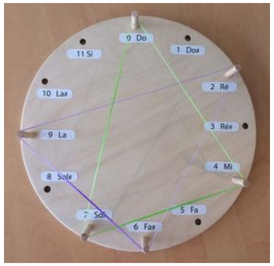


A focus on some theoretical problems in contemporary ‘mathemusical’ research

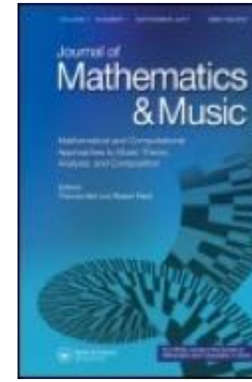
Moreno Andreatta

CNRS / IRMA / Université de Strasbourg
CNRS / IRCAM / Sorbonne Université
http://repmus.ircam.fr/moreno/smir

Maths & Music in Academic Research

Official Journal and MC code (00A65: Mathematics and Music)
https://www.tandfonline.com/journals/tmam20

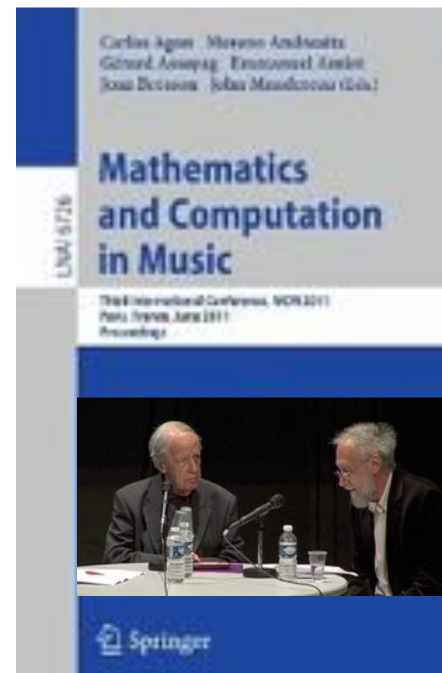
- *Journal of Mathematics and Music*, Taylor & Francis
  - Editors-in-Chief: E. Amiot, J. Yust
  - Associate Editor: D. Conklin
  - Review Editor: M. Andreatta



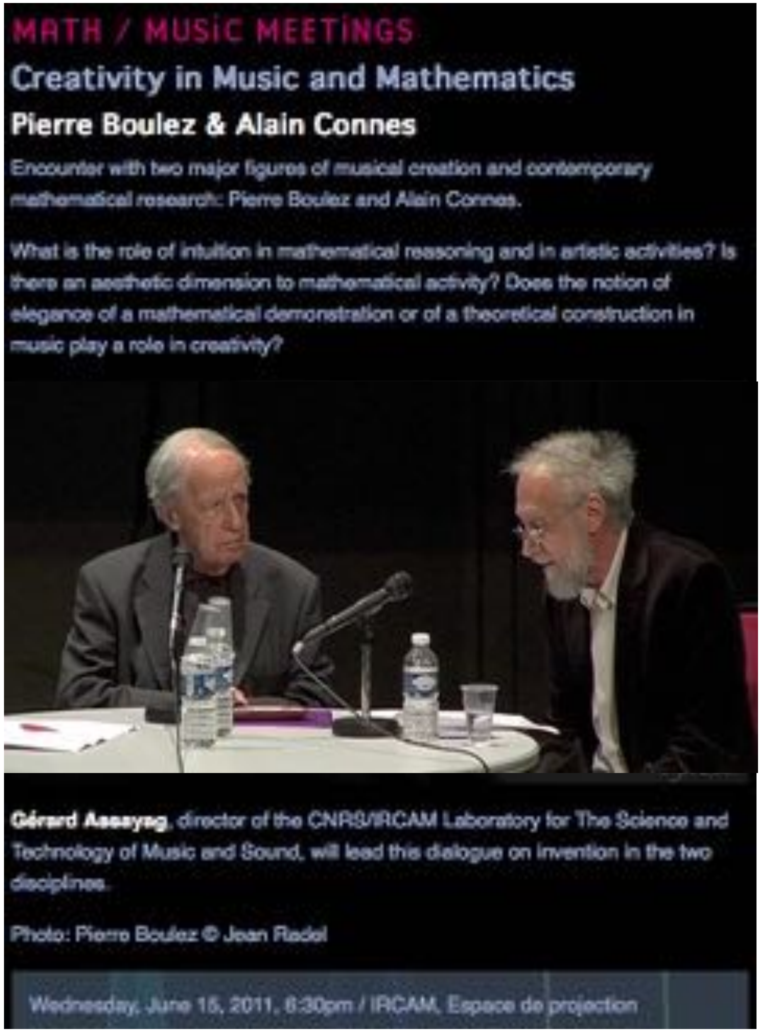
International Society for Mathematics and Computation in Music (SMCM) - http://www.smcm-net.info/

SOCIETY FOR MATHEMATICS AND COMPUTATION IN MUSIC

- 2007 Technische Universität (Berlin, Allemagne)
- 2009 Yale University (New Haven, USA)
- 2011 IRCAM (Paris, France)
- 2013 McGill University (Canada)
- 2015 Queen Mary University (London, United Kingdom)
- 2017 UNAM (Mexico City)
- 2019 Universidad Complutense de Madrid (Spain)
- 2022 Georgia State University (Atlanta, USA)
- 2024 University of Coimbra (Portugal)



The interplay between algebra and geometry in music



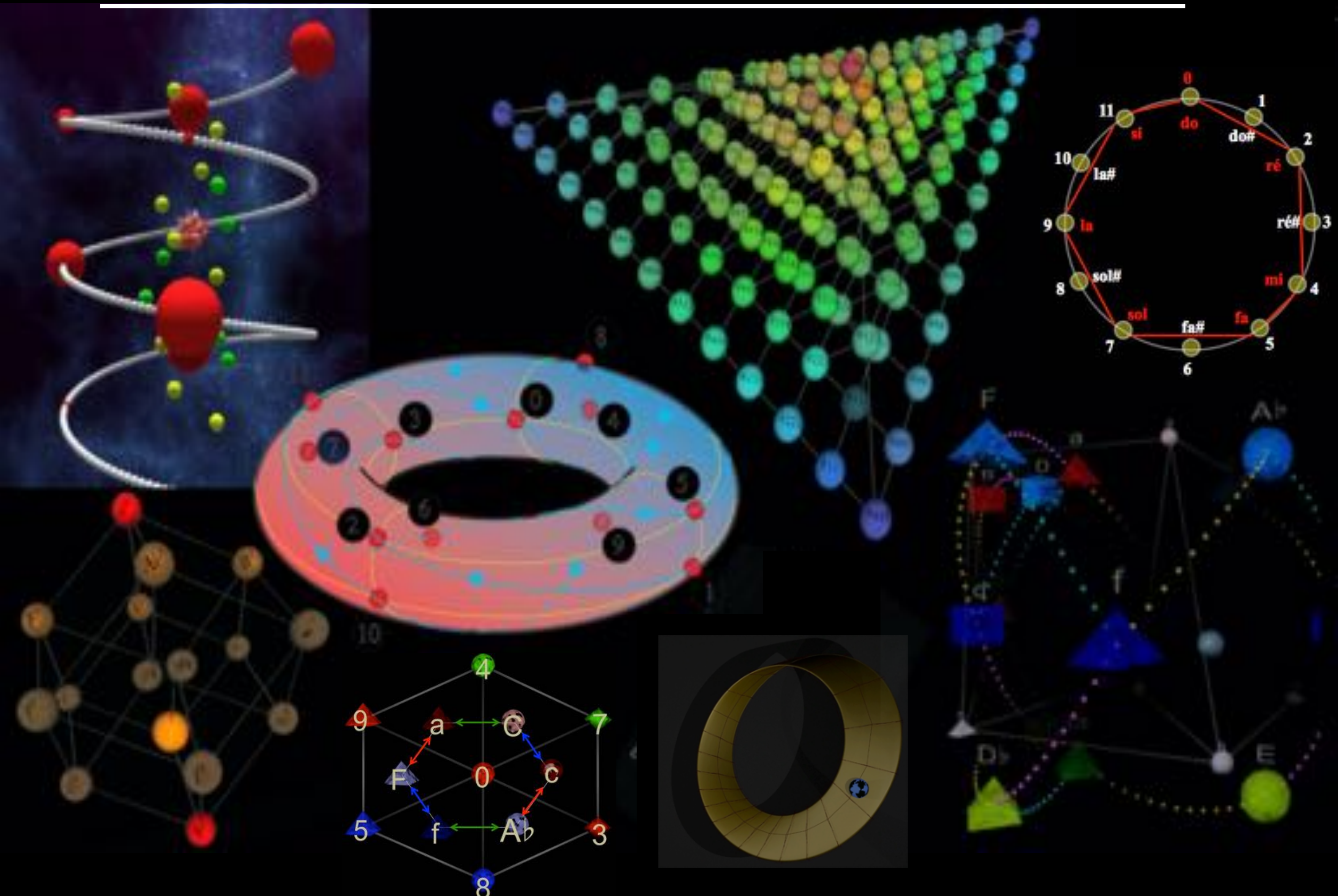
➔ `http://agora2011.ircam.fr`

"Concerning **music**, it takes place in **time**, like **algebra**. In **mathematics**, there is this fundamental duality between, on the one hand, **geometry** – which corresponds to the visual arts, an immediate intuition – and on the other hand **algebra**. This is not visual, it has a temporality. This fits in time, it is a computation, something that is very close to the language, and which has its diabolical precision. [...] **And one only perceives the development of algebra through music**" (A. Connes).

Société Mathématique de France

SMF

The galaxy of mathematical models at the service of music



Some examples of PhD in maths / music / computer science

- **Victoria Callet**, *Topological modeling of Musical Structures and Processes*, PhD in **maths**, Université de Strasbourg (supervised by P. Guillot and M. Andreatta, IRMA)
- **Riccardo Giblas,** *Combinatorics and music,* PhD in **maths** in cotutelle agreement, **University of Padova** (L. Fiorot & A. Tonolo) / **Université de Strasbourg** (M. Andreatta).
- **Paul Lascabettes,** *Mathematical Models for the Discovery of Musical Patterns, Structures and for Performances Analysis,* PhD in **maths** at **Sorbonne University** (supervised by I. Bloch and E. Chew, in collaboration with M. Andreatta)
- **Gonzalo Romero**, *Mathematical Morphology for the Analysis and Generation of Time-Frequency Representations of Music*, PhD in **computer science** at **Sorbonne University** (supervised by I. Bloch and C. Agon, in collaboration with M. Andreatta)
- **Matias Fernandez Rosales,** *Mathematical models in Computer-assisted composition,* PhD in **composition and research**, **HEAR/University of Strasbourg** (supervision: D. D'Adamo, X. Hascher, M. Andreatta)
- **Greta Lanzarotto**, *Fuglede Spectral Conjecture, Musical Tilings and Homometry*, PhD in **maths** in cotutelle agreement, **University of Pavia** (L. Pernazza) / **Université de Strasbourg** (M. Andreatta).
- **Sonia Cannas**, *Représentations géométriques et formalisations algébriques en musicologie computationnelle*, PhD in **maths** in cotutelle agreement, **University of Pavia** (L. Pernazza) / **Université de Strasbourg** (A. Papadopoulos & M. Andreatta), 2019.
- **Grégoire Genuys**, *Théorie de l'homométrie et musique*, PhD in **maths**, **Sorbonne University** / IRCAM (cosupervised with J.-P. Allouche), 2017.
- **Hélianthe Caure**, *Pavages en musique et conjectures ouvertes en mathématiques*, PhD in **computer science**, **Sorbonne University** (cosupervised with J.-P. Allouche), 2016.
- **Mattia Bergomi**, *Dynamical and topological tools for (modern) music analysis,* PhD in **maths** in a cotutelle agreement **Sorbonne University** / **University of Milan** (with G. Haus, 2015).
- **Charles De Paiva**, *Systèmes complexes et informatique musicale*, PhD in **musicology** in a cotutelle agreement, **Sorbonne University** / **UNICAMP**, Brésil, 2016.
- **Louis Bigo**, *Représentation symboliques musicales et calcul spatial*, PhD in **computer science**, **University of Paris Est Créteil** / **IRCAM**, 2013 (with O. Michel and A. Spicher)
- **Emmanuel Amiot**, *Modèles algébriques et algorithmiques pour la formalisation mathématique de structures musicales*, PhD in **computer science**, **Sorbonne University** / **IRCAM**, 2010 (cosupervised with C. Agon)
- **Yun-Kang Ahn**, *L'analyse musicale computationnelle*, PhD in **computer science**, **Sorbonne University** / **IRCAM**, 2009 (cosupervised with Carlos Agon)

Université de Strasbourg

Università degli Studi di Padova

Sorbonne Université

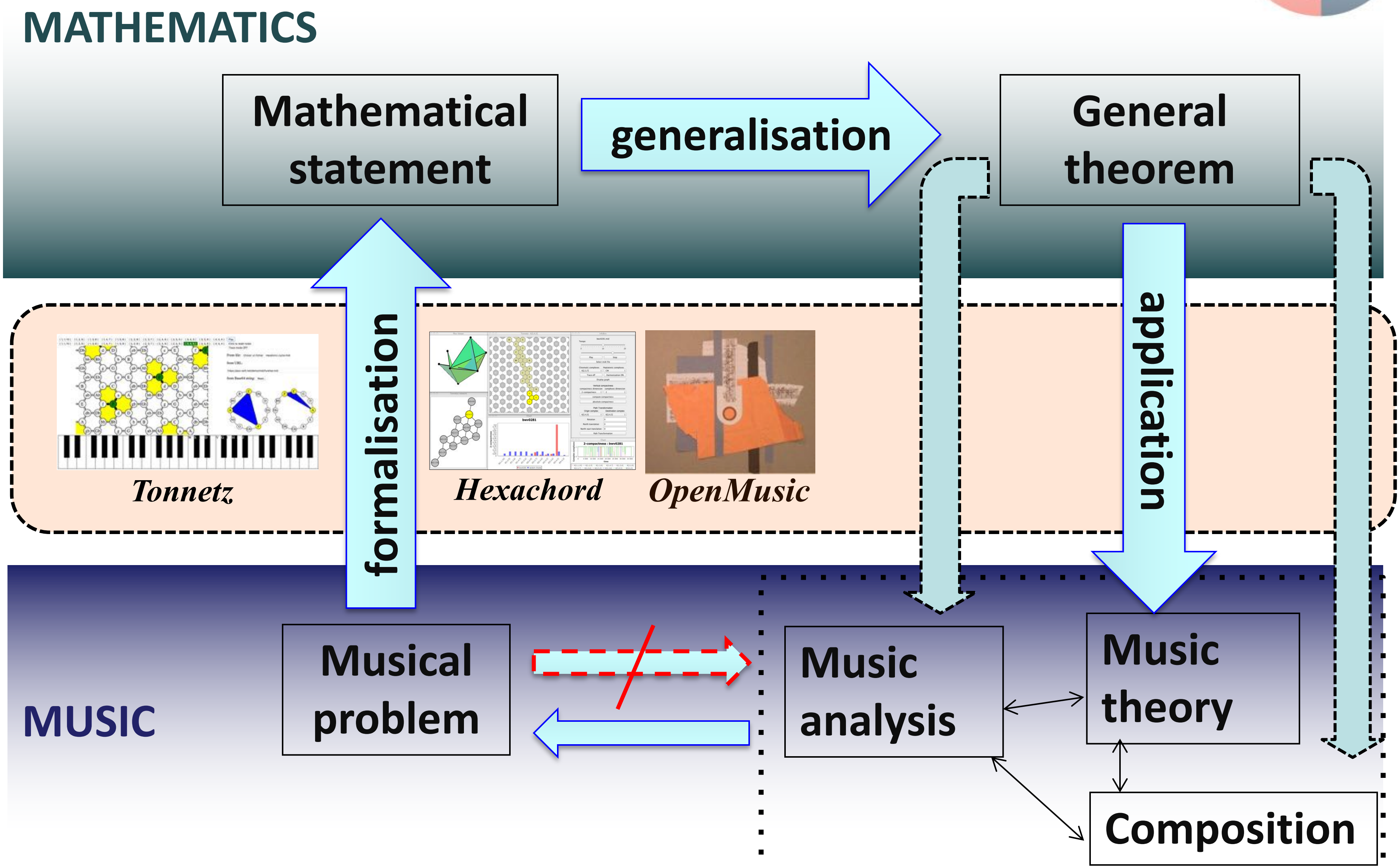
HEAR Haute école des arts du Rhin

UNICAMP

UPEC Université Paris-Est Créteil Val de Marne

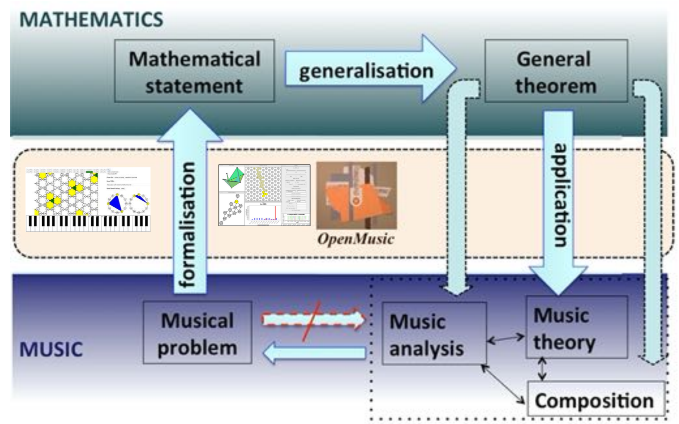
ircam Centre Pompidou

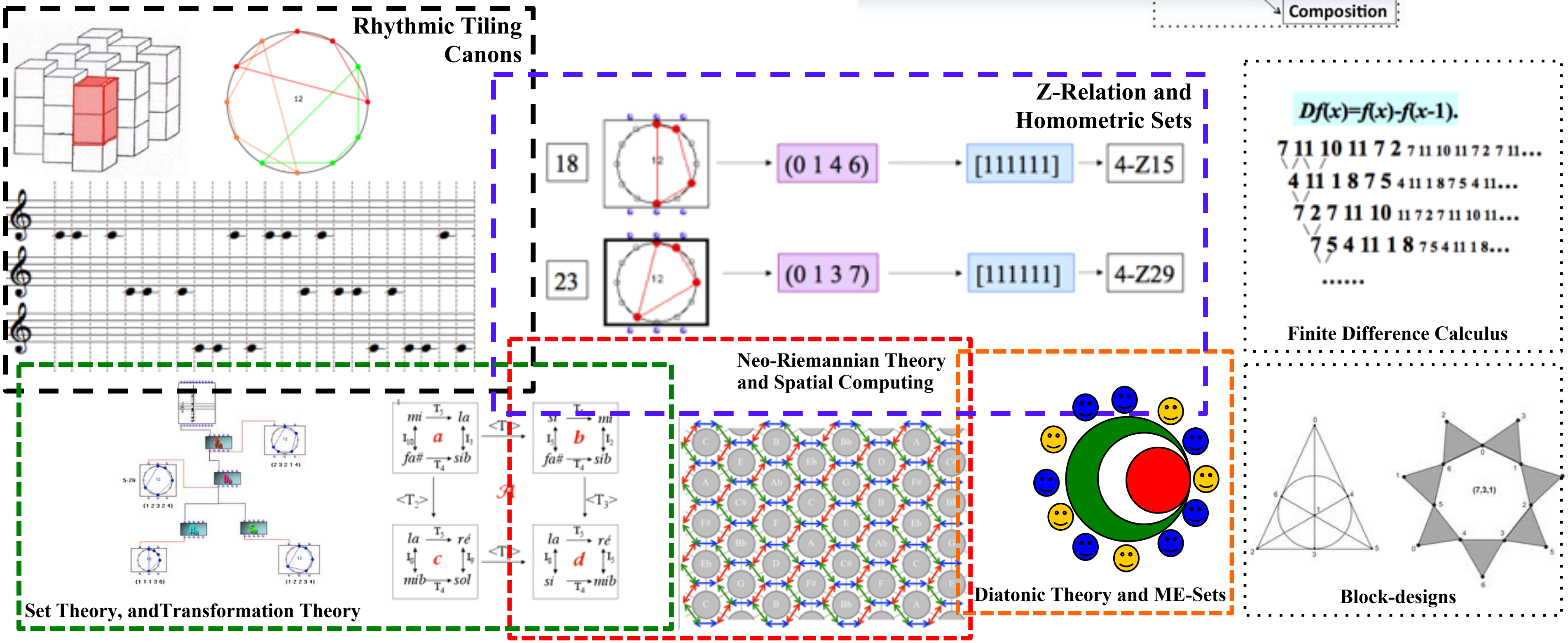
The double movement of a ‘mathemusical’ activity



Some musically-driven mathematical problems

- Tiling Rhythmic Canons
- *Z* relation and homometry
- Transformational Theory
- Music Analysis, Spatial Computing and FCA
- Diatonic Theory and Maximally-Even Sets
- Periodic sequences and Finite Difference Calculus
- Block-designs in composition





➔ M. Andreatta : *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

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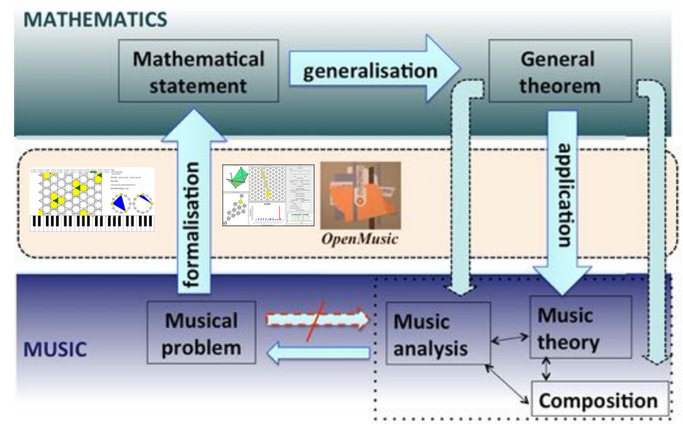
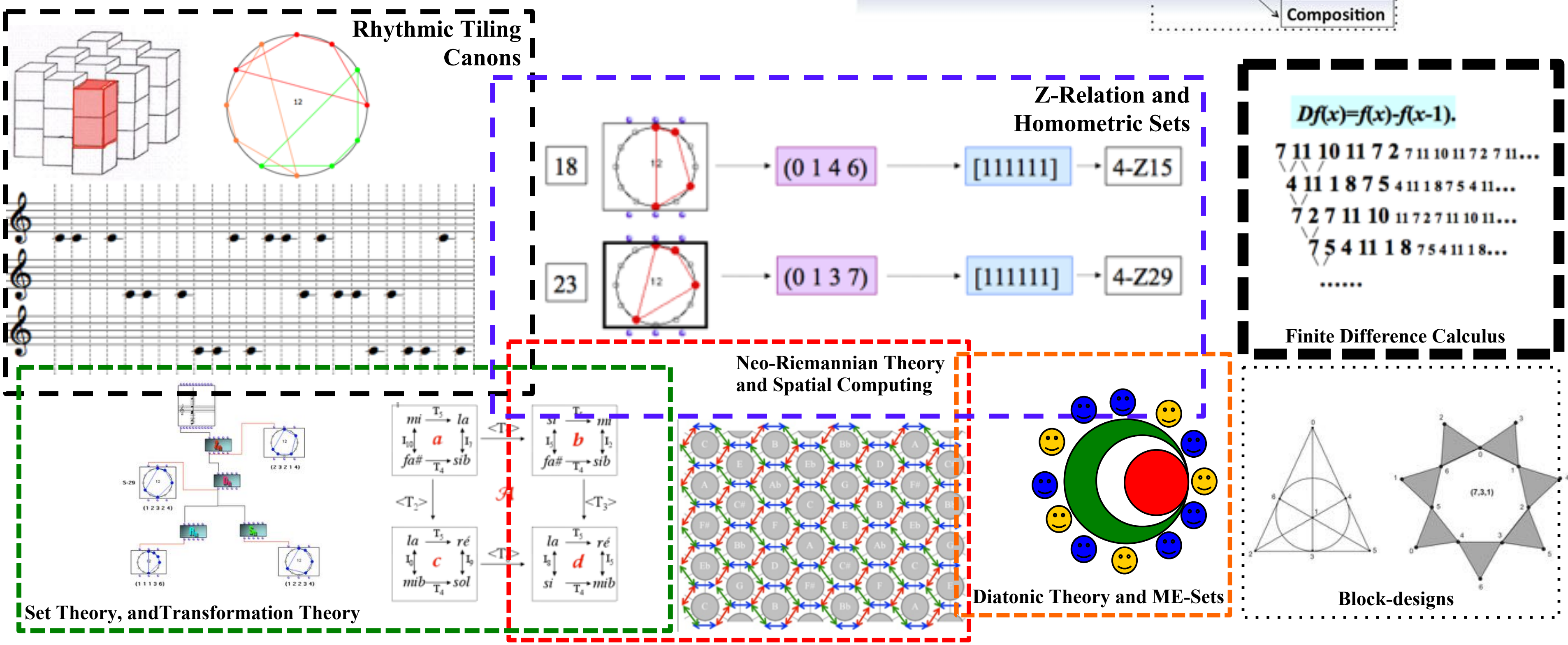




➔ M. Andreatta : *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

Periodic sequences and finite difference calculus

$$Df(x)=f(x)-f(x-1)$$

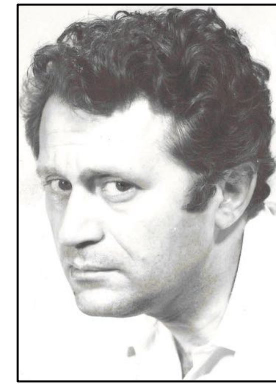
f = 7 11 10 11 7 2 7 11 10 11 7 2 7 11…

Df = 4 11 1 8 7 5 4 11 1 8 7 5 4 11…

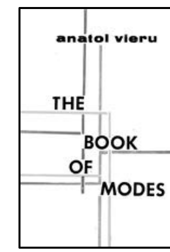
$D^2 f$ = 7 2 7 11 10 11 7 2 7 11 10 11…

$D^3 f$ = 7 5 4 11 1 8 7 5 4 11 1 8…

$D^k f$ = ……



Anatol Vieru



V	0	3	8	7	11	0	11	10	6	9	0	9	1	2	9	8	4	3	6
VIII	0	0	0	0	3	3	7	2	0	0	0	6	3	3	3	4	8	0	0
IV	3	3	4	4	1	11	11	8	3	3	9	4	1	7	11	8	11	3	9
IX	0	0	0	0	0	3	6	[1]	3	3	3	3	9	0	3	6	[10]	6	6
IV	0	10	3	9	10	0	9	7	0	6	7	9	6	4	9	3	4	6	3

Anatol Vieru: *Zone d'oubli* pour alto (1973)

Reducible and reproducible sequences

Reducible sequences:
$\exists\ k \geq 1$ **such that**
$D^k f = 0$

f = 11 6 7 2 3 10 11 6 …

Df = 7 1 7 1 7 1 7 1…

D^2f = 6 6 6 6 6…

D^4f = 0 0 0

Reproducible sequences:
$\exists\ k \geq 1$ **such that**
$D^k f = f$

f = 7 11 10 11 7 2 7 11 …

Df = 4 11 1 8 7 5 4 11 1 …

D^2f = 7 2 7 11 10 11 7 2 7 …

D^3f = 7 5 4 11 1 8 7 5 4 11 1 8…

D^4f = 10 11 7 2 7 11 10 11 …

D^5f = 1 8 7 5 4 11 1 8 …

D^6f = 7 11 10 11 7 2 7 11 …

A decomposition property of any periodic sequence

$Df(x)=f(x)-f(x-1)$.

