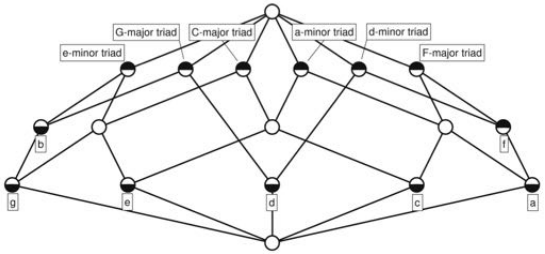
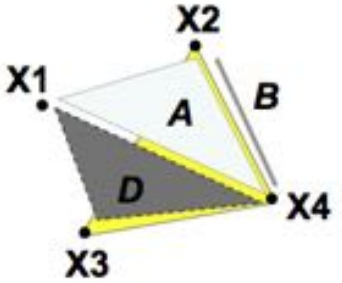
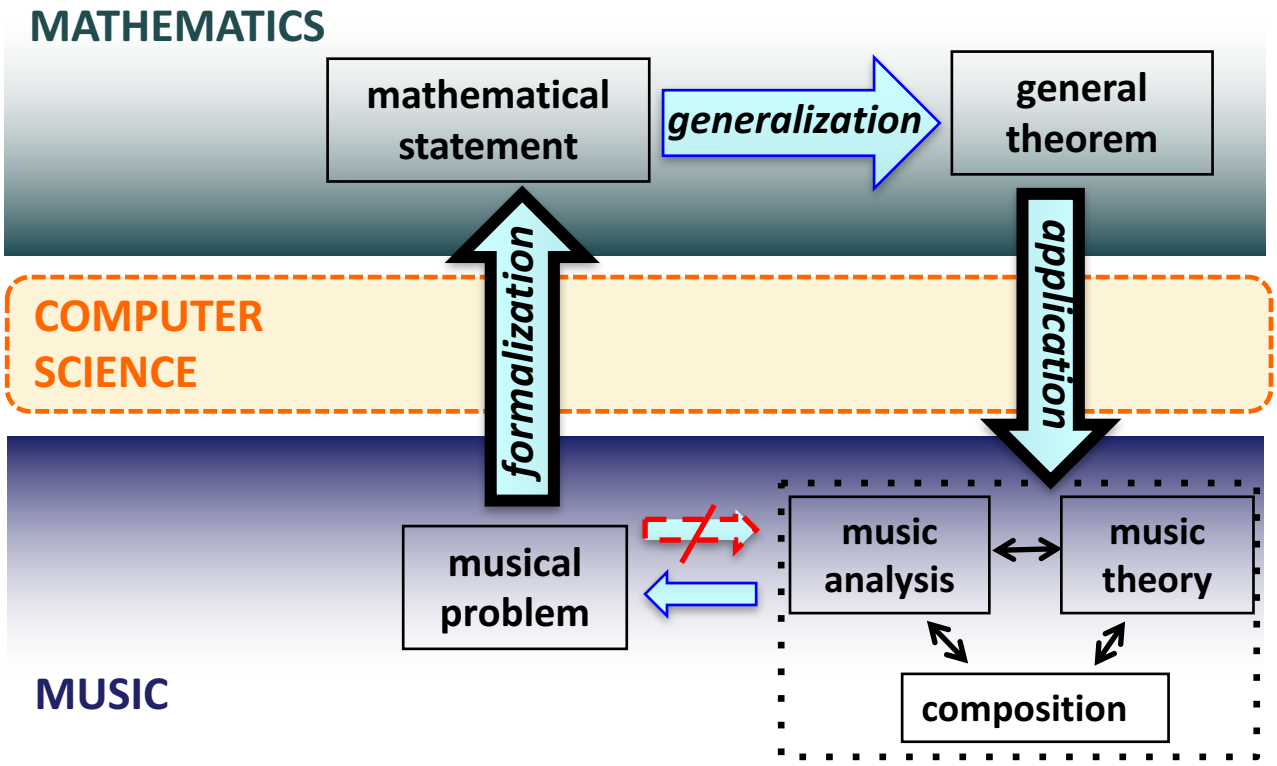




Eléments de recherche mathémusicale : approches algébriques, topologiques et catégorielles

Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS/UPMC & IRMA/GREAM/USIAS
http://repmus.ircam.fr/moreno/smir

MathMusic research at the interface of three disciplines



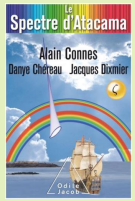
The Structural Music Information Research Project (2017-2019)

Society for Mathematics and Computation in Music

Mathematics

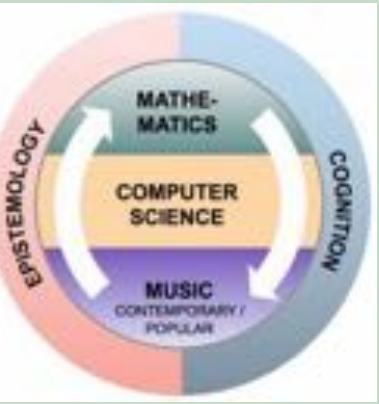
- IRMA (université de Strasbourg)
  - Main hosting institution

- IHES (Paris)
  - Working group on category and topos theory and music (avec A. Connes, P. Cartier, O. Caramello, Ch. Alunni)



- IRCAM (STMS Lab)
  - Master ATIAM Program
  - *OpenMusic*, *Hexachord*, ...

Computational Musicology



Popular music / art music

- GREAM (université de Strasbourg)
  - Course on mathematical and computational models in popular music

- USIAS (institut d'études avancées)
  - *Fellowship*

Epistemology and Cognition

- GDR ESARS (CNRS)
  - Math'n Pop

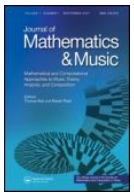
The *Society for Mathematics and Computation in Music*

Conferences:

- 2007 Technische Universität (Berlin, Allemagne)
- 2009 Yale University (New Haven, USA)
- 2011 Ircam (Paris, France)
- 2013 McGill University (Canada)
- 2015 Queen Mary University (Londres)
- 2017 UNAM (Mexico City)

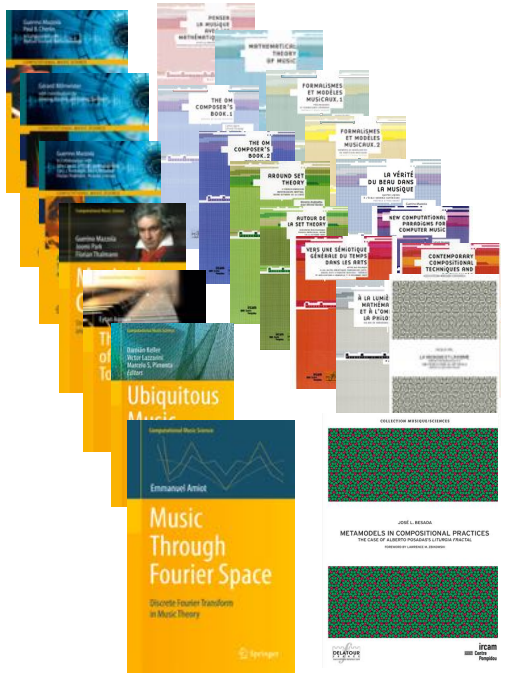
Official Journal:

- *Journal of Mathematics and Music*, Taylor & Francis (Editors : Th. Fiore & C. Callender),



Books Series:

- *Computational Music Sciences* Series, Springer (G. Mazzola & M. Andreatta eds. – 12 books published (since 2009)
- Collection *Musique/Sciences*, Ircam-Delatour France (J.-M. Bardez & M. Andreatta dir. – 16 books published (since 2006)



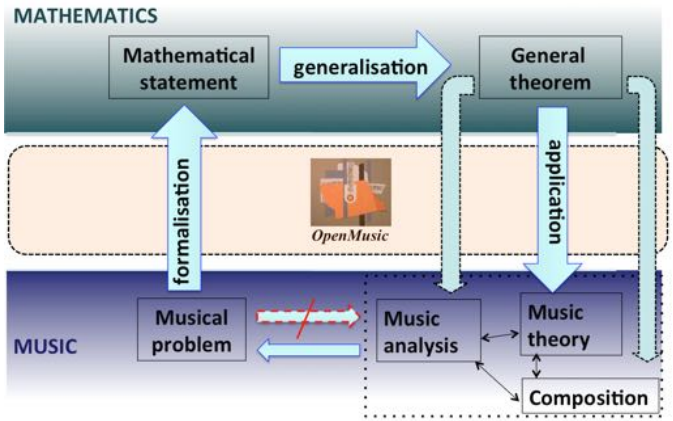
European Training Network on Computational and Mathematical Music Analysis and Generation ("InForMusic")

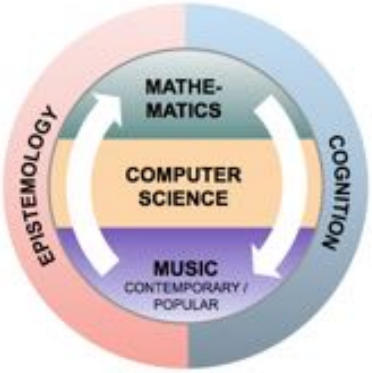
(Aalborg Universitet, City, University of London, Universiteit Utrecht, Aristotelio Panepistimio Thessalonikis, IRISA (UMR 6074), IRMA (UMR 7501), STMS (UMR 9912), Vrije Universiteit Brussel + Sony Europe Ltd, Chordify, Melodrive, Steinberg)

From MISA to SMIR

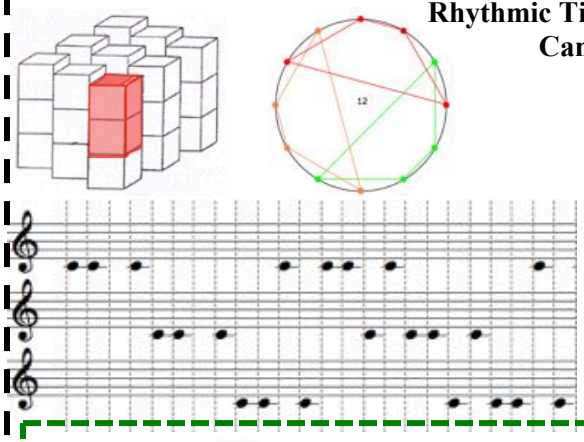
M. Andreatta : *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

- Tiling Rhythmic Canons
- *Z* relation and homometry
- Transformational Theory
- Music Analysis, SC and FCA
- Diatonic Theory and ME-Sets
- Periodic sequences and FDC
- Block-designs in composition

