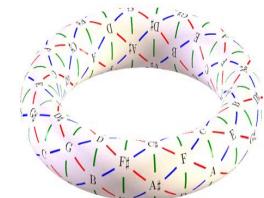
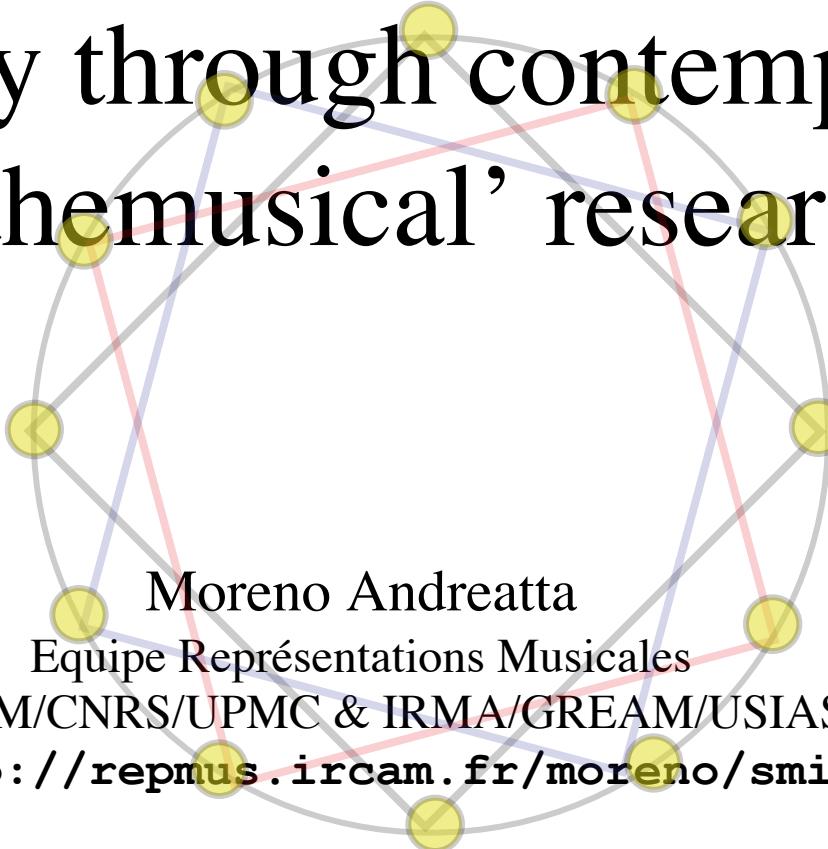
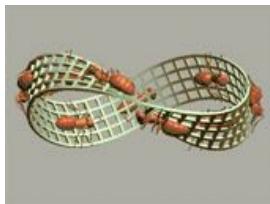
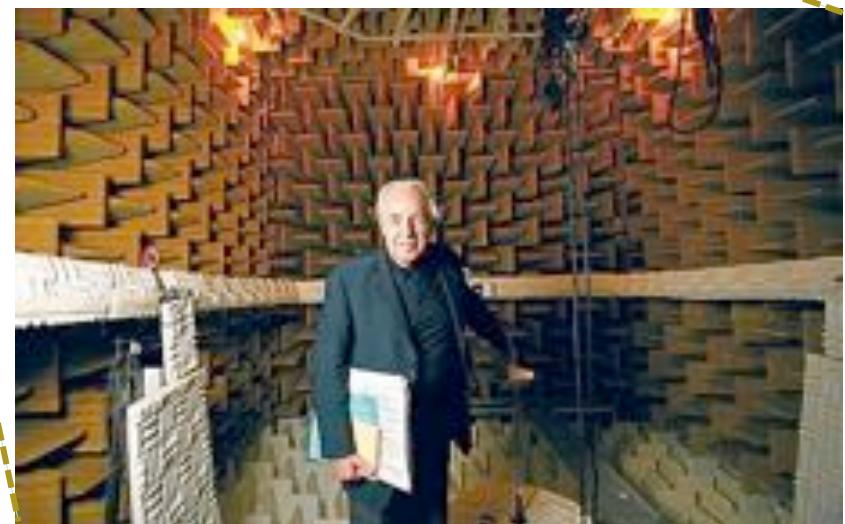
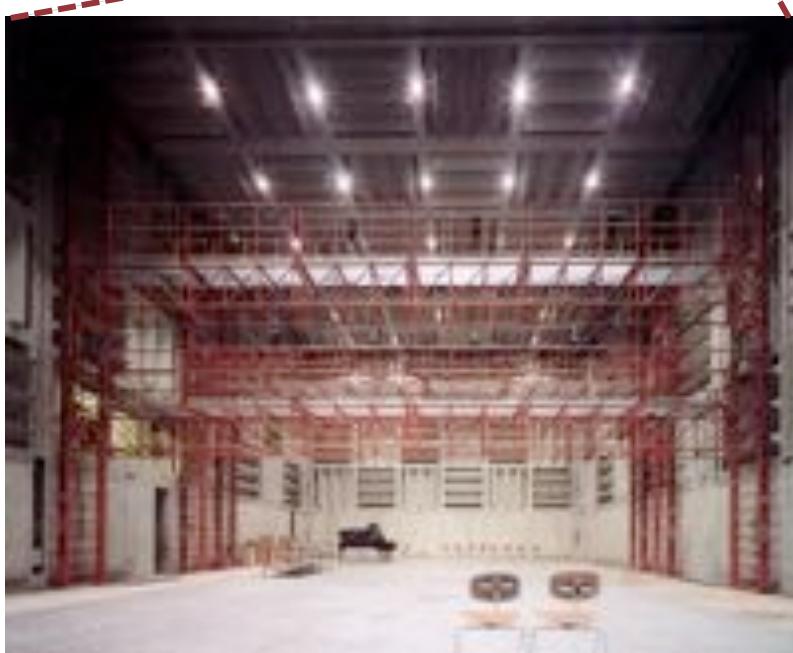
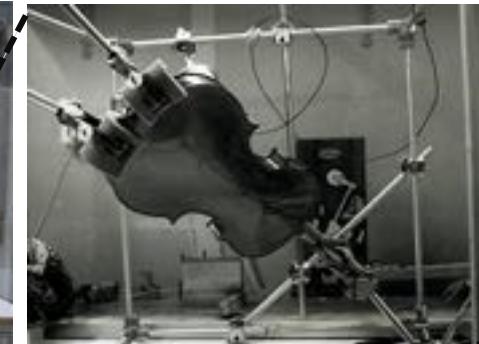
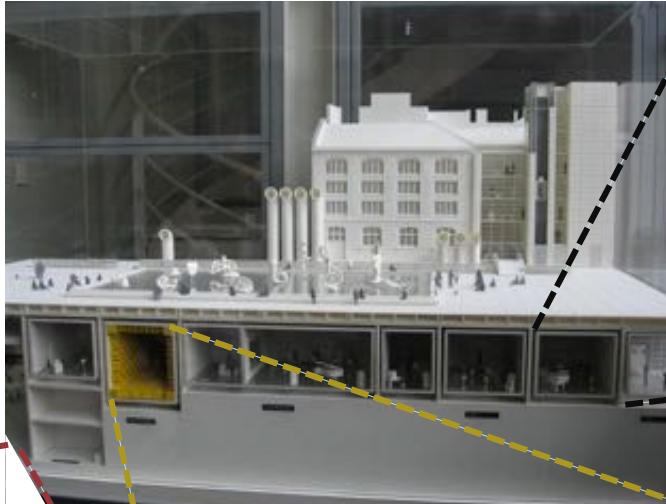


# Strips, clocks and donughts: a journey through contemporary 'mathemusical' research



# The musical and scientific research at IRCAM...

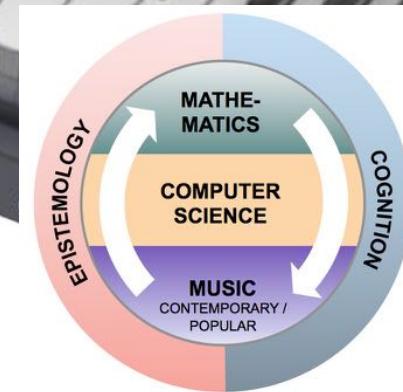
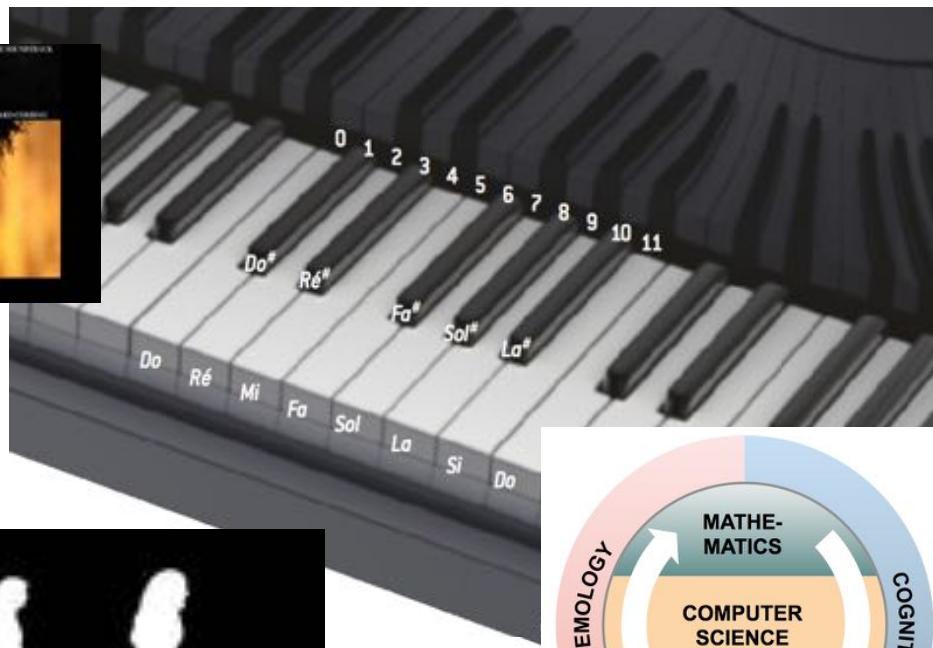
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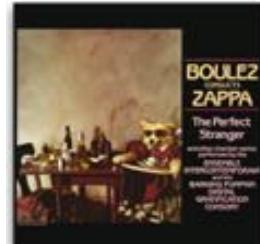
# ... at the interface between art and popular music



MusiqueLab 2



OMAX (computer-aided impro)



# The Society for Mathematics and Computation in Music

## Conferences:

- 2007 Technische Universität (Berlin, Allemagne)
- 2009 Yale University (New Haven, USA)
- 2011 IRCAM (Paris, France)
- 2013 McGill University (Canada)
- 2015 Queen Mary University (Londres)
- 2017 UNAM (Mexico City)



## Official Journal and MC code (00A65: Mathematics and Music)

- *Journal of Mathematics and Music*, Taylor & Francis  
(Editors: Th. Fiore, C. Callender | Associate eds.: E. Amiot, J. Yust)



## Books Series:

- *Computational Music Sciences Series*, Springer (G. Mazzola & M. Andreatta eds. – 12 books published (since 2009))
- *Collection Musique/Sciences*, Ircam-Delatour France (J.-M. Bardez & M. Andreatta dir. – 16 books published (since 2006))

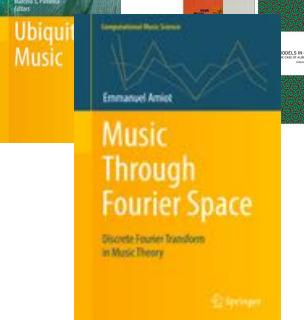


## European Training Network on Computational and Mathematical Music Analysis and Generation (“InForMusic”)

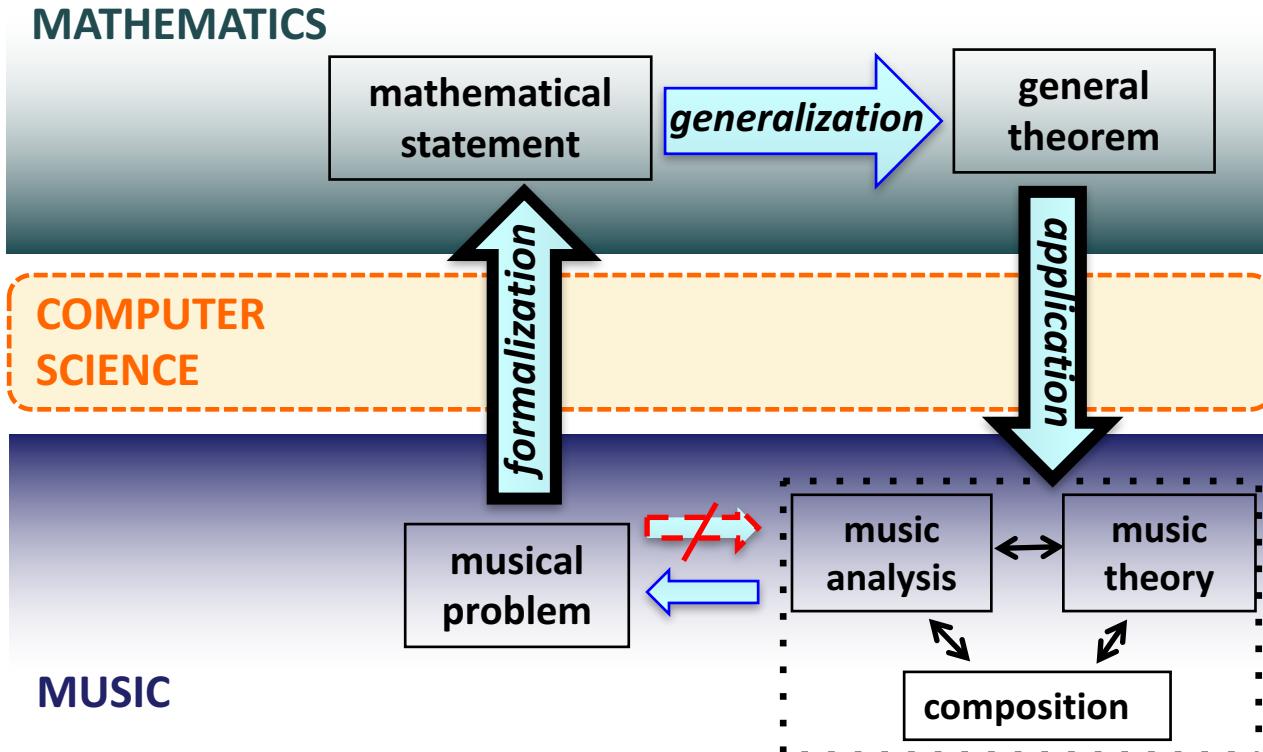
(Aalborg Universitet, City, University of London, Universiteit Utrecht, Aristotelio Panepistimio Thessalonikis, IRISA (UMR 6074), IRMA (UMR 7501), STMS (UMR 9912), Vrije Universiteit Brussel + Sony Europe Ltd, Chordify, Melodrive, Steinberg)



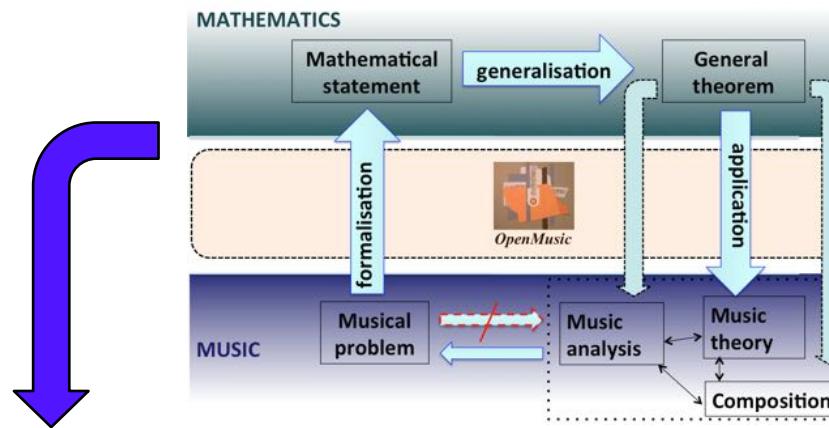
Under reviewing...



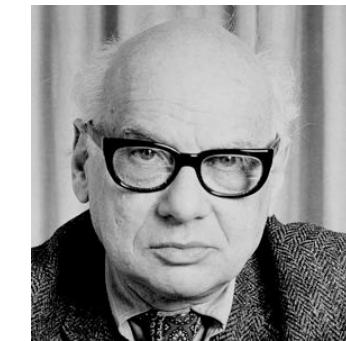
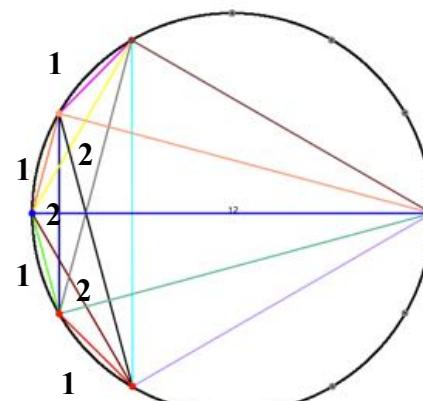
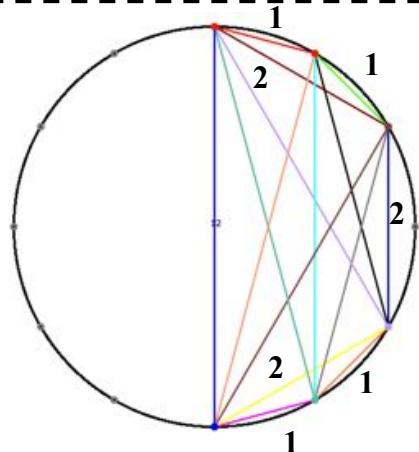
# *'Mathemusical'* research at the interface of three disciplines



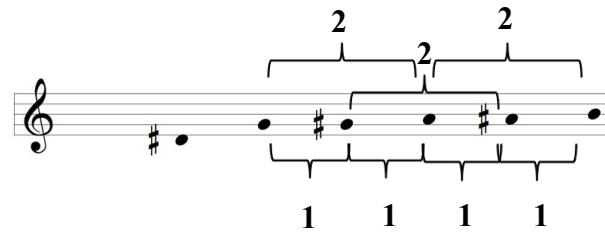
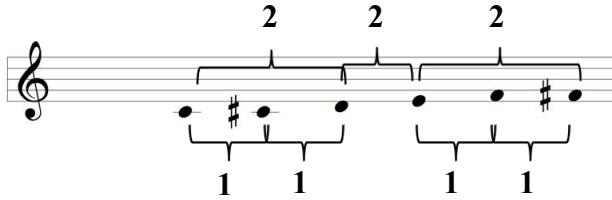
# A historical example of “mathemusical” problem



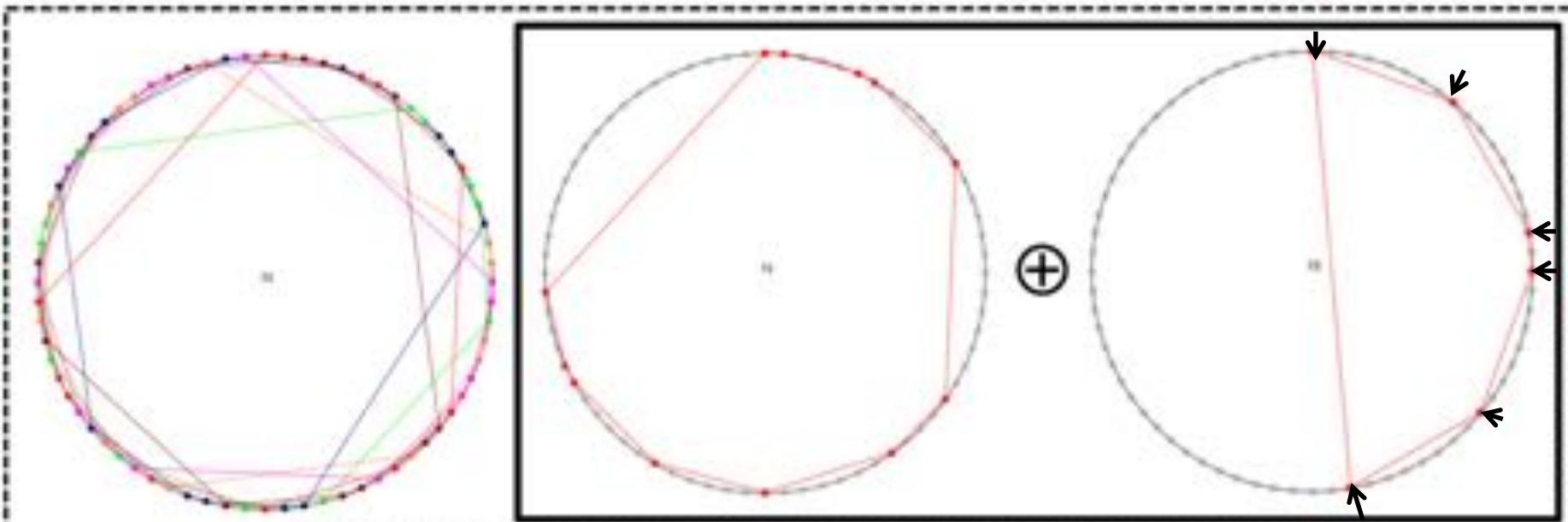
HOMOMETRY  
Hexachord  
Theorem



M. Babbitt



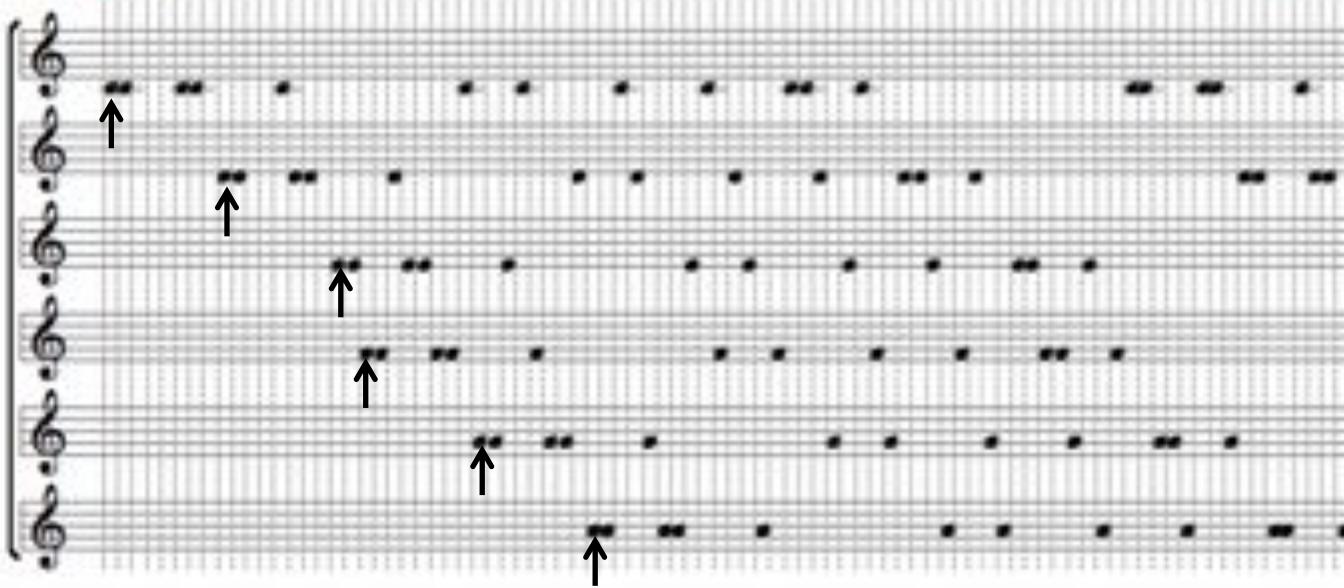
# Aperiodic Rhythmic Tiling Canons (Vuza Canons)