7 11 10 11 7 2 7 11 10 11 7 2 7 11…

4 11 1 8 7 5 4 11 1 8 7 5 4 11…

7 2 7 11 10 11 7 2 7 11 10 11…

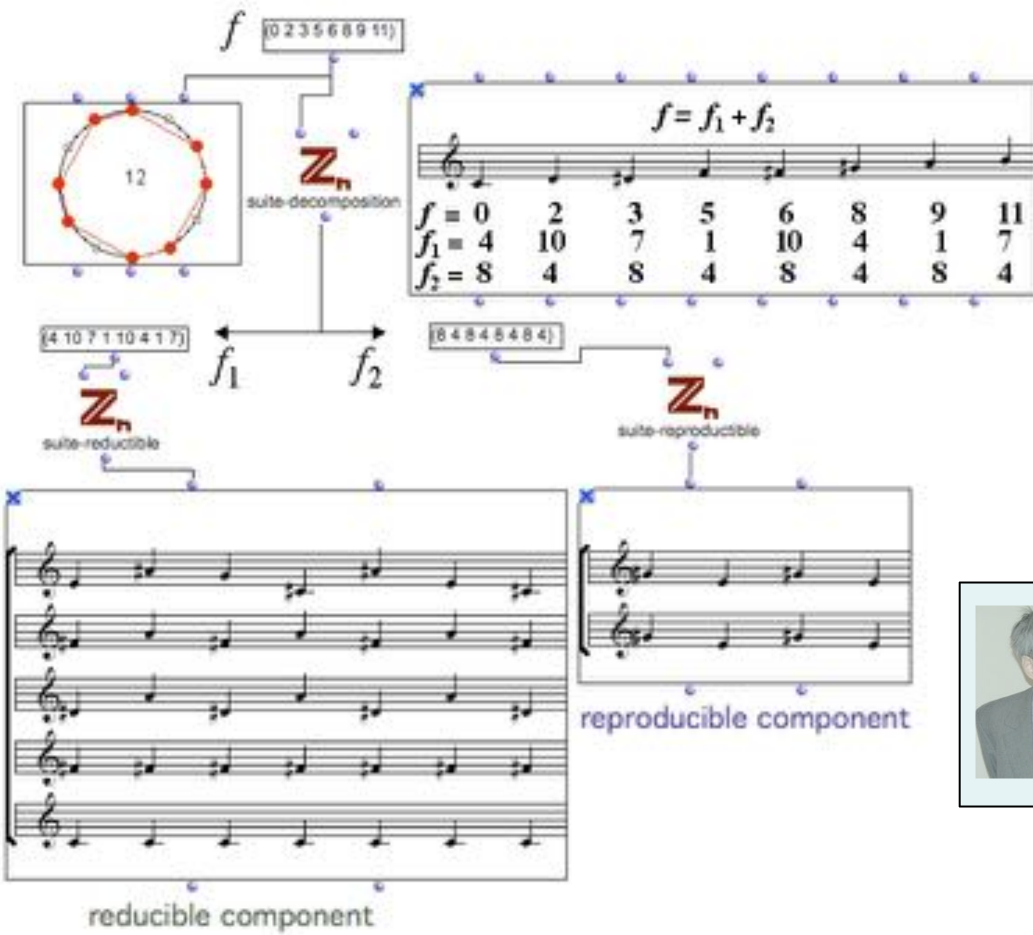
7 5 4 11 1 8 7 5 4 11 1 8…

……

Anatol Vieru: *Zone d'oubli* for viola (1973)

Reducible sequences:
$\exists\, k \geq 1$ such that $D^k f = 0$

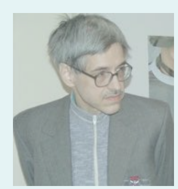
Reproducible sequences:
$\exists\, k \geq 1$ such that $D^k f = f$



- **Decomposition theorem**

(Vuza & Andreatta, *Tatra M.*, 2001)

Every periodic sequence f can be decomposed in a unique way as a sum $f_1 + f_2$ of a reducible sequence f_1 and a reproducible sequence f_2



D. Vuza, M. Andreatta (2001), « On some properties of periodic sequences in Anatol Vieru's modal theory », Tatra Mountains Mathematical Publications, Vol. 23, p. 1-15

A decomposition property of any periodic sequence

Theorem (Vuza & Andreatta, 2001): Any periodic sequence can be decomposed <u>in a unique way</u> as a sum of a *reducible* and a *reproducible* sequence.

Growing by additions and proliferation of values

A^3g = 8 4 8 10 11 1 5 1 2 10 2 4 5 7 11 7 8 4 …

A^2g = 8 4 2 1 2 4 8 1 8 4…

Ag = 8 10 11 1 2 4 5 7 8 10 …

g = 2 1 2 1 2 1 2 1

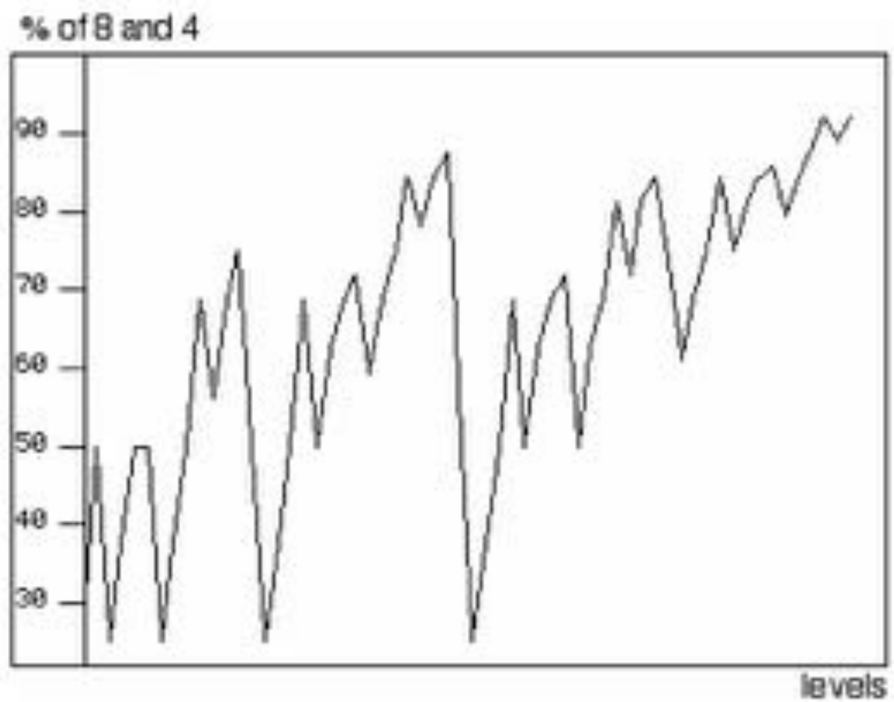


Fig. 11 Initial values equal to 8

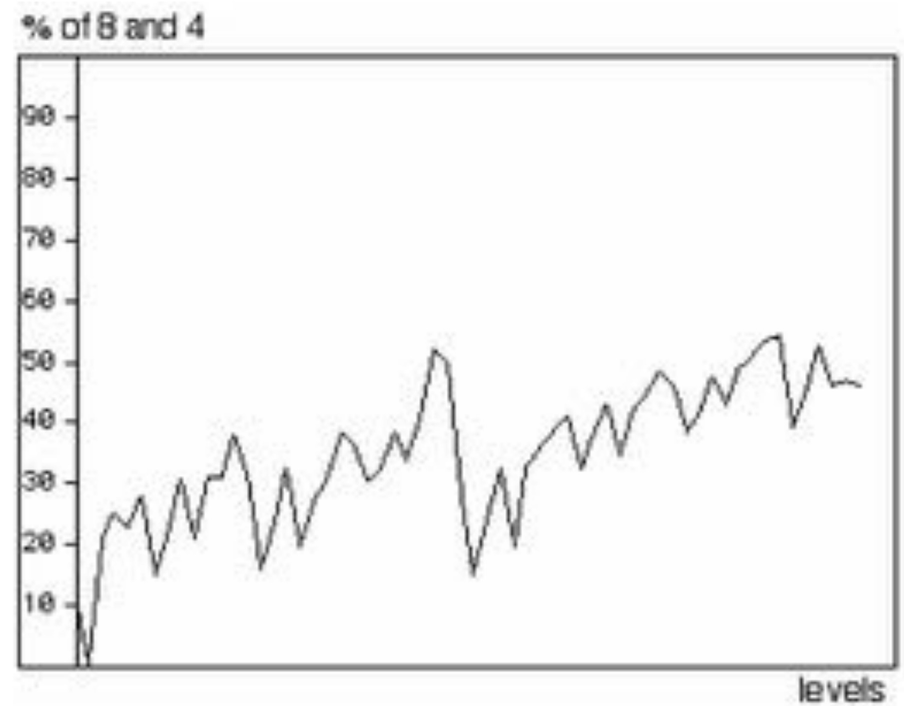


Fig. 15 Initial values equal to 4

The mathematical environment

The periodic sequences

On the $\mathbb{Z}_m$-module $\mathbb{Z}_m^{\mathbb{N}}$, consider the shifting operator:

$$\theta(f)(i) := f(i+1) \quad \forall i \in \mathbb{N}.$$

A sequence $f \in \mathbb{Z}_m^{\mathbb{N}}$ is *periodic* if there exists $k \geq 1$ such that

$$f \in \ker(\theta^k - \mathrm{id}).$$

The minimal k satisfying this condition is the *period* of f. We work in the $\mathbb{Z}_m$ -(sub)module of periodic sequences

$$P_m = \bigcup_{k \geq 1} \ker(\theta^k - \mathrm{id}).$$

The operators

On the module P_m of periodic sequences, we consider two other operators:

- the *discrete derivative*

$$\Delta := \theta - \mathrm{id};$$

- given $c \in \mathbb{Z}_m$, a *finite sum operator* : for each $f \in P_m$,

$$\Sigma_c f(i) := \begin{cases} c \text{ if } i = 1 \\ f(n-1) + \Sigma_c f(n-1) \text{ if } i > 1. \end{cases}$$

We write just Σ to mean Σ_0; we say that $\Sigma^k f$ is the discrete *k-primitive* of f.

Δ-nilpotent and Δ-idempotent sequences

Definitions

- $f \in P_m$ is *Δ-nilpotent* (resp. *Δ-idempotent*) if there exists $k \geq 1$ such that $\Delta^k f = 0$ (resp. $\Delta^k f = f$); the minimal k such that this happens is said nilpotency (resp. idempotency) order of f.
- We denote by I_m^Δ the subset of P_m of Δ-idempotent sequences and by N_m^Δ the subset of Δ-nilpotent sequences.

Fitting Lemma

The *Fitting Lemma* gives the decomposition

$$P_m = N_m^\Delta \oplus I_m^\Delta$$

hence any $f \in P_m$ uniquely determines $f_I \in I_m^\Delta$ and $f_N \in N_m^\Delta$ such that

$$f = f_I + f_N.$$

Riccardo Gilblas

Luisa Fiorot

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Università degli Studi di Padova

Université de Strasbourg

The properties of Δ-nilpotent sequences

The period

Let $f \in P_{p^n}$ be a periodic sequence. Then:

- $f \in N^{\Delta}_{p^n}$ if and only if the period of f is p^m for $m \in \mathbb{N}$;
- if $f \in N^{\Delta}_{p^n}$ with period p^m and nilpotency order η, then $\eta \leq np^m$.

Sum of constants

Sequences in N^{Δ}_{m} are *finite sums* of discrete k-primitives of *constant sequences* ; furthermore, one shows that for any $c \in \mathbb{Z}_m$ and any $i \in \mathbb{N}$:

$$\Sigma^k(c)(i) \equiv_m c\binom{i}{k}.$$

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Kummer's and Luca's generalized Theorems

We need to study the binomial coefficient modulo a power of a prime. The main classical tools in this setting are:

Kummer's Theorem

Given integers $i \geq k \geq 0$ and a prime number p, the p-adic valuation $\nu_p(\binom{i}{k})$ is equal to the number of carries when k is added to $i-k$ in base p.
This result allows us to find out if a binomial coefficient is 0 modulo a power of a prime.

Lucas's generalized Theorem

The generalization of Lucas's Theorem permits to compute explicitly a binomial coefficient modulo a power of a prime. Example: consider $41 = [1112]_3$ and $11 = [0102]_3$; lets us compute the residue class of $\binom{41}{11}$ modulo 9. One gets

$$\binom{[1112]_3}{[0102]_3} \equiv_9 \left\langle {11 \atop 01} \right\rangle \left\langle {1 \atop 1} \right\rangle^{-1} \left\langle {11 \atop 10} \right\rangle \left\langle {1 \atop 0} \right\rangle^{-1} \left\langle {12 \atop 02} \right\rangle \equiv_9 7.$$

Angled parentheses here denote a generalization of the binomial coefficient to the case when the numerator is smaller than the denominator.

TO BE SUBMITTED

Introduction

In his *Book of Modes*, the romanian composer Anatol Vieru studies periodic sequences taking values in $\mathbb{Z}_m = \mathbb{Z}/m\mathbb{Z}$ using finite difference calculus. From the musical point of view, each coefficient of the sequence may represent a pitch class or an interval, as well as a rhythmic beat.

Musical meaning

Starting from the constant sequence (6), corresponding to the triton interval, Vieru collects other periodic sequences by iteratively applying the finite sum operator. Then he decodes from each sequence a musical aspect, giving rise to a composition, *Zone d'Oubli*.

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On the $\mathbb{Z}_m$-module $\mathbb{Z}_m^{\mathbb{N}}$, consider the shifting operator:

$$\theta(f)(i) := f(i+1) \quad \forall i \in \mathbb{N}.$$

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- the *discrete derivative*

$$\Delta := \theta - \mathrm{id};$$

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We write just Σ to mean Σ_0; we say that $\Sigma^k f$ is the discrete *k-primitive* of f.

Some properties of the operators

- For every $c \in \mathbb{Z}_m$ and $f \in P_m$, $\Sigma_c f = \Sigma f + (c)$, where (c) is the constant sequence $(c, c, c, \dots)$.
- For every $c \in \mathbb{Z}_m$,

$$\Delta \circ \Sigma = \mathrm{id}.$$

- The period of Σf is a multiple of the period of f.

Decomposition in p-parts

If $m \in \mathbb{N}$, $m \geq 2$ and its factorization is:

$$m = \prod_{i=1}^{s} p_i^{n_i},$$

the group isomorphism

$$\mathbb{Z}/m\mathbb{Z} \to \bigoplus_{i=1}^{s} \mathbb{Z}/p_i^{n_i}\mathbb{Z}$$

gives rise to an isomorphism of abelian groups

$$P_m \to \bigoplus_{i=0}^{s} P_{p_i^{n_i}}$$
$$f \mapsto (f \bmod p_i^{n_i})_{0 \leq i \leq s}.$$

$f \bmod p_i^{n_i}$ is the p_i-part of f. We can study separately the p_i-parts, so we can restrict to work in $P_{p_i^{n_i}}$.

Δ-nilpotent and Δ-idempotent sequences

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Angled parentheses here denote a generalization of the binomial coefficient to the case when the numerator is smaller than the denominator.

New results

The period of primitives of constants

Let (c) be a non zero constant sequence in P_{p^n} with $c = p^l b$, $p \nmid b$. Let $s \in \mathbb{N}$ and $[a_k a_{k-1} \cdots a_1 a_0]_p$, $a_k \neq 0$, the representation of s in base p. Then the sequence $\Sigma^s(c)$ has period p^{n-l+k}.

Leading coefficient

Given $f \in N_{p^n}^{\Delta}$, there is a constant c among the constants in the decomposition of f which *definitively leads* the period of the primitives of f.

The sequence from Messiaen's second mode of limited transpositions

Vieru noticed that starting from a particular sequence and collecting its primitives, some values prolifer. This observation gave rise to some interest in understanding the motivation of this proliferation. The sequence is $f = (2, 1, 2, 4, 8, 1, 8, 4)$, whose decomposition in Δ-nilpotent and Δ-idempotent part coincides with the decomposition in p-parts:

$$f_I = (2, 1) \in \mathbb{Z}/3\mathbb{Z}, \quad f_N = (2, 1, 0, 0, 1, 0, 0) \in \mathbb{Z}/4\mathbb{Z}.$$

f_I is also Σ-idempotent, so it gives constant contribution to the primitives of f. On the other hand,

$$f_N = \Sigma^4(2) + \Sigma^3(3) + \Sigma^2(2) + \Sigma(3) + (2)$$

is the decomposition in primitives of constant sequences and $\Sigma^3(3)$ is the leading term for the period. Studying the behaviour of the primitives of the constant sequences (2) and (3), we found an algebraic explanation to the proliferation.

Riccardo Gilblas

Luisa Fiorot

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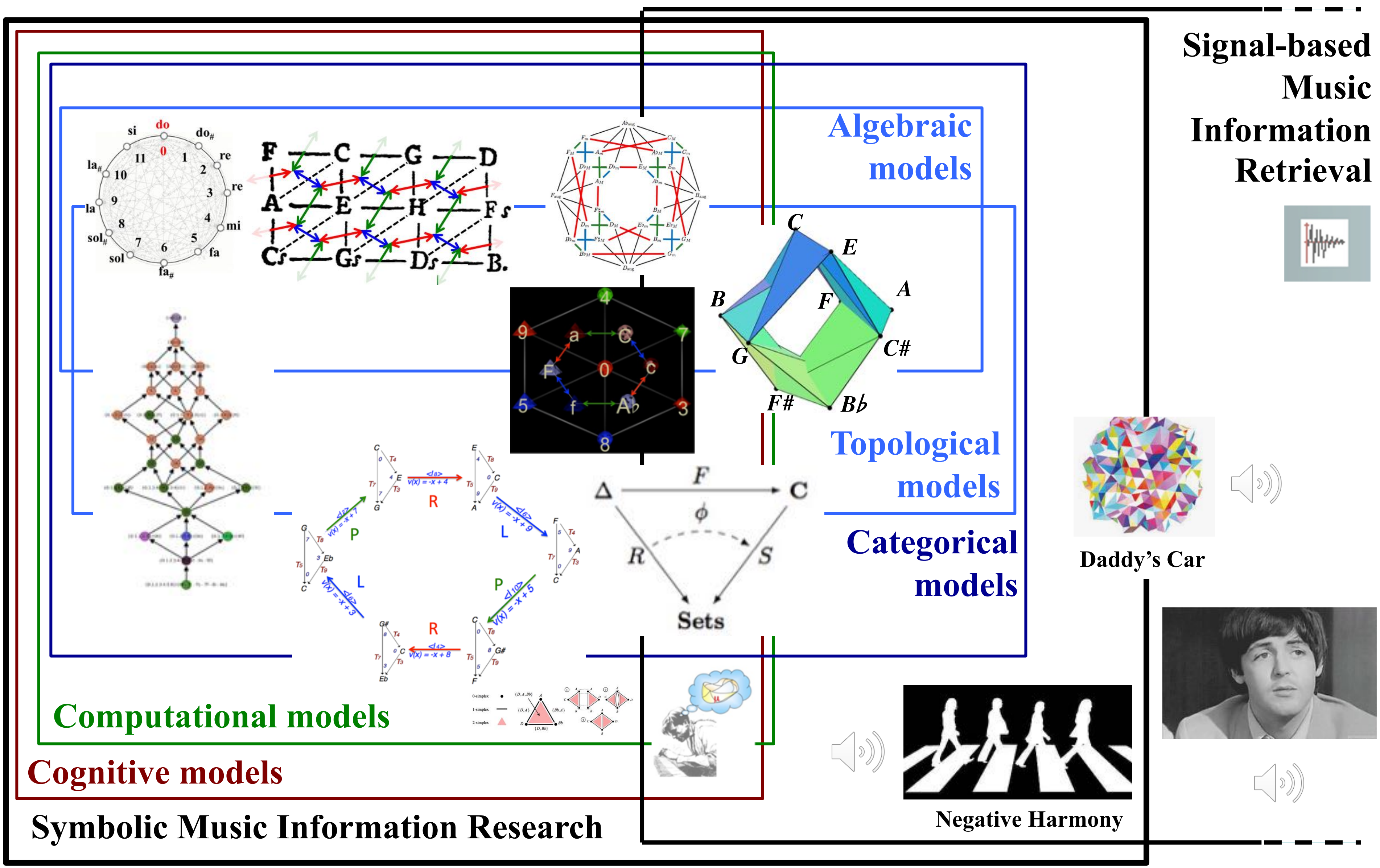
Università degli Studi di Padova

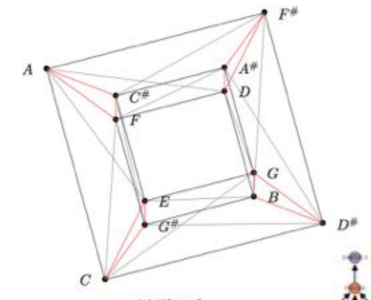
Université de Strasbourg

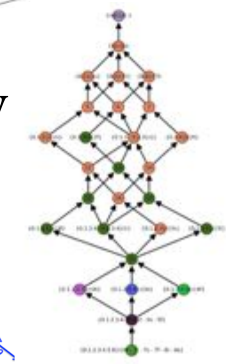
The SMIR Project: advanced maths for the working musicologist



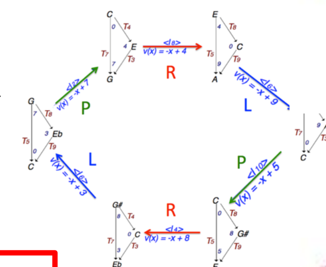
The SMIR Project: Structural Music Information Research

- **Generalized Tonnetze and Persistent Homology**
  - The Tonnetz as a simplicial complex
  - Algebraic classification of the twelve possible Tonnetze
  - Isotropic and anisotropic Tonnetze
  - Application to automatic stylistic classification



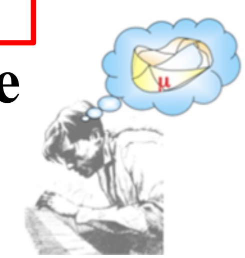


- **Formal Concept Analysis and Mathematical Morphology**
  - Lattice structure of formal concepts
  - Derivation operators (in FCA) and dilation/erosion in MM
  - Application to pattern recognition and extraction



- **Category theory and Transformational Theory**
  - From K-nets to PK-nets
  - Diatonicism
- **‘Mathemusical’ problems and open conjectures**
  - Tiling rhythmic canons and Fuglede Spectral Conjecture
  - Homometric musical structures

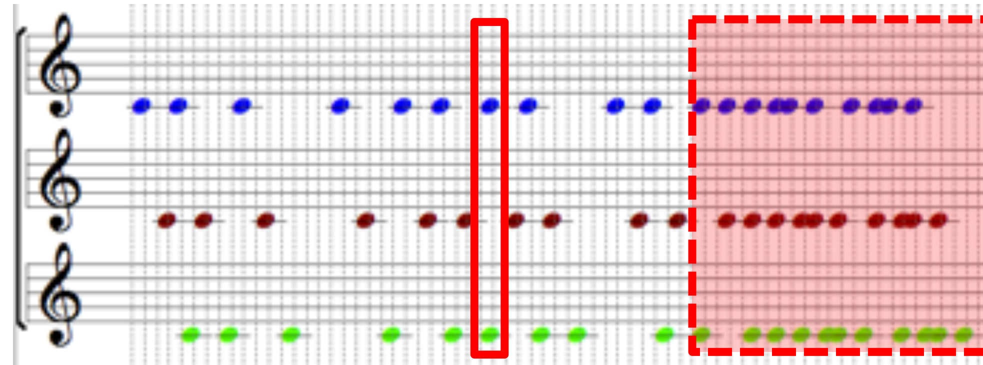
- **Philosophy, Epistemology and Cognitive Science**
  - Geometry-based Neo-structuralism in music analysis
  - Processes and techniques of mathemusical learning



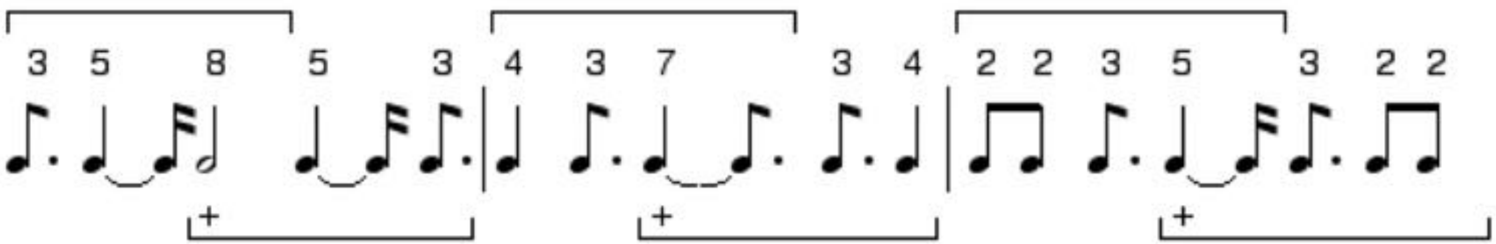
➔ Andreatta, M., « From music to mathematics and backwards: introducing algebra, topology and category theory into computational musicology », in M. Emmer and M. Abate (eds.), *Imagine Math 6* - Springer, 2018

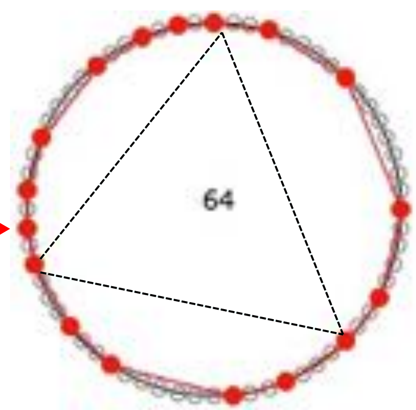
Tiling rhythmic canons as a ‘mathemusical’ problem

***Harawi* (1945)**



***Harawi*: rhythmic reduction**



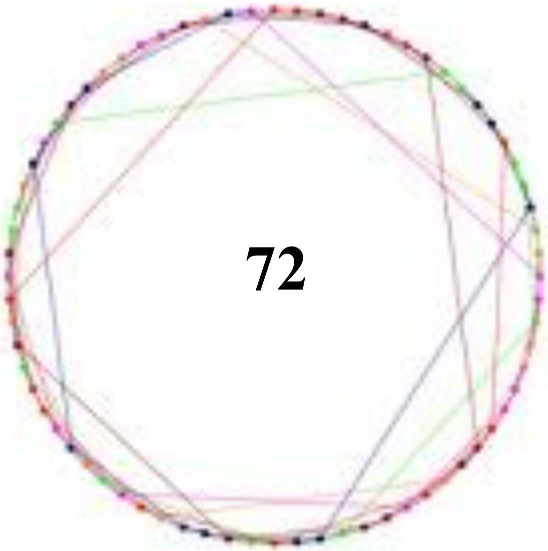


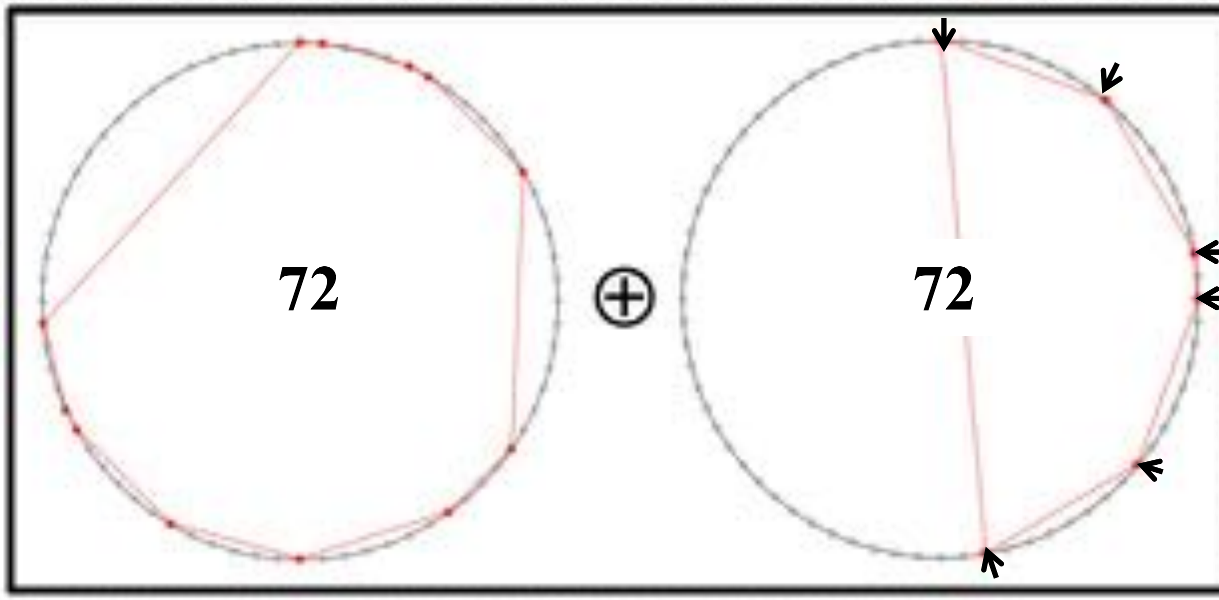
« …il résulte de tout cela que les différentes sonorités se mélangent ou s’opposent de manières très diverses, jamais au même moment ni au même endroit […]. C’est du désordre organisé »

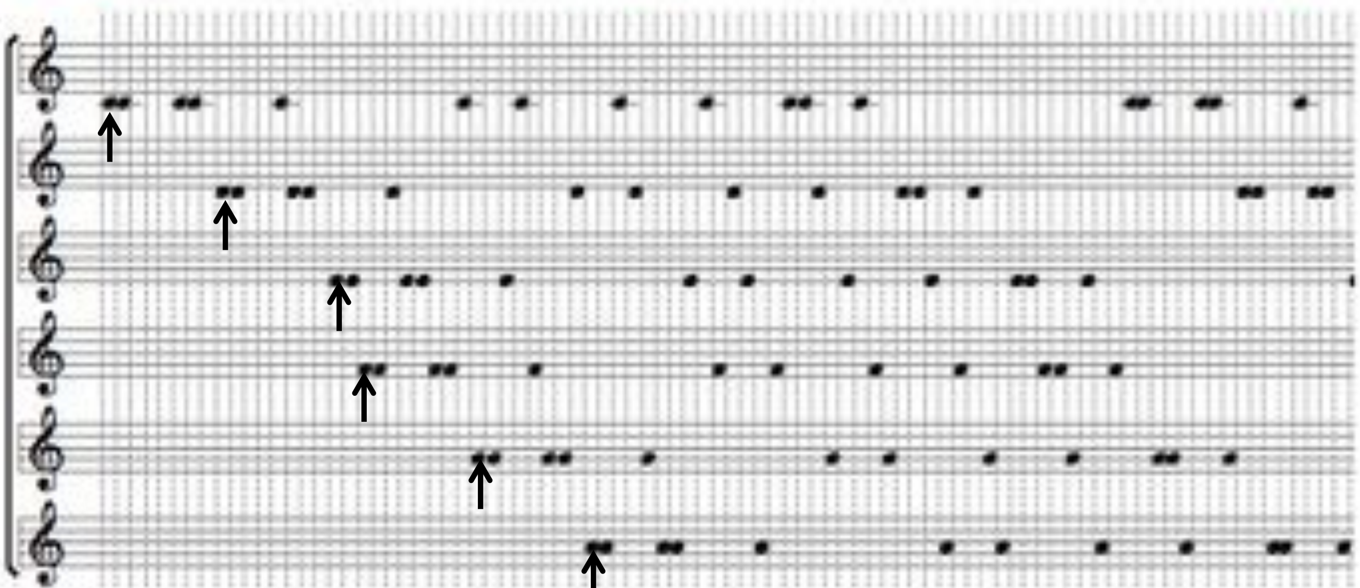
O. Messiaen: *Traité de Rythme, de Couleur et d’Ornithologie,* tome 2, Alphonse Leduc, 1992.

