

From Music to Mathematics and backwards:

some perceptual and cognitive implications of algebraic,
topological and category-theory models in
computational music analysis

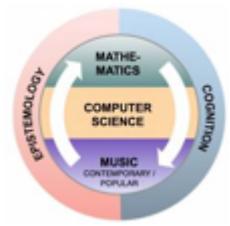
Moreno Andreatta

CNRS / IRMA / Université de Strasbourg

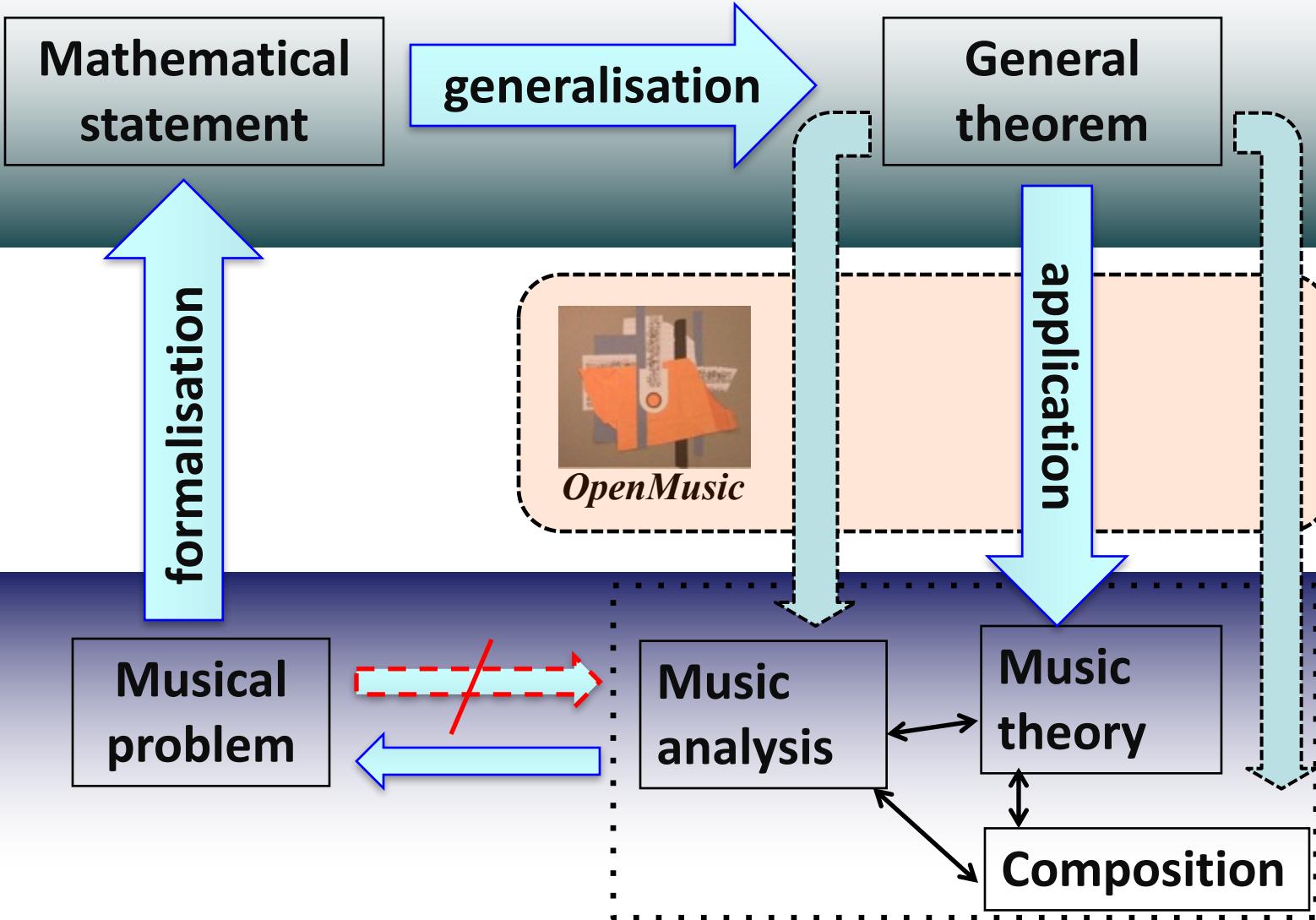
CNRS / IRCAM / Sorbonne Université

<http://repmus.ircam.fr/moreno/smir>

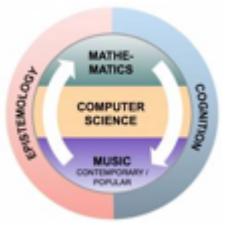
The double movement of a ‘mathemusical’ activity



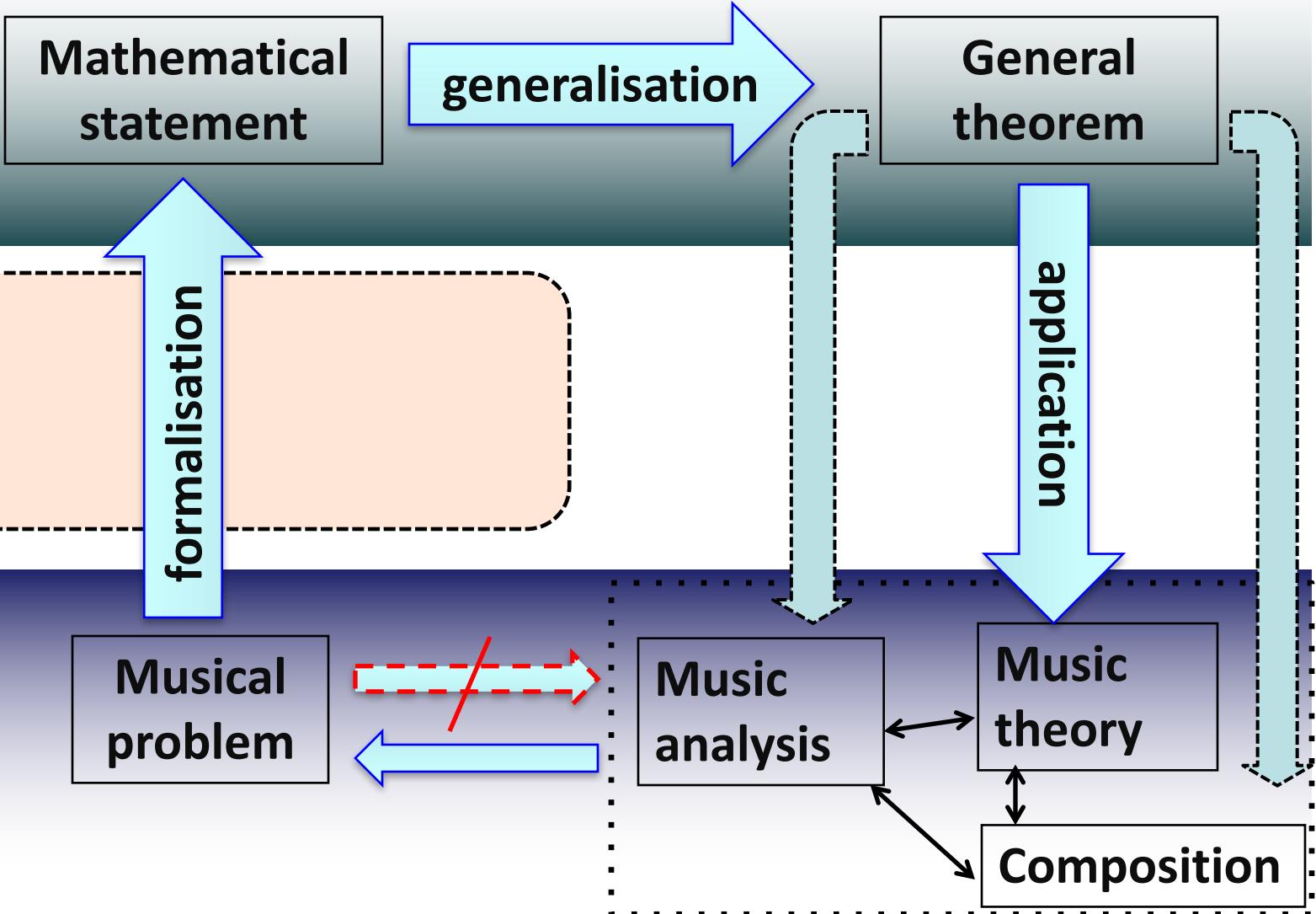
MATHEMATICS



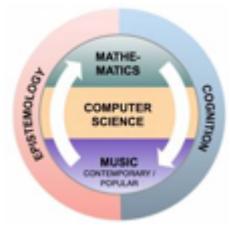
The double movement of a ‘mathemusical’ activity



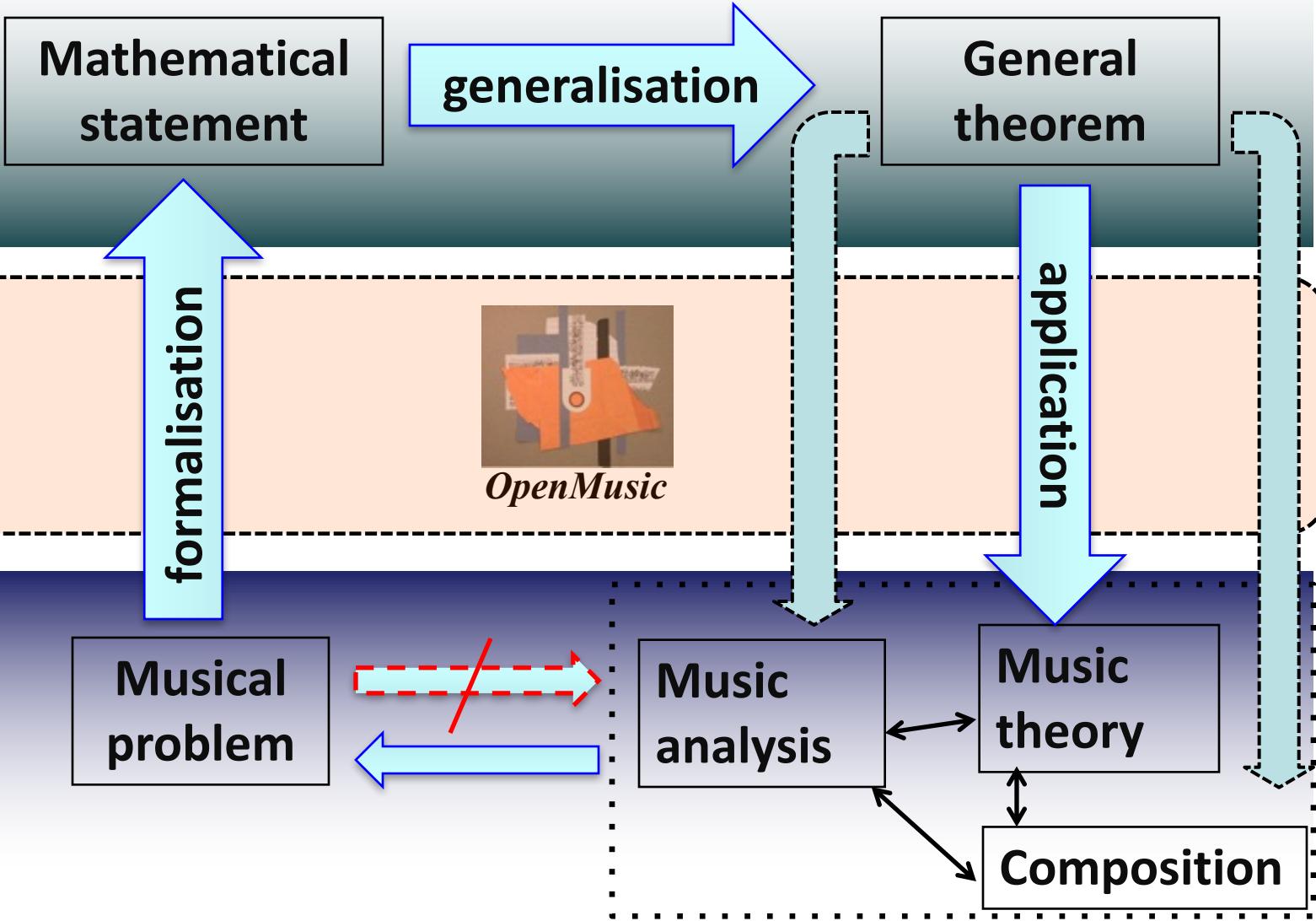
MATHEMATICS



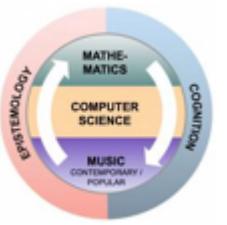
The double movement of a ‘mathemusical’ activity



MATHEMATICS



The double movement of a ‘mathemusical’ activity

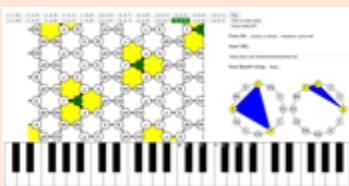


MATHEMATICS

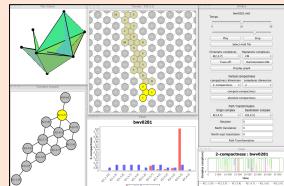
Mathematical statement

generalisation

General theorem



Tonnetz



Hexachord

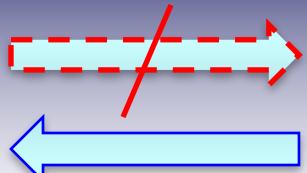


OpenMusic

formalisation

MUSIC

Musical problem



Music analysis

Music theory

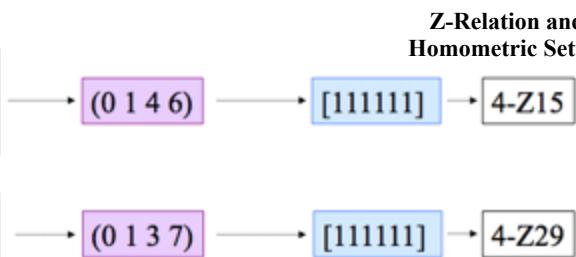
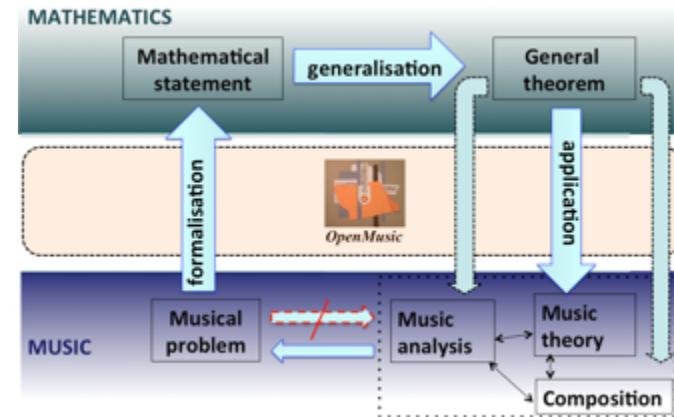
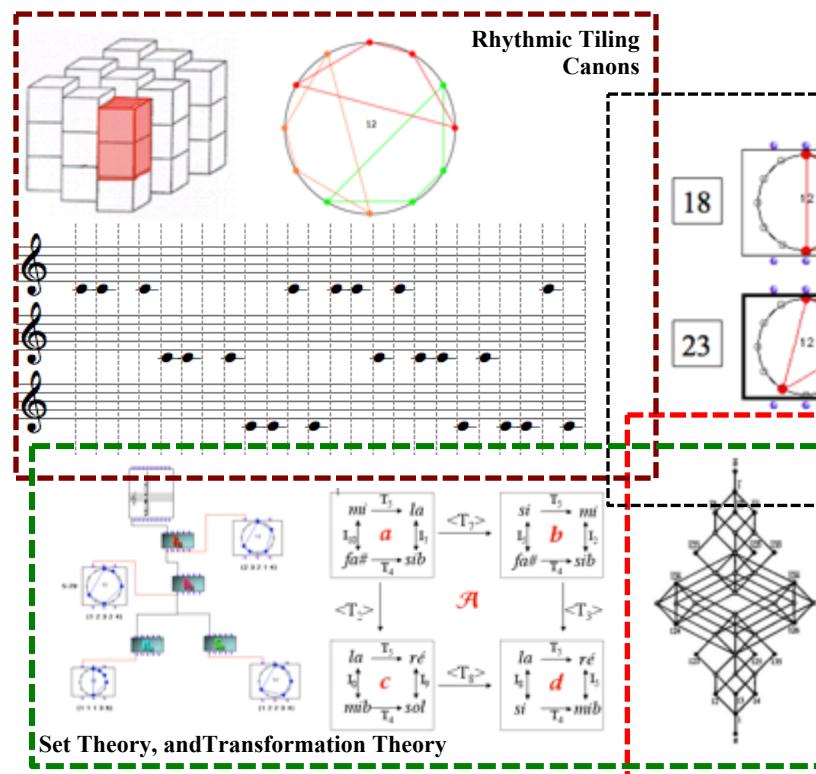
Composition

application

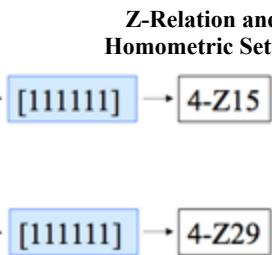
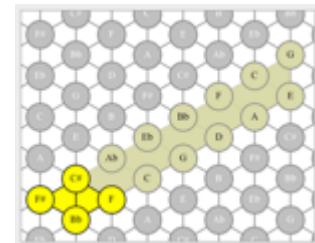
Some examples of ‘mathemusical’ problems