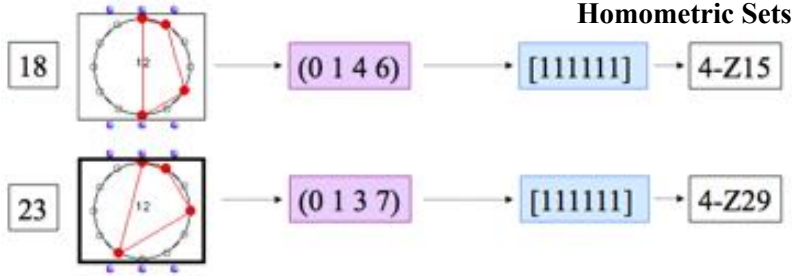


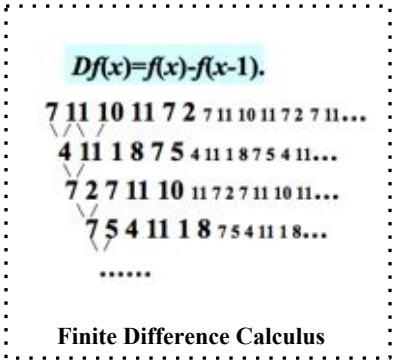


Rhythmic Tiling Canons



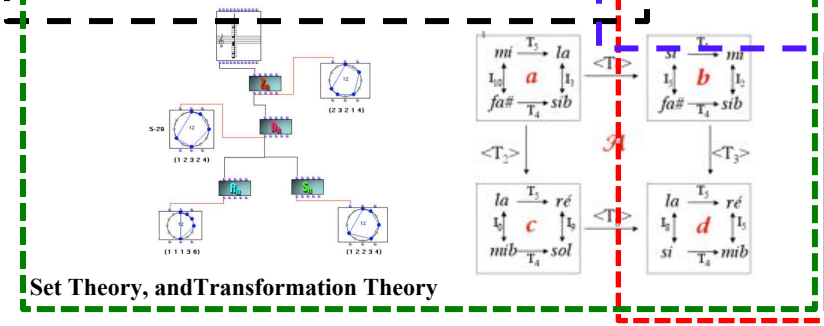
Z-Relation and Homometric Sets



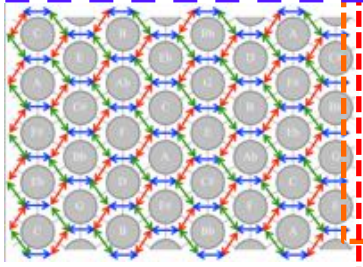


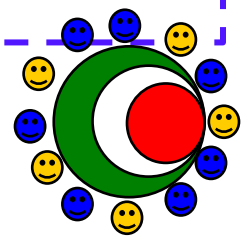
Finite Difference Calculus

Neo-Riemannian Theory and Spatial Computing

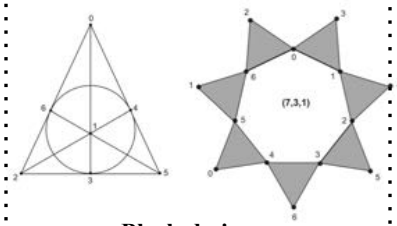


Set Theory, andTransformation Theory





Diatonic Theory and ME-Sets



Block-designs

The 4+1 main axes of the SMIR Project

- **Generalized Tonnetze, Persistent Homology and Style Analysis**
  - Sonia Cannas (PhD candidate, University of Pavia / Strasbourg)
  - Davide Stefani (PhD candidate, UPMC)
  - Corentin Guichaoua (post-doc IRMA/USIAS)
  - Pierre Guillot (Researcher, IRMA)
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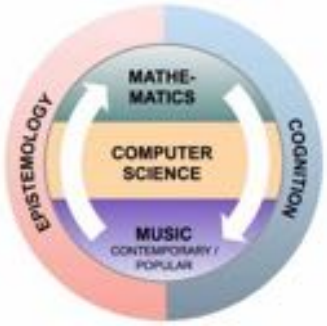
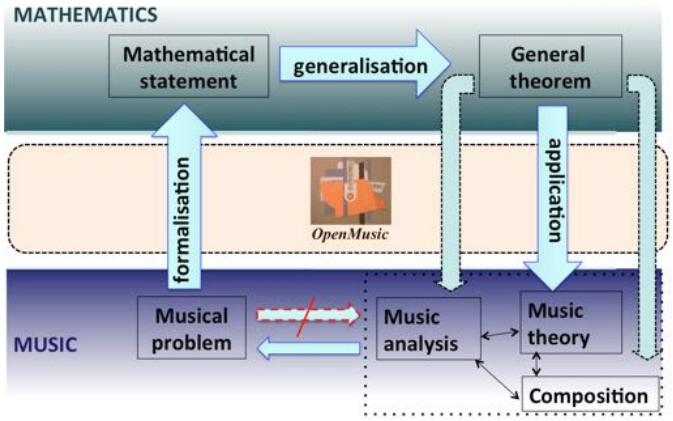
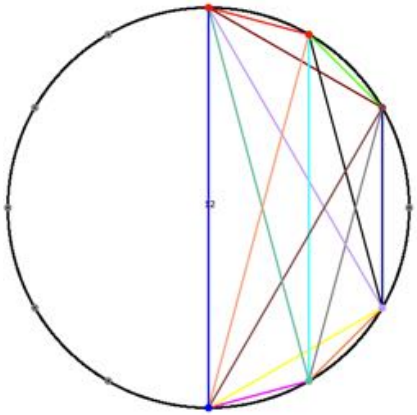
  - Nathalie Herold (post-doc GREAM)



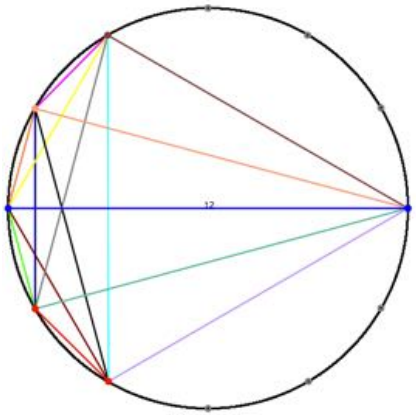
A historical example of “mathemusical” problem

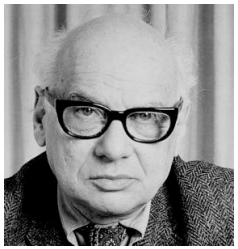




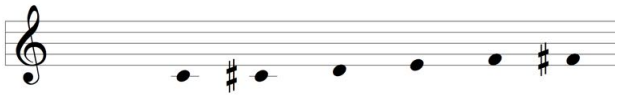
HOMOMETRY

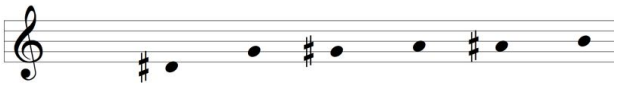
Hexachord Theorem





M. Babbitt





A historic[illegible]l" problem

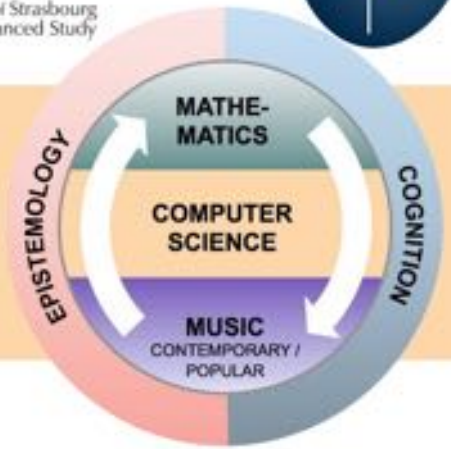
IRMA
Institut de Recherche Mathématique Avancée

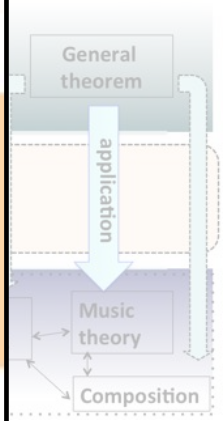
USIAS
University of Strasbourg Institute for Advanced Study

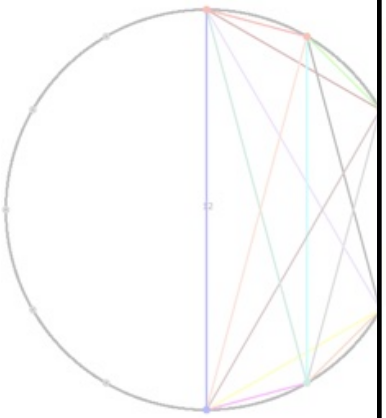
cnrs

IRMA/USIAS

Mathemusical Seminars



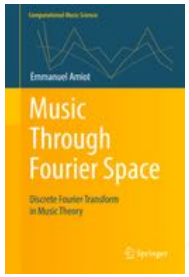




M. Babbitt

"Recent applications of the Discrete Fourier Transform in Music and some still open problems"

Emmanuel Amiot (LAMPS, Université de Perpignan)



26th January 2018, 16:00

Salle de conférences IRMA
7 rue René-Descartes, 67084 Strasbourg
Université de Strasbourg

Website: http://repmus.ircam.fr/moreno/smir/seminars
Info: moreno.andreatta@math.unistra.fr, sonia.cannas01@universitadipavia.it

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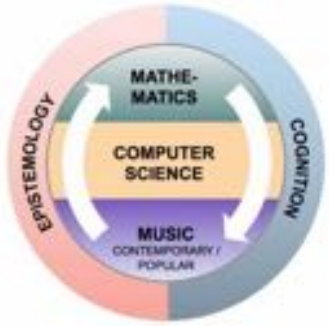
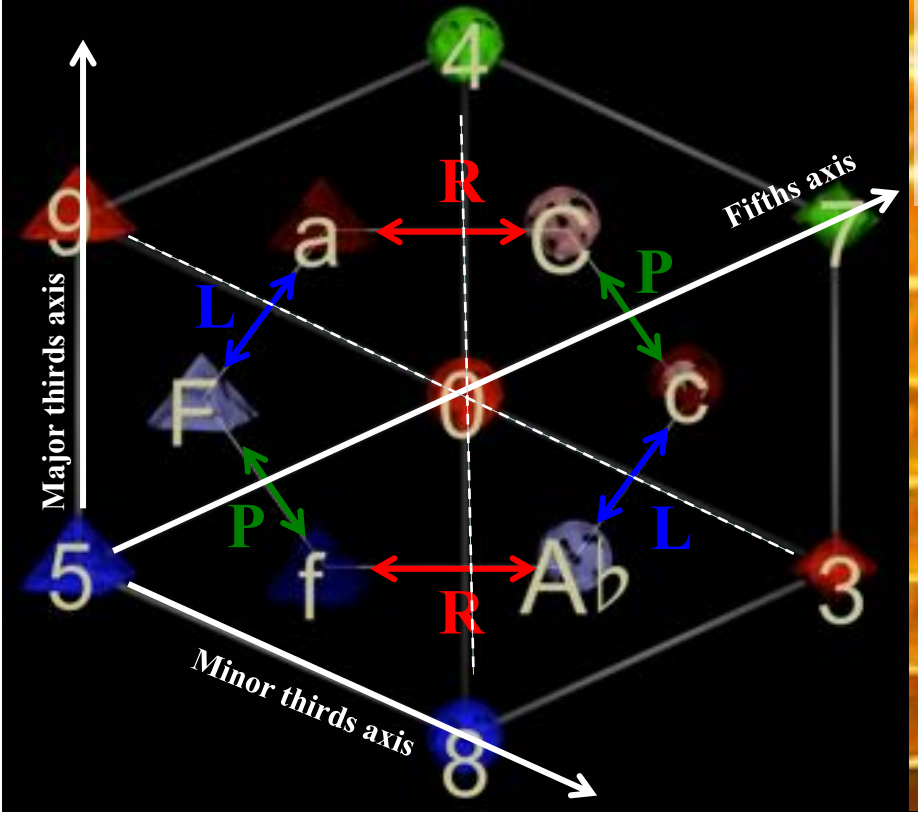
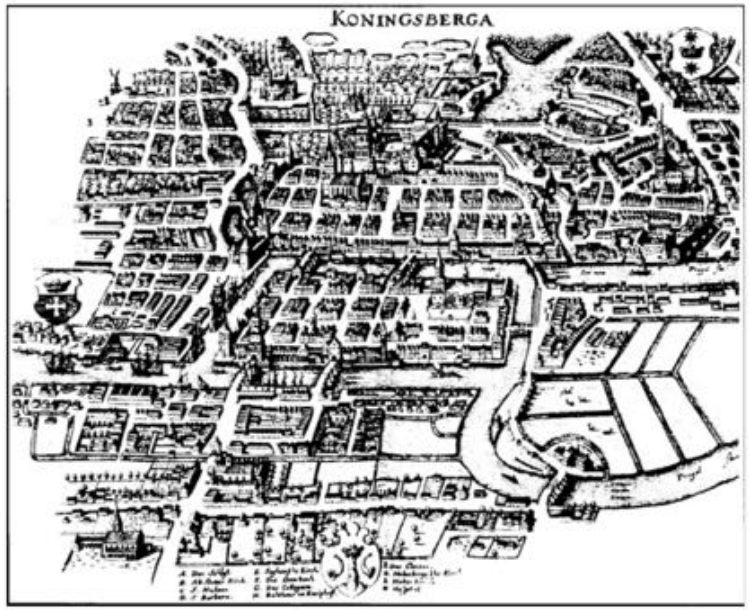
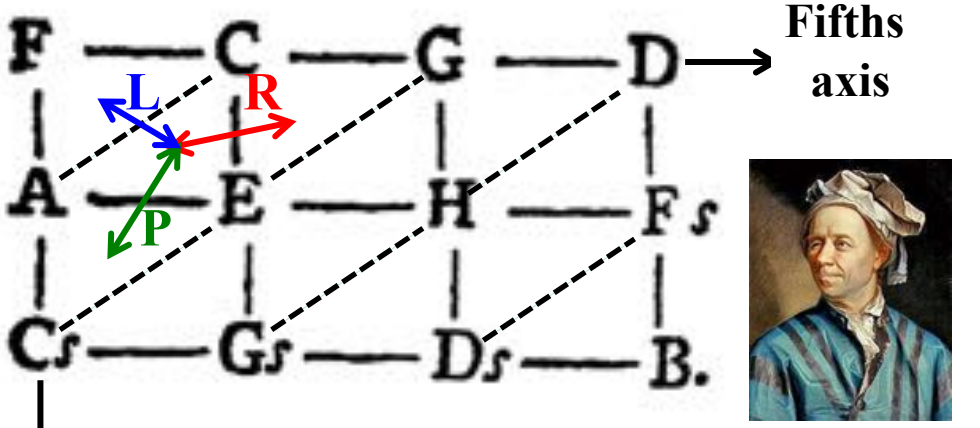


The *Tonnetz*





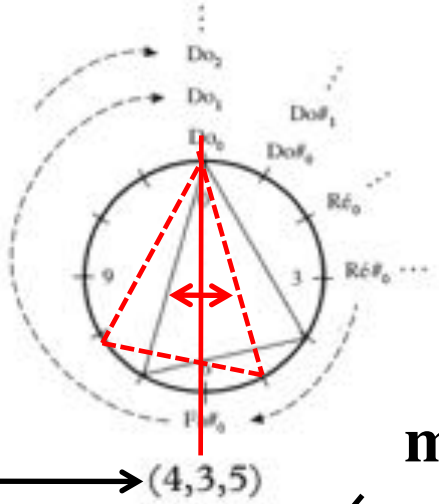


Speculum Musicum
(Euler, 1773)

The *Tonnetz* as a simplicial complex

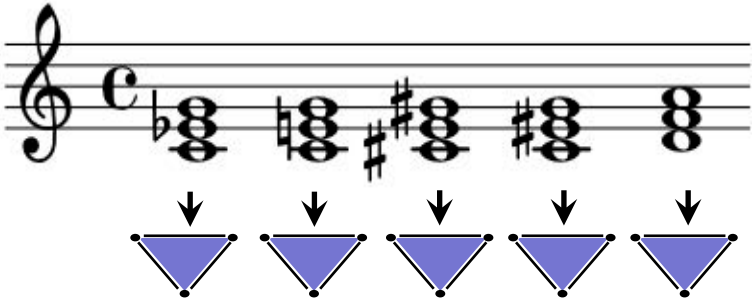
L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- Assembling chords related by some equivalence relation
  - Transposition/inversion: Dihedral group action on $P(Z_n)$

