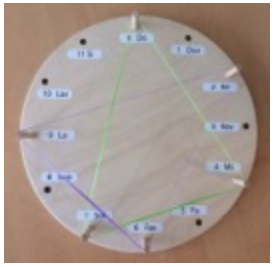
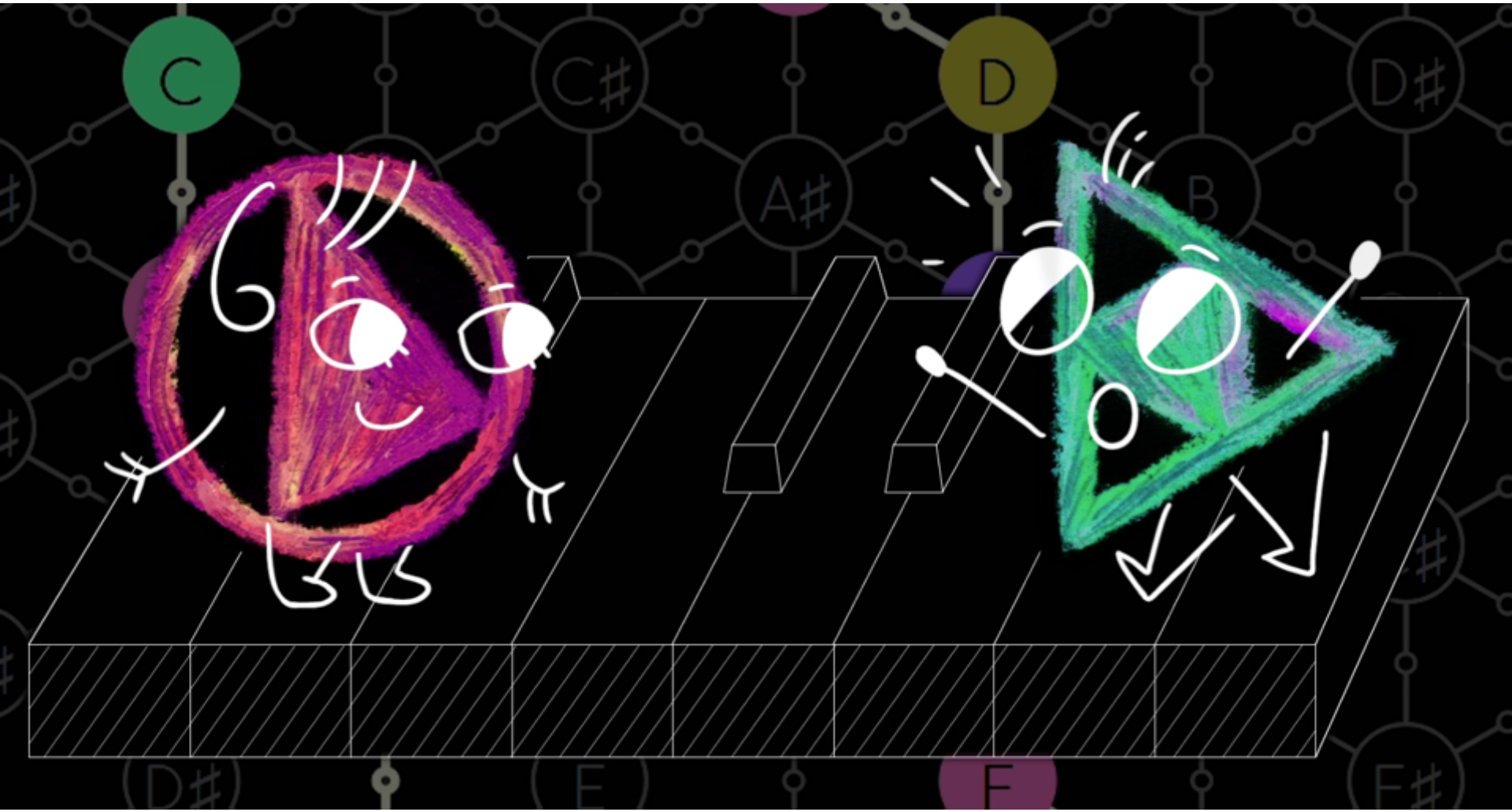


From Music to Mathematics and backwards: introducing algebra, topology and category theory into computational musicology

Moreno Andreatta

CNRS / IRMA / Université de Strasbourg
CNRS / IRCAM / Sorbonne Université
http://repmus.ircam.fr/moreno/smir

« Musique et mathématiques »: a pedagogical film



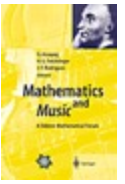
➔ www.morenoandreatta.com

cnrs
advancing the frontiers

AuDiMATH
AUTOUR DE LA DIFFUSION DES MATHÉMATIQUES

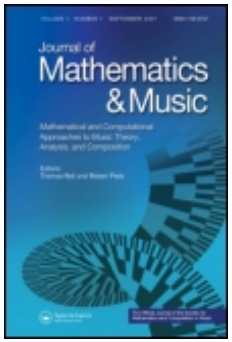
Maths/music: a (very) recent perspective

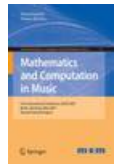
- 1999: 4e Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (G. Assayag, H.G. Feichtinger, J.F. Rodrigues, Springer, 2001)
- 2000-2003: International Seminar on *MaMuTh* (*Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Luis-Puebla eds, epOs, 2004)
- 2001-2016: *MaMuX* Seminar at Ircam and *mamuphi* Seminar at the ENS
- 2003: *The Topos of Music* (G. Mazzola et al.)
- 2006: 'Musique/Sciences' Series (Ircam/Delatour France)
- 2007: Journal of Mathematics and Music (Taylor & Francis) and SMCM
- 2007: First MCM at the TU-Berlin and first Proceedings by Springer
- 2009: *Computational Music Science* (eds: G. Mazzola, M. Andreatta, Springer)
- 2009: MCM at Yale University
- 2010: Mathematics Subject Classification : 00A65 Mathematics and music
- 2011: MCM at Ircam, Paris
- 2013: MCM at McGill University, Canada
- 2015: MCM at Queen Mary University, London
- 2017: MCM at UNAM, Mexico City
- 2019: MCM 2019 at Universidad Complutense, Madrid
- 2022: MCM at Atlanta State University







Future events: music&maths at the TU-Dresden

Über das Projekt

Das Symposium „Music & Mathematics" ist an der Schnittstelle von Musik und Mathematik angesiedelt und bietet eine Plattform für den interdisziplinären Austausch von Musiker*innen und Komponist*innen, Musikwissenschaftler*innen, Informatiker*innen und Mathematiker*innen. Neben klassischen Vorträgen werden daher auch künstlerischen Interventionen zu hören und zu sehen sein. Wir freuen uns auf die spannende Projekte, musikalische Präsentationen und rege Diskussionen in einer interdisziplinären Gruppe.

Die Veranstaltung ist zweiteilig und beginnt am 27. November 2020 mit einem Workshopformat. Für den Vortrag konnten wir Martin Supper zum Thema "Einige Anmerkungen zu Musik und Mathematik von Iannis Xenakis gewinnen. Im Anschluss folgt eine impulsgeleitete Diskussion.

Das Symposium selbst findet am 16. und 17. April 2021 in digitaler Form statt.

➔ **https://www.diejungeakademie.de/aktivitaeten/projekte/music-mathematics/**

SYMPOSIUM, 16.04. UND 17.04.2021

Wir laden ein zum zweitägigen Symposium „Music & Mathematics" am 16. und 17. April 2021. In Vorträgen, Diskussion und musikalischen Beiträgen beleuchten unsere Gäste das Spannungsfeld von Musik und Mathematik. Timo de Wolff, Miriam Akkermann und Dirk Pflüger führen durch das Programm.

Die Veranstaltung findet in englischer Sprache digital über Zoom statt.
Um Anmeldung wird gebeten.

PROGRAMM
Freitag, 16.04.2021 / 13.30 – 21.30 Uhr

13.45 Uhr Welcome
Timo de Wolff, Miriam Akkermann, Dirk Pflüger (Die Junge Akademie)

14.00 – 15.00 Uhr Vortrag und Musik
Maurice Rojas: „Can You Hear Pseudorandomness?"

15.30- 16.30 Uhr Vortrag und Musik
Kathlen Kohn (KTH Stockholm) und Ernst Ulrich Deuker: „The Complex of Non-Chromatic Scales" Musik: Ernst Ulrich Deuker

17.00 – 17.30 Uhr Visuals
Michael Sedlmair (Universität Stuttgart): „Visualizing Music."

17.30 – 18.00 Uhr Visuals
Rainer Groh (TU Dresden): „Reverse engineering of sound visualizations"

19.30 – 21.00 Uhr Keynote und Diskussion
Margaret Schedel (Stony Brook University): „Secret Analogies: Finite and Infinite"

Samstag, 17.04.2021 / 9.30 – 14.00 Uhr

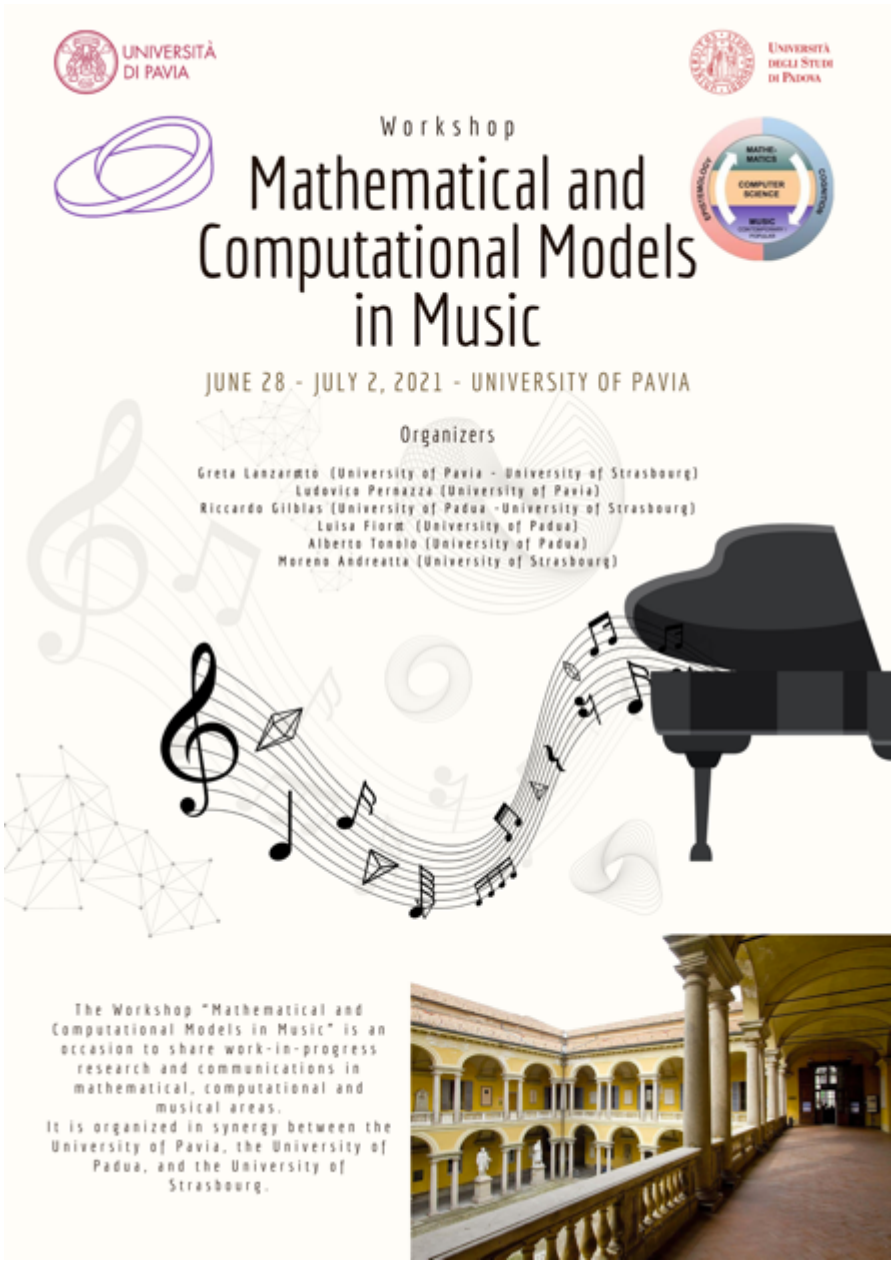
9.30 – 10.30 Uhr Musik
Martin Supper (Universität der Künste Berlin): „From Mycenae-Alpha (1978) to Gendy3 (1991)."
Musik: Iannis Xenakis: Gendy3

10.30 – 11.00 Uhr Vortrag
Stefan E. Schmidt (TU Dresden): „What is an interval in music? Algebraic measurement aspects of how we may think tonal distance."

11.30 – 12.30 Uhr Vortrag
Moreno Andreatta (University of Strasbourg): „From Music to Mathematics and backwards: come perceptual and cognitive implications of algebraic, topological and category-theory models in computational music analysis"

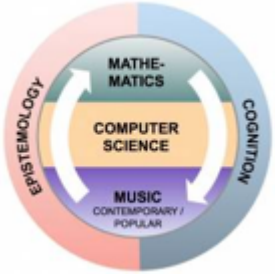
13.00 – 14.00 Uhr Musik
Gilles Baroin (University of Toulouse):
Musik: Gilles Baroin

Future events: music&maths at the University of Pavia



Preliminary list of speakers:

- Giovanni Albini
- Emmanuel Amiot
- Mattia Bergomi
- Anthony Coniglio
- Isaac Del Pozo
- Julio Estrada
- Riccardo Gilblas
- Saba Goodarzi
- James Hughes
- Jeremy Kastine
- Mihalis Kolountzakis
- Greta Lanzarotto
- Jordan Lenchitz
- Maria Mannone
- Mariana Montiel
- Robert W Peck
- Alexandre Popoff
- William Sethares
- Dmitri Tymoczko
- Jason Yust
- …

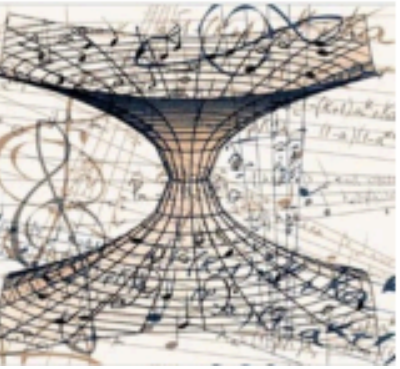


800 ANNI 1222·2022

UNIVERSITÀ DEGLI STUDI DI PADOVA

DIPARTIMENTO MATEMATICA

Dipartimento di Matematica "Tullio Levi-Civita"



The Department of Mathematics "Tullio Levi-Civita" of the University of Padova is offering a thematic PhD position in Mathematic and Music titled:

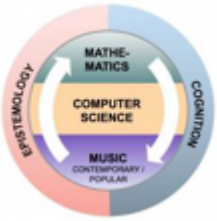
Mathematics and Music:
algebraic, categorical and computational methods
in the maths/music research

NEW 2020-2021

Two doctoral courses (**36h, May-June 2021**):

- Moreno Andreatta: An overview of some research axes in **Mathemusical Research**
- Emmanuel Amiot: **Discrete Fourier Transform** in music analysis: from tilings to musical scales
- Greta Lanzarotto: **Tiling Canons** and Fuglede Spectral Conjecture
- Franck Jedrzejewski: **Homometry** and neo-Riemannian Theory
- Thomas Noll: **Word theory** and its application to scales, modes, chords and rhythms
- Alexandre Popoff: **Category-Theory** formalization of transformational music analysis

The double movement of a ‘mathemusical’ activity



MATHEMATICS

Mathematical statement

generalisation

General theorem

formalisation

application

OpenMusic

MUSIC

Musical problem

Music analysis

Music theory

Composition

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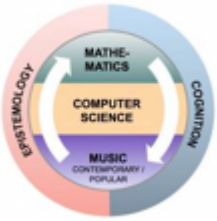

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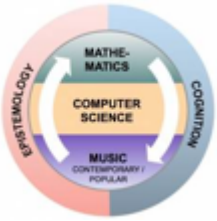

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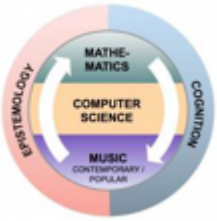

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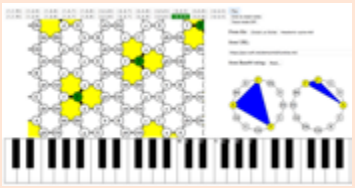
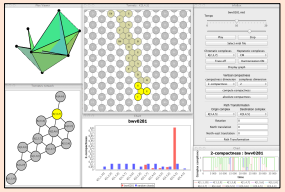


Tonnetz



Hexachord

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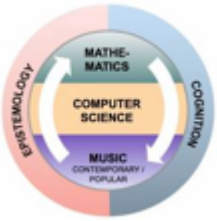

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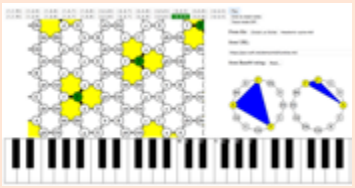
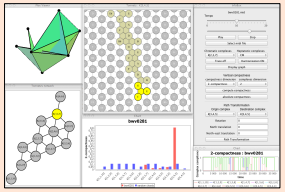


Tonnetz



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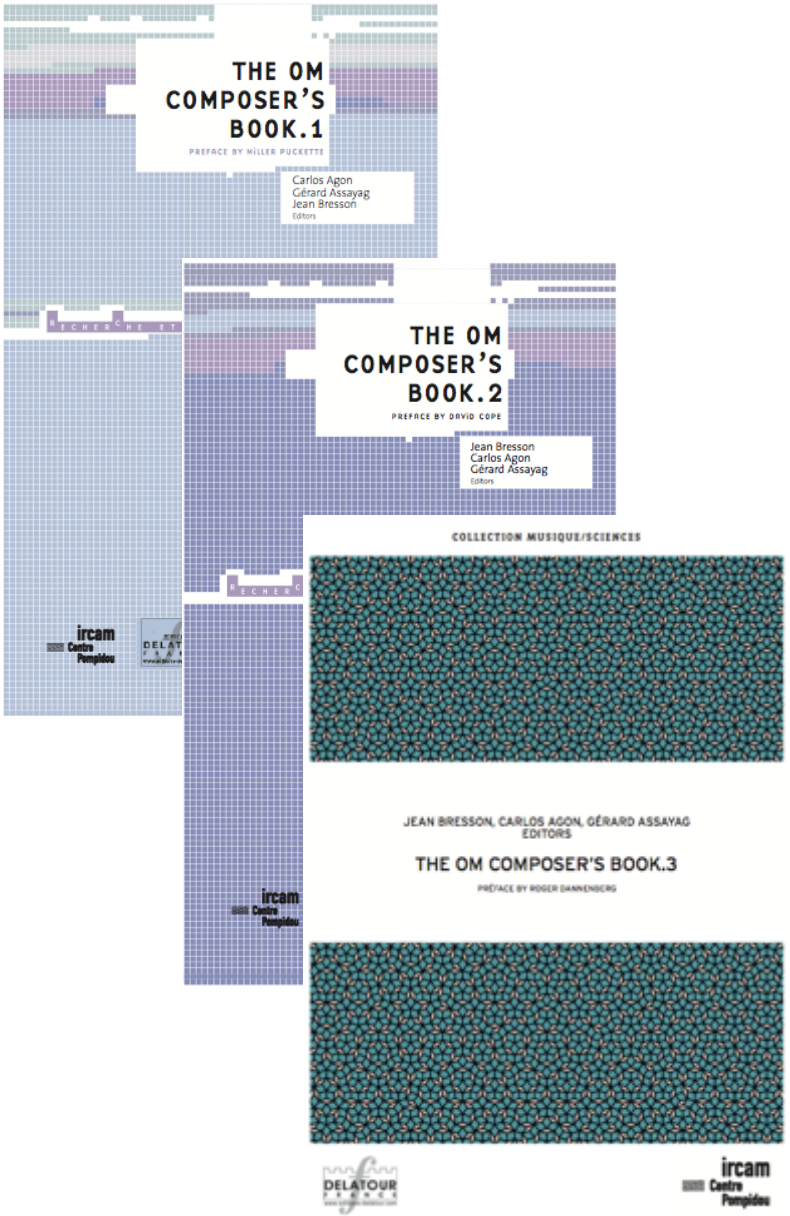
OpenMusic, a Visual Programming Language for computer-aided composition

www.repmus.ircam.fr/openmusic/home

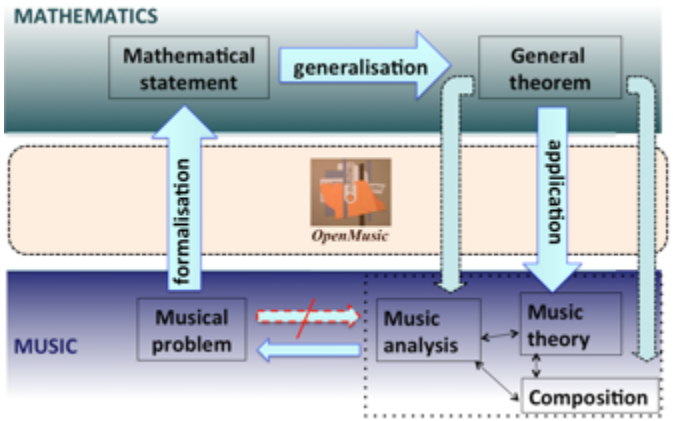


C. Agon, G. Assayag and J. Bresson, *The OM Composer's Book* (3 volumes) "Musique/Sciences" Series, Ircam/Delatour, 2006, 2007 and 2016

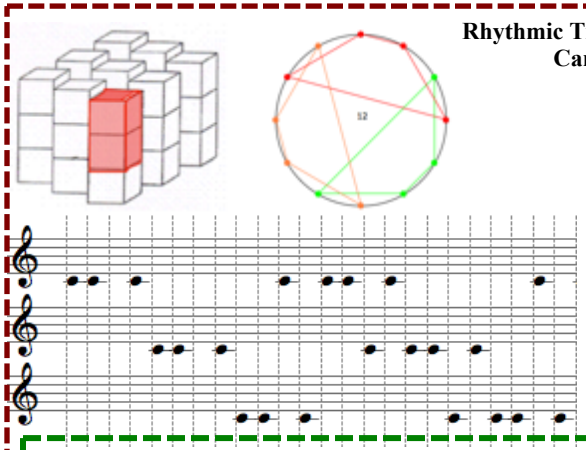
Some examples of ‘mathemusical’ problems

M. Andreatta : *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

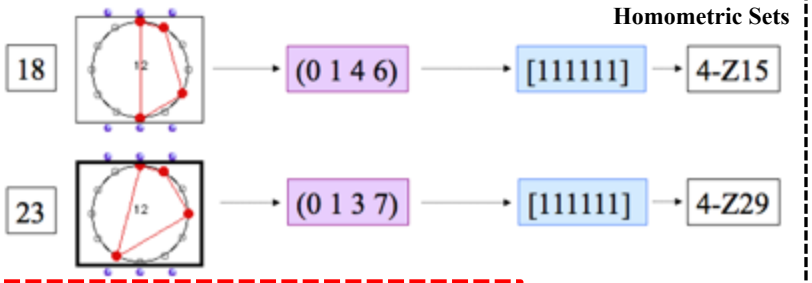
- The construction of Tiling Rhythmic Canons
- The *Z* relation and the theory of homometric sets
- *Set Theory* and Transformational Theory
- Neo-Riemannian Theory, Spatial Computing and FCA
- Diatonic Theory and Maximally-Even Sets
- Periodic sequences and finite difference calculus
- Block-designs and algorithmic composition

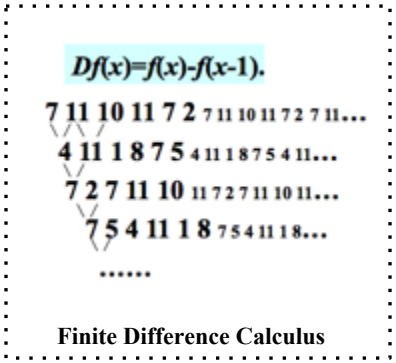


Rhythmic Tiling Canons



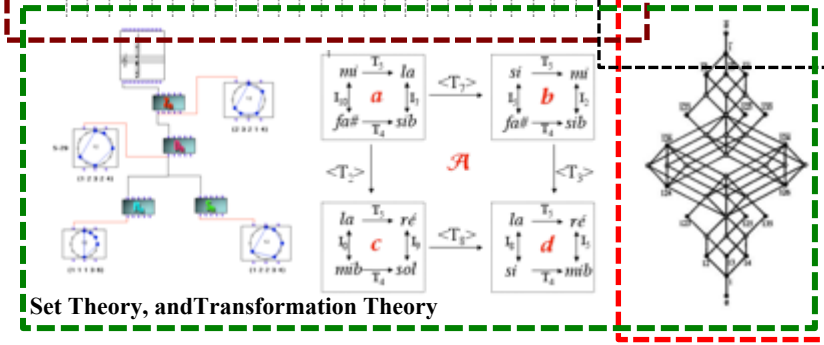
Z-Relation and Homometric Sets

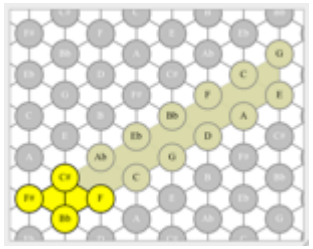


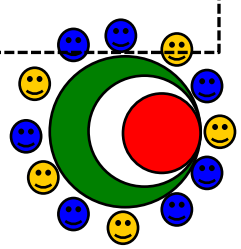


Finite Difference Calculus

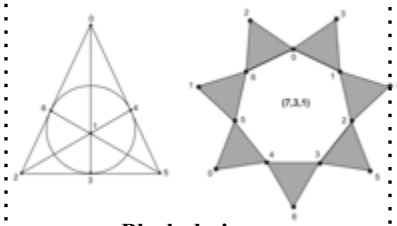
Neo-Riemannian Theory and Spatial Computing





