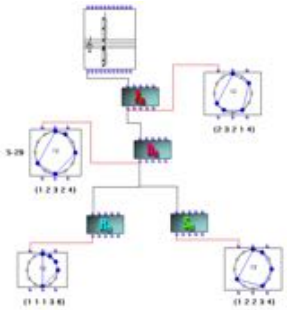
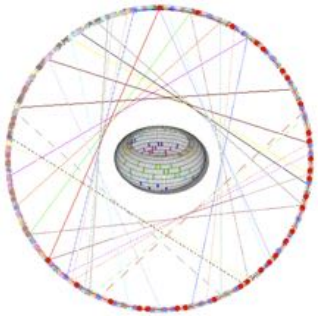
PARCOURS DU MASTER SCIENCES ET TECHNOLOGIES
UPMC • IRCAM • TELECOM PARISTECH

# Some mathematical and computational models for computer-aided music theory, analysis and composition

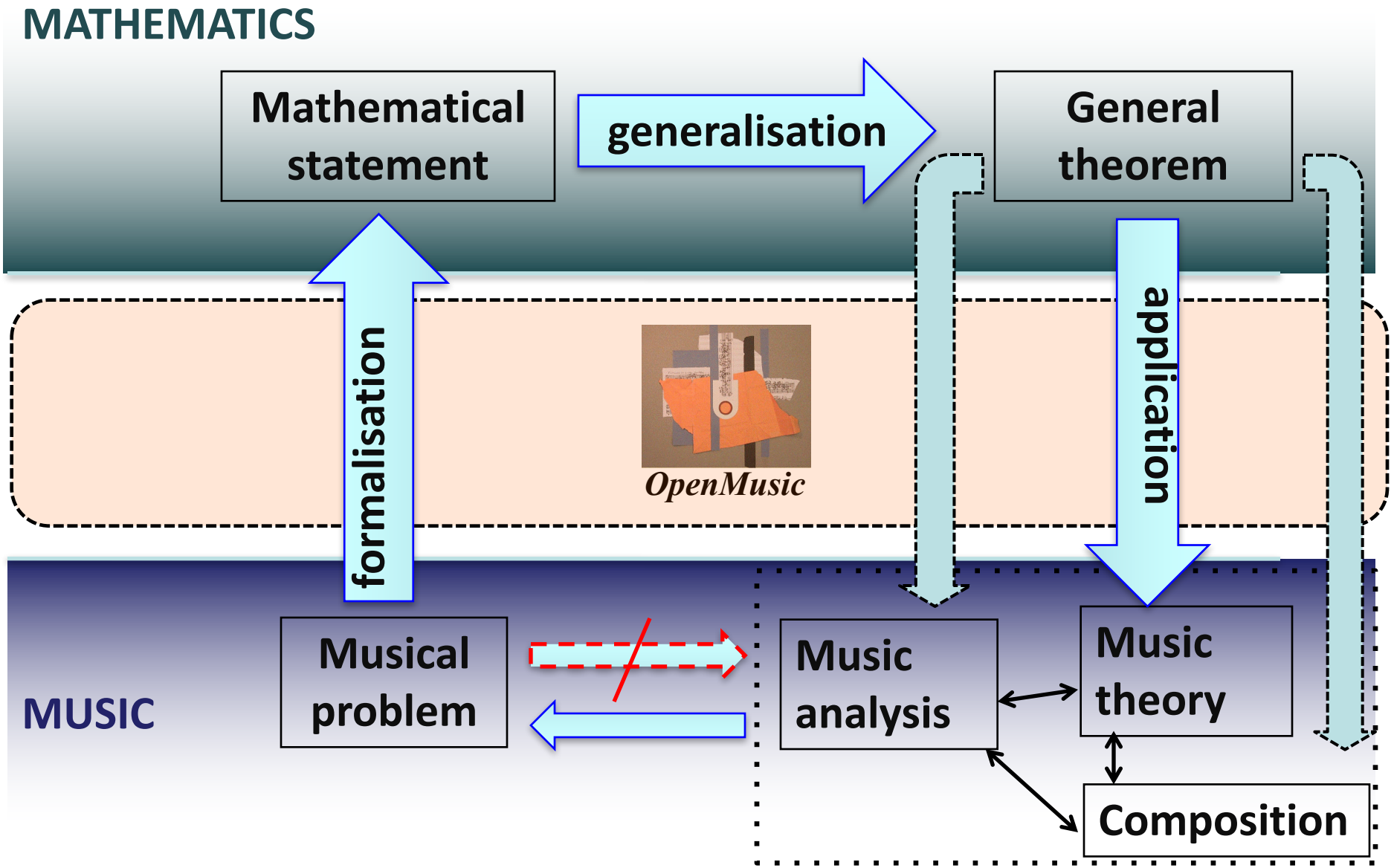



Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS/UPMC & IRMA/USIAS
http://repmus.ircam.fr/moreno/smir