Dan Vuza



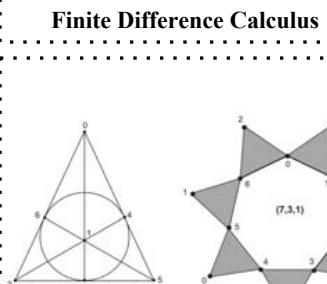
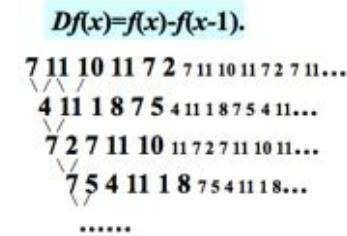
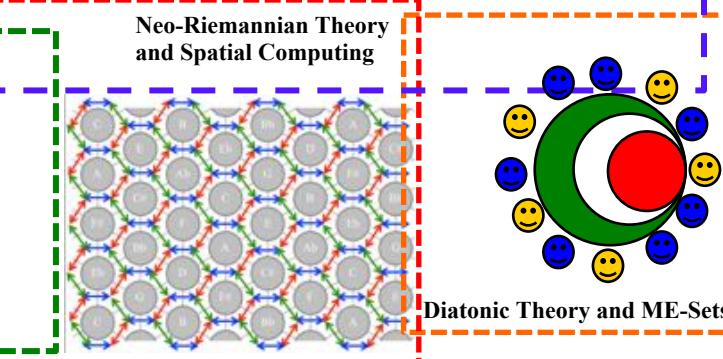
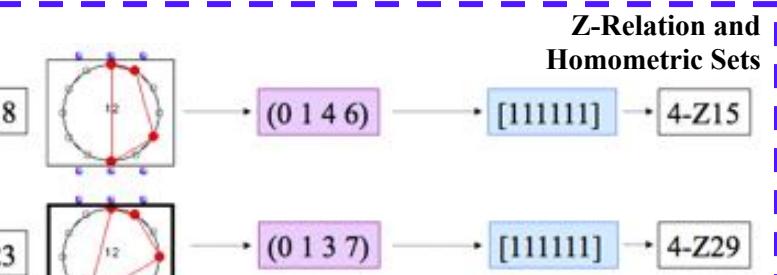
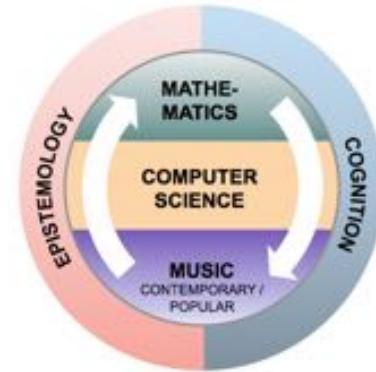
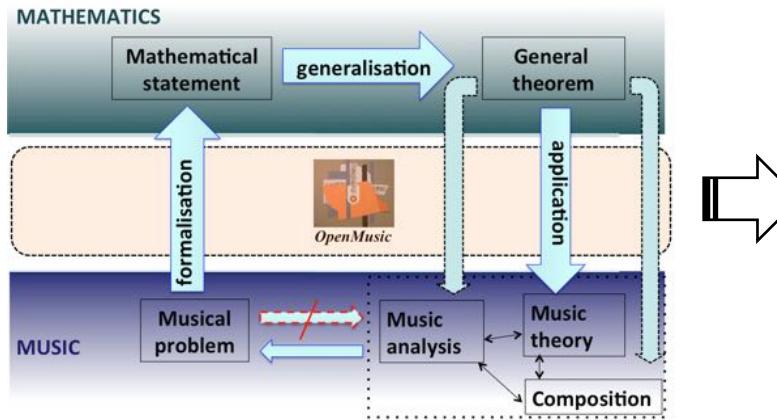
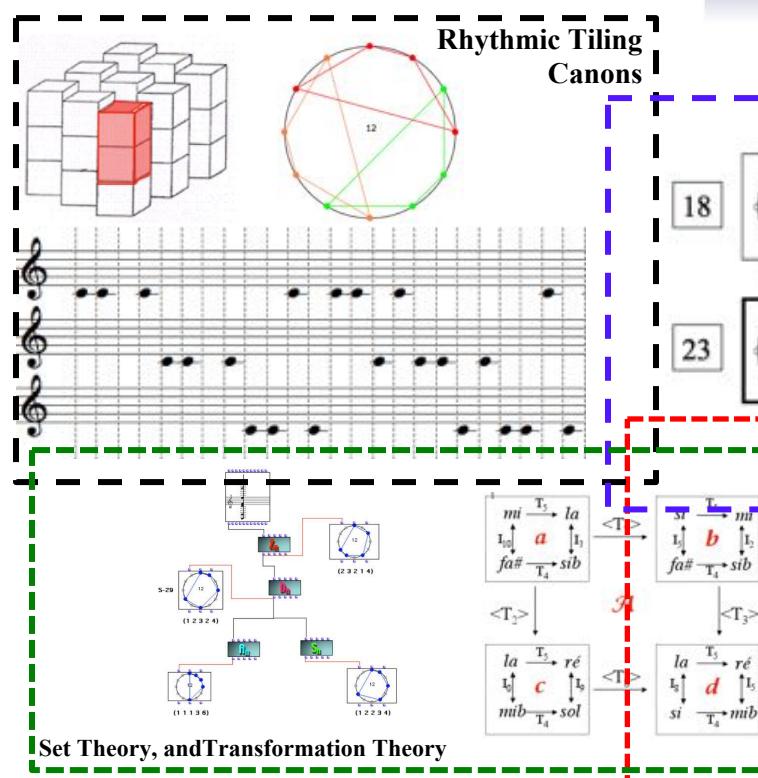
Anatol Vieru



# A short catalogue of mathemusical problems

M. Andreatta : *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

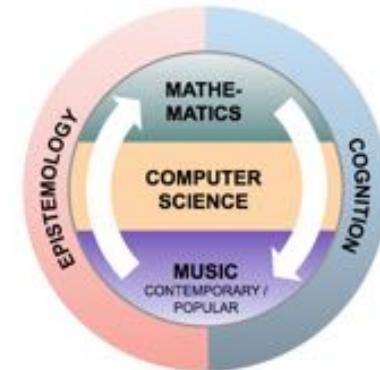
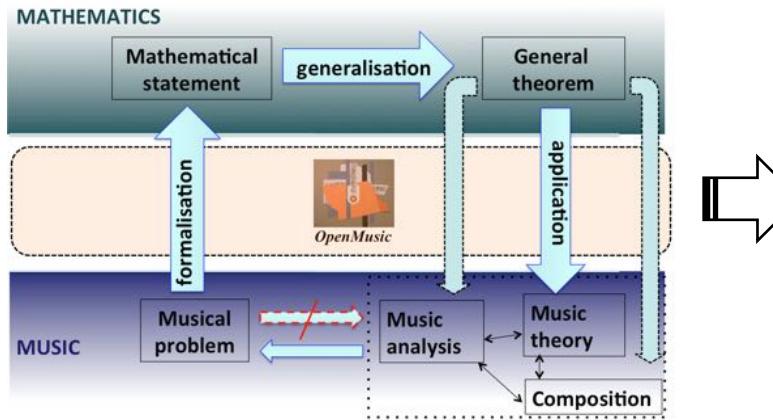
- Tiling Rhythmic Canons
- Z relation and homometry
- Transformational Theory
- Music Analysis, SC and FCA
- Diatonic Theory and ME-Sets
- Periodic sequences and FDC
- Block-designs in composition



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- Tiling Rhythmic Canons
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- Periodic sequences and FDC
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**Rhythmic Tiling Canons**

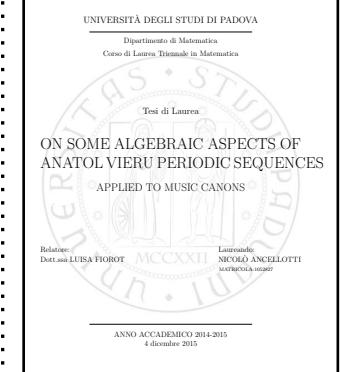
**Z-Relation and Homometric Sets**

**Set Theory, and Transformation Theory**

**Neo-Riemannian Theory and Spatial Computing**

**Diatonic Theory and ME-Sets**

**Finite Difference Calculus**



# The interplay between algebra and geometry in music

MATH / MUSIC MEETINGS

Creativity in Music and Mathematics

Pierre Boulez & Alain Connes

Encounter with two major figures of musical creation and contemporary mathematical research: Pierre Boulez and Alain Connes.

What is the role of intuition in mathematical reasoning and in artistic activities? Is there an aesthetic dimension to mathematical activity? Does the notion of elegance of a mathematical demonstration or of a theoretical construction in music play a role in creativity?



Gérard Assayag, director of the CNRS/IRCAM Laboratory for The Science and Technology of Music and Sound, will lead this dialogue on invention in the two disciplines.

Photo: Pierre Boulez © Jean Radel

Wednesday, June 15, 2011, 6:30pm / IRCAM, Espace de projection

→ <http://agora2011.ircam.fr>



“Concerning music, it takes place in **time**, like **algebra**. In **mathematics**, there is this fundamental duality between, on the one hand, **geometry** – which corresponds to the visual arts, an immediate intuition – and on the other hand **algebra**. This is not visual, it has a temporality. This fits in time, it is a computation, something that is very close to the language, and which has its diabolical precision. [...] **And one only perceives the development of algebra through music**”  
(A. Connes).

→ <http://videotheque.cnrs.fr/>



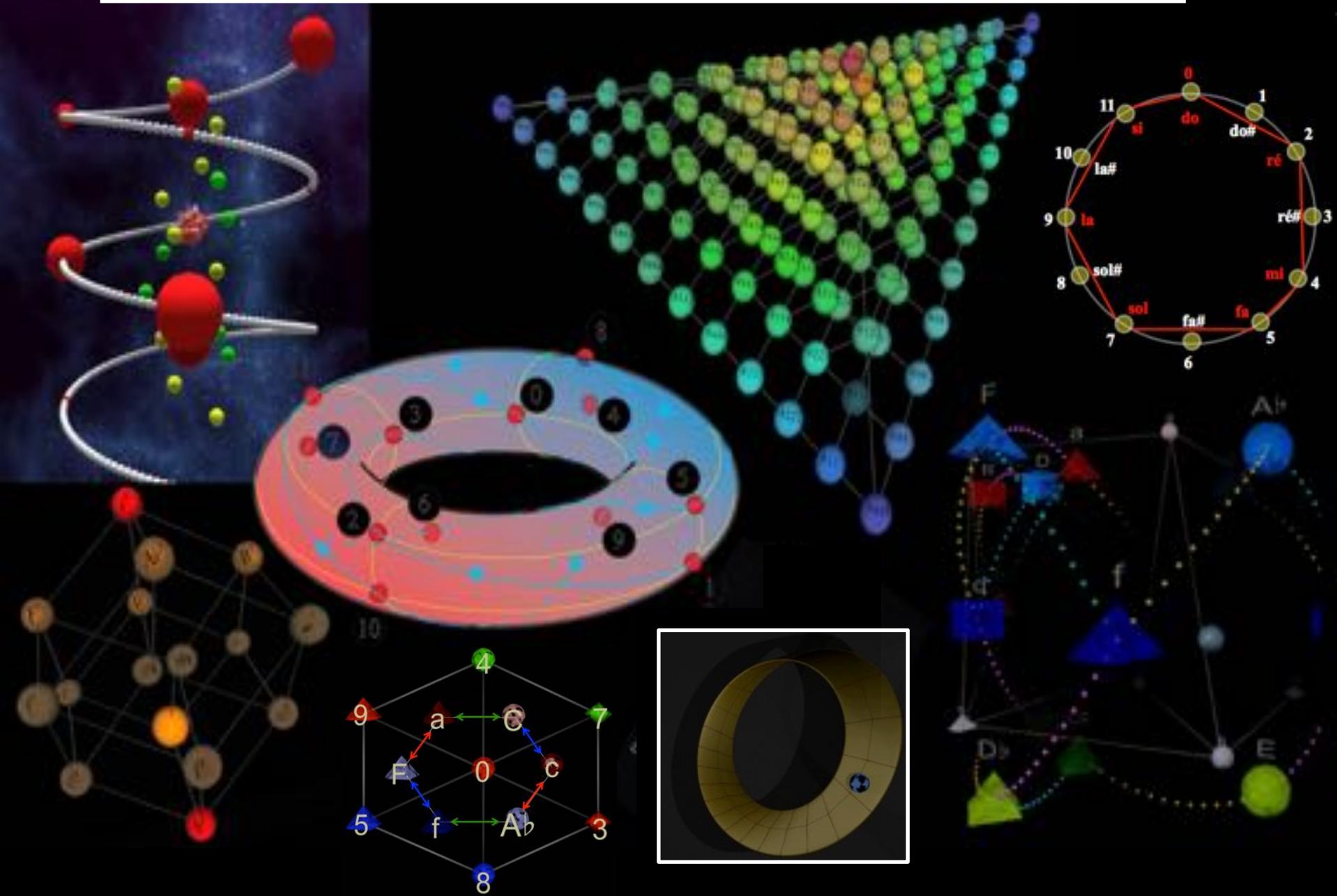
Gérard Assayag, Mathieu Dutour, Yves Guermonpre, Jean Manduchi (Géométrie)

Mathematics and Computation in Music

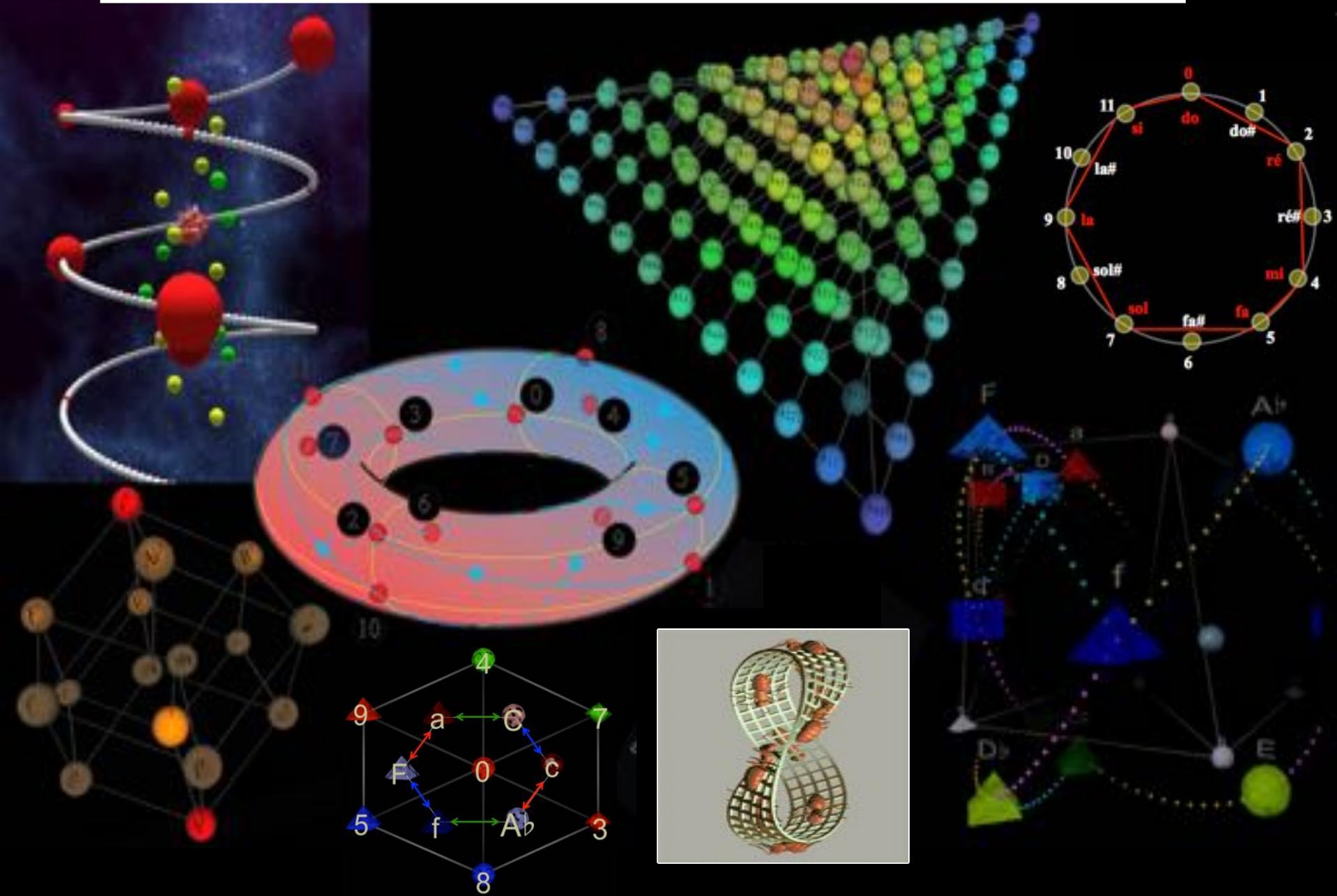
FIRST INTERNATIONAL CONFERENCE, NICE 2011  
PROCEEDINGS

Springer

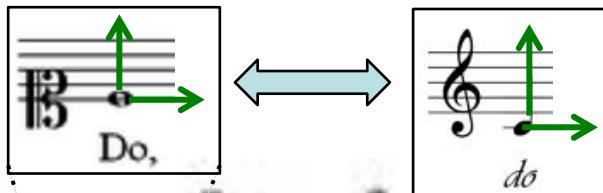
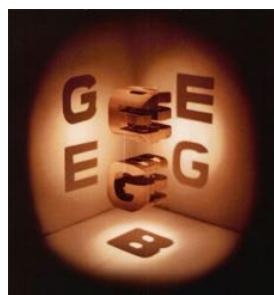
# The galaxy of geometrical models at the service of music



# The galaxy of geometrical models at the service of music

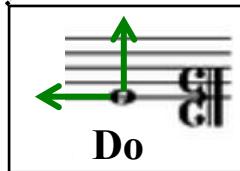
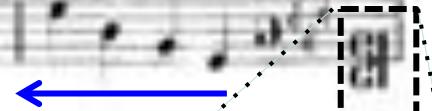
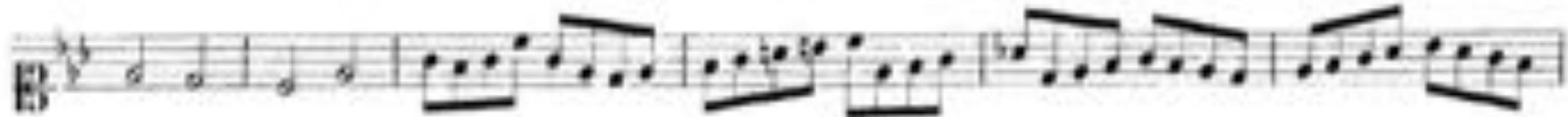
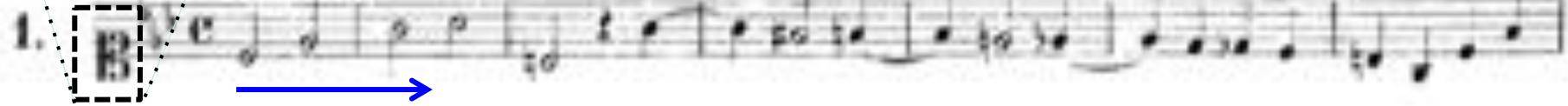


# Bach's enigmatic canons and geometry



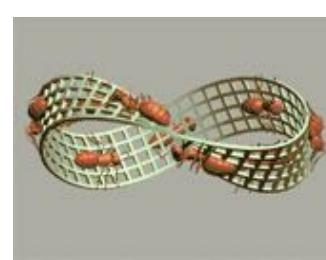
Canones diversi  
super thema regium

Canon a 2.





# My end is my beginning (but twisted!)



Canones diversi

super thema regium

1.

Canon n. 2

Musical score for Canon n. 2, showing three staves of music in common time, key signature of one flat, and treble clef. The music consists of eighth and sixteenth note patterns.

A bracketed section of the musical score, spanning multiple staves, with a blue arrow pointing upwards from the right side of the bracket.

Canones diversi  
super thema regium

1.

Canon n. 2

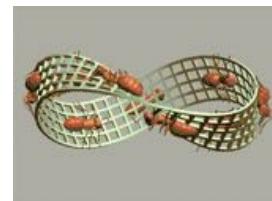
Continuation of the musical score for Canon n. 2, showing two staves of music in common time, key signature of one flat, and treble clef. The music consists of eighth and sixteenth note patterns.

Continuation of the musical score for Canon n. 2, showing two staves of music in common time, key signature of one flat, and treble clef. The music consists of eighth and sixteenth note patterns.

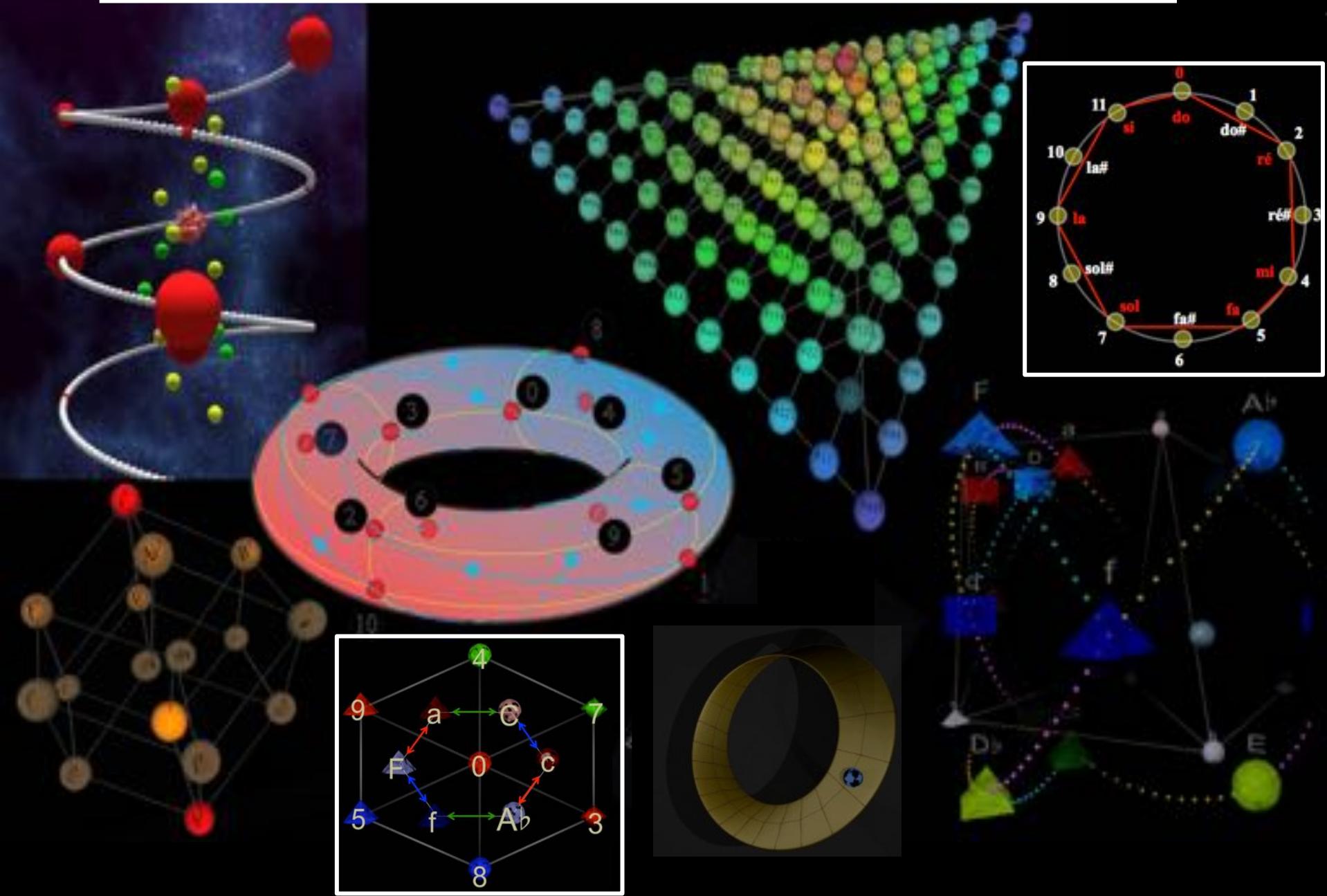


<http://www.josleys.com/Canon/Canon.html>

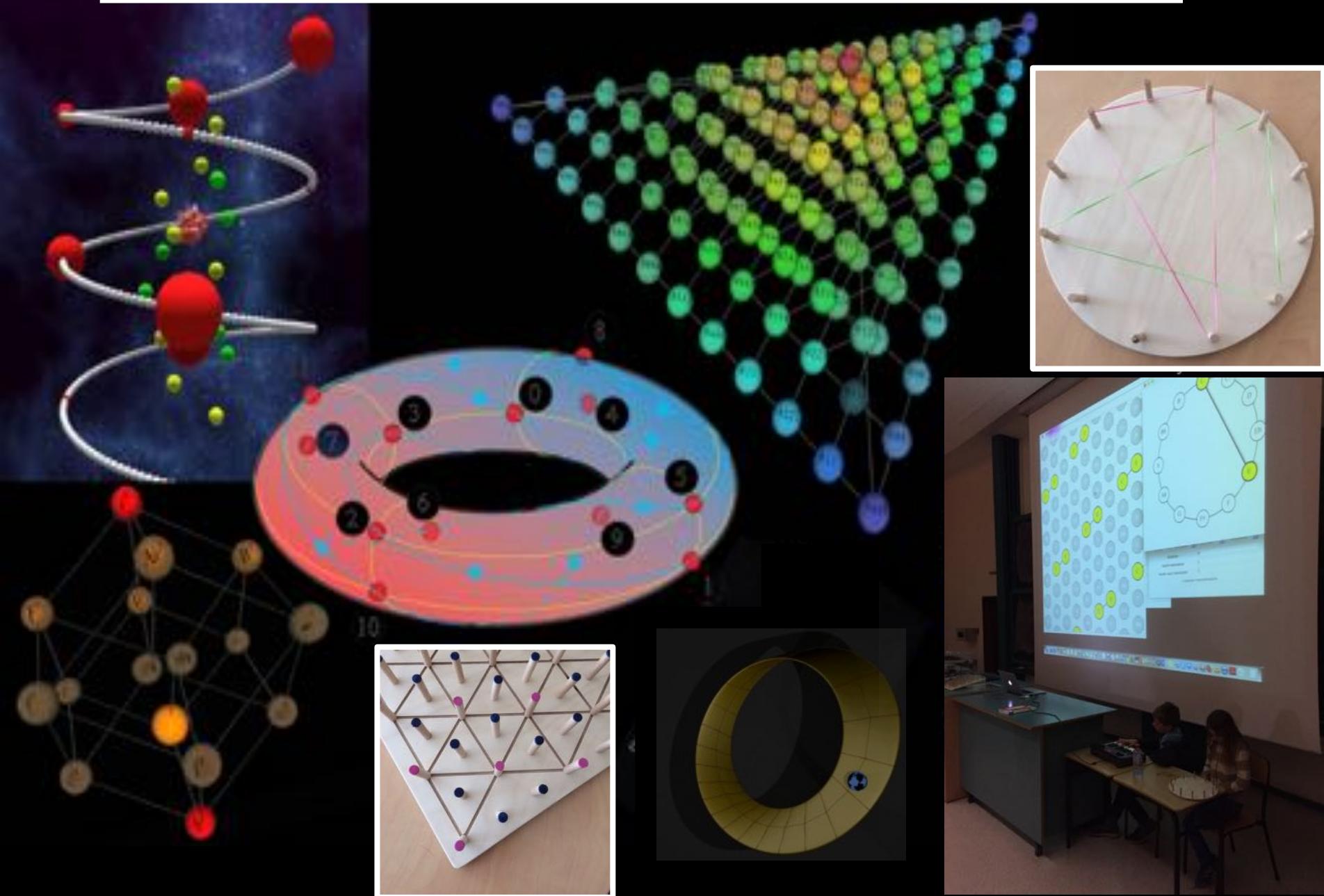
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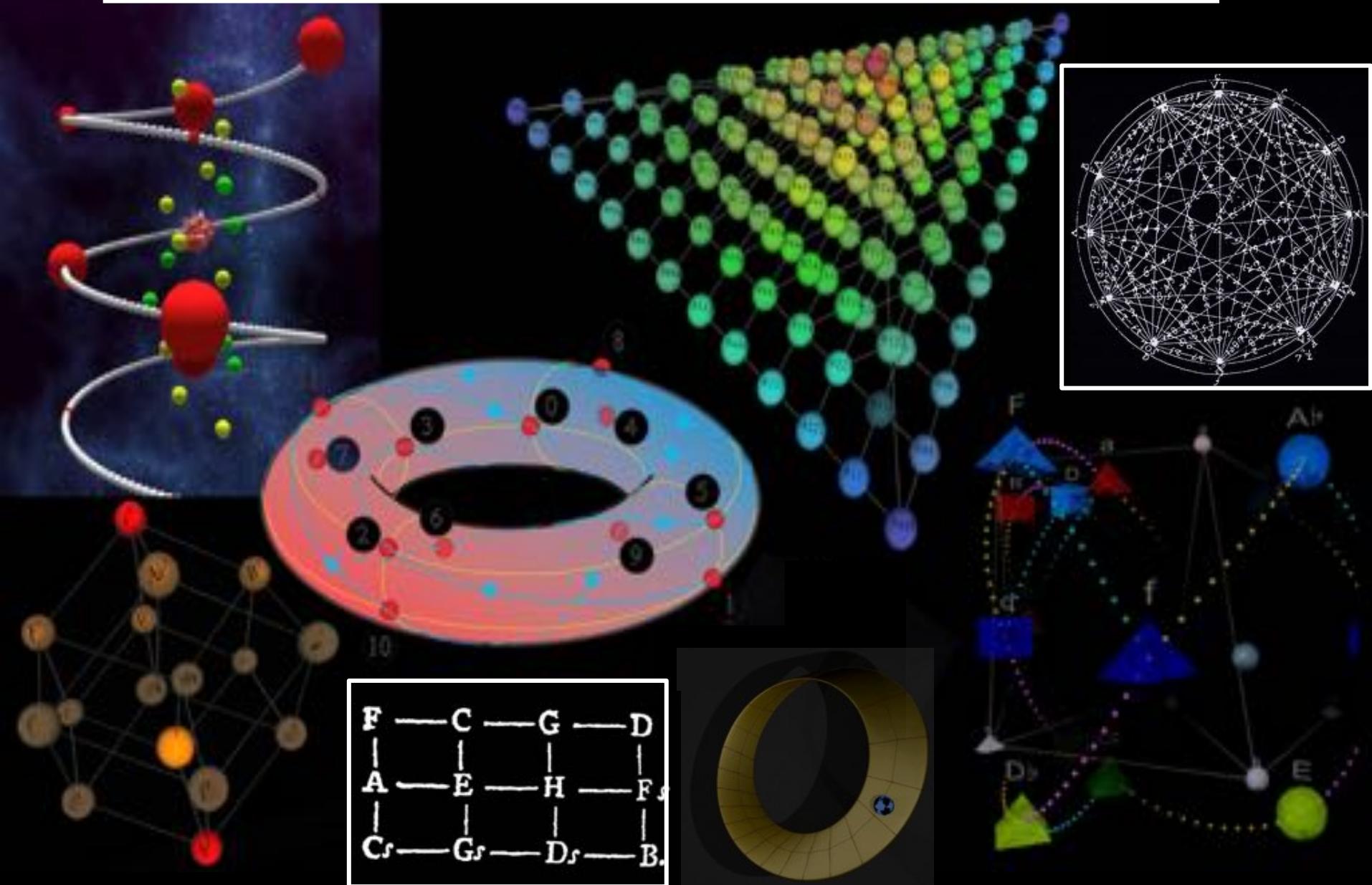
# The galaxy of geometrical models at the service of music



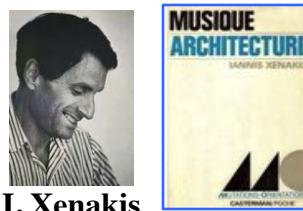
# The galaxy of geometrical models at the service of music



# The galaxy of geometrical models at the service of music



# Music and mathematics: « prima la musica »!



## MUSIC

**500 B.C.** Pitches and lengths of strings are related. Here music gives a marvelous thrust to number theory and geometry.