Aperiodic Rhythmic Tiling Canons (Vuza Canons)

Which is the smallest factorisation of Z_n into two non-periodic subsets?



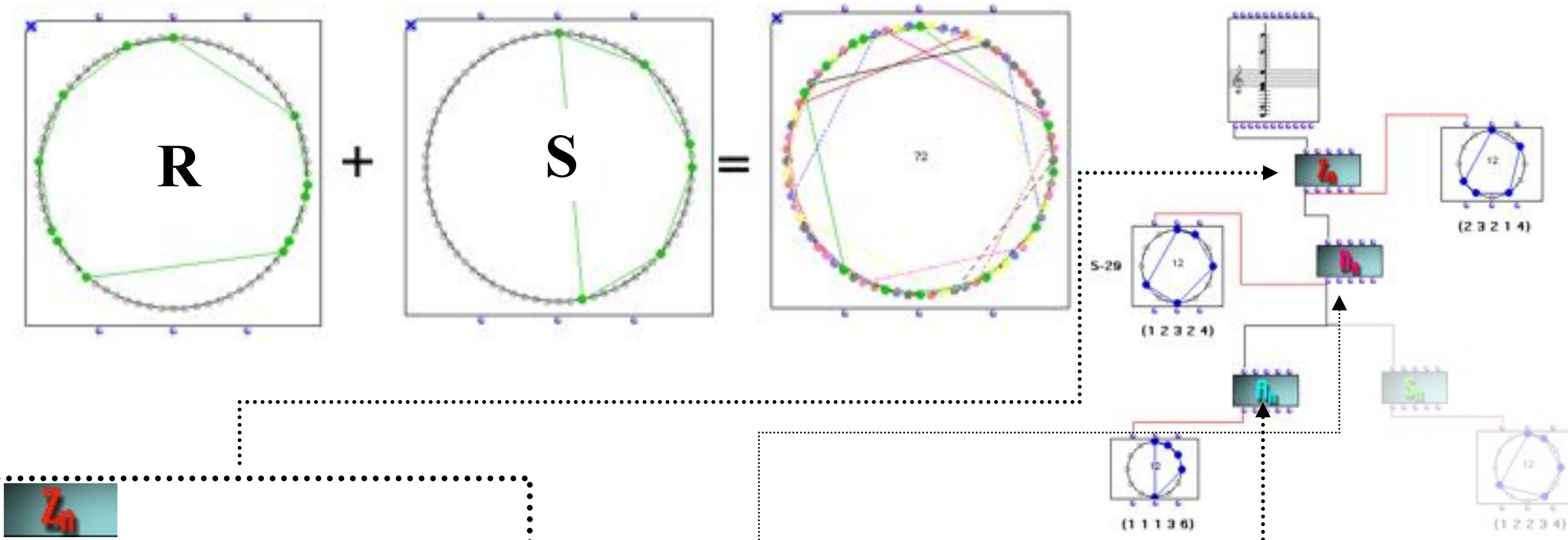




Anatol Vieru

Dan Vuza

Group-based paradigmatic classification of Vuza Canons



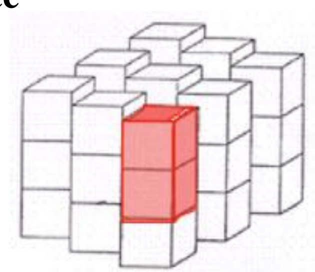
	Z_n	D_n	A_n
R	**(1 3 3 6 11 4 9 6 5 1 3 20)** **(20 3 1 5 6 9 4 11 6 3 3 1)** **(1 4 1 19 4 1 6 6 7 4 13 6)** **(6 13 4 7 6 6 1 4 19 1 4 1)** **(1 5 15 4 5 6 6 3 4 17 3 3)** **(3 3 17 4 3 6 6 5 4 15 5 1)**	**(1 3 3 6 11 4 9 6 5 1 3 20)** **(1 4 1 19 4 1 6 6 7 4 13 6)** **(1 5 15 4 5 6 6 3 4 17 3 3)**	**(1 3 3 6 11 4 9 6 5 1 3 20)** **(1 4 1 19 4 1 6 6 7 4 13 6)**
S	**(8 8 2 8 8 38)** **(16 2 14 2 16 22)** **(14 8 10 8 14 18)**	**(8 8 2 8 8 38)** **(16 2 14 2 16 22)** **(14 8 10 8 14 18)**	**(14 8 10 8 14 18)**

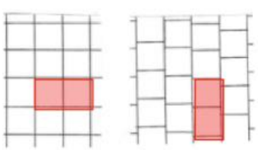
Tijdeman's
'Fundamental Lemma' (1995)
***R* tiles Z_n => *aR* tiles Z_n $\langle a,n\rangle=1$**

Tiling Rhythmic Canons as a ‘mathemusical’ problem

Minkowski/Hajós Problem (1907-1941)

In any simple lattice tiling of the *n*-dimensional Euclidean space by unit cubes, some pair of cubes must share a complete (*n*-1)-dimensional face



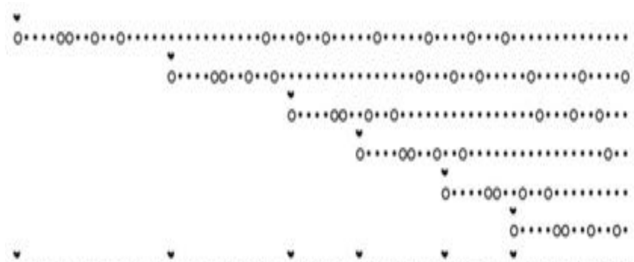


S. Stein, S. Szabó: *Algebra and Tiling*, Carus Math. Mon. 1994

Vieru’s problem and Vuza’s formalization (PNM, 1991)

A Vuza Canon is a factorization of a cyclic group in a direct sum of two non-periodic subsets

$$\mathbf{Z}/n\mathbf{Z} = R \oplus S$$



Link between Minkowski problem and Vuza Canons (Andreatta, 1996)

Hajós groups (*good groups*)

Z/*n*Z with *n* $\in \{p^\alpha, p^\alpha q, pqr, p^2q^2, p^2qr, pqrs\}$ **where** p, q, r, s, **are distinct prime numbers**

Non-Hajós group (*bad groups*)

72
108 120 144 168 180
200 216 240 252 264 270 280 288
300 312 324 336 360 378 392 396
400 408 432 440 450 456 468 480
500 504 520 528 540 552 560 576 588 594
600 612 616 624 648 672 675 680 684 696
700 702 720 728 744 750 756 760 784 792
800 810 816 828 864 880 882 888…

(Sloane’s sequence A102562)

Fuglede Spectral Conjecture

A subset of the *n*-dimensional Euclidean space tiles by translation *iff* it is *spectral*.

(*J. Func. Anal.* 16, 1974)

Vuza Canons and Fuglede Spectral Conjecture

A subset of the *n*-dimensional Euclidean space tiles by translation *iff* it is *spectral*.

(*J. Func. Anal.* 16, 1974)

➔**False in dim. $n \geq 3$**
(Tao, Kolountzakis, Matolcsi, Farkas and Mora)

➔ **Open in dim. 1 et 2**

DEFINITION 6 *A subset A of some vector space (say $\mathbb{R}^n$) is spectral iff it admits a Hilbert base of exponentials, i.e. if any map $f \in L^2(A)$ can be written*

$$f(x) = \sum f_k \exp(2i\pi\lambda_k.x)$$

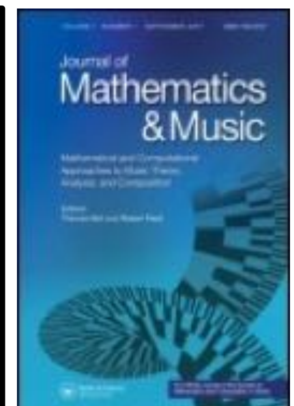
for some fixed family of vectors $(\lambda_k)_{k\in\mathbb{Z}}$ where the maps $e_k : x \mapsto \exp(2i\pi\lambda_k.x)$ are mutually orthogonal (i.e. $\int_A \overline{e_k} e_j = 0$ whenever $k \neq j$).

$\downarrow$ ***n*=1**

DEFINITION 8. *A subset $A \in \mathbb{Z}$ is* spectral *if there exists a spectrum $\Lambda \subset [0,1]$ (i.e., a subset with the same cardinality as A) such that $e^{2i\pi(\lambda_i - \lambda_j)}$ is a root of $A(X)$ for all distinct $\lambda_i, \lambda_j \in \Lambda$.*

Theorem (Amiot, 2009)

- *All non-Vuza canons are spectral.*
- *Fuglede Conjecture is true (or false) iff it is true (or false) for Vuza Canons*



Orders of non-Hajos groups ('Bad groups')

72
108 120 144 168 180
200 216 240 252 264 270 280 288
300 312 324 336 360 378 392 396
400 408 432 440 450 456 468 480
500 504 520 528 540 552 560 576 588 594
600 612 616 624 648 672 675 680 684 696
700 702 720 728 744 750 756 760 784 792
800 810 816 828 864 880 882 888…

(Sloane's sequence A102562)

M. Andreatta & C. Agon (eds), « Tiling Problems in Music », Special Issue of the *Journal of Mathematics and Music*, Vol. 3, Number 2, July 2009 (with contributions by E. Amiot, F. Jedrzejewski, M. Kolountzakis and M. Matolcsi)

Fuglede Spectral conjecture for convex domains is true (in all dimensions)

(Submitted on 28 Apr 2019)

Let Ω be a convex body in $\mathbb{R}^d$. We say that Ω is spectral if the space $L^2(\Omega)$ has an orthogonal basis of exponential functions. There is a conjecture going back to Fuglede (1974) which states that Ω is spectral if and only if it can tile the space by translations. It has long been known that if a convex body Ω tiles then it must be a polytope, and it is also spectral. The converse, however, was proved only in dimensions $d \leq 3$ and under the a priori assumption that Ω is a polytope.
In this paper we prove that for every dimension d, if a convex body $\Omega \subset \mathbb{R}^d$ is spectral then it must be a polytope, and it can tile the space by translations. The result thus settles Fuglede's conjecture for convex bodies in the affirmative. Our approach involves a construction from crystallographic diffraction theory, that allows us to establish a geometric "weak tiling" condition necessary for the spectrality of Ω.

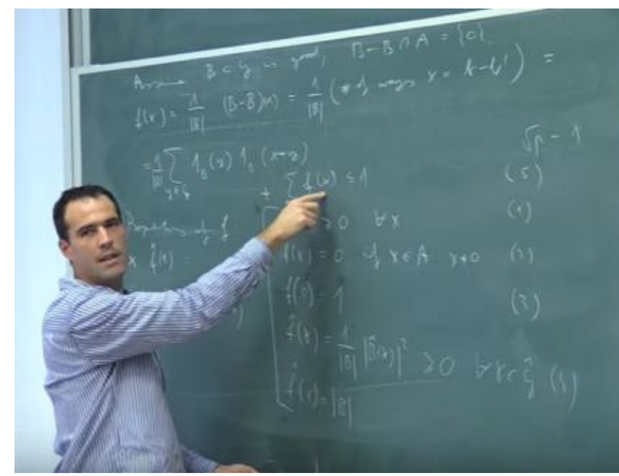
Subjects: **Classical Analysis and ODEs (math.CA)**; Functional Analysis (math.FA); Metric Geometry (math.MG)

MSC classes: 42B10, 52B11, 52C07, 52C22

Cite as: arXiv:1904.12262 **[math.CA]**
(or arXiv:1904.12262v1 **[math.CA]** for this version)

Acta Math., 228 (2022), 385–420
DOI: 10.4310/ACTA.2022.v228.n2.a3

Nir Lev (Bar-Ilan University, Tel-Aviv)



Mate Matolcsi (Rényi Institute, Budapest)

Vuza's Algorithm and other approaches (PhD Greta Lanzarotto)

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ÉCOLE DOCTORALE
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L'INFORMATION ET DE L'INGÉNIEUR
UNIVERSITÉ DE STRASBOURG

Dipartimenti di
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RECHERCHE
MATHÉMATIQUE AVANCÉE

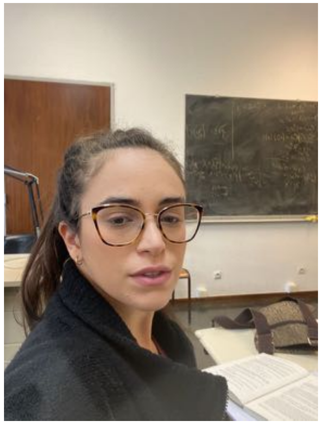
Ph.D. program in MATHEMATICS

Cycle XXXIV

EXTENDED VUZA CANONS

Surname: LANZAROTTO Name: GRETA

Registration number: 847623



Tutor: Prof. PERNAZZA LUDOVICO

Co-tutor: Prof. ANDREATTA MORENO

Coordinator: Prof. COLLI PIERLUIGI

ACADEMIC YEAR 2020/2021

Table 4.5: Number of extended Vuza canons for $n = 900$, with $n_3 \in \{2, 3, 5, 6, 10, 15\}$.

p_1	n_1	p_2	n_2	n_3	L	$\#K$	S_A	$\#B$
2	25	3	3	2	$\{0\}$	1	$\{3,5,25\}$	15600
25	2	3	3	2	$\{0\}$	1	$\{3,4\}$	67783088736
5	10	3	3	2	$\{0\}$	1	$\{3,4,5\}$	15840
10	5	3	3	2	$\{0\}$	1	$\{3,5\}$	235200
2	9	5	5	2	$\{0\}$	1	$\{3,5,9\}$	118080
9	2	5	5	2	$\{0\}$	1	$\{4,5\}$	1302000
3	6	5	5	2	$\{0\}$	1	$\{3,4,5\}$	62160
6	3	5	5	2	$\{0\}$	1	$\{3,5\}$	123840
3	25	2	2	3	$\{0\}$	1	$\{2,5,25\}$	870000
25	3	2	2	3	$\{0\}$	1	$\{2,9\}$	405323290006272
5	15	2	2	3	$\{0\}$	1	$\{2,5,9\}$	585900
15	5	2	2	3	$\{0\}$	1	$\{2,5\}$	3572152200
2	6	5	5	3	$\{0\}$	1	$\{2,5,9\}$	606526200
6	2	5	5	3	$\{0\}$	1	$\{2,5\}$	481892400
3	4	5	5	3	$\{0\}$	1	$\{2,4,5\}$	45859200
4	3	5	5	3	$\{0\}$	1	$\{5,9\}$	21816000
3	15	2	2	5	$\{0\}$	1	$\{2,3,25\}$	304875900000
15	3	2	2	5	$\{0\}$	1	$\{2,3\}$	6199976956848428880
5	9	2	2	5	$\{0\}$	1	$\{2,3,9\}$	14392209600
9	5	2	2	5	$\{0\}$	1	$\{2,25\}$	9397268160000
2	10	3	3	5	$\{0\}$	1	$\{2,3,25\}$	28101810330000
10	2	3	3	5	$\{0\}$	1	$\{2,3\}$	191359865356916256000
5	4	3	3	5	$\{0\}$	1	$\{2,3,4\}$	1290026373120
4	5	3	3	5	$\{0\}$	1	$\{3,25\}$	221859000000
2	3	5	5	6	$\{0\}$	1	$\{5,9\}$	1198799538300000
2	3	5	5	6	$\{0,1\}$	2	$\{2,5,9\}$	619200
2	3	5	5	6	$\{0,1,2\}$	3	$\{3,5,9\}$	480
2	3	5	5	6	$\{0,3\}$	8	$\{2,5,9\}$	2476800
2	3	5	5	6	$\{0,2,4\}$	9	$\{3,5,9\}$	1440
3	2	5	5	6	$\{0\}$	1	$\{4,5\}$	4261202400000
3	2	5	5	6	$\{0,1\}$	2	$\{2,4,5\}$	98400
3	2	5	5	6	$\{0,1,2\}$	3	$\{3,4,5\}$	240
3	2	5	5	6	$\{0,3\}$	8	$\{2,4,5\}$	393600
3	2	5	5	6	$\{0,2,4\}$	9	$\{3,4,5\}$	720
2	5	3	3	10	$\{0\}$	1	$\{3,25\}$	708150388956481968750000
2	5	3	3	10	$\{0,1\}$	2	$\{2,3,25\}$	7733880000
2	5	3	3	10	$\{0,1,2,3,4\}$	5	$\{3,5,25\}$	120
2	5	3	3	10	$\{0,5\}$	32	$\{2,3,25\}$	123742080000
2	5	3	3	10	$\{0,2,4,6,8\}$	25	$\{3,5,25\}$	600
5	2	3	3	10	$\{0\}$	1	$\{3,4\}$	3582592319127622715228816
5	2	3	3	10	$\{0,1\}$	2	$\{2,3,4\}$	12015371520
5	2	3	3	10	$\{0,1,2,3,4\}$	5	$\{3,4,5\}$	96
5	2	3	3	10	$\{0,5\}$	32	$\{2,3,4\}$	192245944320
5	2	3	3	10	$\{0,2,4,6,8\}$	25	$\{3,4,5\}$	480
3	5	2	2	15	$\{0\}$	1	$\{2,25\}$	390586452987600000000000
3	5	2	2	15	$\{0,1,2\}$	3	$\{2,3,25\}$	9540000
3	5	2	2	15	$\{0,1,2,3,4\}$	5	$\{2,5,25\}$	1800
3	5	2	2	15	$\{0,5,10\}$	243	$\{2,3,25\}$	772740000
3	5	2	2	15	$\{0,3,6,9,12\}$	125	$\{2,5,25\}$	45000
5	3	2	2	15	$\{0\}$	1	$\{2,9\}$	2922314149256236917556396032
5	3	2	2	15	$\{0,1,2\}$	3	$\{2,3,9\}$	21792240
5	3	2	2	15	$\{0,1,2,3,4\}$	5	$\{2,5,9\}$	2052
5	3	2	2	15	$\{0,5,10\}$	243	$\{2,3,9\}$	1765171440
5	3	2	2	15	$\{0,3,6,9,12\}$	125	$\{2,5,9\}$	51300

Vuza's Algorithm and other approaches (PhD Greta Lanzarotto)

An Integer Linear Programming Model for Tilings

Gennaro Auricchio *[1], Luca Ferrarini †[1] and Greta Lanzarotto ‡[1,2]

[1]Department of Mathematics, University of Pavia
[2]IRMA, University of Strasbourg

July 8, 2021

Abstract

In this paper, we propose an Integer Linear Model whose solutions are the aperiodic rhythms tiling with a given rhythm A.
We show how this model can be used to efficiently check the necessity of the Coven-Meyerowitz's $(T2)$ condition and also to define an iterative algorithm that finds all the possible tilings of the rhythm A.
To conclude, we run several experiments to validate the time efficiency of this model.

Keywords: Integer Programming, Mathematics and Music, Tiling Problems, Vuza Canons, $(T2)$ Conjecture

AMS: 90C10, 05B45

Greta Lanzarotto

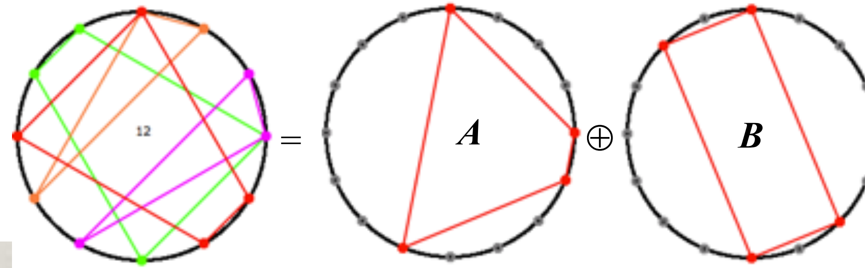
Ludovico Pernazza

Université de Strasbourg

➔ *Journal of Mathematics and Music*, 17(3), Nov. 2023, 514-530.

Tiling Rhythmic Canons and Discrete Fourier Transform

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$



$Z_A=\{1,3,4,5,7,9,11,12,13,15\}$

$Z_B=\{2,6,8,10,14\}$

Tiling

Let $Z_A = \{ t \in \mathbb{Z}_c, F_A(t)=0 \}$

A tiles $\mathbb{Z}_c$ when equivalently:

- There exists B, $A \oplus B = \mathbb{Z}_c$
- $1_A \star 1_B = 1$
- $F_A \times F_B\,(t) = 1 + e^{-2i\pi t/c} + \ldots e^{-2i\pi t(c-1)/c}$ (0 unless t=0)
- $Z_A \cup Z_B = \{1,2 \ldots c-1\}$ AND Card A × Card B = c

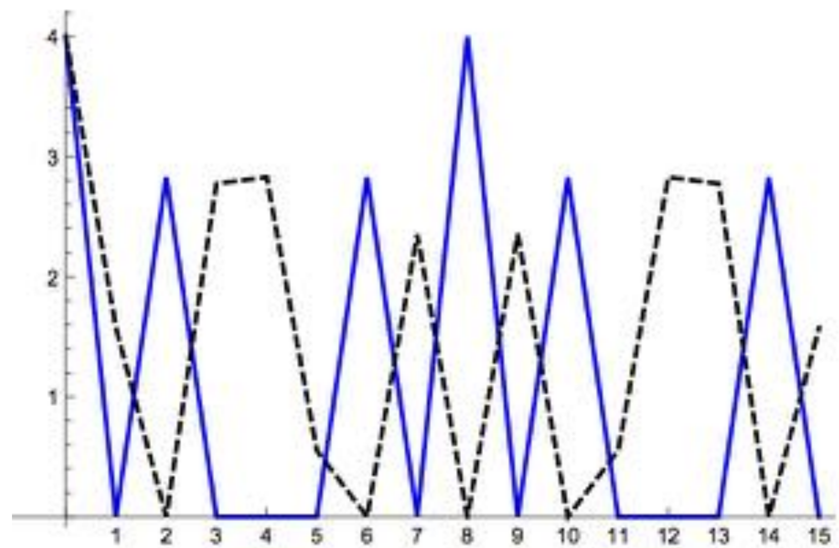
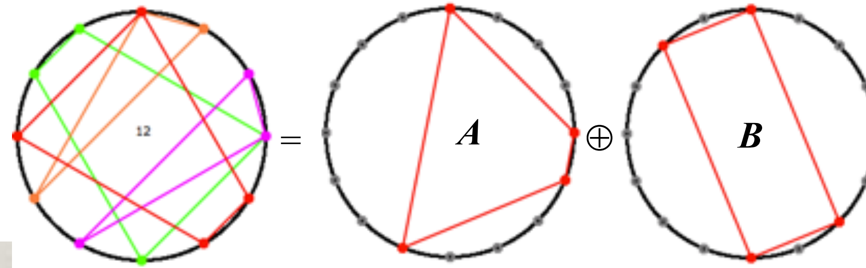


Fig. 3.7. $Z(A)$ and $Z(B)$ cover $\mathbb{Z}_{16} \setminus \{0\}$

E. Amiot, *Music through Fourier Space. Discrete Fourier Transform in Music Theory*, Springer, 2016

Tiling Rhythmic Canons and Discrete Fourier Transform

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$



TILING

Let $Z_A = \{ t \in \mathbb{Z}_c, F_A(t)=0 \}$

A tiles $\mathbb{Z}_c$ when equivalently:

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- $F_A \times F_B(t) = 1 + e^{-2i\pi t/c} + \ldots e^{-2i\pi t(c-1)/c}$ (0 unless t=0)
- $Z_A \cup Z_B = \{1,2 \ldots c-1\}$ AND Card A × Card B = c
- $IC_A \star IC_B = IC(\mathbb{Z}_c)=c$ and Card A × Card B = c

$$Z_A=\{1,3,4,5,7,9,11,12,13,15\}$$
$$Z_B=\{2,6,8,10,14\}$$

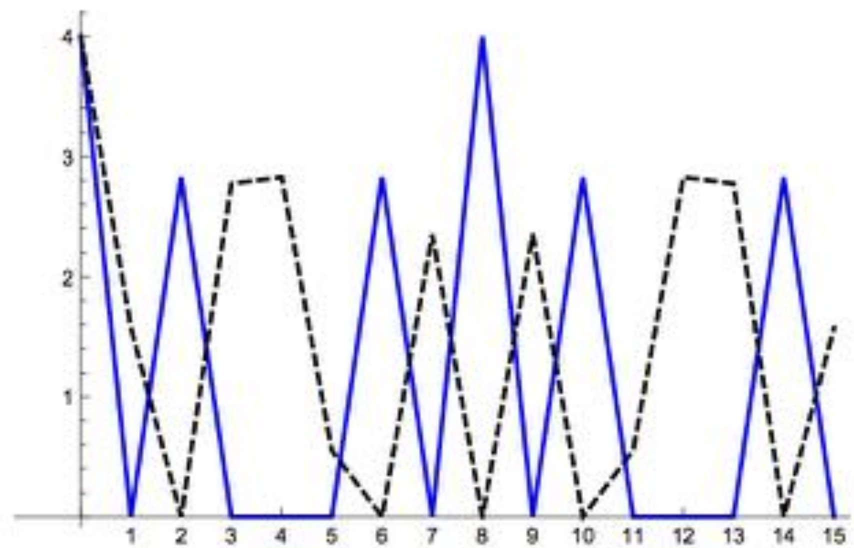
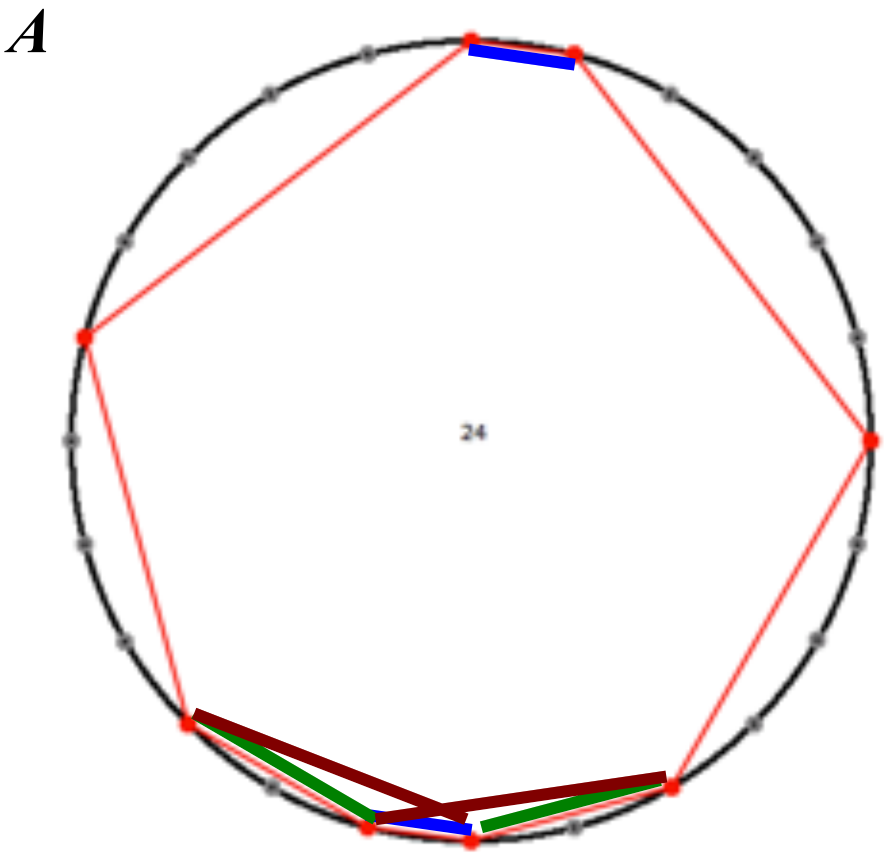


