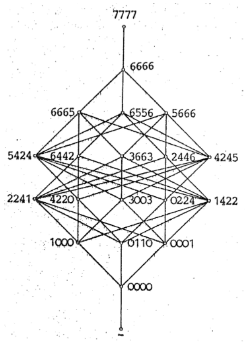
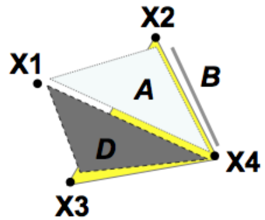


Séminaire *mamuphi*

ENS, 2 février 2013



Analyse formelle des concepts, Q-analyse et programmation spatiale : quelques aspects philosophiques du nœud mathématique/musique/informatique

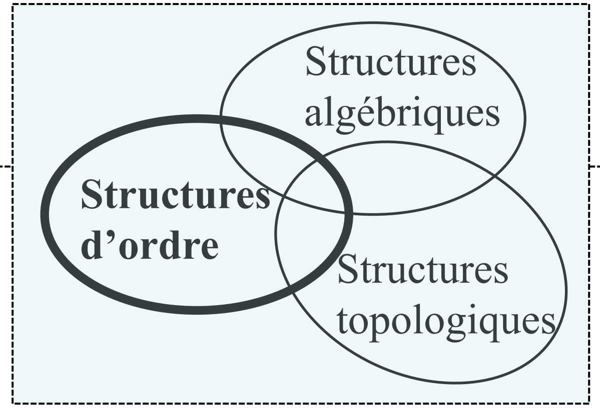
Moreno Andreatta & Jean-Louis Giavitto
Equipe Représentations Musicales
IRCAM/CNRS/UPMC
http://www.ircam.fr/repmus.html

ircam Centre Pompidou

cnrs

UPMC SORBONNE UNIVERSITÉS

L'Analyse Formelle des Concepts : rappel historique



CAMS
Centre d'analyse et de mathématique sociales

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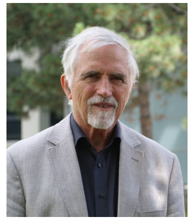
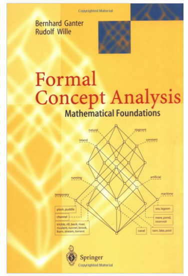
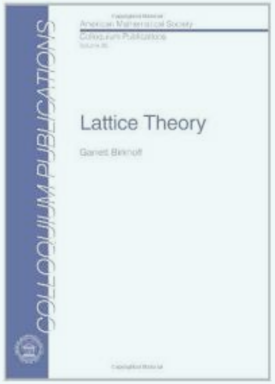
- M. Barbut, « Note sur l'algèbre des techniques d'analyse hiérarchique », in B. Matalon (éd.), L'analyse hiérarchique, Paris, Gauthier-Villars, 1965.
- M. Barbut, B. Monjardet, *Ordre et Classification. Algèbre et Combinatoire*, en deux tomes, 1970
- M. Barbut, L. Frey, « Techniques ordinales en analyse des données », Tome I, *Algèbre et Combinatoire des Méthodes Mathématiques en Sciences de l'Homme*, Paris, Hachette, 1971.
- B. Leclerc, B. Monjardet, « Structures d'ordres et sciences sociales », *Mathématiques et sciences humaines,* 193, 2011, 77-97





- R. Wille, « Mathematische Sprache in der Musiktheorie », in B. Fuchssteiner, U. Kulisch, D. Laugwitz, R. Liedl (Hrsg.): Jahrbuch Überblicke Mathematik. B.I.-Wissenschaftsverlag, Mannheim, 1980, p. 167-184.
- R. Wille, « Restructuring Lattice Theory: An approach based on Hierarchies of Concepts », I. Rival (ed.), *Ordered Sets,* 1982
- R. Wille, « Sur la fusion des contextes individuals », Mathématiques et sciences humaines, tome 85, 1984.
- B. Ganter & R. Wille, *Formal Concept Analysis: Mathematical Foundations*, Springer, Berlin, 1998

L'Analyse Formelle des Concepts : rappel historique



Garrett Birkhoff
(1911-1996)

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• M. Barbut, « Note sur l'algèbre des techniques d'analyse hiérarchique », in B. Matalon (éd.), L'analyse hiérarchique, Paris, Gauthier-Villars, 1965.

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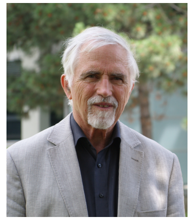

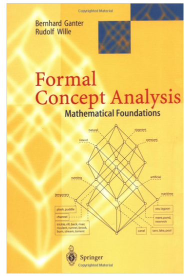

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• B. Ganter & R. Wille, *Formal Concept Analysis: Mathematical Foundations*, Springer, Berlin, 1998

AFC comme « restructuration » de la théorie des treillis

RESTRUCTURING LATTICE THEORY:

AN APPROACH BASED ON HIERARCHIES OF CONCEPTS

Rudolf Wille
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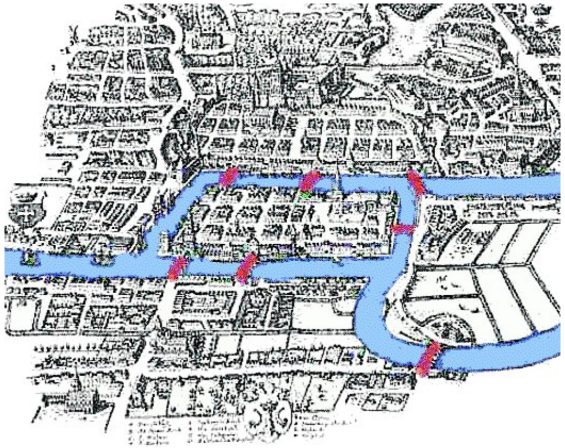
6100 Darmstadt
Federal Republic of Germany

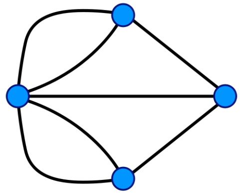
ABSTRACT

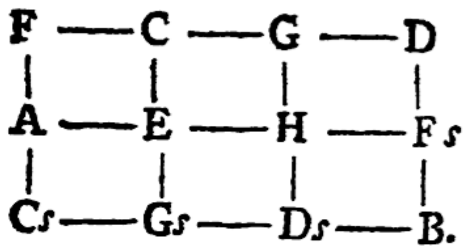
Lattice theory today reflects the general status of current mathematics: there is a rich production of theoretical concepts, results, and developments, many of which are reached by elaborate mental gymnastics; on the other hand, the connections of the theory to its surroundings are getting weaker and weaker, with the result that the theory and even many of its parts become more isolated. Restructuring lattice theory is an attempt to reinvigorate connections with our general culture by interpreting the theory as concretely as possible, and in this way to promote better communication between lattice theorists and potential users of lattice theory.

Dynamique « mathémusicale » dans la FCA

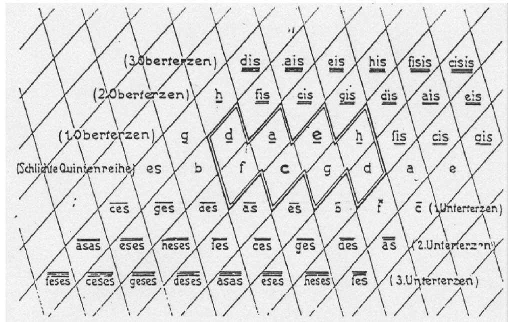


Königsberg et ses ponts

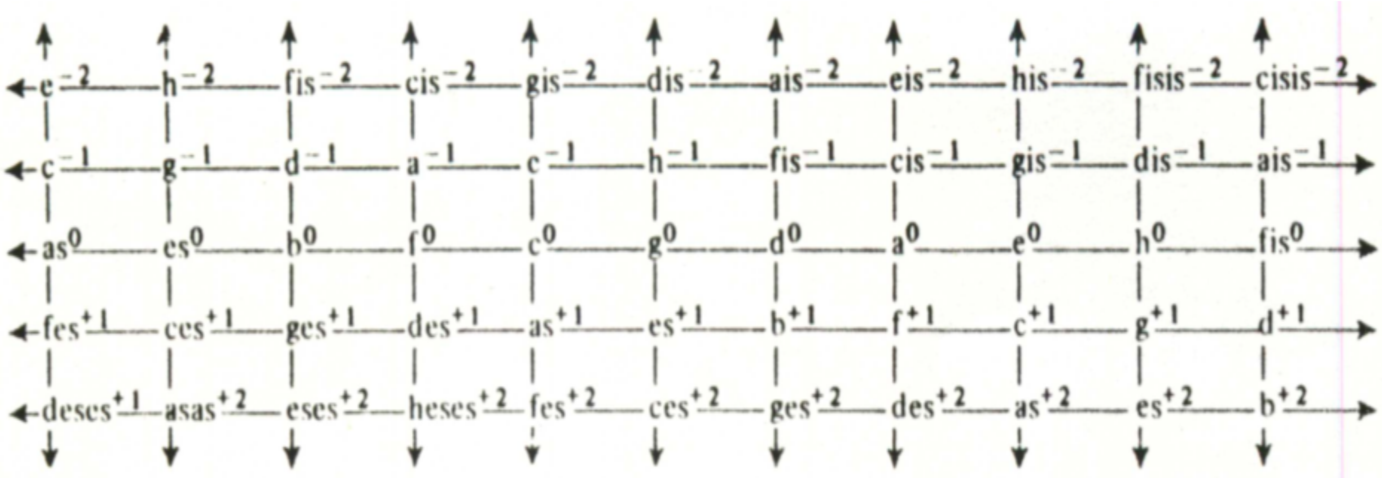




Euler, *Speculum musicum*, 1773

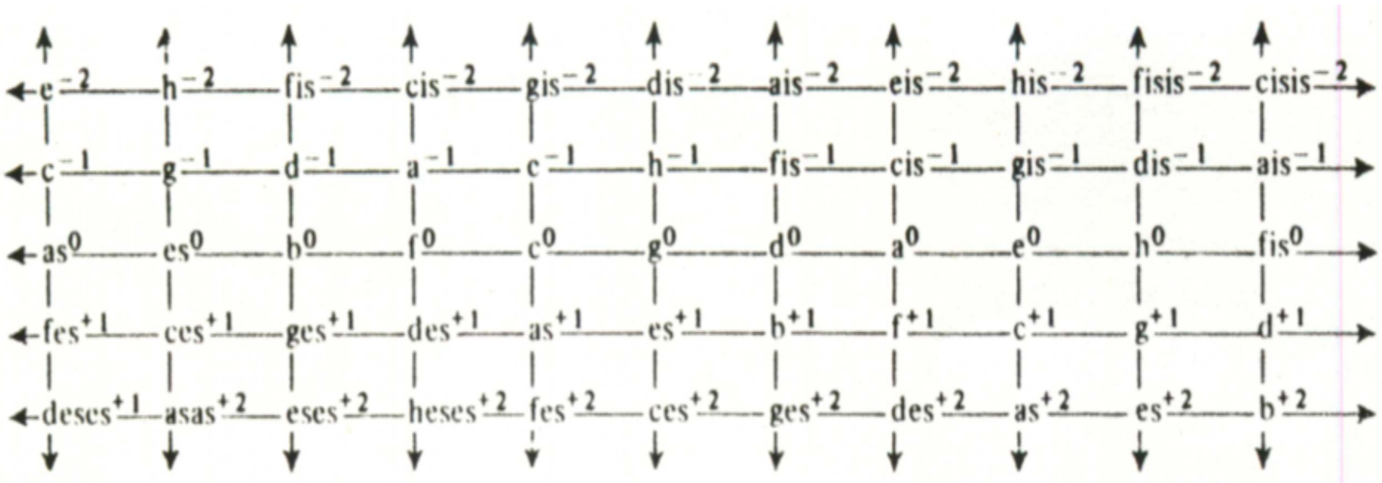


Le Tonnetz d'Hugo Riemann



R. Wille, « Mathematische Sprache in der Musiktheorie », in B. Fuchssteiner, U. Kulisch, D. Laugwitz, R. Liedl (Hrsg.): Jahrbuch Überblicke Mathematik. B.I.-Wissenschaftsverlag, Mannheim, 1980, p. 167-184.

