orbites par rapport à l'action d'un groupe



Lemme de Burnside

$$n = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

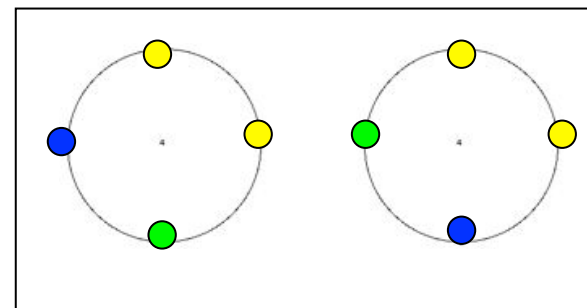
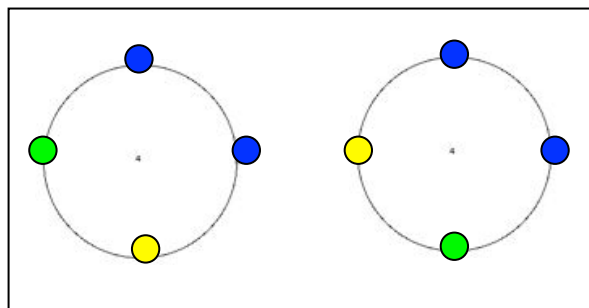
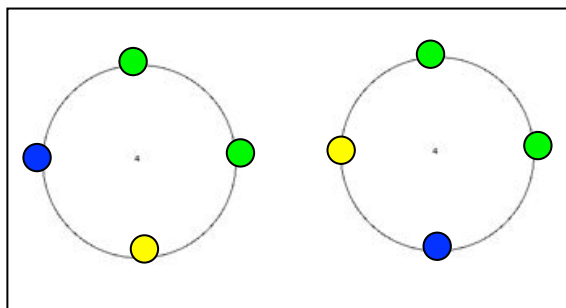
$X^g = \{x \in X : gx = x\}$



Action de D_4

- | | |
|-------------------------------|---------------------------|
| $T_0 = \text{id}$ | $T_0I = \text{inversion}$ |
| $T_1 = \text{rot } 90^\circ$ | $T_1I = \text{inv.}$ |
| $T_2 = \text{rot } 180^\circ$ | $T_2I = \text{inv.}$ |
| $T_3 = \text{rot } 270^\circ$ | $T_3I = \text{inv.}$ |

→ 21=24-3



Énumération d'accords par rapport à l'action du groupe diédrale



Transformation	Cycle representation	No. of cycles	Fixed configs.	Cycle type	Cycle index
T_0	(0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(A)(B)	12	$2^{12} = 4096$	1^{12}	t_1^{12}
T_1	(0 1 2 3 4 5 6 7 8 9 A B)	1	$2^1 = 2$	12^1	t_{12}^1
T_2	(0 2 4 6 8 A)(1 3 5 7 9 B)	2	$2^2 = 4$	6^2	t_6^2
T_3	(0 3 6 9)(1 4 7 A)(2 5 8 B)	3	$2^3 = 8$	4^3	t_4^3
T_4	(0 4 8)(1 5 9)(2 6 A)(3 7 B)	4	$2^4 = 16$	3^4	t_3^4
T_5	(0 5 A 3 8 1 6 B 4 9 2 7)	1	$2^1 = 2$	12^1	t_{12}^1
T_6	(0 6)(1 7)(2 8)(3 9)(4 A)(5 B)	6	$2^6 = 64$	2^6	t_2^6
T_7	(0 7 2 9 4 B 6 1 8 3 A 5)	1	$2^1 = 2$	12^1	t_{12}^1
T_8	(0 8 4)(1 9 5)(2 A 6)(3 B 7)	4	$2^4 = 16$	3^4	t_3^4
T_9	(0 9 6 3)(1 A 7 4)(2 B 8 5)	3	$2^3 = 8$	4^3	t_4^3
T_{10}	(0 A 8 6 4 2)(1 B 9 7 5 3)	2	$2^2 = 4$	6^2	t_6^2
T_{11}	(0 B A 9 8 7 6 5 4 3 2 1)	1	$2^1 = 2$	12^1	t_{12}^1
I	(0)(1 B)(2 A)(3 9)(4 8)(5 7)(6)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
$T_1 I$	(0 1)(2 B)(3 A)(4 9)(5 8)(6 7)	6	$2^6 = 64$	2^6	t_2^6
$T_2 I$	(0 2)(1)(3 B)(4 A)(5 9)(6 8)(7)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
$T_3 I$	(0 3)(1 2)(4 B)(5 A)(6 9)(7 8)	6	$2^6 = 64$	2^6	t_2^6
$T_4 I$	(0 4)(1 3)(2)(5 B)(6 A)(7 9)(8)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
$T_5 I$	(0 5)(1 4)(2 3)(6 B)(7 A)(8 9)	6	$2^6 = 64$	2^6	t_2^6
$T_6 I$	(0 6)(1 5)(2 4)(3)(7 B)(8 A)(9)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
$T_7 I$	(0 7)(1 6)(2 5)(3 4)(8 B)(9 A)	6	$2^6 = 64$	2^6	t_2^6
$T_8 I$	(0 8)(1 7)(2 6)(3 5)(4)(9 B)(A)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
$T_9 I$	(0 9)(1 8)(2 7)(3 6)(4 5)(A B)	6	$2^6 = 64$	2^6	t_2^6
$T_{10} I$	(0 A)(1 9)(2 8)(3 7)(4 6)(5)(B)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
$T_{11} I$	(0 B)(1 A)(2 9)(3 8)(4 7)(5 6)	6	$2^6 = 64$	2^6	t_2^6

Lemme de Burnside

$$n = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

$X^g = \{x \in X : gx = x\}$

Action de D_{12}
(Hook, MTO)




d'accords = $1/12[4096+2+4+8+16+2+64+2+16+8+4+2]=4224/12=352$




d'accords = $1/24[4224+1152] = 224$

Énumération d'accords et dans un système tempéré


(Reiner, 1985)



$$\# \text{ of } k\text{-chords} = \frac{1}{n} \sum_{j|(n,k)} \phi(j) \binom{n/j}{k/j} = \frac{1}{n} \Phi_n(k),$$



$$\# \text{ of } k\text{-chords} = \begin{cases} \frac{1}{2n} \left[\Phi_n(k) + n \binom{(n-1)/2}{[k/2]} \right], & \text{if } n \text{ is odd,} \\ \frac{1}{2n} \left[\Phi_n(k) + n \binom{n/2}{k/2} \right], & \text{if } n \text{ is even and } k \text{ is even,} \\ \frac{1}{2n} \left[\Phi_n(k) + n \binom{(n/2)-1}{[k/2]} \right], & \text{if } n \text{ is even and } k \text{ is odd.} \end{cases}$$

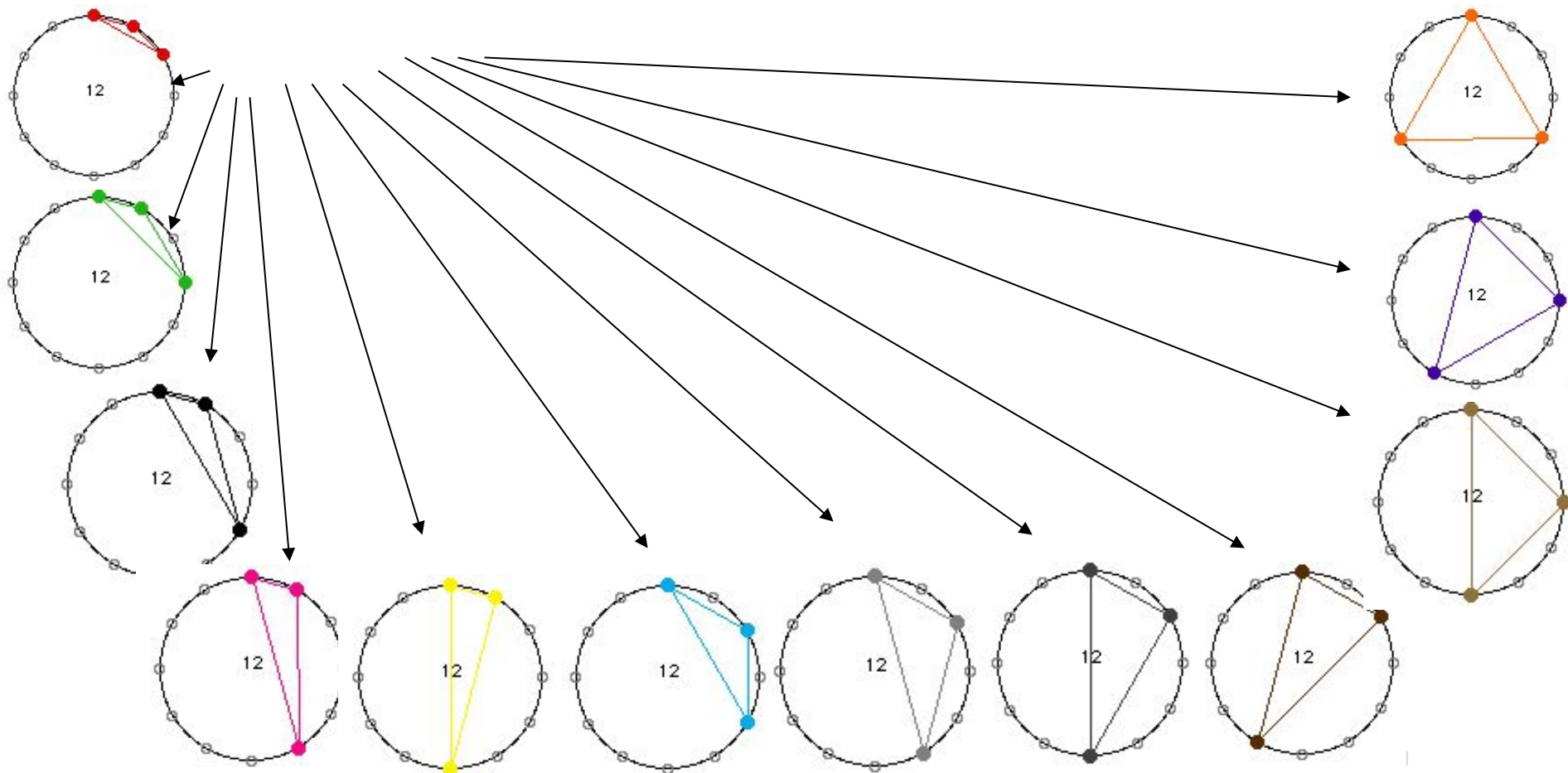
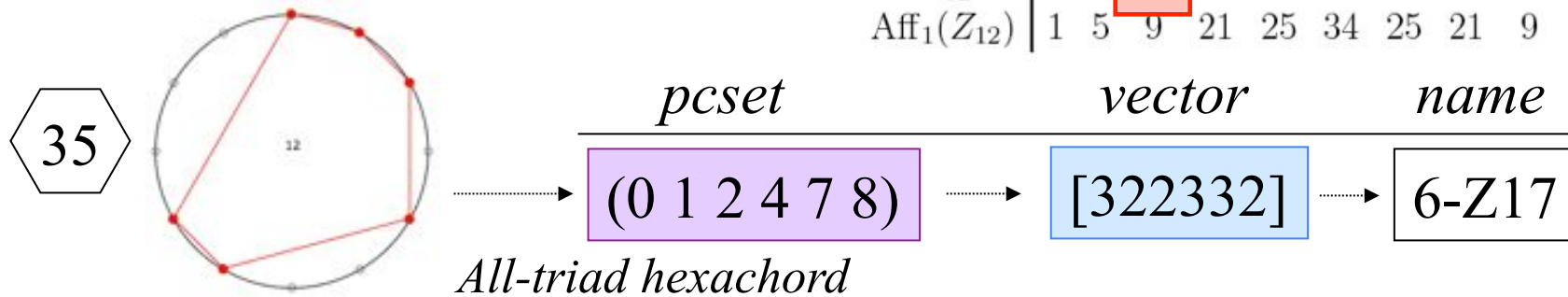