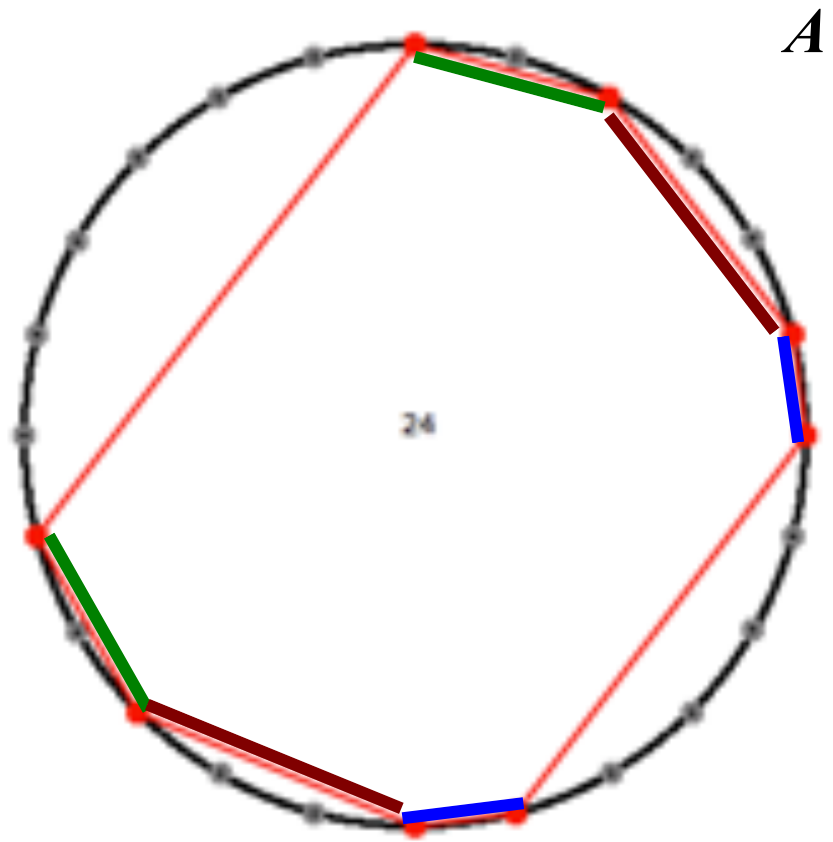
Fig. 3.7. $Z(A)$ and $Z(B)$ cover $\mathbb{Z}_{16} \setminus \{0\}$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

E. Amiot, *Music through Fourier Space. Discrete Fourier Transform in Music Theory*, Springer, 2016

Z-relation (music) and homometry (cristallography)



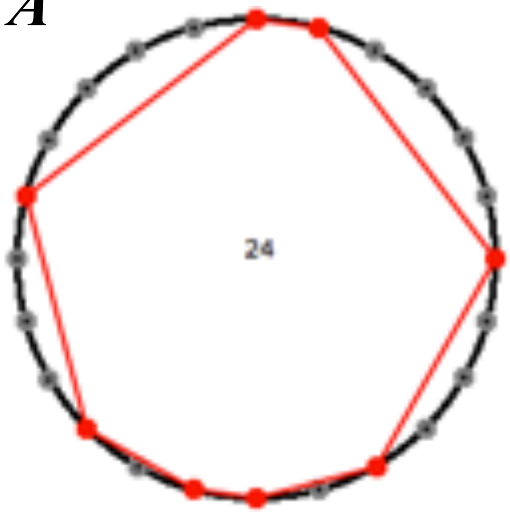


$$IC_A(k) = (1_A \star 1_{-A})(k)$$

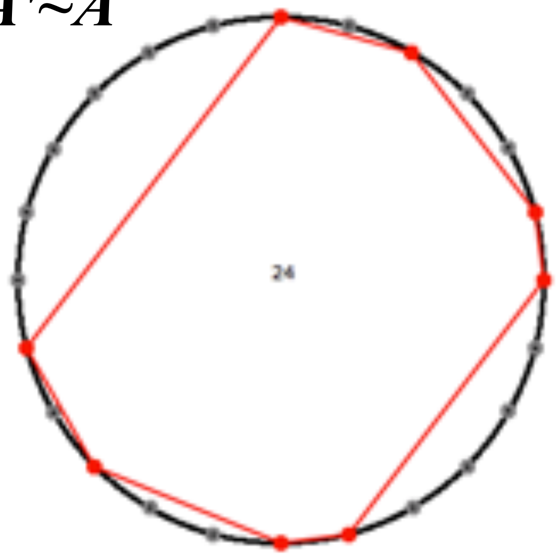
$$A \sim A' \quad \underset{\text{Homometry}}{\overset{\text{Z-relation}}{\Longleftrightarrow}} \quad IC_A(k) = IC_{A'}(k) \quad \Longleftrightarrow \quad |F_A|^2 = |F_{A'}|^2$$

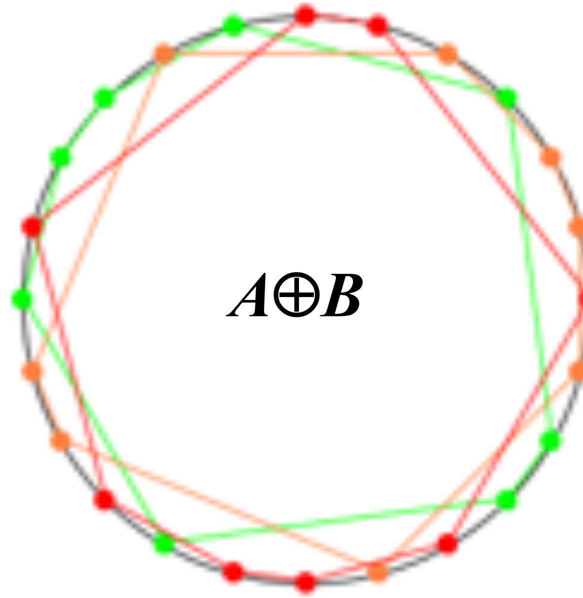
Tiling Rhythmic Canons and Homometry

A

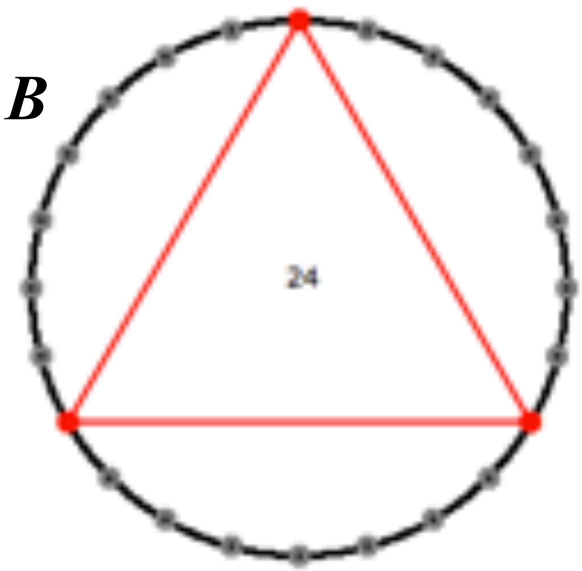


A′~A





B

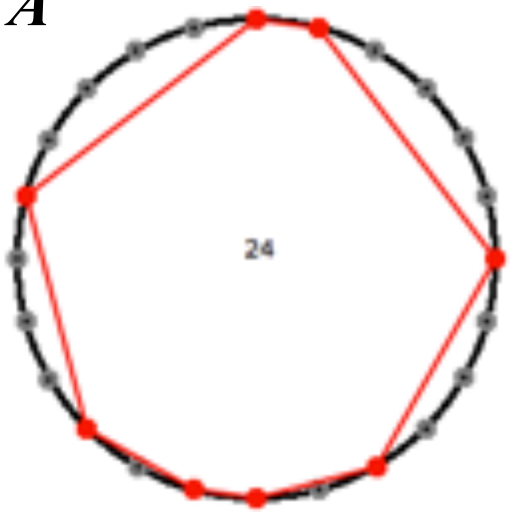


A tiles $\mathbb{Z}_c$ when equivalently:

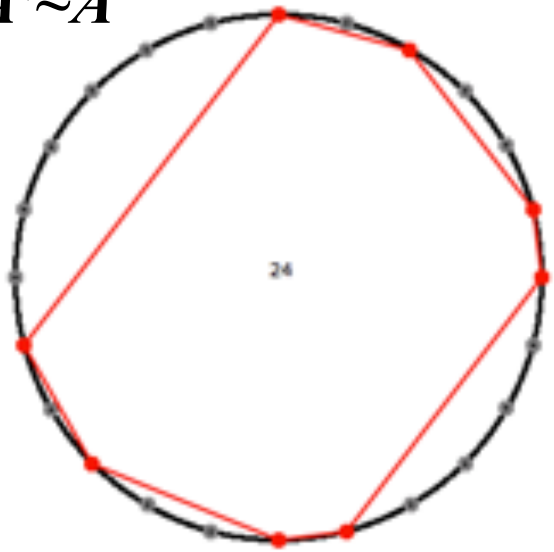
- There exists B, $A \oplus B = \mathbb{Z}_c$
- $1_A \star 1_B = 1$
- $F_A \times F_B\,(t) = 1 + e^{-2i\pi t/c} + \ldots e^{-2i\pi t(c-1)/c}$ (0 unless $t=0$)
- $Z_A \cup Z_B = \{1,2 \ldots c-1\}$ AND Card $A \times$ Card $B = c$
- ➔ $IC_A \star IC_B = IC(\mathbb{Z}_c) = c$ and Card $A \times$ Card $B = c$

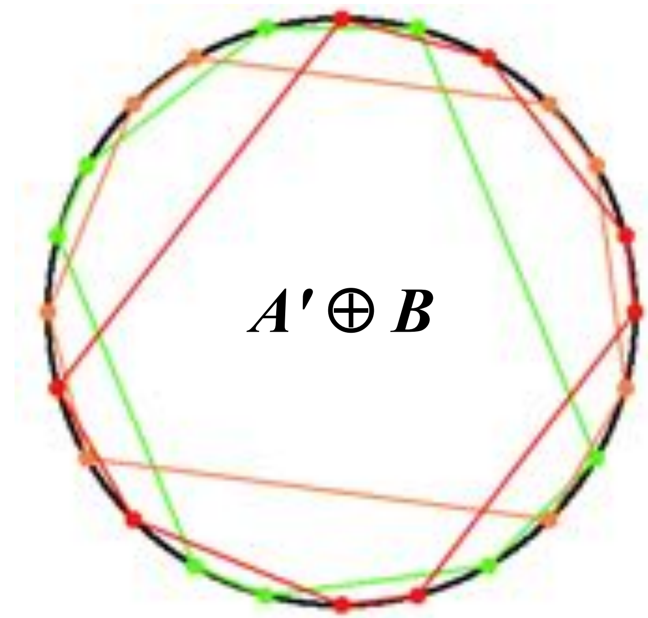
Tiling Rhythmic Canons and Homometry

A

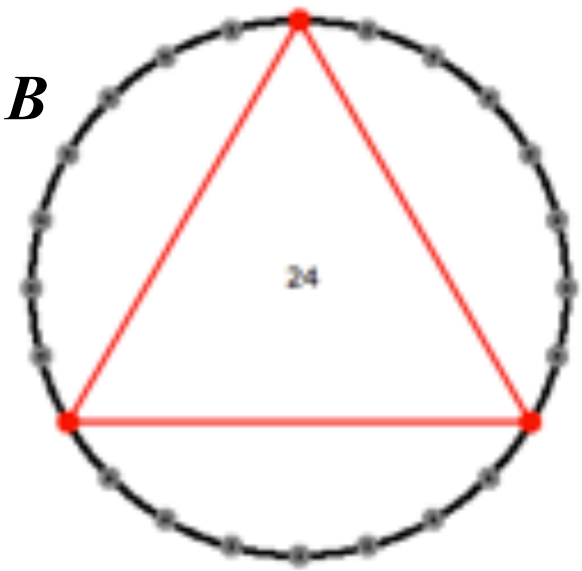


A′~A





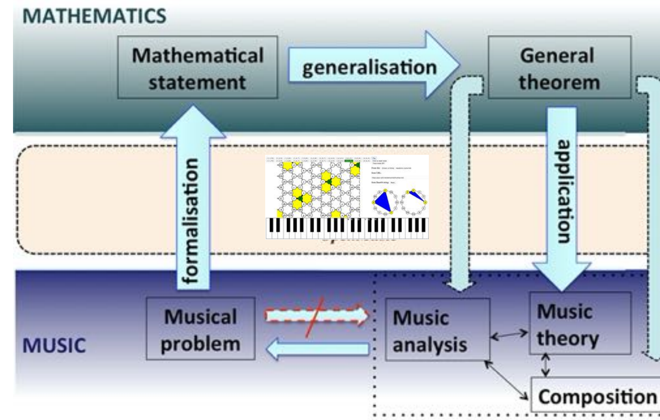
B

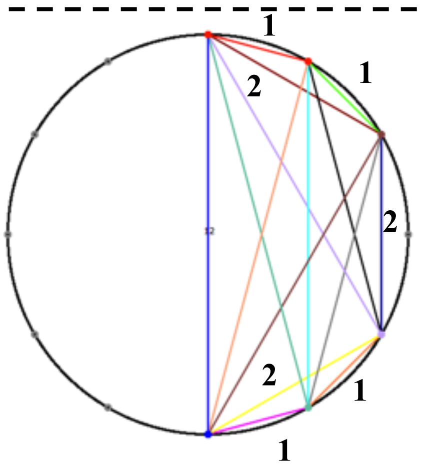


A tiles $\mathbb{Z}_c$ when equivalently:

- There exists B, $A \oplus B = \mathbb{Z}_c$
- $1_A \star 1_B = 1$
- $F_A \times F_B (t) = 1 + e^{-2i\pi t/c} + \dots e^{-2i\pi t(c-1)/c}$ (0 unless t=0)
- $Z_A \cup Z_B = \{1,2 \dots c-1\}$ AND Card A × Card B = c
- ➔ $IC_A \star IC_B = IC(\mathbb{Z}_c) = c$ and Card A × Card B = c

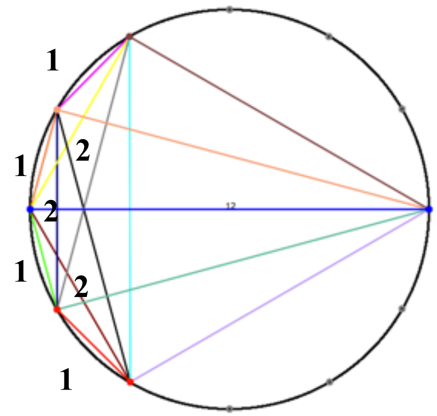
A second example of “mathemusical” problem

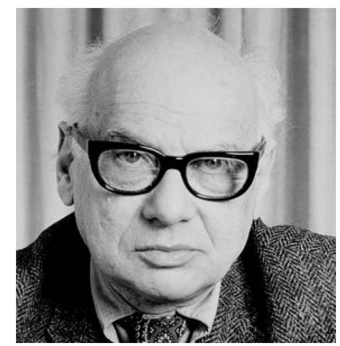




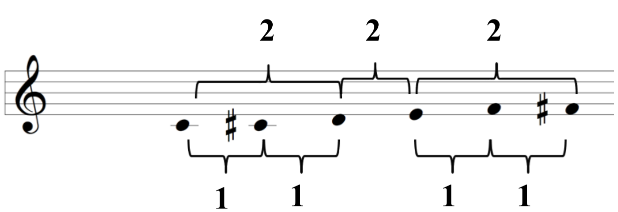
≈

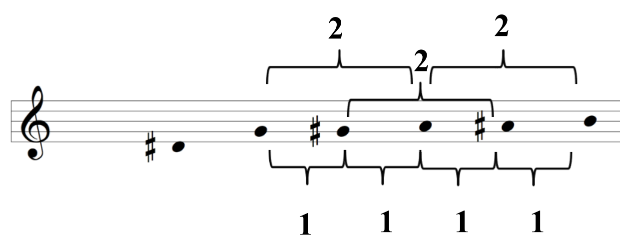
Hexachord Theorem





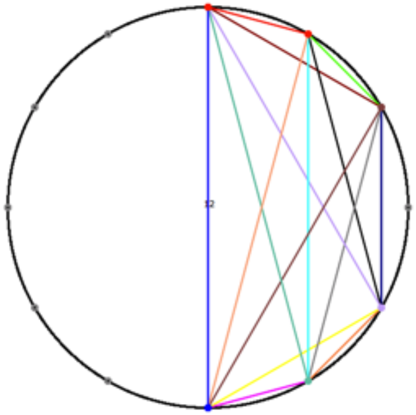
M. Babbitt





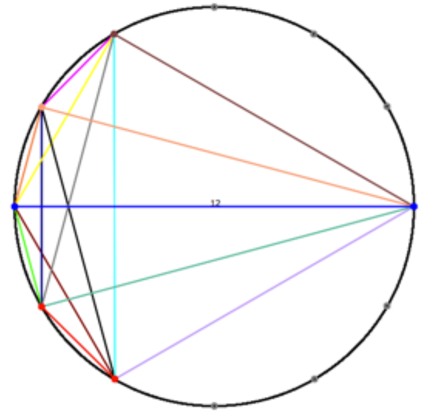
➔ **https://guichaoua.gitlab.io/web-hexachord/hexachordTheorem**

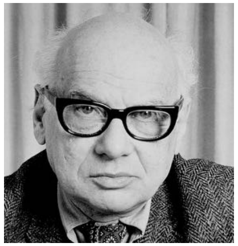
The shortest proof of Babbitt's Theorem?



$\approx$

Hexachord Theorem





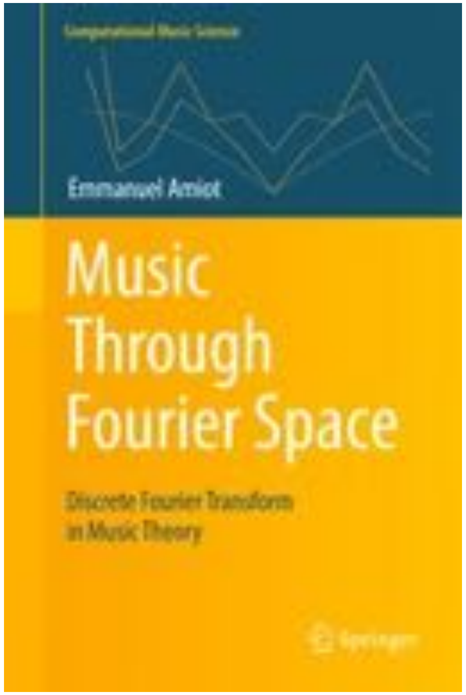
M. Babbitt

$$IC_A = [4, 3, 2, 3, 2, 1\,] = [4, 3, 2, 3, 2, 1\,] = IC_{A'}$$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

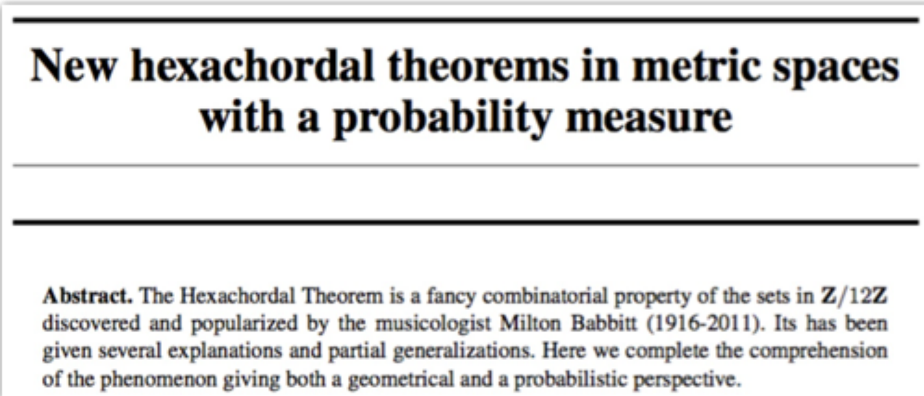
$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$\forall \mathrm{k}\ \mathcal{F}(\mathrm{IC}_{\mathbb{Z}_c \setminus A})(\mathrm{k}) = \mathcal{F}(\mathrm{IC}_A)(\mathrm{k})$$



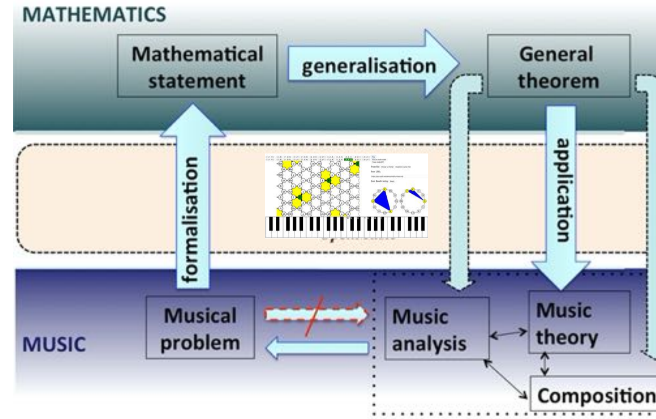
E. Amiot : « Une preuve élégante du théorème de Babbitt par transformée de Fourier discrète », Quadrature, 61, 2006.

Some generalizations of Babbitt's Theorem



New hexachordal theorems in metric spaces with a probability measure

Abstract. The Hexachordal Theorem is a fancy combinatorial property of the sets in $\mathbf{Z}/12\mathbf{Z}$ discovered and popularized by the musicologist Milton Babbitt (1916-2011). Its has been given several explanations and partial generalizations. Here we complete the comprehension of the phenomenon giving both a geometrical and a probabilistic perspective.



Theorem

Let $f : \mathbb{S}^k \mapsto [0,1]$ ($k = 1, 3$ or 7) be such that $\int_{\mathbb{S}^k} f = 1/2$ and $\bar{f} = 1 - f$. Then for every t one has

$$\int f(x)f(x \cdot t)\, dx = \int \bar{f}(x)\bar{f}(x \cdot t)\, dx.$$

Application to $f = \mathbf{1}_A$.

Continuous Hexachordal Theorems

Let $(\mathfrak{X}, d, \mu)$ be a metric measure space of mass 1. We consider two $\mathcal{X}$-valued random variables, X and Y that are μ-uniform and independent.

Definition

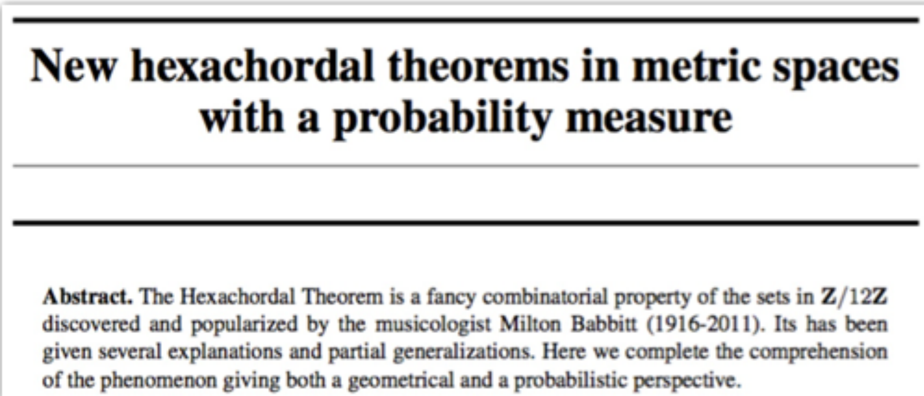
Two sets A and B of mass $1/2$ are (metrically) homometric if

$$\mathbb{P}(d(X,Y) \in \cdot | X \in A, Y \in A) = \mathbb{P}(d(X,Y) \in \cdot | X \in B, Y \in B).$$

Probabilistic version

Nicolas Juillet

Some generalizations of Babbitt's Theorem



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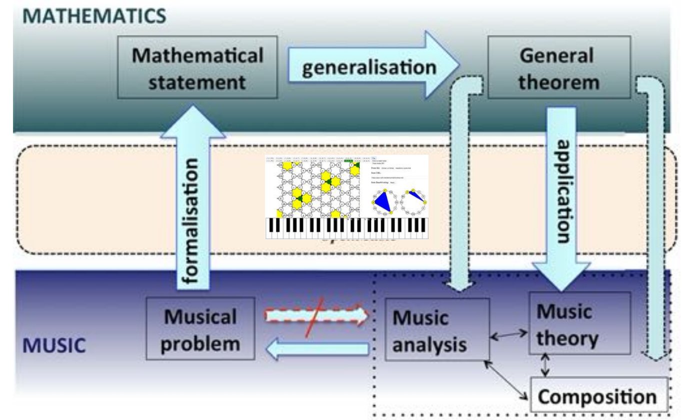


- The finite/compact spaces on which a group is transitively acting (homogeneous spaces).
  - Compact Lie groups
  - Spheres, torii, etc.
  - Cayley graphs of finite groups with generators (Hypercubes, symmetric group, etc.)
  - Other homogeneous graphs (Petersen graph, truncated icosahedron, etc.)