No correspondence in music.

**300 B.C.** [...] Music theory highlights the discovery of the **isomorphism between the logarithms** (musical intervals) and **exponentials** (string lengths) more than 15 centuries before their discovery in mathematics; also a premonition of **group theory** is suggested by Aristoxenos.

**1000 A.D.** Invention of the two-dimensional spatial representation of pitches linked with time by means of staves and points [...] seven centuries (1635-37) before the magnificent analytical geometry of Fermat and Descartes.

**1500** No response or development of the preceding concepts.

**1600** No equivalence, no reaction.

**1648** Invention of musical combinatorics by Marin Mersenne (*Harmonicorum Libri*)

**1700** [...] The fugue, for example, is an **abstract automaton** used two centuries before the birth of the science of automata. Also, there is an **unconscious manipulation of finite groups** (Klein group) in the four variations of a melodic line used in counterpoint.

**1773** A first geometric and graph-theoretic representation of pitches (*Speculum Musicum*)

**1900** Liberation from the tonal yoke. First acceptance of the neutrality of chromatic totality (Loquin [1895], Hauer, Schoenberg).

**1920** First radical formalization of macrostructures through the serial system of Schoenberg.

**1929 and 1937-1939** Susanne K. Langer and Ernst Krenek on the role of axioms in music

**1946** Milton Babbitt on group theory and integral serialism

## MATHS

Discovery of the fundamental importance of natural numbers and the invention of fractions.

Positive irrational numbers [...]

No reaction in mathematics. [...]

No parallel in mathematics.

Zero and negative numbers are adopted. Construction of the set of rationals.

The sets of real numbers and of logarithms are invented.

Probability theory by Bernoulli [*Ars Conjectandi*, 1713]

Number theory is ahead of but has no equivalent yet in temporal structures. [...]

Invention of graph theory

The infinite and transfinite numbers (Cantor). Peano axiomatics. [...] The beautiful measure theory (Lebesgue, ...)

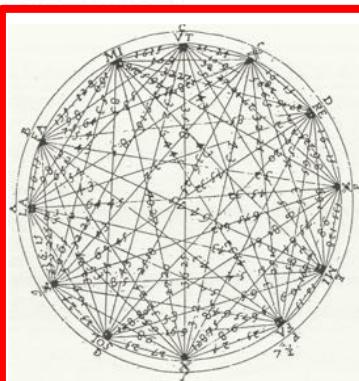
No new development of the number theory.

David Hilbert, *Die Grundlage der Geometrie* (1899)

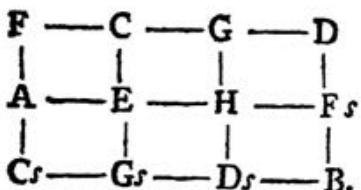
Rudolf Carnap, *The Logical Syntax of Language* (1937)



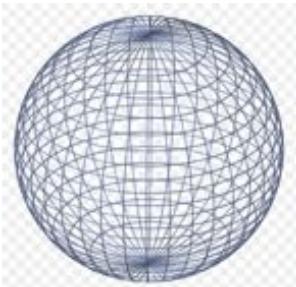
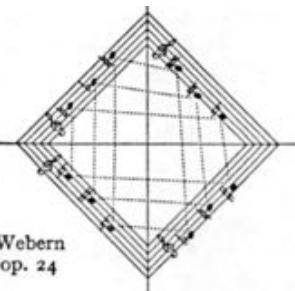
Pythagoras and the monochord,  
VI<sup>th</sup>-V<sup>th</sup> Century B.C.



Mersenne and  
the ‘musical  
clock’, 1648



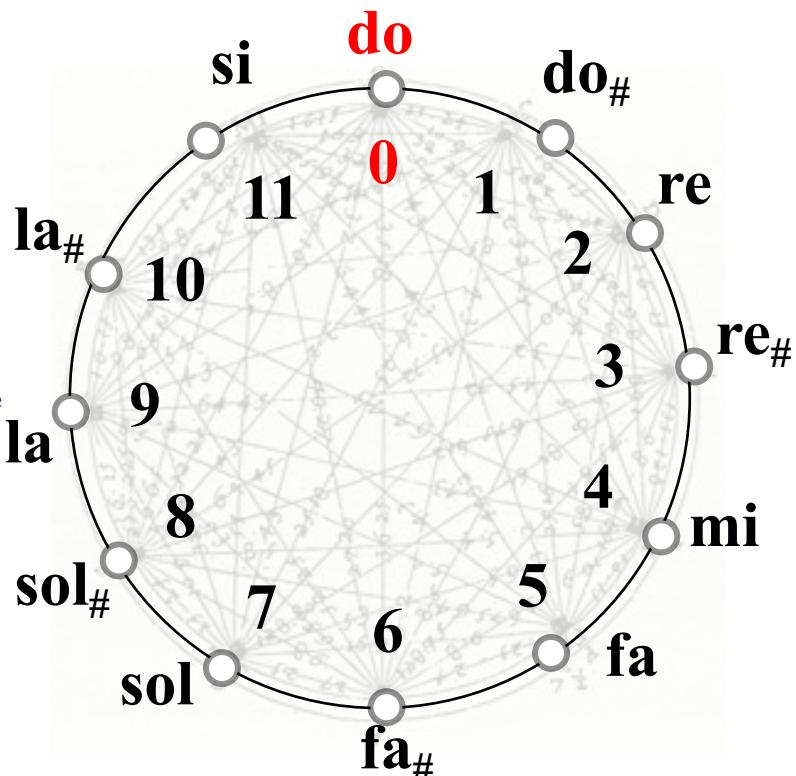
Euler and the  
*Speculum  
musicum*, 1773



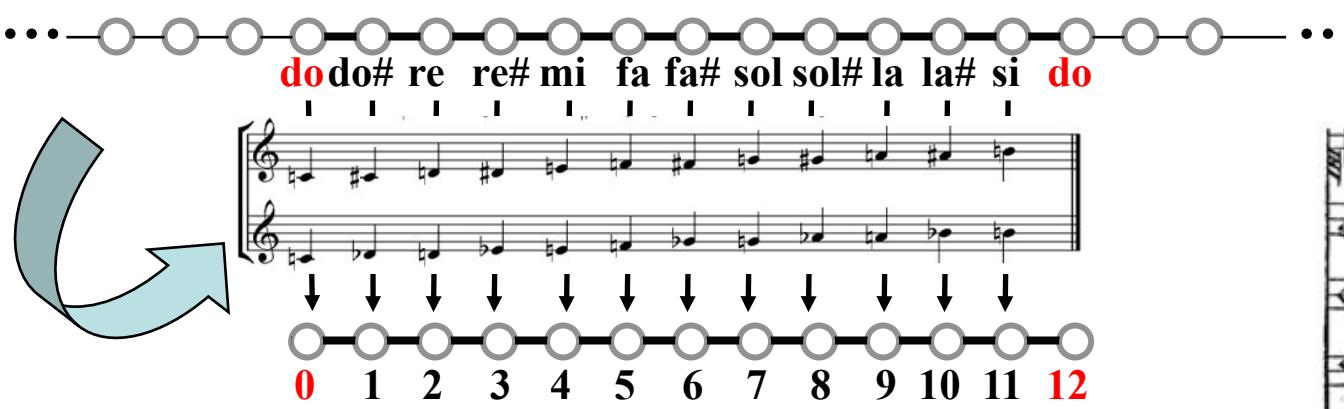
# The circular representation of the pitch space



Marin Mersenne



*Harmonicorum Libri XII, 1648*

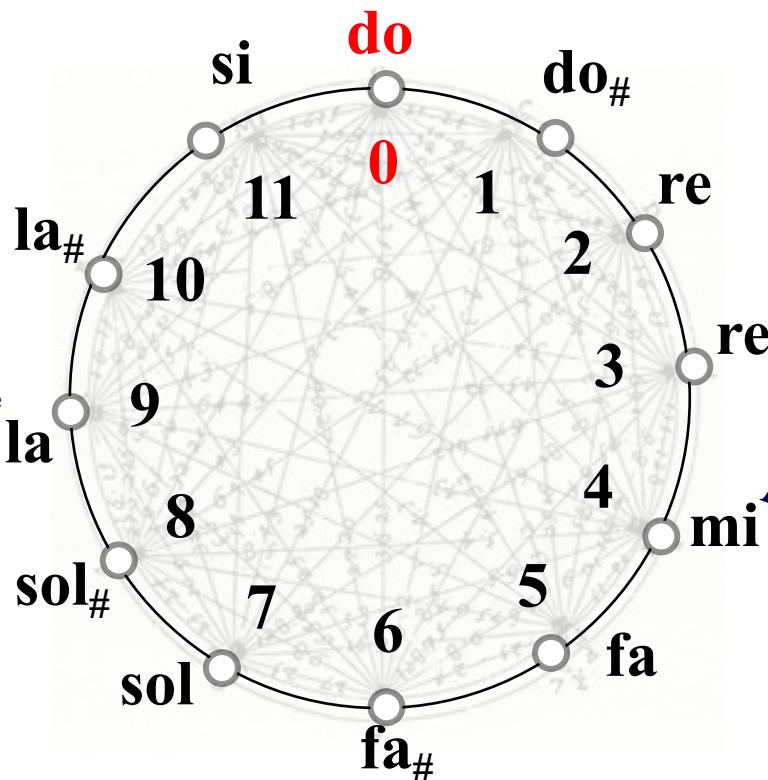


LIBER SEPTIMVS DE CANTIBVS, SEV CANTILENIS, EARVMQ; NVMERO, PARTIBVS, ET SPECIEBV.	
Tabula Combinationis ab 1 ad 12.	
I	1
II	2
III	6
IV	24
V	120
VI	720
VII	1040
VIII	40320
IX	361880
X	3618800
XI	39916800
XII	479004600
XIII	617101800
XIV	377859100
XV	1107674568000
XVI	20912759588000
XVII	311687418096000
XVIII	6404173705788000
XIX	1116410040183000
XX	141350100176640000
XXI	51090941171709440000
XXII	1114000737777607180000

*Varietas, seu Combinatio quatuor notarum.*



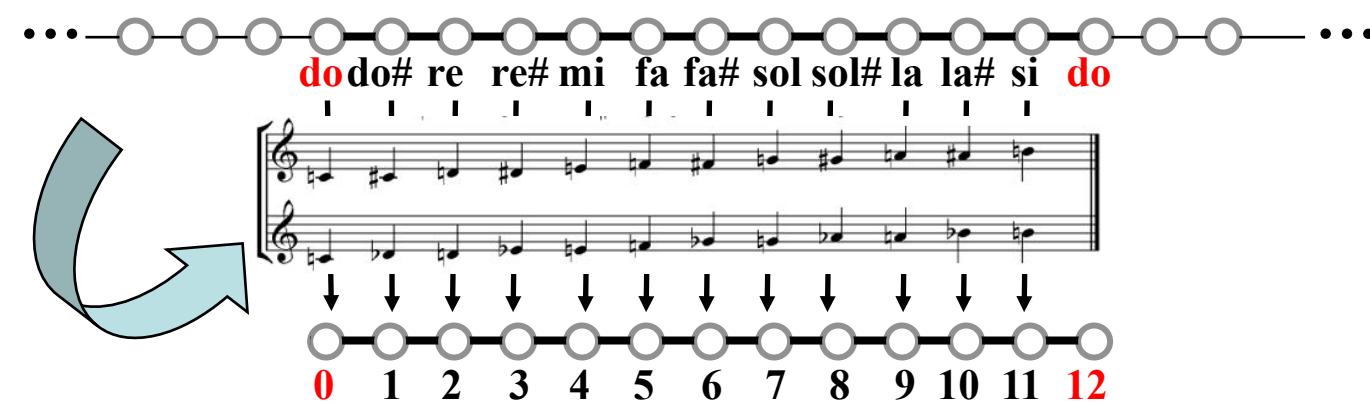
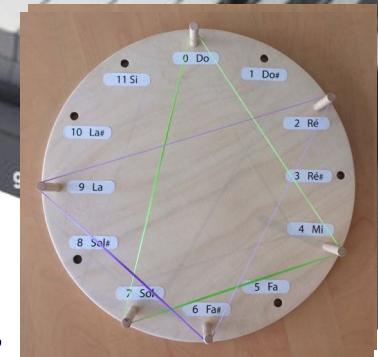
# The circular representation of the pitch space



*Harmonicorum Libri XII, 1648*

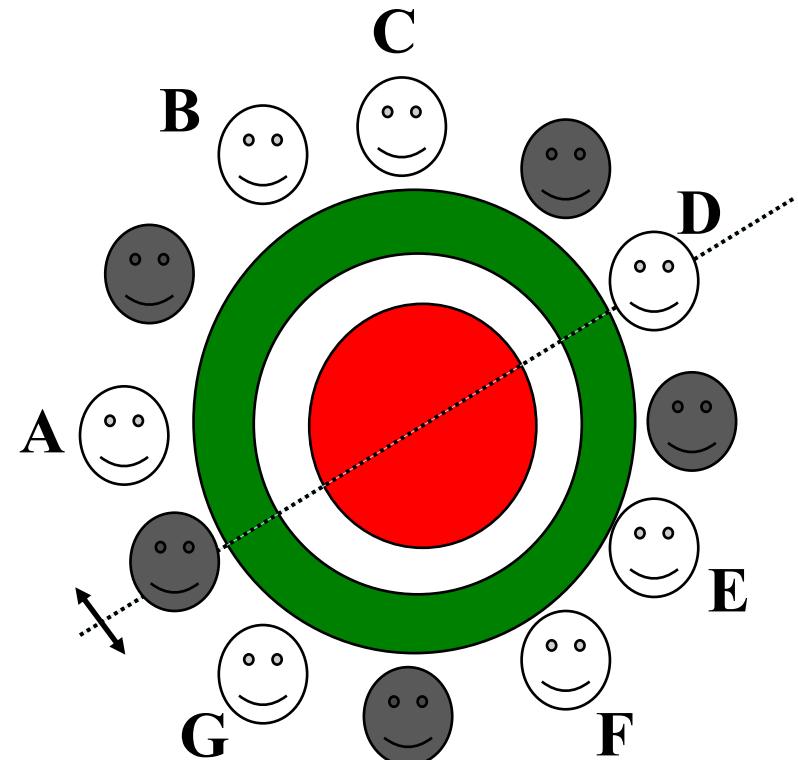
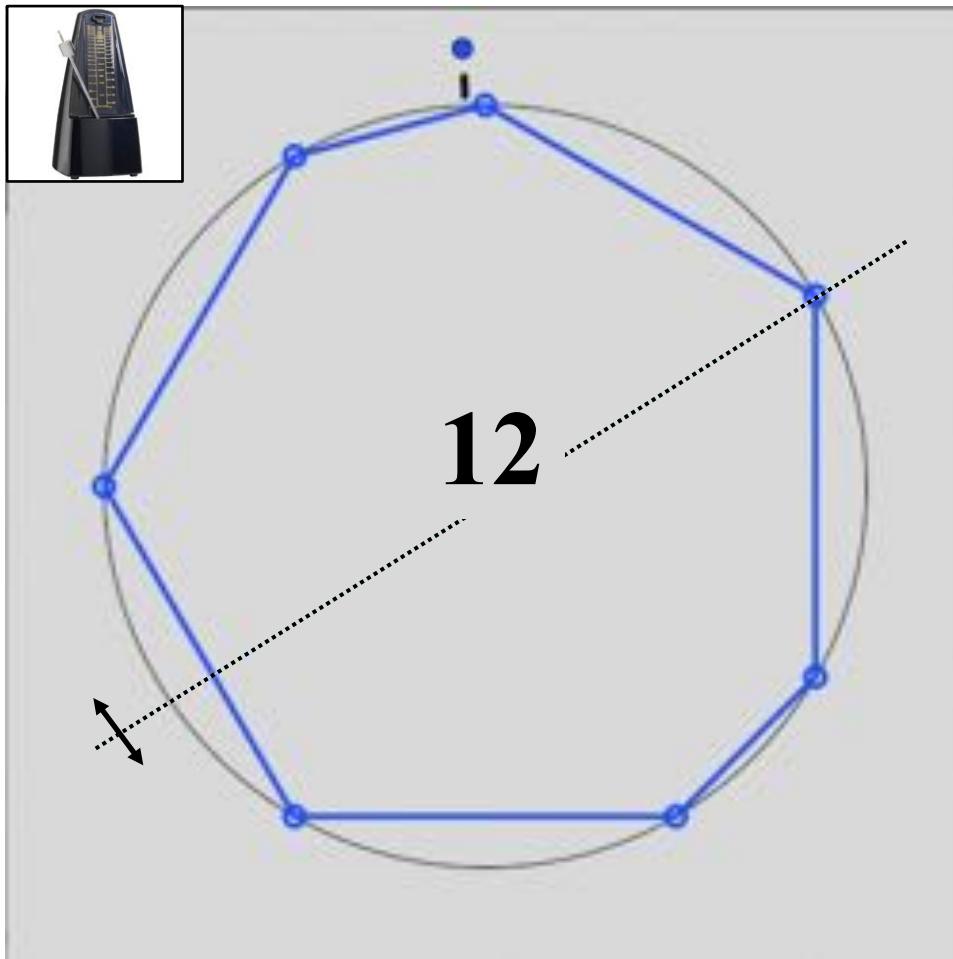


M. Andreatta, C. Agon,  
«La musique mise en algèbre»,  
Pour la Science, 2008



→ DEMO

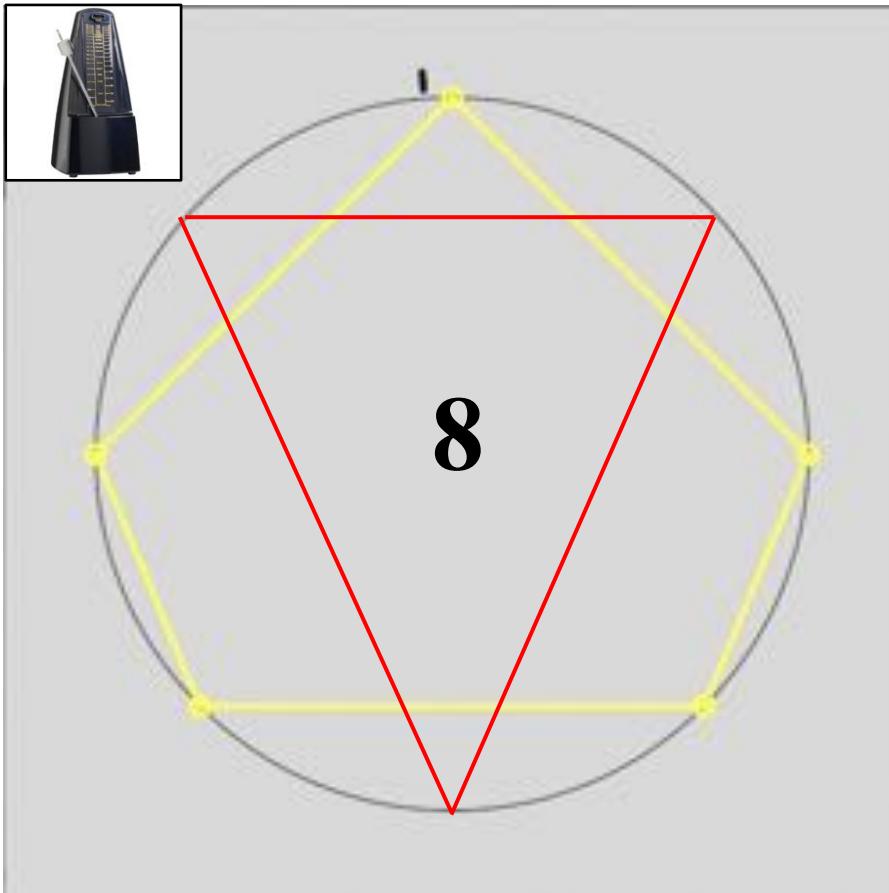
# The circle: a model for periodic rhythms