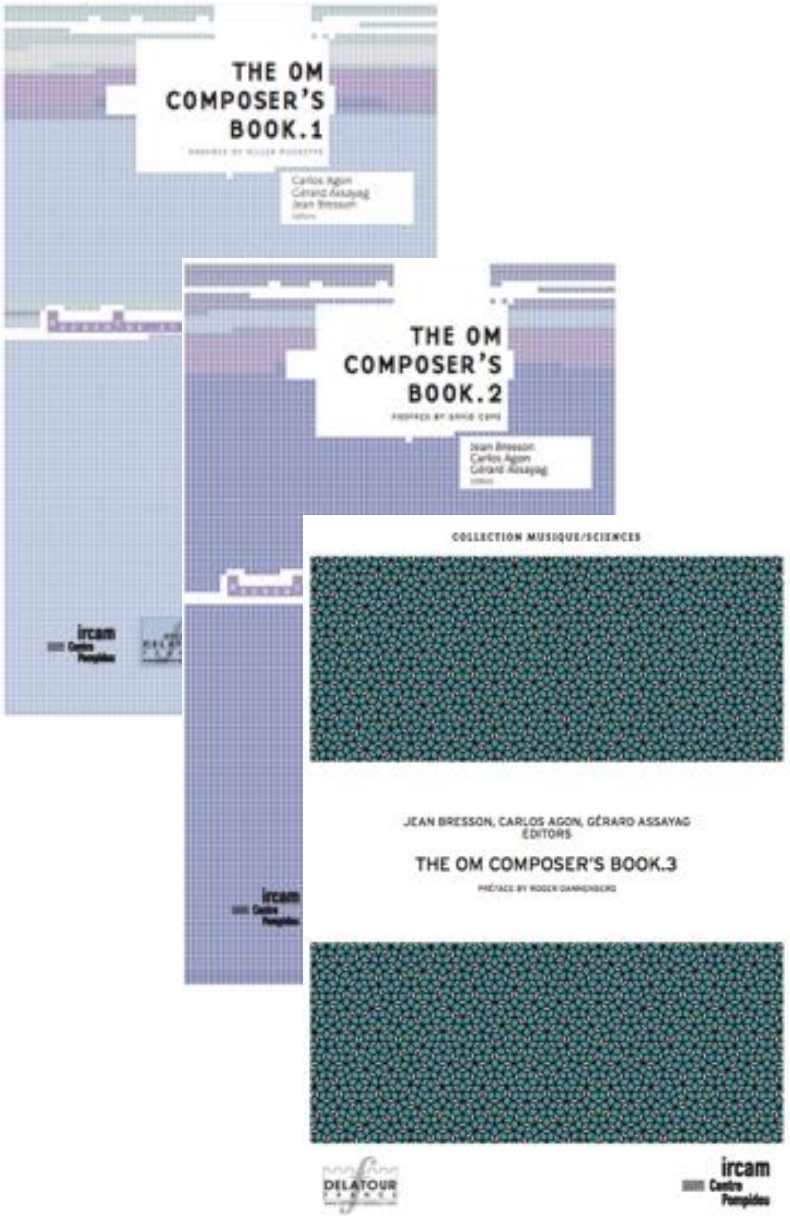
ircam Centre Pompidou
cnrs
IRMA Institut de Recherche Mathématique Avancée
USIAS University of Strasbourg Institute for Advanced Study
UPMC SORBONNE UNIVERSITÉS

# The double movement of a ‘mathemusical’ activity



# OpenMusic, a Visual Programming Language for computer-aided composition

www.repmus.ircam.fr/openmusic/home


**C. Agon, G. Assayag and J. Bresson, *The OM Composer's Book* (3 volumes) "Musique/Sciences" Series, Ircam/Delatour, 2006, 2007 and 2016**

# "MathTools": an algebraic environment within OpenMusic visual programming language

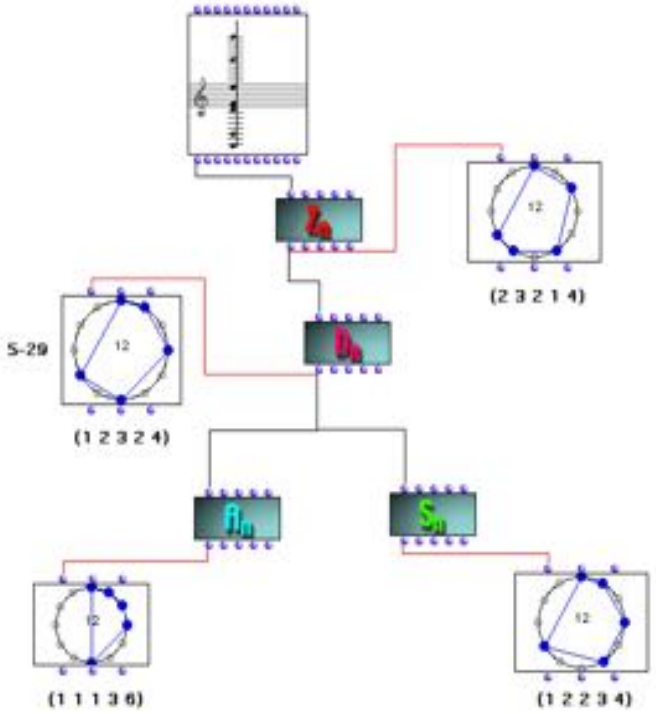
OM=♬+∑

## Computational Music Theory:

- Classification and Enumeration of musical structures
  - Chords/scales, motifs and rhythms:
    - ♬ Catalogues (Costère, Zalewski, Vieru, Forte, Carter, Morris, Mazzola, Estrada, …)
    - ∑ Combinatorial algebra, Polya Enumeration Theory, Burnside Lemma, Discrete Fourier Transform
  - Rhythmic Tiling Canons (by translation, inversion and augmentation)
    - ♬ Messiaen, Vieru, Levy, Johnson, Bloch, Wild, Lanza, Ghisi, …
    - ∑ Group and ring factorization theory and Discrete Fourier Transform (DFT)

## Computational Music Analysis:

- *Set Theory,* Transformational Analysis and Sieve Theory
  - Pitch-class sets, interval vectors and IFUNC, Z-relations:
    - ♬ Carter, Vieru, Xenakis
    - ∑ Group Actions, Homometry, DFT
  - Transformational progression/network, *K-nets*
    - ♬ Generalized Interval Systems (David Lewin)
    - ∑ Group action and category theory