FCA et musicologie computationnelle

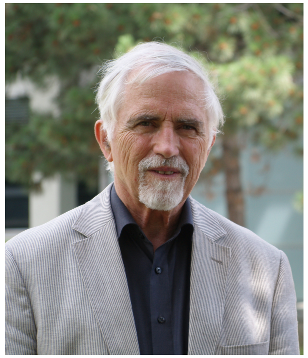


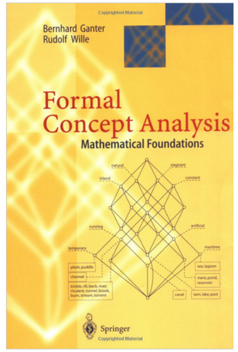
Mutabor **à Darmstadt au début des années 1980**

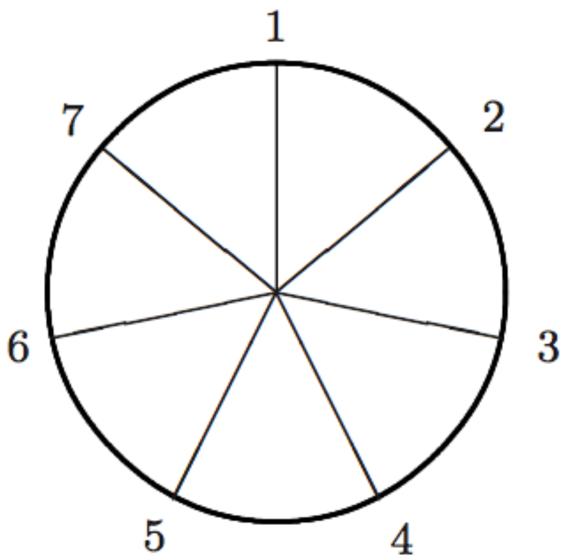
➔ `http://www.math.tu-dresden.de/~mutabor/`

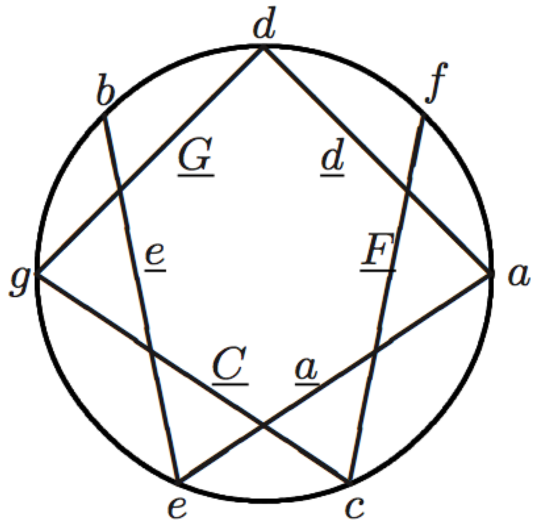
Structures d’ordre en musique

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	×			×		×
d		×			×	
e	×		×			×
f		×		×		
g	×		×		×	
a		×		×		×
b			×		×	





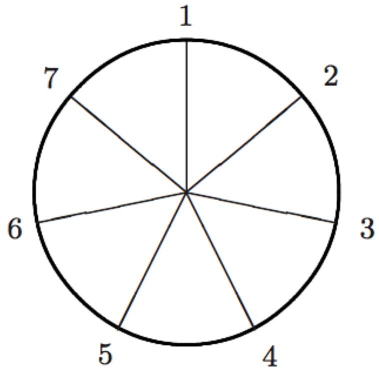


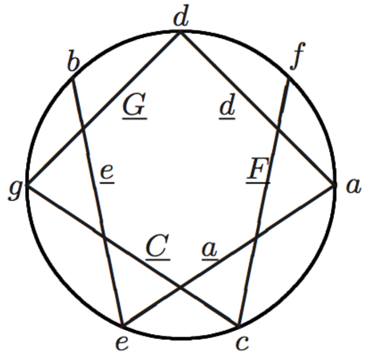


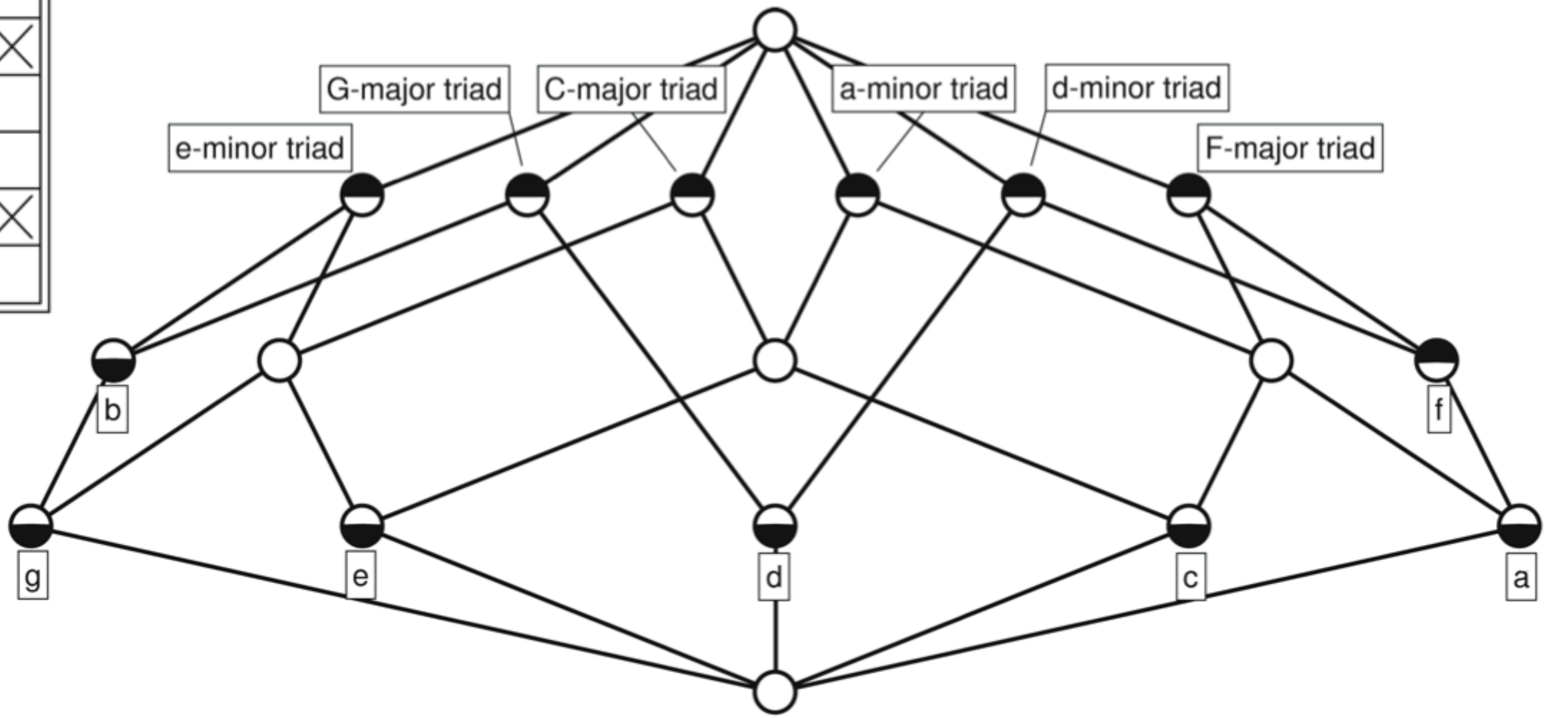
[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007, Springer, 2007]

Un treillis des concepts pour la gamme diatonique

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	×			×		×
d		×			×	
e	×		×			×
f		×		×		
g	×		×		×	
a		×		×		×
b			×		×	







[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007, Springer, 2007]

Des relations binaires au treillis des concepts

2. CONCEPT LATTICES

We start defining a *context* as a triple (G,M,I) where G and M are sets, and I is a binary relation between G and M; the elements of G and M are called *objects* [Gegenstände] and *attributes* [Merkmale], respectively. If gIm for $g \in G$ and $m \in M$ we say: the object g *has* the attribute m. For $A \subseteq G$ and $B \subseteq M$ we define

$$A' := \{m \in M \mid gIm \text{ for all } g \in A\},$$
$$B' := \{g \in G \mid gIm \text{ for all } m \in B\}.$$

The mappings given by $A \mapsto A'$ and $B \mapsto B'$ are said to form a *Galois connection* between the power sets of G and M, i.e. they fulfill the following basic properties (cf. Birkhoff [3; pp. 122-125]).

PROPOSITION. *For a context* (G,M,I):

(1)	$A_1 \subseteq A_2$	*implies*	$A_1' \supseteq A_2'$	*for* $A_1, A_2 \subseteq G$,
(1')	$B_1 \subseteq B_2$	*implies*	$B_1' \supseteq B_2'$	*for* $B_1, B_2 \subseteq M$,
(2)	$A \subseteq A''$	*and*	$A' = A'''$	*for* $A \subseteq G$,
(2')	$B \subseteq B''$	*and*	$B' = B'''$	*for* $B \subseteq M$.