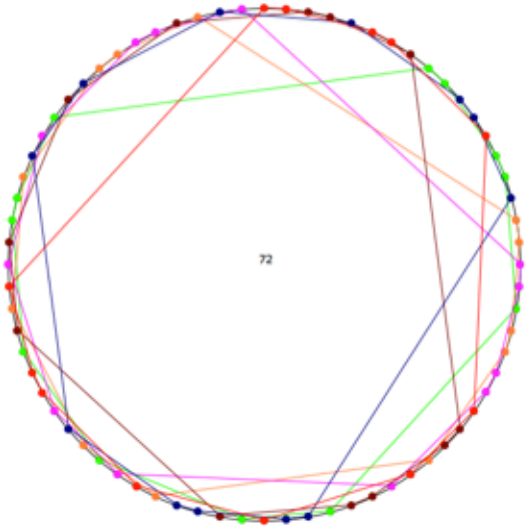


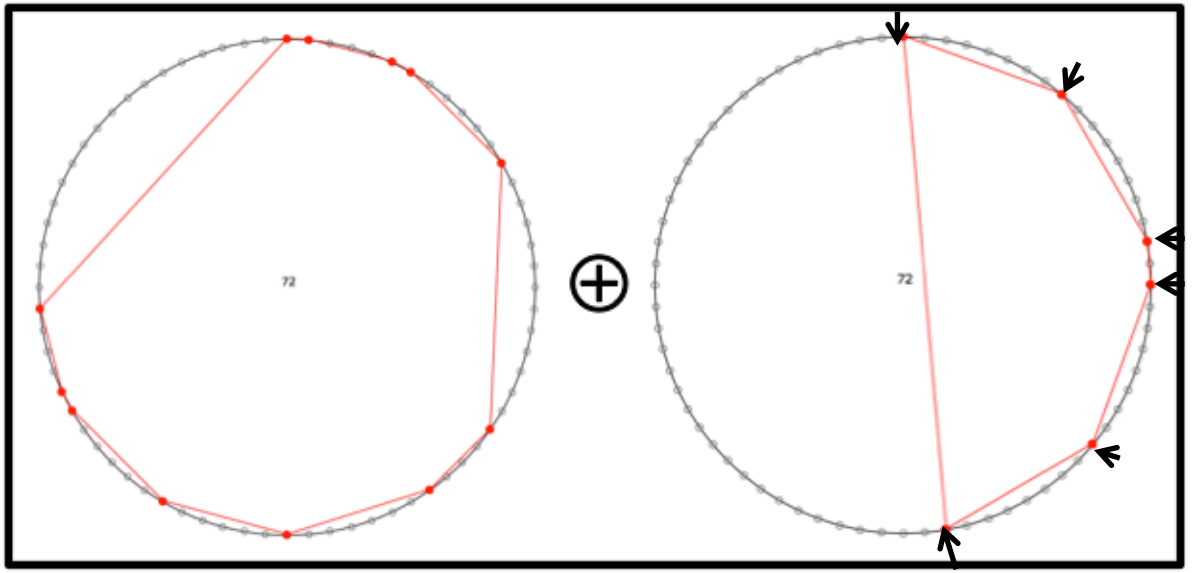
Diatonic Theory and ME-Sets



Block-designs

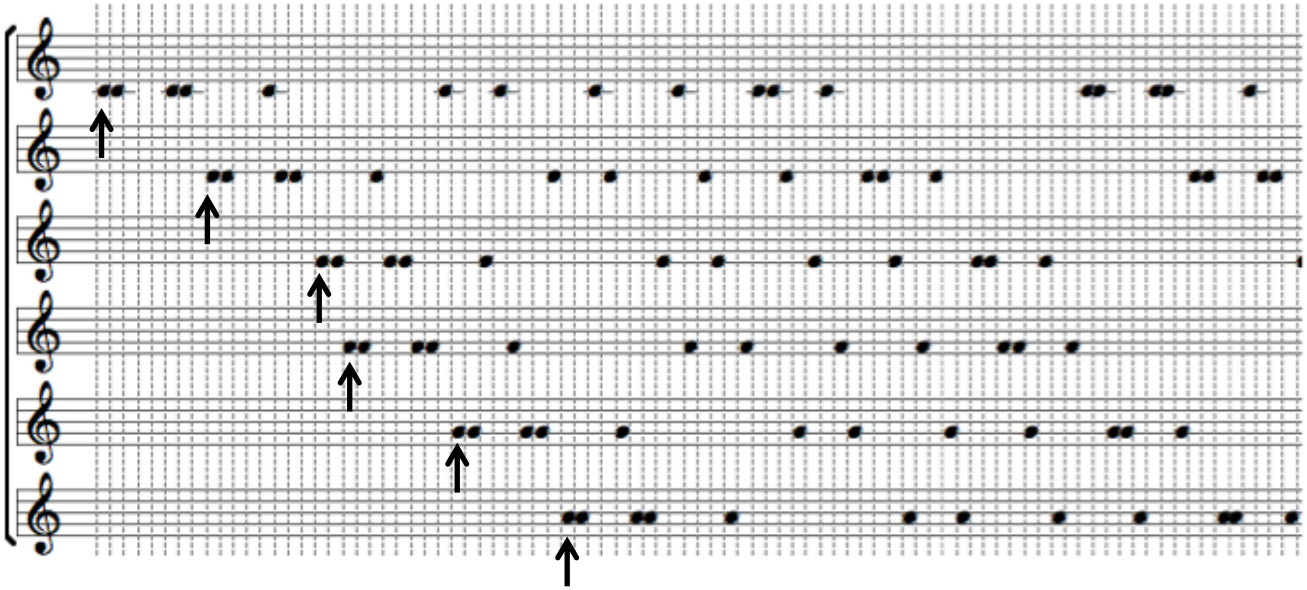
Aperiodic Rhythmic Tiling Canons (Vuza Canons)





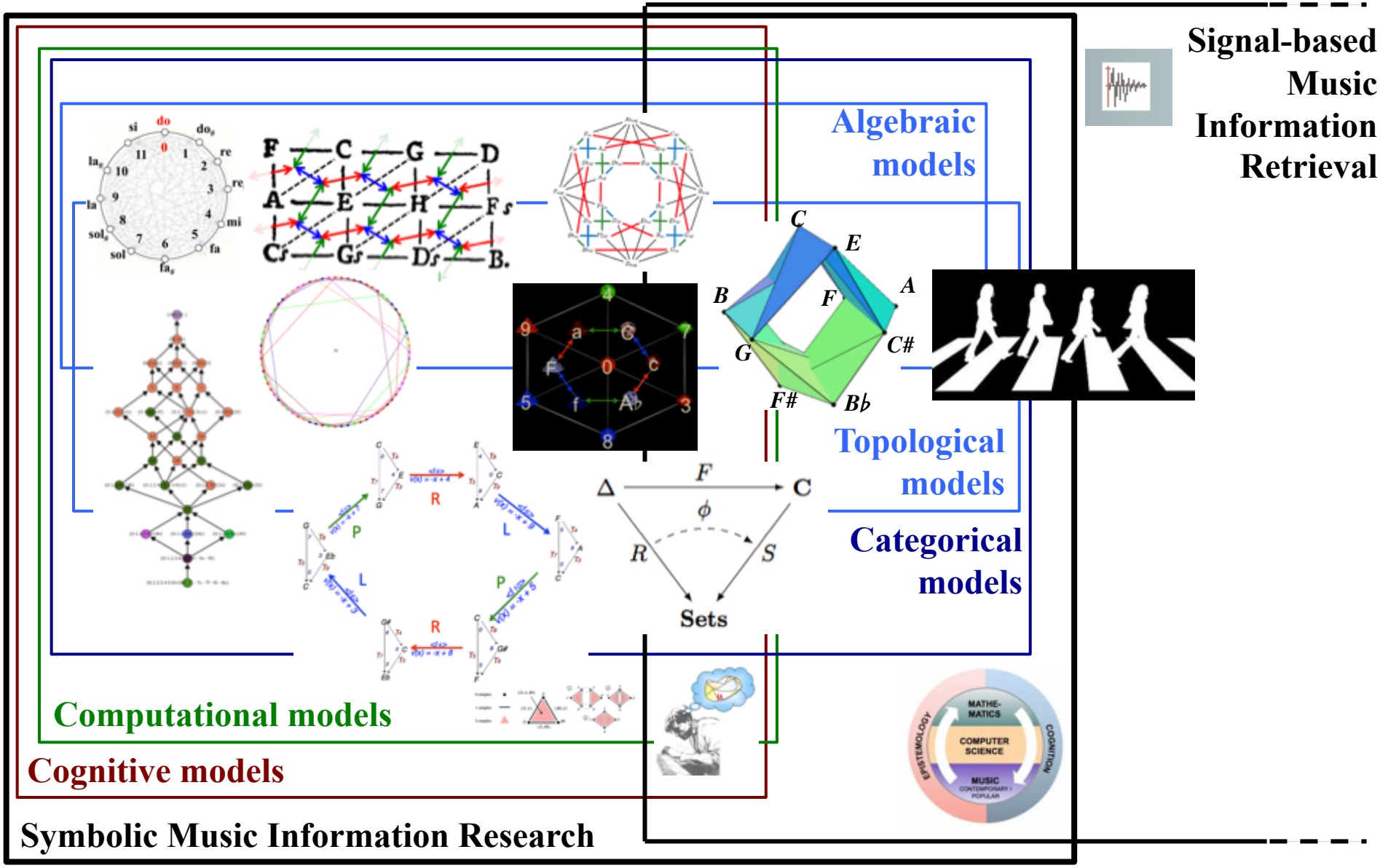
Dan Vuza

Anatol Vieru



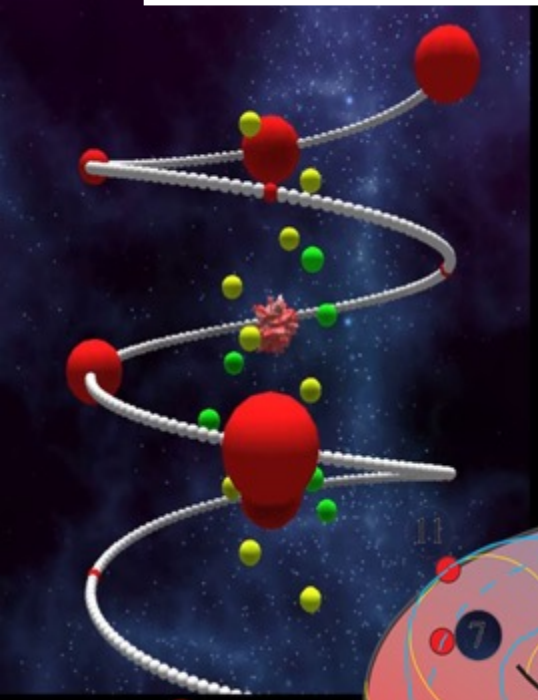
➔ ‘Tiling Problems in Music and Fuglede's Spectral Conjecture’, ongoing PhD, University of Pavia/Unistra

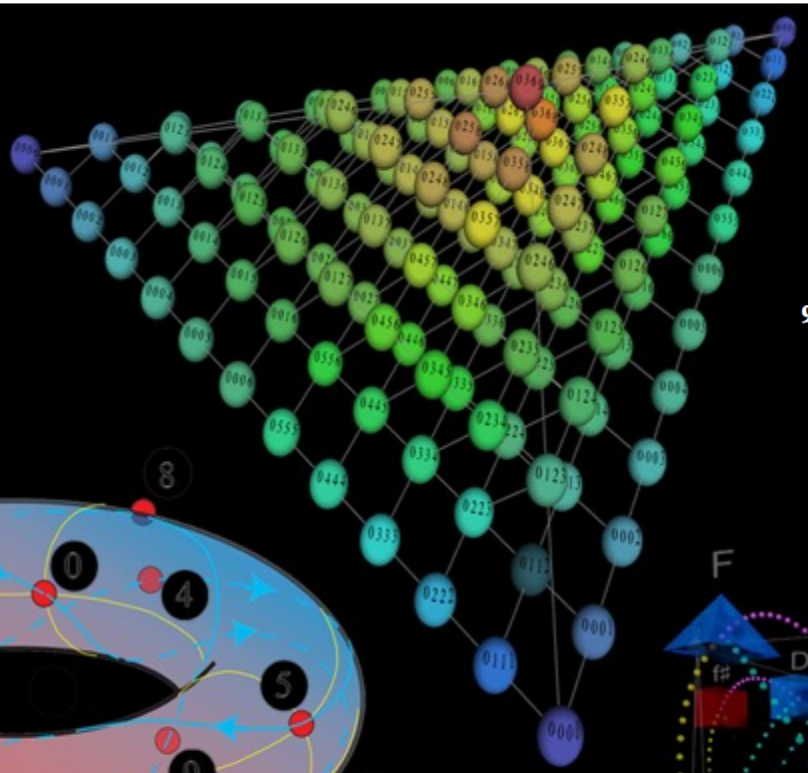
The SMIR Project: advanced maths for the working musicologist

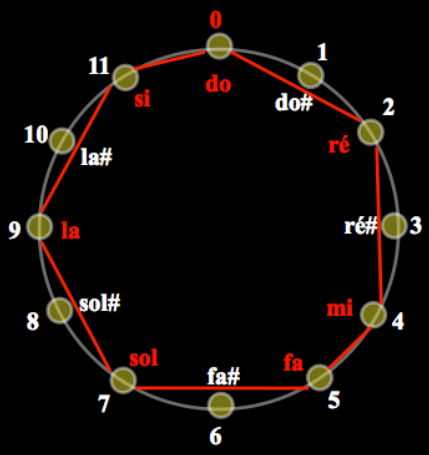


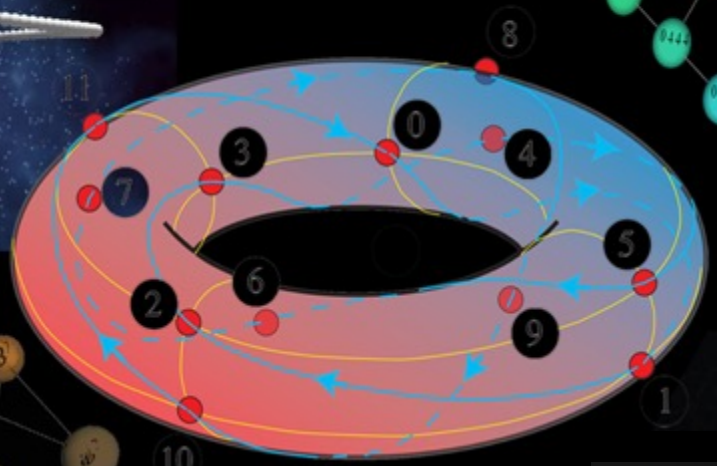
➔ Andreatta, M. (2018), « From music to mathematics and backwards: introducing algebra, topology and category theory into computational musicology », in M. Emmer and M. Abate (eds.), *Imagine Math 6* - Springer, pp. 77-88

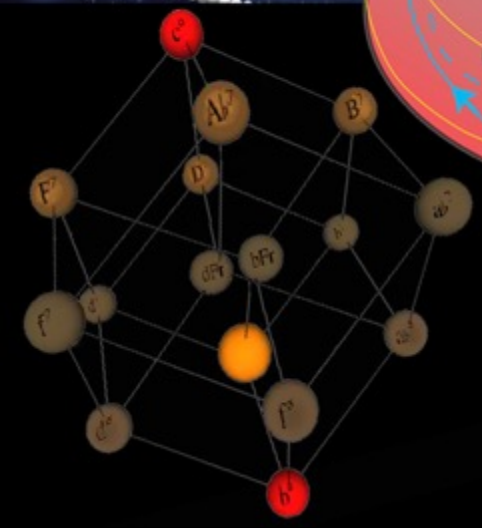
The galaxy of geometrical models at the service of music

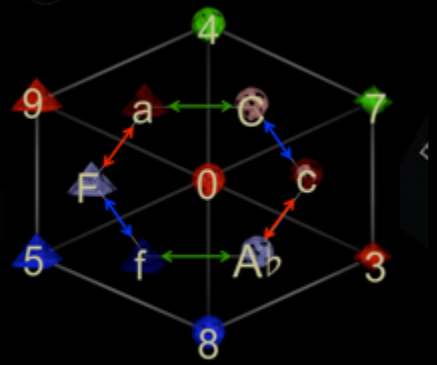


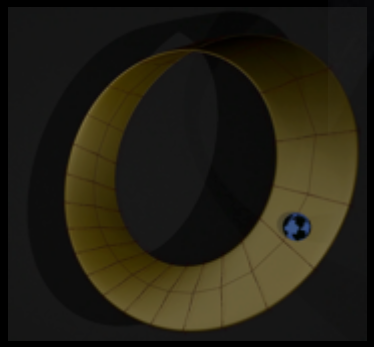


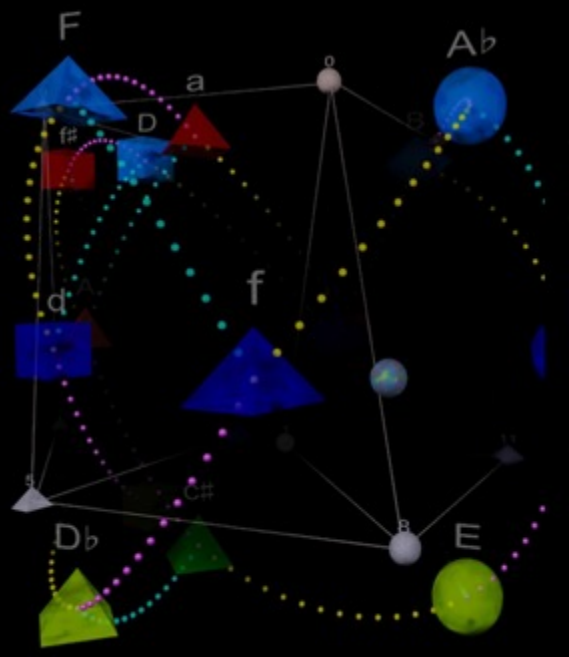












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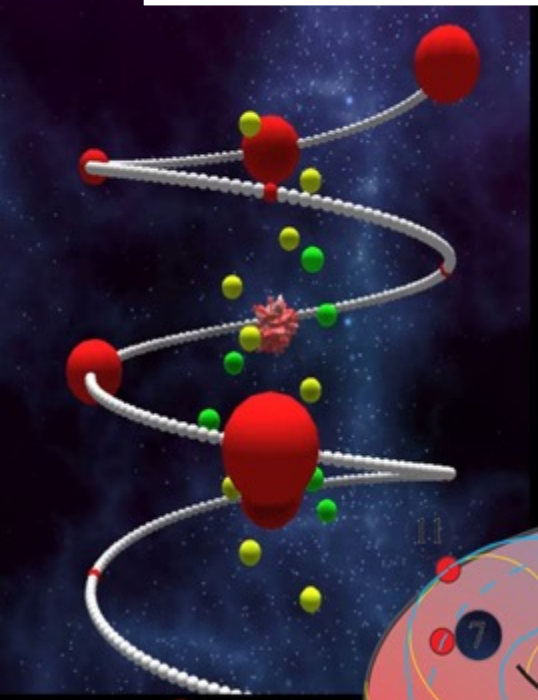
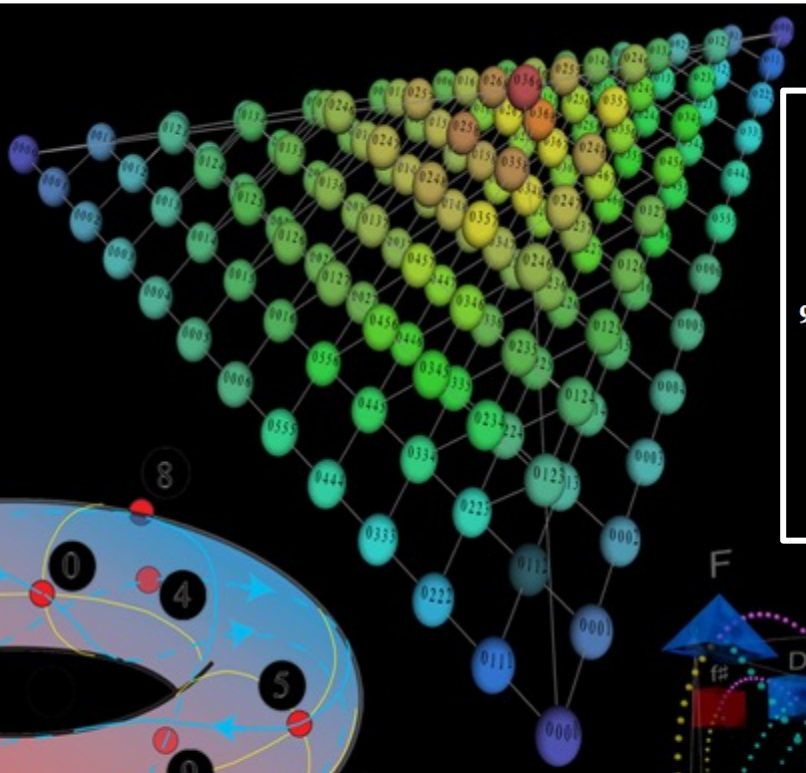
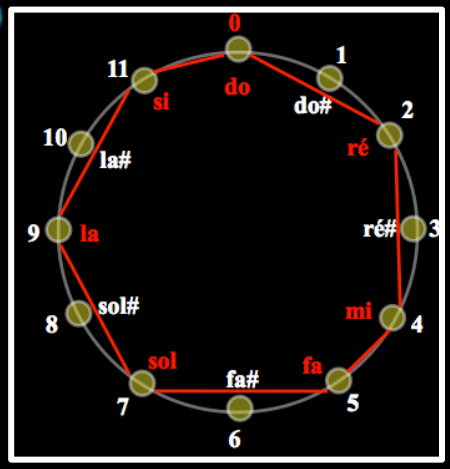
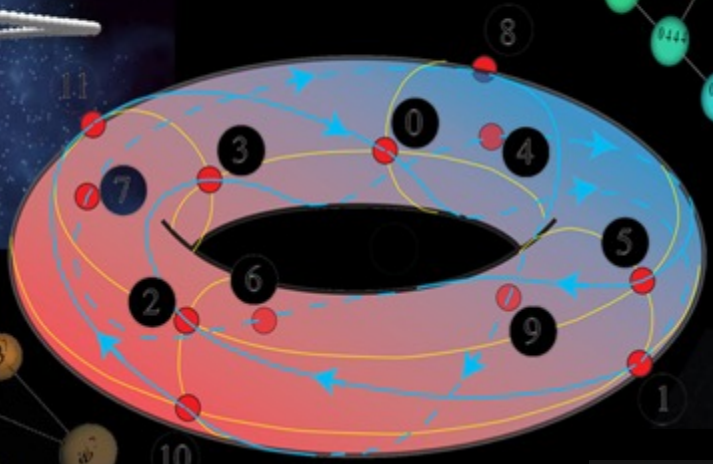
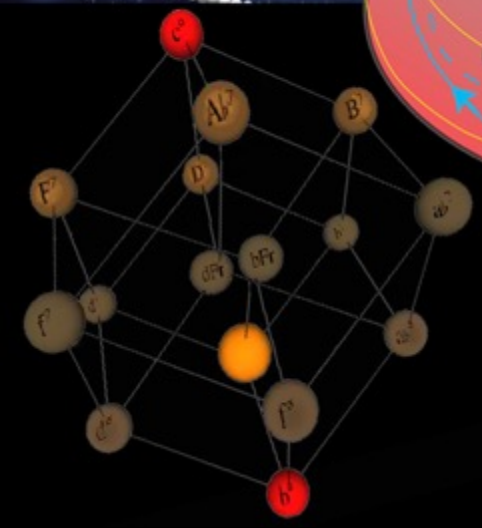
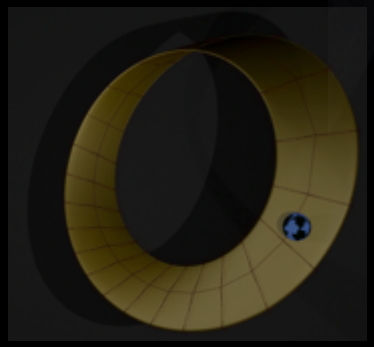