- **Andreatta M., C. Guichaoua, N. Juillet (2023), "Taking music seriously: on the dynamics of 'mathemusical' research with a focus on Hexachordal Theorems", to appear in SIGMA (Symmetry, Integrability and Geometry: Methods and Applications).**
- **Andreatta M., C. Guichaoua, N. Juillet (2023), "New hexachordal theorems in metric spaces with a probability measure", *Rendiconti del Seminario Matematico della Università di Padova*, vol. 150, 2023.**

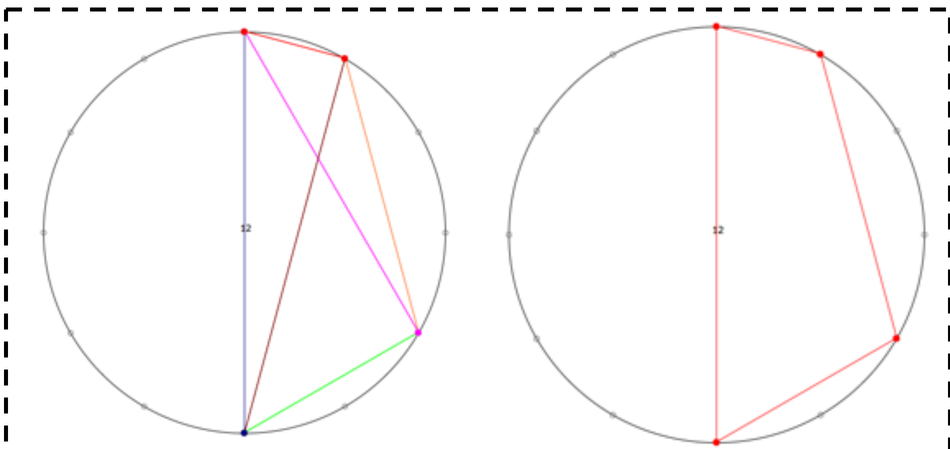
Z-relation, homometry and phase retrieval problem

- Two sets are Z-related if they have the same module of the DFT

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

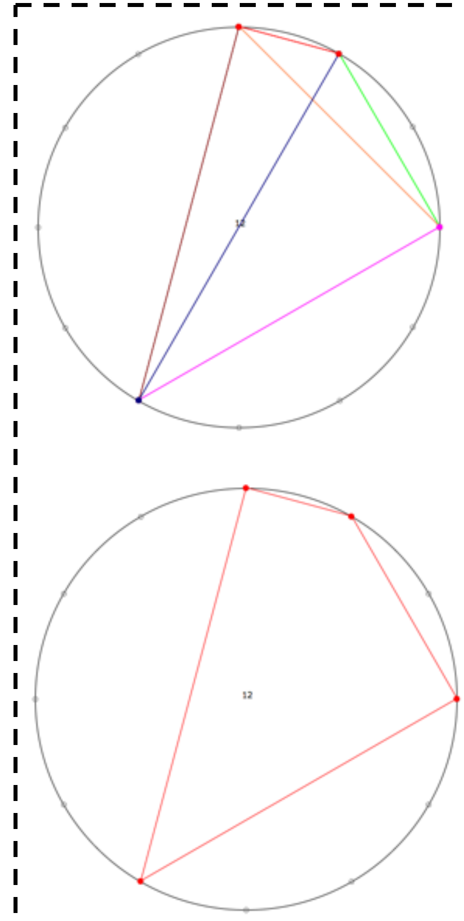
$$\mathcal{F}(IC_A) = \mathcal{F}_A \times \mathcal{F}_{-A} = |\mathcal{F}_A|^2$$



Z-relation

$\longleftrightarrow$

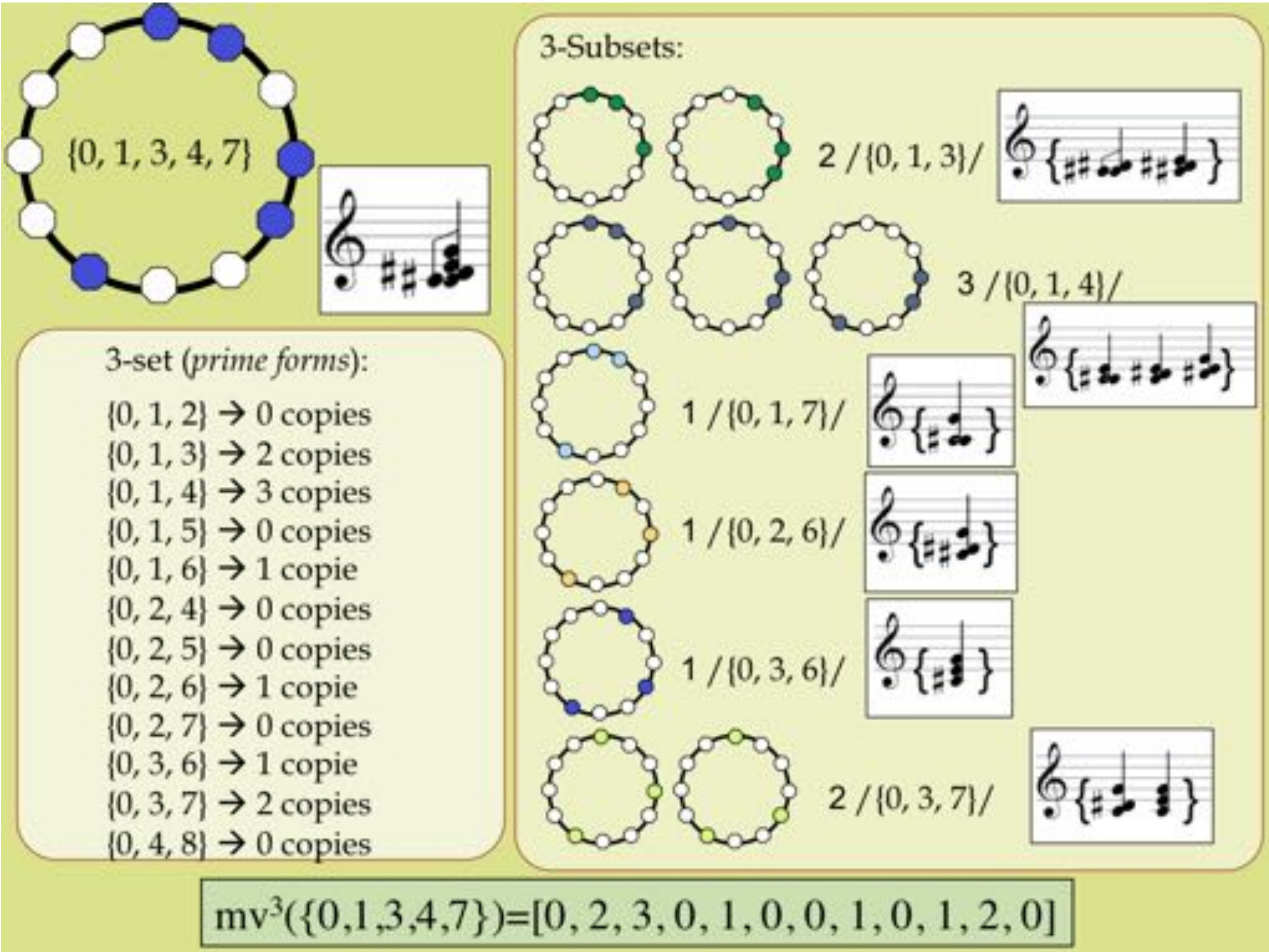
homometry



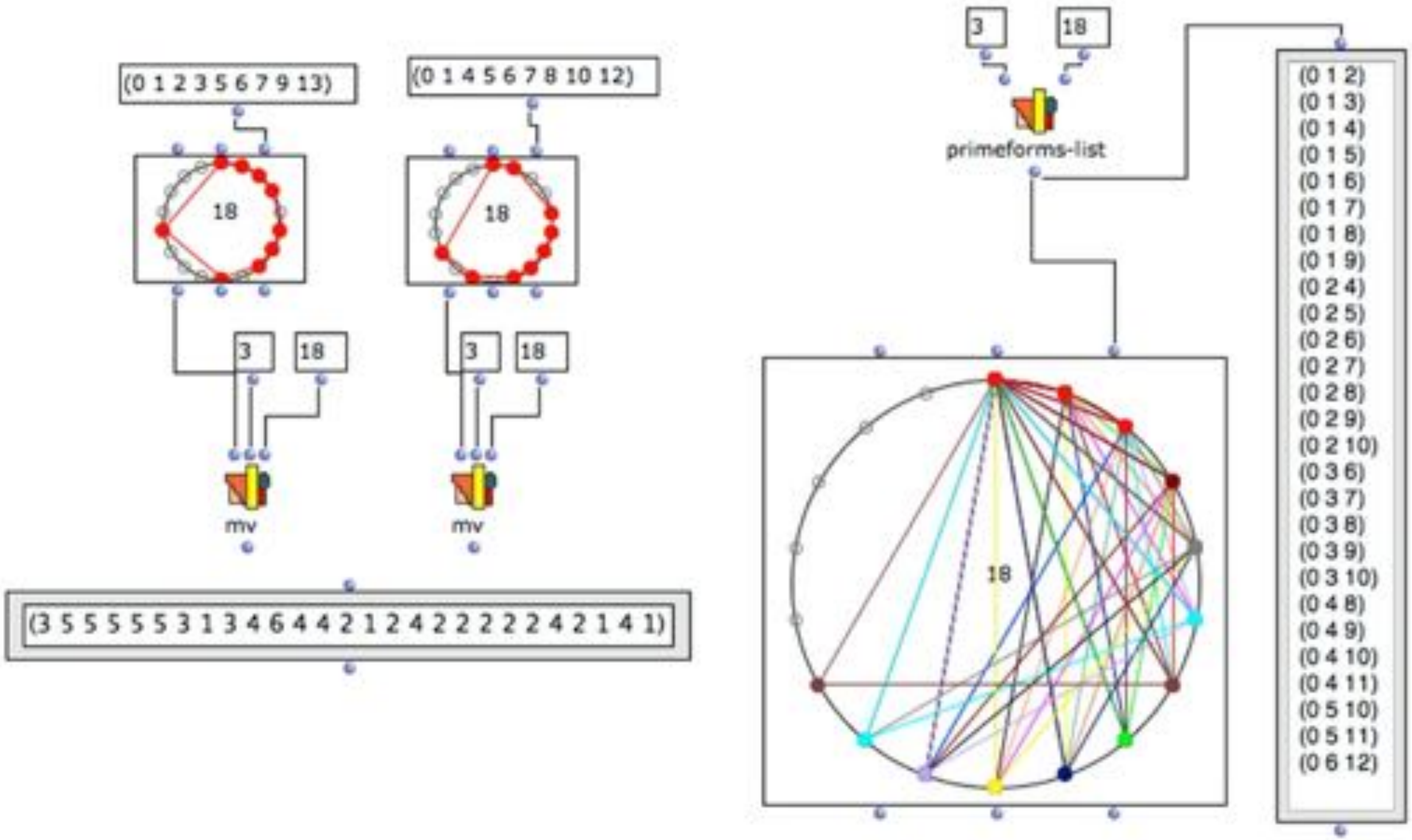
• **Mandereau J., D. Ghisi, E. Amiot, M. Andreatta, C. Agon, (2011), « Z-relation and homometry in musical distributions »,** ***Journal of Mathematics and Music*****, vol. 5, n° 2, p. 83-98.**

• **Mandereau J, D. Ghisi, E. Amiot, M. Andreatta, C. Agon (2011), « Discrete phase retrieval in musical structures »,** ***Journal of Mathematics and Music*****, vol. 5, n° 2, p. 99-116.**

High-order ‘interval’ content: Lewin’s mv^k vector

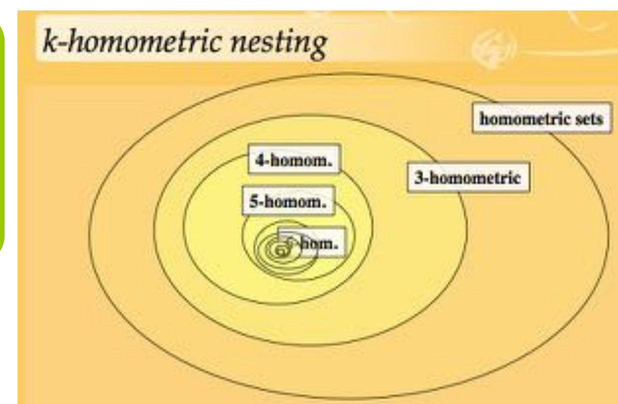


High-order Z-relation and *k*-homometric nesting



$\mathbb{Z}_{18}$ $A = \{0, 1, 2, 3, 5, 6, 7, 9, 13\}$
$B = \{0, 1, 4, 5, 6, 7, 8, 10, 12\}$

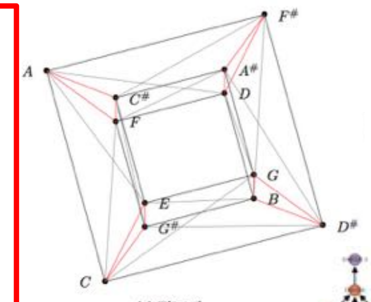
$mv^3(A) = mv^3(B)$ ➔ 3-homometry (➔ 2-homometry)



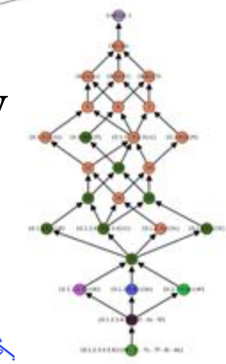
Where does this nesting stop?

The SMIR Project: Structural Music Information Research

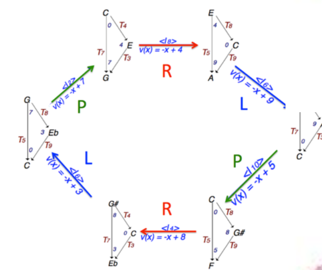
- **Generalized Tonnetze and Persistent Homology**
  - The Tonnetz as a simplicial complex
  - Algebraic classification of the twelve possible Tonnetze
  - Isotropic and anisotropic Tonnetze
  - Application to automatic stylistic classification



- **Formal Concept Analysis and Mathematical Morphology**
  - Lattice structure of formal concepts
  - Derivation operators (in FCA) and dilation/erosion in MM
  - Application to pattern recognition and extraction



- **Category theory and Transformational Theory**
  - From K-nets to PK-nets
  - Diatonicism



- **'Mathemusical' problems and open conjectures**
  - Tiling rhythmic canons and Fuglede Spectral Conjecture
  - Homometric musical structures

- **Philosophy, Epistemology and Cognitive Science**
  - Geometry-based Neo-structuralism in music analysis
  - Processes and techniques of mathemusical learning

Generalized Tonnetze et Persistent Homology

Département de Mathématiques

Mémoire de Master 2

Homologie persistante appliquée à la classification automatique des styles musicaux

Printemps 2023

Maximilien Wang

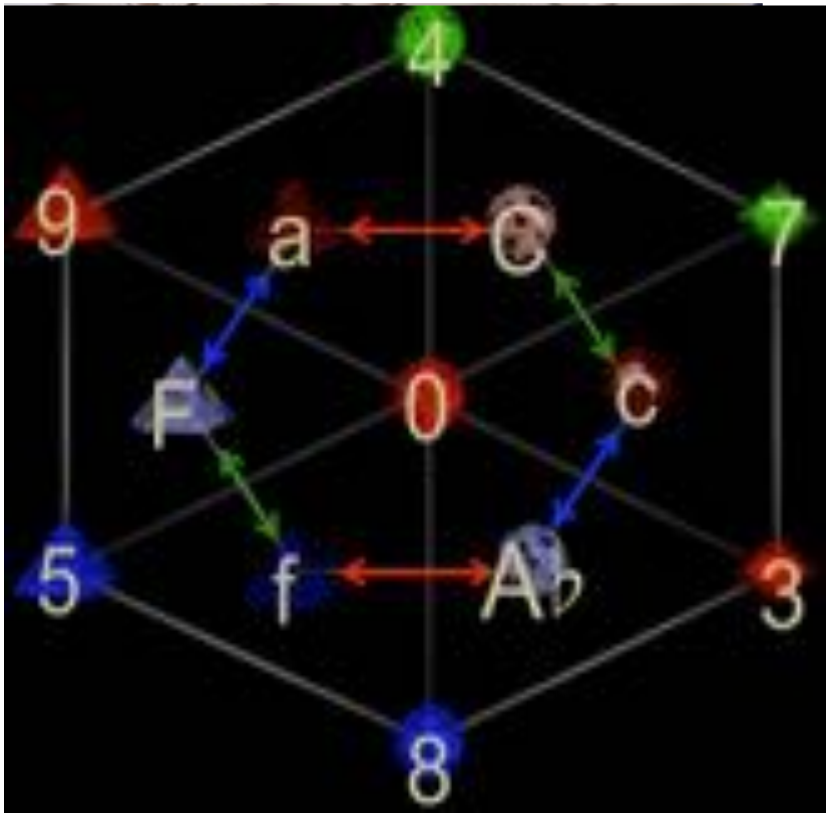
Directeur de mémoire : Moreno Andreatta, IRMA,Université de Strasbourg

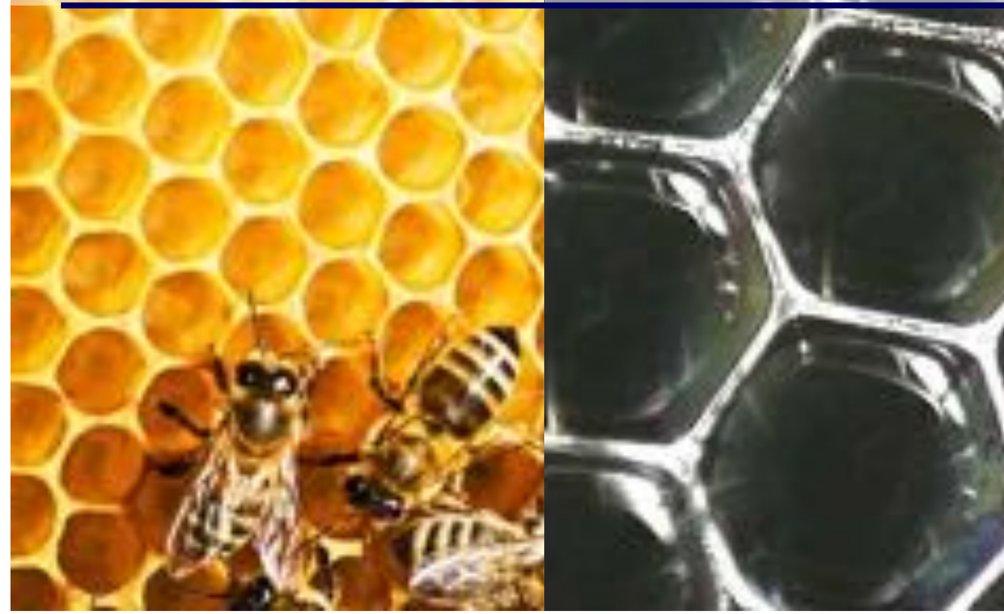
Topological modeling of musical structures and processes

Victoria Callet

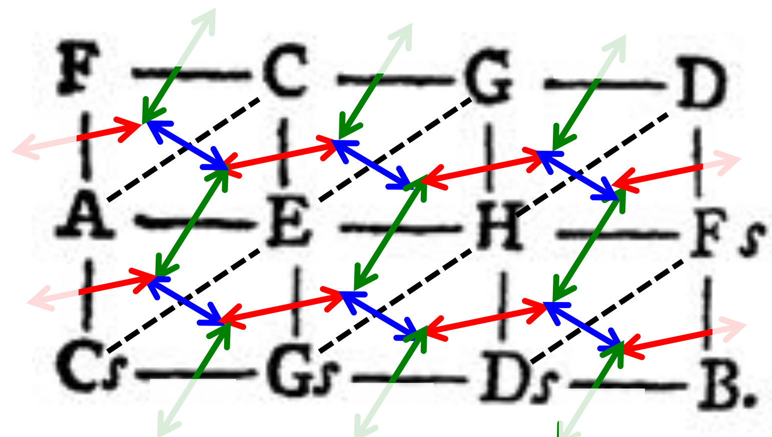
December 2023

The *Tonnetz*
(or 'honeycomb' hexagonal tiling)



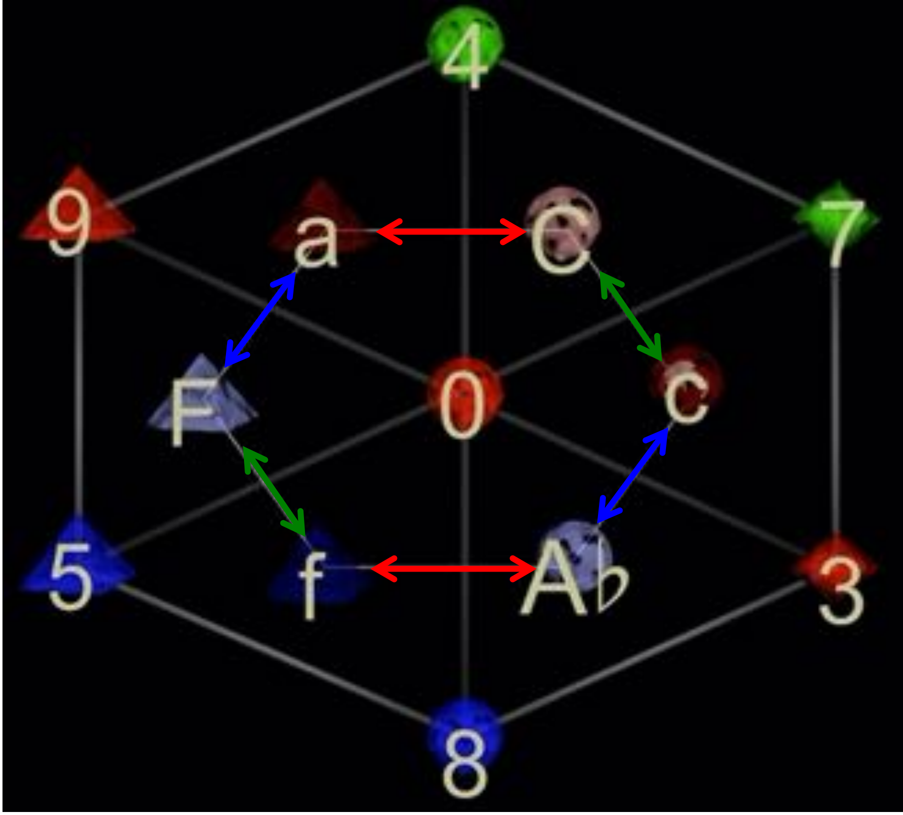


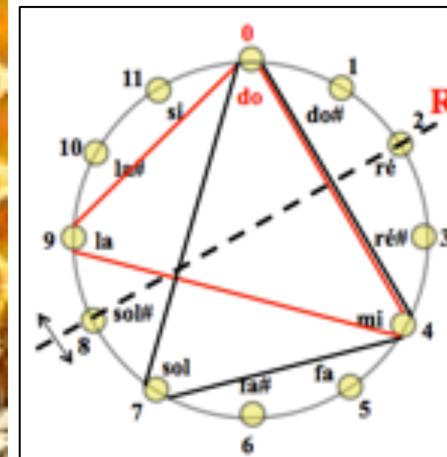
Leonhard Euler

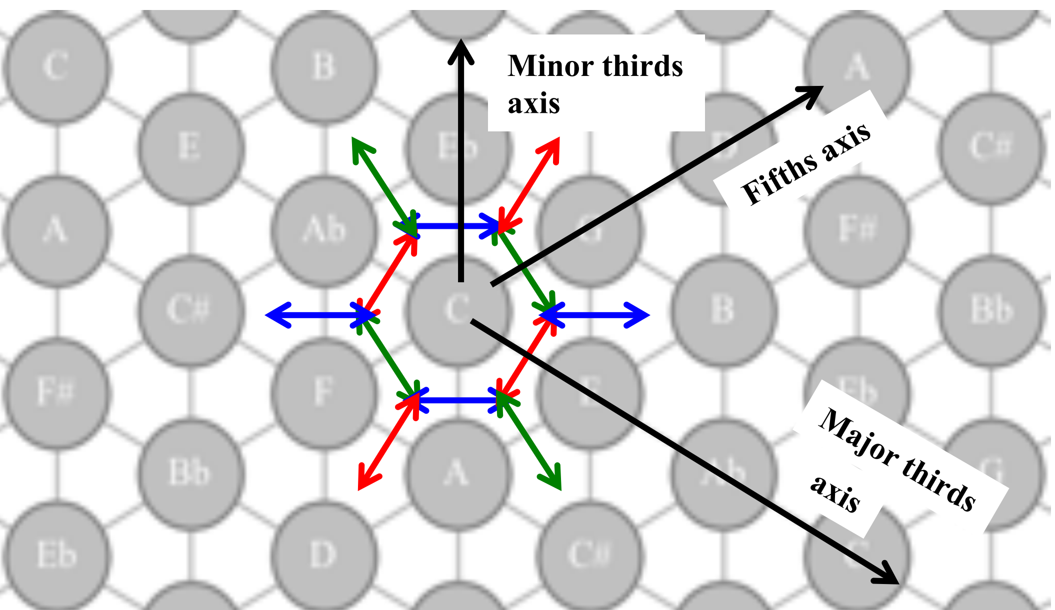


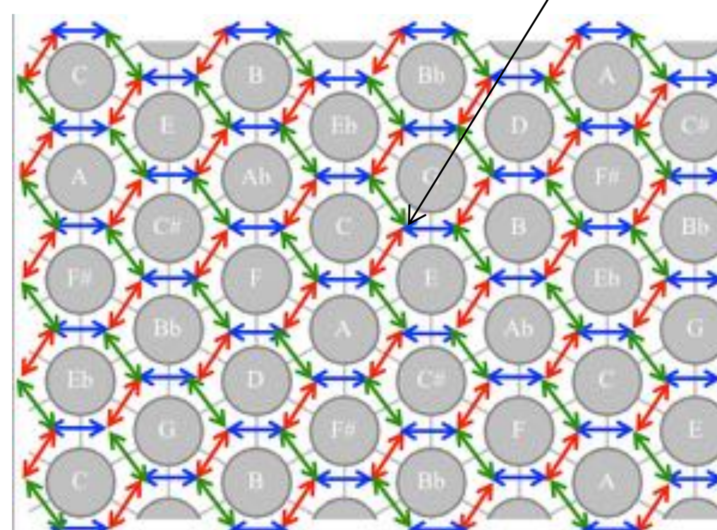
Speculum Musicum (1773)

The *Tonnetz*
(or ‘honeycomb’ hexagonal tiling)

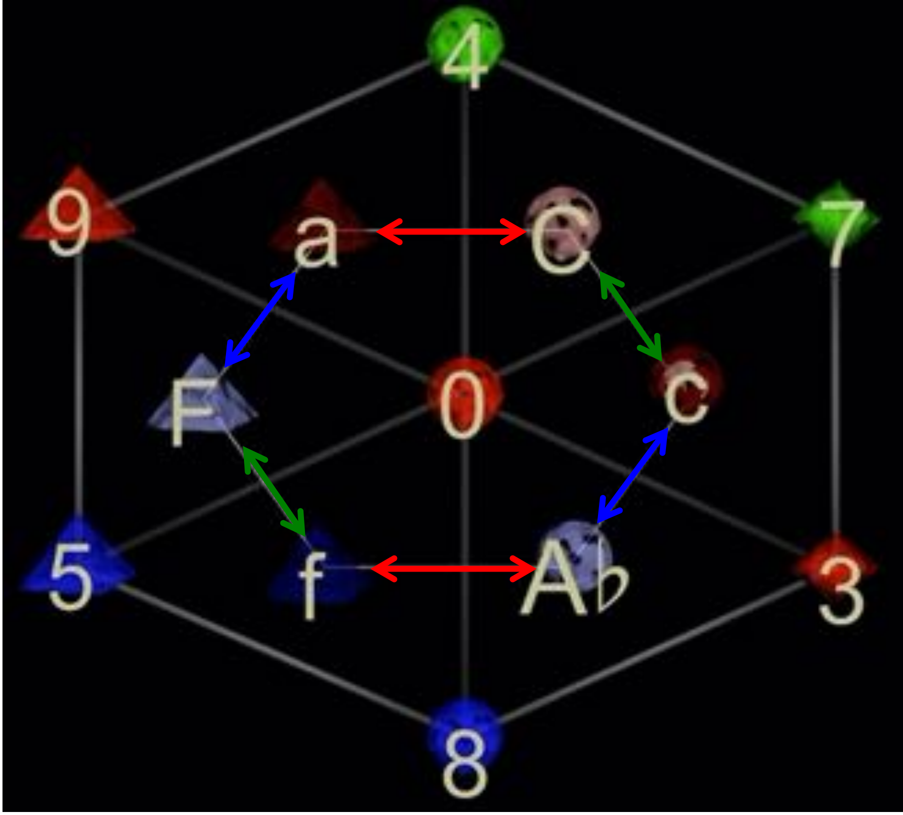


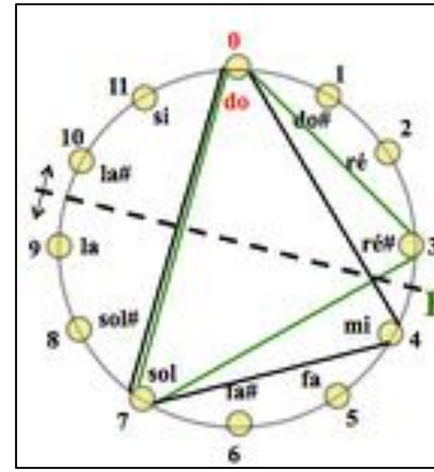


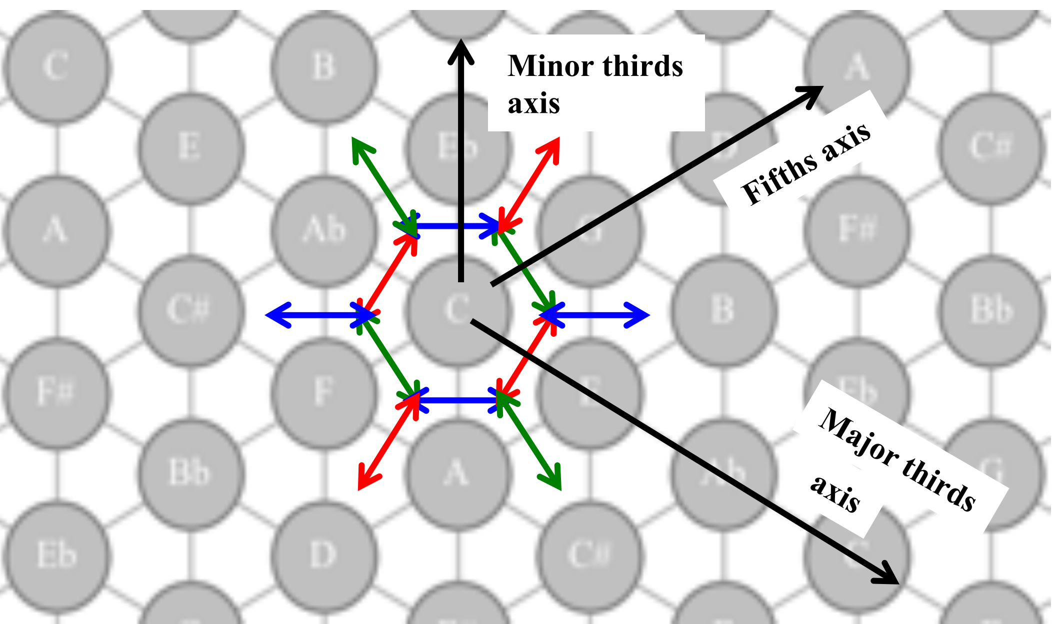


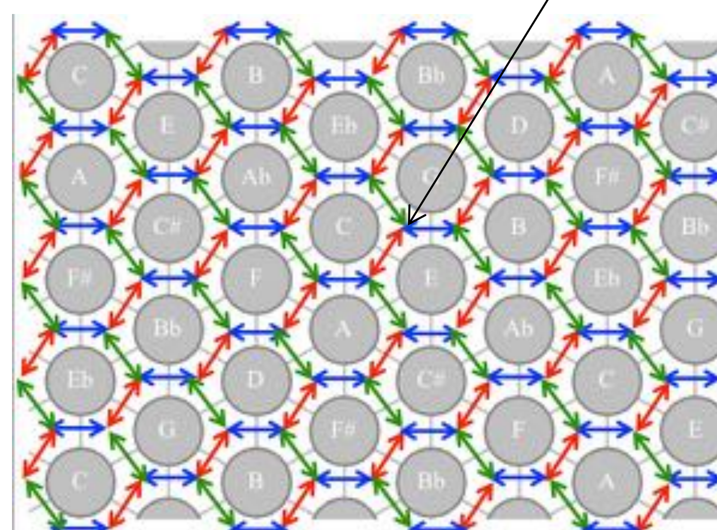


The *Tonnetz*
(or ‘honeycomb’ hexagonal tiling)









