

# Maths in song writing

## Permutations and Hamiltonian Cycles



La Commune Libre d'Aligre

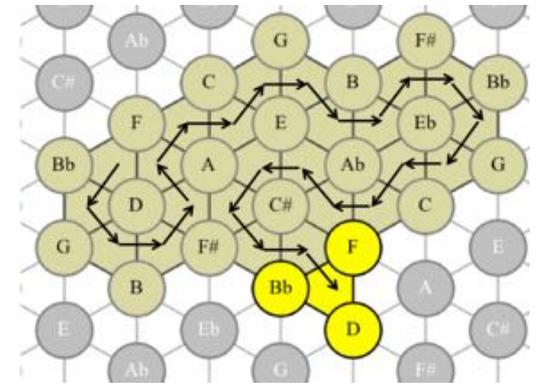
23 March 2017

Moreno Andreatta

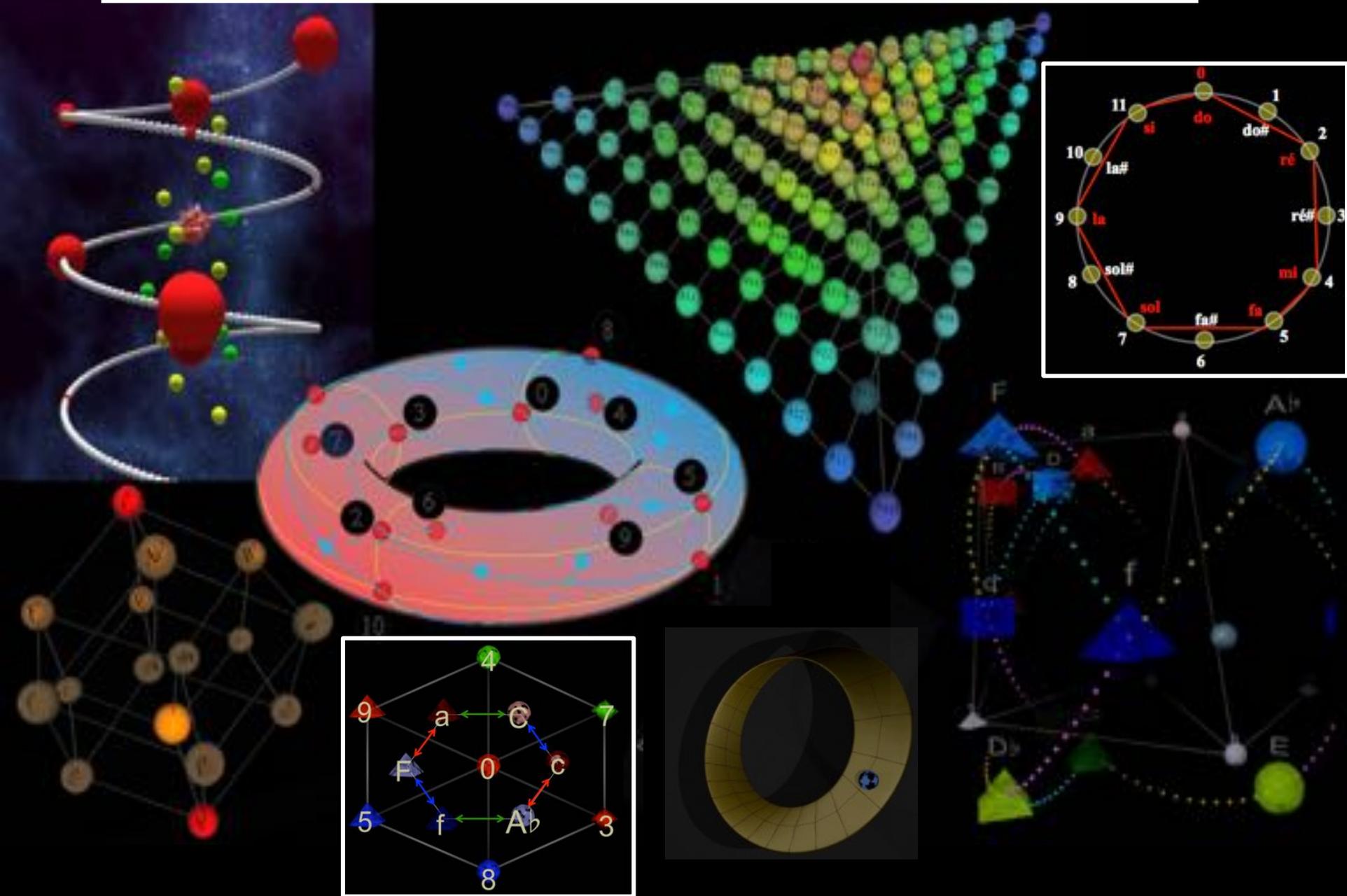
Music Representations Team

IRCAM / CNRS UMR 9912 / UPMC, Paris

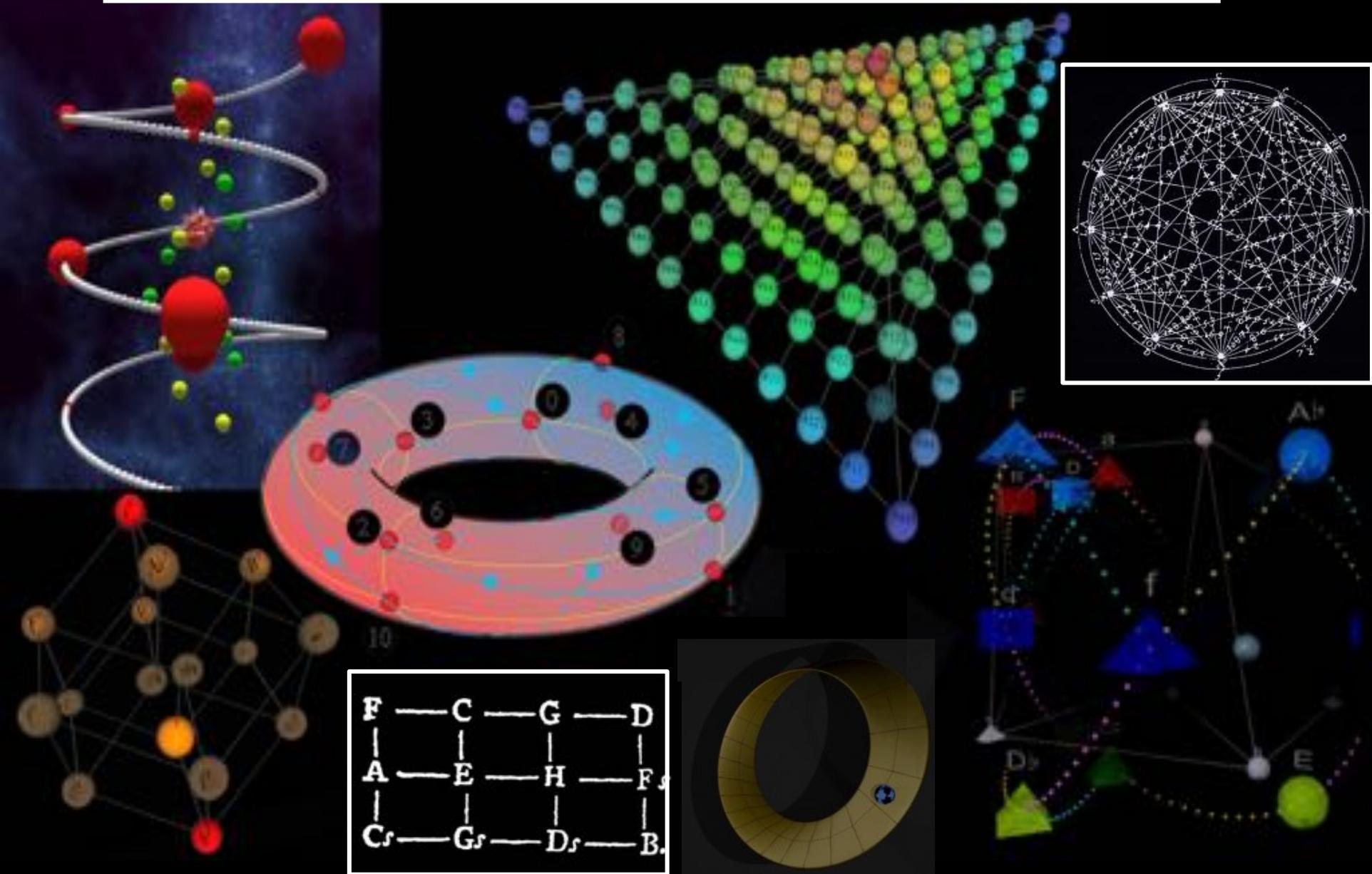
IRMA & GREAM, University of Strasbourg



# The galaxy of geometrical models at the service of music



# The galaxy of geometrical models at the service of music



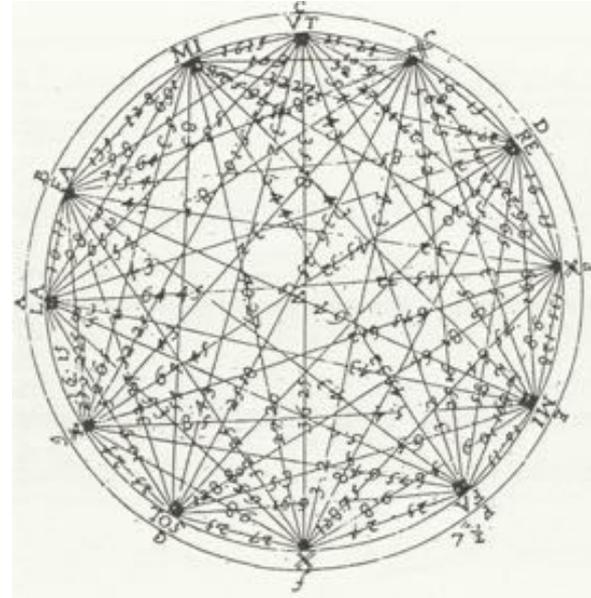
# Mersenne and the birth of (musical) combinatorics

114. Marin Mersenne, *Harmonicorum Libri XII*, 1648

## LIBER SEPTIMVS. DE CANTIBVS, SEV CANTILENIS, EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

*Tabula Combinationis ab 1 ad 22.*

I	1
II	2
III	6
IV	24
V	110
VI	710
VII	3040
VIII	40310
IX	361380
X	3613800
XI	39916800
XII	479001600
XIII	6127016800
XIV	87178120000
XV	1307674368000
XVI	20912789888000
XVII	315687418096000
XVIII	6401373705718000
XIX	121648100408811000
XX	2431301008176640000
XXI	5090941871709440000
XXII.	1114000717777607480000



Marin Mersenne

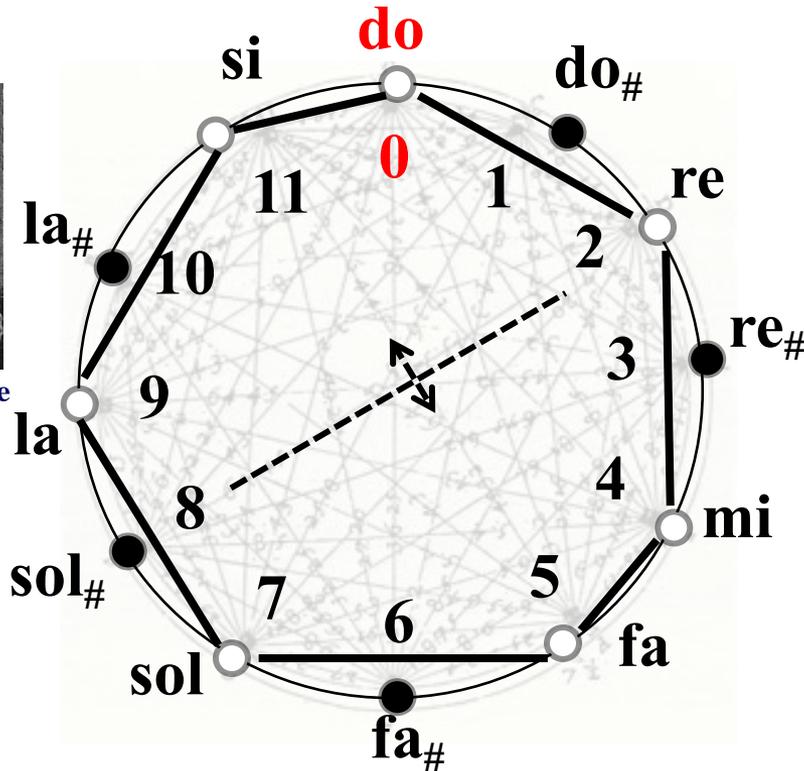
*Varietas, seu Combinatio quatuor notarum.*

A musical score illustrating 24 variations of four notes. The score is arranged in four staves, each containing a sequence of notes. The variations are numbered 1 through 24. The notes are represented by diamond-shaped symbols. The variations show different combinations and permutations of the four notes, demonstrating the concept of musical combinatorics.