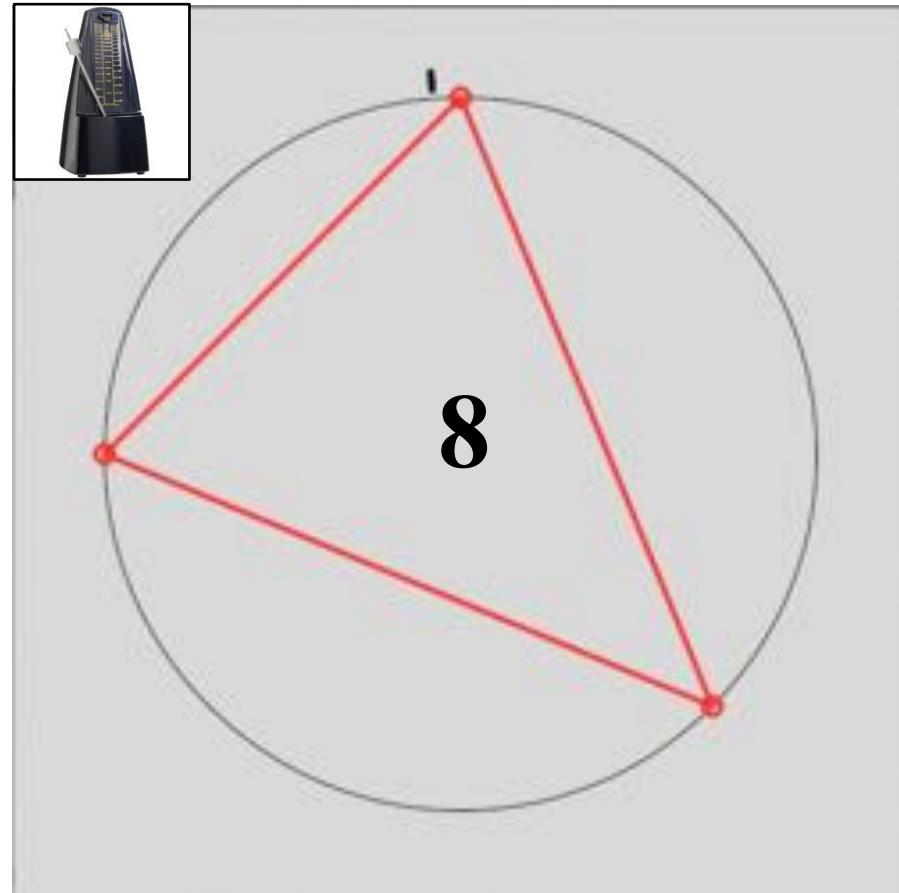
Abadja ou Bembé

# African-cuban ME-rhythms

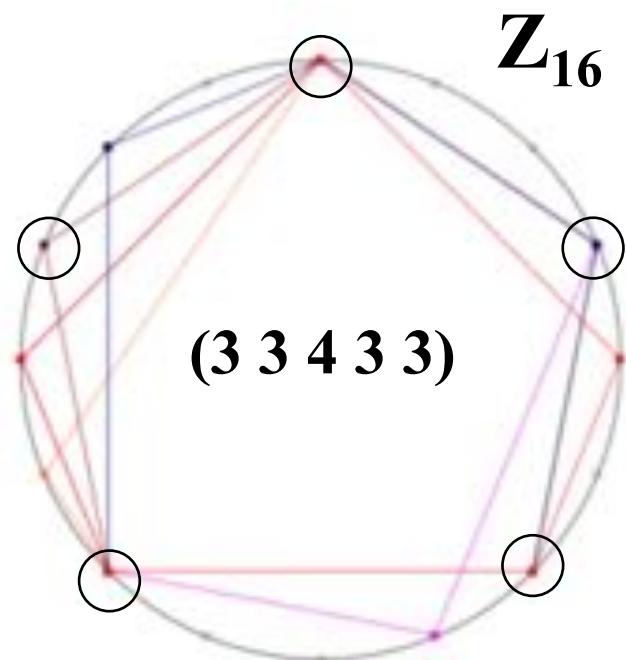
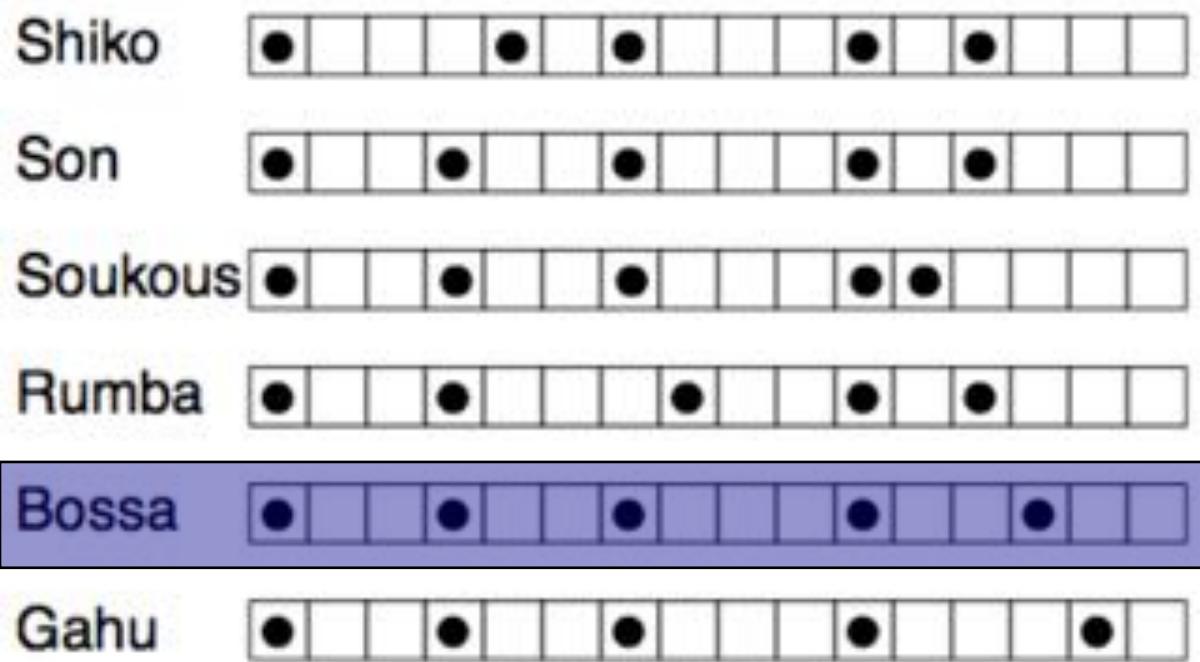
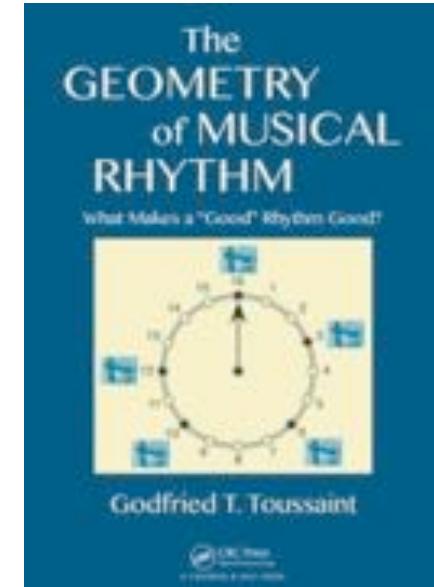
## *El cinquillo*



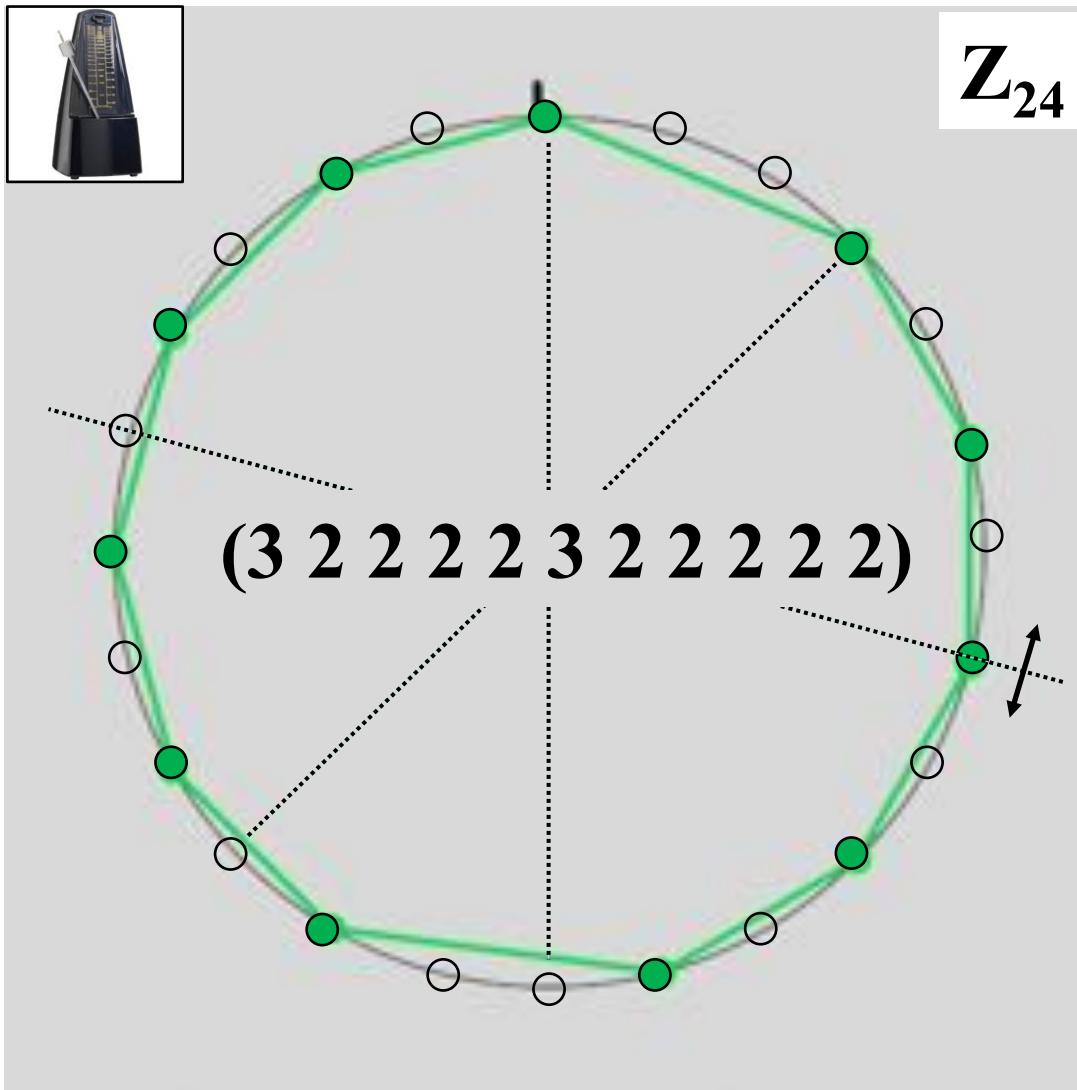
## *El trecillo*



# The geometry of African-Cuban rhythms



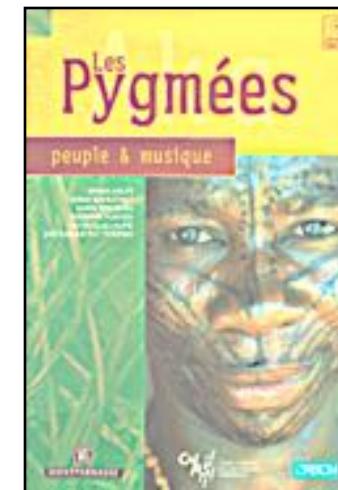
# Odditive property of orally-transmitted practices



Simha Arom



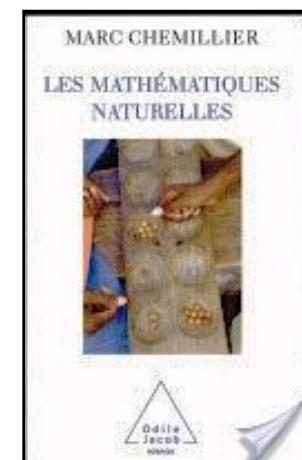
Marc Chemillier



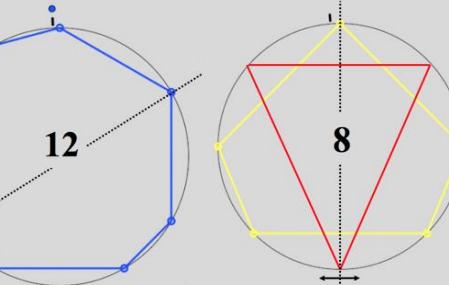
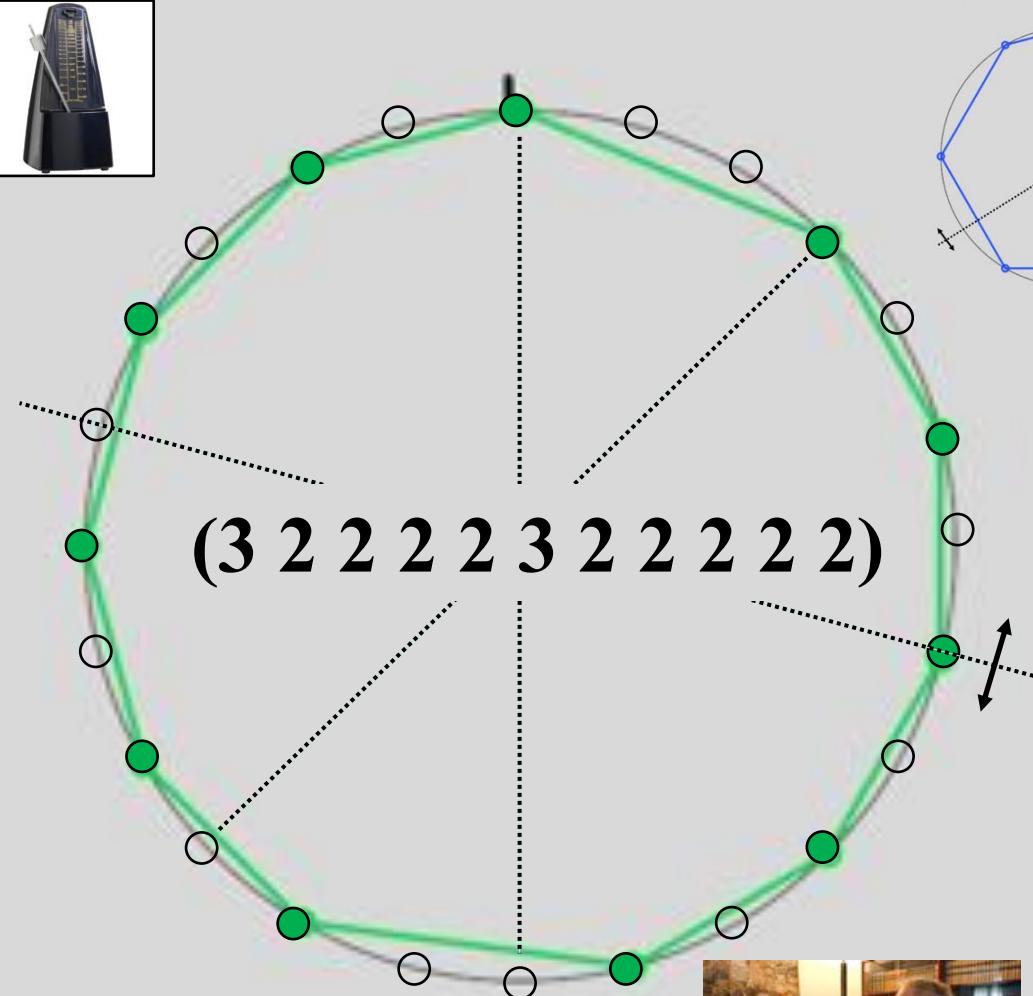
**musimédiane**

publiée avec le concours de la SFAM

revue audiovisuelle et multimédia d'analyse musicale



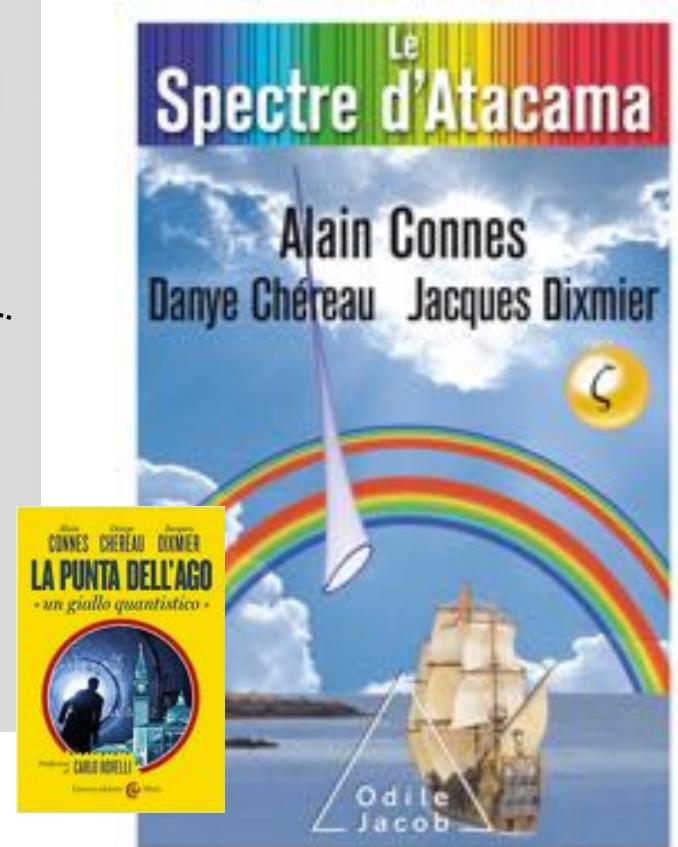
# Olivier Messiaen's non-invertible rhythms



Olivier Messiaen



Alain Connes



# Palindromic structures in Steve Reich's music

## CLAPPING MUSIC

FOR TWO PERFORMERS

J: 164-168

CLAPS CLAPS f

①

②

③

④

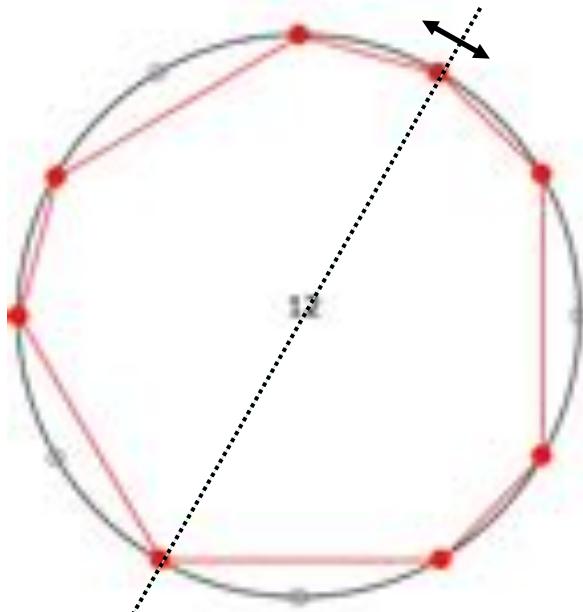
Repeat back, then end

The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer part does not change, it is up to the second performer to move from section to the next. The second performer should try to keep his own slurs but where this is written, i.e., on the first beat of each section (but on the first beat of the group of three claps), so that his slurs will always fall on a new beat of his own underlying pattern.

The choice of a particular clapping sound, i.e., with cupped or flat hands, is left up to the performers. Whichever technique is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall統一 pattern.

*Clapping Music de Steve Reich (1972)*

Steve Reich 1972  
revised 1979



# The circle and its ‘canonic’ rotations

## CLAPPING MUSIC

FOR TWO PERFORMERS

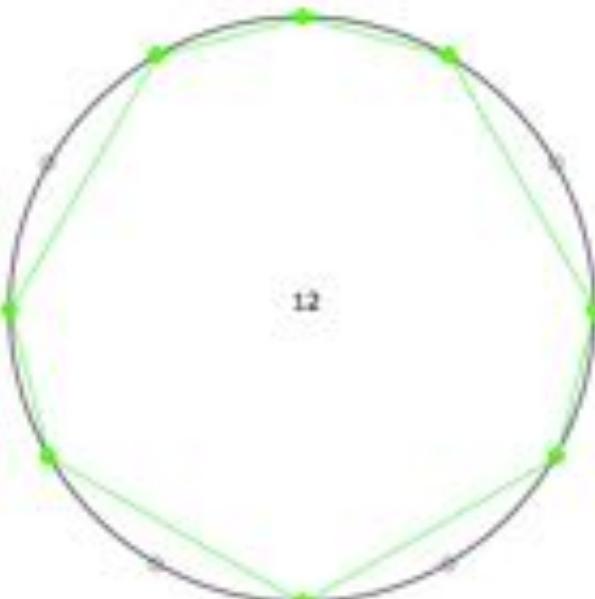
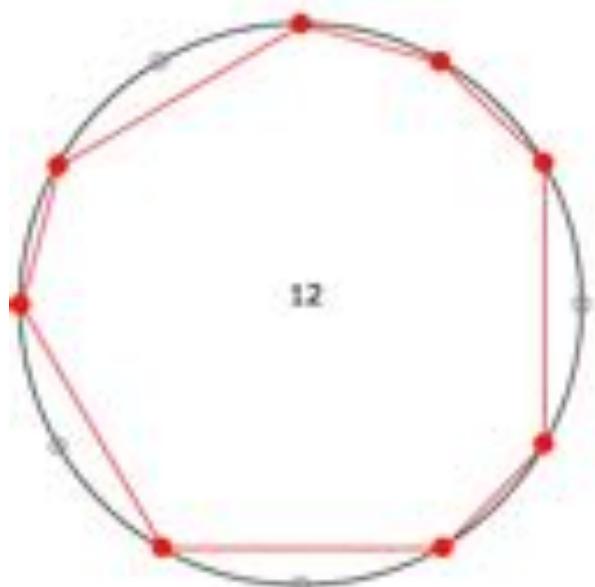
J: 16th-16th

CLAPS 1  
CLAPS 2 f

① ②   
③ ④   
⑤ ⑥   
⑦ ⑧   
⑨ ⑩   
⑪ ⑫   
⑬ ⑭   
⑮ ⑯   
⑰ ⑱   
⑲ ⑳   
⑳ ⑳   
Repeat back to ①, then end

The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer's part does not change, it is up to the second performer to move from section to the next. The second performer should try to keep his own slurs where this is written, i.e. on the first beat of each section (but on the first beat of the group of three slaps), so that his loudest beat always falls on a new beat of his own underlying pattern.

The choice of a particular clapping sound, i.e., with cupped or flat hands, is left up to the performers. Whichever technique is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall統一 pattern.



# The circle and its ‘canonic’ rotations

## CLAPPING MUSIC

FOR TWO PERFORMERS

J: 164-188

CLAP1

CLAP2

f

♩ = 164-188

①

②

③

④

⑤

⑥

⑦

⑧

⑨

⑩

⑪

⑫

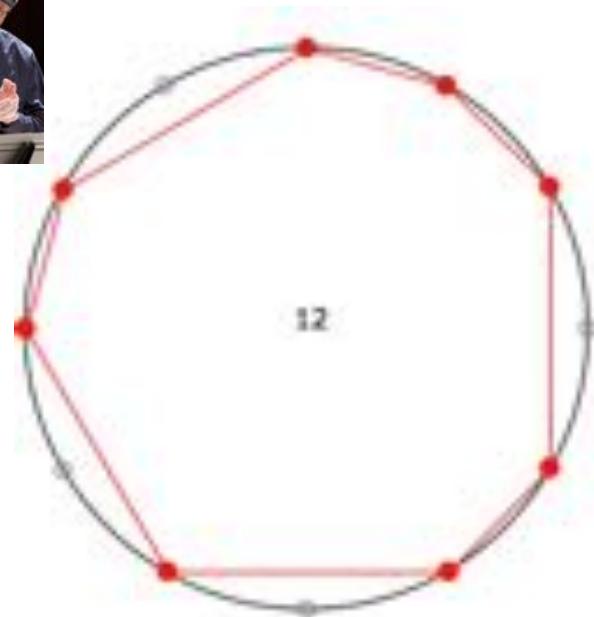
Repeat last 3, then end

The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer part does not change, it is up to the second performer to move from section to the next. The second performer should try to keep his own slurs where this is written, i.e., on the first beat of each section (but on the first beat of the group of three slaps), so that his longest attack always falls on a new beat of his his underlying pattern.

The choice of a particular clapping sound, i.e., with cupped or flat hands, is left up to the performers. Whichever technique is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall統一 pattern.

Clapping Music (1972)

After Bob 12/72  
revised 1979



12345678

(SHIFT)

YOUTUBE.COM/GERUBACH

GERUBACH

Gerubach's Scrolling Score Project  
<http://www.gerubach.com>

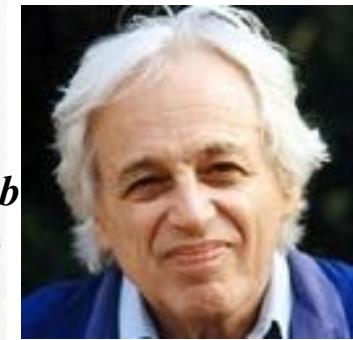
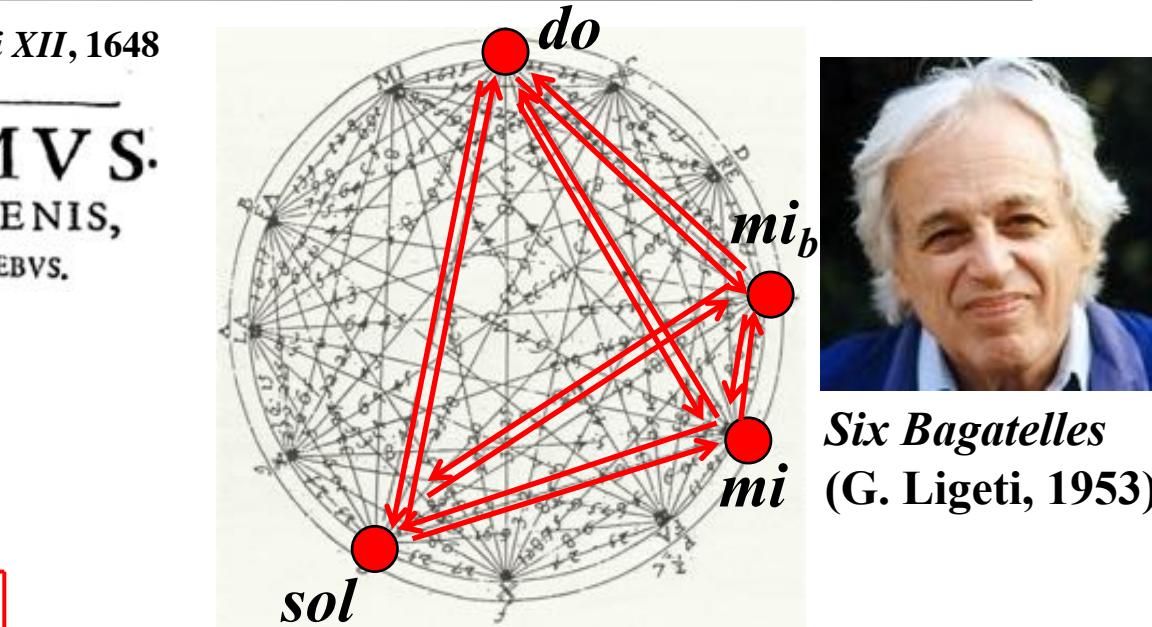
# Permutational melodies in contemporary (art) music

II.4 Marin Mersenne, *Harmonicorum Libri XII*, 1648

## LIBER SEPTIMVS. DE CANTIBVS, SEV CANTILENIS, EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tableta Combinationis ab I ad XXI.

I	I
II	II
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	361800
X	3618000
XI	39916800
XII	479001600
XIII	6117010800
XIV	8717819100
XV	1107674568000
XVI	10922789888000
XVII	311687418296000
XVIII	6401173705718000
XIX	11164100040813000
XX	1433904008176640000
XXI	51090941171709440000
XXII.	1884000737777607680000



*Six Bagatelles*  
(G. Ligeti, 1953)

The musical score consists of 27 staves, each representing a different permutation of the notes do, re, mi, fa, sol, la, si. The staves are numbered 1 through 27. The notation uses diamond-shaped note heads on a standard five-line staff.