The *Tonnetz*
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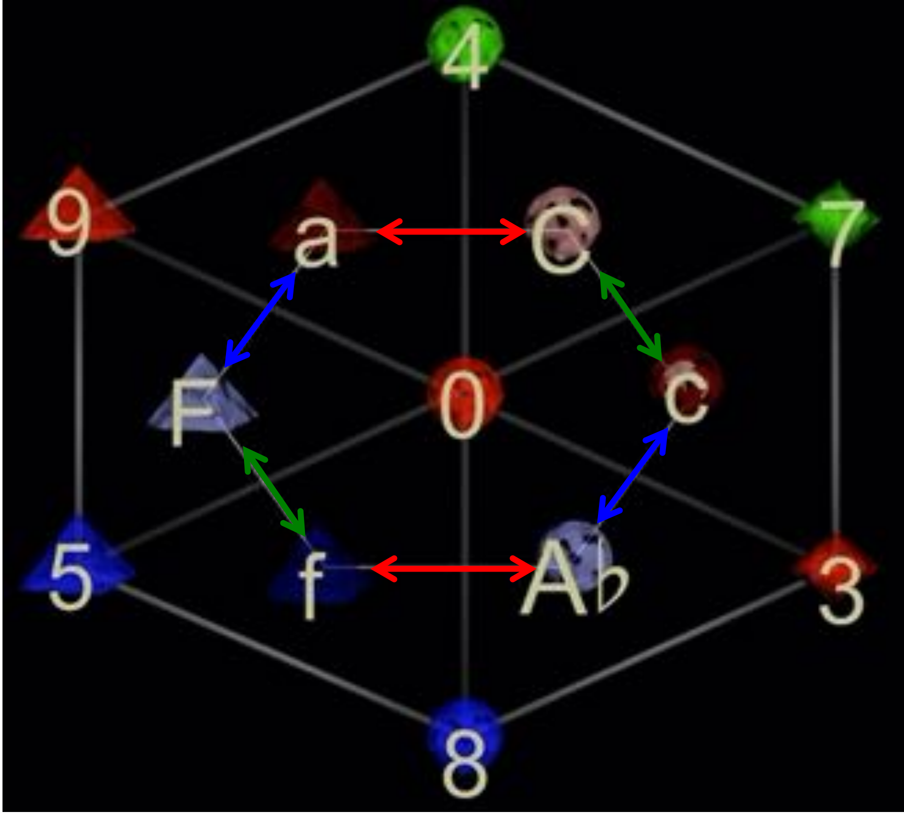
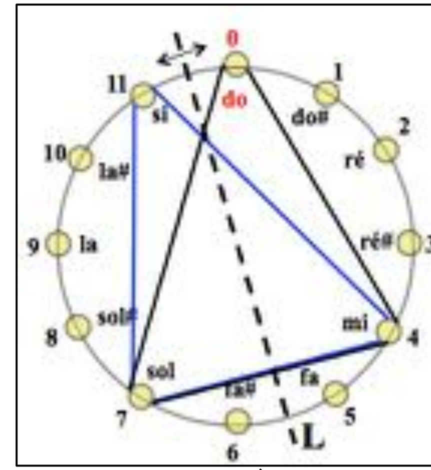
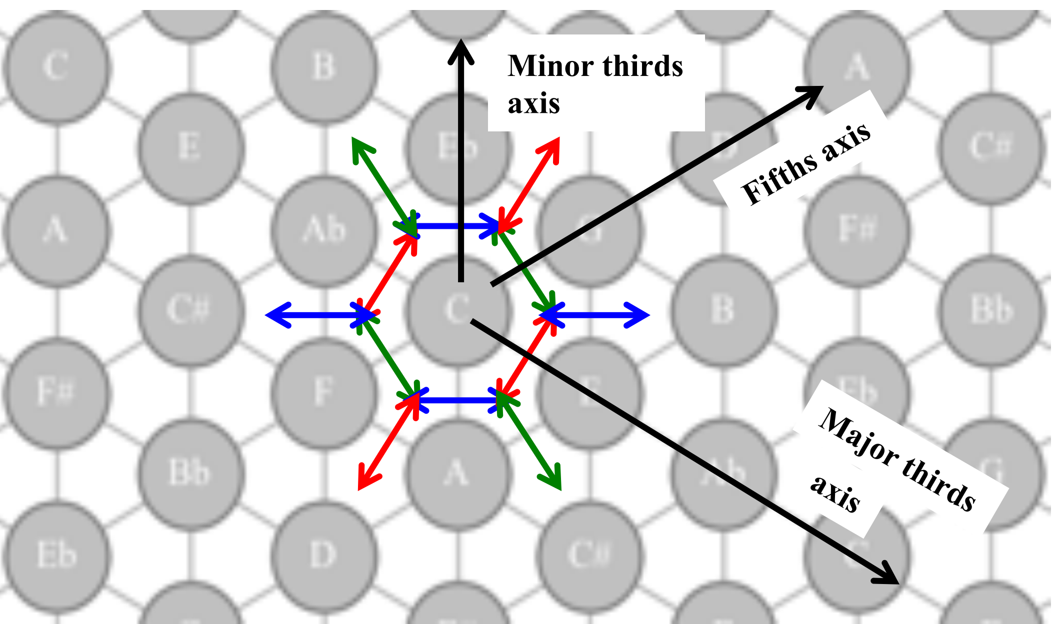
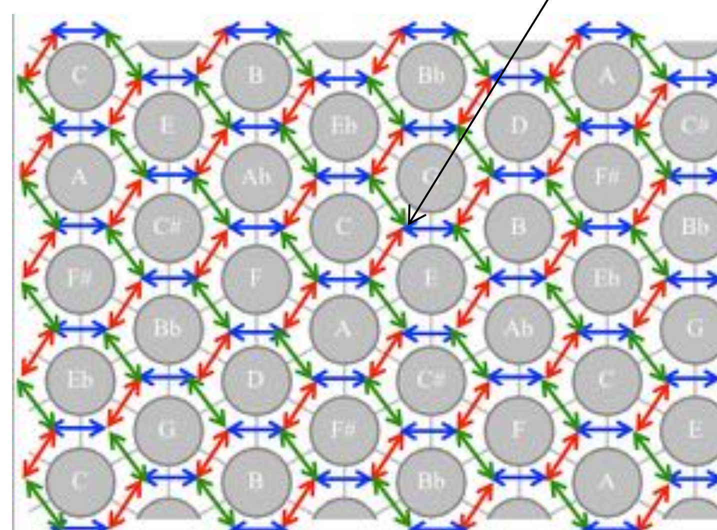
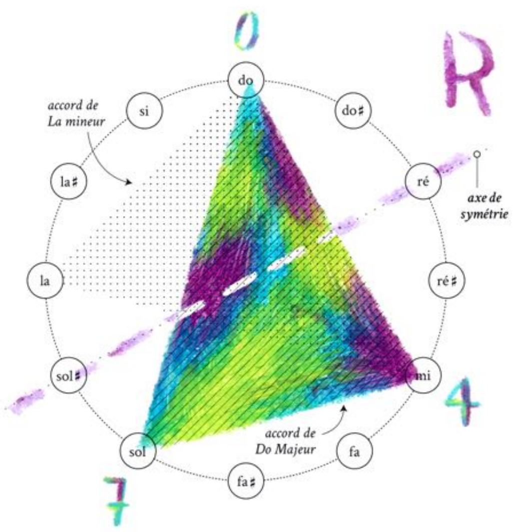


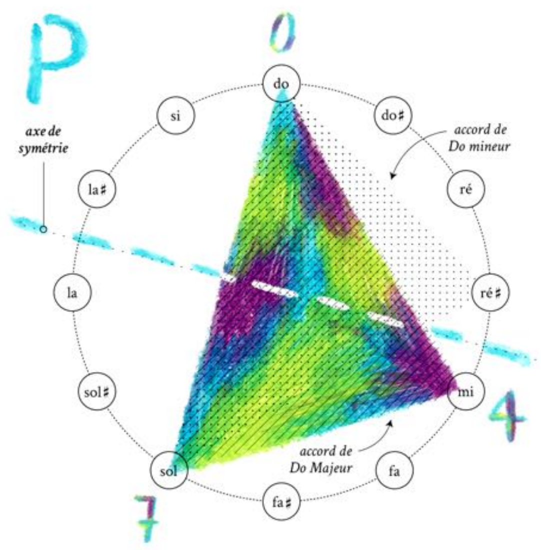


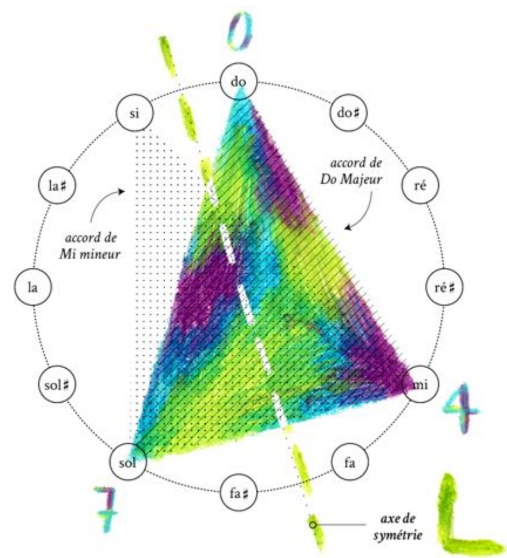




The three main major-minor symmetries







R as RELATIVE | **P as PARALLEL** | **L as LEADING-TONE (EXCHANGE)**

C major ↕ A minor

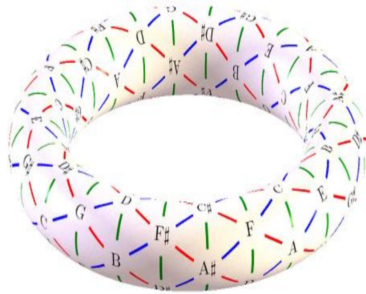
C major ↕ C minor

C major ↕ E minor

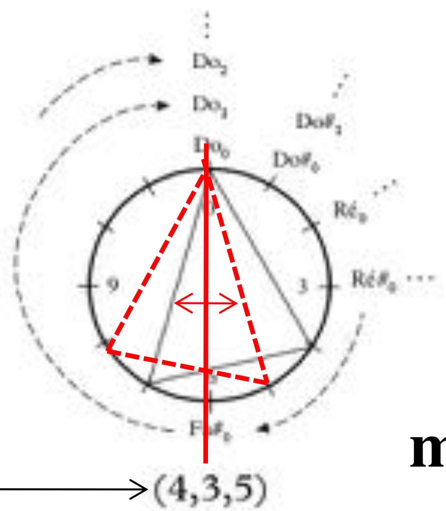
The *Tonnetz* as a simplicial complex

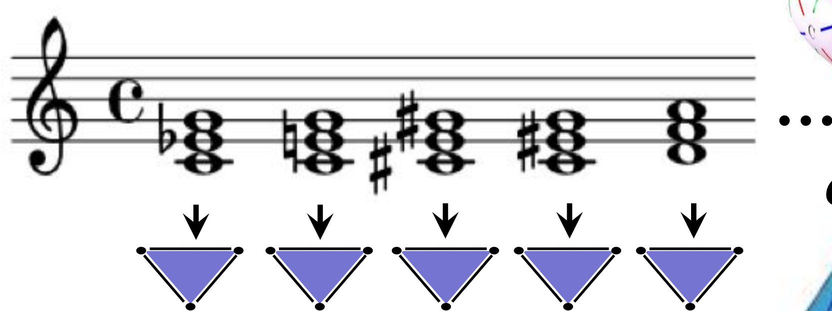
L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- Assembling chords related by some equivalence relation
  – Equivalence up to transposition/inversion:

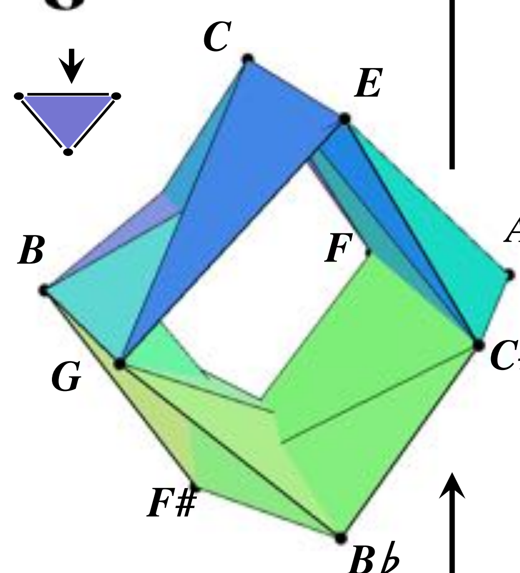


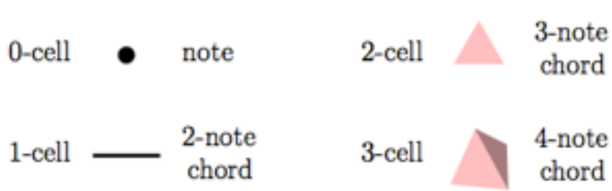
Intervallic structure

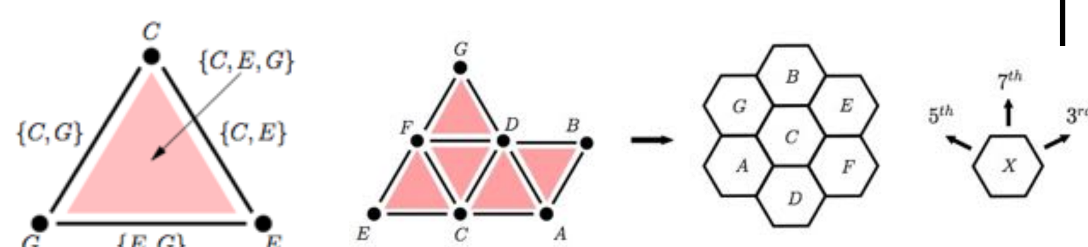




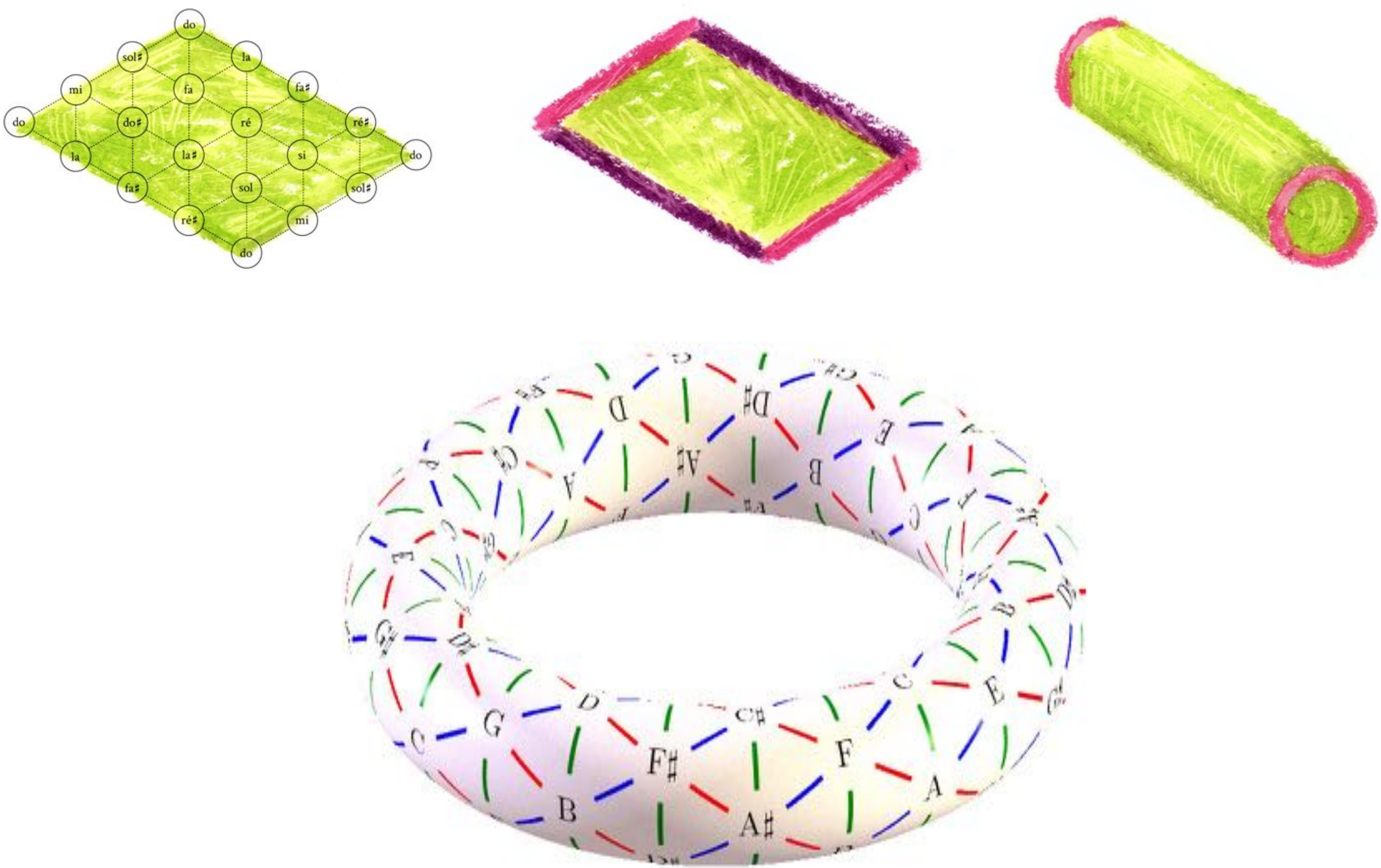
major/minor triads







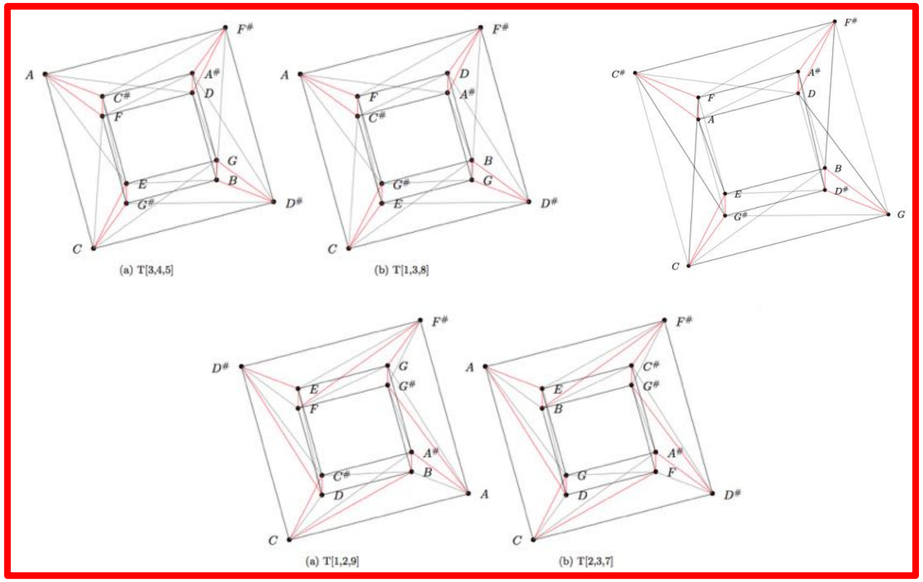
The topological structure of the *Tonnetz*



(Source: www.wikimedia.org/)

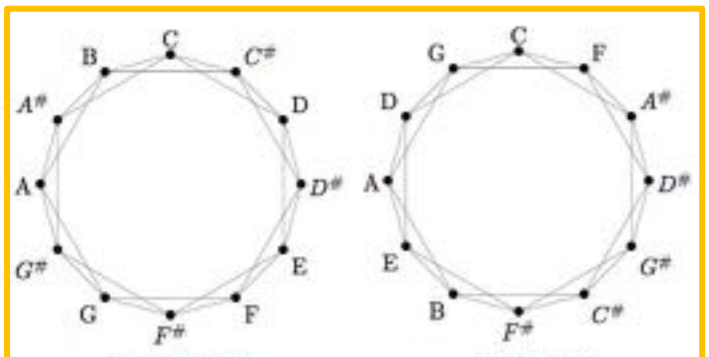
The *Tonnetze* as simplicial complexes

Tonnetz	Betti Numbers		
	β_0	β_1	β_2
$T[1,2,9]$	1	2	1
$T[1,3,8]$	1	2	1
$T[1,4,7]$	1	2	1
$T[2,3,7]$	1	2	1
$T[3,4,5]$	1	2	1
$T[1,1,10]$	1	1	0
$T[2,5,5]$	1	1	0
$T[2,2,8]$	2	2	0
$T[1,5,6]$	1	1	6
$T[2,4,6]$	2	2	6
$T[3,3,6]$	3	0	3
$T[4,4,4]$	4	0	0

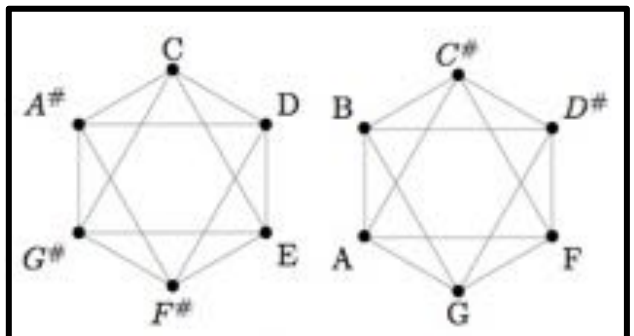

(a) T[3,4,5] (b) T[1,3,8]
(a) T[1,2,9] (b) T[2,3,7]


L. Bigo

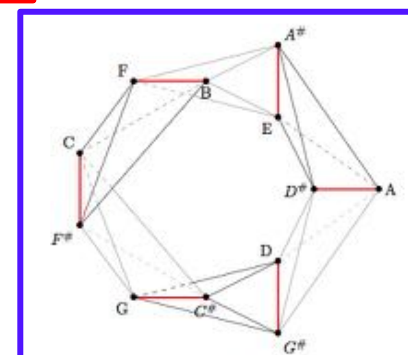

P. Lascabettes

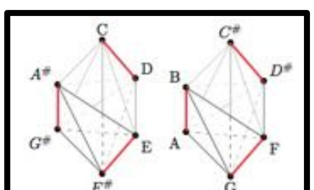


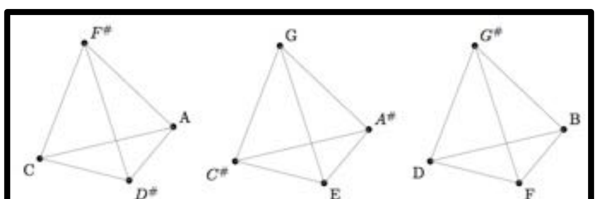
(a) $T[1,1,10]$ (b) $T[2,5,5]$

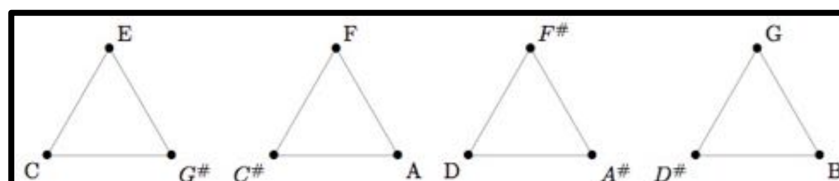


(c) $T[2,2,8]$


(a) $T[1,5,6]$


(b) $T[2,4,6]$


(a) $T[3,3,6]$

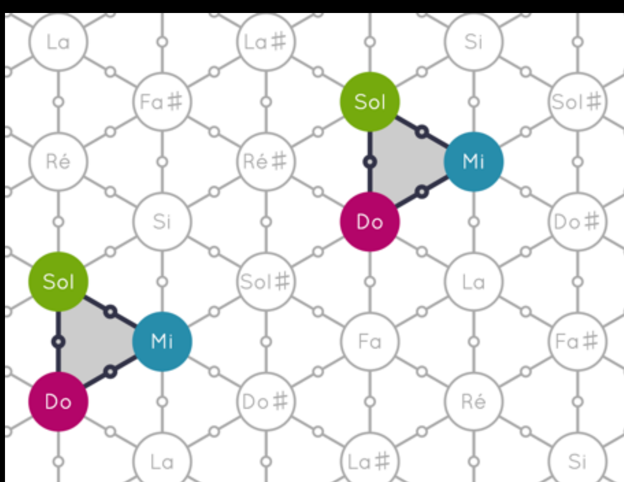


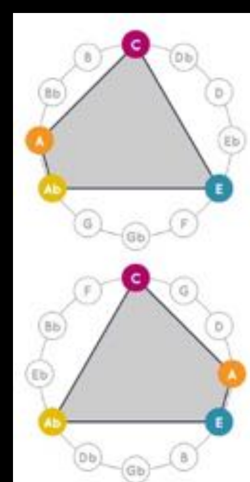
(b) T[4,4,4]

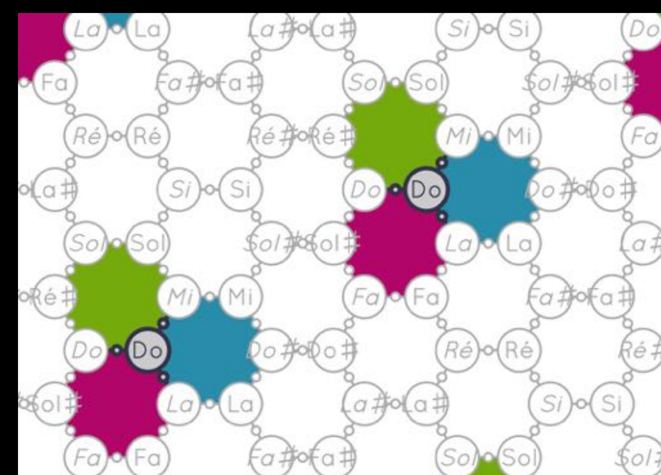
The generalized Tonnetz environment

www.morenoandreatta.com/software/

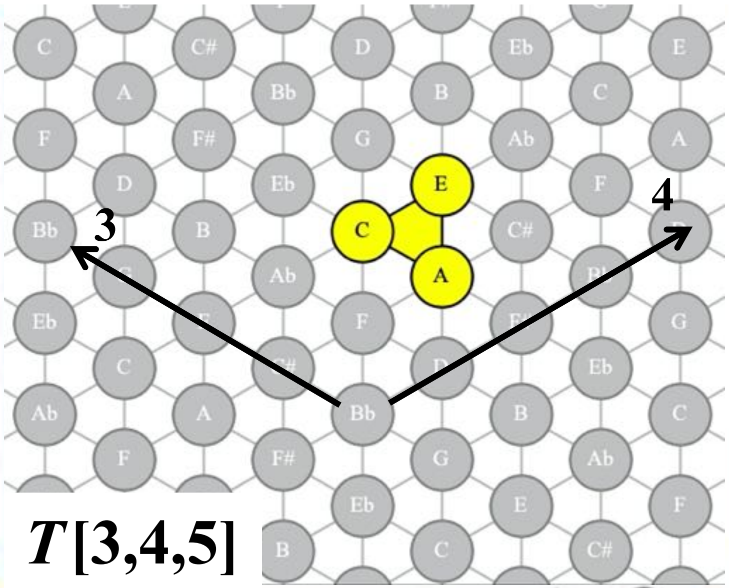
DEMO



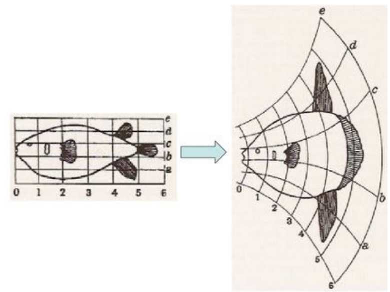


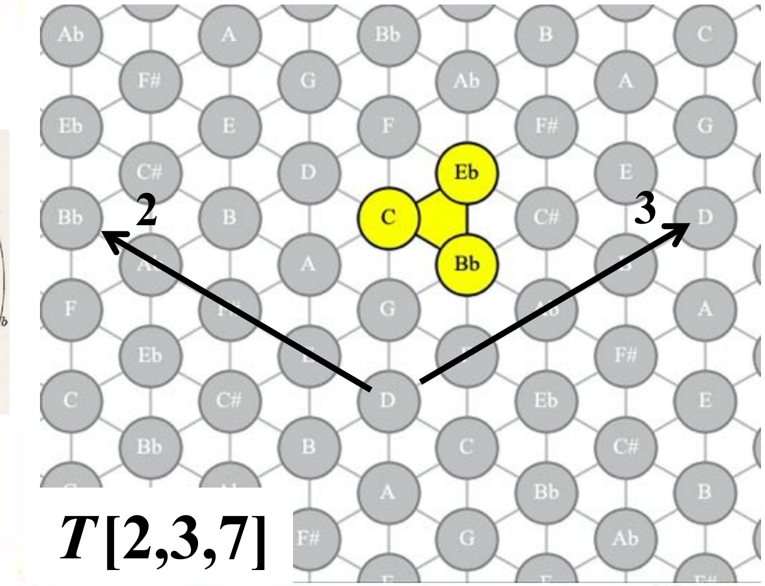


Musical style and space trajectories

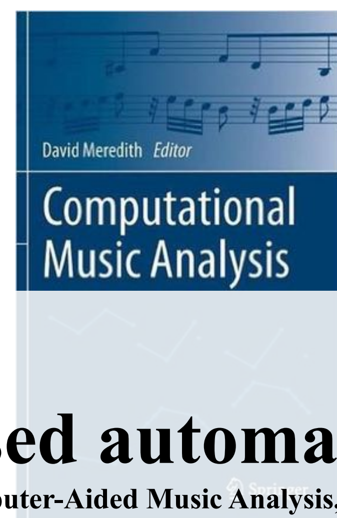


T[3,4,5]