$k =$	0	1	2	3	4	5	6	7	8	9	10	11	12
number	1	1	6	12	29	38	50	38	29	12	6	1	1

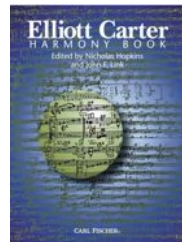
- D. Halsey & E. Hewitt: « Eine gruppentheoretische Methode in der Musik-theorie », *Jahresber. Der Dt. Math.-Vereinigung*, 80, 1978.
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- R.C. Read: « Combinatorial problems in the theory of music », *Discrete Math.*, 1997
- H. Friepertinger: « Enumeration of mosaics », *Discrete Math.*, 1999
- H. Friepertinger: « Enumeration of non-isomorphic canons », *Tatra Mt. Math. Publ.*, 2001
- M. Broué : « Les tonalités musicales vues par un mathématicien », *Le temps des savoirs, Revue de l'Institut Universitaire de France*, 2002
- David J. Hunter & Paul T. von Hippel : « How Rare Is Symmetry in Musical 12-Tone Rows? », *The American Mathematical Monthly*, Vol. 110, No. 2., Feb., 2003
- H. Friepertinger: « Tiling problems in music theory », in *Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Puebla ed., Epos, 2004)
- Rachel W. Hall & P. Klingsberg: « Asymmetric Rhythms, Tiling Canons, and Burnside's Lemma », *Bridge Proceedings*, 2004
- ...

Elliott Carter: 90+ (1994)

$G \setminus k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\text{Aff}_1(\mathbb{Z}_{12})$	1	5	9	21	25	34	25	21	9	5	1	1



Elliott Carter : 90+ (1994)



- **Combinatoire d'accords**
 - Hexacordes
 - Tétracordes
 - Triades
 - Relation Z
- **Séries tous-intervalles**
 - *Link-chords*

 (piano: John Snijders)

mille e novanta auguri a caro Geffredo

90+

Elliott Carter
(1994)

♩ = 96

Piano

(senza pedale)*

* Use pedal only to join one chord to another legato, as in mm. 1-13, 16-21, 36-43, and 45-48.

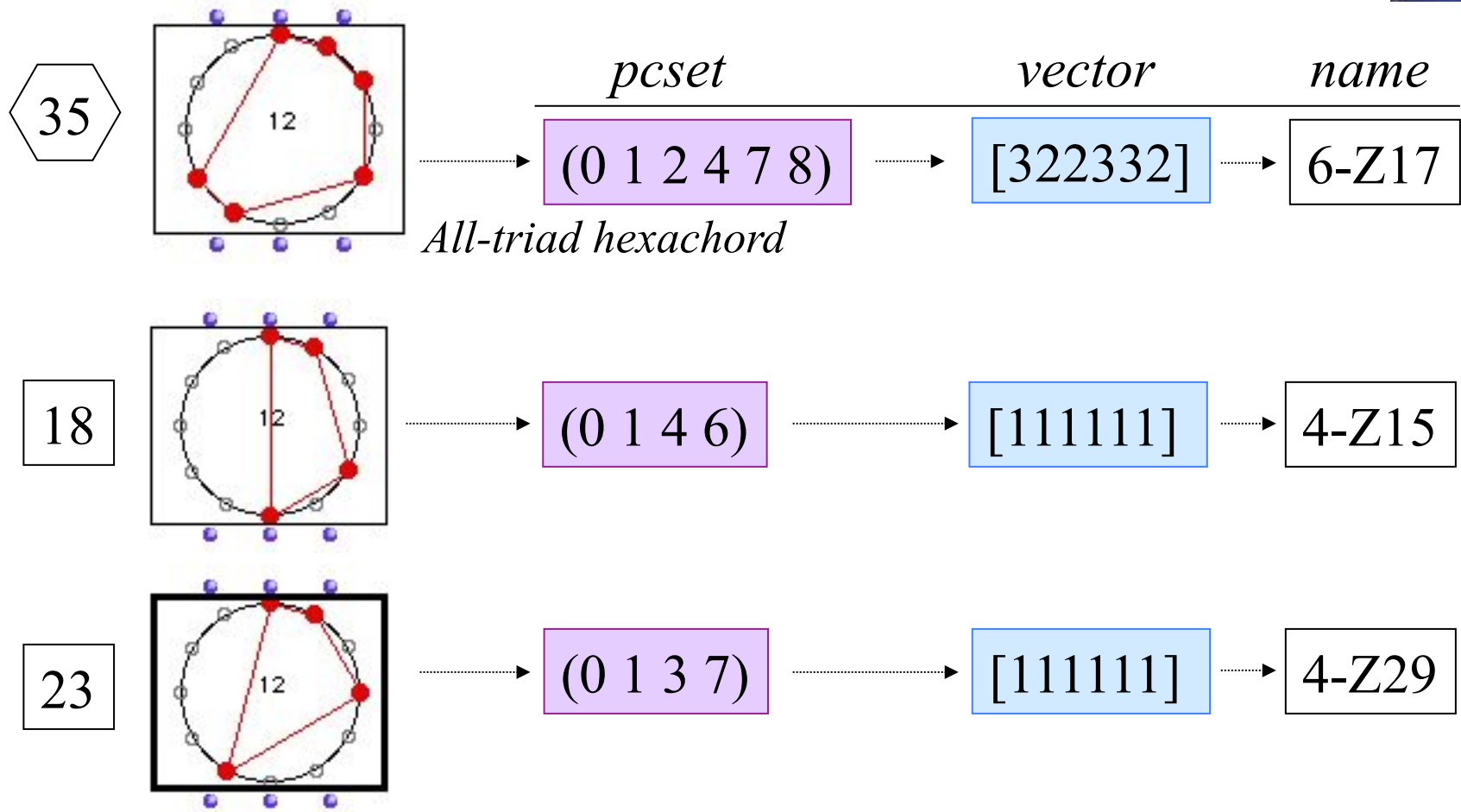
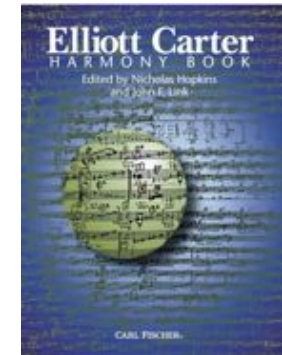
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PSB 503

Printed in U.S.A.

Elliott Carter: 90+ (1994)

« From about 1990, I have reduced my vocabulary of chords more and more to the six note chord n° 35 and the four note chords n° 18 and 23, which encompass all the intervals » (Harmony Book, 2002, p. ix)

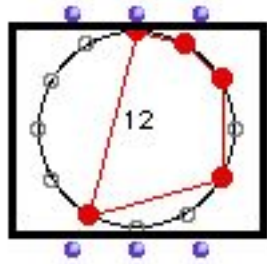


Catalogue des *pitch-class sets* d'Allen Forte (1973)

complémentaire

name	pcs	vector	name	pcs	vector
5-30	0,1,4,6,8	121321	7-30	0,1,2,4,6,8,9	343542
5-31	0,1,3,6,9	114112	7-31	0,1,3,4,6,7,9	336333
5-32	0,1,4,6,9	113221	7-32	0,1,3,4,6,8,9	335442
5-33(12)	0,2,4,6,8	040402	7-33	0,1,2,4,6,8,10	262623
5-34(12)	0,2,4,6,9	032221	7-34	0,1,3,4,6,8,10	254442
5-35(12)	0,2,4,7,9	032140	7-35	0,1,3,5,6,8,10	254361
5-Z36	0,1,2,4,7	222121	7-Z36	0,1,2,3,5,6,8	444342
5-Z37(12)	0,3,4,5,8	212320	7-Z37	0,1,3,4,5,7,8	434541
5-Z38	0,1,2,5,8	212221	7-Z38	0,1,2,4,5,7,8	434442
6-1(12)	0,1,2,3,4,5	543210			
6-2	0,1,2,3,4,6	443211			
5-Z36	0,1,2,4,7	222121	7-Z36	0,1,2,3,5,6,8	444342
6-Z4(12)	0,1,2,4,5,6	432321	6-Z37(12)	0,1,2,3,4,8	
6-5	0,1,2,3,6,7	422232	6-Z38(12)	0,1,2,3,7,8	
6-Z6(12)	0,1,2,5,6,7	421242			
6-7(6)	0,1,2,6,7,8	420243			
6-8(12)	0,2,3,4,5,7	343230			
6-9	0,1,2,3,5,7	342231			
6-Z10	0,1,3,4,5,7	333321	6-Z39	0,2,3,4,5,8	
6-Z11	0,1,2,4,5,7	333231	6-Z40	0,1,2,3,5,8	
6-Z12	0,1,2,4,6,7	332232	6-Z41	0,1,2,3,6,8	
6-Z13(12)	0,1,3,4,6,7	324222	6-Z42(12)	0,1,2,3,6,9	