M. Andreatta : *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

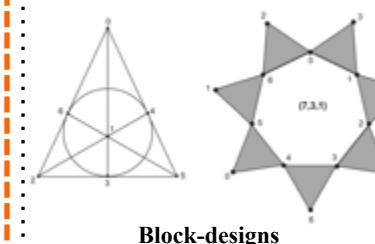
- The construction of Tiling Rhythmic Canons
- The Z relation and the theory of homometric sets
- Set Theory and Transformational Theory
- Neo-Riemannian Theory, Spatial Computing and FCA
- Diatonic Theory and Maximally-Even Sets
- Periodic sequences and finite difference calculus
- Block-designs and algorithmic composition



Neo-Riemannian Theory and Spatial Computing

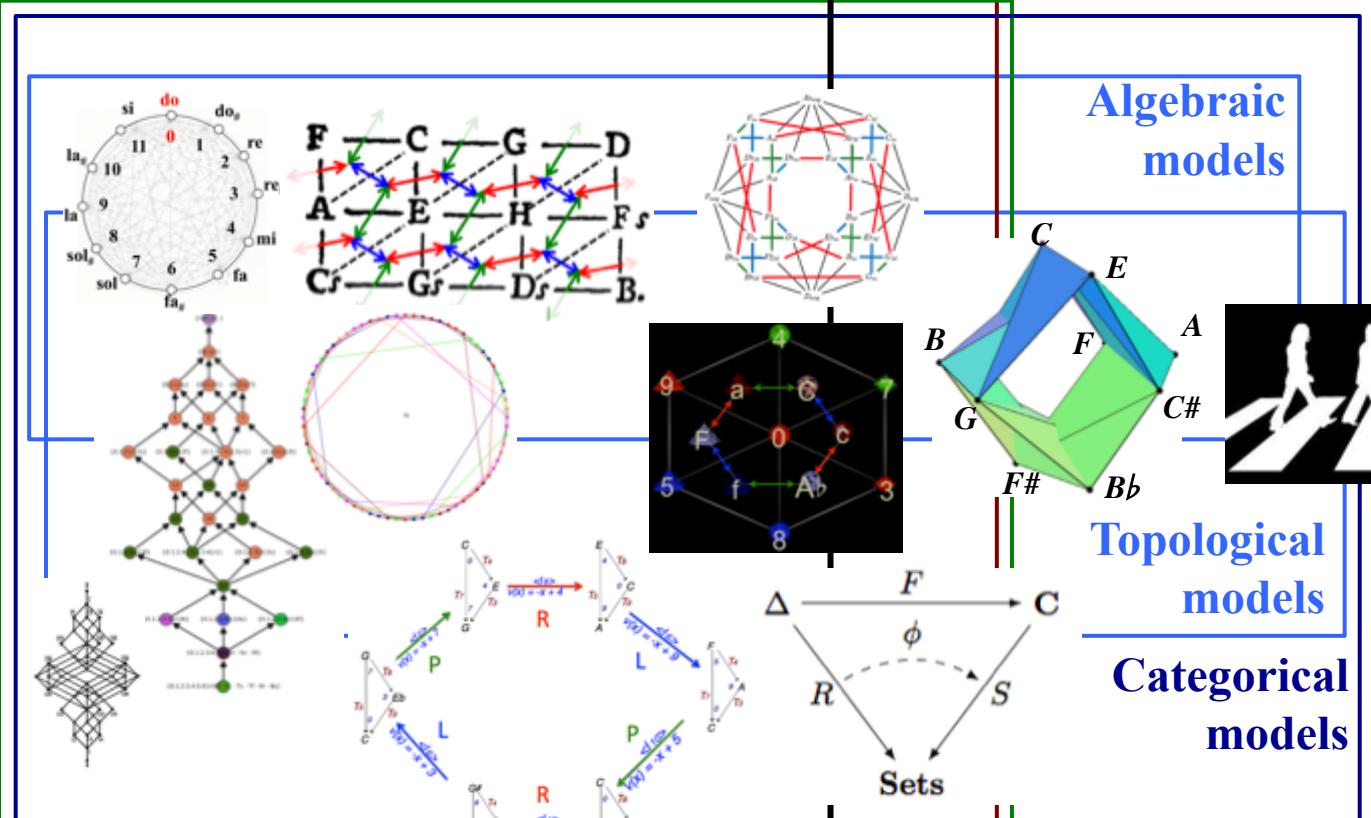


Diatonic Theory and ME-Sets



The SMIR Project: advanced maths for the working musicologist

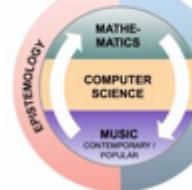
Signal-based
Music
Information
Retrieval



Computational models

Cognitive models

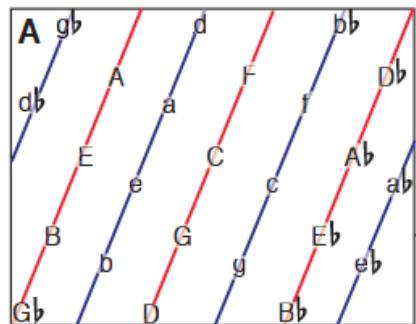
Symbolic Music Information Research



Signal-based
Music
Information
Retrieval

→ Andreatta, M. (2018), « From music to mathematics and backwards: introducing algebra, topology and category theory into computational musicology », in M. Emmer and M. Abate (eds.), *Imagine Math 6* - Springer, pp. 77-88

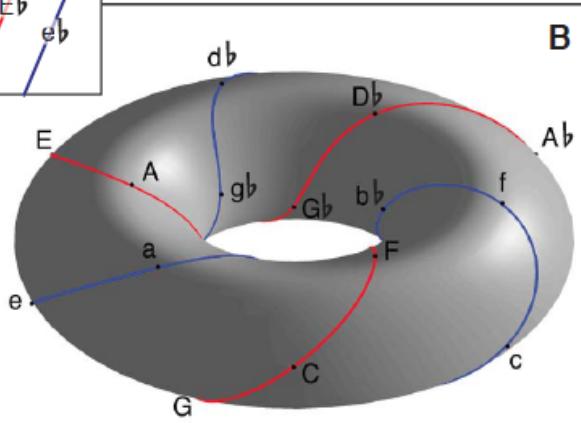
Neurosciences and Mathemusical Learning



PERSPECTIVES: NEUROSCIENCE

Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl



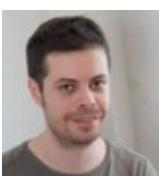
Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each

wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor).

(B) Musical keys as points on the surface of a torus.



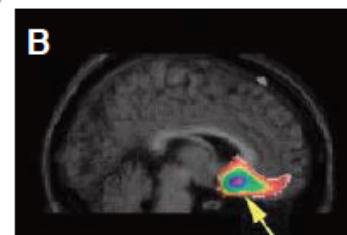
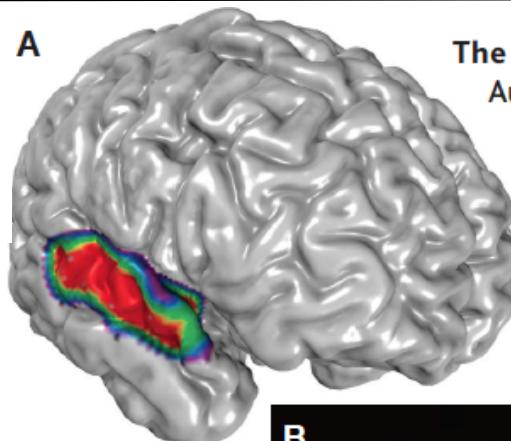
E. Bisesti



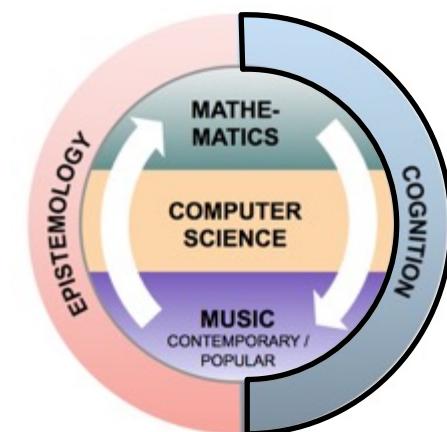
J.-L. Besada



C. Guichaoua

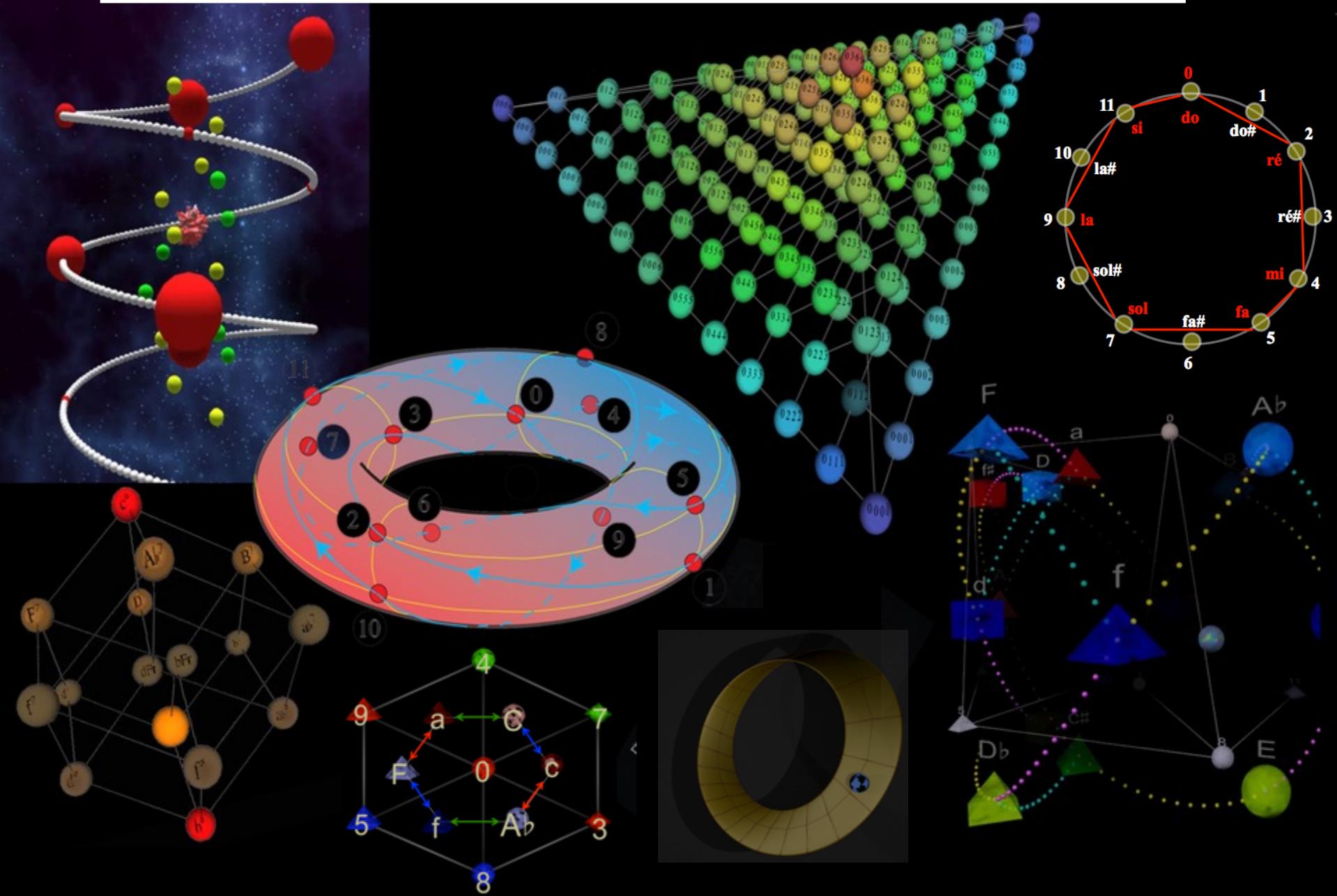


The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

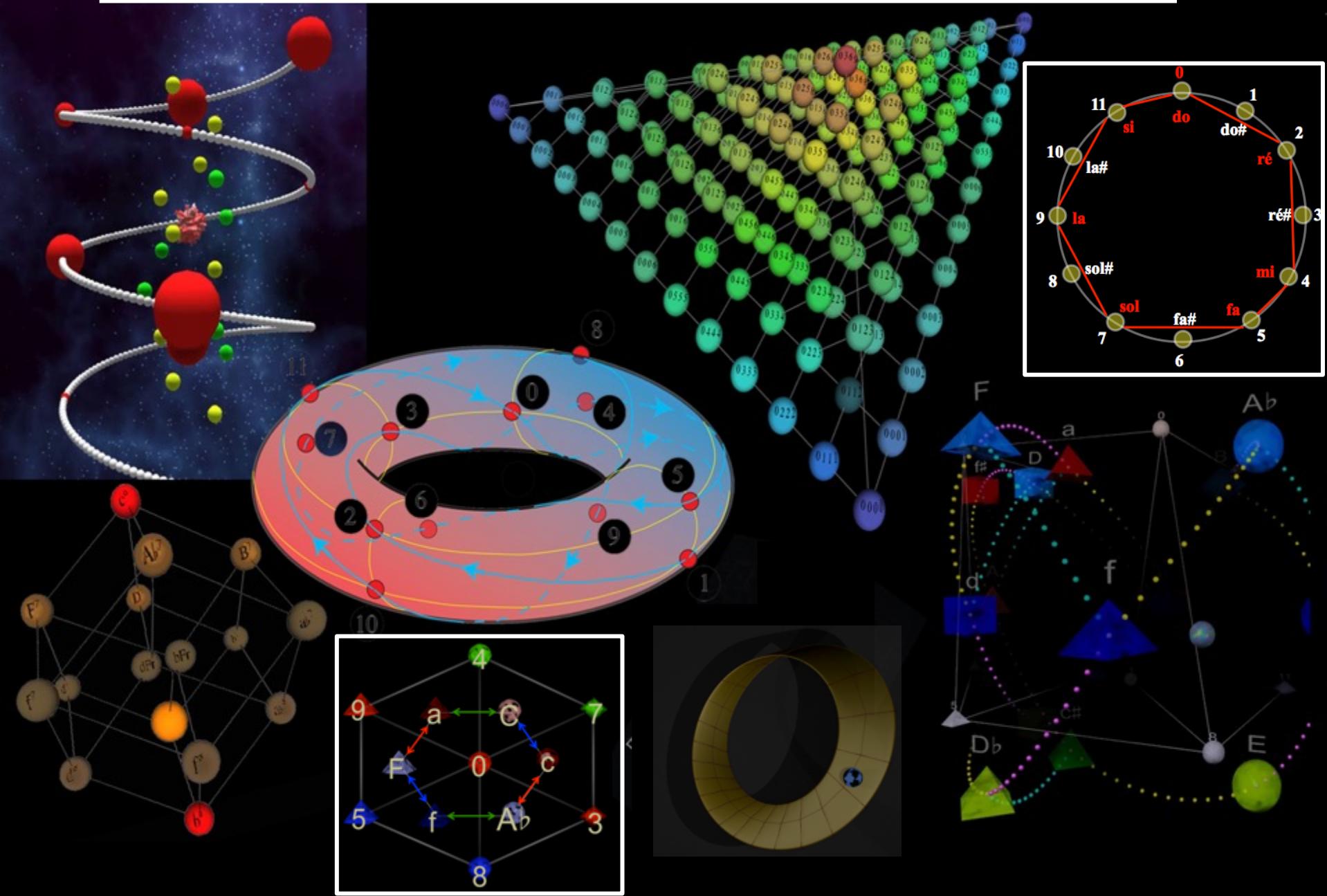


 The Tonnetz Environment: A Web Environment for Computer-Aided “Mathemusical” Learning and Research (paper to be presented at *Computer Supported Music Education 2021*, online)