T[2,3,7]

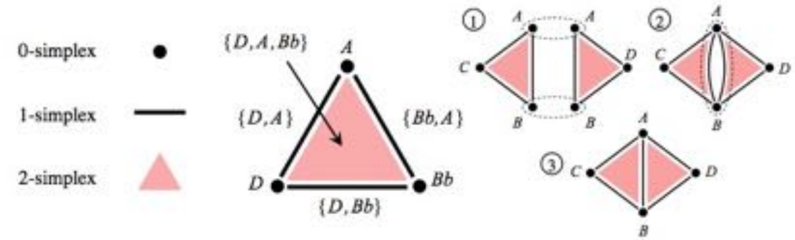


Towards a geometry-based automatic musical style analysis

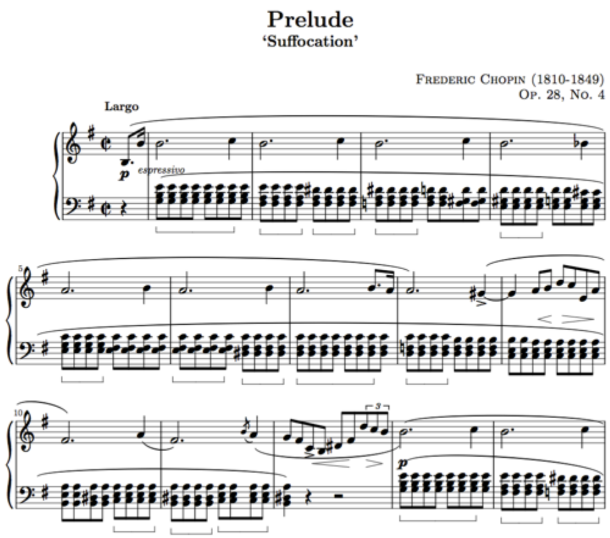
Bigo L., M. Andreatta (2015), Topological Structures in Computer-Aided Music Analysis, in D. Meredith (ed.), *Computational Music Analysis*, Springer

Towards a topological signature of a musical piece
A structural approach in Music Information Retrieval

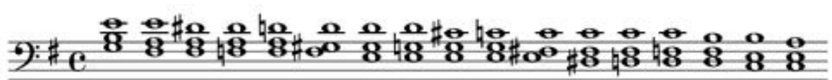
The simplices and their self-assembly



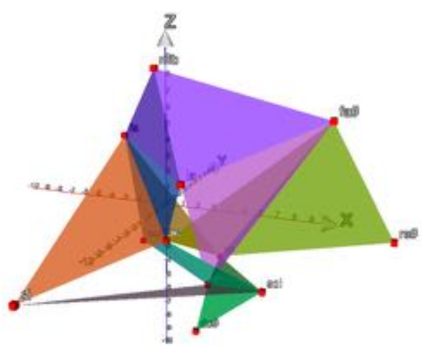
The score

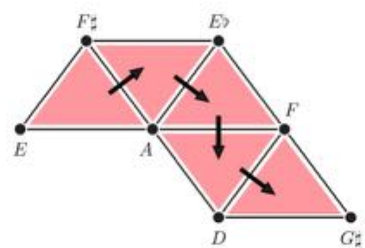


Score reduction



The simplicial complex generated by the piece



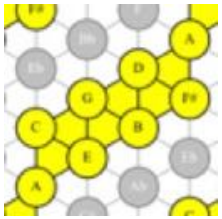


A specific trajectory in the complex

Topological signature?

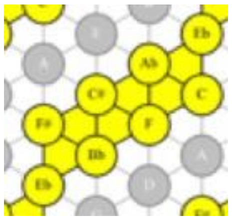
Towards an anisotropic *Tonnetz*

Mattia Bergomi

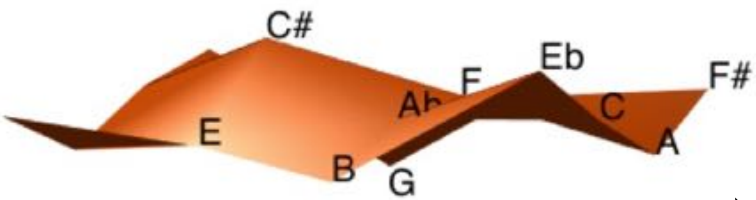


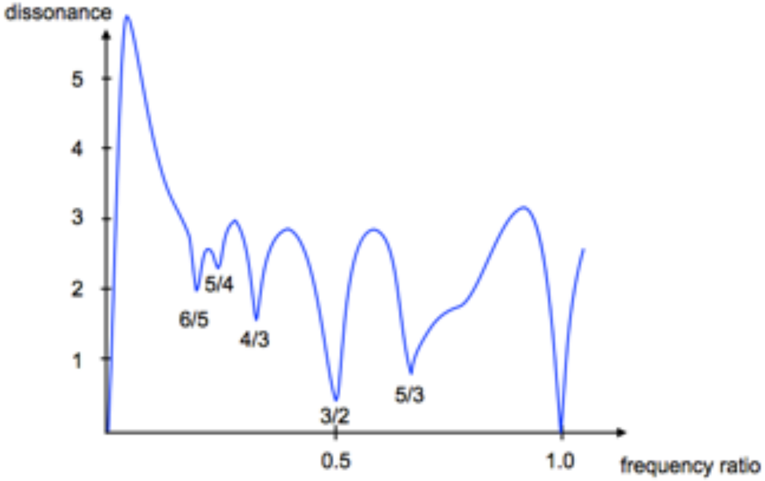
Ionian mode

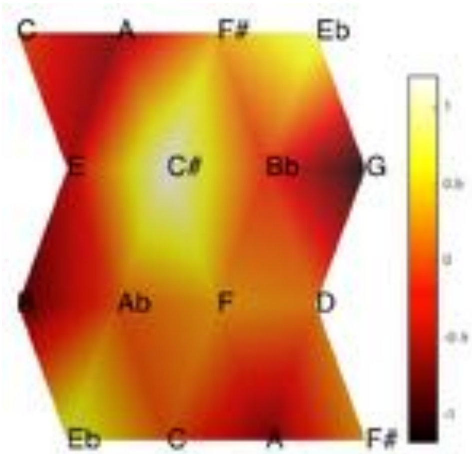
=

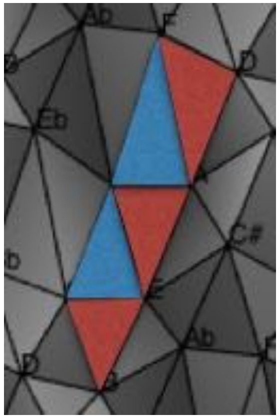


Locrian mode



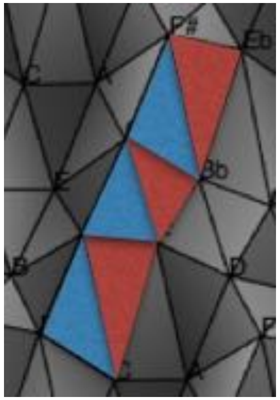






Ionian mode

≠



Locrian mode

Persistent homology and music

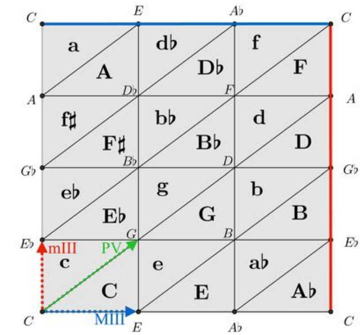

Mattia Bergomi

Homological persistence in time series: an application to music classification

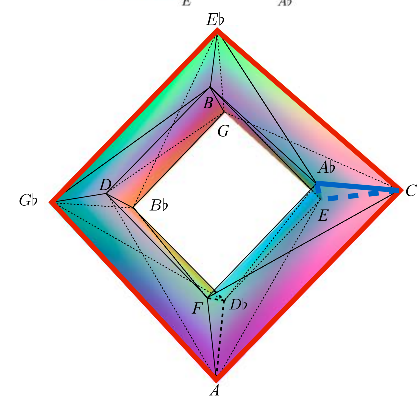
Mattia G. Bergomi[a*] and Adriano Baratè[b]

[a]Veos Digital, Milan, Italy; [b]Music and Computer Science Laboratory, University of Milan, Milan, Italy

(*Received 22 May 2019; accepted 2 June 2020*)



Meaningful low-dimensional representations of dynamical processes are essential to better understand the mechanisms underlying complex systems, from music composition to learning in both biological and artificial intelligence. We suggest to describe time-varying systems by considering the evolution of their geometrical and topological properties in time, by using a method based on persistent homology. In the static case, persistent homology allows one to provide a representation of a manifold paired with a continuous function as a collection of multisets of points and lines called persistence diagrams. The idea is to fingerprint the change of a variable-geometry space as a time series of persistence diagrams, and afterwards compare such time series by using dynamic time warping. As an application, we express some music features and their time dependency by updating the values of a function defined on a polyhedral surface, called the *Tonnetz*. Thereafter, we use this time-based representation to automatically classify three collections of compositions according to their style, and discuss the optimal time-granularity for the analysis of different musical genres.

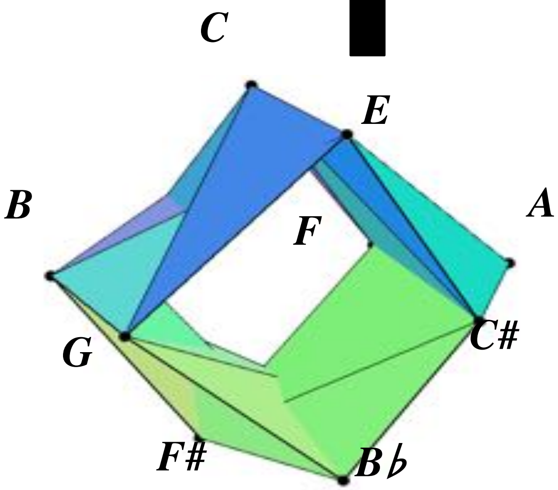


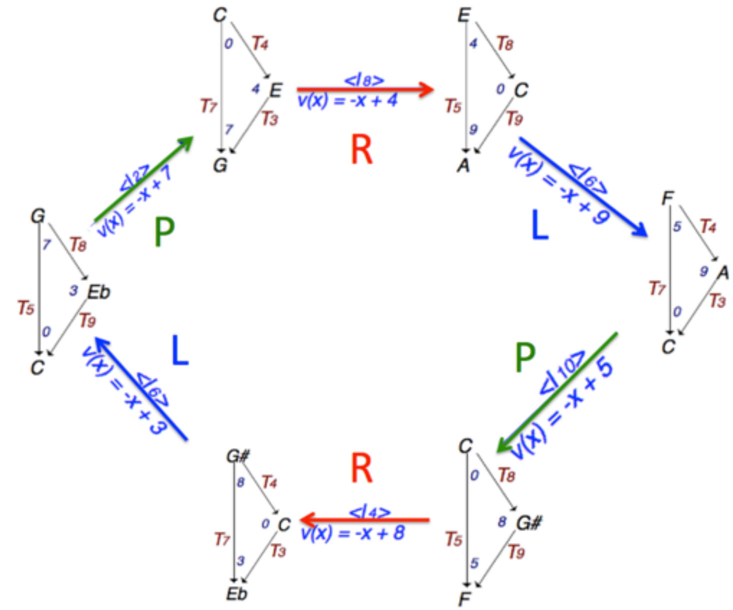
Keywords: *Tonnetz*; topology; time-series analysis; persistent homology; dynamic time warping; classification; style

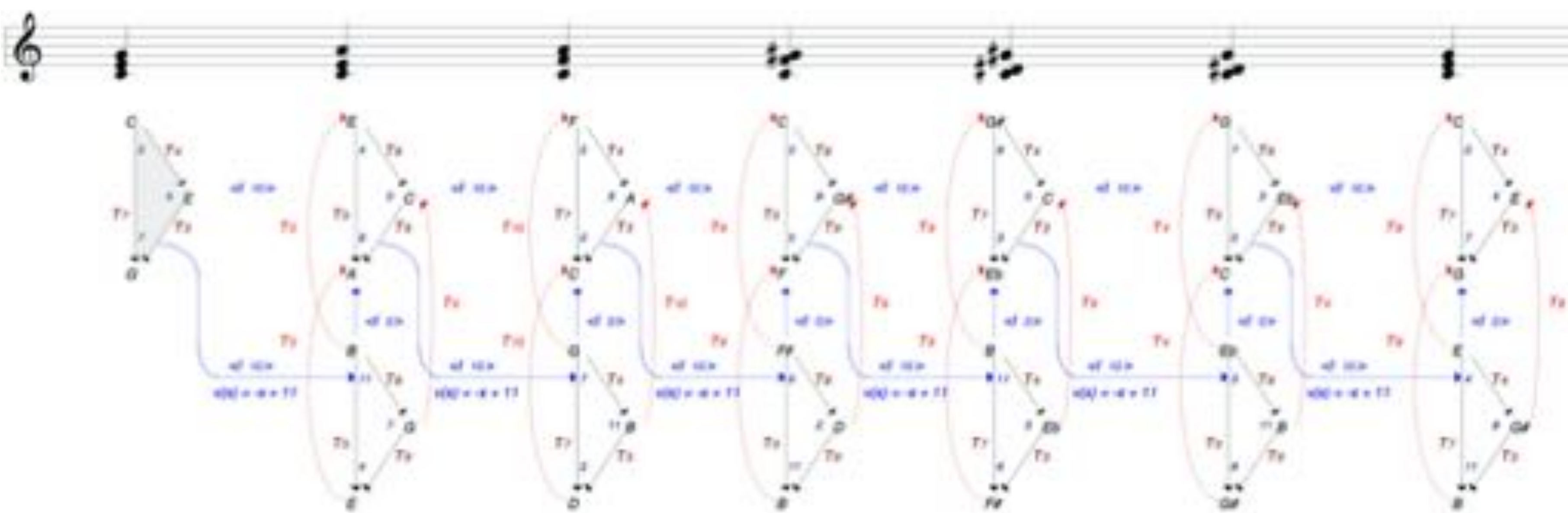
- **Mattia Bergomi, *Dynamical and topological tools for (modern) music analysis,* Sorbonne/LIM Milan, 2015.**
- **Mattia Bergomi, "Homological persistence in time series: an application to music classification", Journal of Mathematics and Music, Vol. 14, Nr. 2, pp. 204-221, 2020 (Special Issue on Geometry and Topology in Music; Guest Editors: M. Andreatta, E. Amiot, and J. Yust).**

➔ **Victoria Callet**, *Modélisation topologique de structures et processus musicaux*, ongoing PhD, Université de Strasbourg

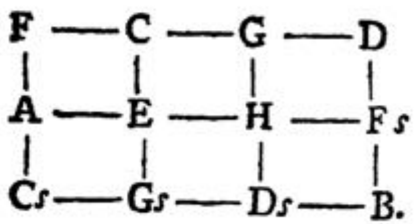
Mixing Algebra, Topology and Category Theory



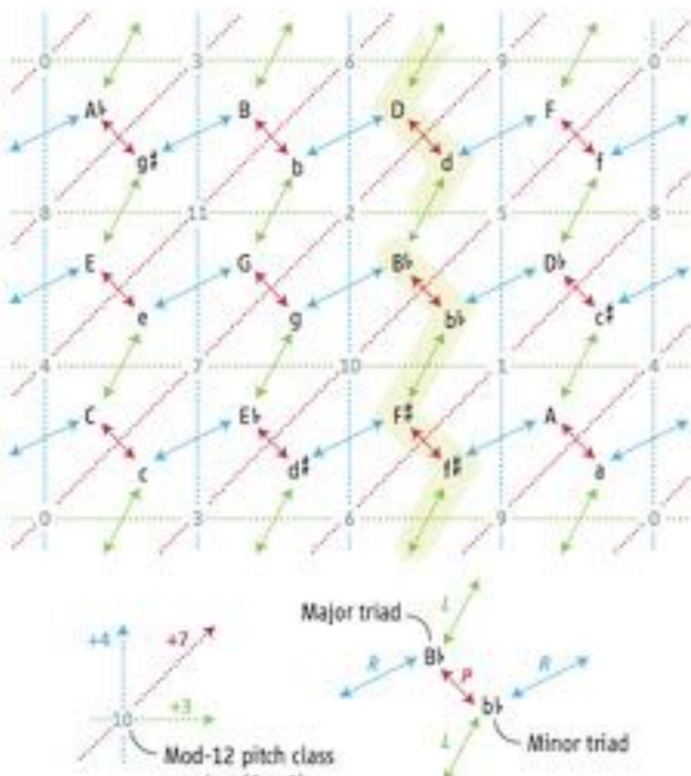




The Tonnetz and the dihedral group as dual actions



Euler : *Speculum musicum*, 1773



[Hook, *Science*, 2006]

[Thomas Noll 2004]

$\rho = \langle L,R \mid L^2 = (LR)^{12} = 1$
$LRL = L(LR)^{-1} \rangle$

$$\rho \subseteq C_{Sym}(D_{12})$$

$$D_{12} \subseteq C_{Sym}(\rho)$$

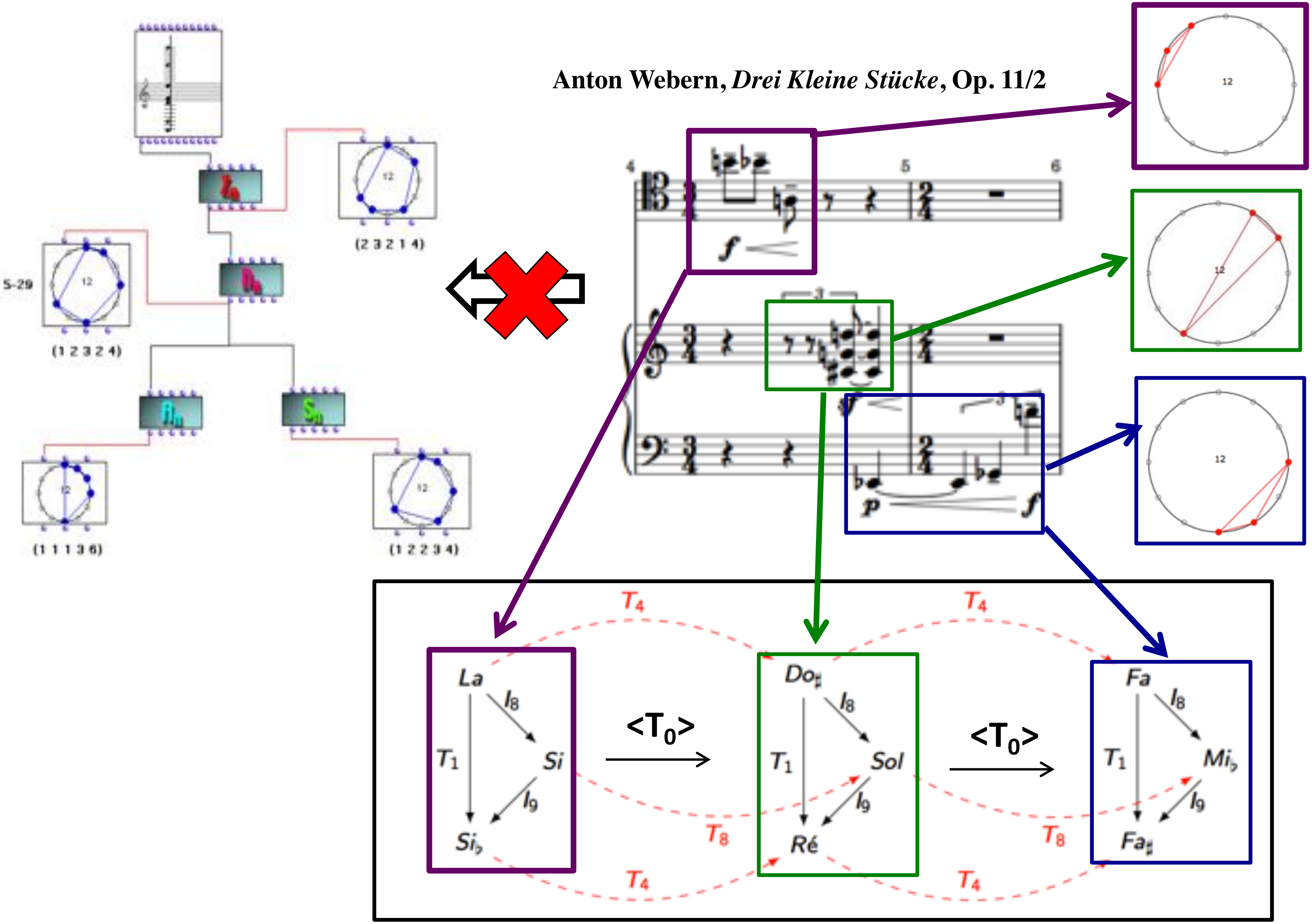
$D_{12} = \langle I, T \mid I^2 = T^{12} = 1$
$ITI = I(IT)^{-1} \rangle$

$$\begin{tikzcd} \circ \arrow[r, "f"] \arrow[d, "g"] & \circ \arrow[d, "g"] \\ \circ \arrow[r, "f"] & \circ \end{tikzcd}$$

$\forall f \in D_{12}$
$\forall g \in \rho$

Crans A., Fiore T., and Satyendra R. "Musical Actions of Dihedral Groups." *The American Mathematical Monthly*, Vol. 116 (2009), No. 6: 479 – 495

Limitations of a paradigmatic action-based approach



A category-based approach of transformational analysis

$$\begin{tikzcd} C \arrow[r, "T_3"] \arrow[d, leftrightarrow, "I_2"'] & E\flat \arrow[d, "T_7M_7"] \\ D \arrow[r, "T_2"] & E \end{tikzcd} \quad \begin{tikzcd} E\flat \arrow[r, "T_3"] \arrow[d, leftrightarrow, "I_2"'] & F\sharp \arrow[d, "T_7M_7"] \\ B \arrow[r, "T_2"] & C\sharp \end{tikzcd} \quad \begin{tikzcd} F\sharp \arrow[r, "T_3"] \arrow[d, leftrightarrow, "I_2"'] & A \arrow[d, "T_7M_7"] \\ G\sharp \arrow[r, "T_2"] & B\flat \end{tikzcd} \quad \begin{tikzcd} A \arrow[r, "T_3"] \arrow[d, leftrightarrow, "I_2"'] & C \arrow[d, "T_7M_7"] \\ F \arrow[r, "T_2"] & G \end{tikzcd}$$

$$\begin{tikzcd} \square \arrow[rr, "F"] \arrow[dr, "R"'] & \phi & \square \arrow[dl, "S"] \\ & \square & \end{tikzcd}$$

Definition 1 *Let* $\mathbf{C}$ *be a category, and* S *a functor from* $\mathbf{C}$ *to the category* **Sets**. *Let* Δ *be a small category and* R *a functor from* Δ *to* **Sets**. *A PK-net of form* R *and of support* S *is a 4-tuple* (R, S, F, ϕ), *in which*

- F *is a functor from* Δ *to* $\mathbf{C}$,
- *and* ϕ *is a natural transformation from* R *to* SF.