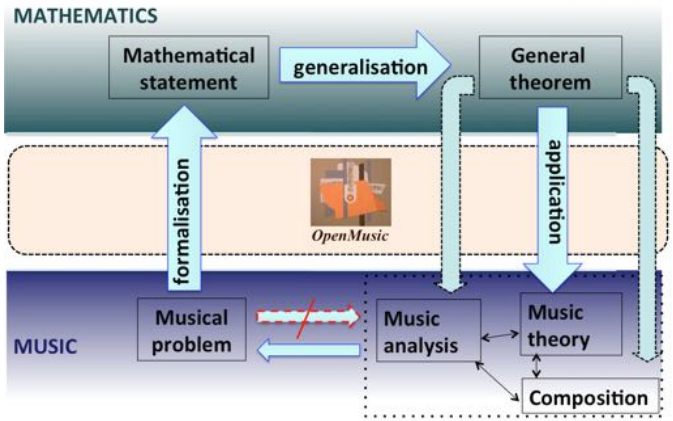

## Computer Aided-Composition:

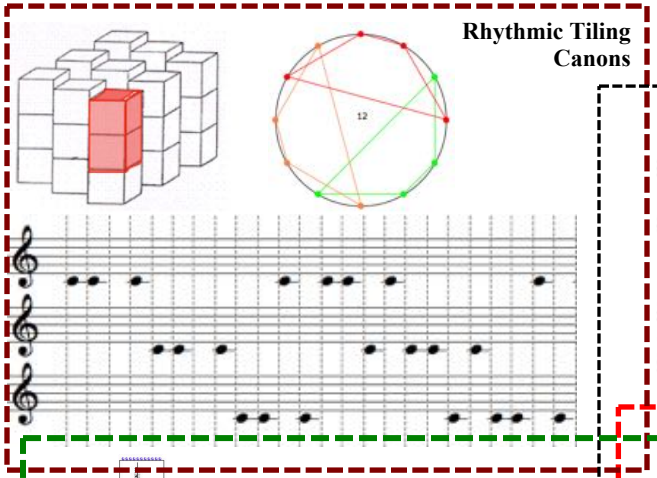
- Scales, modes, chords manipulation
- Rhythmic structures organizations

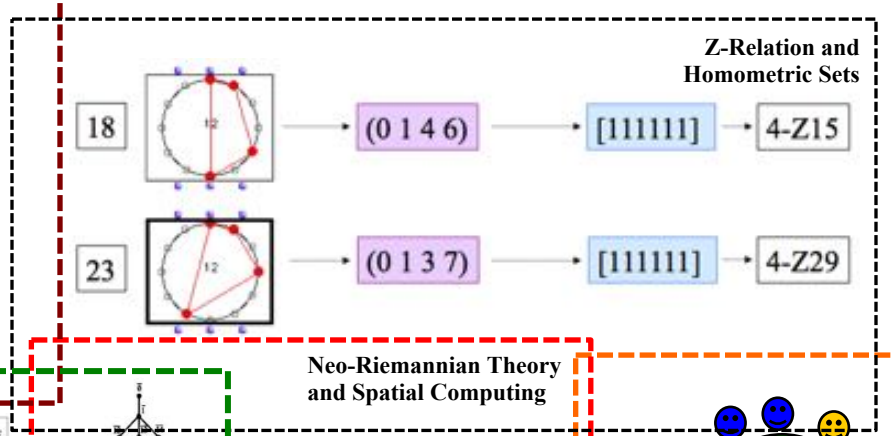
# Some examples of ‘mathemusical’ problems

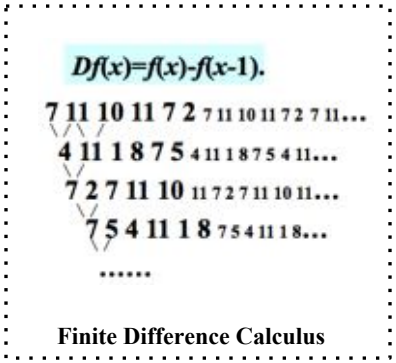
**M. Andreatta: *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010**

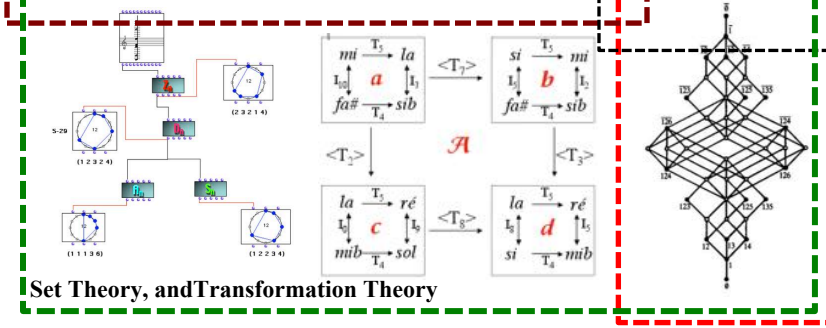
- The construction of Tiling Rhythmic Canons
- The *Z* relation and the theory of homometric sets
- *Set Theory* and Transformational Theory
- Neo-Riemannian Theory, Spatial Computing and FCA
- Diatonic Theory and Maximally-Even Sets
- Periodic sequences and finite difference calculus
- Block-designs and algorithmic composition