Vecteur d'intervalles et relation Z



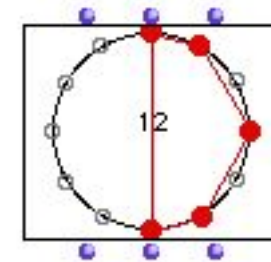
5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140
5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211

5-Z36	0,1,2,4,7	222121
6-Z4(12)	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442

6-Z36	0,1,2,3,4,7
6-Z37(12)	0,1,2,3,4,8
6-Z38(12)	0,1,2,3,7,8

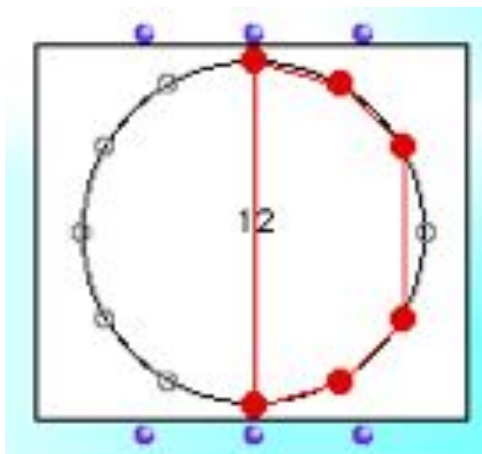
6-Z39	0,2,3,4,5,8
6-Z40	0,1,2,3,5,8
6-Z41	0,1,2,3,6,8
6-Z42(12)	0,1,2,3,6,9



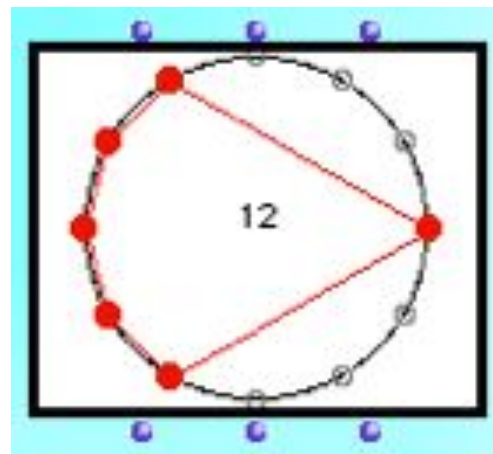
5-Z12

Théorème de l'hexacorde (ou théorème de Babbitt)

(Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, ..., Amiot [2006])



A



A'

$$IV(A) = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = IV(A')$$

Un hexacorde et son complémentaire ont le même vecteur d'intervalles

Elliott Carter : 90+ (1994) : combinatoire tetra/tricordale

The image displays a musical score for Elliott Carter's piece '90+' (1994) with various annotations and chord diagrams. The score is divided into four systems, with specific measures highlighted by colored boxes and ovals. To the right, a grid of 18 chord diagrams (circles with 12 points and internal lines) illustrates the combinatorial structure. The diagrams are arranged in three rows of six. The first row contains six diagrams with colored borders: blue, cyan, green, magenta, orange, and brown. The second row contains six diagrams with colored borders: lime green, light blue, purple, and three with no border. The third row contains six diagrams with no border, the last one containing a question mark. Below the score, two large chord diagrams are shown in dashed boxes, and two more are shown in solid and dashed boxes at the bottom right.

« Making and Using a Pcset Network for Stockhausen's Klavierstück III »



Trois interprétations :



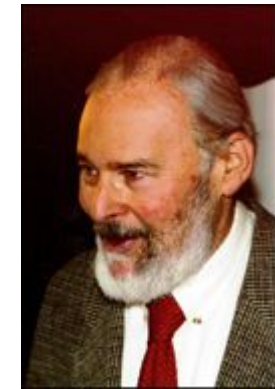
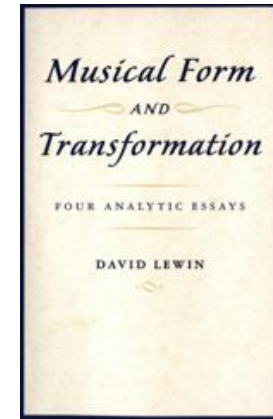
Henck



Kontarsky

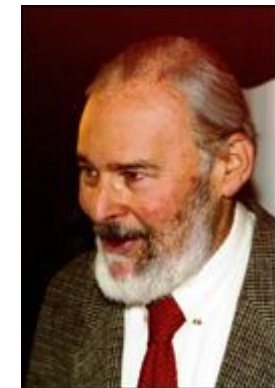
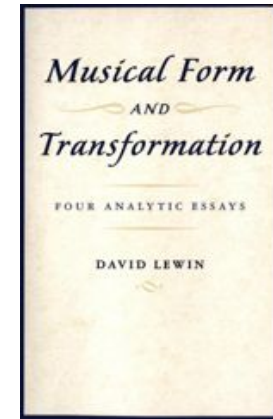


Tudor



« Making and Using a Pcset Network for Stockhausen's Klavierstück III »

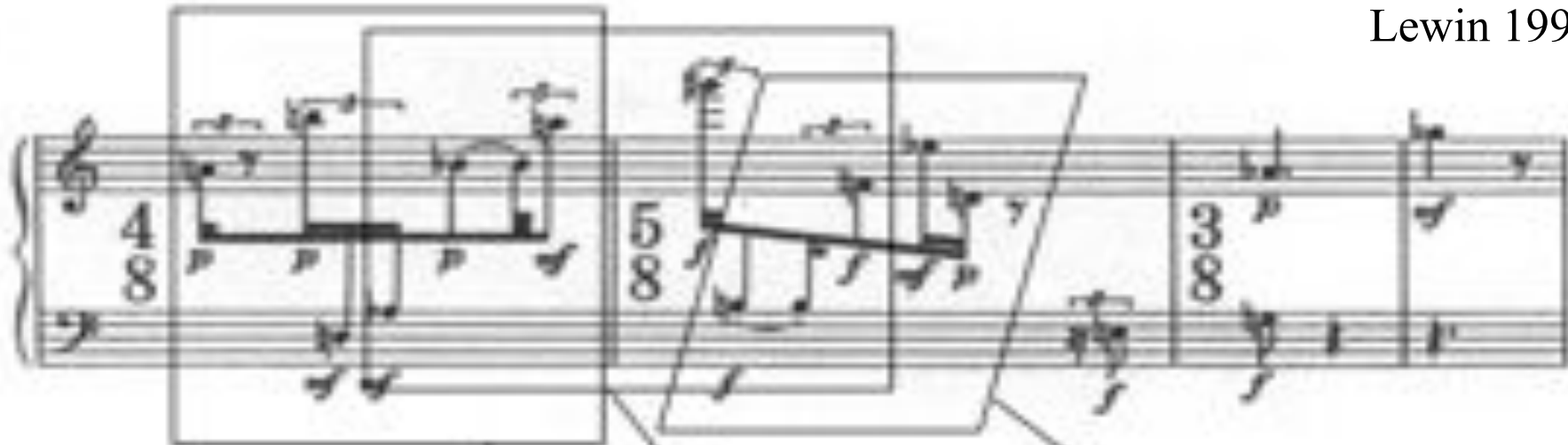
The image shows a musical score for Stockhausen's Klavierstück III. The score is in 4/8, 5/8, and 3/8 time signatures. It features various dynamics such as *p*, *mf*, *f*, and *mf*. Three sections of the score are highlighted with colored boxes: a red box around the first section, a green box around the second section, and a blue box around the third section. Below each highlighted section, there is a red, green, and blue arrow respectively, each pointing to a question mark. Below the question marks are three identical pentachord diagrams, each consisting of a circle with 12 points on its circumference and the number 12 in the center.



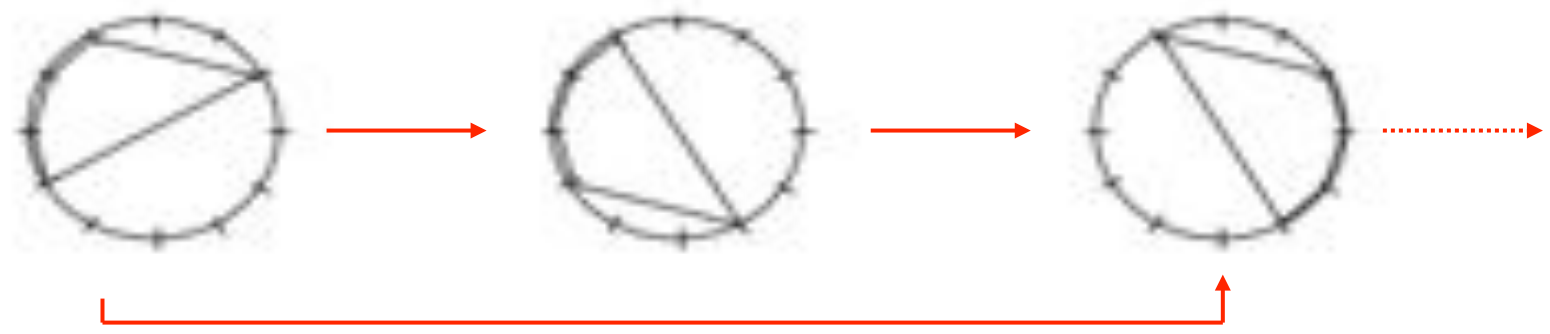
« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

« *Making and Using a Pcset Network for Stockhausen's Klavierstück III* »

