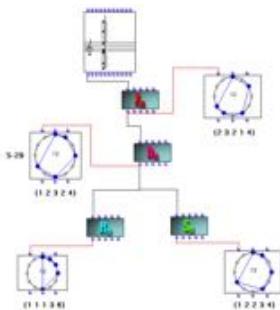




Some mathematical and computational models for computer-aided music theory, analysis and composition



Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS/UPMC & IRMA/USIAS
<http://repmus.ircam.fr/moreno/smri>

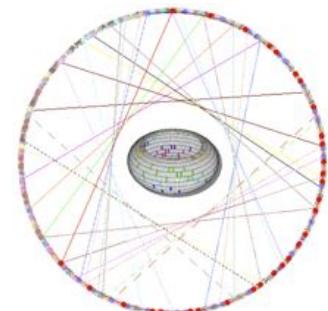
ircam
Centre
Pompidou



IRMA
Institut de Recherche
Mathématique Avancée

USIAS
University of Strasbourg
Institute for Advanced Study

UPMC
SORBONNE UNIVERSITÉS



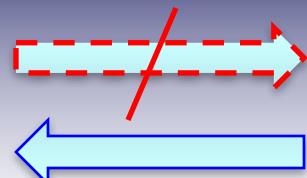
The double movement of a ‘mathemusical’ activity

MATHEMATICS



MUSIC

Musical problem



Music analysis

Music theory

Composition

OpenMusic, a Visual Programming Language for computer-aided composition

www.repmus.ircam.fr/openmusic/home

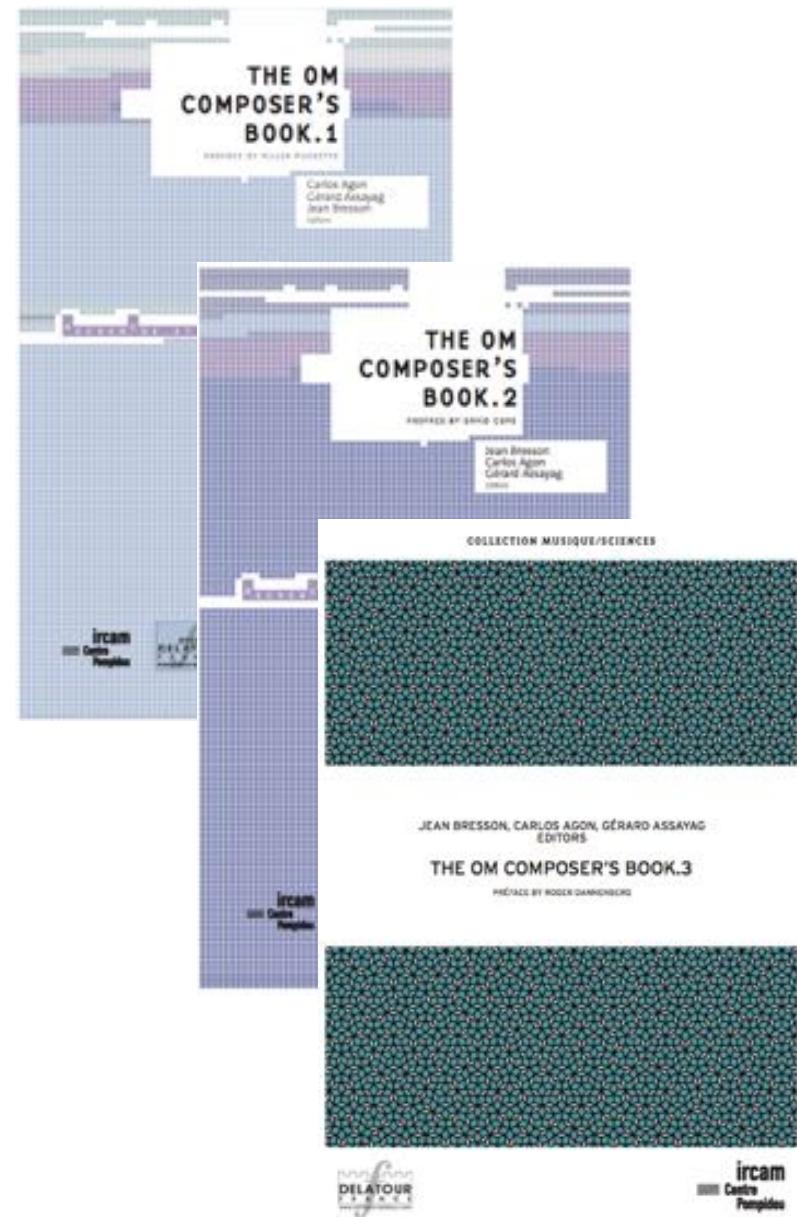
OpenMusic

(c) Ircam - Centre Pompidou



Dedicated to the memory of Gérard Grisey (French composer, 1946-1998)

Design and development : G. Assayag, A. Agon and J. Bresson
with help from C. Ruette, D. Delerue, Use Malibahare (Grame)
Musical expertise by : M. Andreatta, J. Balon, J. Finberg, K. Haddad,
C. Matherne, M. Mart, T. Murali, O. Sandler, M. Stroppa, H. Tutschku.
Artwork : A. Muhesen.



**C. Agon, G. Assayag and J. Bresson, *The OM Composer's Book* (3 volumes)
“Musique/Sciences” Series, Ircam/Delatour, 2006, 2007 and 2016**

“MathTools”: an algebraic environment within OpenMusic visual programming language



Computational Music Theory:

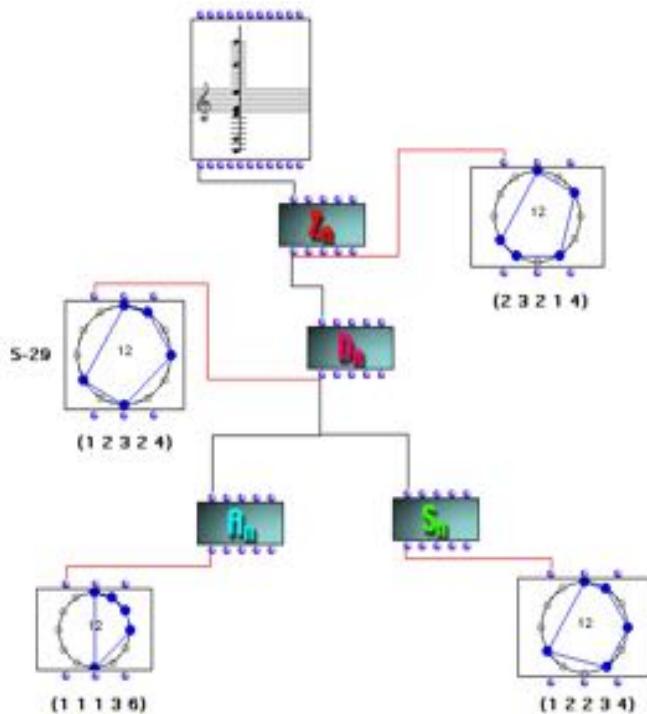
- Classification and Enumeration of musical structures
- Chords/scales, motifs and rhythms:
 - ♪ Catalogues (Costère, Zalewski, Vieru, Forte, Carter, Morris, Mazzola, Estrada, ...)
 - Σ Combinatorial algebra, Polya Enumeration Theory, Burnside Lemma, Discrete Fourier Transform
- Rhythmic Tiling Canons (by translation, inversion and augmentation)
 - ♪ Messiaen, Vieru, Levy, Johnson, Bloch, Wild, Lanza, Ghisi, ...
 - Σ Group and ring factorization theory and Discrete Fourier Transform (DFT)

Computational Music Analysis:

- *Set Theory*, Transformational Analysis and Sieve Theory
- Pitch-class sets, interval vectors and IFUNC, Z-relations:
 - ♪ Carter, Vieru, Xenakis
 - Σ Group Actions, Homometry, DFT
- Transformational progression/network, *K-nets*
 - ♪ Generalized Interval Systems (David Lewin)
 - Σ Group action and category theory

Computer Aided-Composition:

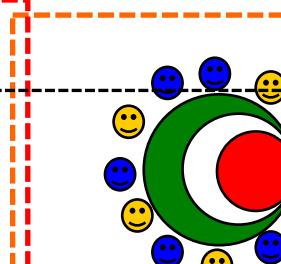
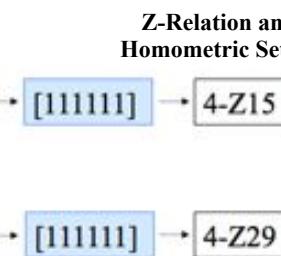
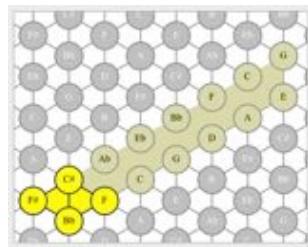
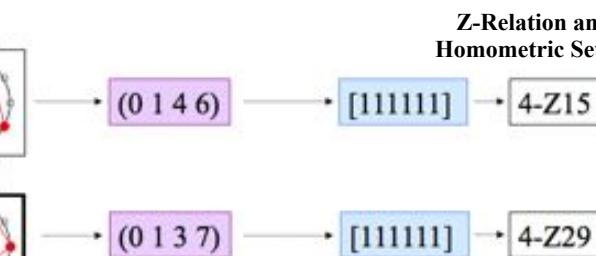
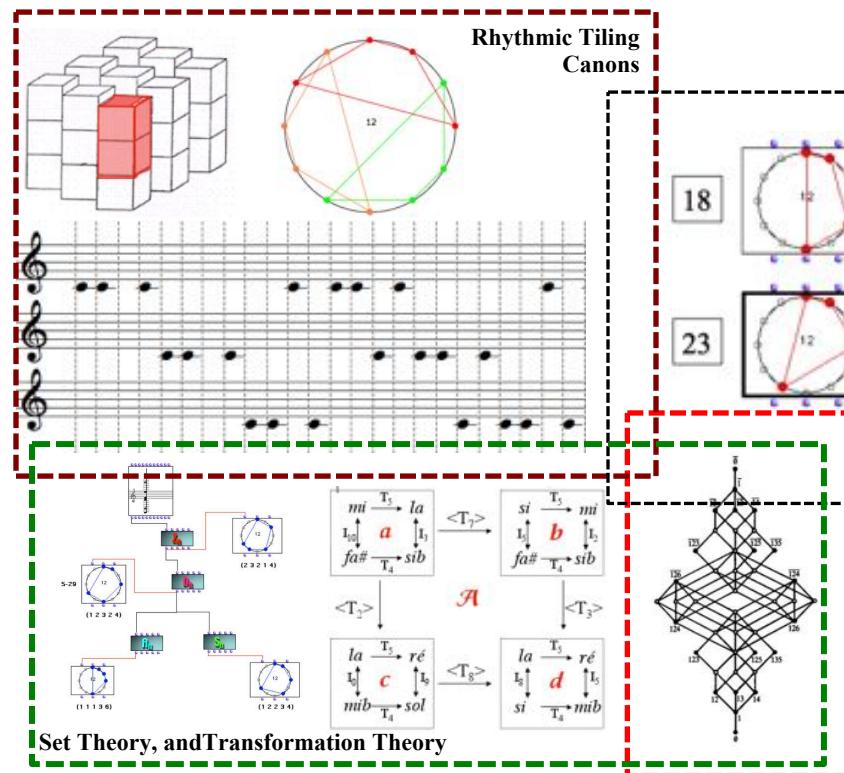
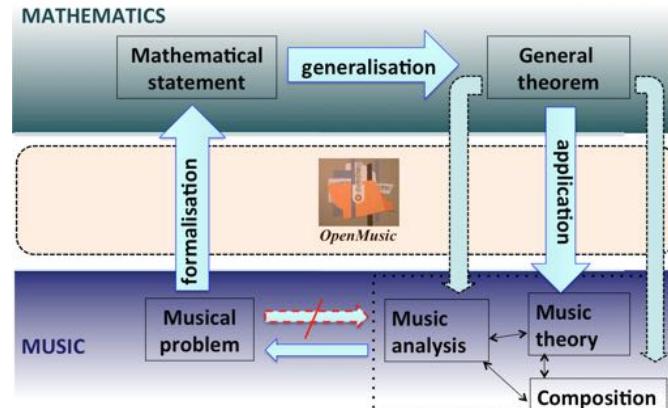
- Scales, modes, chords manipulation
- Rhythmic structures organizations



Some examples of ‘mathemusical’ problems

M. Andreatta: *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

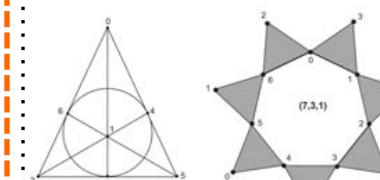
- The construction of Tiling Rhythmic Canons
 - The Z relation and the theory of homometric sets
 - *Set Theory* and Transformational Theory
 - Neo-Riemannian Theory, Spatial Computing and FCA
 - Diatonic Theory and Maximally-Even Sets
 - Periodic sequences and finite difference calculus
 - Block-designs and algorithmic composition



$$Df(x) = f(x) - f(x-1)$$

7 11 10 11 7 2 7 11 10 11 7 2 7 11...
 4 11 1 8 7 5 4 11 1 8 7 5 4 11...
 7 2 7 11 10 11 7 2 7 11 10 11...
 7 5 4 11 1 8 7 5 4 11 1 8...

Finite Difference Calculus



Block-designs

The interplay between algebra and geometry in music

MATH / MUSIC MEETINGS

Creativity in Music and Mathematics

Pierre Boulez & Alain Connes

Encounter with two major figures of musical creation and contemporary mathematical research: Pierre Boulez and Alain Connes.

What is the role of intuition in mathematical reasoning and in artistic activities? Is there an aesthetic dimension to mathematical activity? Does the notion of elegance of a mathematical demonstration or of a theoretical construction in music play a role in creativity?



Gérard Assayag, director of the CNRS/IRCAM Laboratory for The Science and Technology of Music and Sound, will lead this dialogue on invention in the two disciplines.

Photo: Pierre Boulez © Jean Radel

Wednesday, June 15, 2011, 6:30pm / IRCAM, Espace de projection

→ <http://agora2011.ircam.fr>



“Concerning **music**, it takes place in **time**, like **algebra**. In **mathematics**, there is this fundamental duality between, on the one hand, **geometry** – which corresponds to the visual arts, an immediate intuition – and on the other hand **algebra**. This is not visual, it has a temporality. This fits in time, it is a computation, something that is very close to the language, and which has its diabolical precision. [...] **And one only perceives the development of algebra through music**” (A. Connes).

Société
Mathématique
de France



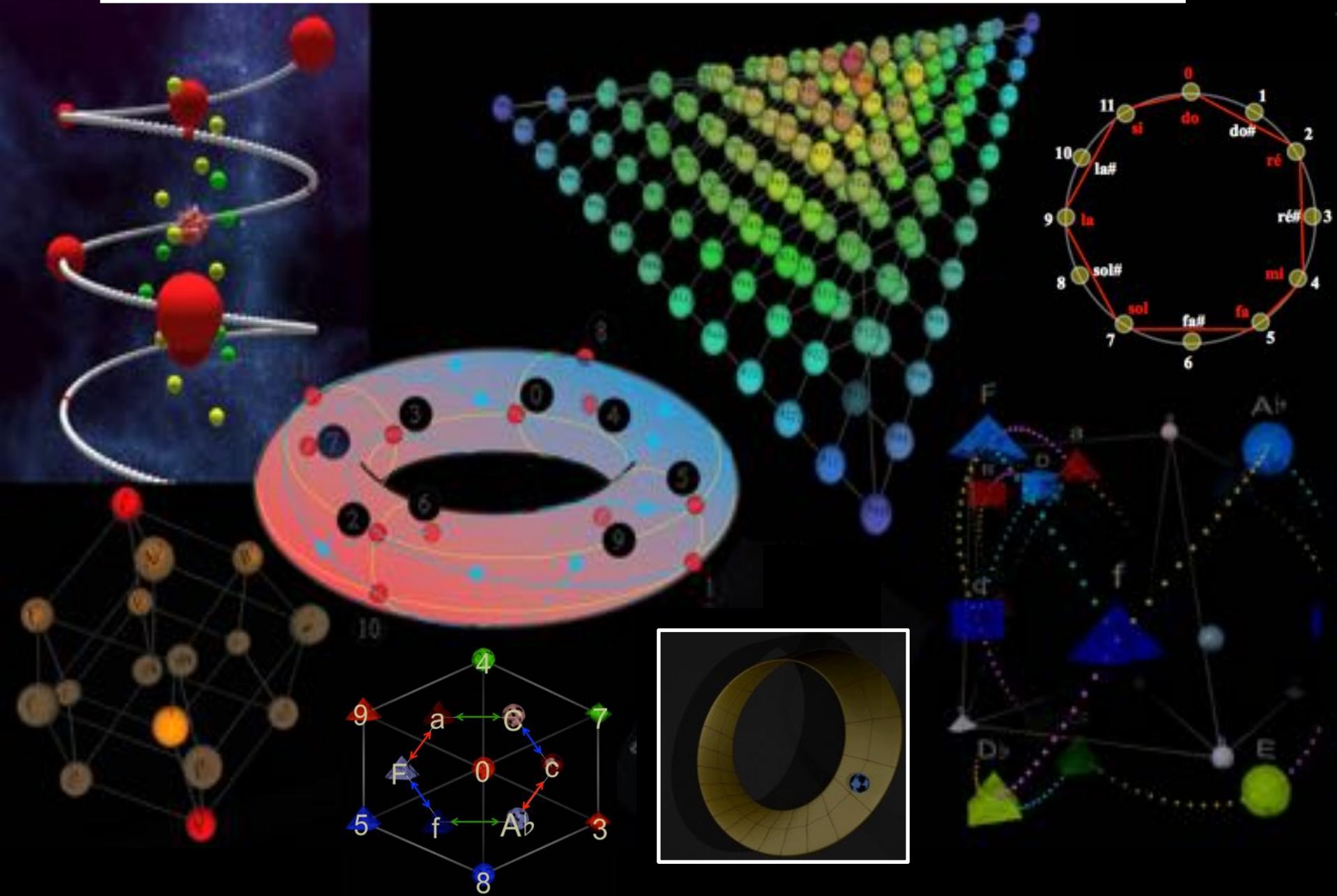
Carles Alsina · Moreno-Armella
Gérard Assayag · Emmanuel Candès
José Burillo · John Mandelieu (éd.)

Mathematics
and Computation
in Music

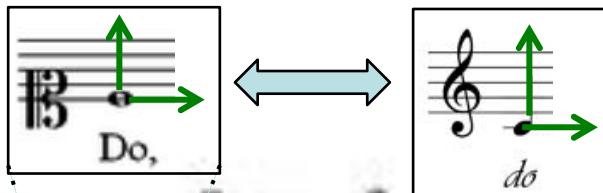
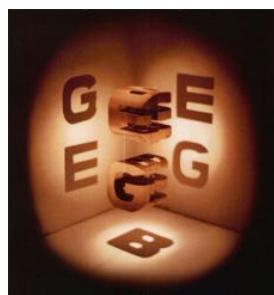
Third International Conference, MON 2011
Paris, France, June 2011
Proceedings

Springer

The galaxy of geometrical models at the service of music

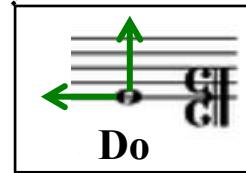
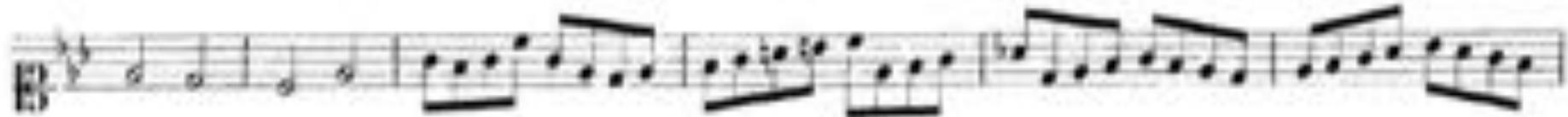
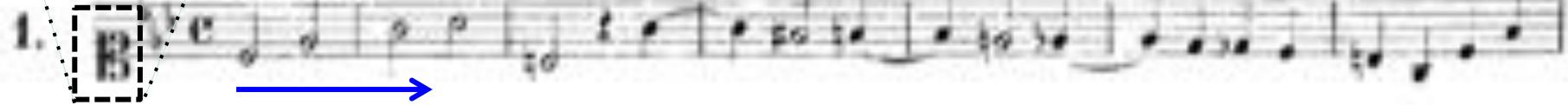


Bach's enigmatic canons and geometry



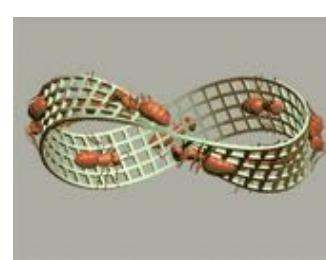
Canones diversi
super thema regium

Canon a 2.





My end is my beginning (but twisted!)



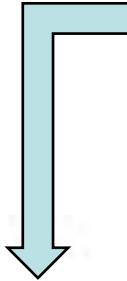
Canones diversi

super thema regium

1.

Canon n. 2

Musical score for Canon n. 2, first system. The score consists of three staves of music in common time, key signature of one flat, and treble clef. The music features various note heads and stems, with some notes connected by horizontal lines. Measures 1 through 8 are shown.



Canones diversi
super thema regium

Canon n. 2

4.

Musical score for Canon n. 2, fourth system. The score consists of four staves of music in common time, key signature of one flat, and treble clef. The music features various note heads and stems, with some notes connected by horizontal lines. Measures 1 through 8 are shown.

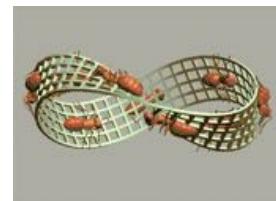
A horizontal bracket with three vertical arrows pointing upwards from the bottom right towards the musical score, indicating a repeating section or variation.



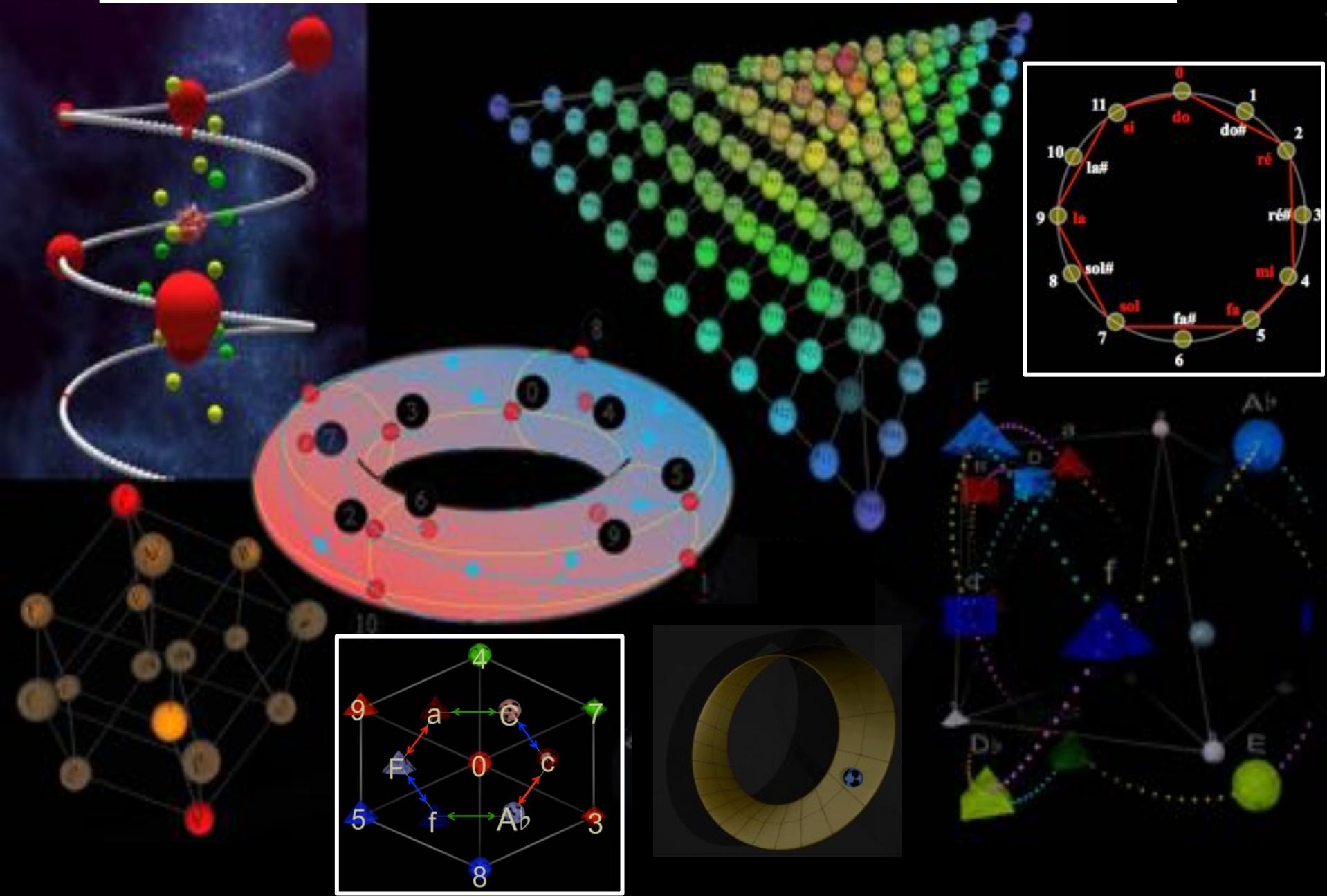


<http://www.josleys.com/Canon/Canon.html>

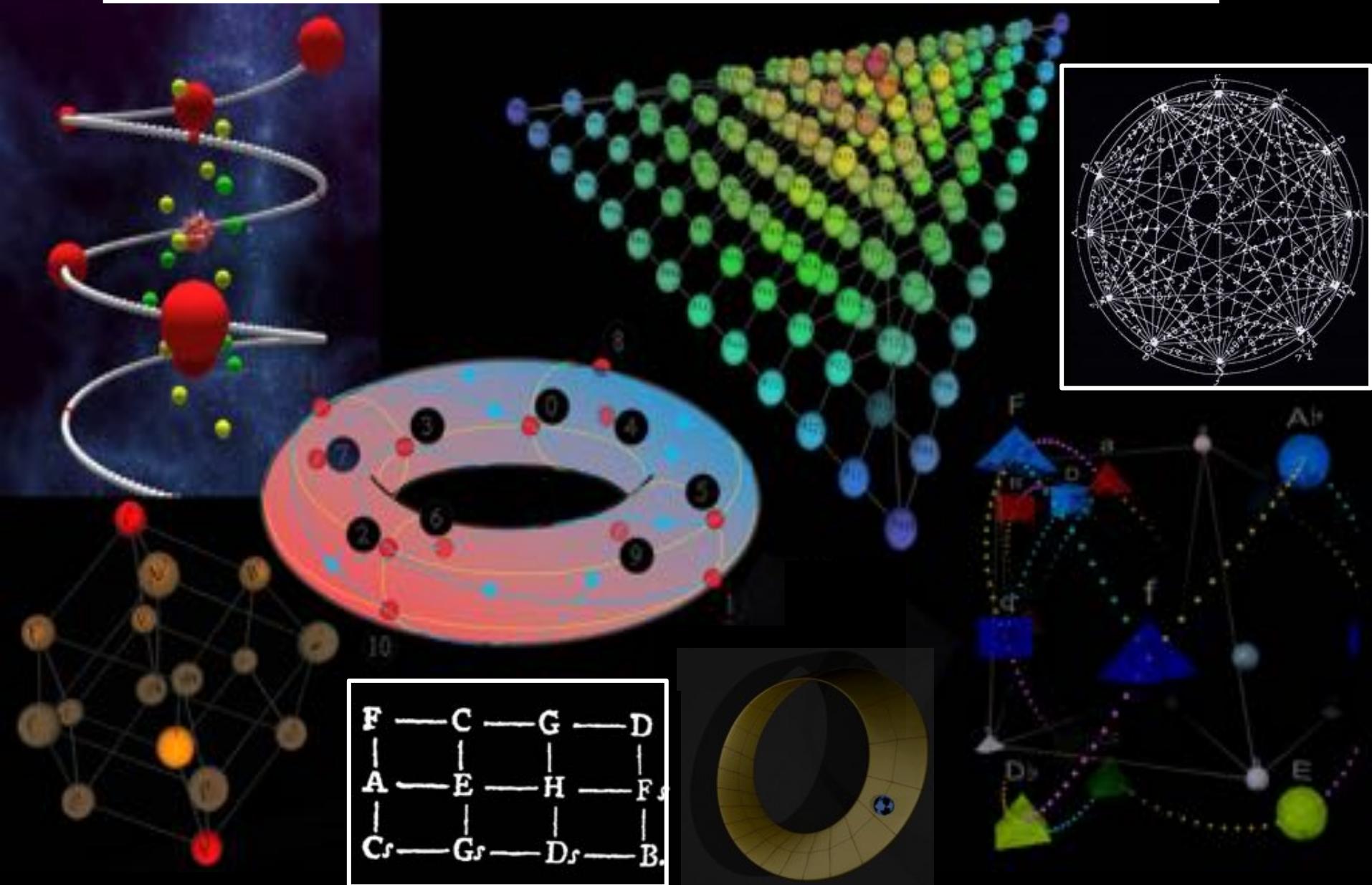
[min. 1'14"]



The galaxy of geometrical models at the service of music

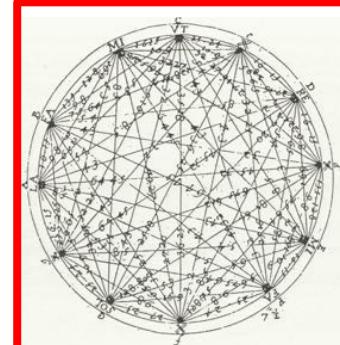


The galaxy of geometrical models at the service of music

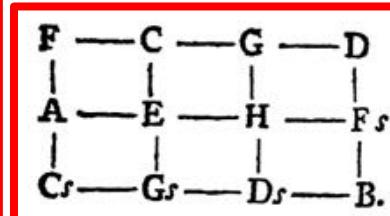


Music and mathematics: « prima la musica »!

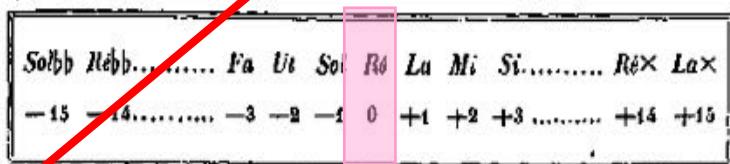
MUSIQUE	MATHS
500 av. J. C. Relation hauteur/longueur corde. La musique est source d'inspiration pour la théorie des nombres et la géométrie. <i>Pas de correspondance musicale.</i>	Nombres naturels et rationnels.
300 a.J. Invention (théorique) de la gamme chromatique tempérée égale par Aristoxénos de Tarente) et prémonition de la théorie des groupes. Isomorphismes entre les logarithmes (intervalles musicaux) et les exponentiels (longueur d'une corde).	Nombres irrationnels, théorème de Pythagore. Les mathématiques ne réagissent pas.
1000 ap. J.C. Invention de la représentation bidimensionnelle des hauteurs.	Aucune correspondance.
1500 Aucune reprise des concepts précédents.	Nombres négatifs. Construction des rationnels.
1600 Aucune relation.	Nombres réels et les logarithmes. Invention des repères cartésiens.
1648 Marin Mersenne : invention de la combinatoire musicale (<i>Harmonicorum Libri</i>)	Systématisation du calcul des probabilités par Bernoulli (<i>Arz Conjectandi</i> , 1713)
1700 La fugue comme un automate abstrait. Manipulation inconsciente du groupe de Klein.	Nombres complexes (Euler, Gauss), les quaternions (Hamilton), continuité (Cauchy), structure de groupe (Galois, Abel).
1773 Leonhard Euler : représentation géométrique des hauteurs (<i>Speculum Musicum</i>)	Invention de la théorie des graphes
1855 Camille Durutte : analyse harmonique, rythmique et mélodique	Développement en série d'une fonction (Wronski)
1900 Libération de la prison de la tonalité (Loquin, Hauer, Schoenberg).	Nombres infinis et transfinis (Cantor). Axiomatique de Peano. Théorie de la mesure (Lebesgue, Borel).
1920 Formalisation radicale des macrostructures à travers le système sériel (Schoenberg).	Aucun développement de la théorie des nombres. Logique (contradictions de la théorie des ensembles).
1937-1939 Ernst Krének : les axiomes en musique	David Hilbert, <i>Les fondements de la géométrie</i> (1899)
1946 Milton Babbitt : théorie des groupes et système dodécaphonique	Rudolf Carnap, <i>The Logical Syntax of Language</i> (1937)



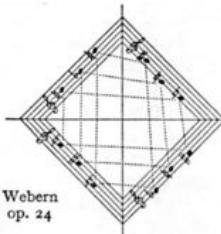
Mersenne,
*Harmonicorum
Libri XII*, 1648



Euler, *Speculum
musicum*, 1773



Durutte, *Technie, ou lois générales du système harmonique* (1855)



S	I	R	RI
S	I	R	RI
I	I	S	RI
R	R	RI	S
RI	RI	R	I

Krenek and Babbitt, twelve-tone method, axiomatics and Klein group



Iannis Xenakis

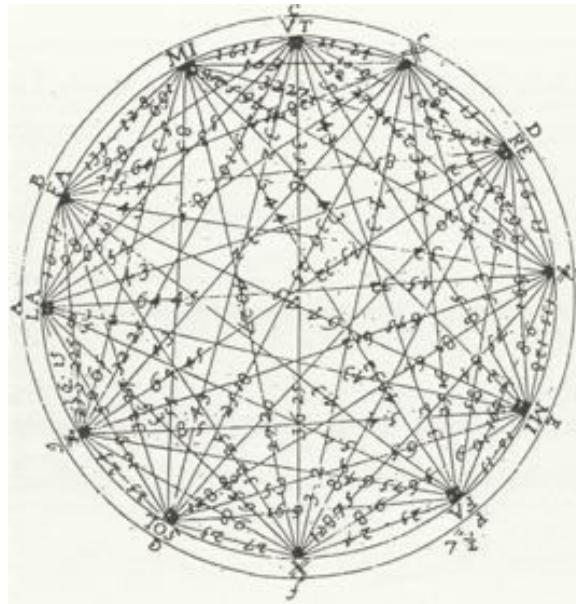
Mersenne and the birth of (musical) combinatorics

II4 Marin Mersenne, *Harmonicorum Libri XII*, 1648

LIBER SEPTIMVS. DE CANTIBVS, SEV CANTILENIS, EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabela Combinationis ab I ad XXII.

I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6117020800
XIV	8717819200
XV	1107674368000
XVI	10933789888000
XVII	311687418296000
XVIII	6401573705718000
XIX	11164100040818000
XX	1433904008176640000
XXI	31090941171709440000
XXII.	384000737777607680000

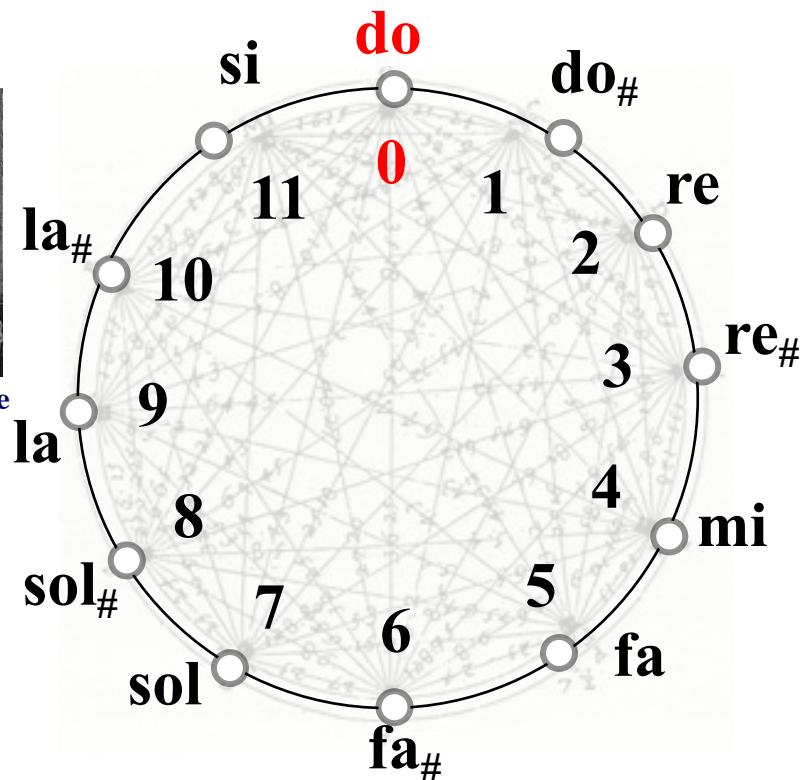


Marin Mersenne

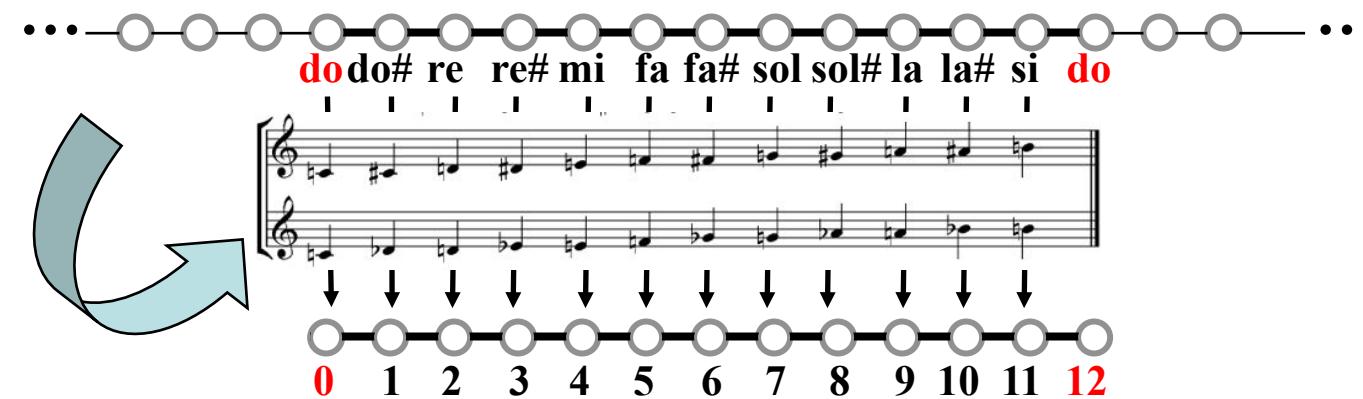
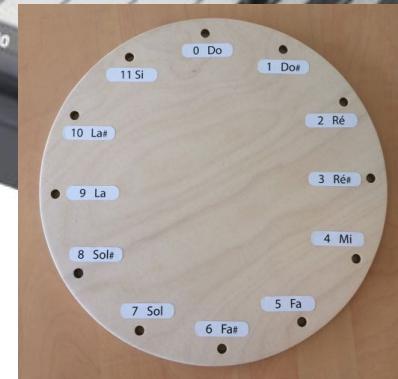
Varietas, seu Combinatio quatuor notarum.



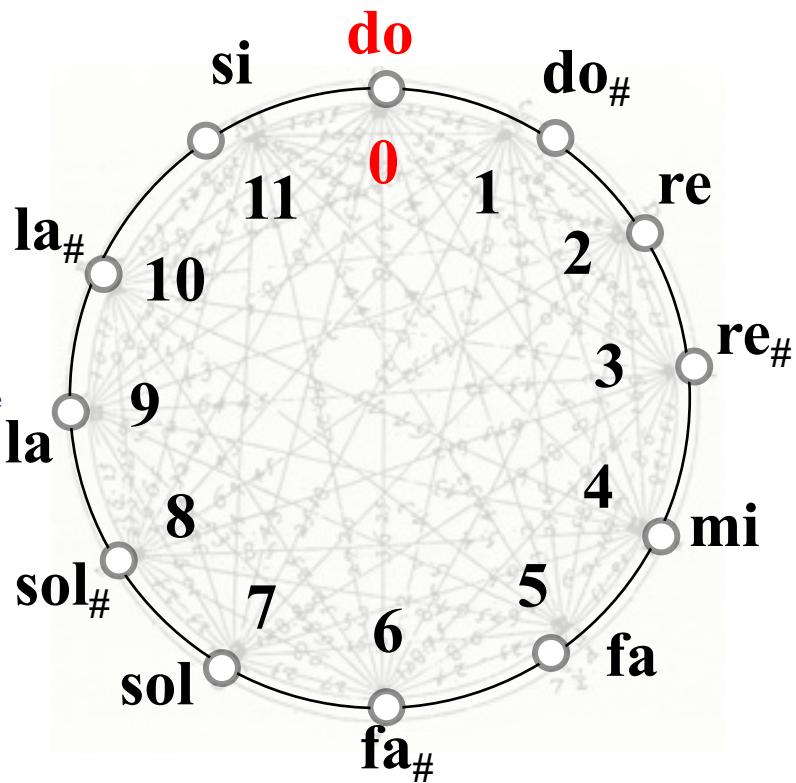
The circular representation of the pitch space



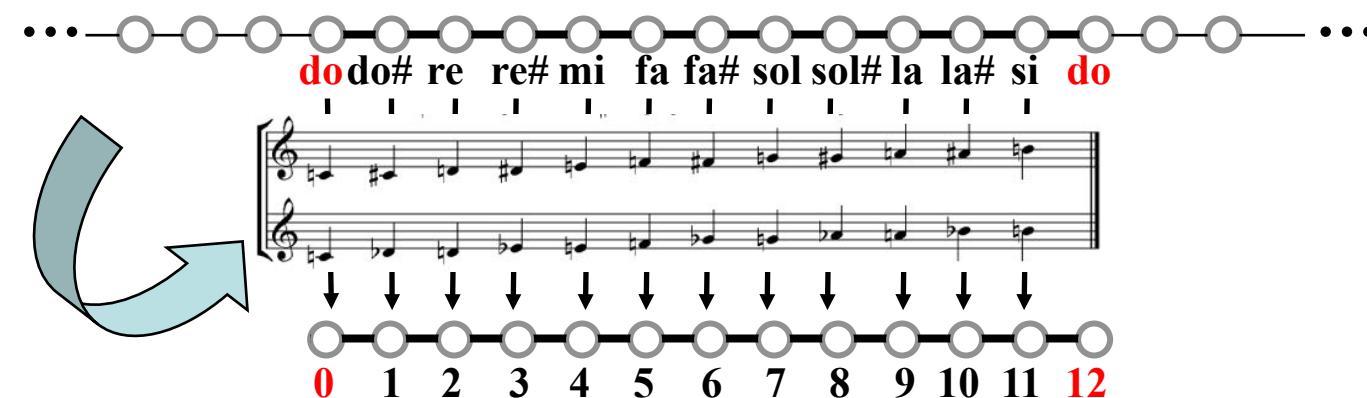
Harmonicorum Libri XII, 1648



The circular representation of the pitch space



Harmonicorum Libri XII, 1648

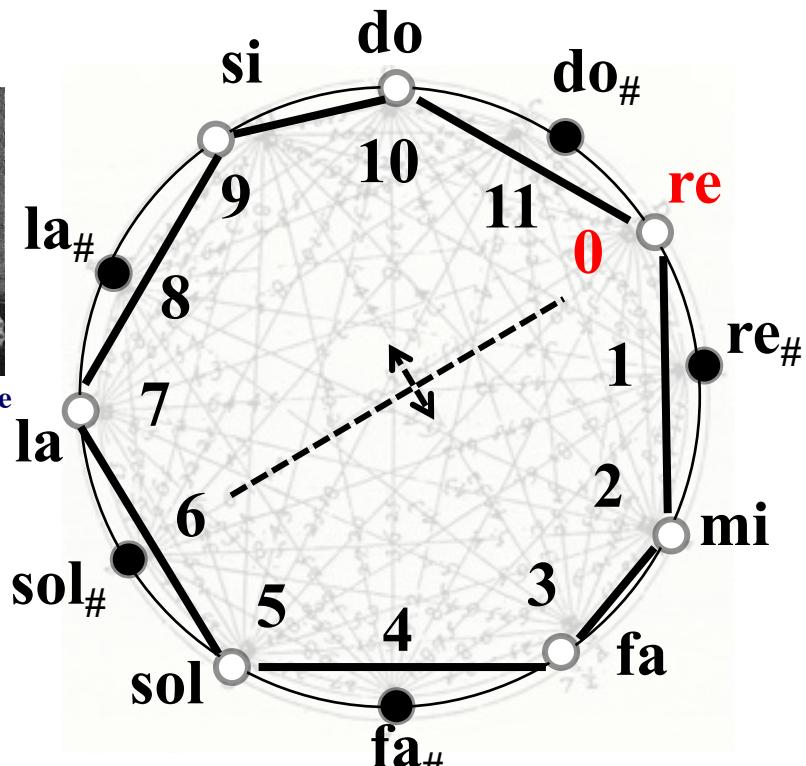


→ DEMO

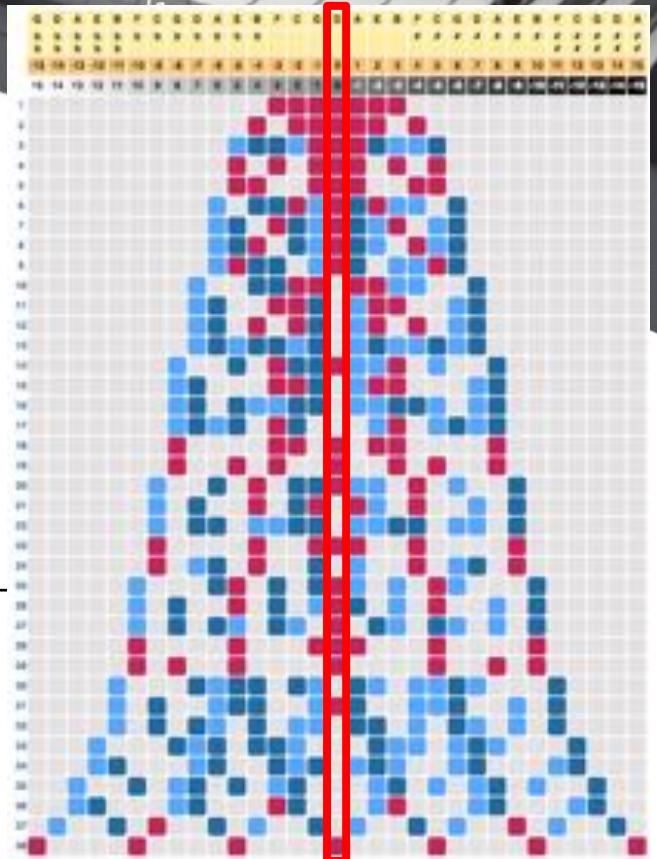
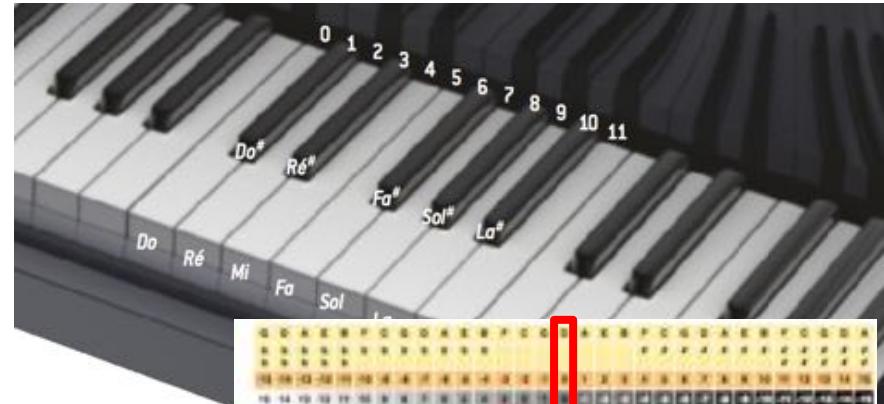
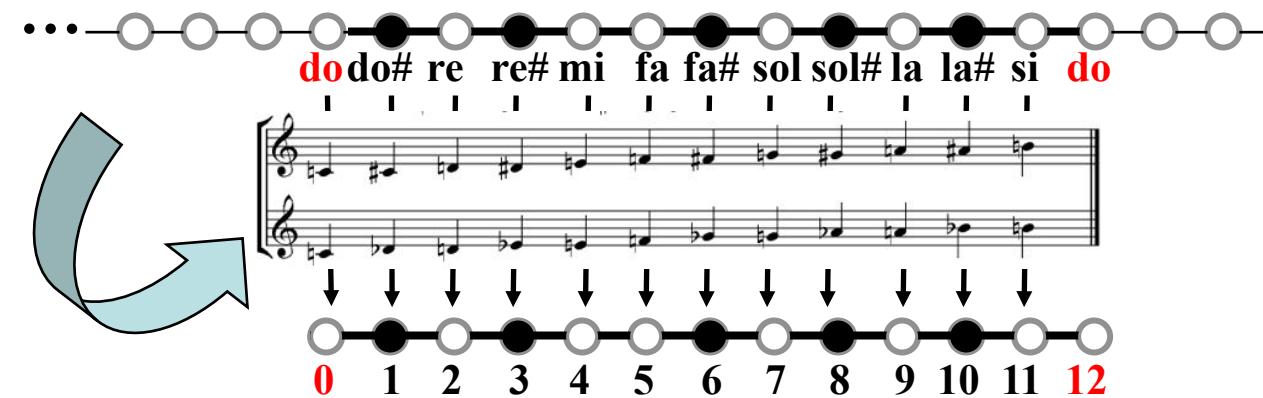
The circular representation of the pitch space



Marin Mersenne



Harmonicorum Libri XII, 1648



<http://www.cloche-diatonique.ch/>

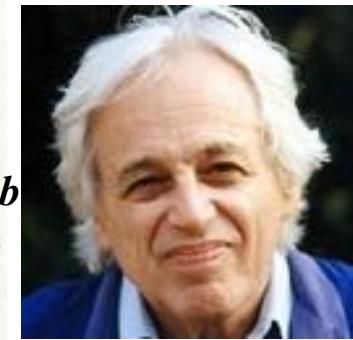
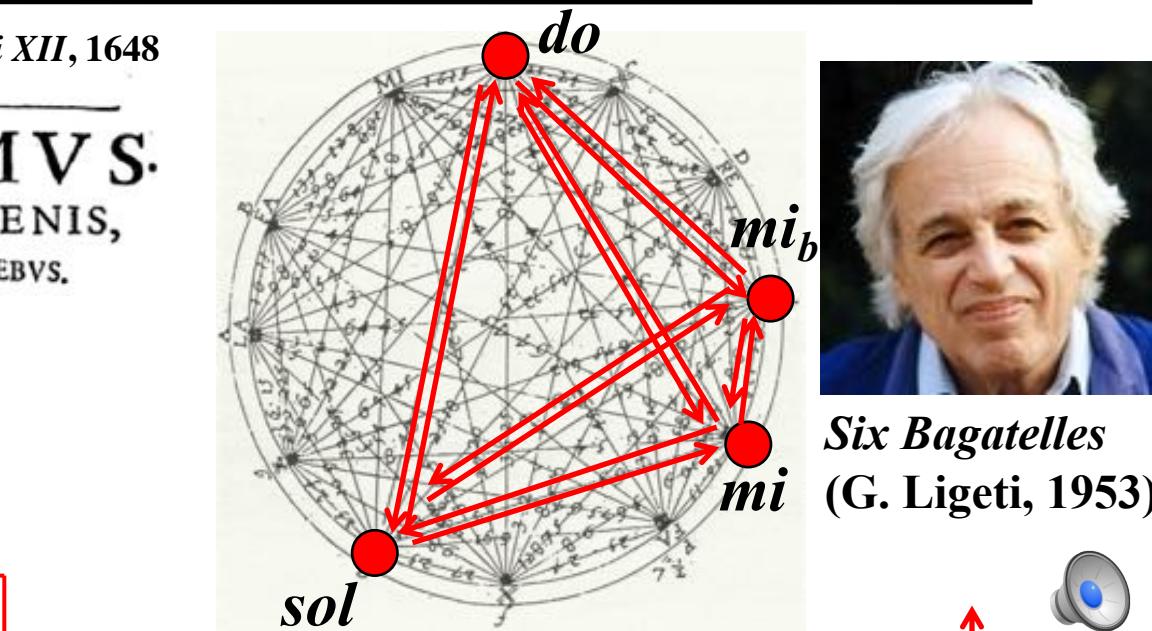
Permutational melodies in contemporary (art) music

II.4 Marin Mersenne, *Harmonicorum Libri XII*, 1648

LIBER SEPTIMVS. DE CANTIBVS, SEV CANTILENIS, EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tableta Combinationis ab I ad XXI.