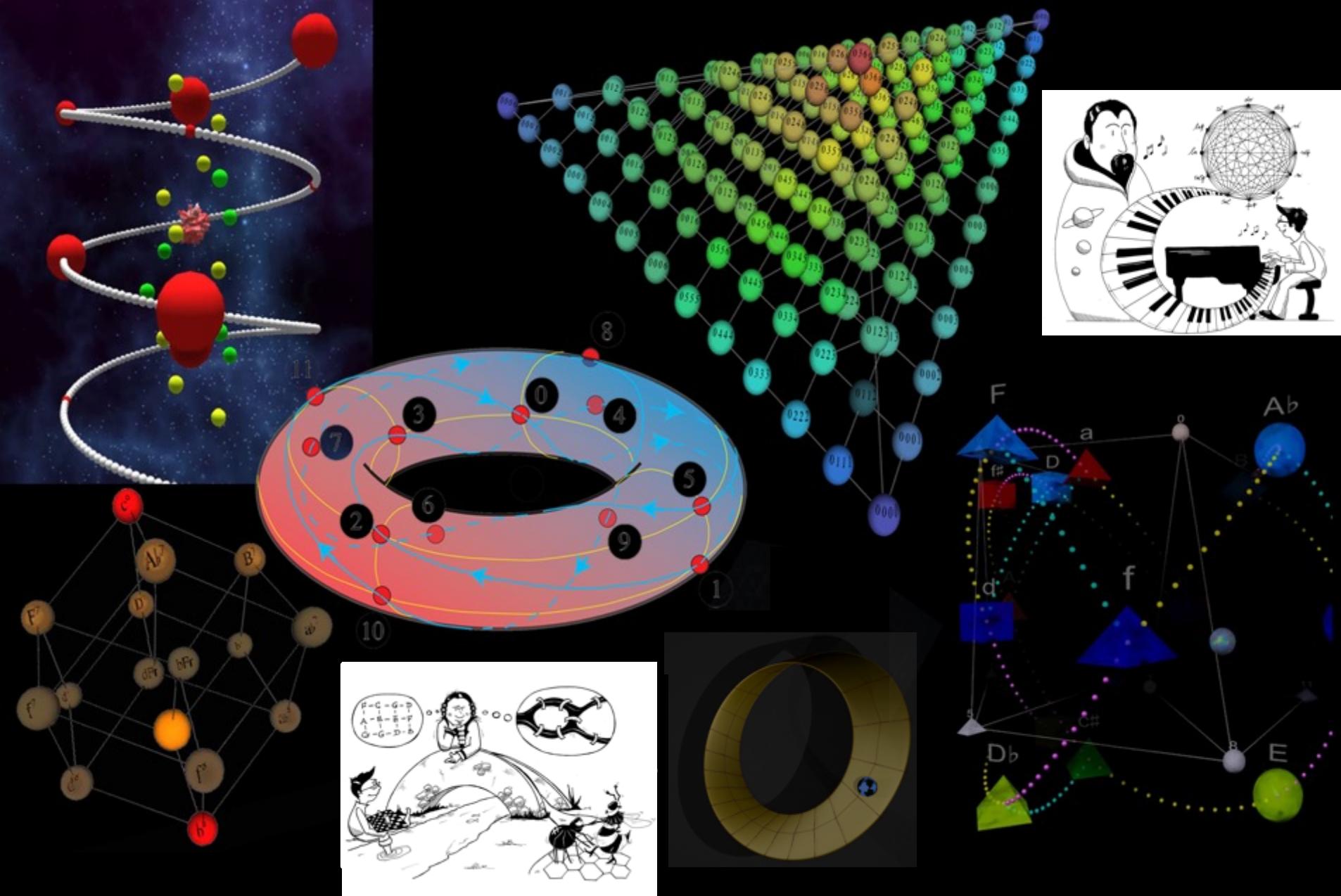
The galaxy of geometrical models at the service of music



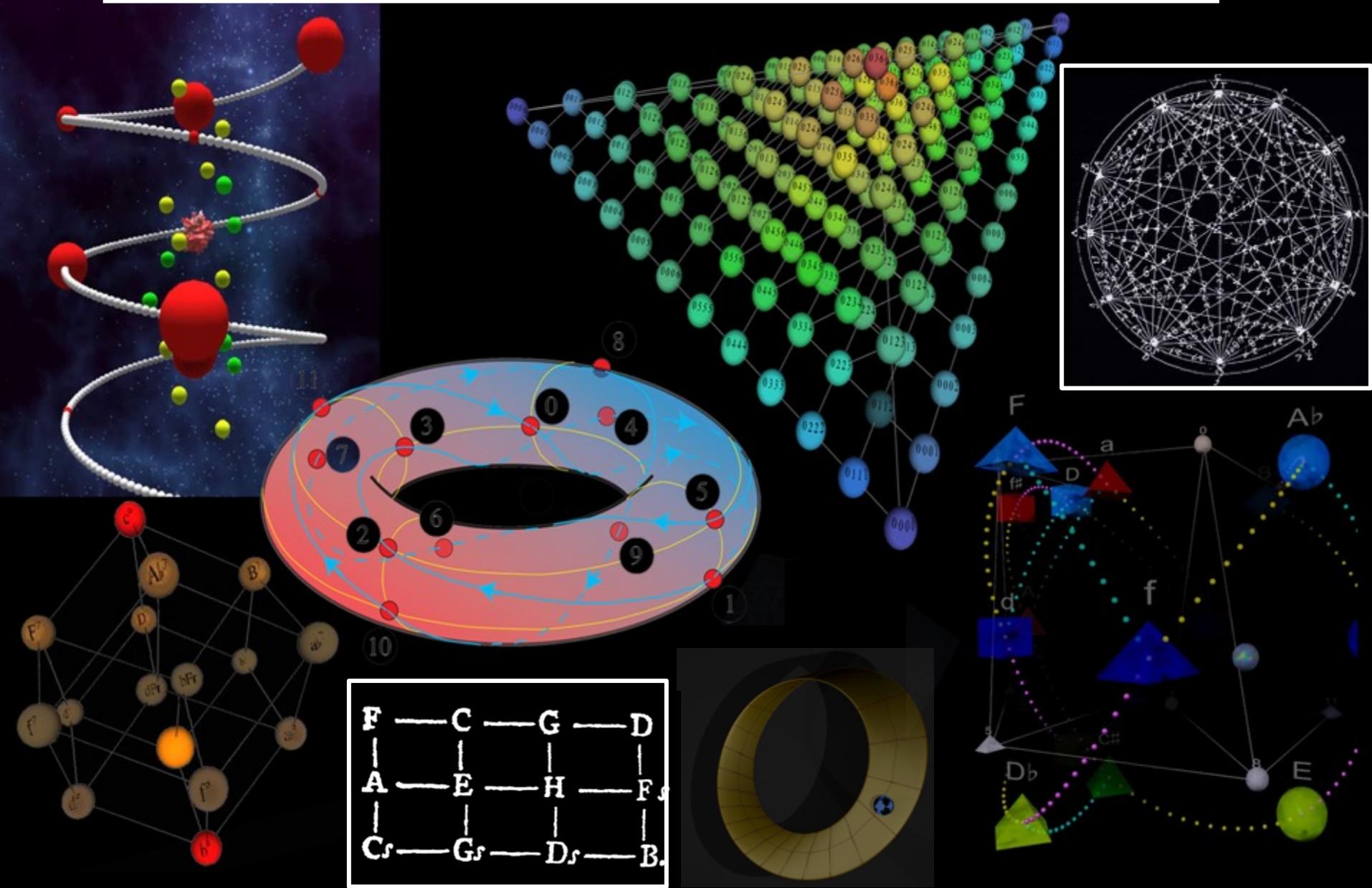
The galaxy of geometrical models at the service of music



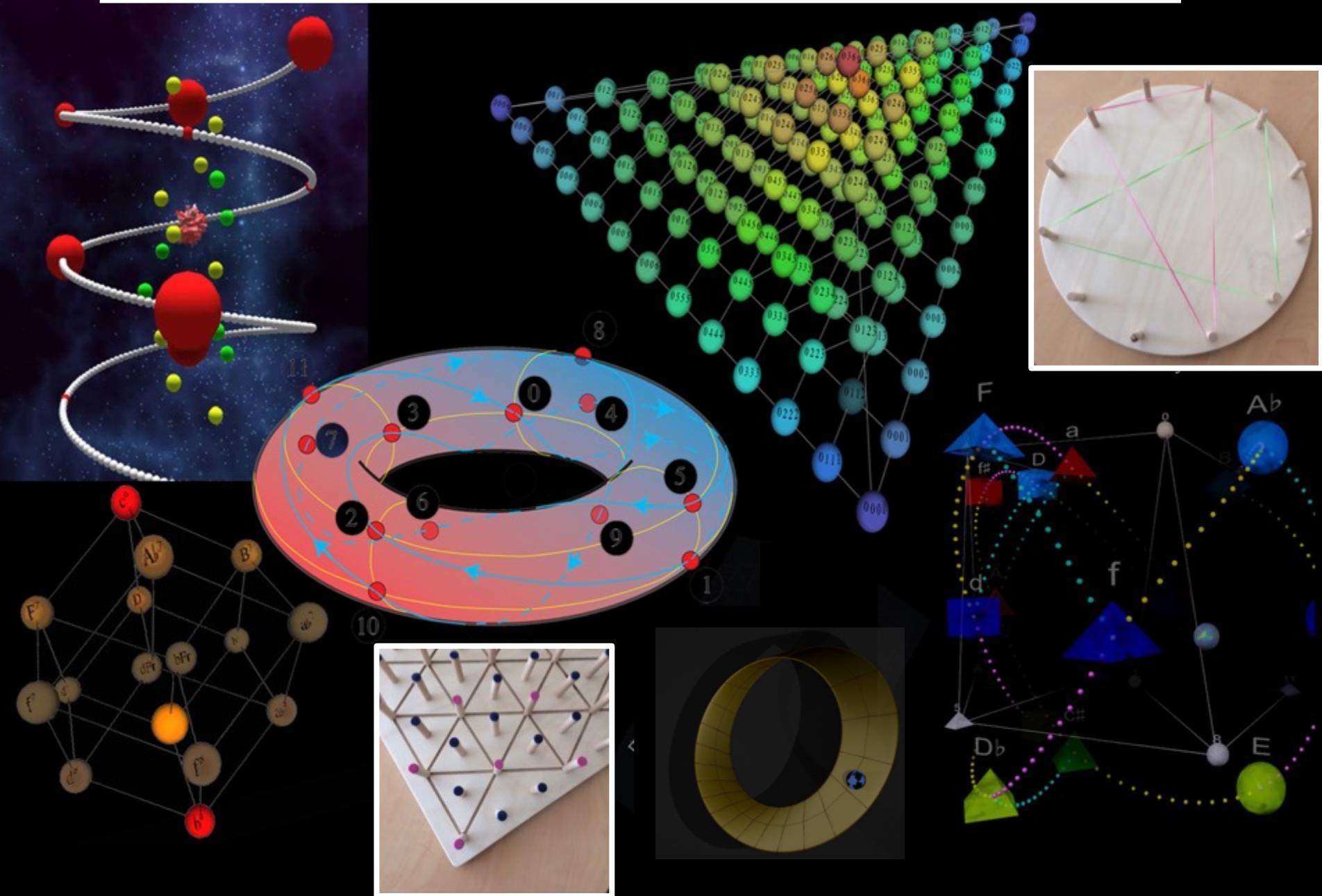
The galaxy of geometrical models at the service of music