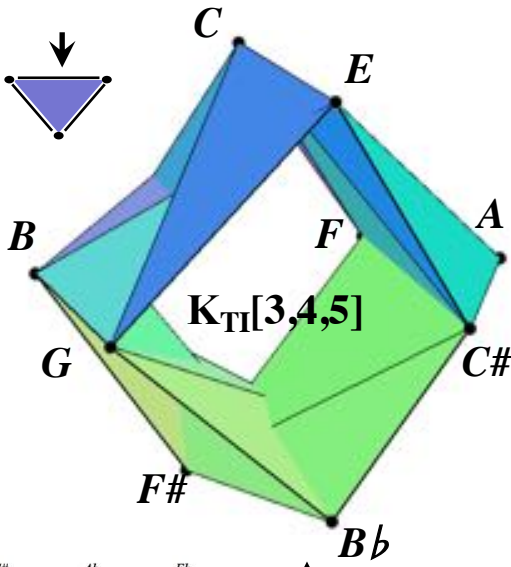


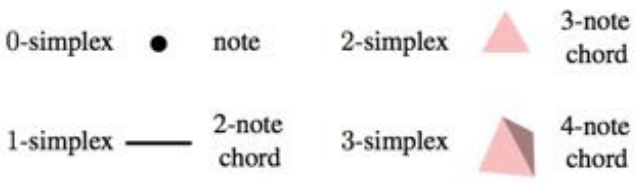
Intervallic structure

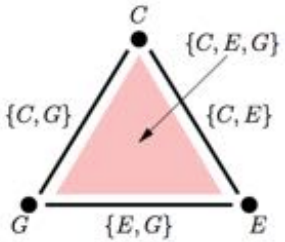
(4,3,5)

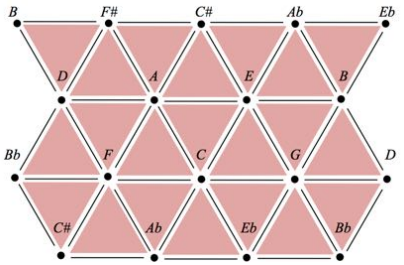


major/minor triads





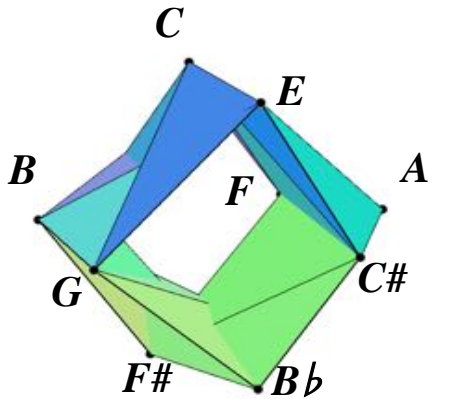


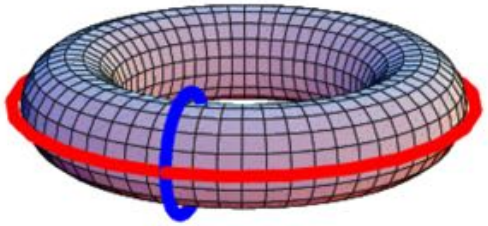


Classifying Chord Complexes

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- Complexes enumeration in the chromatic system





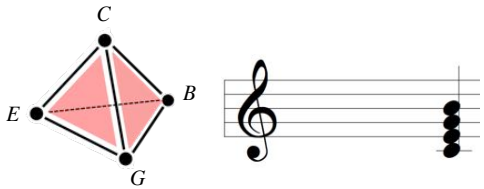
$\beta_0=1$
$\beta_1=2$
$\beta_2=1$

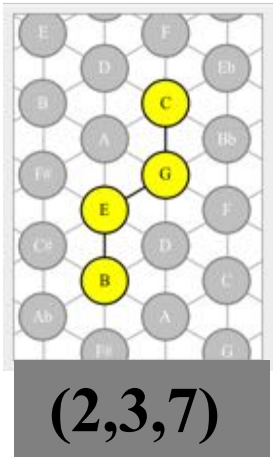
d	complexe	taille	b_n	p-v	χ
-	$\mathcal{K}_\emptyset$	0	0		0
0	$\mathcal{K}_{TI}[0]$	0	[0]		0
1	$\mathcal{K}_{TI}[1,11]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[2,10]$	12	[2, 2]		0
	$\mathcal{K}_{TI}[3,9]$	12	[3, 3]		0
	$\mathcal{K}_{TI}[4,8]$	12	[4, 4]		0
	$\mathcal{K}_{TI}[5,7]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[6,6]$	6	[6, 0]		6
2	$\mathcal{K}_{TI}[1,1,10]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[1,2,9]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,3,8]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,4,7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,5,6]$	24	[1, 1, 6]		6
	$\mathcal{K}_{TI}[2,2,8]$	12	[2, 2, 0]		0
	$\mathcal{K}_{TI}[2,3,7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[2,4,6]$	24	[2, 2, 6]		6
	$\mathcal{K}_{TI}[2,5,5]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[3,3,6]$	12	[3, 0, 3]		6
	$\mathcal{K}_{TI}[3,4,5]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[4,4,4]$	4	[4, 0, 0]		4
	$\mathcal{K}_{TI}[1,1,1,9]$	12	[1, 1, 0, 0]	x	0
	$\mathcal{K}_{TI}[1,1,2,8]$	24	[1, 1, 12, 0]		12
	$\mathcal{K}_{TI}[1,1,3,7]$	24	[1, 2, 13, 0]		12
	$\mathcal{K}_{TI}[1,1,4,6]$	24	[1, 1, 18, 0]		18
	$\mathcal{K}_{TI}[1,1,5,5]$	12	[1, 1, 6, 0]		6

Classifying Chord Complexes

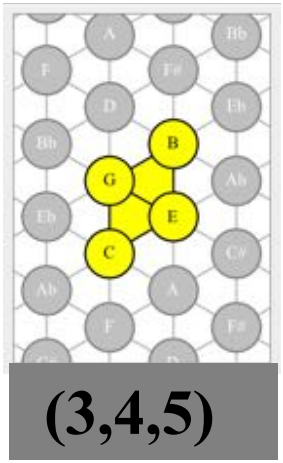
L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- The search of the optimal space





(2,3,7)



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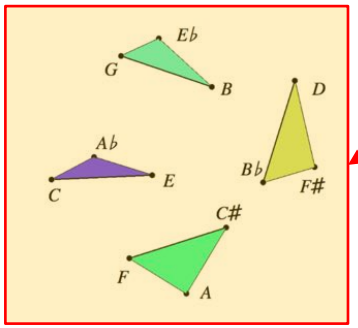
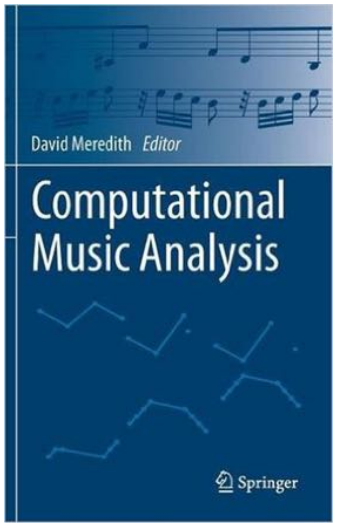
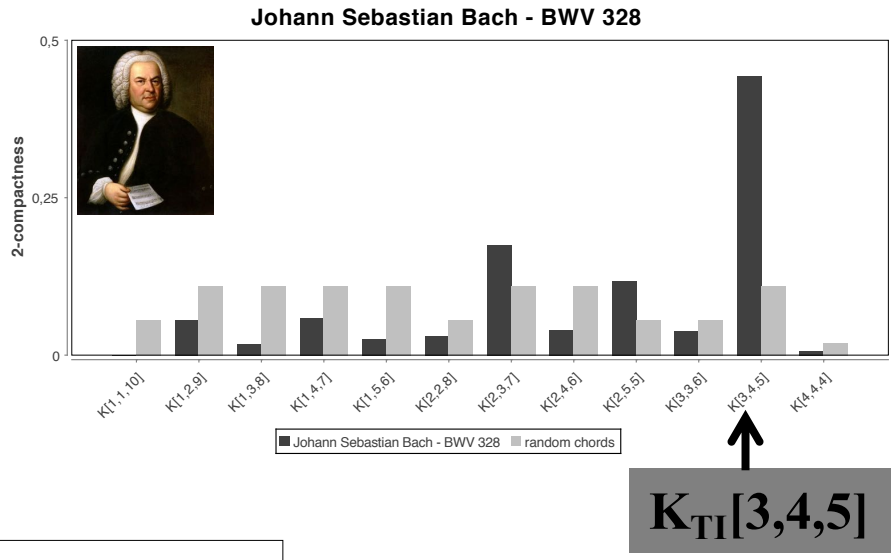


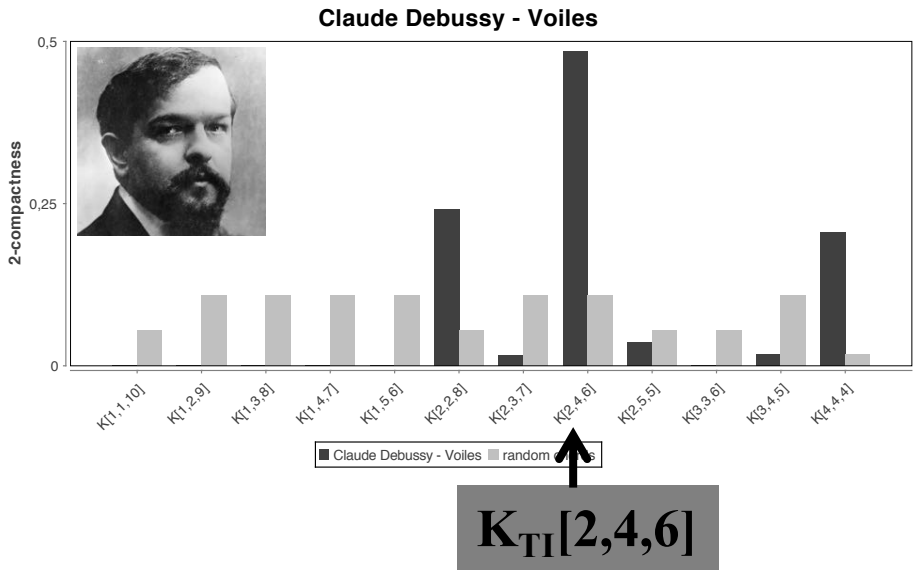
d	complexe	taille	b_n	p-v	χ
-	$\mathcal{K}_{\emptyset}$	0	0		0
0	$\mathcal{K}_{TI}[0]$	0	[0]		0
1	$\mathcal{K}_{TI}[1,11]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[2,10]$	12	[2, 2]		0
	$\mathcal{K}_{TI}[3,9]$	12	[3, 3]		0
	$\mathcal{K}_{TI}[4,8]$	12	[4, 4]		0
	$\mathcal{K}_{TI}[5,7]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[6,6]$	6	[6, 0]		6
2	$\mathcal{K}_{TI}[1,1,10]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[1,2,9]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,3,8]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,4,7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,5,6]$	24	[1, 1, 6]		6
	$\mathcal{K}_{TI}[2,2,8]$	12	[2, 2, 0]		0
	$\mathcal{K}_{TI}[2,3,7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[2,4,6]$	24	[2, 2, 6]		6
	$\mathcal{K}_{TI}[2,5,5]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[3,3,6]$	12	[3, 0, 3]		6
	$\mathcal{K}_{TI}[3,4,5]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[4,4,4]$	4	[4, 0, 0]		4
	$\mathcal{K}_{TI}[1,1,1,9]$	12	[1, 1, 0, 0]	x	0
	$\mathcal{K}_{TI}[1,1,2,8]$	24	[1, 1, 12, 0]		12
	$\mathcal{K}_{TI}[1,1,3,7]$	24	[1, 2, 13, 0]		12
	$\mathcal{K}_{TI}[1,1,4,6]$	24	[1, 1, 18, 0]		18
	$\mathcal{K}_{TI}[1,1,5,5]$	12	[1, 1, 6, 0]		6

The geometric character of musical logic

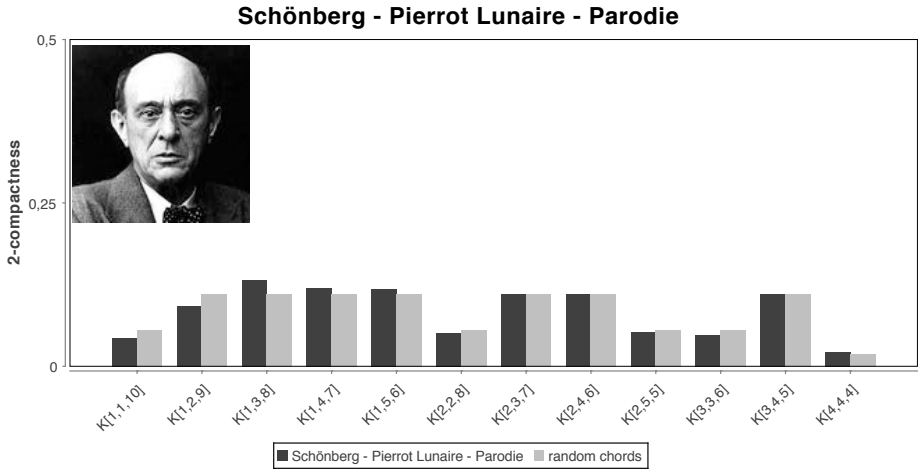




$K_{TI}[3,4,5]$



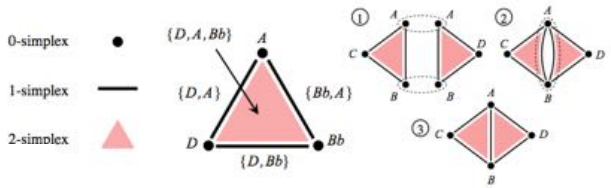
$K_{TI}[2,4,6]$



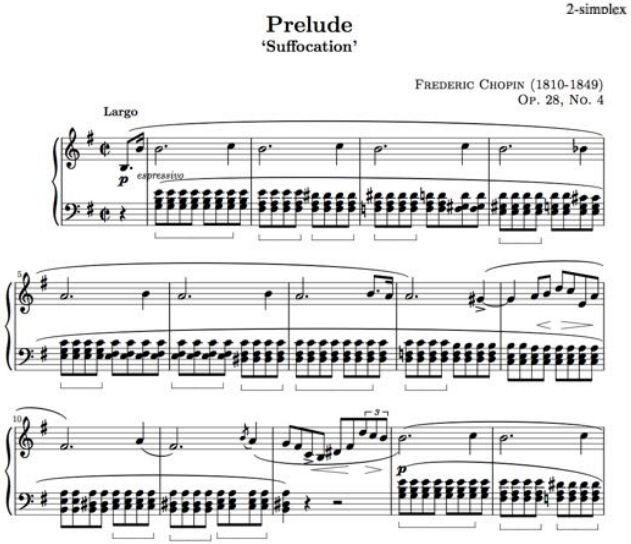
Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015