[R. Wille, « Restructuring Lattice Theory: An approach based on Hierarchies of Concepts », I. Rival (ed.), *Ordered Sets,* 1982]

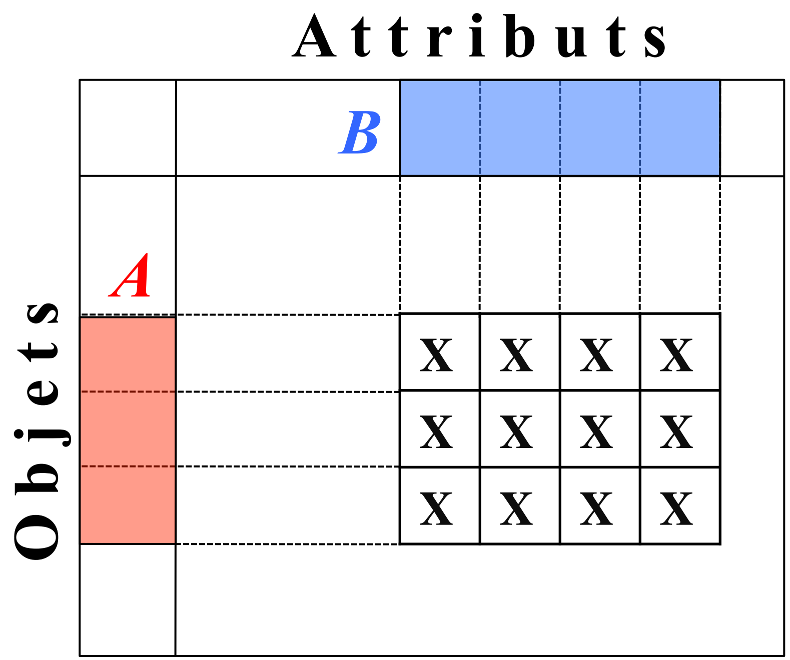
Des relations binaires au treillis des concepts

Now, a *concept* [Begriff] of the context (G,M,I) may be defined as a pair (A,B) where $A \subseteq G$, $B \subseteq M$, $A' = B$, and $B' = A$; A and B are called the *extent* and the *intent* of the concept (A,B), respectively. The hierarchy of concepts is captured by the definition

$$(A_1,B_1) \le (A_2,B_2) \;:\Leftrightarrow\; A_1 \subseteq A_2 \;(\Leftrightarrow B_1 \supseteq B_2)$$

for concepts (A_1,B_1) and (A_2,B_2) of (G,M,I); (A_1,B_1) is called the *subconcept* of (A_2,B_2), and (A_2,B_2) is called the *superconcept* of (A_1,B_1).

$$A' := \{m \in M \mid gIm \text{ for all } g \in A\}$$
$$B' := \{g \in G \mid gIm \text{ for all } m \in B\}$$



A = **extension** **du concept *(A,B)***

B = **intension** **du concept *(A,B)***

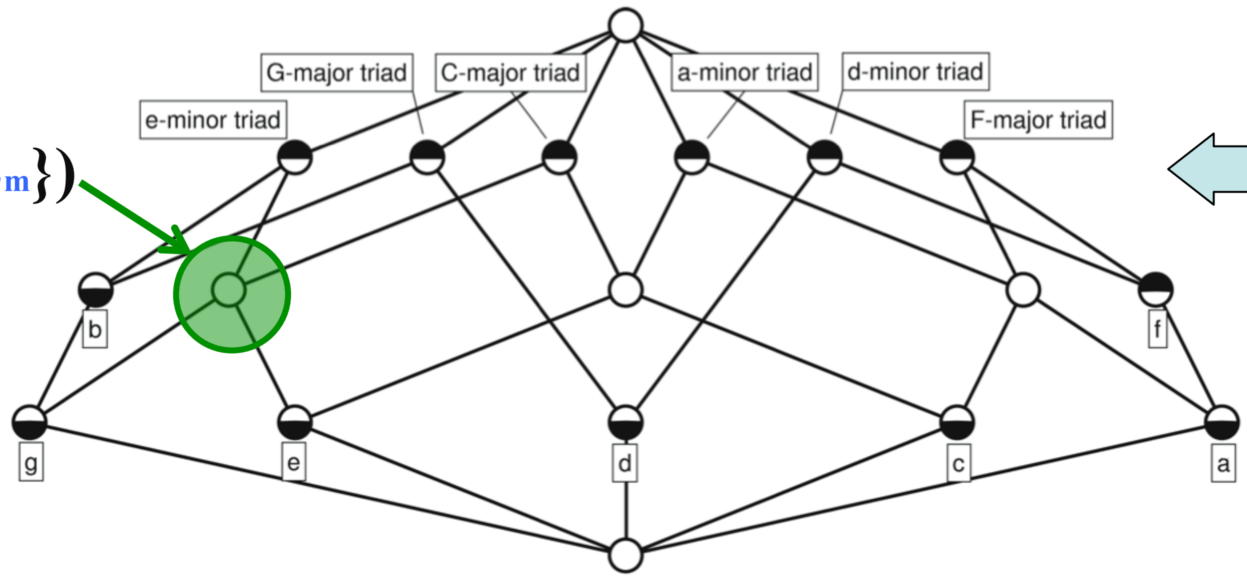
[R. Wille, « Restructuring Lattice Theory: An approach based on Hierarchies of Concepts », I. Rival (ed.), *Ordered Sets,* 1982]

Un treillis des concepts pour la gamme diatonique

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	X			X		X
d		X			X	
e	X		X			X
f		X		X		
g	X		X		X	
a		X		X		X
b			X		X	

	CM	Em				
mi	X	X				
sol	X	X				

({***mi, sol***},{**CM, Em**})



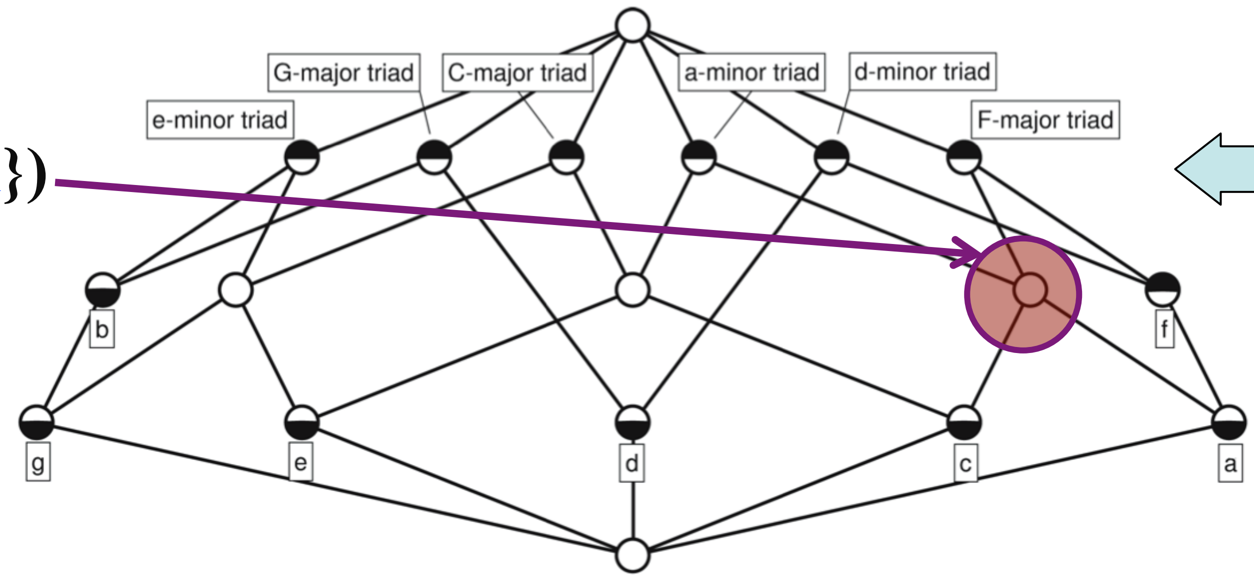
[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007, Springer, 2007]

Un treillis des concepts pour la gamme diatonique

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	X			X		X
d		X			X	
e	X		X			X
f		X		X		
g	X		X		X	
a		X		X		X
b			X		X	

	CM	Em	Am	FM		
mi	X	X				
sol	X	X				
la			X	X		
do			X	X		

({*la, do*},{FM, Am})



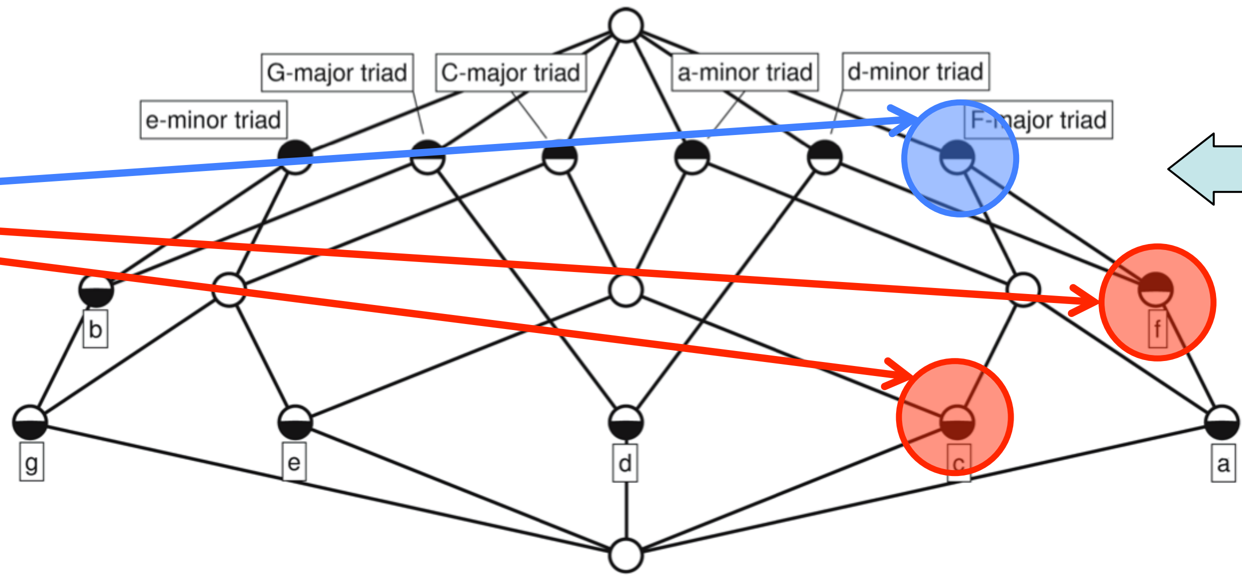
[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007, Springer, 2007]

Opérateur de clôture « ’’ »

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	X			X		X
d		X			X	
e	X		X			X
f		X		X		
g	X		X		X	
a		X		X		X
b			X		X	

	CM	Em	Am	FM		
mi	X	X				
sol	X	X				
la			X	X		
do			X	X		
fa				X		

{*do*, *fa*}’’={FM}’



