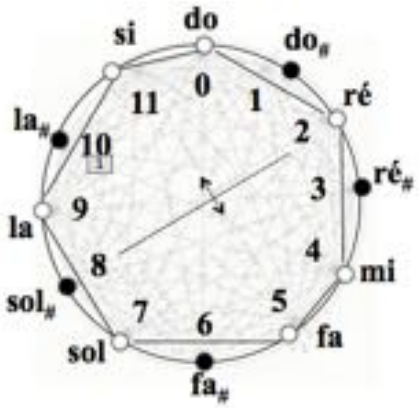
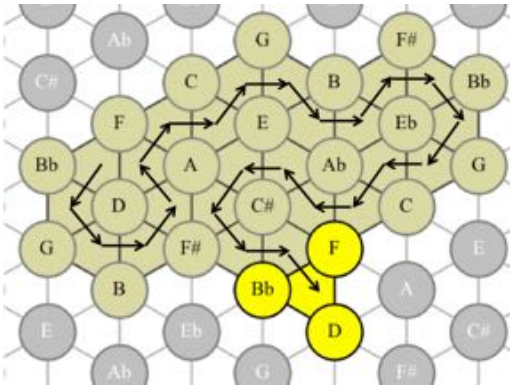


Université Populaire ANTONY



Musique et mathématiques

en musique savante et dans la chanson

Conservatoire Darius Milhaud d'Antony

4 Mai 2017

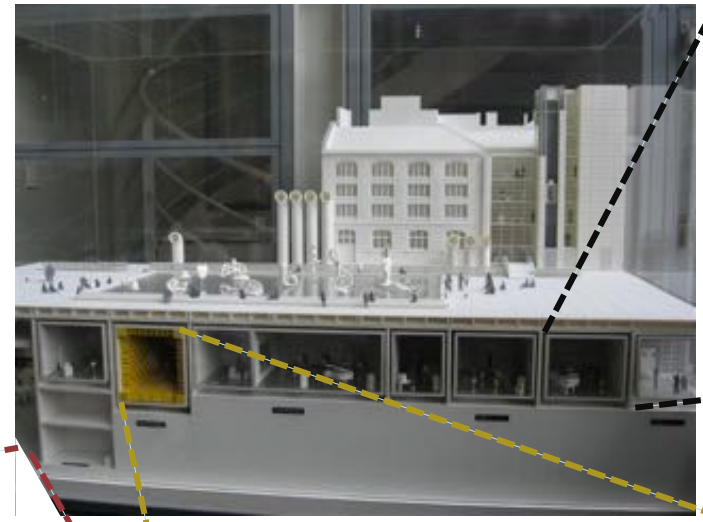
Moreno Andreatta
Music Representations Team
IRCAM / CNRS UMR 9912 / UPMC, Paris
IRMA & GREAM, University of Strasbourg

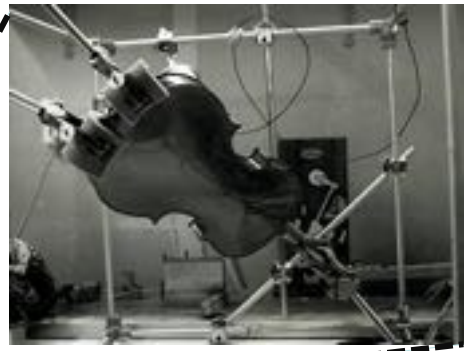
ircam Centre Pompidou

cnrs

UPMC SORBONNE UNIVERSITÉS

La recherche musicale et scientifique à l'IRCAM...

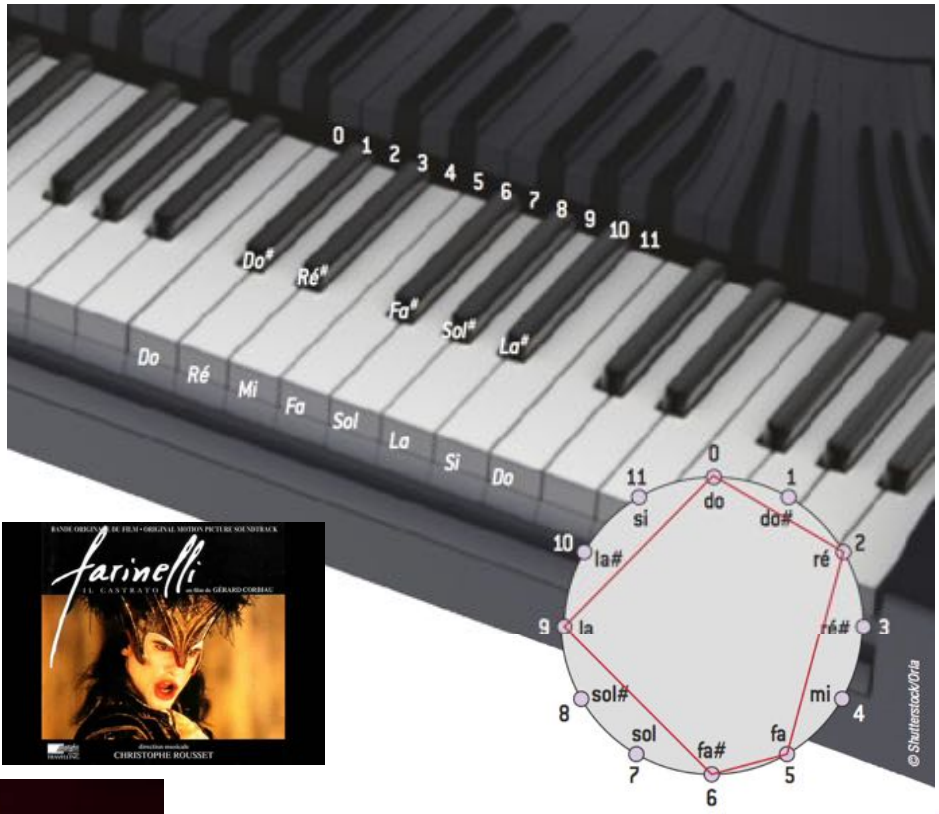




www.ircam.fr

... entre musique savante et *popular music*

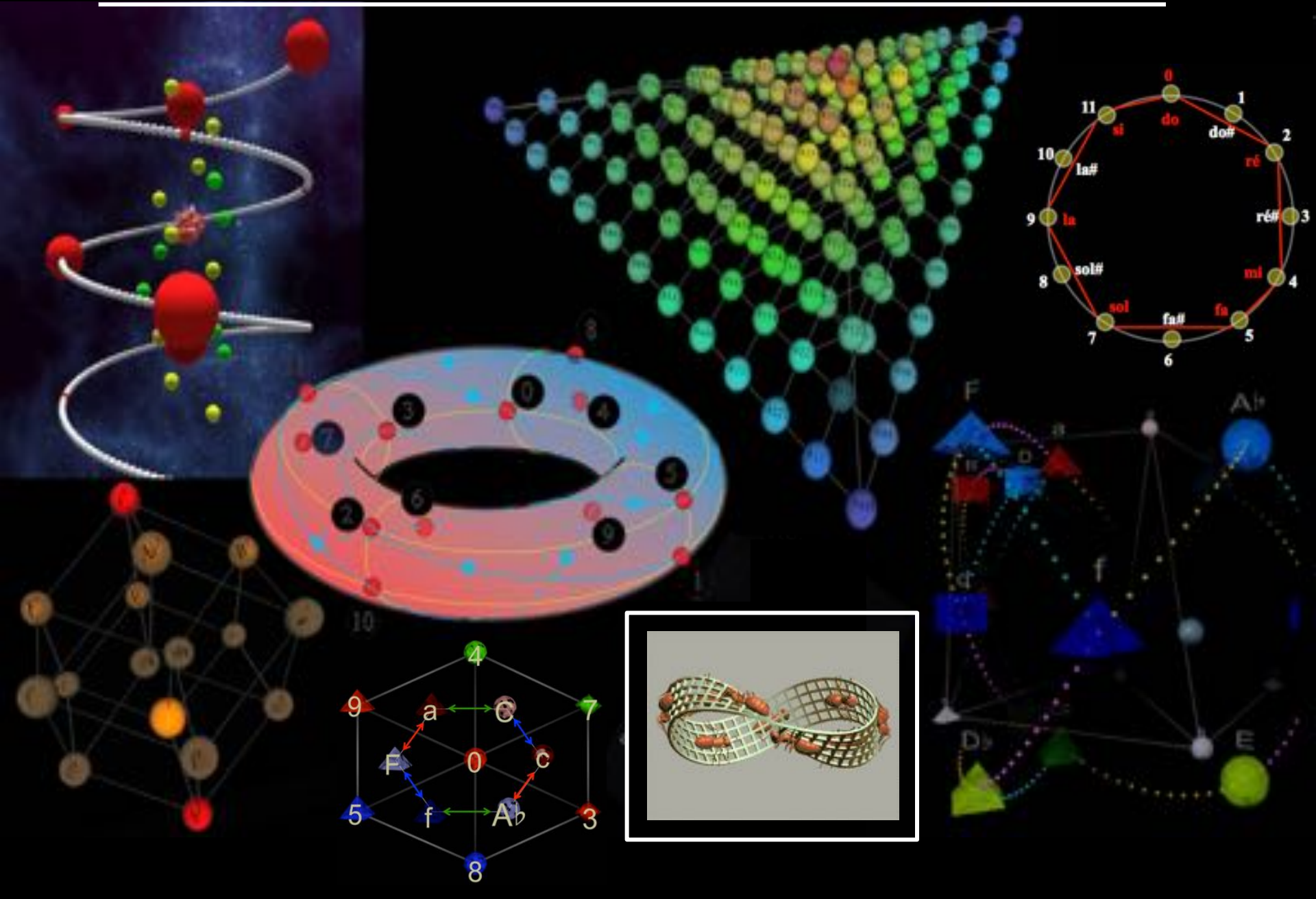
MusiqueLab 2



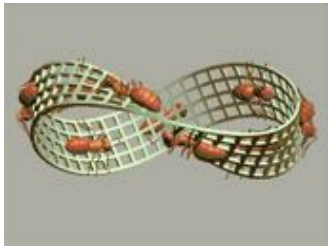
OMAX (logiciel d'improvisation)

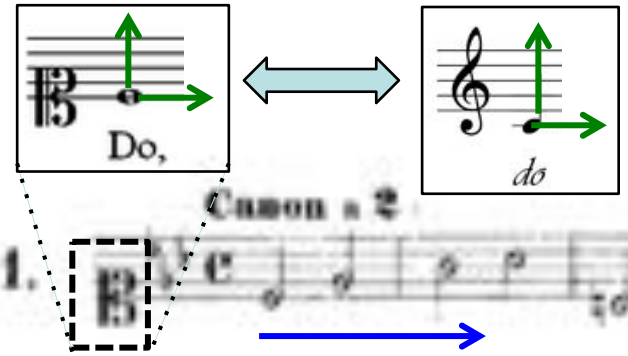
www.ircam.fr

La galaxie des modèles géométriques au service de la musique



Canons énigmatiques chez Bach et géométrie



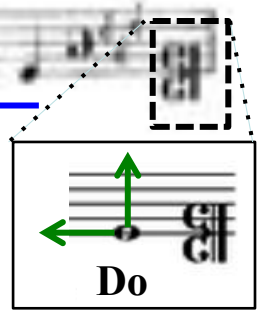


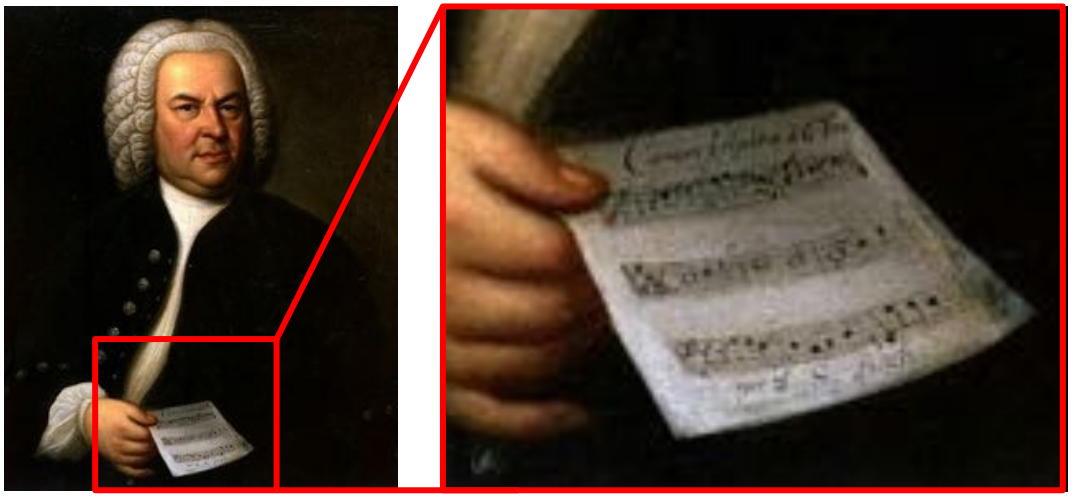
Canones diversi

super thema regium

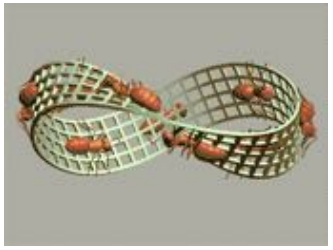
Canon a 2

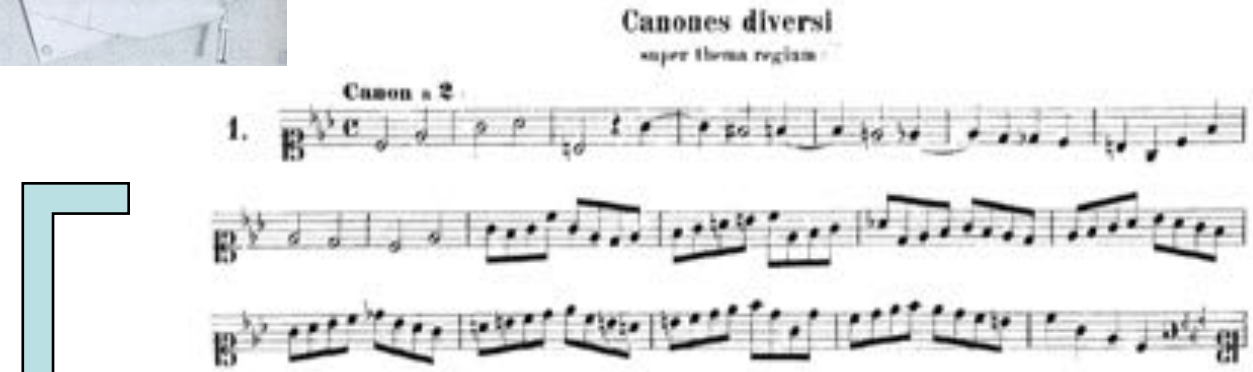
1.

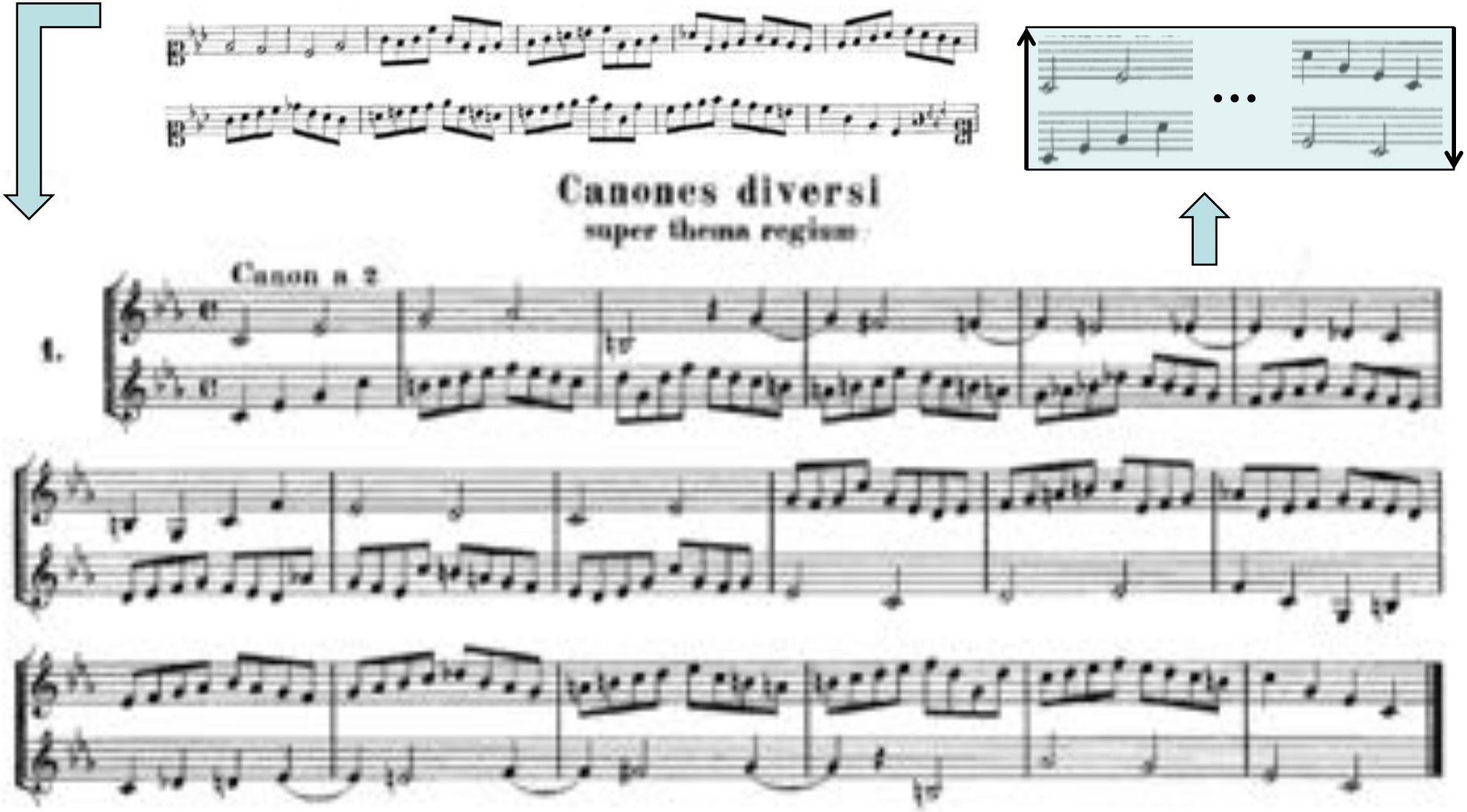




Ma fin est mon début (renversé !)