Towards a topological signature of a musical piece

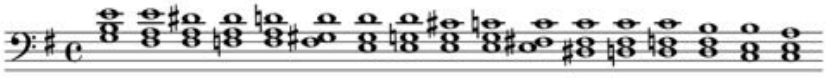
The simplices and their self-assembly



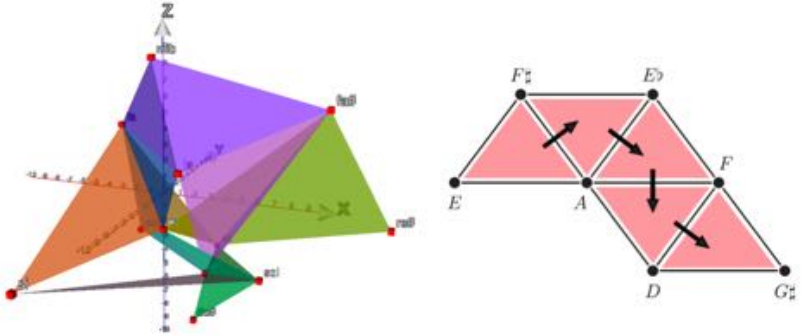
The score



Score reduction



The simplicial complex generated by the piece



A specific trajectory in the complex

Persistent homology

Topological signature

Towards a topological signature of a musical piece

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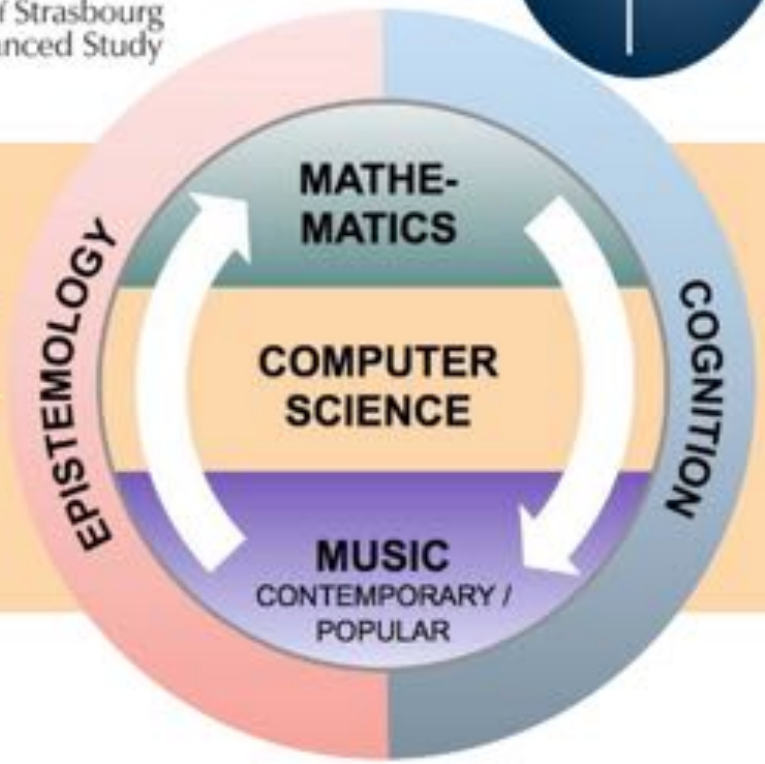
IRMA/USIAS

Mathemusical Seminars



➡ **SMIR Team (?)**

The s… complex generated by the piece

A speeche… trajectory in the complex

The 4+1 main axes of the SMIR Project

- **Generalized Tonnetze, Persistent Homology and Style**
  - Sonia Cannas (PhD candidate, University of Pavia / Strasbourg)
  - Davide Stefani (PhD candidate, UPMC)
  - Corentin Guichaoua (post-doc IRMA/USIAS)
  - Pierre Guillot (Researcher, IRMA)
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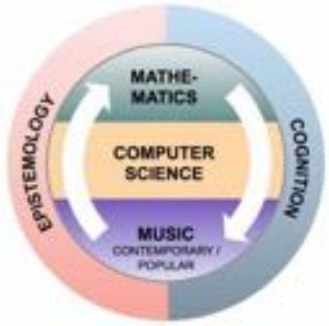
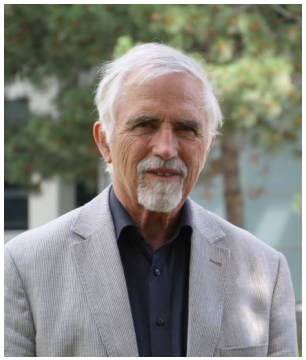
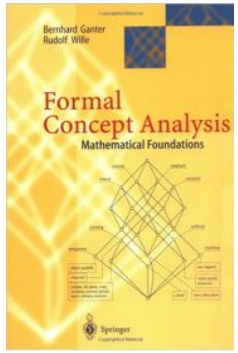


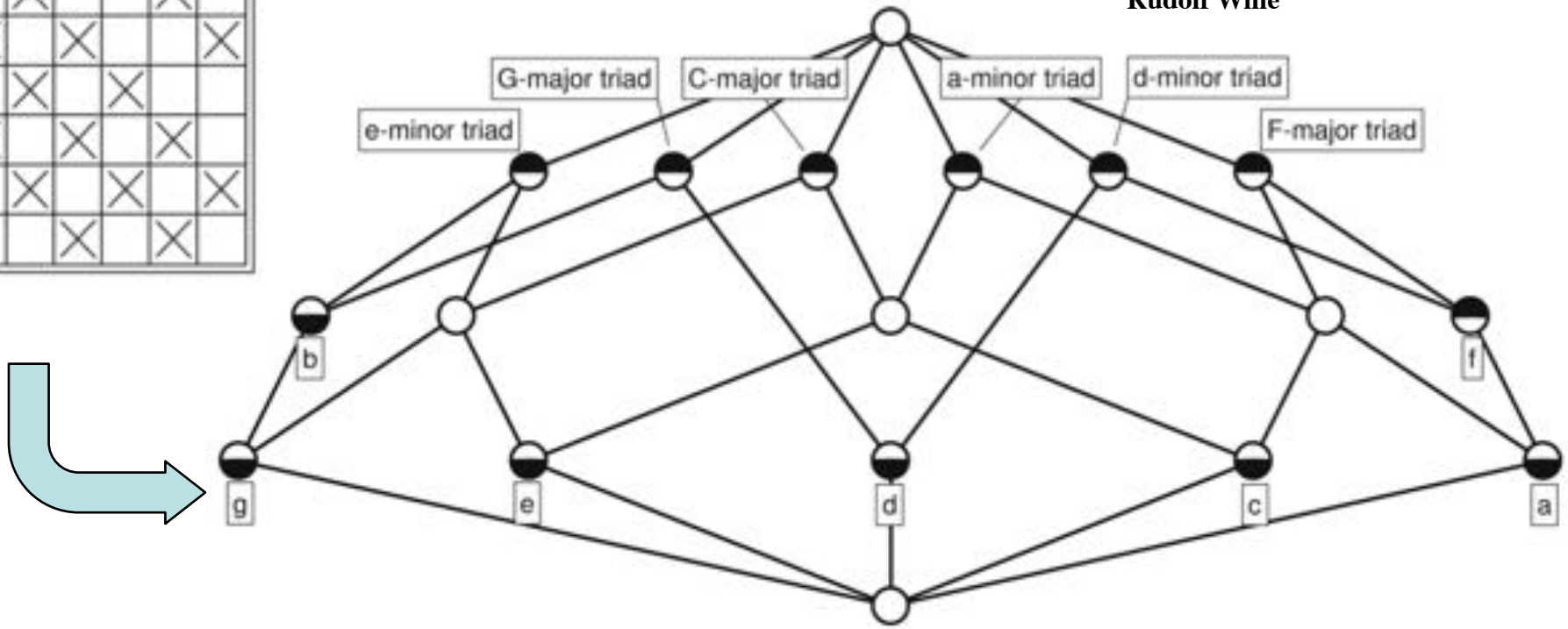
A concept lattice for musical structures

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	X			X		X
d		X			X	
e	X		X			X
f		X		X		
g	X		X		X	
a		X		X		X
b			X		X	



Rudolf Wille



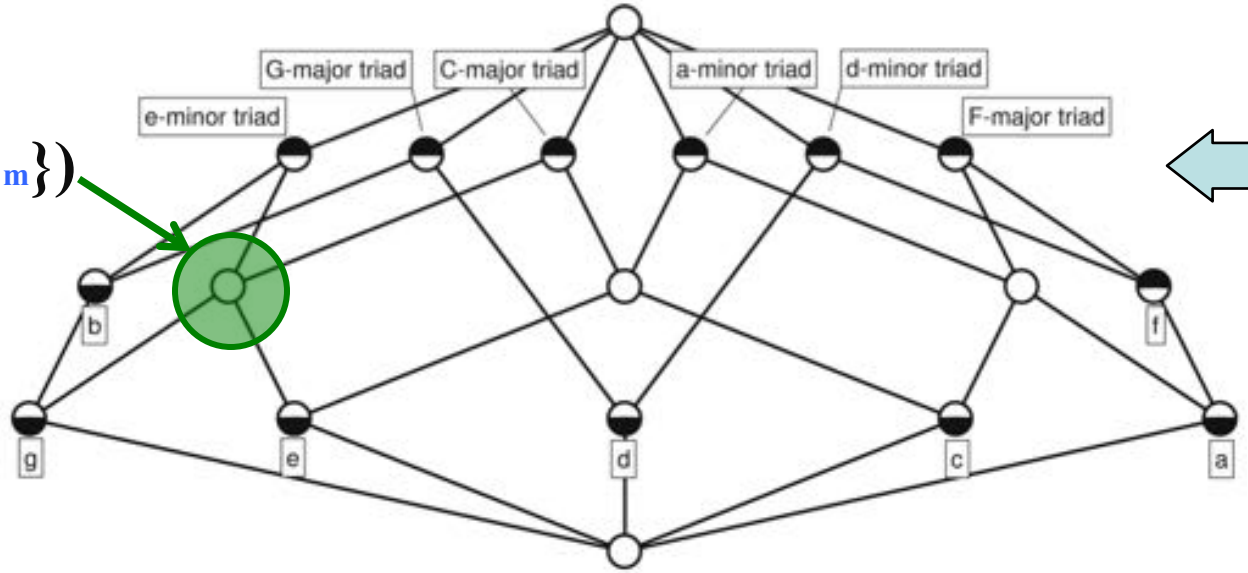


[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007, Springer, 2007]

A concept lattice for the diatonic scale

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	X			X		X
d		X			X	
e	X		X			X
f		X		X		
g	X		X		X	
a		X		X		X
b			X		X	

	C_M	E_m				
mi	X	X				
sol	X	X				



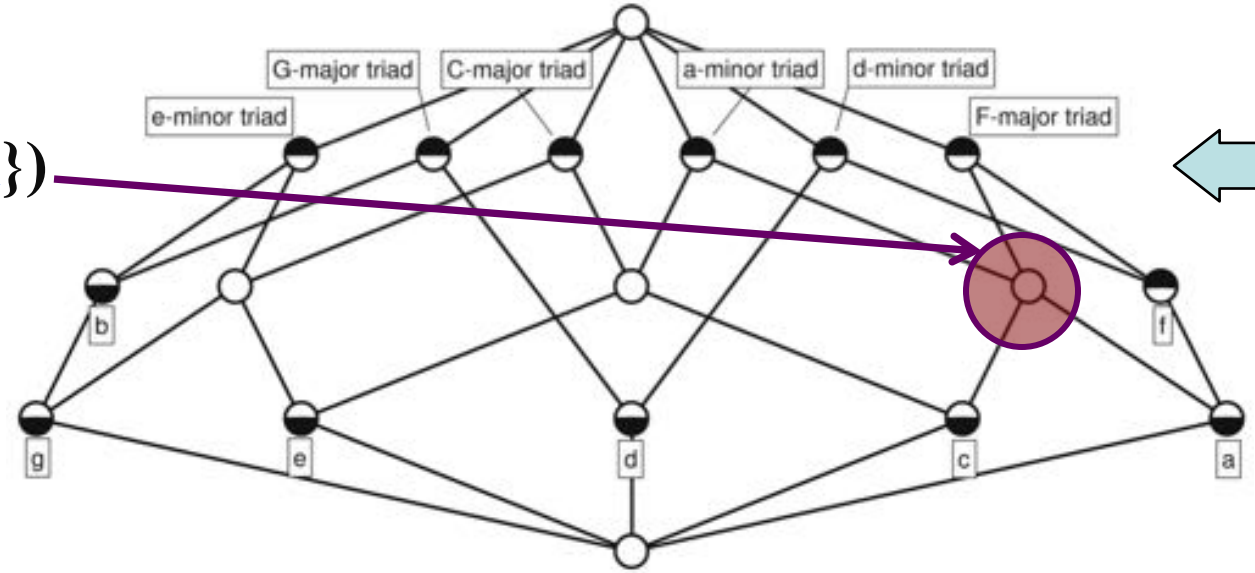
({*mi, sol*},{C_M, E_m})

[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007, Springer, 2007]

A concept lattice for the diatonic scale

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	X			X		X
d		X			X	
e	X		X			X
f		X		X		
g	X		X		X	
a		X		X		X
b			X		X	