The galaxy of geometrical models at the service of music

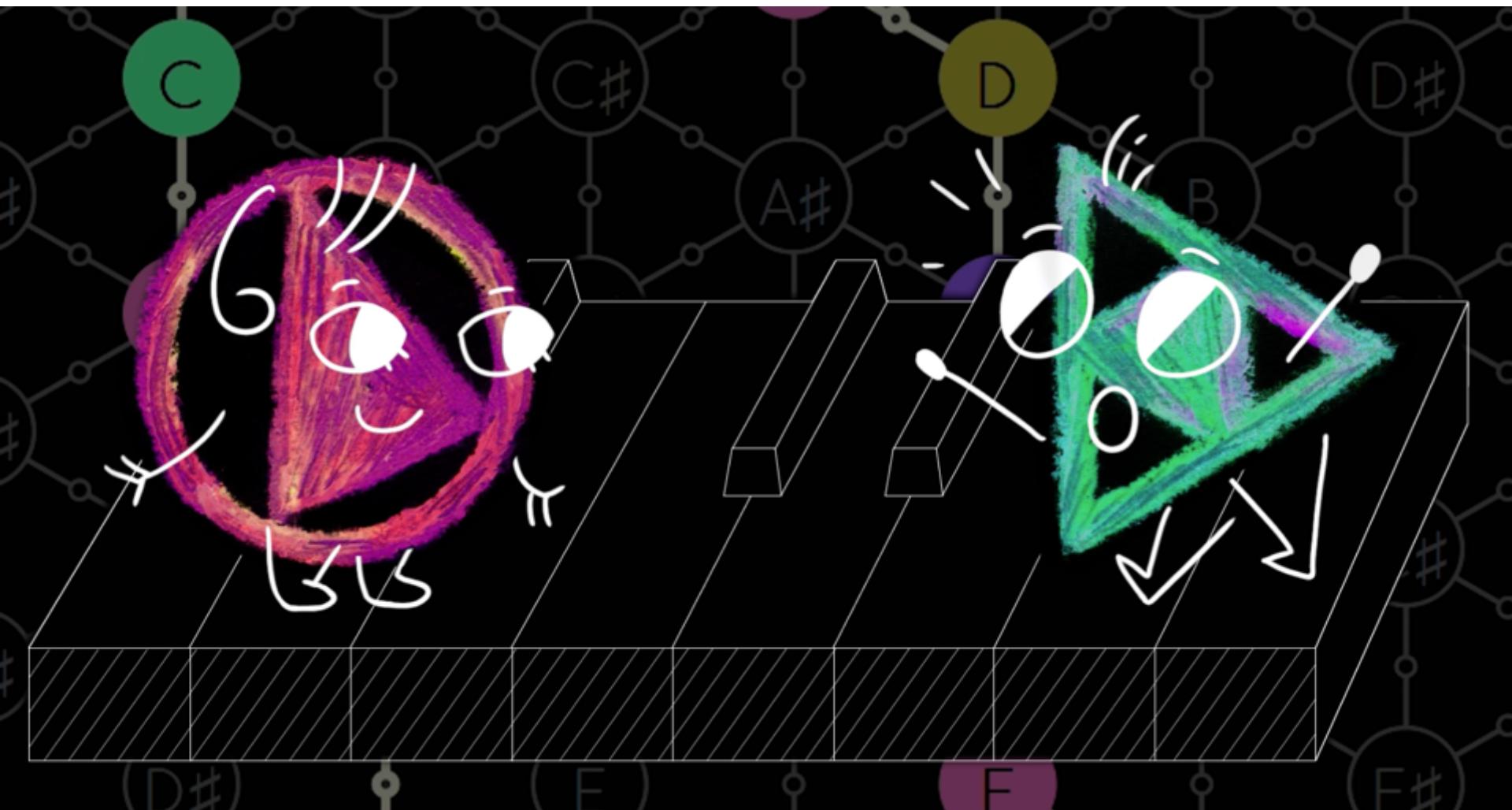


The galaxy of geometrical models at the service of music





« Musique et mathématiques »: a pedagogical film

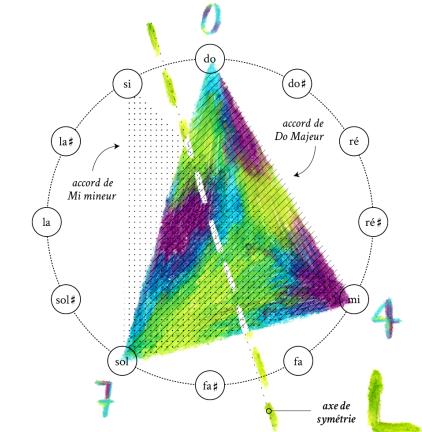
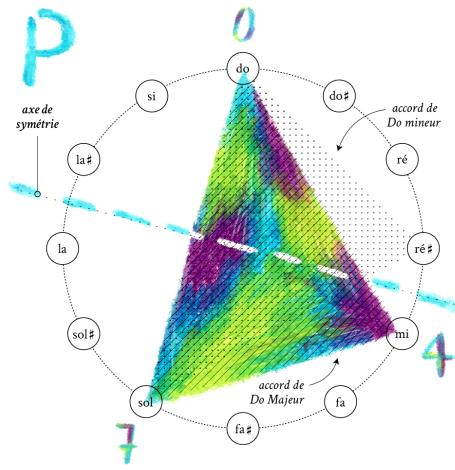
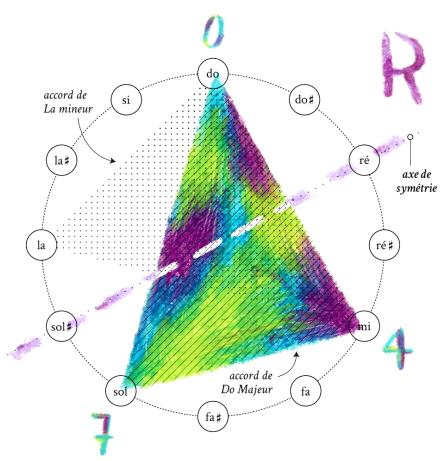
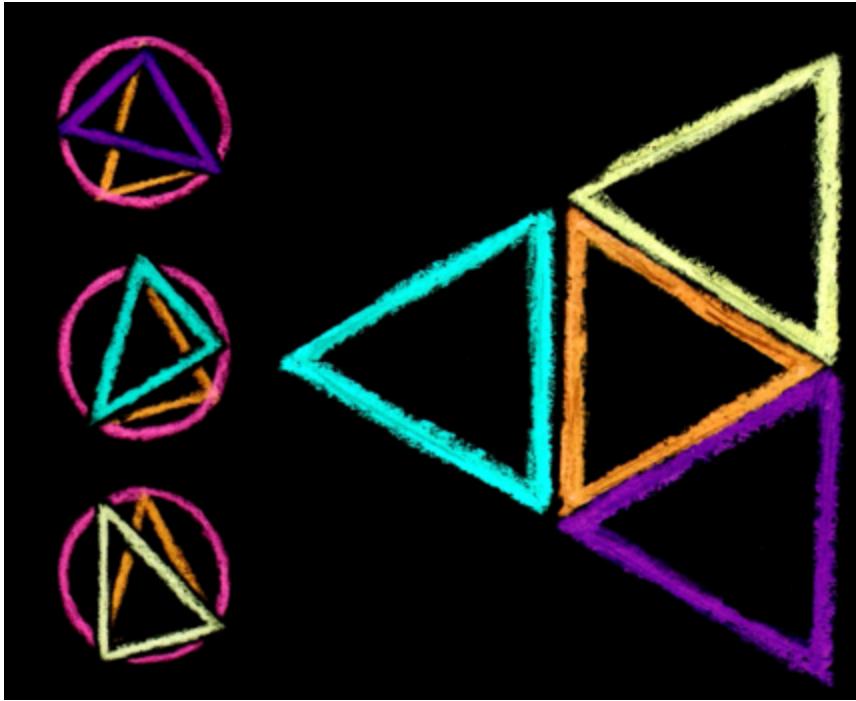


→ www.morenoandreatta.com



AuDiMATH
AUTOUR DE LA DIFFUSION
DES MATHÉMATIQUES

The three main symmetries in the Tonnetz

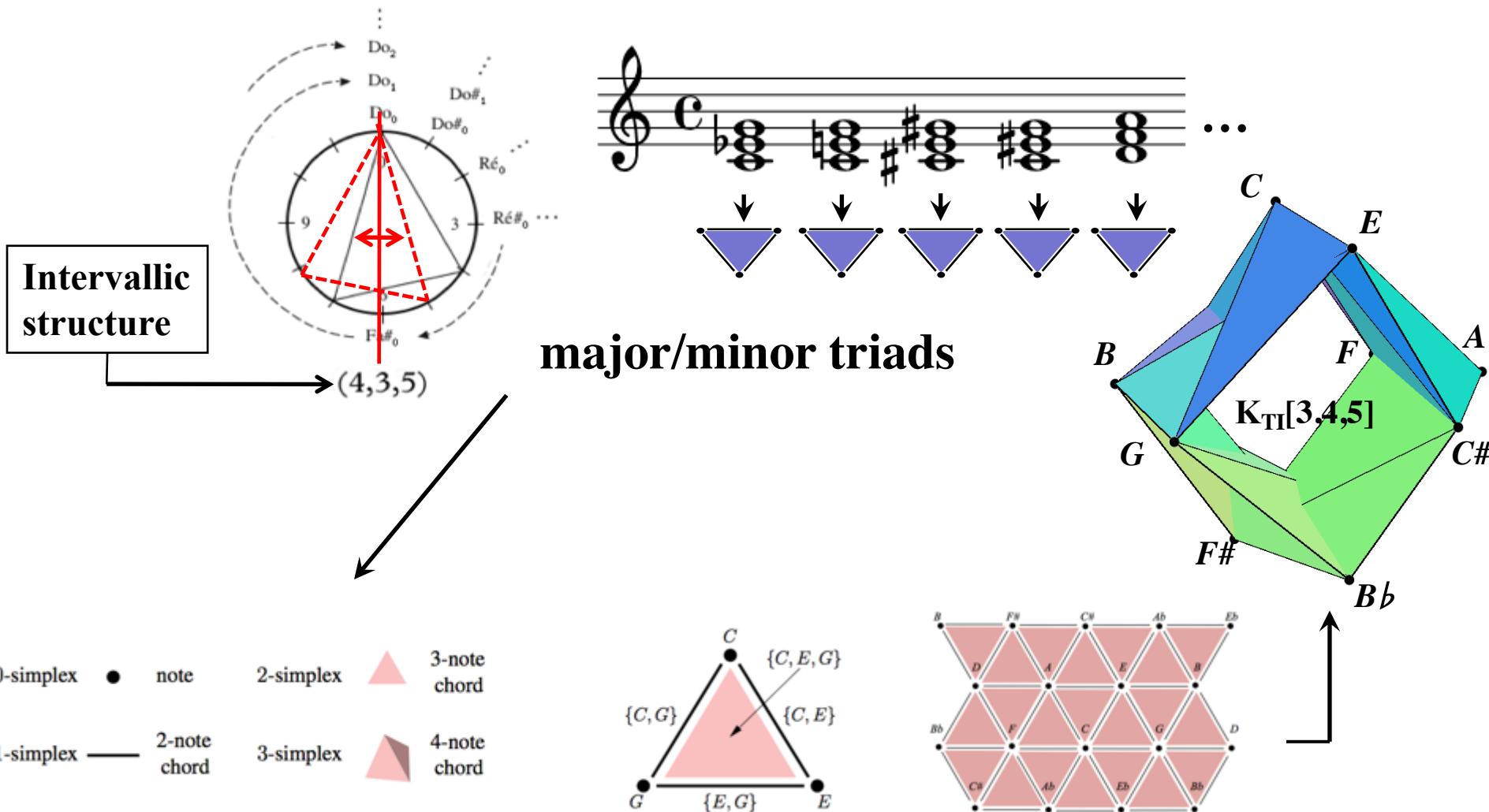


The Tonnetz as a simplicial complex

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013



- Assembling chords related by some equivalence relation
 - Transposition/inversion: Dihedral group action on $P(\mathbb{Z}_n)$



Mixing Algebra, Topology and Category Theory...



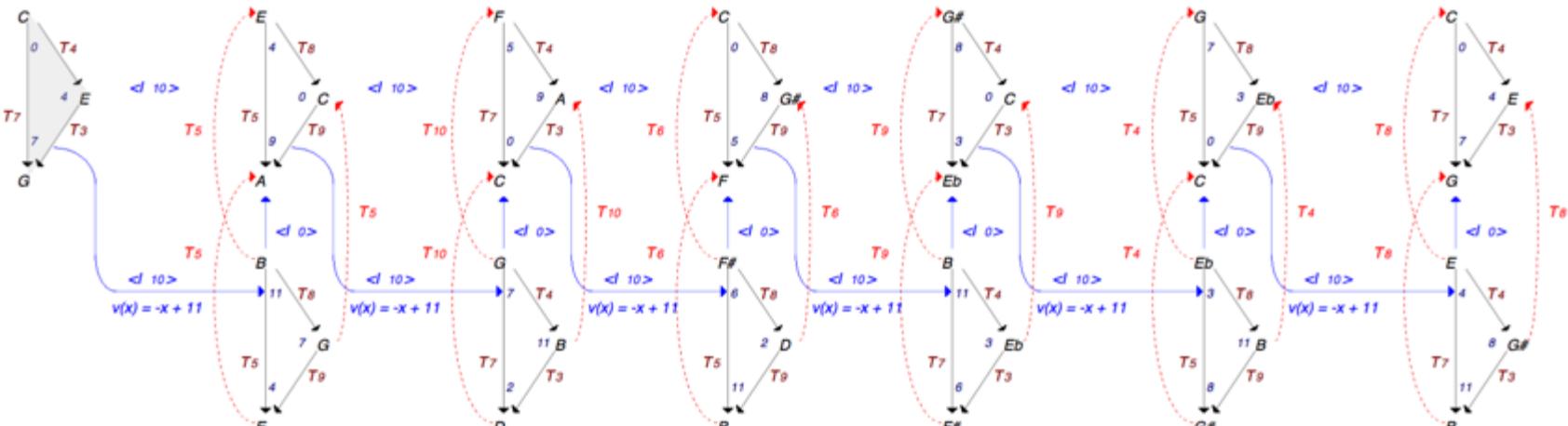
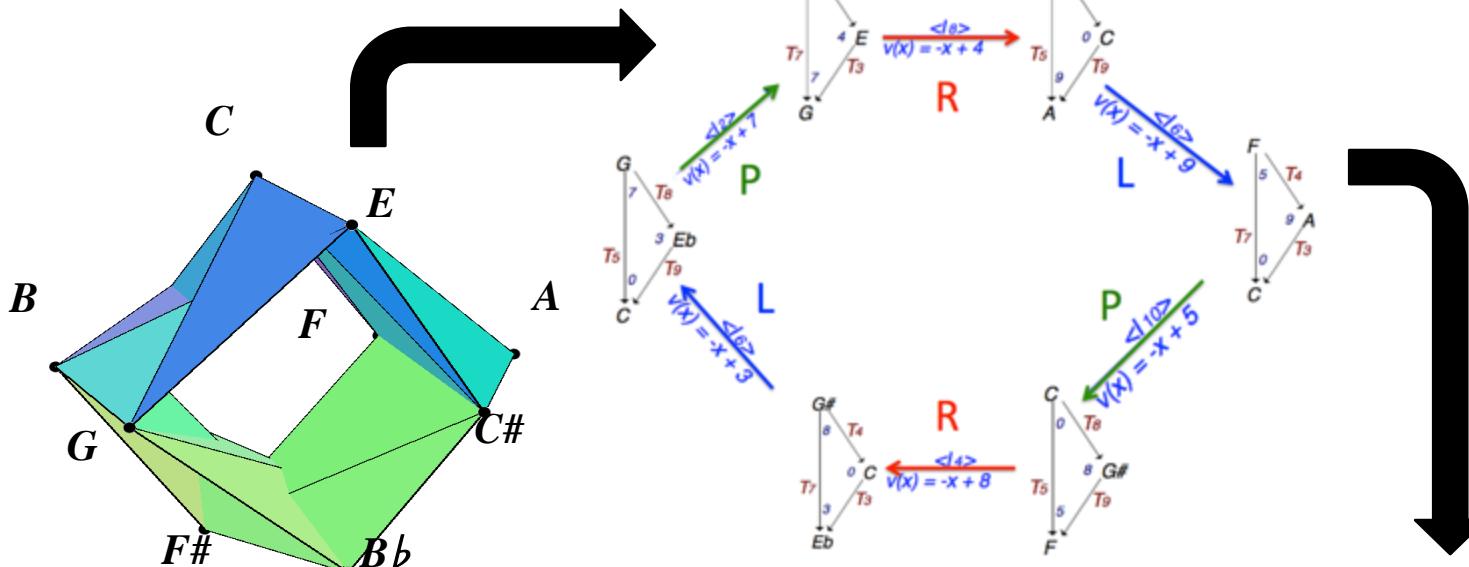
A. Popoff