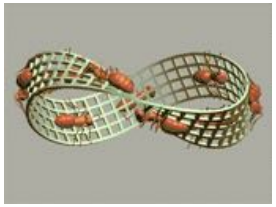




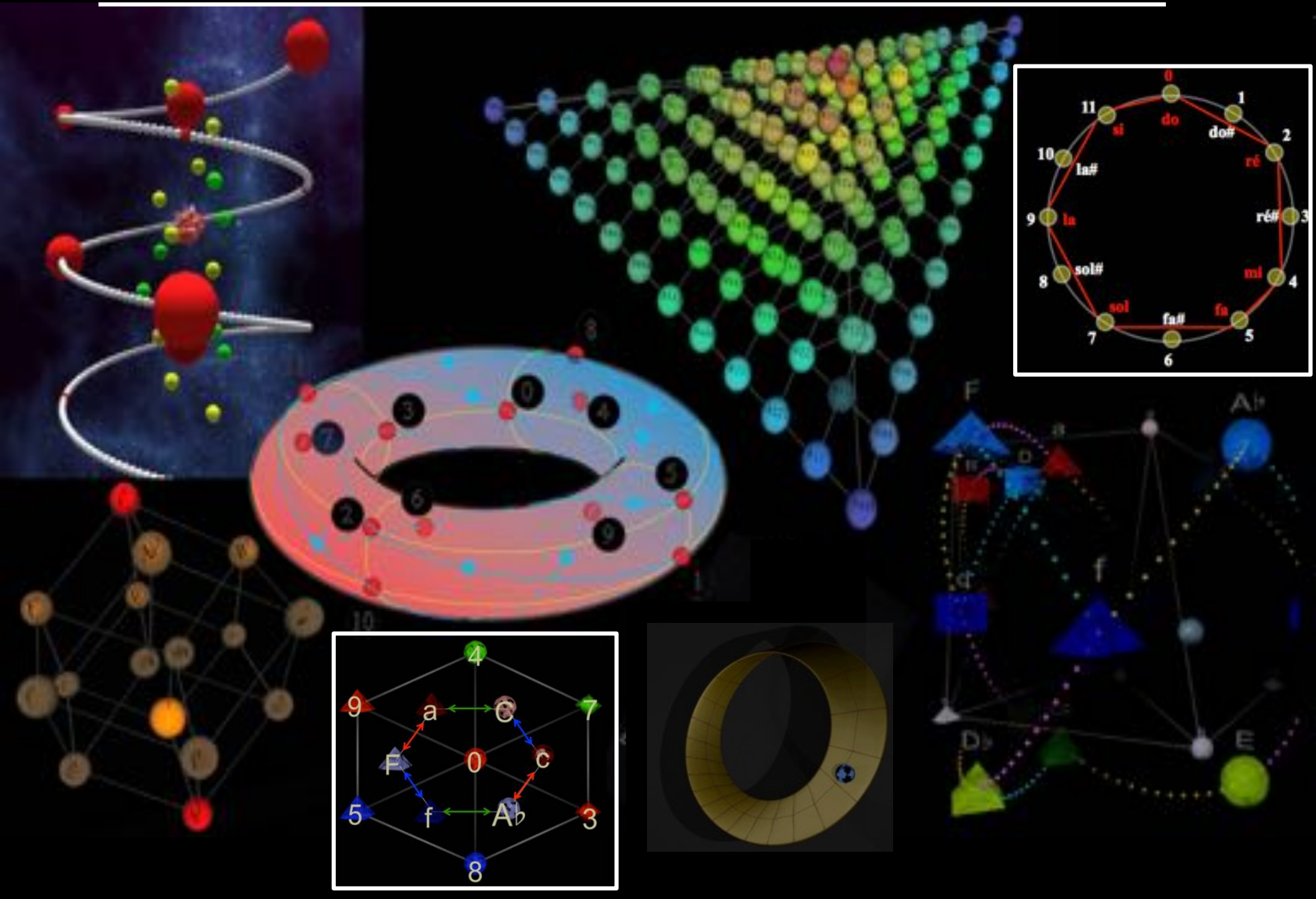


http://www.josleys.com/Canon/Canon.html

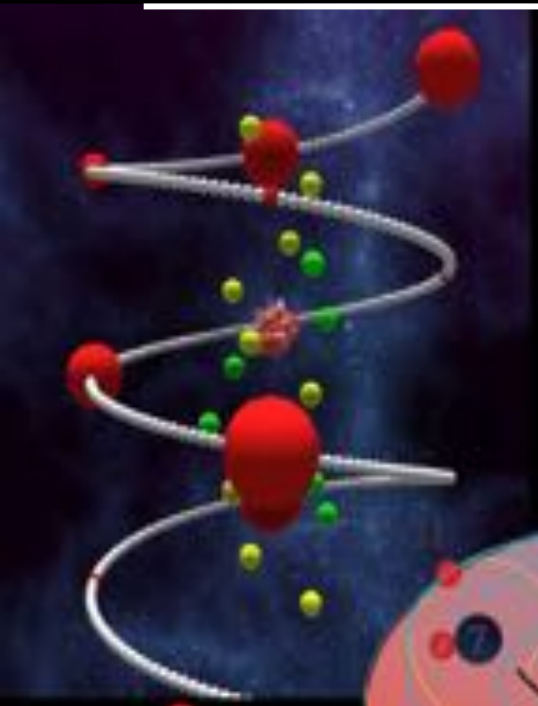
[min. 1’14”]

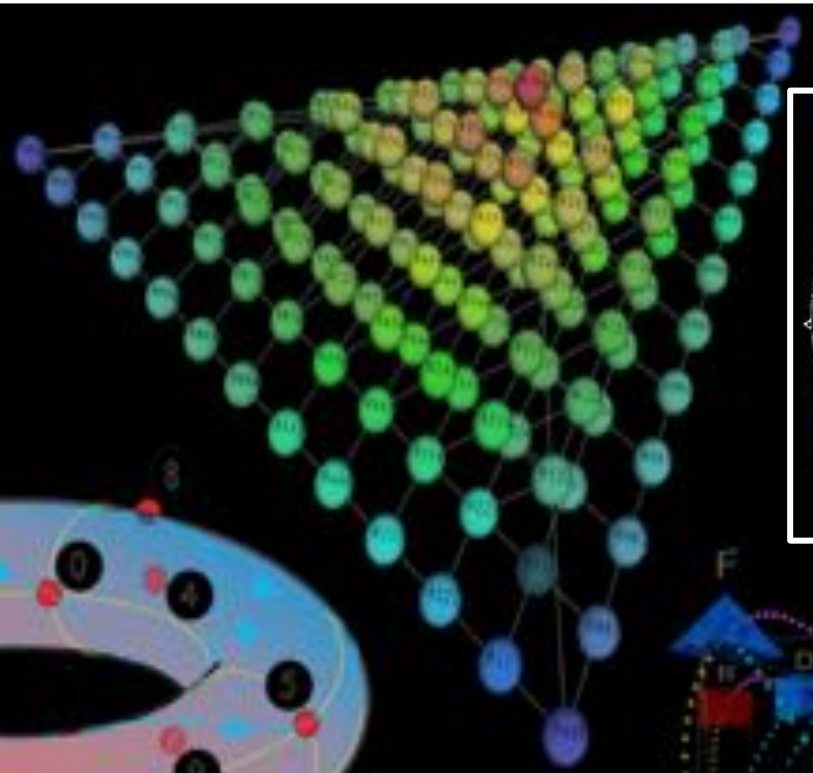


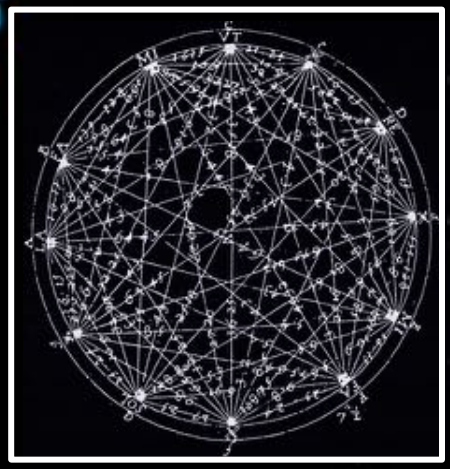
La galaxie des modèles géométriques au service de la musique

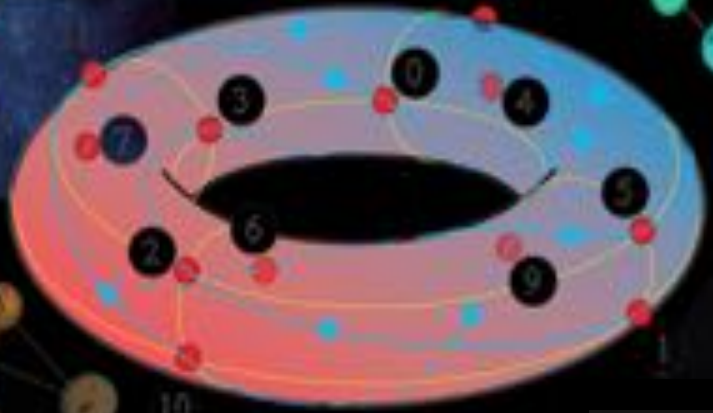


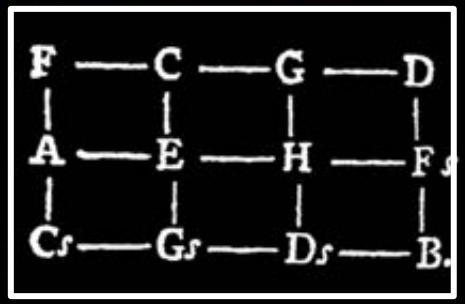
La galaxie des modèles géométriques au service de la musique

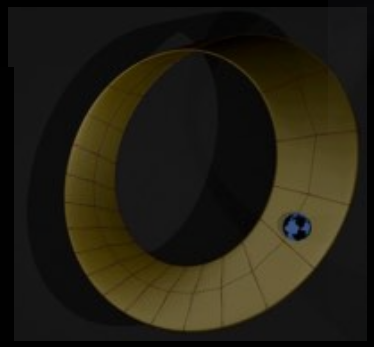






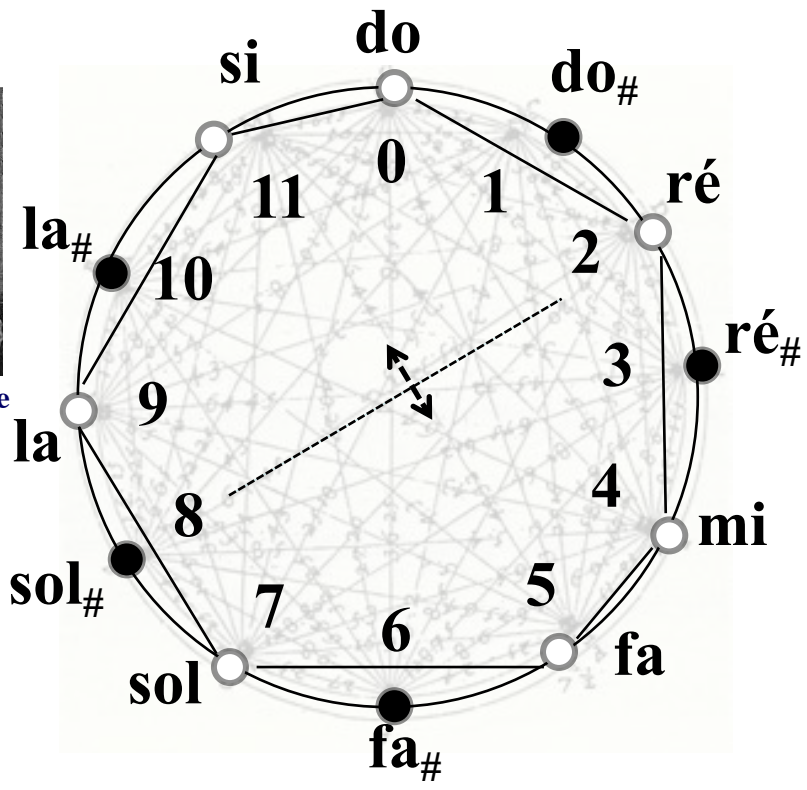




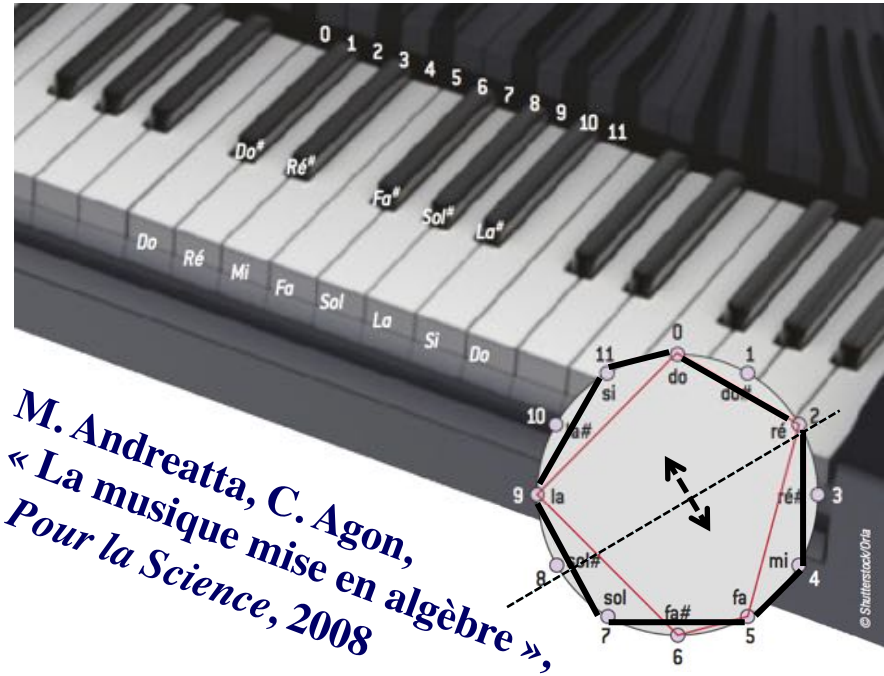


La représentation circulaire des notes de musique

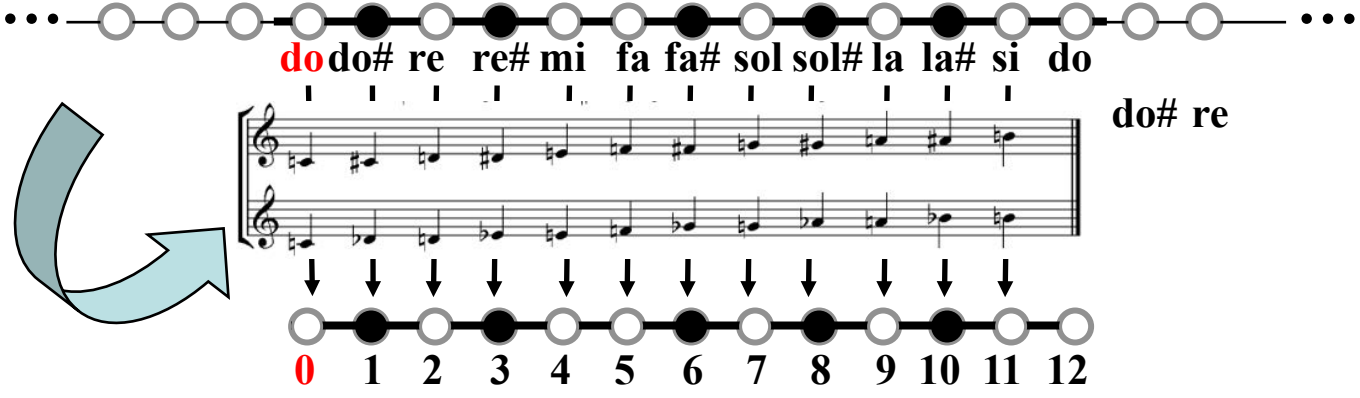
Marin Mersenne



Harmonicorum Libri XII, 1648



M. Andreatta, C. Agon, « La musique mise en algèbre », *Pour la Science*, 2008



Mersenne et la naissance de la combinatoire

114 **Marin Mersenne, *Harmonicorum Libri XII*, 1648**

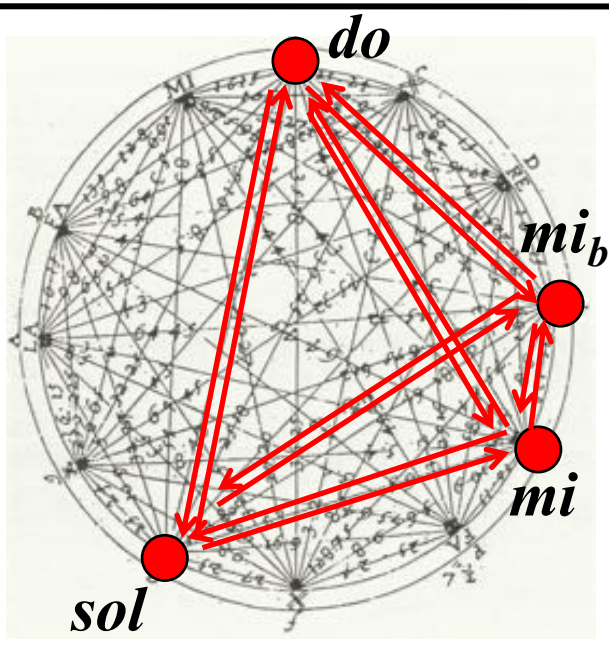
LIBER SEPTIMVS.

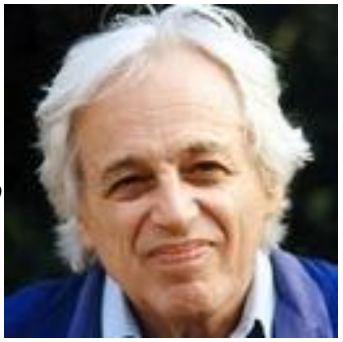
DE CANTIBVS, SEV CANTILENIS,

EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 22.

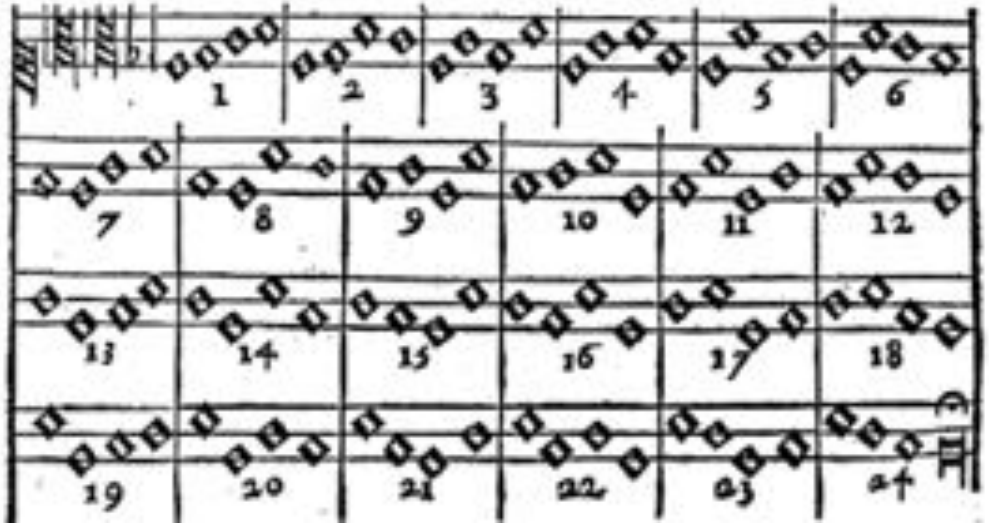
I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6227020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000





Six Bagatelles
(G. Ligeti, 1953)

Varietas, seu Combinatio quatuor notarum.