Lewin 1993

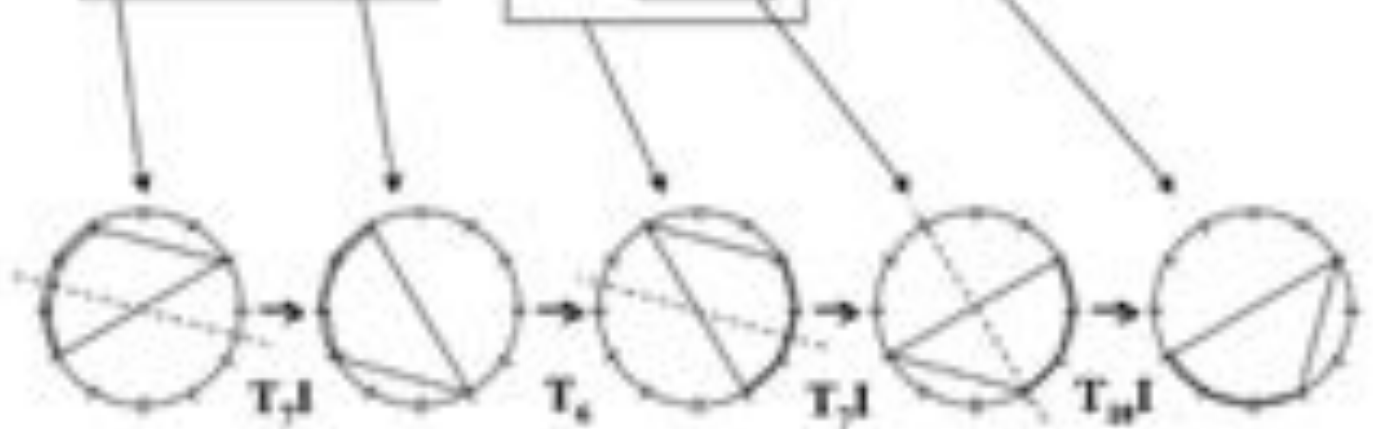


SI: (1, 1, 1, 3, 6) (6, 3, 1, 1, 1) (6, 3, 1, 1, 1)
IFUNC: [5 3 2 2 1 1 1 1 2 2 3] [5 3 2 2 1 1 1 1 2 2 3] [5 3 2 2 1 1 1 1 2 2 3]
VI: [3 2 2 1 1 1] [3 2 2 1 1 1] [3 2 2 1 1 1]



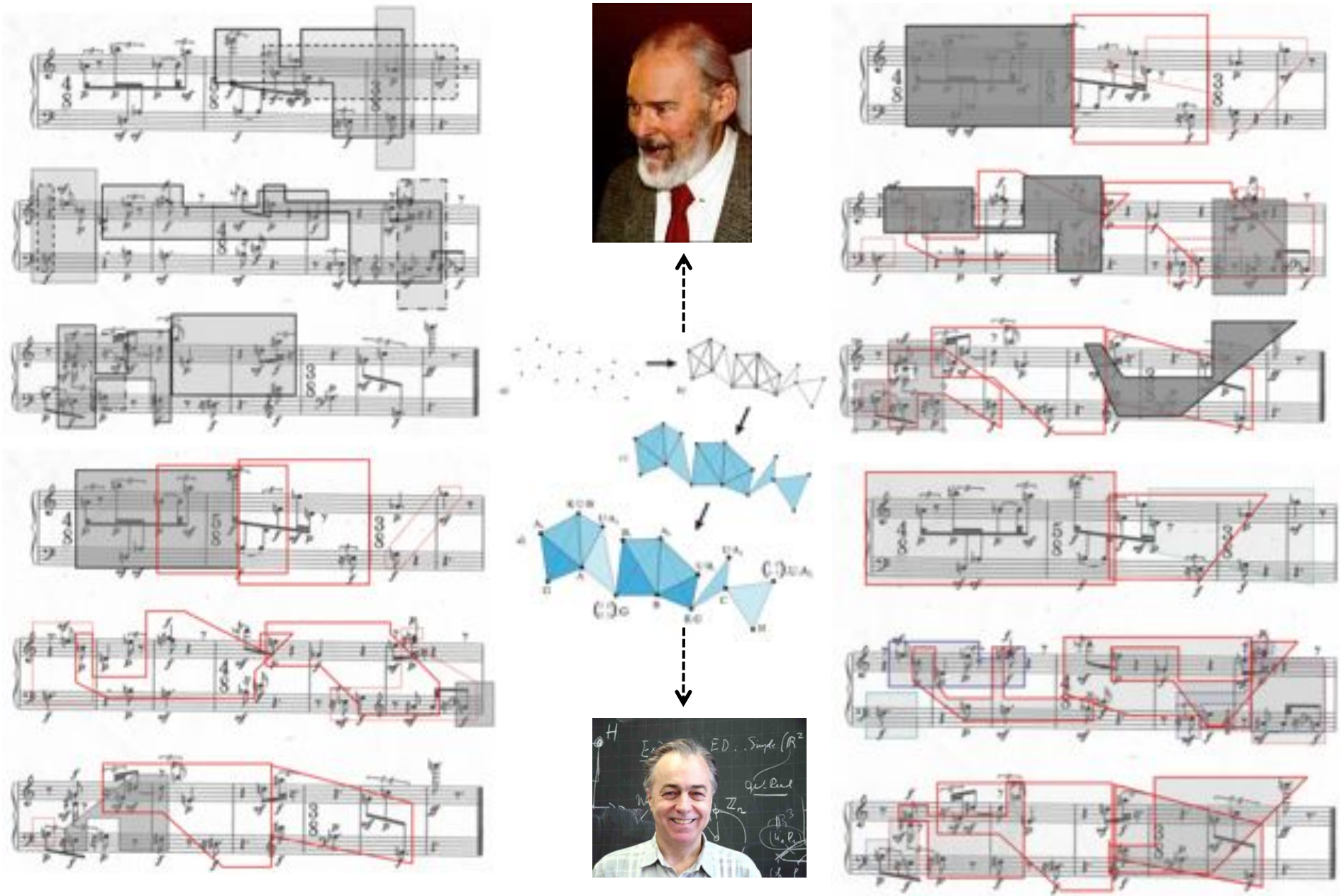
Segmentation par « imbrication »: progression transformationnelle

Stockhausen: *Klavierstück III* (Analisi di D. Lewin)

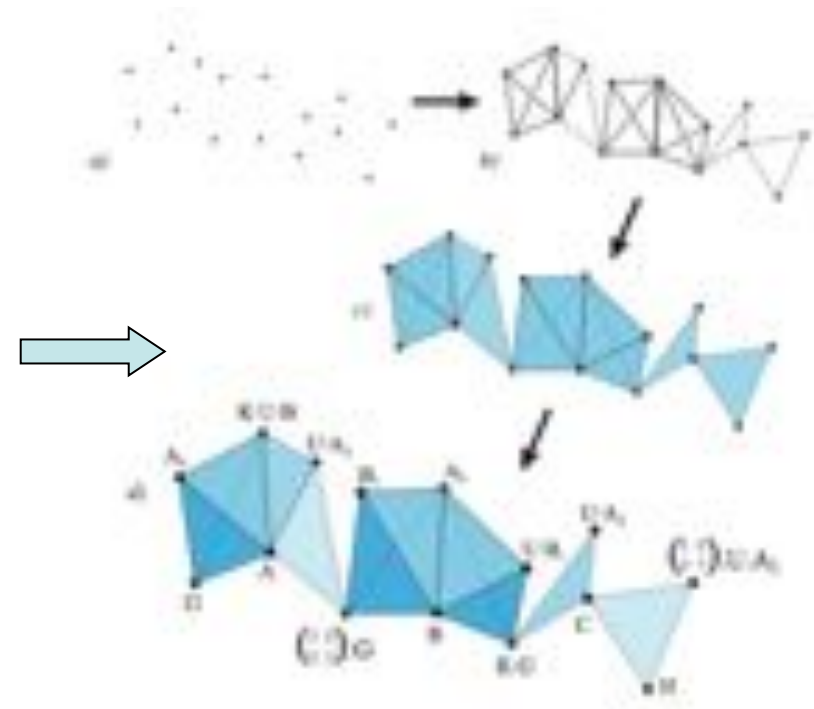
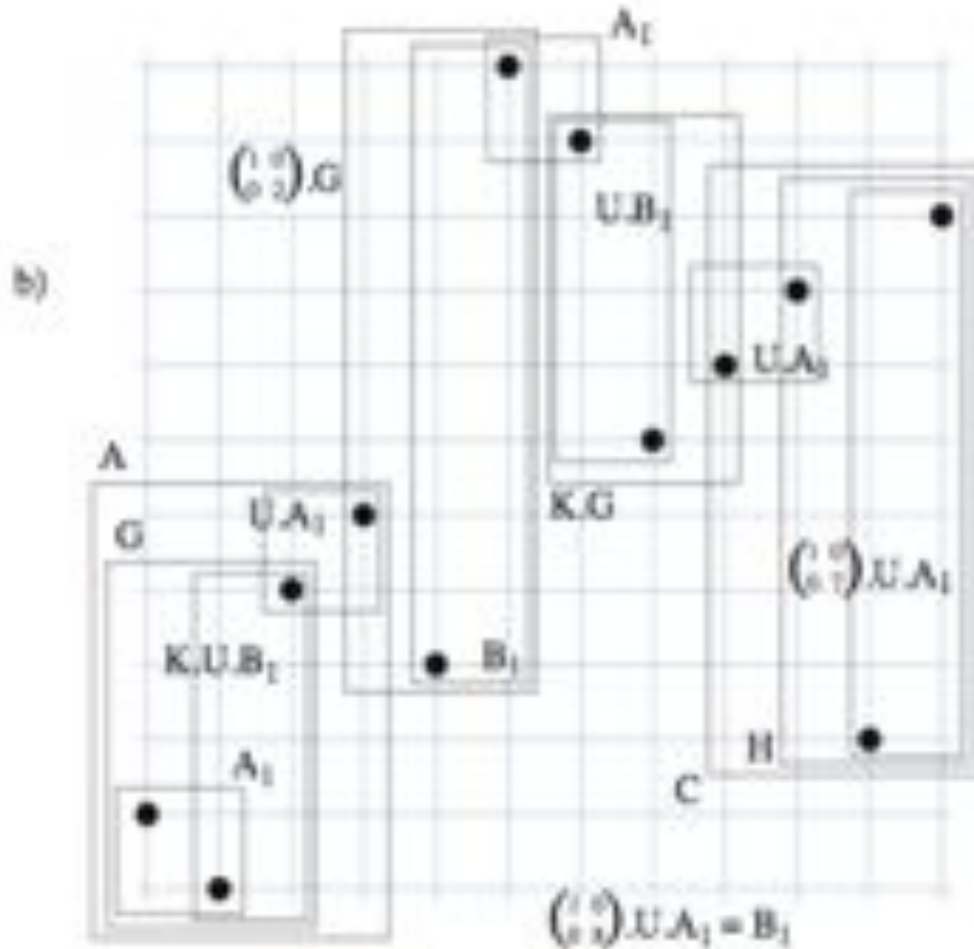
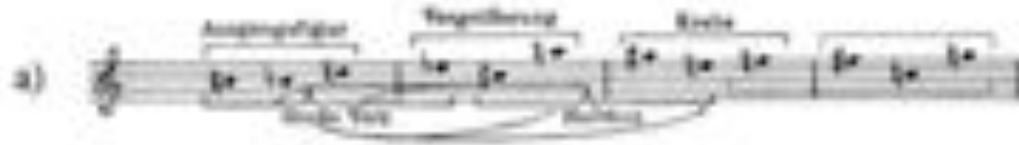