A. Ehresmann

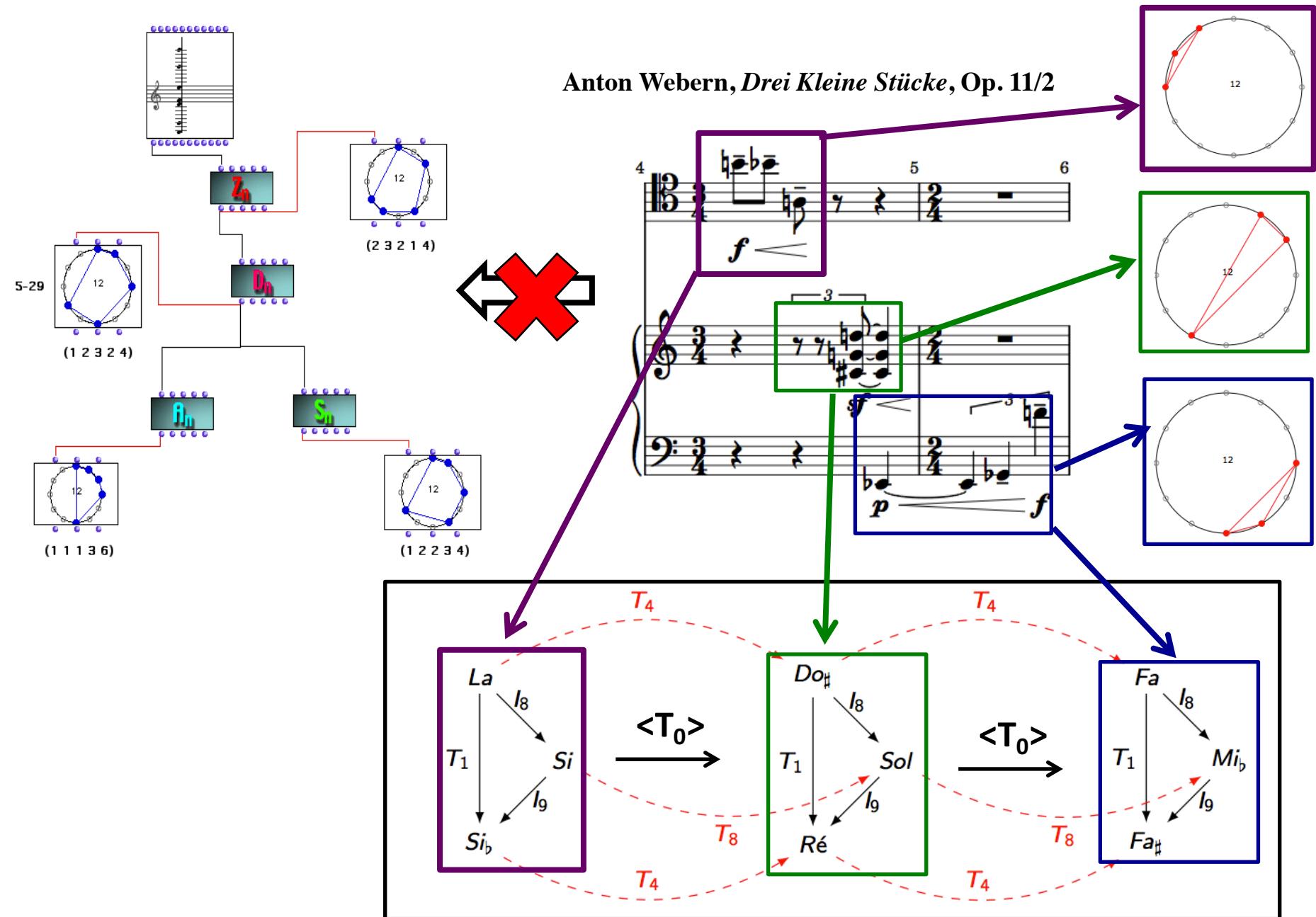


C. Agon

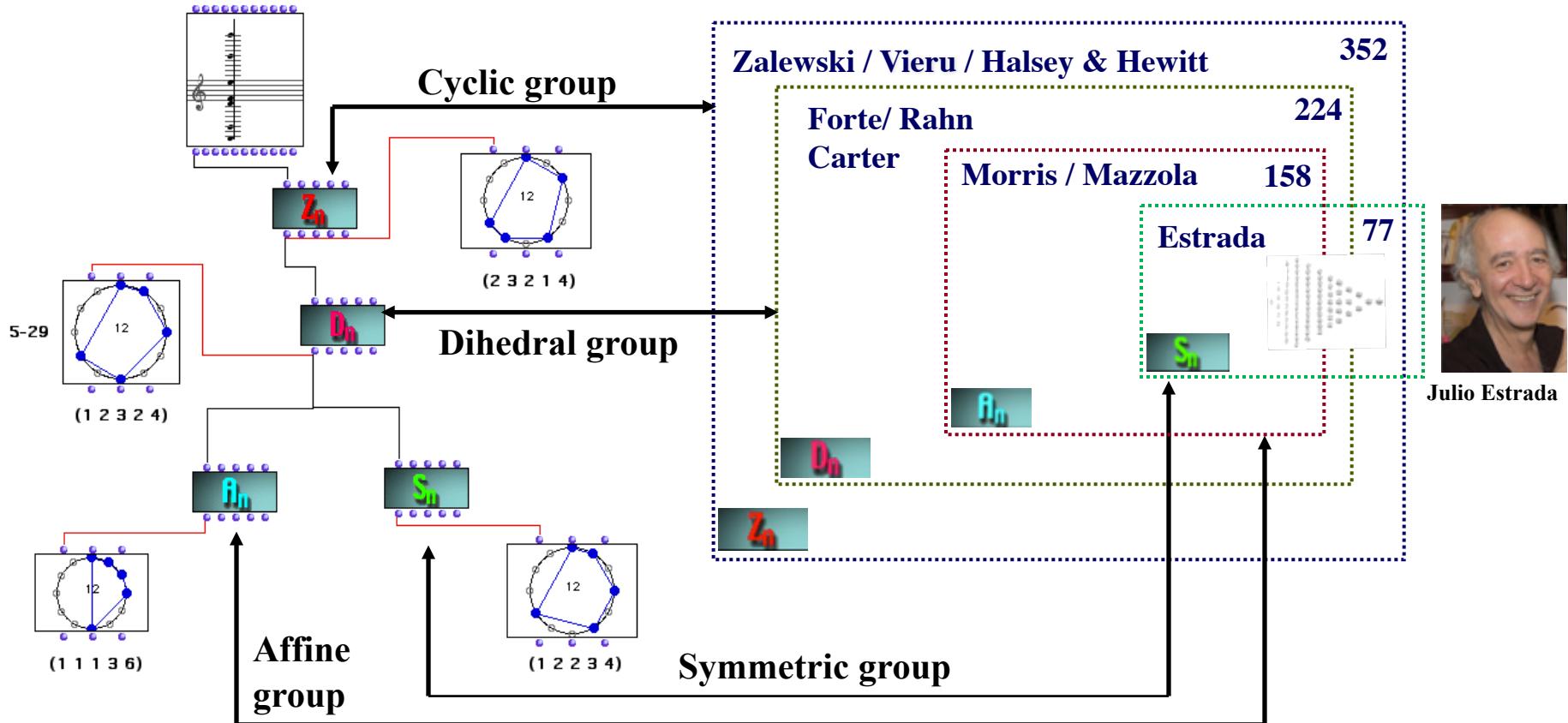


...in a computational environment

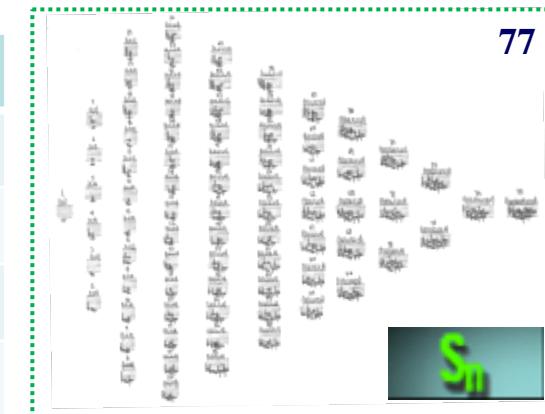
The categorical vs paradigmatic action-based approach



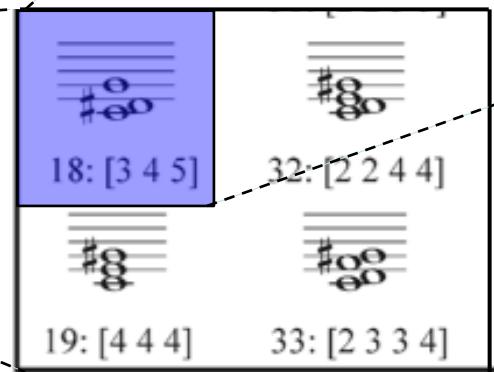
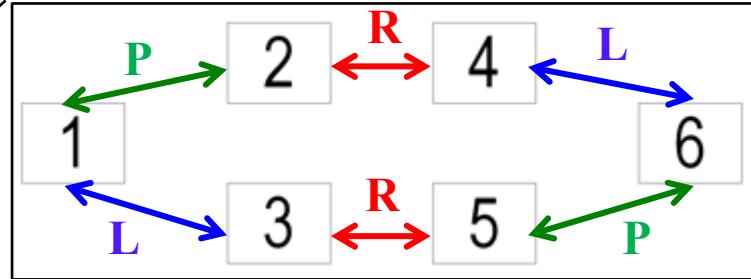
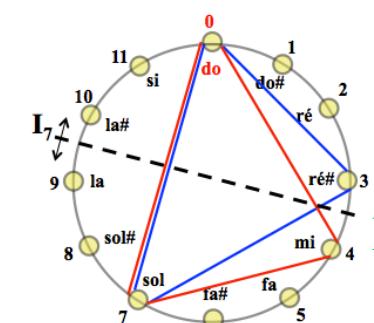
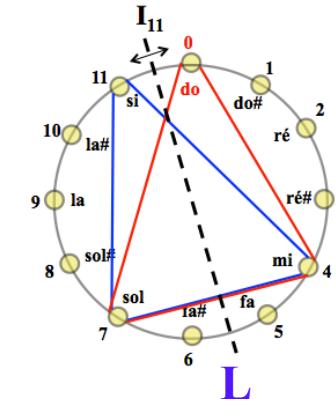
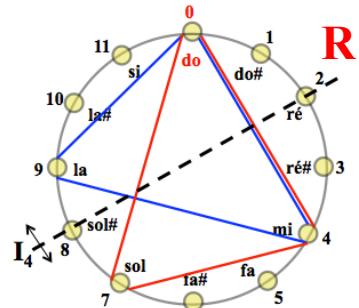
Combining Algebra and the theory of Lattices



	1	2	3	4	5	6	7	8	9	10	11	12
Z_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1
S_n	1	6	12	15	13	11	7	5	3	2	1	1



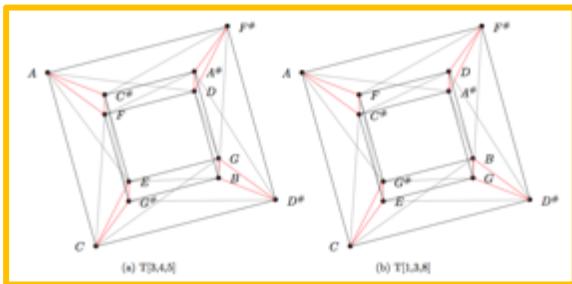
Permutohedron and Tonnetz: a structural inclusion



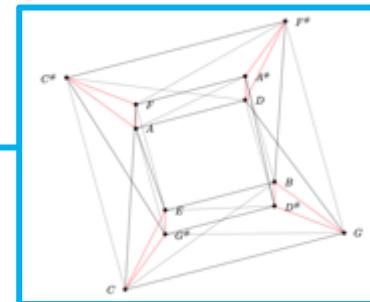
1	=	(3 4 5)
2	=	(4 3 5)
3	=	(3 5 4)
4	=	(4 5 3)
5	=	(5 3 4)
6	=	(5 4 3)

Classifying Tonnetze as Simplicial Chord Complexes

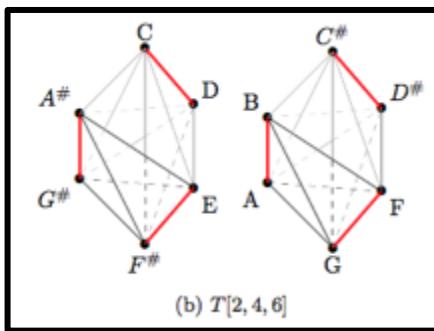
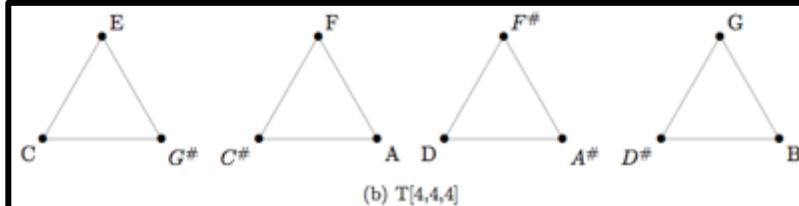
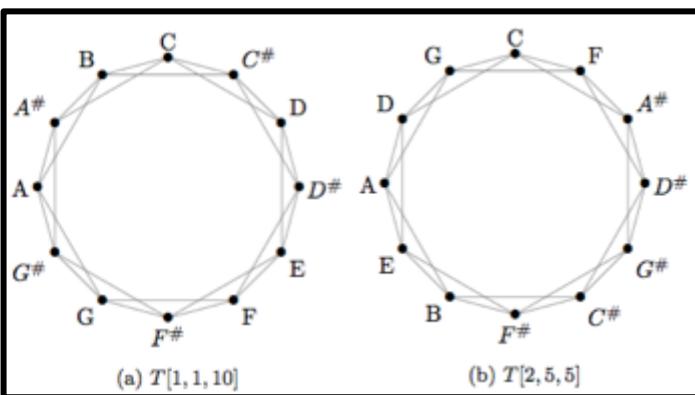
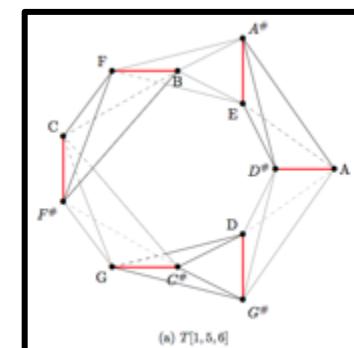
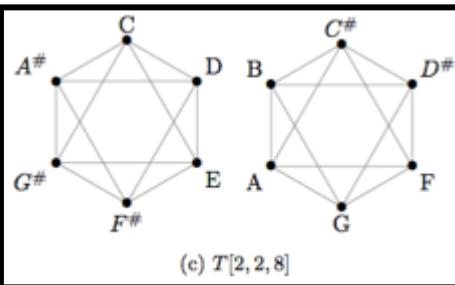
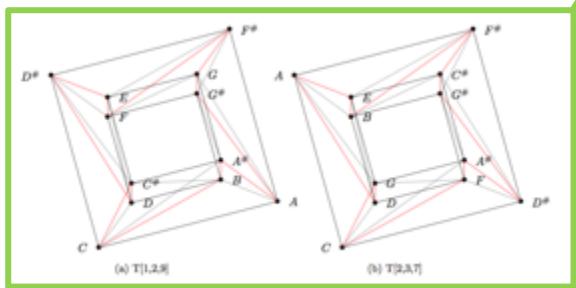
P. Lascabettes, *Homologie Persistante Appliquée à la Reconnaissance de Genres Musicaux*, Master 1, ENS Saclay



Tonnetz	Nombres de Betti		
	β_0	β_1	β_2
$T[1, 2, 9]$	1	2	1
$T[1, 3, 8]$	1	2	1
$T[1, 4, 7]$	1	2	1
$T[2, 3, 7]$	1	2	1
$T[3, 4, 5]$	1	2	1
$T[1, 1, 10]$	1	1	0
$T[2, 5, 5]$	1	1	0
$T[2, 2, 8]$	2	2	0
$T[1, 5, 6]$	1	1	6
$T[2, 4, 6]$	2	2	6
$T[3, 3, 6]$	3	0	3
$T[4, 4, 4]$	4	0	0