# Permutational melodies in song writing

*Se telefonando*, 1966 (Maurizio Costanzo/Ennio Morricone) / Mina

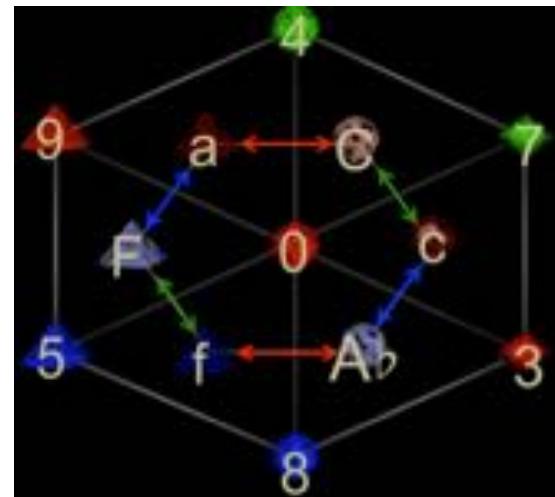


(min. 0'53")



Ennio Morricone

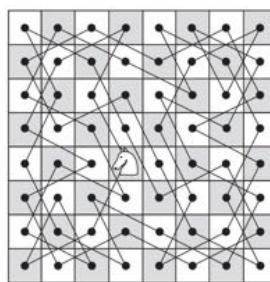
The harmonic space



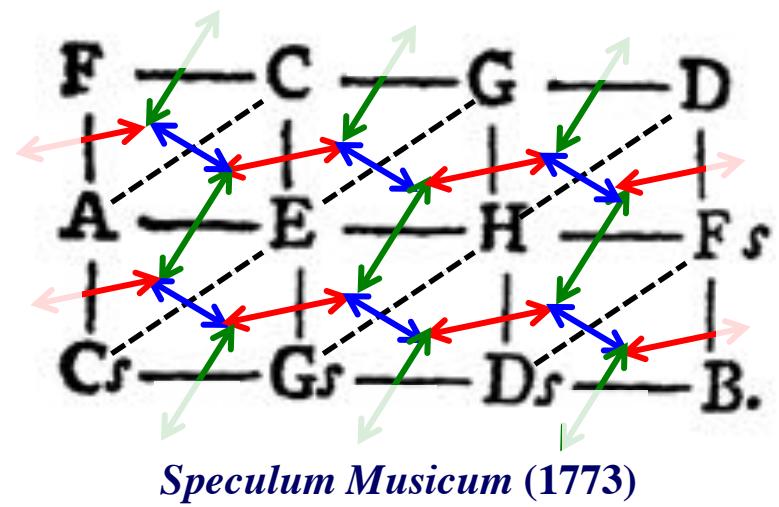
C	c	<b>C<sub>#</sub></b>	c <sub>#</sub>	D	<b>d</b>
E <sub>b</sub>	e <sub>b</sub>	E	e	F	f
F <sub>#</sub>	f <sub>#</sub>	G	<b>g</b>	G <sub>#</sub>	g <sub>#</sub>
A	a	<b>B<sub>b</sub></b>	<b>b<sub>b</sub></b>	<b>B</b>	<b>b</b>

**Chord enumeration**

# The Tonnetz (Network of Tones)



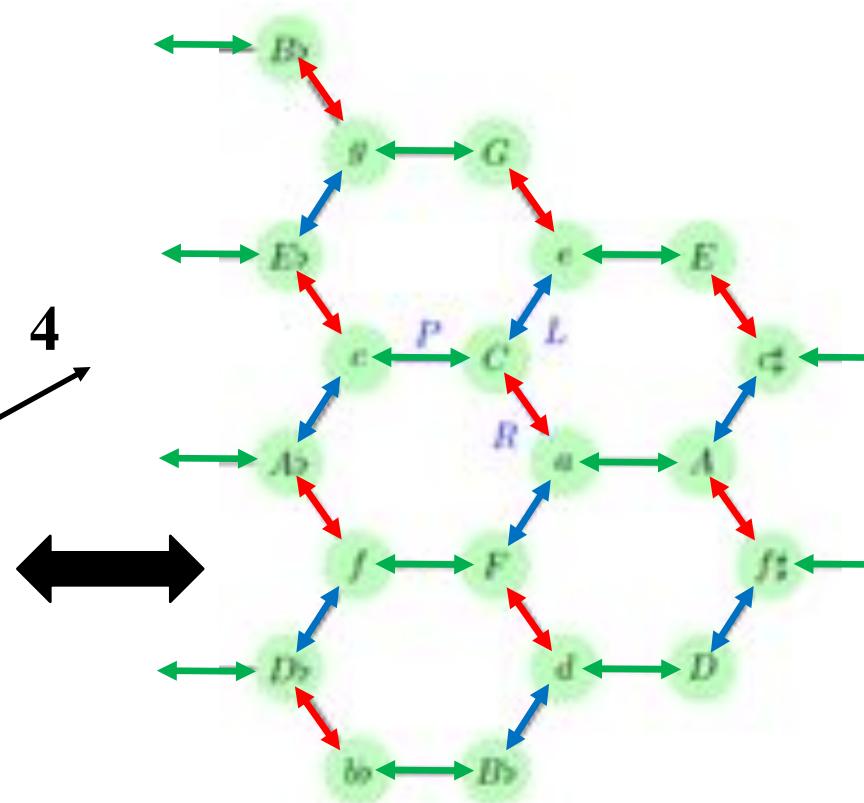
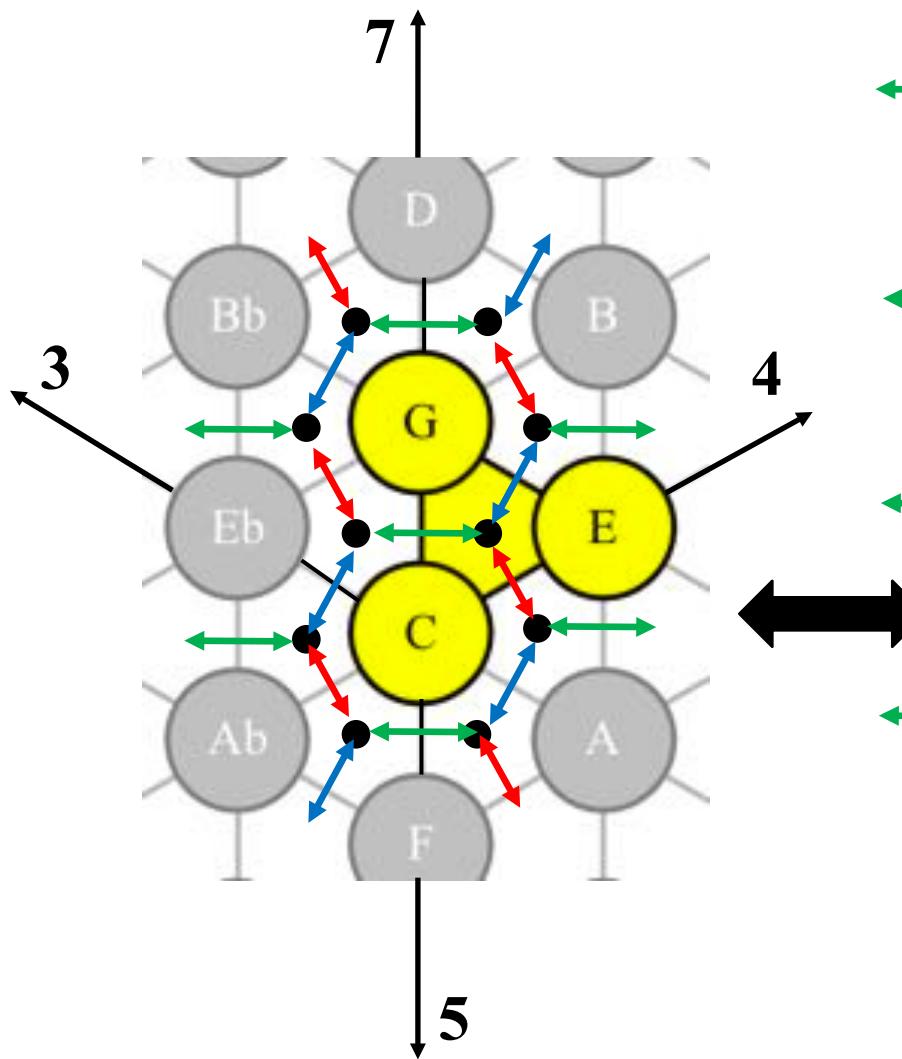
Leonhard Euler



*Speculum Musicum (1773)*

→ DEMO

# From the Tonnetz to the dual one (and vice-versa)



[Sonia Cannas, 2018]

# Harmonic Progressions

In Paolo Conte

*Sotto le Stelle del Jazz*



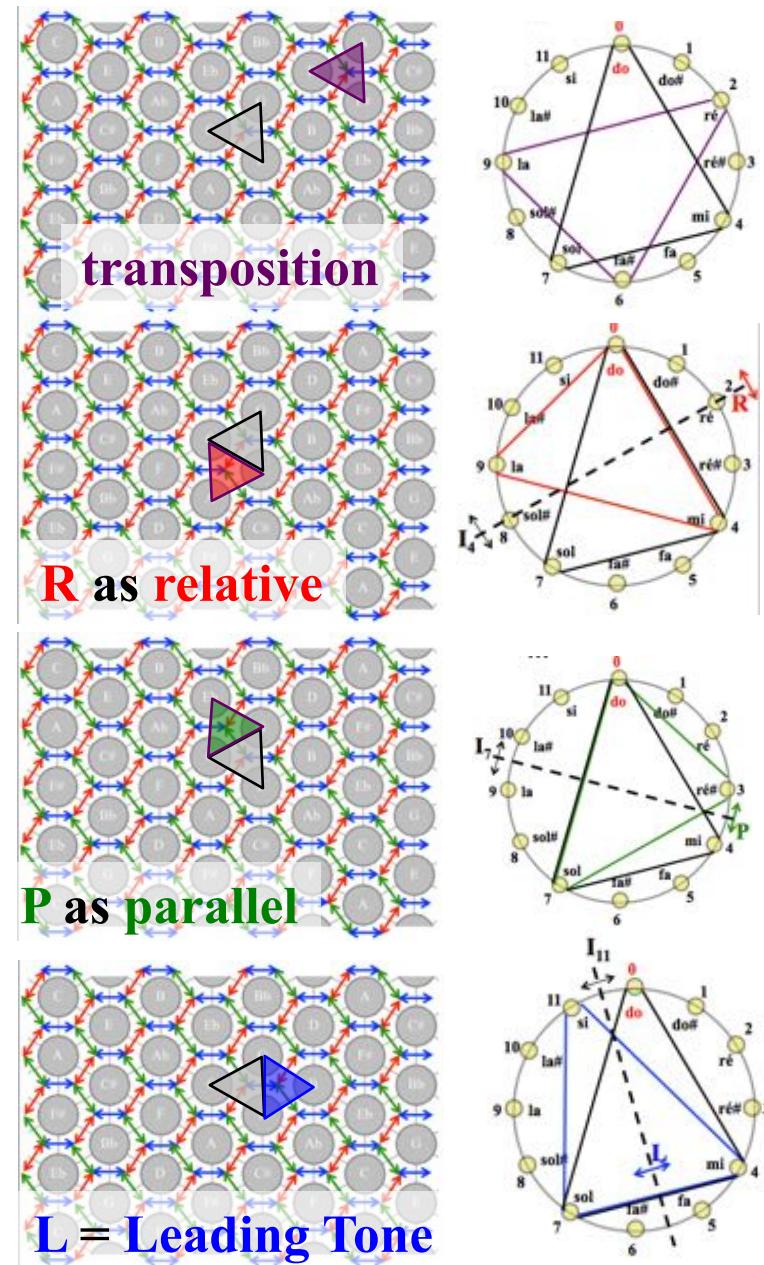
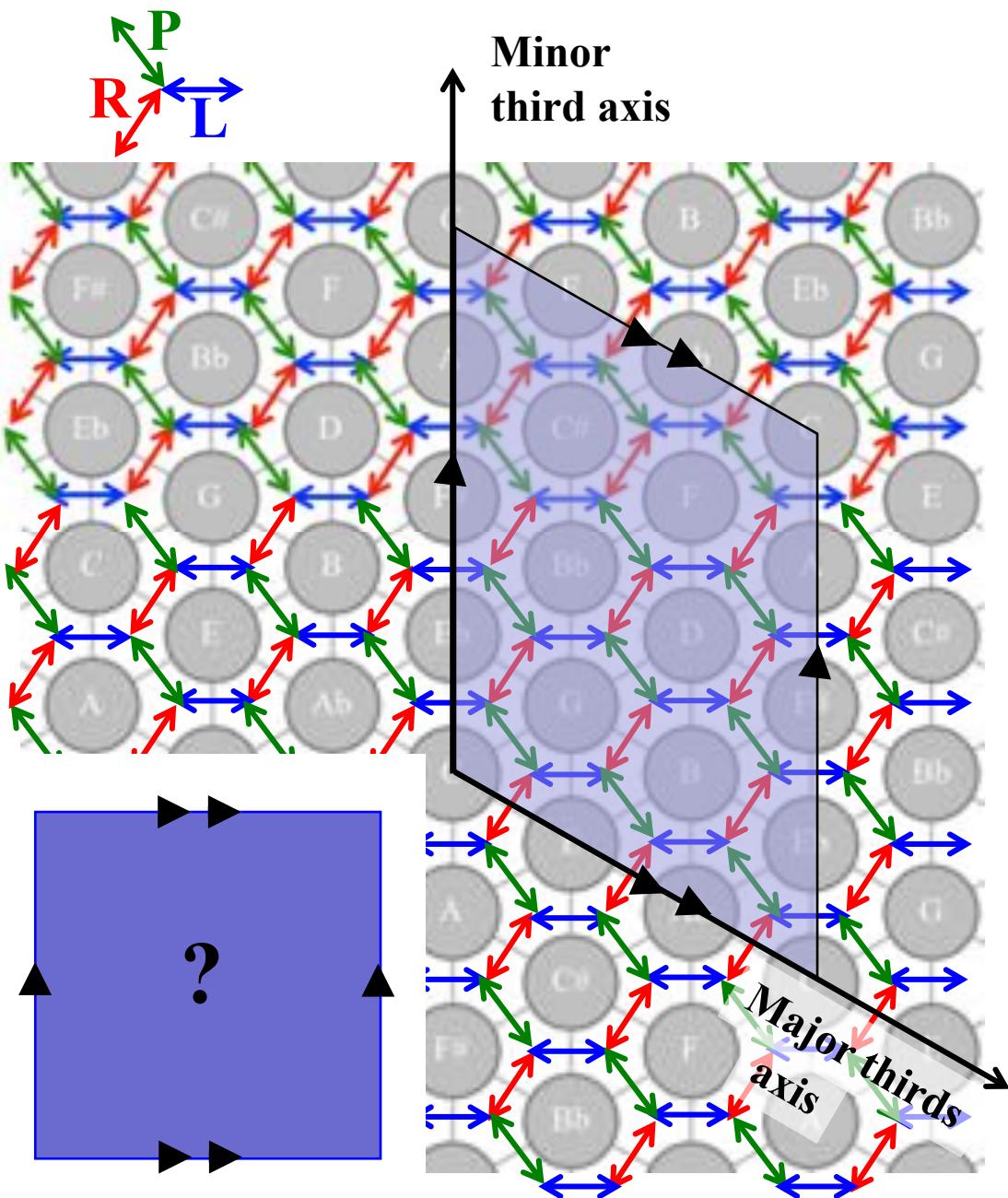
Supervision Moreno Andreatta  
Modelisation Gilles Baroin 2016



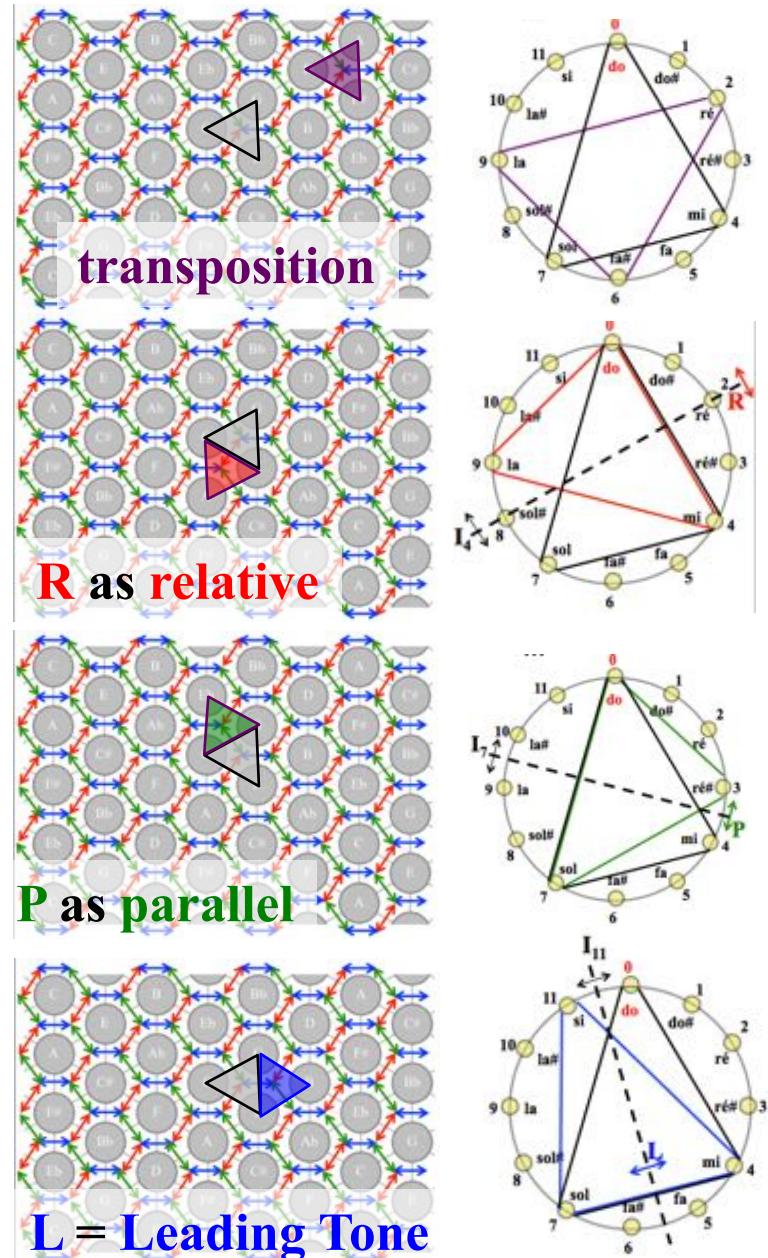
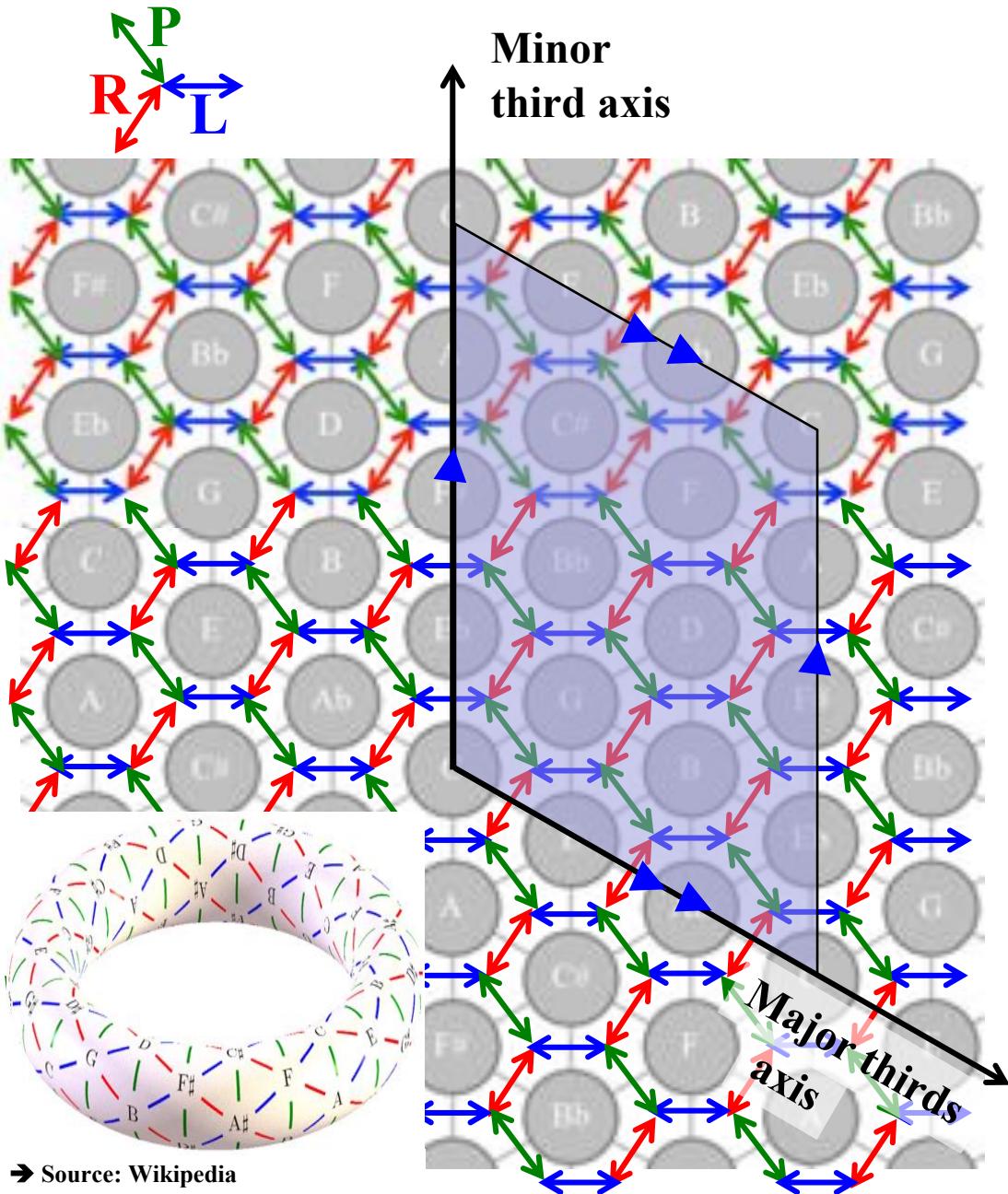
Gilles Baroin

→ [www.mathemusic.net](http://www.mathemusic.net)

# The Tonnetz, its symmetries and its topological structure



# The Tonnetz, its symmetries and its topological structure

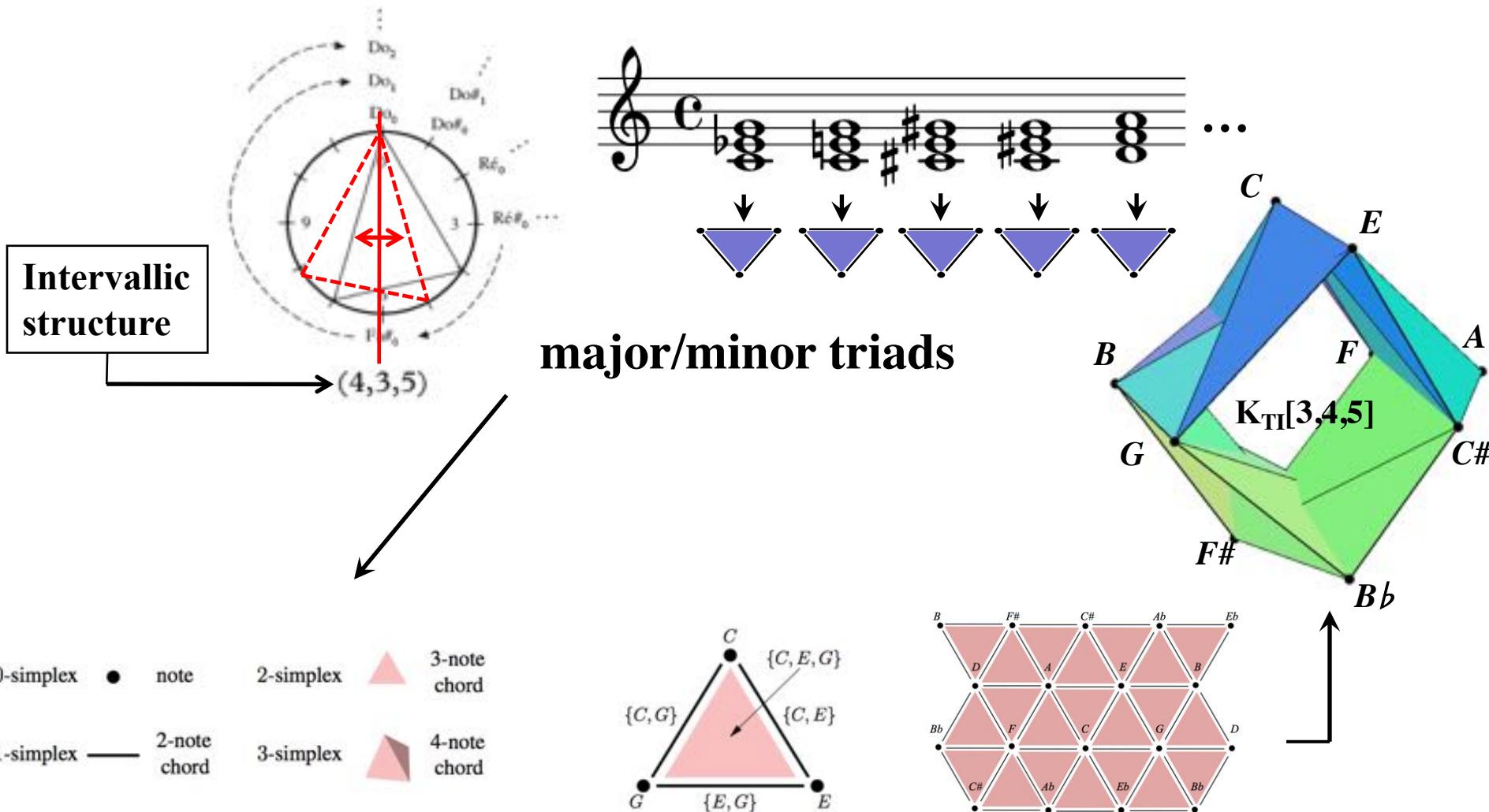


# The Tonnetz as a simplicial complex

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013



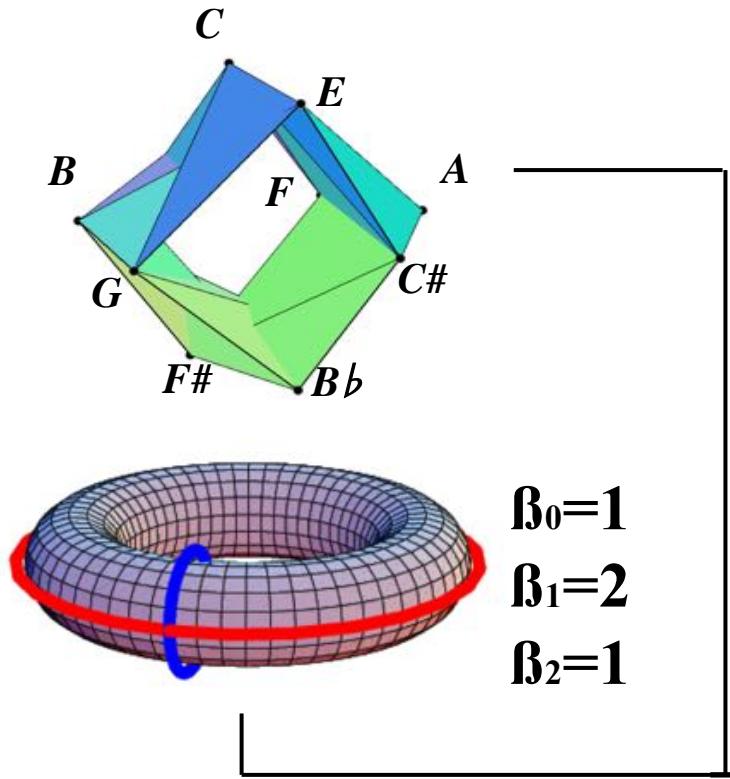
- Assembling chords related by some equivalence relation
  - Transposition/inversion: Dihedral group action on  $P(\mathbb{Z}_n)$