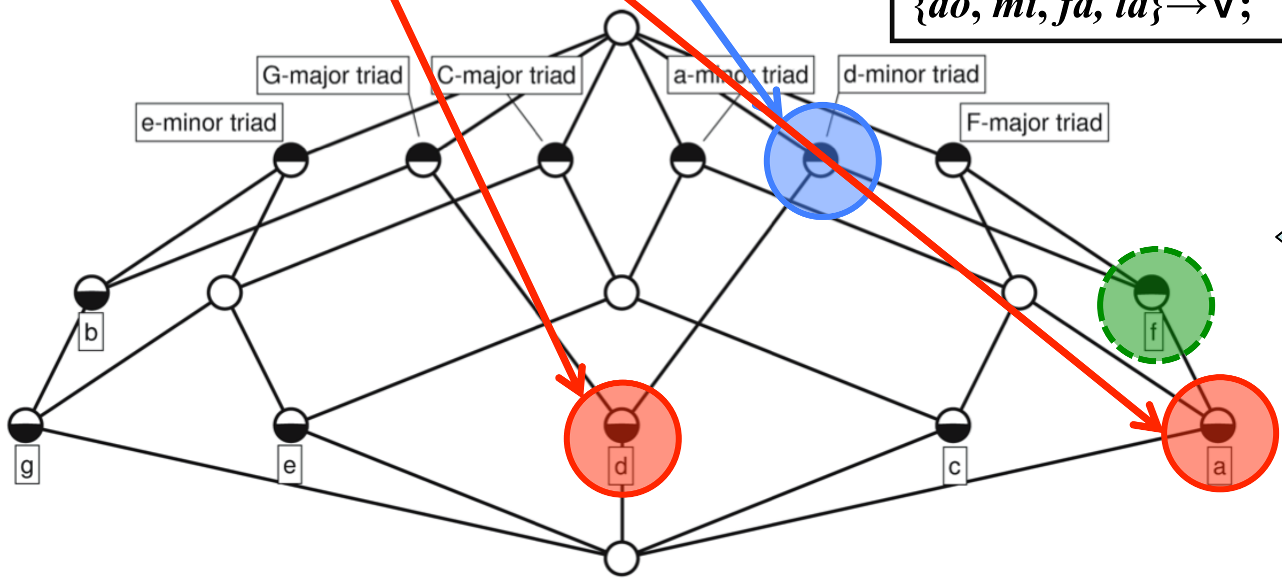
[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007, Springer, 2007]

Opérateur de clôture « ’’ » comme implication logique

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	X			X		X
d		X			X	
e	X		X			X
f		X		X		
g	X		X		X	
a		X		X		X
b			X		X	

{*ré*,*la*}→{*fa*};

{*ré*, *la*}’’={D_m}’



Base de Duquenne-Guiges

{*ré*, *la*}→{*fa*};
{*do*, *sol*}→{*mi*};
{*mi*, *la*}→{*do*};
{*ré*, *sol*}→{*si*};
{*fa*}→{*la*};
{*si*}→{*sol*};
{*sol*, *la*}→∀;
{*ré*, *mi*}→∀;
{*do*, *ré*}→∀;
{*do*, *mi*, *sol*, *si*}→∀;
{*do*, *mi*, *fa*, *la*}→∀;

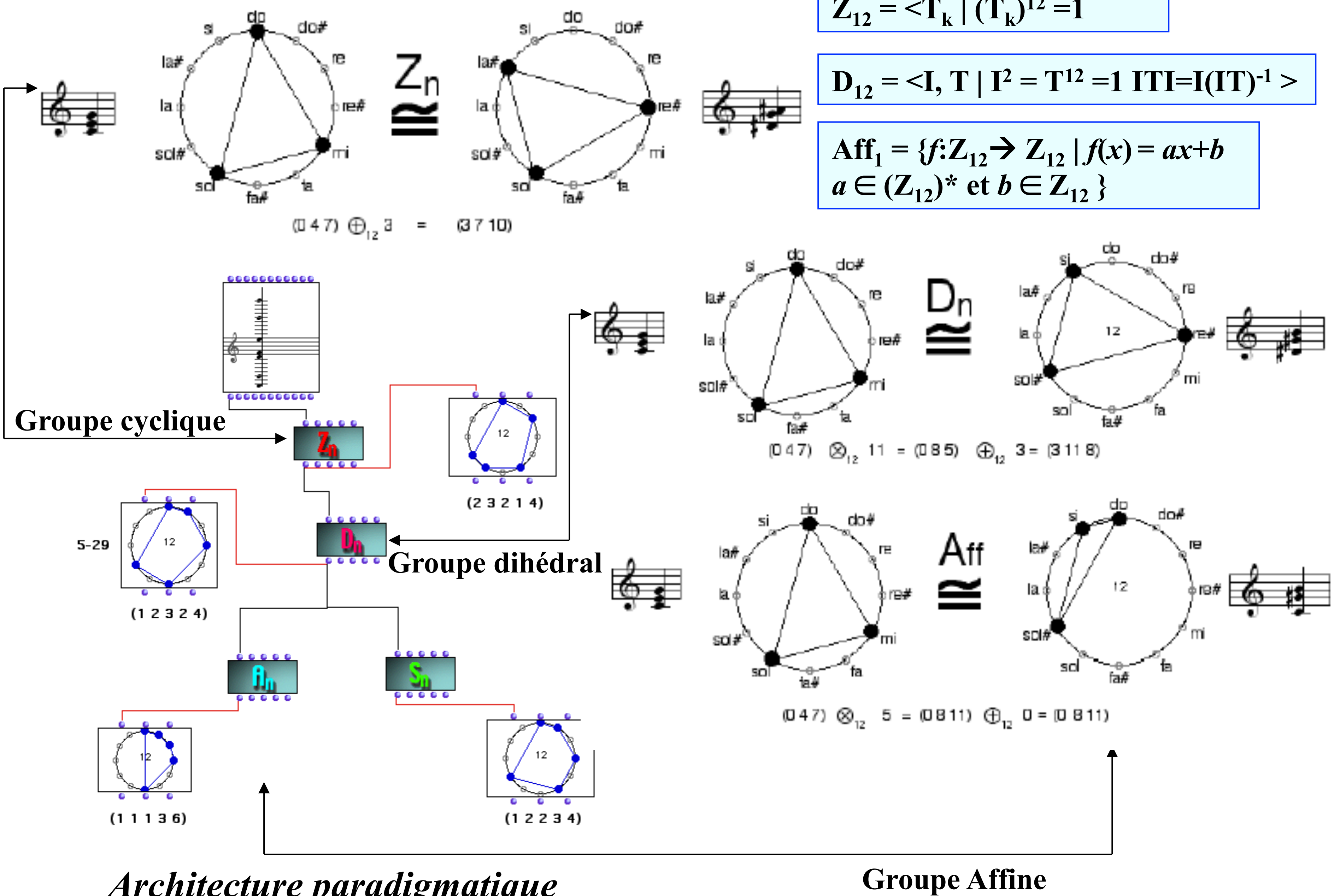
J-L. Guigues, V. Duquenne, « Familles minimales d'implications informatives résultant d'un tableau de données binaries », *Math. Sci. Humaines* 95, 1986.

Classification paradigmatique des structures musicales

$Z_{12} = <T_k \mid (T_k)^{12} = 1$

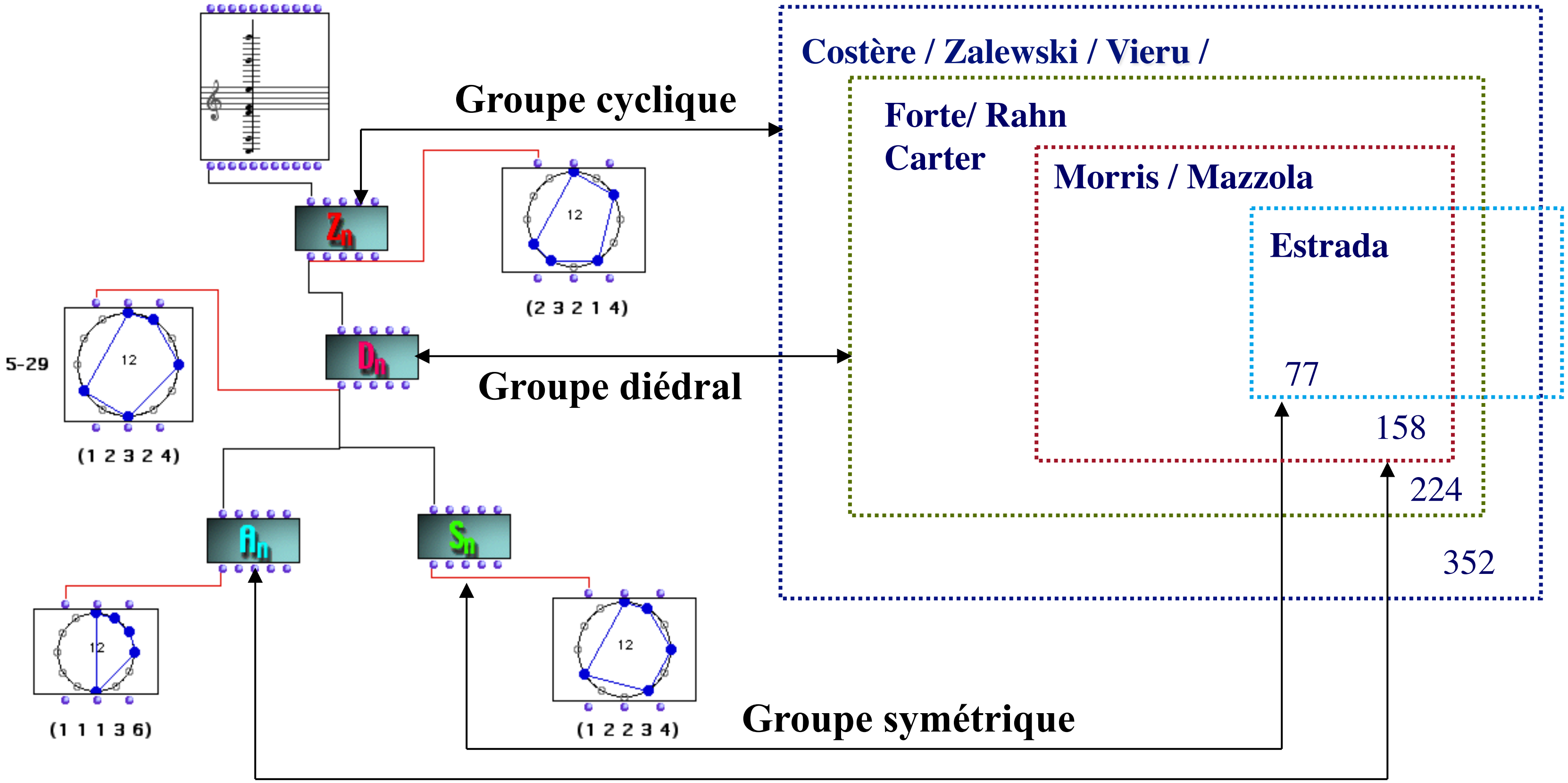
$D_{12} = <I, T \mid I^2 = T^{12} = 1 \;\; ITI = I(IT)^{-1} >$

$Aff_1 = \{f: Z_{12} \rightarrow Z_{12} \mid f(x) = ax+b$
$a \in (Z_{12})^*$ et $b \in Z_{12} \}$