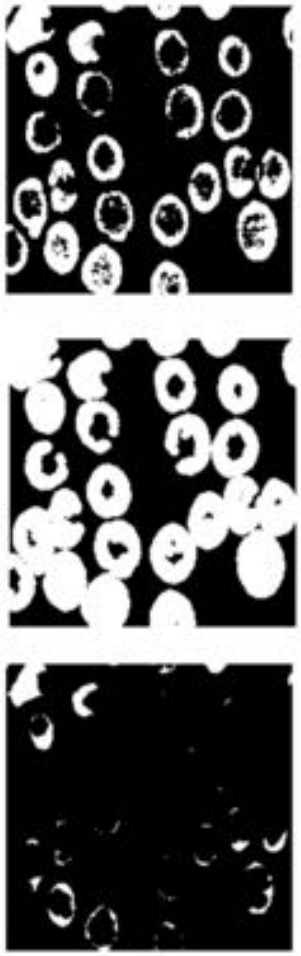
	C_M	E_m	A_m	F_M		
mi	X	X				
sol	X	X				
la			X	X		
do			X	X		

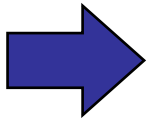


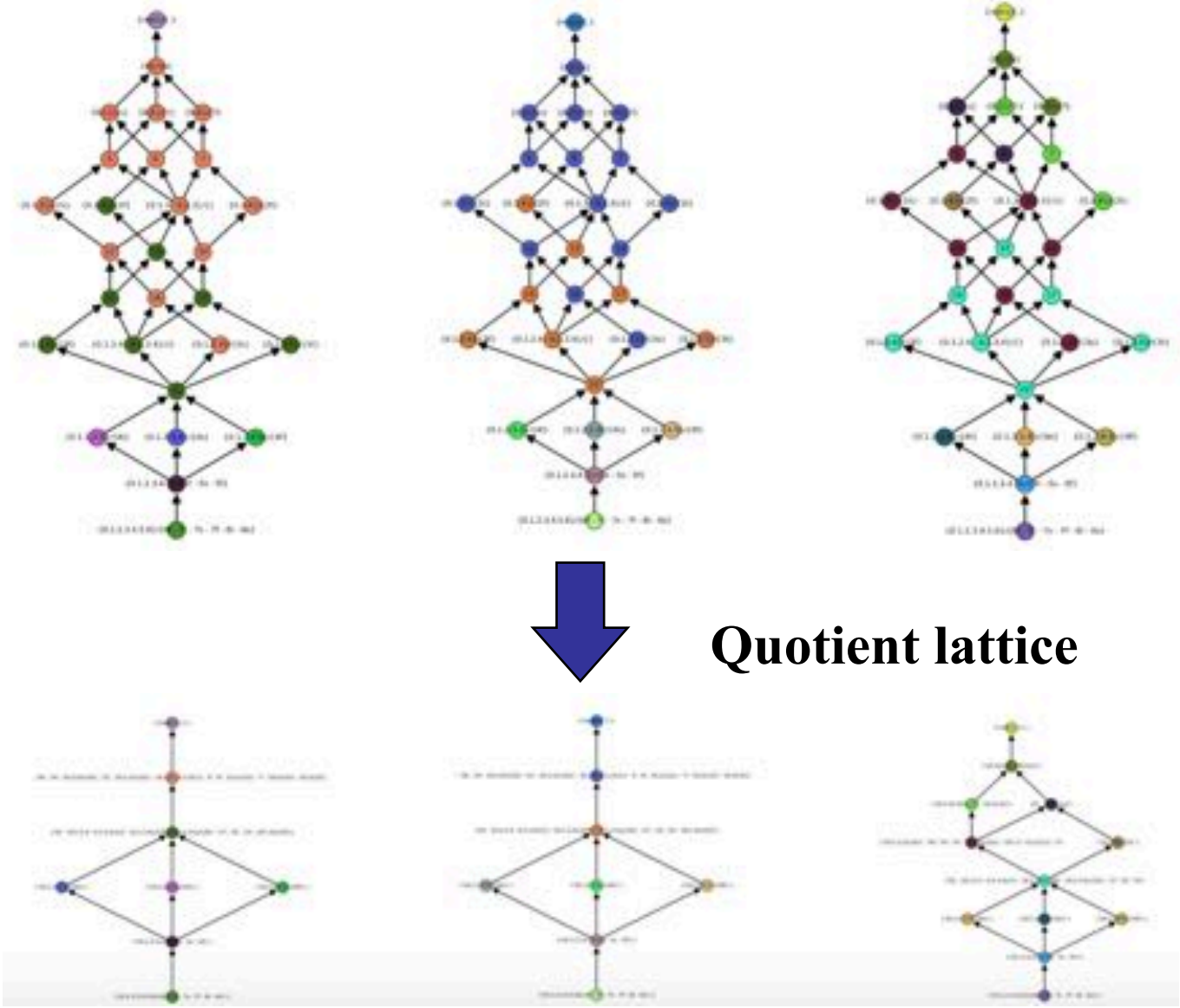
({*la, do*},{F_M, A_m})

[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007, Springer, 2007]

Concept lattices & mathematical morphology







Quotient lattice

Pierre M. Relaño, Morphologie mathématique, FCA et musicologie computationnelle, Master 2, 2017 (codir. I. Bloch (LTCI/Télécom ParisTech), J. Atif (LAMSADE, Université Paris Dauphine), C. Agon (CNRS-IRCAM-UPMC) and M. Andreatta (CNRS-IRCAM-UPMC)

Concept lattices & mathematical morphology

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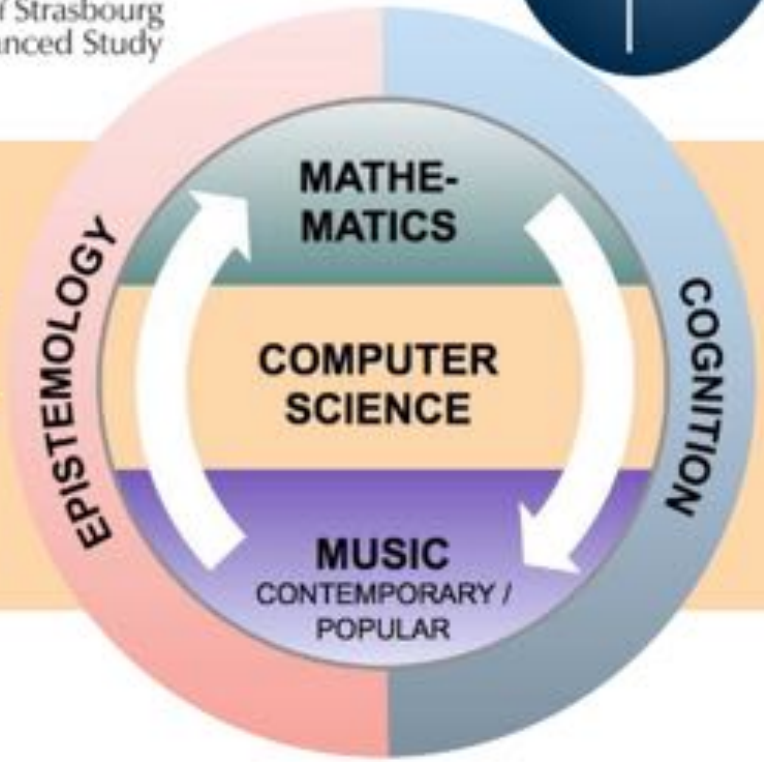

Isabelle Bloch
Jamal Atif

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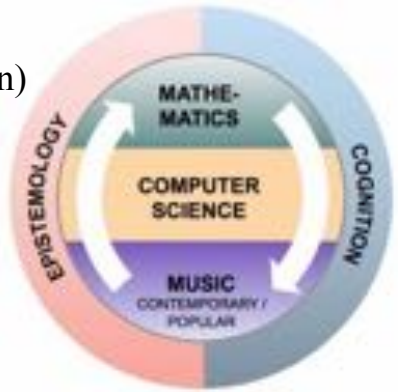
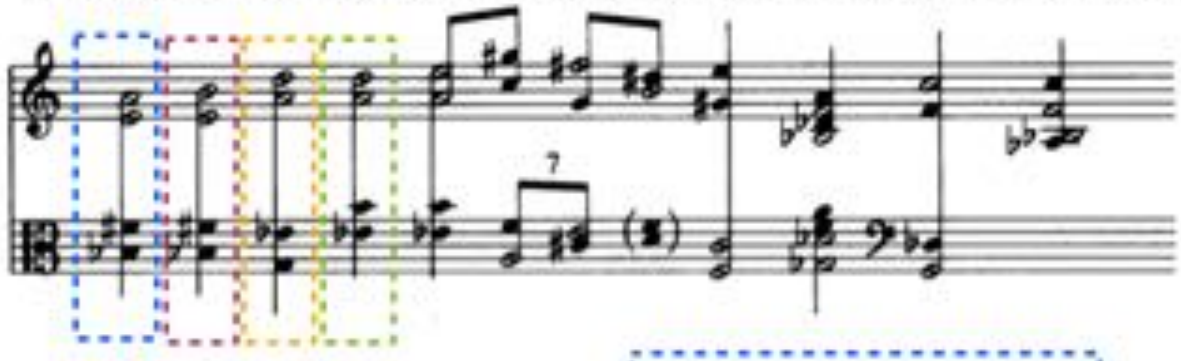
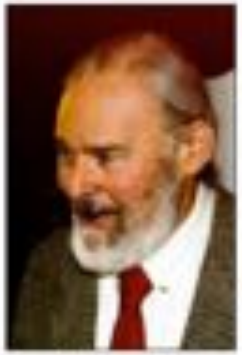


K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994

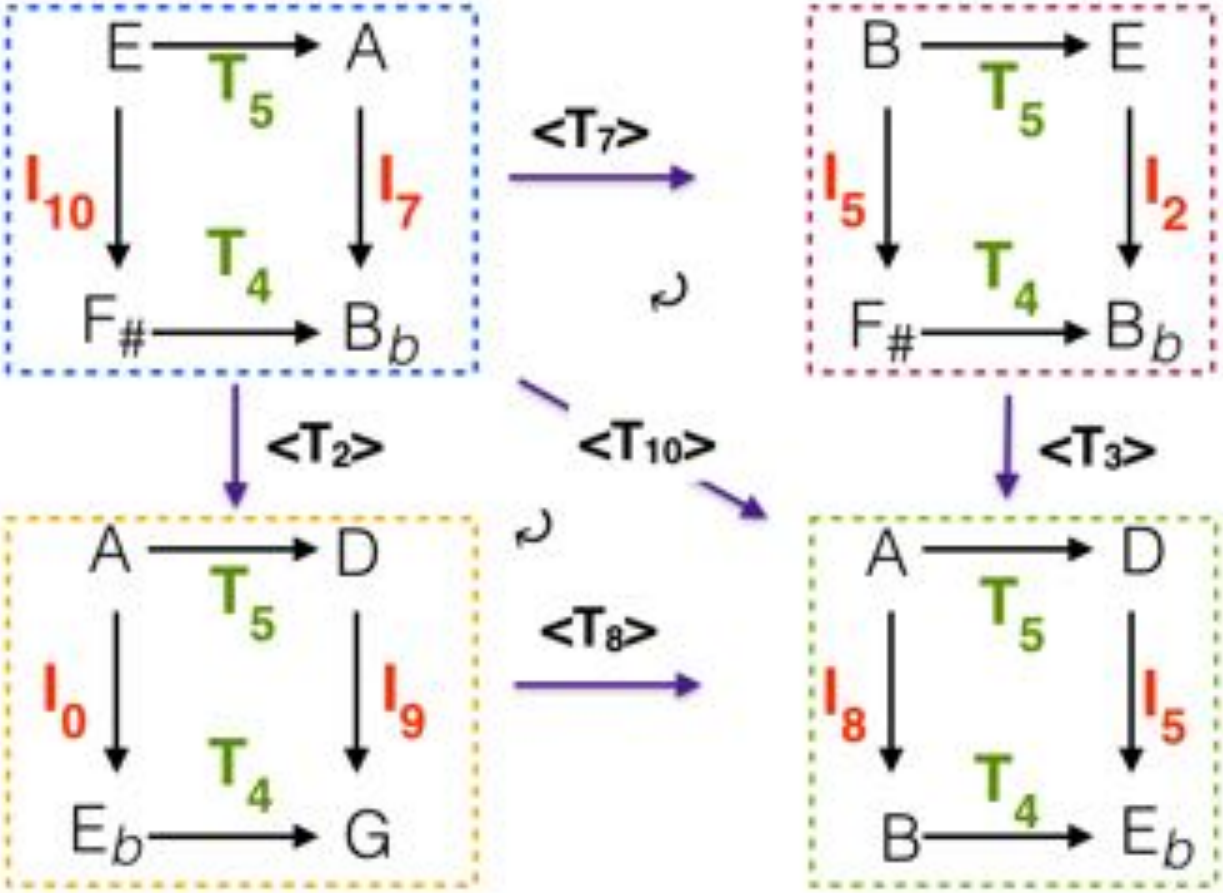




D. Lewin

H. Klumpenhouwer

$\langle T_k \rangle : T_m \rightarrow T_m$
$I_m \rightarrow I_{k+m}$



$\langle T_k \rangle \cdot \langle T_m \rangle = \langle T_{k+m} \rangle$

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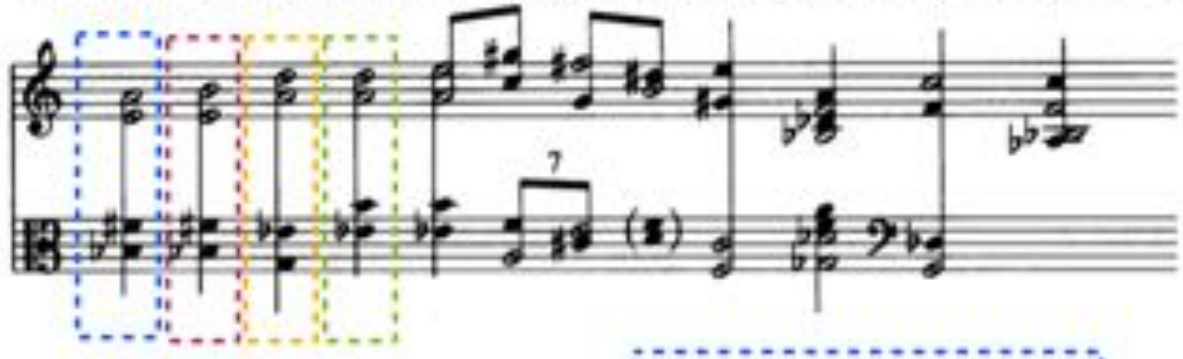