# Classifying Chord Complexes

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- Complexes enumeration in the chromatic system

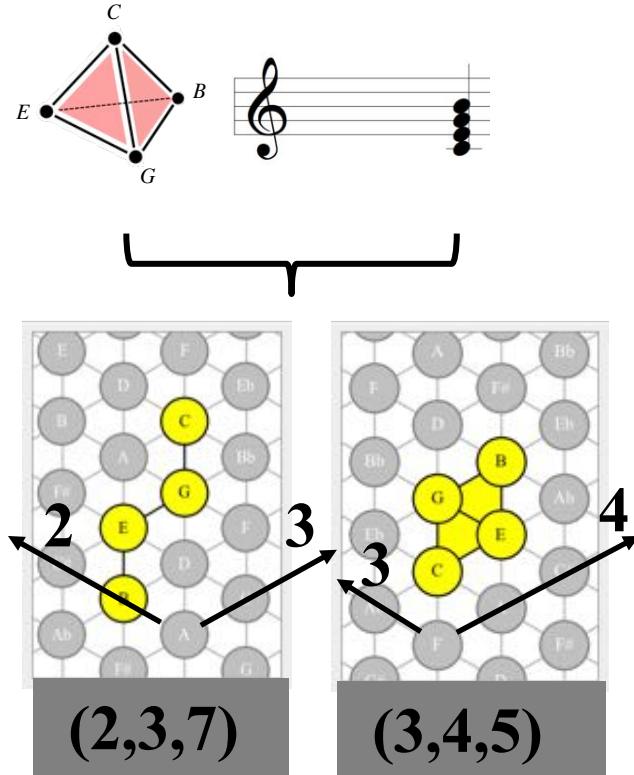


$d$	complexe	taille	$b_n$	p-v	$\chi$
-	$\mathcal{K}_\emptyset$	0	[0]		0
0	$\mathcal{K}_{TI}[0]$	0	[0]		0
	$\mathcal{K}_{TI}[1, 11]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[2, 10]$	12	[2, 2]		0
	$\mathcal{K}_{TI}[3, 9]$	12	[3, 3]		0
	$\mathcal{K}_{TI}[4, 8]$	12	[4, 4]		0
	$\mathcal{K}_{TI}[5, 7]$	12	[1, 1]	x	0
1	$\mathcal{K}_{TI}[6, 6]$	6	[6, 0]		6
	$\mathcal{K}_{TI}[1, 1, 10]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[1, 2, 9]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1, 3, 8]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1, 4, 7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1, 5, 6]$	24	[1, 1, 6]		6
	$\mathcal{K}_{TI}[2, 2, 8]$	12	[2, 2, 0]		0
	$\mathcal{K}_{TI}[2, 3, 7]$	24	[1, 2, 1]	x	0
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	$\mathcal{K}_{TI}[2, 5, 5]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[3, 3, 6]$	12	[3, 0, 3]		6
	$\mathcal{K}_{TI}[3, 4, 5]$	24	[1, 2, 1]	x	0
2	$\mathcal{K}_{TI}[4, 4, 4]$	4	[4, 0, 0]		4
	$\mathcal{K}_{TI}[1, 1, 1, 9]$	12	[1, 1, 0, 0]	x	0
	$\mathcal{K}_{TI}[1, 1, 2, 8]$	24	[1, 1, 12, 0]		12
	$\mathcal{K}_{TI}[1, 1, 3, 7]$	24	[1, 2, 13, 0]		12
	$\mathcal{K}_{TI}[1, 1, 4, 6]$	24	[1, 1, 18, 0]		18
	$\mathcal{K}_{TI}[1, 1, 5, 5]$	12	[1, 1, 6, 0]		6

# Classifying Chord Complexes

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- The search of the optimal space

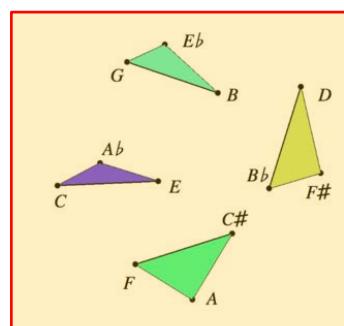
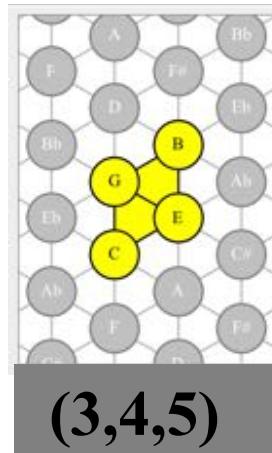
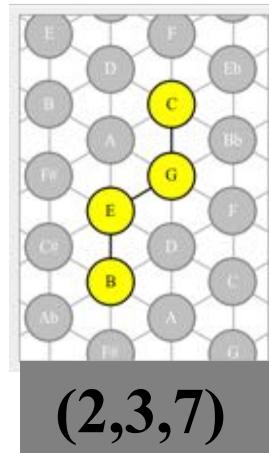
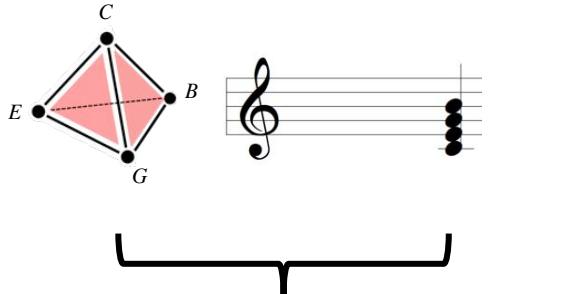


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	$\mathcal{K}_{TI}[5, 7]$	12	[1, 1]	x	0
1	$\mathcal{K}_{TI}[6, 6]$	6	[6, 0]		6
	$\mathcal{K}_{TI}[1, 1, 10]$	12	[1, 1, 0]	x	0
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	$\mathcal{K}_{TI}[1, 1, 4, 6]$	24	[1, 1, 18, 0]		18
	$\mathcal{K}_{TI}[1, 1, 5, 5]$	12	[1, 1, 6, 0]		6

# Classifying Chord Complexes

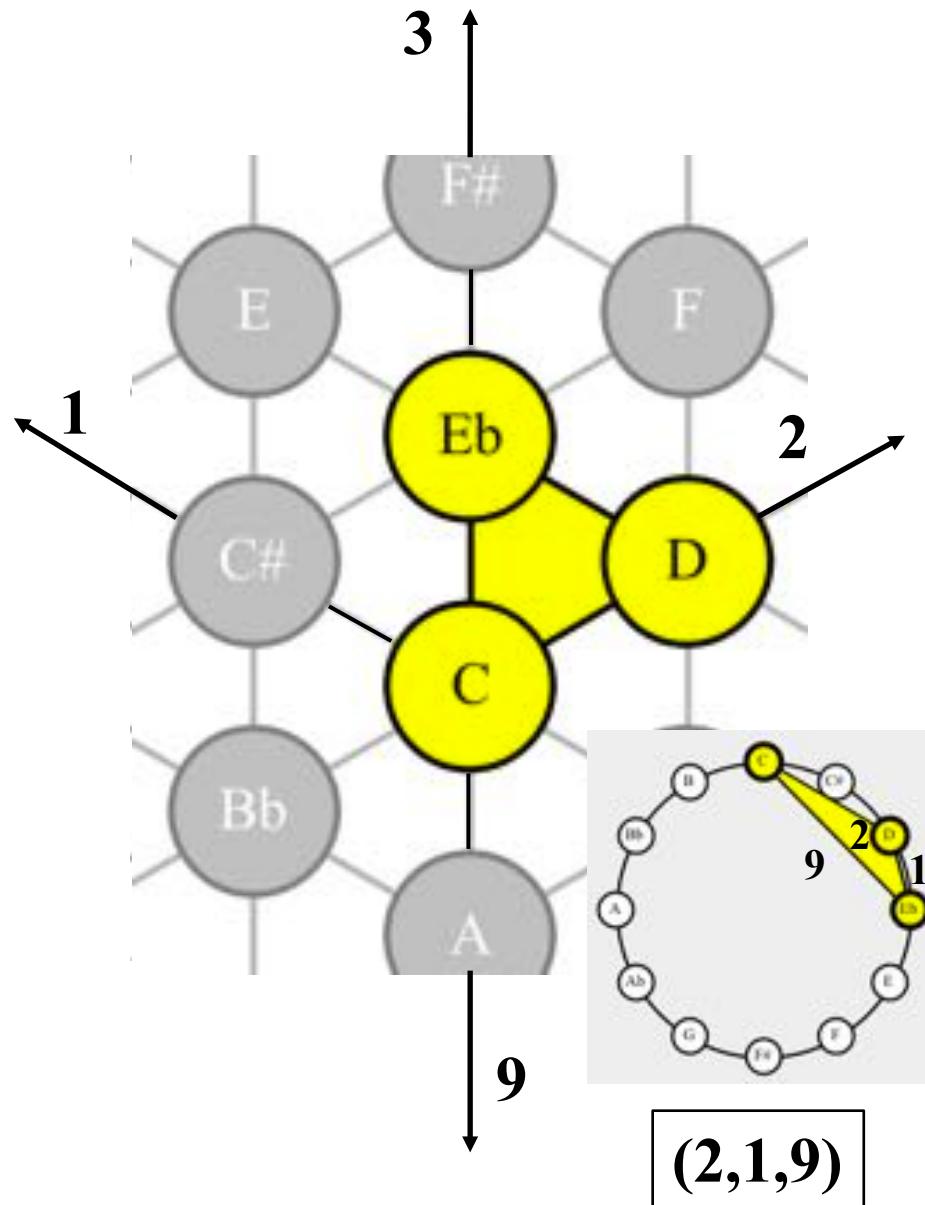
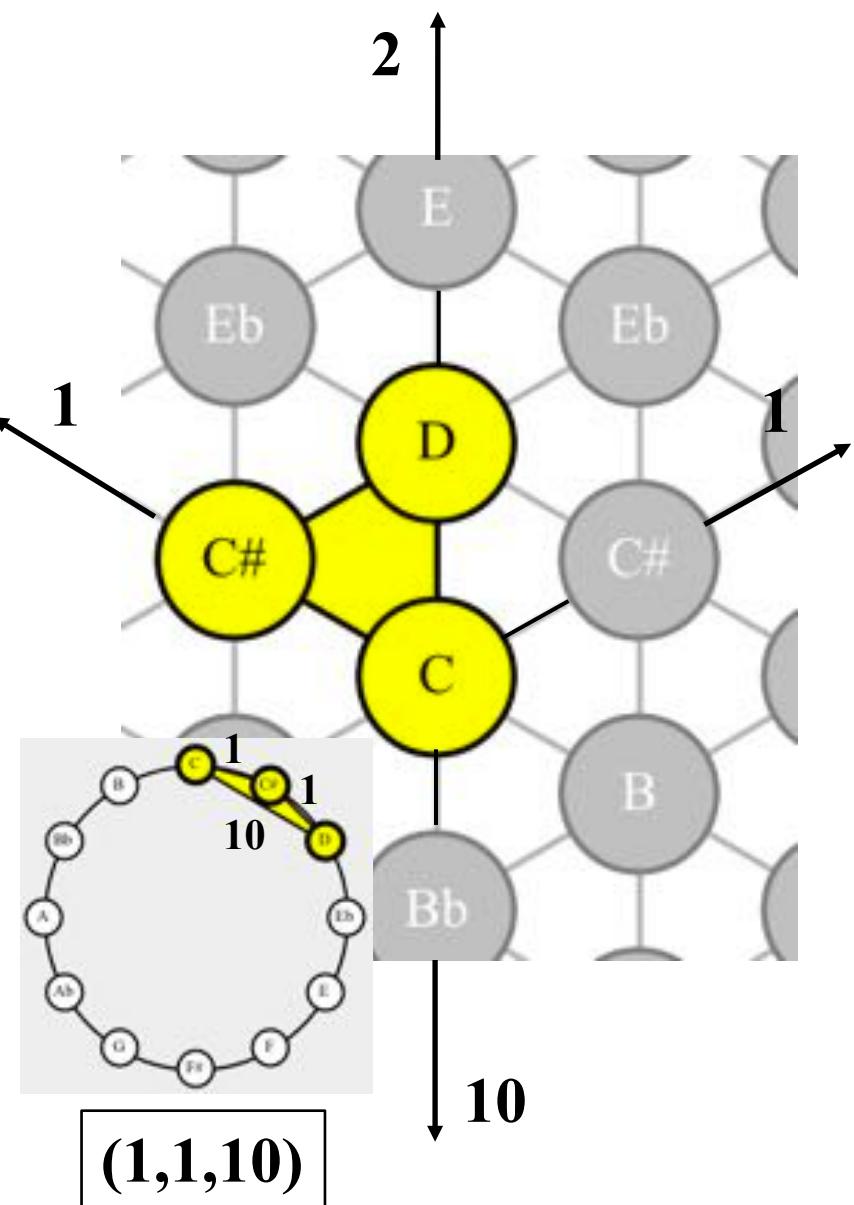
L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- The search of the optimal space

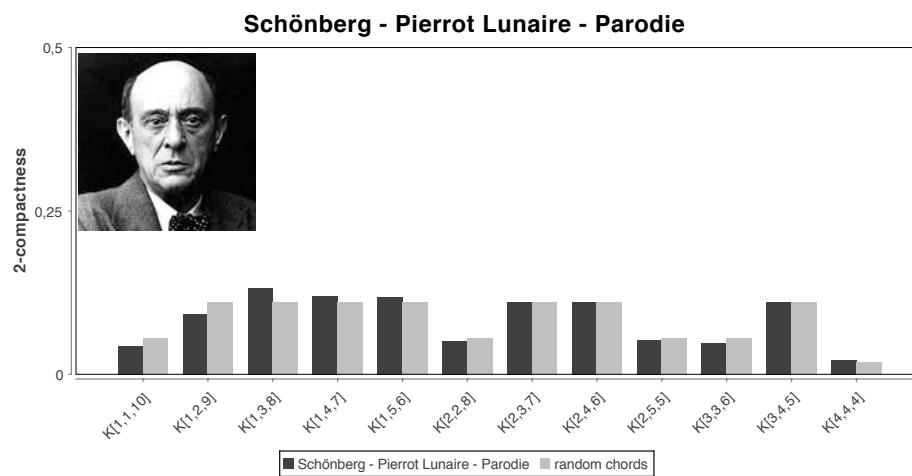
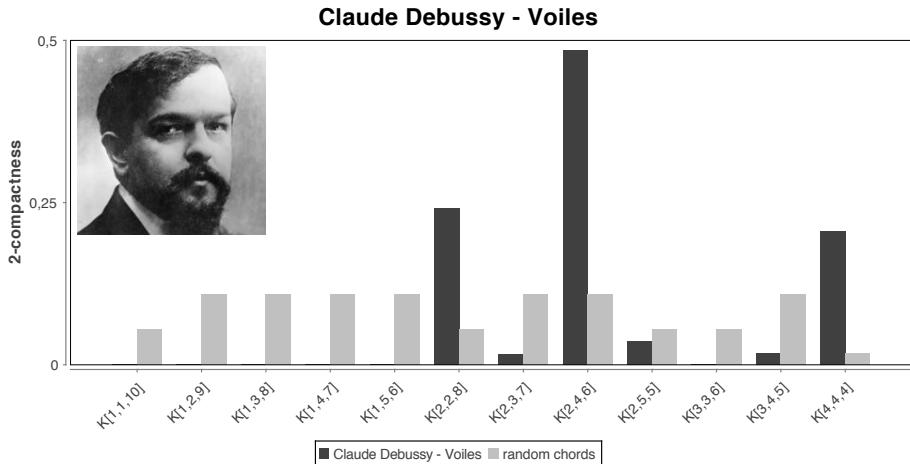
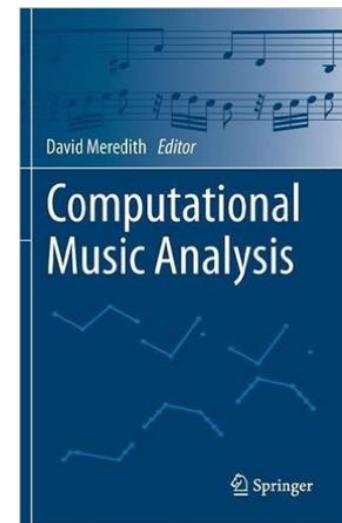
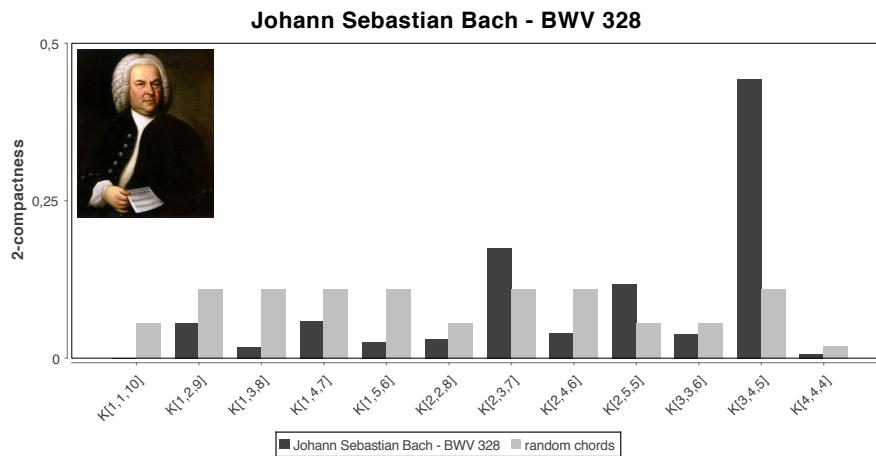
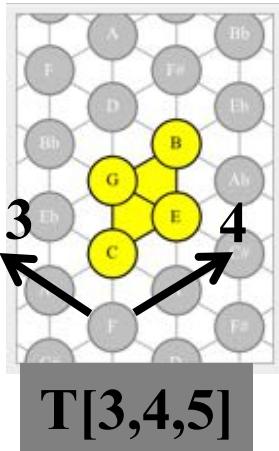
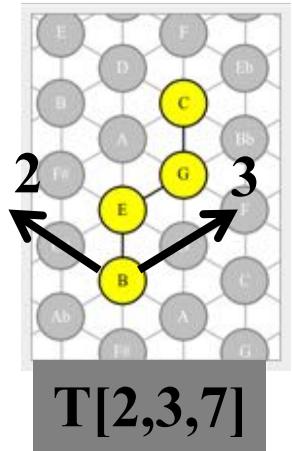


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# The panoply of *Tonnetze* at the service of the analyst

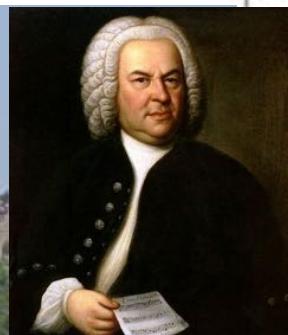
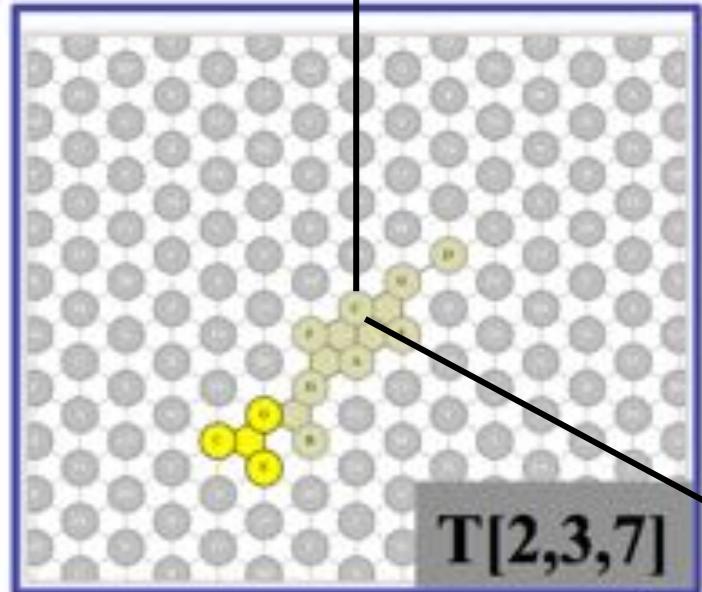
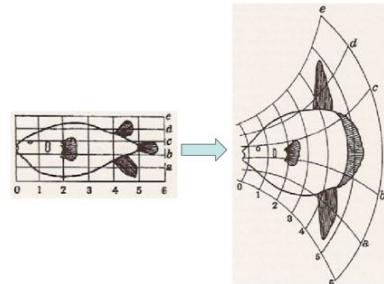
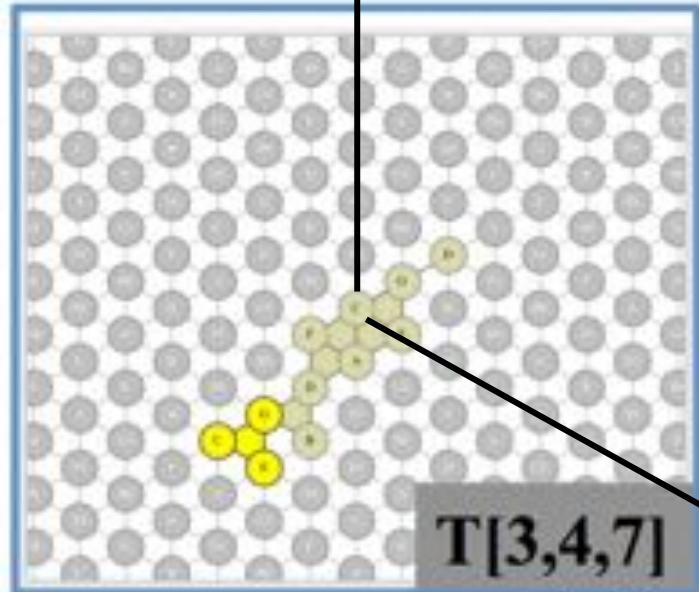


# The geometric character of musical logic



Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015

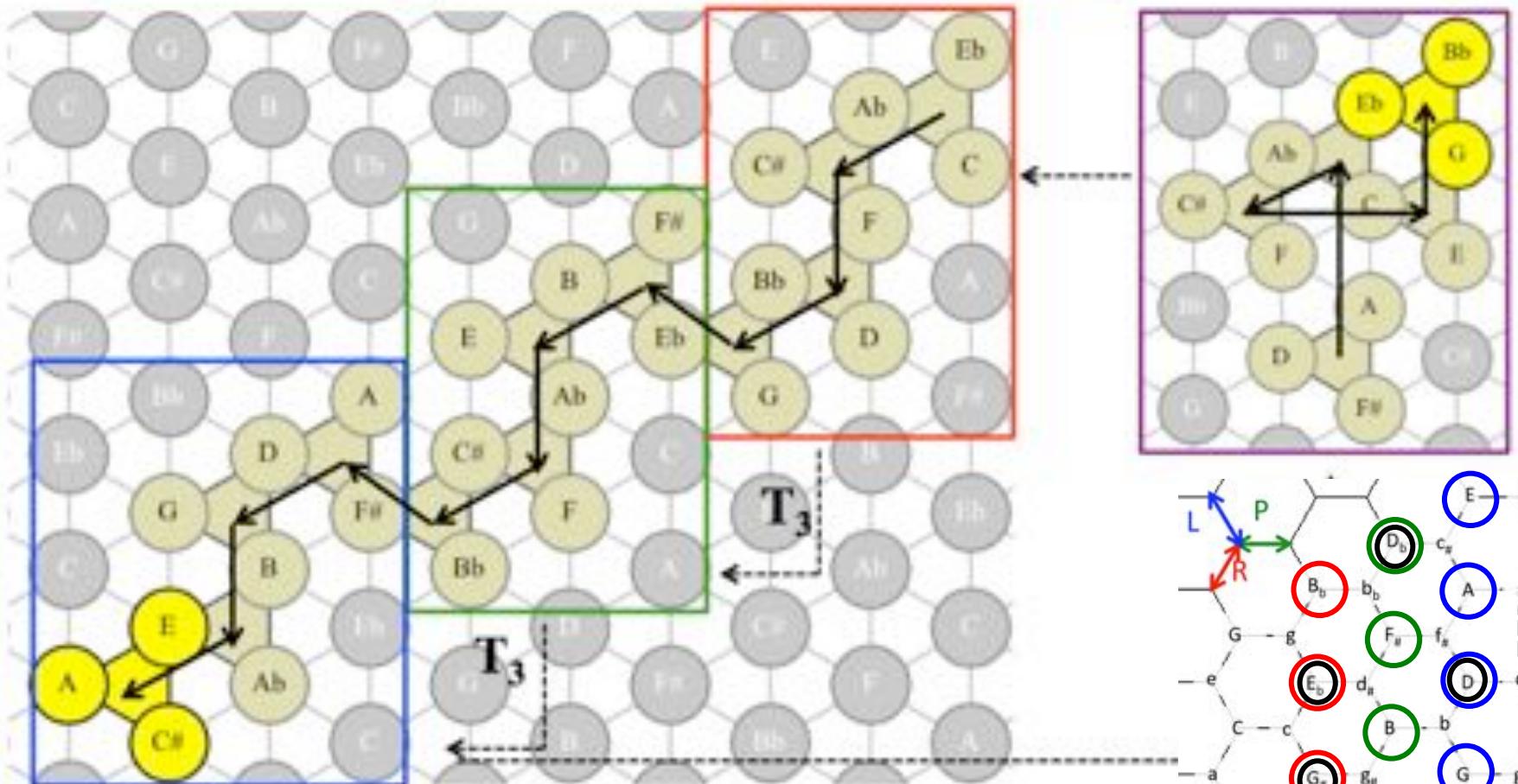
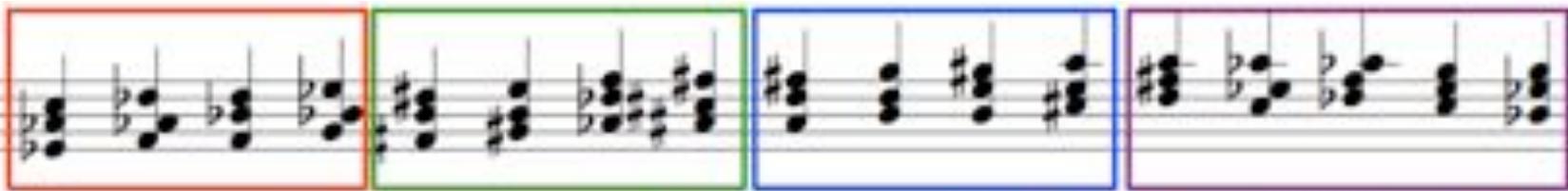
# The musical style...is the space!