Une chanson permutationnelle

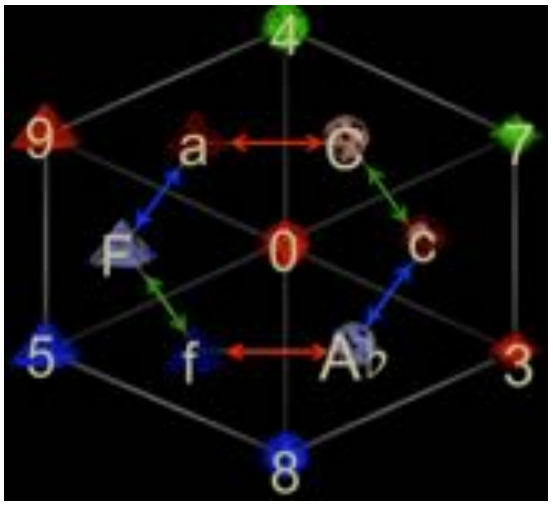
***Se telefonando*, 1966 (Maurizio Costanzo/Ennio Morricone) / Mina**

(min. 0'53")

***Je changerais d'avis*, (Françoise Hardy)**

Ennio Morricone

L'espace harmonique



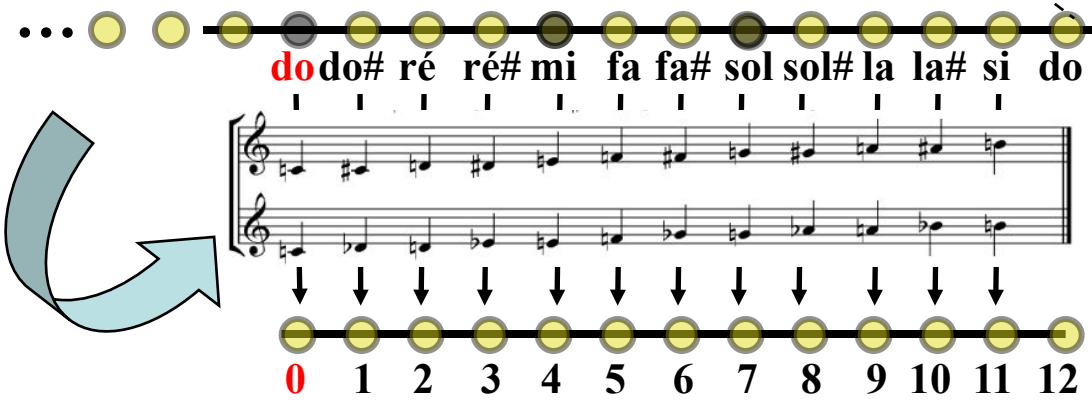
C	c	$C_{\#}$	$c_{\#}$	D	d
E_b	e_b	E	e	F	f
$F_{\#}$	$f_{\#}$	G	g	$G_{\#}$	$g_{\#}$
A	a	B_b	b_b	B	b

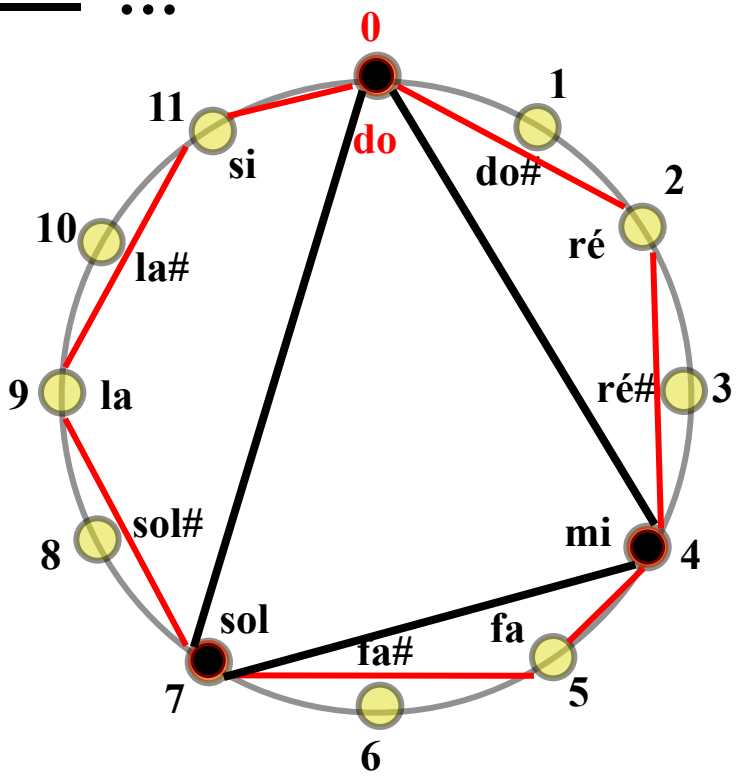
Chord enumeration

→demo

Les transpositions sont des additions...

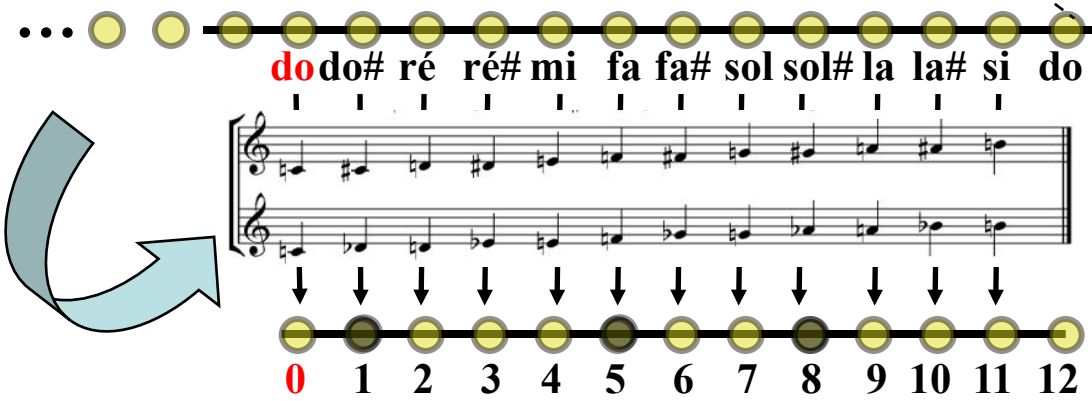
***Do* maj = {0,2,4,5,7,9,11} +1**

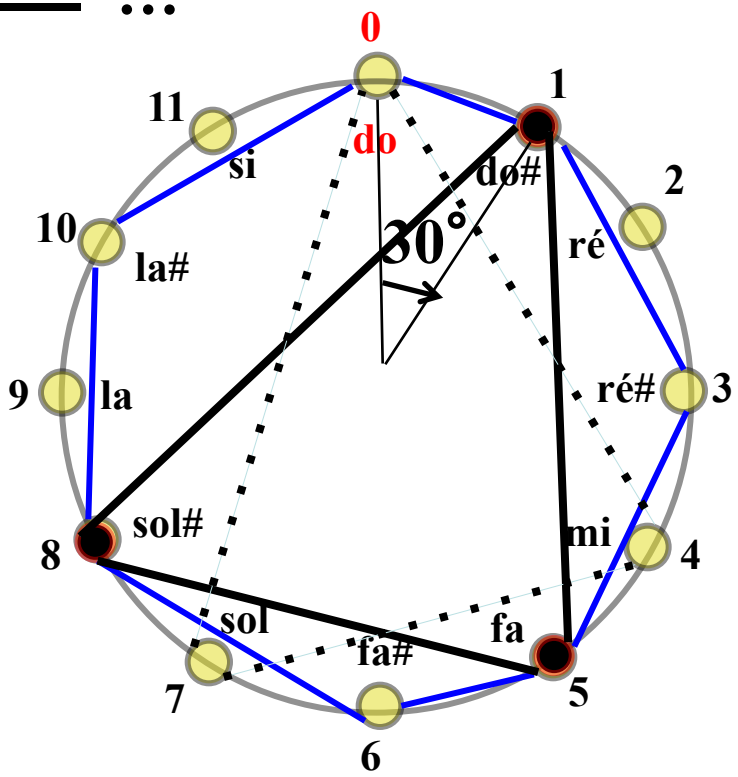




Les transpositions sont des additions...

***Do#* maj = {1,3,5,6,8,10,0}**





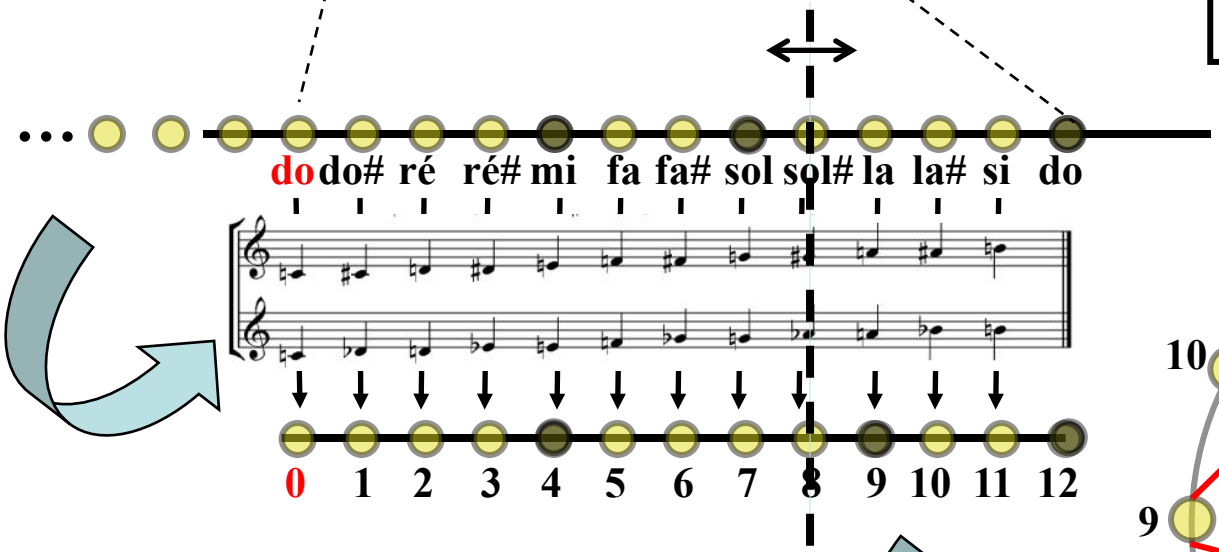
... ou des rotations !

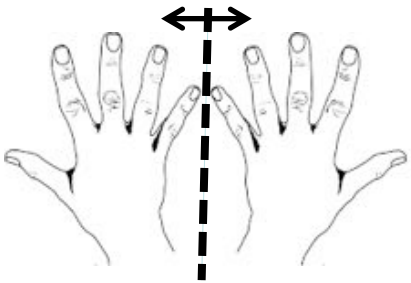
Les inversions sont des symétries axiales

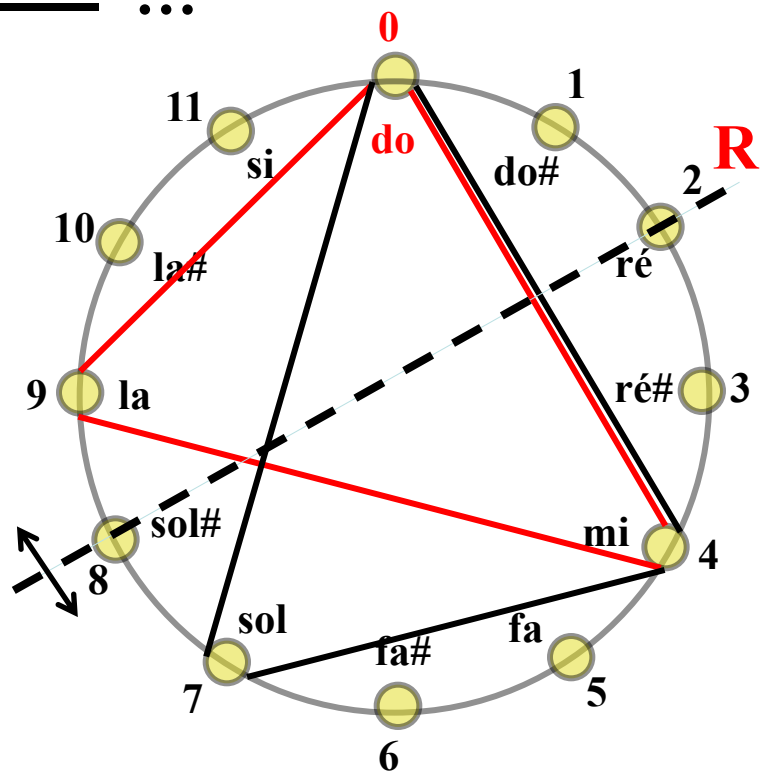
R comme relatif

Do maj = {0,4,7}

La min = {0,4,9}



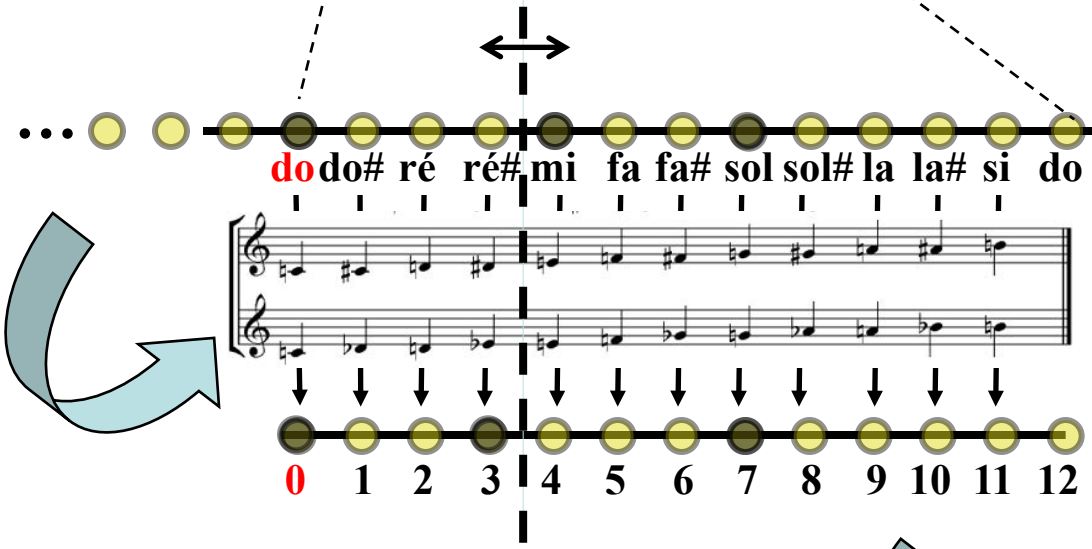


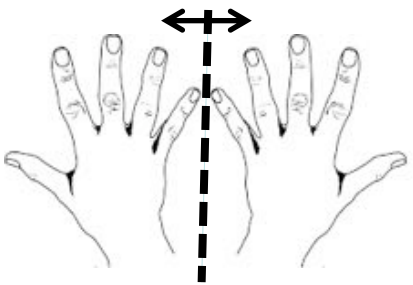


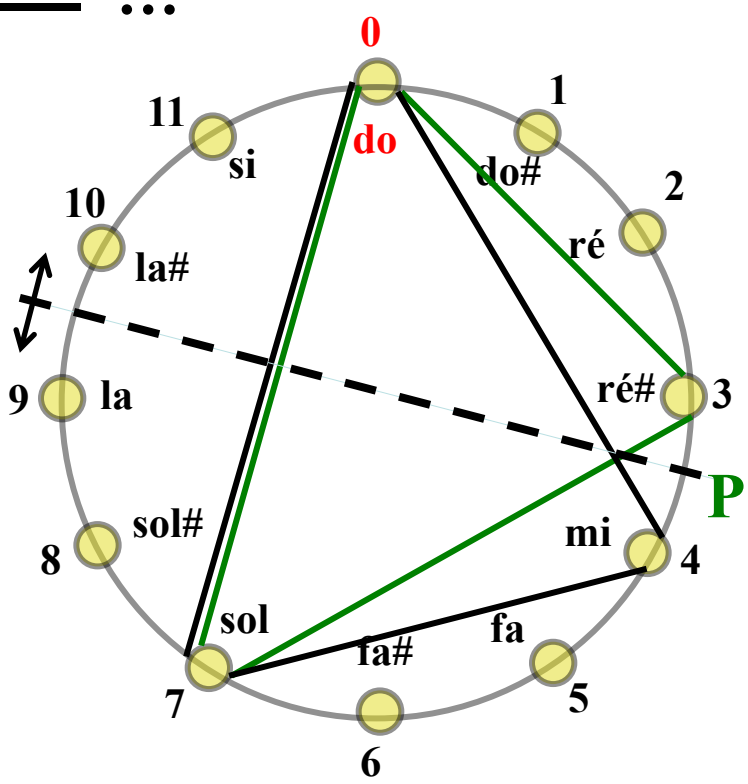
Les inversions sont des symétries axiales

P comme parallèle

Do maj	= {0,4,7}
Do min	= {0,3,7}



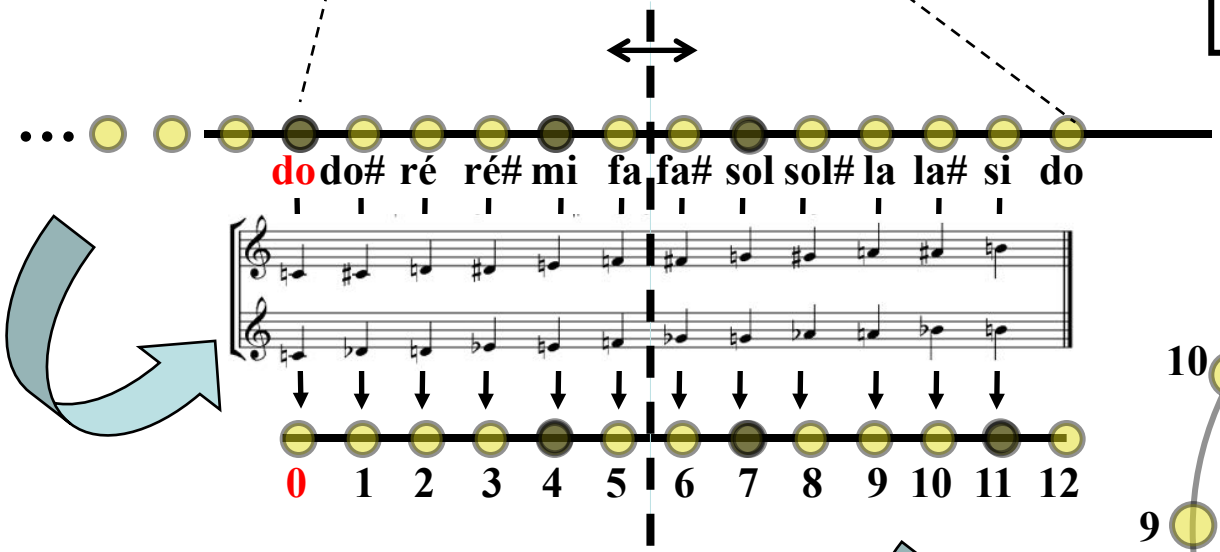


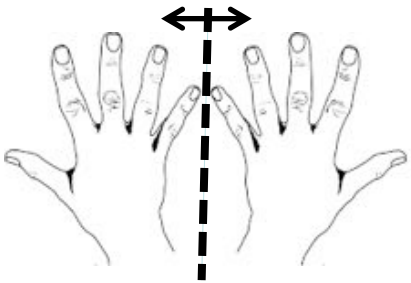


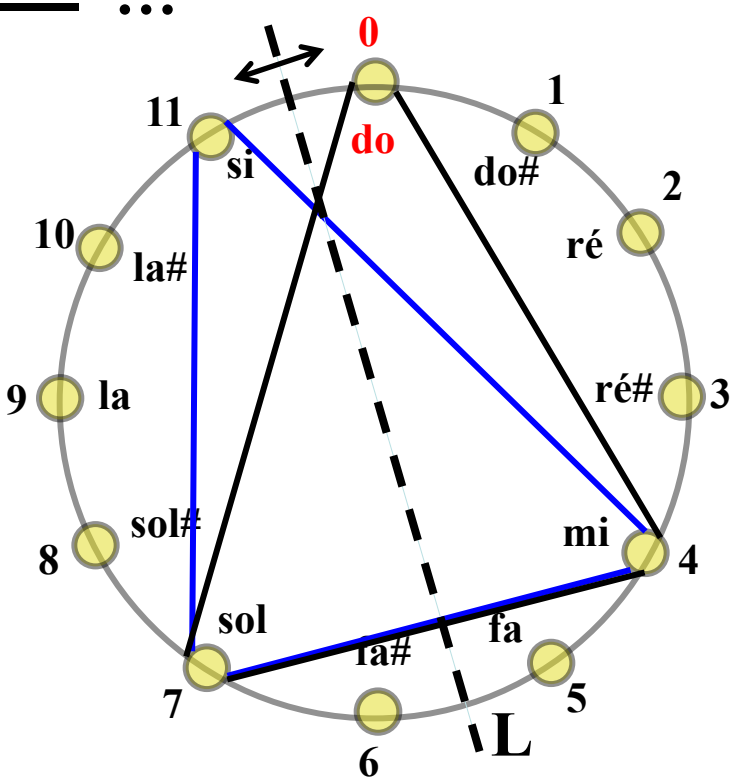
Les inversions sont des symétries axiales