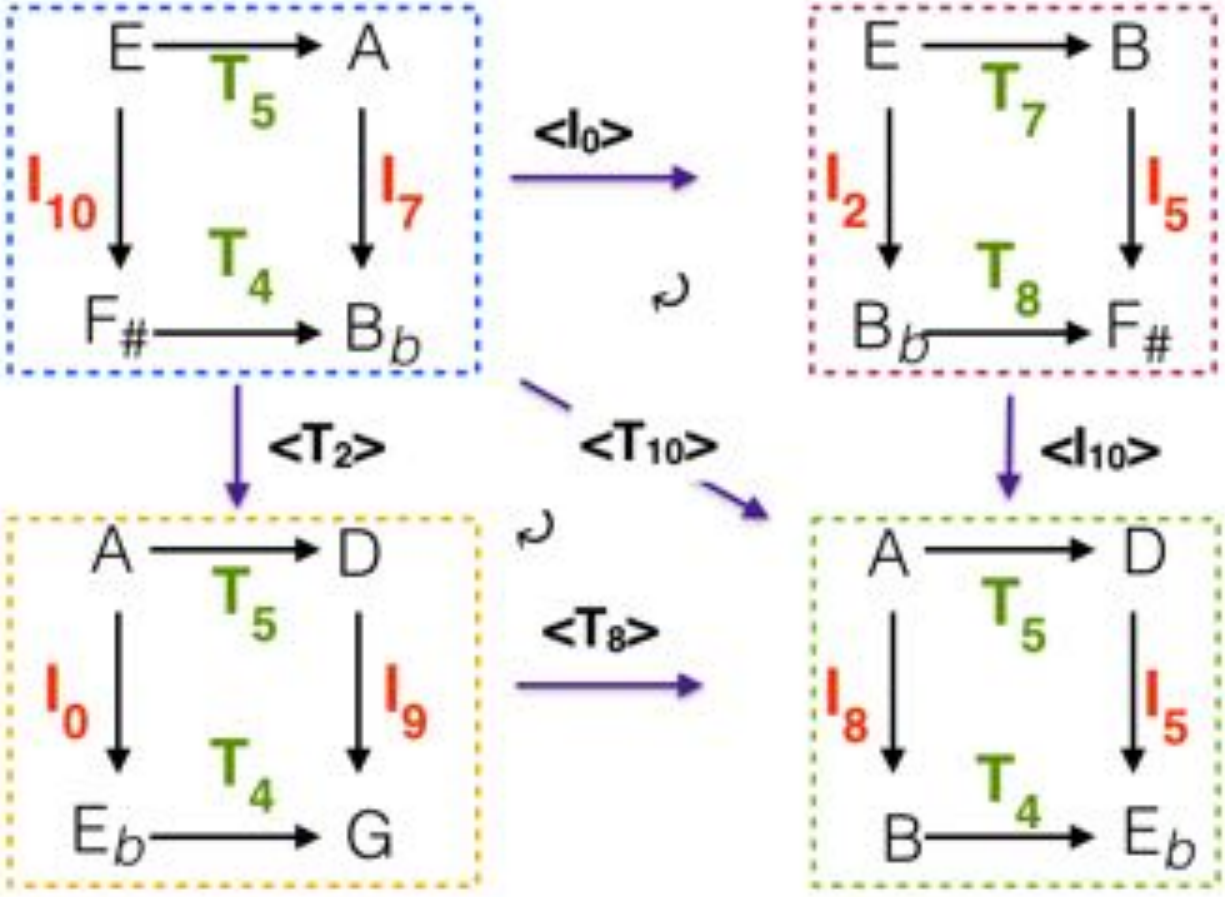
D. Lewin

H. Klumpenhouwer

$\langle T_k \rangle : T_m \rightarrow T_m$
$I_m \rightarrow I_{k+m}$

$\langle I_k \rangle : T_m \rightarrow T_{-m}$
$I_m \rightarrow I_{k-m}$

$\langle T_k \rangle \cdot \langle T_m \rangle = \langle T_{k+m} \rangle$
$\langle I_k \rangle \cdot \langle I_m \rangle = \langle T_{m-k} \rangle$



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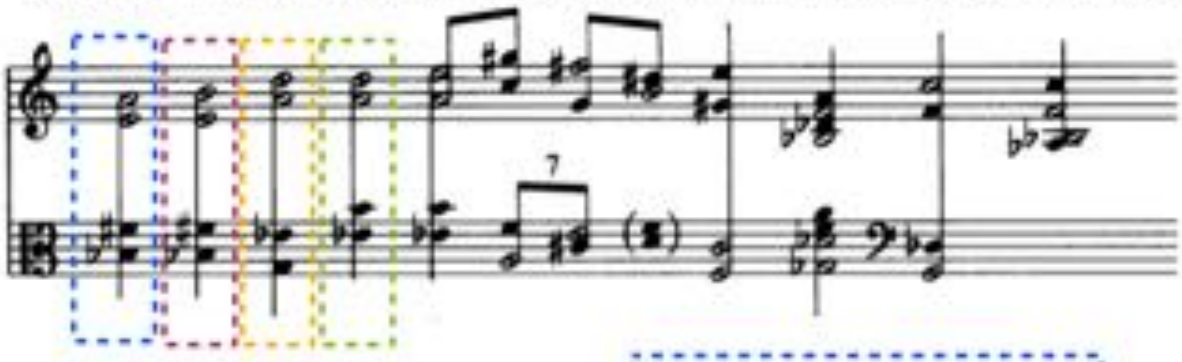

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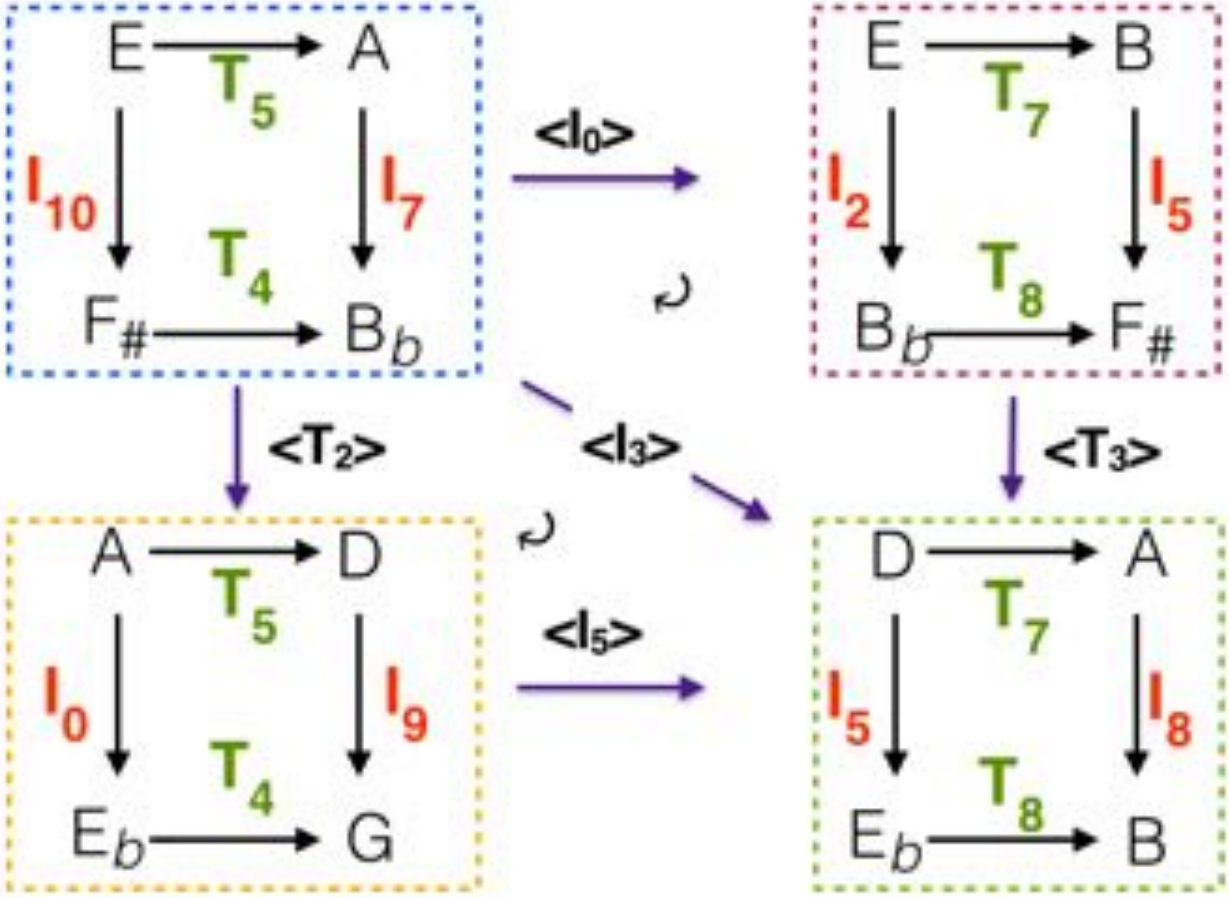
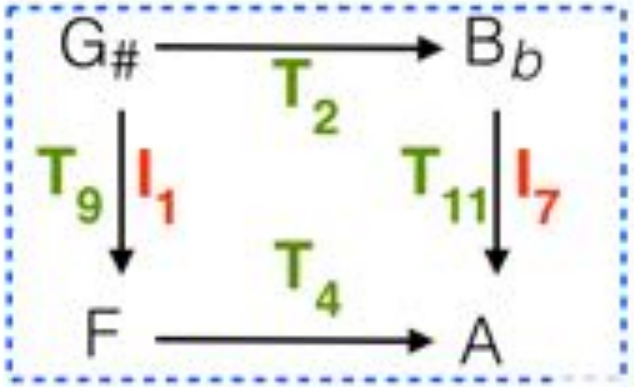
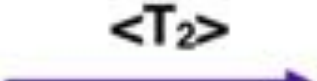
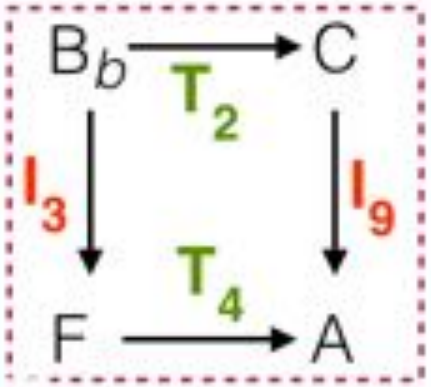
$\langle T_k \rangle \cdot \langle T_m \rangle = \langle T_{k+m} \rangle$
$\langle T_k \rangle \cdot \langle I_m \rangle = \langle I_{m-k} \rangle$
$\langle I_m \rangle \cdot \langle T_k \rangle = \langle I_{k+m} \rangle$
$\langle I_k \rangle \cdot \langle I_m \rangle = \langle T_{m-k} \rangle$

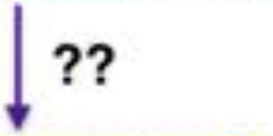


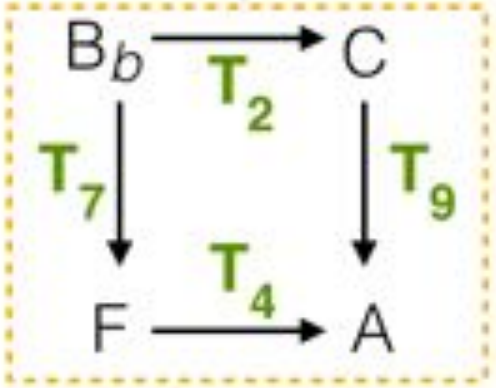
Some theoretical difficulties with the isographic relations

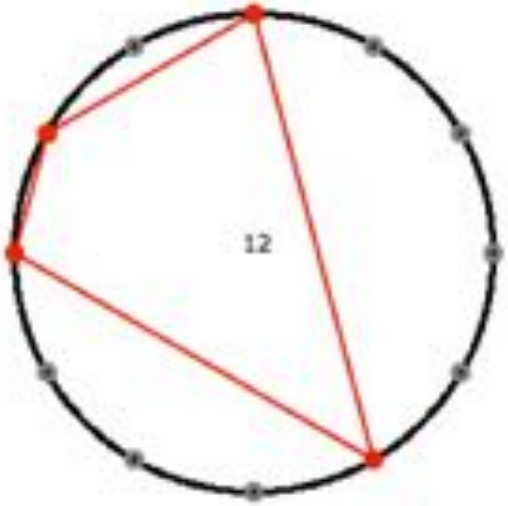








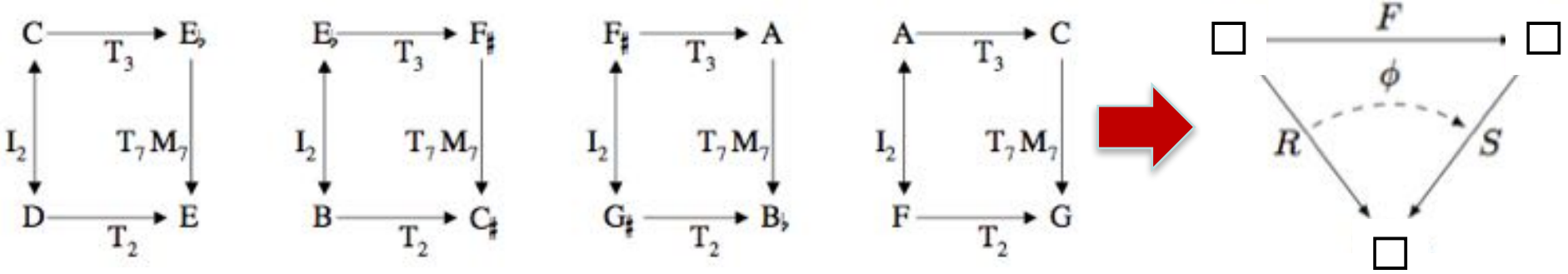




CONCLUSION

There are K-Nets which are not always isographic to a given one, i.e. the isographic relations are highly sensitives to the transformations used to label the arrows.
Is it possible to overstep this theoretical limitation? Which new definition of K-nets allows one to do that?