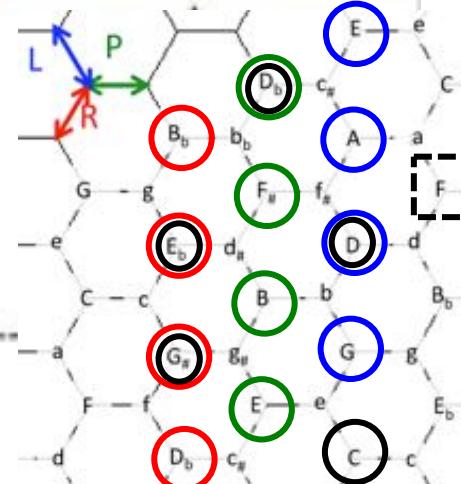


# Symmetries in Paolo Conte's *Madeleine*

La<sub>b</sub> Re<sub>b</sub> Si<sub>b</sub> Mi<sub>b</sub> Si Mi Re<sub>b</sub> Fa<sub>#</sub> Re Sol Mi La Re La<sub>b</sub> Re<sub>b</sub> Do Mi<sub>b</sub>



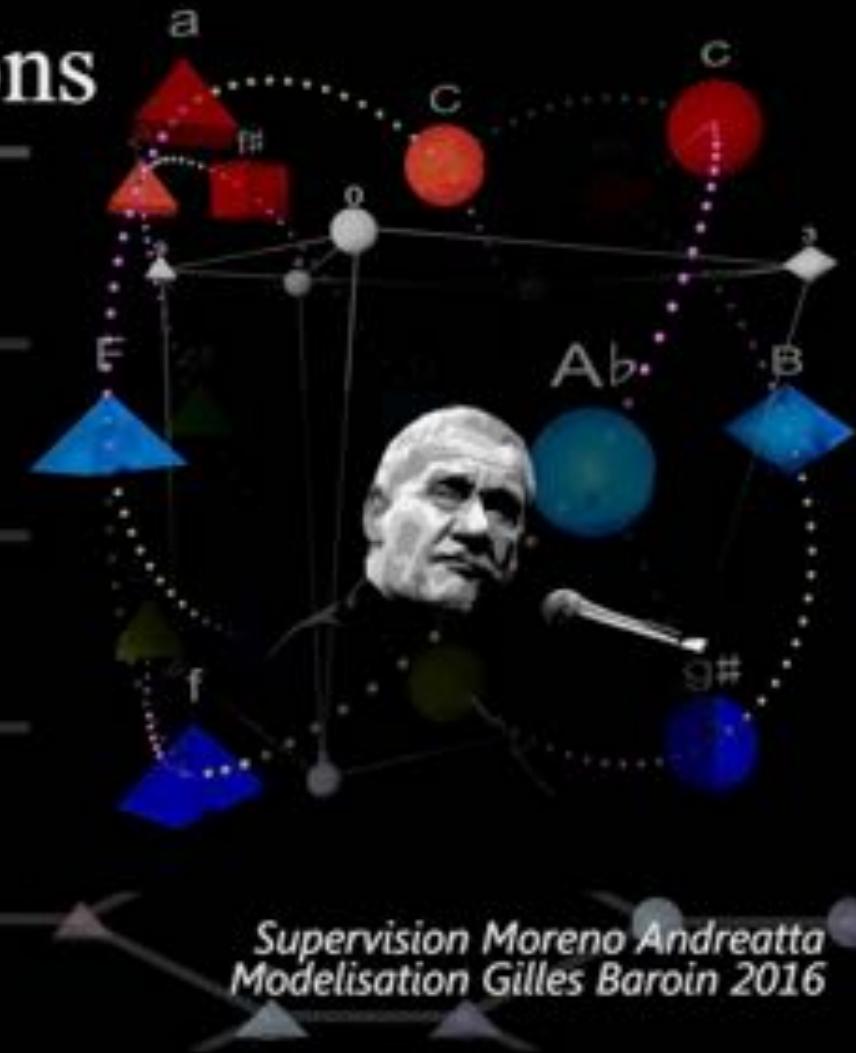
Almost total covering of the major-chords space



# Harmonic Progressions

In Paolo Conte

Madeleine



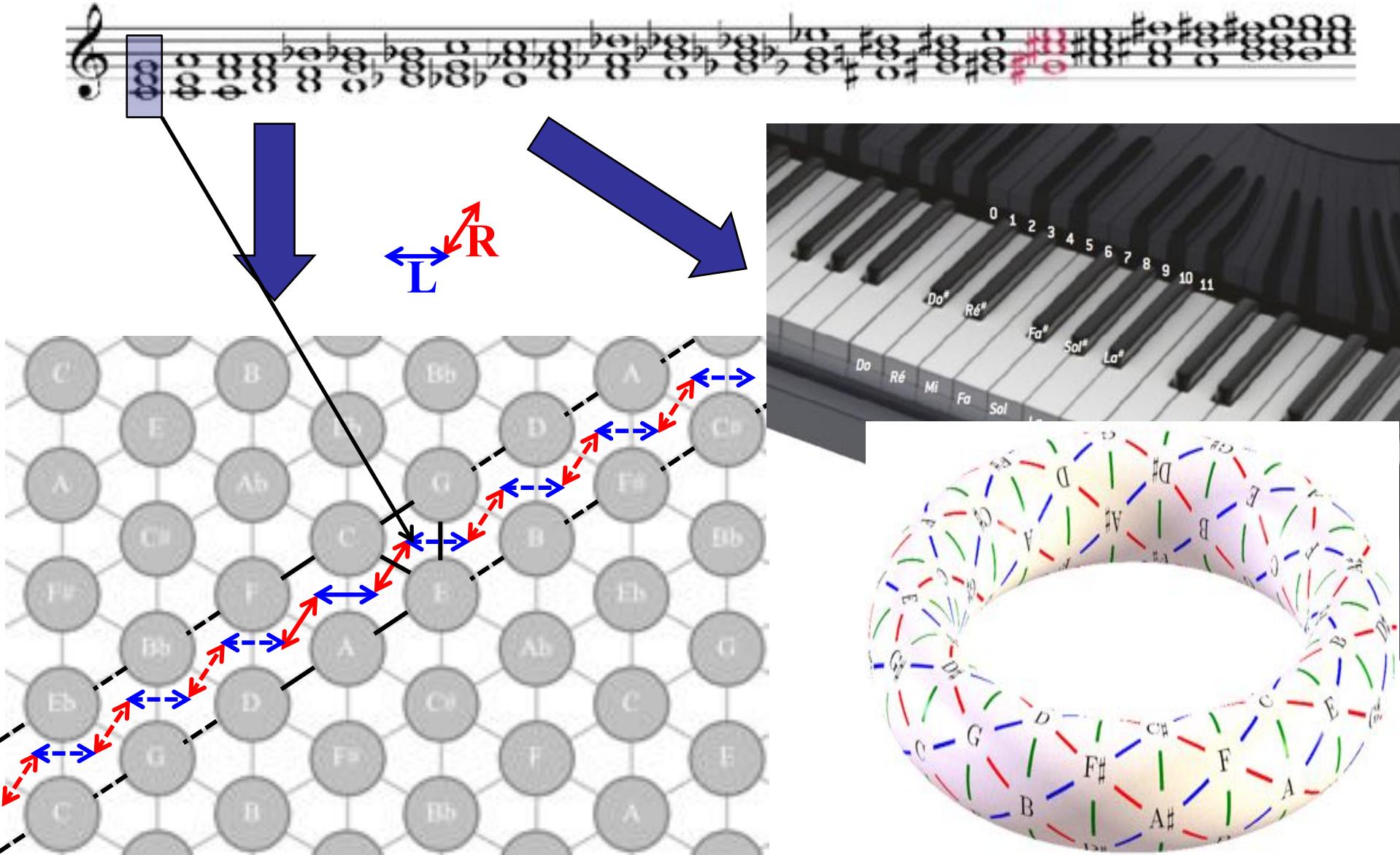
Supervision Moreno Andreatta  
Modélisation Gilles Baroin 2016



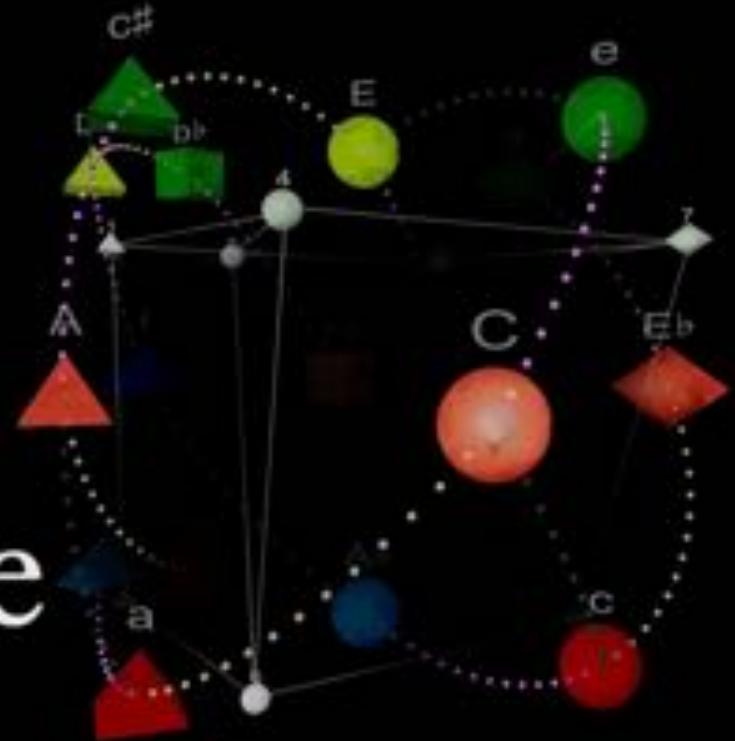
Gilles Baroin

→ [www.mathemusic.net](http://www.mathemusic.net)

# Harmonic progressions as spatial trajectories



# Beethoven and the Hypersphere *(and the Tonnetz)*



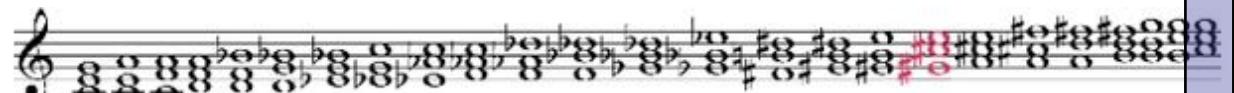
Gilles Baroin 2016  
[www.MatheMusic.net](http://www.MatheMusic.net)



Gilles Baroin

→ [www.mathemusic.net](http://www.mathemusic.net)

# Reading Beethoven backwards

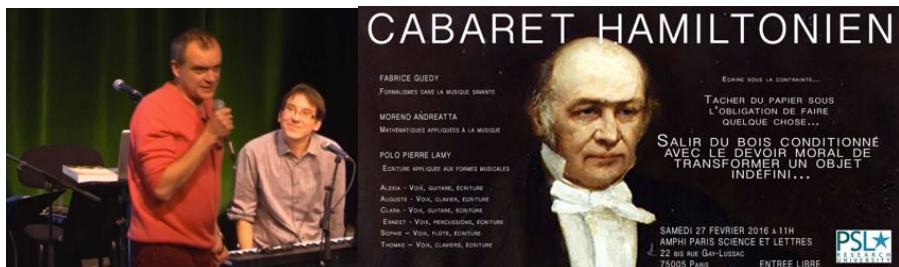
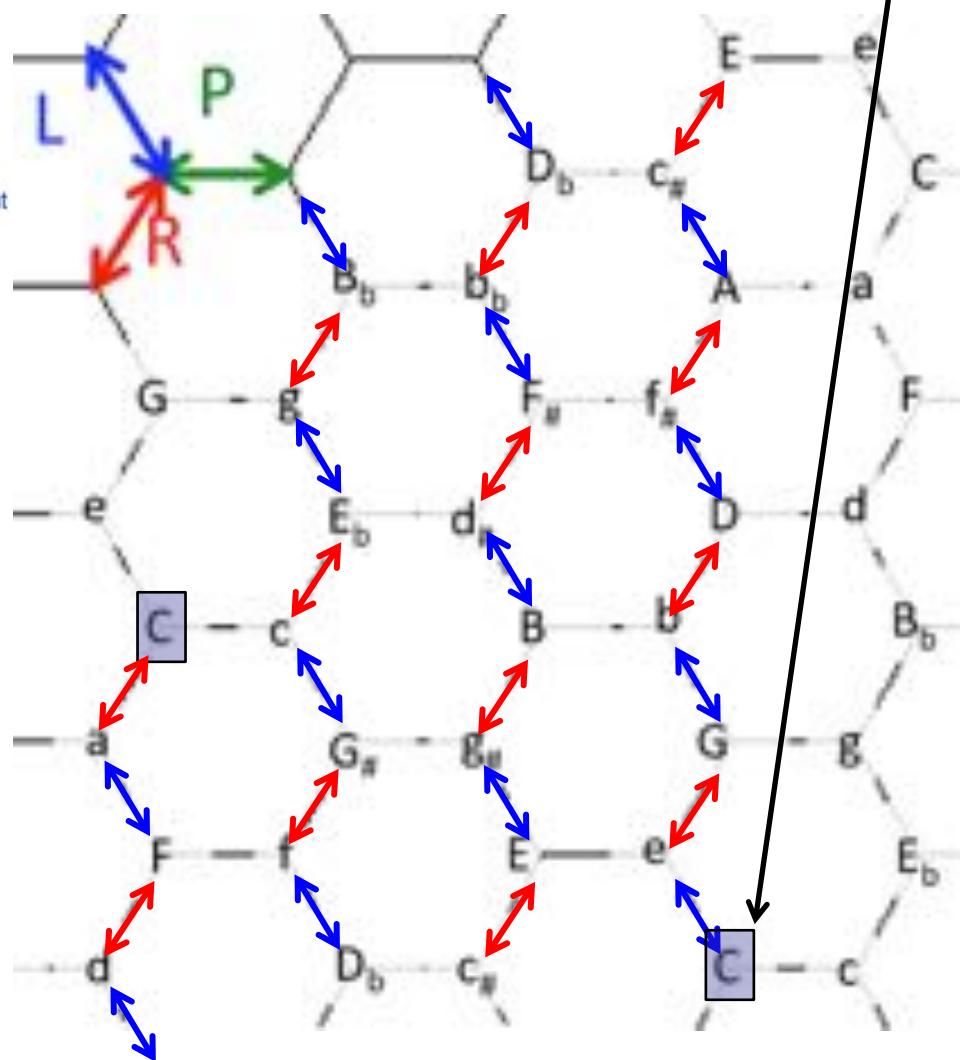


← time

## Le Blé en Herbe

(Polo/Moreno/Dieu)

- |  |  |
|--|--|
| Plonger comme un enfant, cheveux au vent | Croiser matin dans l'herbe folle       |
| Sous l'océan du blé en herbe             | Deux tourterelles qui s'envolent       |
| Marée d'épis couleur d'amande            | Suivre les jeux des hirondelles        |
| Qui tendent à caresser le ciel           | Sur le paysage éternel                 |
| Algues tendres de mille plages           | Nager comme un enfant, cheveux au vent |
| Frôlant le ventre des nuages             | Sous l'océan                           |
| Cheveux de pluie, dos de poissons        | Du blé en herbe                        |
| Qui frissonnent à l'unisson              | Marée de fruits au goût amer           |
| Suivre le bord des continents            | Acide et salée comme la mer            |
| Dans l'océan du blé en herbe             | Vers l'îlot d'un petit village         |
| Pêcher le corail du pavot                | Vers un château d'eau sur la plage     |
| Dans le sang des coquelicots             | Quand tout s'éteint avant l'orage      |
|  | Quand se lève le vent du large         |
|  | Sur le blé vert                        |



# The collection of 124 Hamiltonian Cycles

ACTIONS

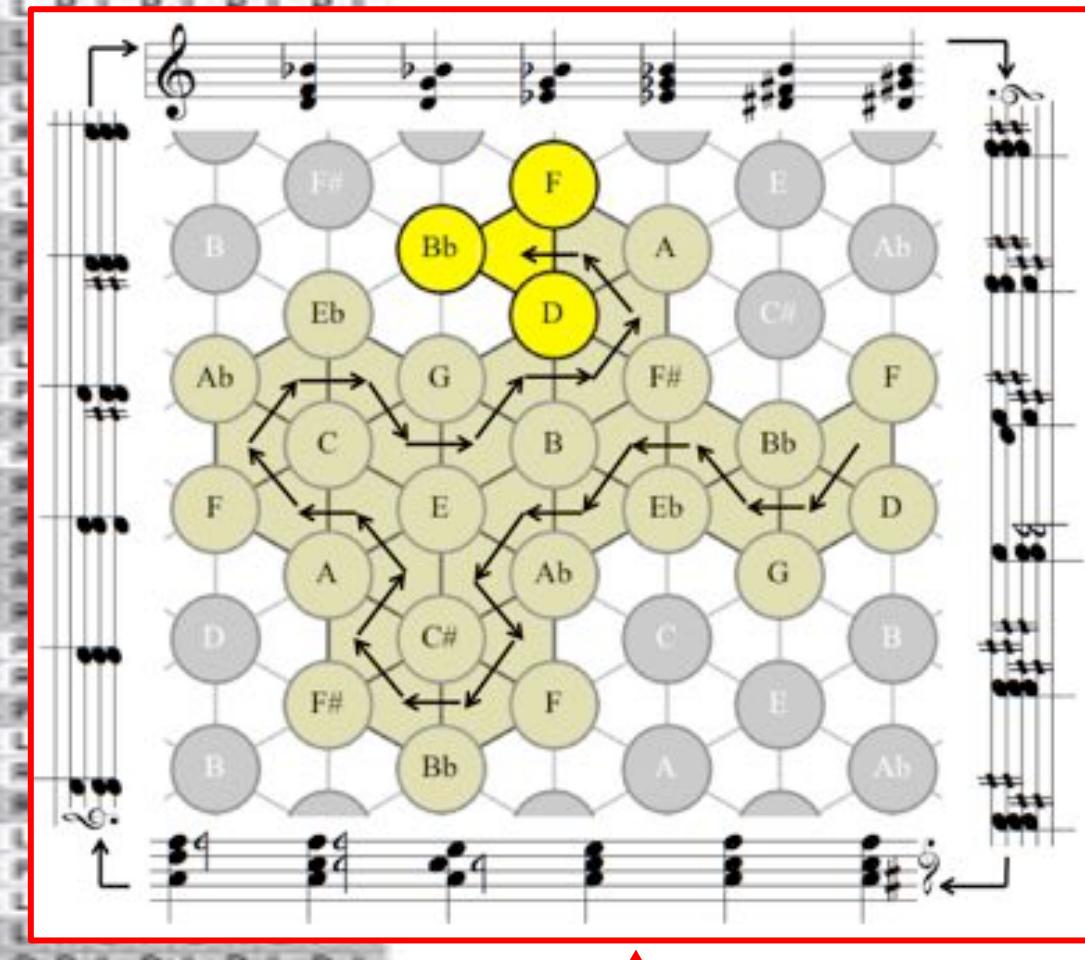
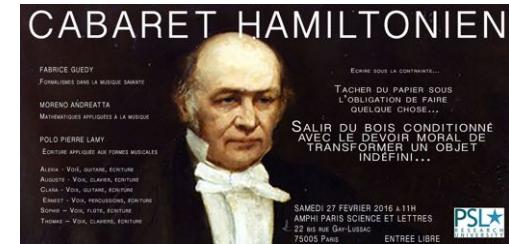
Math'n'pop

**Aprile (d'après Gabriele D'Annunzio)**

#41 LRLRLLRLLRLRLLRLLRLLRLLR  
#42 LPLPLRLPLPRLPLPRLPLPRLPLP  
#43 PLRLPPLPLPRLPRLPLPRLPRLPLP

#44 PLPLRLPLPRLPRLPLPRLPRLPLP  
#45 LPLPLRLPRLPRLPRLPRLPRLPRLP  
#46 PLPLPRLPLPLPRLPRLPRLPRLPRLP  
#47 PLPLPRLPLPLPRLPRLPRLPRLPRLP  
#48 PLPLPRLPLPLPRLPRLPRLPRLPRLP  
#49 PLPLPRLPLPLPRLPRLPRLPRLPRLP  
#50 PLPLPRLPLPLPRLPRLPRLPRLPRLP  
#51 PRPLPLPRLPLPLPRLPRLPRLPRLP  
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**Tangente**  
L'aventure mathématique



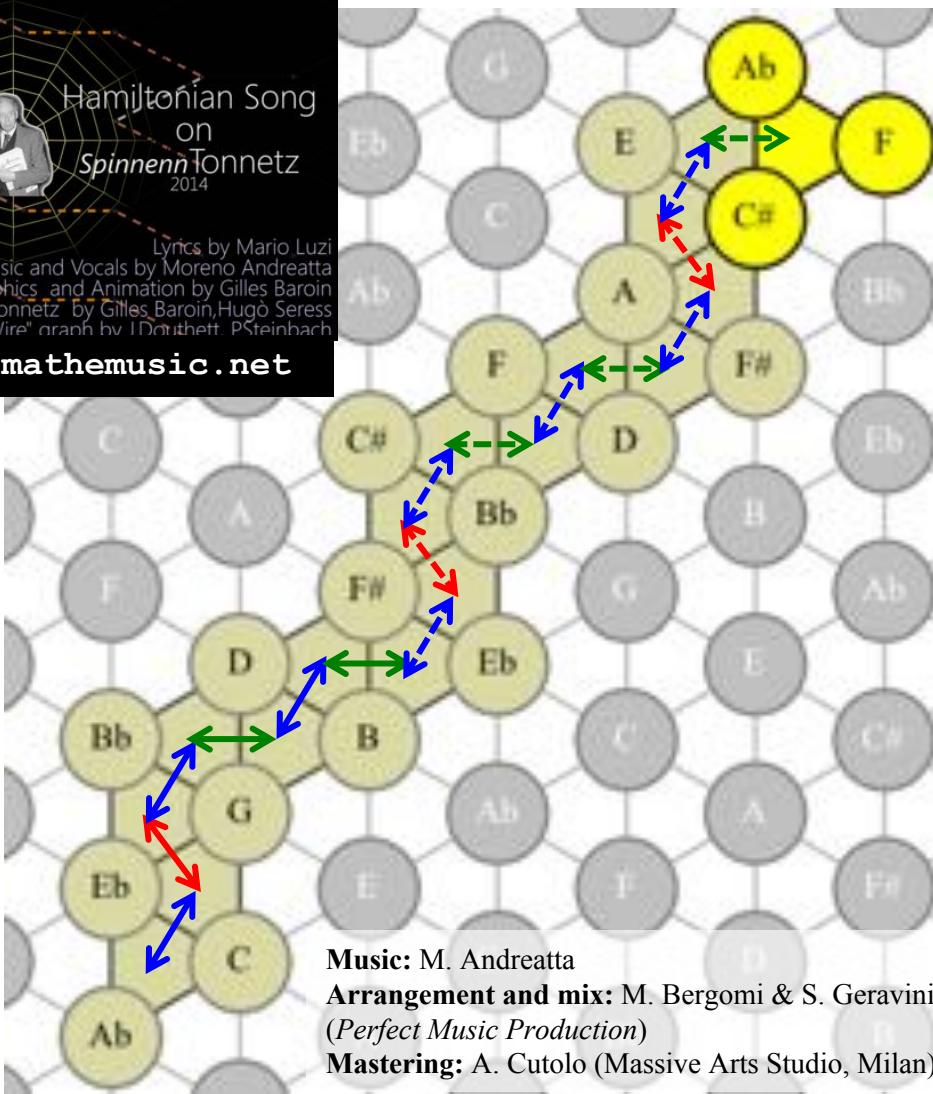
# Hamiltonian cycles with inner periodicity

8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR



L P L P L R ...  
 P L P L R L ...  
 L P L R L P ...  
 PL R L P L ...  
**L R L P L P ...**  
 R L P L P L ...

Luzi



La sera non è più la tua canzone  
 (Mario Luzi, 1945, in *Poesie sparse*)

La sera non è più la tua canzone,  
 è questa roccia d'ombra traforata  
 dai lumi e dalle voci senza fine,  
 la quiete d'una cosa già pensata.

Ah questa luce viva e chiara viene  
 solo da te, sei tu così vicina  
 al vero d'una cosa conosciuta,  
 per nome hai una parola ch'è passata  
 nell'intimo del cuore e s'è perduta.

Caduto è più che un segno della vita,  
 riposi, dal viaggio sei tornata  
 dentro di te, sei scesa in questa pura  
 sostanza così tua, così romita  
 nel silenzio dell'essere, (compiuta).

L'aria tace ed il tempo dietro a te  
 si leva come un'arida montagna  
 dove vaga il tuo spirito e si perde,  
 un vento raro scivola e ristagna.