L = Leading Tone

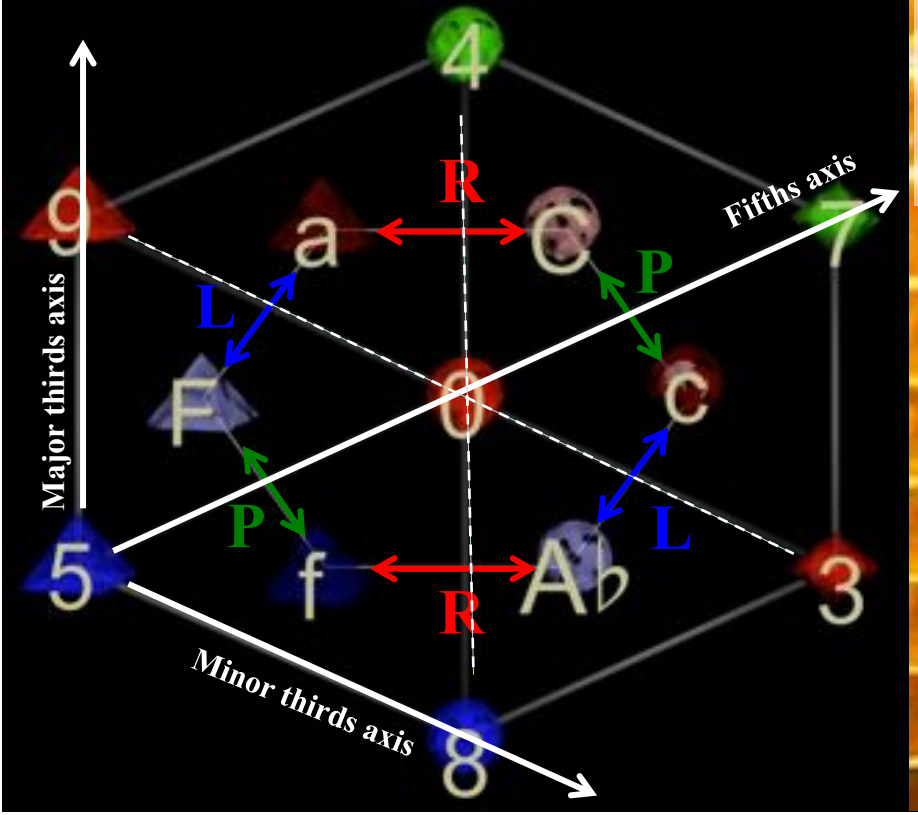
Do maj	= {0,4,7}
Mi min	= {4,7,11}

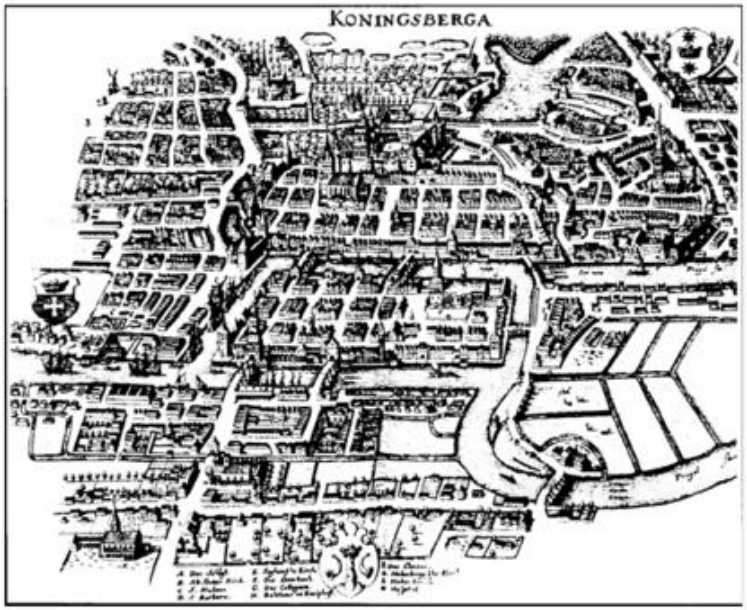


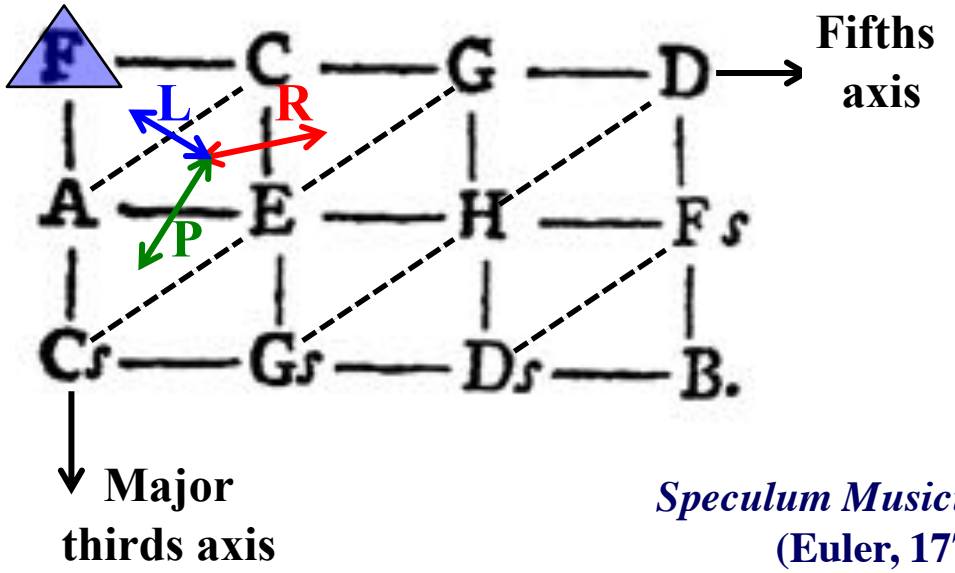




Le *Tonnetz* (ou nid musical d'abeilles)



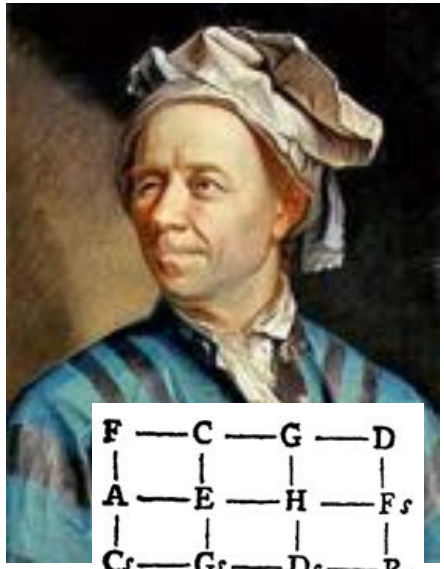




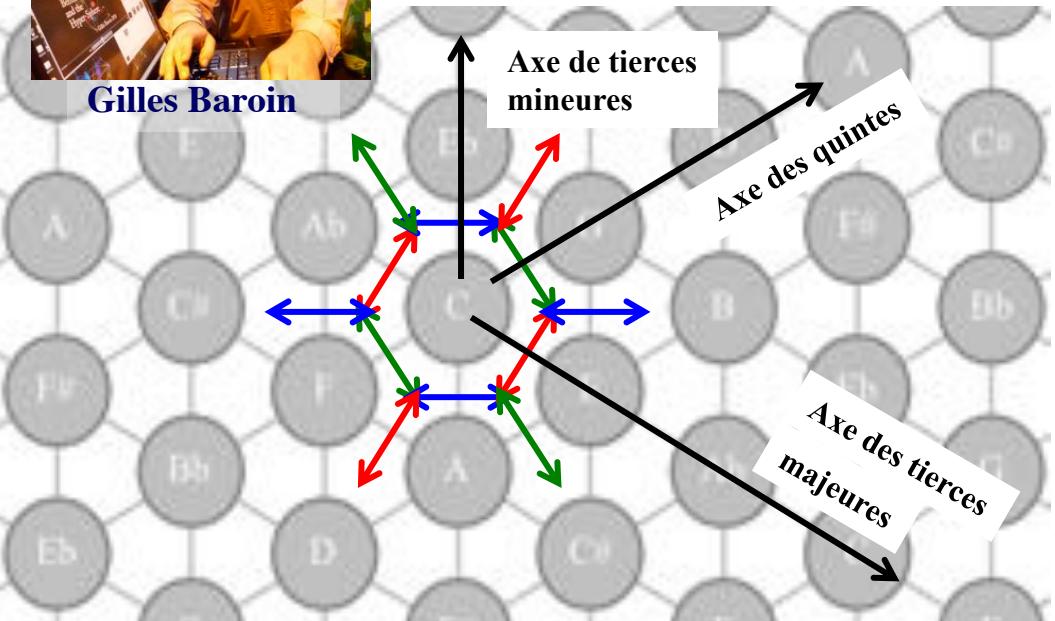
Speculum Musicum (Euler, 1773)

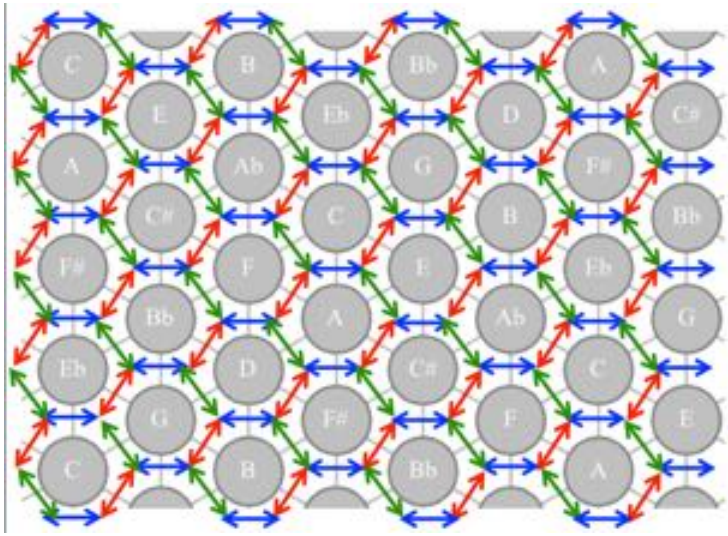
Le *Tonnetz* (ou nid musical d'abeilles)

Gilles Baroin



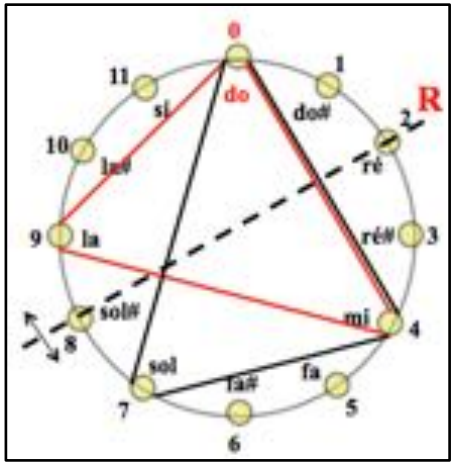
Speculum Musicum (Euler, 1773)

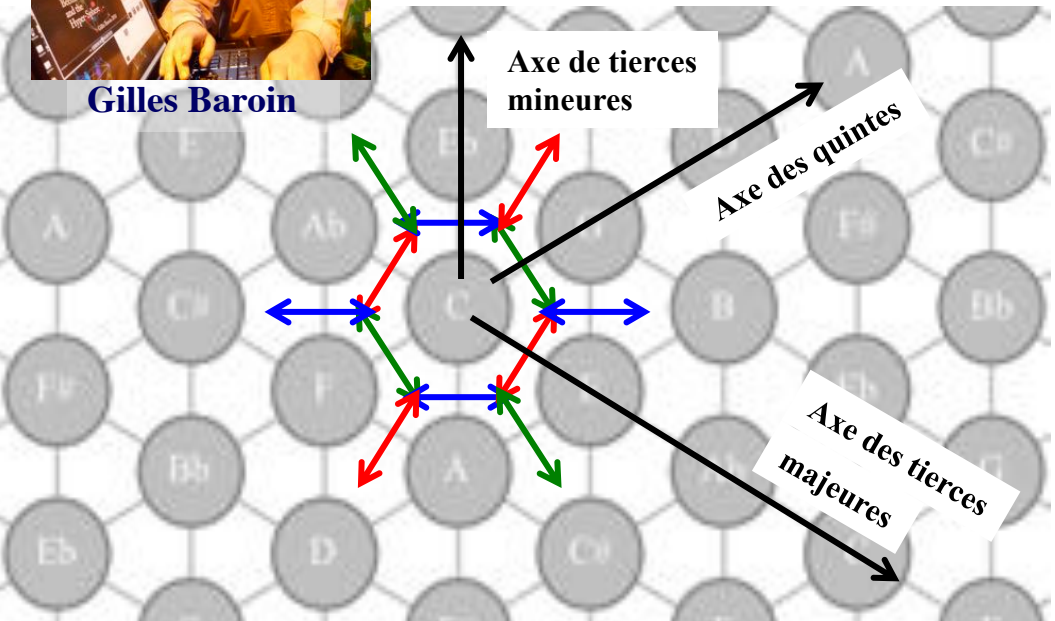


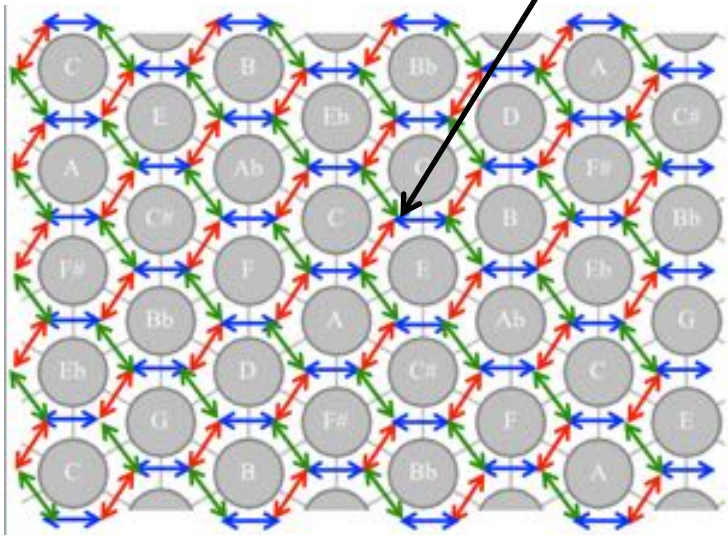


Le *Tonnetz* (ou nid musical d'abeilles)

Gilles Baroin

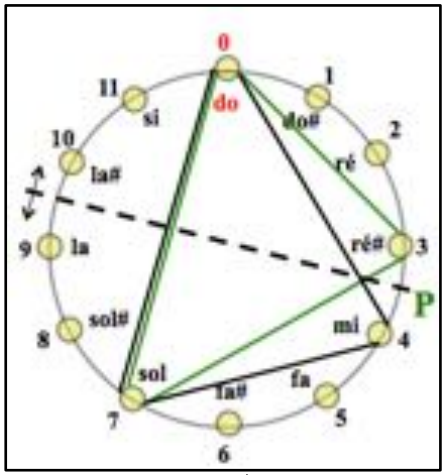


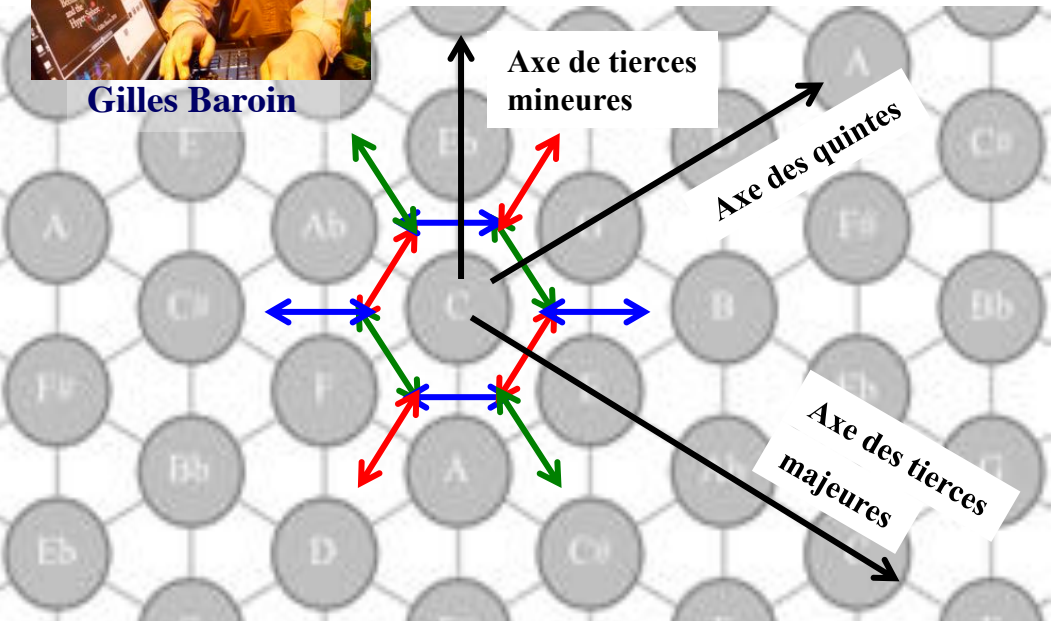


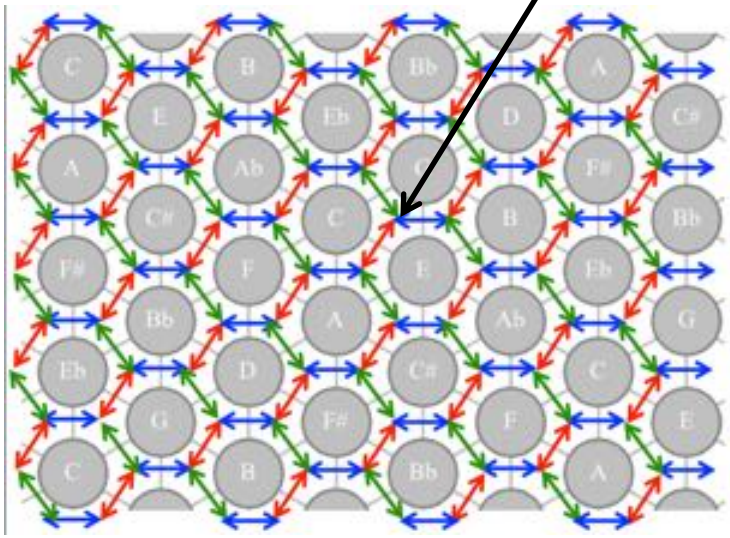


Le *Tonnetz* (ou nid musical d'abeilles)