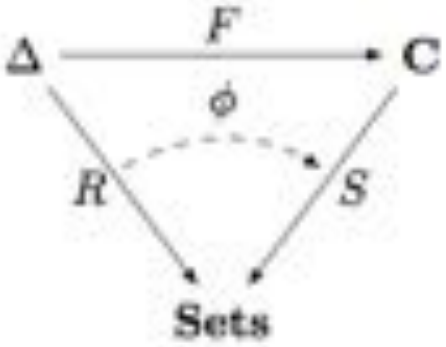
From K-Nets to category-based PK-Nets



Definition 1 *Let* **C** *be a category, and* S *a functor from* **C** *to the category* **Sets**. *Let* Δ *be a small category and* R *a functor from* Δ *to* **Sets**. *A PK-net of form* R *and of support* S *is a 4-tuple* (R, S, F, ϕ), *in which*

- F *is a functor from* Δ *to* **C**,
- *and* ϕ *is a natural transformation from* R *to* SF.

The definition of a PK-net is summed up by the following diagram:



Popoff A., M. Andreatta, A. Ehresmann, « A Categorical Generalization of Klumpenhouwer Networks », MCM 2015, Queen Mary University, Springer, p. 303-314

From K-Nets to category-based PK-Nets

IRMA
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USIAS
University of Strasbourg Institute for Advanced Study

cnrs

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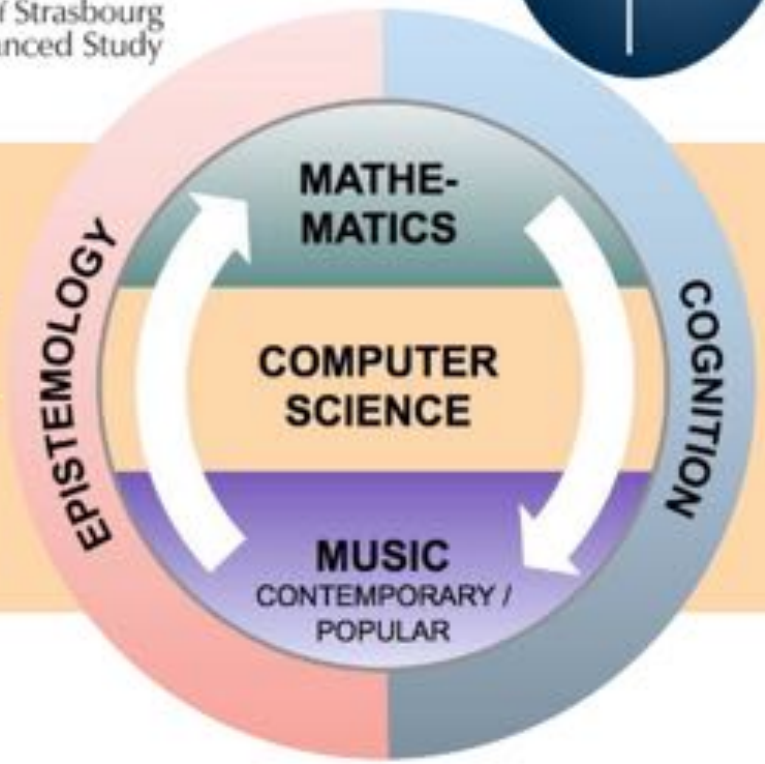
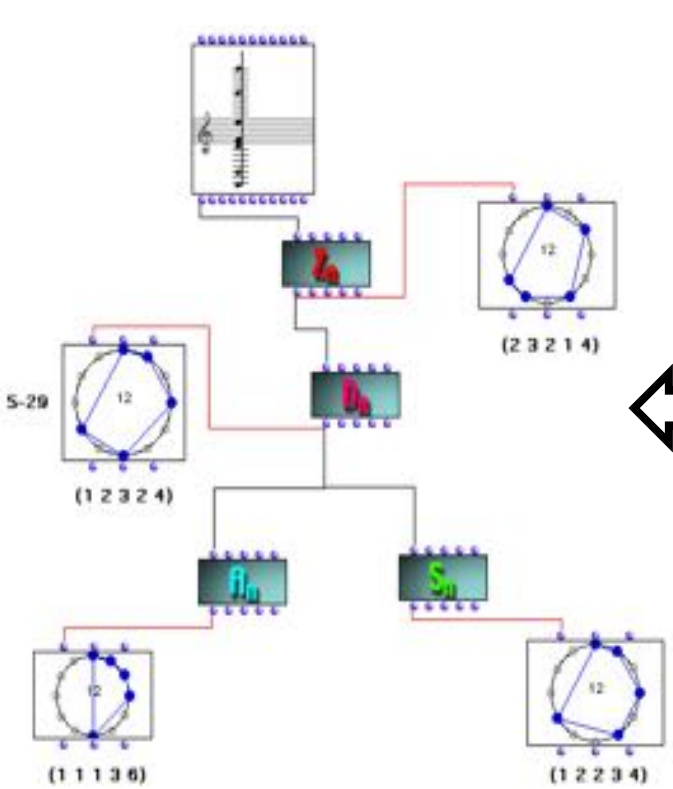
Mathemusical Seminars

23-02-2018

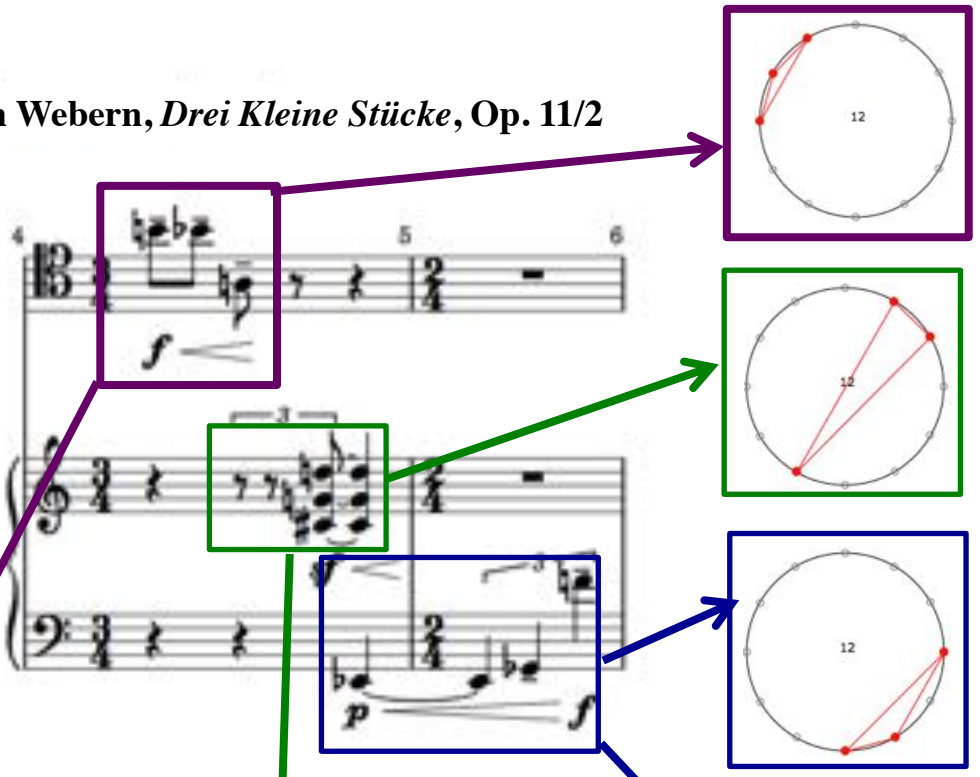


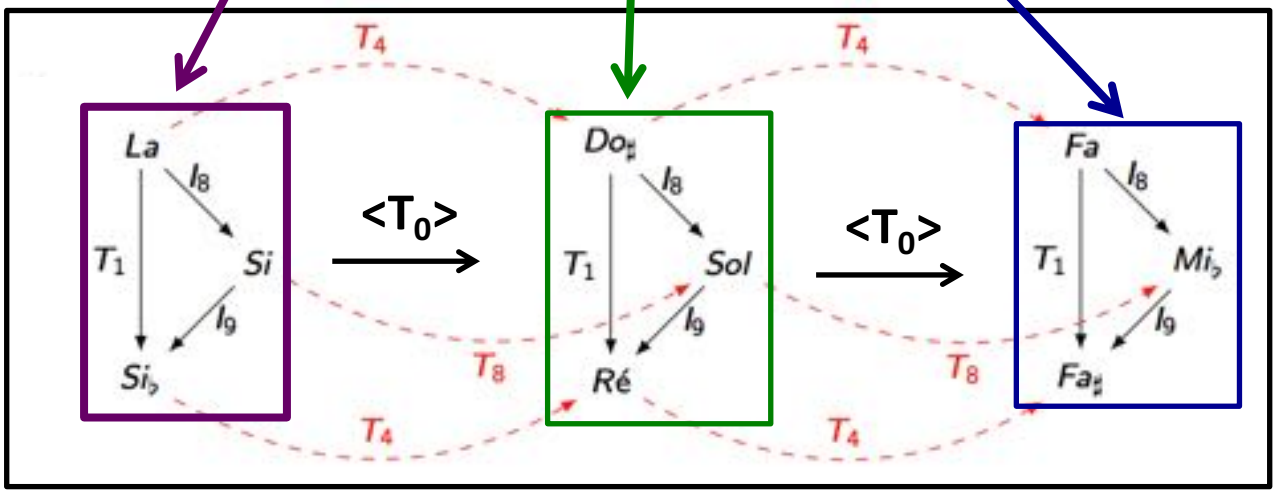
Andrée Ehresmann
Alexandre Popoff

Limitations of a paradigmatic action-based approach

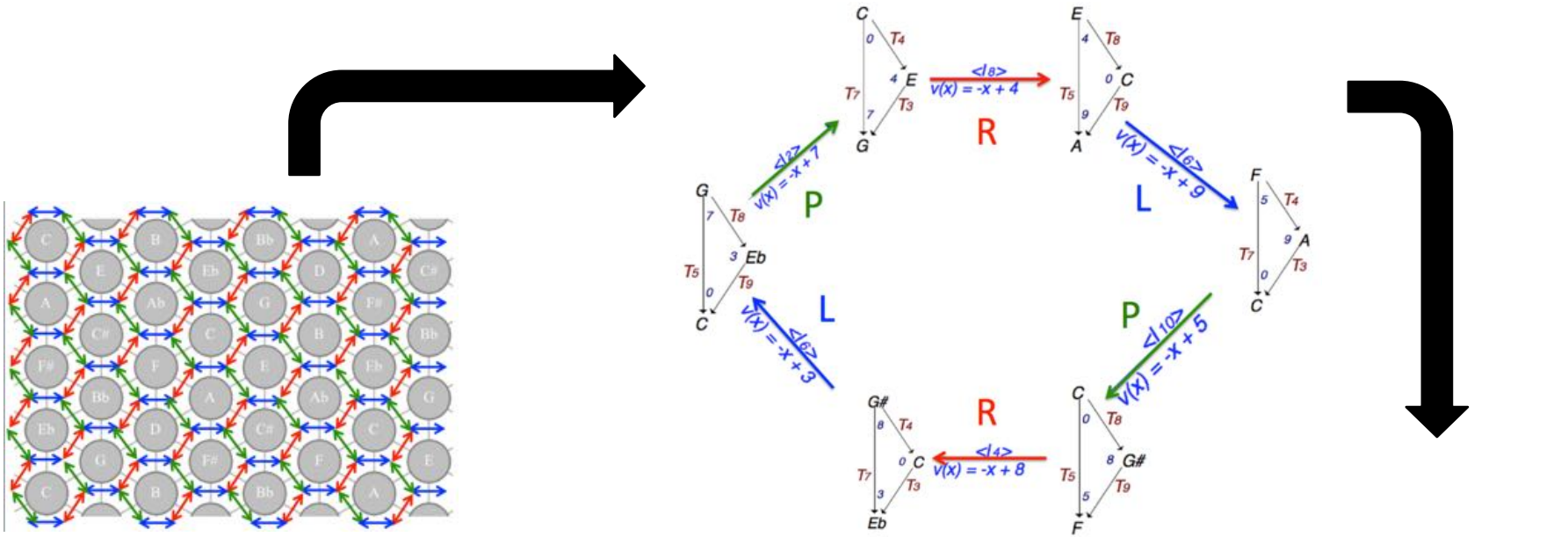


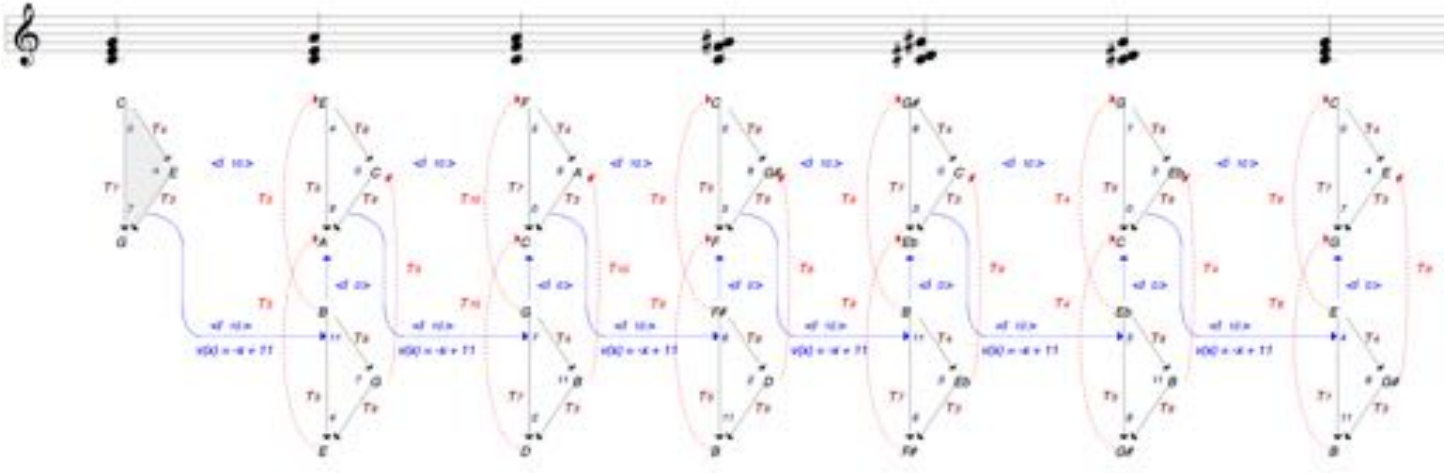
Anton Webern, *Drei Kleine Stücke*, Op. 11/2