I	I
II	II
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	361800
X	3618000
XI	39916800
XII	479001600
XIII	6117010800
XIV	87178191200
XV	1107674568000
XVI	10922789888000
XVII	311687418296000
XVIII	6401173705718000
XIX	11164100040813000
XX	1433904008176640000
XXI	51090941171709440000
XXII.	1884000737777607680000

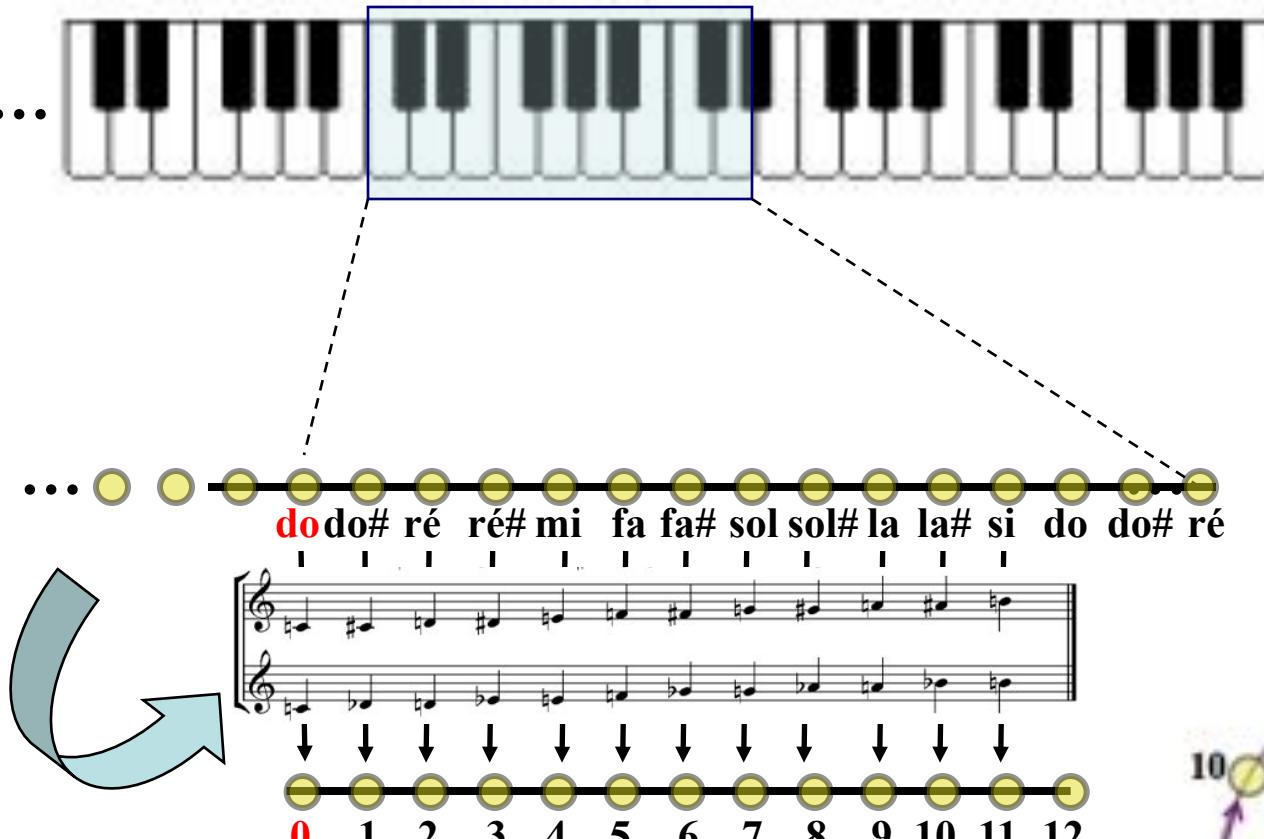


Six Bagatelles
(G. Ligeti, 1953)

A musical score titled 'Varietas, seu Combinatio quatuor notarum.' It consists of two staves of music. The top staff has 12 numbered measures (1 through 12). The bottom staff has 12 numbered measures (13 through 24). The music is composed of short, rhythmic patterns of eighth and sixteenth notes. A red arrow points from the number 24 in the first column of the table on the left to the number 24 in the first column of the score on the right.



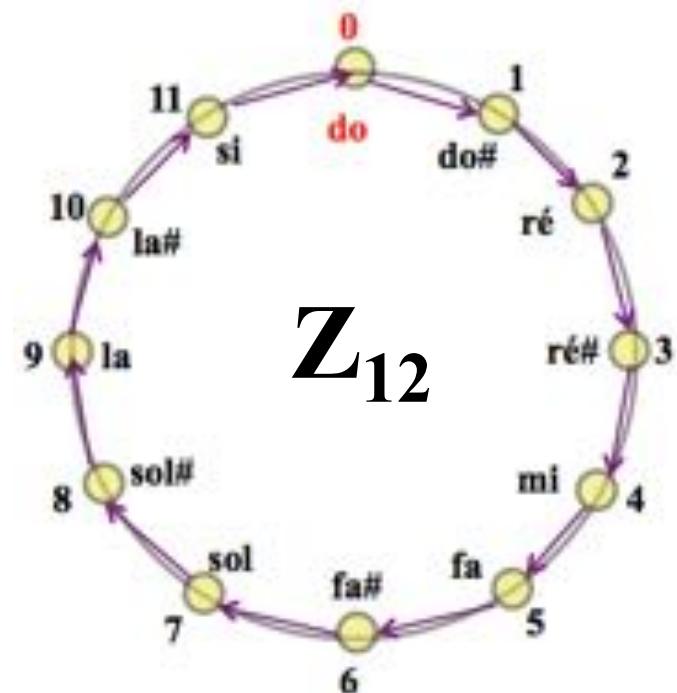
The equal tempered space is a cyclic group



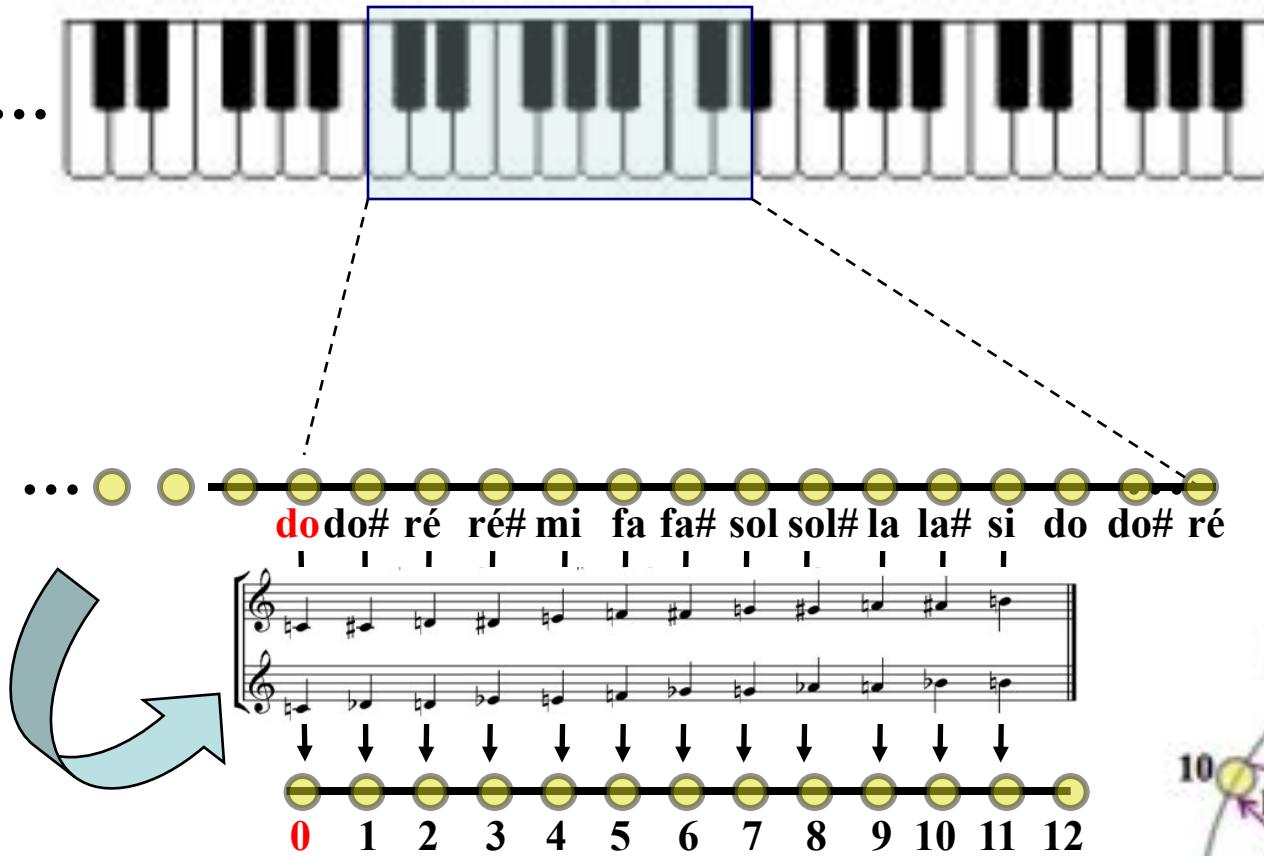
$$Z_{12} = \langle T_1 \mid (T_1)^{12} = T_0 \rangle$$

The generators of the cyclic group of order 12 are the transpositions T_1 , T_5 , T_7 et T_{11} where

$$T_k: x \rightarrow x+k \bmod 12$$



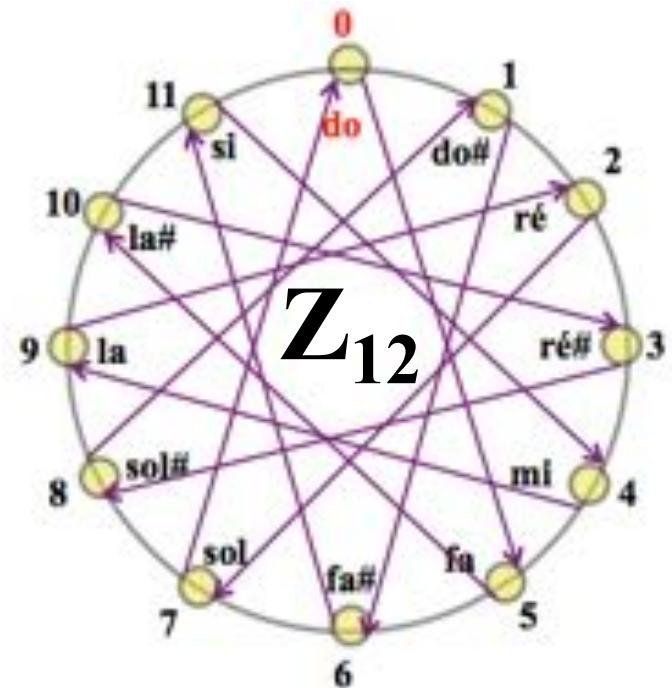
The equal tempered space is a cyclic group



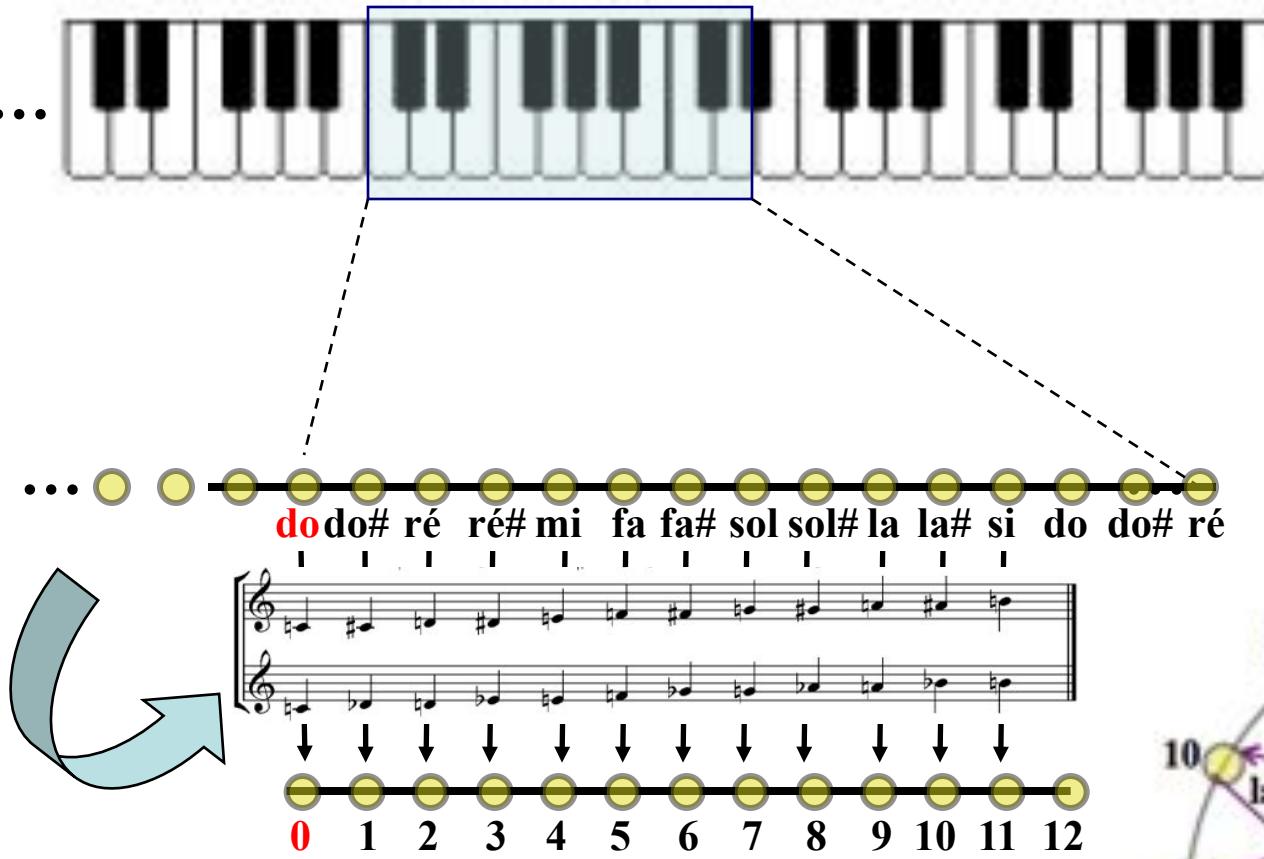
$$\begin{aligned}Z_{12} &= \langle T_1 \mid (T_1)^{12} = T_0 \rangle = \\&= \langle T_5 \mid (T_5)^{12} = T_0 \rangle\end{aligned}$$

The generators of the cyclic group of order 12 are the transpositions T_1 , T_5 , T_7 et T_{11} where

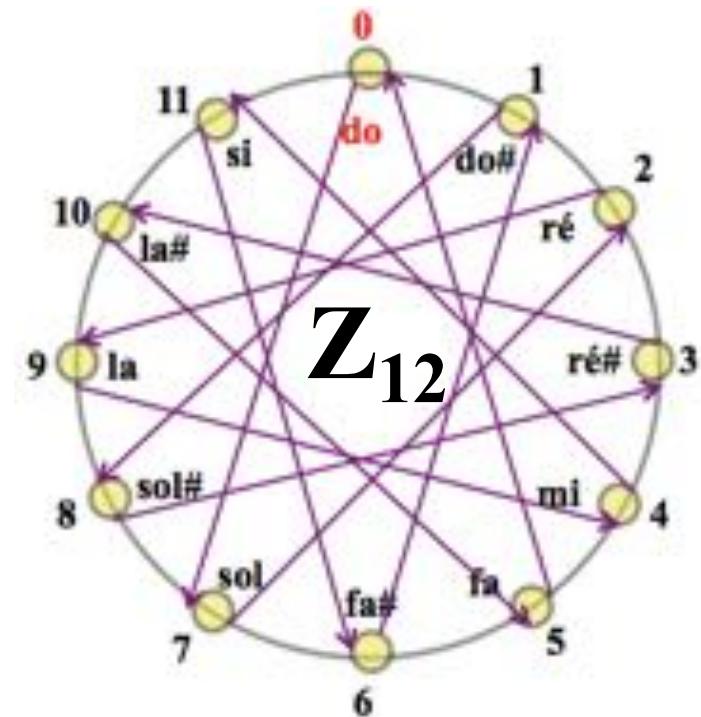
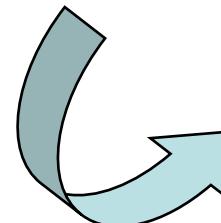
$$T_k: x \rightarrow x+k \bmod 12$$



The equal tempered space is a cyclic group



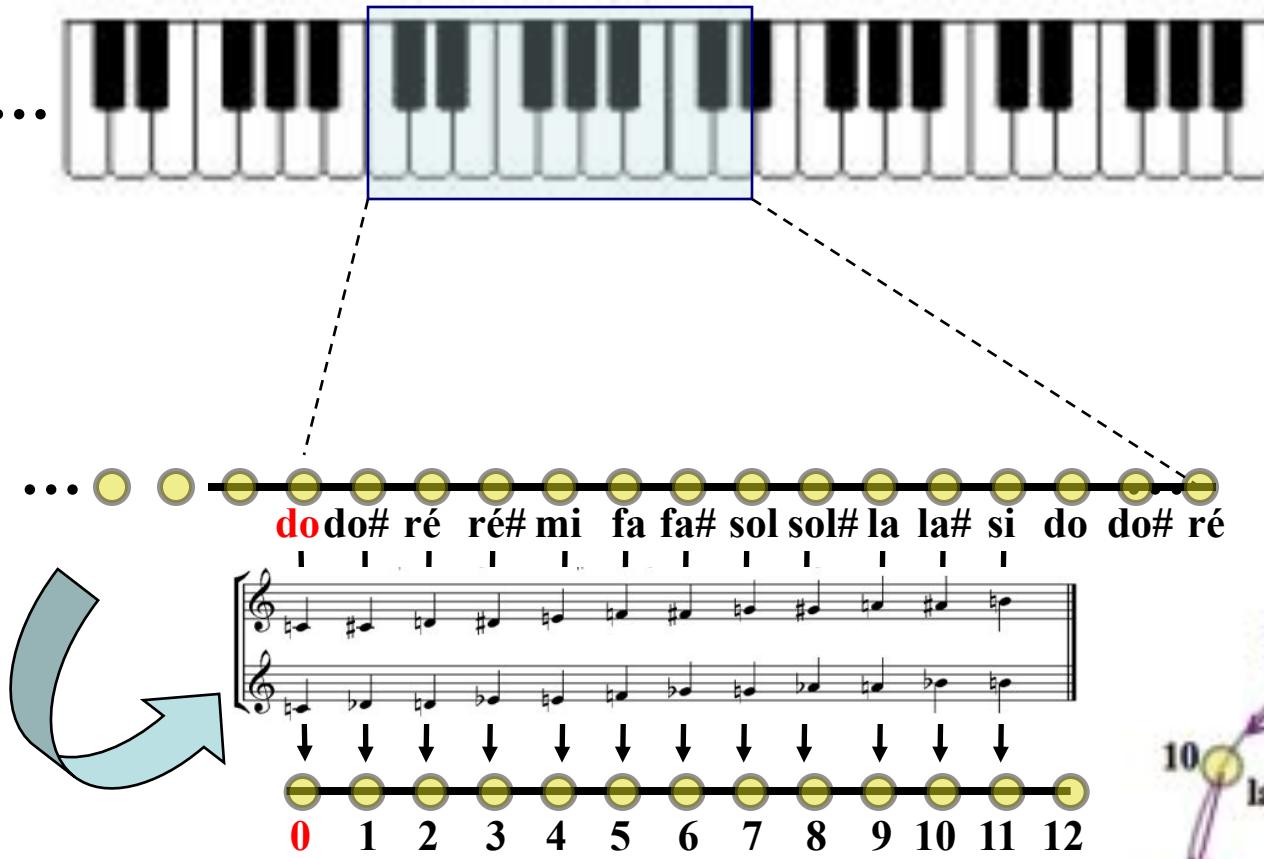
$$\begin{aligned} Z_{12} &= \langle T_1 \mid (T_1)^{12} = T_0 \rangle = \\ &= \langle T_5 \mid (T_5)^{12} = T_0 \rangle = \\ &= \langle T_7 \mid (T_7)^{12} = T_0 \rangle \end{aligned}$$



The generators of the cyclic group of order 12 are the transpositions T_1 , T_5 , T_7 et T_{11} where

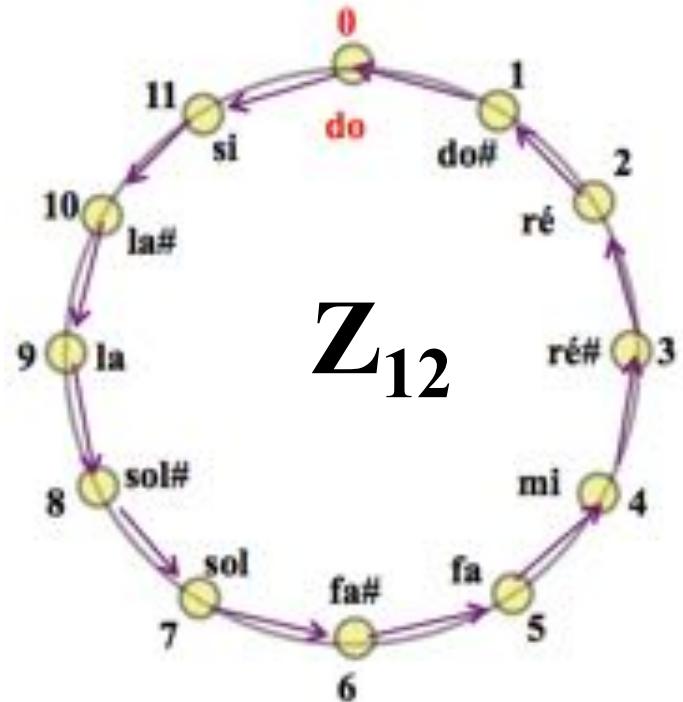
$$T_k: x \rightarrow x+k \bmod 12$$

The equal tempered space is a cyclic group

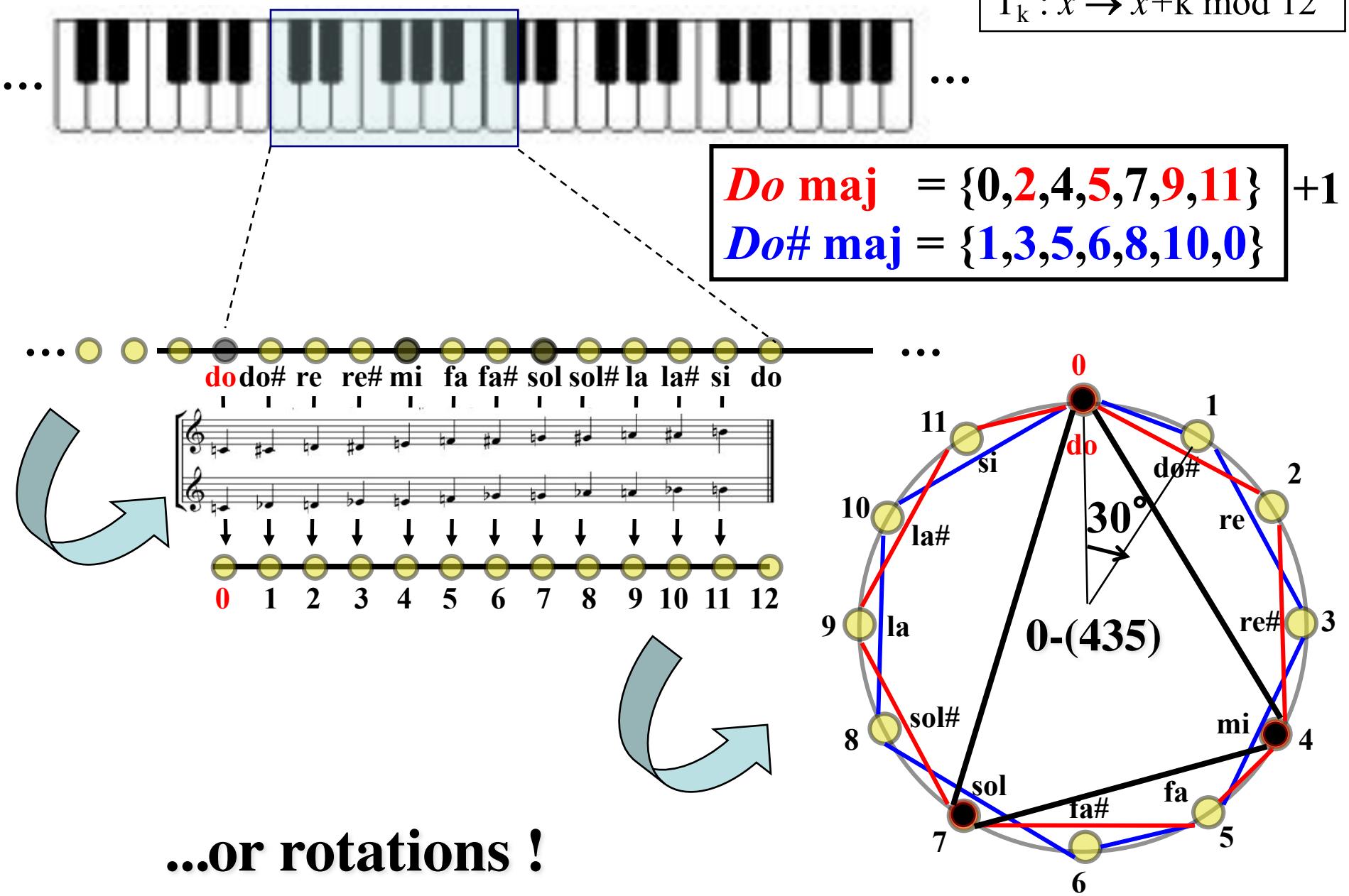


$$\begin{aligned}
 Z_{12} &= \langle T_1 \mid (T_1)^{12} = T_0 \rangle = \\
 &= \langle T_5 \mid (T_5)^{12} = T_0 \rangle = \\
 &= \langle T_7 \mid (T_7)^{12} = T_0 \rangle = \\
 &= \langle T_{11} \mid (T_{11})^{12} = T_0 \rangle
 \end{aligned}$$

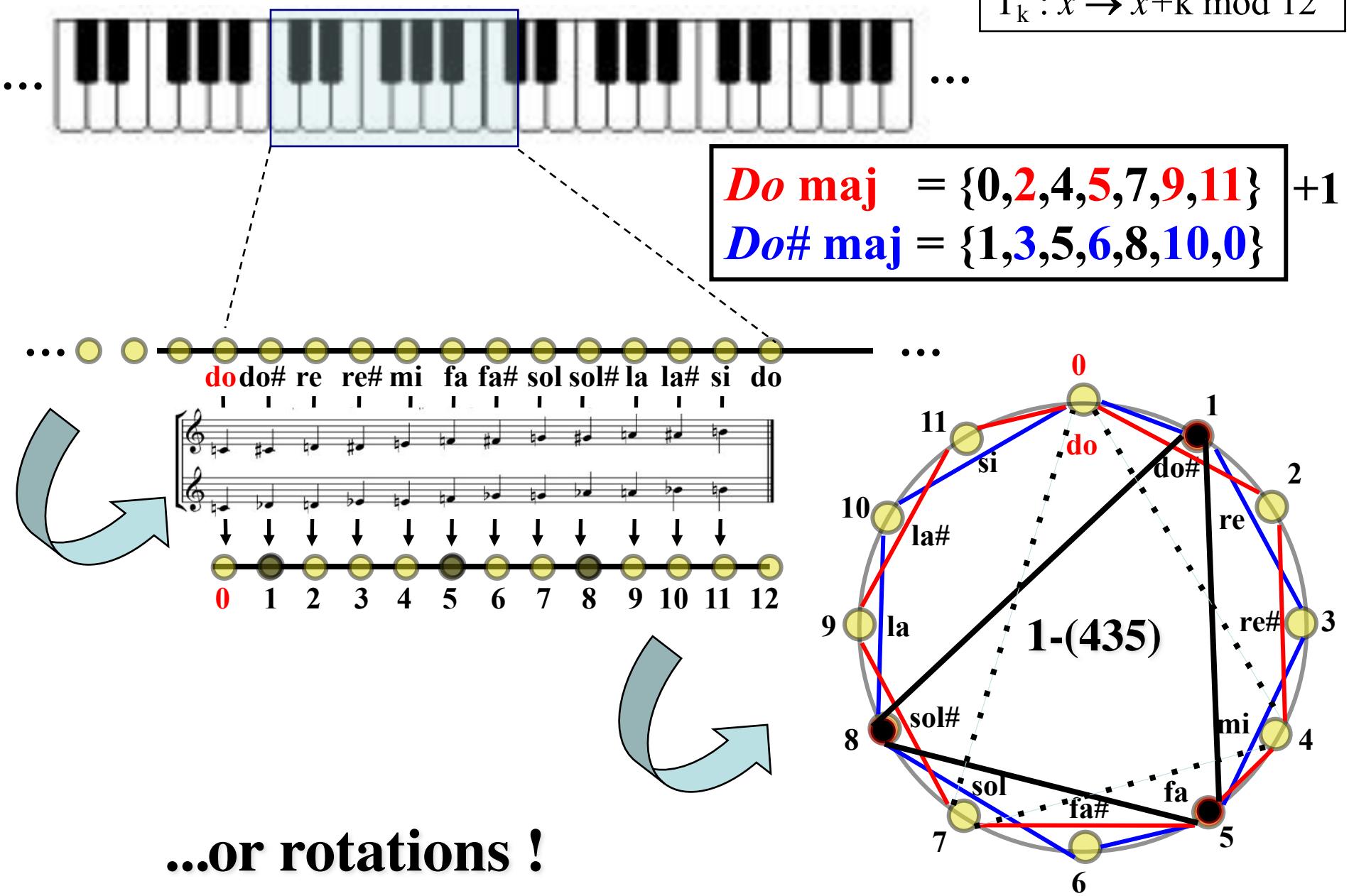
The generators of the cyclic group of order 12 are the transpositions T_1 , T_5 , T_7 et T_{11}
where
 $T_k: x \rightarrow x+k \bmod 12$



Musical transpositions are additions...



Musical transpositions are additions...



Musical inversions are differences...

... or axial symmetries!

The diagram illustrates musical inversions and axial symmetries in a 12-note system, mapping notes from a piano keyboard to a circle of 12 notes labeled 0 through 11.

Piano Keyboard: Shows a segment of a piano keyboard with black and white keys. A blue box highlights a segment of three black keys (C# to E) and four white keys (F to G). Dashed arrows point from this segment to the 12-note circle and the musical staff below.

12-Note Circle: A circle with 12 points labeled 0 through 11. Points 0, 4, and 7 are highlighted in red. Points 0, 4, and 9 are highlighted in blue. A red arrow labeled $I_4(x) = 4 - x$ connects point 0 to point 4, and point 4 to point 7. A blue arrow connects point 0 to point 9. Arrows also point from the 12-note circle to the musical staff and the piano keyboard.

Musical Staff: A musical staff with two staves. The top staff shows notes: do, do#, re, re#, mi, fa, fa#, sol, sol#, la, la#, si, do. The bottom staff shows notes: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Arrows point from the 12-note circle to the notes on the staff.

Equation: $I : x \rightarrow -x \bmod 12$

Set Definitions:

- Do maj** = {0, 4, 7}
- La min** = {0, 4, 9}

A large blue arrow points from the 12-note circle to the piano keyboard at the bottom left.

Musical inversions are differences...

... or axial symmetries!

The diagram illustrates musical inversions and axial symmetries through various representations of the twelve-tone system.

Piano Keyboard: A piano keyboard is shown with a blue box highlighting the notes C, D, E, F, G, A, B (Do, Do#, Re, Re#, Mi, Fa, Sol). Dashed arrows point from this box to the 12-tone circle and the musical staff.

12-Tone Circle: A circle with 12 points labeled 0 through 11. Points 0, 4, and 7 are highlighted in red. Points 0, 3, and 7 are highlighted in blue. The red set is labeled "Do maj" and the blue set is labeled "Do min".

Mathematical Rule: $I : x \rightarrow -x \bmod 12$ and $I_7(x) = 7-x$.

Musical Staff: A musical staff shows the same 12 notes: Do, Do#, Re, Re#, Mi, Fa, Fa#, Sol, Sol#, La, La#, Si, Do. Arrows point from the 12-tone circle to the staff.

Curved Arrows: Curved arrows indicate a mapping between the 12-tone circle and the musical staff, showing how notes map under the inversion rule I .

12-Tone Circle with Labels: The 12-tone circle is shown again with labels: 0 (do), 1 (do#), 2 (re), 3 (re#), 4 (mi), 5 (fa), 6 (fa#), 7 (sol), 8 (sol#), 9 (la), 10 (la#), 11 (si), 12 (do). A red arrow connects 0 to 7, and a blue arrow connects 0 to 4. A dashed arrow labeled I_7 points from 0 to 11.

Musical inversions are differences...

... or axial symmetries!

The diagram illustrates musical inversions and axial symmetries through various representations of the twelve-tone system.

Piano Keyboard: A piano keyboard is shown with a blue box highlighting the notes C, D, E, G, A, and B. This set of notes corresponds to the major mode in the circle of fifths.

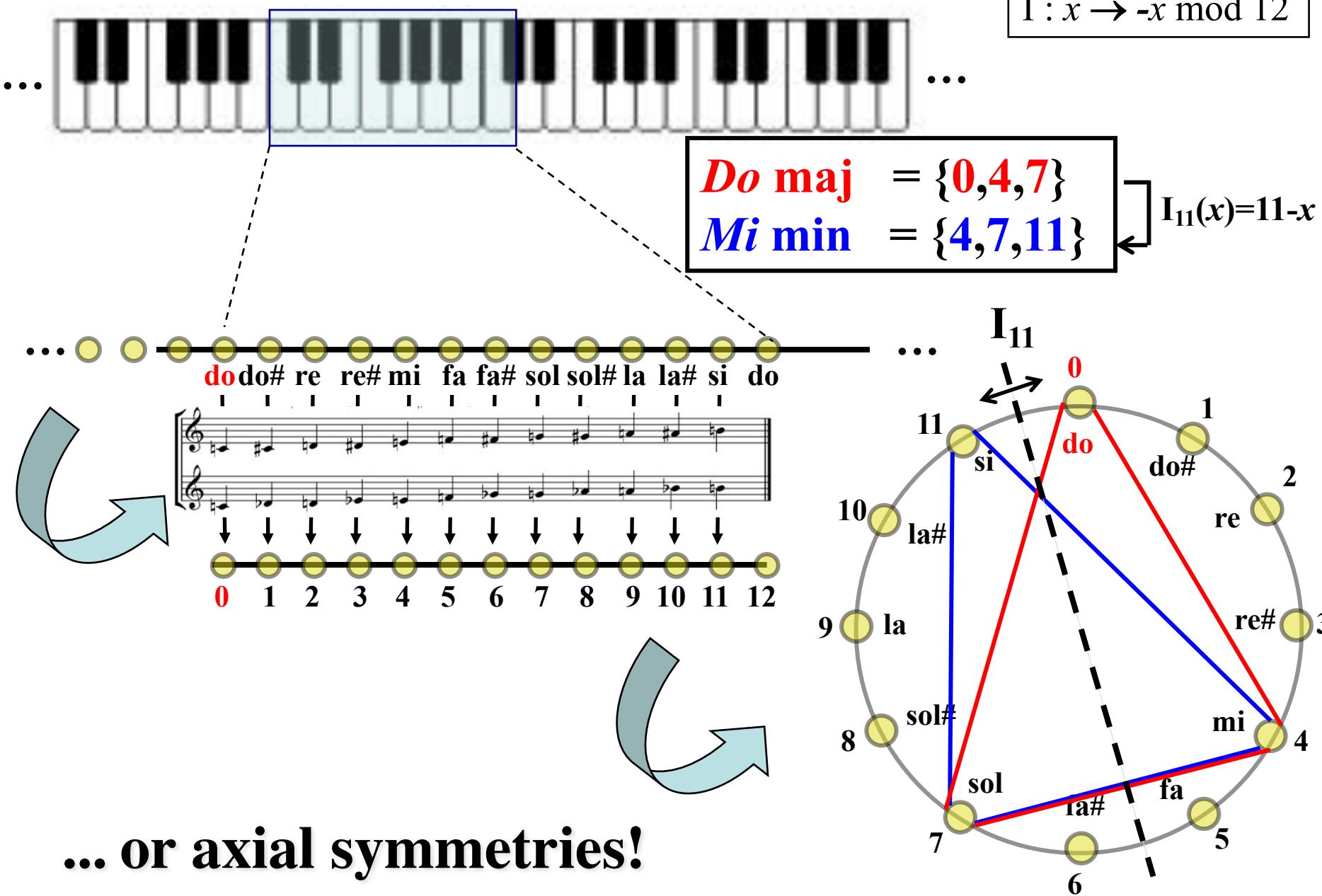
Number Line: A horizontal number line from 0 to 12 represents the twelve tones. Yellow circles mark the notes: do (0), do# (1), re (2), re# (3), mi (4), fa (5), fa# (6), sol (7), sol# (8), la (9), la# (10), si (11), and do (12). Arrows point from the notes on the piano keyboard to their corresponding numbers on the line. A blue arrow points from the number line to the circle of fifths.

Circle of Fifths: A circle with 12 points labeled 0 through 11. Points 0, 4, and 7 are highlighted in red. Points 0, 3, and 7 are highlighted in blue. The red points represent the notes in the major mode (C, G, D, A, E, B). The blue points represent the notes in the minor mode (C, F#, G, D, A, E). The mapping between the number line and the circle is shown by dashed lines connecting the numbered points to their corresponding points on the circle. The formula $I_7(x) = 7 - x$ is given, indicating the inversion operation.

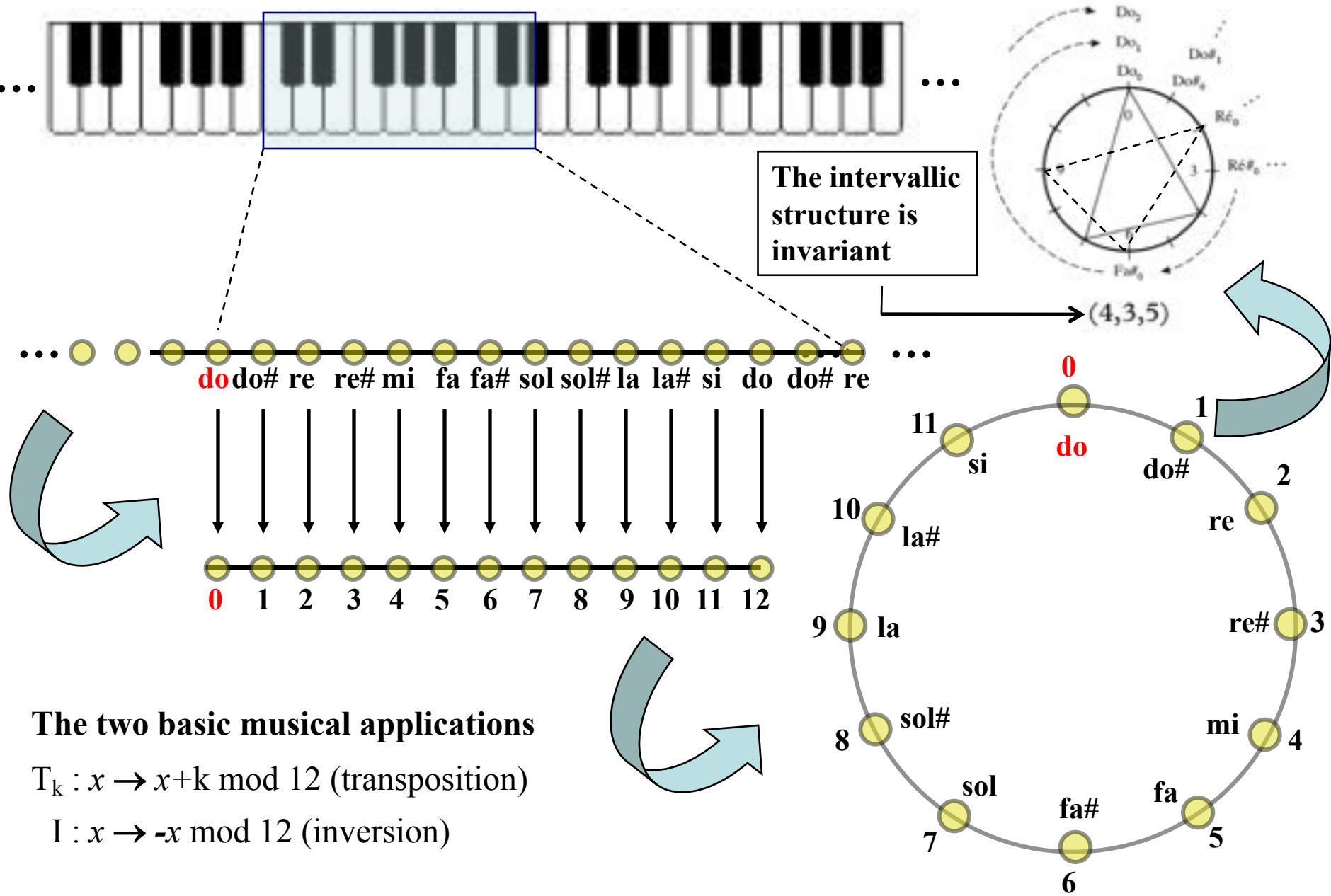
Text Boxes:

- $I : x \rightarrow -x \bmod 12$
- Do maj** = {0, 4, 7}
- Do min** = {0, 3, 7}
- $I_7(x) = 7 - x$

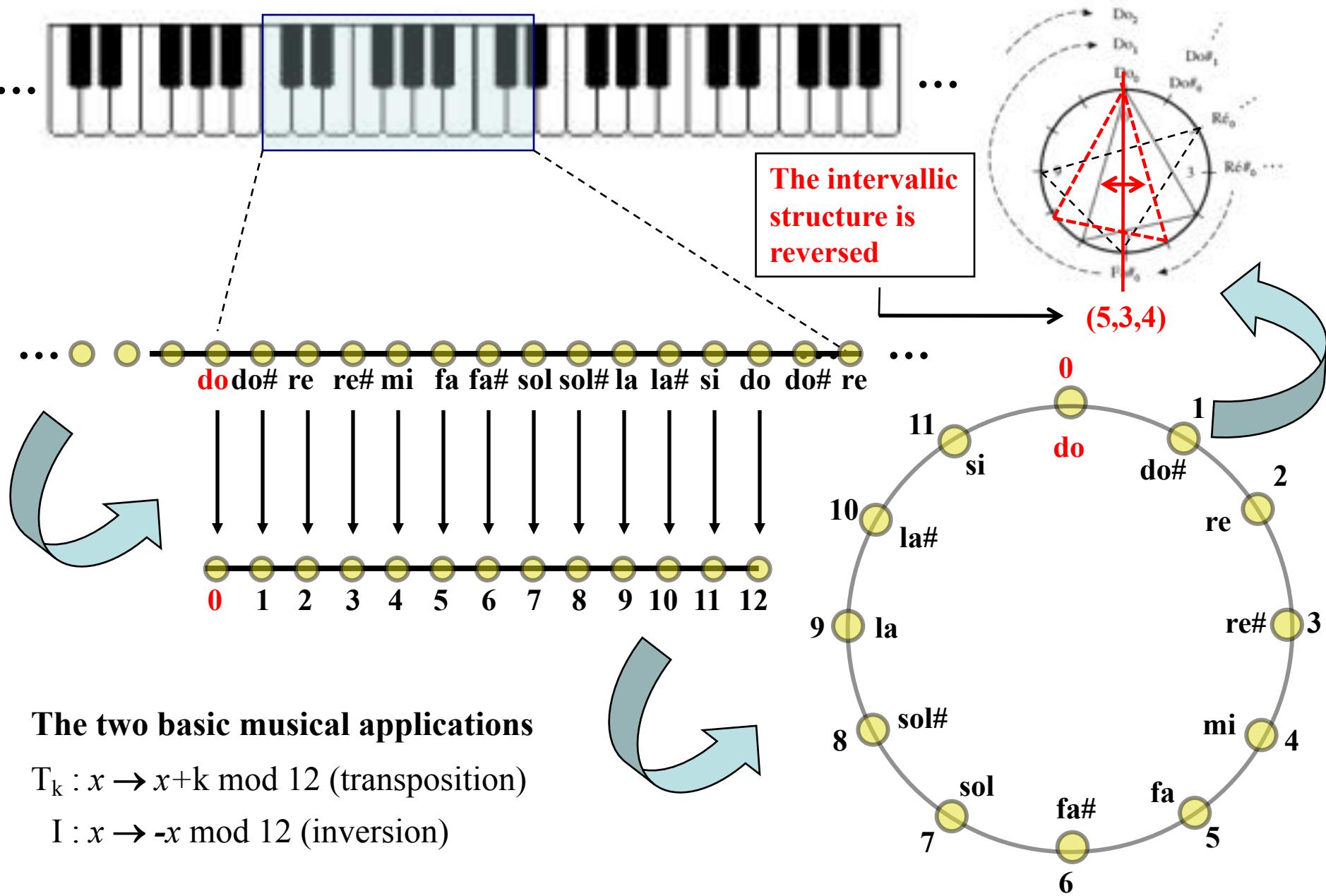
Musical inversions are differences...