Gilles Baroin

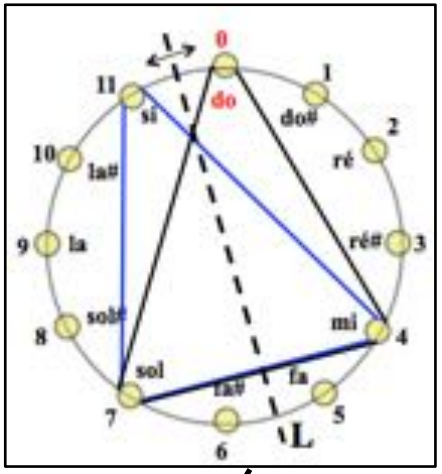


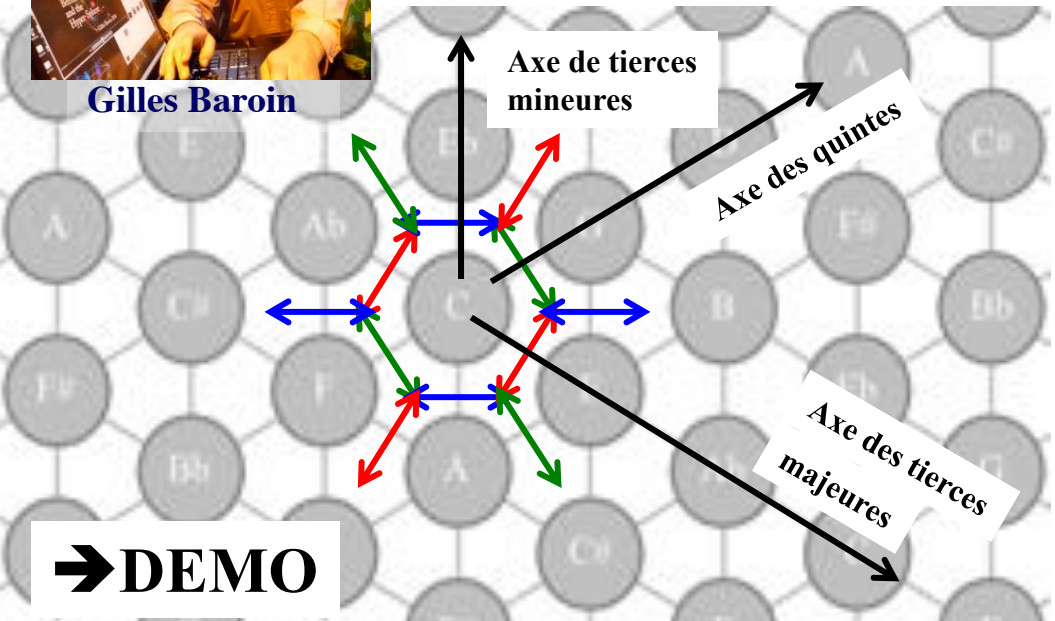




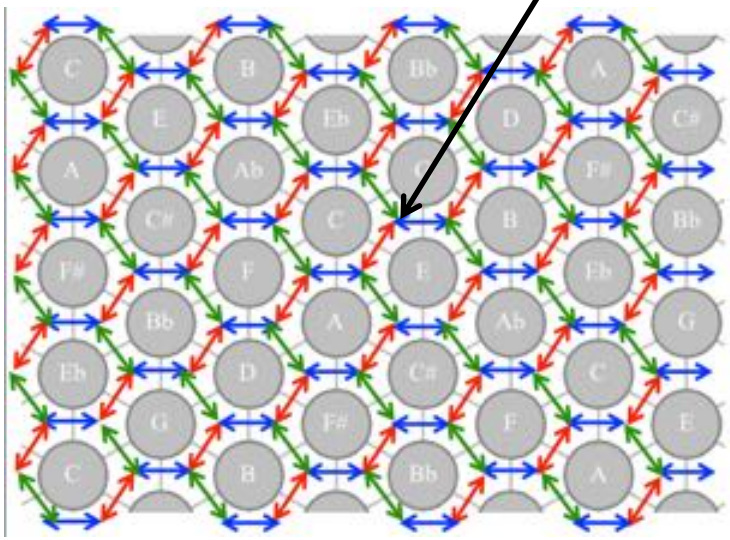
Le *Tonnetz* (ou nid musical d'abeilles)

Gilles Baroin





➔DEMO

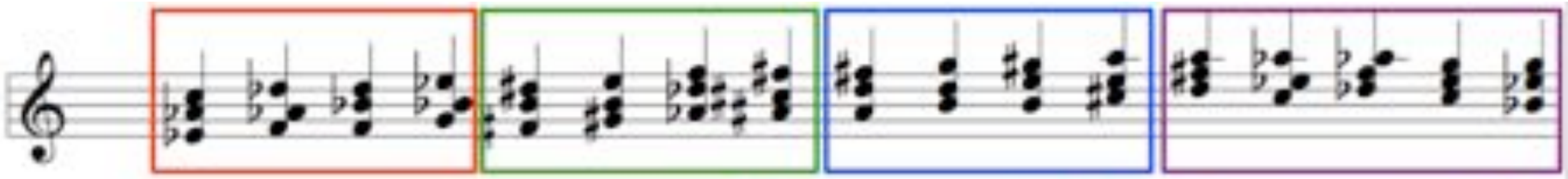


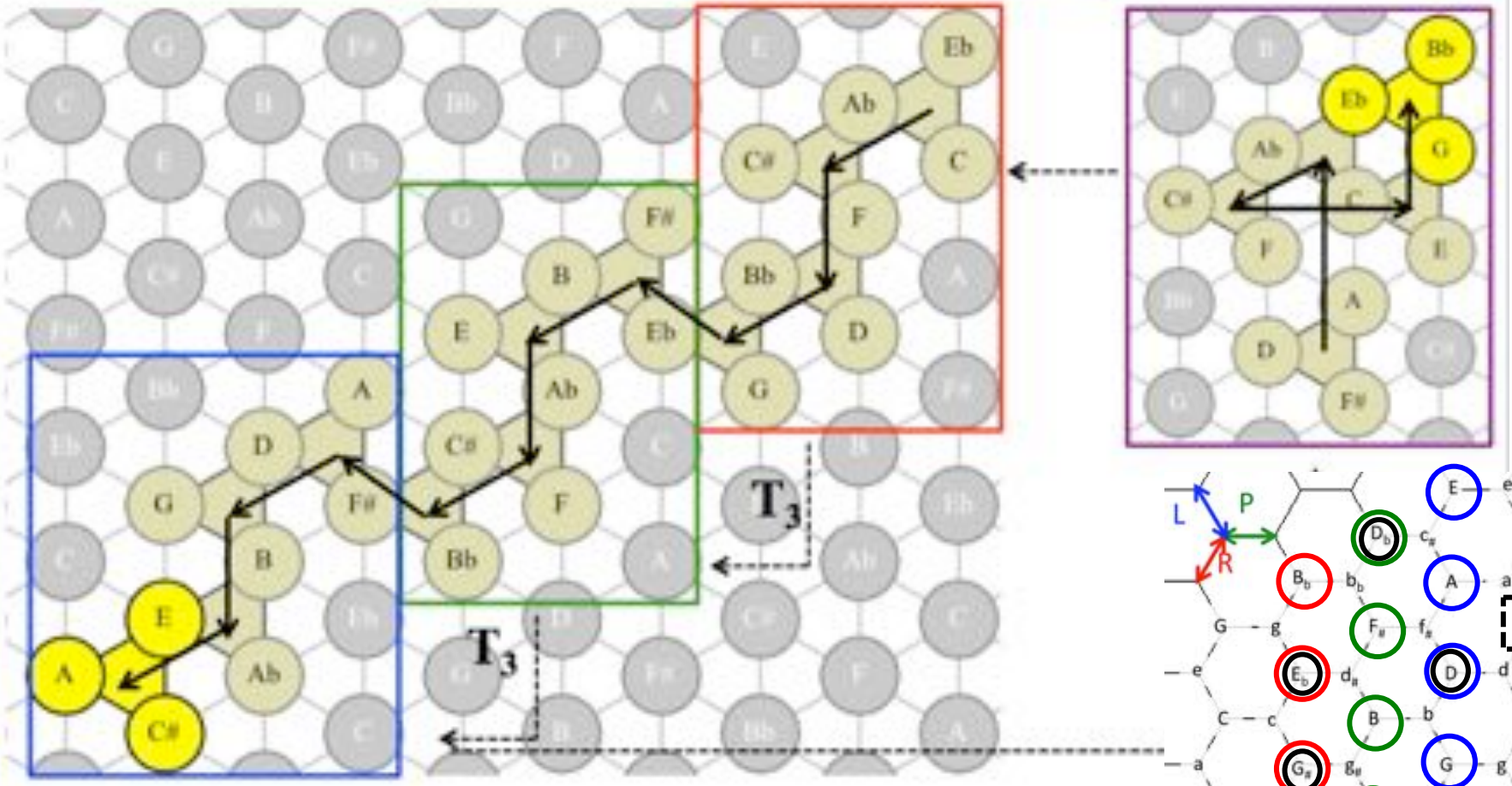
➔ **https://www.youtube.com/user/MatheMusic4D**

➔ https://www.youtube.com/user/MatheMusic4D

Symétries dans *Madeleine* de Paolo Conte

La$_b$ Re$_b$ Si$_b$ Mi$_b$ Si Mi Re$_b$ Fa$_\#$ Re Sol Mi La Re La$_b$ Re$_b$ Do Mi$_b$



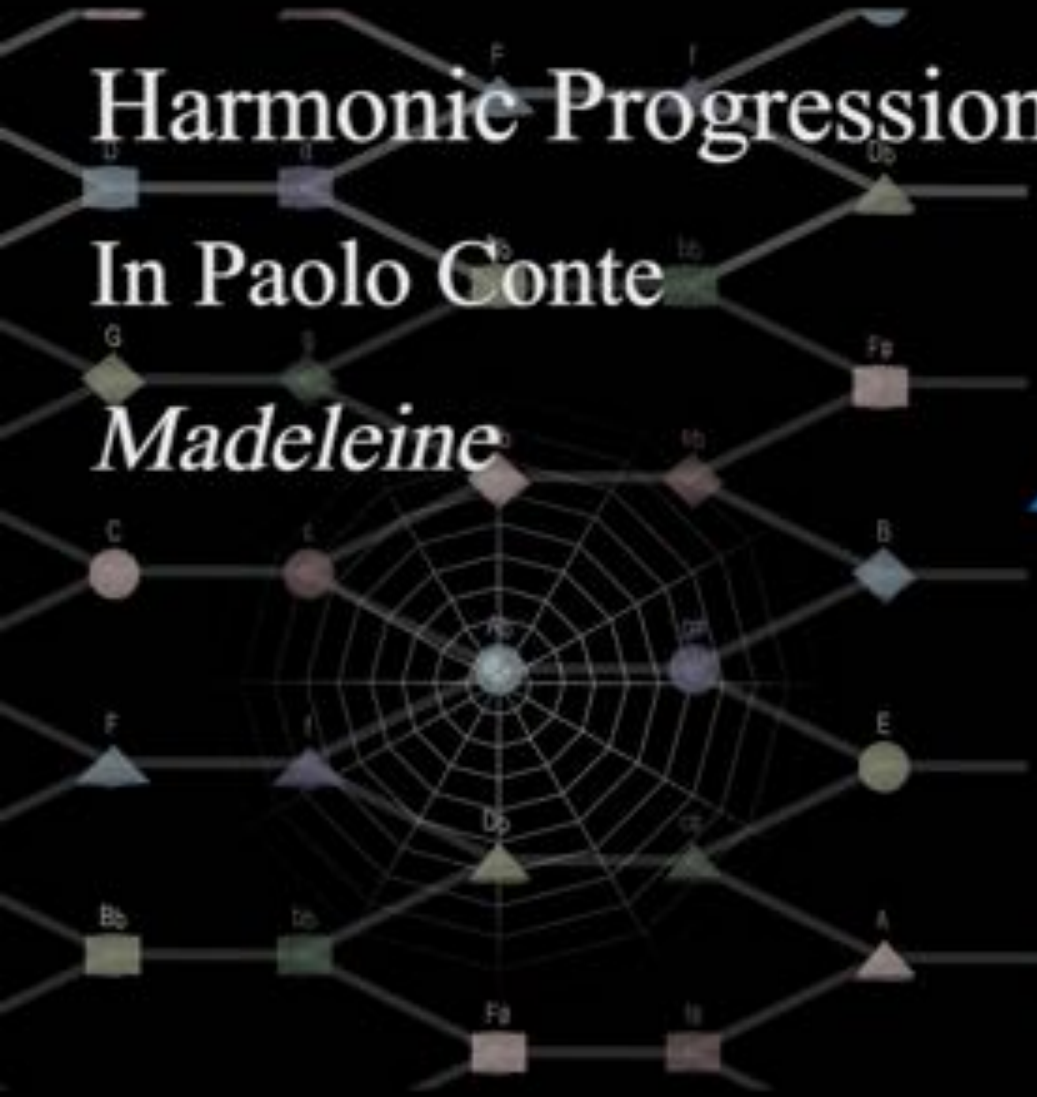


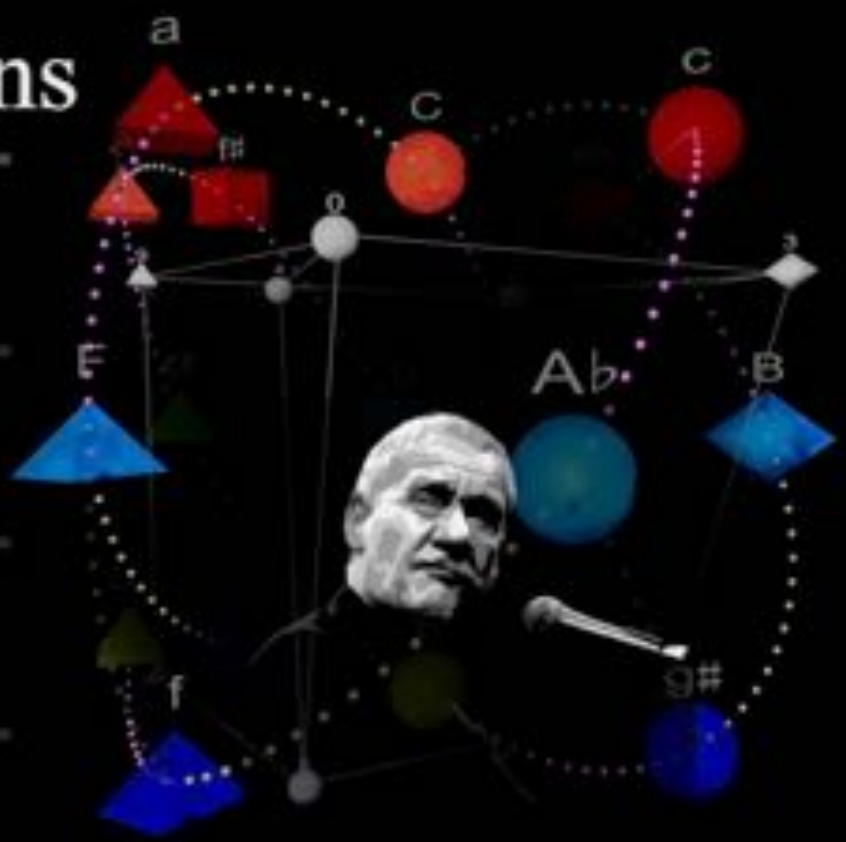
‘Recouvrement’ de l’espace des accords majeurs

Harmonic Progressions

In Paolo Conte

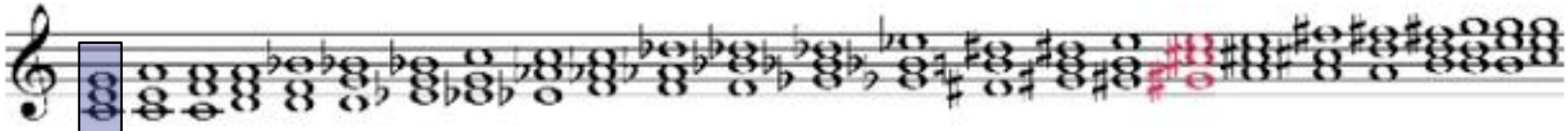
Madeleine

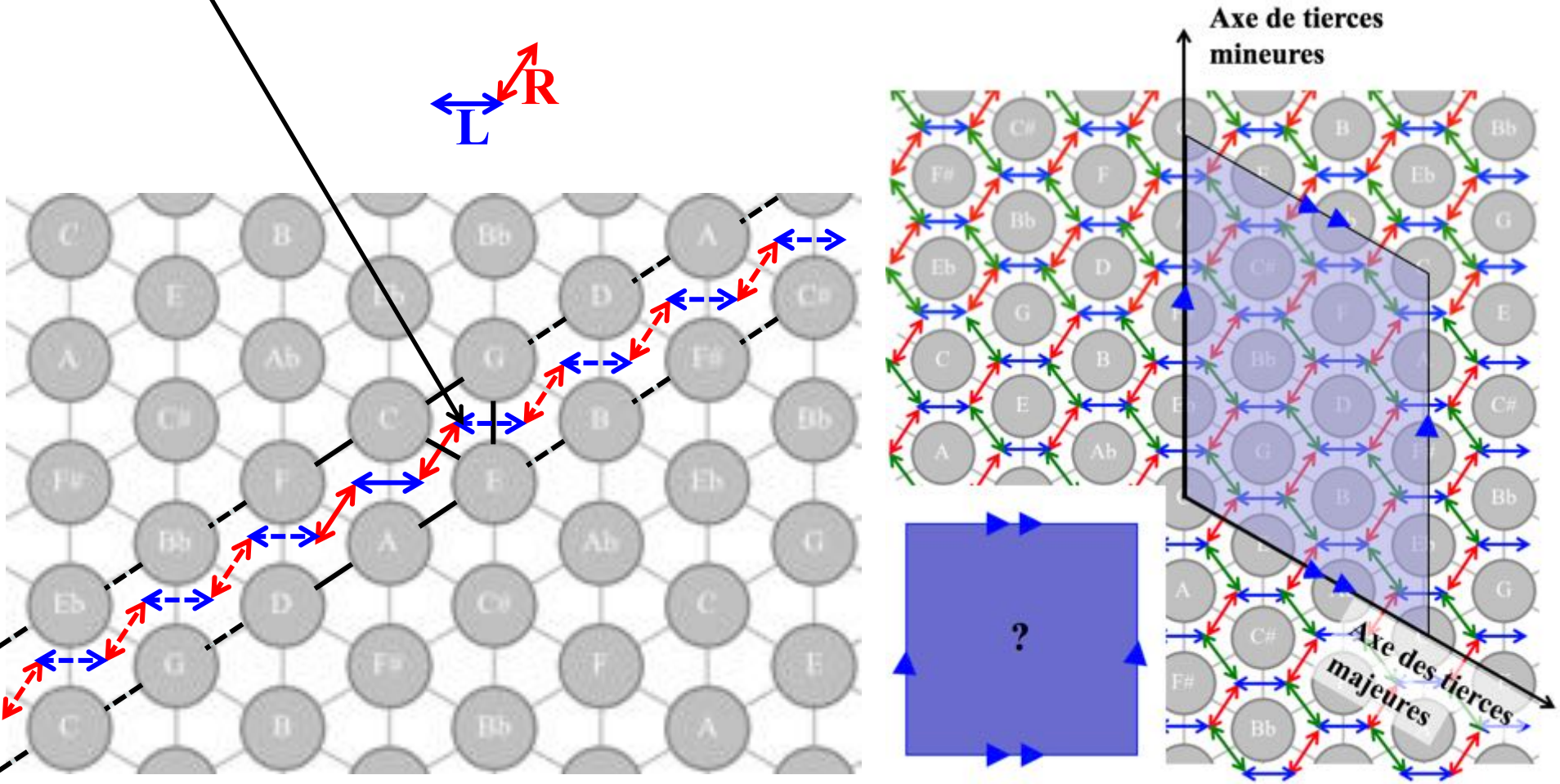




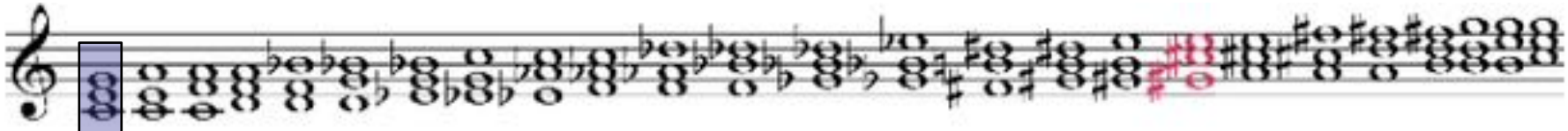
Supervision Moreno Andreatta
Modelisation Gilles Baroin 2016