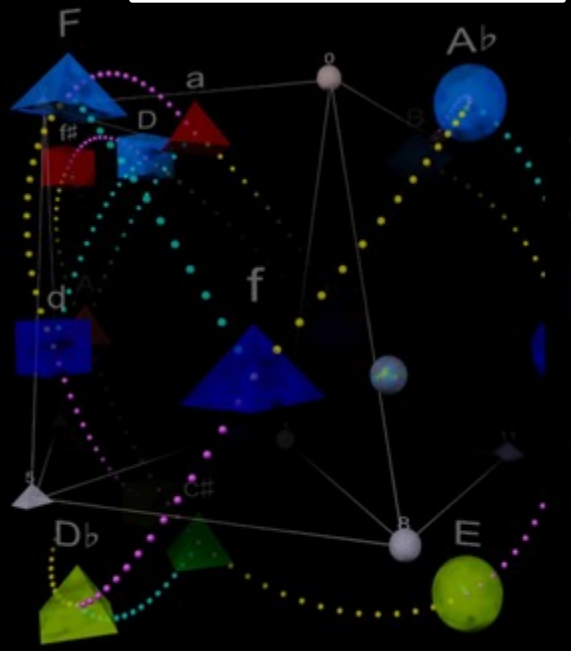




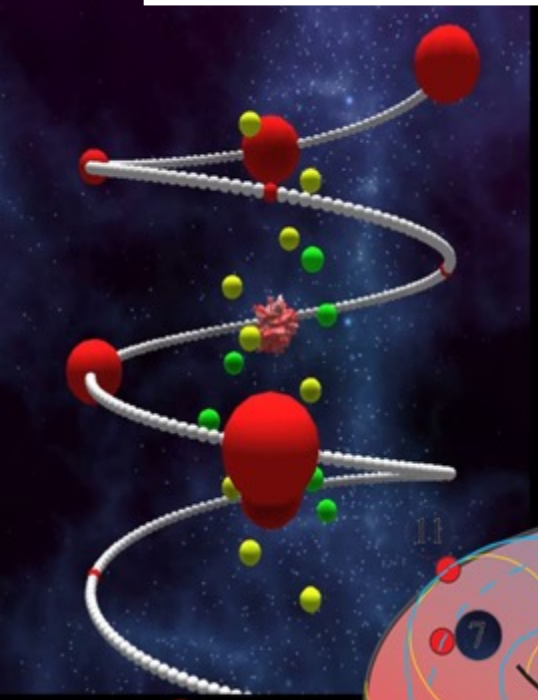


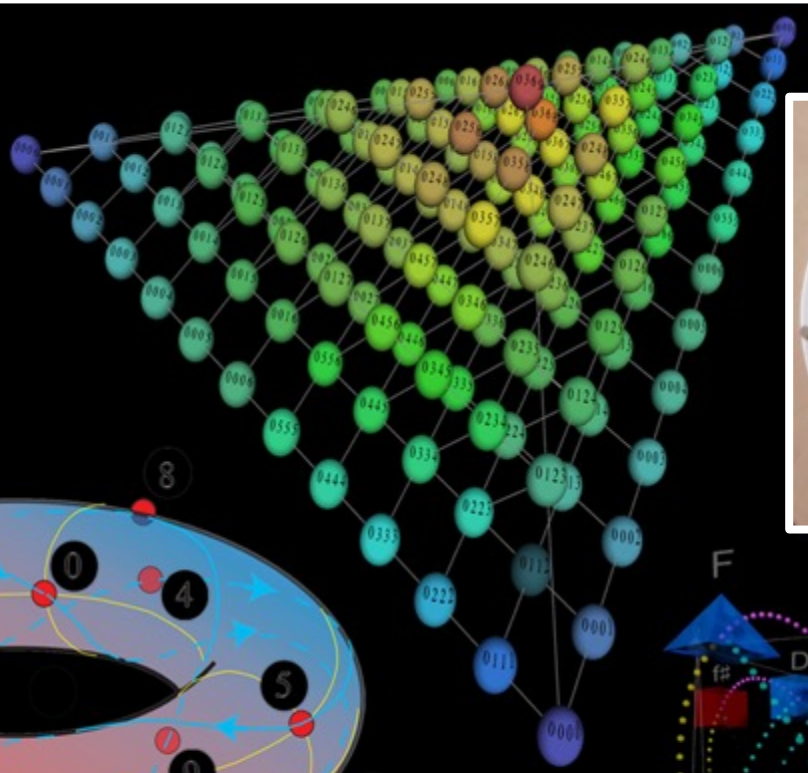


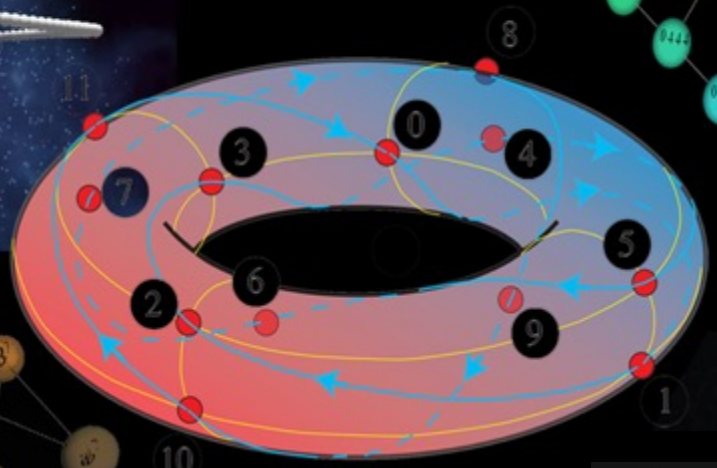


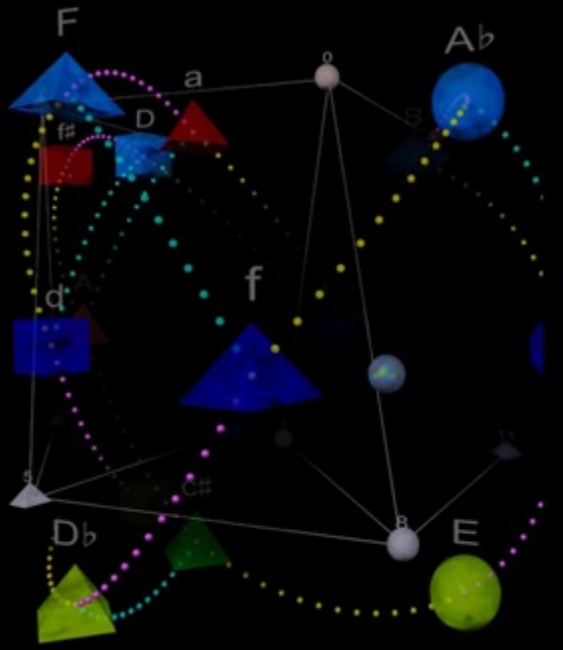


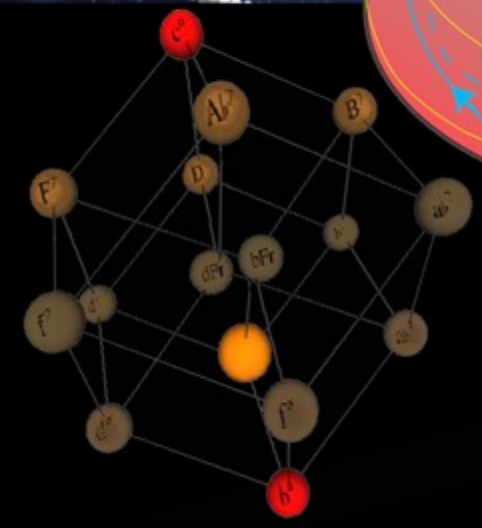
The galaxy of geometrical models at the service of music

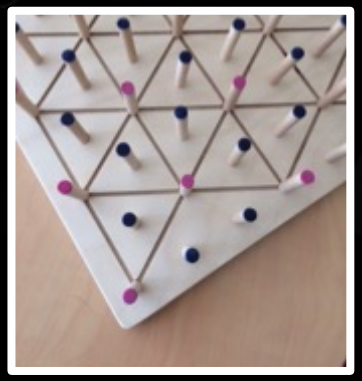


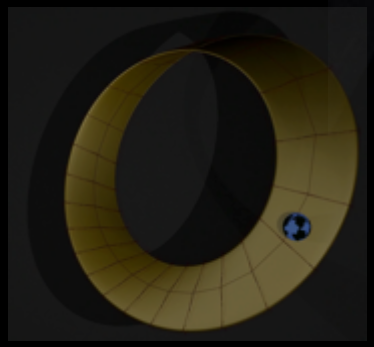




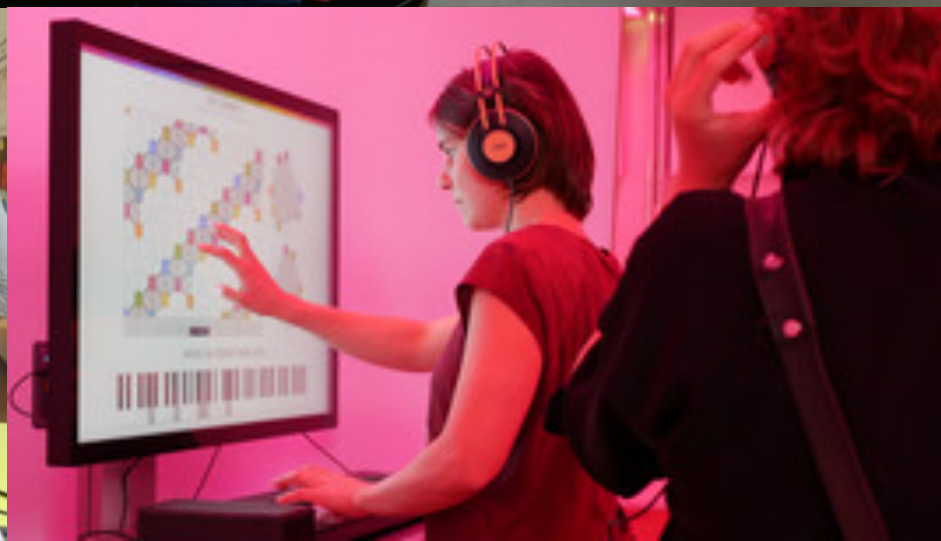









Rakuten
M-AUDIO



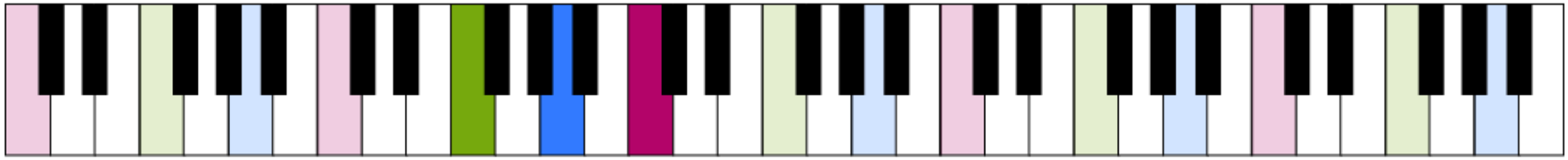
la.la.lab
THE MATHS OF MUSIC

IMAGINARY
open mathematics

HEIDELBERG
LAUREATE FORUM
FOUNDATION

May. 17, 2019 to Dec. 2020

https://imaginary.org/

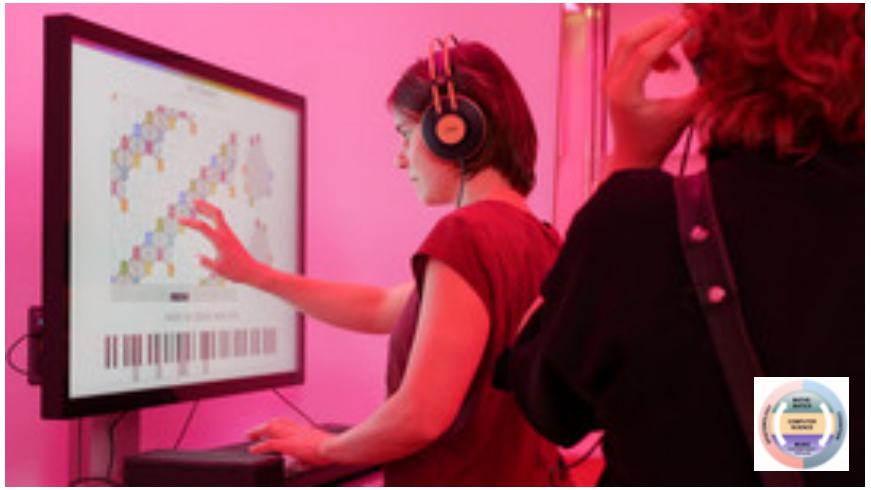


La.La.Lab brings the visitor to an interactive exploration and discovery of music from a mathematical perspective. The exhibition pivots over three axis:

- **Music theory.** Learning what tools build music, and how these tools are used to create art. Basic concepts and historical comments.

- **Current research.** The latest trends of research in the connection of maths and music. Artificial Intelligence, theoretical and new instruments, classification and composition tools.

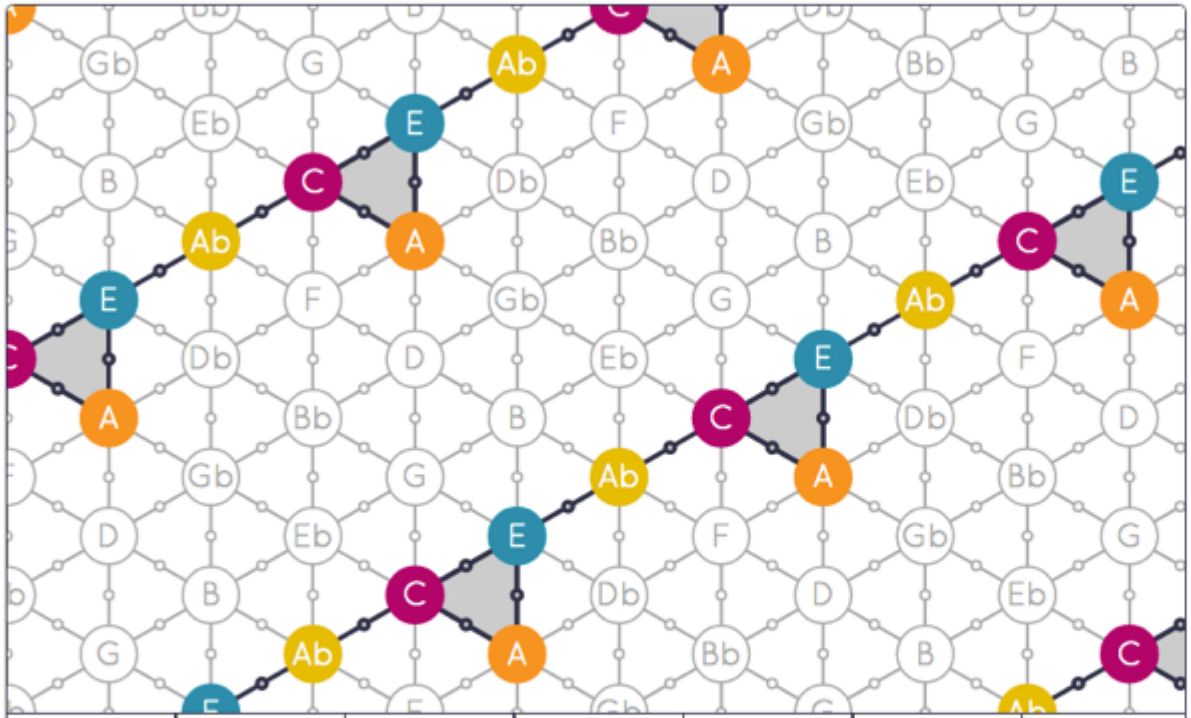
- **Art and entertainment.** A joyful display of artworks from artists and mathematicians in the field. Talks/concerts at scheduled events

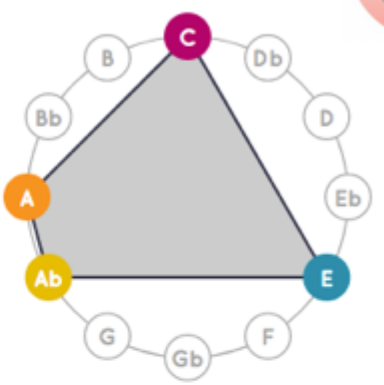


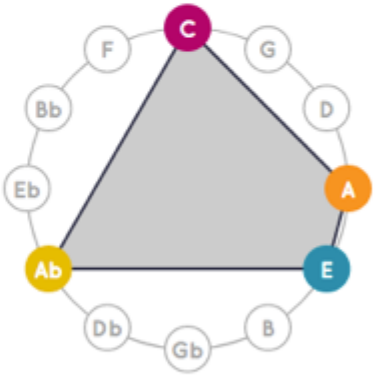
The Tonnetz web environment (developer: C. Guichaoua)

THE TONNETZ

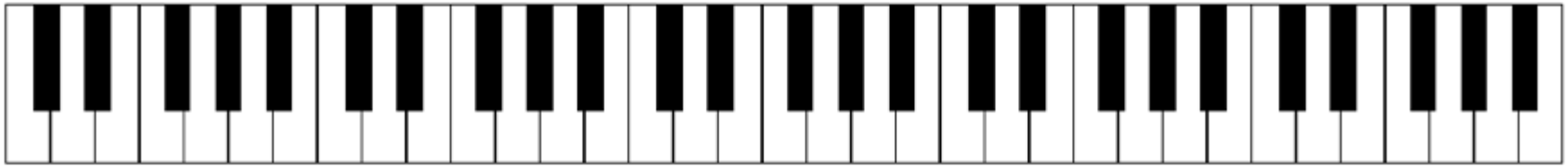
ONE KEY – MANY REPRESENTATIONS





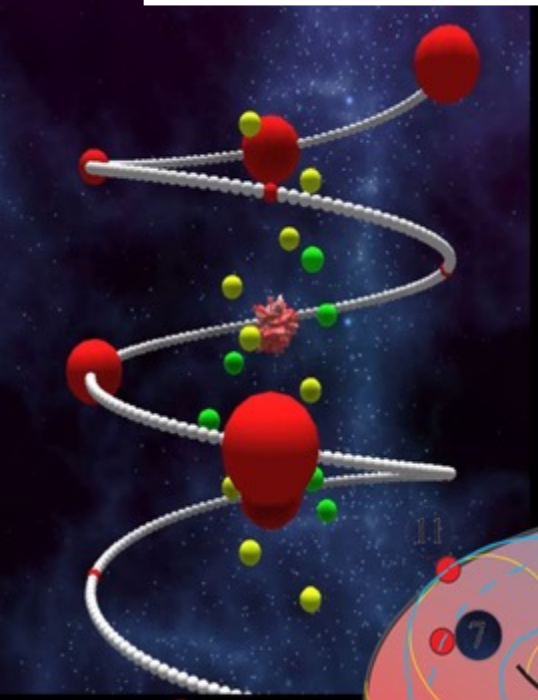


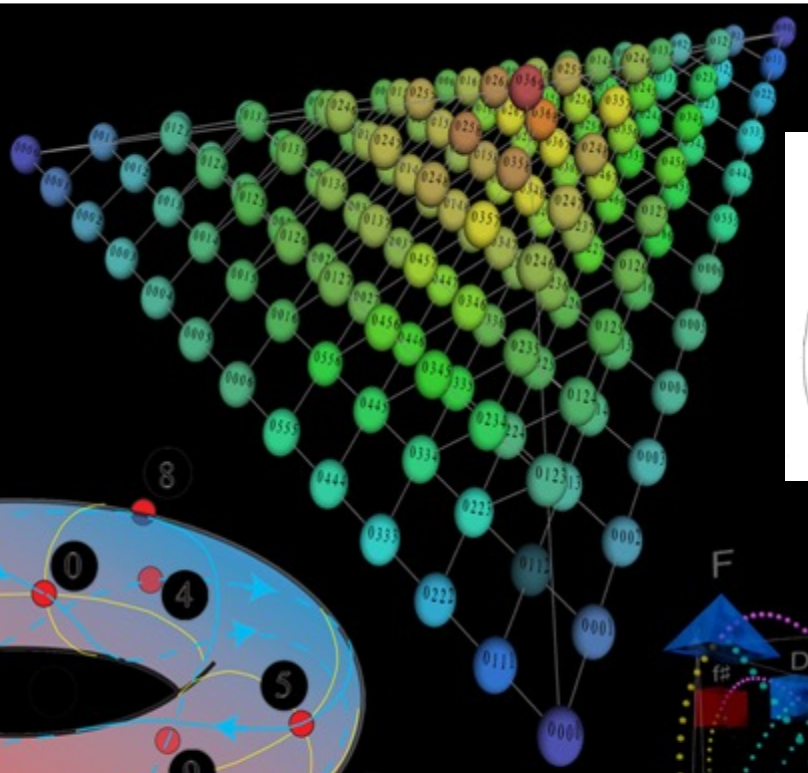
Load Midi File | Play | Start Recording | Rotate 180° | Translate

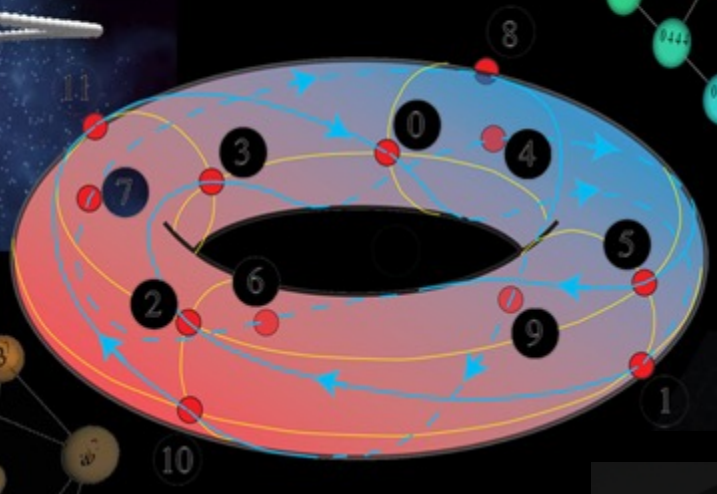


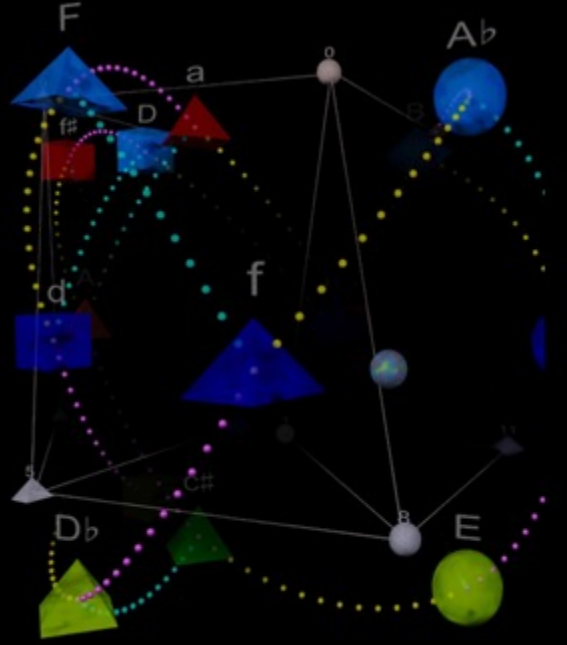
➔ **https://guichaoua.gitlab.io/web-hexachord/**

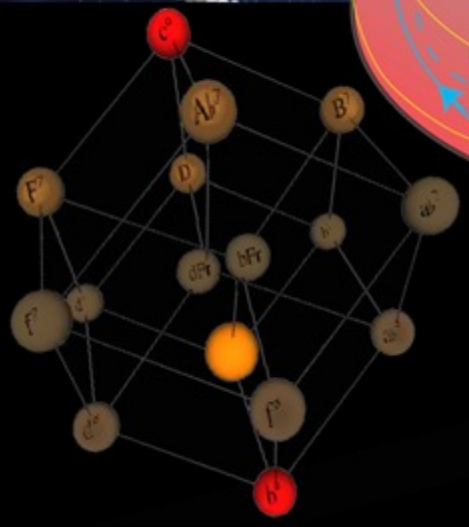
The galaxy of geometrical models at the service of music