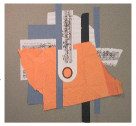


Architecture paradigmatique

Classification paradigmatique des structures musicales

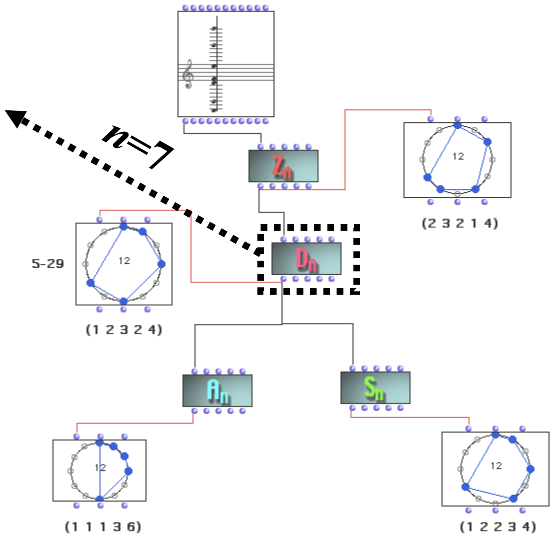


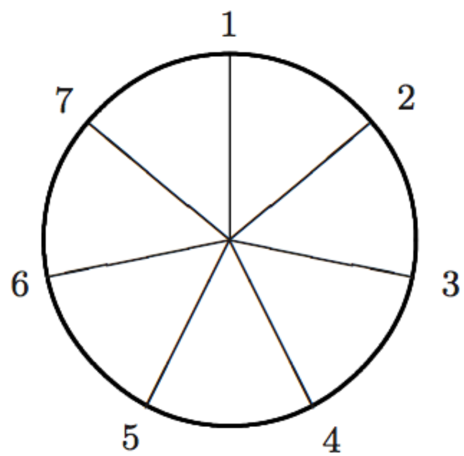
Architecture paradigmatique



AFC et classification paradigmatique

	{}	{1}	{1,2}	{1,3}	{1,4}	{1,2,3}	{1,3,7}	{1,2,5}	{1,5,7}	{1,3,5}	{1,3,5,7}	{1,2,3,5}	{1,3,4,7}	{1,3,4,5}	{1,2,3,7}	{1,3,4,5,7}	{1,2,3,5,7}	{1,2,3,4,5}	{1,2,3,4,5,7}	{1,2,3,4,5,6,7}
rest { }	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
unison {1}		×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
second {1,2}			×			×	×	×	×		×	×	×	×	×	×	×	×	×	×
third {1,3}				×		×	×		×	×	×	×	×	×	×	×	×	×	×	×
fourth {1,4}					×		×	×	×	×	×	×	×	×	×	×	×	×	×	×
fifth-seventh-free ninth chord {1,2,3}						×						×		×	×	×	×	×	×	×
fifth-free seventh chord {1,3,7}							×				×	×	×		×	×	×	×	×	×
third-seventh-free ninth chord {1,2,5}								×				×	×	×		×	×	×	×	×
third-free seventh chord {1,5,7}									×		×		×	×	×	×	×	×	×	×
triad {1,3,5}										×	×	×		×		×	×	×	×	×
compact seventh chord {1,3,5,7}											×					×	×		×	×
seventh-free ninth chord {1,2,3,5}												×				×	×	×	×	×
fifth-ninth-free eleventh chord {1,3,4,7}													×			×		×	×	×
seventh-ninth-free eleventh chord {1,3,4,5}														×		×	×	×	×	×
fifth-free ninth chord {1,2,3,7}															×		×	×	×	×
ninth-free eleventh chord {1,3,4,5,7}																×			×	×
compact ninth chord {1,2,3,5,7}																	×		×	×
seventh-free eleventh chord {1,2,3,4,5}																		×	×	×
compact eleventh chord {1,2,3,4,5,7}																			×	×
compact thirteenth chord {1,2,3,4,5,6,7}																				×





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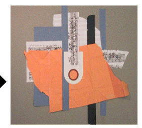
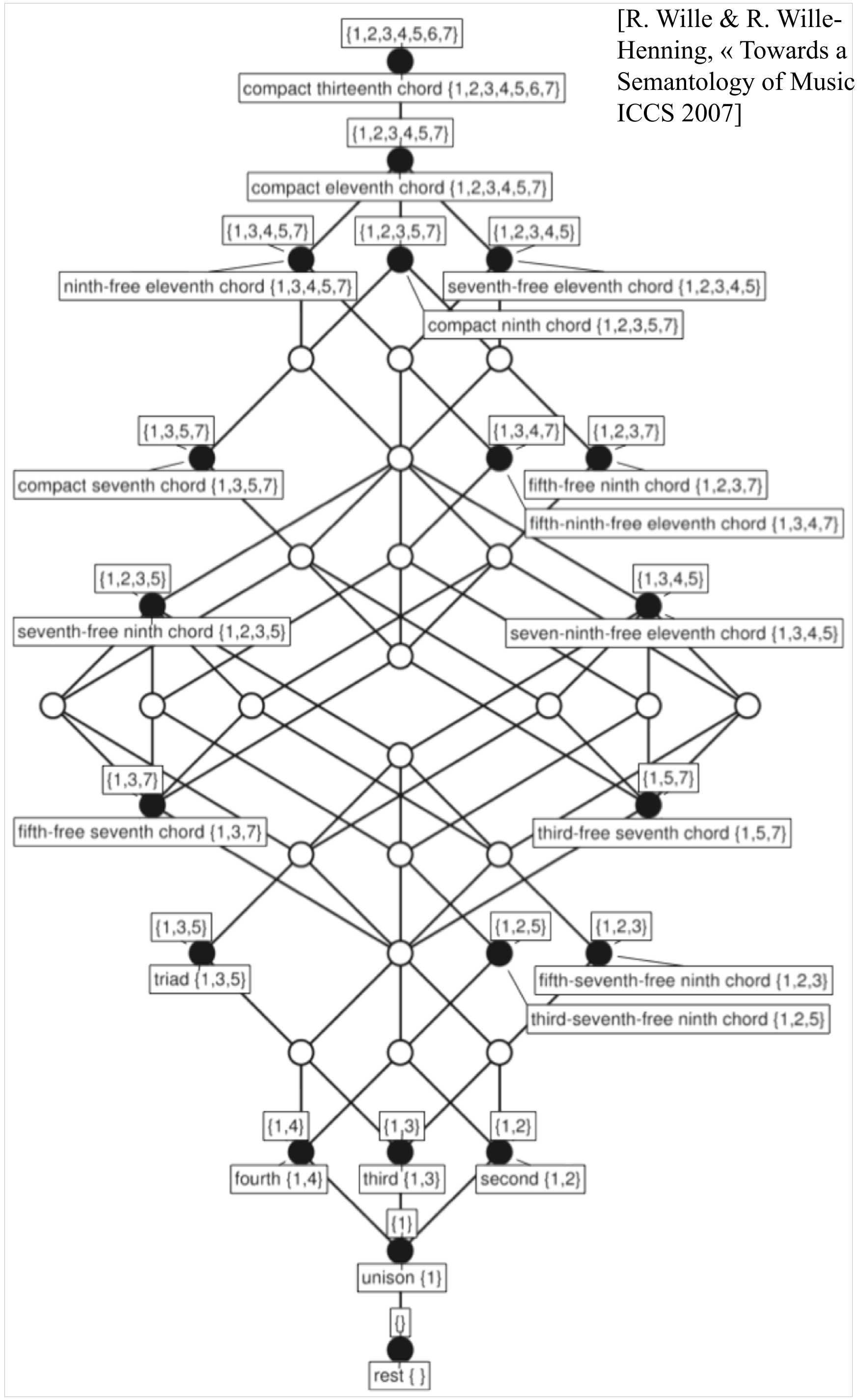


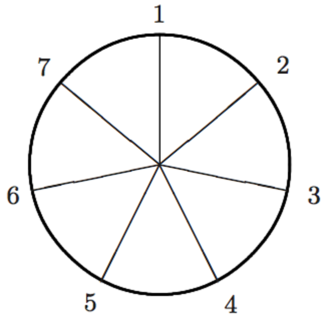
Le treillis de concepts formels

{}	{1}	{1,2}	{1,3}	{1,4}	{1,2,3}	{1,3,7}	{1,2,5}	{1,5,7}	{1,3,5}	{1,3,5,7}	{1,2,3,5}	{1,3,4,7}	{1,3,4,5}	{1,2,3,7}	{1,3,4,5,7}	{1,2,3,5,7}	{1,2,3,4,5}	{1,2,3,4,5,7}	{1,2,3,4,5,6,7}
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
		X			X	X	X	X		X	X	X	X	X	X	X	X	X	X
			X		X	X		X	X	X	X	X	X	X	X	X	X	X	X
				X		X	X	X	X	X	X	X	X	X	X	X	X	X	X
					X						X		X	X	X	X	X	X	X
						X				X	X	X		X	X	X	X	X	X
							X				X	X	X		X	X	X	X	X
								X		X		X	X	X	X	X	X	X	X
									X	X	X		X		X	X	X	X	X
										X					X	X		X	X
											X				X	X	X	X	X
												X			X		X	X	X
													X		X	X	X	X	X
														X		X	X	X	X
															X			X	X
																X		X	X
																	X	X	X
																		X	X
																			X



[R. Wille & R. Wille-Henning, « Towards a Semantology of Music », ICCS 2007]