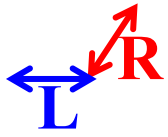
Progressions harmoniques comme trajectoires spatiales

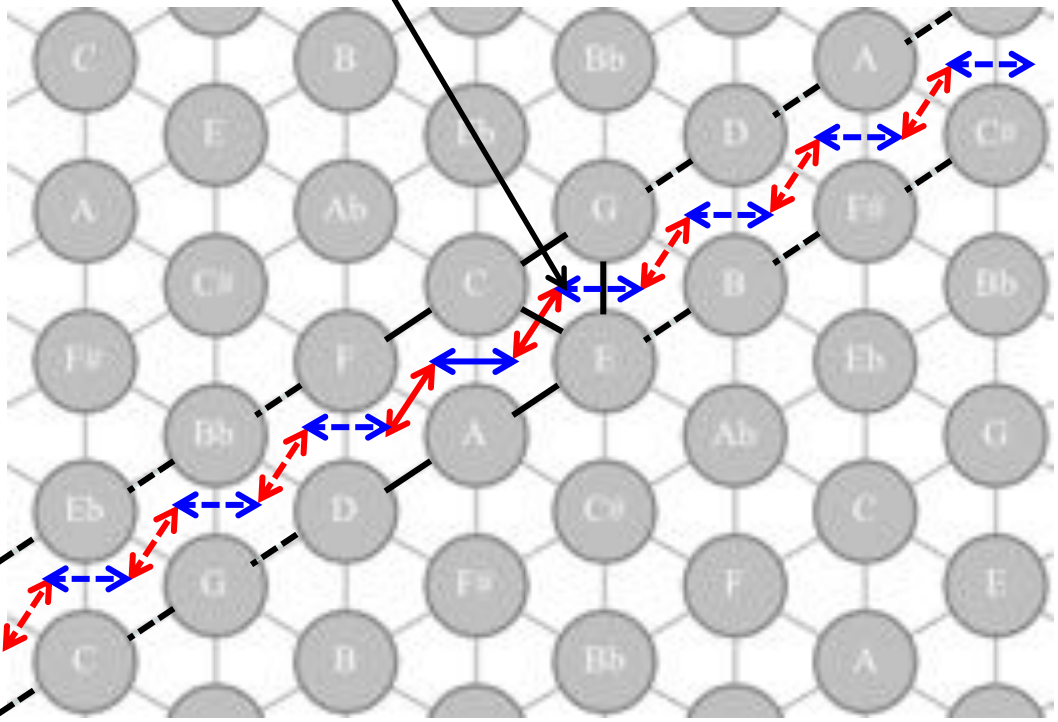


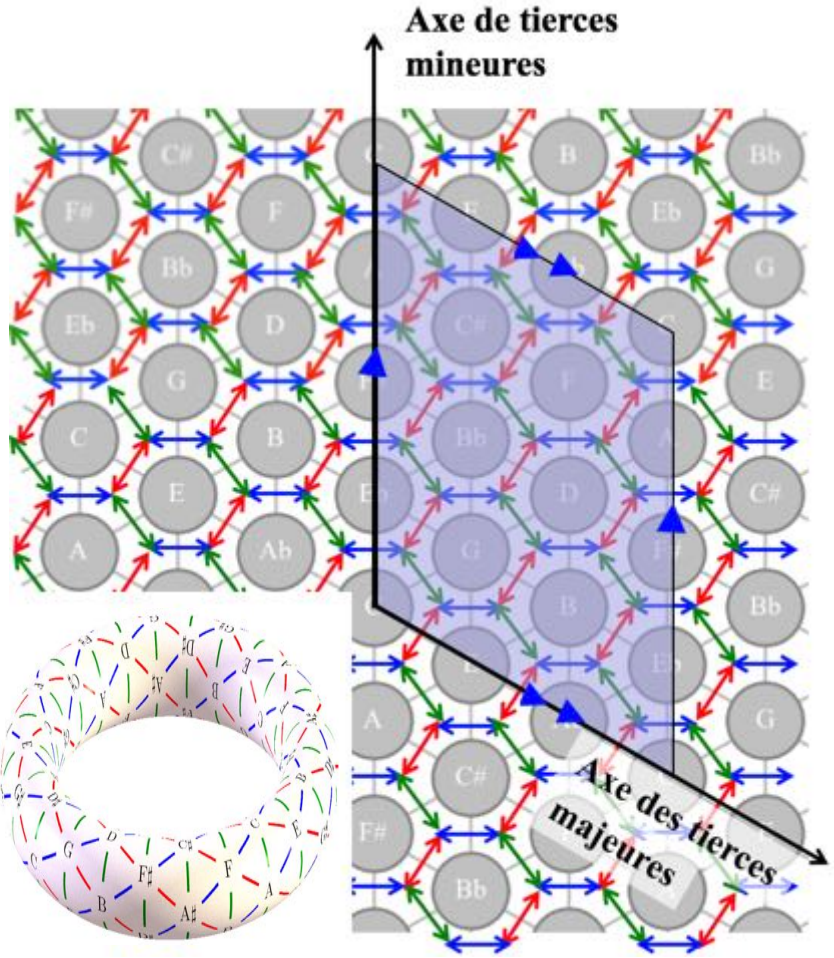


Progressions harmoniques comme trajectoires spatiales

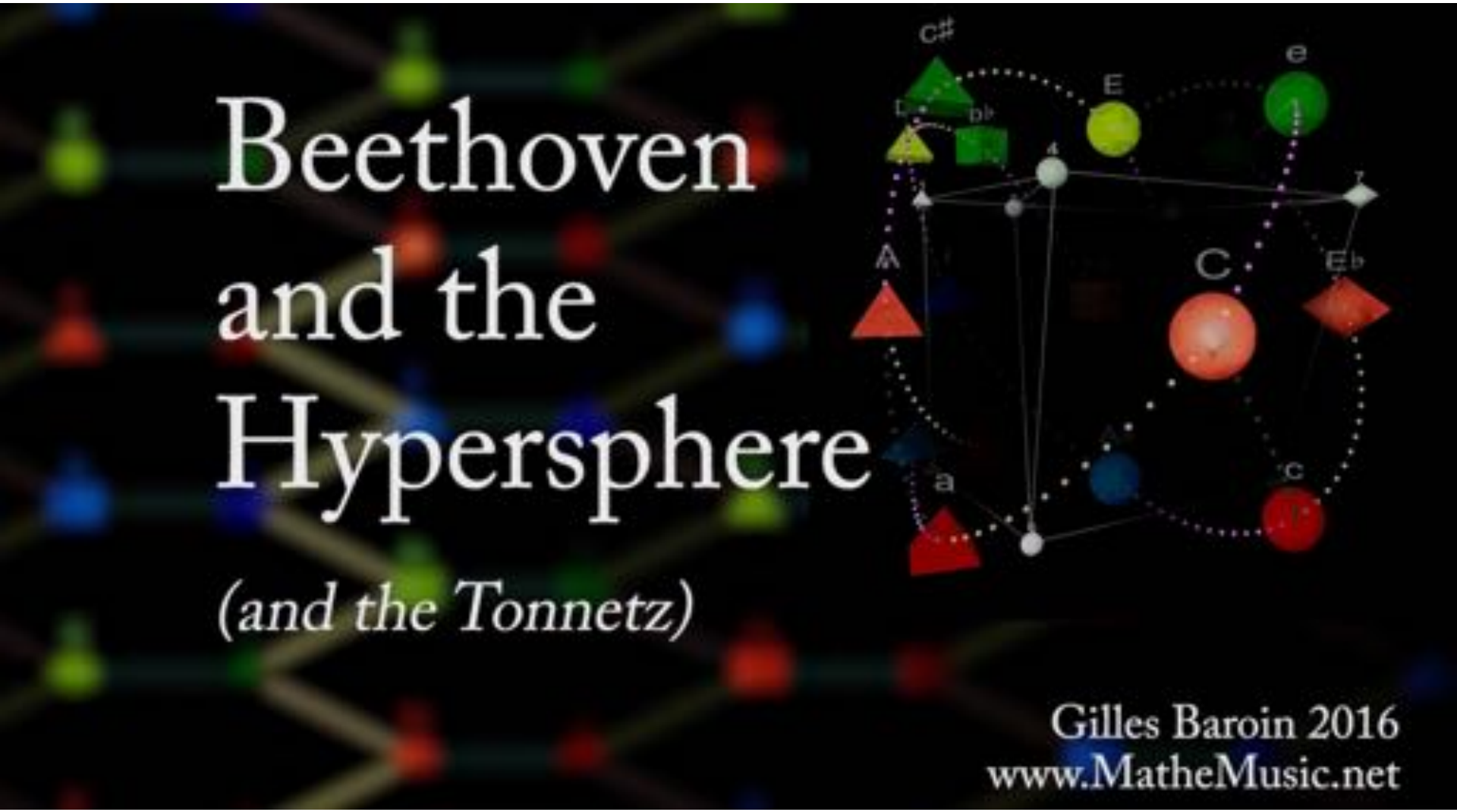






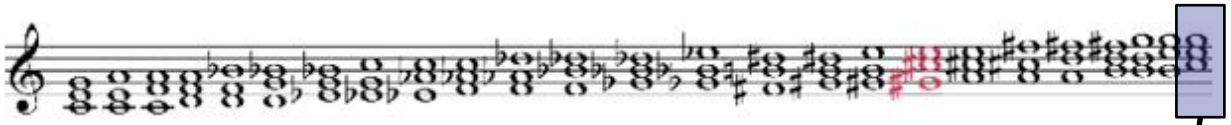


➔ Source : wikimedia.org



➔ **https://www.youtube.com/user/MatheMusic4D**

Beethoven de droite à gauche…



time

Le Blé en Herbe

(Polo/Moreno/Dieu)

Plonger comme un enfant, cheveux au vent
Sous l'océan du blé en herbe
Marée d'épis couleur d'amande
Qui tendent à caresser le ciel

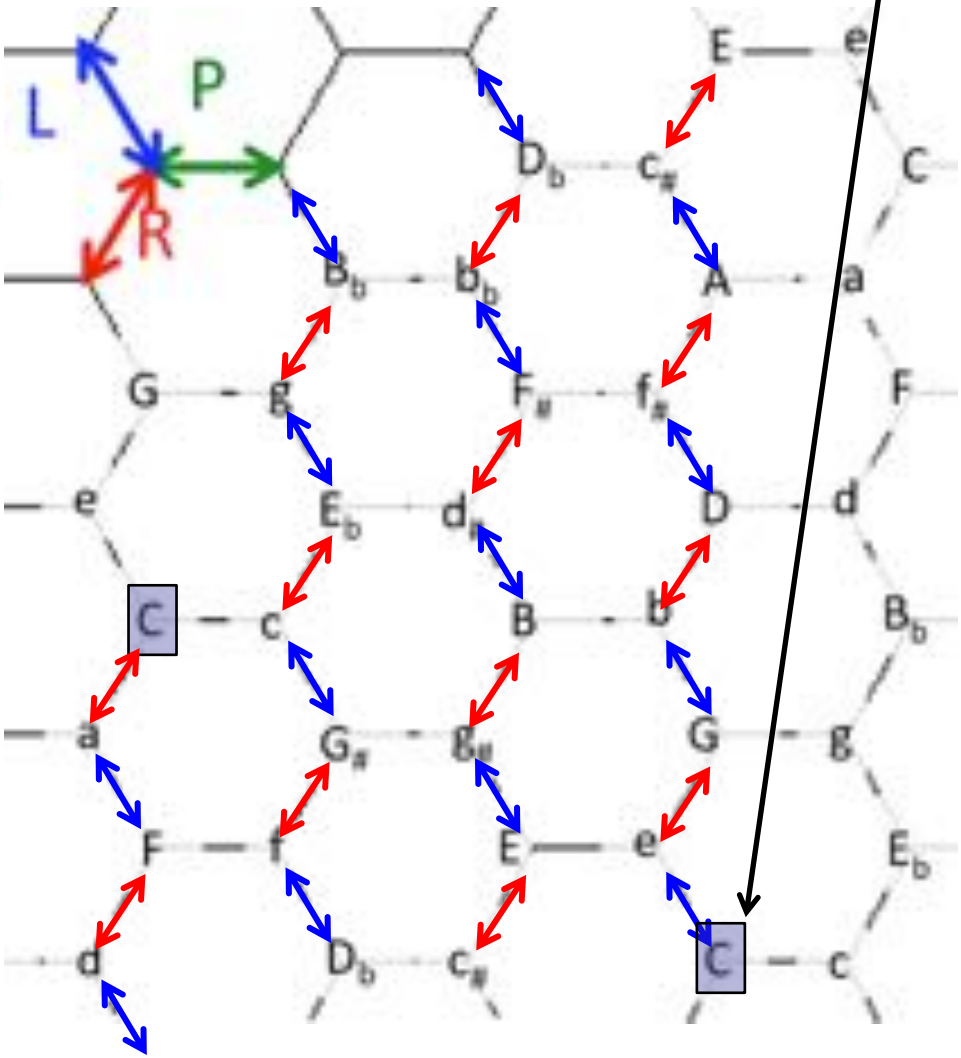
Algues tendres de mille plages
Frôlant le ventre des nuages
Cheveux de pluie, dos de poissons
Qui frissonnent à l'unisson

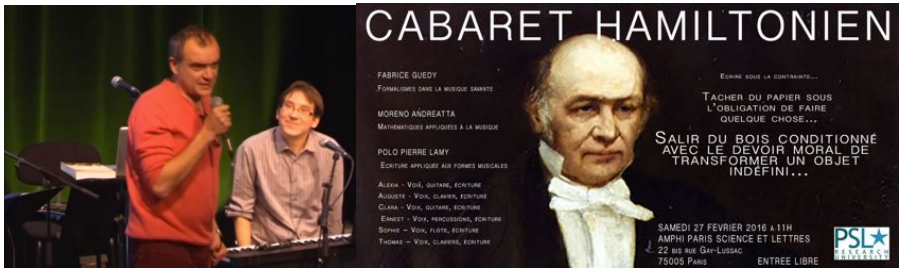
Suivre le bord des continents
Dans l'océan du blé en herbe
Pécher le corail du pavot
Dans le sang des coquelicots

Croiser matin dans l'herbe folle
Deux tourterelles qui s'envolent
Suivre les jeux des hirondelles
Sur le paysage éternel
Nager comme un enfant, cheveux au vent
Sous l'océan
Du blé en herbe

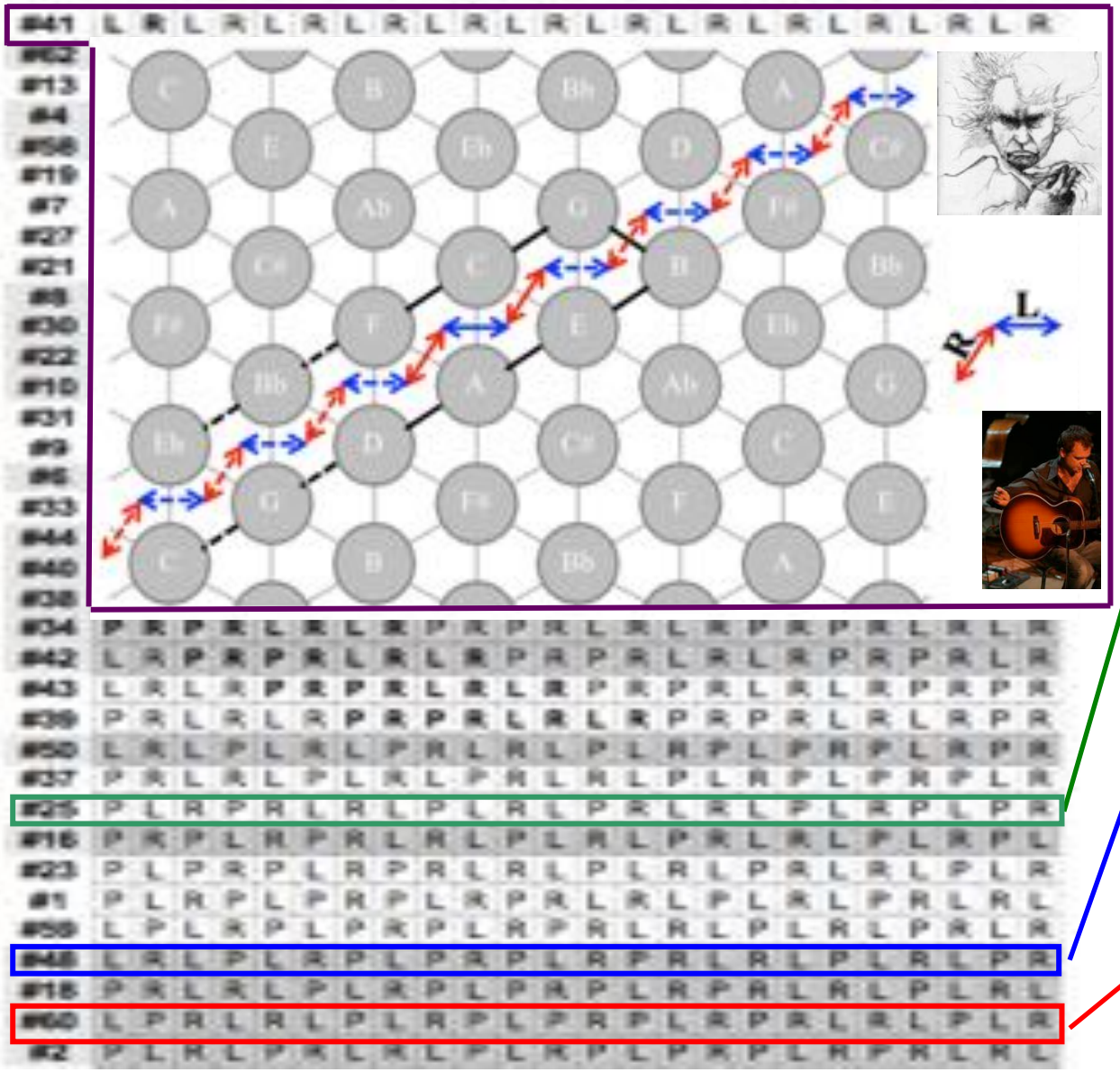
Marée de fruits au goût amer
Acide et salée comme la mer

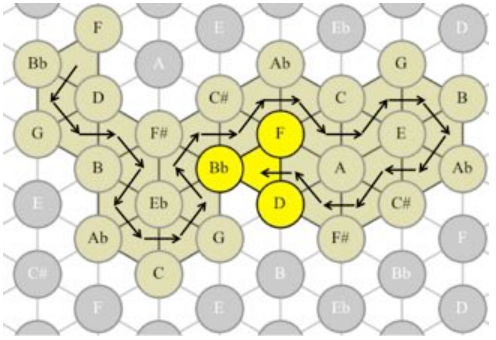
Vers l'îlôt d'un petit village
Vers un château d'eau sur la plage
Quand tout s'éteint avant l'orage
Quand se lève le vent du large
Sur le blé vert