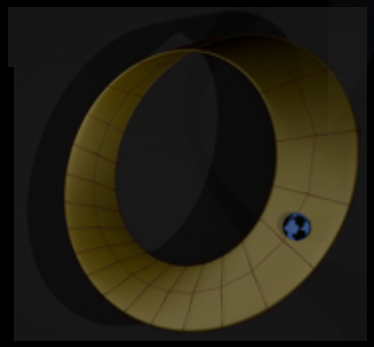




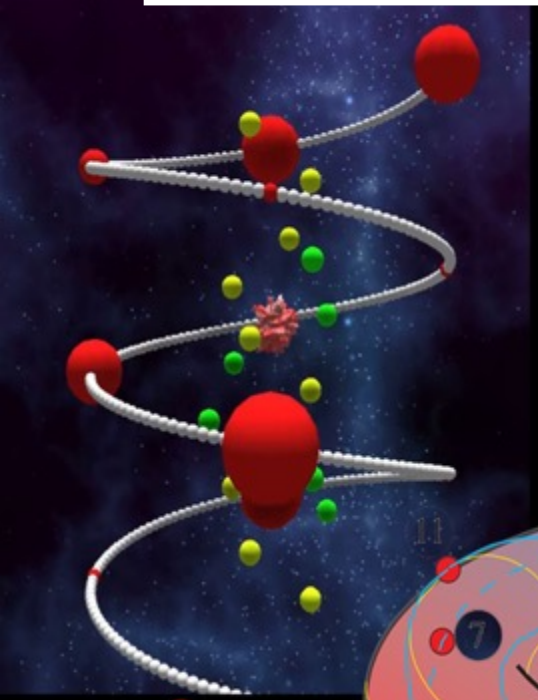


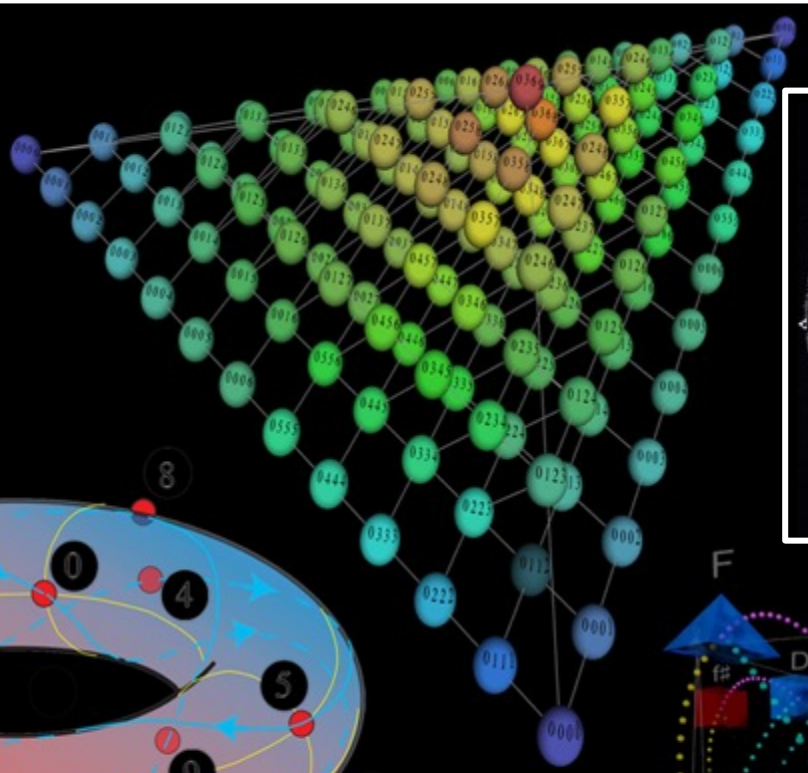


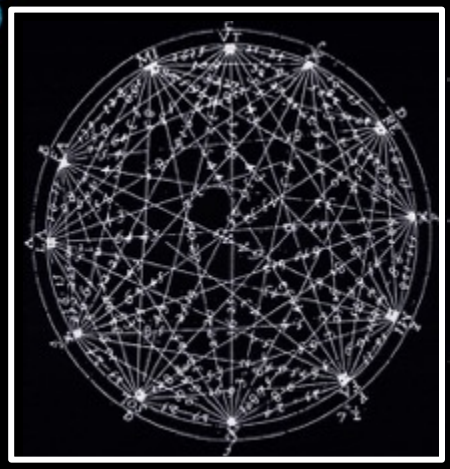


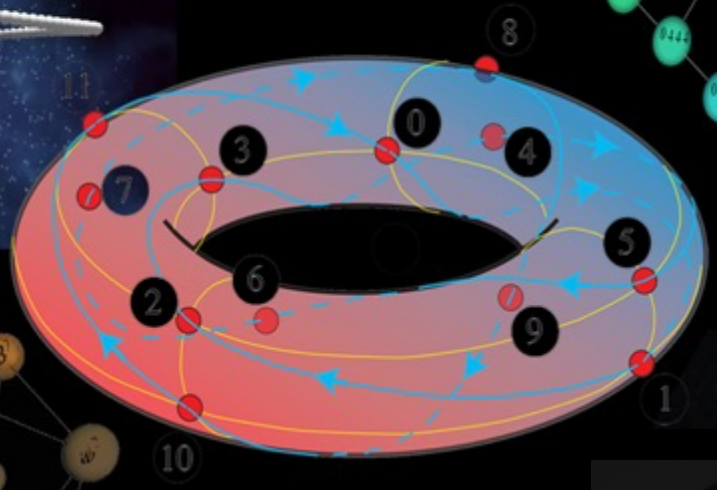


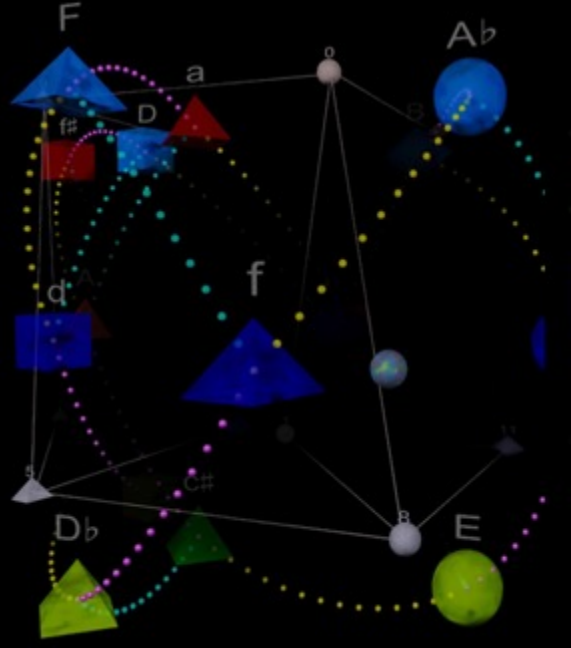
The galaxy of geometrical models at the service of music

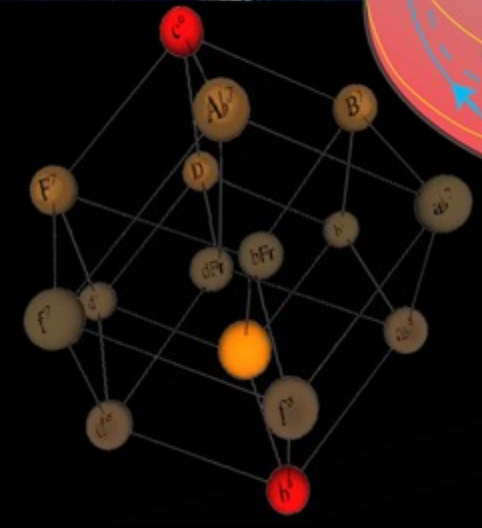


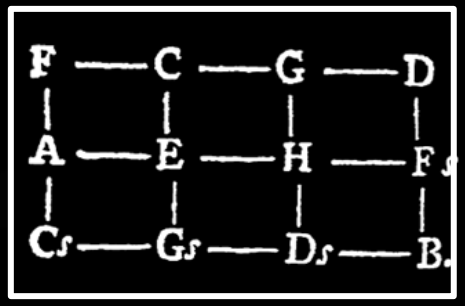


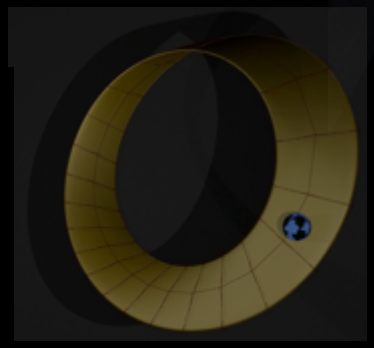






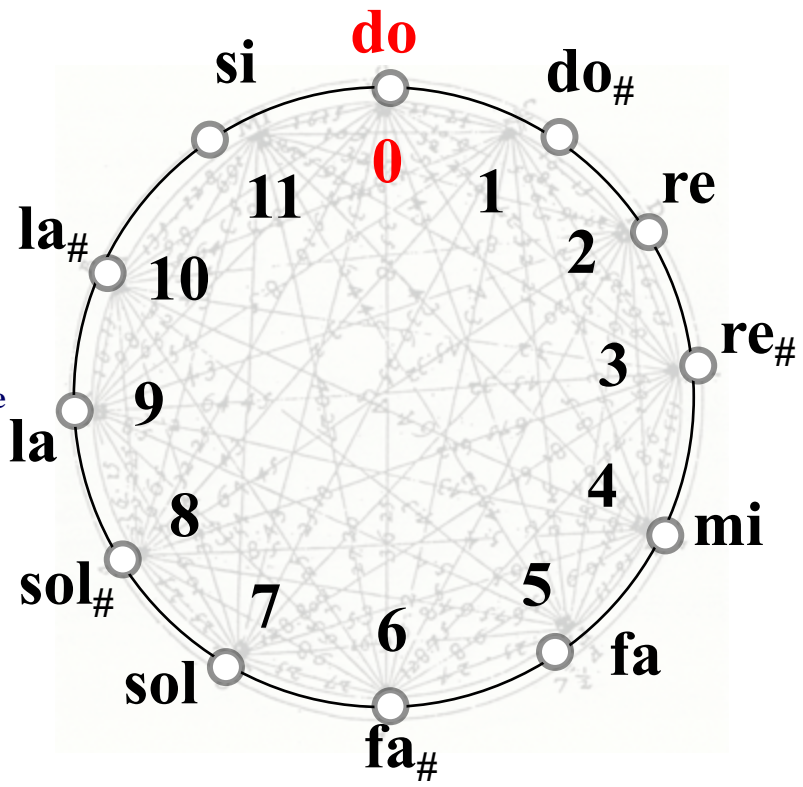




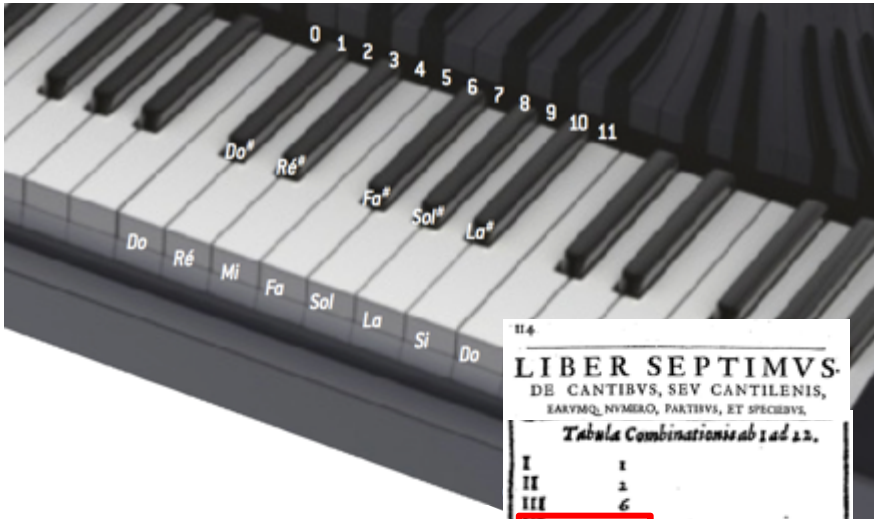


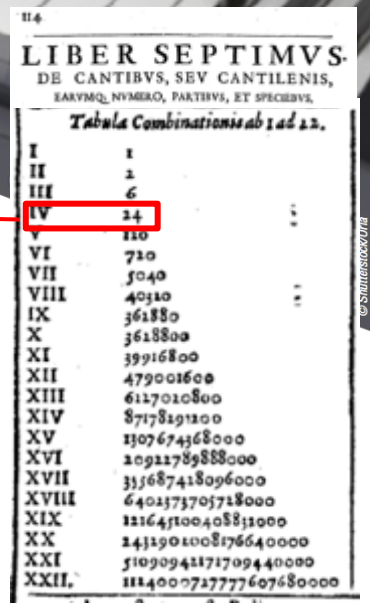
The circular representation of the pitch space

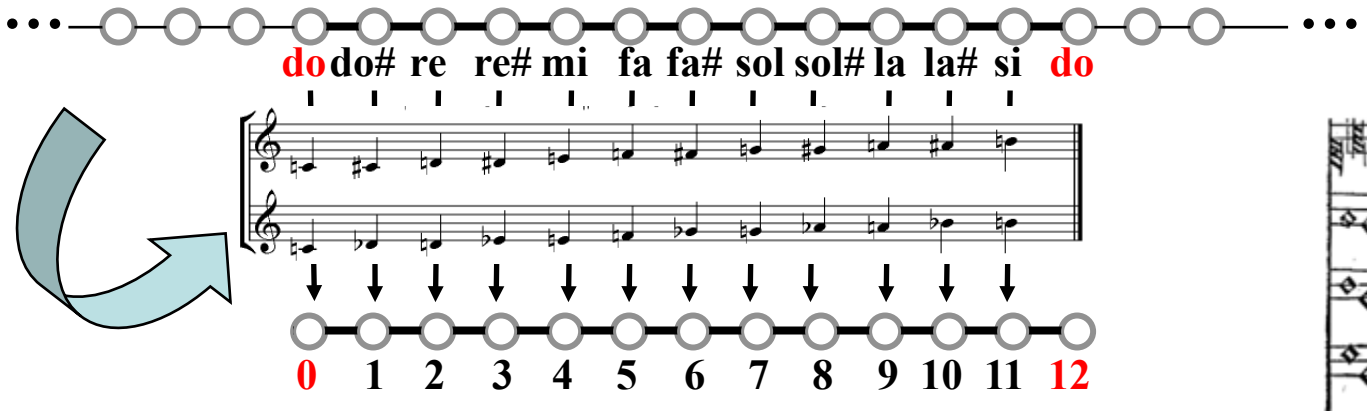
Marin Mersenne

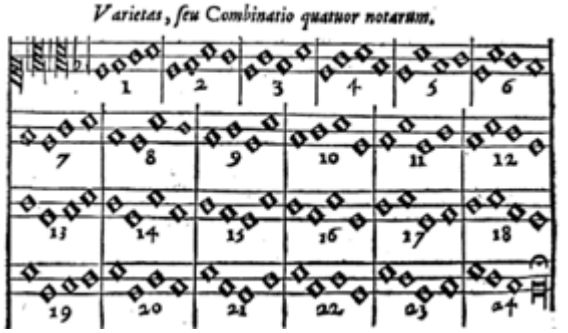


Harmonicorum Libri XII, 1648









Permutational melodies in contemporary (art) music

114 **Marin Mersenne, *Harmonicorum Libri XII*, 1648**

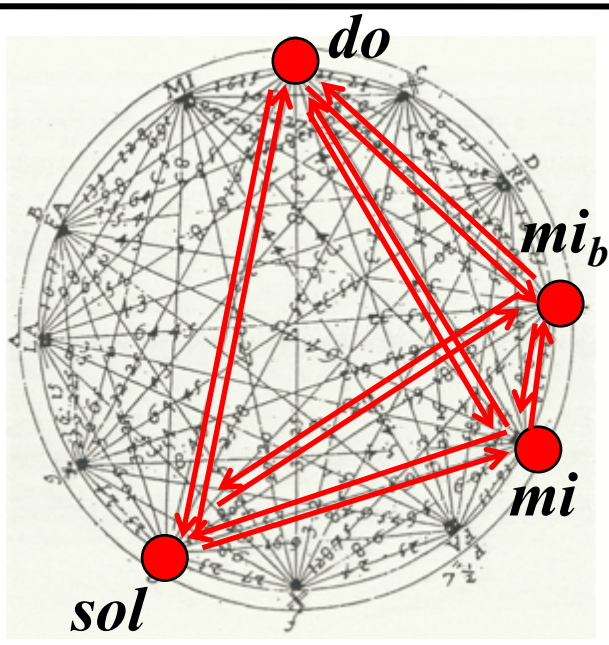
LIBER SEPTIMVS.

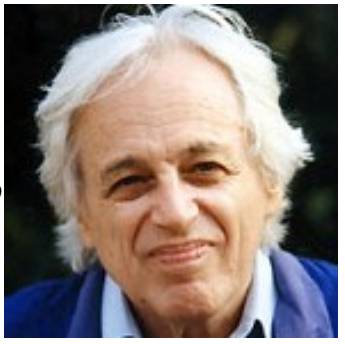
DE CANTIBVS, SEV CANTILENIS,

EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 22.

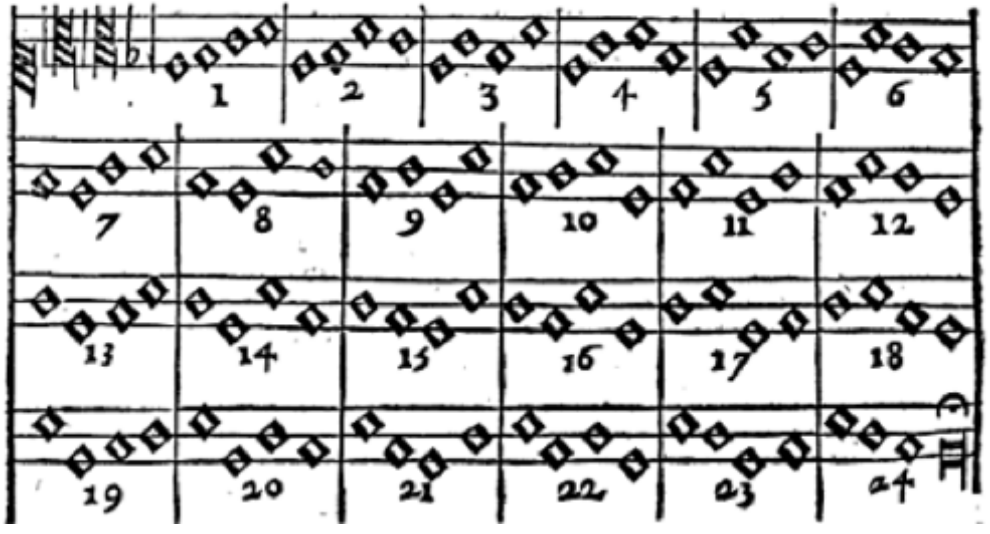
I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6127020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000





Six Bagatelles
(G. Ligeti, 1953)

Varietas, seu Combinatio quatuor notarum.

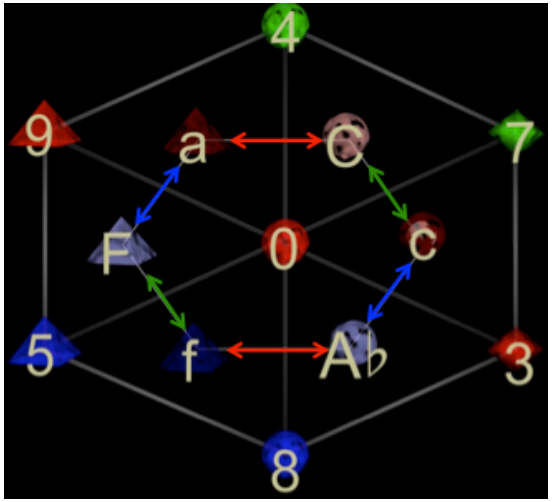


Permutational melodies in song writing

***Se telefonando*, 1966 (Maurizio Costanzo/Ennio Morricone) / Mina**

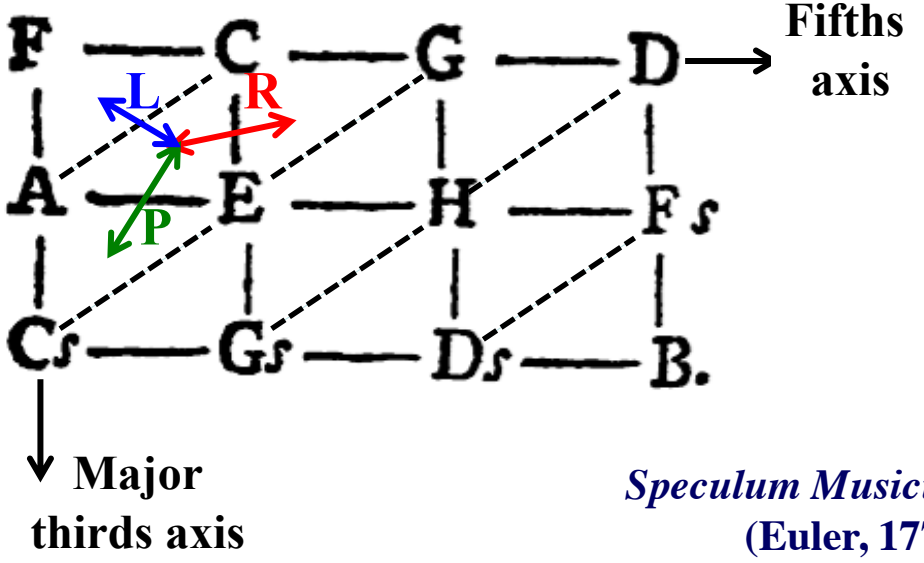
Ennio Morricone (1928-2020)

The harmonic space



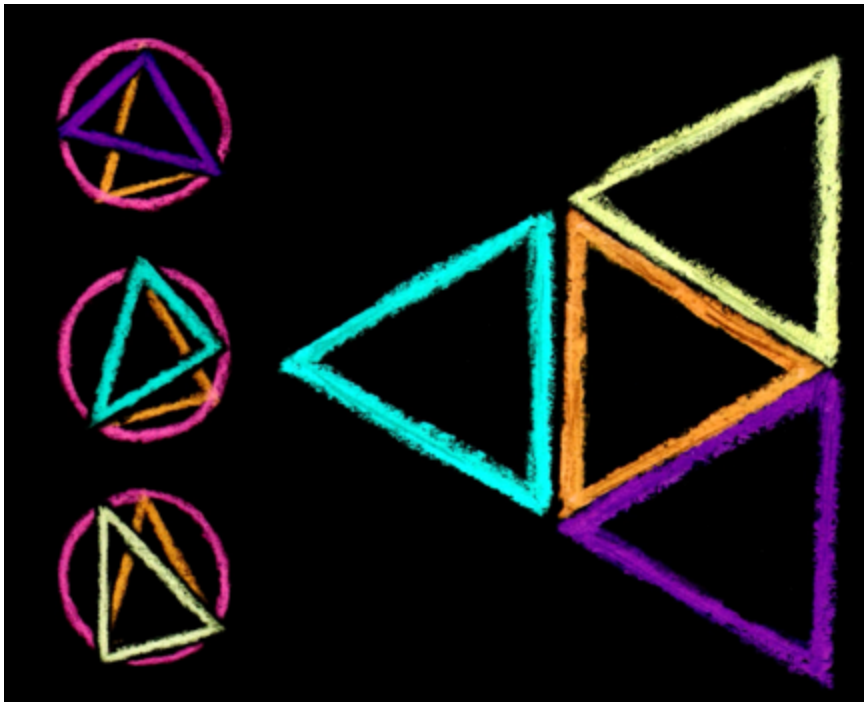
Chord enumeration

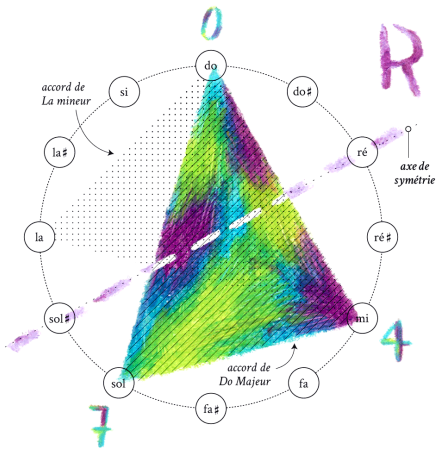
The *Tonnetz*
(or 'honeycomb' hexagonal tiling)

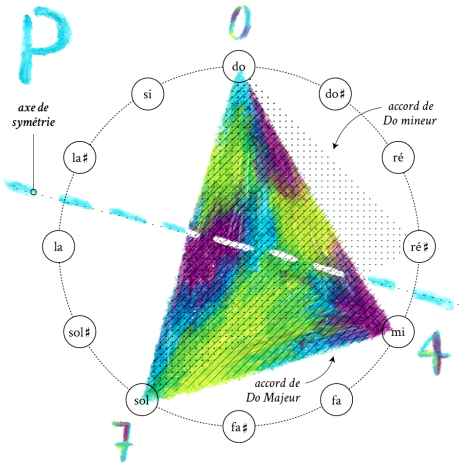


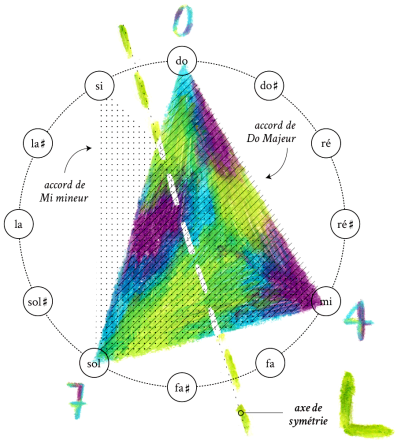
Speculum Musicum
(Euler, 1773)