Base d’implications de Duquenne-Guigues

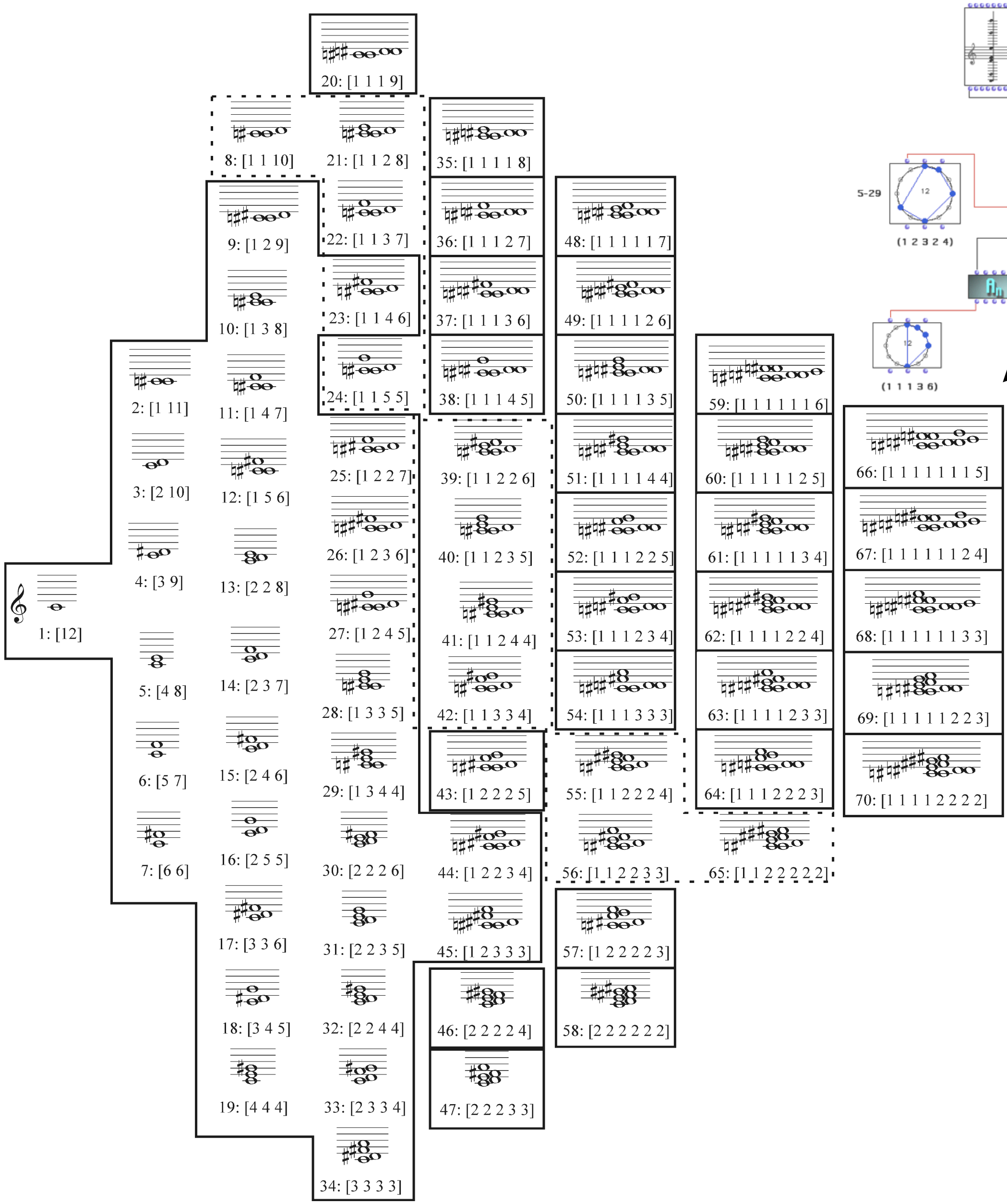


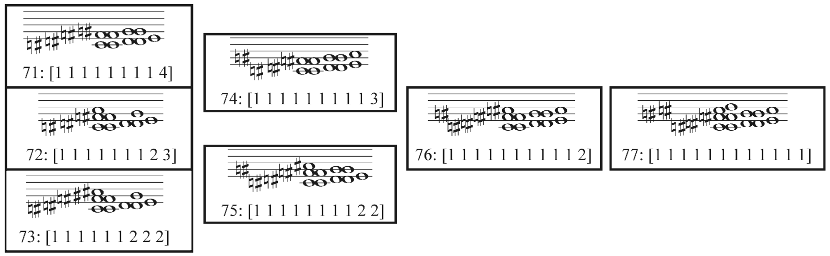
{}	{1}	{1,2}	{1,3}	{1,4}	{1,2,3}	{1,3,7}	{1,2,5}	{1,5,7}	{1,3,5}	{1,3,5,7}	{1,2,3,5}	{1,3,4,7}	{1,3,4,5}	{1,2,3,7}	{1,3,4,5,7}	{1,2,3,5,7}	{1,2,3,4,5}	{1,2,3,4,5,7}	{1,2,3,4,5,6,7}
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
		X			X	X	X	X		X	X	X	X	X	X	X	X	X	X
			X		X	X		X	X	X	X	X	X	X	X	X	X	X	X
				X		X	X	X	X	X	X	X	X	X	X	X	X	X	X
					X						X		X	X	X	X	X	X	X
						X				X	X	X		X	X	X	X	X	X
							X				X	X	X		X	X	X	X	X
								X		X		X	X	X	X	X	X	X	X
									X	X	X		X		X	X	X	X	X
										X					X	X		X	X
											X				X	X	X	X	X
												X			X		X	X	X
													X		X	X	X	X	X
														X		X	X	X	X
															X			X	X
																X		X	X
																	X	X	X
																		X	X
																			X

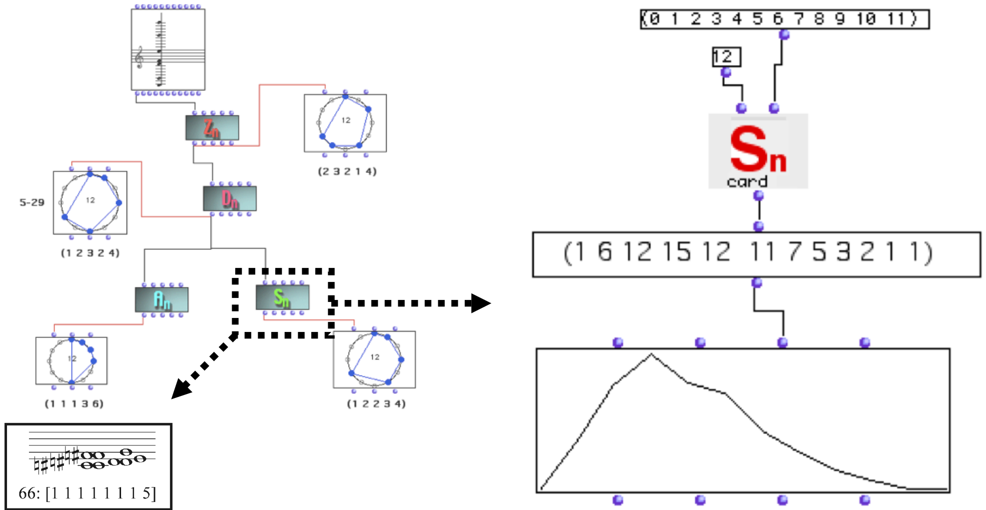
→{ };
{1,2}→{1};
{1,3}→{1};
{1,4}→{1};
{1,2,3}→{1,2},{1,3};
{1,3,7}→{1,2},{1,3},{1,4};
{1,2,5}→{1,2},{1,4};
{1,5,7}→{1,2},{1,3},{1,4};
{1,3,5}→{1,3},{1,4};
{1,2,3},{1,2,5}→{1,3,5};
{1,2,3},{1,3,5}→{1,2,5};
{1,2,5},{1,3,5}→{1,2,3};
{1,3,5,7}→{1,3,5},{1,3,7},{1,5,7};
{1,2,3},{1,2,5},{1,3,5},{1,3,7}→{1,2,3,5,};
{1,2,3,5}→{1,2,3},{1,2,5},{1,3,5},{1,3,7};
{1,3,4,7}→{1,2,5},{1,3,7},{1,5,7};
{1,2,3},{1,2,5},{1,3,5},{1,5,7}→{1,3,4,5};
{1,3,4,5}→{1,2,3},{1,2,5},{1,3,5},{1,5,7};
{1,2,3,7}→{1,2,3},{1,3,7},{1,5,7};
{1,3,4,5},{1,3,5,7}→{1,3,4,5,7};
{1,3,4,5,7}→{1,3,4,5},{1,3,5,7};
{1,2,3,7},{1,3,5,7}→{1,2,3,5,7};
{1,2,3,5,7}→{1,2,3,7},{1,3,5,7};
{1,2,3,7},{1,3,4,7}→{1,2,3,4,5,7};
{1,2,3,4,5,7}→{1,2,3,7},{1,3,4,7};
{1,2,3,4,5},{1,2,3,5,7},{1,3,4,5,7}→
{1,2,3,4,5,7};
{1,2,3,4,5,7}→
{1,2,3,4,5},{1,2,3,5,7},{1,3,4,5,7};
{1,2,3,4,5,6,7}→{1,2,3,4,5,7}.

J-L. Guigues, V. Duquenne, « Familles minimales d'implications informatives résultant d'un tableau de données binaries », *Math. Sci. Humaines* 95, 1986.

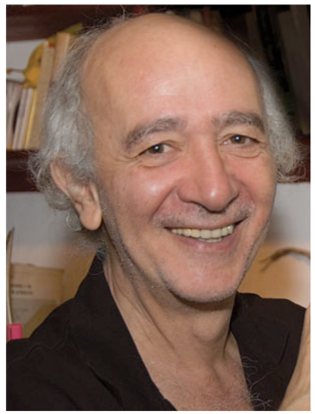
Le permutoèdre des « partitions » de Julio Estrada