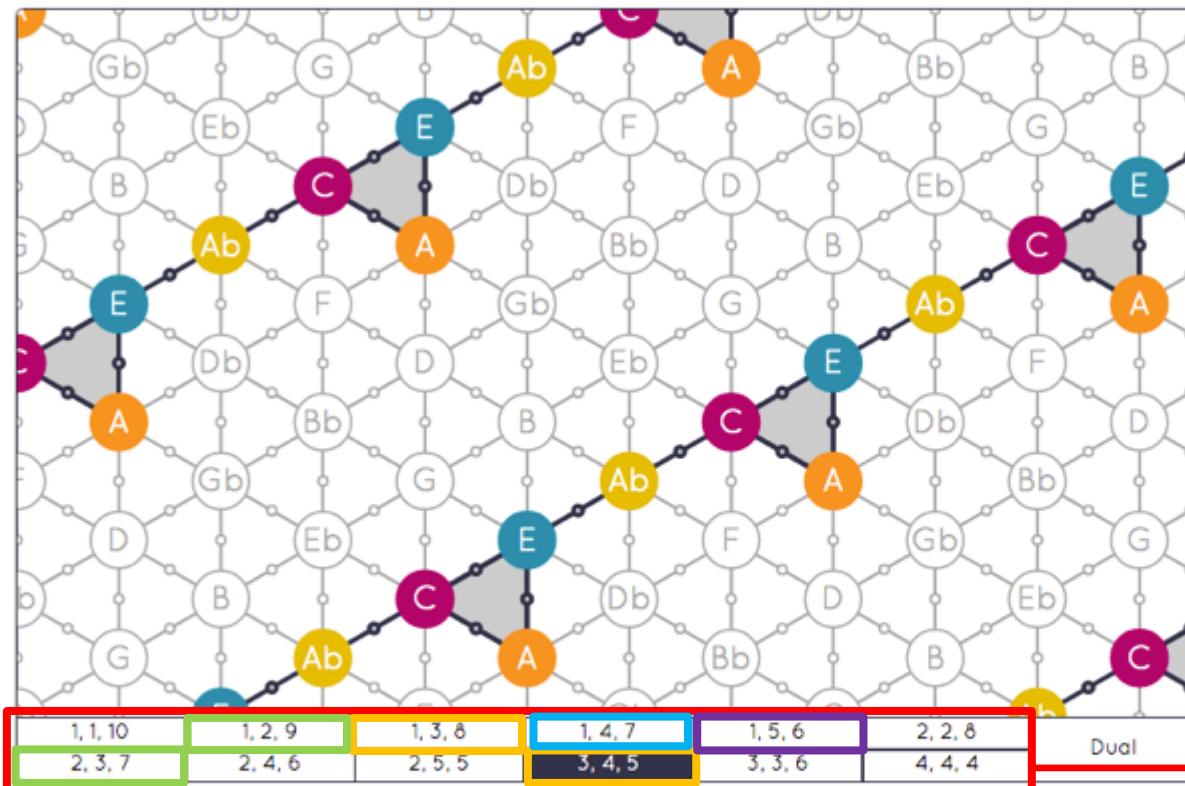


Paul Lascabettes



THE TONNETZ

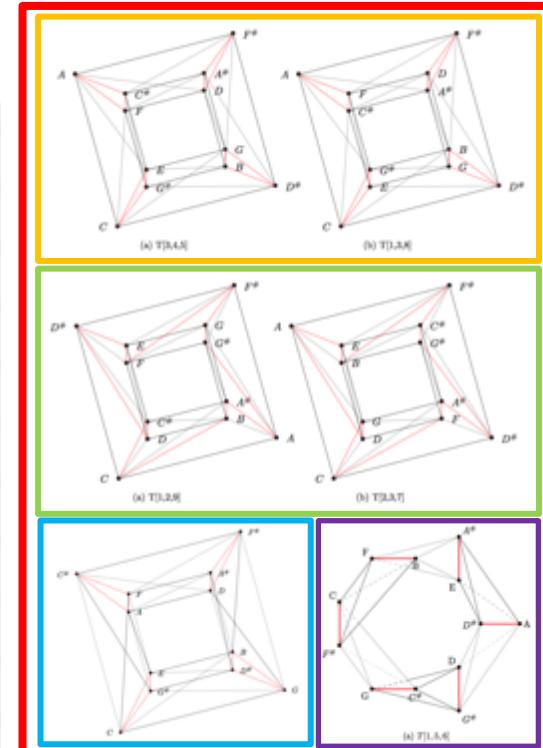
ONE KEY – MANY REPRESENTATIONS



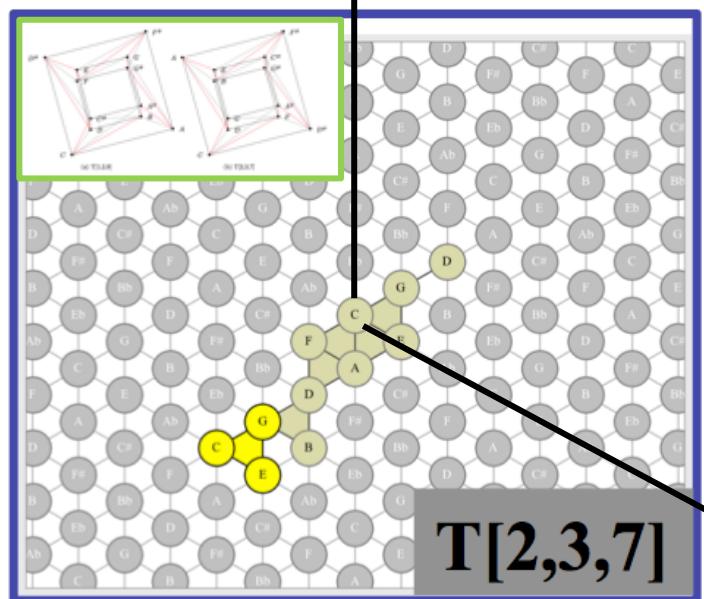
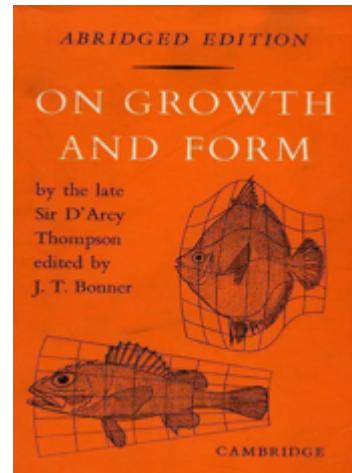
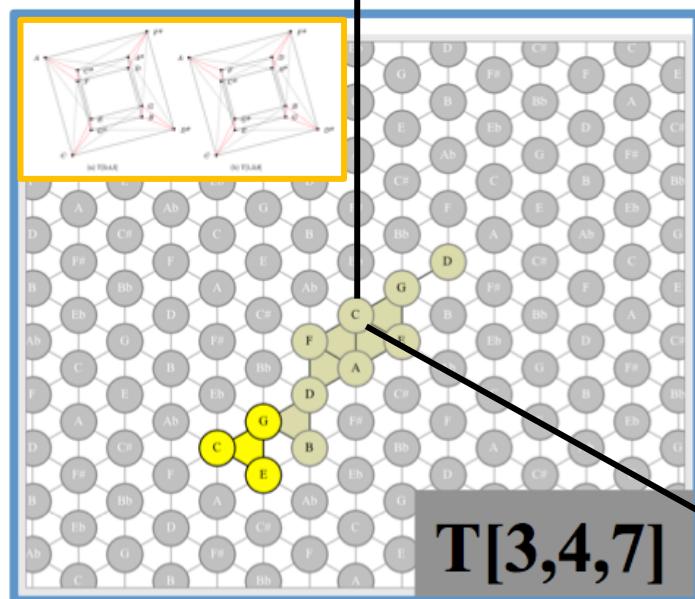
Load Midi File Play Start Recording Rotate 180° Translate



→ <https://guichaoua.gitlab.io/web-hexachord/>



The style...is the space!



Spatial music analysis via *Hexachord*

Plex Viewer

Tonnetz : $K[3,4,5]$

Computer Music Journal

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Dynamical Systems and Simplicial Chord Spaces

bwv0281.mid

Tempo

Play Stop

Select midi file

Chromatic complexes Heptatonic complexes

$K[2,3,7]$ CM

Trace off Harmonization ON

Display graph

Vertical compactness

compactness dimension complexes dimension

2-compactness 2

compute compactness

absolute compactness

Path Transformation

Origin complex Destination complex

$K[3,4,5]$ $K[3,4,5]$

Rotation 0

North translation 0

North-east translation 0

Path Transformation

Chart

bwv0281

2-compactness

0,8
0,75
0,7
0,65
0,6
0,55
0,5
0,45
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0

K[1,1,10] K[1,2,9] K[1,3,8] K[1,4,7] K[1,5,6] K[2,2,8] K[2,3,7] K[2,4,6] K[2,5,5] K[3,3,6] K[3,4,5] K[4,4,4]

■ bwv0281 ■ random chords

Complex compliance

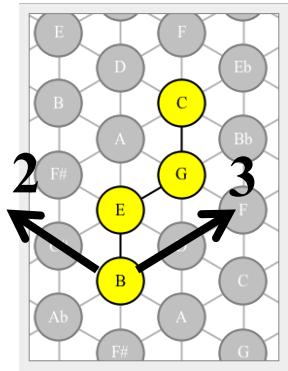
1
0

0 5 000 10 000 15 000 20 000 25 000 30 000 35 000 time

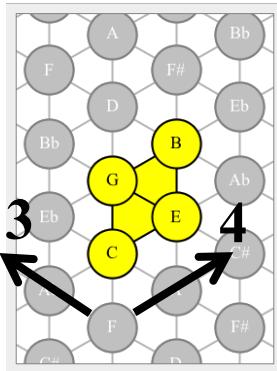
— K[1,1,10] — K[1,2,9] — K[1,3,8] — K[1,4,7] — K[1,5,6] — K[2,2,8]
— K[2,3,7] — K[2,4,6] — K[2,5,5] — K[3,3,6] — K[3,4,5] — K[4,4,4]

→ <http://www.lacl.fr/~lbigo/hexachord>

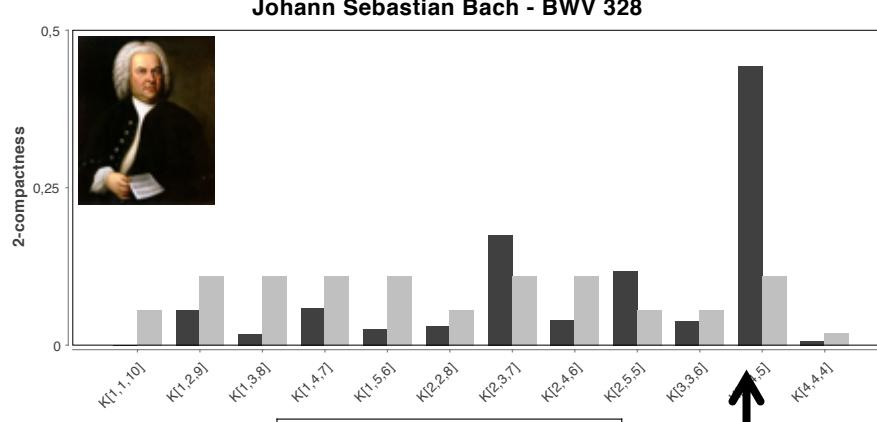
The geometric character of musical style



(2,3,7)

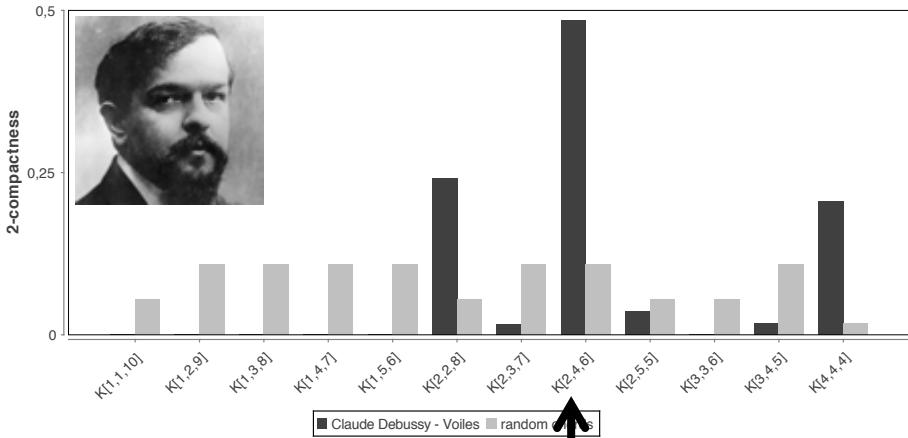


(3,4,5)

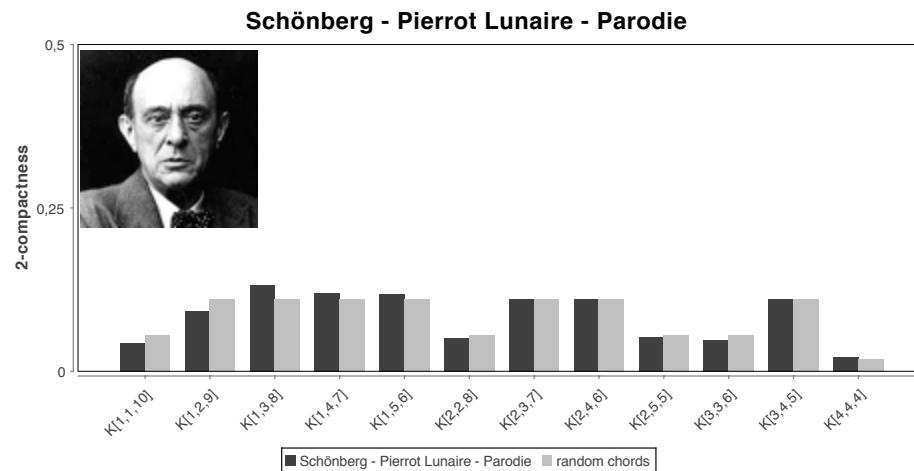


(3,4,5)

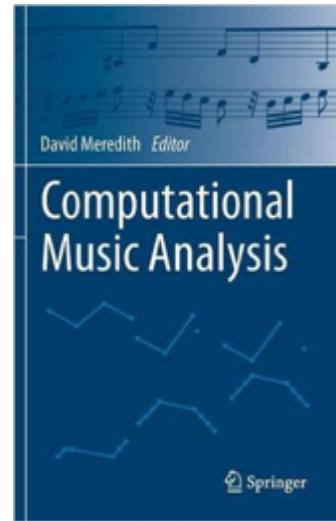
Claude Debussy - Voiles



(2,4,6)



→ Bigo L., M. Andreatta, "Musical analysis with simplicial chord spaces", in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015