Vers une modélisation informatique de l'analyse transformationnelle

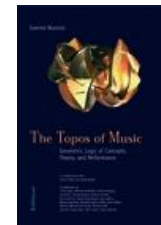
YunKang Ahn, L'analyse musicale computationnelle, thèse, Université de Paris VI / Ircam, déc 2009

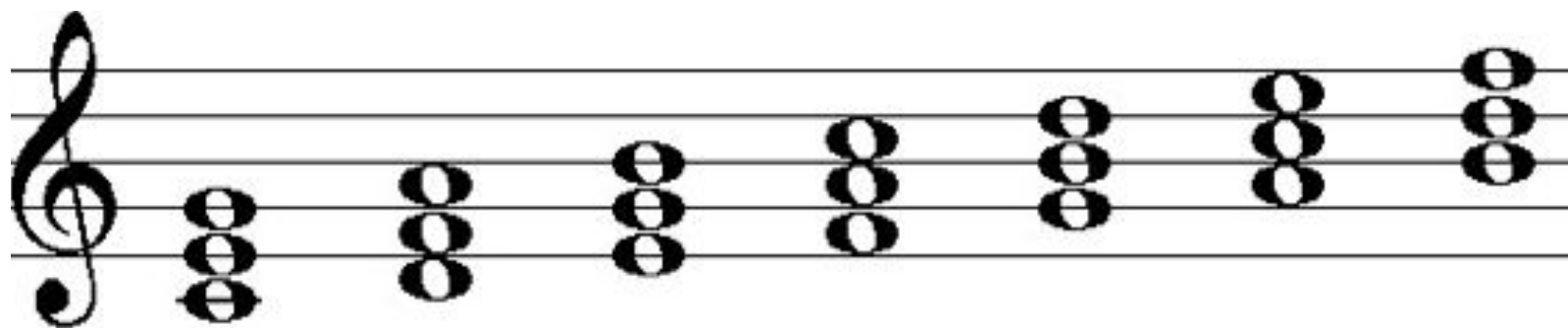


Nerf topologique et analyse musicale



G. Mazzola : *The Topos of Music*,
 ch. 13 - "What are
 global compositions ?"





I

II

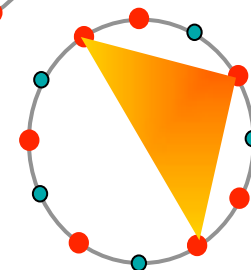
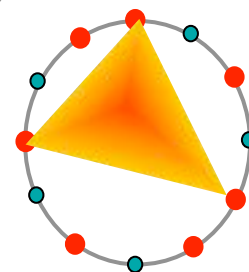
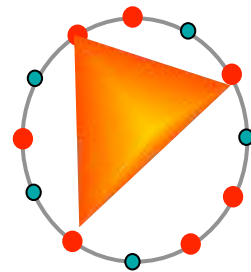
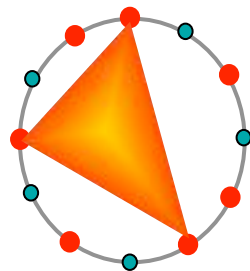
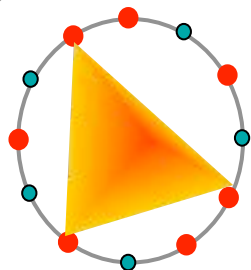
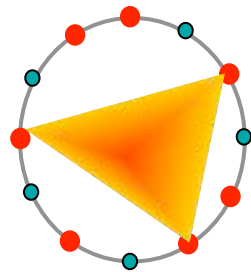
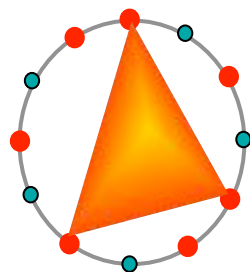
III

IV

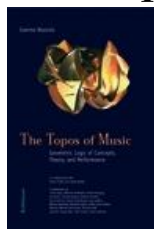
V

VI

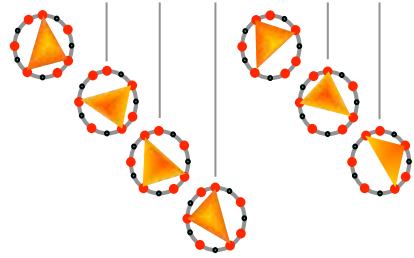
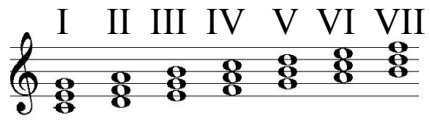
VII



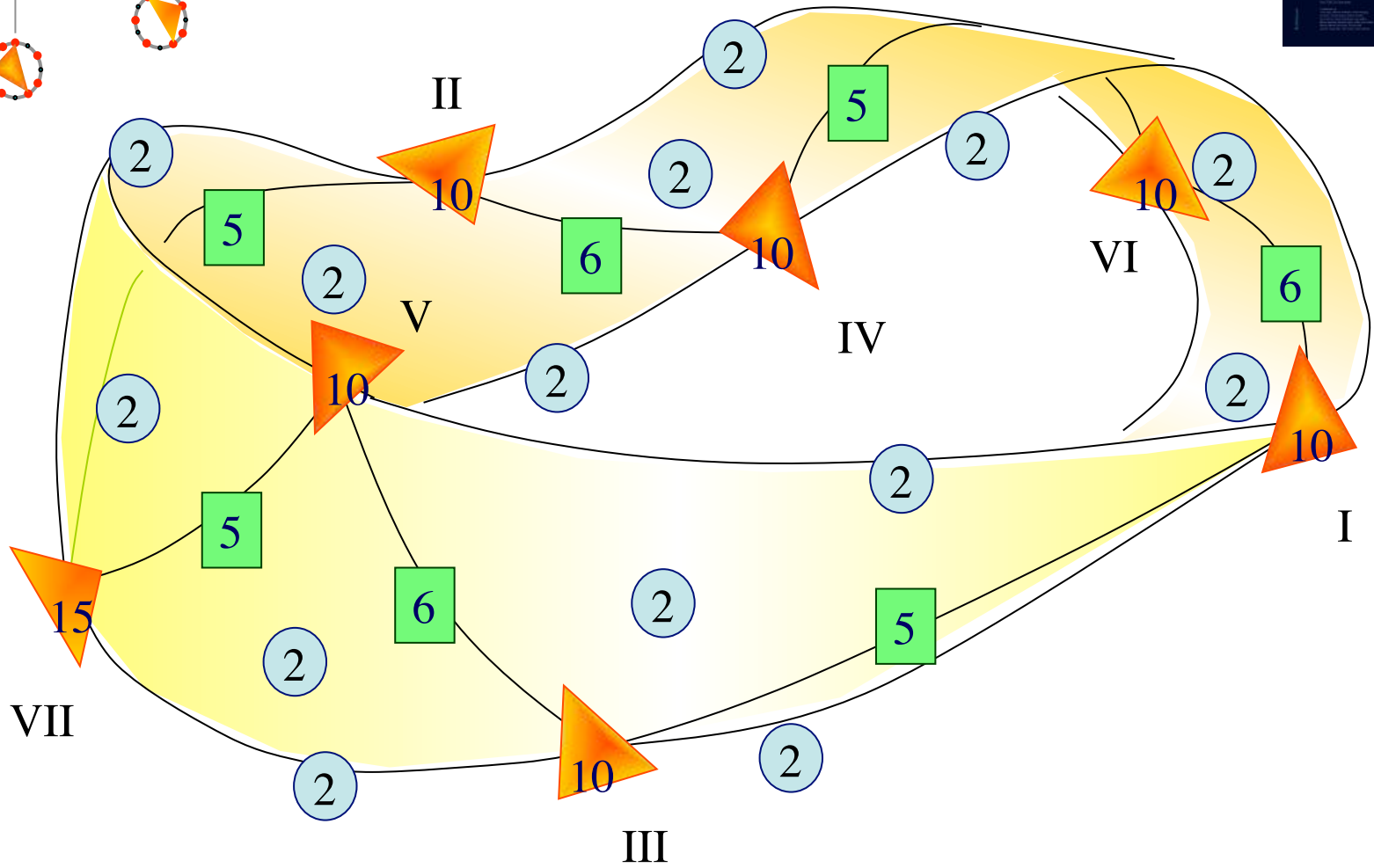
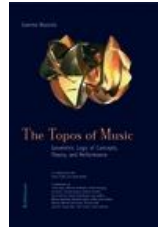
Un atlas pour la gamme diatonique...