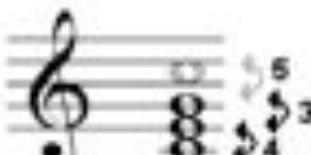
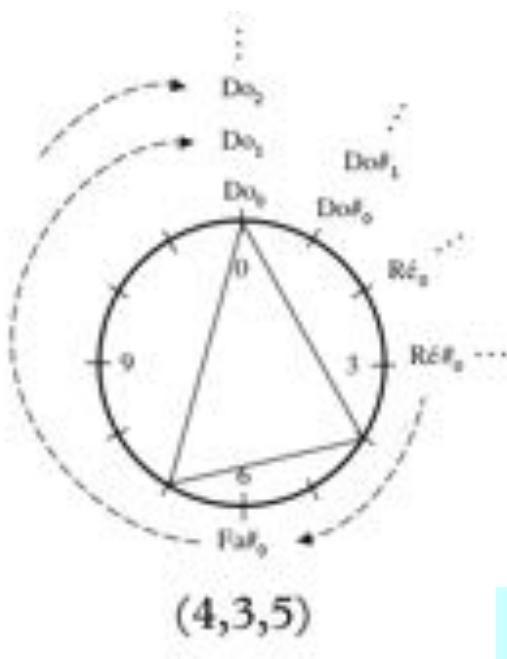
Circular representation and intervallic structure



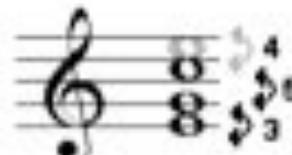
Circular representation and intervallic structure



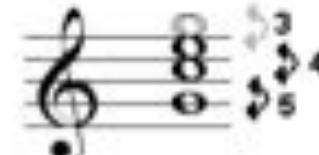
Circular representation and intervallic structure



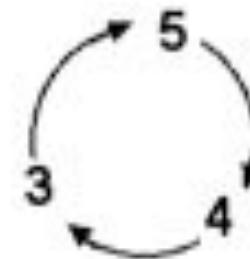
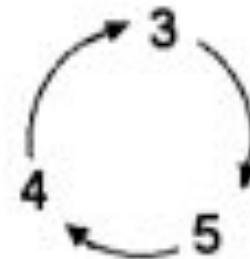
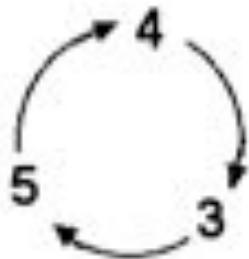
(4 3 5)



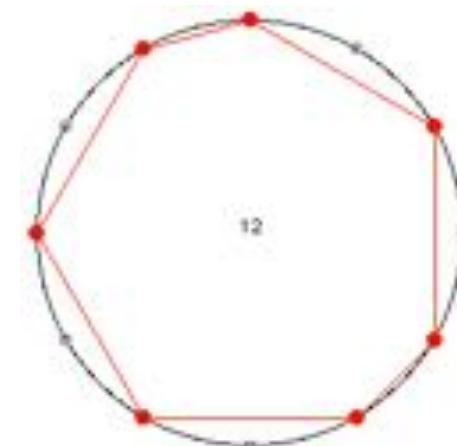
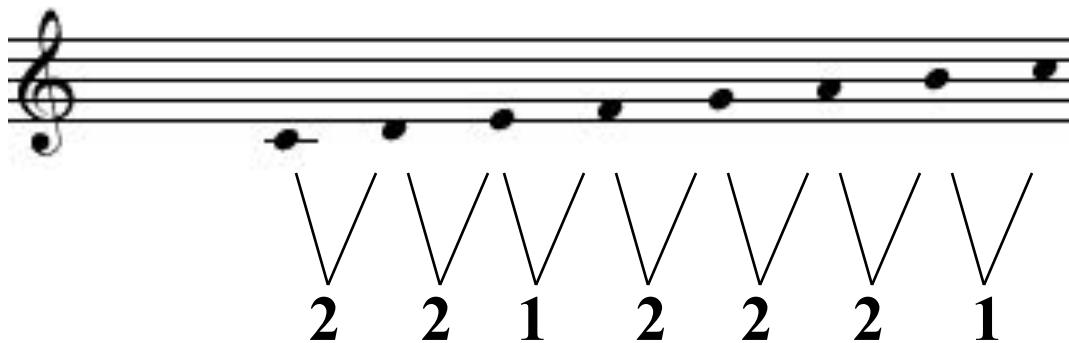
(3 5 4)



(5 4 3)



The « inversions » of a chord are all circular permutations on an intervallic structure



The pitch-rhythm *mystic* isomorphic correspondence



“These modes [of Limited Transpositions] realize in the vertical direction (transposition) what non-retrogradable rhythms realize in the horizontal direction (retrogradation). In fact, these modes *cannot be transposed* beyond a certain number of transpositions without falling again into the same notes, enharmonically speaking; likewise, these rhythms *cannot be read in a retrograde sense* without one’s finding again exactly the same order of values as in the right sense. These modes cannot be transposed because they are—without polytonality—in the modal atmosphere of several keys at once and contain in themselves small transpositions; these rhythms cannot be retrograded because they contain in themselves small retrogradations. These modes are divisible into symmetrical groups; these rhythms, also, with this difference: the symmetry of the rhythmic groups is a retrograde symmetry. Finally, the last note of each group of these modes is always *common* with the first of the following group; and the groups of these rhythms frame a central value *common* to each group. **The analogy is now complete**”.

- O. Messiaen, *Technique de mon langage musical*, Alphonse Leduc, 1944
- O. Messiaen, *Traité de rythme, de couleurs et d'ornithologie*, Alphonse Leduc, 1949-1992

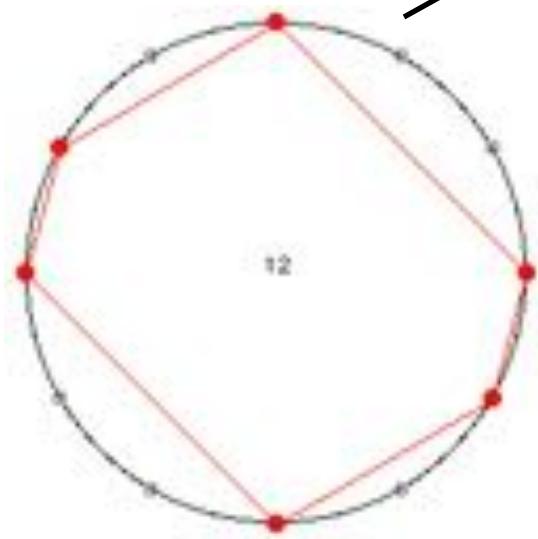
Messiaen's Modes of Limited Transpositions

Schoenberg: Serenade Op.24, Mouvement 5

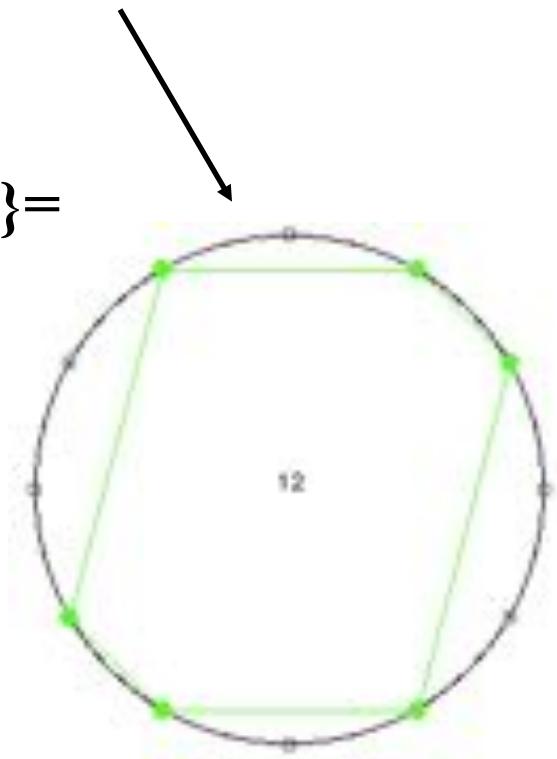
$A = \{9, 10, 0, 3, 4, 6\}$ $\{5, 7, 8, 11, 1, 2\}$

$$\begin{aligned} T6\{9,10,0,3,4,6\} &= \\ &= \{6+9, 6+10, 6, 6+3, 6+4, 6+6\} = \\ &= \{3, 4, 6, 9, 10, 0\} \end{aligned}$$

$$T6(A) = A$$

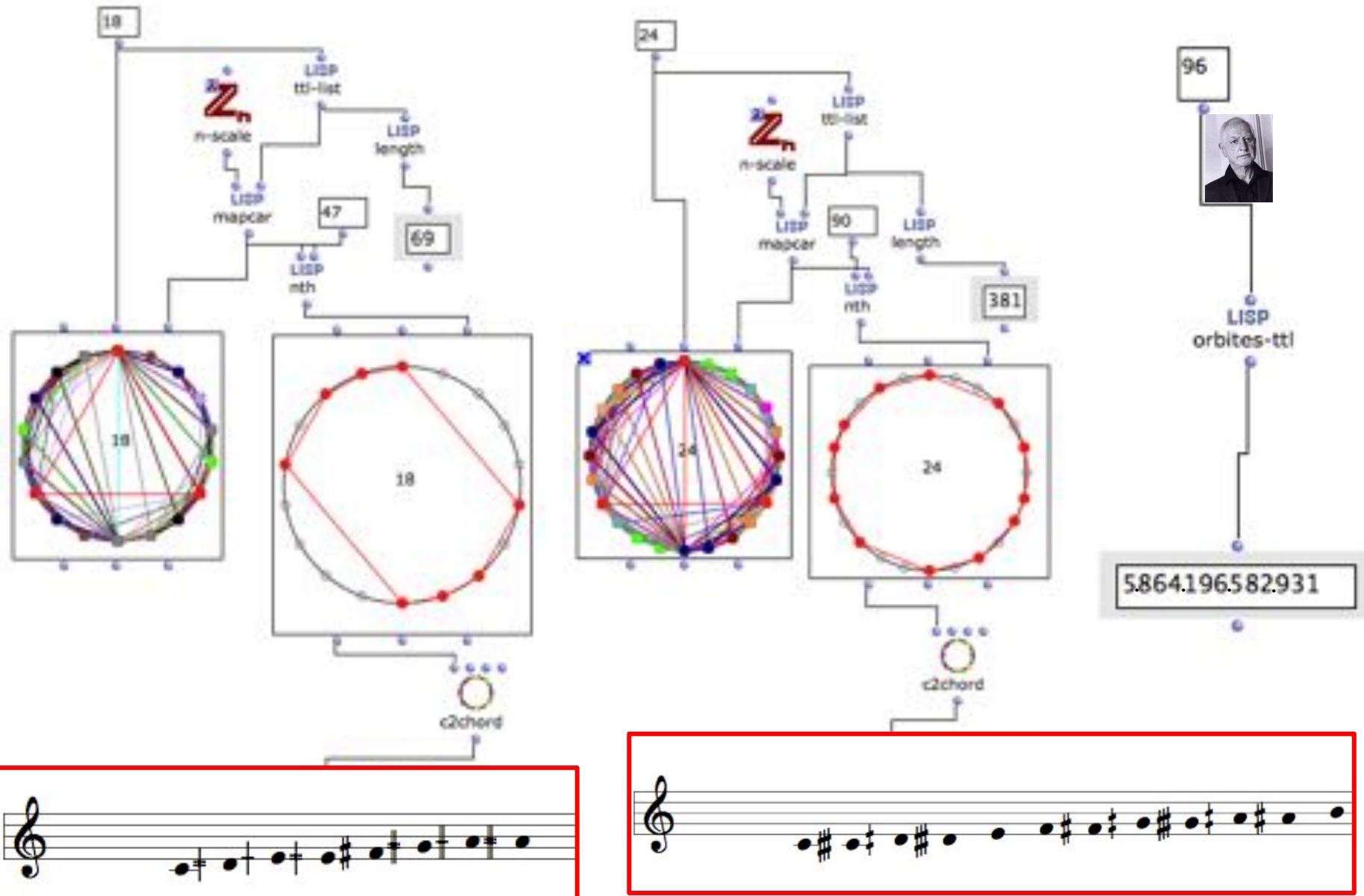


(3, 1, 2, 3, 1, 2)

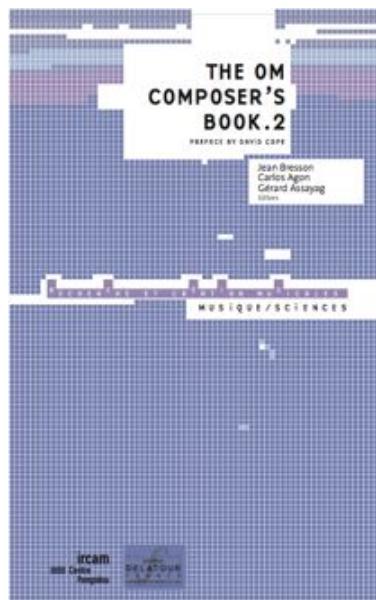
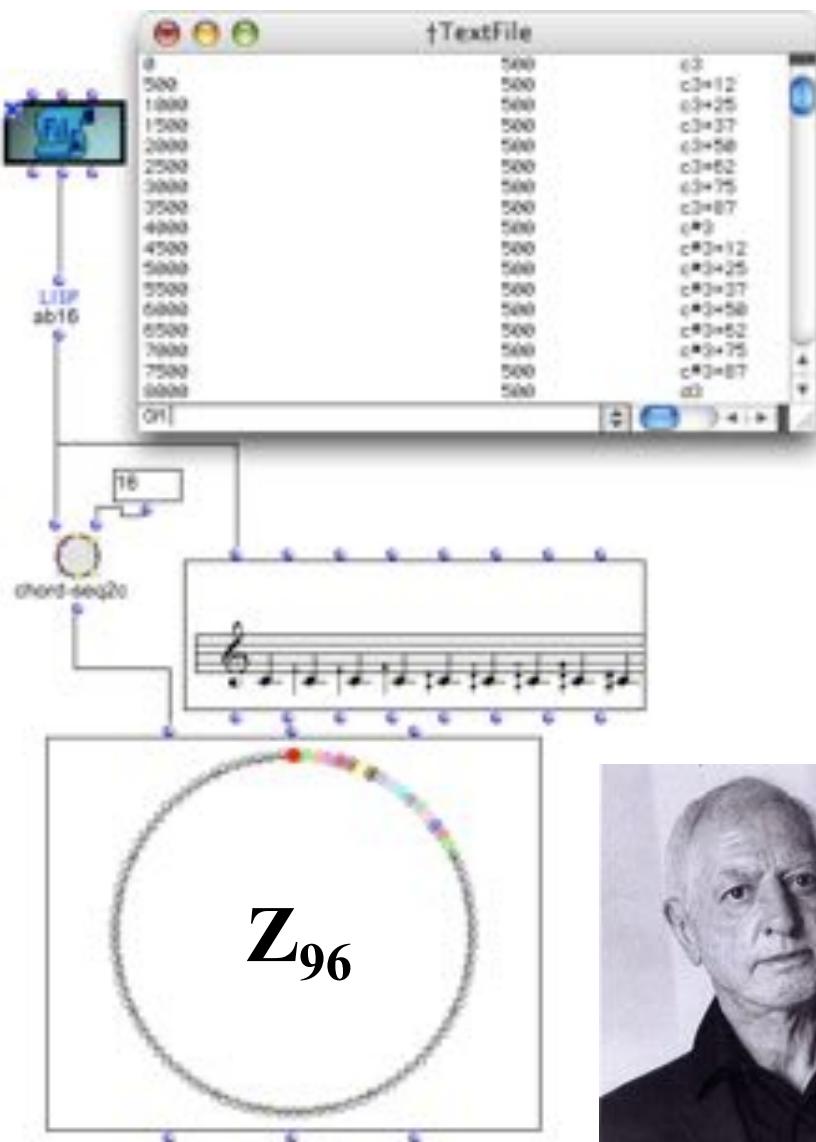
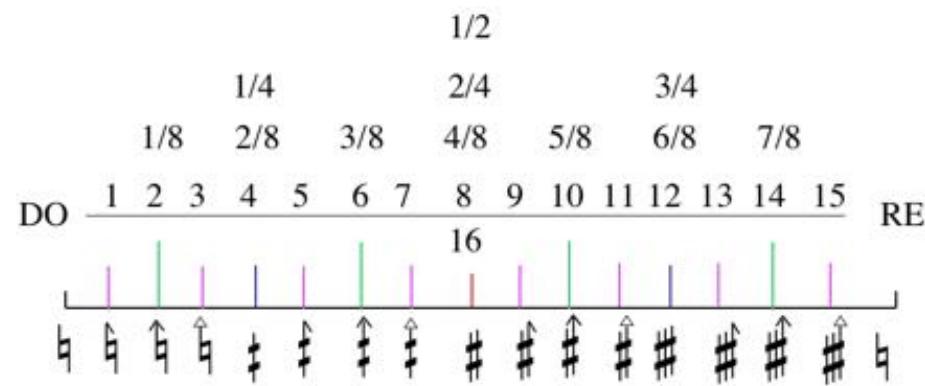


(2, 1, 3, 2, 1, 3)

Messiaen's Modes and their microtonal extensions



Microtonal Composition



Microtonality

A. Bancquart, M. Andreatta, et C. Agon, « Microtonal Composition », The OM Composer's Book 2, éd. Jean Bresson, Carlos Agon, Gérard Assayag (Ircam/Delatour France, Sampzon), 2008, p. 279-302.



Alain Bancquart

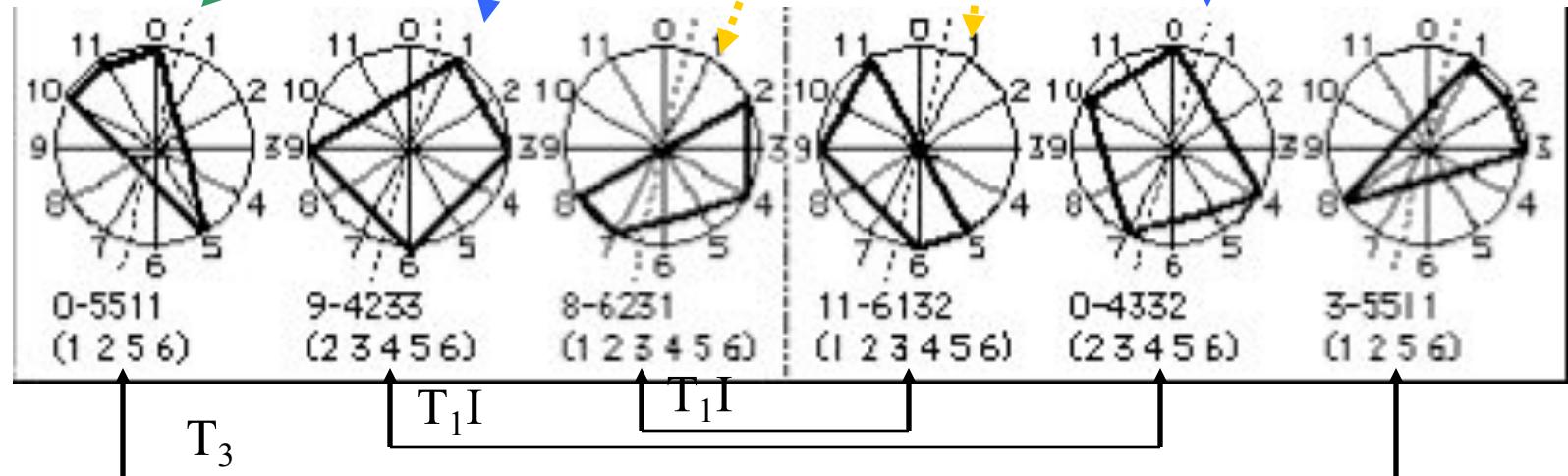


« Entités formelles pour l'analyse musicale »

Marcel Mesnage (1998)

A. Schoenberg : *Klavierstück Op. 33a*, 1929

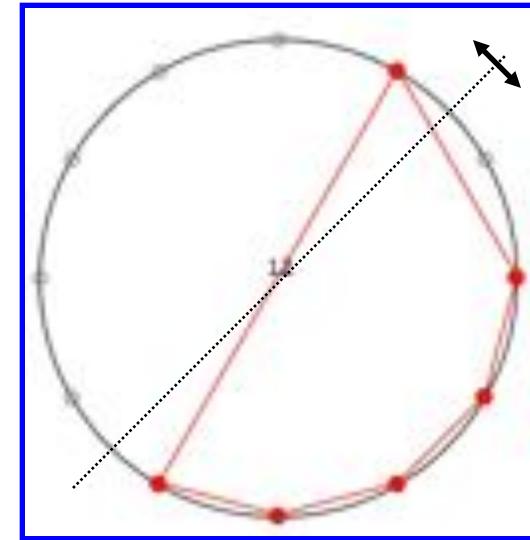
The musical score consists of two staves. The top staff is in treble clef and 4/4 time, starting with a C major chord. The bottom staff is in bass clef and 4/4 time, starting with a G major chord. Several melodic fragments are highlighted with colored boxes: a blue box on the top staff, an orange box on the top staff, a green dashed box on the bottom staff, another blue box on the bottom staff, and a green dashed box on the top staff. Arrows point from these boxes to the corresponding circle graphs below.



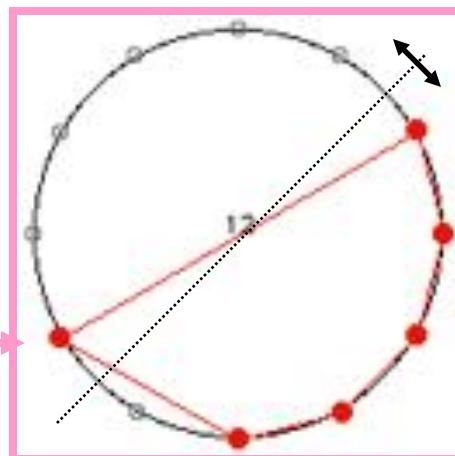
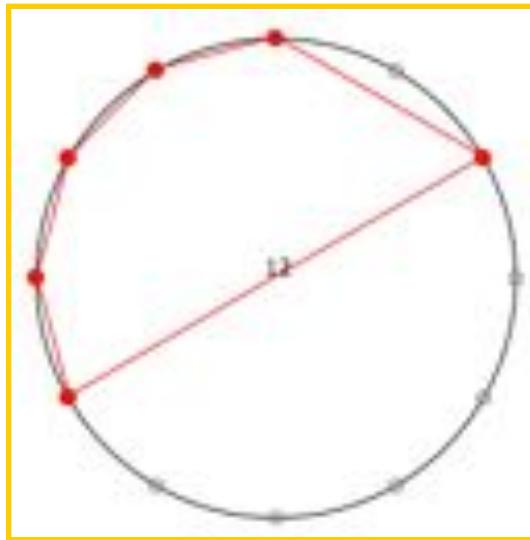
Serialism and hexachordal combinatoriality

Schoenberg: Suite Op.25, Minuetto

A musical score excerpt from Schoenberg's Suite Op.25, Minuetto. The top staff shows a treble clef, a key signature of one sharp, and a common time signature. The bottom staff shows a bass clef, a key signature of one flat, and a common time signature. Two hexachords are highlighted: one in pink (measures 5-6) and one in green (measures 9-12). The hexachords are composed of numbered circles (e.g., 5, 6, 8, 9, 10, 11, 12) indicating specific notes or pitch classes.



Double combinatoriality



Hexachordal Combinatoriality in Messiaen

- Mode de valeurs et d'intensités (1950)



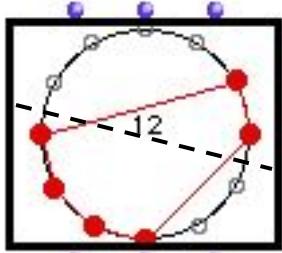
Voici le mode:

I

$ppp\ ppp\ ff\ f\ mf\ ff\ f\ mf\ ff\ pp\ ff\ p$

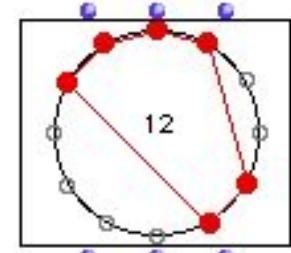
(la Division I est utilisée dans la portée supérieure du Piano)

The diagram shows a hexachordal cycle with red dots representing notes and blue dots representing rests. A red box highlights a segment of the cycle, corresponding to the notes shown in the piano score above.



$$\{3, 2, 9, 8, 7, 6\} \longrightarrow \{4, 5, 10, 11, 0, 1\}$$

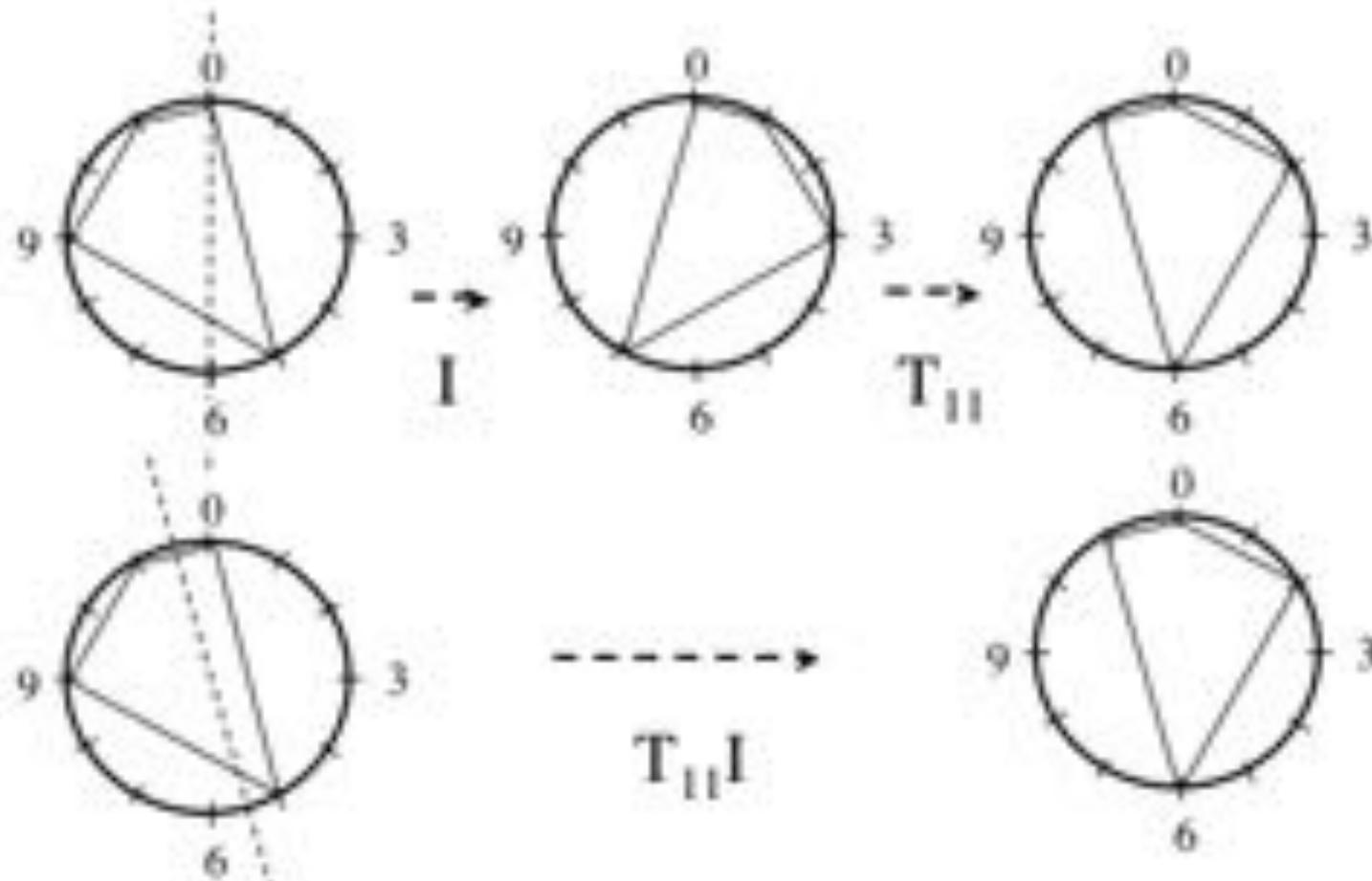
$T_7I : x \rightarrow 7-x$



Transposition and Inversion

I: $x \rightarrow 12-x$

$T_k: x \rightarrow k+x$



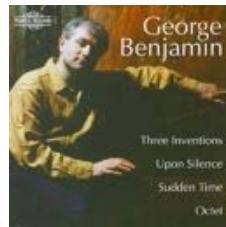
$T_{11}I: x \rightarrow 11-x$

{0, 5, 9, 11}

{11, 6, 3, 0}

Combinatorics and modal compositional processes

On a combinatorial problem posed by G. Benjamin



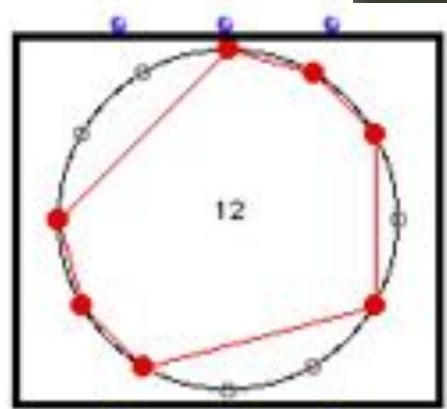
The scale :



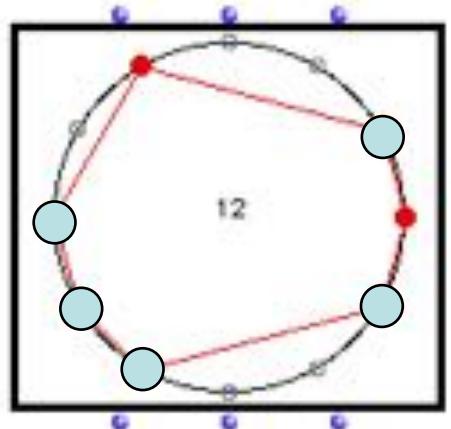
Is it unique ?

How to find 7-notes scales preserving :

- 1) 4 notes by inversion
- 2) 5 notes by fifth transposition
- 3) 5 notes once the inversion is transposed by a fourth

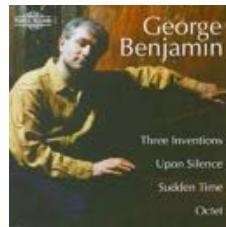


T_7



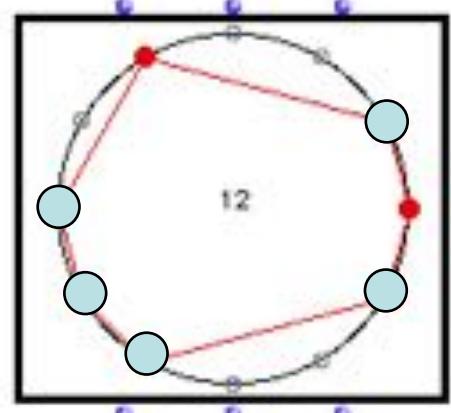
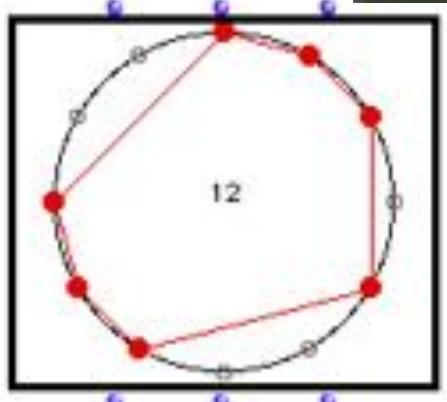
Combinatorics and modal compositional processes

On a combinatorial problem posed by G. Benjamin



How to find 7-notes scales preserving :

- 1) 4 notes by inversion
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pcset

Interval Content

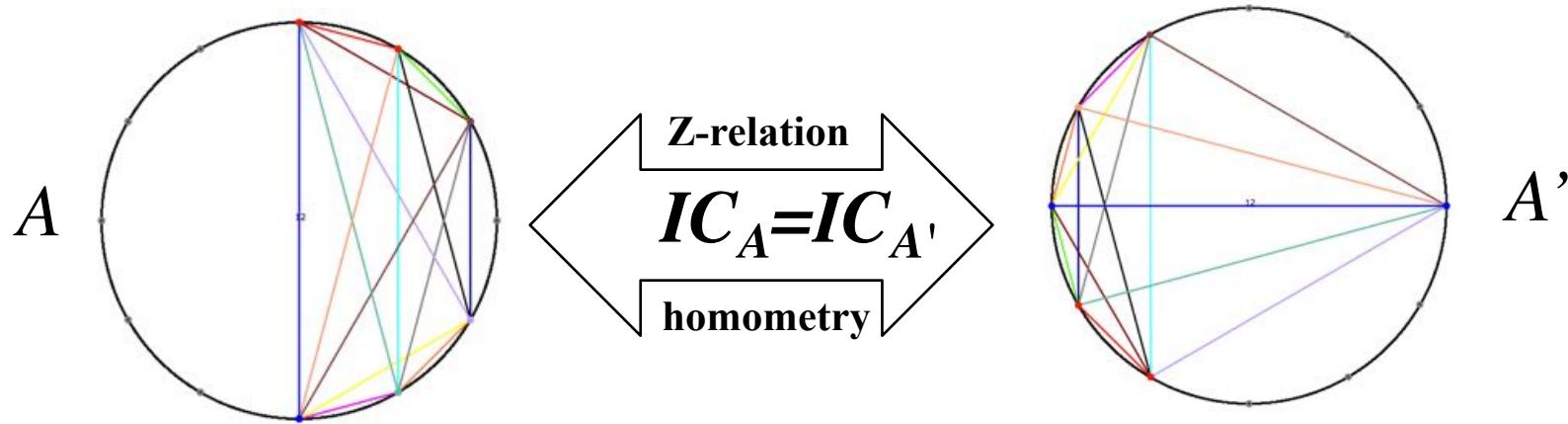
(0 1 3 4 6 8 10)	↔	[7 4 3 3 4 5 4 5 4 3 3 4]
	↑	
	T_0	...



Interval content

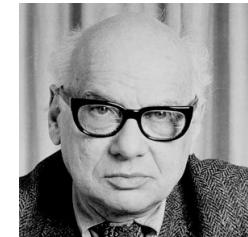
Z-relation and Babbitt's Hexachord Theorem

The *interval content* IC_A of a chord provides the multiplicity of occurrences of its intervals



$$IC_A = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = IC_{A'}$$

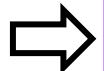
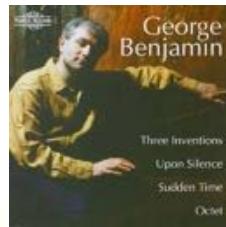
Babbitt's Hexachord Theorem:
A hexacord and its complement have the same interval content



Milton Babbitt

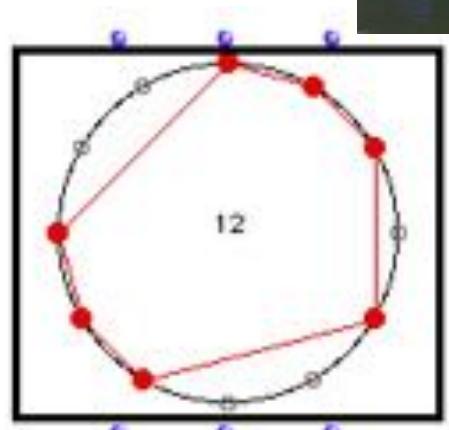
Combinatorics and modal compositional processes

On a combinatorial problem posed by G. Benjamin

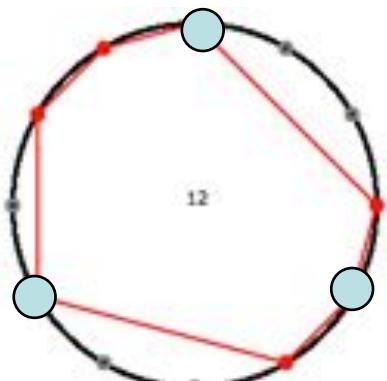


How to find 7-notes scales preserving :

- 1) 4 notes by inversion
- 2) 5 notes by fifth transposition
- 3) 5 notes once the inversion is transposed by a fourth

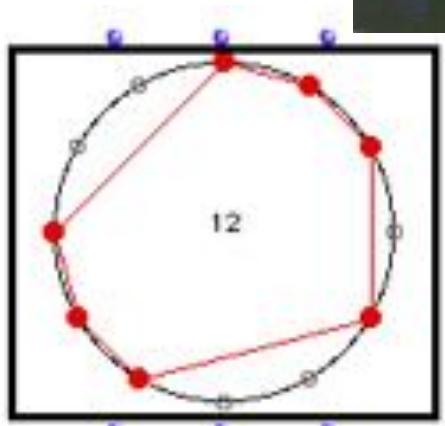
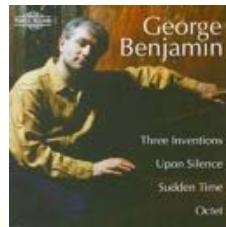


I
↓



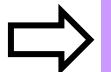
Combinatorics and modal compositional processes

On a combinatorial problem posed by G. Benjamin

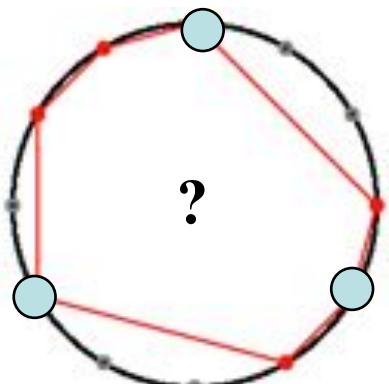


I
↓

How to find 7-notes scales preserving :

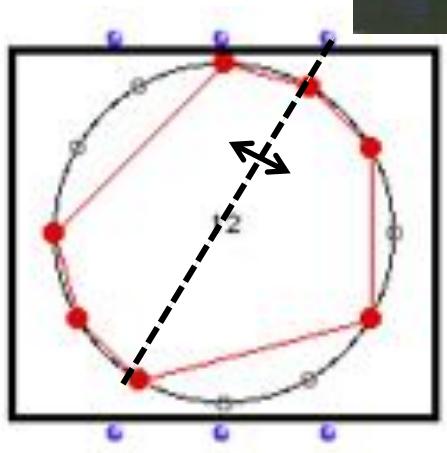
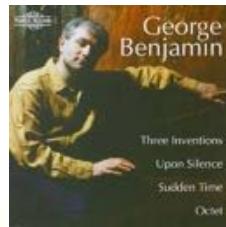


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- 2) 5 notes by fifth transposition
- 3) 5 notes once the inversion is transposed by a fourth



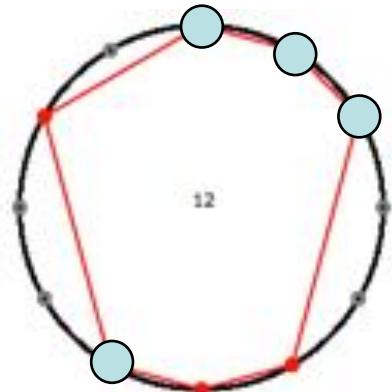
Combinatorics and modal compositional processes

On a combinatorial problem posed by G. Benjamin



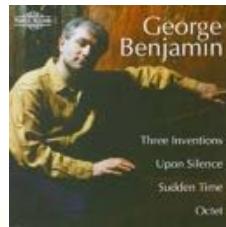
How to find 7-notes scales preserving :

-
- 1) 4 notes by inversion
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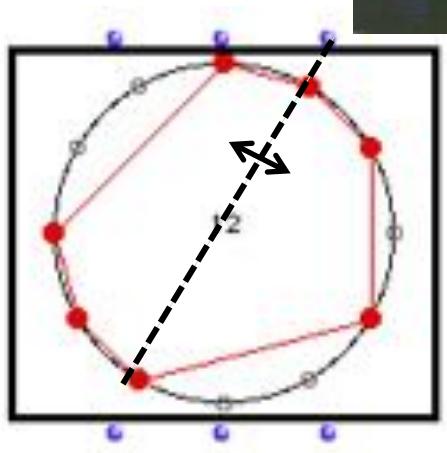
Combinatorics and modal compositional processes

On a combinatorial problem posed by G. Benjamin

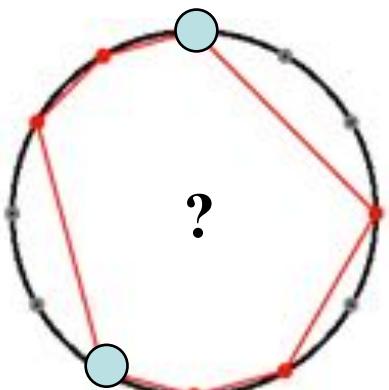


How to find 7-notes scales preserving :

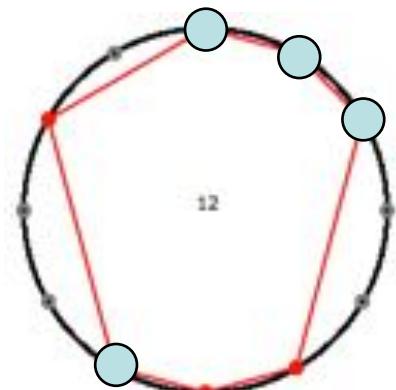
- 1) 4 notes by inversion
- 2) 5 notes by fifth transposition
- 3) 5 notes once the inversion is transposed by a fourth



I_2

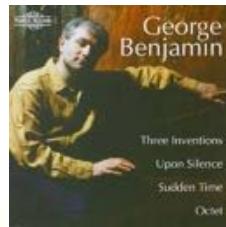


T_5



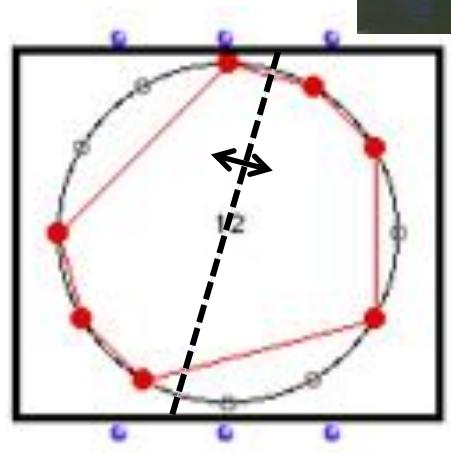
Combinatorics and modal compositional processes

On a combinatorial problem posed by G. Benjamin

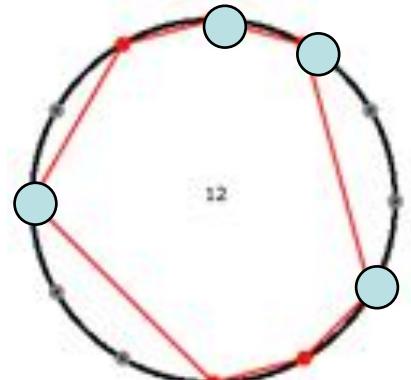
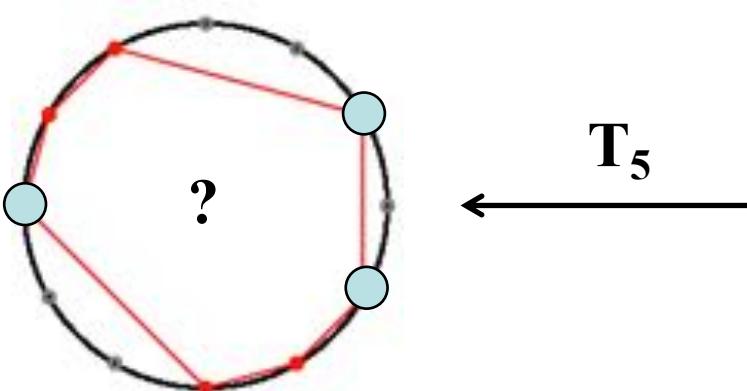


How to find 7-notes scales preserving :

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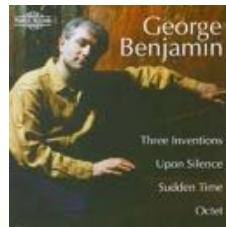


I_1



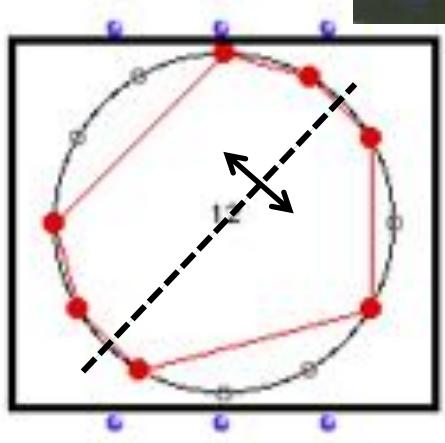
Combinatorics and modal compositional processes

On a combinatorial problem posed by G. Benjamin

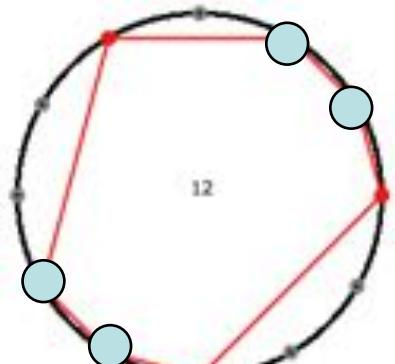
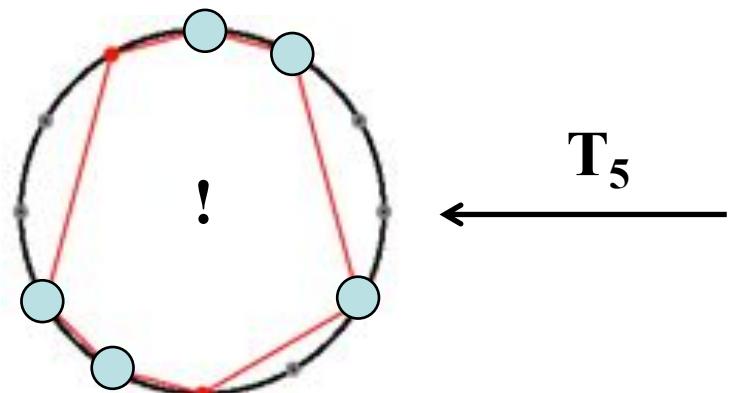


How to find 7-notes scales preserving :

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- 2) 5 notes by fifth transposition
- 3) 5 notes once the inversion is transposed by a fourth

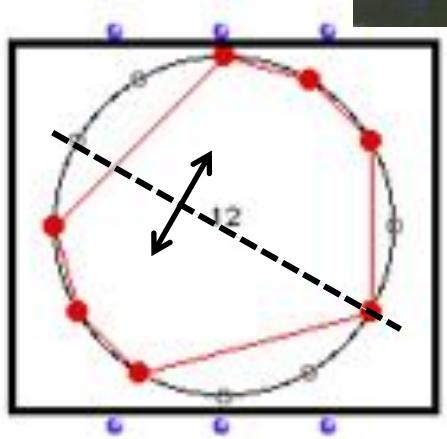
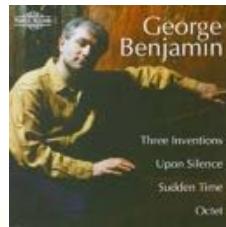


↓
 I_3



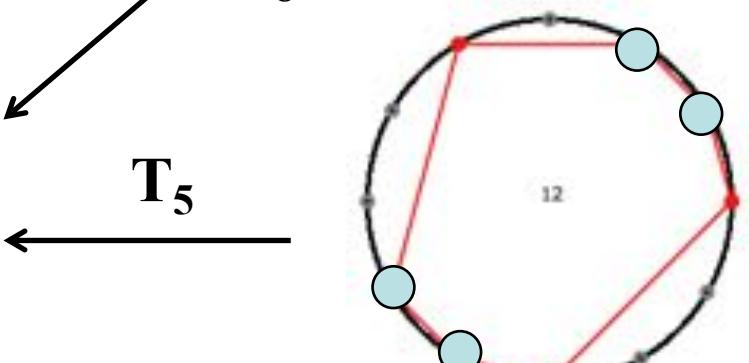
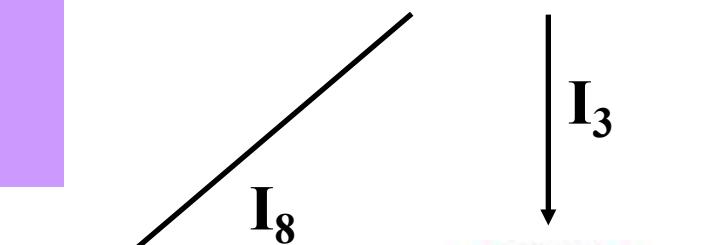
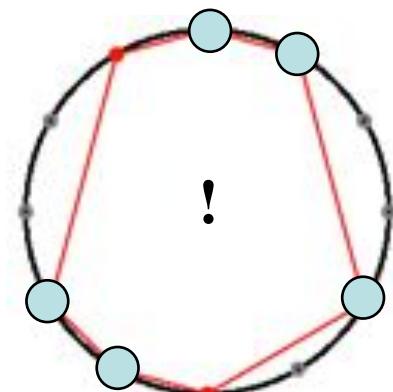
Combinatorics and modal compositional processes

On a combinatorial problem posed by G. Benjamin

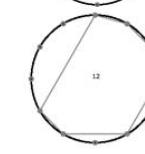
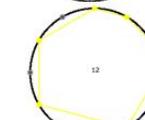
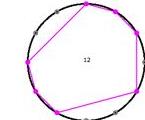
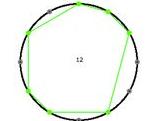
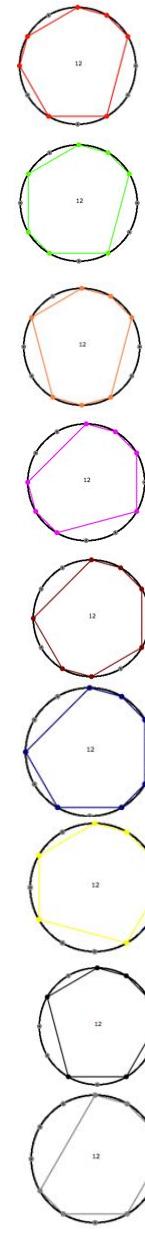


How to find 7-notes scales preserving :

- 1) 4 notes by inversion
- 2) 5 notes by fifth transposition
- 3) 5 notes once the inversion is transposed by a fourth



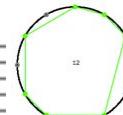
The family of solutions of Benjamin's problem



The catalogue of all solutions (up to transposition)



(1132212)



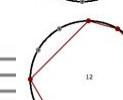
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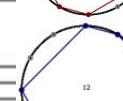
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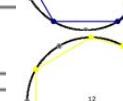
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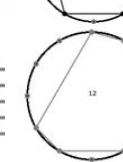
(1121223)



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(1112232)



(1112214)

Intervallic
structure

The « cognitive » pitch/rhythm isomorphisms

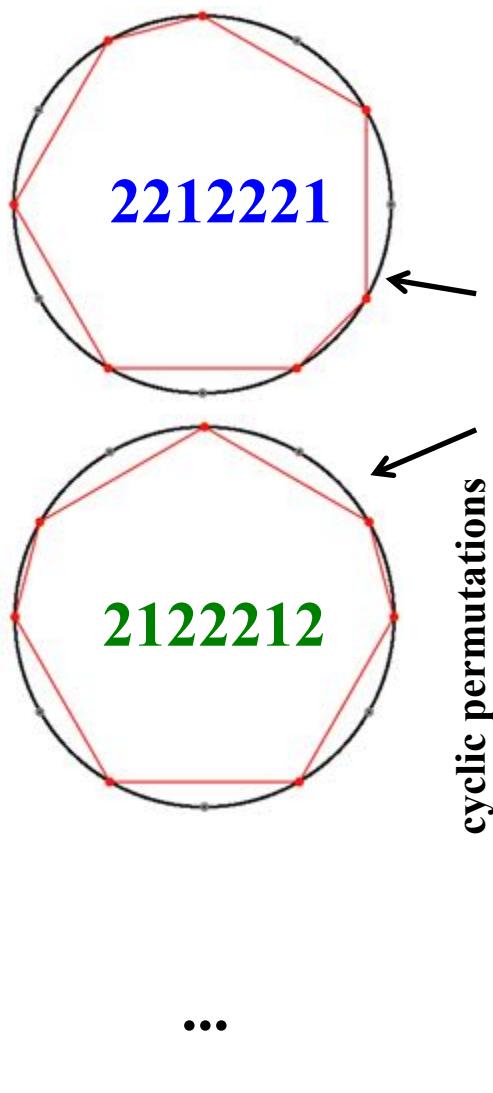


TABLE I
Comparison of $M = 7$, $L = 12$ patterns for pitch (scales) and rhythm (time-lines)

pattern	pitch domain name and notation (in C)	rhythm domain notation	examples from West Africa	references
1. 2212221	major scale (Ionian) CDEFGAB	↓ ↓ ↓ ↓ ↓	Ewe (Atsiabek, Sogba, Atsia) also Yoruba	Jones (1959), C. K. Ladzekpo, S. K. Ladzekpo and Pantaleoni, Locke
2. 2122212	Dorian CDE [†] FGAB [†]	↓ ↓ ↓ ↓ ↓	Bemba—Northern Rhodesia	Jones (1965), (Ekwueme)
3. 1222122	Phrygian CD [†] E [†] FGA [†] B [†]	↓↓↓↓↓	—	—
4. 2221221	Lydian CDEF#GAB	↓↓↓↓↓	Ga-Adangme (common) also common Haitian pattern, Akan (Ab fo)	C. K. Ladzekpo, Combs (1974), R. Hill, Asiamah
5. 2212212	Mixolydian CDEFGAB [†]	↓↓↓↓↓	Yoruba sacred music from Ekiti	King
6. 2122122	Aeolian CDE [†] FGA [†] B [†]	↓↓↓↓↓	Ashanti (Ab fo , Mpre)	Koetting
7. 1221222	Locrian CD [†] E [†] FG [†] A [†] B [†]	↓↓↓↓↓	Ghana*	Nketia (1963a)
8. 2121222	(#2 Locrian) CDE [†] FG [†] A [†] B [†]	↓↓↓↓↓↓	Ashanti (Asedua)	C. K. Ladzekpo
9. 2112123	— CDD#EF#GA	↓↓↓↓↓↓	Akan (juvenile song)	Nketia (1963b)

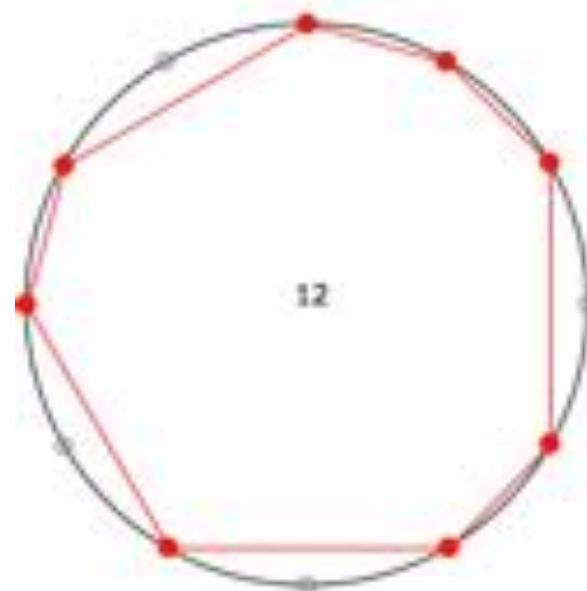
* clap pattern

† mute stroke on bell

Cyclic permutations in minimalistic music

CLAPPING MUSIC

FOR TWO PERFORMERS



The performance begins and ends with both performers in unison at bar ①. The number of segments of each bar should be fixed at twelve segments per bar. Since the first performer's part does not change, it is up to the second performer to escape from one bar to the next! The second performer should try to keep his pulse constant while it is written, i.e. on the first beat of each measure (not on the first beat of the group of three slaps), so that his

The choice of a particular clapping sound, i.e. with cupped or flat hands, is left up to the performers. Whichever technique is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall rhythmic pattern.