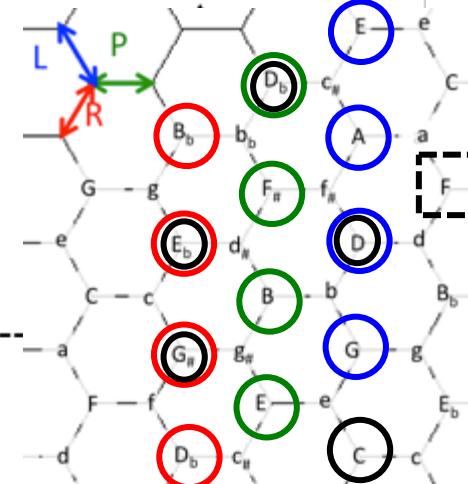
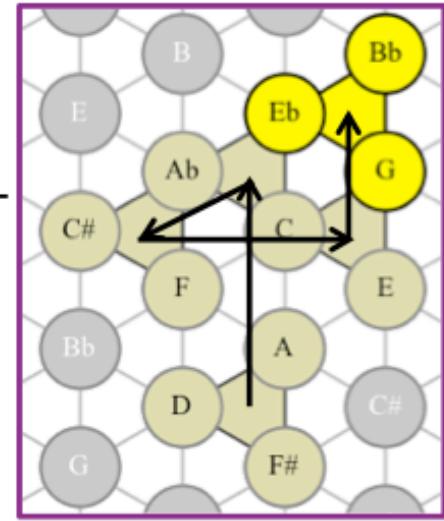
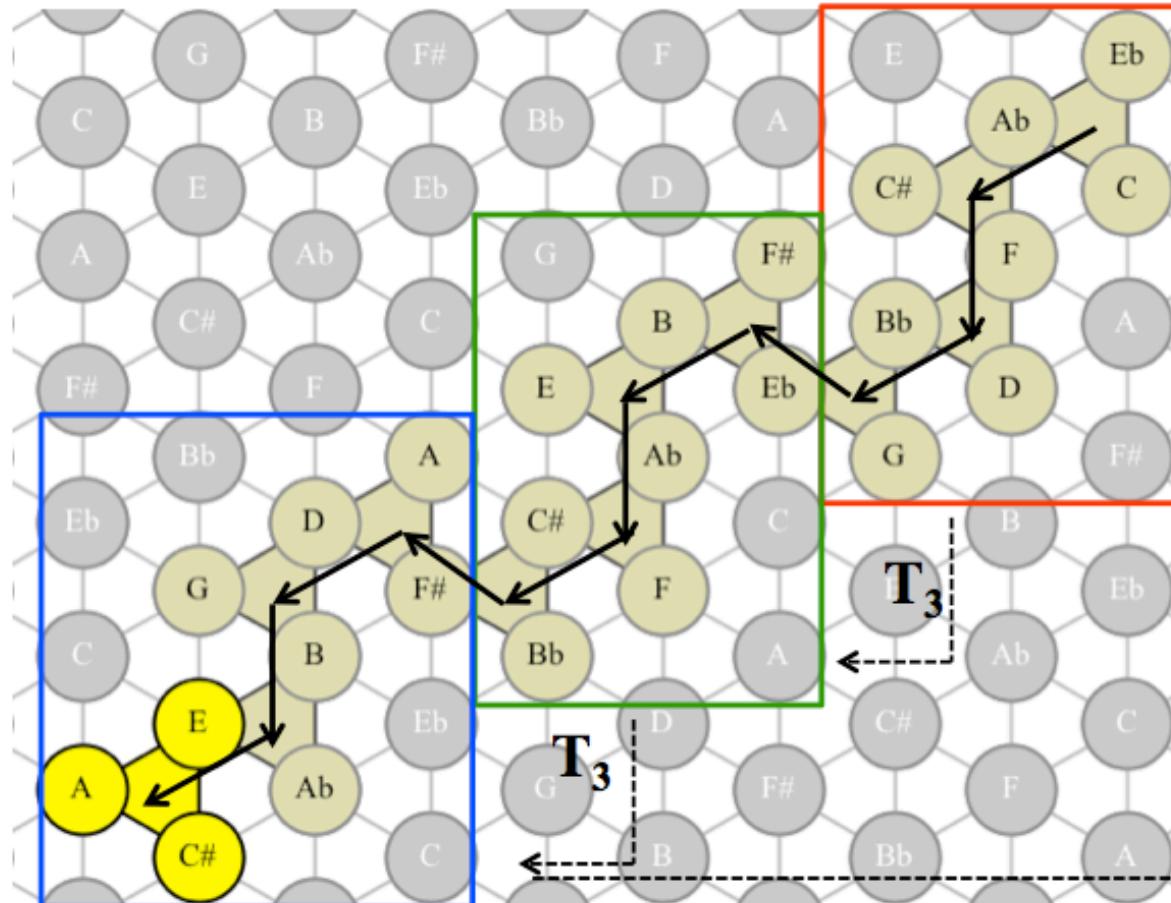




Symmetries in Paolo Conte's *Madeleine*

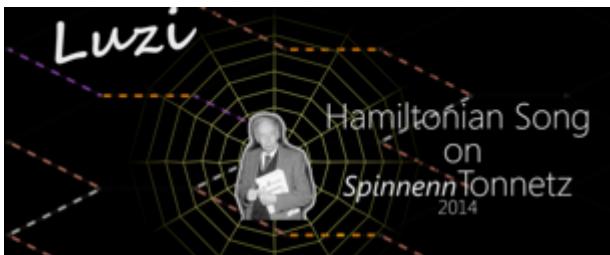
La_b Re_b Si_b Mi_b Si Mi Re_b Fa_# Re Sol Mi La Re La_b Re_b Do Mi_b

Four musical staves are shown, each consisting of a treble clef, a key signature, and four measures of music. The staves are color-coded: red, green, blue, and purple. The red staff contains chords La_b, Re_b, Si_b, and Mi_b. The green staff contains Si, Mi, Re_b, and Fa_#. The blue staff contains Re, Sol, Mi, and La. The purple staff contains Re, La_b, Re_b, Do, and Mi_b.

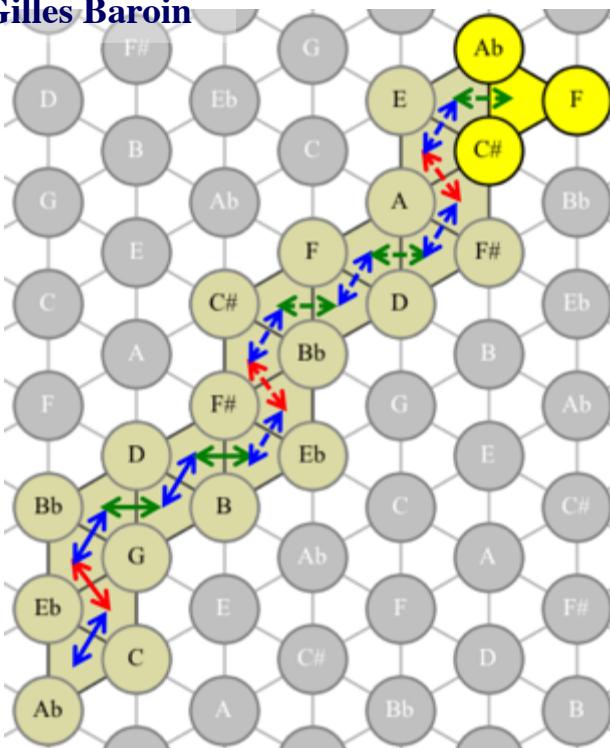


Almost total covering of the major-chords space

La sera non è più la tua canzone (after Mario Luzi)



Gilles Baroin

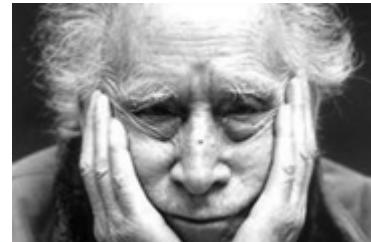


**La sera non è più la tua canzone,
è questa roccia d'ombra traforata
dai lumi e dalle voci senza fine,
la quiete d'una cosa già pensata.**

**Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d'una cosa conosciuta,
per nome hai una parola ch'è passata
nell'intimo del cuore e s'è perduta.**

**Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell'essere, (compiuta).**

**L'aria tace ed il tempo dietro a te
si leva come un'arida montagna
dove vaga il tuo spirito e si perde,
un vento raro scivola e ristagna.**



M. Luzi (1914-2005)

*Le soir n'est plus ta chanson,
c'est ce rochet d'ombre transpercé
par les lumières et les voix sans fin,
la paix d'une chose déjà pensée.*

*Ah, cette lumière vive et claire vient
uniquement de toi, tu es si proche
du vrai d'une chose connue,
tu as pour nom une parole qui est passée
dans l'intimité du cœur où elle s'est
perdue .*

*Tombé est plus qu'un signe de la vie,
tu te reposes, du voyage tu es revenue
à l'intérieur de toi même, tu es
descendue dans cette
pure substance qui est si tienne,
si éloignée
dans le silence de l'être, achevée.*

*L'air se tait et le temps derrière toi
se lève tel une montagne aride
où plane ton esprit et se perd,
un vent rare glisse et stagne.*

(tr. Antonia Soulez, philosophe et poète)

Music: M. Andreatta
Arrangement and mix: M. Bergomi & S. Geravini
(Perfect Music Production)
Mastering: A. Cutolo (Massive Arts Studio, Milan)

Reading Beethoven backwards

Le Blé en Herbe



(Polo/Moreno/Dieu)

Plonger comme un enfant, cheveux au vent

Croiser matin dans l'herbe folle

Sous l'océan du blé en herbe

Deux tourterelles qui s'envolent

Marée d'épis couleur d'amande

Suivre les jeux des hirondelles

Qui tendent à caresser le ciel

Sur le paysage éternel

Algues tendres de mille plages

Nager comme un enfant, cheveux au vent

Frôlant le ventre des nuages

Sous l'océan

Cheveux de pluie, dos de poissons

Du blé en herbe

Qui frissonnent à l'unisson

Marée de fruits au goût amer

Suivre le bord des continents

Acide et salée comme la mer

Dans l'océan du blé en herbe

Vers l'ilôt d'un petit village

Pêcher le corail du pavot

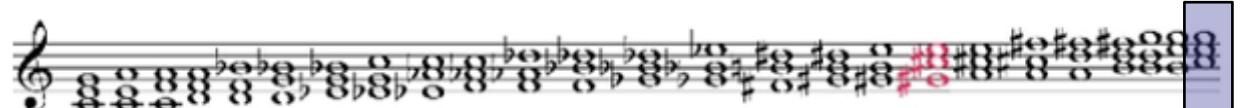
Vers un château d'eau sur la plage

Dans le sang des coquelicots

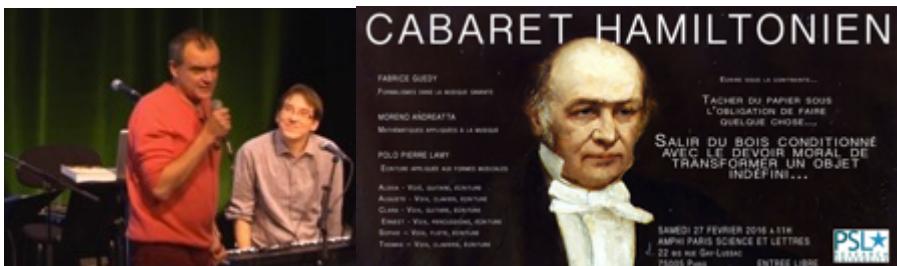
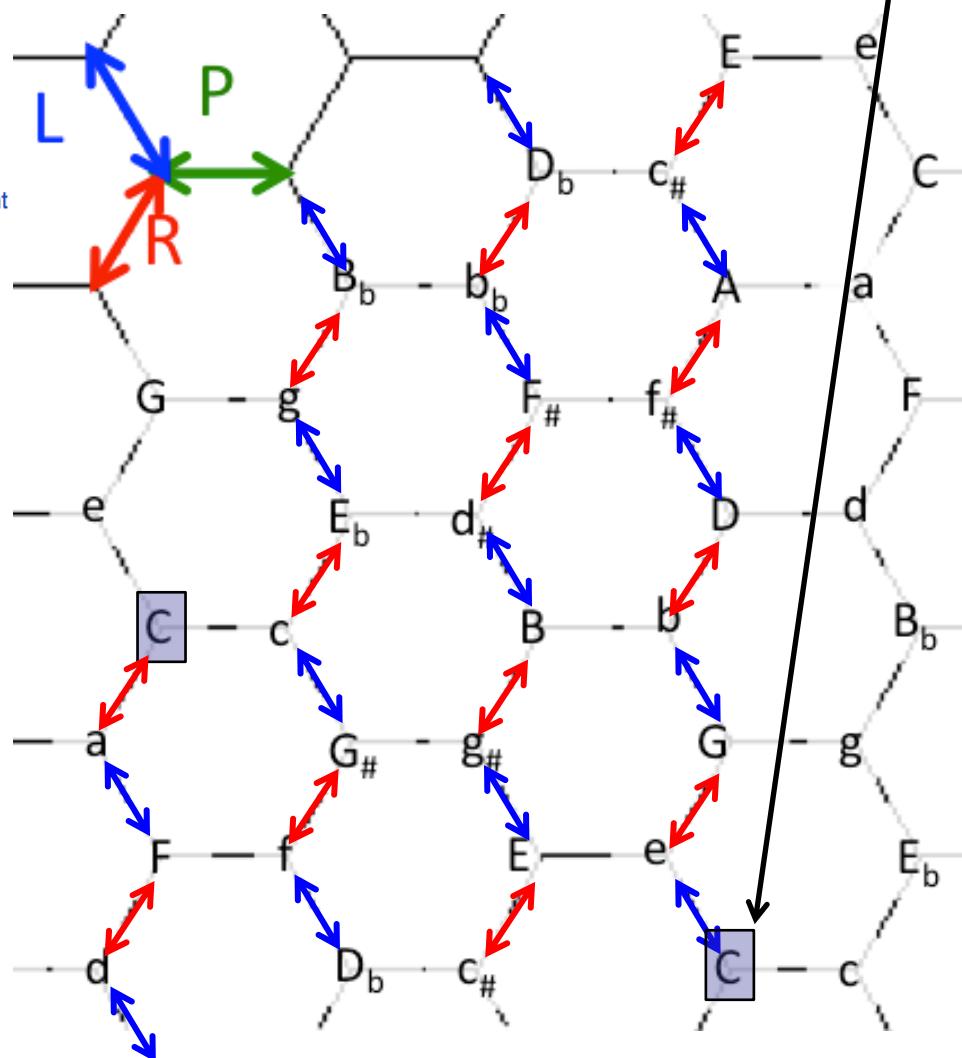
Quand tout s'éteint avant l'orage

Quand se lève le vent du large

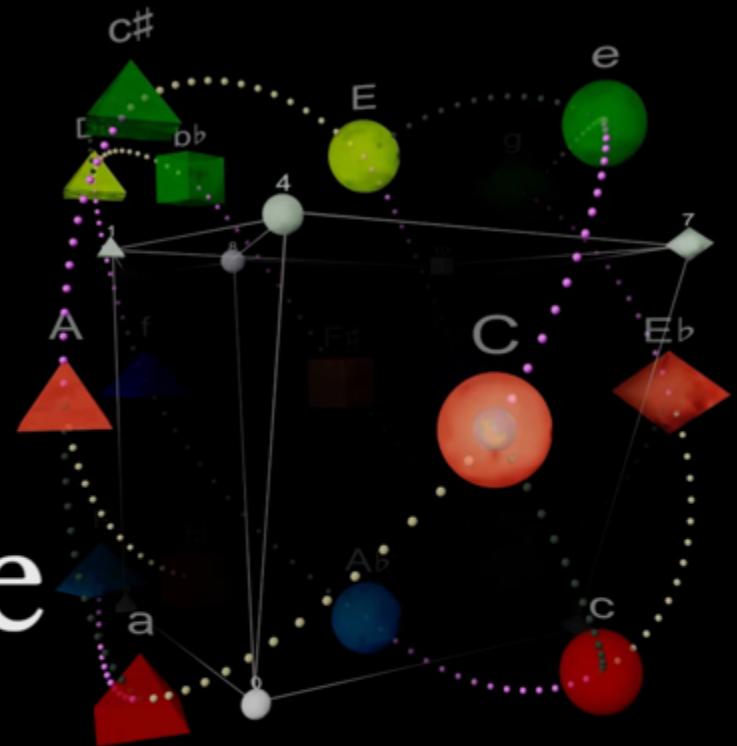
Sur le blé vert



← time



Beethoven and the Hypersphere *(and the Tonnetz)*



Gilles Baroin 2016
www.MatheMusic.net



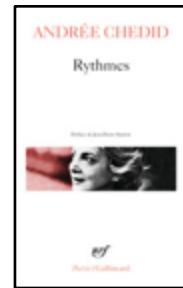
Gilles Baroin



From poetry to song writing:

hamiltonian compositional strategies

A part (Andrée Chedid, poème tiré du recueil *Rhymes Collection Poésie/Gallimard* (n. 527), Gallimard, 2018)