Computer-aided categorical construction of the Tonnetz

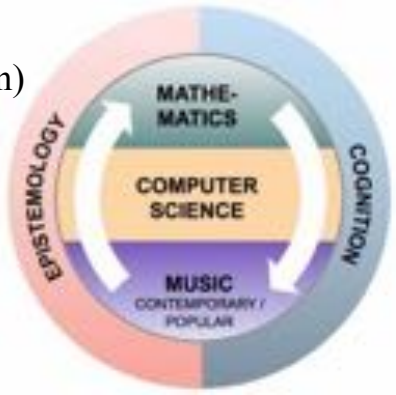




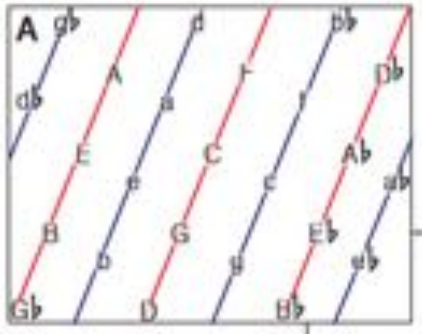
- Popoff A., C. Agon, M. Andreatta, A. Ehresmann (2016), « From K-Nets to PK-Nets: A Categorical Approach », PNM, 54(1)
- Popoff A., M. Andreatta, A. Ehresmann, « Relational PK-Nets for Transformational Music Analysis » (forthcoming in the JMM)

The 4+1 main axes of the SMIR Project

- **Generalized Tonnetze, Persistent Homology and Style**
  - Sonia Cannas (PhD candidate, University of Pavia / Strasbourg)
  - Davide Stefani (PhD candidate, UPMC)
  - Corentin Guichaoua (post-doc IRMA/USIAS)
  - Pierre Guillot (Researcher, IRMA)
- **Mathematical Morphology and Formal Concept Analysis**
  - Isabelle Bloch (Télécom ParisTech)
  - Jamal Atif (University of Dauphine)
  - Carlos Agon (CNRS-IRCAM-UPMC)
  - Jean-Louis Giavitto (CNRS-IRCAM-UPMC)
  - Pierre Relaño (former Master student & composer, Conservatory of Strasbourg)
- **Category theory and Transformational Music Analysis**
  - **Andrée Ehresmann** (mathematician)
  - **Alexandre Popoff** (mathematical music theorist)
  - **Tom Fiore** (University of Michigan, Editor-in-chief of the JMM)
  - **Thomas Noll** (ESMuC, Barcelona, former Editor-in-chief JMM)
- **'Mathemusical' problems and open conjectures**
  - Emmanuel Amiot (mathematician and mathematical music theorist, LAMPS Perpignan)
  - Franck Jedrzejewski (mathematical music theorist, CEA)
  - Alain Connes and his maths/music working group (IHES)
- **Epistemology and Cognitive Musicology**
  - **José-Luis Besada** (post-doc GREAM/IRMA)
  - **Nathalie Herold** (post-doc GREAM)



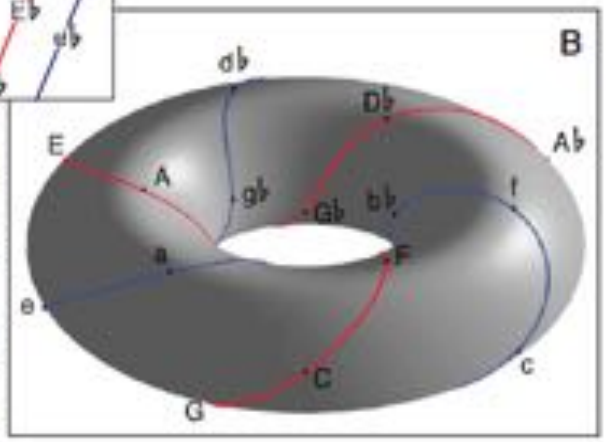
Neurosciences and Tonnetz



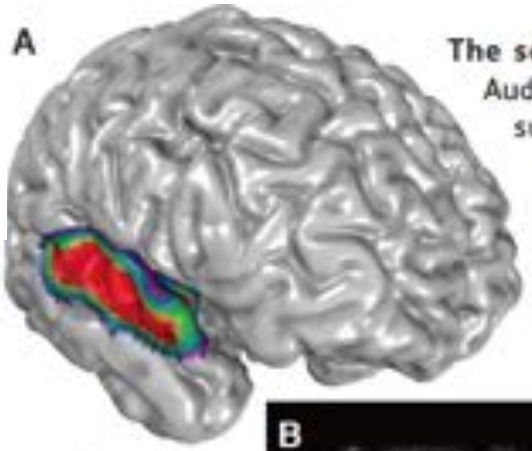
PERSPECTIVES: NEUROSCIENCE

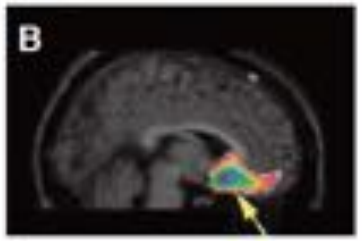
Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl

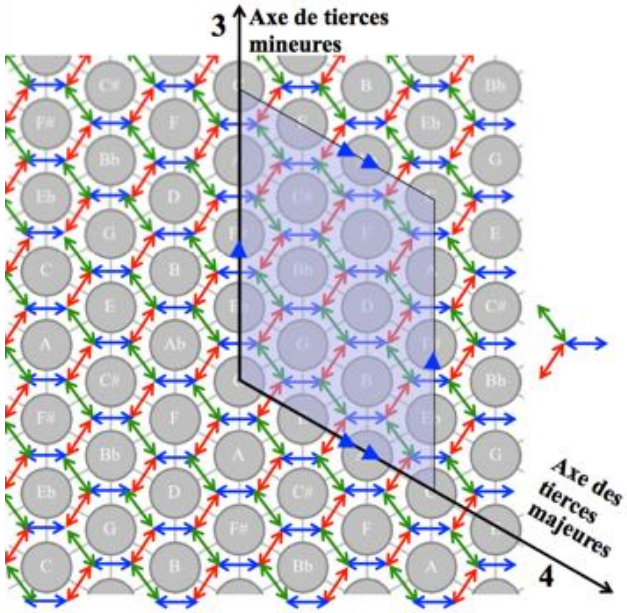


Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (B) Musical keys as points on the surface of a torus.





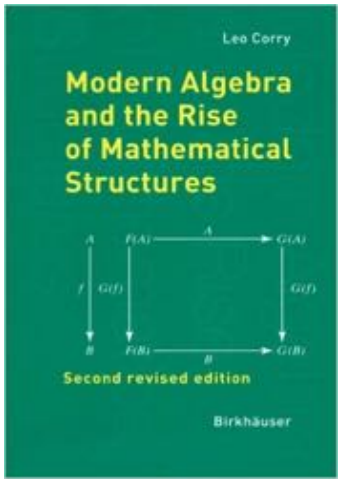
The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

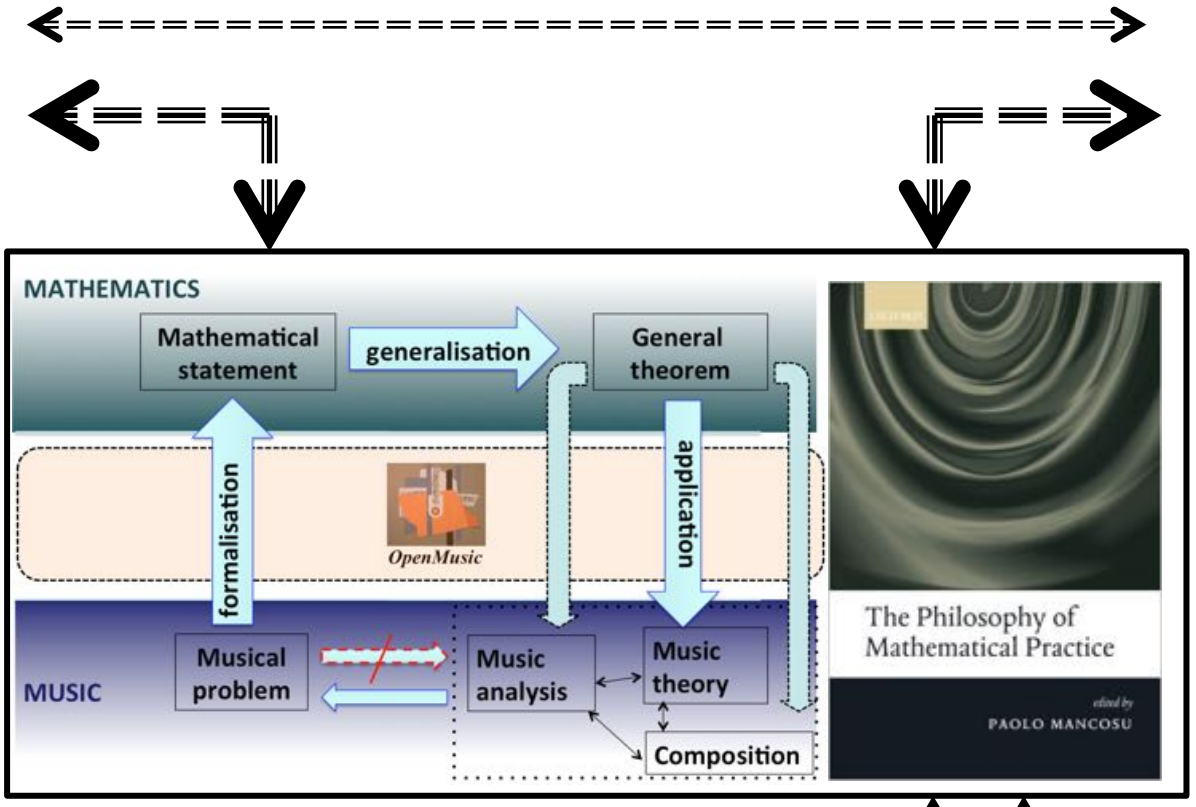


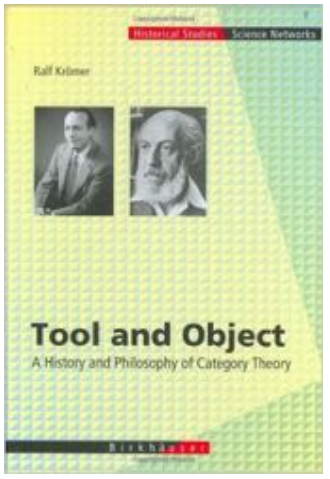
Acotto E. et M. Andreatta (2012), « Between Mind and Mathematics. Different Kinds of Computational Representations of Music », *Mathematics and Social Sciences*, n° 199, 2012(3), p. 9-26.

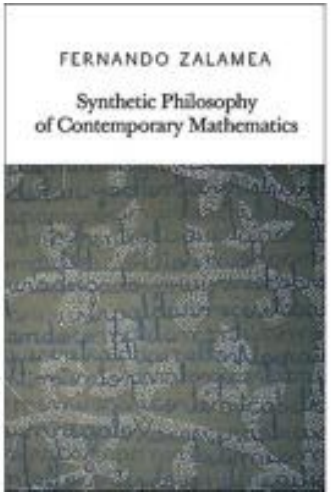
EMPG European Mathematical Psychology Group

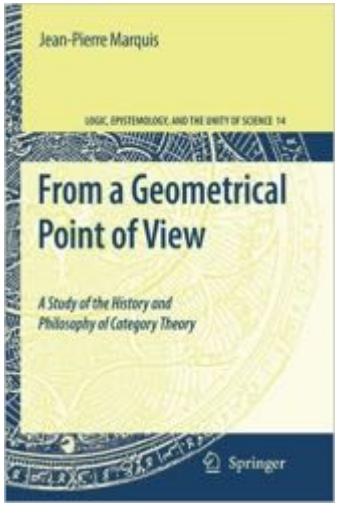
Which type of philosophy for the *mathemusical* practice?



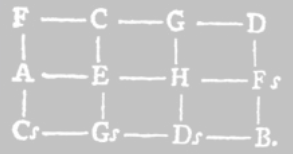








*A **synthetic** vision allows us to link together apparently distant strata of mathematics and culture, helping us to break down many artificial barriers. Not only can today's mathematics be appreciated through epistemic, ontic, phenomenological and aesthetic modes, but in turn, it should help to transform **philosophy.***



THANK YOU FOR YOUR ATTENTION