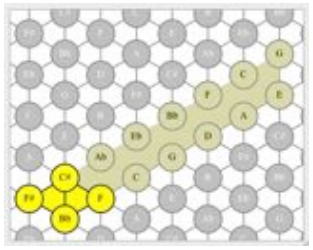



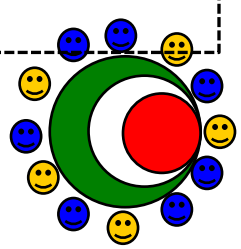



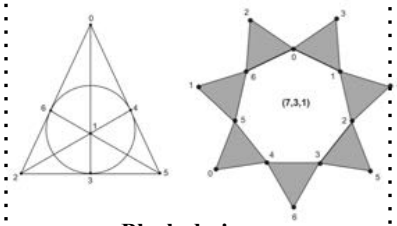



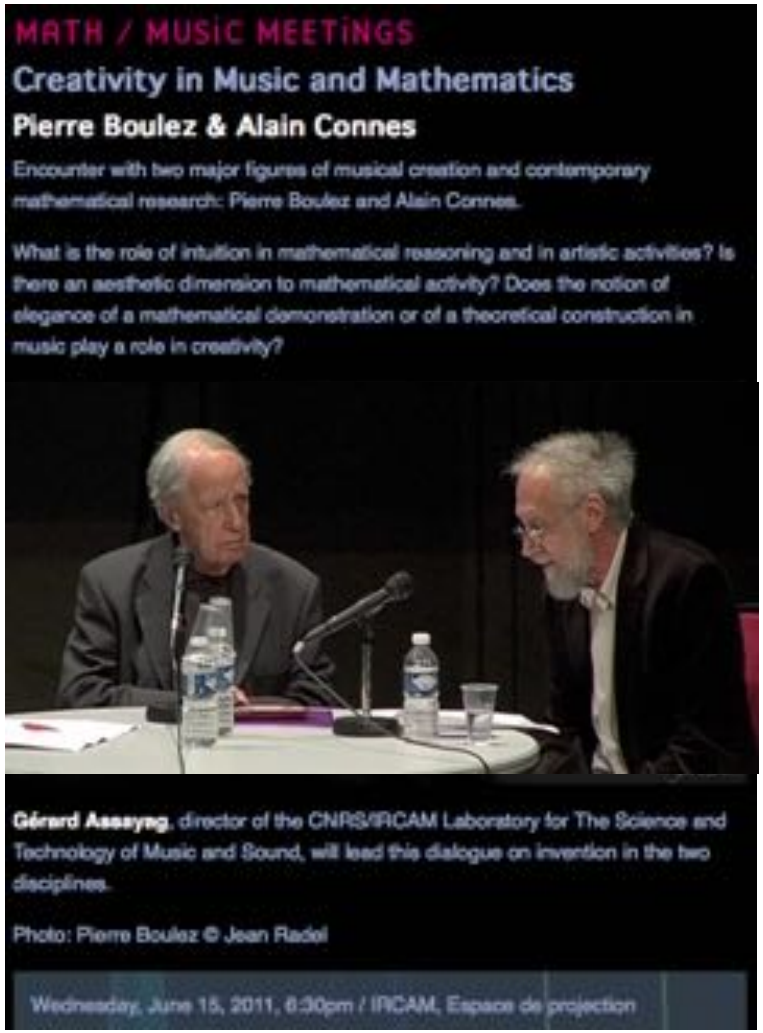


# The interplay between algebra and geometry in music




➔ `http://agora2011.ircam.fr`

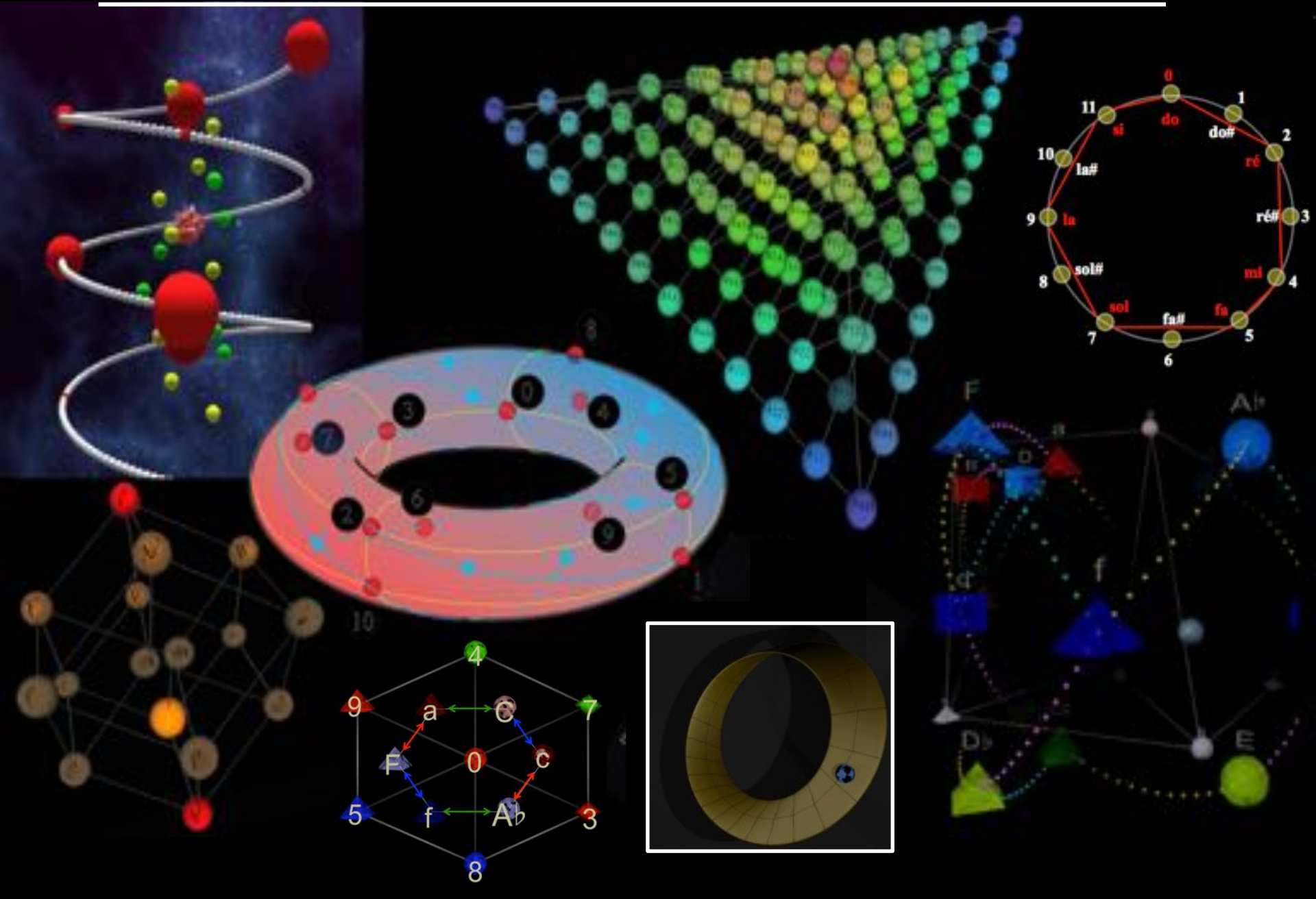


"Concerning **music**, it takes place in **time**, like **algebra**. In **mathematics**, there is this fundamental duality between, on the one hand, **geometry** – which corresponds to the visual arts, an immediate intuition – and on the other hand **algebra**. This is not visual, it has a temporality. This fits in time, it is a computation, something that is very close to the language, and which has its diabolical precision. [...] **And one only perceives the development of algebra through music**" (A. Connes).