Cyclic permutations in minimalistic music

CLAPPING MUSIC

FOR TWO PERFORMERS

J: 164-168

CLAPS 1
CLAPS 2 f

①

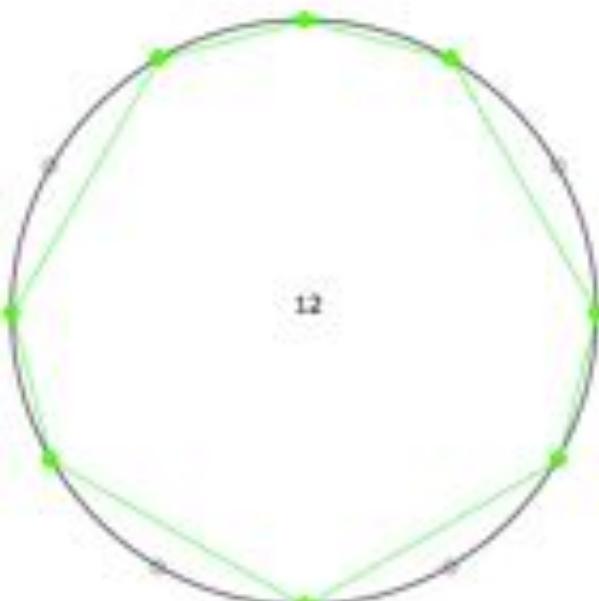
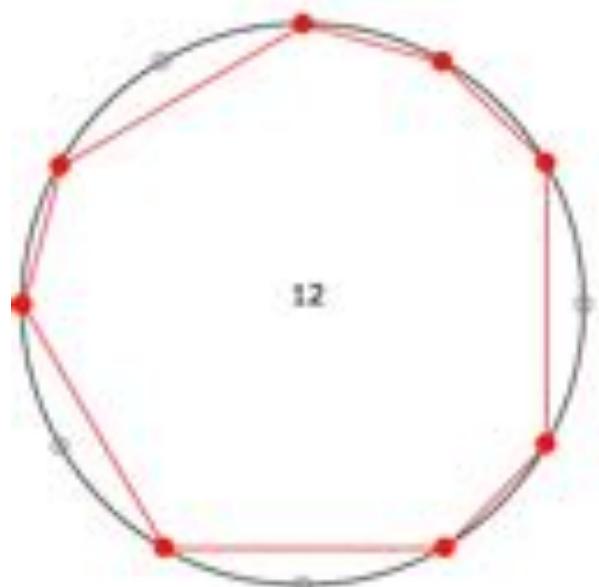
②

③

④ Repeat back, then end

The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer part does not change, it is up to the second performer to move from section to the next. The second performer should try to keep his own slumber where it is written, i.e., on the first beat of each section (but on the first beat of the group of three claps), so that his loudest beat always falls on a new beat of his own underlying pattern.

The choice of a particular clapping sound, i.e., with cupped or flat hands, is left up to the performers. Whichever technique is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall統一 pattern.



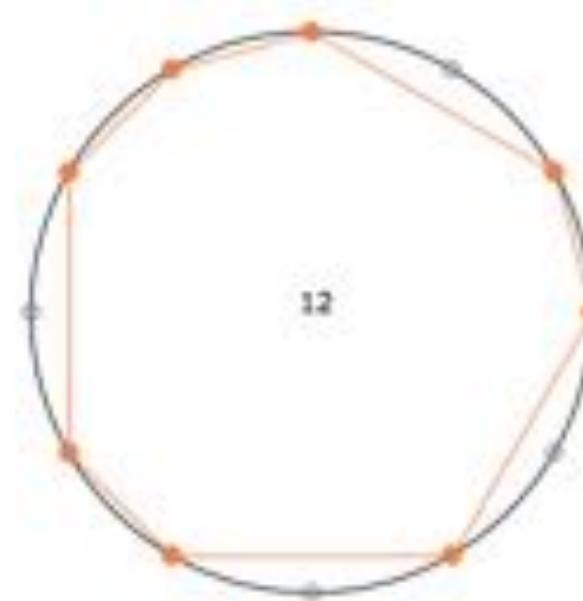
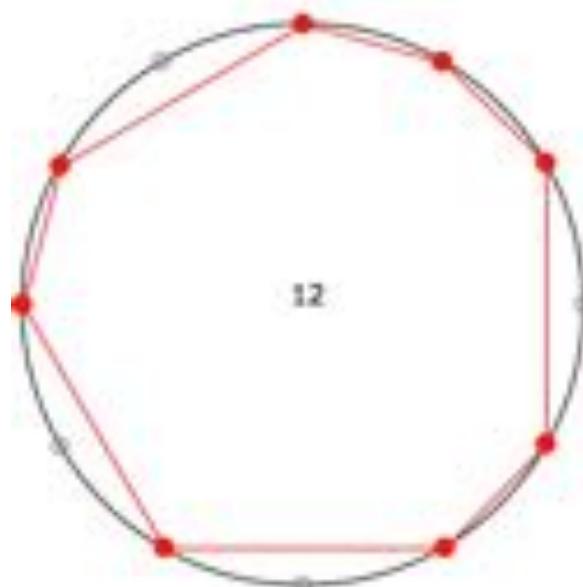
Cyclic permutations in minimalistic music

CLAPPING MUSIC

FOR TWO PERFORMERS

The performance begins and ends with both performers in unison at bar ①. The number of segments of each bar should be fixed at twelve segments per bar. Since the first performer's part does not change, it is up to the second performer to move from one bar to the next. The second performer should try to keep his own count when it is written, i.e. on the first beat of each measure (not on the first beat of the group of three slaps), so that his count will always fall on a new beat of his underlying pattern.

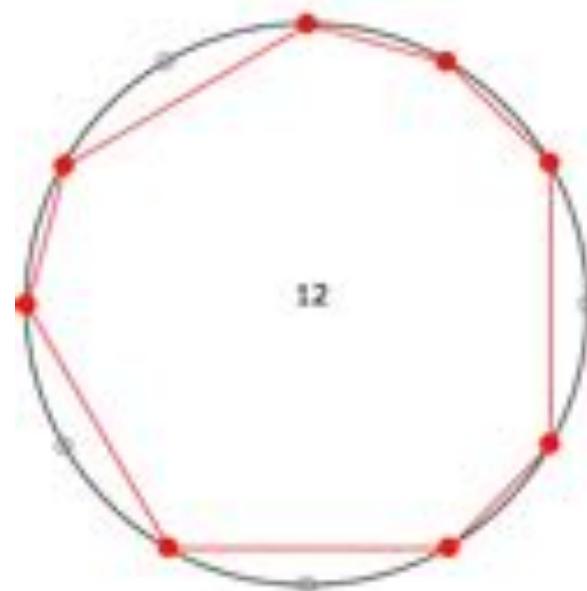
The choice of a particular clapping sound, i.e., with cupped or flat hands, is left up to the performers. Whichever technique is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall rhythmic pattern.



Cyclic permutations in minimalistic music

CLAPPING MUSIC

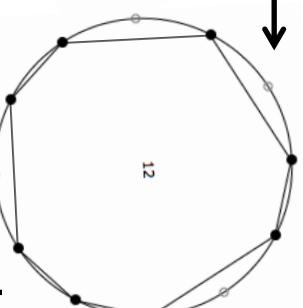
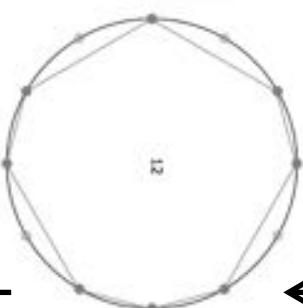
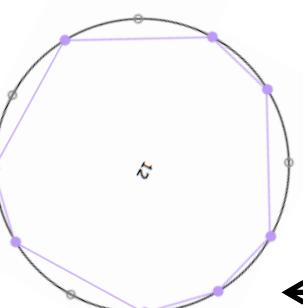
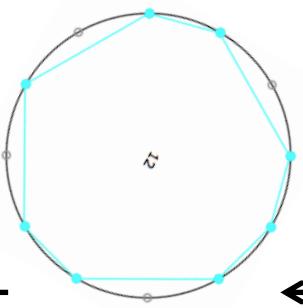
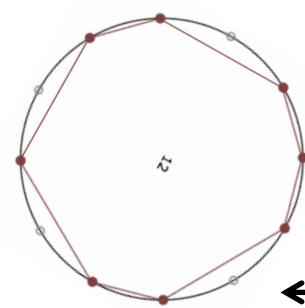
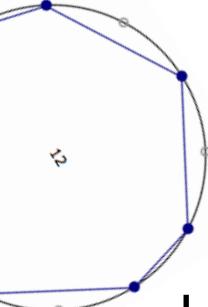
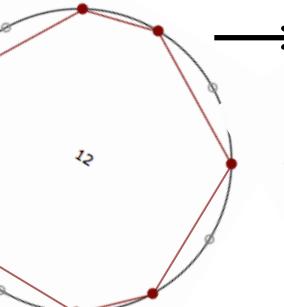
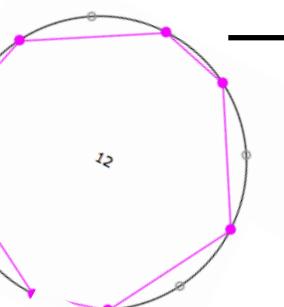
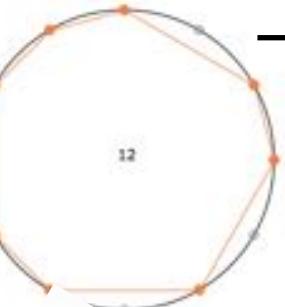
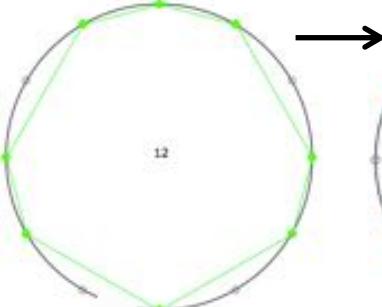
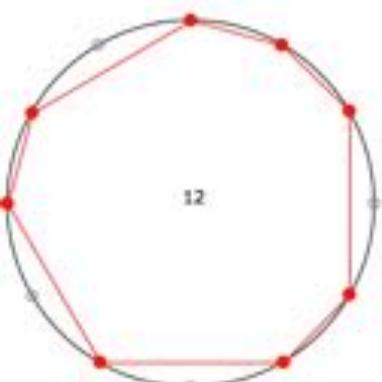
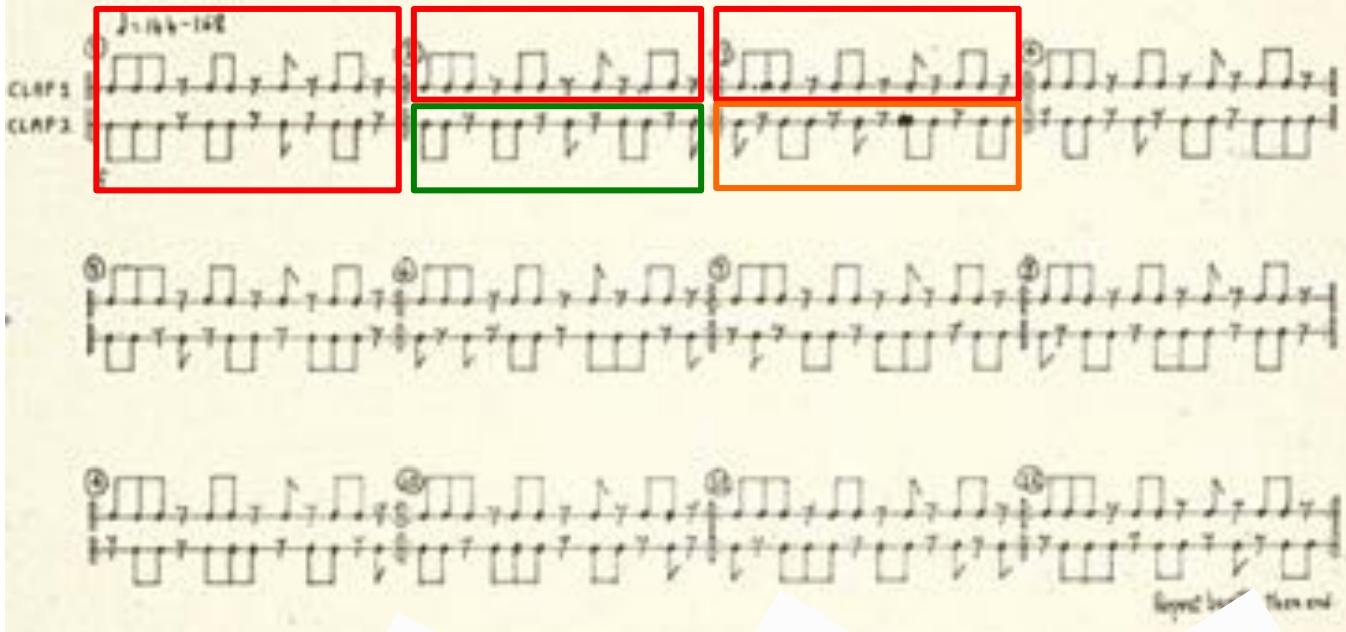
FOR TWO PERFORMERS



The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer's part does not change, it is up to the second performer to repeat from one bar to the next. The second performer should try to keep his pulse consistent where it is written, i.e. on the first beat of each measure (not on the first beat of the group of three slaps), so that his accent will always fall on a new beat of his underlying pattern.

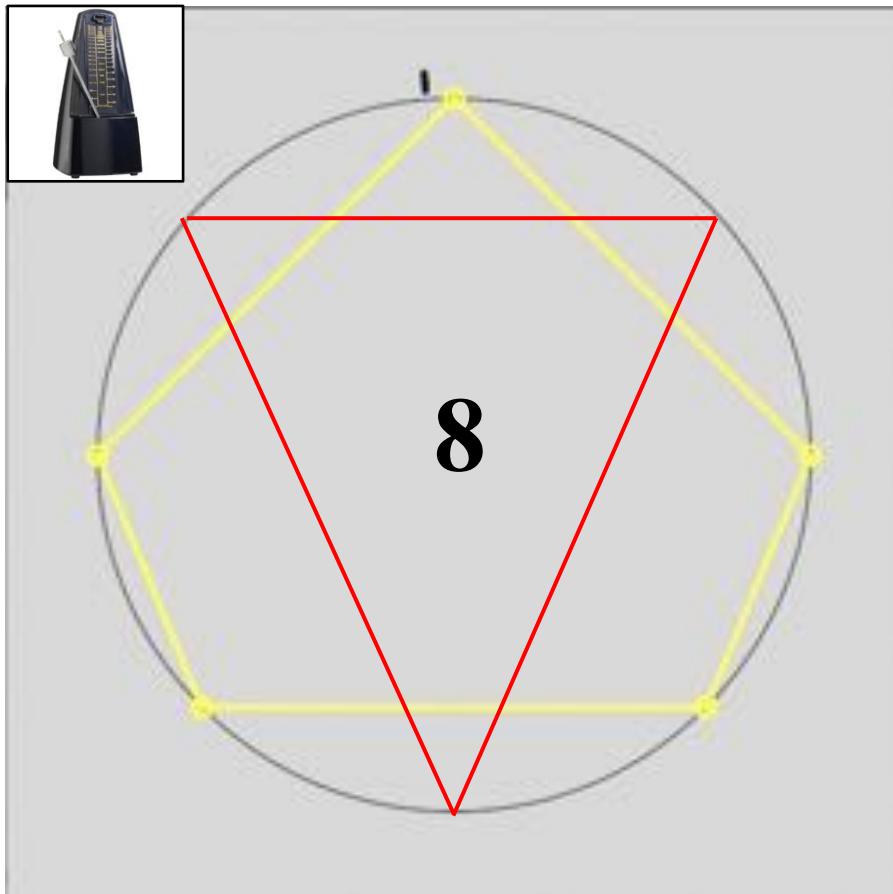
The choice of a particular clapping sound, i.e. with cupped or flat hands, is left up to the performers. Whichever technique is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall rhythmic pattern.

Cyclic permutations in minimalistic music

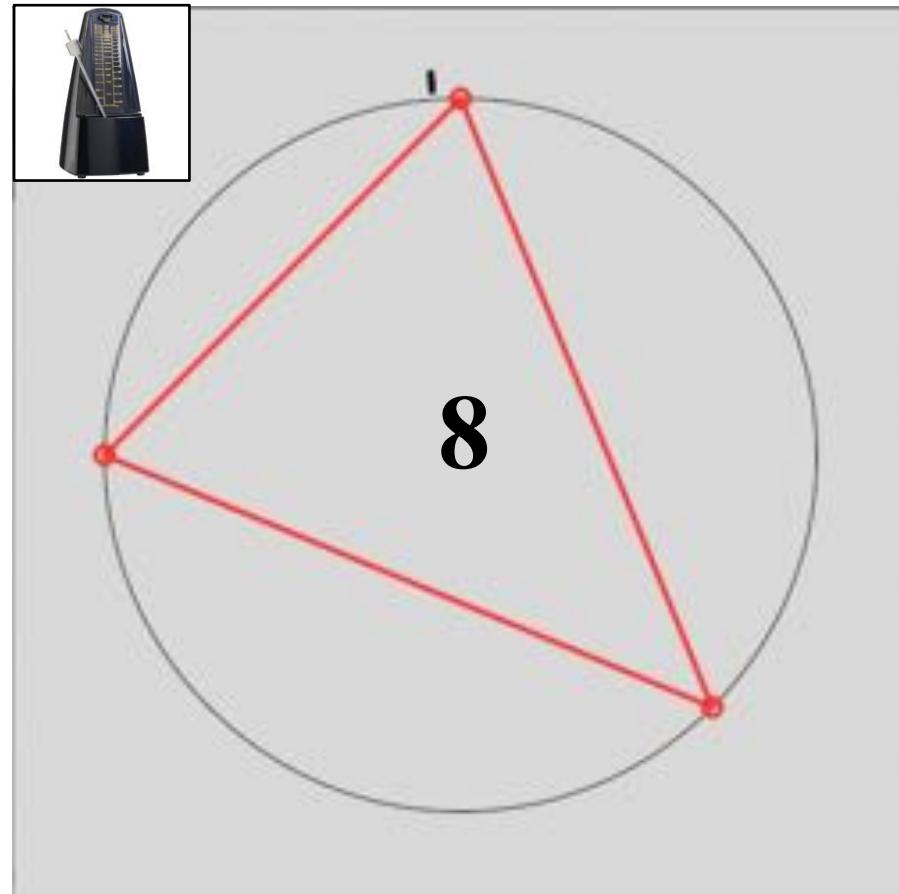


Clapping African-cuban ME-rhythms?

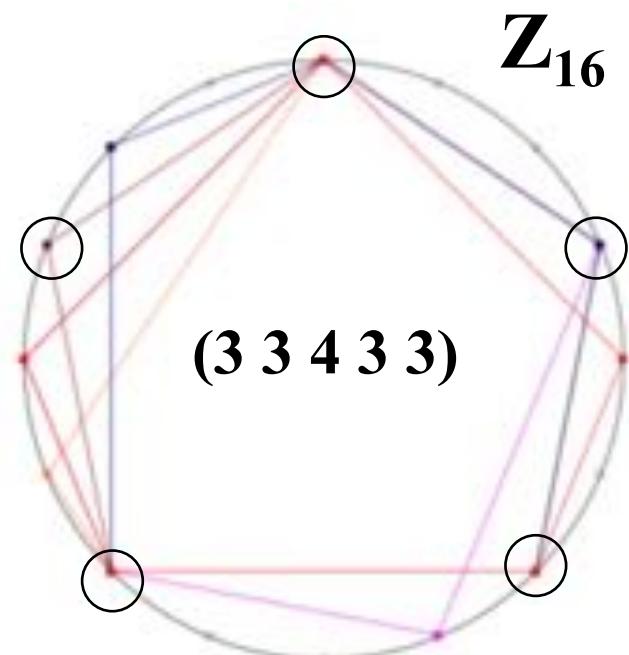
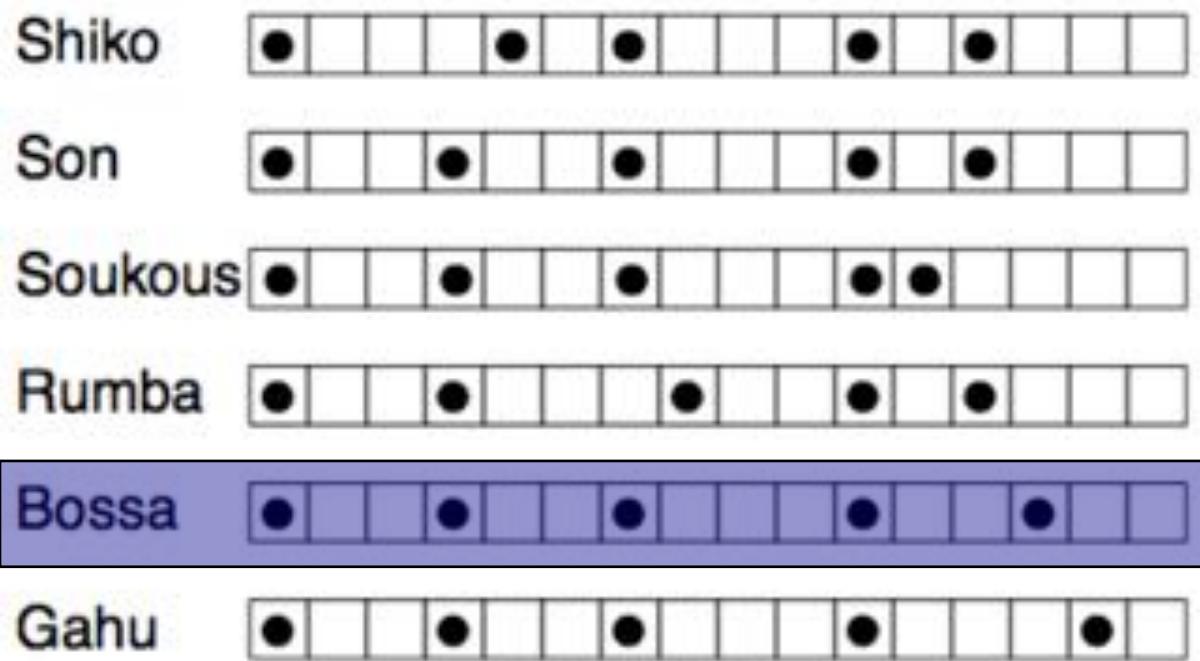
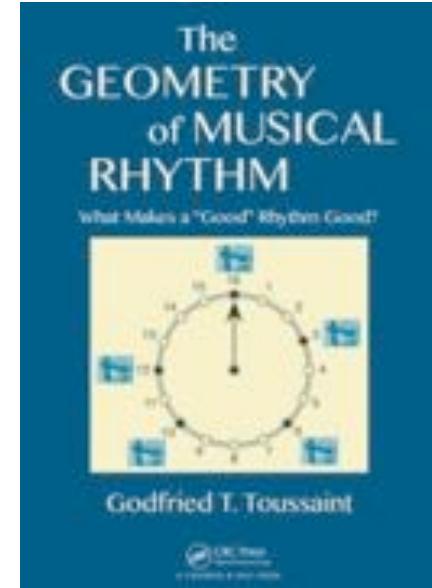
El cinquillo



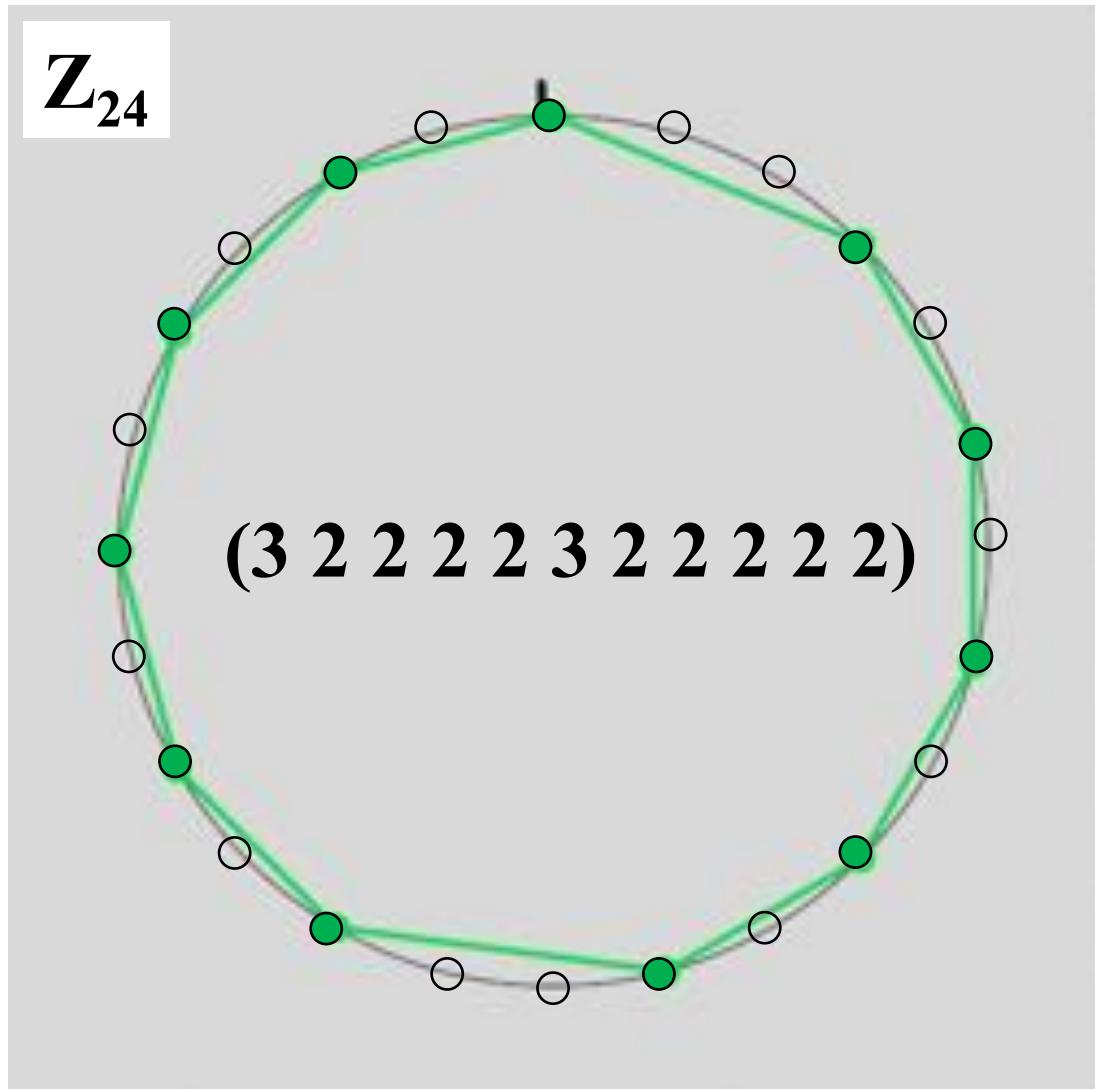
El trecillo



Clapping African-cuban (ME-)rhythms



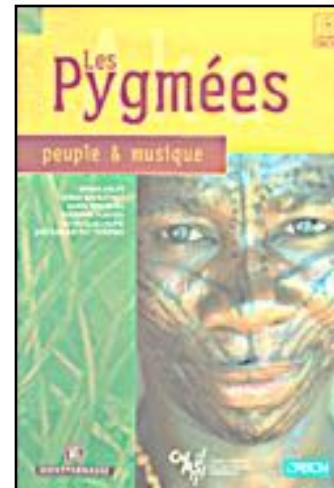
Clapping odditive rhythms?



Simha Arom



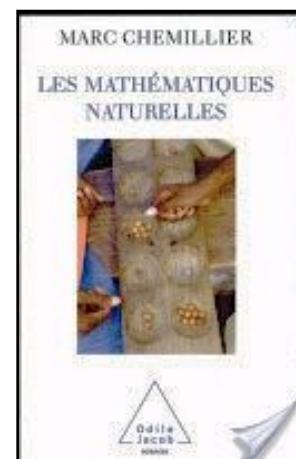
Marc Chemillier



musimédiane

publiée avec le concours de la SFAM

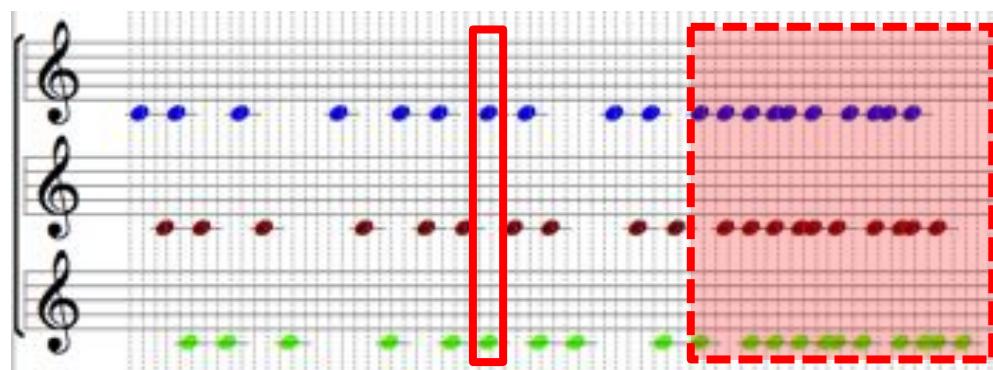
revue audiovisuelle et multimédia d'analyse musicale



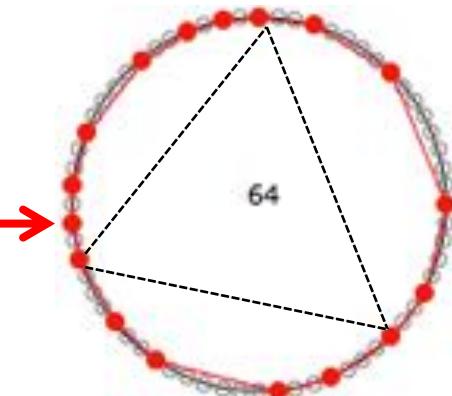
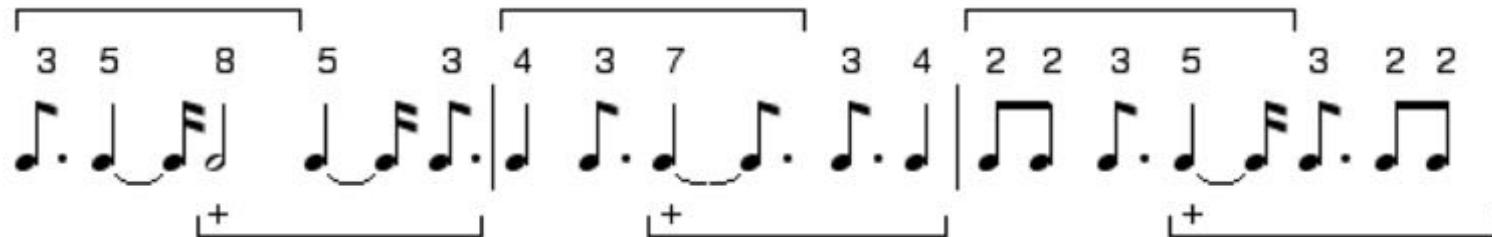
Periodic rhythmic sequences and tiling canons



🔊 **Harawi (1945)**

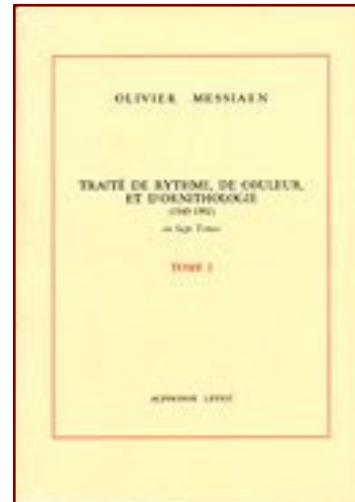


Harawi: rhythmic reduction

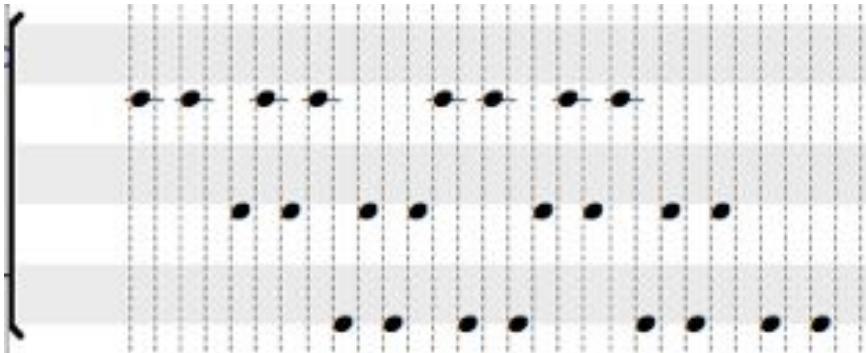
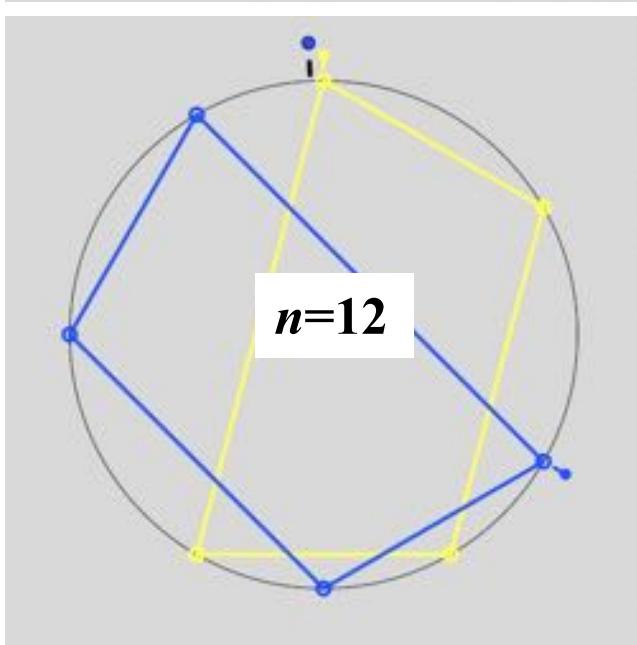
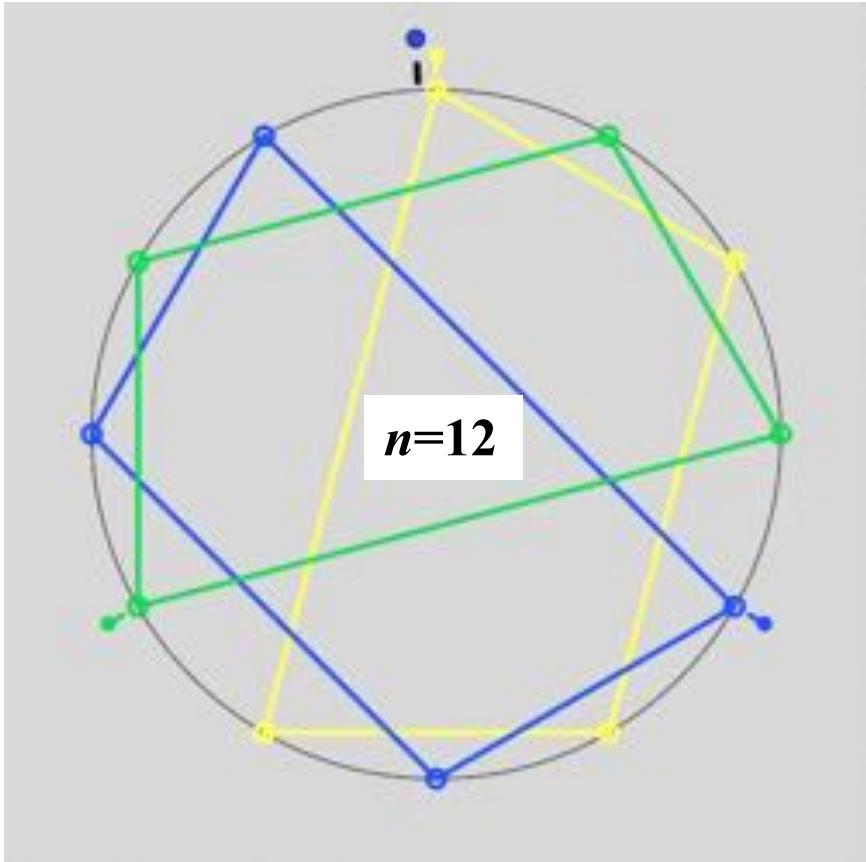
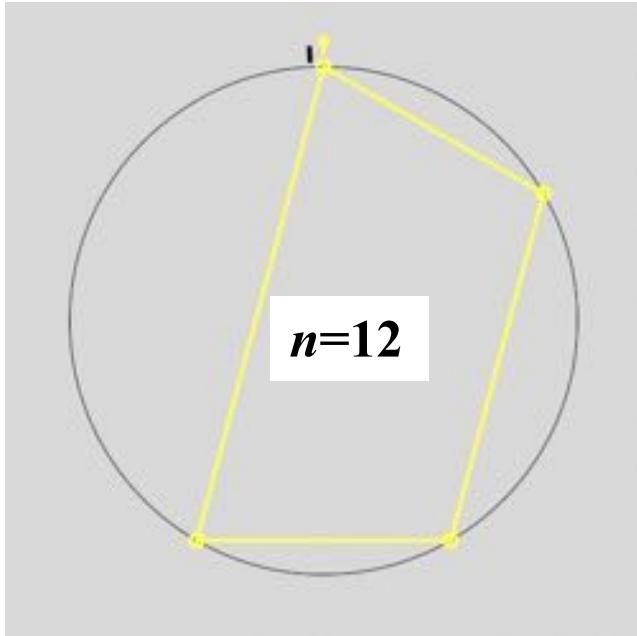


« ...il résulte de tout cela que les différentes sonorités se mélangent ou s'opposent de manières très diverses, **jamais au même moment ni au même endroit** [...]. C'est du désordre organisé »

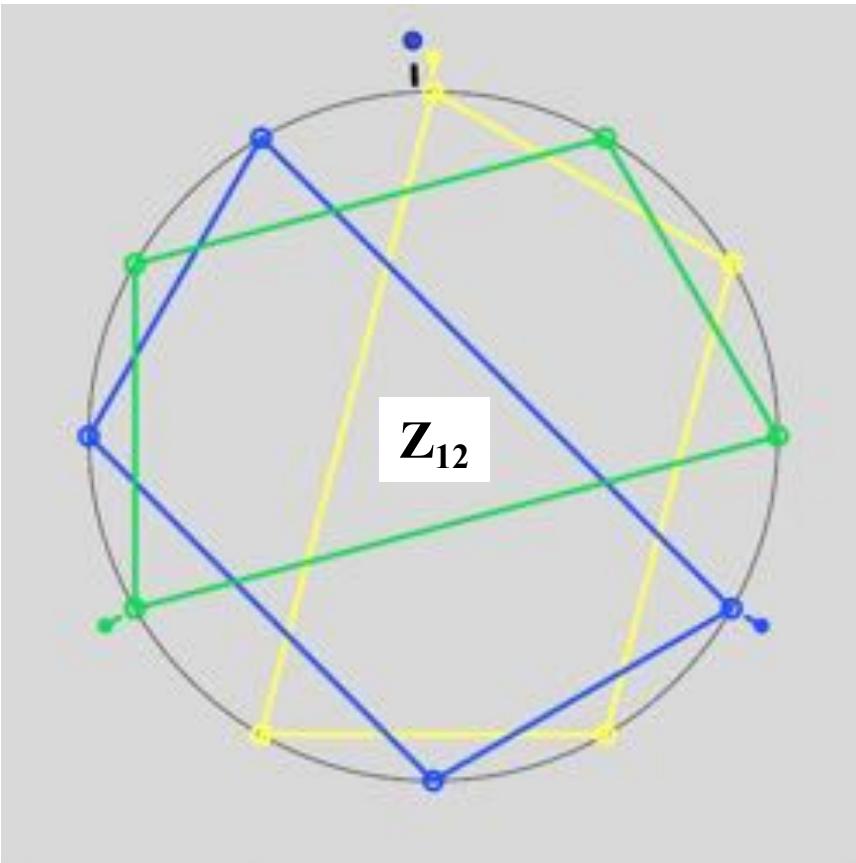
O. Messiaen: *Traité de Rythme, de Couleur et d'Ornithologie*, tome 2, Alphonse Leduc, 1992.