G. Mazzola, *The Topos of Music*



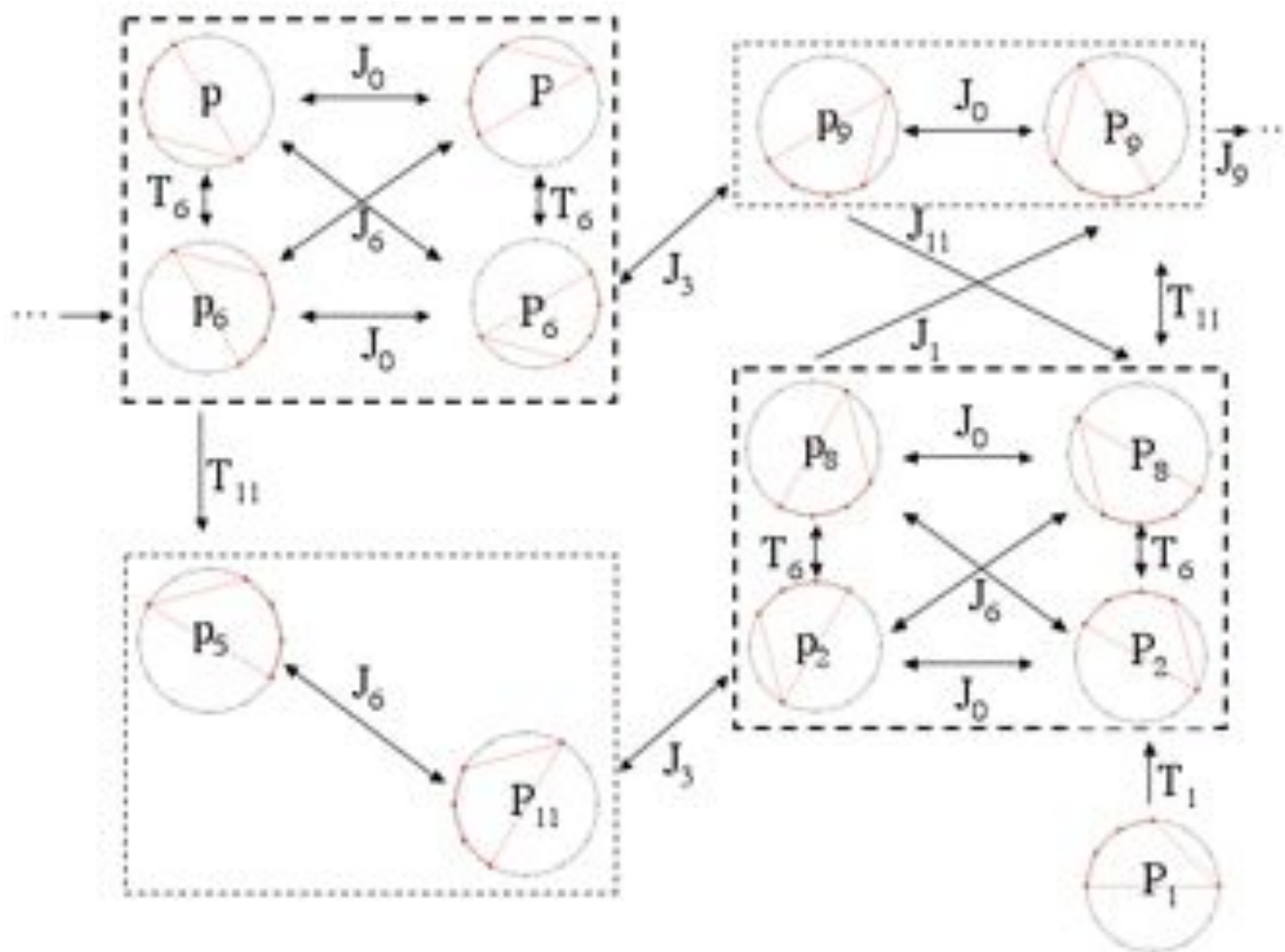
G. Mazzola, *The Topos of Music*



...et le nerf topologique associé

Reseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)



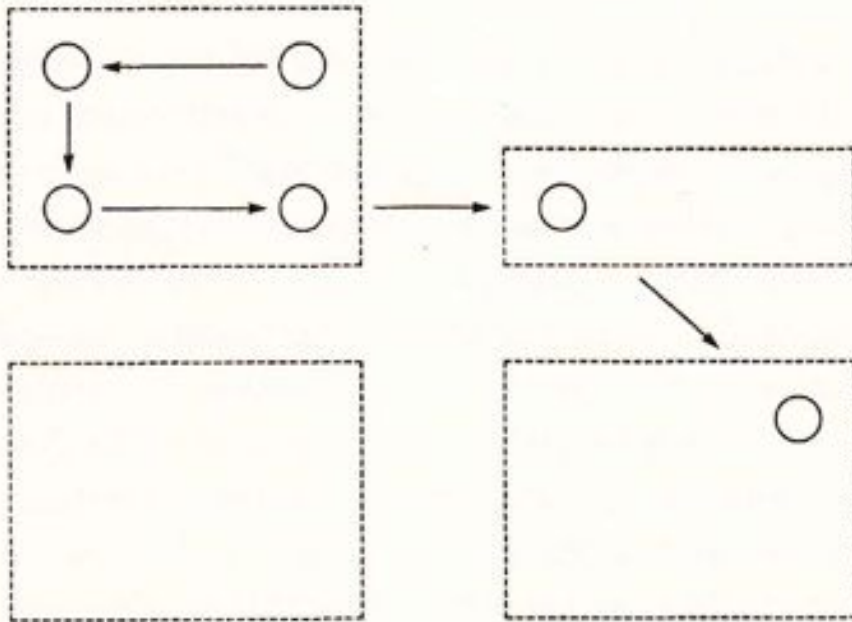
« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their **potentially functional interrelationships**, in a very compactly organized little **spatial configuration**. »

« [...] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - **it makes the abstract space of such a world accessible to our sensibilities**. That is to say that the story projects what one would traditionally call *form*. »

Parcours multiples d'écoute dans un réseau transformationnel

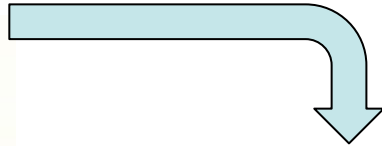
Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Pass 1 (mm. 1-5).

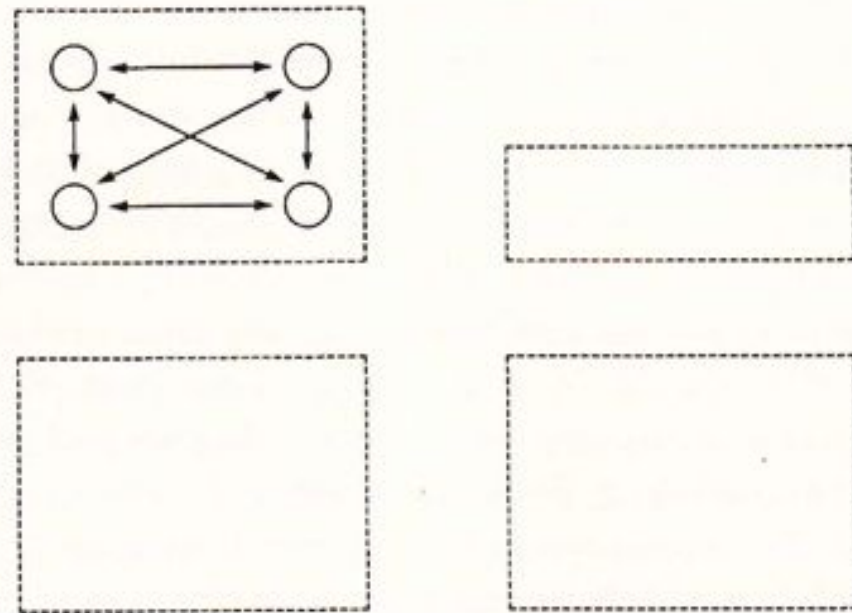


a

horizontal arrows within boxes = J0; between boxes = J3 or J9
 vertical arrows within boxes = T6; between boxes = Te or T1
 diagonal arrows within boxes = J6; between boxes = Je or J1

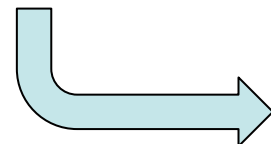


Pass 2 (mm. 5-8) goes back and elaborates the beginning area of pass 1.



b

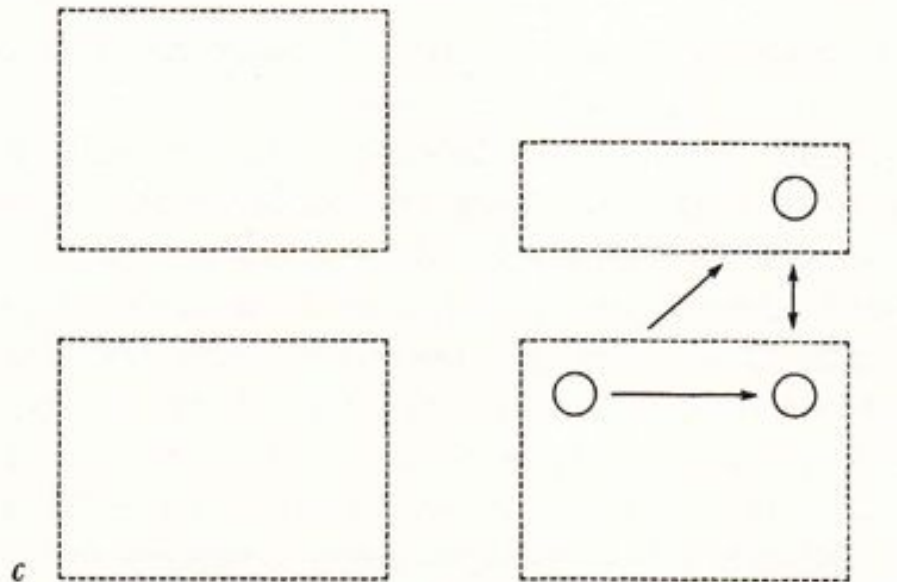
horizontal arrows within boxes = J0; between boxes = J3 or J9
 vertical arrows within boxes = T6; between boxes = Te or T1
 diagonal arrows within boxes = J6; between boxes = Je or J1



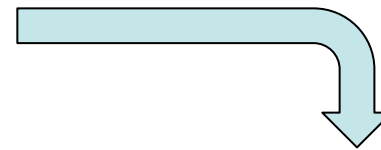
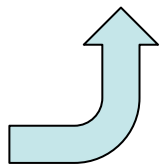
Parcours multiples d'écoute dans un réseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