The three main symmetries in the Tonnetz

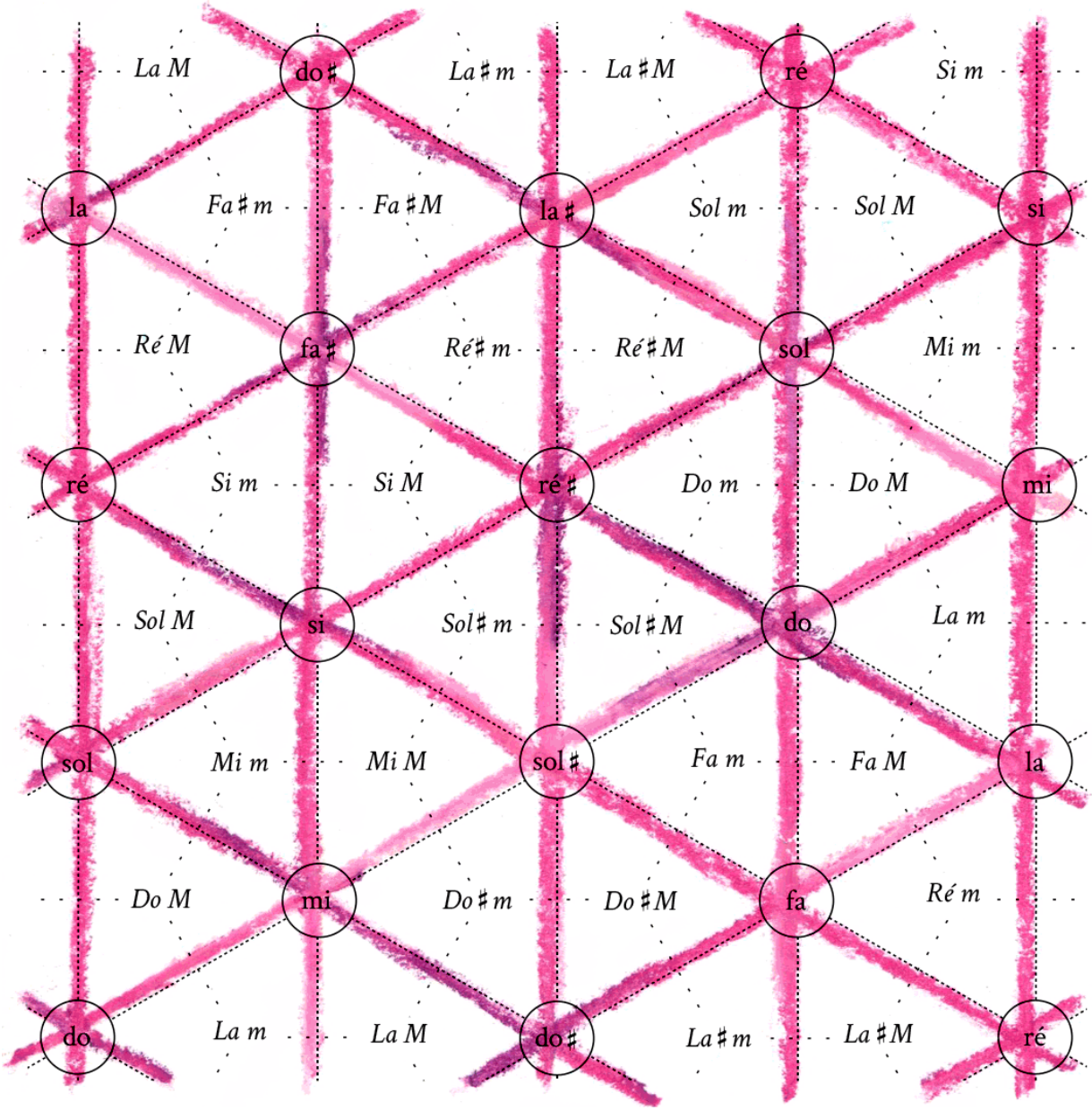


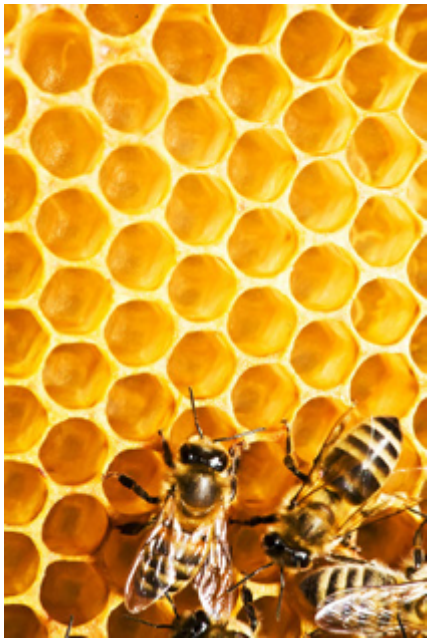




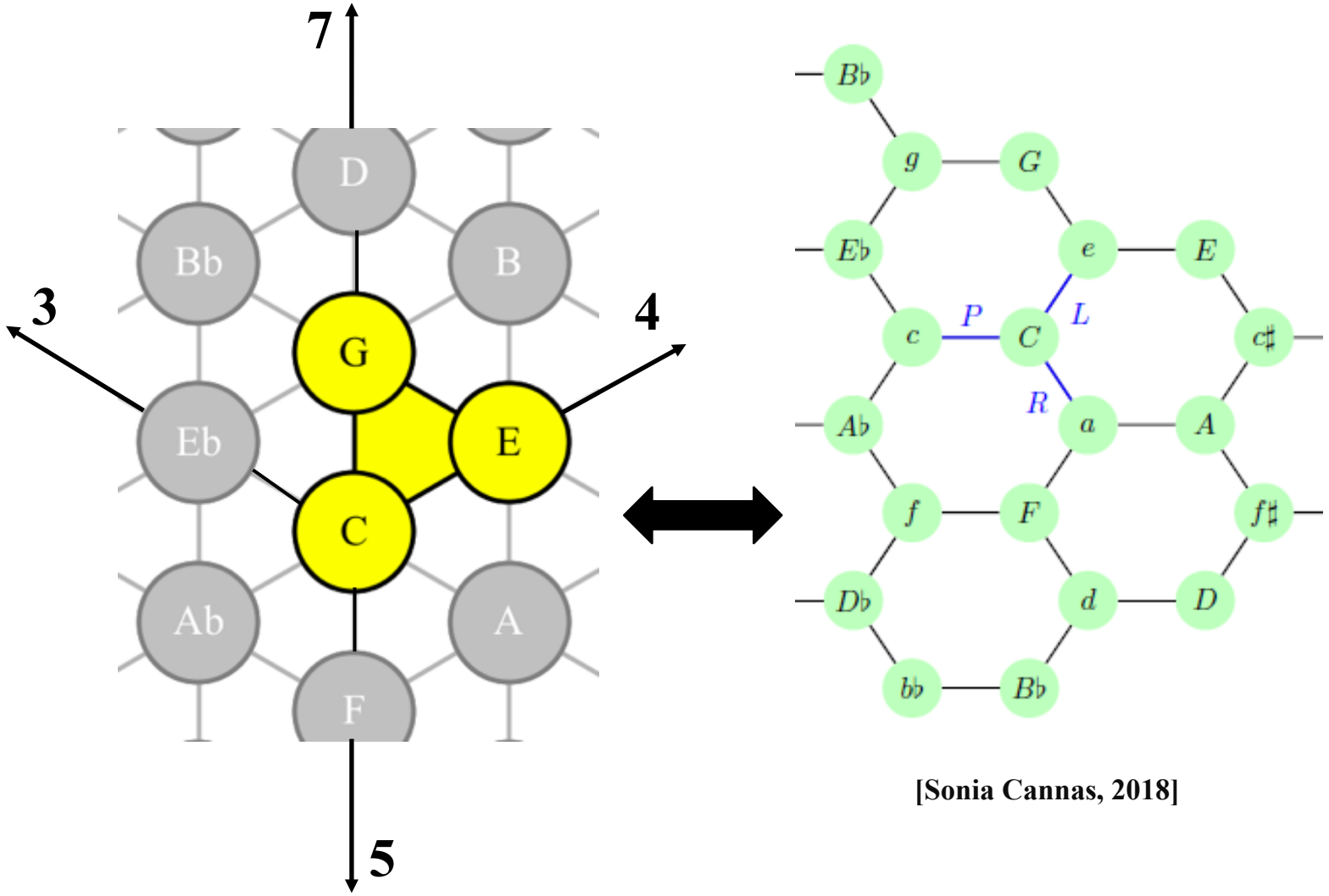


From the Tonnetz to the dual one (and vice-versa)



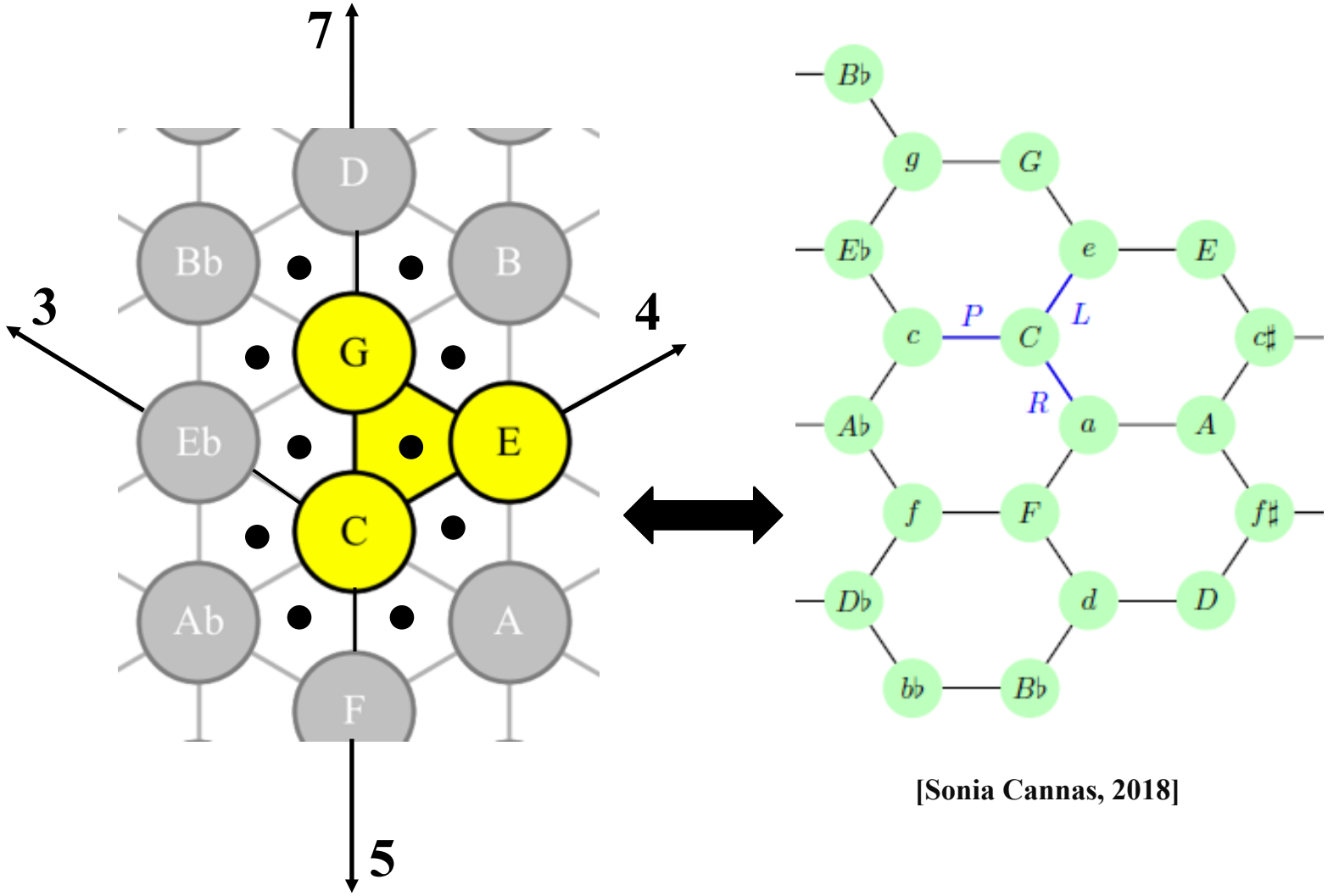


From the Tonnetz to the dual one (and vice-versa)



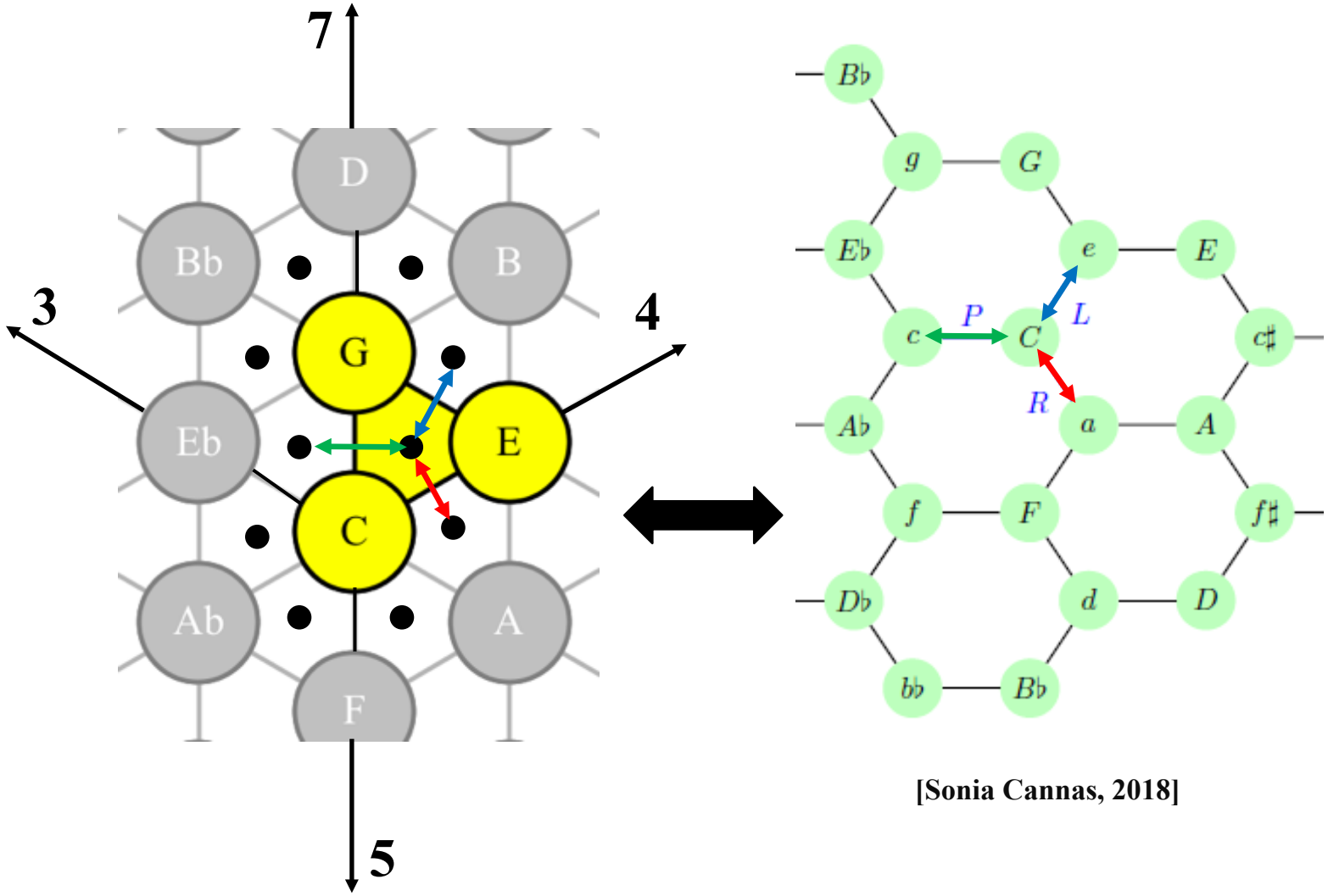
[Sonia Cannas, 2018]

From the Tonnetz to the dual one (and vice-versa)



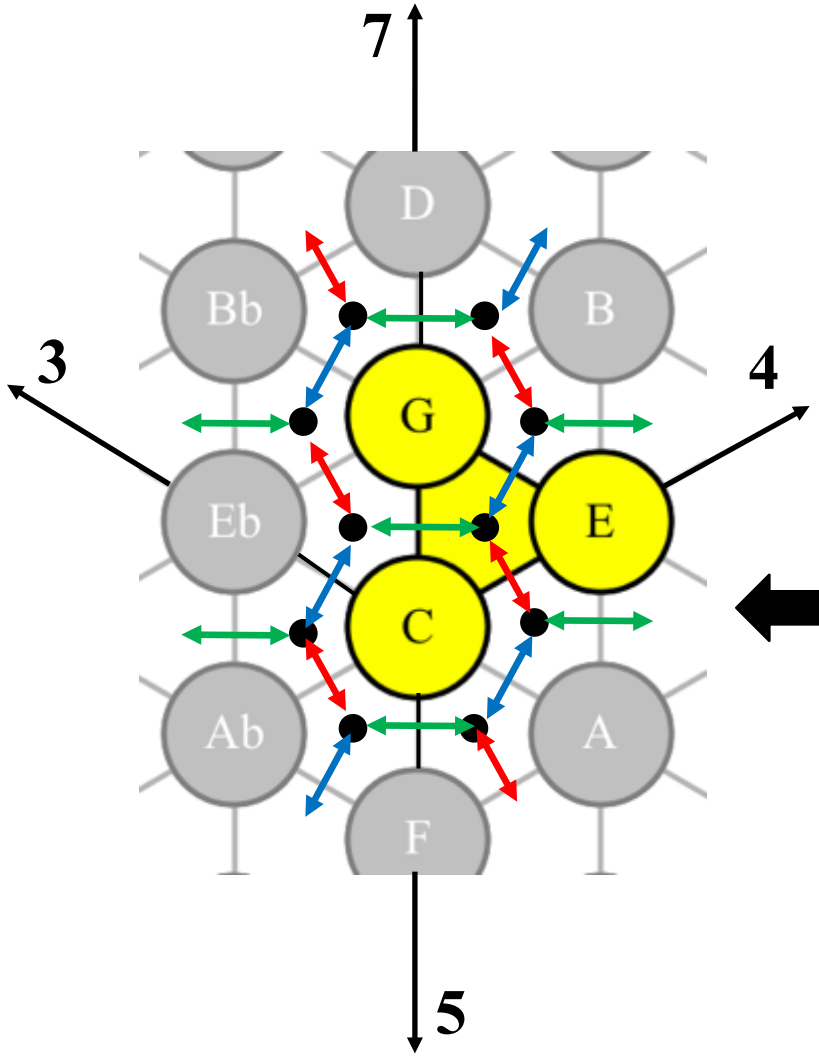
[Sonia Cannas, 2018]

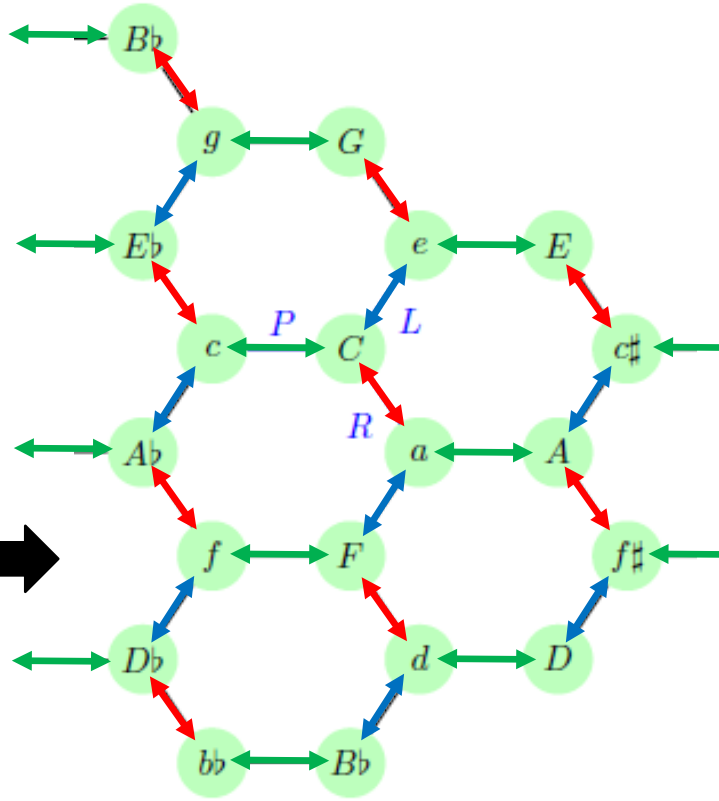
From the Tonnetz to the dual one (and vice-versa)



[Sonia Cannas, 2018]

From the Tonnetz to the dual one (and vice-versa)

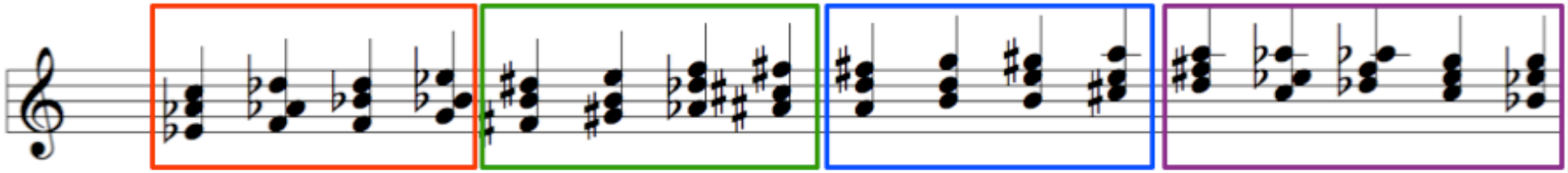


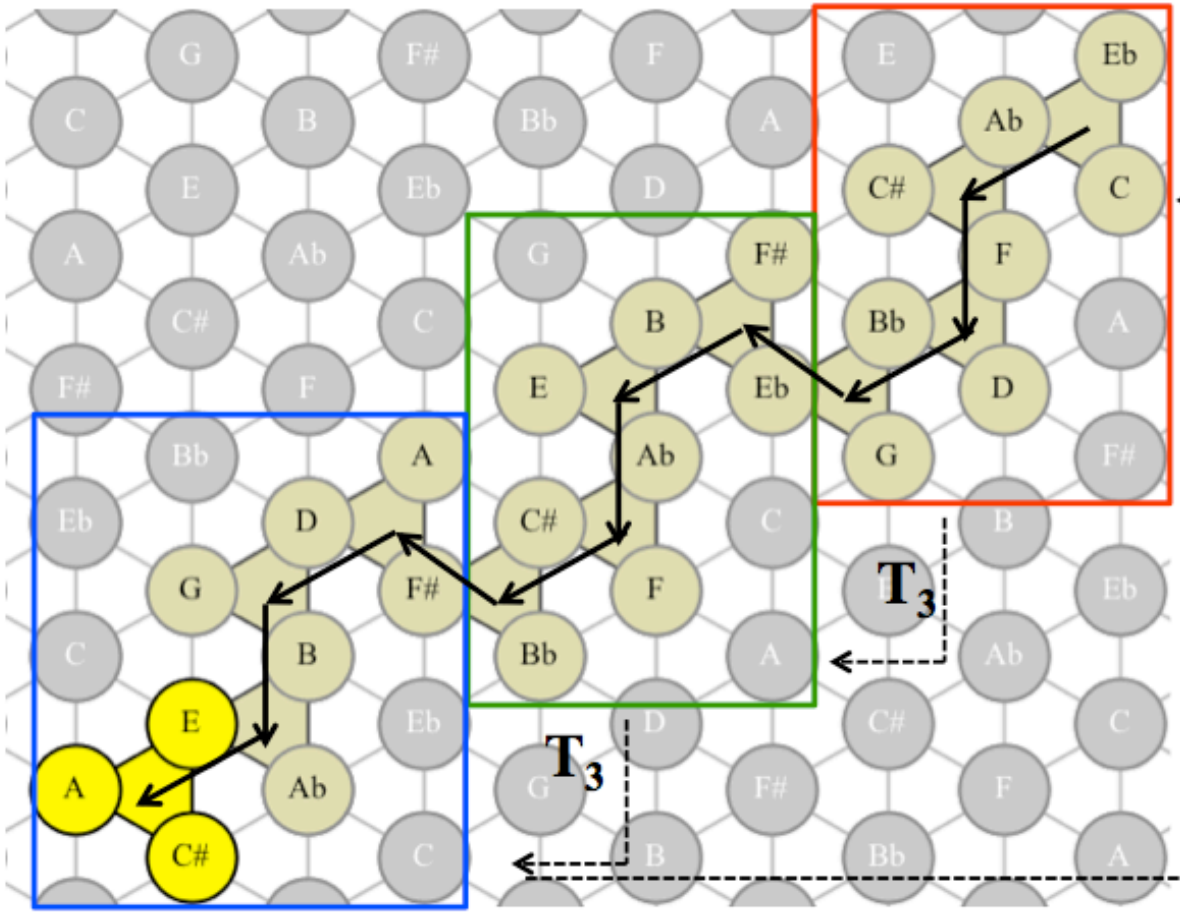


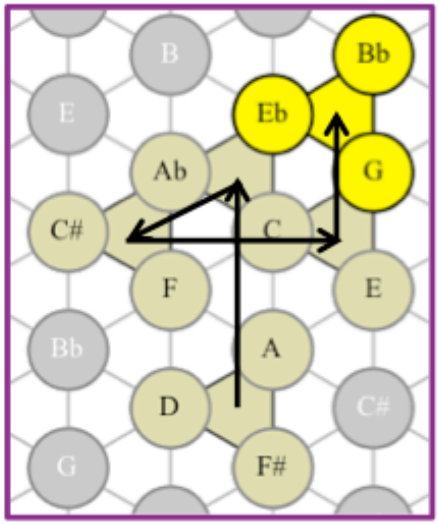
[Sonia Cannas, 2018]