Tiling the time axis with translates of one tile



Formalizing the tiling process as set-theoretical operations



$$A_1 = \{0, 2, 5, 7\}$$

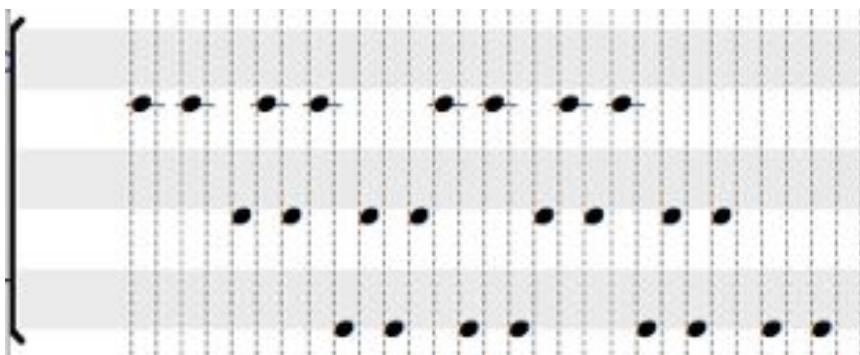
T_4
↓

$$A_2 = \{4, 6, 9, 11\}$$

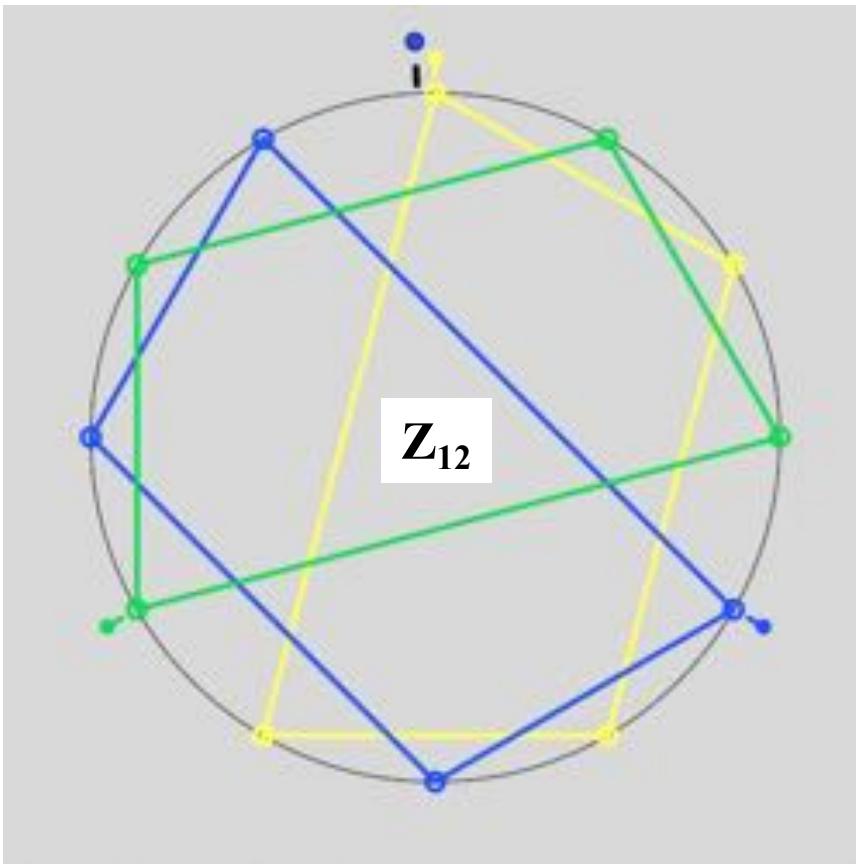
T_4
↓

$$A_3 = \{8, 10, 1, 3\}$$

$$Z_{12} = A_1 \cup A_2 \cup A_3$$



Formalizing the tiling process as a direct sum of subsets



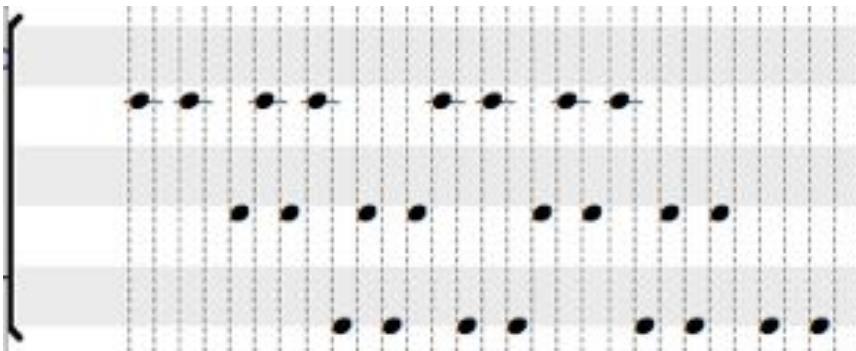
$$A_1 = \{0, 2, 5, 7\}$$

T₄
↓

$$A_2 = \{4, 6, 9, 11\}$$

T₄
↓

$$A_3 = \{8, 10, 1, 3\}$$



$$Z_{12} = A_1 \cup A_2 \cup A_3$$

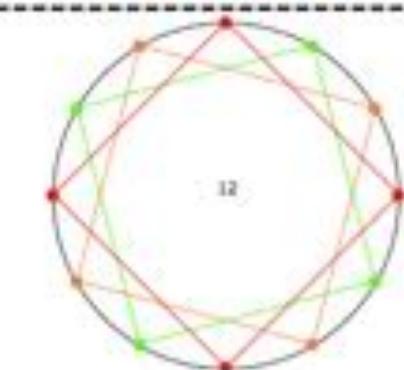
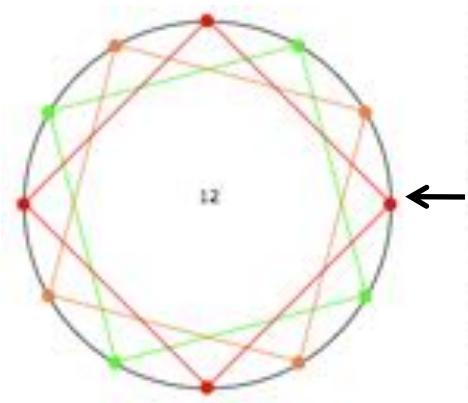
$$Z_{12} = A \oplus B$$

$$A = \{0, 2, 5, 7\}$$

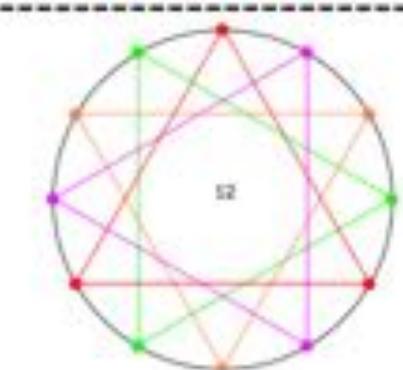
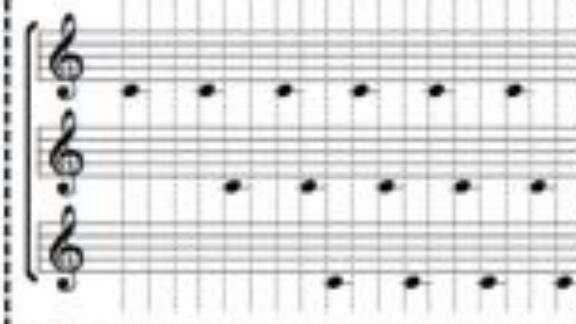
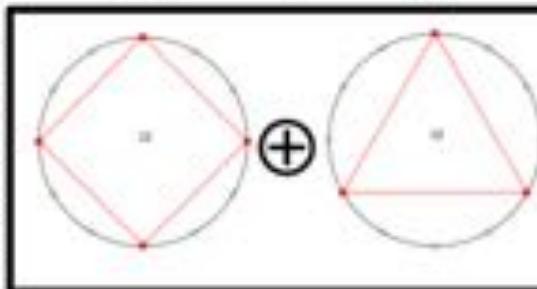
$$B = \{0, 4, 8\}$$

Type 1: tiling rhythmic canons with max regularity

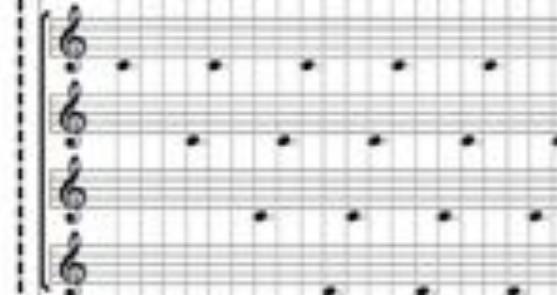
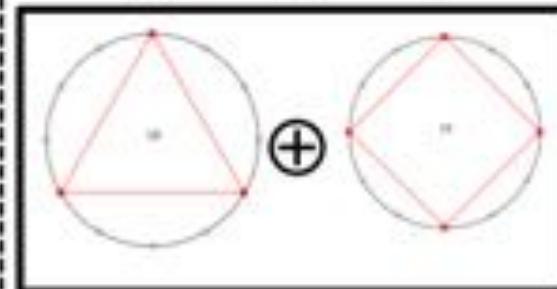
$A < \mathbb{Z}_n$
 $B < \mathbb{Z}_n$



$$\mathbb{Z}_{12} = \mathbb{Z}_4 \oplus \mathbb{Z}_3$$

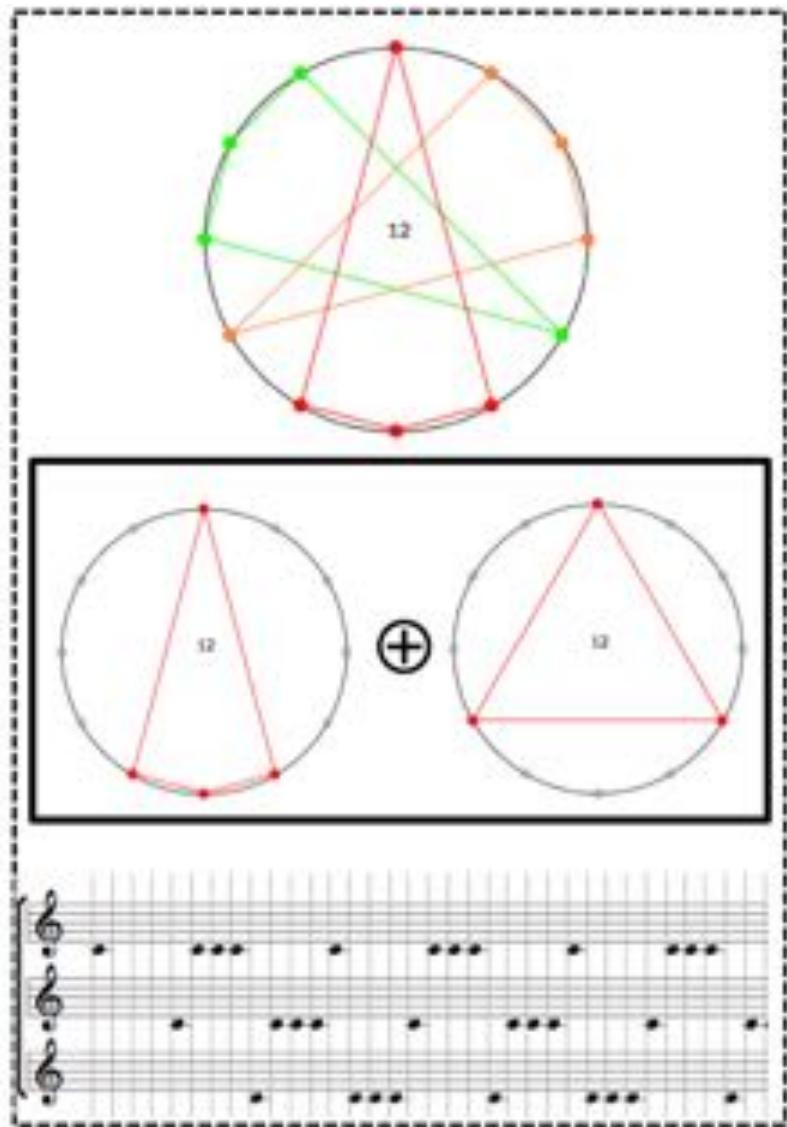


$$\mathbb{Z}_{12} = \mathbb{Z}_3 \oplus \mathbb{Z}_4$$

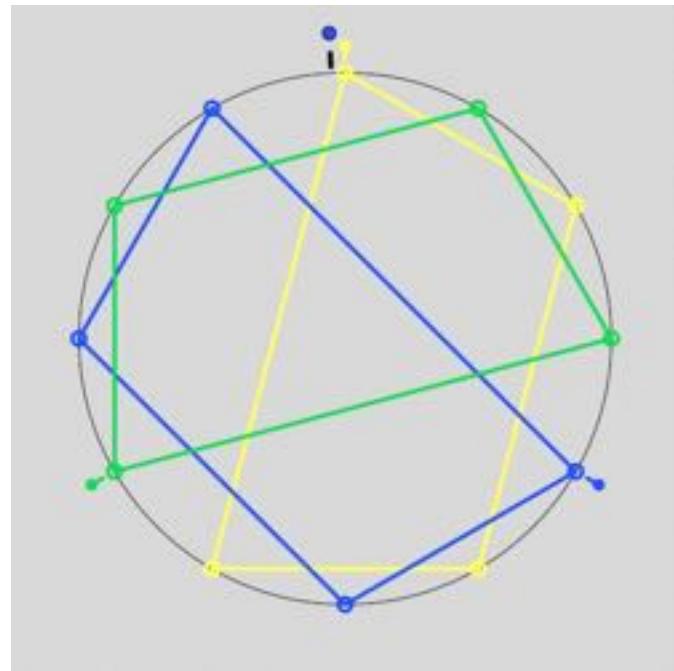
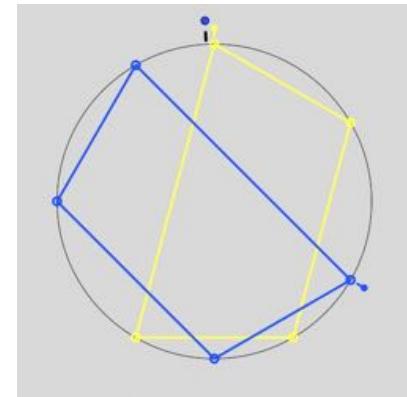
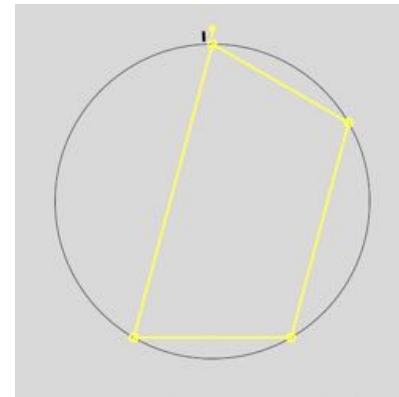


duality

Type 2: Rhythmic Tiling Canons with less regularity

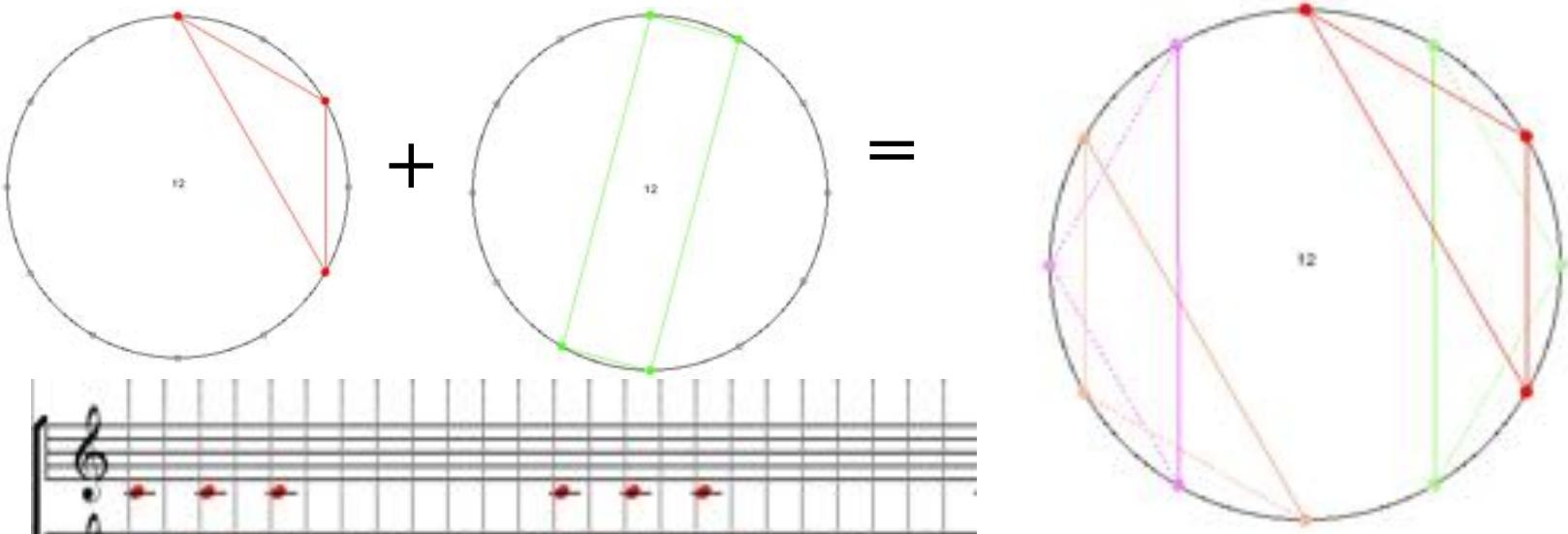


$\{0,5,6,7\} \oplus \{0,4,8\}$



$\{0,2,5,7\} \oplus \{0,4,8\}$

Type 3: rhythmic tiling canons with no regular entries

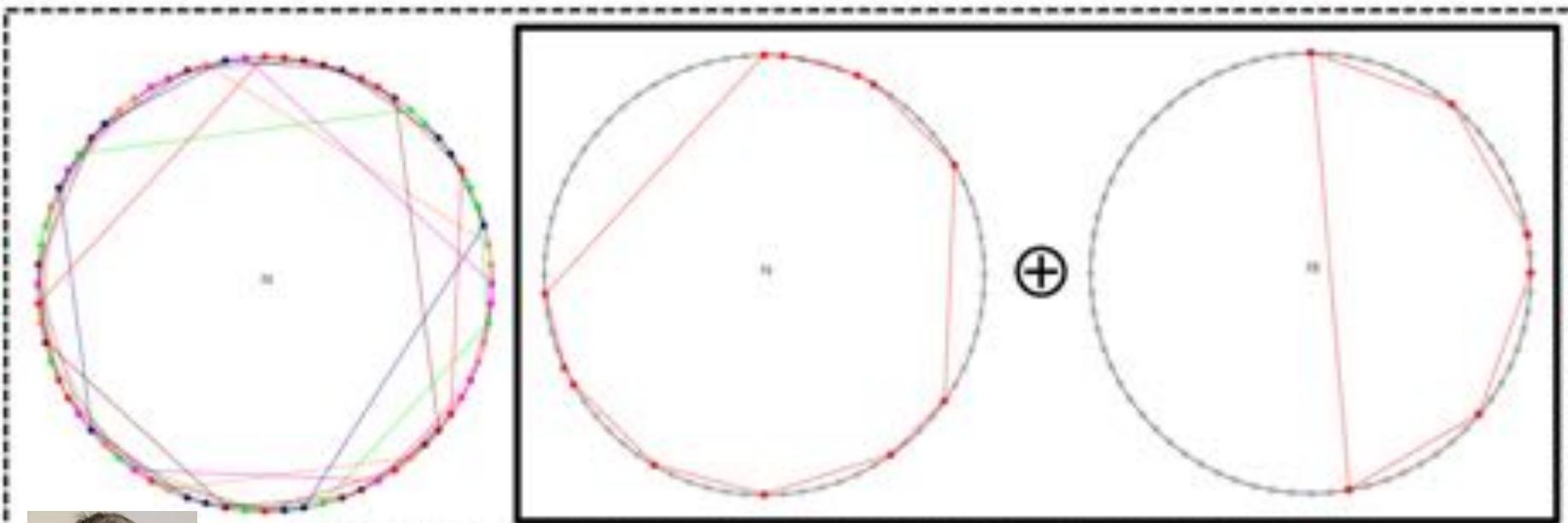


transpositional
combination

$$\{0,2,4\} \oplus \{0,1,6,7\} = Z_{12} = (2 \ 2 \ 8) \bullet (1 \ 5 \ 1 \ 5)$$

One of the two factors is a Messiaen's mode of limited transposition

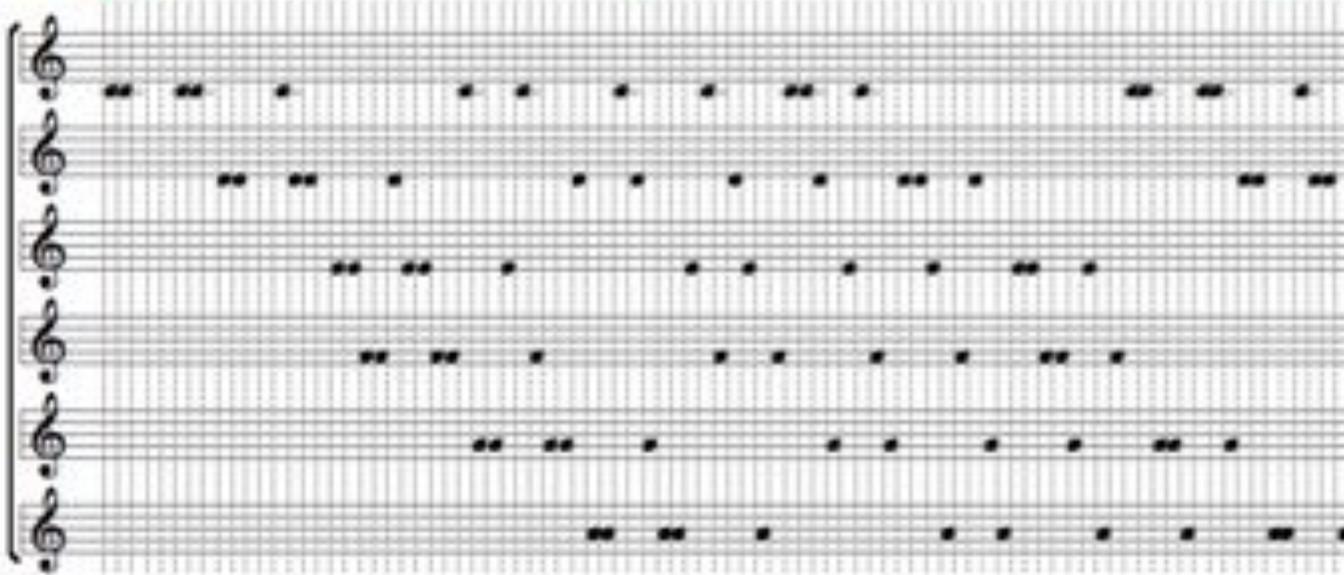
Aperiodic Rhythmic Tiling Canons (Vuza Canons)



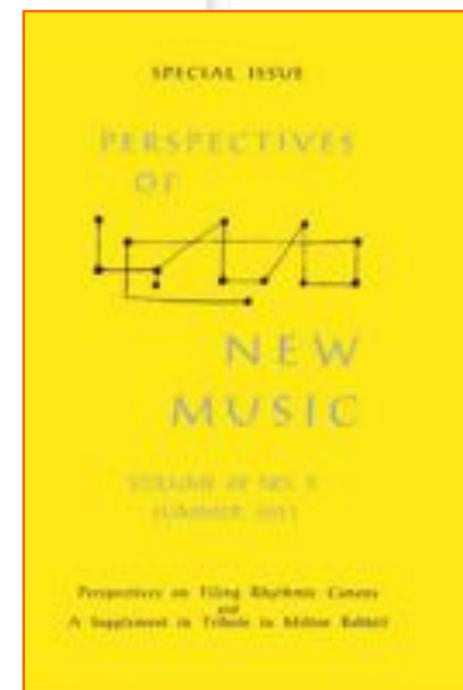
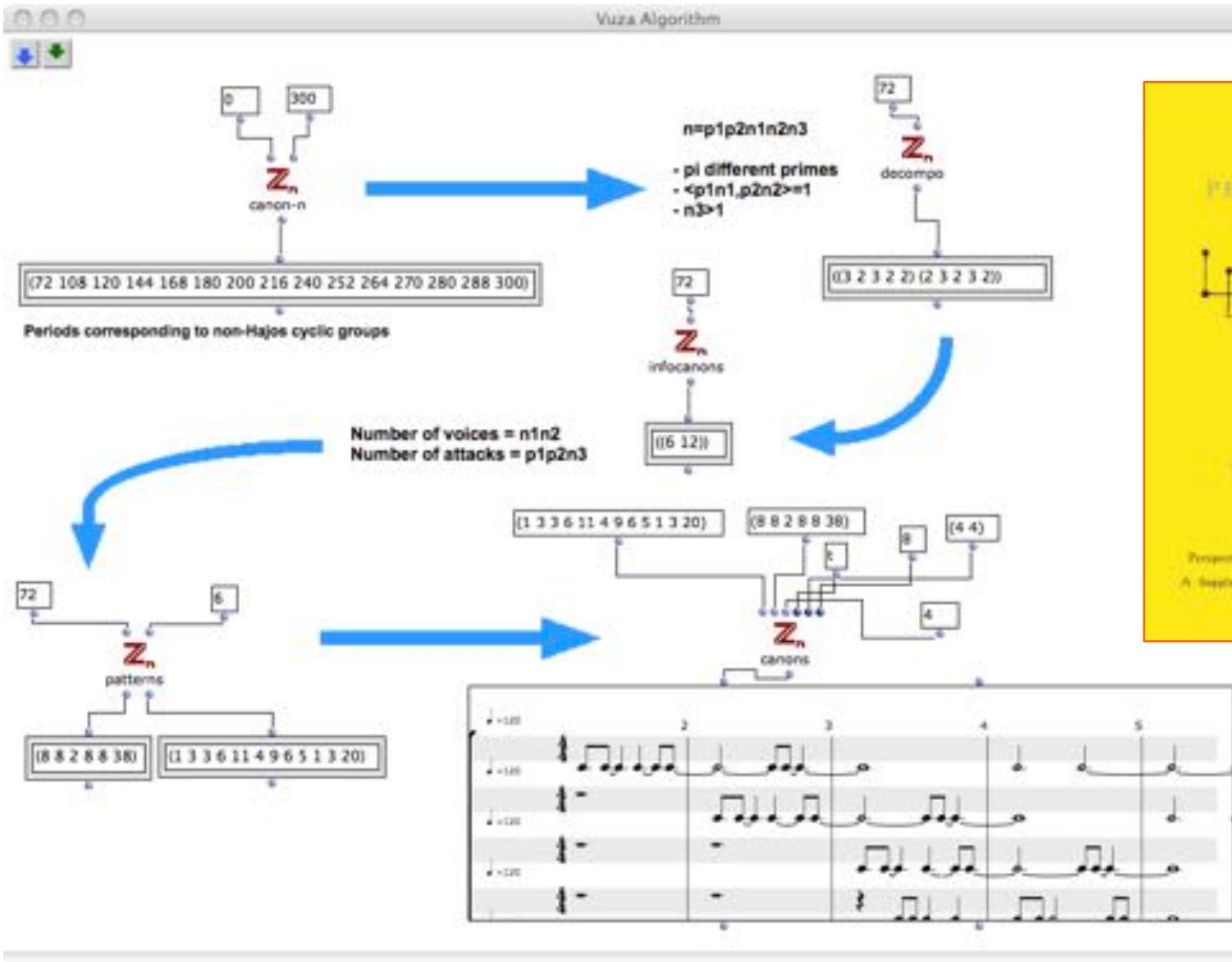
Dan Vuza



Anatol Vieru

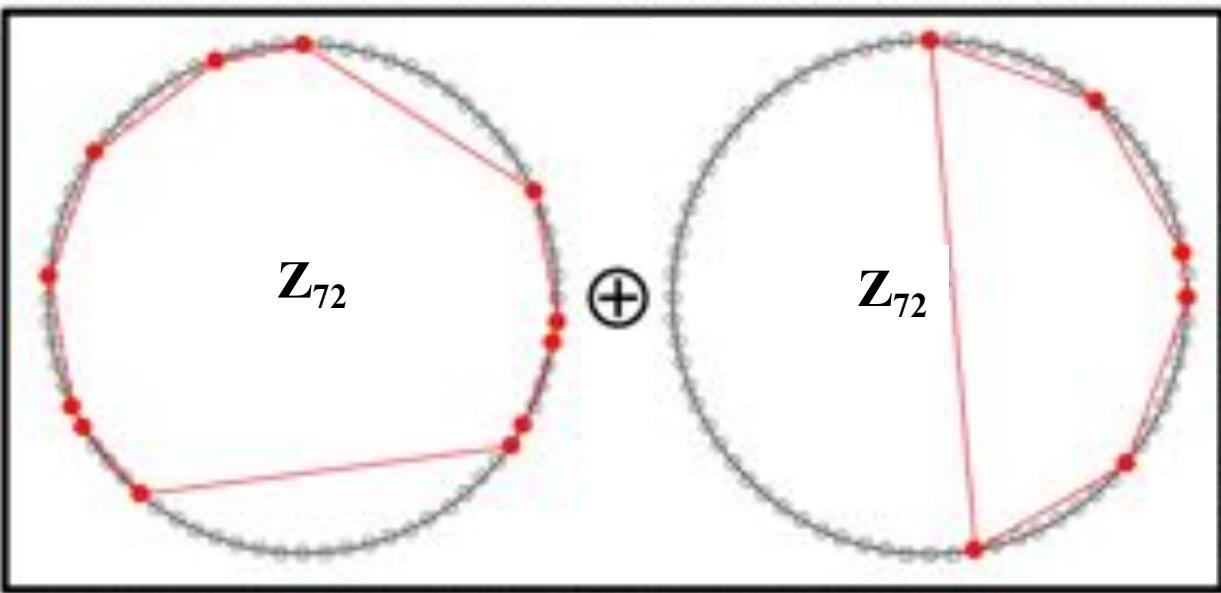
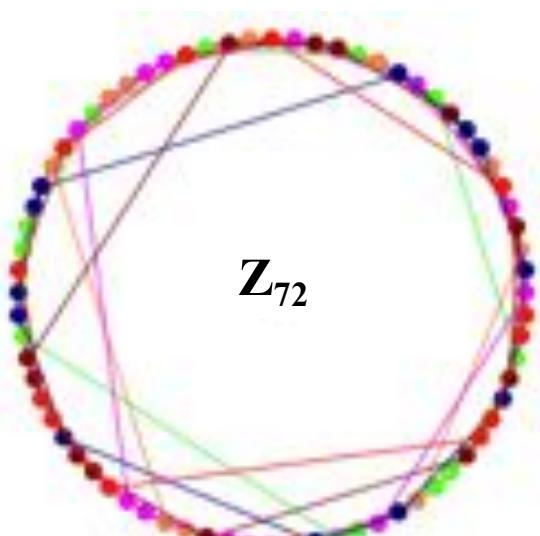


Vuza Canons in OpenMusic ‘MathTool’ environment



Vuza Algorithm

Microtonal Vuza Canons

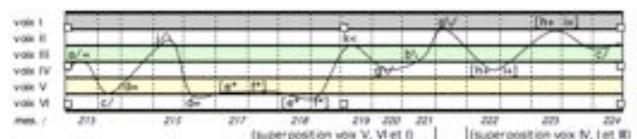
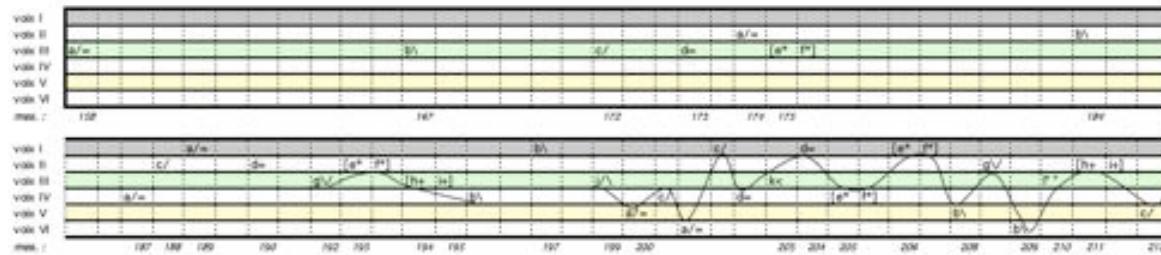


Dan Vuza



Anatol Vieru

Some compositional applications of the Vuza Canons model



a/ = montée vers accord puis "mise en pulsation"
bl. : "mise en pulsation" superposé à un gliss. descendant
cl. : montée vers accord (tête de a/ =)
de = "mise en pulsation" en diminuendo (fin de a/ =)
[e/P] : accord mis en "cross rhythm I" (durée double)
g/V : gliss. descendant puis ascendant
[v/e] : accord mis en "cross rhythm II" (durée double)
[V/v] : gliss. ascendant puis descendant avec accent
ke : son à l'envers
P : deux impacts brefs et piano

Coïncidences (1999)

Coincidences - Fabien Levy : déroulement du canon (mes. 158 à 226)
(chaque impact fait 3 temps)



F. Lévy



M. Lanza

La bataille de caresme et de charnage

(pour violoncelle et accompagnement, 2012)

A piece based on Monk (2007) ("Well You Need'n't")



G. Bloch

La notte poco prima della foresta

(opéra de chambre pour acteur, mezzo-soprano, baryton, ensemble et électronique, 2009)



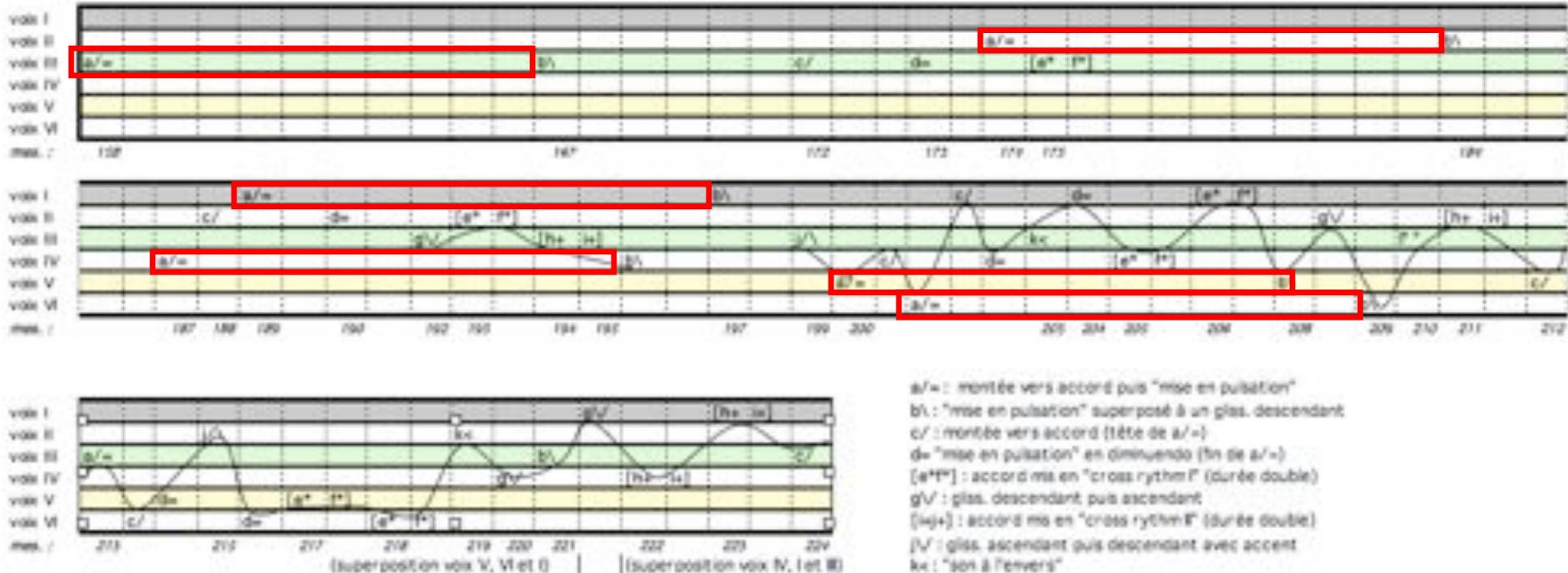
D. Ghisi

Fabien Lévy

Morphological Tiling Canons



- *Coïncidences* (pour 33 musiciens, 1999-2007)



a/+> : montée vers accord puis "mise en pulsation"
bl. : "mise en pulsation" superposé à un gliss. descendant
c/> : montée vers accord (tête de a/+>)
de : "mise en pulsation" en diminuendo (fin de a/+>)
[e**] : accord mis en "cross rhythm F" (durée double)
g/v : gliss. descendant puis ascendant
(h+ h*) : accord mis en "cross rhythm F" (durée double)
(h') : gliss. ascendant puis descendant avec accent
ka : "son à l'envers"
f" : deux impacts courts et piano

Coincidences - Fabien Levy : déroulement du canon (mes. 158 à 226)
(chaque impact fait 3 temps)



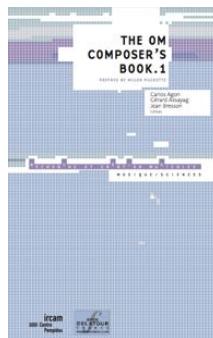
Tokyo Symphony Orchestra, Dir.: Kazuyoshi Akiyama, 05/09/2007, Suntory Hall, Tokyo, Japon

Georges Bloch

Several compositional strategies

- Metrical organization of a tiling canon
- Self-similarity processes
- Metrical modulations between canons

- *Projet Beyeler* (2001)
- *Projet Hitchcock*
- *Visite des tours de la cathédrale de Reims*
- *Noël des Chasseurs*
- *Canons à marcher*
- *Canon à eau*
- *Harawun* (2004)
- *L'Homme du champ* (2005)
- ***A piece based on Monk* (2007)**
- *Peking Duck Soup* (2008)



- *A piece based on Monk*, 2007 (« Well You Need'n't »)



G. Bloch, « Vuza Canons into the museum », *The OM Composers' Book*, 2006

Mauro Lanza

Vuza Canons and local periodicities



- *La descrizione del diluvio* (Ricordi, 2007-2008)



"6 voices are live and 8 are in the electronic part. The choice of the notes and the durations is made in order to stress some quasi-periodicities of the underlying Vuza canon and this gives to each voice a much more "redundant" character".

Mauro Lanza

Vuza Canons and local periodicities



- *La descrizione del diluvio* (Ricordi, 2007-2008)

“The choice of the notes and the durations is made in order to stress some quasi-periodicities of the underlying Vuza canon [...]”

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)

Mauro Lanza

Vuza Canons and local periodicities



No. 1 "Aria"

- *La descrizione del diluvio* (Ricordi, 2007-2008)

Electronic

Local Dynamics:

J-80

4

4

Savory

34

30

Tenn

卷之三

零四

General Dynamics: 1999 - 2000

poco a poco creciendo fino a mitad de (ppp-mp)



GMEM – Festival « Les Musiques » –Marseille, 25 avril 2008

Mauro Lanza



LA BATAILLE DE CARESME ET DE CHARNAGE

per violoncello e accompagnamento (2012)

(min. 10'15")



190 $\frac{3}{4}$

Canone sulla 2^a corda

$\frac{4}{4}$

1

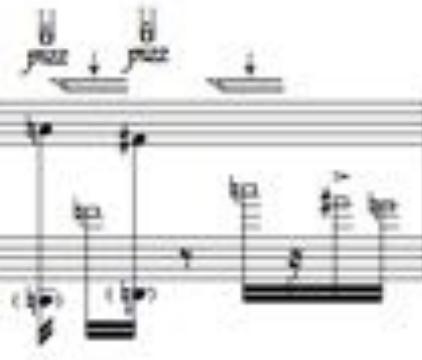


p



194

1



La notte poco prima della foresta (2009)

(opéra de chambre pour acteur, mezzo-soprano, baryton, ensemble et électronique)

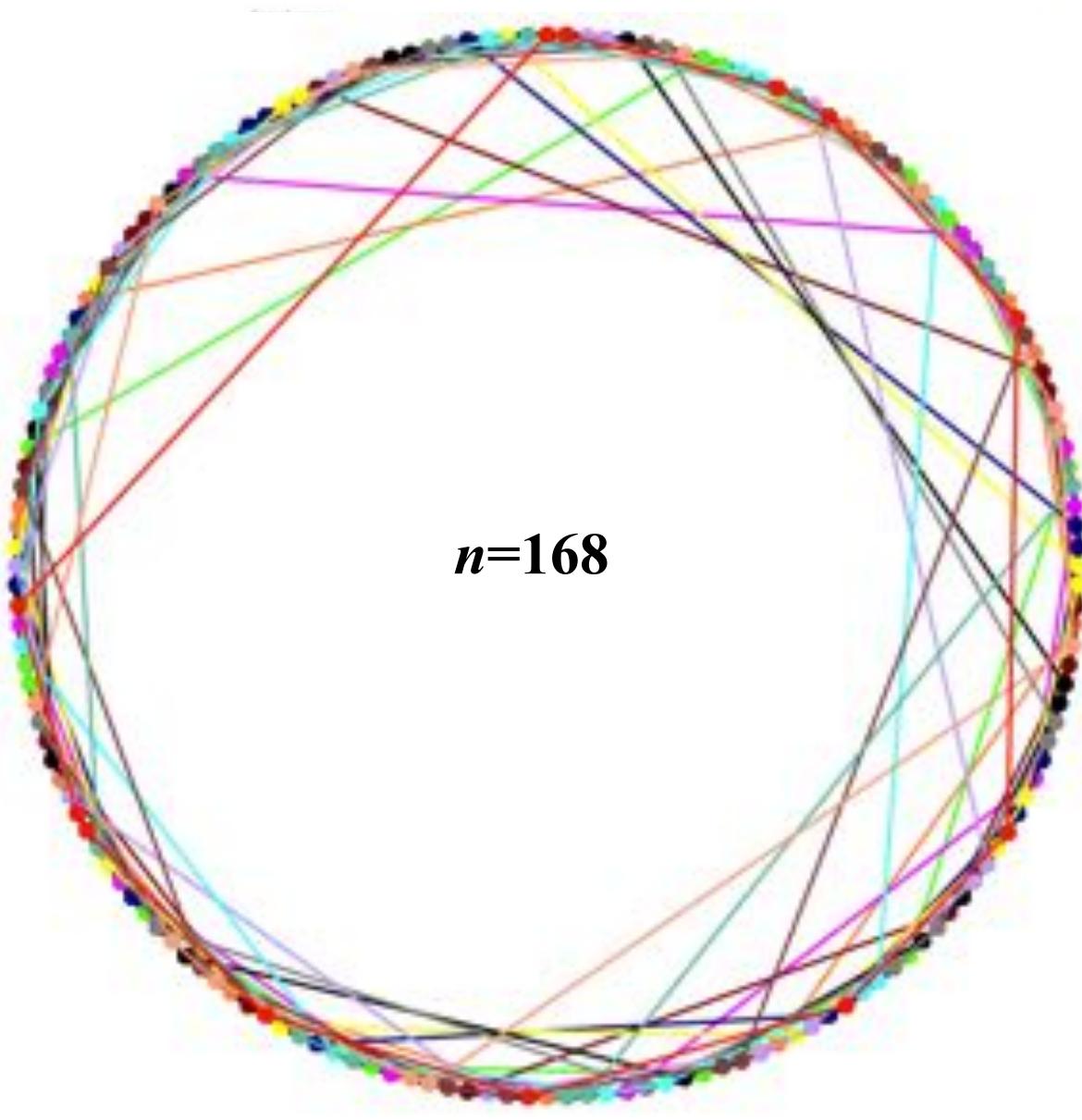


D. Ghisi

457 AC

Flute
Clarinet
Bassoon
Trombone
Alto
Bass
Bassoon

mama ti amo.
ma + ma



D. Ghisi

p
f
ff
p
mama ti amo.
pp
mama ti amo.
pp

La notte poco prima della foresta (2009)

(opéra de chambre pour acteur, mezzo-soprano, baryton, ensemble et électronique)



457

AC

D. Ghisi

Sprechgesang
sostenuto, dolce (senza sforzo)

Fl.

Cl.

Pt.

Pt.

Vcl.

An.

Mx.

Bc.

p ma - ma. /mai/ /mai/

p pp p /mai/ /mai/

p /mai/ /mai/

p /mai/ /mai/

p /mai/ /mai/

mama ti amo. mama ti amo. su tutti i muri, di modo che non potesse non vederlo. mama ti amo.

p ma - ma. /mai/ /mai/

p /mai/ /mai/

p /mai/ /mai/

Vuza Canons and electronic music



« Cinq canons de Vuza » de Sébastien Roux (Diffusion intégrale et portrait)

le dimanche 10 avril 2016

06:57



Sébastien Roux
(ATIAM 2001-2002)



Recent pieces using Vuza Canons

Als Gregor und Griselda

Kanon für 6 Stimmen (auch Amateure) (deutsche Fassung)

Fabien Lévy

*Auftrag des Römlingen Festivals
à Melba Fenouil*

Dieser Kanon folgt das mathematische Prinzip des "Vusa-Kanons". Ich bedanke Moreno Andreatta, von Icam, für seine Hilfe, um diesen Vusa-Kanon zu rechnen, und Melba Fenouil für die deutsche Fassung des Textes.

Phonetisches Alphabet

Konsonanten

- b** : Bad
s : Sellerie
k : Käse
d : Datum
f : Fön
g : Sagen
j : jolie
l : Lärm
m : Mutti
n : Nein
p : agness
P : Punkt
r : Relevant
t : Tanken
v : Wagon, Witwe
x : Süsse, sagen
K : Kirsche

Volkstüm

- a : Papa
a : Kartoffeln
a : wieder
a : Fünf
a : Fleur
e : Merit
z : fern
o : Oktopus
a : Ort
i : wieder
u : du, Suppe
y : lägen

Nasale Vokallen

- é** : rang, avant
è : rein, brin, pain
ɔ : bon, ten
œ : brun, un

Halbe-Konsonanten (oder halbe-Vokale)

- J : jodeln
 w : fouet (/fwε/), veir (/vwax/)
 q : Suite (/syjət/)

Un poco più animato (ca. $\text{♩} = 180$)

S. 1 Schwap nein sich arm
S. 2 ze [se] lang
S. 3 an hin ter [ter]
S. 4 pen schwapp ein aalt
S. 5 der o [o] naß glat in
S. 6 Schwipp ta-ucht Aal der [der]

S. 1 dunk-len (mascot)
S. 2 fe gurr Täub-chen
S. 3 [f] sie cu-cu-rru! wie 'ne
S. 4 ein maunzt sie (mascot) Kat-ze
S. 5 Irc Miao aus-ser
S. 6 wie rru! Rand



From permuatational songs to the Tonnetz

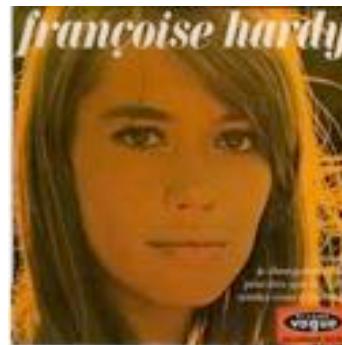
Se telefonando, 1966 (Maurizio Costanzo/Ennio Morricone) / Mina



(min. 0'53")



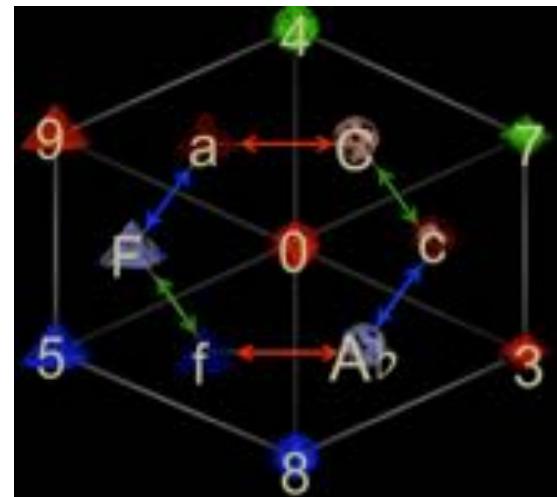
Ennio Morricone



Je changerais d'avis, 1966
(Françoise Hardy)

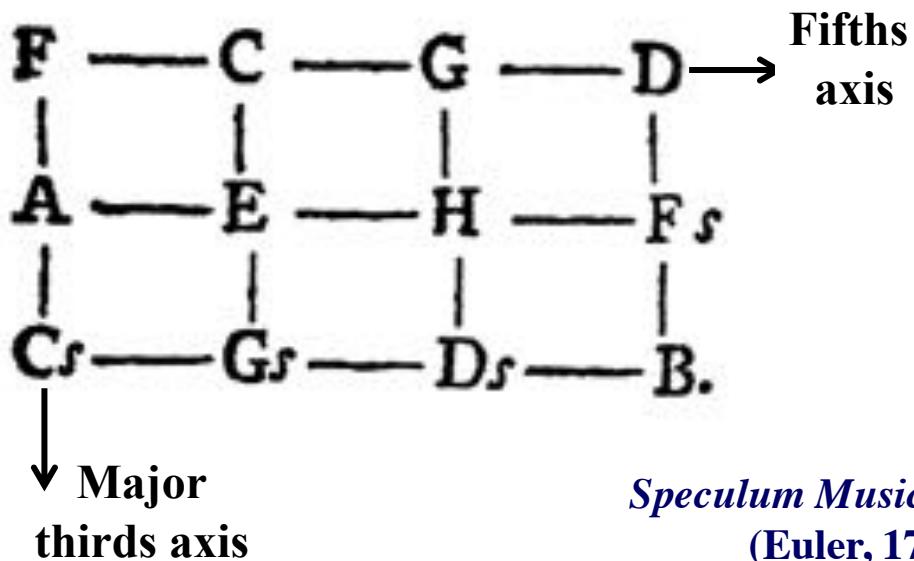
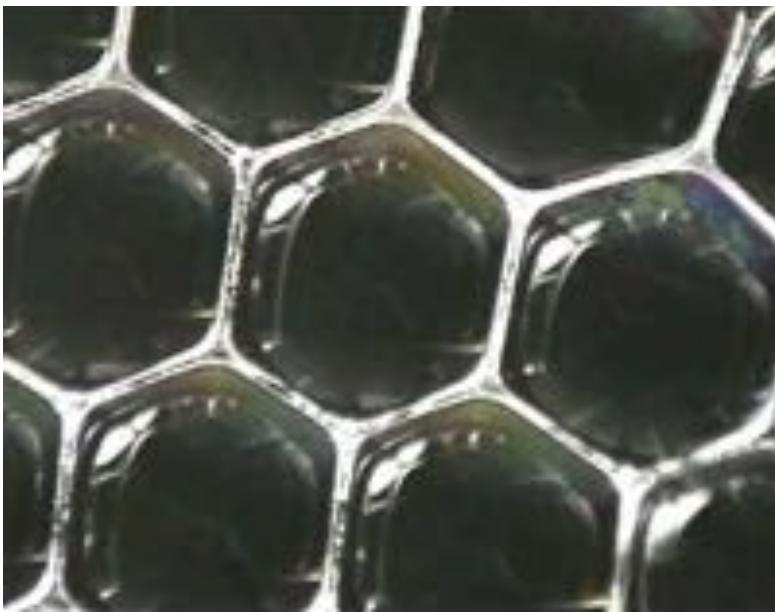