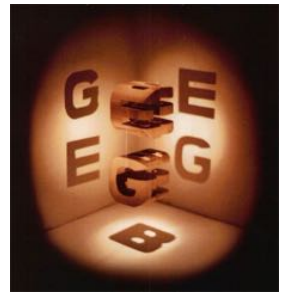
Société Mathématique de France

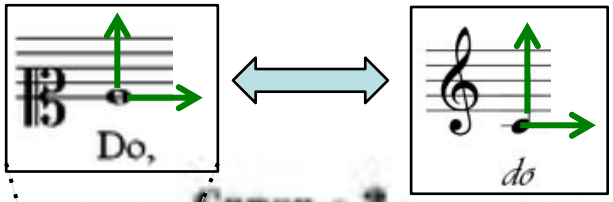
SMF

# The galaxy of geometrical models at the service of music

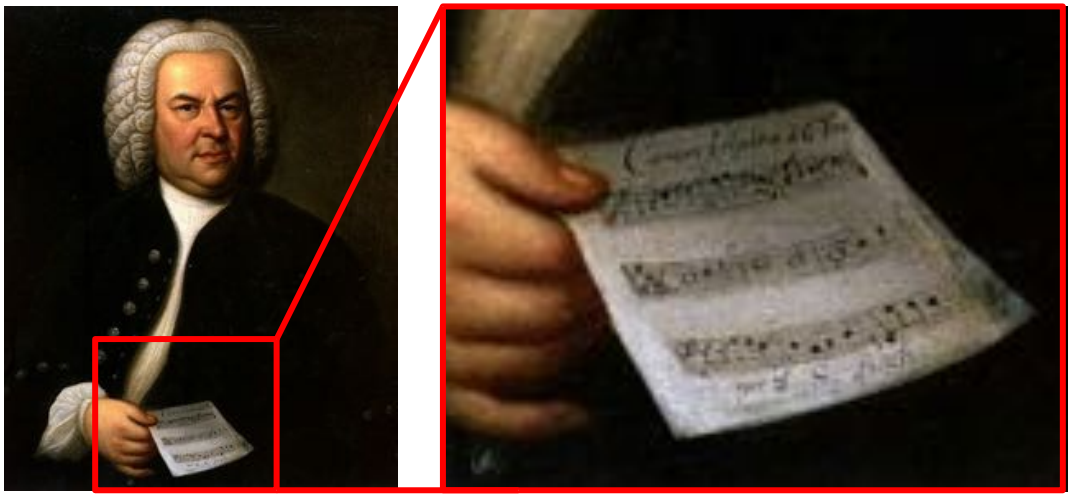


# Bach's enigmatic canons and geometry

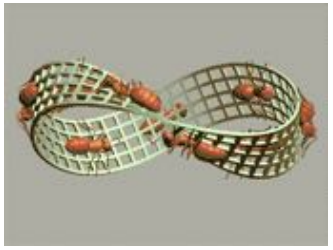