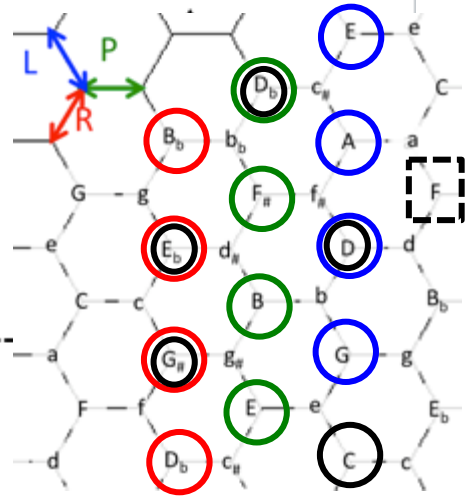
Symmetries in Paolo Conte's *Madeleine*

La_b Re_b Si_b Mi_b Si Mi Re_b $Fa_\#$ Re Sol Mi La Re La_b Re_b Do Mi_b





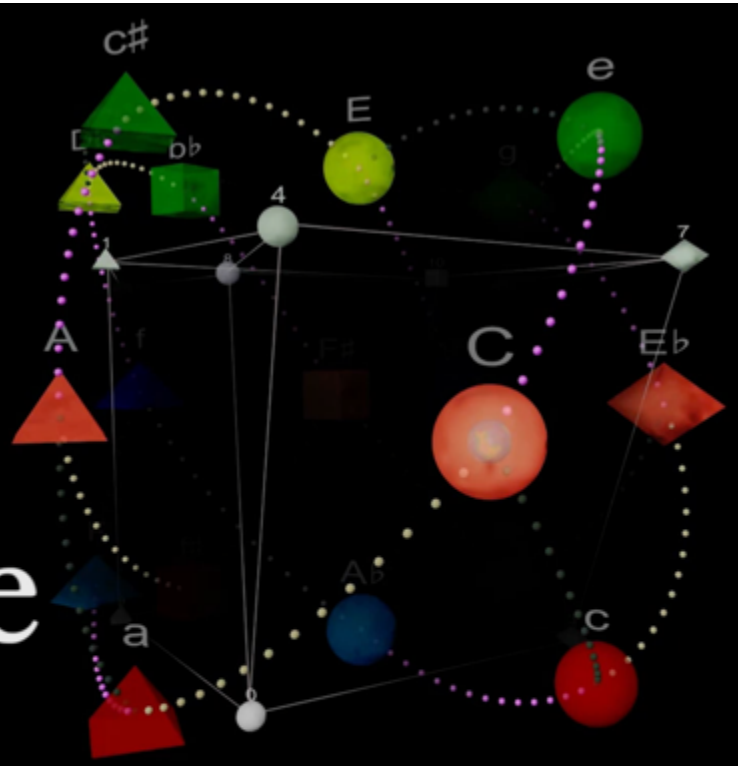




Almost total covering of the major-chords space

Beethoven and the Hypersphere

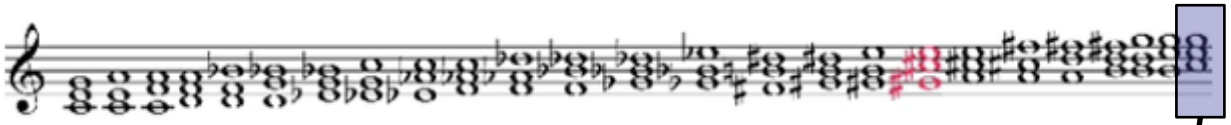
(and the Tonnetz)



Gilles Baroin 2016
www.MatheMusic.net

Gilles Baroin

Reading Beethoven backwards



Le Blé en Herbe

(Polo/Moreno/Dieu)

Plonger comme un enfant, cheveux au vent
Sous l'océan du blé en herbe
Marée d'épis couleur d'amande
Qui tendent à caresser le ciel

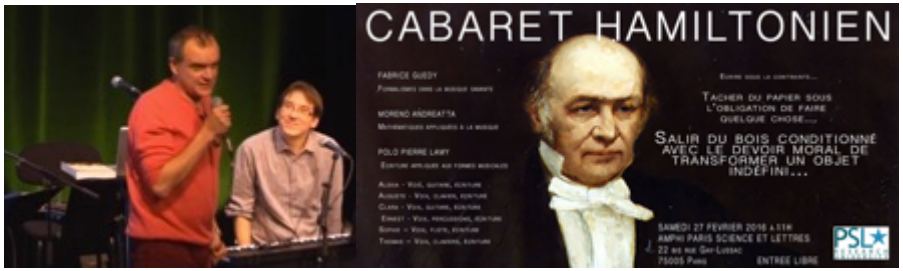
Algues tendres de mille plages
Frôlant le ventre des nuages
Cheveux de pluie, dos de poissons
Qui frissonnent à l'unisson

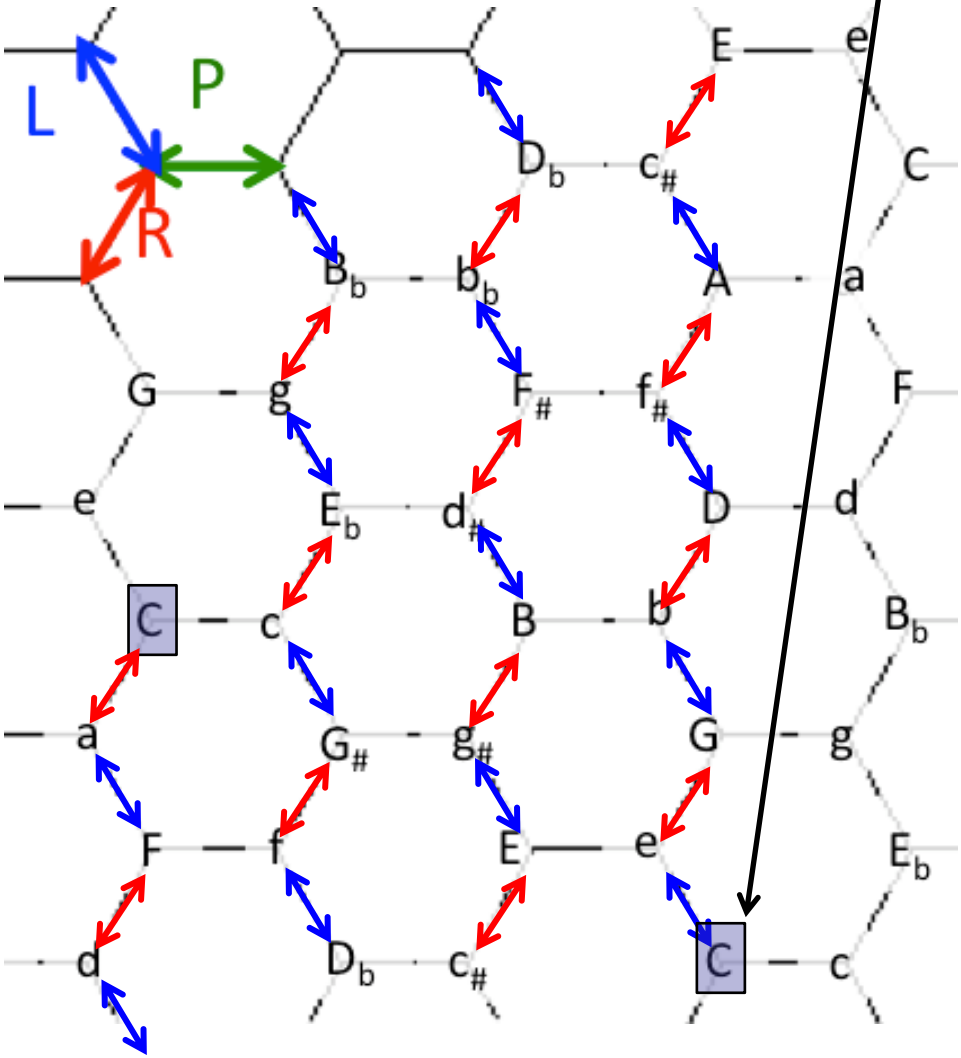
Suivre le bord des continents
Dans l'océan du blé en herbe
Pécher le corail du pavot
Dans le sang des coquelicots

Croiser matin dans l'herbe folle
Deux tourterelles qui s'envolent
Suivre les jeux des hirondelles
Sur le paysage éternel
Nager comme un enfant, cheveux au vent
Sous l'océan
Du blé en herbe

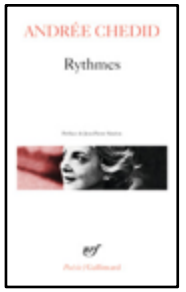
Marée de fruits au goût amer
Acide et salée comme la mer

Vers l'ilôt d'un petit village
Vers un château d'eau sur la plage
Quand tout s'éteint avant l'orage
Quand se lève le vent du large
Sur le blé vert





From poetry to song writing: hamiltonian compositional strategies



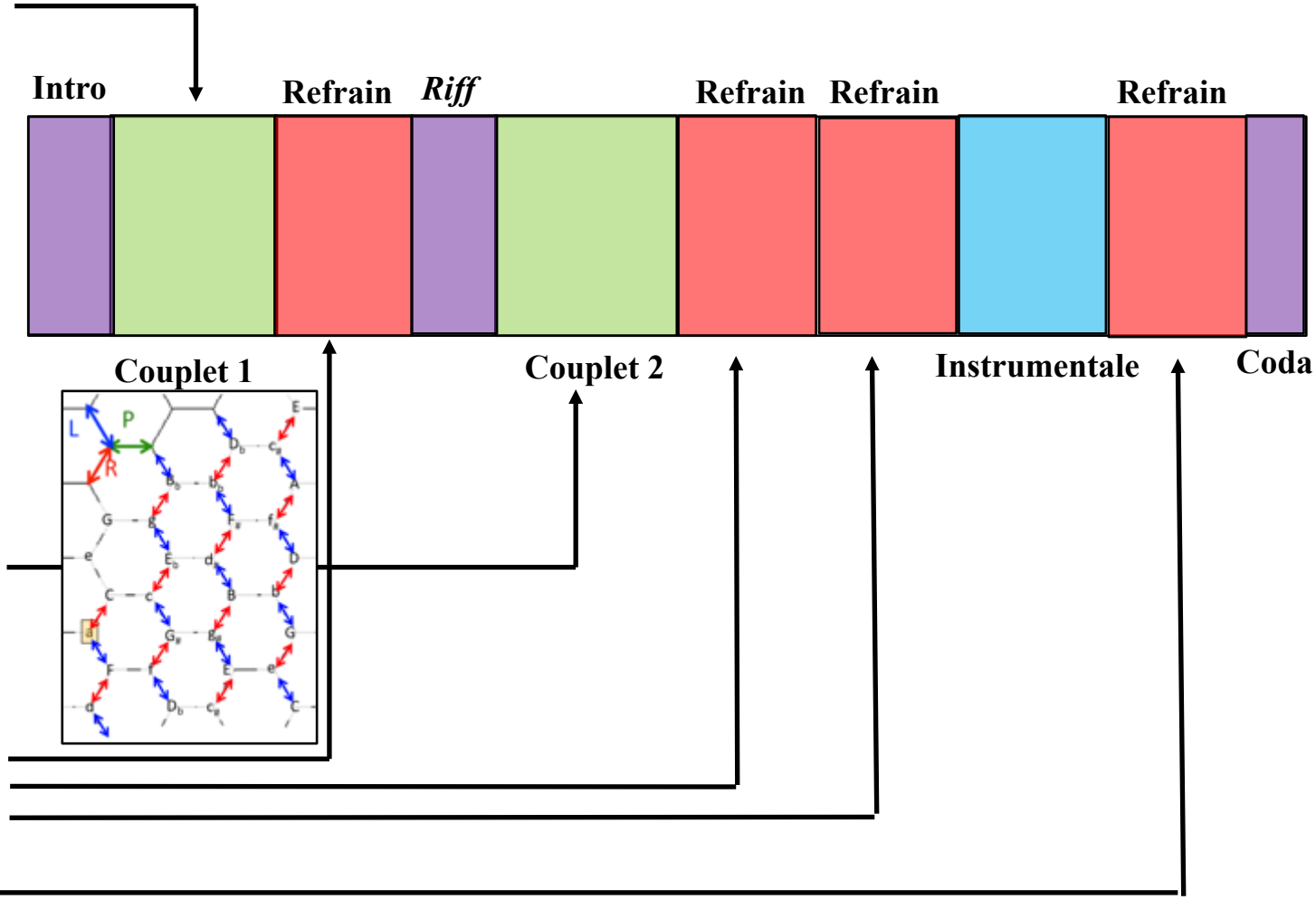
A part (Andrée Chedid, poème tiré du recueil *Rhymes* Collection Poésie/Gallimard (n. 527), Gallimard, 2018)

À part le temps
Et ses rouages
À part la terre
En éruptions
À part le ciel
Pétrisseur de nuages
À part l'ennemi
Qui génère l'ennemi

À part le désamour
Qui ronge l'illusion
À part la durée
Qui moisit nos visages

À part les fléaux
À part la tyrannie
À part l'ombre et le crime
Nos batailles nos outrages

Je te célèbre ô Vie
Entre cavités et songes
Intervalle convoité
Entre le vide et le rien

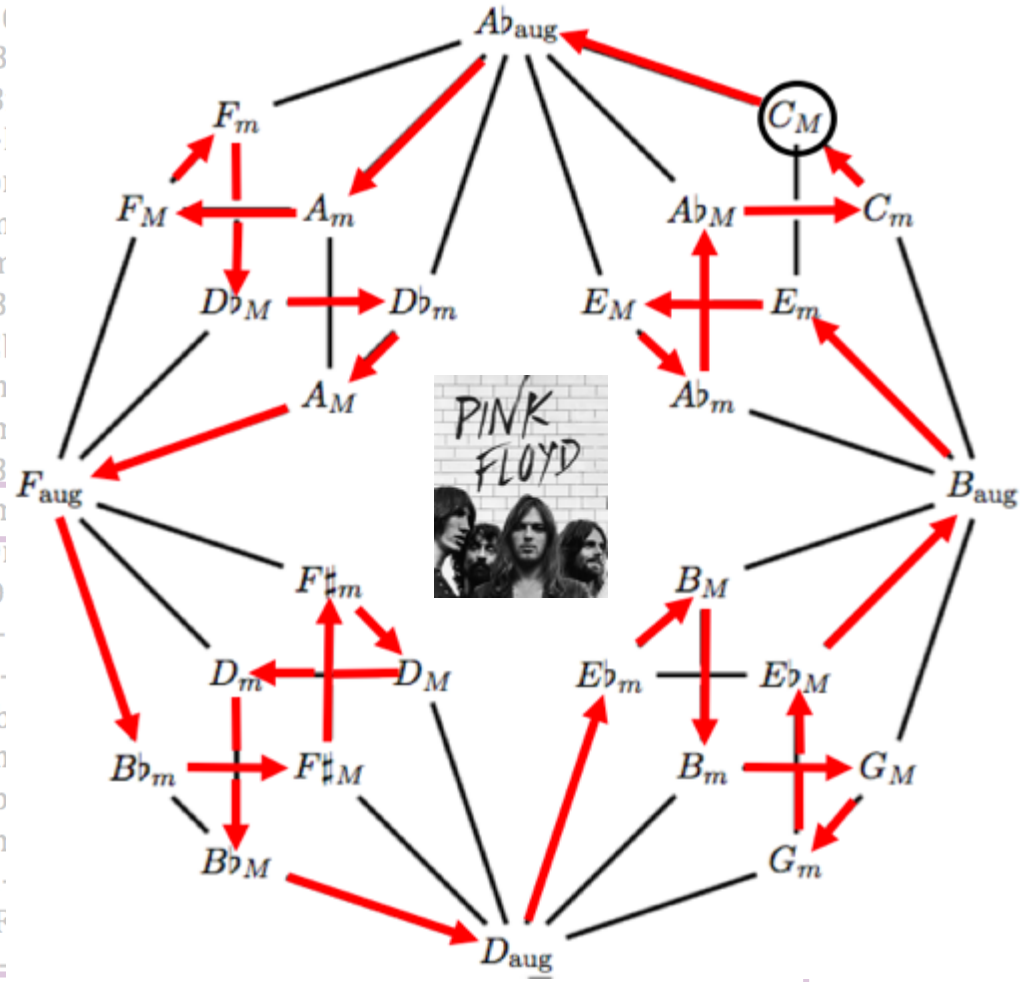


Exploring Hamiltonian trajectories in song writing

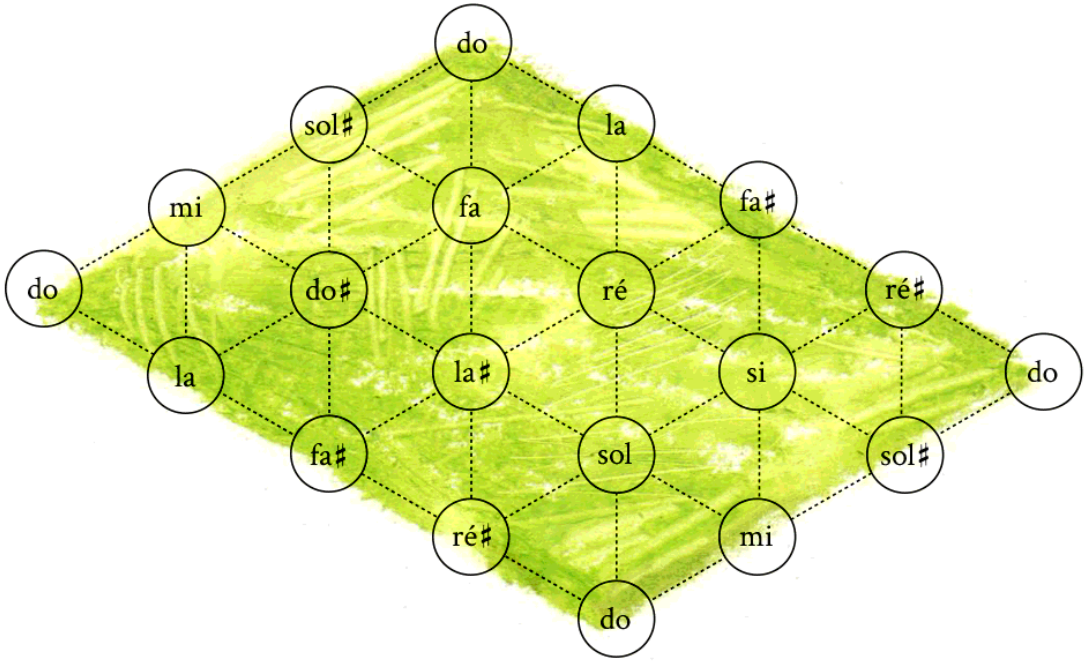
1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPRPR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPRLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRLRPRP
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPRL
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRLRLRP
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm--RLRPRPRP
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em--RLRPRPRL
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLRP
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm--RLRLRPRP
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL

Exploring Hamiltonian trajectories in song writing

1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-B
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Eb
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-B
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-E
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-B
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-D
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-D
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bb
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bb
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL



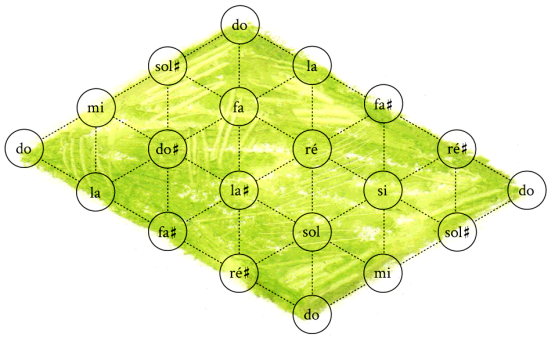
The topological structure of the Tonnetz



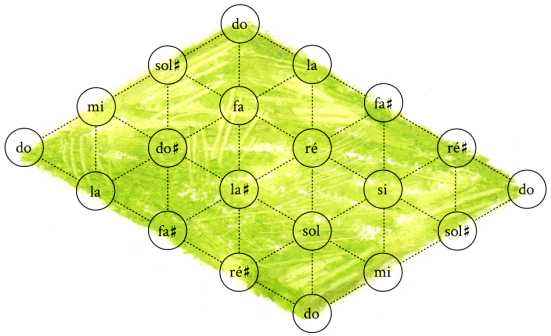
The topological structure of the Tonnetz

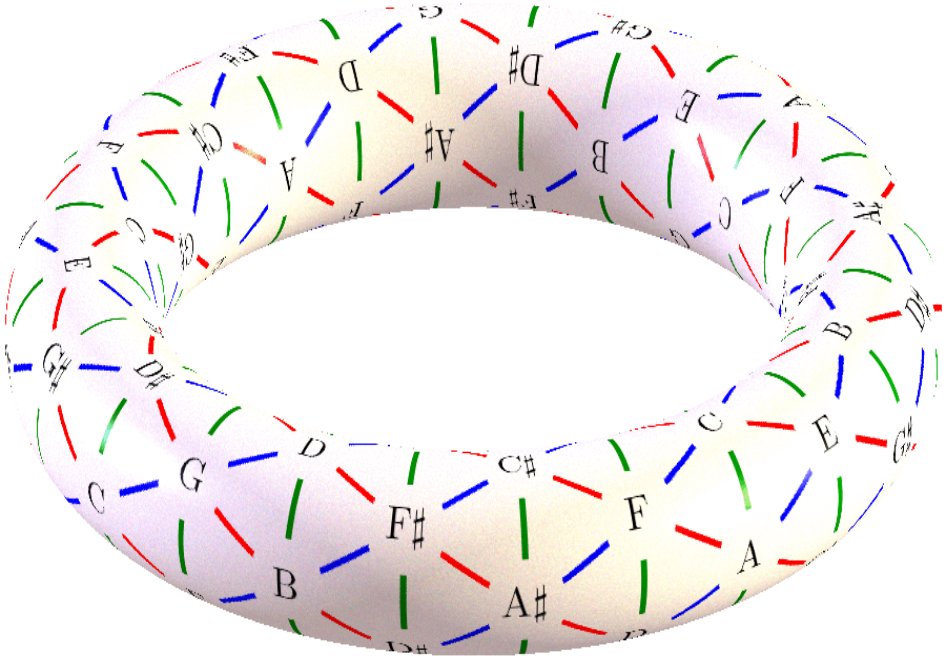
The topological structure of the Tonnetz

The topological structure of the Tonnetz



The topological structure of the Tonnetz





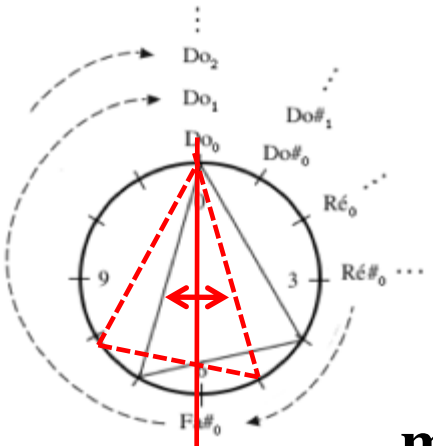
➔ Source : https://upload.wikimedia.org/wikipedia/commons/6/67/TonnetzTorus.gif