The harmonic space



C	c	C_#	c _#	D	d
E _b	e_b	E	e	F	f
F _#	f _#	G	g	G _#	g _#
A	a	B_b	b_b	B	b

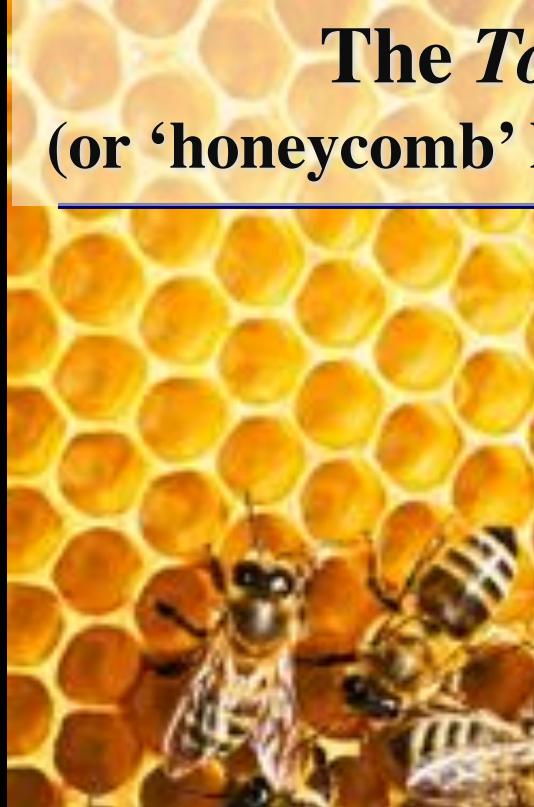
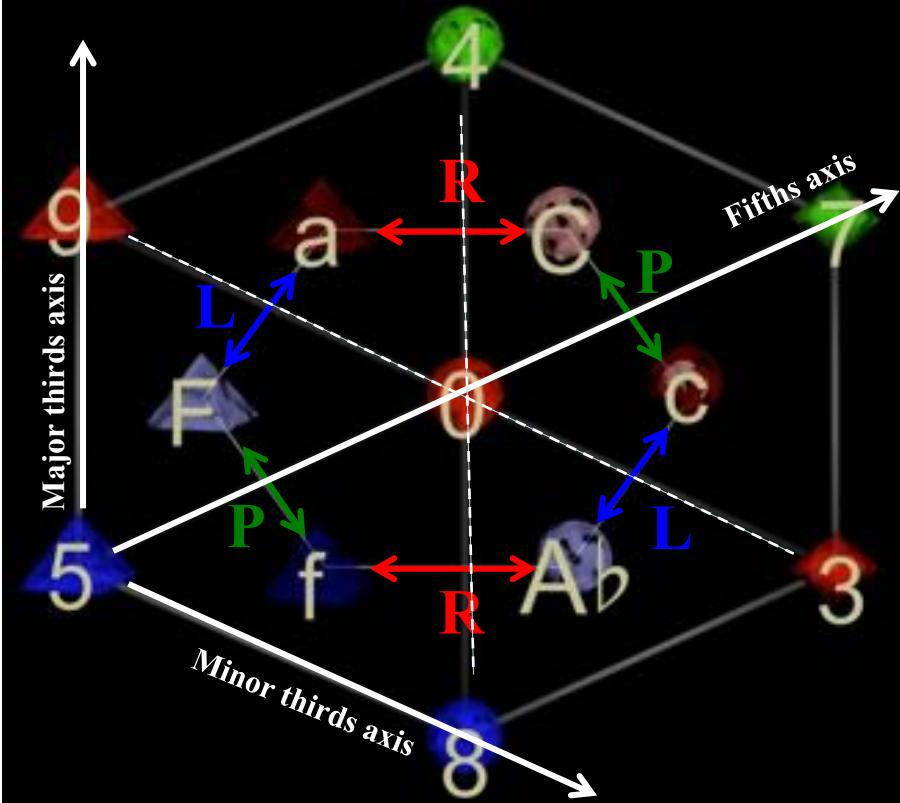
Chord enumeration



The Tonnetz (or ‘honeycomb’ hexagonal tiling)

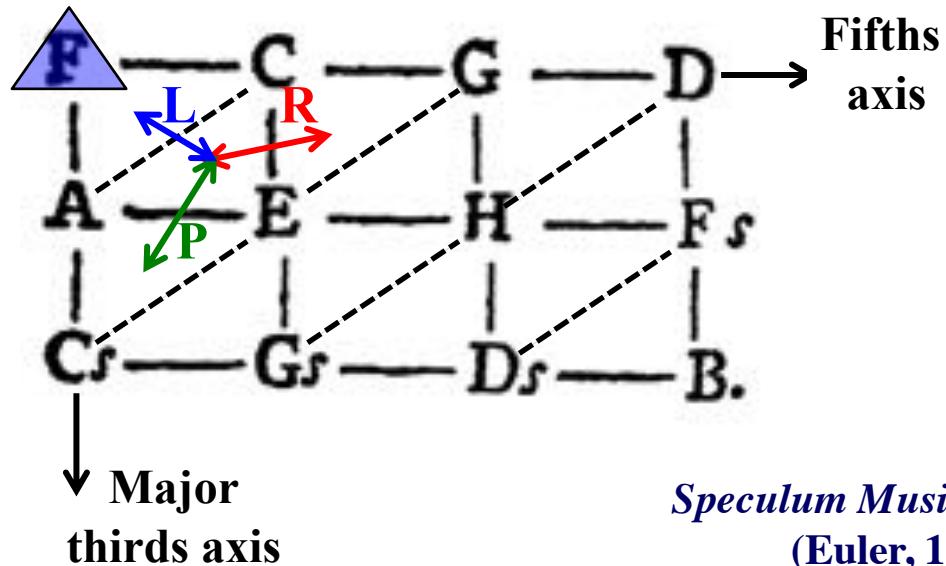
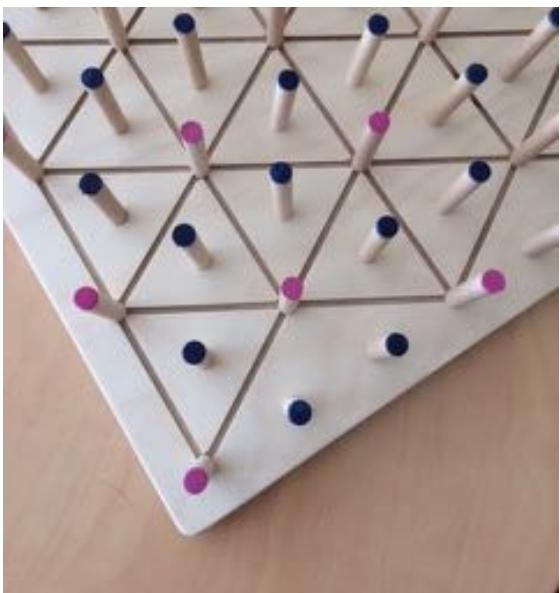


Speculum Musicum
(Euler, 1773)



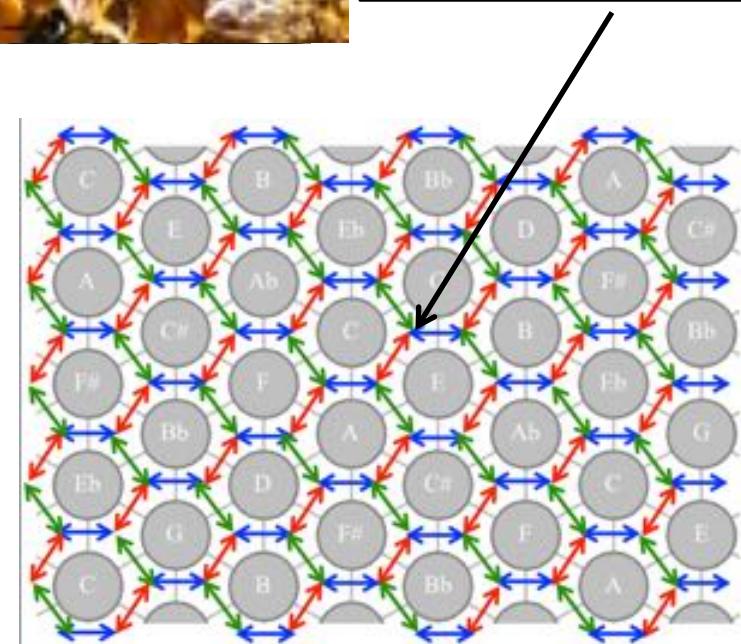
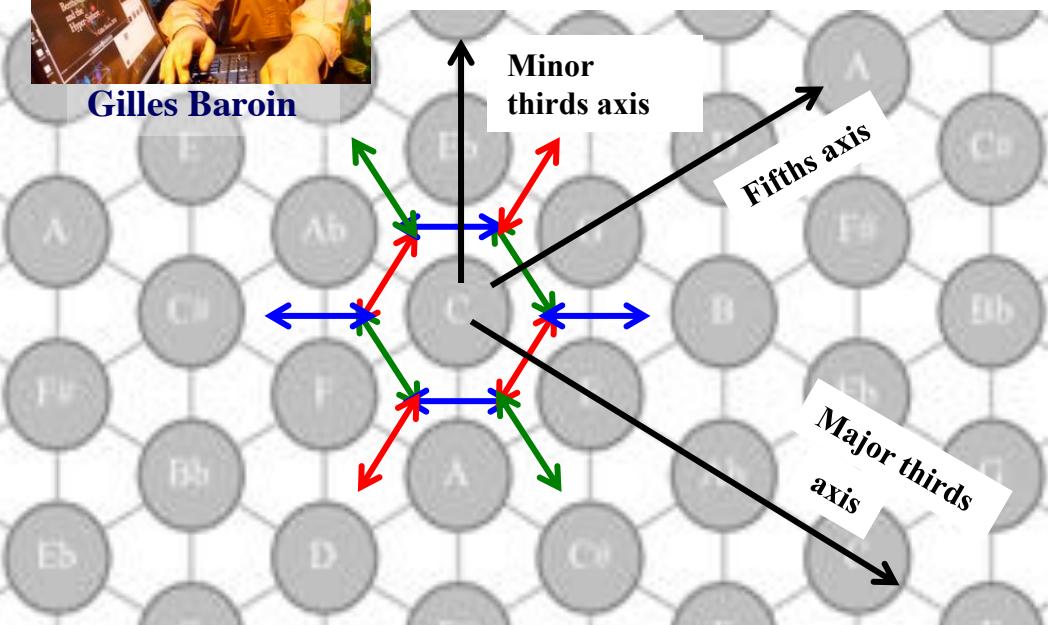
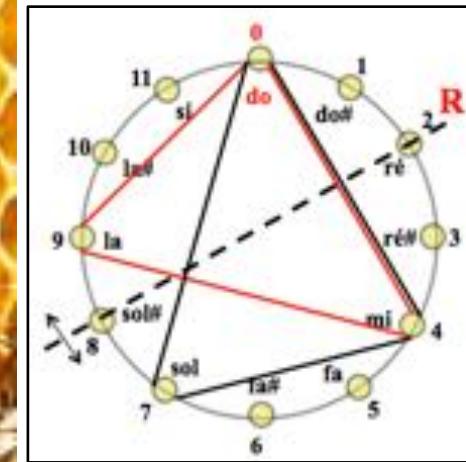
The Tonnetz

(or ‘honeycomb’ hexagonal tiling)

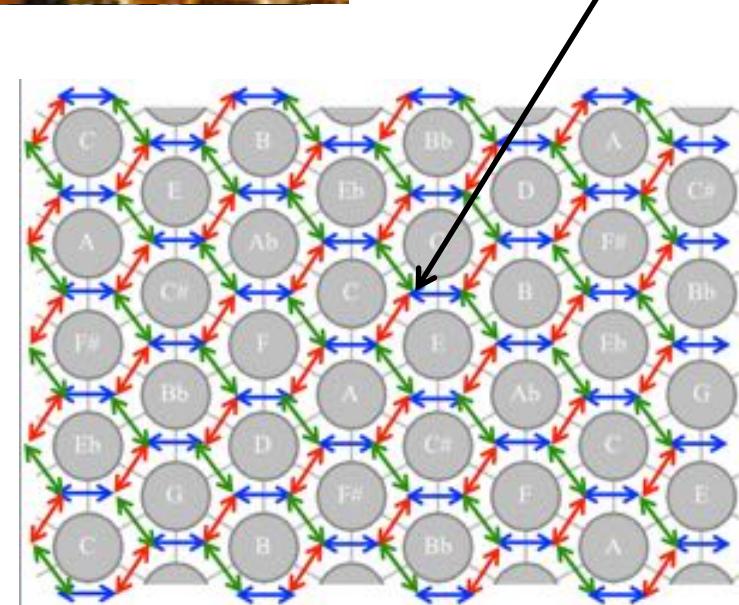
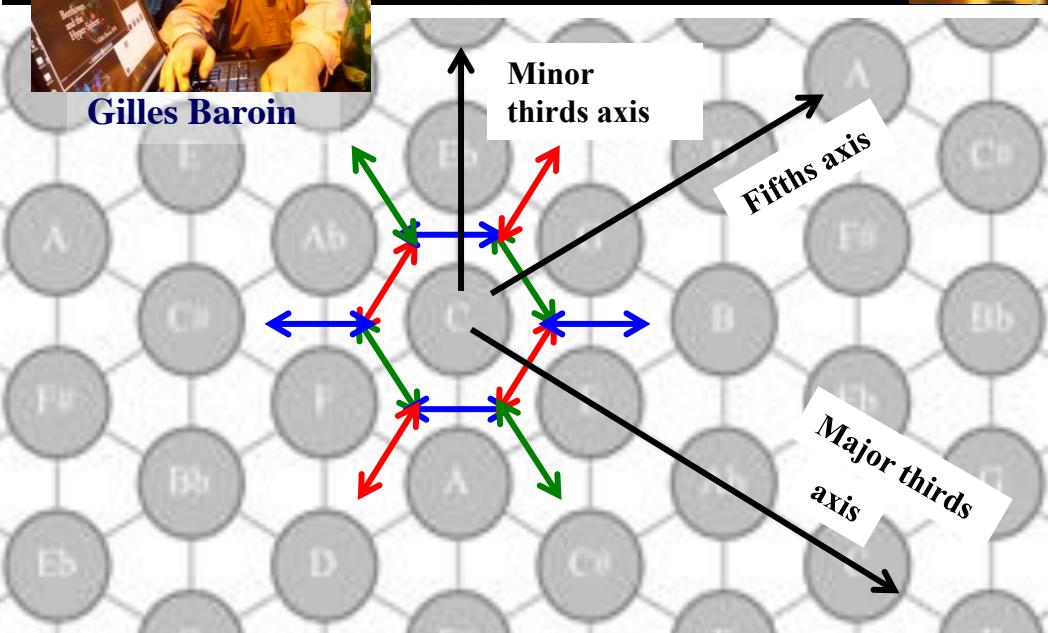
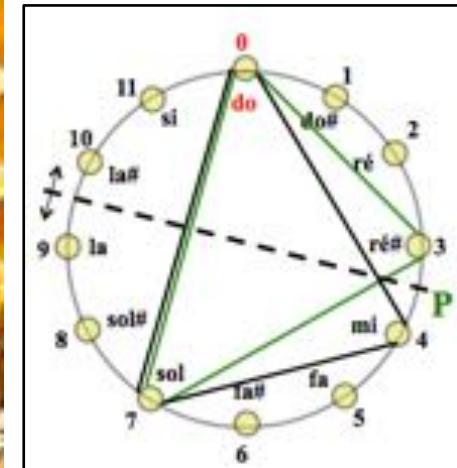


Speculum Musicum
(Euler, 1773)

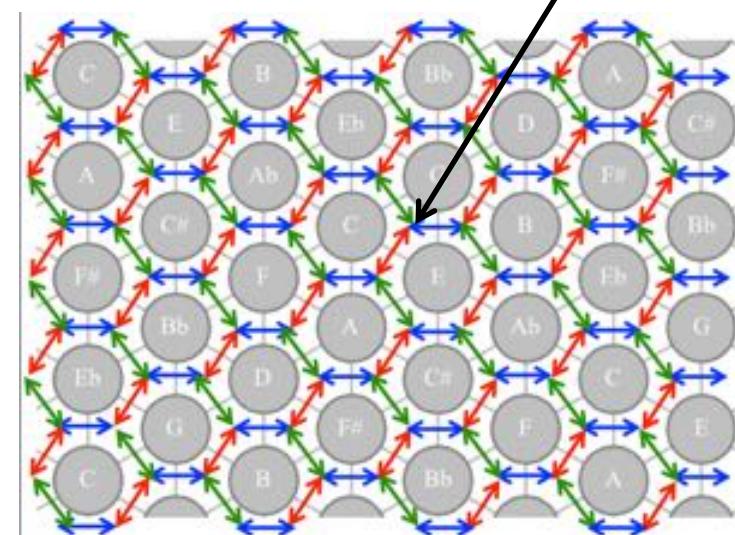
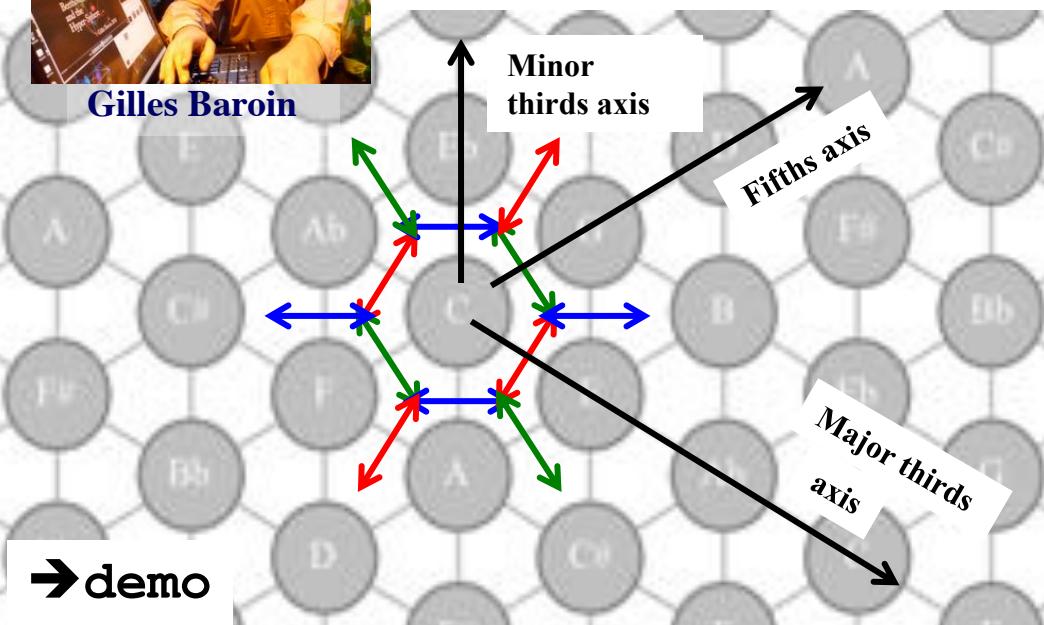
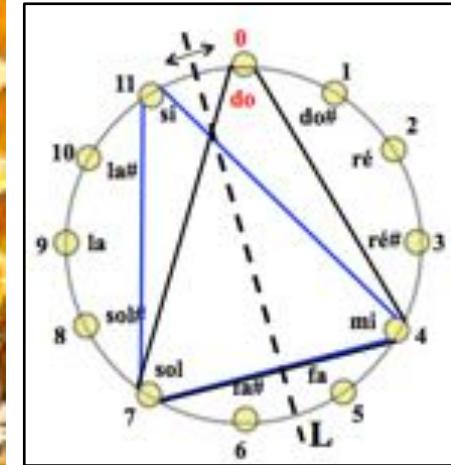
The Tonnetz (or hexagonal tiling honeycomb)



The Tonnetz (or hexagonal tiling honeycomb)



The Tonnetz (or hexagonal tiling honeycomb)





Two Dimensions

Traditional Chordal Space

© Gilles Baroin 2011



Gilles Baroin

Harmonic Progressions

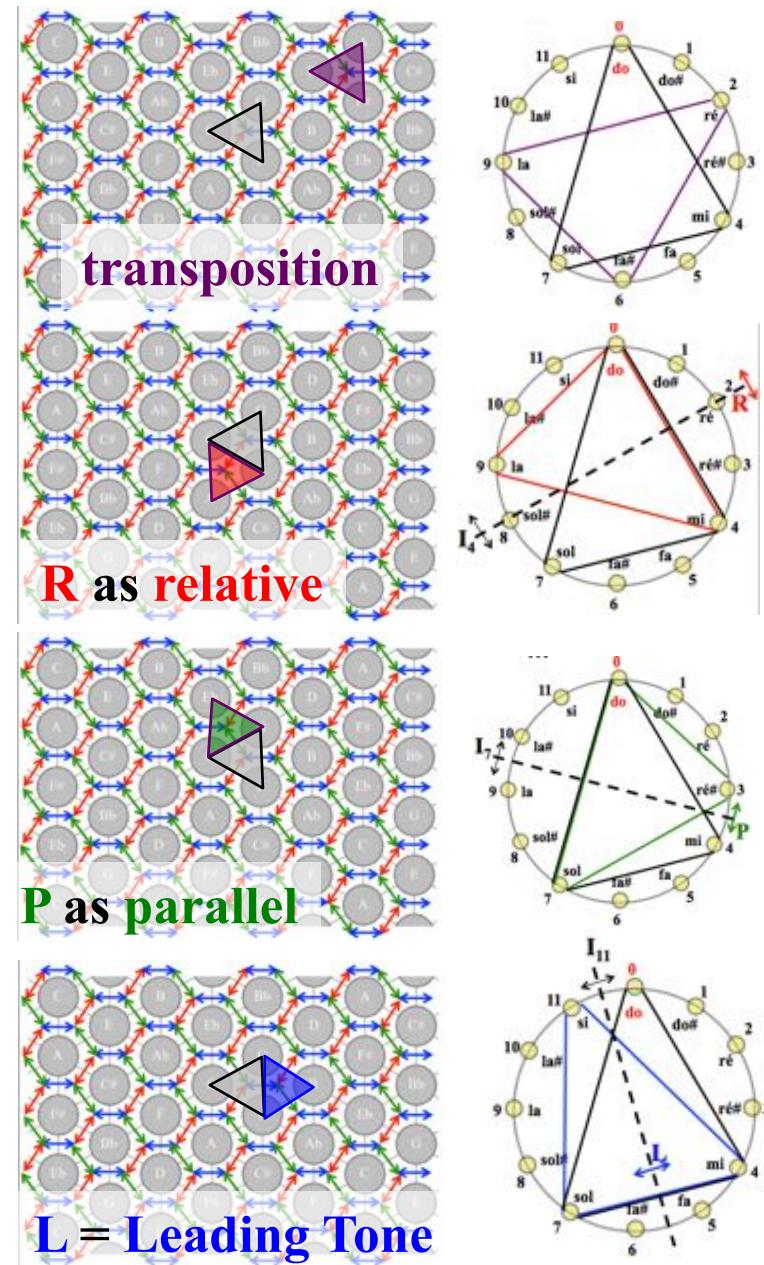
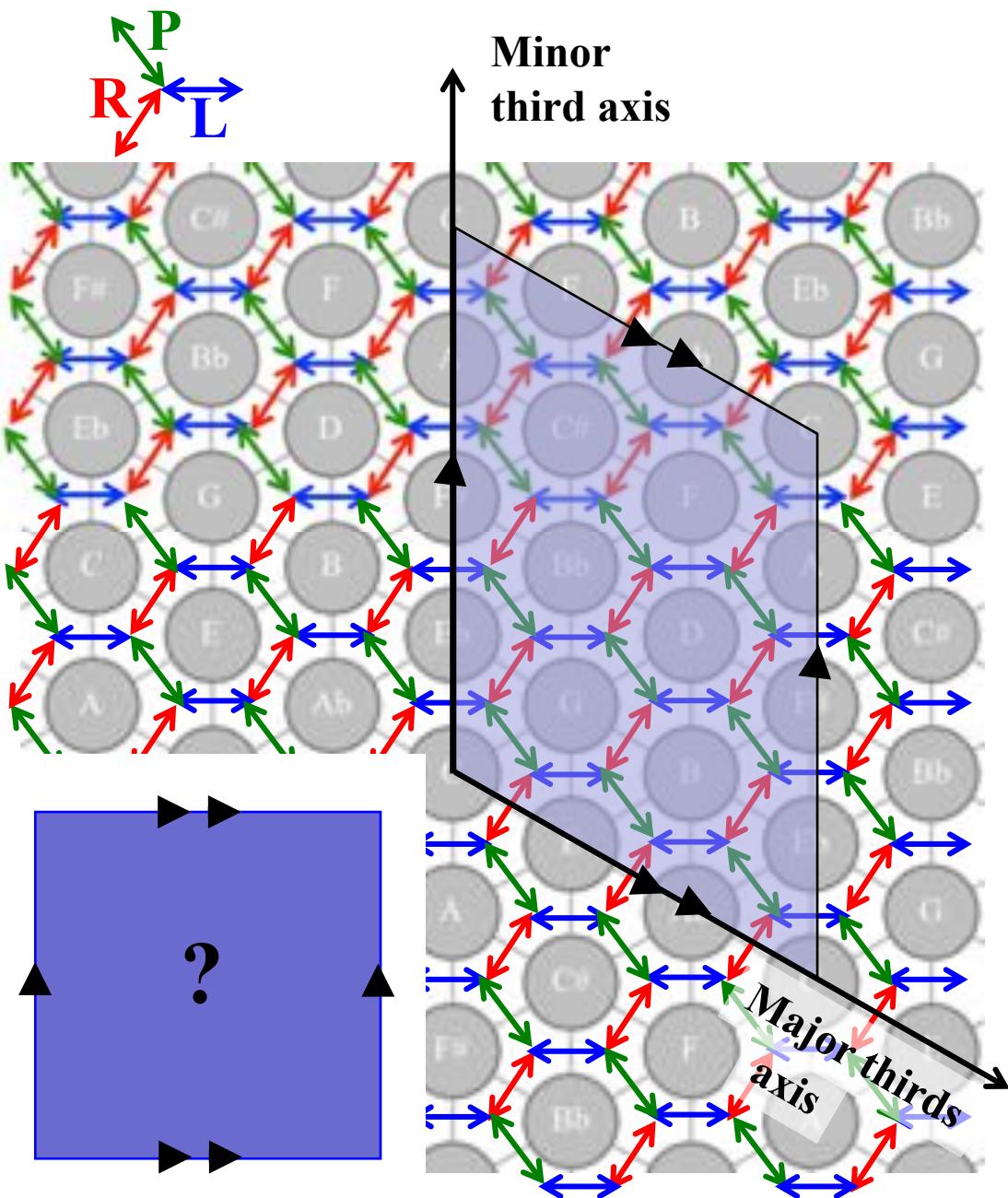
In Paolo Conte

Sotto le Stelle del Jazz

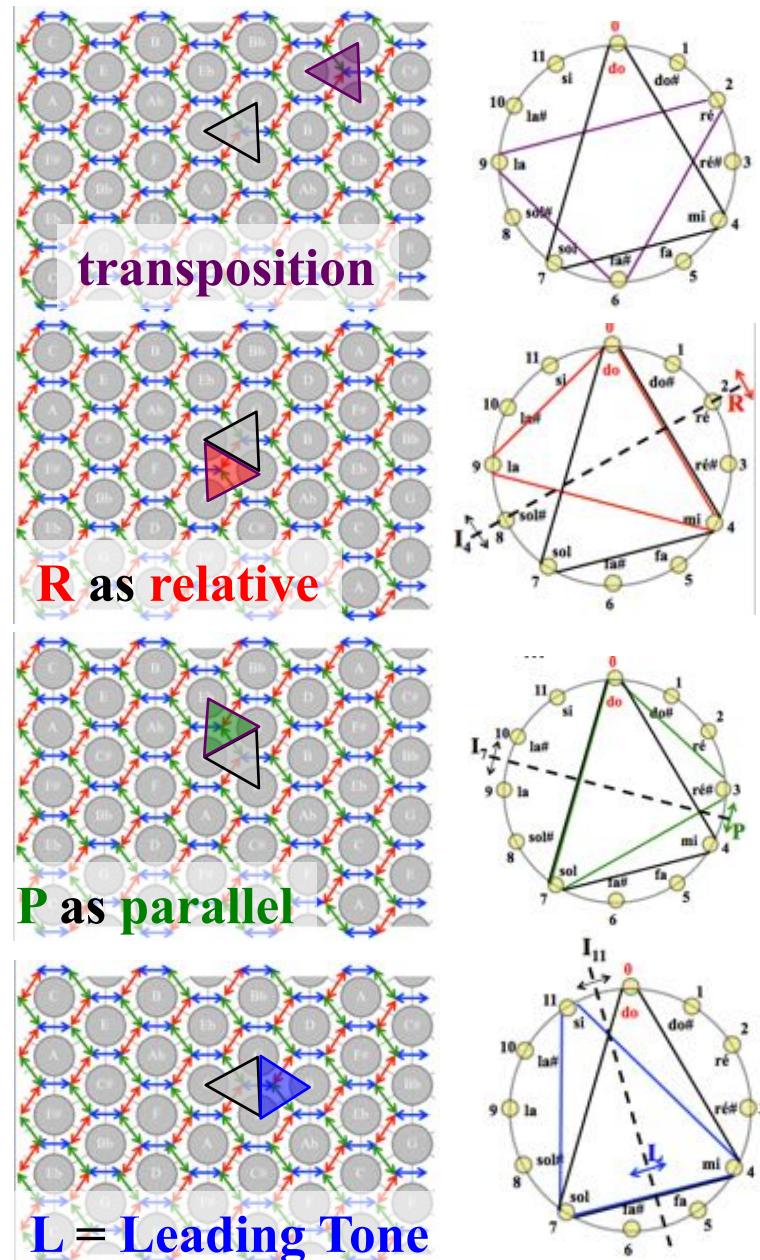
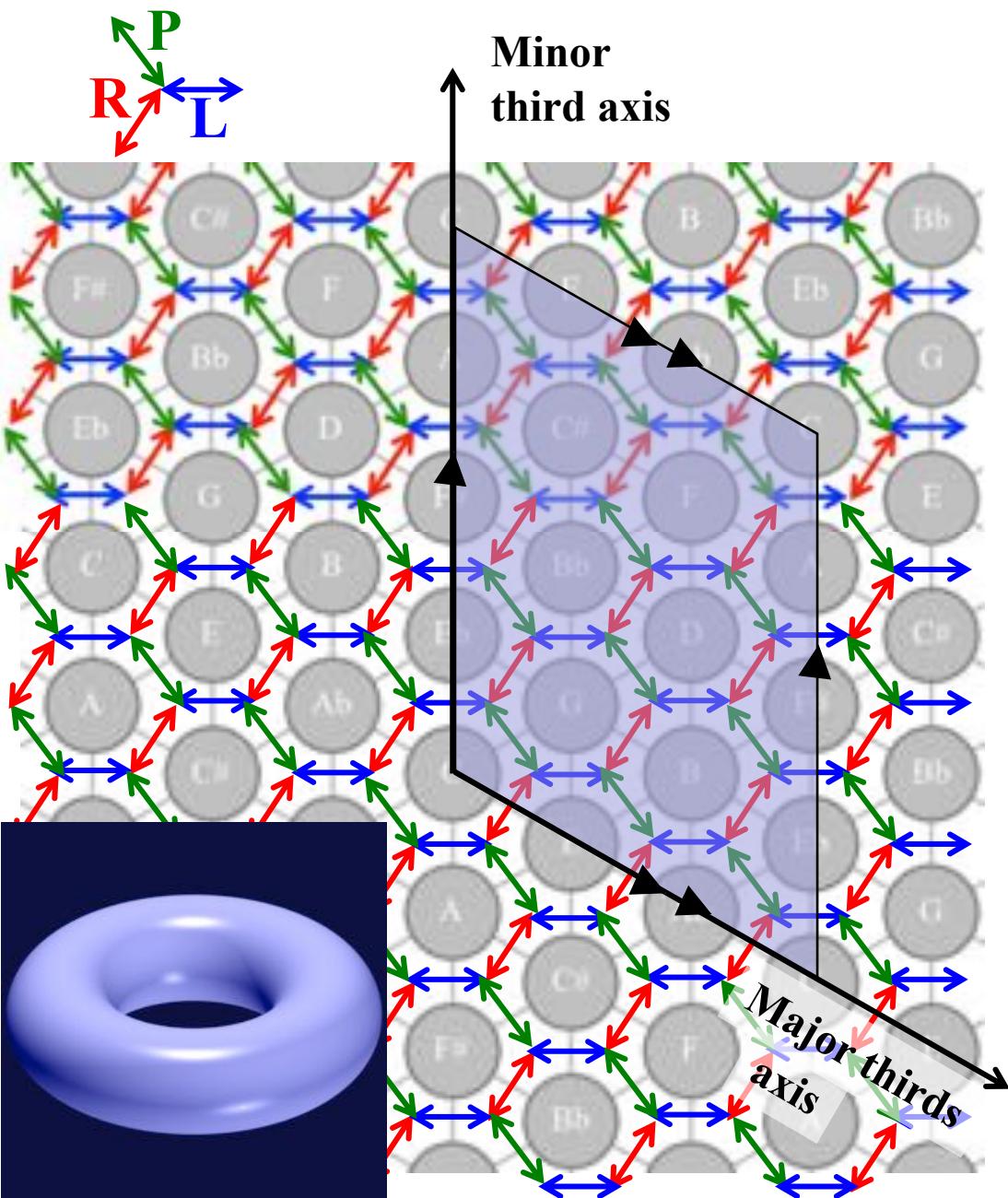


*Supervision Moreno Andreatta
Modelisation Gilles Baroin 2016*

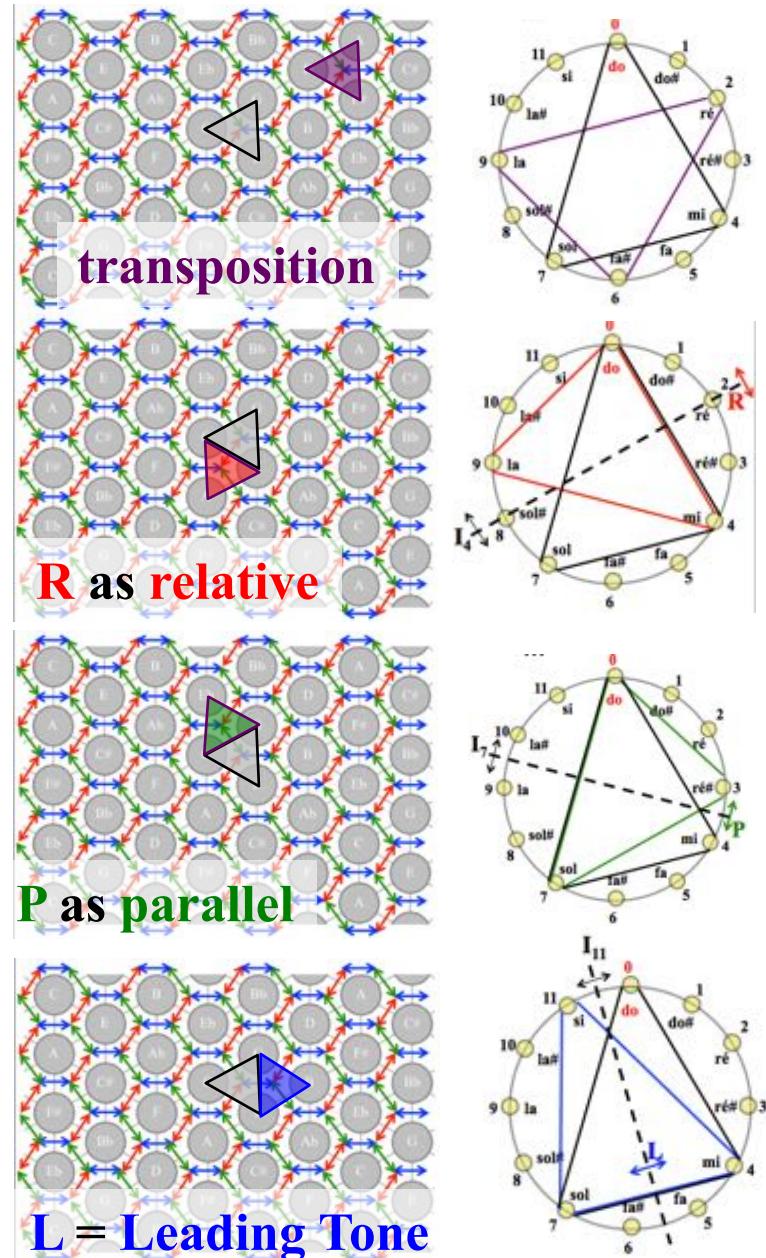
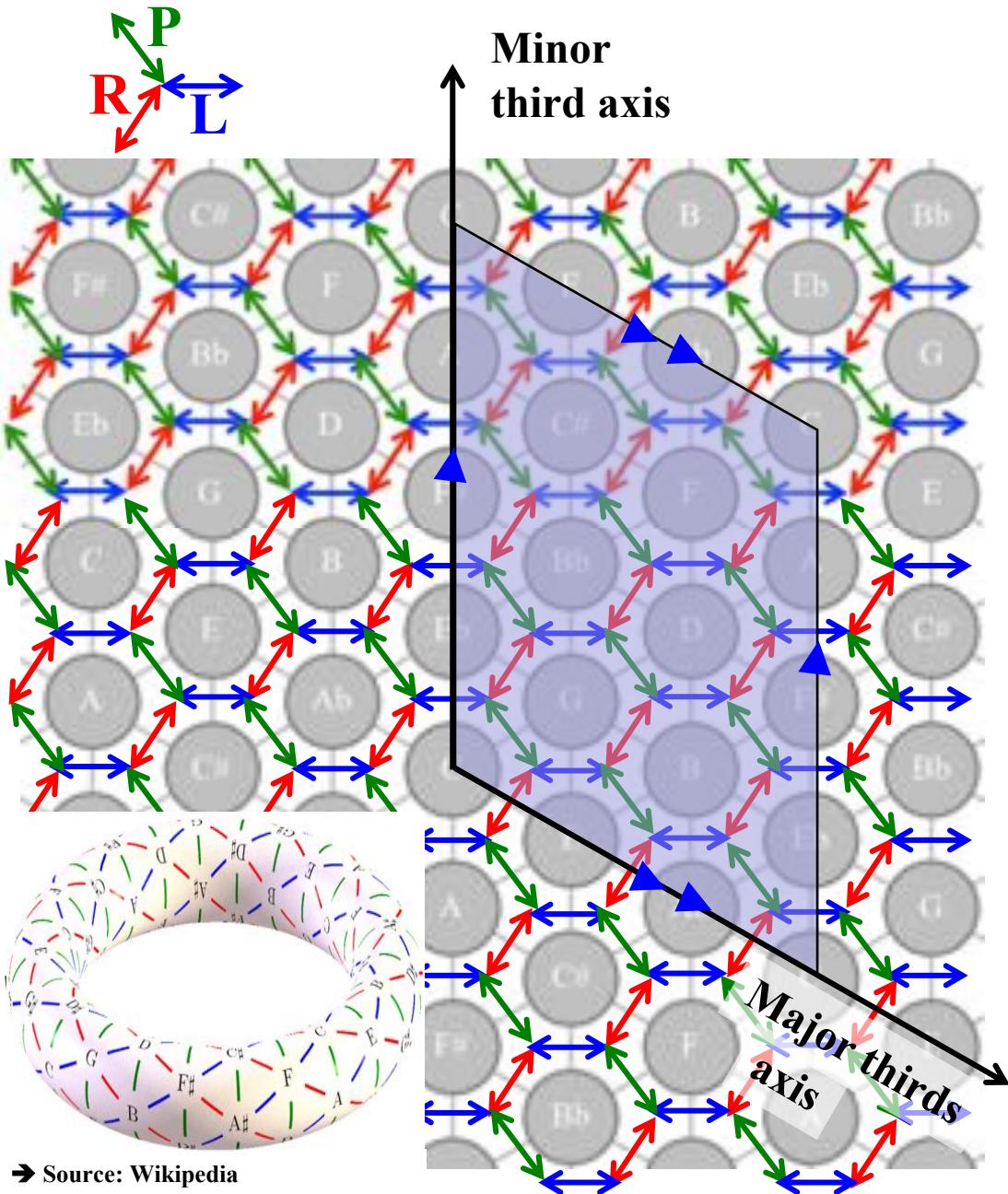
The Tonnetz, its symmetries and its topological structure



The Tonnetz, its symmetries and its topological structure



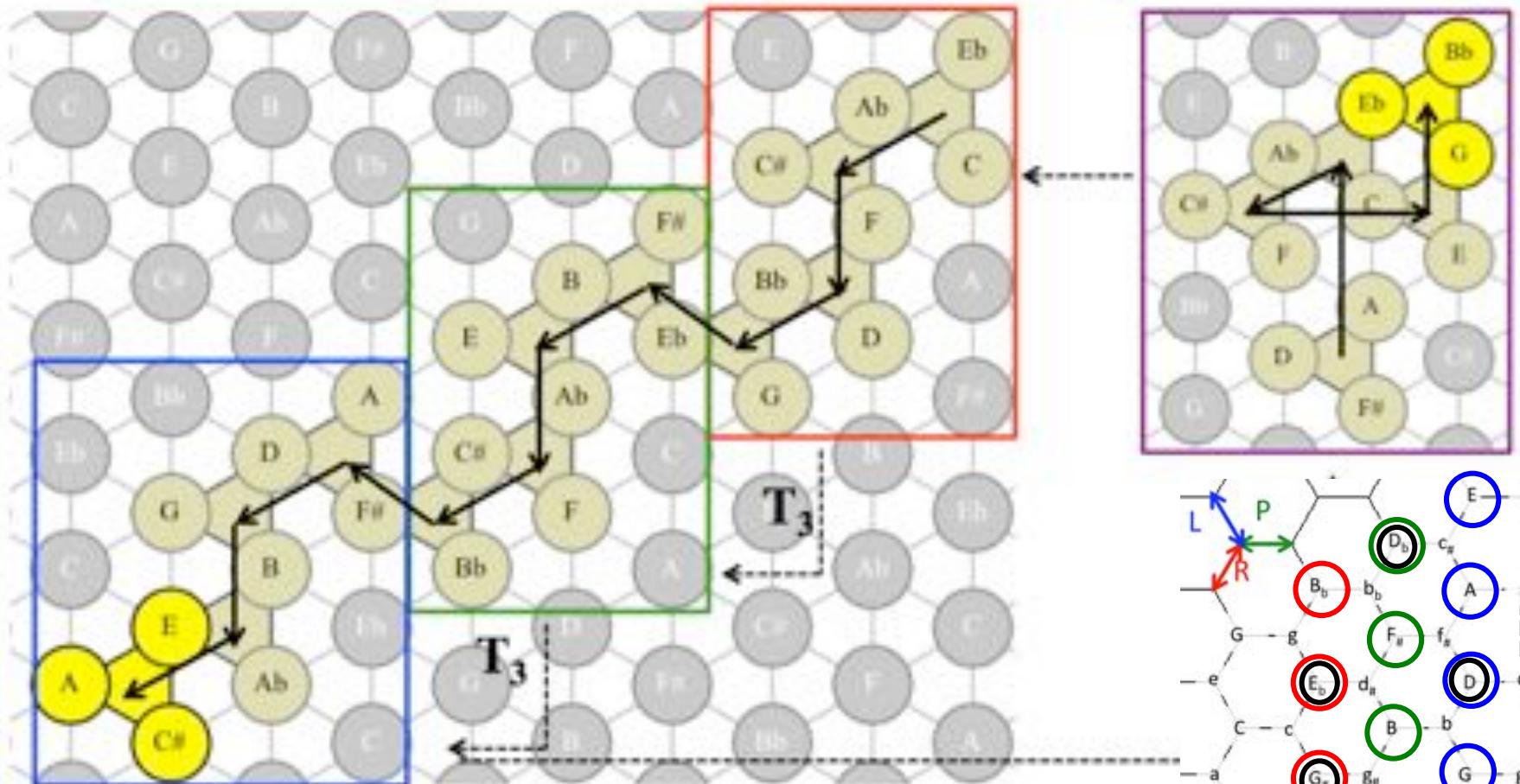
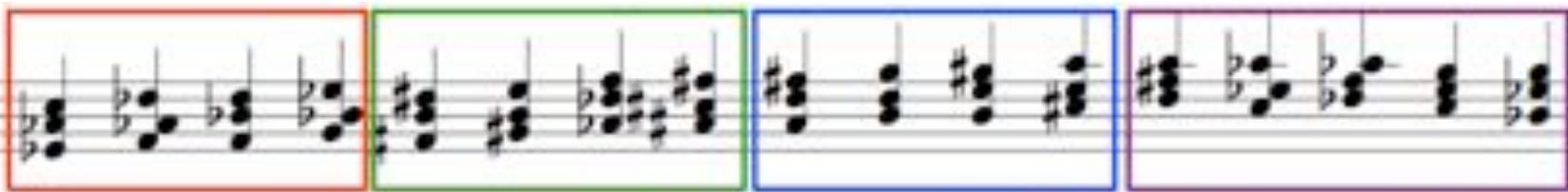
The Tonnetz, its symmetries and its topological structure



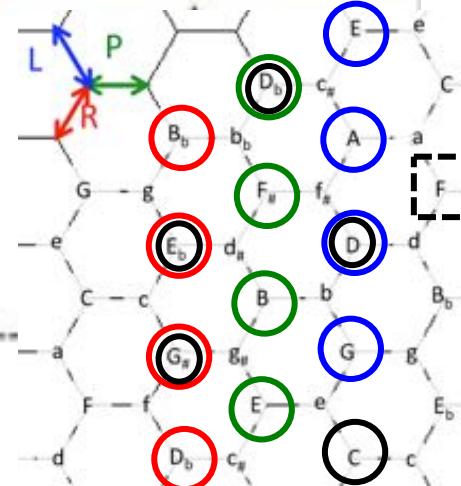


Symmetries in Paolo Conte's *Madeleine*

La_b Re_b Si_b Mi_b Si Mi Re_b Fa_# Re Sol Mi La Re La_b Re_b Do Mi_b



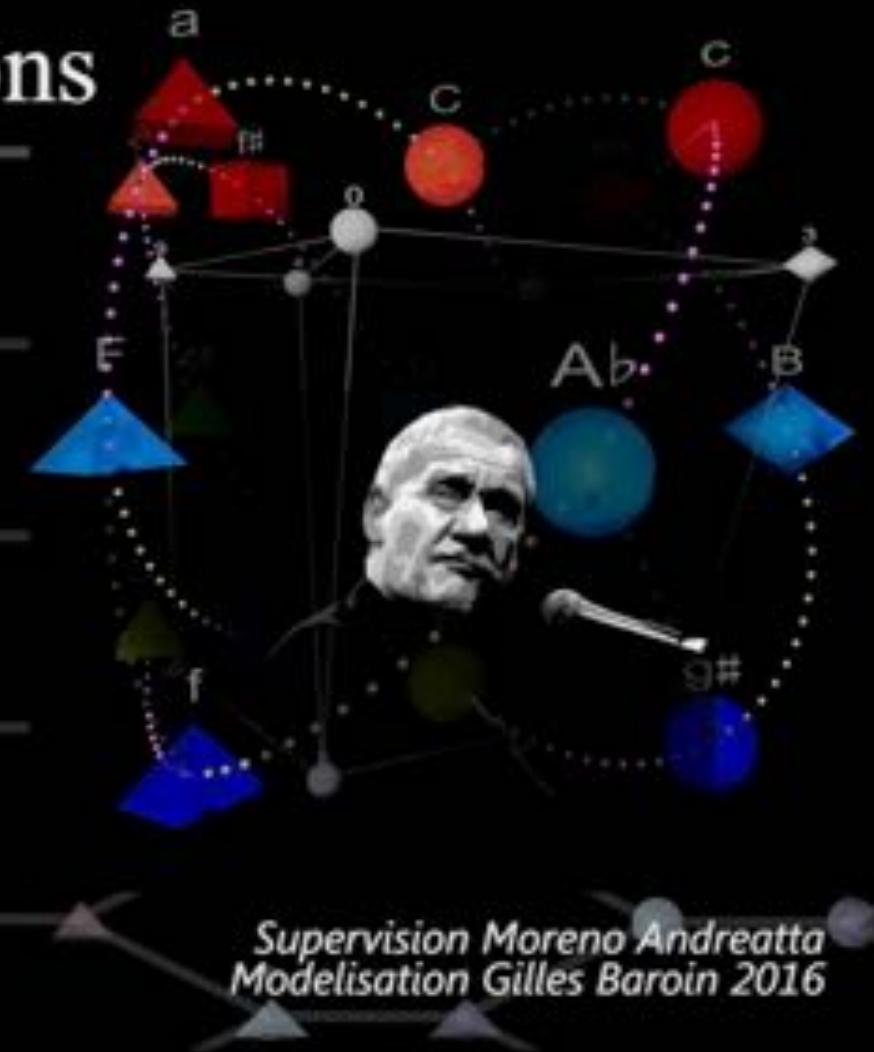
Almost total covering of the major-chords space



Harmonic Progressions

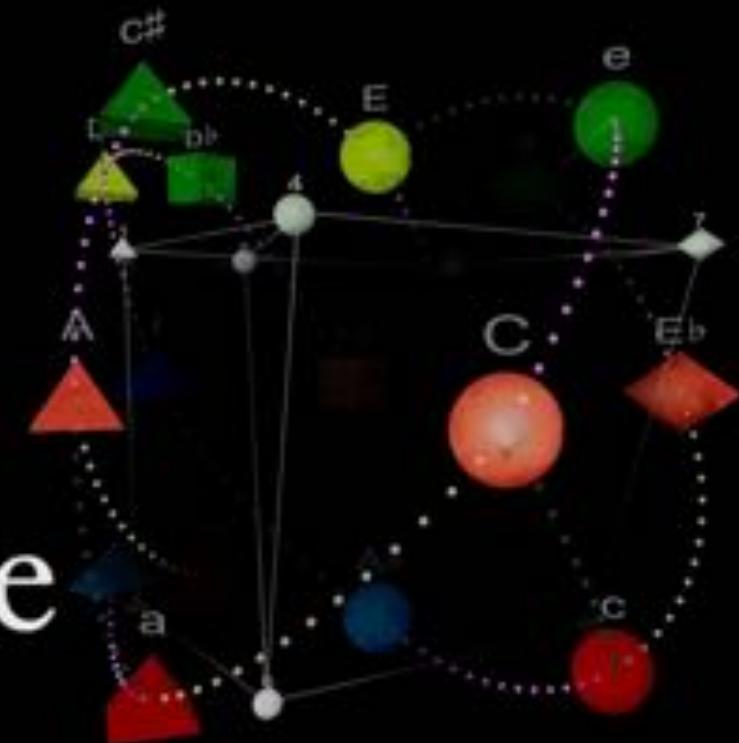
In Paolo Conte

Madeleine



Supervision Moreno Andreatta
Modélisation Gilles Baroin 2016

Beethoven and the Hypersphere *(and the Tonnetz)*



Gilles Baroin 2016
www.MatheMusic.net

Reading Beethoven backwards

Le Blé en Herbe



(Polo/Moreno/Dieu)