# The circular representation of the pitch space



Marin Mersenne

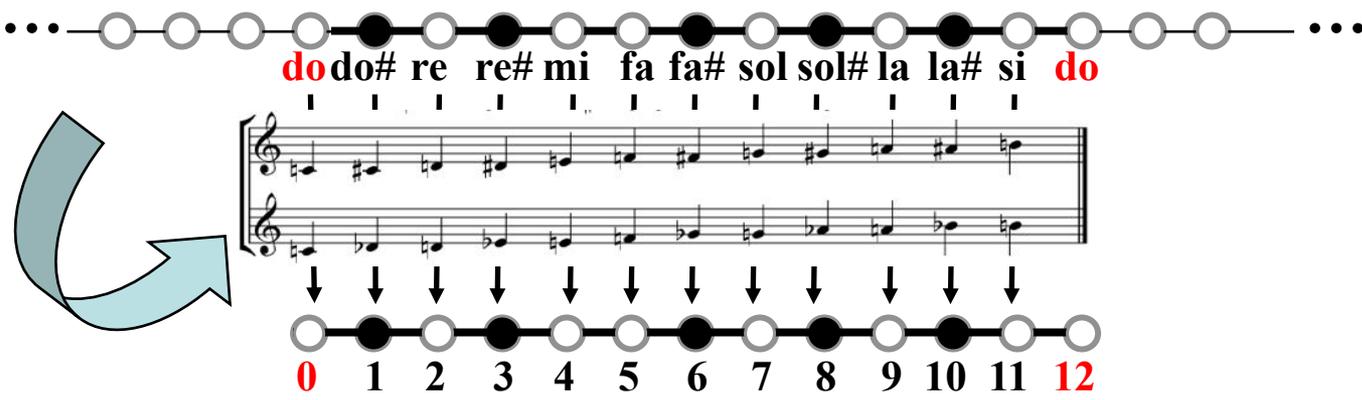


*Harmonicorum Libri XII*, 1648

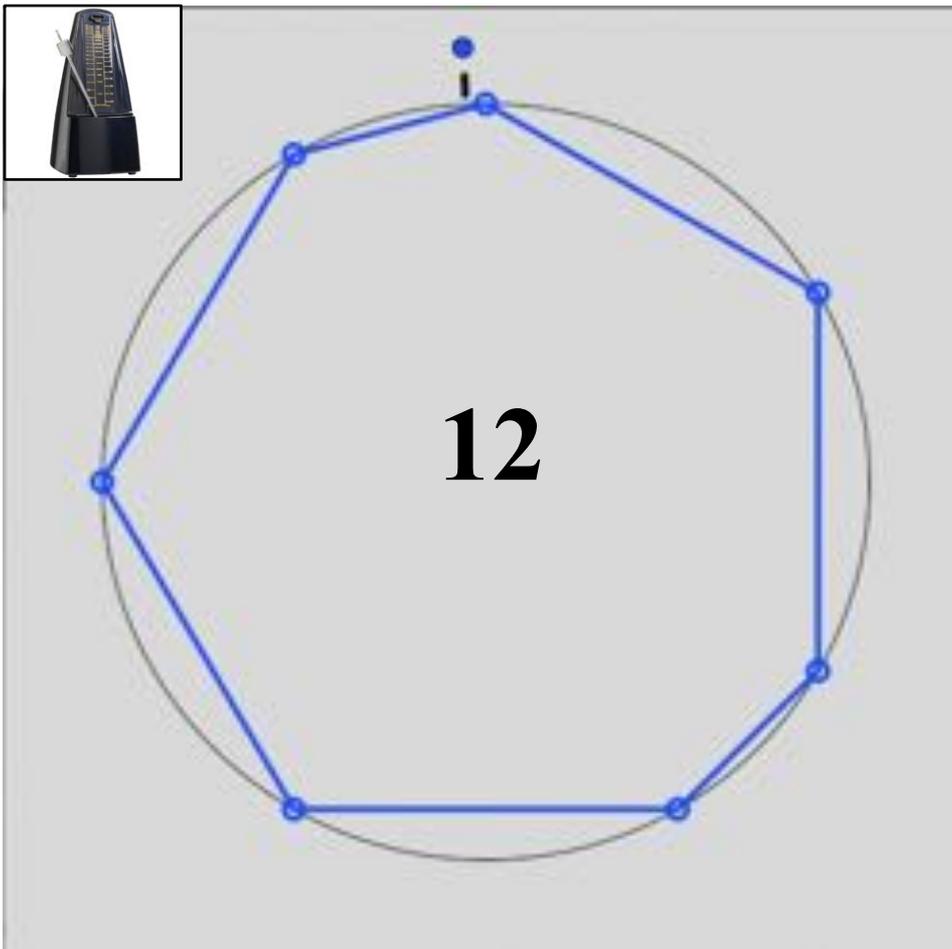


Tabula Combinationum ab 1 ad 12.

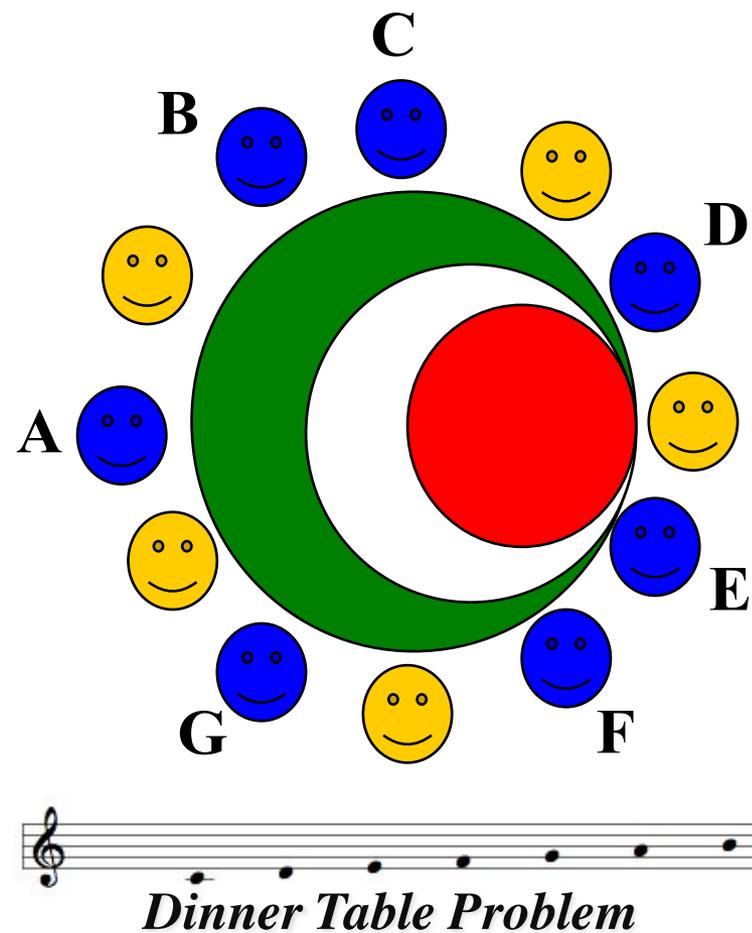
I	1
II	2
III	4
IV	14
V	110
VI	710
VII	3040
VIII	40340
IX	361880
X	3618800
XI	39916800
XII	479001600
XIII	6127016800
XIV	87178120000
XV	1307674560000
XVI	20921785880000
XVII	3366874180960000
XVIII	64021737057180000
XIX	121645100408810000
XX	24129010081766400000
XXI	510909411717094400000
XXII	11140007177776076800000



# The Pitch/Rhythm Isomorphism

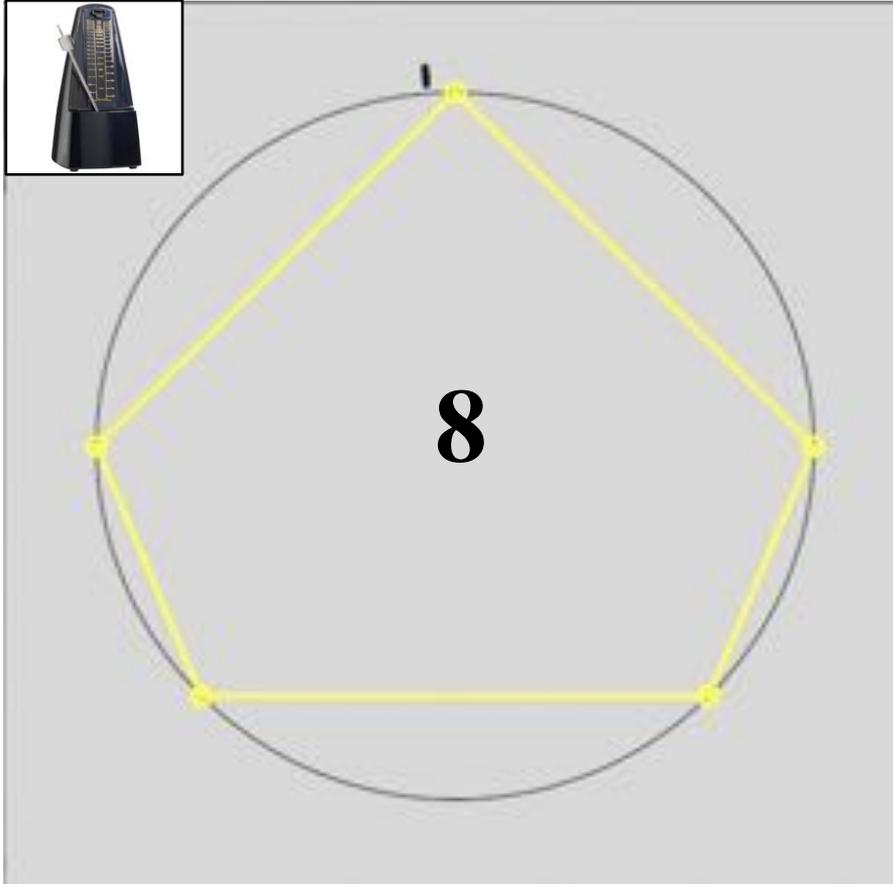


Abadja ou Bembé

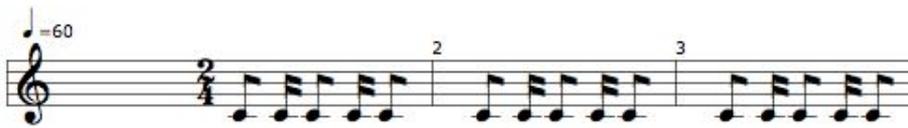


# African-cuban ME-rhythms

## *El cinquillo*



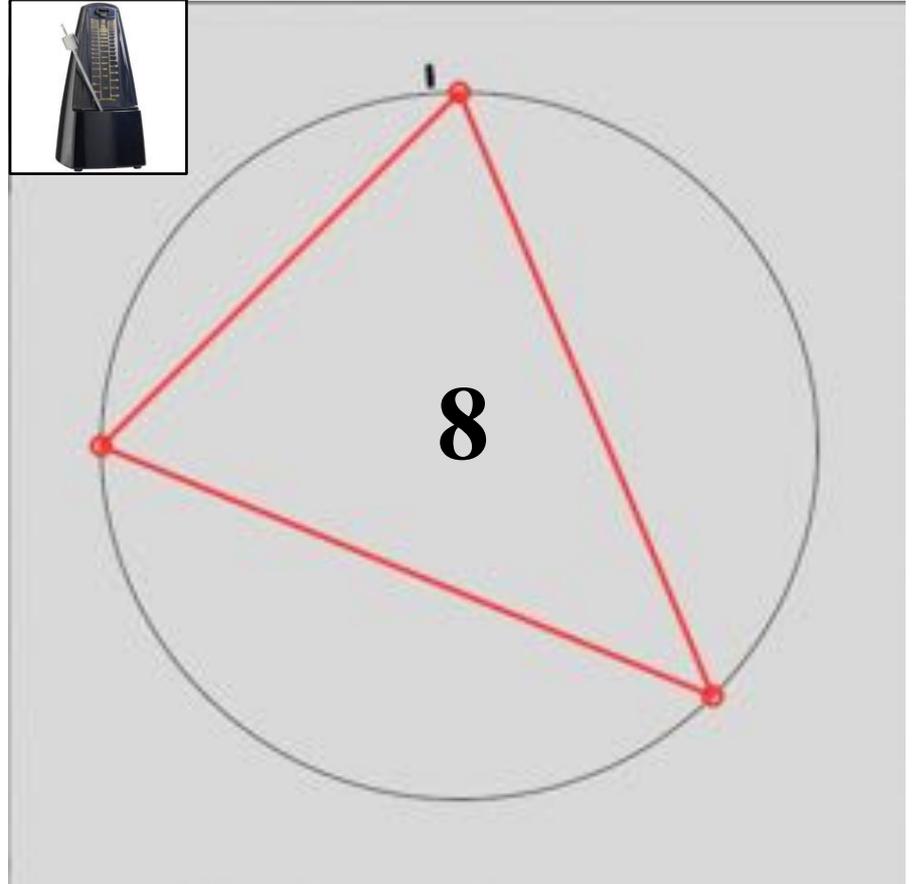
♩ = 60



2 2 3

Musical notation for El cinquillo in 2/4 time. The tempo is marked as ♩ = 60. The notation consists of three measures: the first measure contains two eighth notes, the second measure contains two eighth notes, and the third measure contains three eighth notes. The notes are beamed together in each measure.