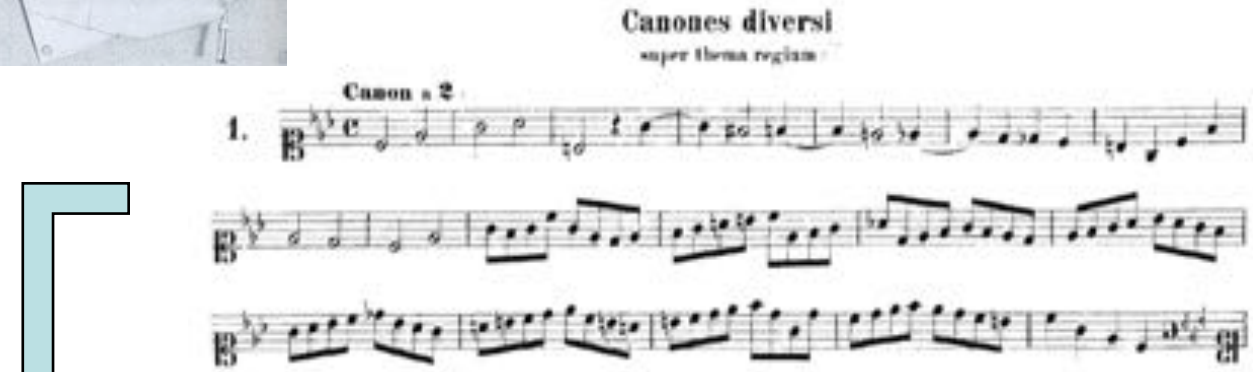



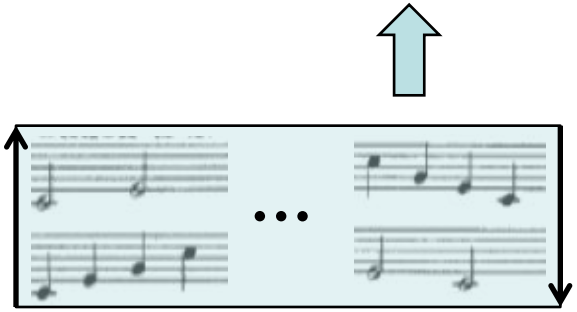




# My end is my beginning (but twisted!)





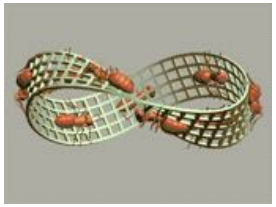
# BACH

## Musikalisches Opfer

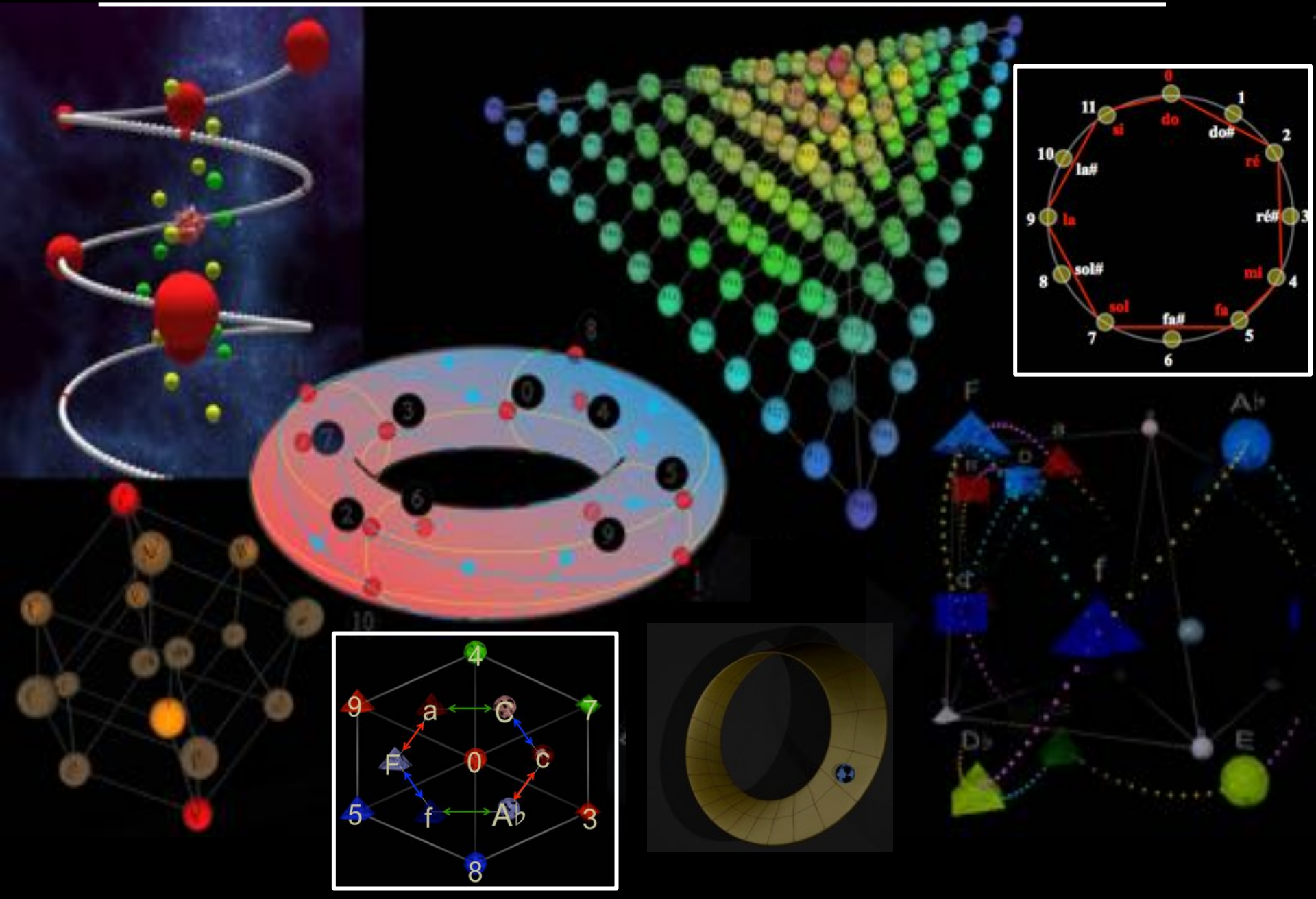
Musical Offering - Offrande Musicale

http://www.josleys.com/Canon/Canon.html

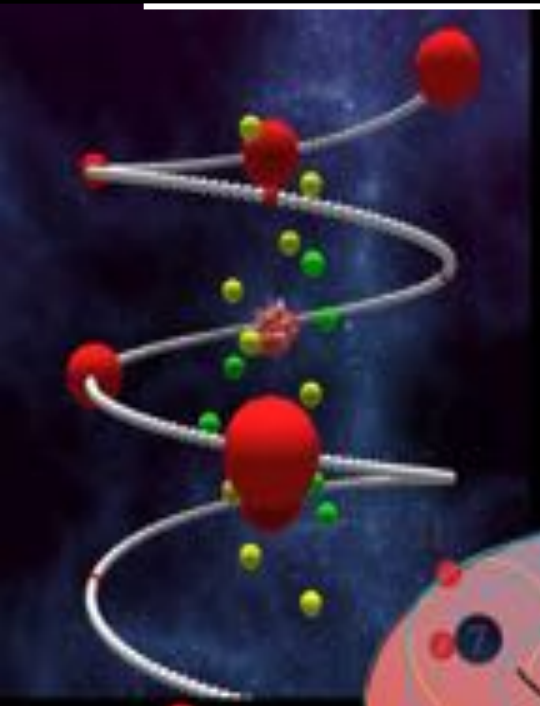