Plonger comme un enfant, cheveux au vent

Croiser matin dans l'herbe folle

Sous l'océan du blé en herbe

Deux tourterelles qui s'envolent

Marée d'épis couleur d'amande

Suivre les jeux des hirondelles

Qui tendent à caresser le ciel

Sur le paysage éternel

Algues tendres de mille plages

Nager comme un enfant, cheveux au vent

Frôlant le ventre des nuages

Sous l'océan

Cheveux de pluie, dos de poissons

Du blé en herbe

Qui frissonnent à l'unisson

Marée de fruits au goût amer

Suivre le bord des continents

Acide et salée comme la mer

Dans l'océan du blé en herbe

Vers l'îlot d'un petit village

Pêcher le corail du pavot

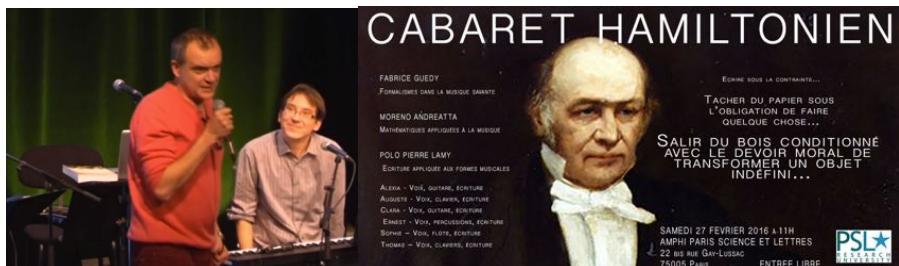
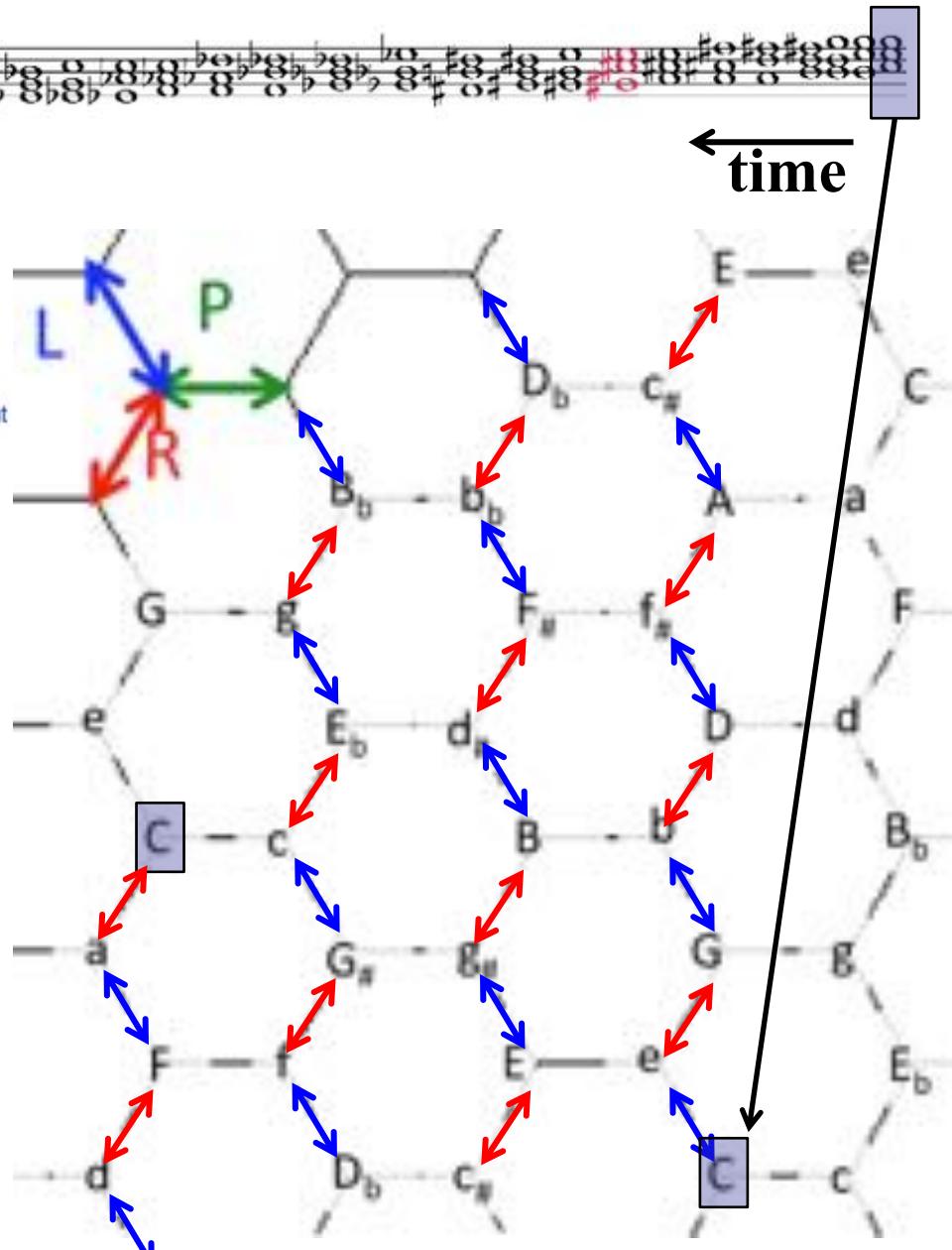
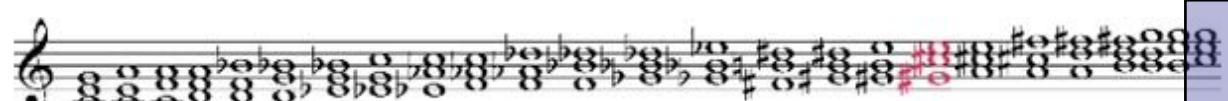
Vers un château d'eau sur la plage

Dans le sang des coquelicots

Quand tout s'éteint avant l'orage

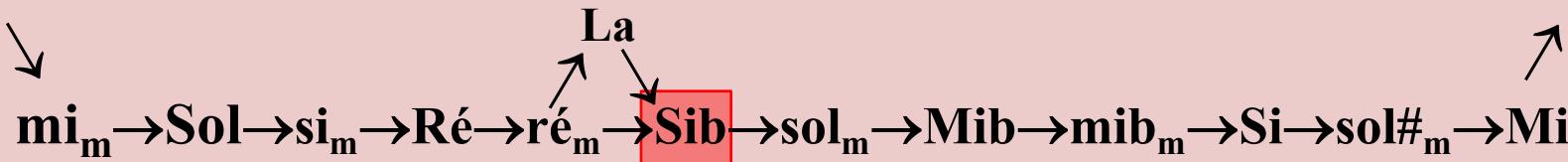
Quand se lève le vent du large

Sur le blé vert



Aprile, a Hamiltonian « decadent » song

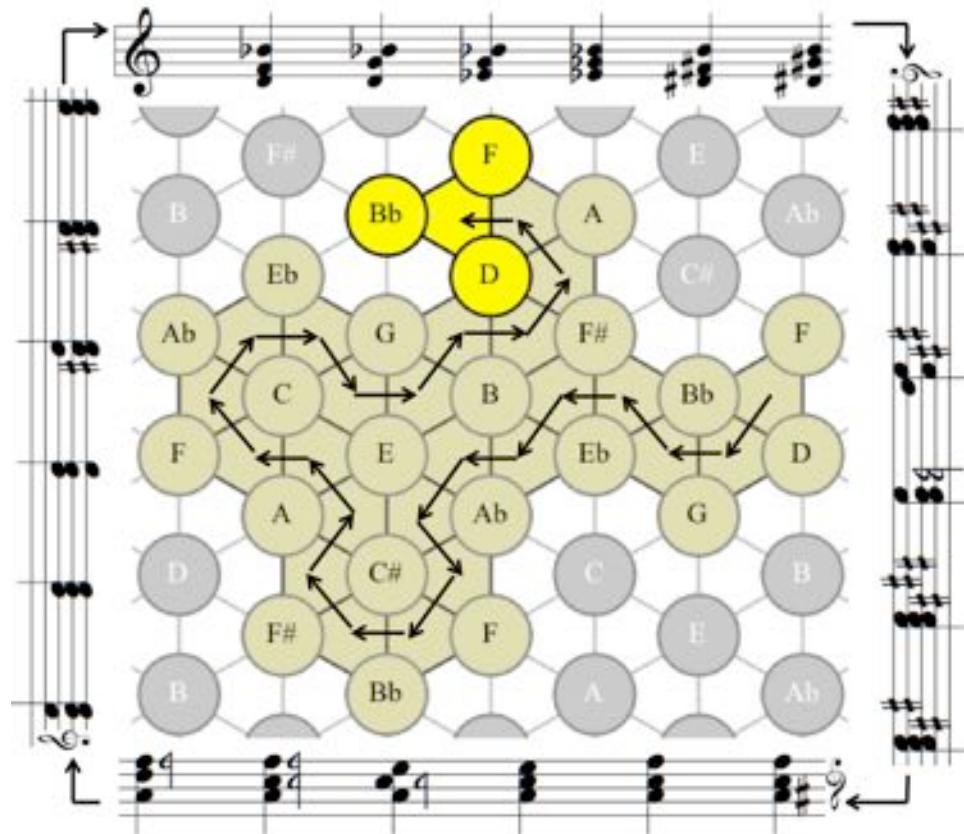
Do←**do**_m←**Sol#**←**fa**_m←**Fa**←**la**_m←**La**←**fa#**_m←**Fa#**←**sib**_m←**Do#**←**do#**_m



*Socchiusa è la finestra, sul giardino.
Un'ora passa lenta, sonnolenta.
Ed ella, ch'era attenta, s'addormenta
A quella voce che già si lamenta,
Che si lamenta in fondo a quel giardino.*

*Non è che voce d'acque su la pietra:
E quante volte, quante volte udita!
Quell'amore e quell'ora in quella vita
S'affondan come ne l'onda infinita
Stretti insieme il cadavere e la pietra.*

*Ella stende l'angoscia sua nel sonno.
L'angoscia è forte, e il sonno è così lieve!
(Par i' luce d'aprile quasi una neve
che sia tiepida.)
Ed ella certo deve soffrire,
Vagamente, anche nel sonno.*



ACTIONS

Math'n'pop

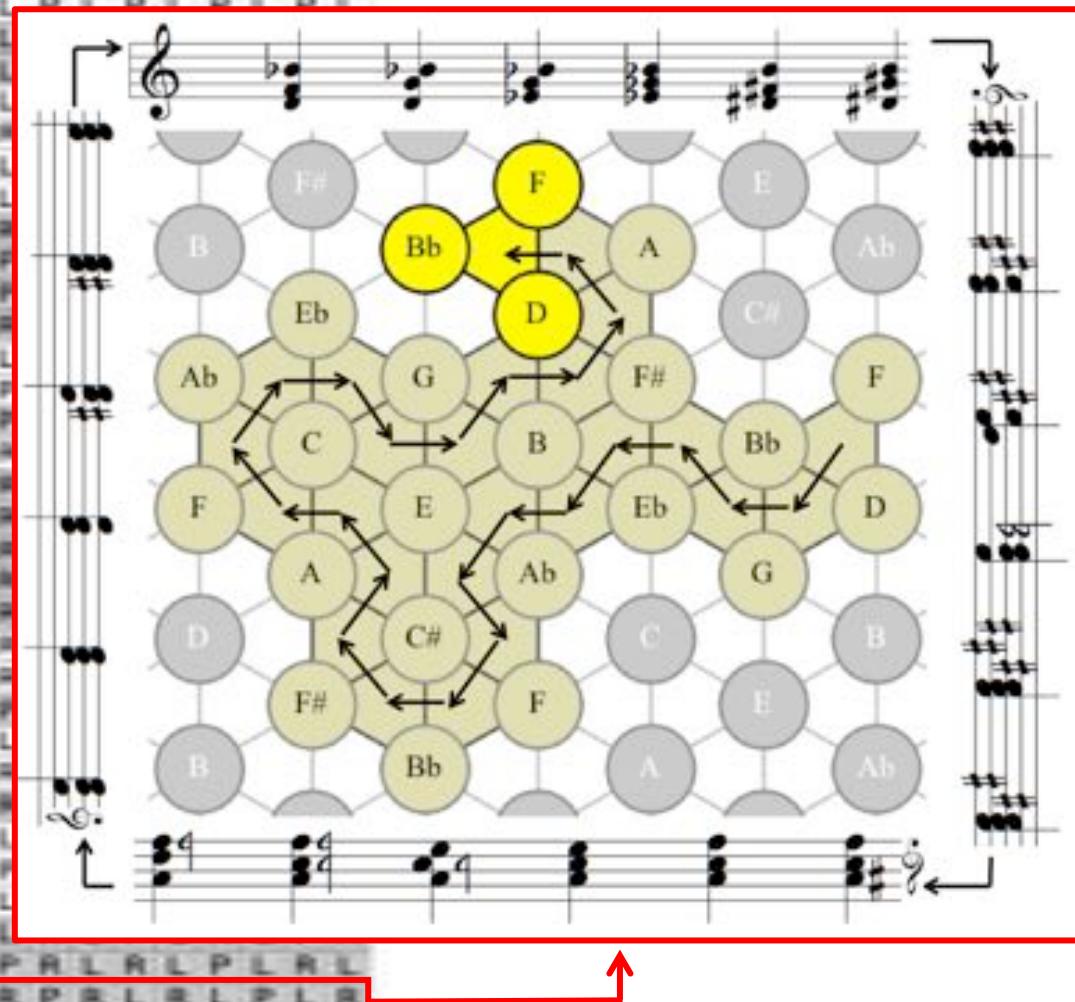
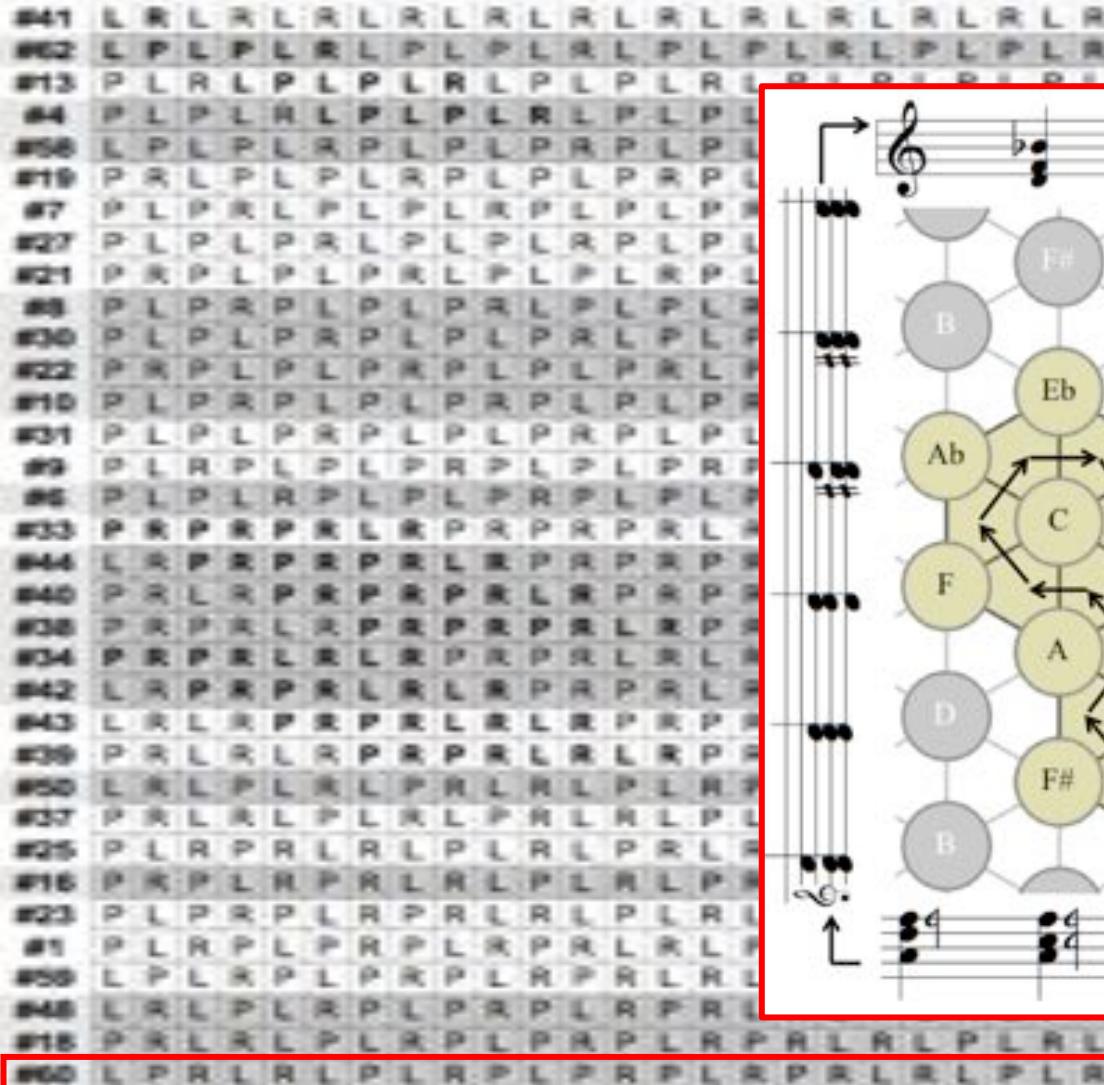
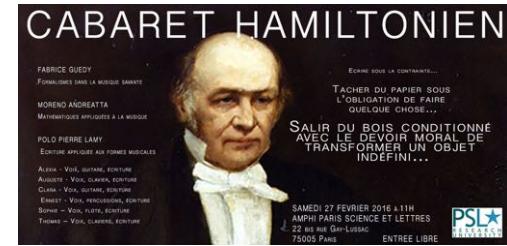
G. D'Annunzio (1863-1938)

The collection of 124 Hamiltonian Cycles

ACTIONS

Math'n'pop

Aprile (d'après Gabriele D'Annunzio)



Aprile (d'après Gabriele D'Annunzio)



Tangente
L'aventure mathématique

pour mieux comprendre
le monde

M. Andreatta, « Math'n pop : symétries et cycles hamiltoniens en chanson », *Tangente*

Aprile

4D & 2D Visualizations
Hamiltonian Cycles
M.Andreatta, G.Baroin 2013

Lyrics: Gabriele d'Annunzio

Music and Vocals: Moreno Andreatta

Hypersphere and Ideogramms: Gilles Baroin

Original "Chicken Wire" graph: J.Douthett, P.Steinbach



<http://www.mathemusic.net>

The collection of 28 « redundant » Hamiltonian Cycles

1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPRPR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLRLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRLRPRP
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPRPL
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRLRLRP
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm--RLRPRPRP
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em--RLRPRPRL
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLRP
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm--RLRLRPRP
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL



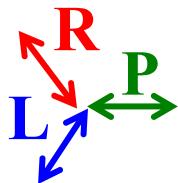
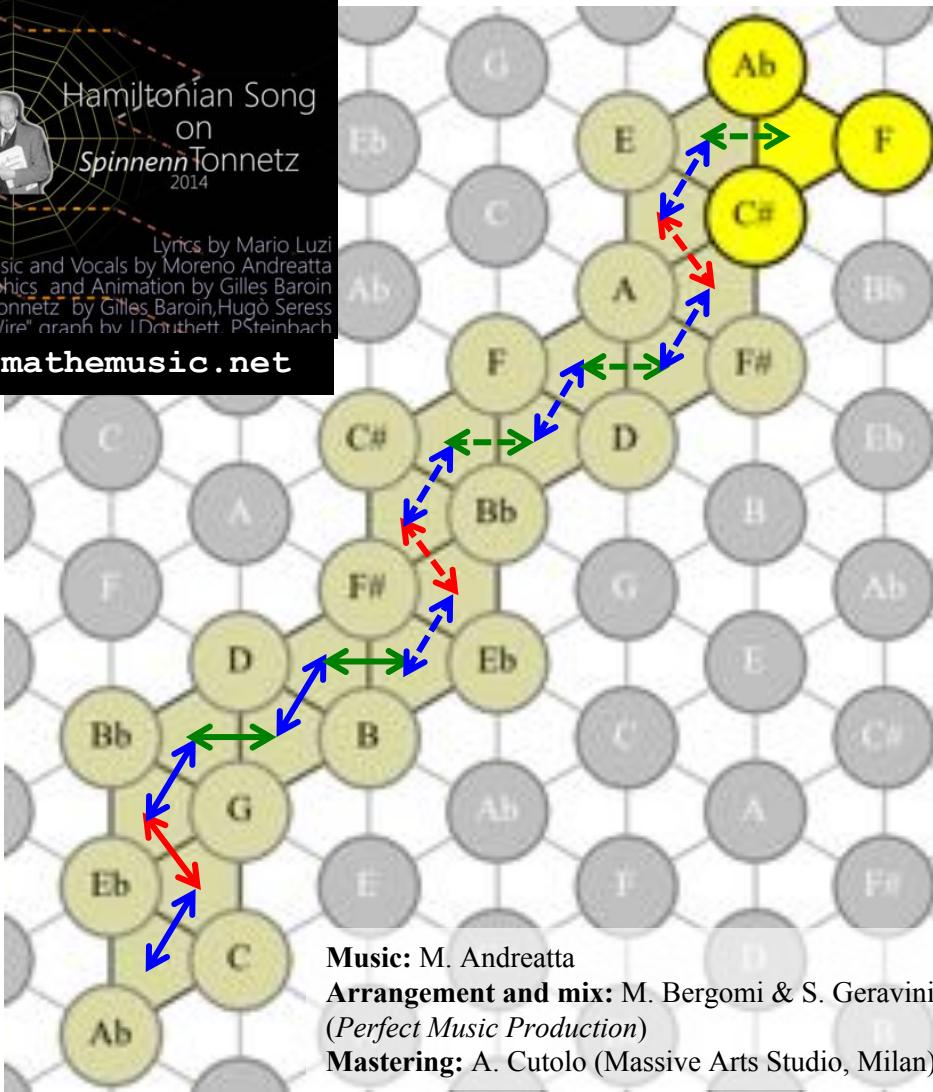
Hamiltonian Cycles with inner periodicities

8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR



L P L P L R ...
 P L P L R L ...
 L P L R L P ...
 PL R L P L ...
L R L P L P ...
 R L P L P L ...

Luzi



La sera non è più la tua canzone
(Mario Luzi, 1945, in *Poesie sparse*)

La sera non è più la tua canzone,
è questa roccia d'ombra traforata
dai lumi e dalle voci senza fine,
la quiete d'una cosa già pensata.

Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d'una cosa conosciuta,
per nome hai una parola ch'è passata
nell'intimo del cuore e s'è perduta.

Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell'essere, (compiuta).

L'aria tace ed il tempo dietro a te
si leva come un'arida montagna
dove vaga il tuo spirito e si perde,
un vento raro scivola e ristagna.



Luzi



Hamiltonian Song
on
SpinnenTonnetz
2014

Lyrics by Mario Luzi

Music and Vocals by Moreno Andreatta

Graphics and Animation by Gilles Baroin

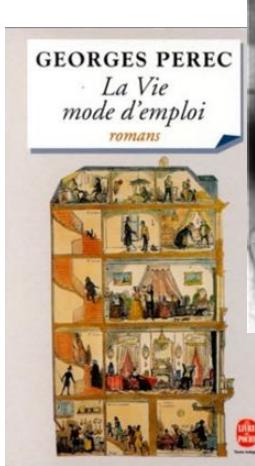
SpinnenTonnetz by Gilles Baroin, Hugo Seress

Original "Chicken Wire" graph by J.Douthett, P.Steinbach

The use of constraints in arts



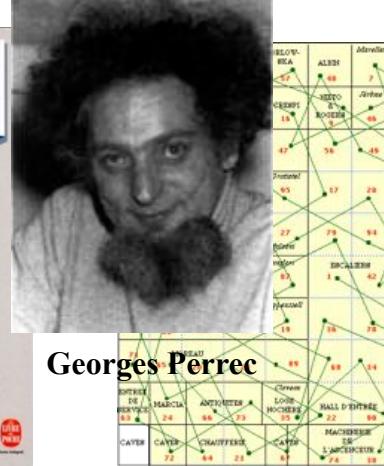
Cent mille milliards de poèmes, 1961



La vie mode d'emploi,



Georges Perec



*Georges
Perec*

Roman

La disparition

Les Lettres Nouvelles

Denoël



Raymond Queneau

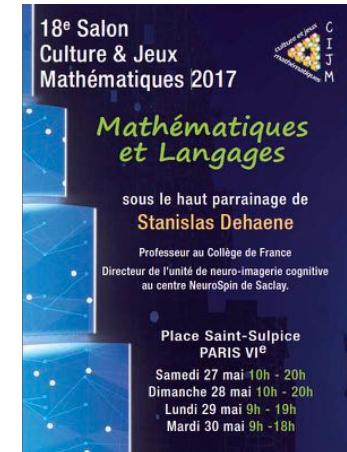
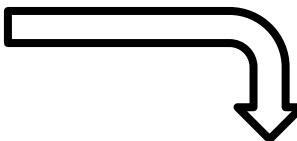


Italo Calvino
Il castello dei destini incrociati, 1969



LN

From the OuLiPo to the OuMuPo (ouvroir de musique potentielle)



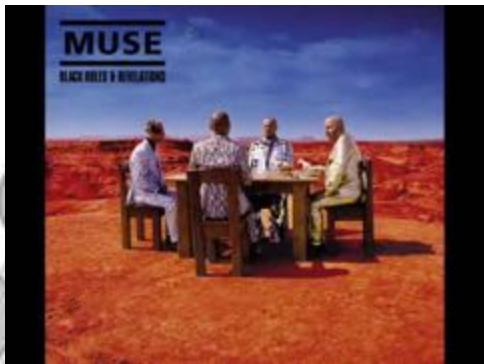
<http://oumupo.org/>

{ BnF

9 novembre 2017



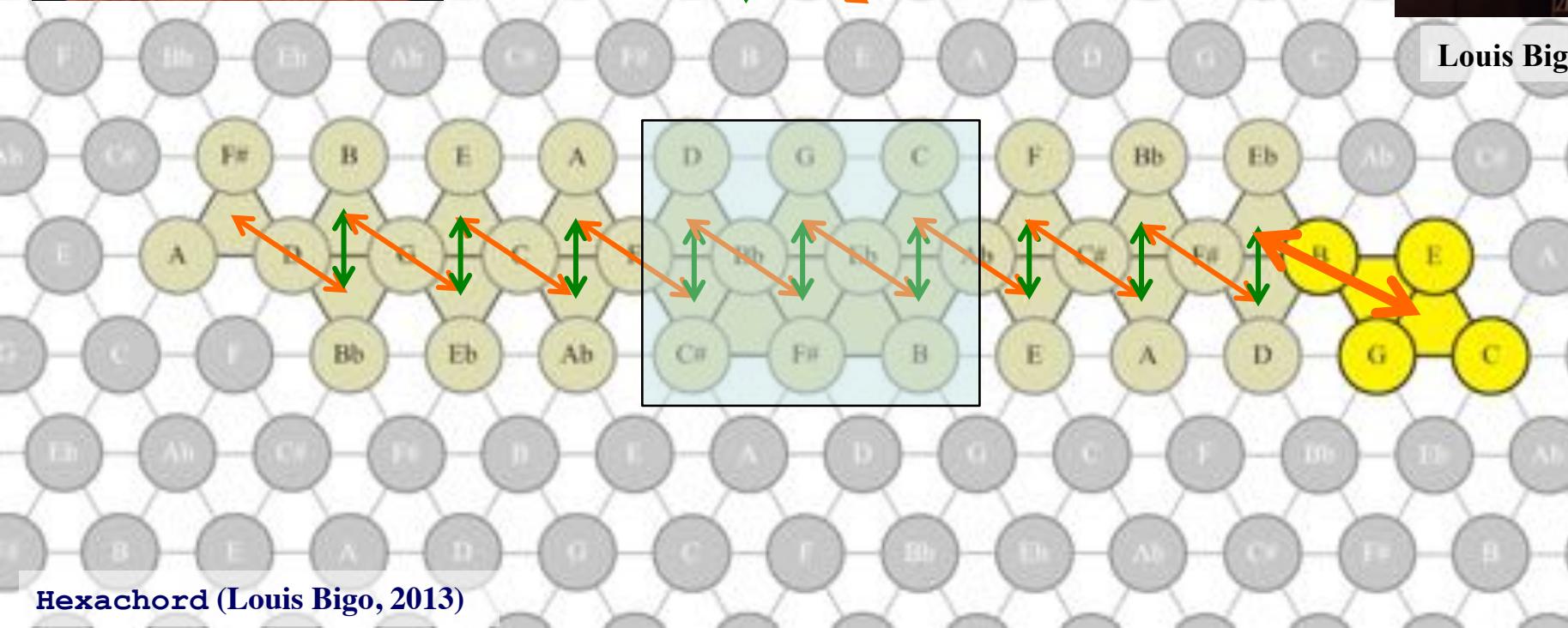
Symmetries and algorithmic processes in *Muse*



“Take a bow” (*Black Holes and Revelations*, 2006)



Louis Bigo

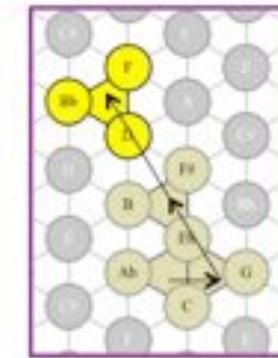
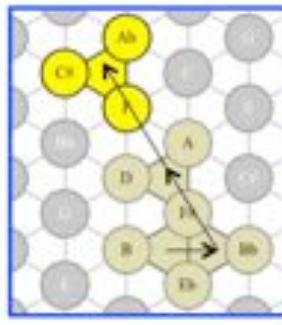
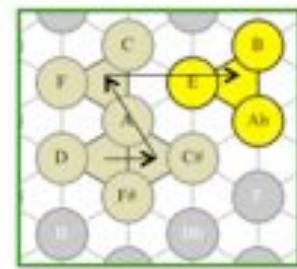
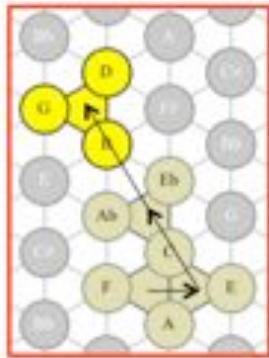
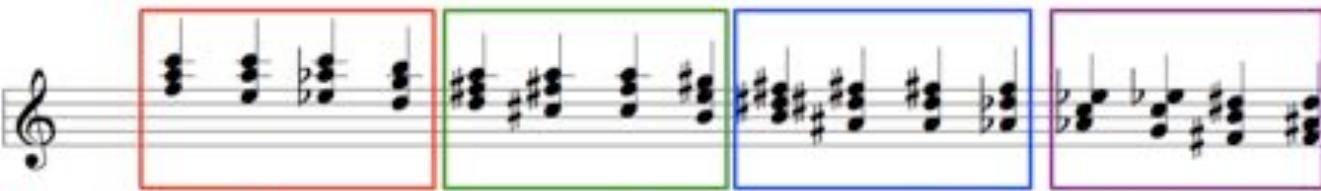


Hexachord (Louis Bigo, 2013)

Temporal axis

Symétries dans la musique de Frank Zappa

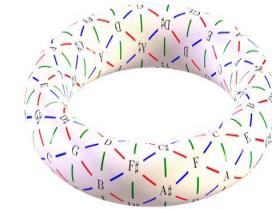
Fa la_m La_b Sol Re fa#_m Fa Mi Si la#_m Re Re_b La_b do_m Si Si_b



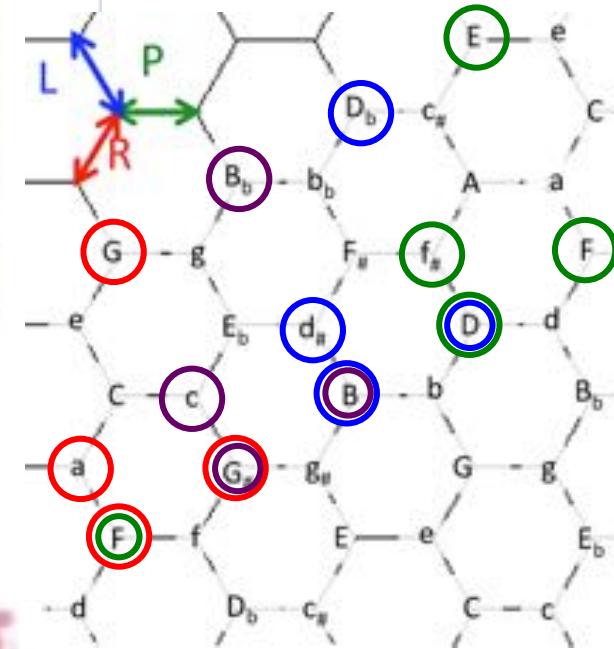
T₋₃

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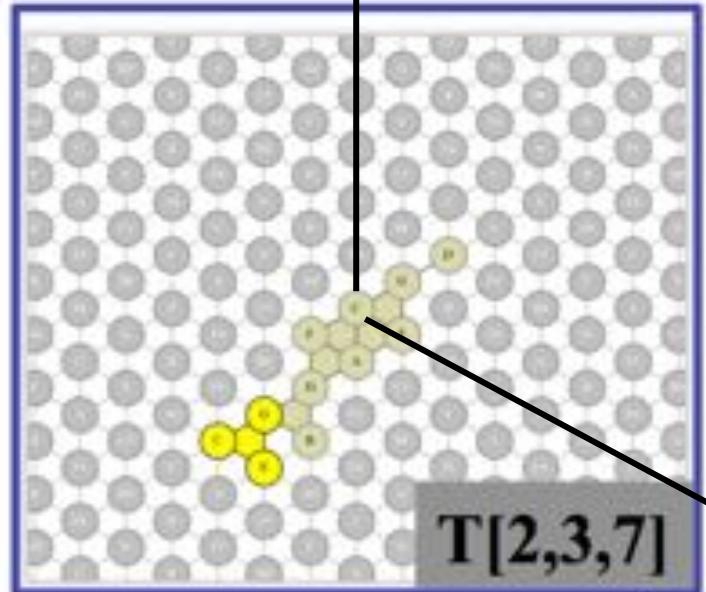
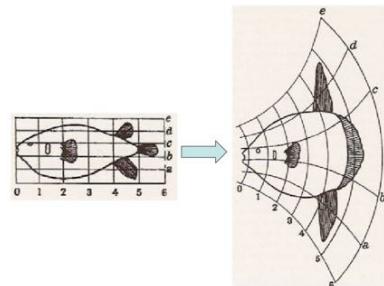
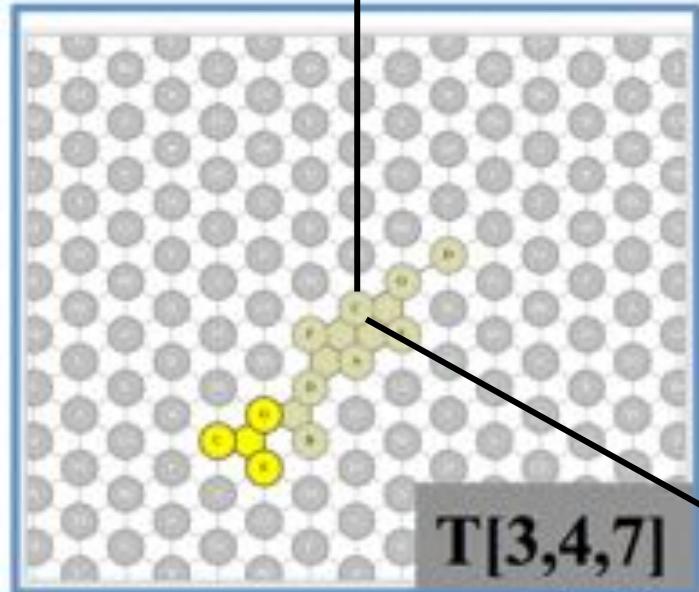


→ Source: Wikipedia

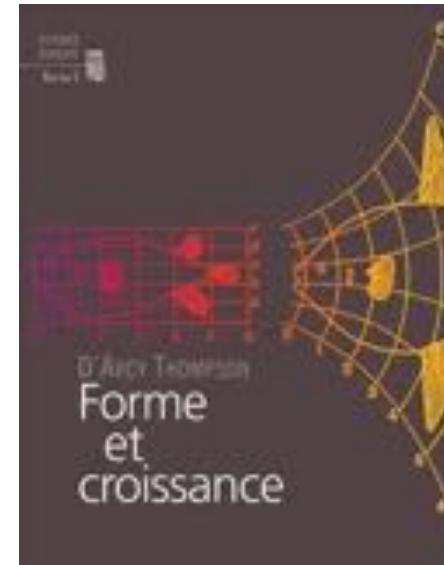
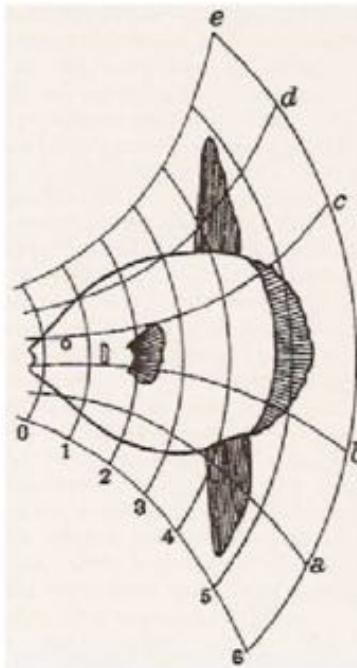
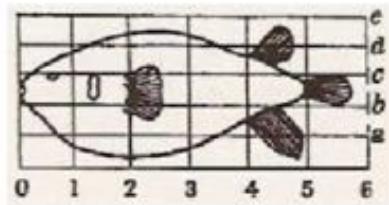


« Easy Meat » - 1981 (Frank Zappa)
min. 1'44" – 2'39"

The musical style...is the space!

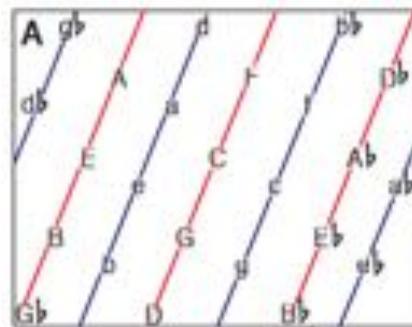


The morphological genealogy of structuralism

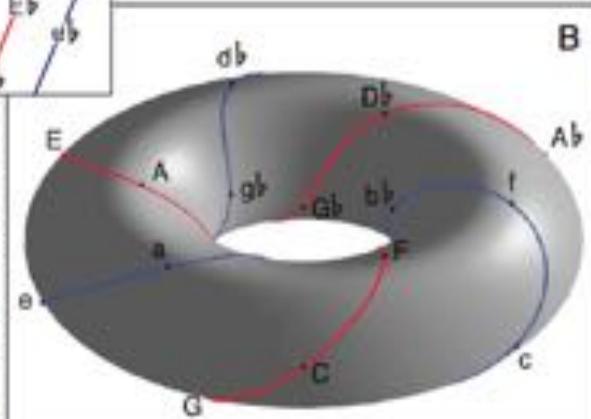


« [La notion de **transformation**] me vient d'un ouvrage qui a joué pour moi un rôle décisif et que j'ai lu pendant la guerre aux États Unis: *On Growth and Form*, en deux volumes, de D'Arcy Wentworth Thompson, paru pour la première fois en 1917. L'auteur, naturaliste écossais, (...) interprétrait comme des transformations les différences visibles entre les espèces ou organes animaux ou végétaux au sein d'un même genre. Ce fut une illumination, d'autant que j'allais vite m'apercevoir que cette façon de voir s'inscrivait dans une longue tradition: derrière Thompson, il y avait la botanique de Goethe, et derrière Goethe, Albert Dürer avec son Traité de la proportion du corps humain » (Lévi-Strauss et Eribon, 1988).

Tonnetz and cognitive neurosciences



Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each

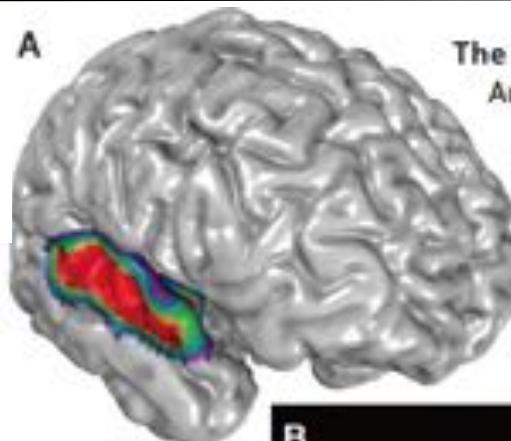


wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (B) Musical keys as points on the surface of a torus.

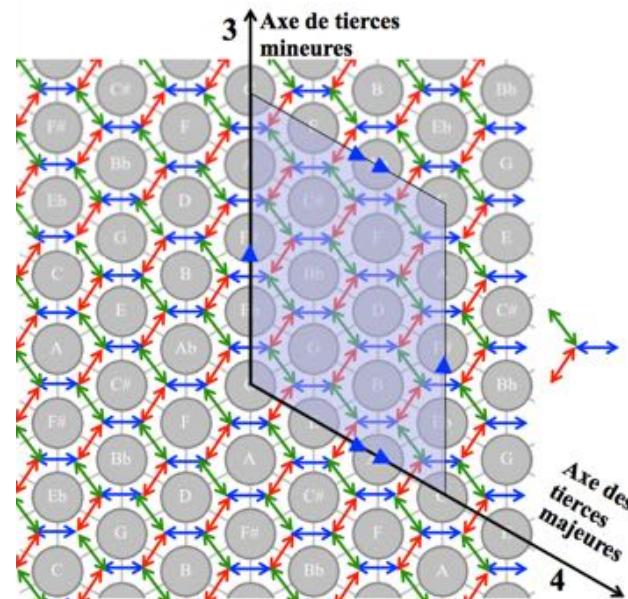
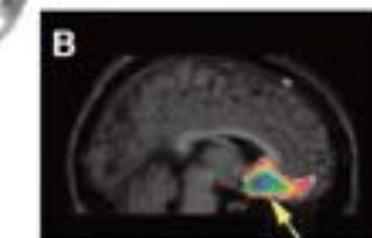
PERSPECTIVES: NEUROSCIENCE

Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl



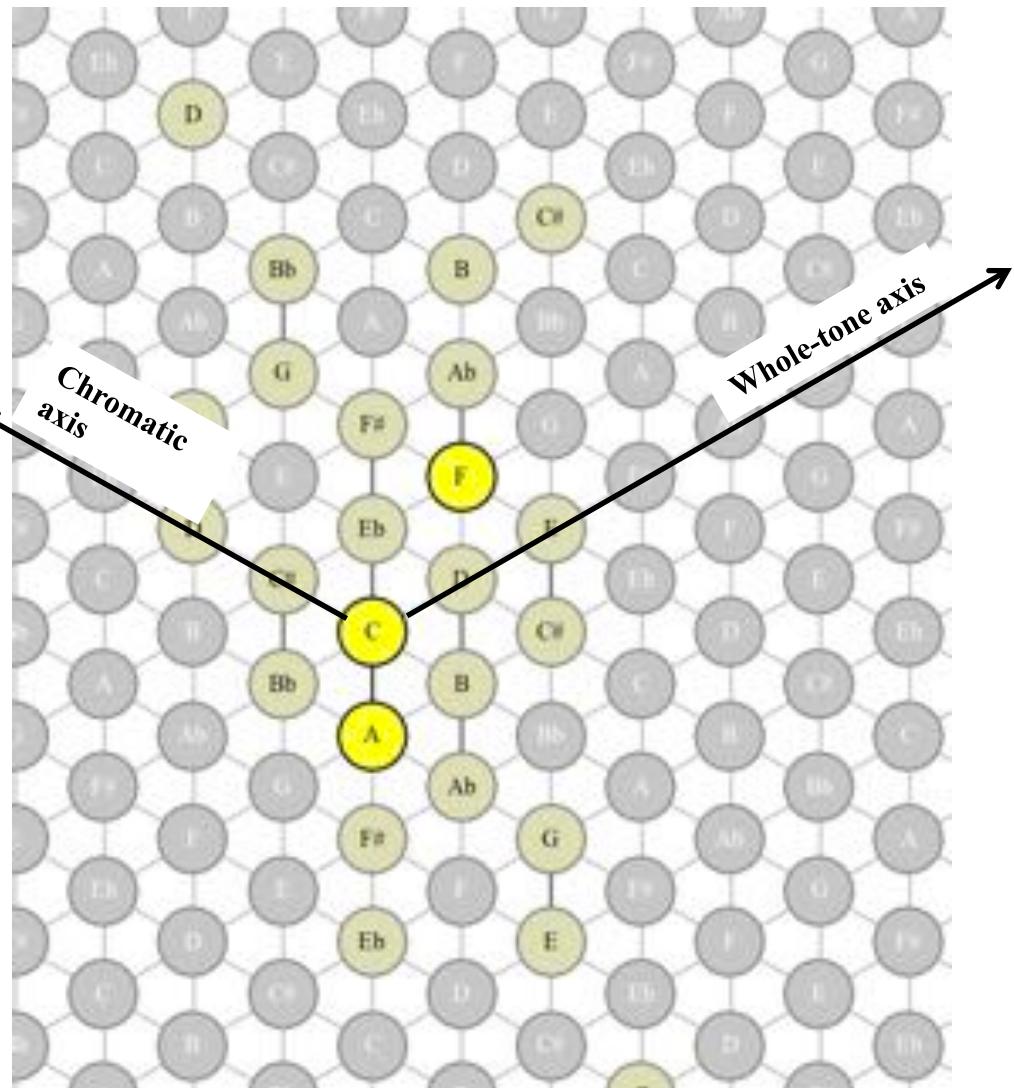
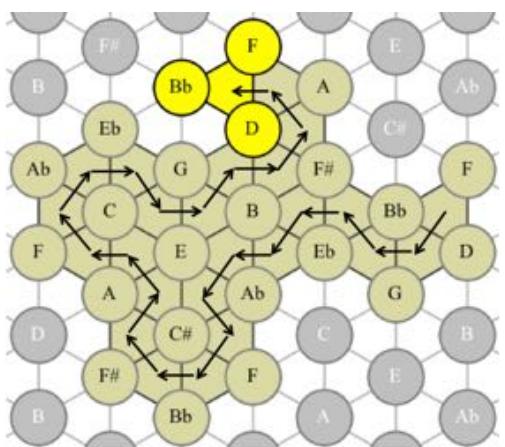
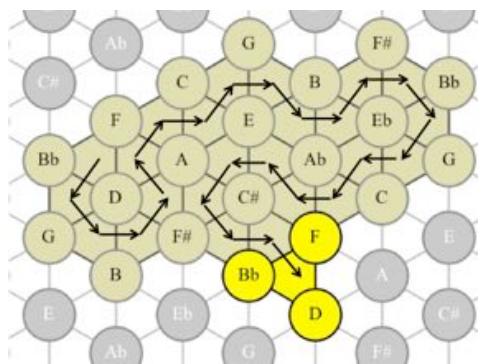
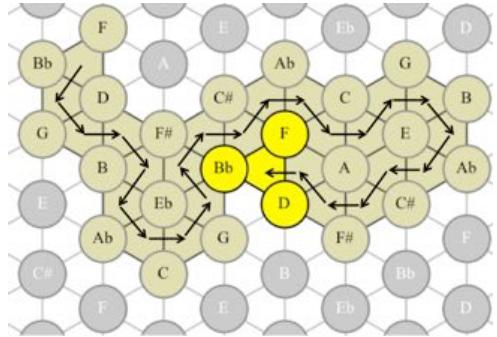
The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.