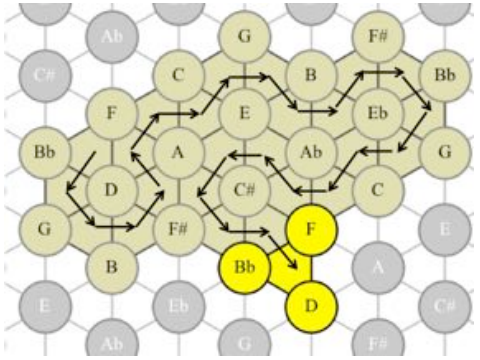


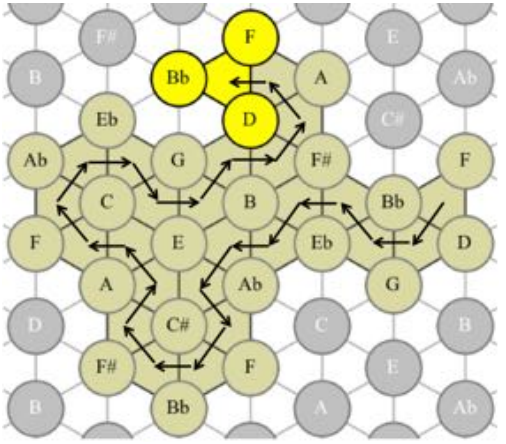


Cycles hamiltoniens sans périodicité interne









M. Andreatta, « Math'n pop : symétries et cycles hamiltoniens en chanson », *Tangente*

Aprile (d'après Gabriele D'Annunzio)

Tangente pour mieux comprendre le monde
L'aventure mathématique

Do←do$_m$←Sol#←fa$_m$←Fa←la$_m$←La←fa#$_m$←Fa#←sib$_m$←Do#←do#$_m$
↘ La ↗
mi$_m$→Sol→si$_m$→Ré→ré$_m$→Sib→sol$_m$→Mib→mib$_m$→Si→sol#$_m$→Mi

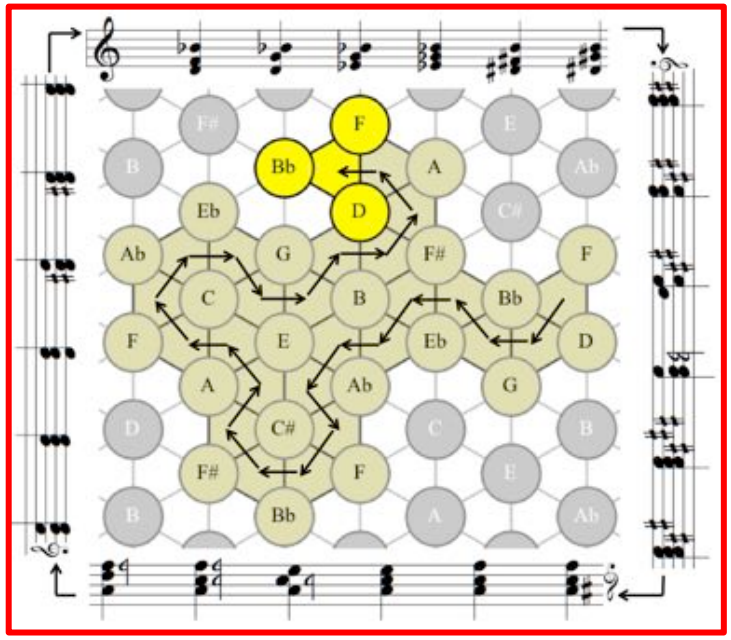
Do→mi$_m$→Mi→sol#$_m$→Si→ré#$_m$→Re#→do$_m$→Lab→fa$_m$→Do#→do#$_m$
↖ ↙
la$_m$←Fa←ré$_m$←Ré←si$_m$←Sol←sol$_m$←Sib←sib$_m$←Fa#←fa#$_m$←La

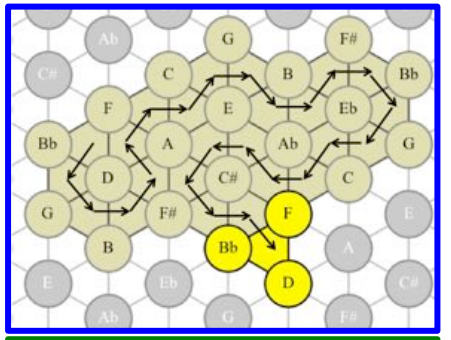
Mi←mi$_m$←Do←la$_m$←Fa←fa$_m$←Reb←sib$_m$←Fa#←mib$_m$←Mib←do$_m$
↘ La ↗
do#$_m$→La→fa#$_m$→Ré→ré$_m$→Sib→sol$_m$→Sol→si$_m$→Si→sol#$_m$→Sol#

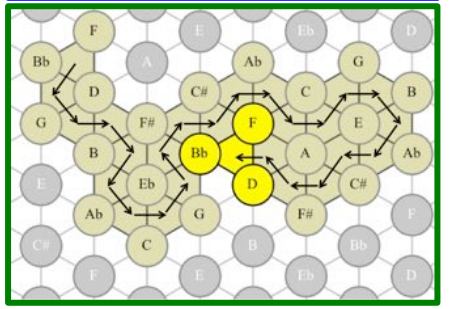
La fenêtre est entr'ouverte, sur le jardin.
Une heure passe, lente, somnolente.
Et elle, d'abord attentive, finit par s'endormir.
À cette voix qui là-bas se lamente,
Qui se lamente au fond de ce jardin.

Ce n'est qu'une voix d'eau sur la pierre,
Et combien de fois, combien de fois entendue !
Cet amour et cette heure s'abîment dans cette vie
Comme s'abîment dans l'onde sans fin
Le cadavre et la pierre liés ensemble.

Elle détend son angoisse dans le sommeil.
Mais l'angoisse est forte, et le sommeil est si léger !
(La lumière d'avril ressemble presque à une neige
qui serait tiède.)
Et certes elle doit souffrir,
Vaguement souffrir, aussi dans le sommeil.

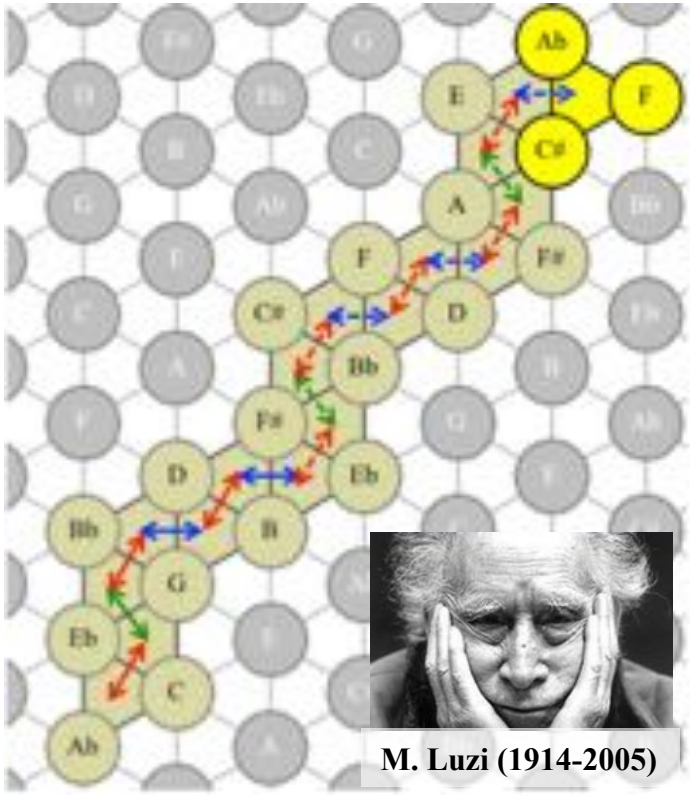






➔ **https://www.youtube.com/user/MatheMusic4D**

La sera non è più la tua canzone : chanson hamiltonienne ‘redondante’



M. Luzi (1914-2005)

La sera non è più la tua canzone,
è questa roccia d’ombra traforata
dai lumi e dalle voci senza fine,
la quiete d’una cosa già pensata.

Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d’una cosa conosciuta,
per nome hai una parola ch’è passata
nell’intimo del cuore e s’è perduta.

Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell’essere, (compiuta).

L’aria tace ed il tempo dietro a te
si leva come un’arida montagna
dove vaga il tuo spirito e si perde,
un vento raro scivola e ristagna.

Le soir n’est plus ta chanson,
c’est ce rochet d’ombre transpercé
par les lumières et les voix sans fin,
la paix d’une chose déjà pensée.

Ah, cette lumière vive et claire vient
uniquement de toi, tu es si proche
du vrai d’une chose connue,
tu as pour nom une parole qui est passée
dans l’intimité du cœur où elle s’est
perdue .

Tombé est plus qu’un signe de la vie,
tu te reposes, du voyage tu es revenue
à l’intérieur de toi même, tu es
descendue dans cette
pure substance qui est si tienne,
si éloignée
dans le silence de l’être, achevée.

L’air se tait et le temps derrière toi
se lève tel une montagne aride
où plane ton esprit et se perd,
un vent rare glisse et stagne.

(tr. Antonia Soulez, philosophe et poète)

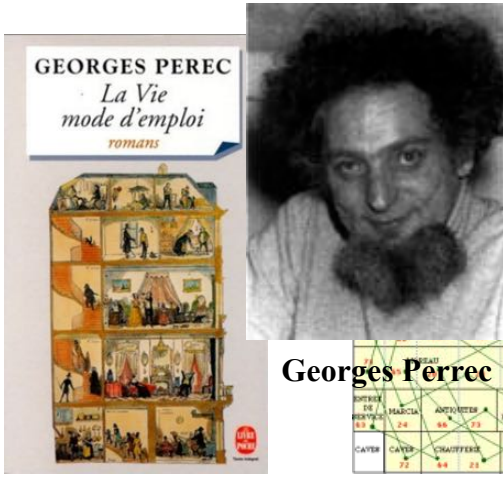
Musique : M. Andreatta
Arrangements et mixage : M. Bergomi & S. Geravini (*Perfect Music Production*)
Mastering : A. Cutolo (Massive Arts Studio, Milan)

min. 1’19”

➔ **https://www.youtube.com/user/MatheMusic4D**

Les contraintes dans l'art : l'OuLiPo (Ouvroir de Littérature Potentielle)

Cent mille milliards de poèmes, 1961



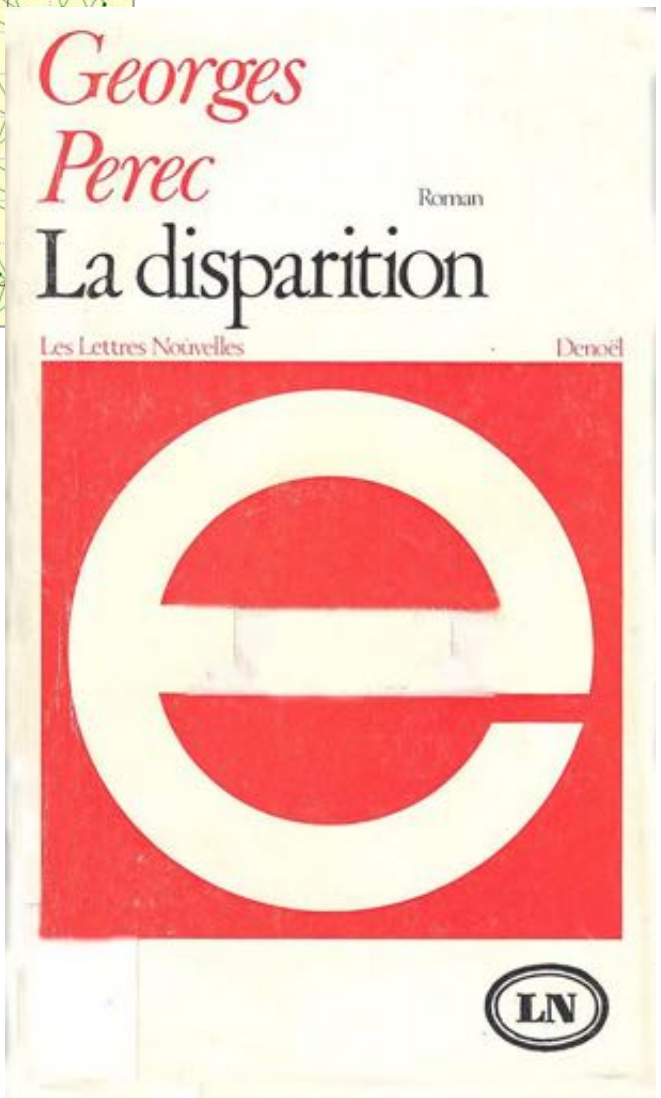
Georges Perrec

La vie mode d'emploi,

Raymond Queneau

Italo Calvino

Le Château des destins croisés, 1969

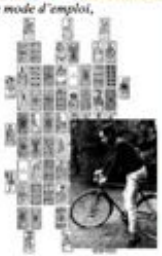


De l'OuLiPo à l'OuMuPo (ouvroir de musique potentielle)

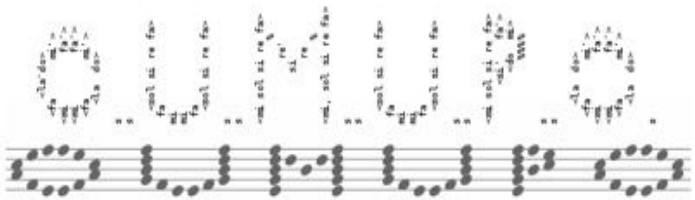
Cent mille milliards de poèmes, 1961

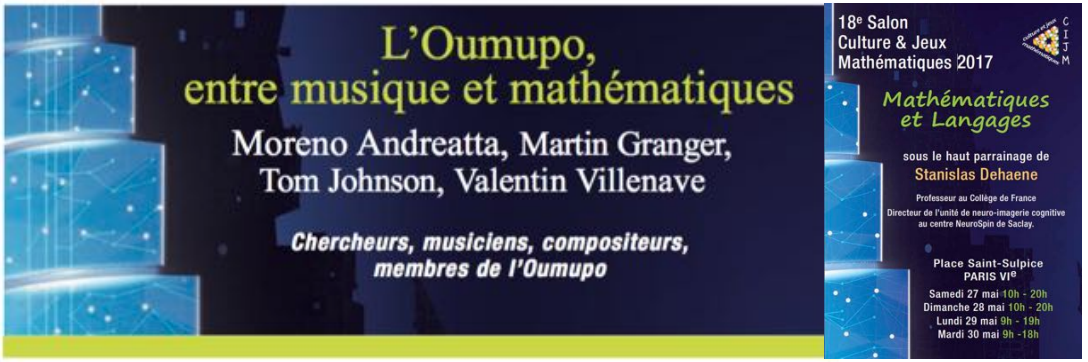
La vie mode d'emploi,

Raymond Queneau



Italo Calvino
Le Château des destins croisés, 1969





Valentin Villenave

Mike Solomon

Jean-François Piette

Martin Granger

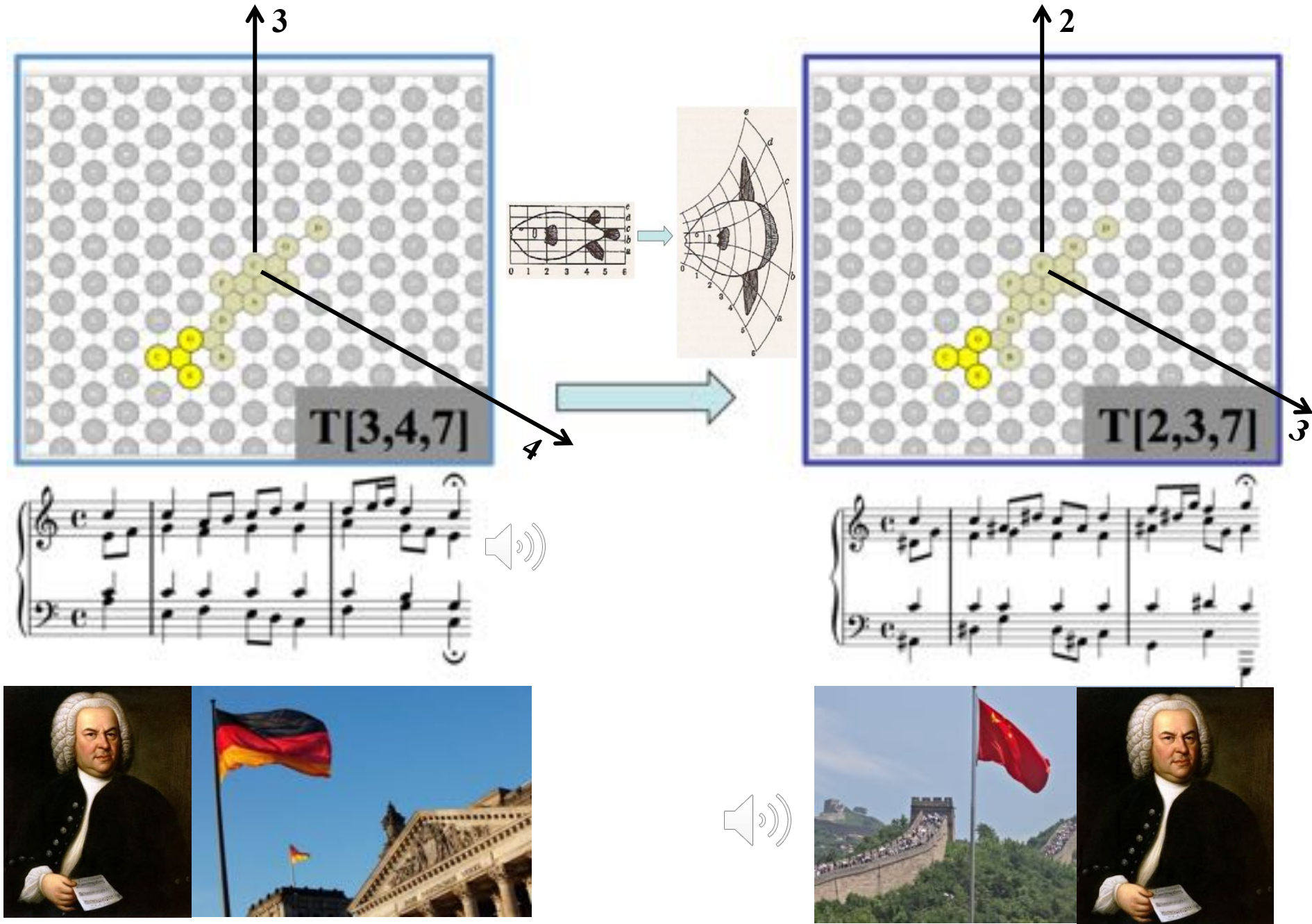
Joseph Boisseau

Moreno Andreatta

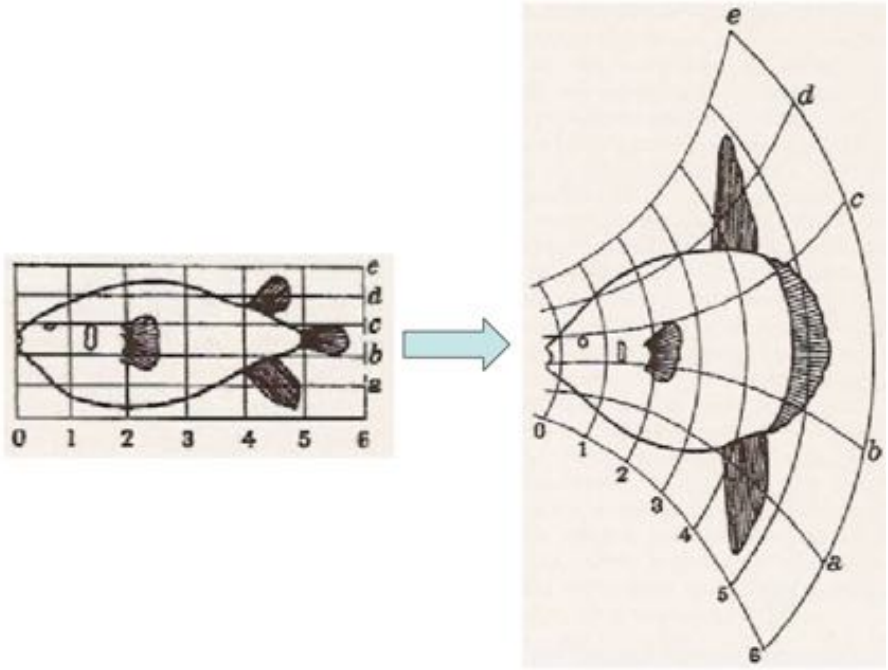
Tom Johnson

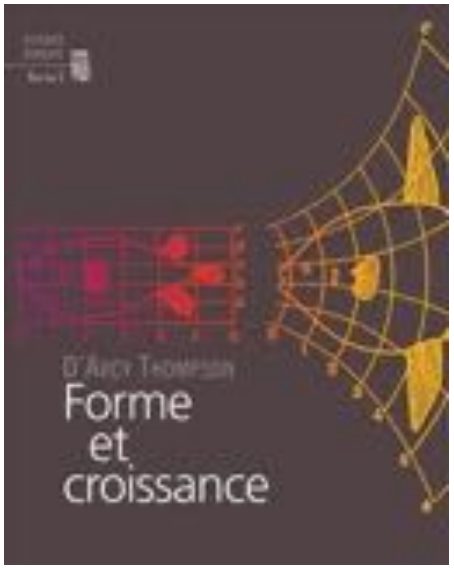
http://oumupo.org/

Que-ce que le style en musique ?