**[min. 1’14”]**

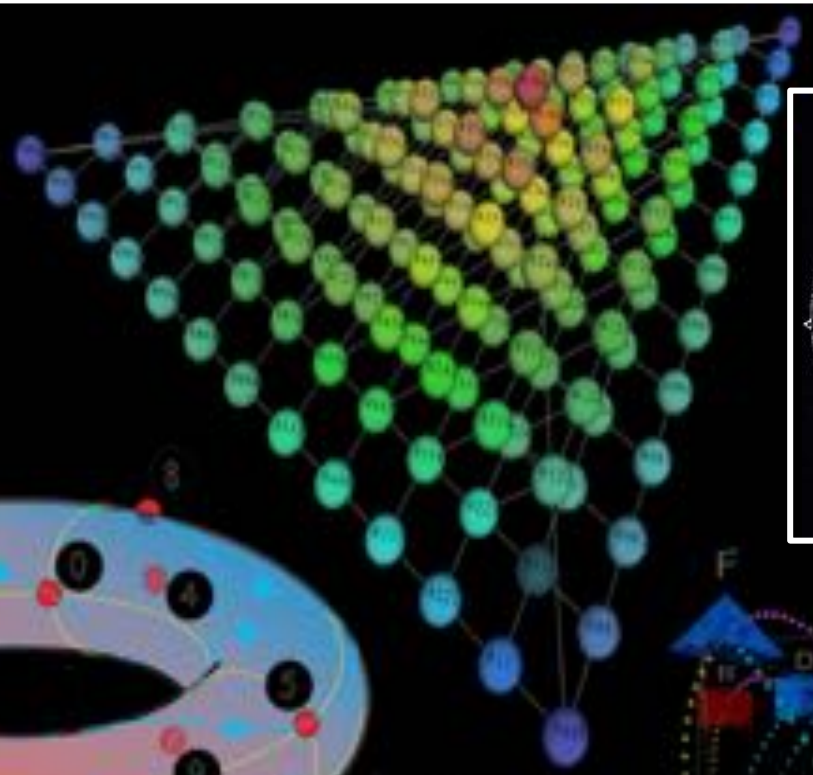


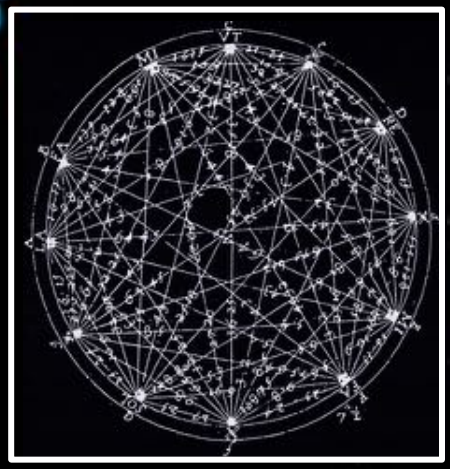
# The galaxy of geometrical models at the service of music

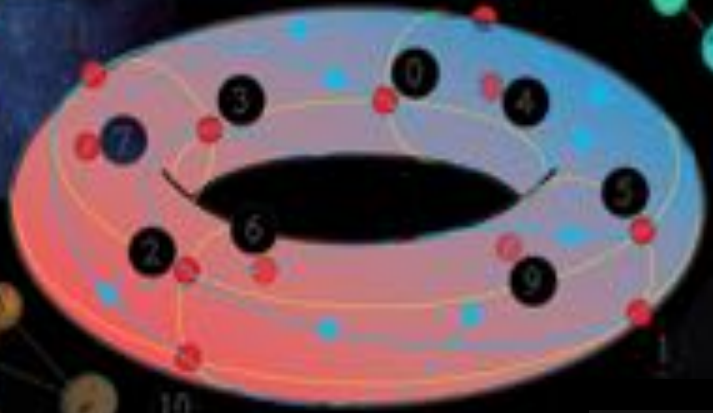


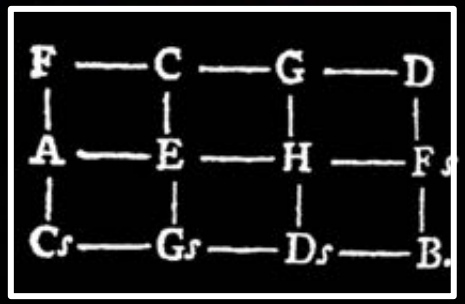
# The galaxy of geometrical models at the service of music