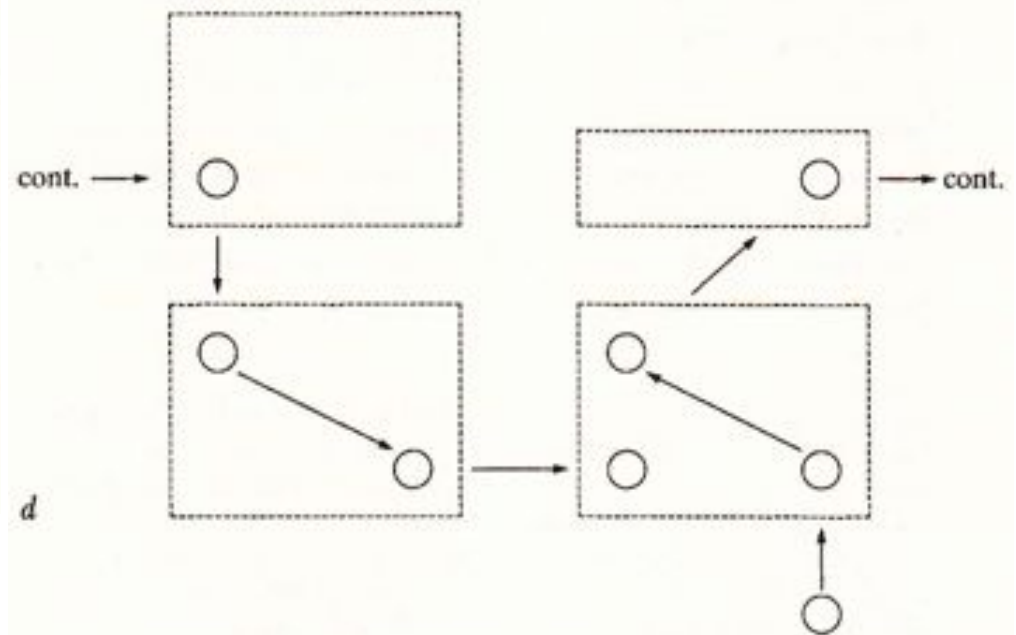
Pass 3 (mm. 8-10) picks up and elaborates the ending area of pass 1.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1



Pass 4 (mm. 9-16) expands the p8 + P8 area of pass 3 to activate P2 and p2 as well. P2 is the "essential" incipit of pass 4; p2 is the end of the pass, and of the piece.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Exercices d'écoute : « do you hear it? » vs « can you hear it? »

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

The image displays three systems of musical notation for Stockhausen's *Klavierstück III*. Each system consists of a treble and bass clef staff with notes and rests. Above the notes, interval numbers and pitch class labels are provided for each measure.

System 1 (measures 1-6):
m. 1: P0
m. 1-2: p0
m. 2: p6
m. 2-3: P6
m. 2-5: p9
m. 2-5: P8

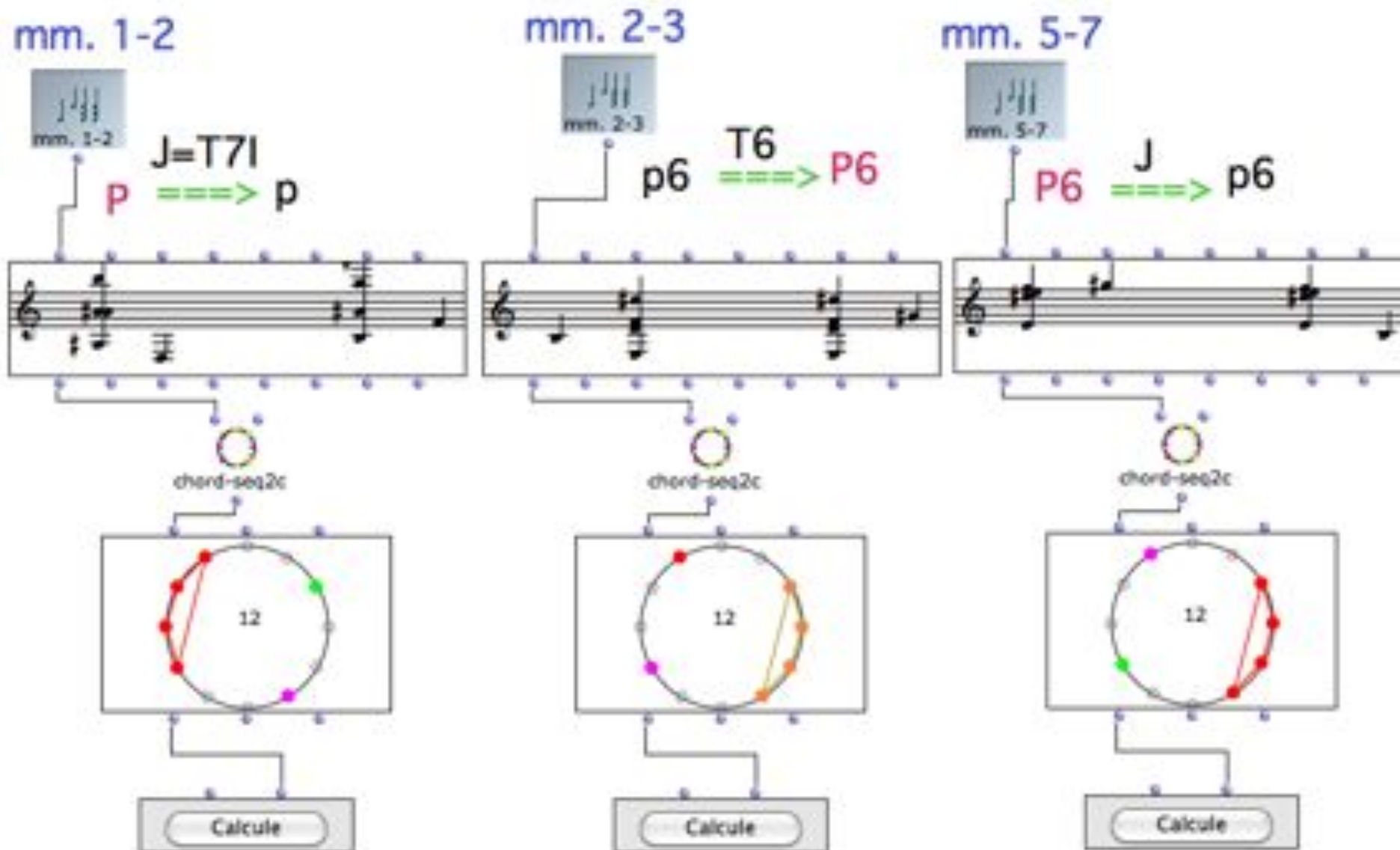
System 2 (measures 5-7, 8-10, 8-10, 8-10):
m. 5-7: P6
m. 5-7: p6
m. 5-7: P0
m. 5-7: p0
m. 8-10: p8
m. 8-10: P8
m. 8-10: P9

System 3 (measures 9-11, 10-11, 11-12, 11-12, 11-13, 12-13, 13-14, 13-15):
m. 9-11: P1
m. 9-11: P2
m. 10-11: p8
m. 11-12: P9
m. 11-12: p6
m. 11-13: p5
m. 12-13: Pc
m. 13-14: p2
m. 13-15: p2

Example 2.7. An ear-training aid for listening to P/p forms and their inter-relations.

« I take the question ‘Can you hear it’ to mean something like this: After studying the analysis in examples 2.5 and 2.6, do you find it possible to focus your **aural attention** upon aspects of the acoustic signal that seem to engage the signifiers of that analysis? [...] For me, the interesting questions involve the extent and ways in which I am satisfied and dissatisfied when **focusing my aural attention** in that manner. It is important to ask those questions about any systematic analysis of any musical composition ».

Computer-Aided Transformational Analysis in OpenMusic



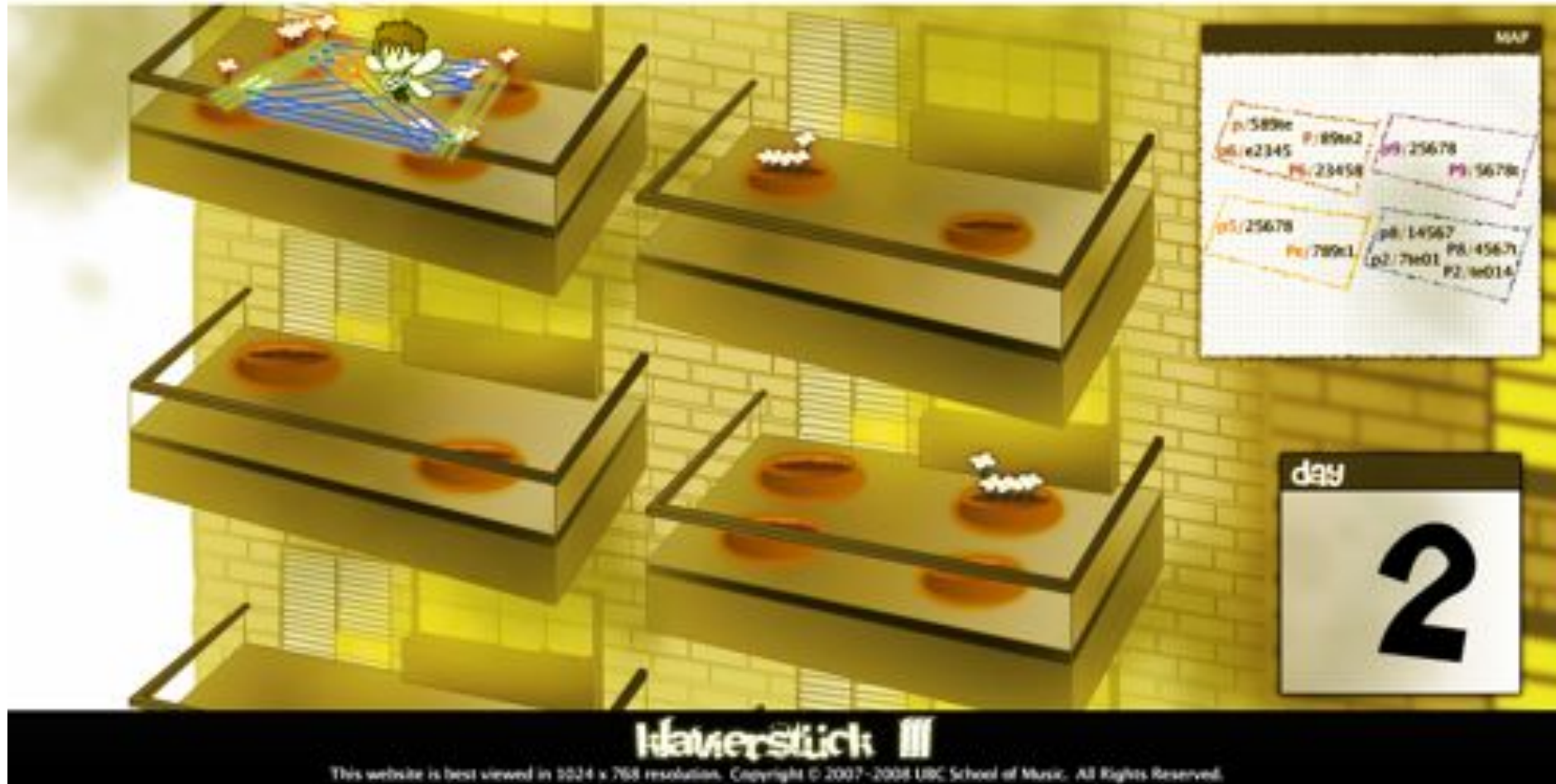
➔ <http://recherche.ircam.fr/equipes/repmus/OpenMusic/>