# Luzi



## Hamiltonian Song on *SpinnenTonnetz* 2014

Lyrics by Mario Luzi

Music and Vocals by Moreno Andreatta

Graphics and Animation by Gilles Baroin

*SpinnenTonnetz* by Gilles Baroin, Hugò Seress

Original "Chicken Wire" graph by J.Douthett, P.Steinbach



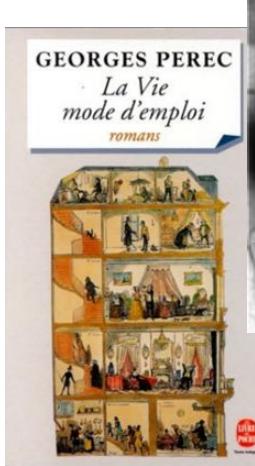
Gilles Baroin

→ [www.mathemusic.net](http://www.mathemusic.net)

# The use of constraints in arts



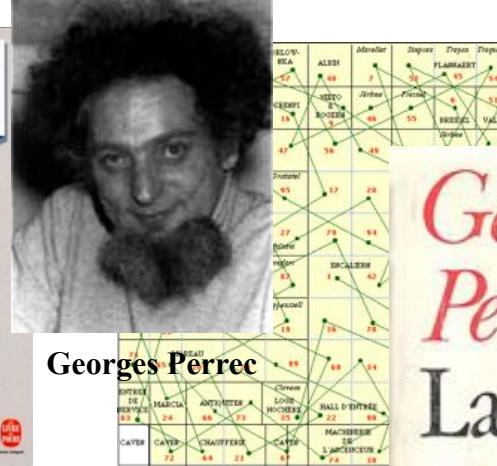
*Cent mille milliards de poèmes*, 1961



*La vie mode d'emploi*,



Georges Perec



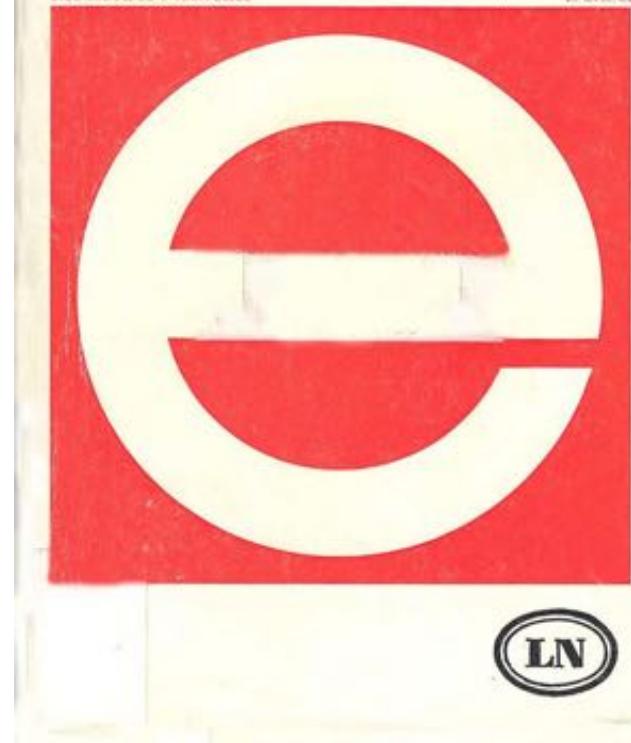
*Georges  
Perec*

Roman

*La disparition*

Les Lettres Nouvelles

Denoël



Raymond Queneau



Italo Calvino

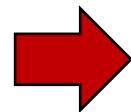
*Il castello dei destini  
incrociati*, 1969

LN

# From the OuLiPo to the OuMuPo (ouvroir de musique potentielle)



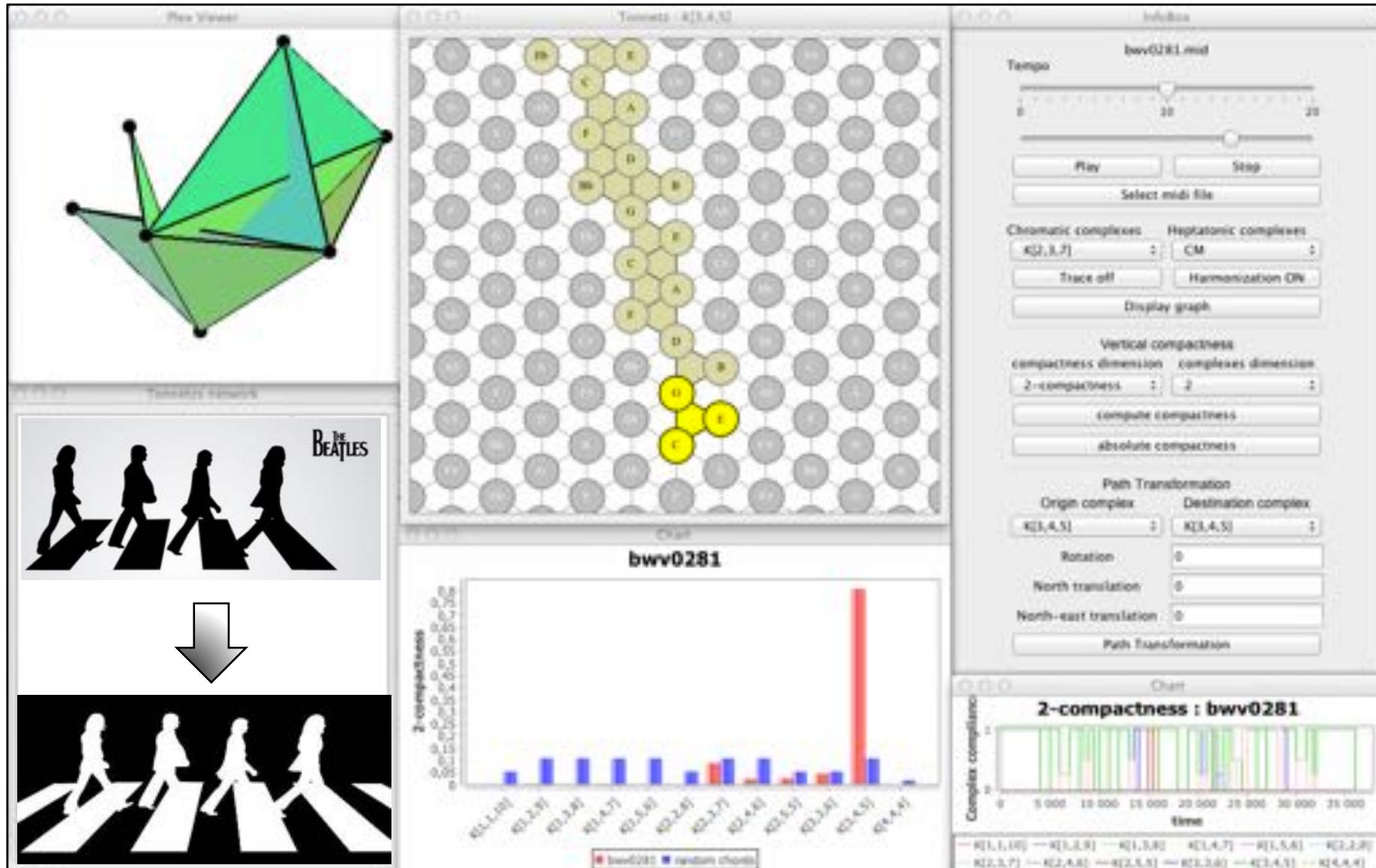
<http://oumupo.org/>



M. Andreatta et al., « Music, mathematics and language: chronicles from the Oumupo sandbox », in Kapoula, Z., Volle, E., Renault, J., Andreatta, M. (Eds.), *Exploring Transdisciplinarity in Art and Sciences*, Springer, 2018

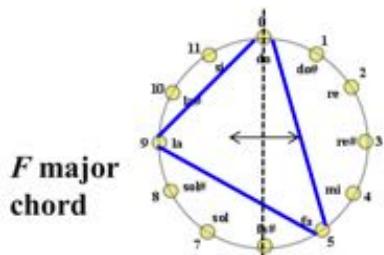
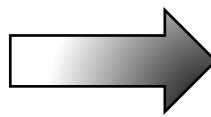


# Keeping the space...but changing the trajectory!

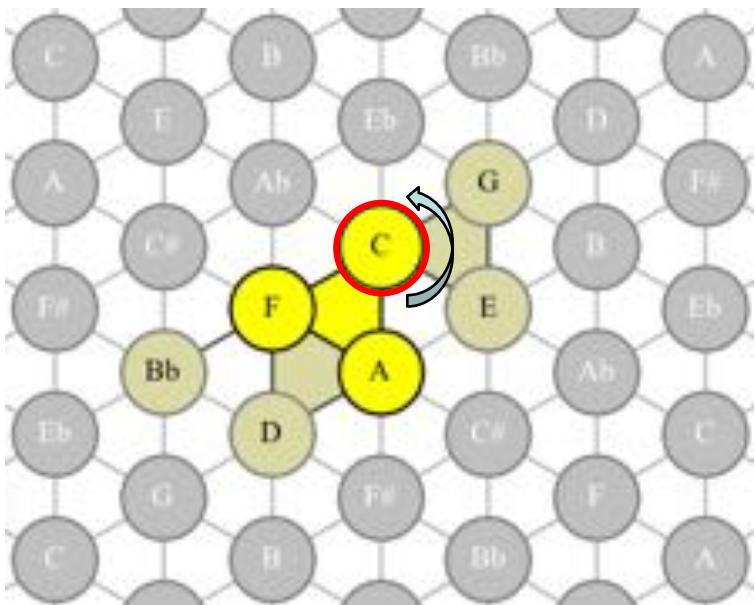
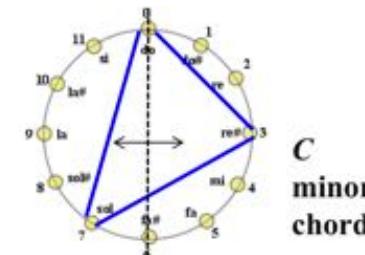


→ <http://www.lacl.fr/~lbigo/hexachord>

# Keeping the space...but changing the trajectory!

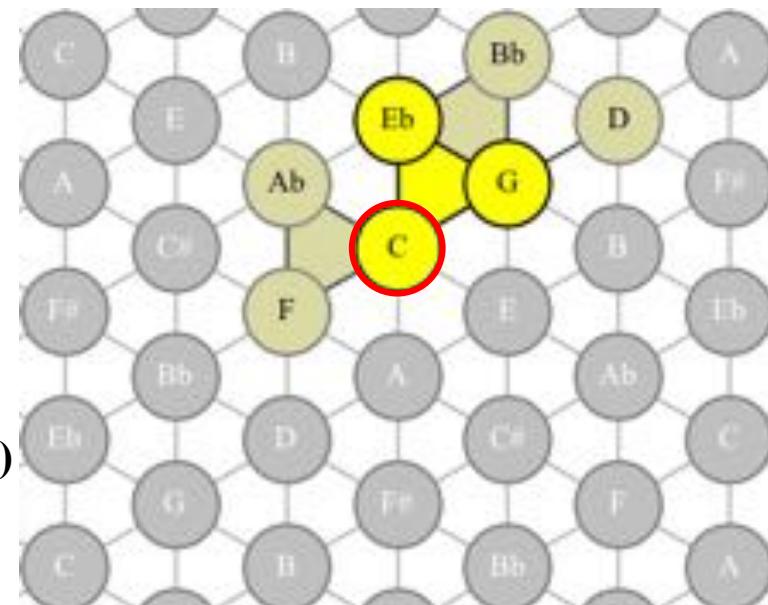


inversion



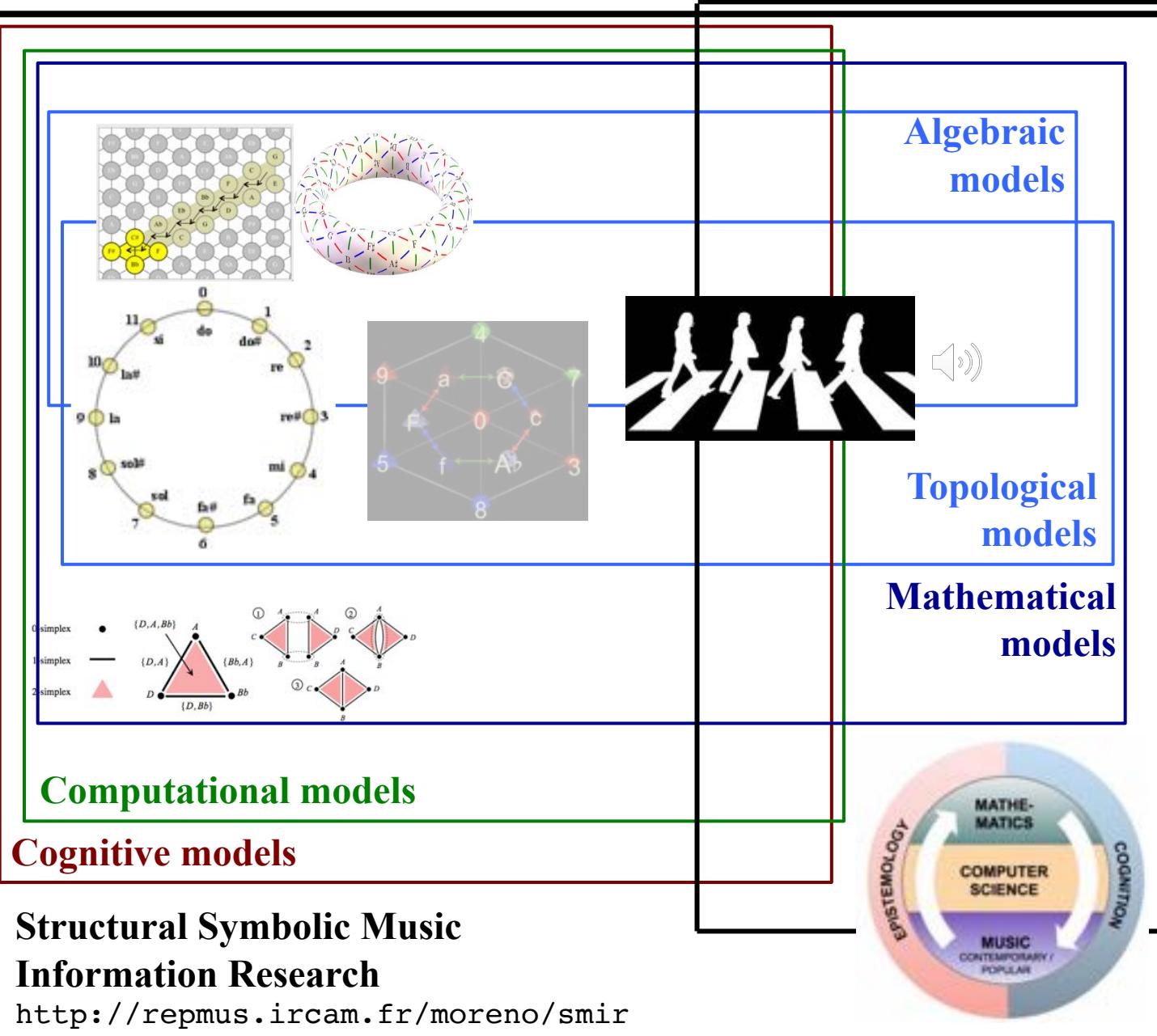
Beatles, Hey Jude (orig. version)

Rotation  
(autour du *do*)

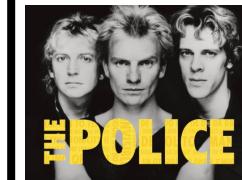


Beatles, Hey Jude (transformed version)

# The SMIR Project: Structural Music Information Research



Signal-based  
Music  
Information  
Retrieval



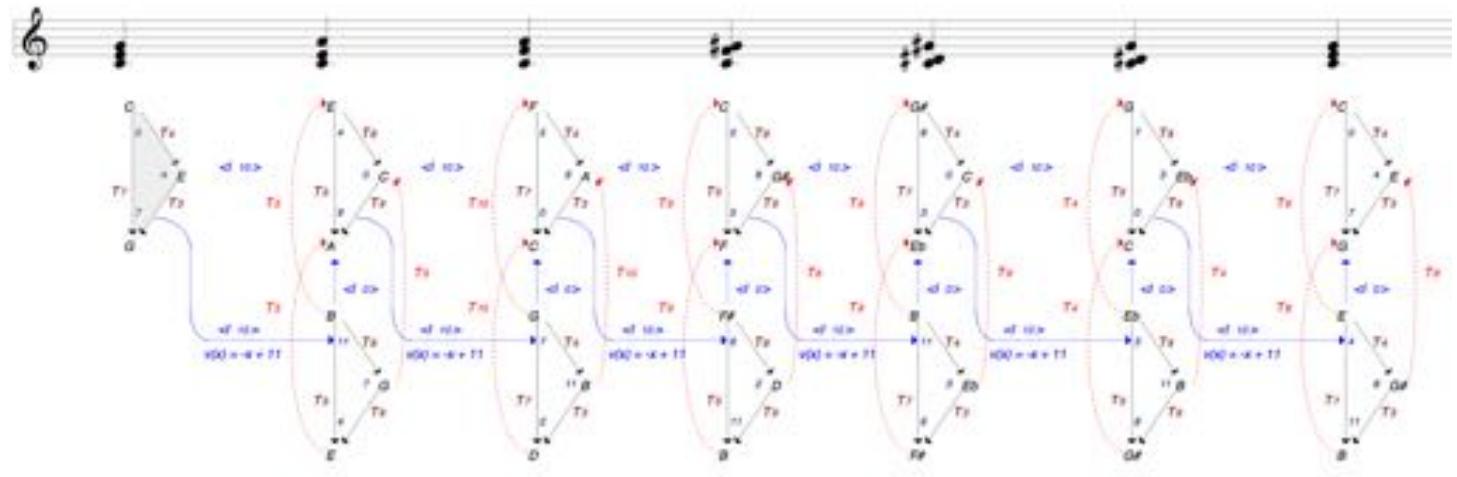
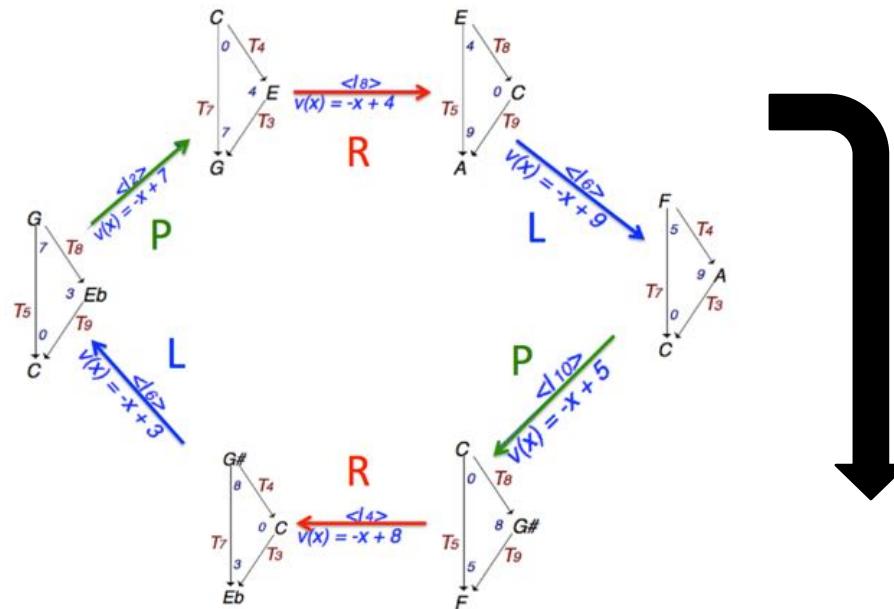
Oleg Berg



## Structural Symbolic Music Information Research

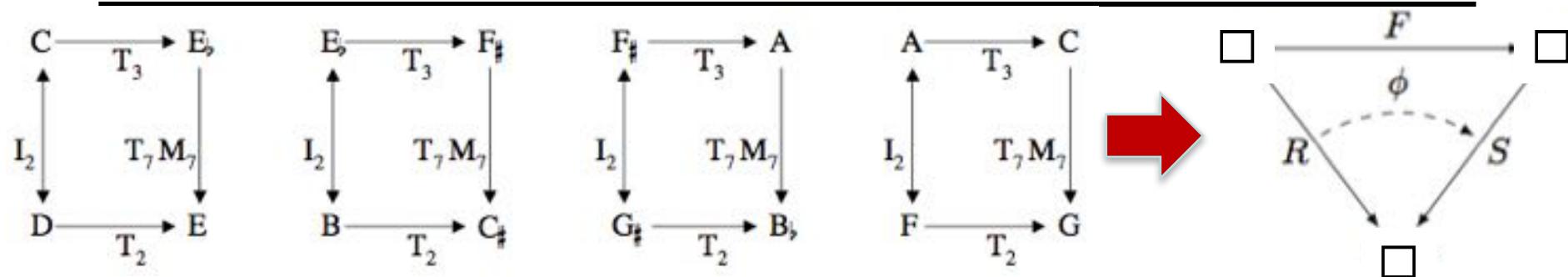
<http://repmus.ircam.fr/moreno/smir>

# Topological vs categorical construction of the Tonnetz



- Popoff A., C. Agon, M. Andreatta, A. Ehresmann (2016), « From K-Nets to PK-Nets: A Categorical Approach », PNM, 54(1)
- Popoff A., M. Andreatta, A. Ehresmann, « Relational PK-Nets for Transformational Music Analysis » (forthcoming in the JMM)

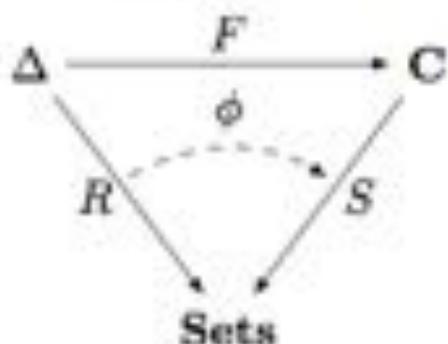
# From K-Nets to category-based PK-Nets



**Definition 1** Let  $\mathbf{C}$  be a category, and  $S$  a functor from  $\mathbf{C}$  to the category Sets. Let  $\Delta$  be a small category and  $R$  a functor from  $\Delta$  to Sets. A PK-net of form  $R$  and of support  $S$  is a 4-tuple  $(R, S, F, \phi)$ , in which

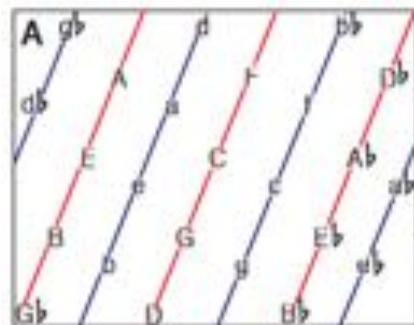
- $F$  is a functor from  $\Delta$  to  $\mathbf{C}$ ,
- and  $\phi$  is a natural transformation from  $R$  to  $SF$ .

The definition of a PK-net is summed up by the following diagram:



Popoff A., M. Andreatta, A. Ehresmann,  
« A Categorical Generalization of  
Klumpenhouwer Networks », MCM 2015,  
Queen Mary University, Springer, p. 303-314

# Some cognitive implications of mathemusical research

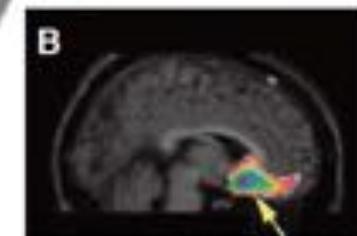
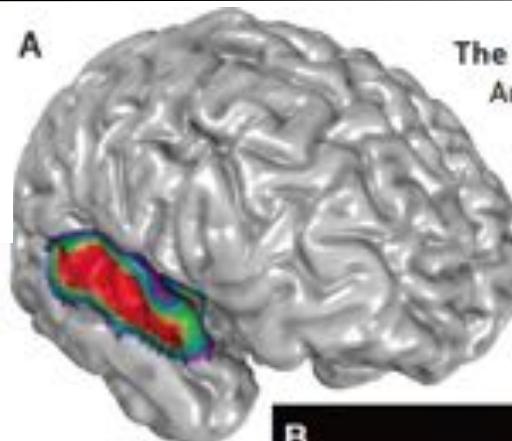
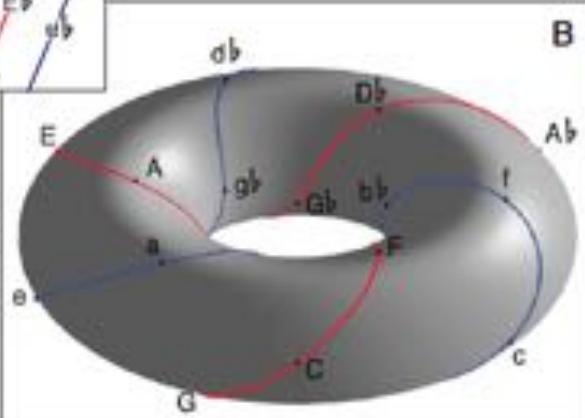


Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (B) Musical keys as points on the surface of a torus.

PERSPECTIVES: NEUROSCIENCE

## Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl

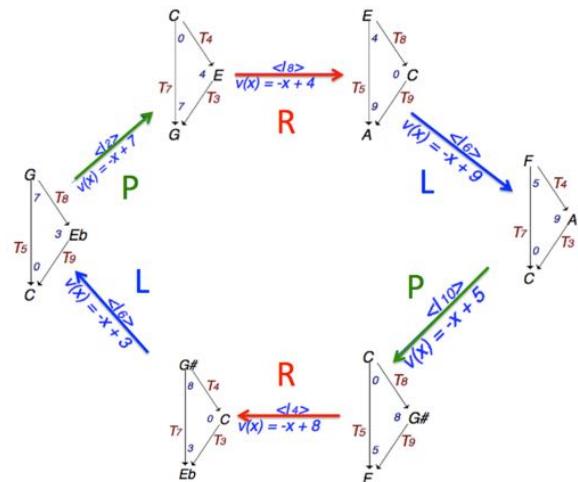


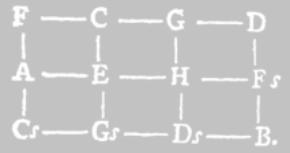
The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

« La théorie des catégories est une théorie des constructions mathématiques, qui est macroscopique, et procède d'étage en étage. Elle est un bel exemple d'abstraction réfléchissante, cette dernière reprenant elle-même un principe constructeur présent dès le stade sensorimoteur. Le style catégoriel qui est ainsi à l'image d'un aspect important de la genèse des facultés cognitives, est un style adéquat à la description de cette genèse »



J. Piaget





# THANK YOU FOR YOUR ATTENTION