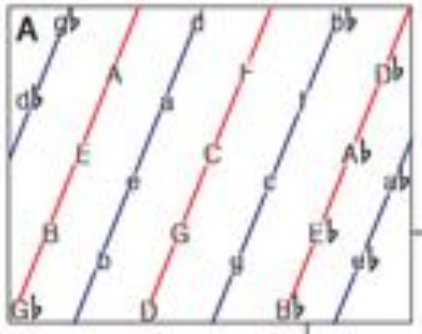
Transformations géométriques et transformations musicales





« [La notion de **transformation**] me vient d'un ouvrage qui a joué pour moi un rôle décisif et que j'ai lu pendant la guerre aux États Unis: *On Growth and Form*, en deux volumes, de D'Arcy Wentworth Thompson, paru pour la première fois en 1917. L'auteur, naturaliste écossais, (...) interprétait comme des transformations les différences visibles entre les espèces ou organes animaux ou végétaux au sein d'un même genre. Ce fut une illumination, d'autant que j'allais vite m'apercevoir que cette façon de voir s'inscrivait dans une longue tradition: derrière Thompson, il y avait la botanique de Goethe, et derrière Goethe, Albert Dürer avec son Traité de la proportion du corps humain » (Lévi-Strauss et Eribon, 1988).

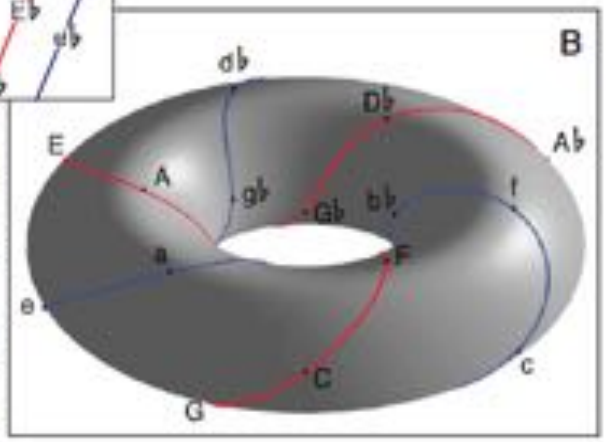
Tonnetz et neurosciences cognitives



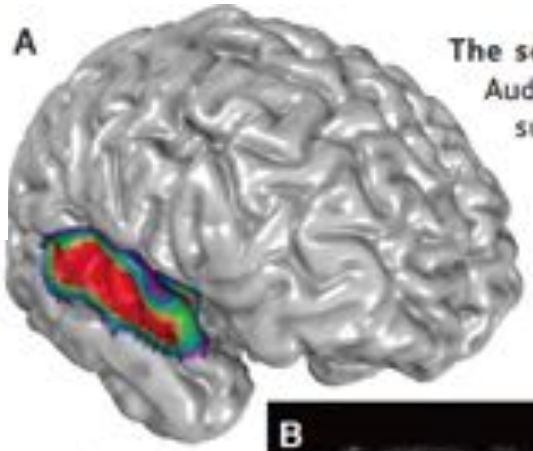
PERSPECTIVES: NEUROSCIENCE

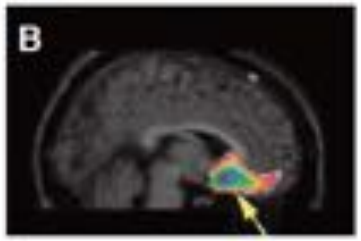
Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl

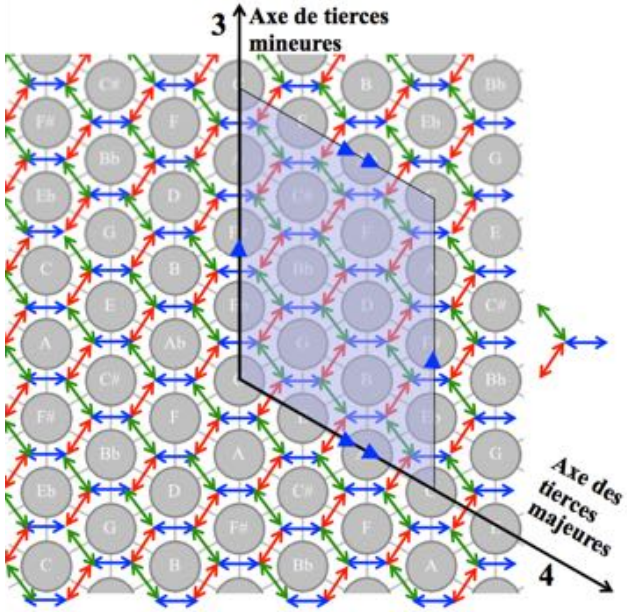


Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (B) Musical keys as points on the surface of a torus.





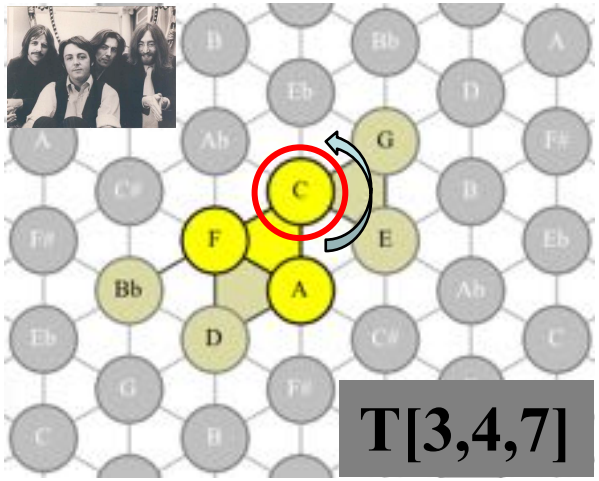
The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

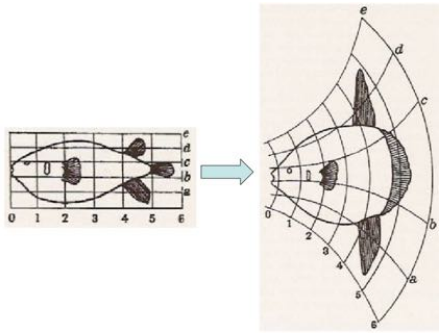


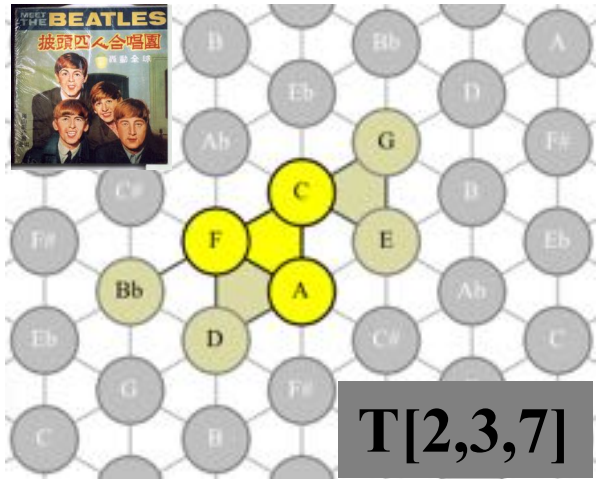
Acotto E. et M. Andreatta (2012), « Between Mind and Mathematics. Different Kinds of Computational Representations of Music », *Mathematics and Social Sciences*, n° 199, 2012(3), p. 9-26.

EMPG European Mathematical Psychology Group

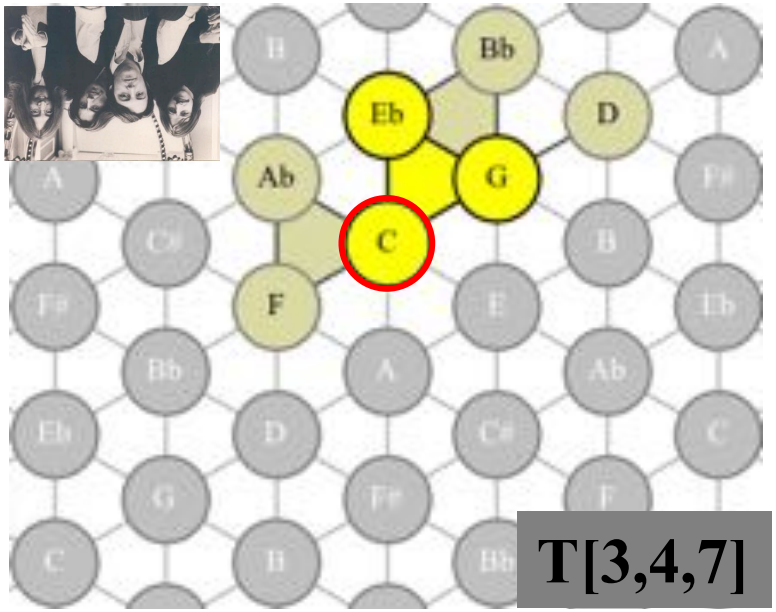
Transformations géométriques sur les Beatles



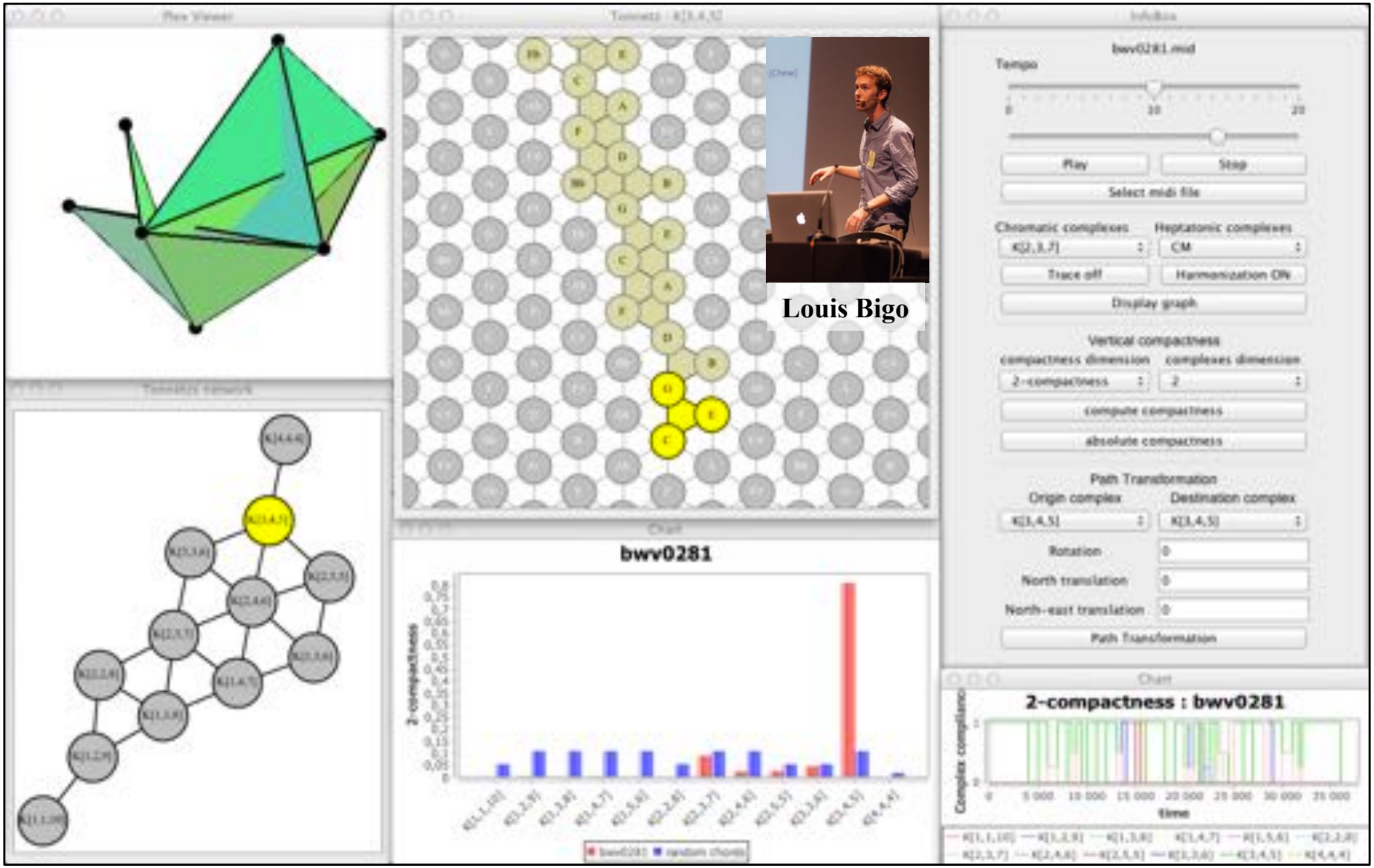




Rotation
(autour du *do*)



Composer et analyser la musique avec *Hexachord*



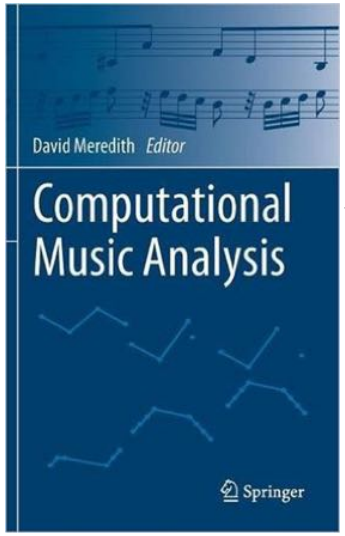
Louis Bigo

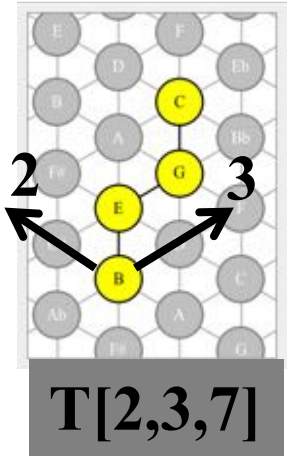
➔`http://www.lacl.fr/~lbigo/hexachord`

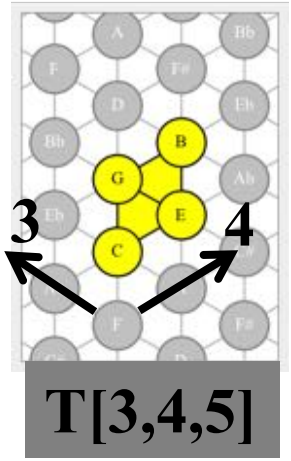
Le caractère spatial de la « logique musicale »

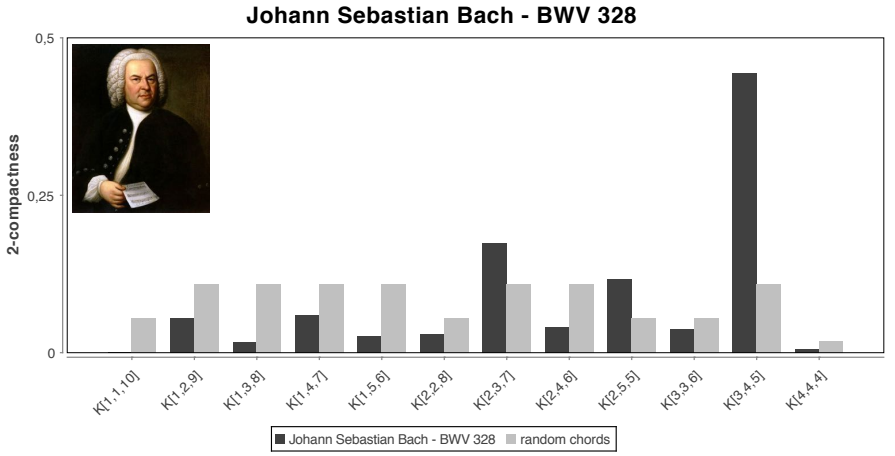
Louis Bigo

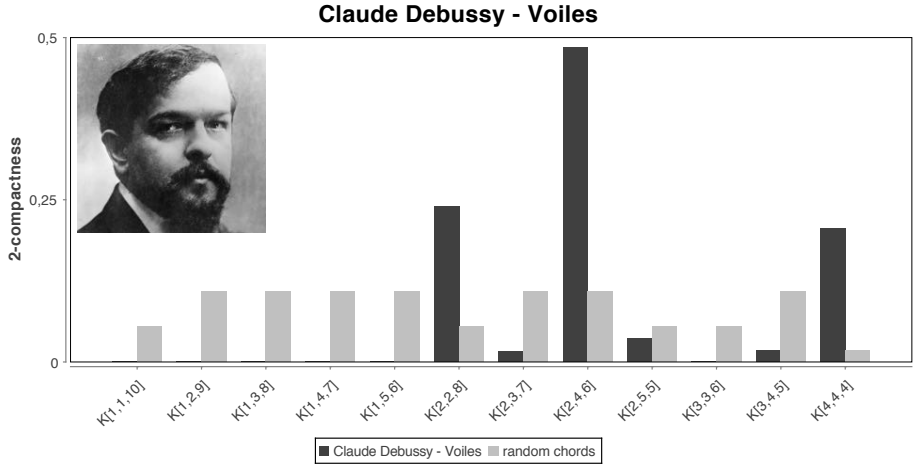
Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015

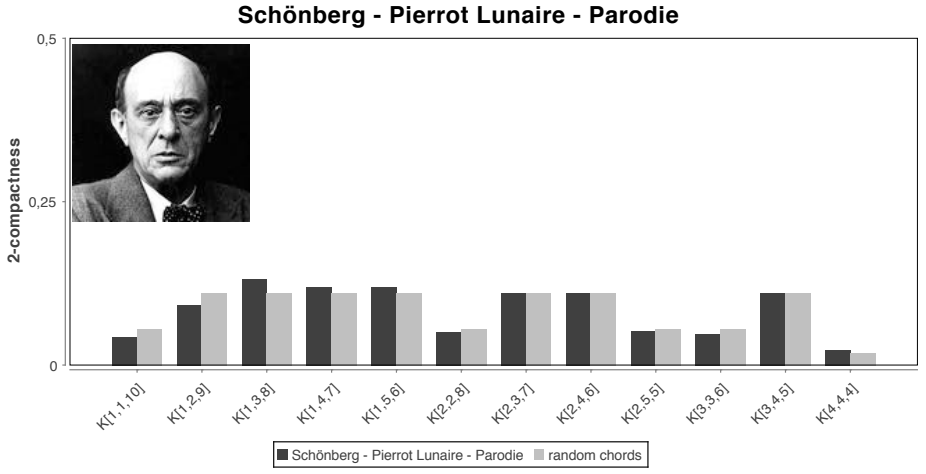













Merci de votre attention!