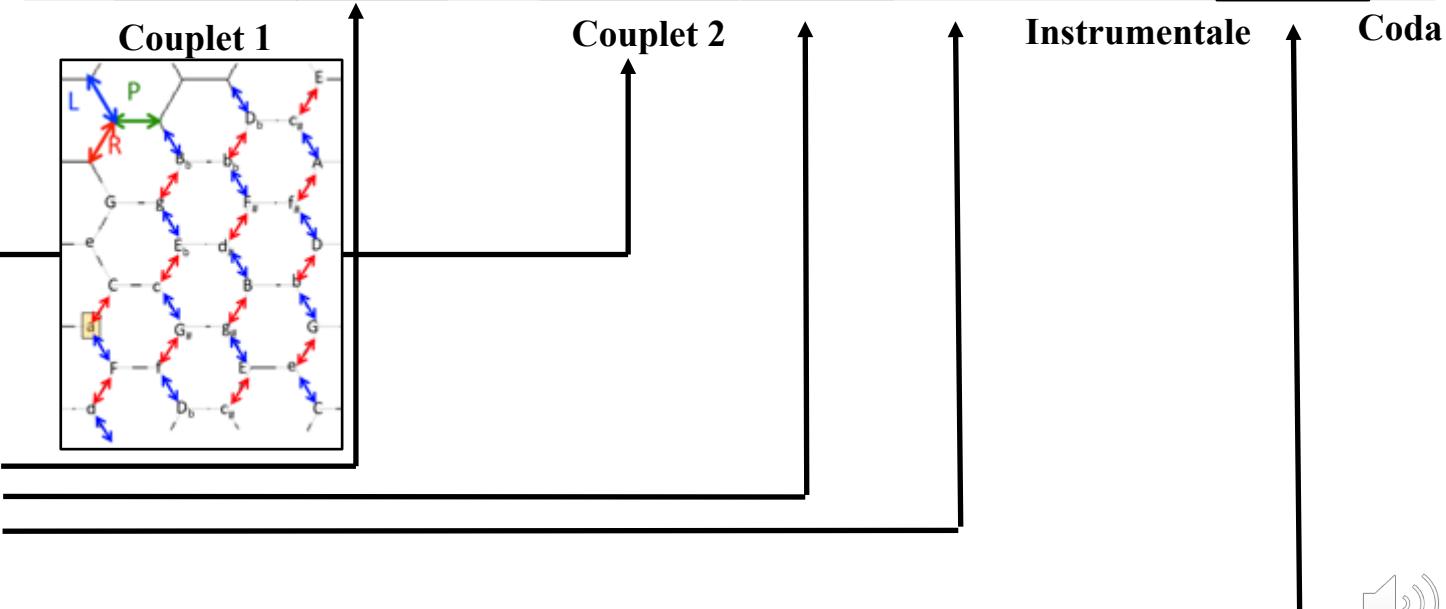
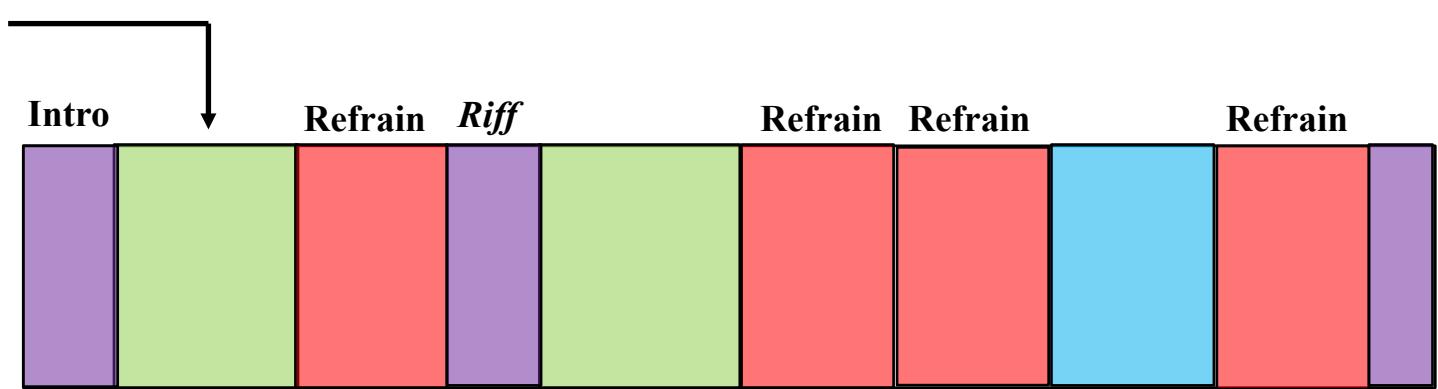


À part le temps
Et ses rouages
À part la terre
En éruptions
À part le ciel
Pétrisseur de nuages
À part l'ennemi
Qui génère l'ennemi

À part le désamour
Qui ronge l'illusion
À part la durée
Qui moisit nos visages

À part les fléaux
À part la tyrannie
À part l'ombre et le crime
Nos batailles nos outrages

Je te célèbre ô Vie
Entre cavités et songes
Intervalle convoité
Entre le vide et le rien

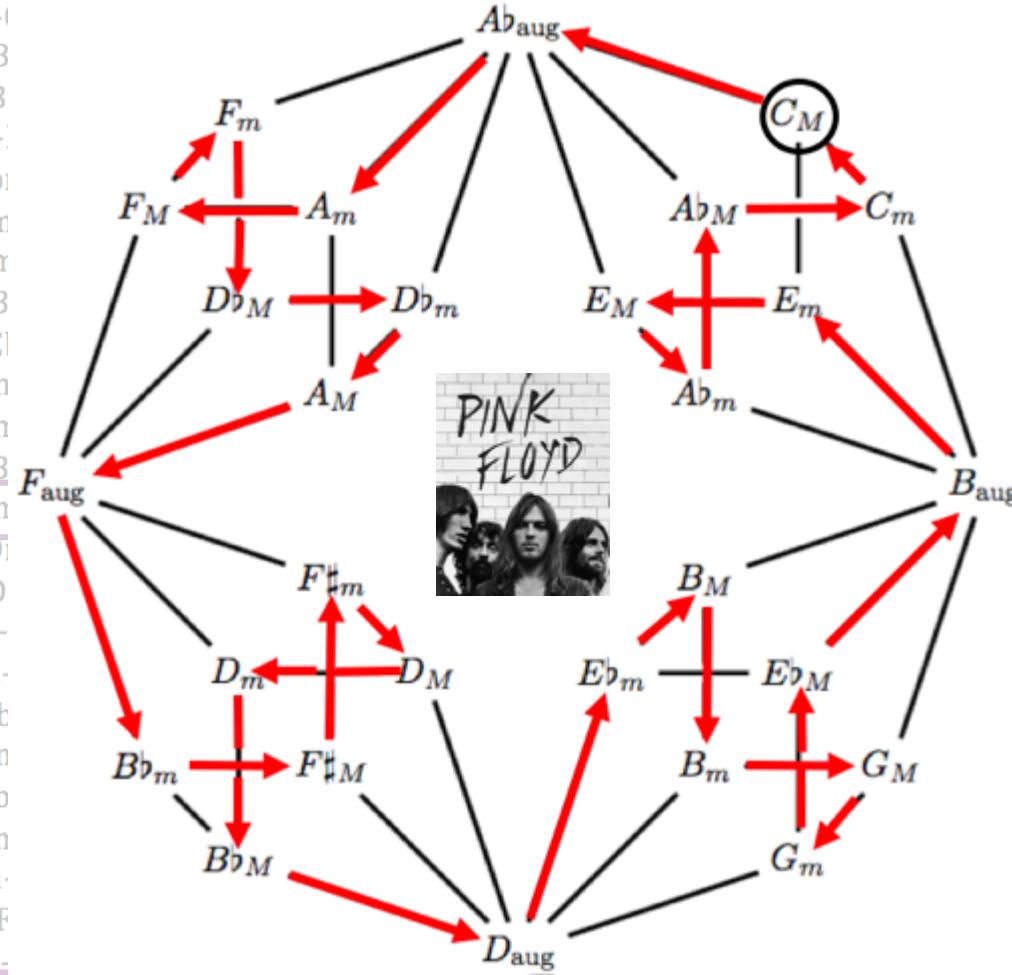


Exploring Hamiltonian trajectories in song writing

1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPRPR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLRLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPRLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRLRPRP
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPRL
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRLRLRP
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm--RLRPRPRP
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em--RLRPRPRL
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLRP
27. C-Am-F-Dm-Bh-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bhm-F#-Ebm-Eb-Cm--RLRLRPRP
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL

Exploring Hamiltonian trajectories in song writing

1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-
↓
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B
↓
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-B
↓
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-
↓
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Eb
↓
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm
↓
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#n
↓
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-B
↓
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-E^l
↓
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm
↓
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#n
↓
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-B
↓
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebn
↓
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-D
↓
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-D
↓
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-
↓
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-
↓
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bb
↓
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbn
↓
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bb
↓
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbn
↓
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-
↓
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F
↓
27. C-Am-F-Dm-Bh-Gm-G-Em-E-C#m-A-F#m-
↓
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL



Rotation of the trajectory and Negative Harmony

The image displays three software windows side-by-side:

- Plex Viewer:** Shows a geometric representation of musical data on a hexagonal grid.
- Tonnetz : K[3,4,5]:** A Tonnetz diagram for the key of K[3,4,5]. It features nodes labeled with musical notes (e.g., C, Eb, G, D, B, A, F, E, Ab, Bb, F#, C#) and chords. A yellow node 'C' is highlighted with a red circle and a rotation arrow, indicating a transformation. A large downward arrow points from this window to the one below it.
- InfoBox:** A control panel for a MIDI file named "bwv0281.mid". It includes settings for tempo (0 to 20), play/stop buttons, and dropdown menus for chromatic complexes (K[2,3,7]) and heptatonic complexes (CM). A section for "Path Transformation" is highlighted with a black border, showing fields for "Rotation" (0), "North translation" (0), and "North-east translation" (0).

Below the top row, there are two images of the Beatles crossing Abbey Road:

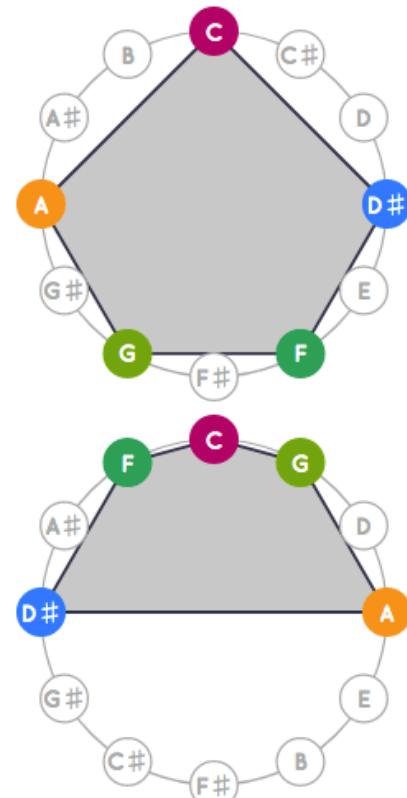
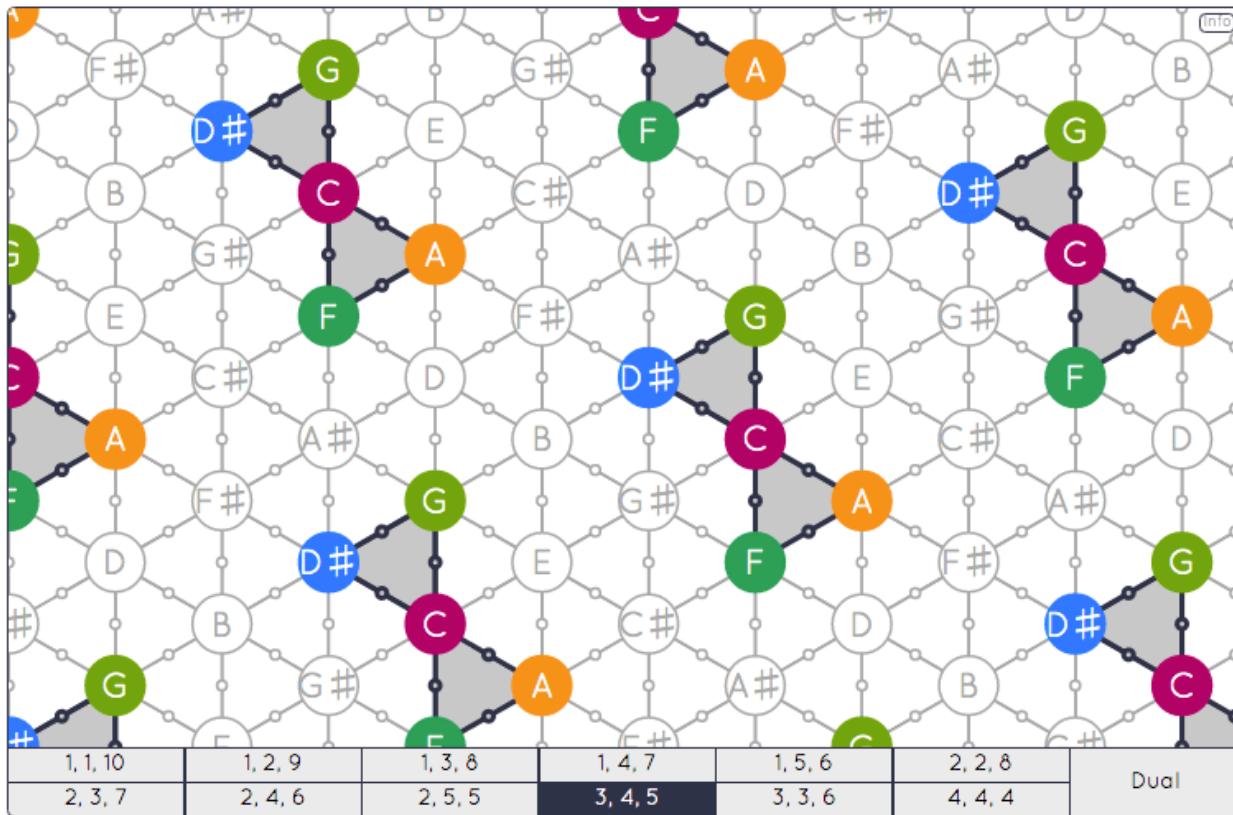
- The top image is a grayscale version of the original photo.
- The bottom image is a high-contrast, black-and-white version of the same scene.
- A large downward arrow points from the middle image to the bottom image.
- A speaker icon is positioned between the two images, suggesting a comparison of different audio representations or transformations.

→ <http://www.lacl.fr/~lbigo/hexachord>

THE TONNETZ

ONE KEY – MANY REPRESENTATIONS

→ DEMO



Load Midi File Play Start Recording Export

Rotate 180° ↪ A ⇌ A
 A ⇌ A#
 A ⇌ B
 A ⇌ C
 A ⇌ C#
 A ⇌ D
 A ⇌ D#
 A ⇌ E
 A ⇌ F
 A ⇌ F#
 A ⇌ G
 A ⇌ G#

Translate 1 1 1 1 1 1 1

12 rotations
 (=12 inversions)

A piano keyboard icon is at the bottom.

Thank you for your attention!