## *El trecillo*



♩ = 60



4 2

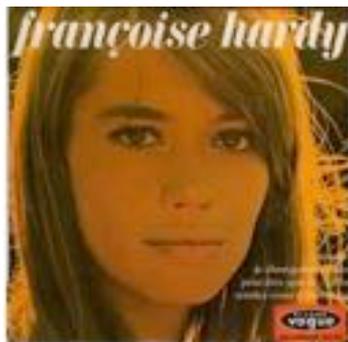
Musical notation for El trecillo in 4/4 time. The tempo is marked as ♩ = 60. The notation consists of two measures: the first measure contains four quarter notes, and the second measure contains two quarter notes. The notes are beamed together in each measure.

# Permutational melodies in song writing

*Se telefonando*, 1966 (Maurizio Costanzo/Ennio Morricone) / Mina



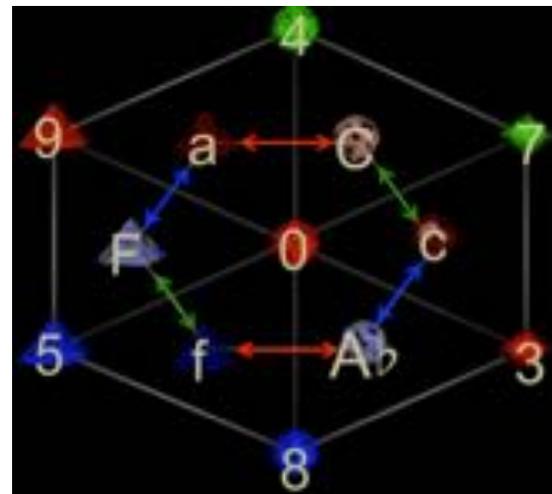
(min. 0'53'')



*Je changerais d'avis*, 1966  
(Françoise Hardy)



The harmonic space



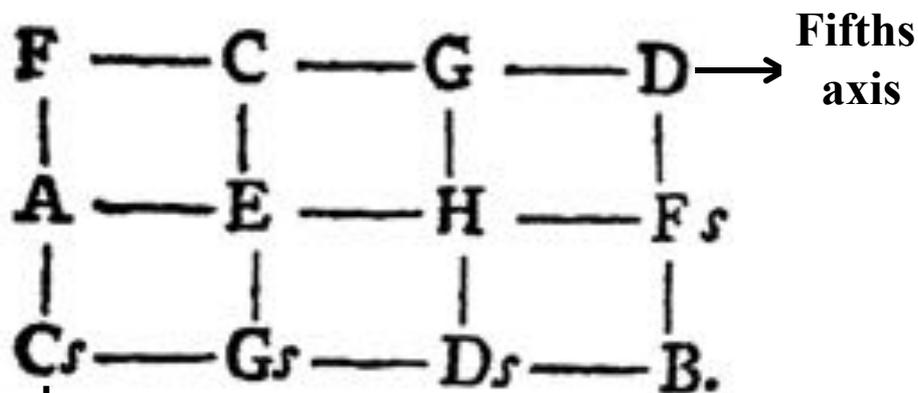
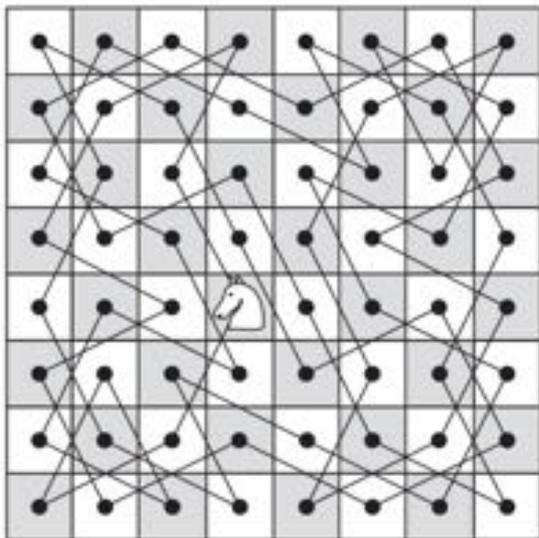
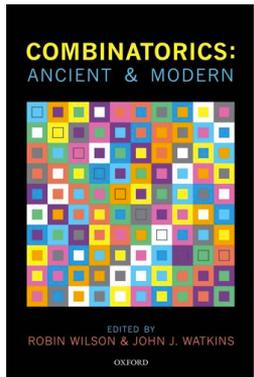
Ennio Morricone



C	c	<b>C#</b>	<b>c#</b>	D	<b>d</b>
<b>E<sub>b</sub></b>	<b>e<sub>b</sub></b>	E	e	<b>F</b>	f
<b>F#</b>	<b>f#</b>	G	<b>g</b>	<b>G#</b>	<b>g#</b>
A	a	<b>B<sub>b</sub></b>	<b>b<sub>b</sub></b>	<b>B</b>	b

**Chord enumeration**

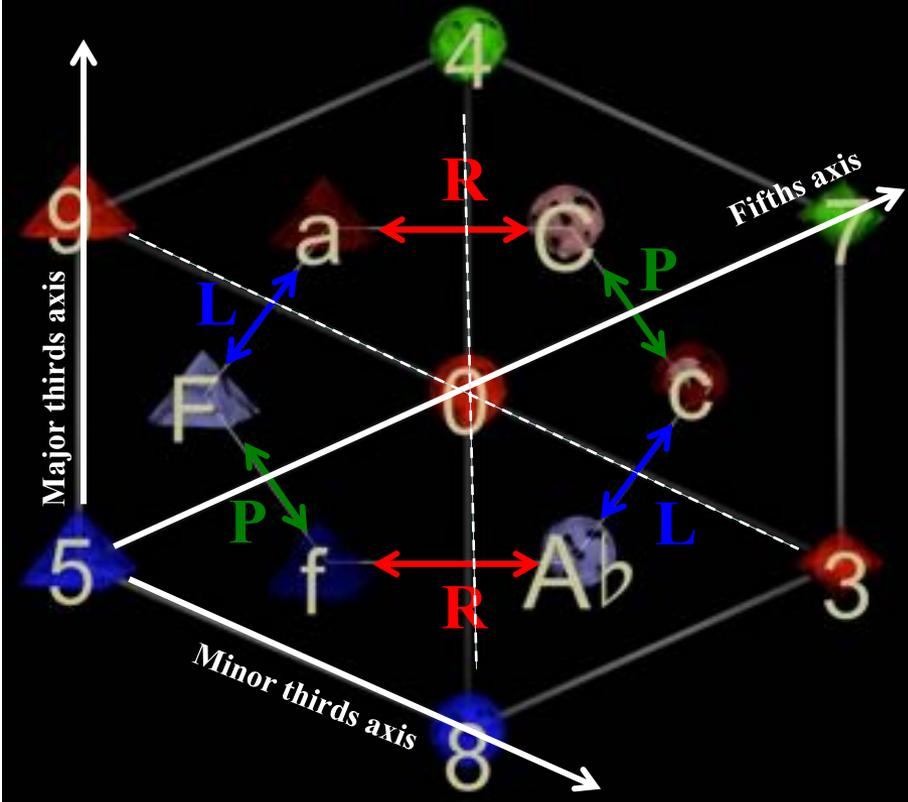
# The *Tonnetz* (or 'honeycomb' hexagonal tiling)



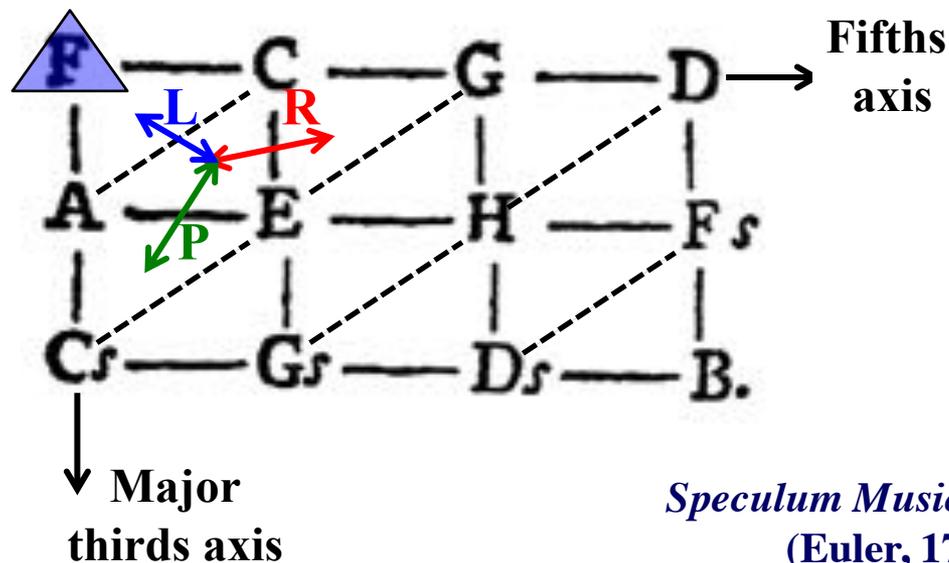
↓ **Major thirds axis**

*Speculum Musicum*  
(Euler, 1773)

# The *Tonnetz* (or 'honeycomb' hexagonal tiling)

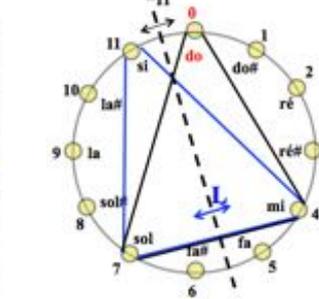
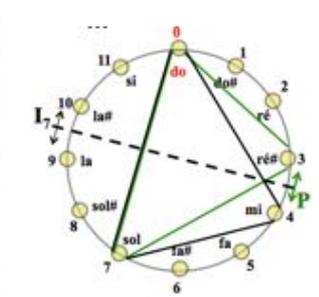
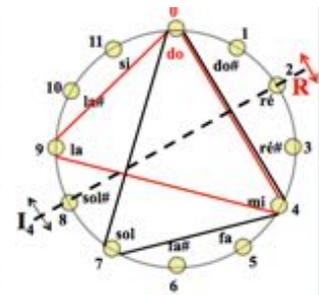
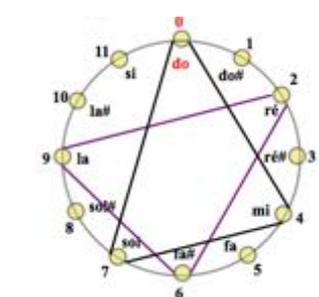
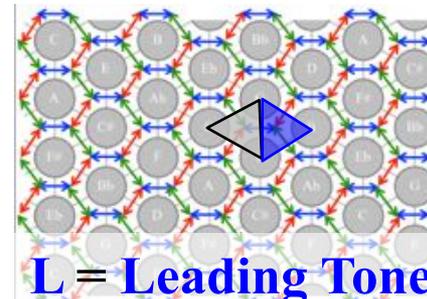
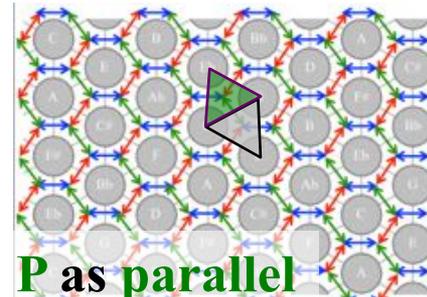
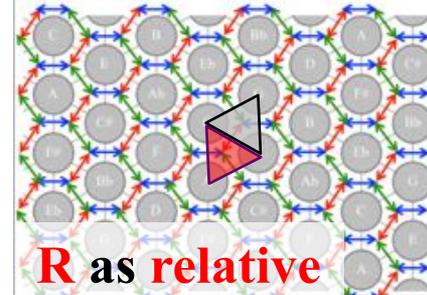
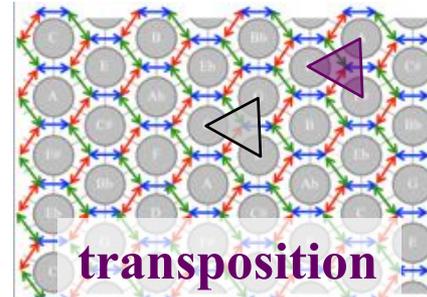
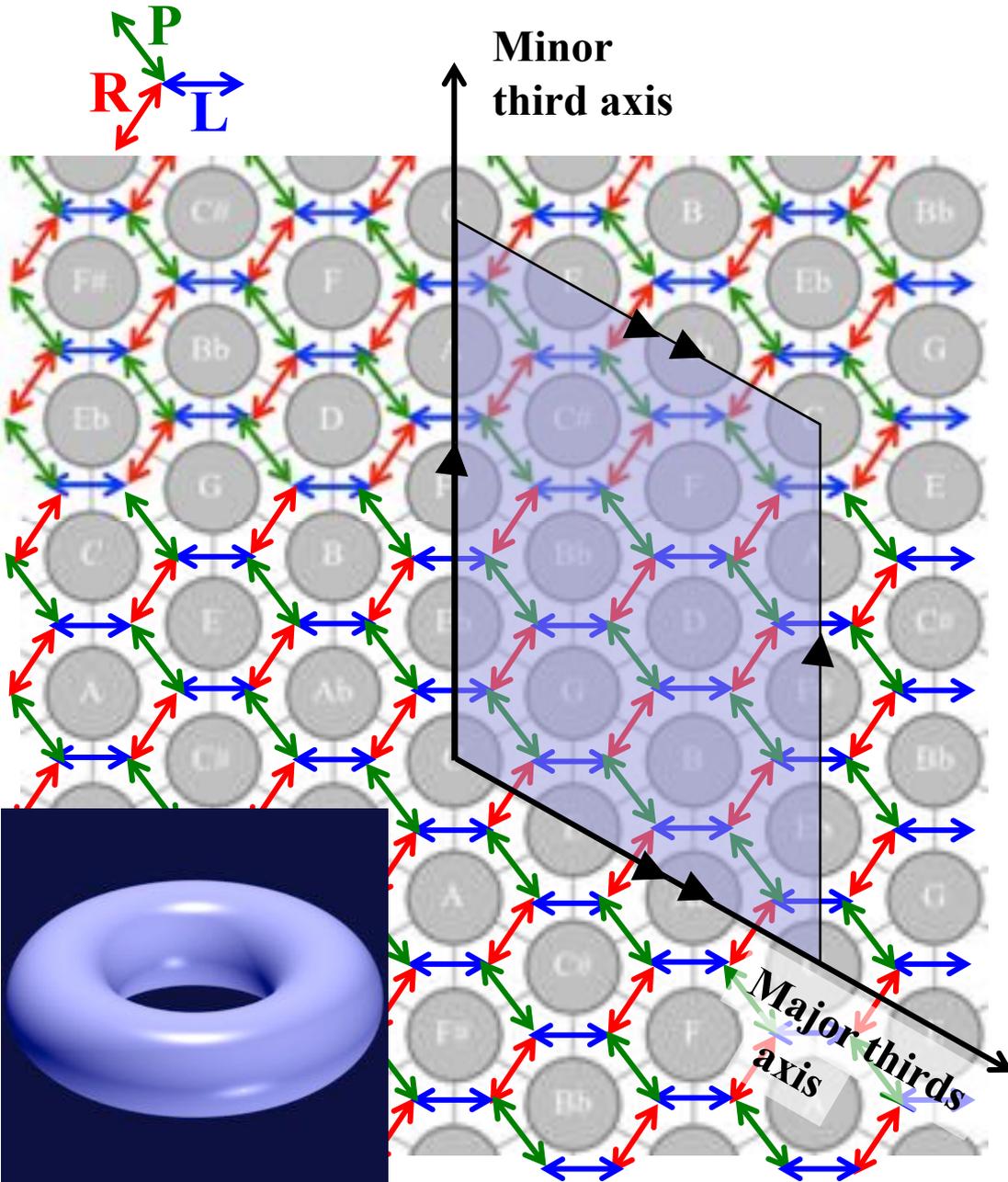