The *Tonnetz* as a simplicial complex

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

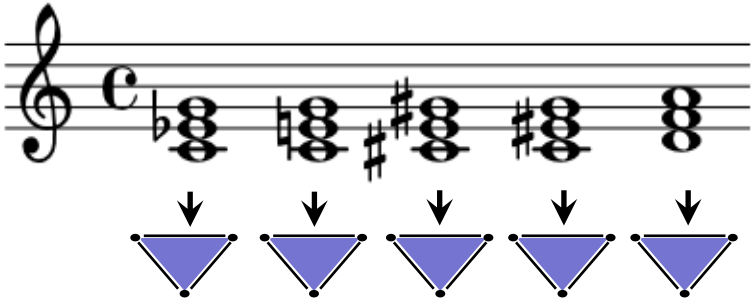
Louis Bigo

- Assembling chords related by some equivalence relation
  - Transposition/inversion: Dihedral group action on $P(Z_n)$

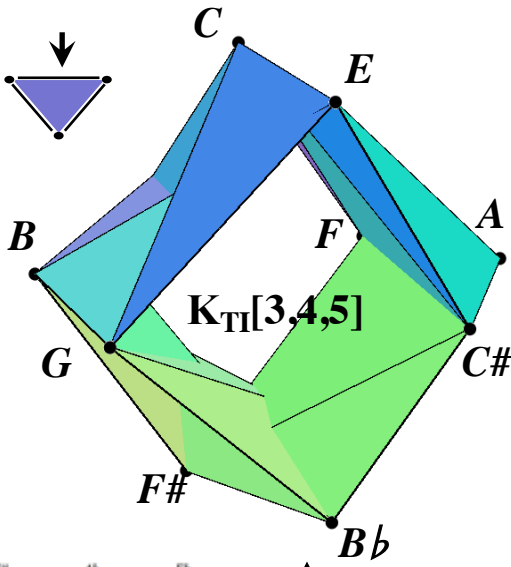


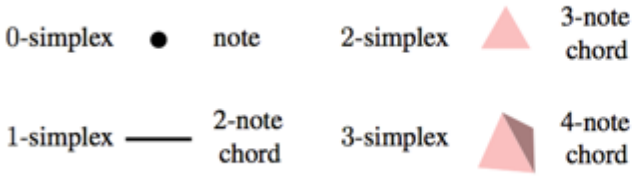
Intervallic structure

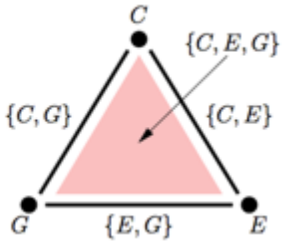
(4,3,5)

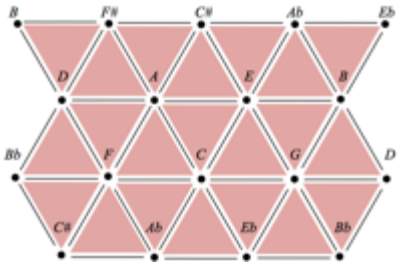


major/minor triads

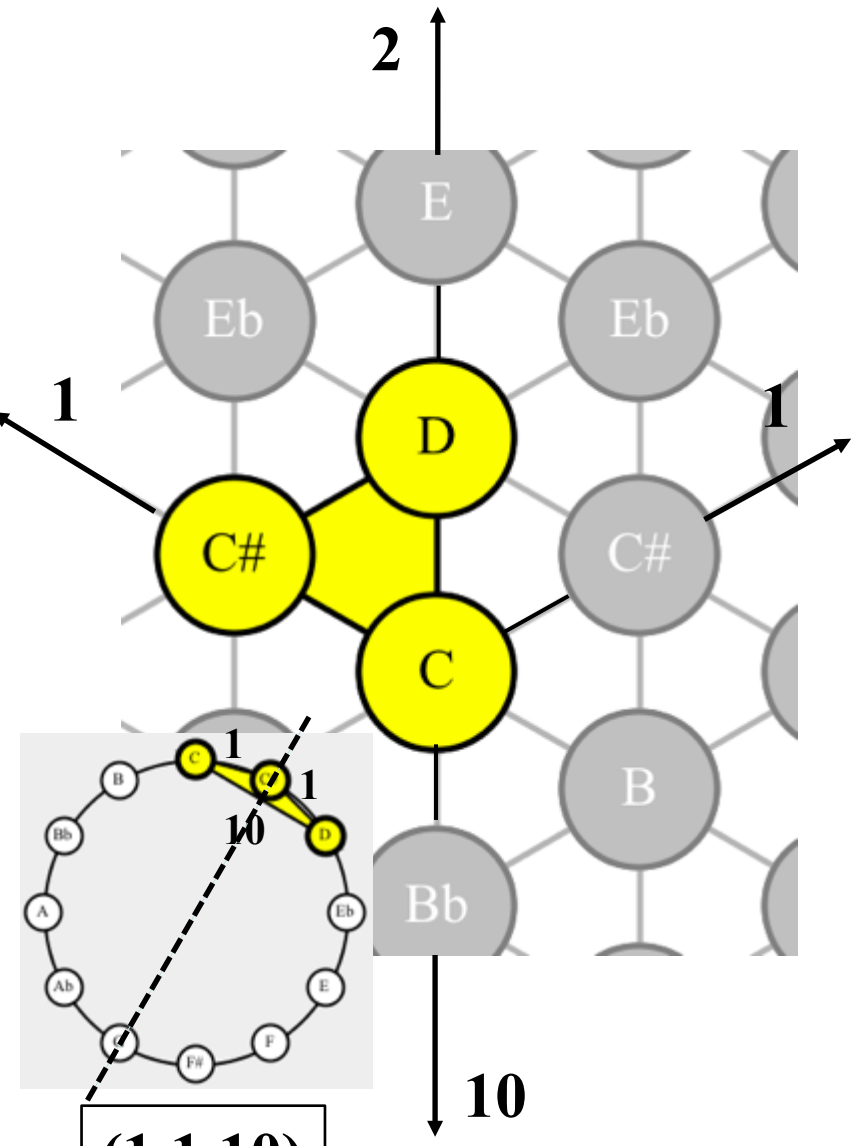


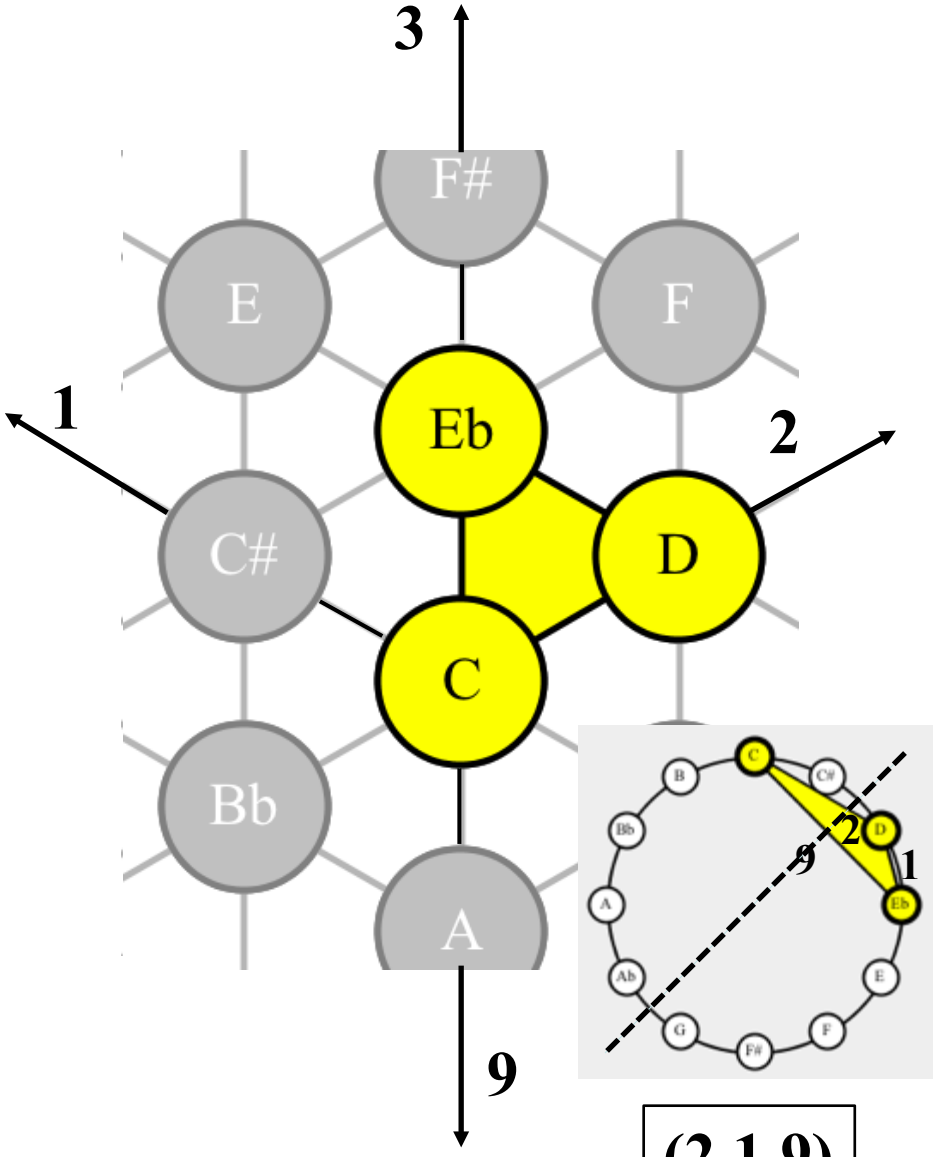






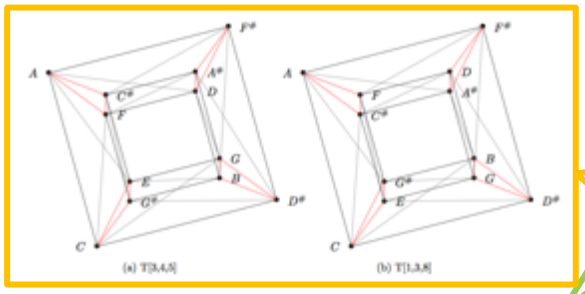
The panoply of *Tonnetze* at the service of the analyst

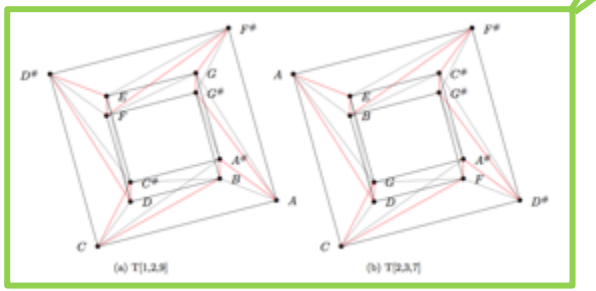




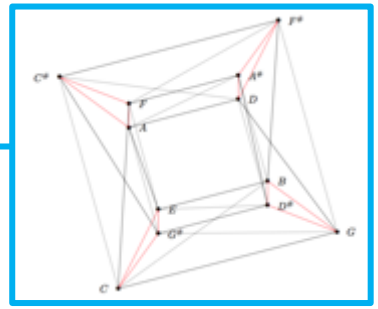
Classifying Tonnetze as Simplicial Chord Complexes

P. Lascabettes, ***Homologie Persistante Appliquée à la Reconnaissance de Genres Musicaux*****, Master 1, ENS Saclay**

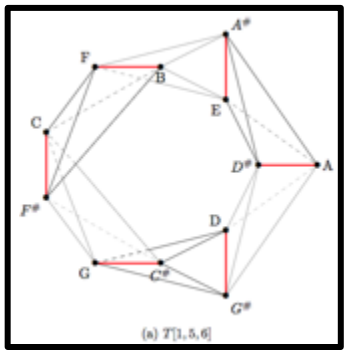




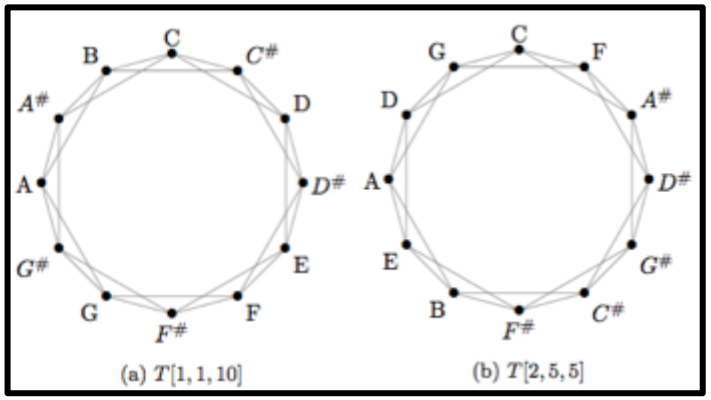
Tonnetz	Nombres de Betti		
	β_0	β_1	β_2
$T[1,2,9]$	1	2	1
$T[1,3,8]$	1	2	1
$T[1,4,7]$	1	2	1
$T[2,3,7]$	1	2	1
$T[3,4,5]$	1	2	1
$T[1,1,10]$	1	1	0
$T[2,5,5]$	1	1	0
$T[2,2,8]$	2	2	0
$T[1,5,6]$	1	1	6
$T[2,4,6]$	2	2	6
$T[3,3,6]$	3	0	3
$T[4,4,4]$	4	0	0



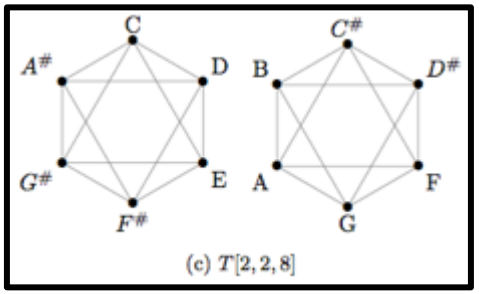
Paul Lascabettes



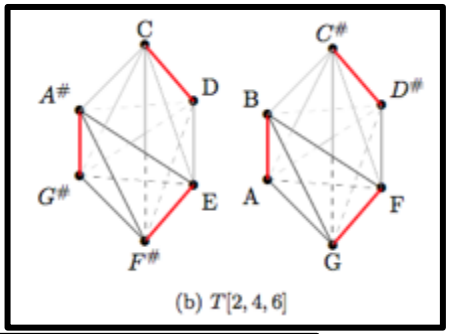
(a) $T[1,5,6]$



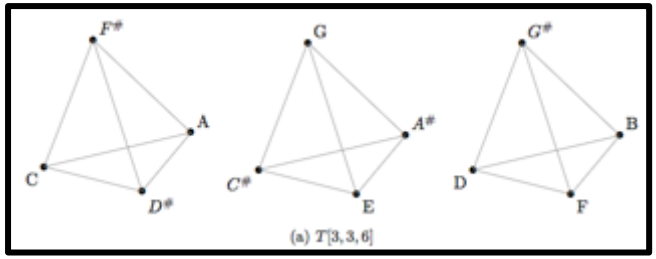
(a) $T[1,1,10]$ (b) $T[2,5,5]$



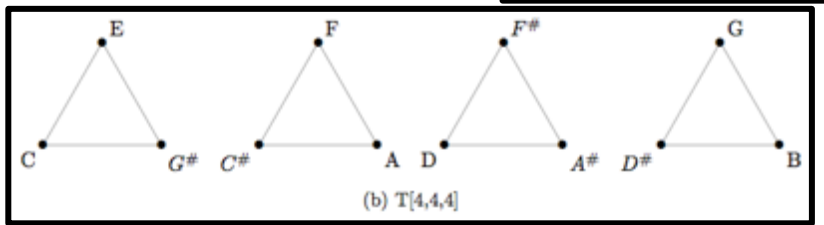
(c) $T[2,2,8]$



(b) $T[2,4,6]$



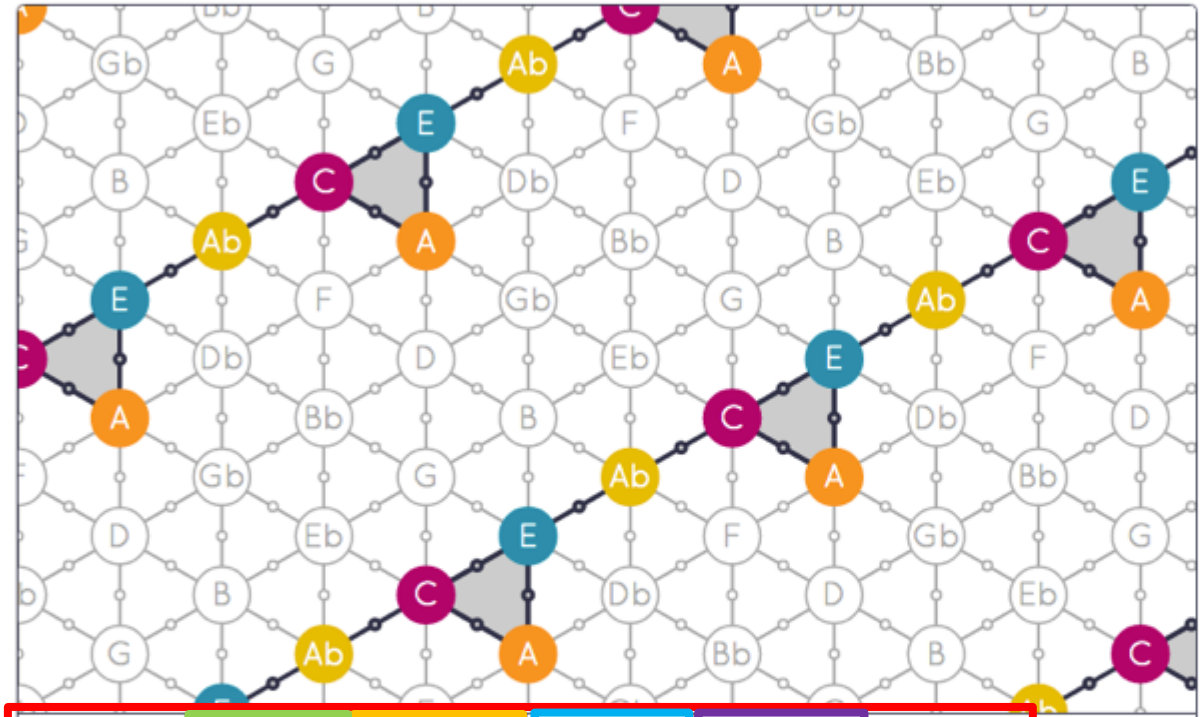
(a) $T[3,3,6]$



(b) T[4,4,4]

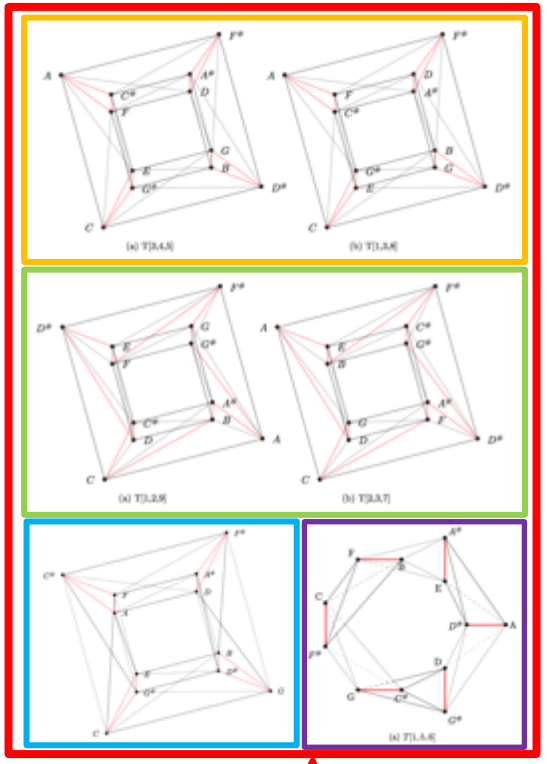
THE TONNETZ

ONE KEY – MANY REPRESENTATIONS

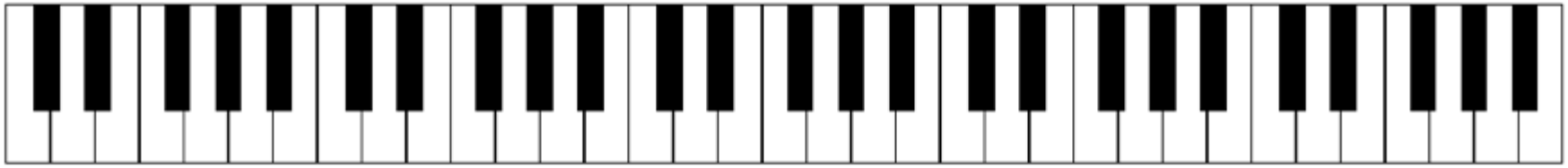


1, 1, 10	1, 2, 9	1, 3, 8	1, 4, 7	1, 5, 6	2, 2, 8
2, 3, 7	2, 4, 6	2, 5, 5	3, 4, 5	3, 3, 6	4, 4, 4

Dual

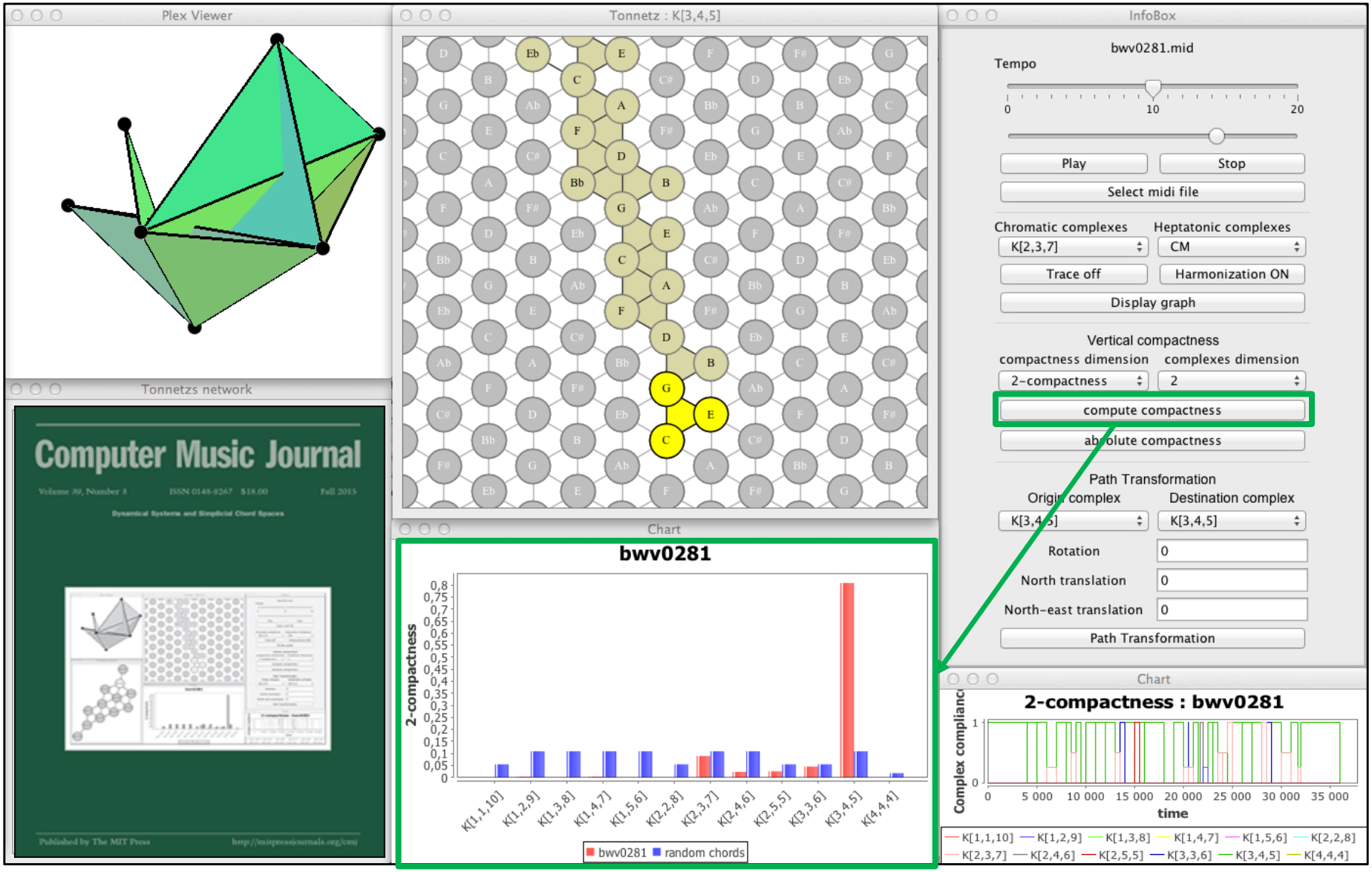


Load Midi File | Play | Start Recording | Rotate 180° | Translate



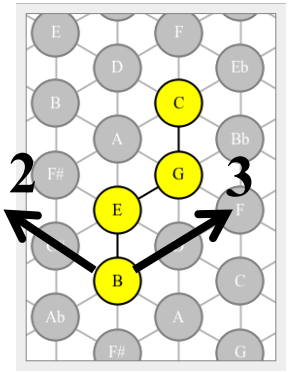
➔ **https://guichaoua.gitlab.io/web-hexachord/**