➔ OpenMusic

Visualisations multimédia de l'analyse transformationnelle

R. Attas : *Metaphors in Motion: Agents and Representation in Transformational Analysis*, *MTO*, 15(1), 2009
<http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html>

Animation 1. Klavierstück III



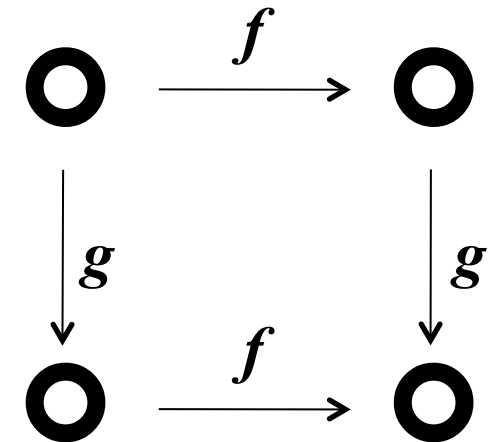
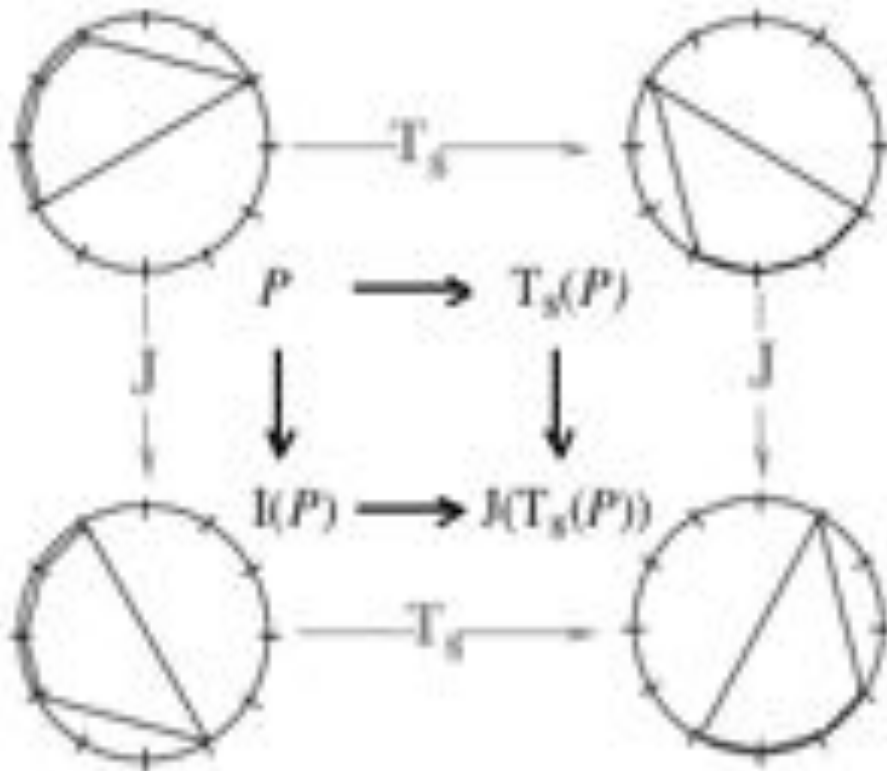
Visualisations multimédia de l'analyse transformationnelle

R. Attas : Metaphors in Motion: Agents and Representation in Transformational Analysis, *MTO*, 15(1), 2009
<http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html>

Animation 2. Grow Your Own Pentachord

The screenshot displays the 'Grow Your Own Pentachord' interface. At the top left, there is a 'PLAYBACK SPEED' control with a slider. In the center, a 3D illustration shows a brown pot with soil and several white flowers with yellow centers. To the left of the pot are two rows of 'T WANDS' and 'J WANDS'. The 'T WANDS' row contains three yellow wands labeled T1, T8, and Te. The 'J WANDS' row contains six blue wands labeled J0, J1, J3, J6, J9, and Jn. To the right of the pot is a 'PENTACHORD MAP' showing a network of nodes (P, T1, T8, Te, J0, J1, J3, J6, J9, Jn) connected by arrows. A red dot on the map is labeled '= You are here'. Below the map is a large black letter 'P' with '89te2' next to it, and the text 'CURRENT PENTACHORD' below that. At the bottom right, there are three buttons: 'INSTRUCTIONS', 'POT LAYOUT', and 'GUIDE'. At the bottom center, the text 'grow your own pentachord!' is displayed in a stylized font. At the very bottom, a small line of text reads: 'This website is best viewed in 1024 x 768 resolution. GROW YOUR OWN PENTACHORD! Copyright © 2007-2008 UBC School of Music. All Rights Reserved.'

Action de groupe et commutativité des diagrammes

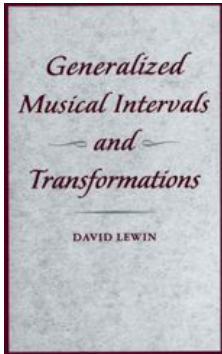


Tout diagramme commute

$$\forall f, g \in \langle T, J \rangle$$

Le groupe des 24 transformations $\sigma = \{T_0, T_1, \dots, T_{11}, T_0J, T_1J, \dots, T_{11}J\}$ est commutatif et opère de manière simplement transitive sur l'espace S des 24 formes du pentacorde de base (i.e. l'ensemble de ses 12 transpositions et de ses 12 inversions)

$\Rightarrow (S, \sigma, \text{int})$ est un GIS



Système d'Intervalles Généralisés - Système Généralisé d'Intervalles

David Lewin's *Generalized Interval System* [GMIT, 1987]

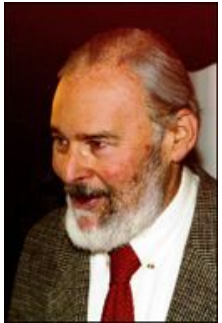
$$\text{GIS} = (S, G, \text{int})$$

S = ensemble

(G, \bullet) = groupe d'intervalles

int = fonction intervallique

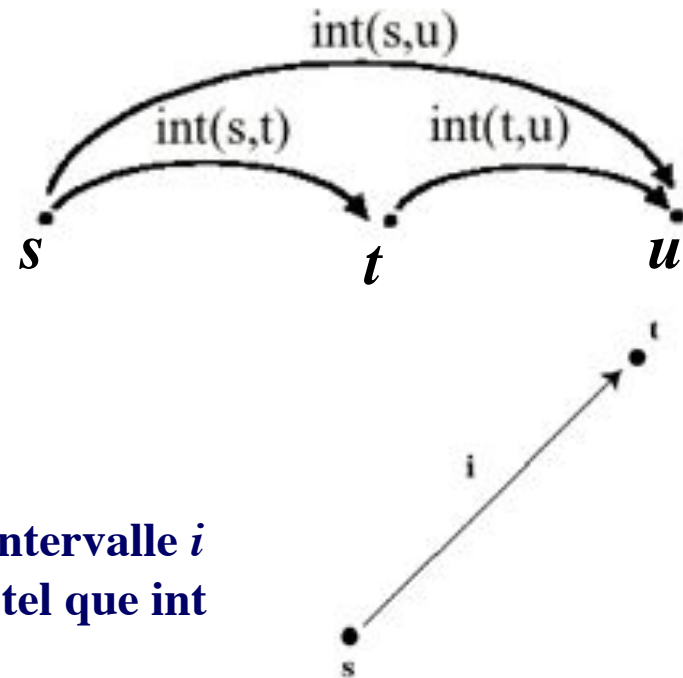
$$S \times S \xrightarrow{\text{int}} G$$



1. Pour tout objets s, t, u dans S :

$$\text{int}(s,t) \bullet \text{int}(t,u) = \text{int}(s,u)$$

2. Pour tout objet s dans S et tout intervalle i dans G il y a un seul objet t dans S tel que $\text{int}(s,t) = i$



- $S = \{\dots, do, do_{\#} = ré_b, ré, \dots, si, do', \dots\}$, $G = \mathbf{Z}$, $\text{int}(do, ré) = 2$, $\text{int}(fa, do) = -5$ etc.
- $S = \{\dots, do, ré, mi, fa, sol, la, si, do', \dots\}$, $G = \mathbf{Z}$, $\text{int}(do, ré) = 1$, $\text{int}(fa, do) = -3$ etc.
- $S = G = \mathbf{Z}_{12} = \{do, do_{\#} = ré_b, ré, \dots, si\}$, $\text{int}(do, ré) = 2$, $\text{int}(fa, do) = 7$ etc.