Gilles Baroin



*Speculum Musicum*  
(Euler, 1773)

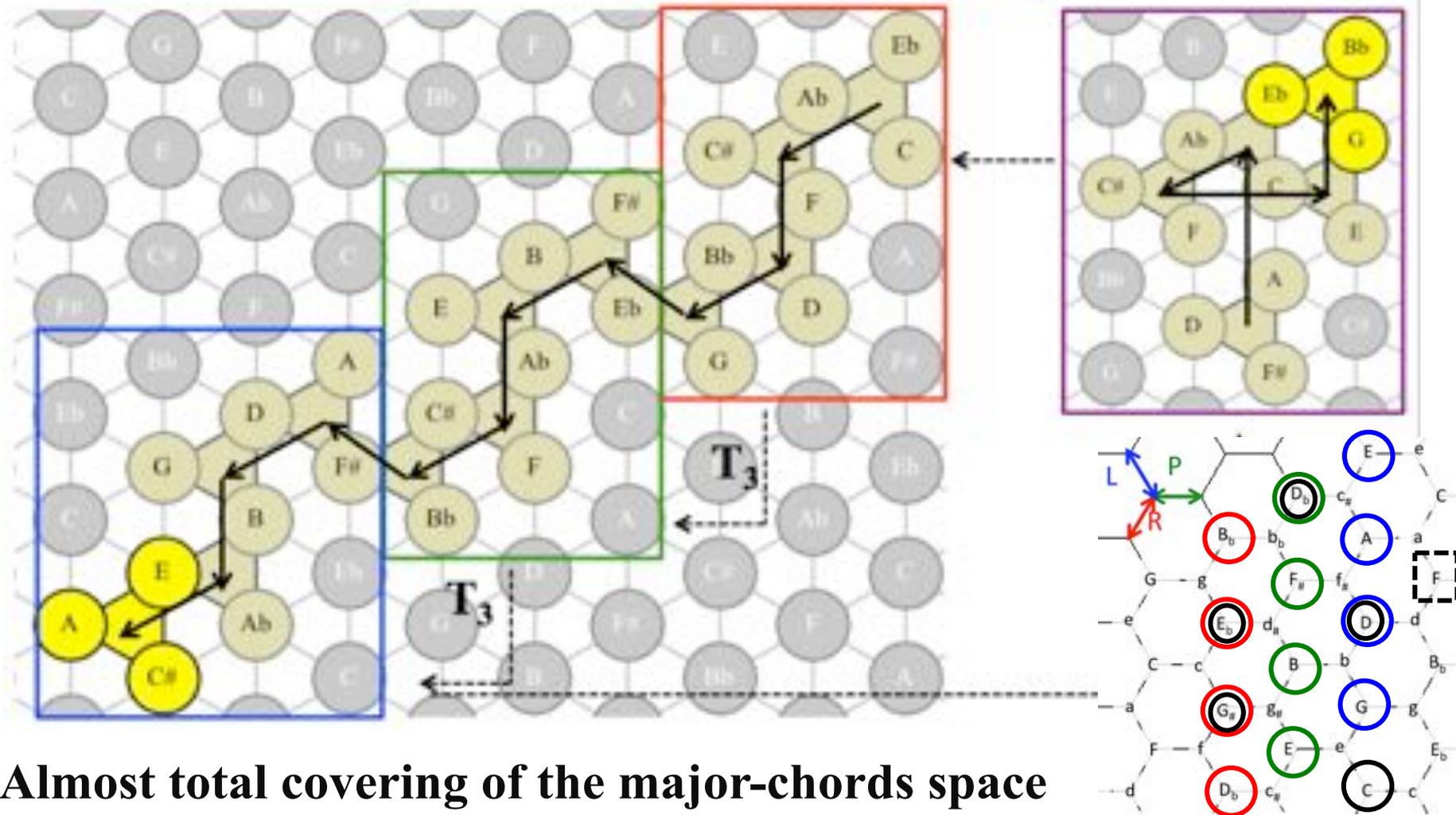
# The *Tonnetz*, its symmetries and its topological structure





# Symmetries in Paolo Conte's *Madeleine*

La<sub>b</sub> Re<sub>b</sub> Si<sub>b</sub> Mi<sub>b</sub> Si Mi Re<sub>b</sub> Fa<sub>#</sub> Re Sol Mi La Re La<sub>b</sub> Re<sub>b</sub> Do Mi<sub>b</sub>



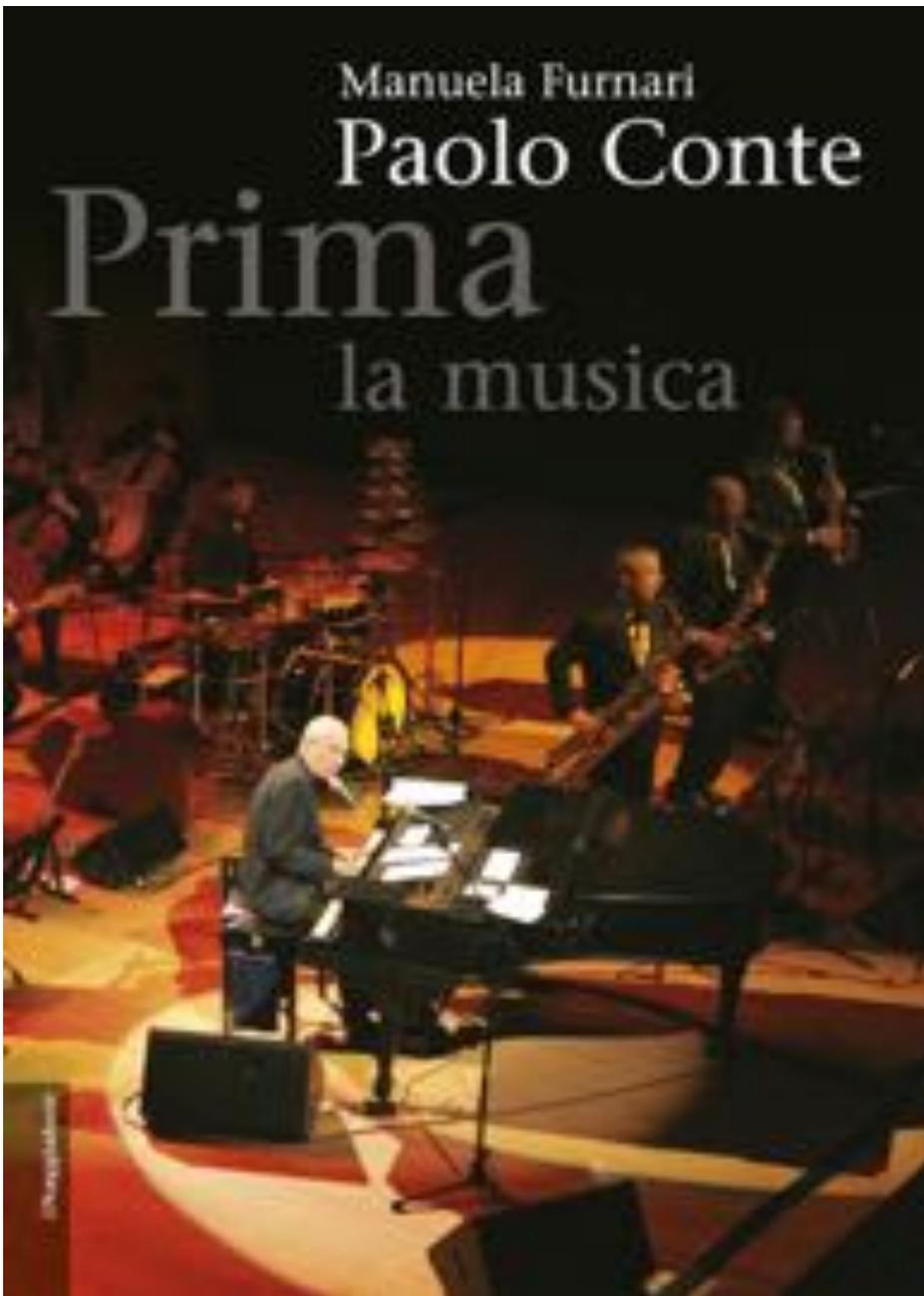
Almost total covering of the major-chords space

# The place of harmonic structures in Paolo Conte's music

Manuela Furnari  
Paolo Conte  
**Prima**  
la musica

Scrivere prima la musica e poi il testo mi obbliga ad un procedimento preso in prestito dal cinema. **Una canzone è forse più cinema che poesia o musica: è il sogno di qualcosa che vorrebbe muoversi. La camminata orizzontale dell'armonia.**

L'abilità nel muoversi armonicamente e enarmonicamente appartiene a tutta la musica. Uno dei limiti del jazz è quello di insistere sempre sullo stesso giro armonico. Io penso che l'**orecchio** dopo un po' [...] **abbia bisogno di spostarsi** [...] da questa continua attenzione sulla **stessa tonalità.**





# Two Dimensions Traditional Chordal Space

© Gilles Baroin 2011

## Harmonic Progressions

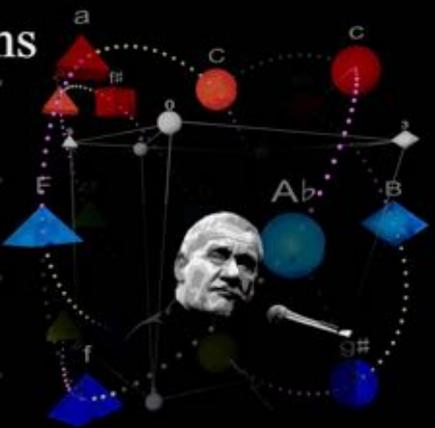
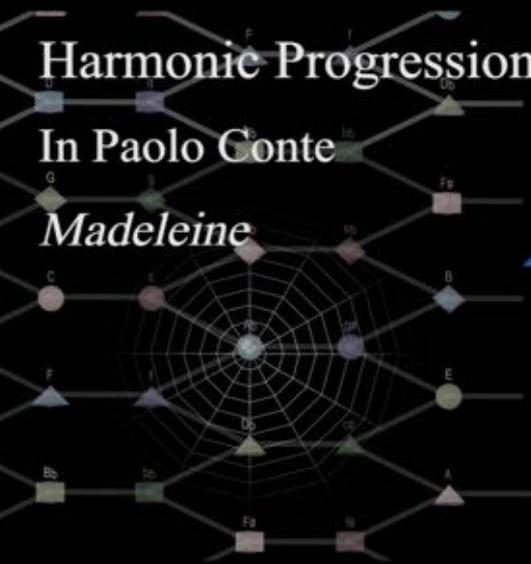
In Paolo Conte