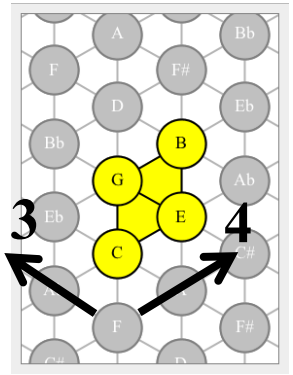
Spatial music analysis via *Hexachord*

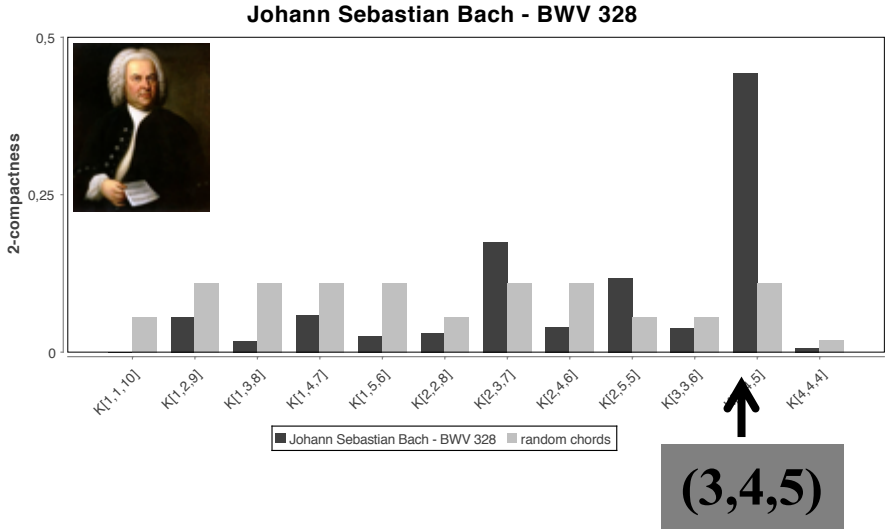


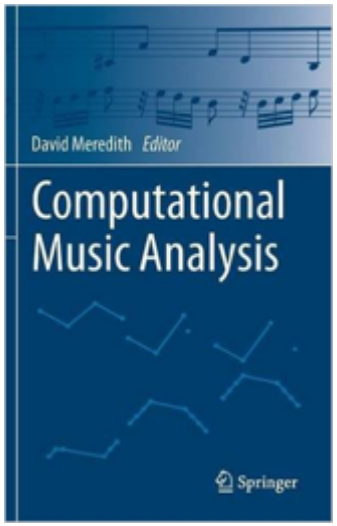
➔`http://www.lacl.fr/~lbigo/hexachord`

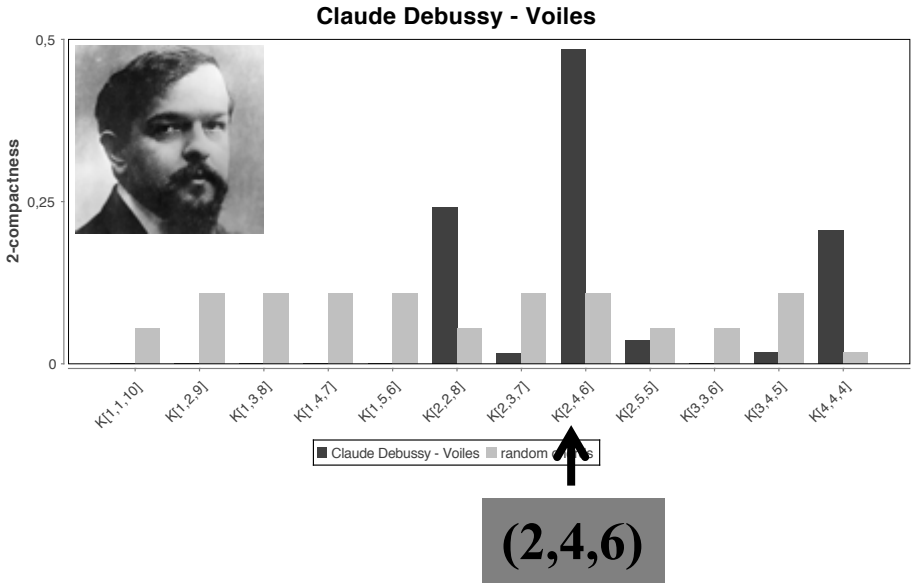
The geometric character of musical style

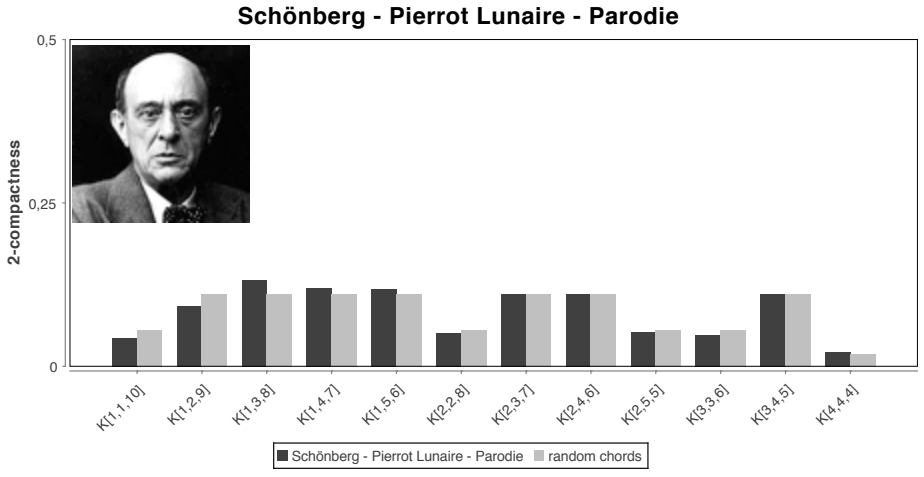






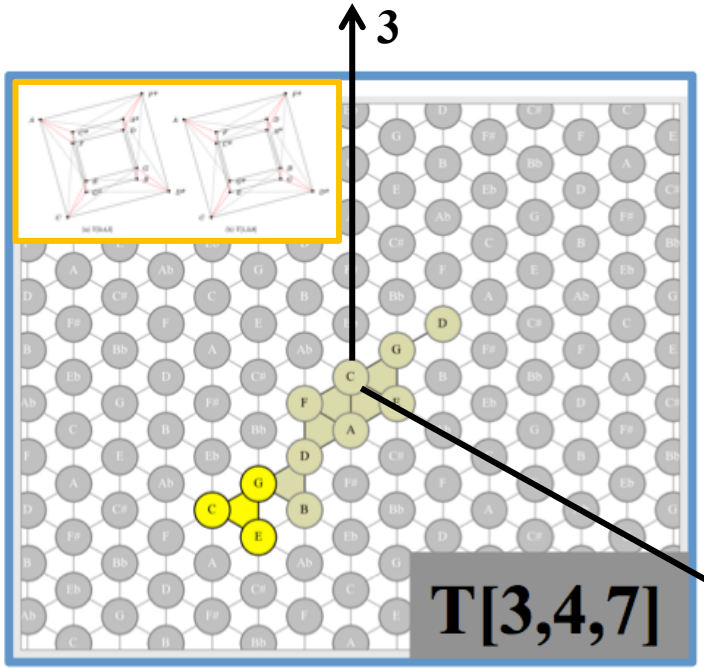


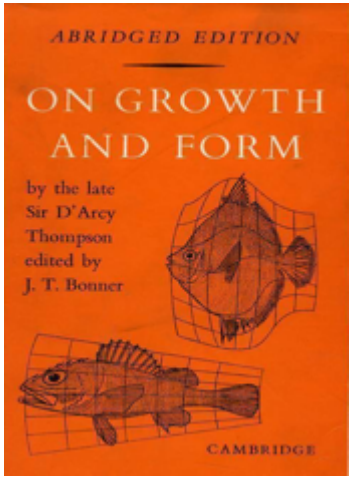


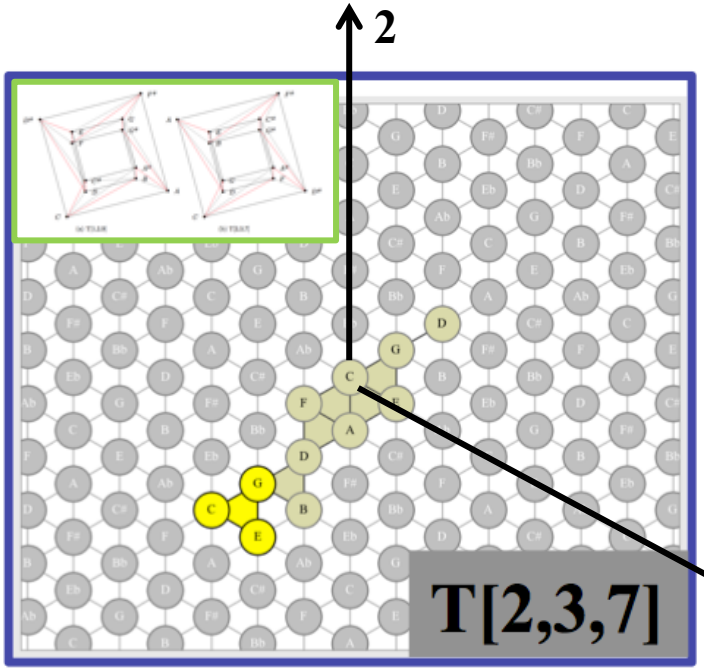


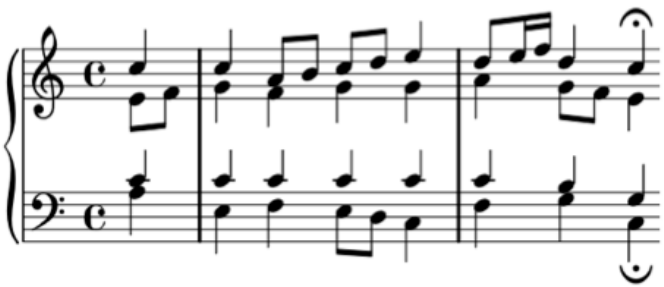
➔ **Bigo L., M. Andreatta, "Musical analysis with simplicial chord spaces", in D. Meredith (ed.),** ***Computational Music Analysis*****, Springer, 2015**

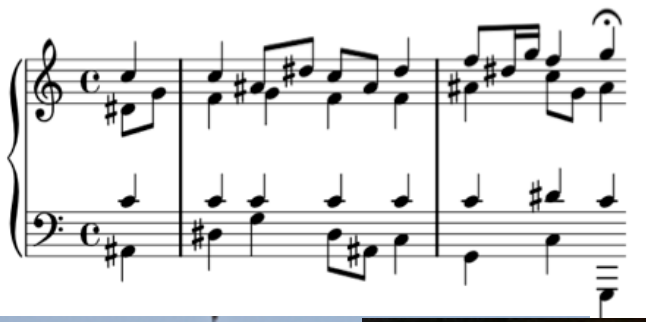
The style...is the space!



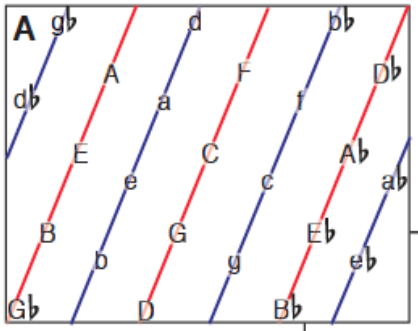








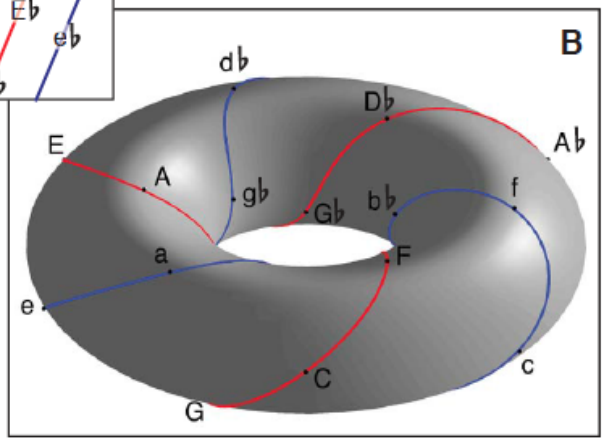
Neurosciences and *Mathemusical* Learning: ProAppMaMu



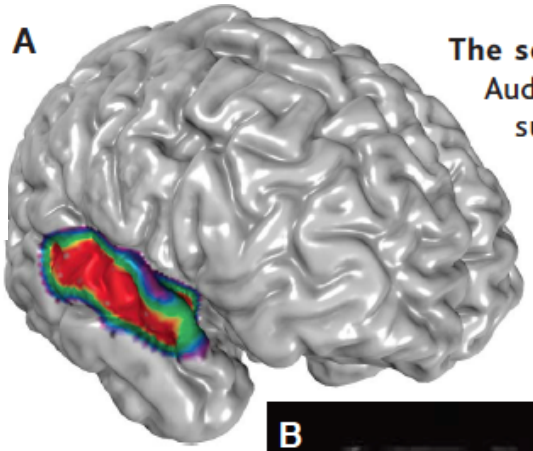
PERSPECTIVES: NEUROSCIENCE

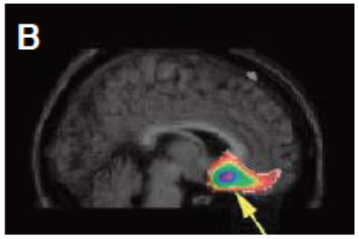
Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl



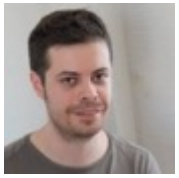
Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). **(B)** Musical keys as points on the surface of a torus.



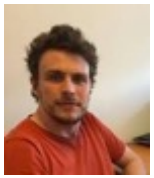


The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. **(B)** Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

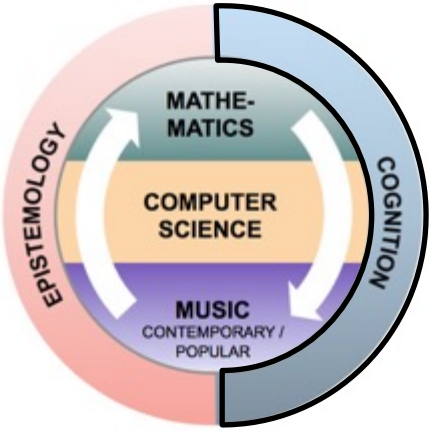
E. Bisesi



J.-L. Besada

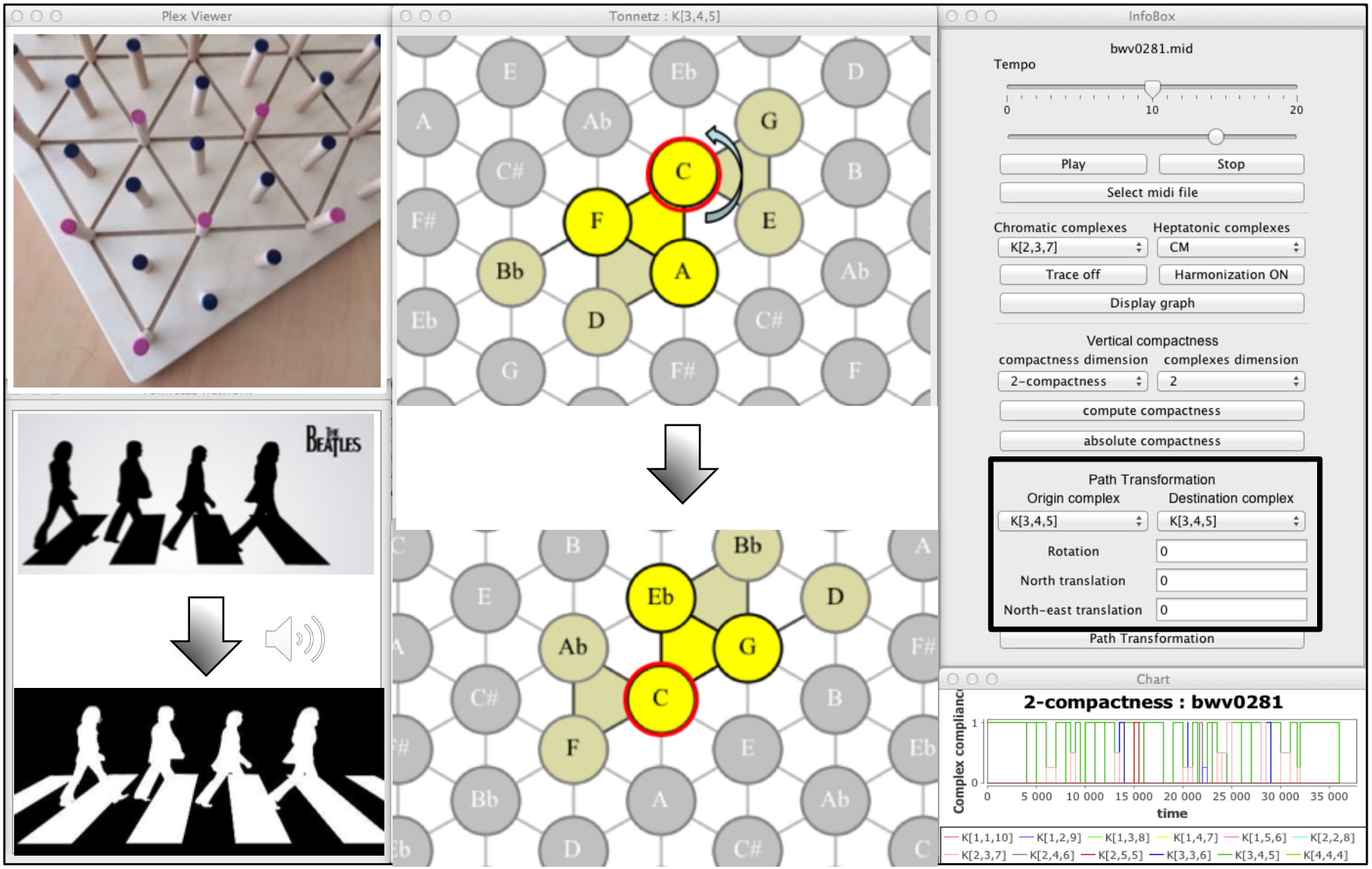


C. Guichaoua



The Tonnetz Environment: A Web Environment for Computer- Aided "Mathemusical" Learning and Research (paper to be presented at *Computer Supported Music Education 2021*, online)

Rotation of the trajectory and Negative Harmony



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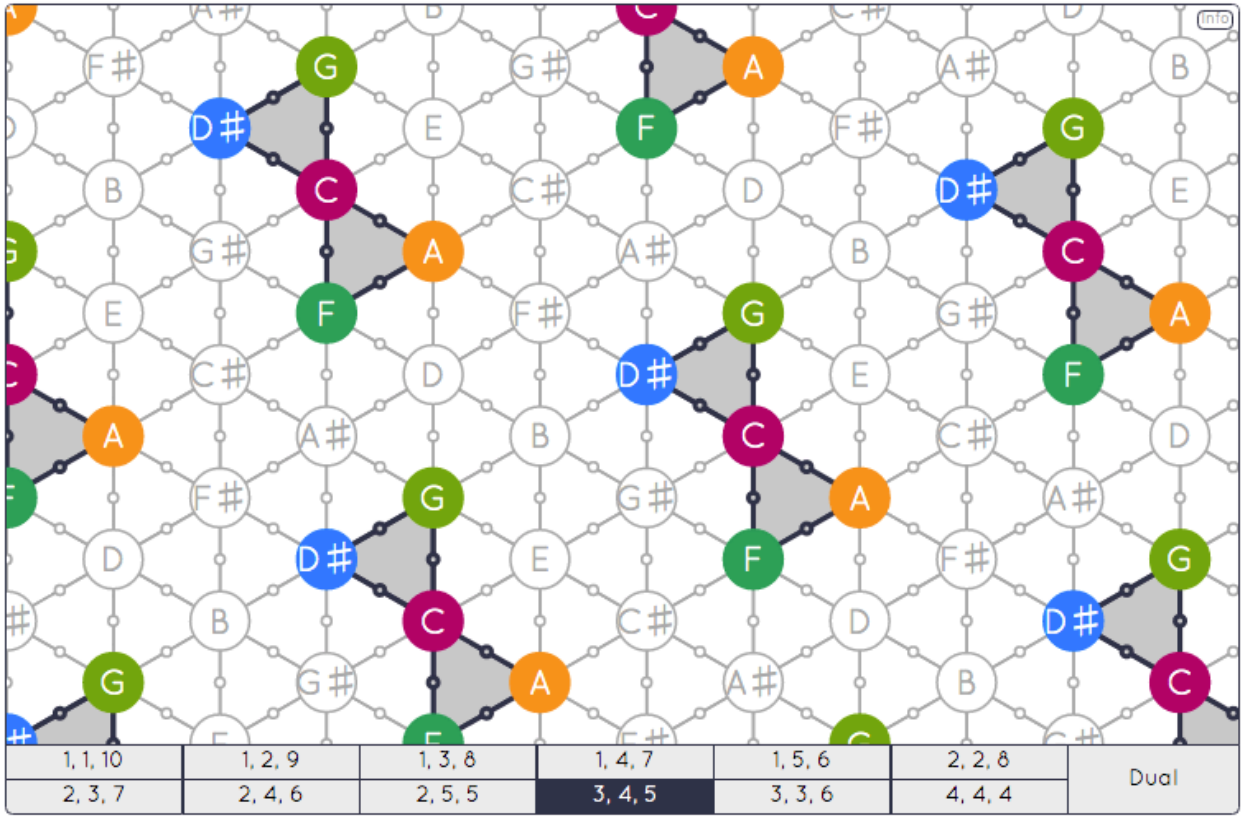
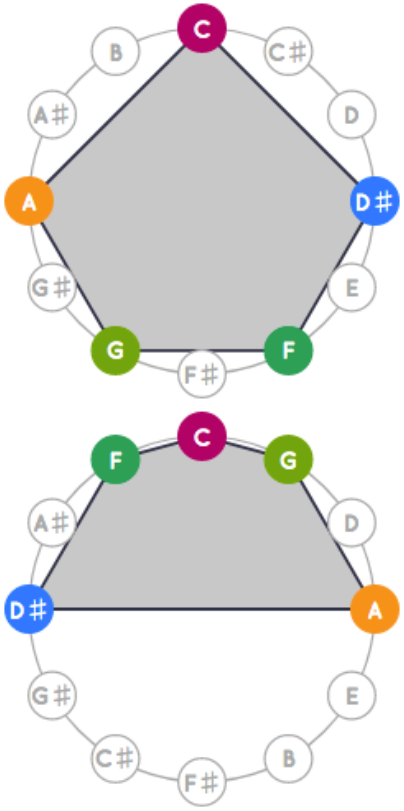
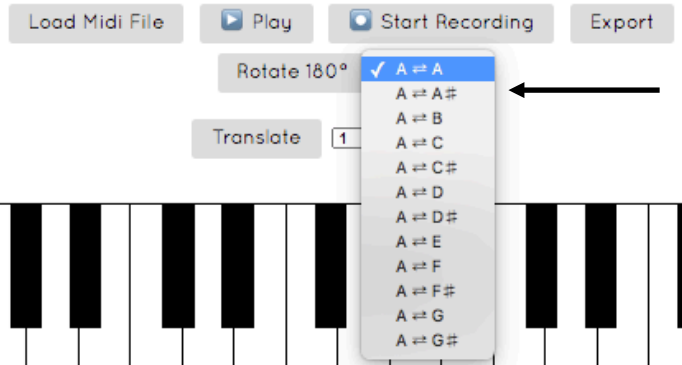
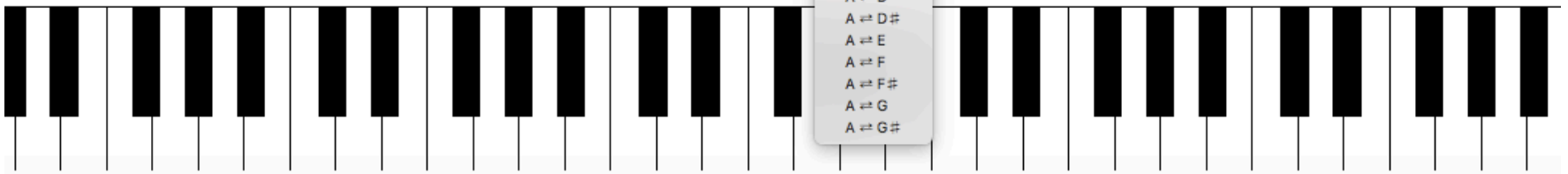






Twelve possible rotations!




Thank you for your attention!