The definition of a PK-net is summed up by the following diagram:

$$\begin{tikzcd} \Delta \arrow[rr, "F"] \arrow[dr, "R"'] & \phi & \mathbf{C} \arrow[dl, "S"] \\ & \mathbf{Sets} & \end{tikzcd}$$

Popoff A., M. Andreatta, A. Ehresmann, « A Categorical Generalization of Klumpenhouwer Networks », MCM 2015, Queen Mary University, Springer, p. 303-314

The catalogue of the 124 Hamiltonian Cycles

List of 124 Hamiltonian Cycles (Bigo/Andreatta, February 2016)

1. C-Cm-Ab-Abm-E-Em-G-Gm-Eb-Ebm-B-Bm-D-Dm-Bb-Bbm-F#-F#m-A-C#m-C#-Fm-F-Am-- PLPLPRPLPLPRPLPLPRLPLPLR
2. C-Cm-Ab-Abm-E-Em-G-Gm-Eb-Ebm-B-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Fm-C#-C#m-A-Am-- PLPLPRPLPLPRLPLPLRPLPLPR
3. C-Cm-Ab-Abm-E-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Bbm-F#-F#m-D-Dm-F-Fm-C#-C#m-A-Am-- PLPLPRLPLPLRPLPLPRPLPLPR
4. C-Cm-Ab-Abm-E-C#m-C#-Fm-F-Am-A-F#m-F#-Bbm-Bb-Dm-D-Bm-B-Ebm-Eb-Gm-G-Em-- PLPLRPLPLPRPLPLPRPLPLPRL
5. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em-- PLPLRL
6. C-Cm-Ab-Abm-B-Bm-G-Gm-Eb-Ebm-F#-F#m-D-Dm-Bb-Bbm-C#-Fm-F-Am-A-C#m-E-Em-- PLPRPLPLPRPLPLPRLPLPLRPL
7. C-Cm-Ab-Abm-B-Bm-G-Gm-Eb-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-Am-F-Fm-C#-C#m-E-Em-- PLPRPLPLPRLPLPLRPLPLPRPL
8. C-Cm-Ab-Abm-B-Bm-G-Em-E-C#m-A-F#m-D-Dm-Bb-Gm-Eb-Ebm-F#-Bbm-C#-Fm-F-Am-- PLPRPLRPRLRLPLRLPRLRLPLR
9. C-Cm-Ab-Abm-B-Ebm-Eb-Gm-G-Bm-D-Dm-Bb-Bbm-F#-F#m-A-Am-F-Fm-C#-C#m-E-Em-- PLPRLPLPLRPLPLPRPLPLPRPL
10. C-Cm-Ab-Abm-B-Ebm-Eb-Gm-Bb-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Bbm-C#-Fm-F-Am-- PLPRLPLRLPRLRPRLRPLRLPLR
11. C-Cm-Ab-Abm-B-Ebm-Eb-Gm-Bb-Dm-F-Fm-C#-Bbm-F#-F#m-D-Bm-G-Em-E-C#m-A-Am-- PLPRLPLRLRPLRLPLRLRPRLPR
12. C-Cm-Ab-Fm-F-Am-A-C#m-C#-Bbm-Bb-Dm-D-F#m-F#-Ebm-Eb-Gm-G-Bm-B-Abm-E-Em-- PLRPLPLPRPLPLPRPLPLPRLPL
13. C-Cm-Ab-Fm-F-Am-A-F#m-F#-Bbm-C#-C#m-E-Abm-B-Ebm-Eb-Gm-Bb-Dm-D-Bm-G-Em-- PLRPLPRPLRPRLRLPLRLPRLRL
14. C-Cm-Ab-Fm-F-Am-A-F#m-D-Dm-Bb-Gm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-G-Em-- PLRPLPRLPLRLPRLRPRLRPLRL
15. C-Cm-Ab-Fm-F-Dm-Bb-Gm-Eb-Ebm-B-Abm-E-Em-G-Bm-D-F#m-F#-Bbm-C#-C#m-A-Am-- PLRPRLRLPLRLPRLRLPLRPLPR
16. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em-- PLRLPL
17. C-Cm-Ab-Fm-C#-C#m-A-F#m-F#-Bbm-Bb-Gm-Eb-Ebm-B-Abm-E-Em-G-Bm-D-Dm-F-Am-- PLRLPLRPLPRLPLRLPRLRPRLR
18. C-Cm-Ab-Fm-C#-C#m-A-F#m-D-Bm-B-Abm-E-Em-G-Gm-Eb-Ebm-F#-Bbm-Bb-Dm-F-Am-- PLRLPLRLRPRLPRPLPRLPLRLR
19. C-Cm-Ab-Fm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am-F-Dm-Bb-Bbm-F#-Ebm-Eb-Gm-G-Em-- PLRLPRLRPRLRPLRLPLRPLPRL
20. C-Cm-Ab-Fm-C#-C#m-E-Abm-B-Ebm-Eb-Gm-Bb-Bbm-F#-F#m-A-Am-F-Dm-D-Bm-G-Em-- PLRLPRLRLPLRPLPRPLRPRLRL
21. C-Cm-Ab-Fm-C#-Bbm-Bb-Dm-F-Am-A-C#m-E-Abm-B-Bm-D-F#m-F#-Ebm-Eb-Gm-G-Em-- PLRLRPLRLPLRLRPRLPRPLPRL
22. C-Cm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Ebm-F#-F#m-D-Dm-F-Am-A-C#m-E-Abm-B-Bm-G-Em-- PLRLRPRLPRPLPRLPLRLRPLRL
23. C-Cm-Eb-Ebm-B-Bm-G-Gm-Bb-Bbm-F#-F#m-D-Dm-F-Am-A-C#m-C#-Fm-Ab-Abm-E-Em-- PRPLPLPRPLPLPRLPLPLRPLPL
24. C-Cm-Eb-Ebm-B-Bm-G-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-C#m-A-Am-F-Fm-Ab-Abm-E-Em-- PRPLPLPRLPLPLRPLPLPRPLPL
25. C-Cm-Eb-Ebm-B-Bm-D-F#m-F#-Bbm-C#-Fm-Ab-Abm-E-C#m-A-Am-F-Dm-Bb-Gm-G-Em-- PRPLPRLPLRLRPLRLPLRLRPRL
26. C-Cm-Eb-Ebm-B-Abm-Ab-Fm-C#-Bbm-F#-F#m-D-Bm-G-Gm-Bb-Dm-F-Am-A-C#m-E-Em-- PRPLRPRLRLPLRLPRLRLPLRPL

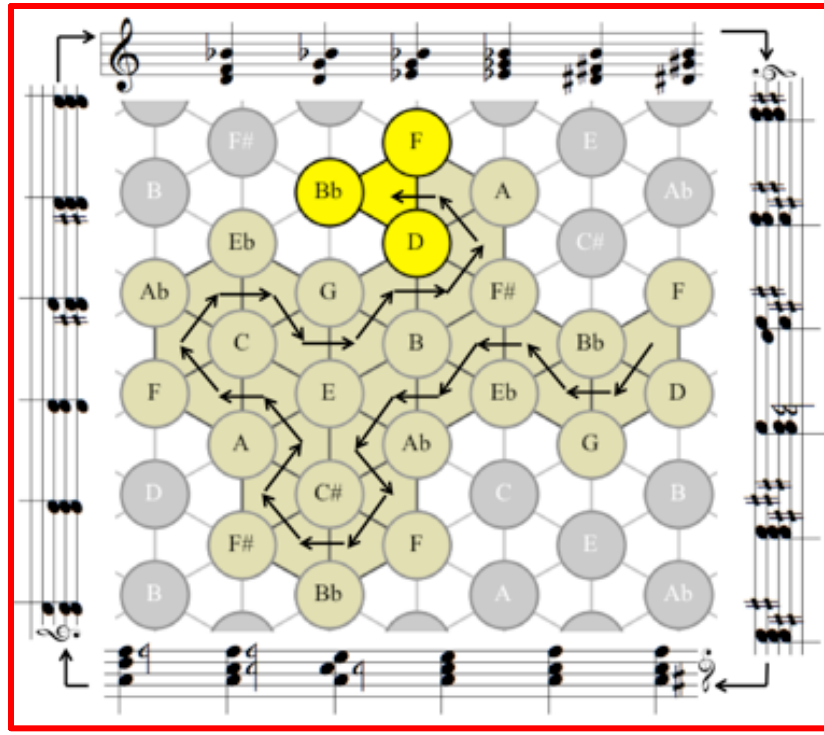
27. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am-- PRPRPRLR
28. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am-- PRPRLRPR
29. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am-- PRPRLRLR
30. C-Cm-Eb-Gm-G-Bm-B-Ebm-F#-F#m-D-Dm-Bb-Bbm-C#-C#m-A-Am-F-Fm-Ab-Abm-E-Em-- PRLPLPLRPLPLPRPLPLPRPLPL
31. C-Cm-Eb-Gm-G-Bm-D-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Dm-F-Am-A-C#m-E-Em-- PRLPLRLPRLRPRLRPLRLPLRPL
32. C-Cm-Eb-Gm-G-Bm-D-F#m-A-Am-F-Dm-Bb-Bbm-F#-Ebm-B-Abm-Ab-Fm-C#-C#m-E-Em-- PRLPLRLRPLRLPLRLRPRLPRPL
33. C-Cm-Eb-Gm-G-Em-E-Abm-Ab-Fm-C#-C#m-A-F#m-D-Bm-B-Ebm-F#-Bbm-Bb-Dm-F-Am-- PRLPRPLPRLPLRLRPLRLPLRLR
34. C-Cm-Eb-Gm-Bb-Bbm-F#-Ebm-B-Bm-G-Em-E-Abm-Ab-Fm-C#-C#m-A-F#m-D-Dm-F-Am-- PRLRPLRLPLRPLPRLPLRLPRLR
35. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am-- PRLRPRPR
36. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-E-C#m-A-Am-F-Dm-D-F#m-F#-Ebm-B-Bm-G-Em-- PRLRPRLRPLRLPLRPLPRLPLRL
37. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am-- PRLR
38. C-Cm-Eb-Gm-Bb-Dm-D-F#m-A-Am-F-Fm-Ab-Abm-E-C#m-C#-Bbm-F#-Ebm-B-Bm-G-Em-- PRLRLPLRPLPRPLRPRLRLPLRL
39. C-Cm-Eb-Gm-Bb-Dm-D-F#m-A-C#m-C#-Bbm-F#-Ebm-B-Bm-G-Em-E-Abm-Ab-Fm-F-Am-- PRLRLPLRLPRLRLPLRPLPRPLR
40. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am-- PRLRLRPR
41. C-Em-E-Abm-Ab-Cm-Eb-Ebm-B-Bm-G-Gm-Bb-Bbm-F#-F#m-D-Dm-F-Fm-C#-C#m-A-Am-- LPLPLRPLPLPRPLPLPRPLPLPR
42. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am-- LPLPLR
43. C-Em-E-Abm-Ab-Fm-F-Am-A-C#m-C#-Bbm-Bb-Dm-D-F#m-F#-Ebm-B-Bm-G-Gm-Eb-Cm-- LPLPRPLPLPRPLPLPRLPLPLRP
44. C-Em-E-Abm-Ab-Fm-F-Am-A-C#m-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-G-Bm-B-Ebm-Eb-Cm-- LPLPRPLPLPRLPLPLRPLPLPRP
45. C-Em-E-Abm-Ab-Fm-C#-C#m-A-Am-F-Dm-D-F#m-F#-Bbm-Bb-Gm-G-Bm-B-Ebm-Eb-Cm-- LPLPRLPLPLRPLPLPRPLPLPRP
46. C-Em-E-Abm-B-Bm-G-Gm-Eb-Ebm-F#-F#m-D-Dm-Bb-Bbm-C#-C#m-A-Am-F-Fm-Ab-Cm-- LPLRPLPLPRPLPLPRPLPLPRLP
47. C-Em-E-Abm-B-Bm-G-Gm-Bb-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-C#-C#m-A-F#m-D-Dm-F-Am-- LPLRPLPRPLRPRLRLPLRLPRLR
48. C-Em-E-Abm-B-Bm-G-Gm-Bb-Dm-D-F#m-A-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Am-- LPLRPLPRLPLRLPRLRPRLRPLR
49. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm-- LPLRLP
50. C-Em-E-Abm-B-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Bm-D-F#m-F#-Bbm-C#-C#m-A-Am-- LPLRLPRLRPRLRPLRLPLRPLPR
51. C-Em-E-Abm-B-Ebm-Eb-Cm-Ab-Fm-C#-C#m-A-F#m-F#-Bbm-Bb-Gm-G-Bm-D-Dm-F-Am-- LPLRLPRLRLPLRPLPRPLRPRLR
52. C-Em-E-Abm-B-Ebm-F#-F#m-D-Bm-G-Gm-Eb-Cm-Ab-Fm-F-Dm-Bb-Bbm-C#-C#m-A-Am-- LPLRLRPLRLPLRLRPRLPRPLPR
53. C-Em-E-Abm-B-Ebm-F#-F#m-A-C#m-C#-Bbm-Bb-Dm-D-Bm-G-Gm-Eb-Cm-Ab-Fm-F-Am-- LPLRLRPRLPRPLPRLPLRLRPLR

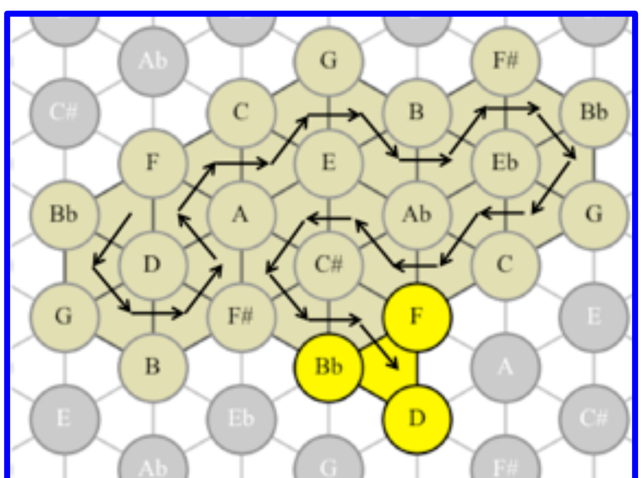
The catalogue of the 124 Hamiltonian Cycles

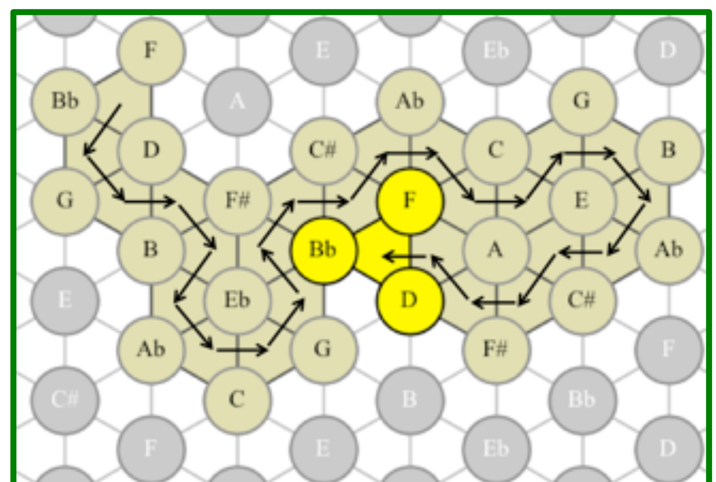
54. C-Em-E-C#m-C#-Fm-F-Am-A-F#m-F#-Bbm-Bb-Dm-D-Bm-G-Gm-Eb-Ebm-B-Abm-Ab-Cm--LPRPLPLPRPLPLPRLPLPLRPLP
55. C-Em-E-C#m-C#-Fm-F-Am-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-Eb-Gm-G-Bm-B-Abm-Ab-Cm--LPRPLPLPRLPLPLRPLPLPRPLP
56. C-Em-E-C#m-C#-Fm-F-Dm-Bb-Bbm-F#-Ebm-B-Abm-Ab-Cm-Eb-Gm-G-Bm-D-F#m-A-Am--LPRPLPRLPLRLRPLRLPLRLRPR
57. C-Em-E-C#m-C#-Fm-Ab-Abm-B-Ebm-F#-Bbm-Bb-Dm-F-Am-A-F#m-D-Bm-G-Gm-Eb-Cm--LPRPLRPRLRLPLRLPRLRLPLRP
58. C-Em-E-C#m-A-Am-F-Fm-C#-Bbm-Bb-Dm-D-F#m-F#-Ebm-Eb-Gm-G-Bm-B-Abm-Ab-Cm--LPRLPLPLRPLPLPRPLPLPRPLP
59. C-Em-E-C#m-A-Am-F-Dm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-D-Bm-G-Gm-Eb-Cm--LPRLPLRLPRLRPRLRPLRLPLRP
60. C-Em-E-C#m-A-Am-F-Dm-Bb-Gm-G-Bm-D-F#m-F#-Bbm-C#-Fm-Ab-Abm-B-Ebm-Eb-Cm--LPRLPLRLRPLRLPLRLRPRLPRP
61. C-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Cm-Eb-Gm-G-Bm-D-Dm-Bb-Bbm-C#-Fm-F-Am--LPRLRPRLRPLRLPLRPLPRLPLR
62. C-Em-E-C#m-A-F#m-D-Dm-Bb-Gm-G-Bm-B-Abm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-Fm-F-Am--LPRLRLPLRPLPRPLRPRLRLPLR
63. C-Em-G-Gm-Eb-Ebm-B-Bm-D-Dm-Bb-Bbm-F#-F#m-A-Am-F-Fm-C#-C#m-E-Abm-Ab-Cm--LRPLPLPRPLPLPRPLPLPRLPLP
64. C-Em-G-Gm-Eb-Ebm-F#-F#m-D-Bm-B-Abm-E-C#m-A-Am-F-Dm-Bb-Bbm-C#-Fm-Ab-Cm--LRPLPRPLRPRLRLPLRLPRLRLP
65. C-Em-G-Gm-Eb-Ebm-F#-Bbm-Bb-Dm-F-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Fm-Ab-Cm--LRPLPRLPLRLPRLRPRLRPLRLP
66. C-Em-G-Gm-Eb-Cm-Ab-Abm-E-C#m-C#-Fm-F-Dm-Bb-Bbm-F#-Ebm-B-Bm-D-F#m-A-Am--LRPLRLPLRPLPRLPLRLPRLRPR
67. C-Em-G-Gm-Eb-Cm-Ab-Abm-E-C#m-A-F#m-F#-Ebm-B-Bm-D-Dm-Bb-Bbm-C#-Fm-F-Am--LRPLRLPLRLRPRLPRPLPRLPLR
68. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
69. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
70. C-Em-G-Gm-Bb-Dm-D-Bm-B-Ebm-Eb-Cm-Ab-Abm-E-C#m-A-F#m-F#-Bbm-C#-Fm-F-Am--LRPRLPRPLPRLPLRLRPLRLPLR
71. C-Em-G-Gm-Bb-Dm-F-Fm-C#-Bbm-F#-F#m-D-Bm-B-Ebm-Eb-Cm-Ab-Abm-E-C#m-A-Am--LRPRLRPLRLPLRPLPRLPLRLPR
72. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
73. C-Em-G-Gm-Bb-Dm-F-Am-A-C#m-E-Abm-Ab-Fm-C#-Bbm-F#-F#m-D-Bm-B-Ebm-Eb-Cm--LRPRLRLPLRLPRLRLPLRPLPRP
74. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
75. C-Em-G-Bm-B-Ebm-F#-F#m-D-Dm-F-Fm-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Abm-E-C#m-A-Am--LRLPLRPLPRPLRPRLRLPLRLPR
76. C-Em-G-Bm-B-Ebm-F#-F#m-D-Dm-F-Am-A-C#m-E-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--LRLPLRPLPRLPLRLPRLRPRLRP
77. C-Em-G-Bm-B-Ebm-F#-Bbm-Bb-Gm-Eb-Cm-Ab-Abm-E-C#m-C#-Fm-F-Dm-D-F#m-A-Am--LRLPLRLPRLRLPLRPLPRPLRPR
78. C-Em-G-Bm-B-Ebm-F#-Bbm-C#-C#m-E-Abm-Ab-Fm-F-Am-A-F#m-D-Dm-Bb-Gm-Eb-Cm--LRLPLRLRPRLPRPLPRLPLRLRP
79. C-Em-G-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Gm-Bb-Dm-D-F#m-A-Am-F-Fm-Ab-Cm--LRLPRLRPRLRPLRLPLRPLPRLP
80. C-Em-G-Bm-B-Abm-E-C#m-A-Am-F-Dm-D-F#m-F#-Ebm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Cm--LRLPRLRLPLRPLPRPLRPRLRLP
81. C-Em-G-Bm-D-Dm-Bb-Gm-Eb-Ebm-B-Abm-E-C#m-C#-Bbm-F#-F#m-A-Am-F-Fm-Ab-Cm--LRLRPLRLPLRLRPRLPRPLPRLP
82. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
83. C-Em-G-Bm-D-Dm-F-Am-A-F#m-F#-Bbm-Bb-Gm-Eb-Ebm-B-Abm-E-C#m-C#-Fm-Ab-Cm--LRLRPRLPRPLPRLPLRLRPLRLP
84. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
85. C-Am-A-C#m-C#-Fm-F-Dm-D-F#m-F#-Bbm-Bb-Gm-G-Bm-B-Ebm-Eb-Cm-Ab-Abm-E-Em--RPLPLPRPLPLPRPLPLPRLPLPL
86. C-Am-A-C#m-C#-Fm-F-Dm-D-F#m-F#-Bbm-Bb-Gm-Eb-Ebm-B-Bm-G-Em-E-Abm-Ab-Cm--RPLPLPRPLPLPRLPLPLRPLPLP
87. C-Am-A-C#m-C#-Fm-F-Dm-Bb-Bbm-F#-F#m-D-Bm-B-Ebm-Eb-Gm-G-Em-E-Abm-Ab-Cm--RPLPLPRLPLPLRPLPLPRPLPLP
88. C-Am-A-C#m-C#-Bbm-Bb-Dm-F-Fm-Ab-Cm-Eb-Gm-G-Bm-D-F#m-F#-Ebm-B-Abm-E-Em--RPLPRPLRPRLRLPLRLPRLRLPL
89. C-Am-A-C#m-C#-Bbm-F#-F#m-D-Bm-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-B-Abm-E-Em--RPLPRLPLRLPRLRPRLRPLRLPL
90. C-Am-A-C#m-C#-Bbm-F#-F#m-D-Bm-G-Em-E-Abm-B-Ebm-Eb-Gm-Bb-Dm-F-Fm-Ab-Cm--RPLPRLPLRLRPLRLPLRLRPRLP
91. C-Am-A-C#m-E-Em-G-Bm-D-F#m-F#-Bbm-C#-Fm-F-Dm-Bb-Gm-Eb-Ebm-B-Abm-Ab-Cm--RPLRPRLRLPLRLPRLRLPLRPLP
92. C-Am-A-C#m-E-Abm-Ab-Cm-Eb-Ebm-B-Bm-D-F#m-F#-Bbm-C#-Fm-F-Dm-Bb-Gm-G-Em--RPLRLPLRPLPRLPLRLPRLRPRL
93. C-Am-A-C#m-E-Abm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-Fm-F-Dm-D-F#m-F#-Ebm-B-Bm-G-Em--RPLRLPLRLRPRLPRPLPRLPLRL
94. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
95. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPRLRP
96. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
97. C-Am-A-F#m-D-Dm-F-Fm-C#-C#m-E-Abm-Ab-Cm-Eb-Gm-Bb-Bbm-F#-Ebm-B-Bm-G-Em--RPRLPRPLPRLPLRLRPLRLPLRL
98. C-Am-A-F#m-D-Bm-B-Ebm-F#-Bbm-Bb-Dm-F-Fm-C#-C#m-E-Abm-Ab-Cm-Eb-Gm-G-Em--RPRLRPLRLPLRPLPRLPLRLPRL
99. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRLRPRP
100. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPRL
101. C-Am-A-F#m-D-Bm-G-Gm-Eb-Cm-Ab-Abm-B-Ebm-F#-Bbm-Bb-Dm-F-Fm-C#-C#m-E-Em--RPRLRLPLRLPRLRLPLRPLPRPL
102. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRLRLRP
103. C-Am-F-Fm-C#-C#m-A-F#m-F#-Bbm-Bb-Dm-D-Bm-B-Ebm-Eb-Gm-G-Em-E-Abm-Ab-Cm--RLPLPLRPLPLPRPLPLPRPLPLP
104. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
105. C-Am-F-Fm-C#-Bbm-Bb-Dm-D-Bm-B-Ebm-F#-F#m-A-C#m-E-Abm-Ab-Cm-Eb-Gm-G-Em--RLPLRPLPRPLRPRLRLPLRLPRL
106. C-Am-F-Fm-C#-Bbm-Bb-Dm-D-Bm-G-Gm-Eb-Cm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em--RLPLRPLPRLPLRLPRLRPRLRPL
107. C-Am-F-Fm-C#-Bbm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-Bb-Gm-Eb-Ebm-B-Abm-Ab-Cm--RLPLRLPRLRPRLRPLRLPLRPLP

The catalogue of the 124 Hamiltonian Cycles

108. C-Am-F-Fm-C#-Bbm-F#-F#m-A-C#m-E-Abm-Ab-Cm-Eb-Ebm-B-Bm-D-Dm-Bb-Gm-G-Em-
-RLPLRLPRLRLPLRPLPRPLRPRL
109. C-Am-F-Fm-C#-Bbm-F#-Ebm-Eb-Gm-Bb-Dm-D-F#m-A-C#m-E-Em-G-Bm-B-Abm-Ab-Cm-
-RLPLRLRPLRLPLRLRPRLPRPLP
110. C-Am-F-Fm-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Abm-B-Bm-G-Gm-Bb-Dm-D-F#m-A-C#m-E-Em-
-RLPLRLRPRLPRPLPRLPLRLRPL
111. C-Am-F-Fm-Ab-Abm-E-Em-G-Bm-B-Ebm-F#-Bbm-C#-C#m-A-F#m-D-Dm-Bb-Gm-Eb-Cm-
-RLPRPLPRLPLRLRPLRLPLRLRP
112. C-Am-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-A-F#m-D-Dm-Bb-Gm-G-Bm-B-Abm-E-Em-
-RLPRLRPRLRPLRLPLRPLPRLPL
113. C-Am-F-Fm-Ab-Cm-Eb-Gm-G-Bm-D-Dm-Bb-Bbm-C#-C#m-A-F#m-F#-Ebm-B-Abm-E-Em-
-RLPRLRLPLRPLPRPLRPRLRLPL
114. C-Am-F-Dm-D-F#m-A-C#m-C#-Fm-Ab-Abm-E-Em-G-Bm-B-Ebm-F#-Bbm-Bb-Gm-Eb-Cm-
-RLRPLRLPLRPLPRLPLRLPRLRP
115. C-Am-F-Dm-D-F#m-A-C#m-C#-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-Bb-Gm-G-Bm-B-Abm-E-Em-
-RLRPLRLPLRLRPRLPRPLPRLPL
116. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm-
-RLRPRPRP
117. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-
-RLRPRPRL
118. C-Am-F-Dm-D-Bm-G-Gm-Bb-Bbm-F#-F#m-A-C#m-C#-Fm-Ab-Cm-Eb-Ebm-B-Abm-E-Em-
-RLRPRLPRPLPRLPLRLRPLRLPL
119. C-Am-F-Dm-D-Bm-G-Em-E-Abm-B-Ebm-Eb-Gm-Bb-Bbm-F#-F#m-A-C#m-C#-Fm-Ab-Cm-
-RLRPRLRPLRLPLRPLPRLPLRLP
120. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm-
-RLRP
121. C-Am-F-Dm-Bb-Bbm-F#-Ebm-Eb-Gm-G-Em-E-Abm-B-Bm-D-F#m-A-C#m-C#-Fm-Ab-Cm-
-RLRLPLRPLPRPLRPRLRLPLRLP
122. C-Am-F-Dm-Bb-Bbm-F#-Ebm-B-Bm-D-F#m-A-C#m-C#-Fm-Ab-Abm-E-Em-G-Gm-Eb-Cm-
-RLRLPLRLPRLRLPLRPLPRPLRP
123. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm-
-RLRLRPRP
124. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em-
-RL



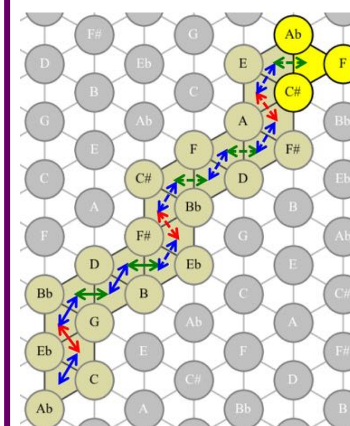




Exploring Hamiltonian trajectories in song writing

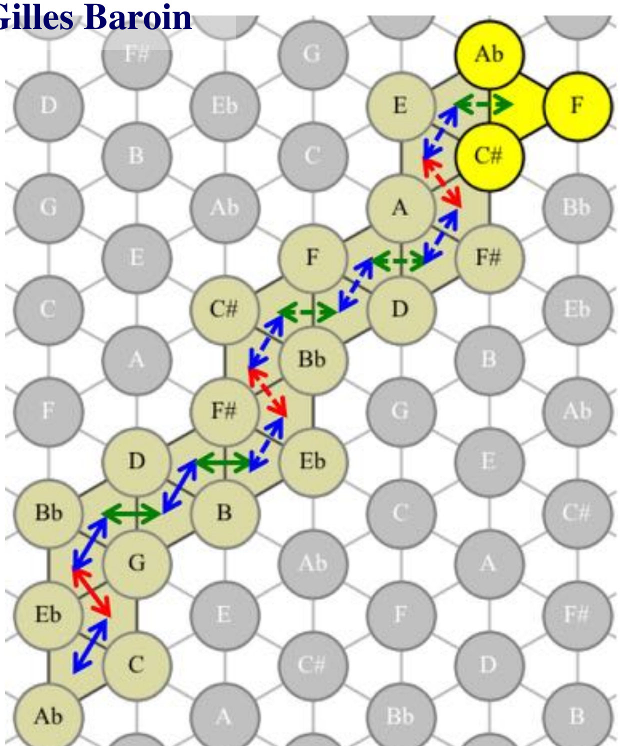
1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPRPR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR →
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPRLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRLRPRP
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPRL
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRLRLRP
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm--RLRPRPRP
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em--RLRPRPRL
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLRP
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm--RLRLRPRP
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL

LPLPLR…
PLPLRL…
LPLRLP…
PLRLPL…
LRLPLP…



La sera non è più la tua canzone (after Mario Luzi)

Gilles Baroin



La sera non è più la tua canzone,
è questa roccia d'ombra traforata
dai lumi e dalle voci senza fine,
la quiete d'una cosa già pensata.

Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d'una cosa conosciuta,
per nome hai una parola ch'è passata
nell'intimo del cuore e s'è perduta.

Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell'essere, (compiuta).

L'aria tace ed il tempo dietro a te
si leva come un'arida montagna
dove vaga il tuo spirito e si perde,
un vento raro scivola e ristagna.

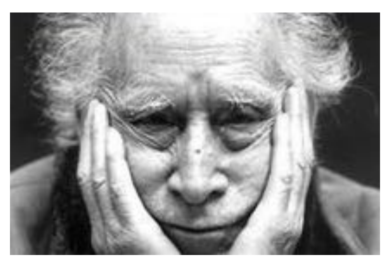
Le soir n'est plus ta chanson,
c'est ce rochet d'ombre transpercé
par les lumières et les voix sans fin,
la paix d'une chose déjà pensée.

Ah, cette lumière vive et claire vient
uniquement de toi, tu es si proche
du vrai d'une chose connue,
tu as pour nom une parole qui est passée
dans l'intimité du cœur où elle s'est
perdue .

Tombé est plus qu'un signe de la vie,
tu te reposes, du voyage tu es revenue
à l'intérieur de toi même, tu es
descendue dans cette
pure substance qui est si tienne,
si éloignée
dans le silence de l'être, achevée.

L'air se tait et le temps derrière toi
se lève tel une montagne aride
où plane ton esprit et se perd,
un vent rare glisse et stagne.

(tr. Antonia Soulez, philosophe et poète)



M. Luzi (1914-2005)

Music: M. Andreatta
Arrangement and mix: M. Bergomi & S. Geravini
(*Perfect Music Production*)
Mastering: A. Cutolo (Massive Arts Studio, Milan)


Concert « Poèmes en chanson »
à l'Impromptu
17 novembre 2023
LIVRE + CD
La BANDE des MOTS
variétés : littérature et chanson
Billet De Femme – P. Obispo
Marceline Desbordes-Valmore
BOB DYLAN
BAUDELAIRE SONG PROJECT
BAUDELAIRE
Treize poèmes
Verlaine & Rimbaud
Quand la littérature prend l'air
CHANSON & POÉSIE
Thank you and…
… see you maybe tomorrow evening at 8pm at the Impromptu!