*Sotto le Stelle del Jazz*



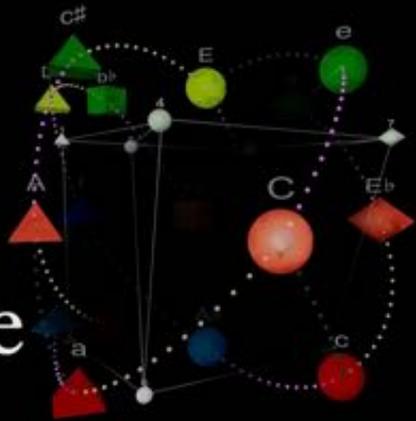
Supervision Moreno Andreatta  
Modelisation Gilles Baroin 2016

## Harmonic Progressions In Paolo Conte *Madeleine*



Supervision Moreno Andreatta  
Modelisation Gilles Baroin 2016

# Beethoven and the Hypersphere (and the Tonnetz)



Gilles Baroin 2016  
www.MatheMusic.net

<https://www.youtube.com/user/MatheMusic4D>

# Reading Beethoven backwards

## Le Blé en Herbe

(Polo/Moreno/Dieu)

Plonger comme un enfant, cheveux au vent

Sous l'océan du blé en herbe

Marée d'épis couleur d'amande

Qui tendent à caresser le ciel

Algues tendres de mille plages

Frôlant le ventre des nuages

Cheveux de pluie, dos de poissons

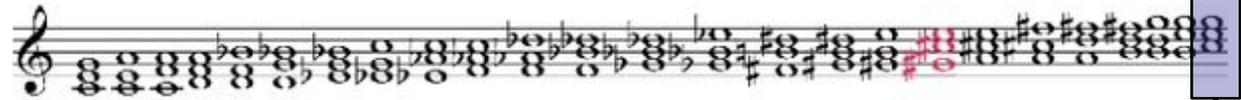
Qui frissonnent à l'unisson

Suivre le bord des continents

Dans l'océan du blé en herbe

Pêcher le corail du pavot

Dans le sang des coquelicots



← time

Croiser matin dans l'herbe folle

Deux tourterelles qui s'envolent

Suivre les jeux des hirondelles

Sur le paysage éternel

Nager comme un enfant, cheveux au vent

Sous l'océan

Du blé en herbe

Marée de fruits au goût amer

Acide et salée comme la mer

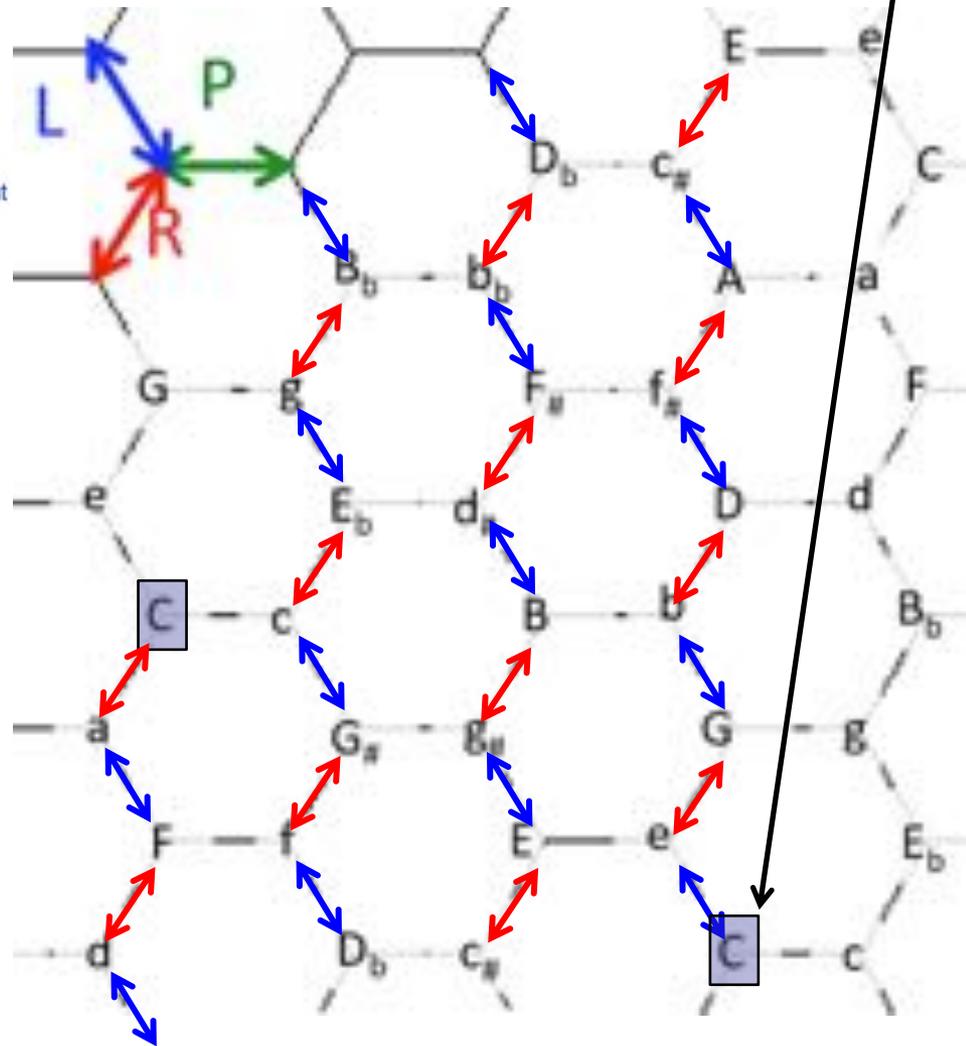
Vers l'îlot d'un petit village

Vers un château d'eau sur la plage

Quand tout s'éteint avant l'orage

Quand se lève le vent du large

Sur le blé vert





### CABARET HAMILTONIEN

FABRICE QUÉDY  
Formalises dans la musique savante

MORENO ANDREATTA  
Mentoratrices appliquées à la musique

POLO PIERRE LAMY  
Écriture appliquée aux formes musicales

Alexis - Voix, guitare, écriture  
Augustin - Voix, claviers, écriture  
Clara - Voix, guitare, écriture  
Emilie - Voix, mandoline, écriture  
Séverine - Voix, alto, écriture  
Thomas - Voix, claviers, écriture

Écrire sous la contrainte...  
TACHER DU PAPIER SOUS  
L'OBLIGATION DE FAIRE  
QUELQUE CHOSE...

SALIR DU BOIS CONDITIONNÉ  
AVEC LE DEVOIR MORAL DE  
TRANSFORMER UN OBJET  
INDEFINI...

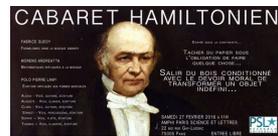
SAMEDI 27 FÉVRIER 2016 à 11h  
AMPHI PARIS SCIENCE ET LETTRES  
22 rue de St-Louis  
75005 Paris

ENTRÉE LIBRE



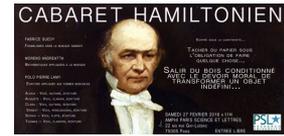
# The collection of 28 « redundant » Hamiltonian Cycles

1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPRPR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPRLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRLRPRP
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPRL
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRLRLRP
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm--RLRPRPRP
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em--RLRPRPRL
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLRP
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm--RLRLRPRP
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL



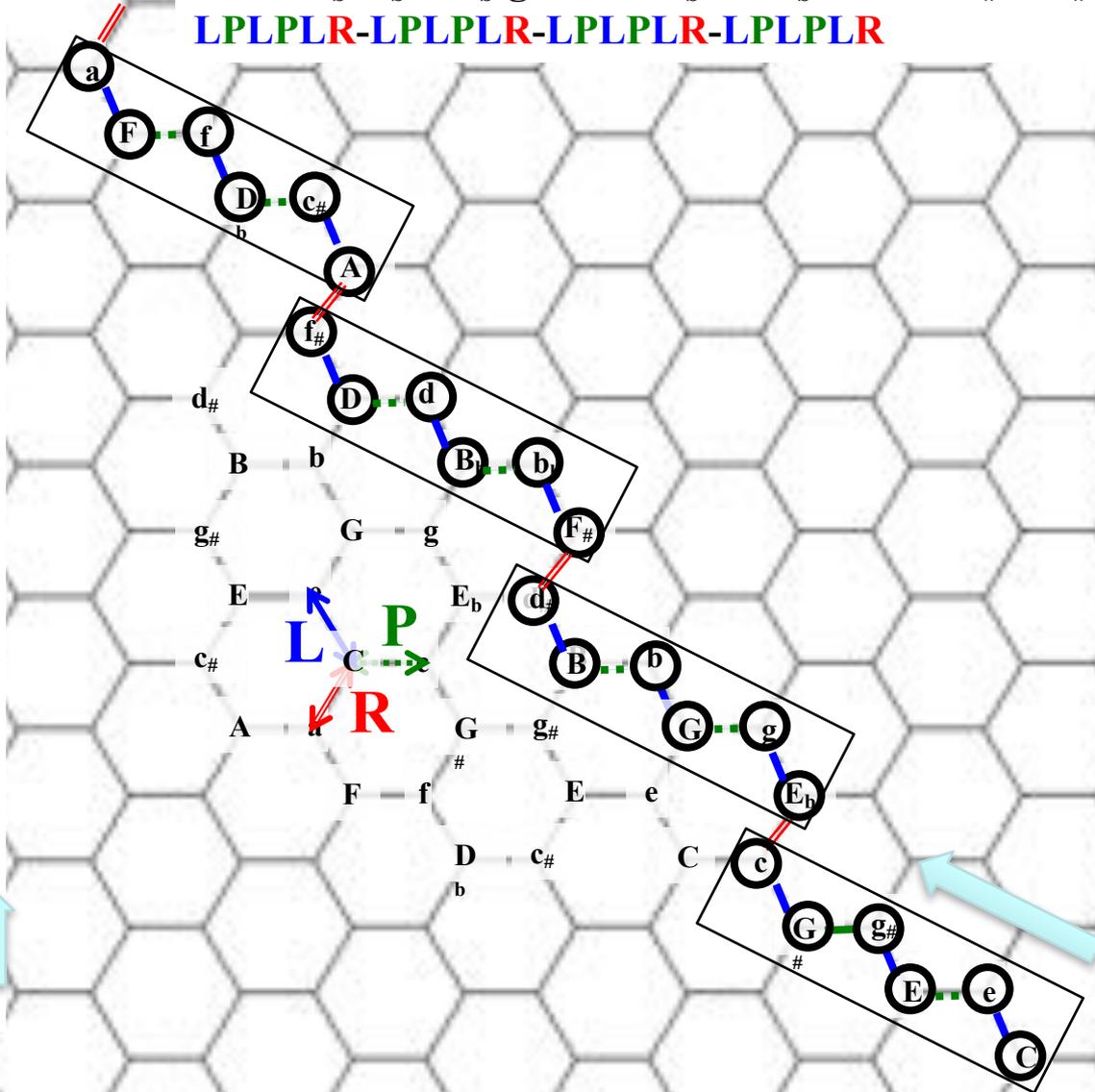
# BALLADE MARABOUT COLLECTIVE

A lire de haut en bas, puis de bas en haut, sans pause.