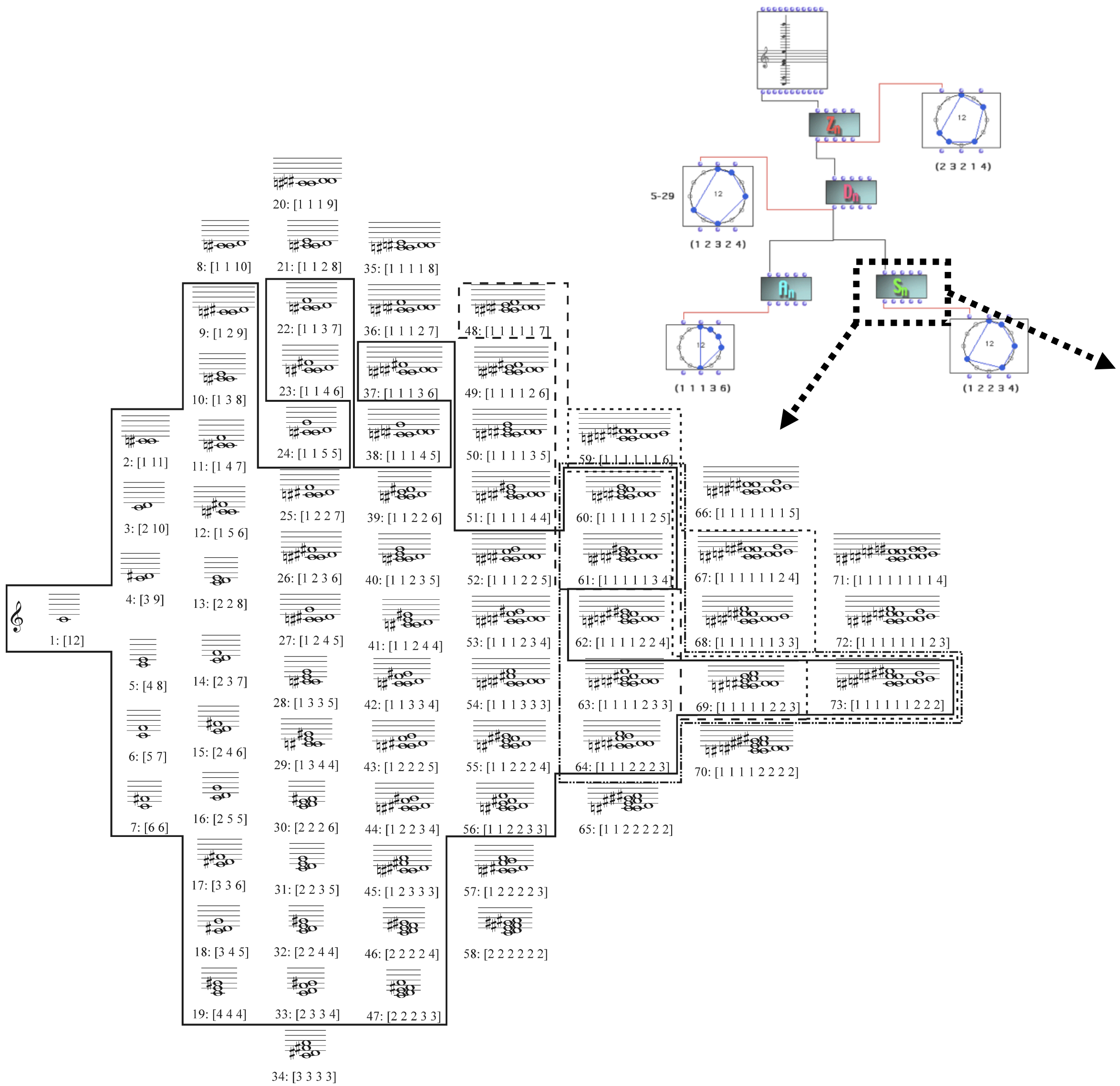


L. Van Beethoven,
Quatuor n° 17



Julio Estrada

Permutoèdre comme catalogue d'accords



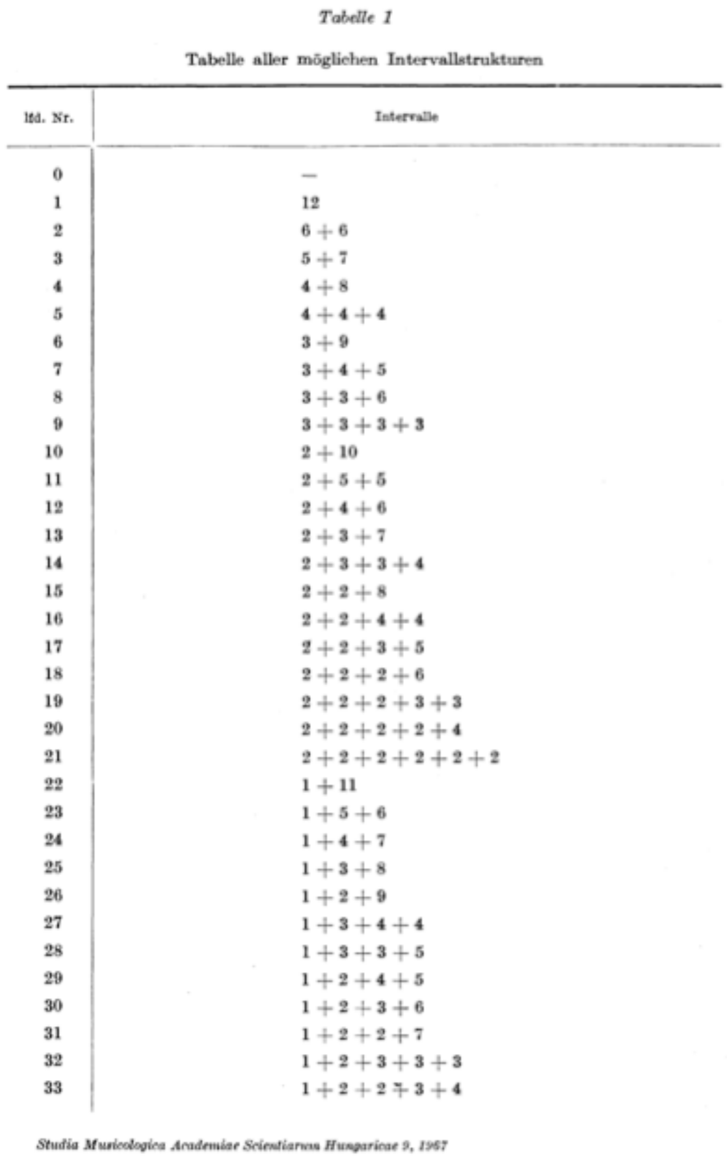


Tabelle 1

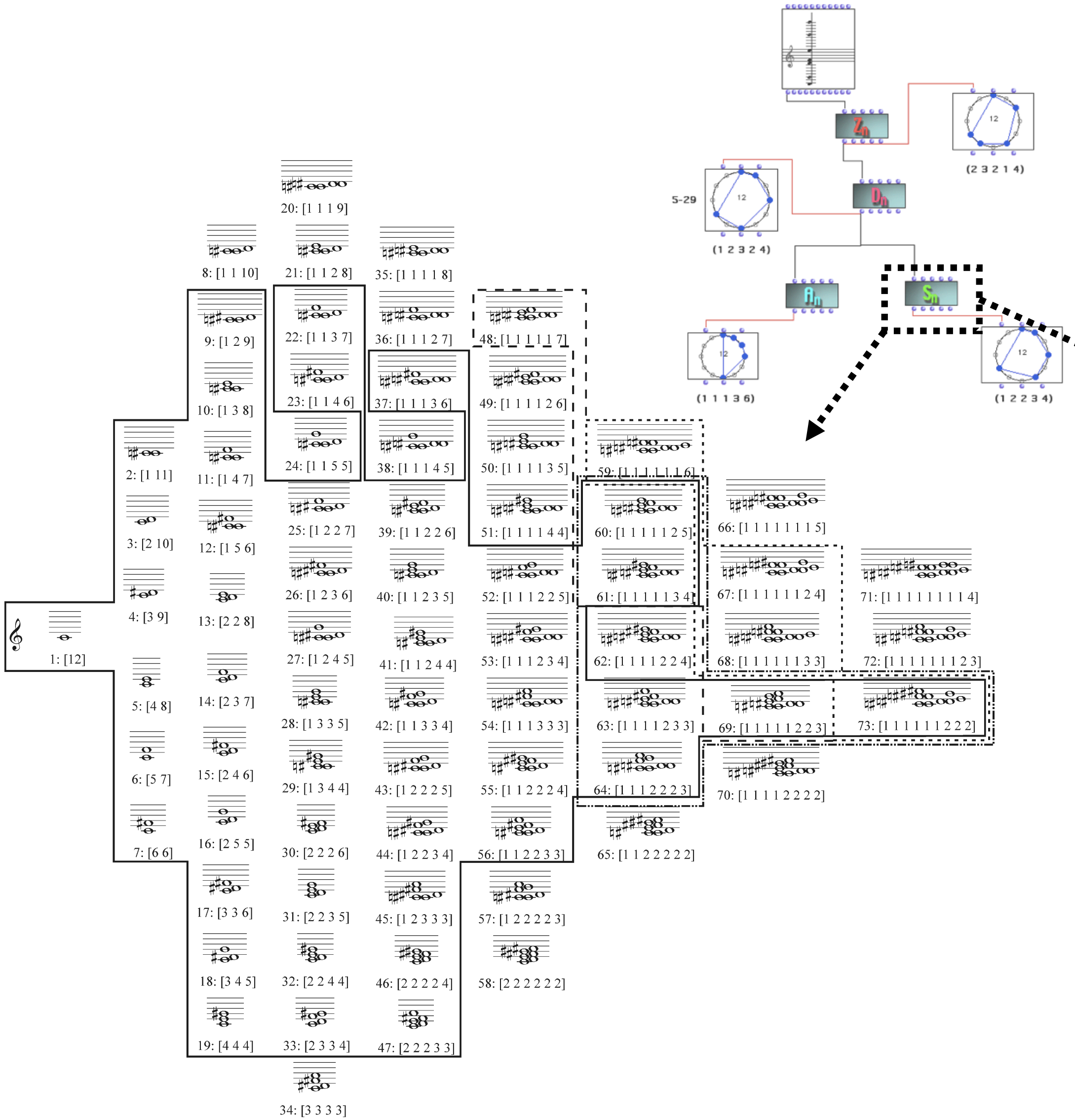
Tabelle aller möglichen Intervallstrukturen

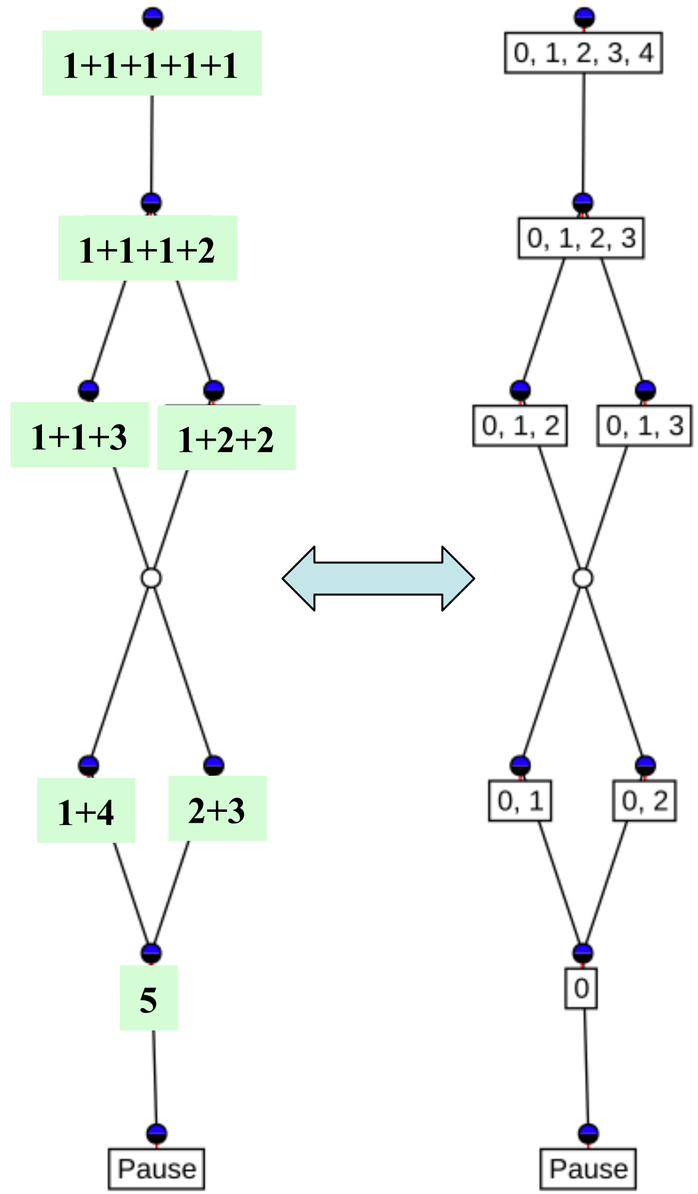
lfd. Nr.	Intervalle
0	—
1	12
2	6 + 6
3	5 + 7
4	4 + 8
5	4 + 4 + 4
6	3 + 9
7	3 + 4 + 5
8	3 + 3 + 6
9	3 + 3 + 3 + 3
10	2 + 10
11	2 + 5 + 5
12	2 + 4 + 6
13	2 + 3 + 7
14	2 + 3 + 3 + 4
15	2 + 2 + 8
16	2 + 2 + 4 + 4
17	2 + 2 + 3 + 5
18	2 + 2 + 2 + 6
19	2 + 2 + 2 + 3 + 3
20	2 + 2 + 2 + 2 + 4
21	2 + 2 + 2 + 2 + 2 + 2
22	1 + 11
23	1 + 5 + 6
24	1 + 4 + 7
25	1 + 3 + 8
26	1 + 2 + 9
27	1 + 3 + 4 + 4
28	1 + 3 + 3 + 5
29	1 + 2 + 4 + 5
30	1 + 2 + 3 + 6
31	1 + 2 + 2 + 7
32	1 + 2 + 3 + 3 + 3
33	1 + 2 + 2 + 3 + 4