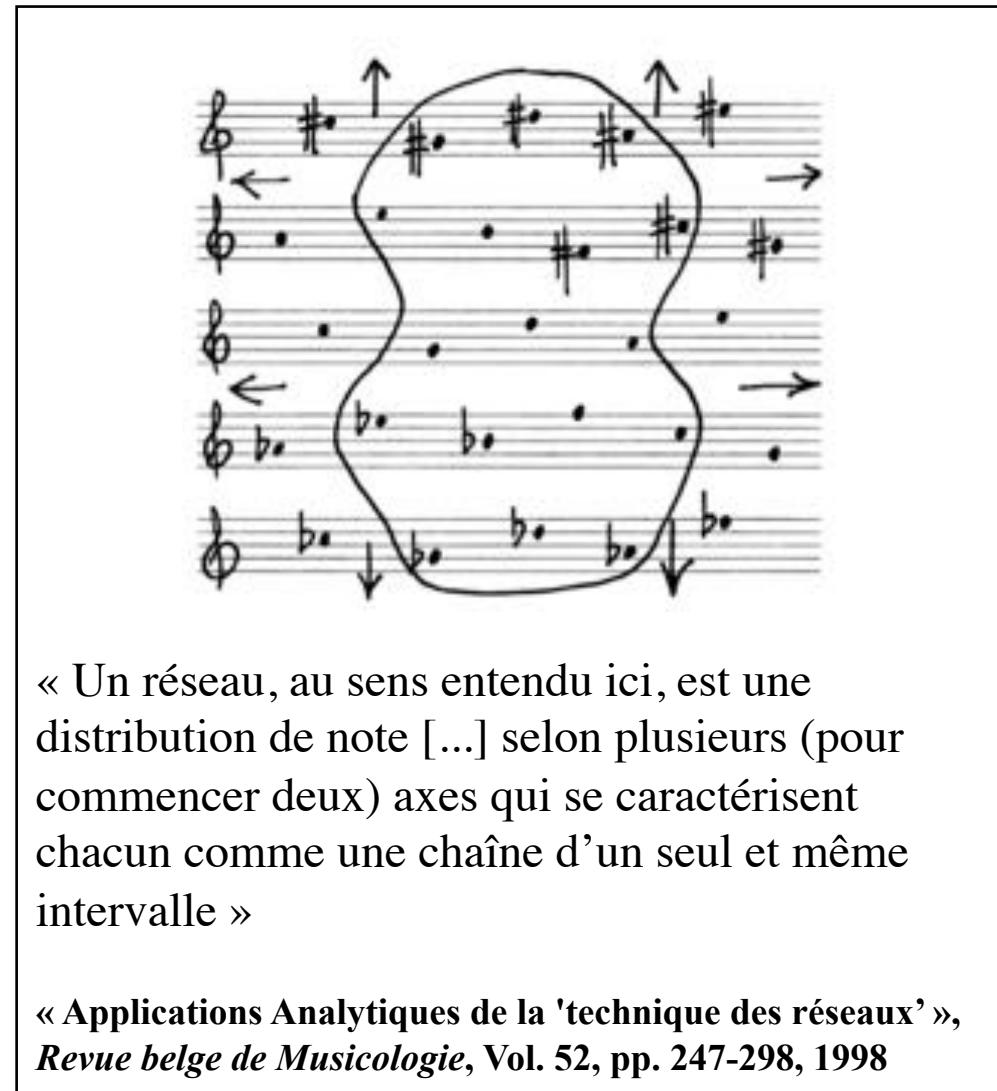
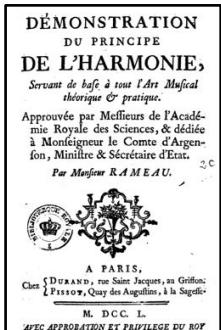
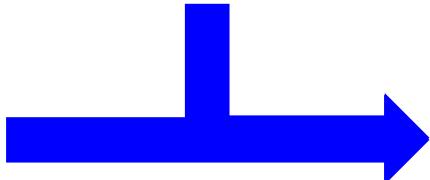
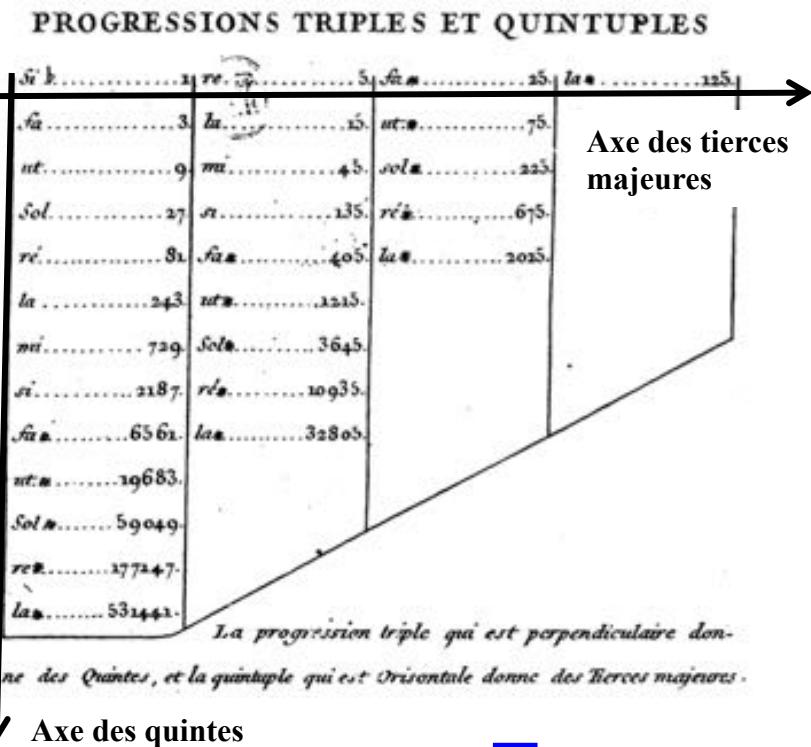
Acotto E. et M. Andreatta (2012),
« Between Mind and Mathematics.
Different Kinds of Computational
Representations of Music »,
Mathematics and Social Sciences, n°
199, 2012(3), p. 9-26.



Different embeddings of Hamiltonian cycles



Henri Pousseur's 'Network Theory' and Rameau's heritage



« Un réseau, au sens entendu ici, est une distribution de note [...] selon plusieurs (pour commencer deux) axes qui se caractérisent chacun comme une chaîne d'un seul et même intervalle »

« Applications Analytiques de la 'technique des réseaux' »,
Revue belge de Musicologie, Vol. 52, pp. 247-298, 1998

Henri Pousseur's 'Network Theory' and Rameau's heritage

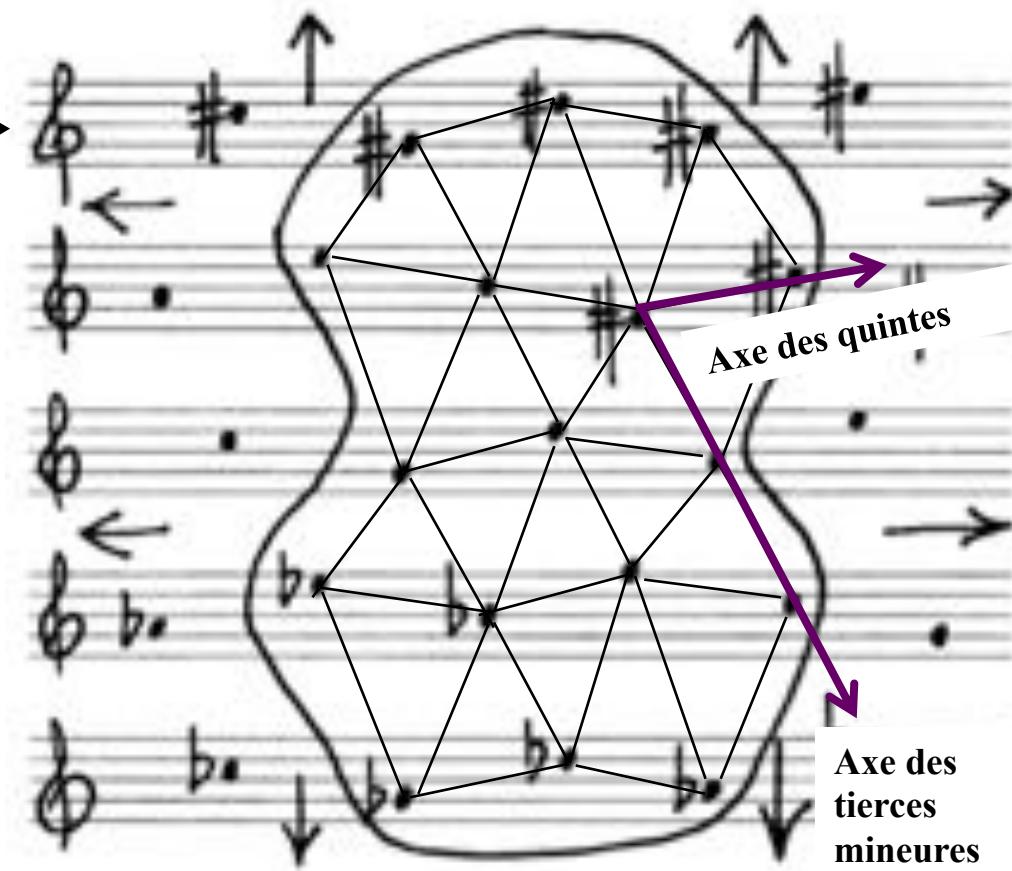
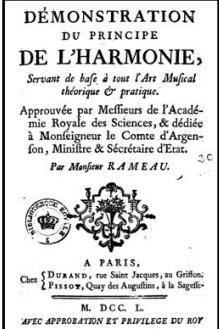
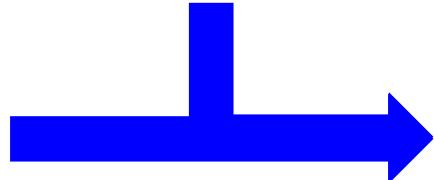
PROGRESSIONS TRIPLES ET QUINTUPLES

<i>si b.</i>	21	<i>re</i>	5	<i>fa m.</i>	25	<i>la s.</i>	125
<i>fa</i>	3	<i>la</i>	15	<i>ut</i>	75		
<i>ut</i>	9	<i>mi</i>	45	<i>sola</i>	225		
<i>Sol</i>	27	<i>si</i>	135	<i>ré</i>	675		
<i>ré</i>	81	<i>fa s.</i>	405	<i>la</i>	2025		
<i>la</i>	243	<i>ut s.</i>	1215				
<i>mi</i>	729	<i>Sola</i>	3645				
<i>si</i>	2187	<i>ré s.</i>	10935				
<i>fa s.</i>	6561	<i>la s.</i>	32805				
<i>ut s.</i>	19683						
<i>Sola</i>	59049						
<i>ré s.</i>	177247						
<i>la s.</i>	531441						

Axe des tierces majeures

La progression triple qui est perpendiculaire donne des Quintes, et la quintuple qui est horizontale donne des Tierces majeures.

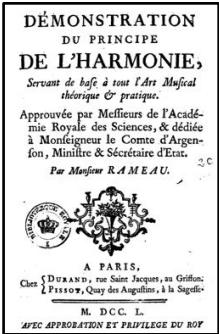
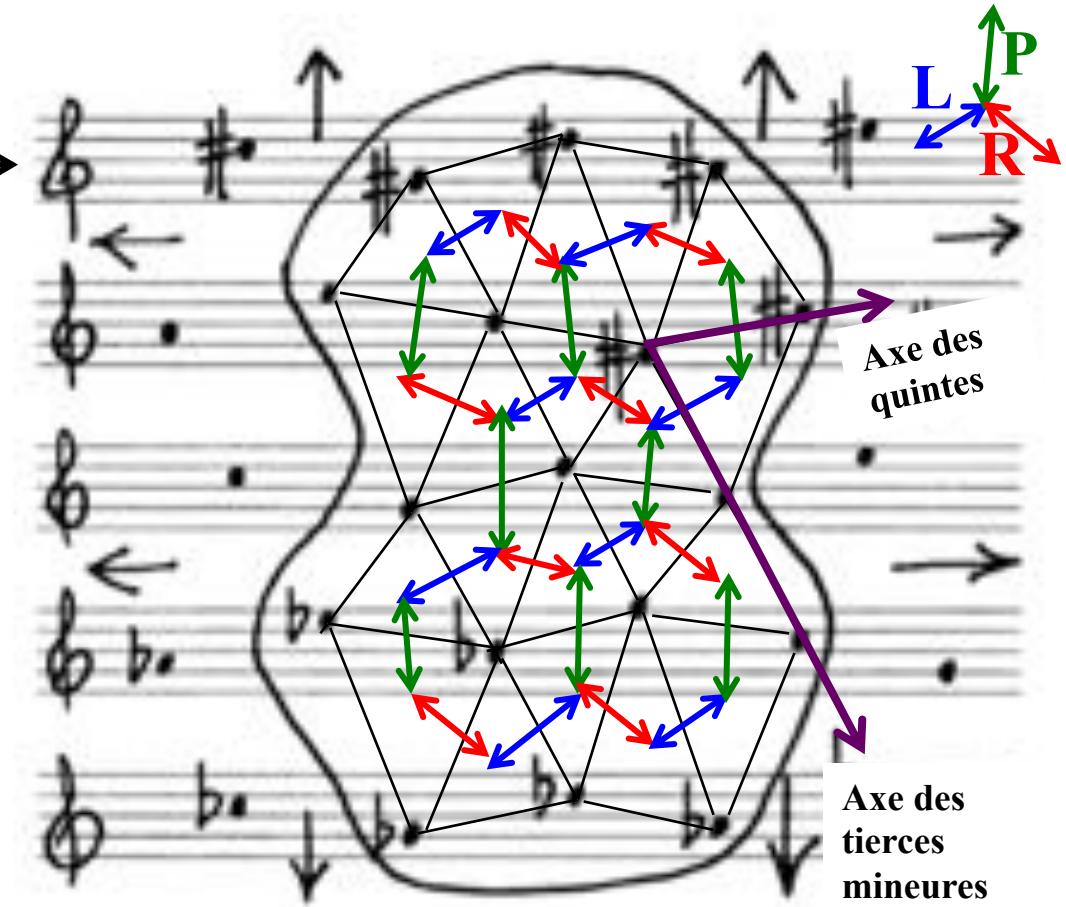
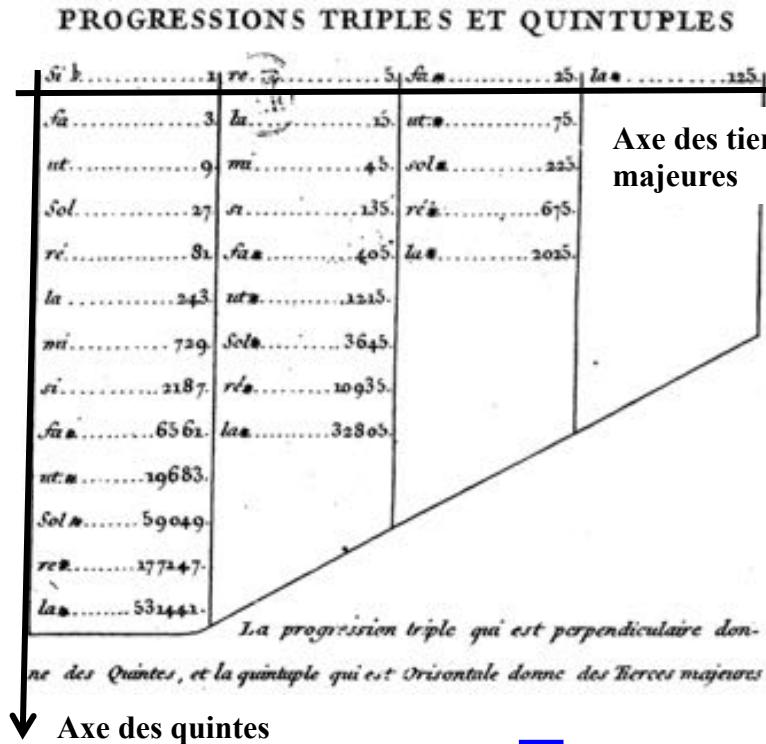
Axe des quintes



- « L'apothéose de Rameau. Essai sur la question harmonique, *Musiques Nouvelles. Revue d'esthétique*, 21, 105-172, 1968

- « Applications Analytiques de la 'technique des réseaux' », *Revue belge de Musicologie*, Vol. 52, pp. 247-298, 1998

Henri Pousseur's 'Network Theory' and Rameau's heritage



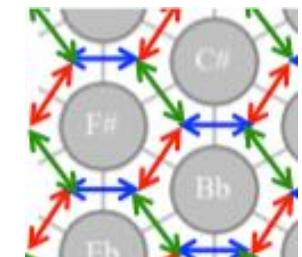
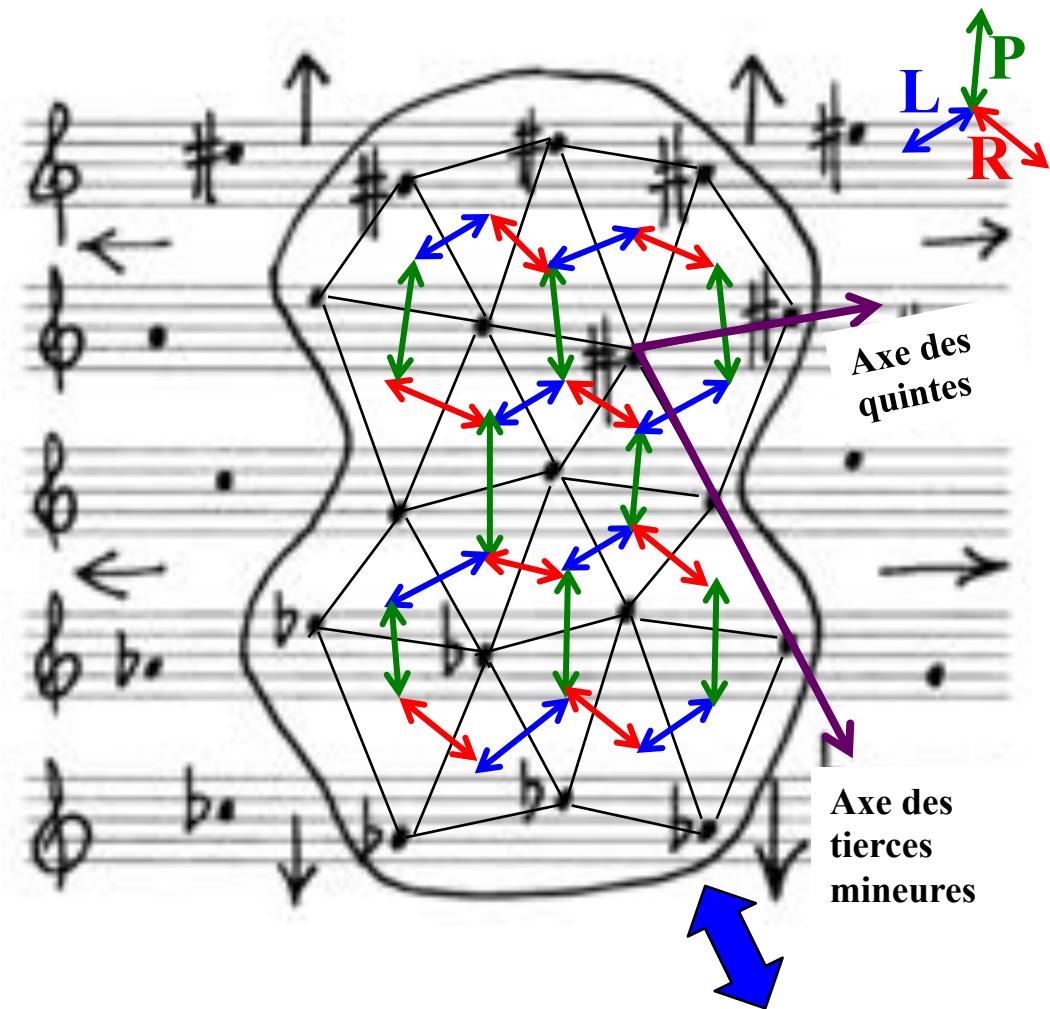
- « L'apothéose de Rameau. Essai sur la question harmonique, *Musiques Nouvelles. Revue d'esthétique*, 21, 105-172, 1968

- « Applications Analytiques de la 'technique des réseaux' », *Revue belge de Musicologie*, Vol. 52, pp. 247-298, 1998

Henri Pousseur's 'Network Theory' and Rameau's heritage

« Il ne faut toutefois pas oublier que le principe même de la méthode réside dans la volonté de construire le lacis de telle sorte que les relations musicales élémentaires effectives, donc 'en-temps', (analysées ou composées, mélodiques ou accordiques) soient les plus **serrées** possibles, s'expriment principalement entre notes **voisines** du réseau, dans un sens ou dans l'autre.

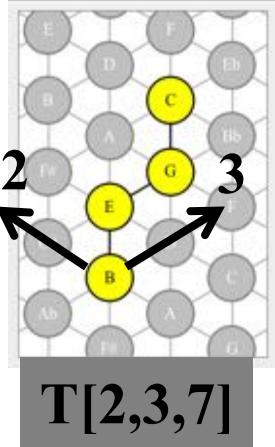
Ajoutons encore que l'on peut passer de certains réseaux à certains autres en faisant simplement 'basculer' les axes [...] ce qui modifie les rapports de proximité structurelle entre les notes et donc la hiérarchie de leurs intervalles ».



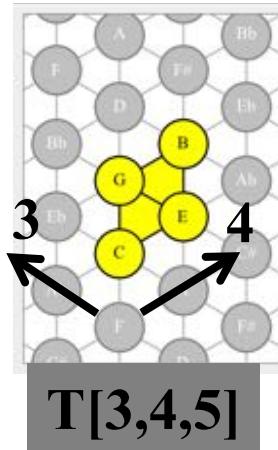
- « Applications Analytiques de la 'technique des réseaux' », *Revue belge de Musicologie*, Vol. 52, pp. 247-298, 1998

The spatial character of the « musical style »

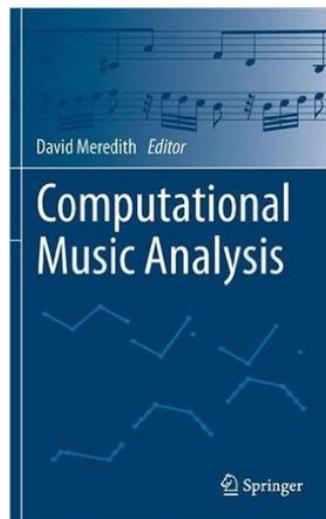
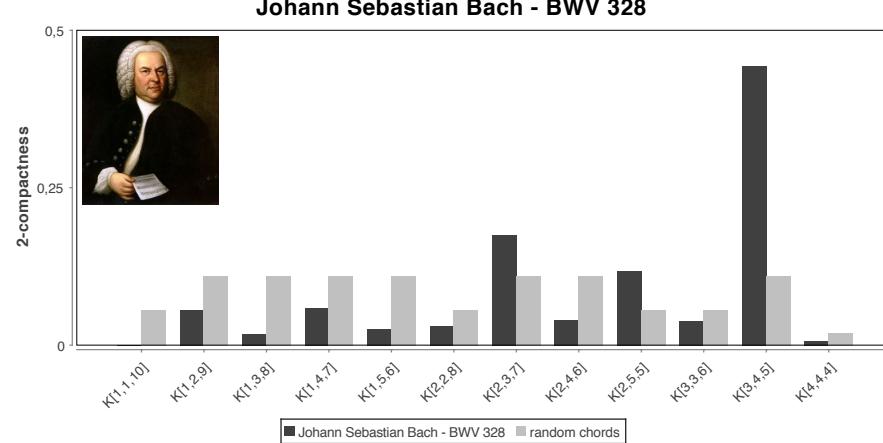
1



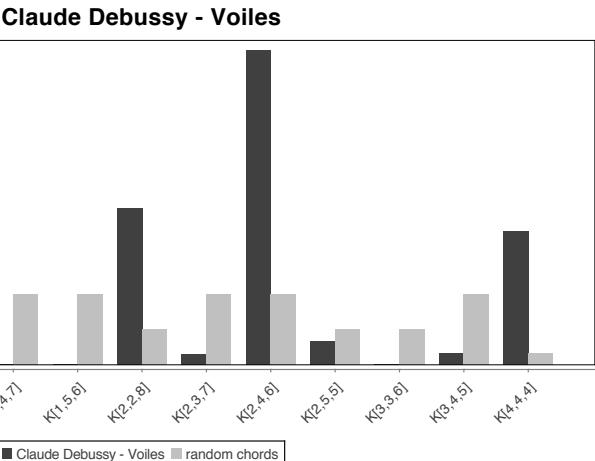
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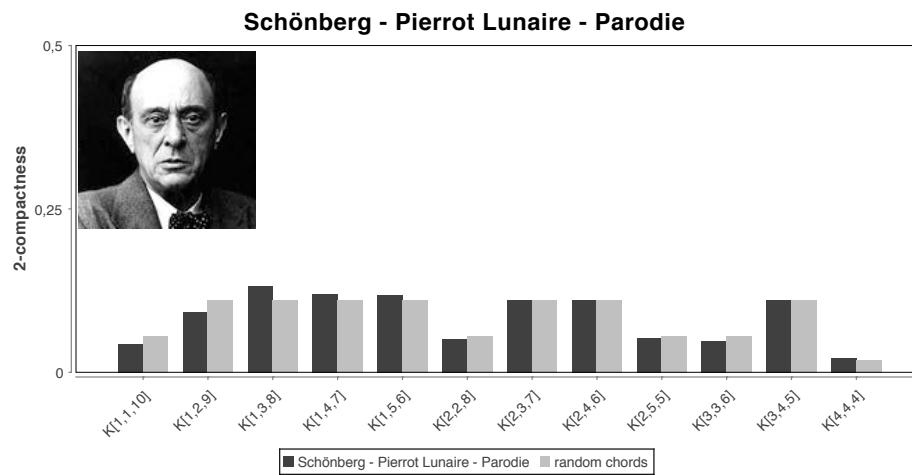
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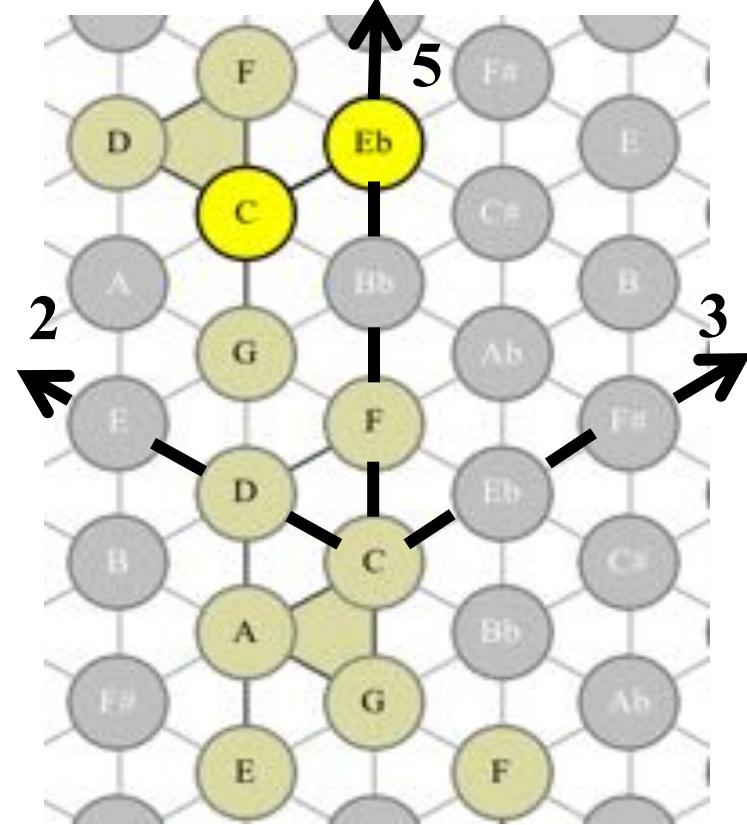
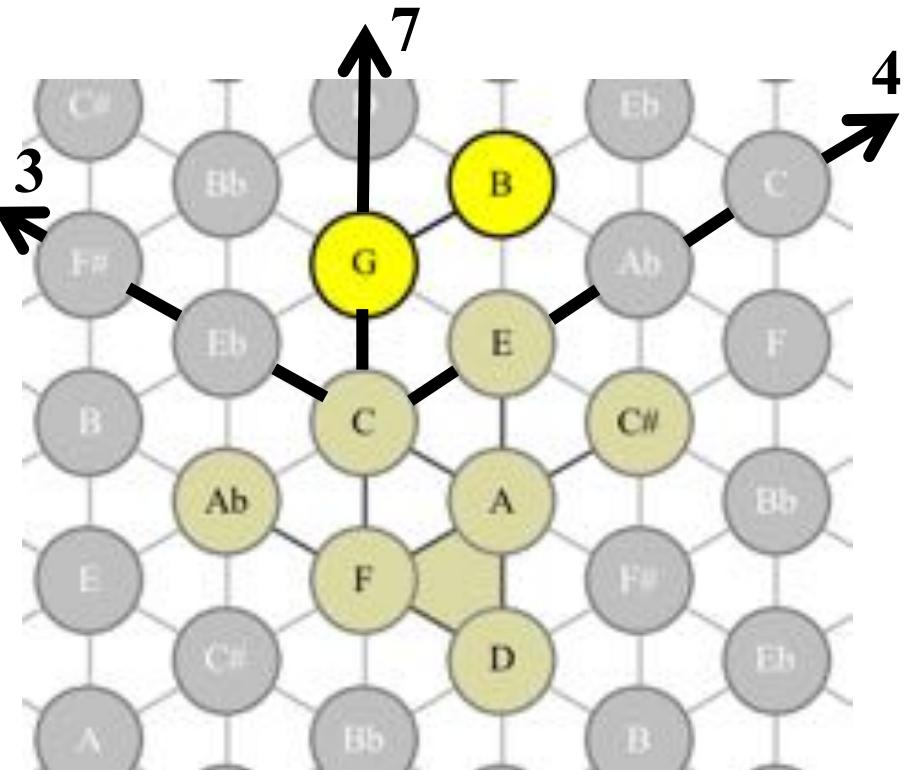
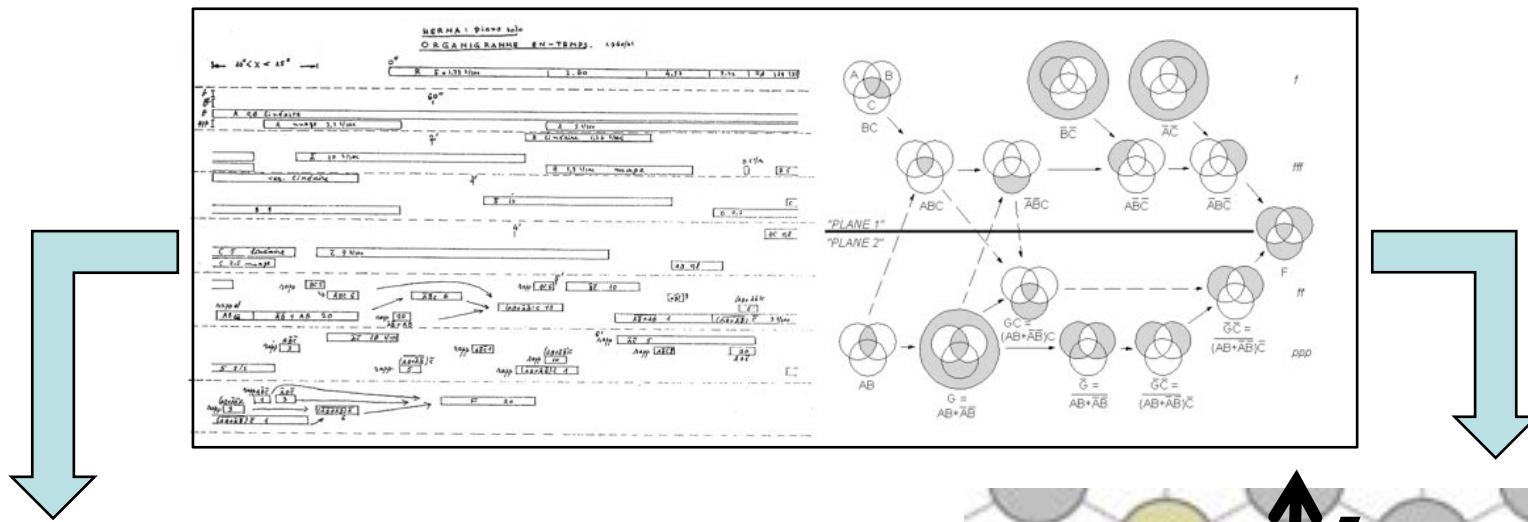
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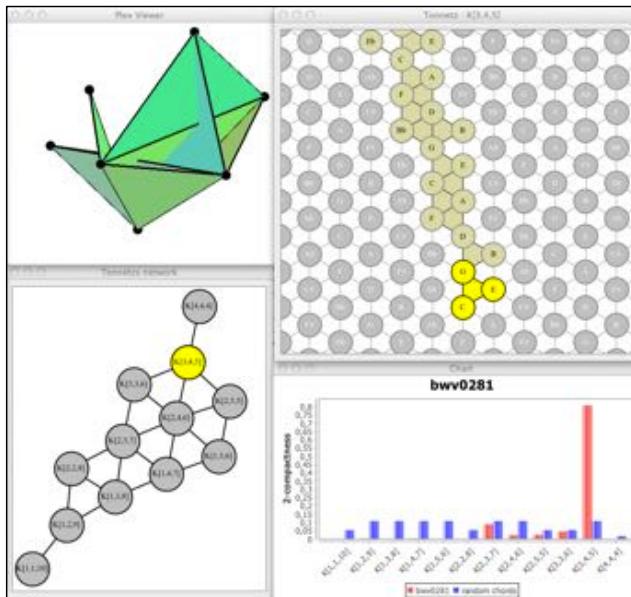
5



A Tonnetz-based analysis of *Herma* by Iannis Xenakis



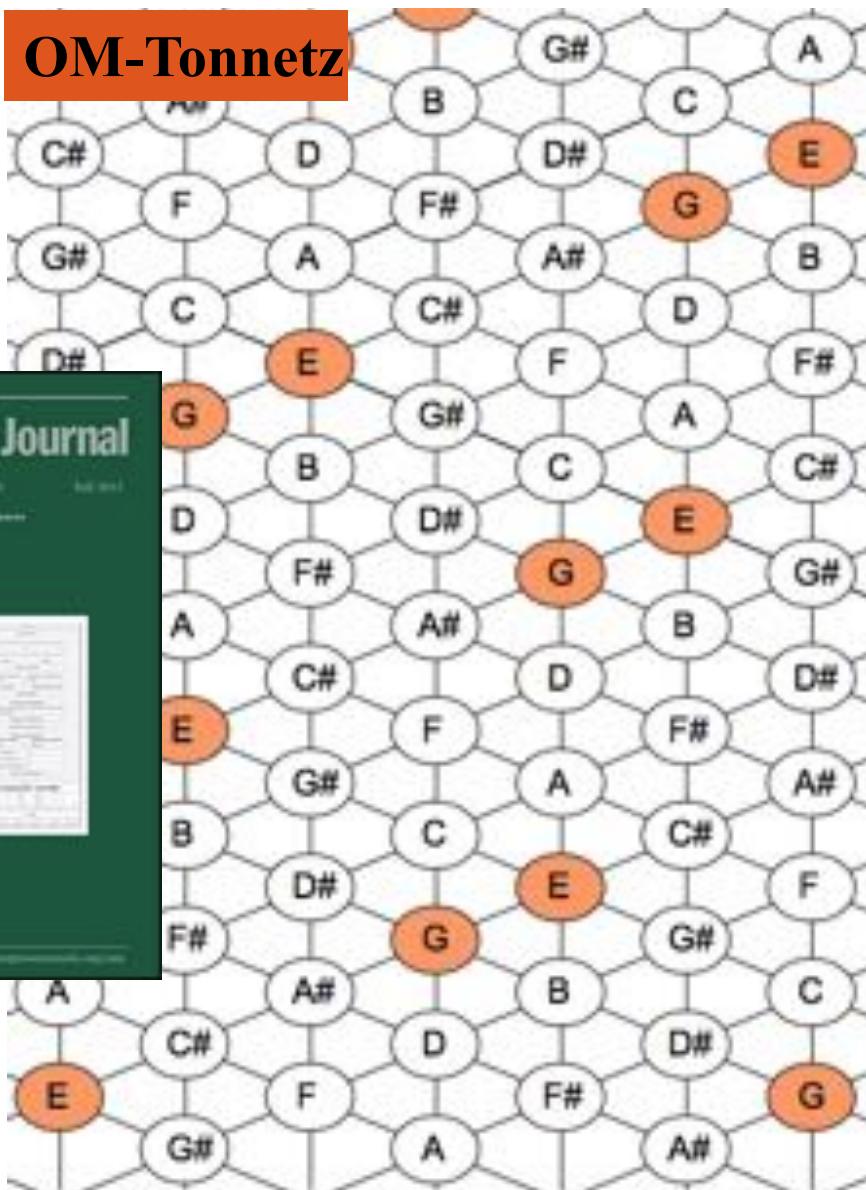
Three environments for a Tonnetz-based Analysis



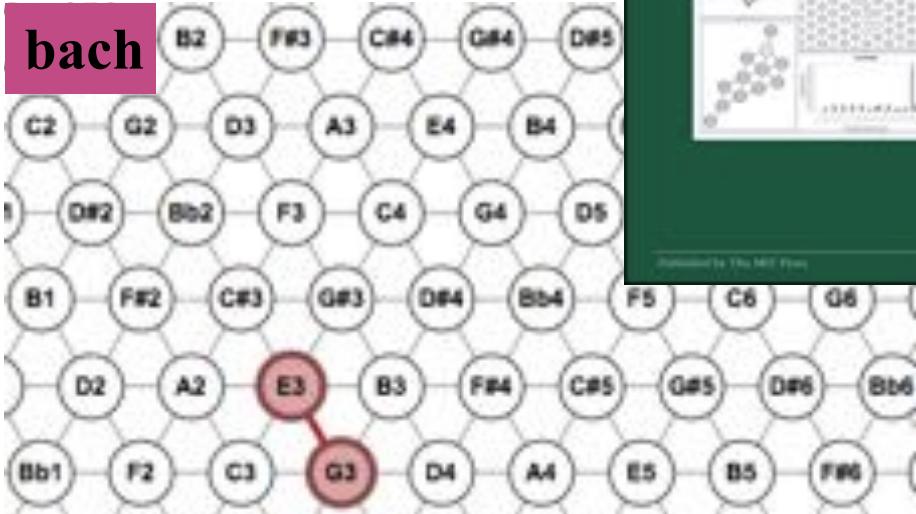
Hexachord



OM-Tonnetz

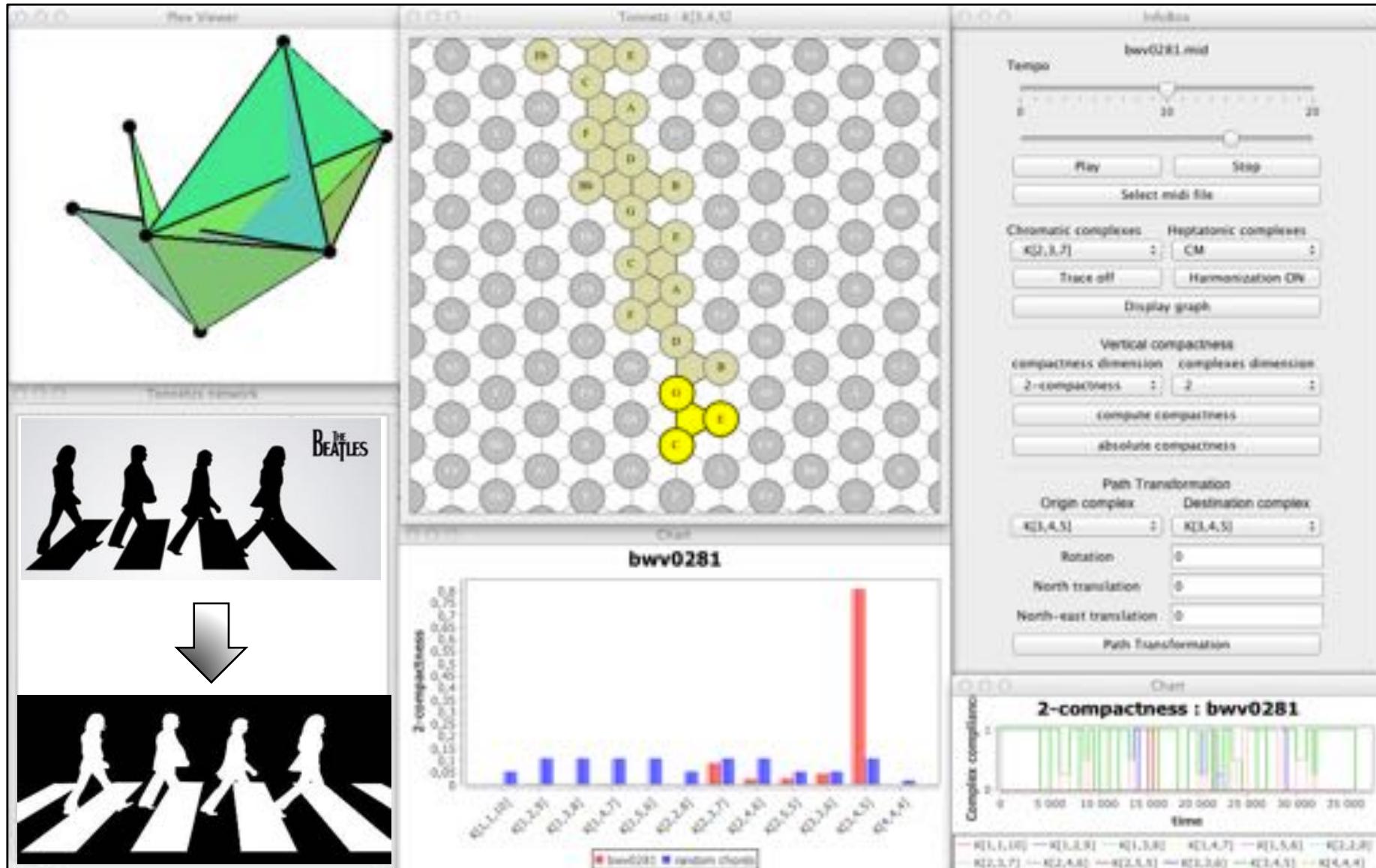


bach



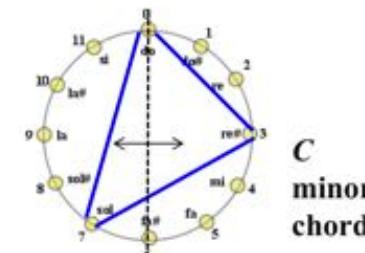
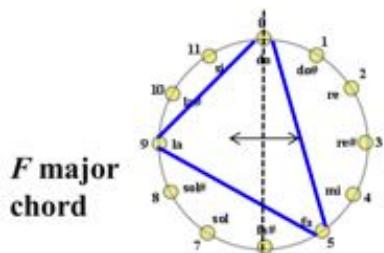
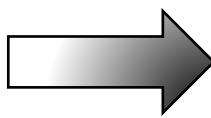
→ demo

Keeping the space...but changing the trajectory!

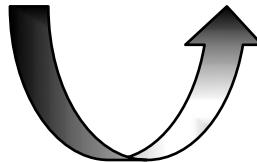
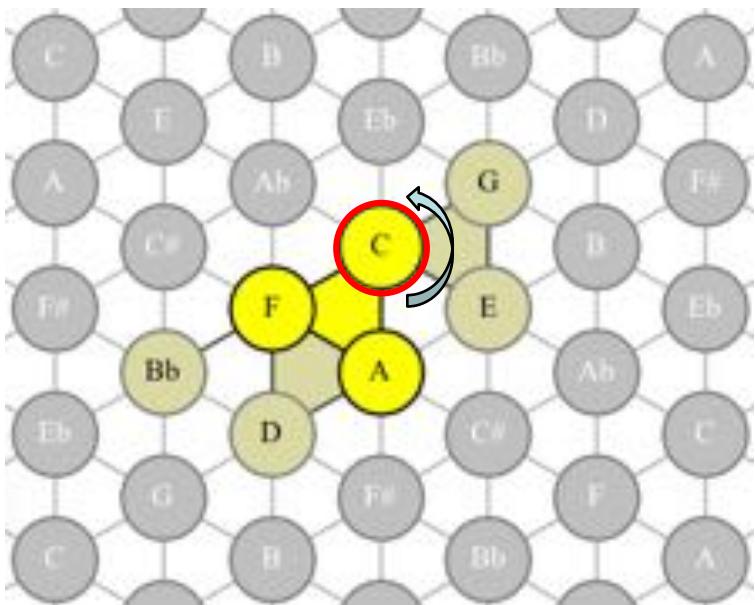


→ <http://www.lacl.fr/~lbigo/hexachord>

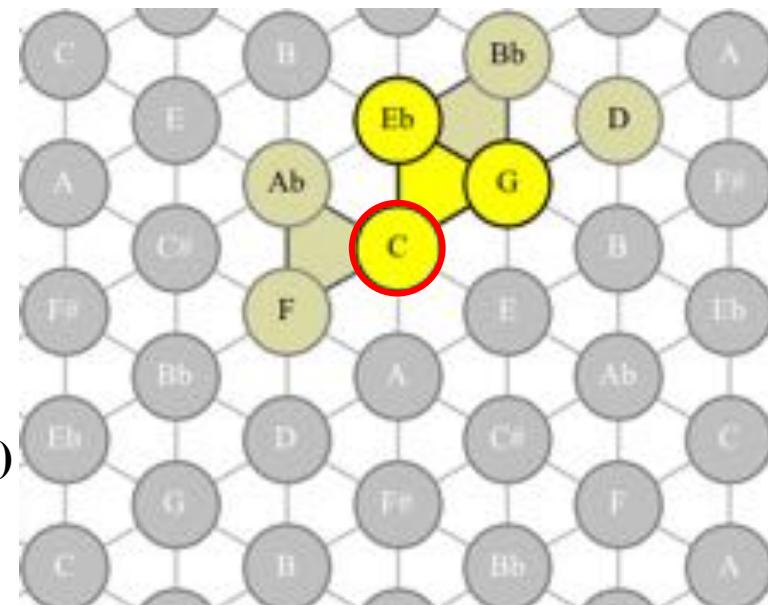
Keeping the space...but changing the trajectory!



C
minor
chord



Rotation
(autour du *do*)



Rotational symmetry applied to traditional Brazilian music

Sonoroso

K. Ximbinho

Brejeiro

Maxixe

Introd.

A

E7

A

Dm

Dm

E \flat

A

E7

A

E7



Doce de Côco

Jacob do
(Jacob E)

G6 *C/E* *G6* *Dm7* *E7* *Am* *Am/G* *E*

Am7 *Am7* *D7* *G7M* *G \flat 7* *F \sharp 7* *F6* *Bm7* *E7*

Am *E7/B* *Am* *Cm7/E \flat* *D7* *Gm* *Gm5+ Gm6* *Gm* *Gm5+*

Gm6 Gm5+ *G7* *G7* *Cm* *Cm* *E \flat 9* *E \flat 9*

G/D *E7* *Am7* *D7* *G6* *Bm7 B \flat 9* *Am7*

Aeroporto do Galeão

Altamiro Carrilho

F9 *%* *F9*

D7 *D \sharp* *Gm9* *Gm9*

Apanhei-te Cavaquinho

Ernesto Nazareth
Baldoman

D7(9) *G6* *B7(9)* *E m*

C6

G6

D7

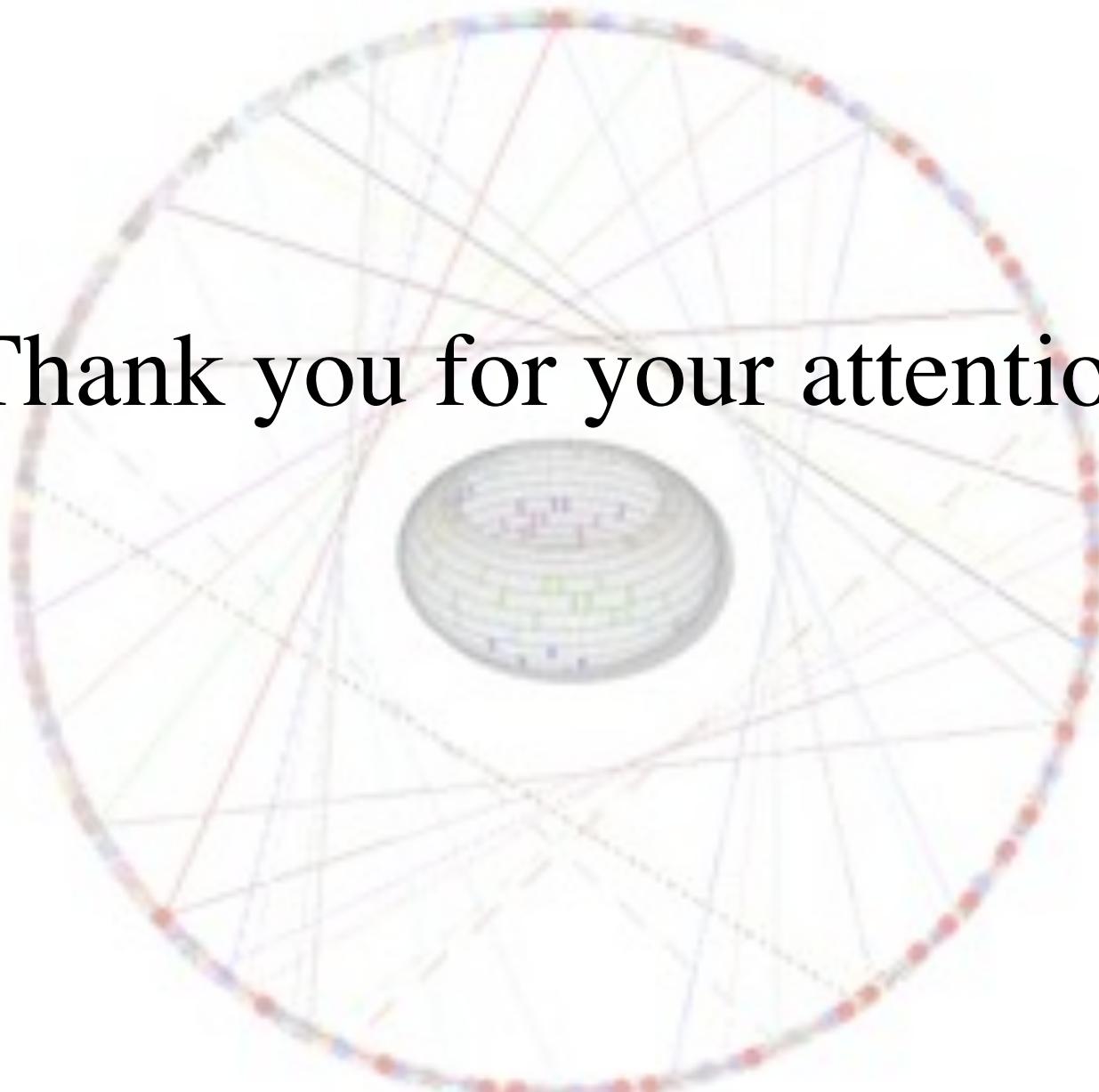
Piano *Grazioso* *P*

Escovado
Tango Brasileiro

Ernesto Nazareth
1905

8 \flat

Fine



Thank you for your attention!