C-e-E-a<sub>b</sub>-A<sub>b</sub>-c-E<sub>b</sub>-g-G-b-B-e<sub>b</sub>-F<sub>#</sub>-b<sub>b</sub>-B<sub>b</sub>-d-D-f<sub>#</sub>-A-c<sub>#</sub>-C<sub>#</sub>-f-F-a  
**LPLPLR-LPLPLR-LPLPLR-LPLPLR**

Lassitudes enfantines  
 Enfantine arithmétique  
 Arithmétique enchantée  
 Chante le flot de nos vies  
  
 La dorure d'un matin  
 D'un matin tombé du lit  
 D'un lit bleu décoloré  
 Colorié comme un oiseau  
  
 Miséreux trappeur des bois  
 Dans les bois poison mortel  
 Mortel ennui qui te scie  
 Qui te scie jusques au sang  
  
 Si tu crois, pense à ta tombe  
 Ta tombe est à profaner  
 Fanée comme une fleur morte  
 Morte qui pourras rêver  
  
 Silencieux miroirs des cimes  
 Décimes tes vains espoirs  
 Poire juteuse presque pourrie  
 Risques-toi à te goûter  
  
 Fabrique un monde arc en ciel  
 Ton ciel est plein de lueurs  
 De lueurs et de clameurs  
 Meurs comme crèvent les chiens





# Aprile, a Hamiltonian « decadent » song

Do ← do<sub>m</sub> ← Sol# ← fa<sub>m</sub> ← Fa ← la<sub>m</sub> ← La ← fa#<sub>m</sub> ← Fa# ← sib<sub>m</sub> ← Do# ← do#<sub>m</sub>

mi<sub>m</sub> → Sol → si<sub>m</sub> → Ré → ré<sub>m</sub> → Sib → sol<sub>m</sub> → Mib → mib<sub>m</sub> → Si → sol#<sub>m</sub> → Mi

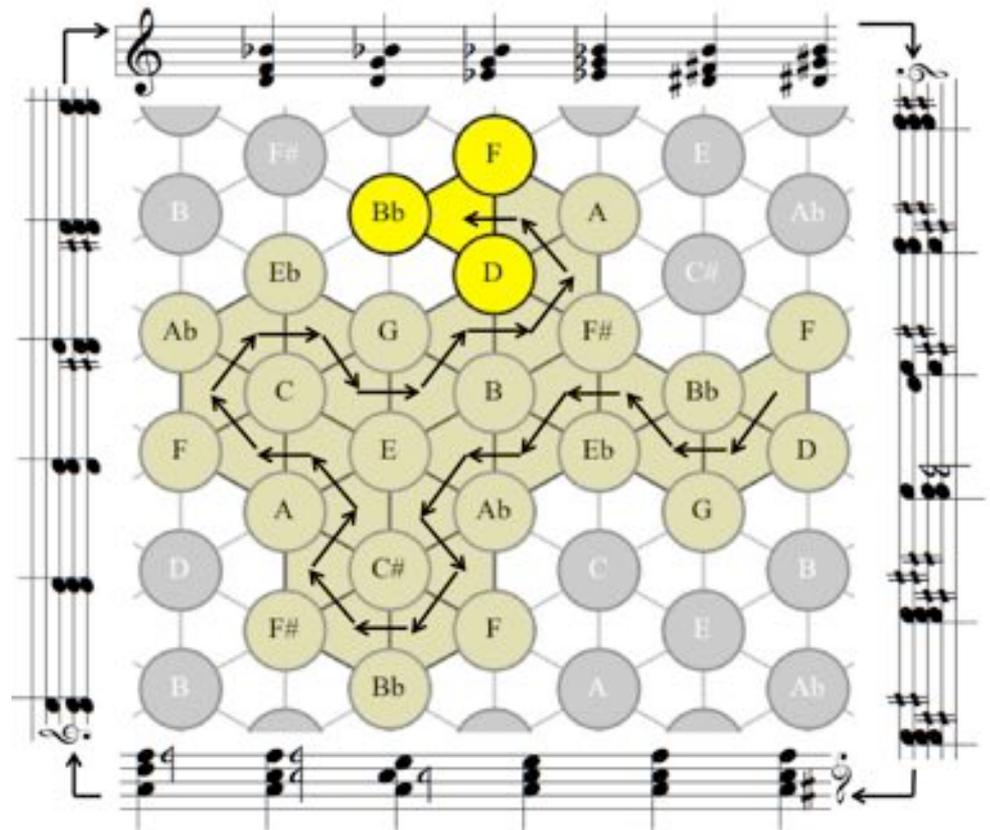
*Socchiusa è la finestra, sul giardino.  
Un'ora passa lenta, sonnolenta.  
Ed ella, ch'era attenta, s'addormenta  
A quella voce che già si lamenta,  
Che si lamenta in fondo a quel giardino.*

*Non è che voce d'acque su la pietra:  
E quante volte, quante volte udita!  
Quell'amore e quell'ora in quella vita  
S'affondan come ne l'onda infinita  
Stretti insieme il cadavere e la pietra.*

*Ella stende l'angoscia sua nel sonno.  
L'angoscia è forte, e il sonno è così lieve!  
(Par la luce d'april quasi una neve  
che sia tiepida.)  
Ed ella certo deve soffrire,  
Vagamente, anche nel sonno.*



G. D'Annunzio (1863-1938)



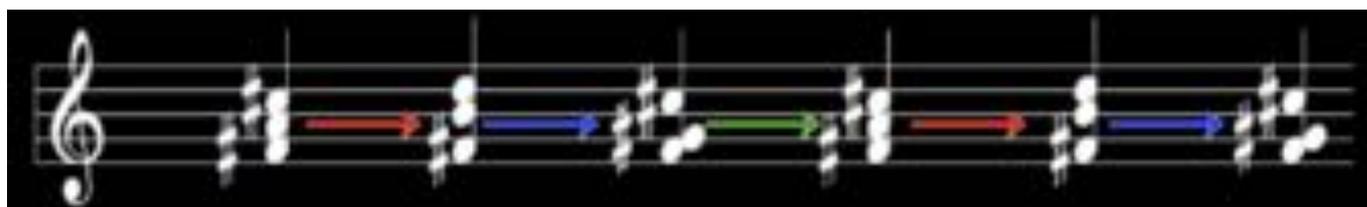
**ACTIONS**

**Math'n'pop**

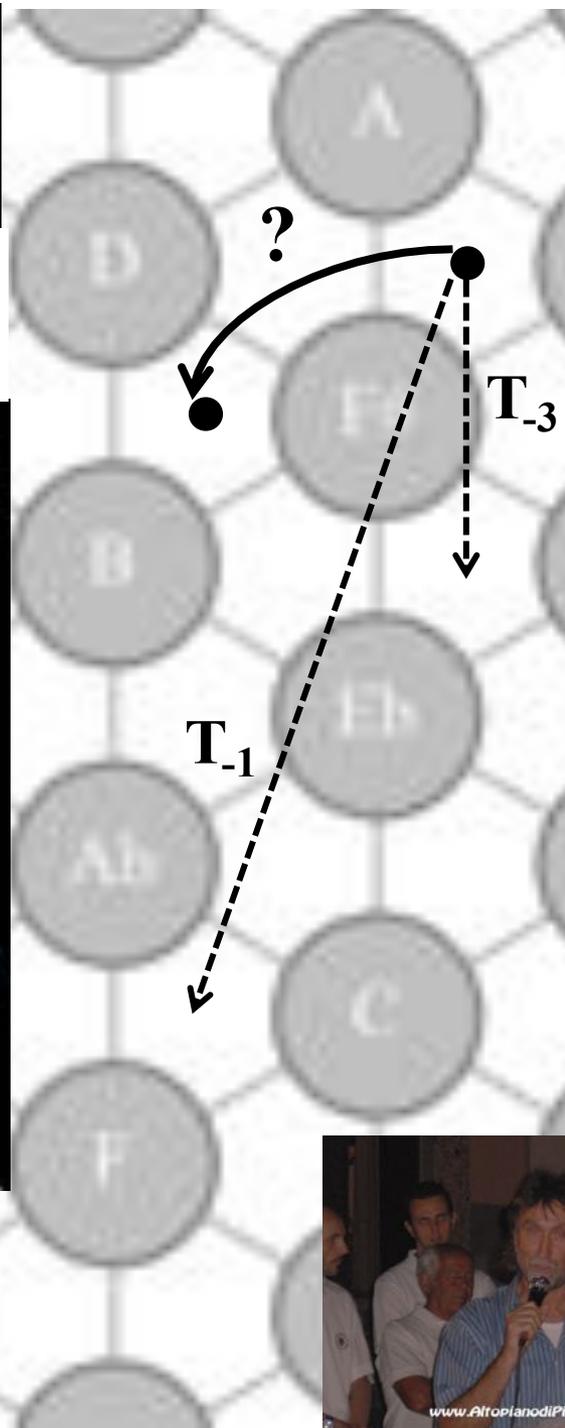
**Aprile (d'après Gabriele D'Annunzio)**

# DOMENEDIO

(« chanson ouverte », based on a poetry by Livio Andeatta)



*Find all Hamiltonian Mathemusical Paths that modulate through all minor triads from f# min to b min, using only -1 or -3*



T[1,3,4]

1

4

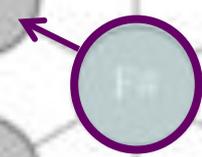
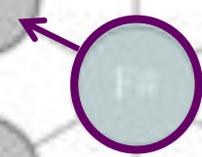
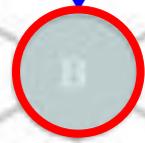
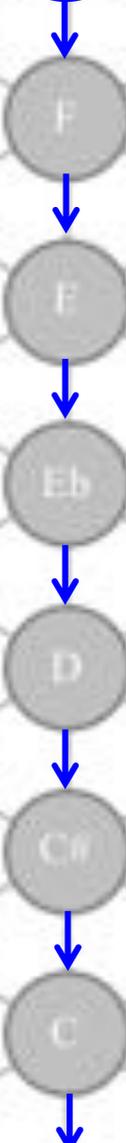
3

# DOMENEDIO



*Find all Hamiltonian Mathematical Paths that modulate through all minor triads from f# min to b min, using only -1 or -3*

**Notation:**  
C = Do minor  
C# = Do# minor  
.  
.  
.  
B = Si minor

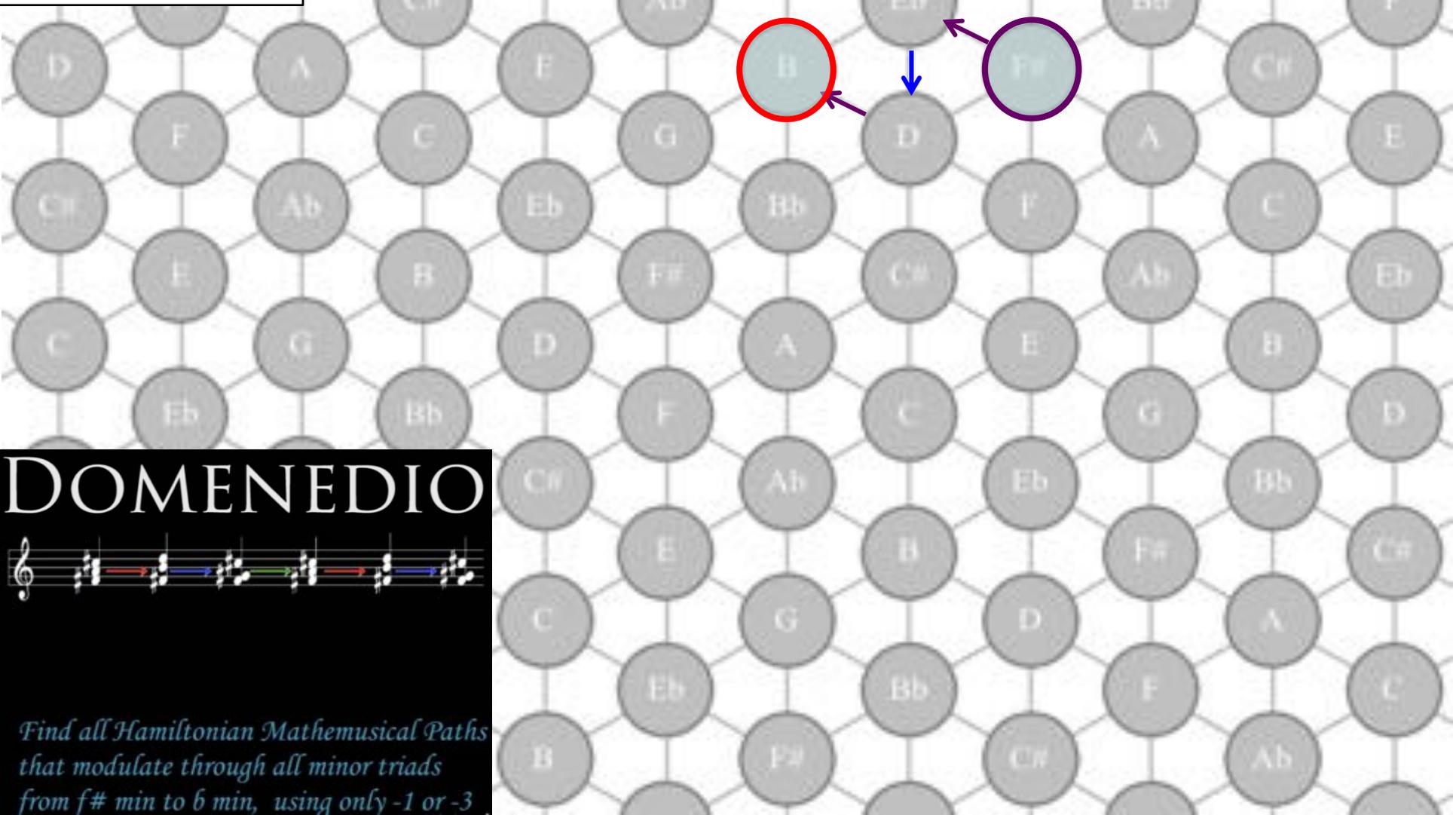


# Notation :

**C** = Do mineur

**C#** = Do# mineur

...