Studia Musicologica Academiae Scientiarum Hungaricae 9, 1967

W. Reckziegel, "Musikanalyse und Wissenschaft." Studia Musicologica 9(1-2), 1967, p. 163-186

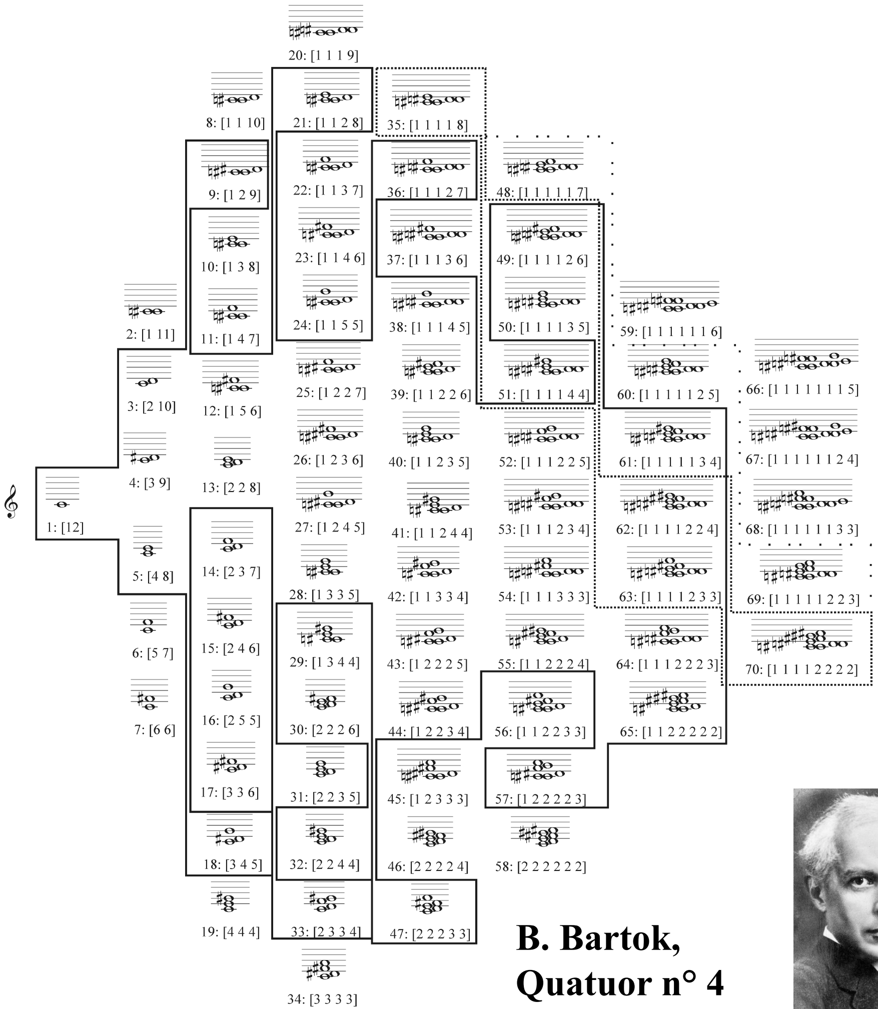
Permutoèdre comme treillis de concepts formels

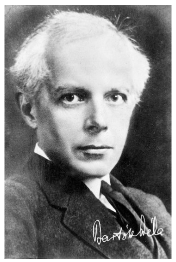




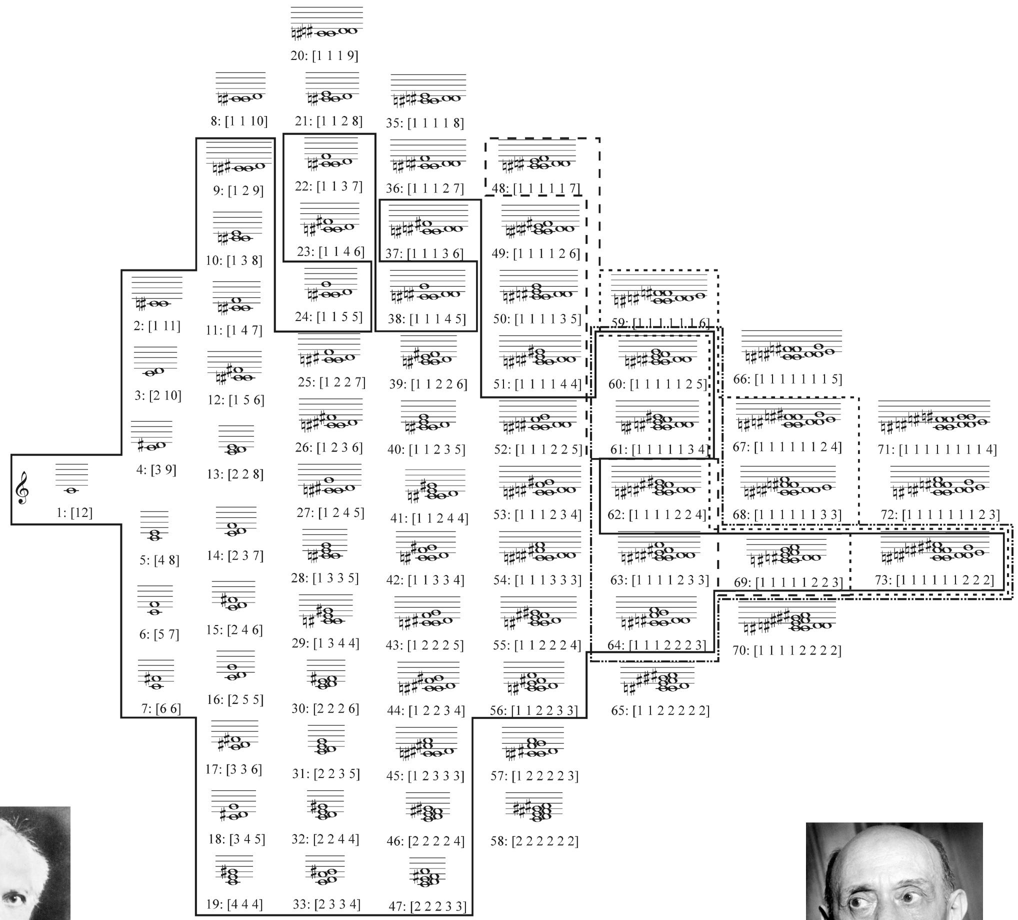
T. Schlemmer, M. Andreatta, « Using Formal Concept Analysis to represent Chroma Systems », submitted to MCM 2013

Analyse comme déplacement dans un espace conceptuel

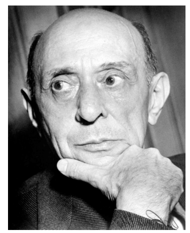




B. Bartok,
Quatuor n° 4
(3e mouvement)



A. Schoenberg,
***Six pièces* op. 19**

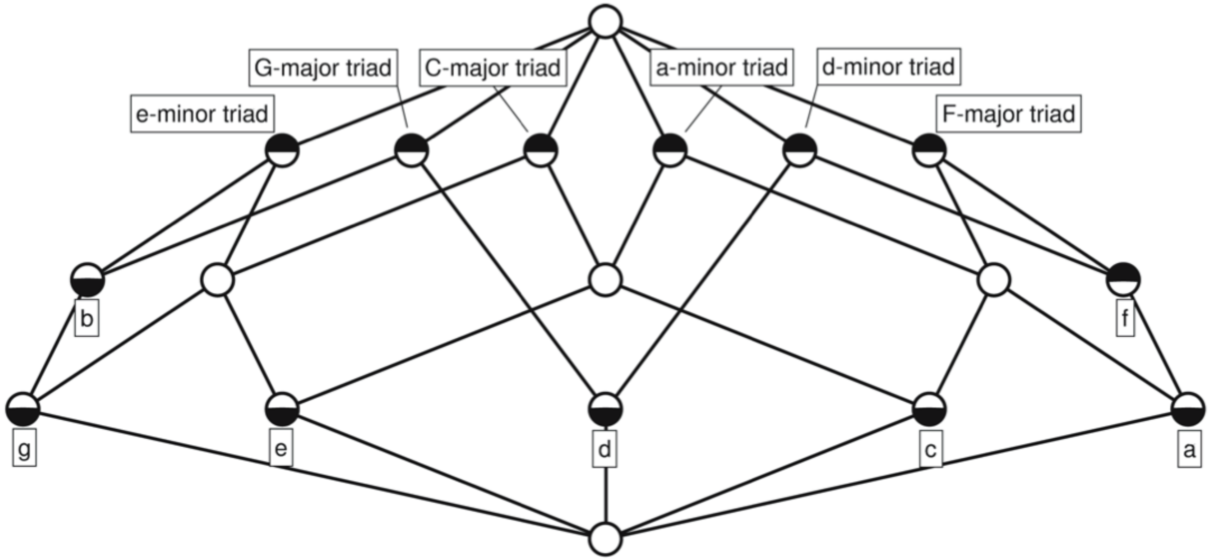


Julio Estrada, Modèles mathématiques en composition et analyse musicales, intervention au Master ATIAM / Cursus de Composition, Ircam, 21 Novembre 2012:

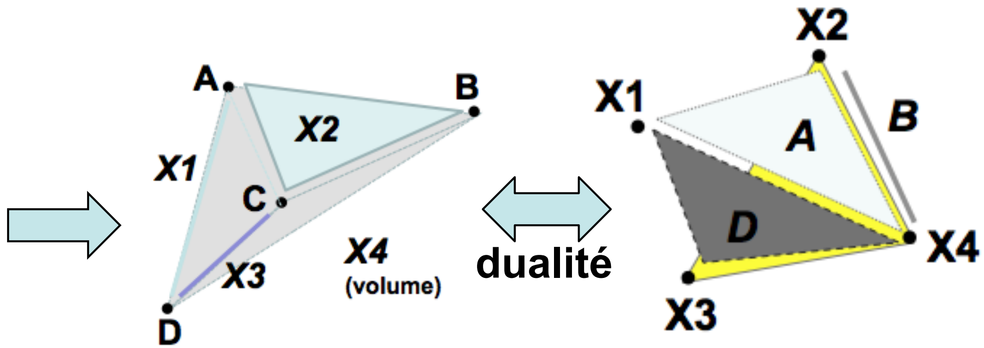
➔ Vidéo bientôt disponible en ligne sur **`http://www.atiam.ircam.fr/`**

Analyse formelle des concepts et topologie : la Q-analyse

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad
c	X			X		X
d		X			X	
e	X		X			X
f		X		X		
g	X		X		X	
a		X		X		X
b			X		X	



	A	B	C	D
X1	1	0	0	1
X2	1	1	1	0
X3	0	0	1	1
X4	1	1	1	1



Un treillis topologique de la gamme diatonique

	C-major triad	d-minor triad	e-minor triad	F-major triad	G-major triad	a-minor triad	accord diminué
c	X			X		X	
d		X			X		x
e	X		X			X	
f		X		X			x
g	X		X		X		
a		X		X		X	
b			X		X		x