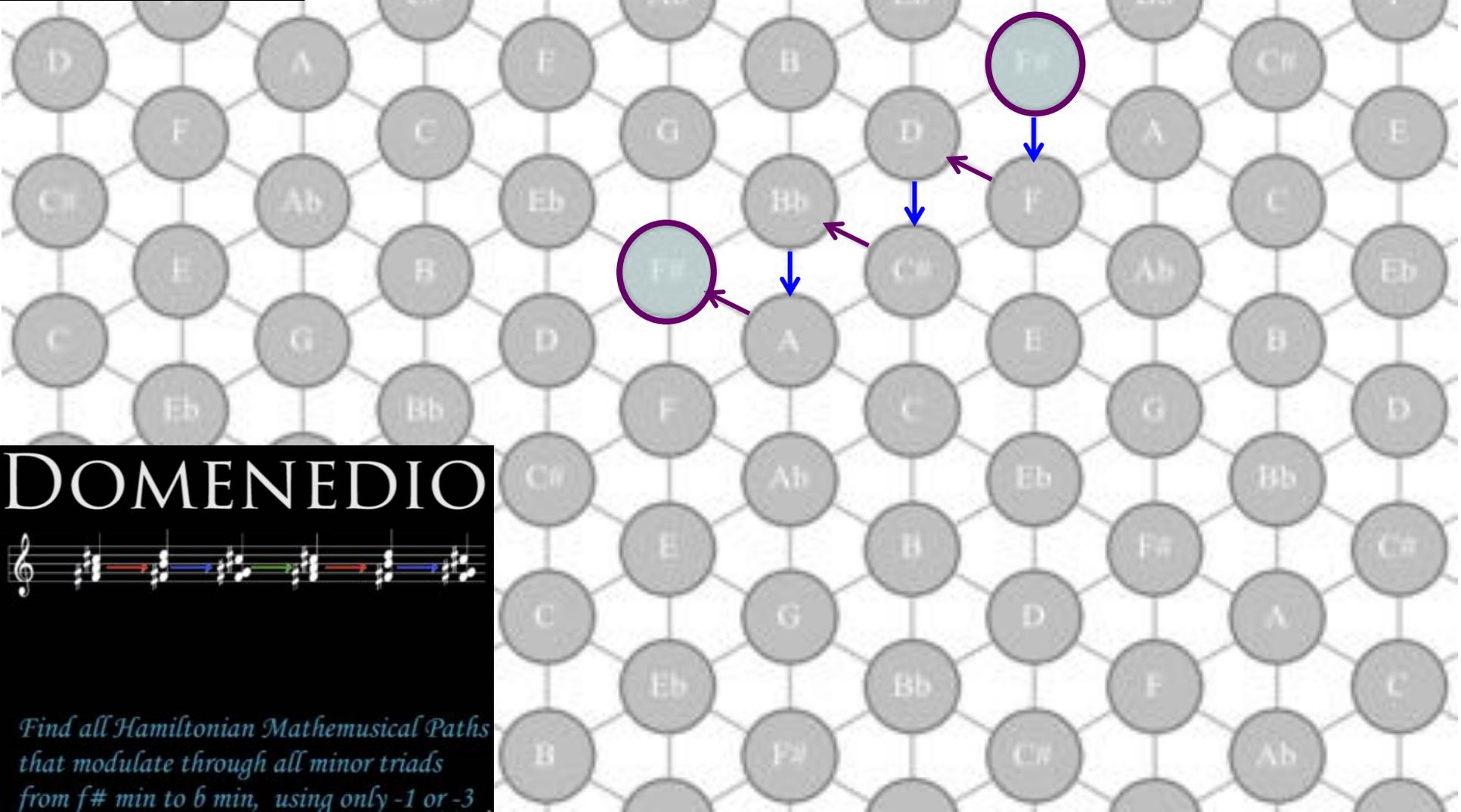


# DOMENEDIO



*Find all Hamiltonian Mathematical Paths that modulate through all minor triads from f# min to b min, using only -1 or -3*

**Notation :**  
**C = Do mineur**  
**C# = Do# mineur**  
...

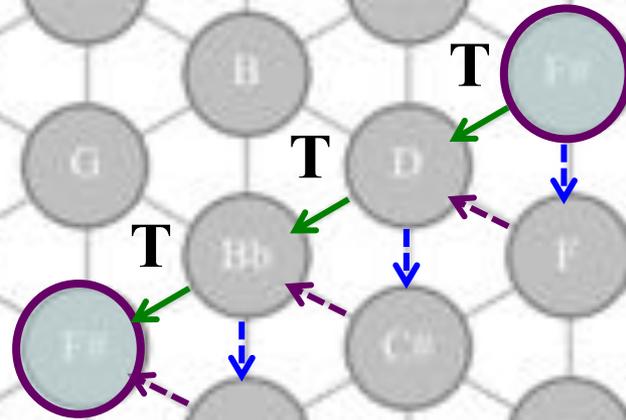


# DOMENEDIO

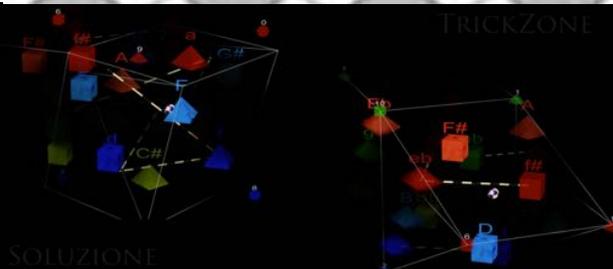
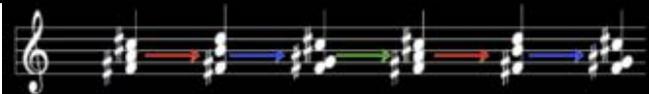


*Find all Hamiltonian Mathematical Paths that modulate through all minor triads from f# min to b min, using only -1 or -3*

# The "T" operator (as "trick")



DOMENEDIO



TRICKZONE

**Notation:**

- C = Do minor
- C# = Do# minor
- ...

*Find all Hamiltonian Mathematical Paths  
that modulate through all minor triads  
from f# min to b min, using only -1 or -3*

<http://www.mathemusic.net>



Luzi



Hamiltonian Song  
on  
Spinnennetz  
2014

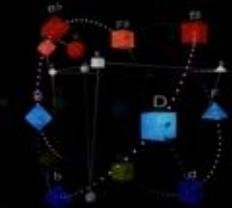
Lyrics by Mario Luzi  
Music and Vocals by Moreno Andreatta  
Graphics and Animation by Gilles Baroin  
SpinnenTonnetz by Gilles Baroin, Hugò Seress  
Original "Chicken Wire" graph by J.Douthett, P.Steinbach

Aprile  
Hamiltonian Song



G. D'Annunzio  
(1863-1938)

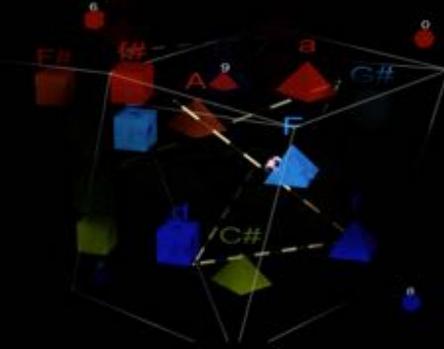
Mathematical  
2D & 4D Visualizations



Composition, Performance: Moreno Andreatta  
Hyperspheres & Animations: Gilles Baroin  
Spinnen-Tonnetz: Hugò Seress & G.B  
Lyrics by Gabriele D'Annunzio  
www.MatheMusic.net



Find all Hamiltonian Mathematical Paths  
that modulate through all minor triads  
from f# min to b min, using only -1 or -3



SOLUZIONE



TRICKZONE

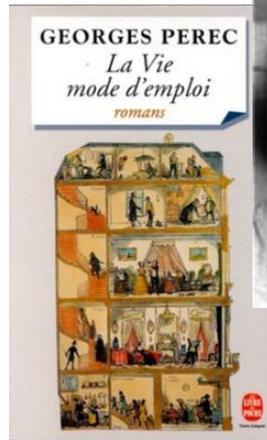
TRICKZONE

<https://www.youtube.com/user/MatheMusic4D>

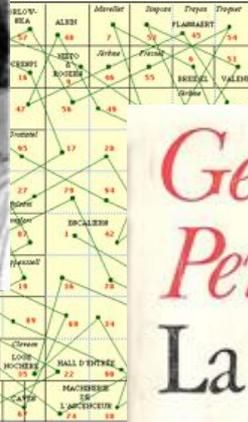
# The use of constraints in arts



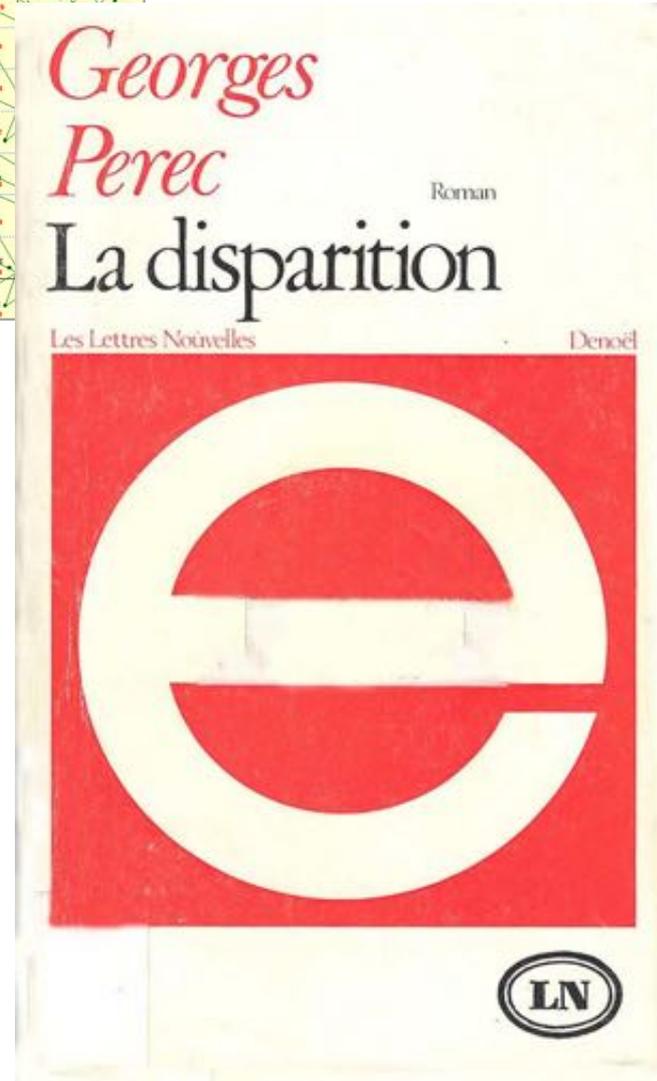
*Cent mille milliards de poèmes, 1961*



**Georges Perec**



**OuLiPo (Ouvroir de Littérature Potentielle)**



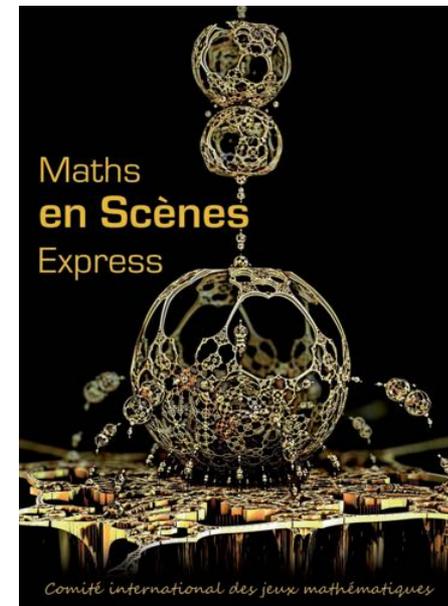
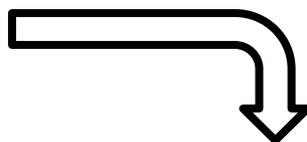
**Raymond Queneau**



**Italo Calvino**

*Il castello dei destini incrociati, 1969*

# From the OuLiPo to the OuMuPo (ouvroir de musique potentielle)



<http://oumupo.org/>



Valentin Villenave



Mike Solomon



Jean-François  
Piette



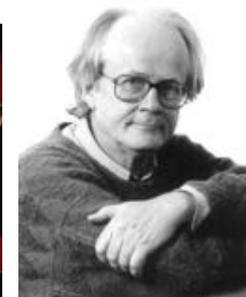
Martin  
Granger



Joseph Boisseau

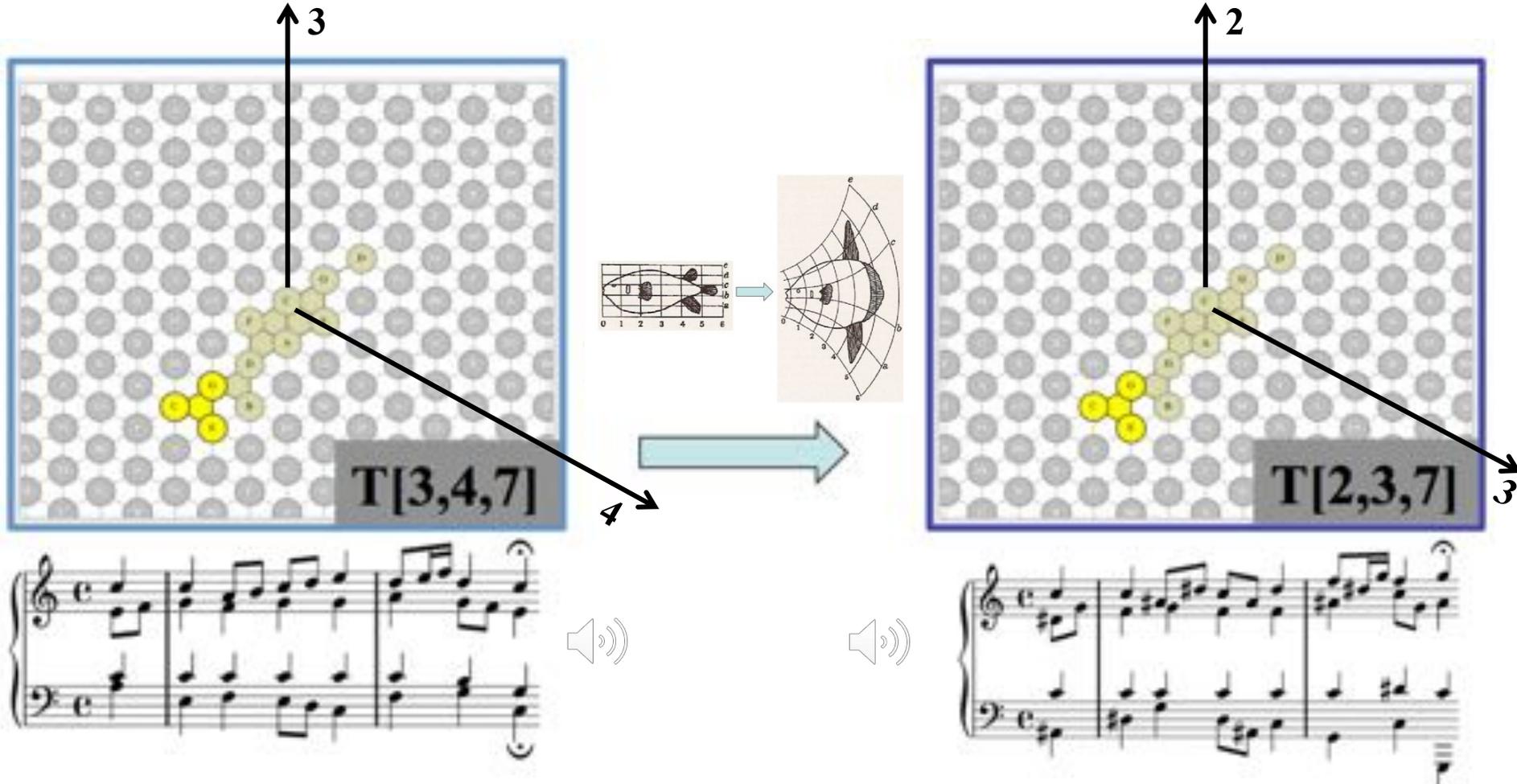


Moreno Andreatta



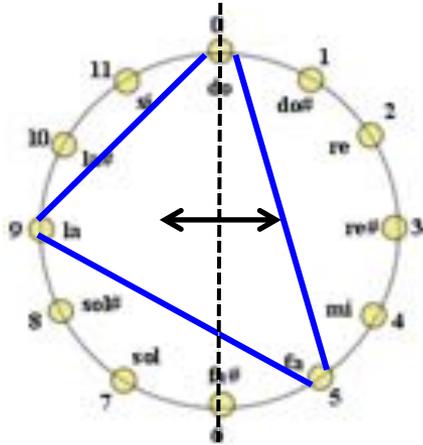
Tom Johnson

# The musical style...is the space!

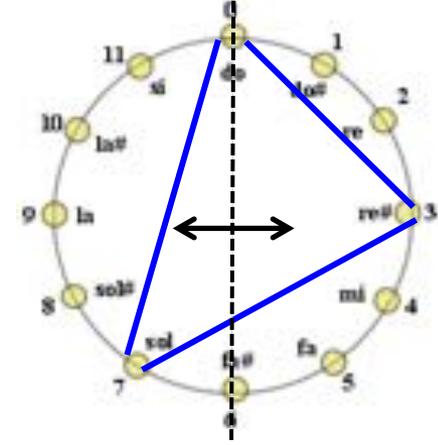


# Keeping the space...but changing the trajectory!

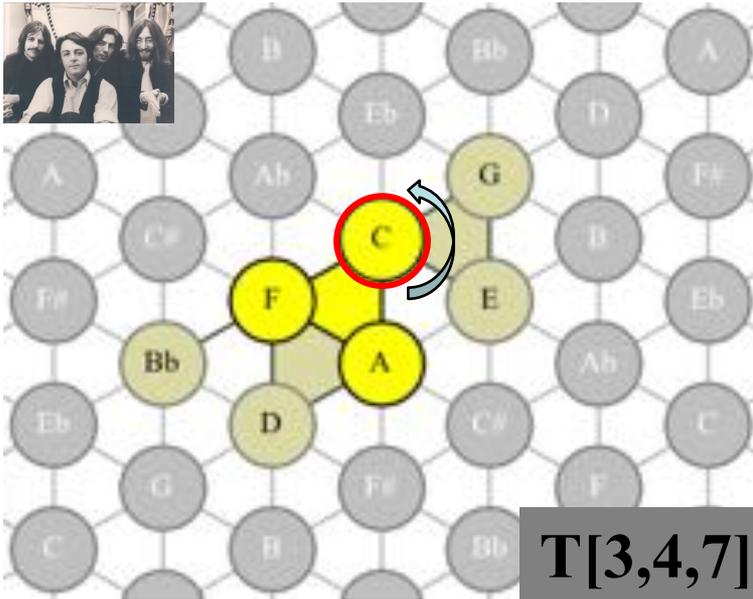
*F* major  
chord



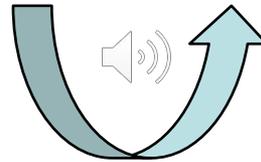
inversion



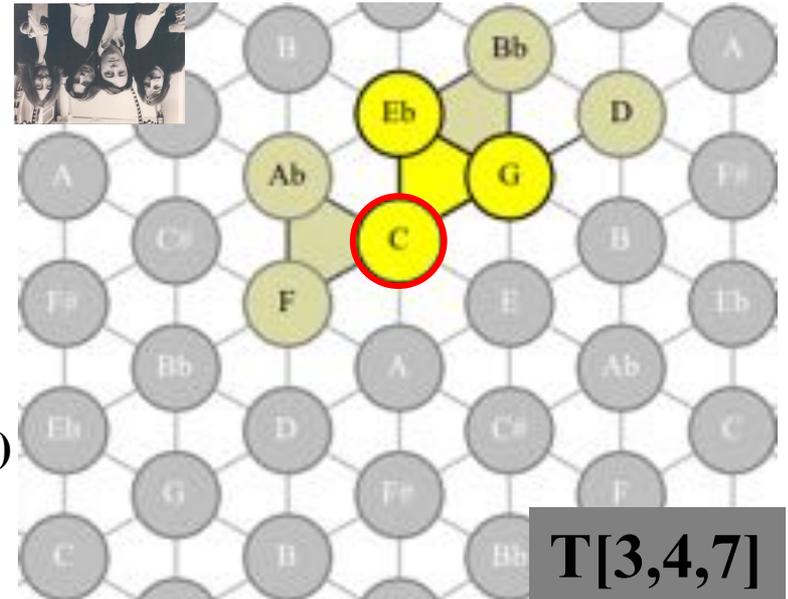
*C*  
minor  
chord



Beatles, Hey Jude  
(orig. version)



Rotation  
(autour du *do*)



Beatles, Hey Jude  
(transformed version)

# A final permutational song: one sentence, one note!

## Una volta soltanto una storia d'amore finisce (M. Andreatta)

Una volta una storia d'amore  
soltanto una storia

Una storia d'amore  
soltanto una storia d'amore

Una storia  
soltanto una storia

Una storia d'amore  
soltanto

Una volta soltanto  
una storia d'amore soltanto

Un amore soltanto una volta  
soltanto una storia d'amore soltanto

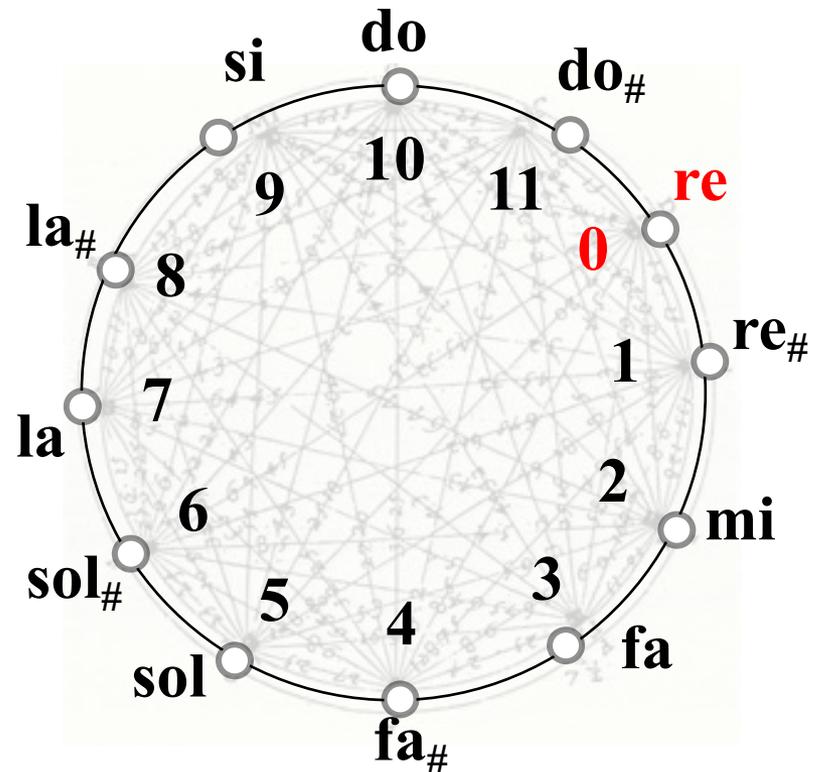
Una storia d'amore  
soltanto una volta una storia

Un amore  
una volta soltanto

Un amore finisce  
soltanto

Una volta una storia d'amore finisce  
Un amore soltanto

Una volta soltanto una storia d'amore finisce



# A final permutational song: one sentence, one note!

## Una volta soltanto una storia d'amore finisce (M. Andreatta)

Una volta una storia d'amore  
soltanto una storia

Una storia d'amore  
soltanto una storia d'amore

Una storia  
soltanto una storia

Una storia d'amore  
soltanto

Una volta soltanto  
una storia d'amore soltanto

Un amore soltanto una volta  
soltanto una storia d'amore soltanto

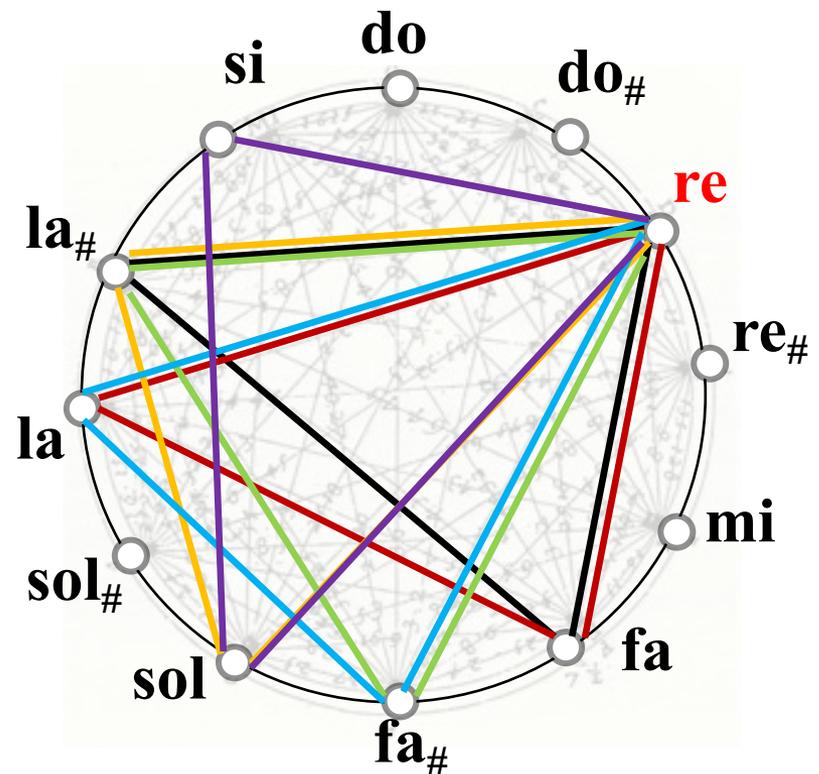
Una storia d'amore  
soltanto una volta una storia

Un amore  
una volta soltanto

Un amore finisce  
soltanto

Una volta una storia d'amore finisce  
Un amore soltanto

Una volta soltanto una storia d'amore finisce



**Thank you for your attention!**