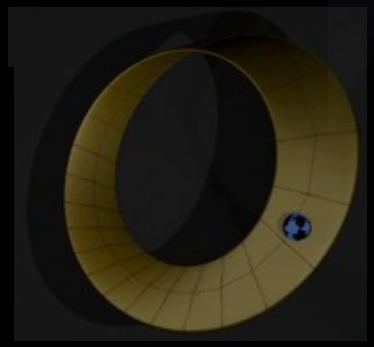








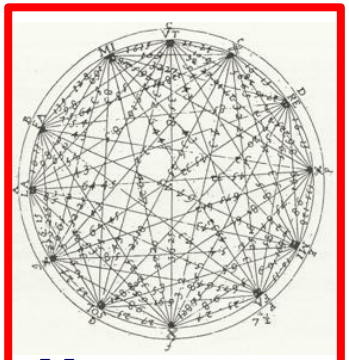


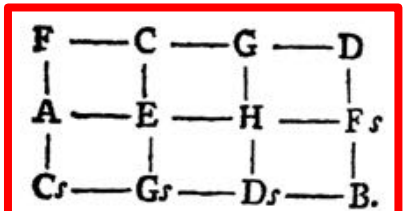
# Music and mathematics: « prima la musica »!

| MUSIQUE | MATHS |
|---|---|
| **500 av. J. C.** Relation hauteur/longueur corde. La musique est source d'inspiration pour la théorie des nombres et la géométrie. | Nombres naturels et rationnels. |
| *Pas de correspondance musicale.* | Nombres irrationnels, théorème de Pythagore. |
| **300 a.J.** Invention (théorique) de la gamme chromatique tempérée égale par Aristoxénos de Tarente) et **prémonition de la théorie des groupes**. Isomorphismes entre les logarithmes (intervalles musicaux) et les exponentiels (longueur d'une corde). | Les mathématiques ne réagissent pas. |
| **1000 ap. J.C.** Invention de la représentation bidimensionnelle des hauteurs. | *Aucune correspondance.* |
| **1500** *Aucune reprise des concepts précédents.* | Nombres négatifs. Construction des rationnels. |
| **1600** *Aucune relation.* | Nombres réels et les logarithmes. Invention des repères cartésiens. |
| **1648** Marin Mersenne : invention de la combinatoire musicale (*Harmonicorum Libri*) | Systématisation du calcul des probabilités par Bernoulli (*Ars Conjectandi*, 1713) |
| **1700** La fugue comme un automate abstrait. Manipulation inconsciente du groupe de Klein. | Nombres complexes (Euler, Gauss), les quaternions (Hamilton), continuité (Cauchy), structure de groupe (Galois, Abel). |
| **1773** Leonhard Euler : représentation géométrique des hauteurs (*Speculum Musicum*) | Invention de la théorie des graphes |
| **1855** Camille Durutte : analyse harmonique, rythmique et mélodique | Développement en série d'une fonction (Wronski) |
| **1900** Libération de la prison de la tonalité (Loquin, Hauer, Schoenberg). | Nombres infinis et transfinis (Cantor). Axiomatique de Peano. Théorie de la mesure (Lebesgue, Borel). |
| **1920** Formalisation radicale des macrostructures à travers le système sériel (Schoenberg). | *Aucun développement de la théorie des nombres.* Logique (contradictions de la théorie des ensembles). |
| **1937-1939** Ernst Krenek : les axiomes en musique | David Hilbert, *Les fondements de la géométrie* (1899) |
| **1946** Milton Babbitt : théorie des groupes et système dodécaphonique | Rudolf Carnap, *The Logical Syntax of Language* (1937) |

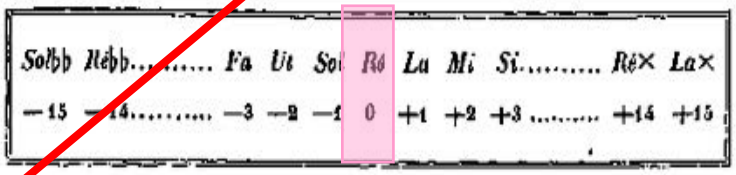
Iannis Xenakis



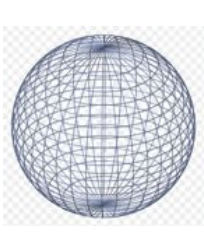
**Mersenne, *Harmonicorum Libri* XII, 1648**

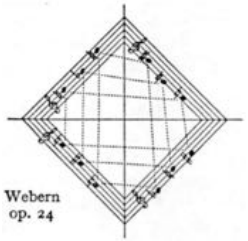


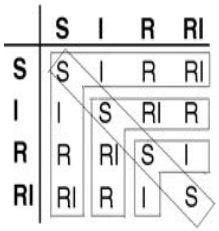

**Euler, *Speculum musicum*, 1773**




**Durutte, *Technie, ou lois générales du système harmonique* (1855)**






**Krenek and Babbitt, twelve-tone method, axiomatics and Klein group**

# Mersenne and the birth of (musical) combinatorics

114 **Marin Mersenne, *Harmonicorum Libri XII*, 1648**

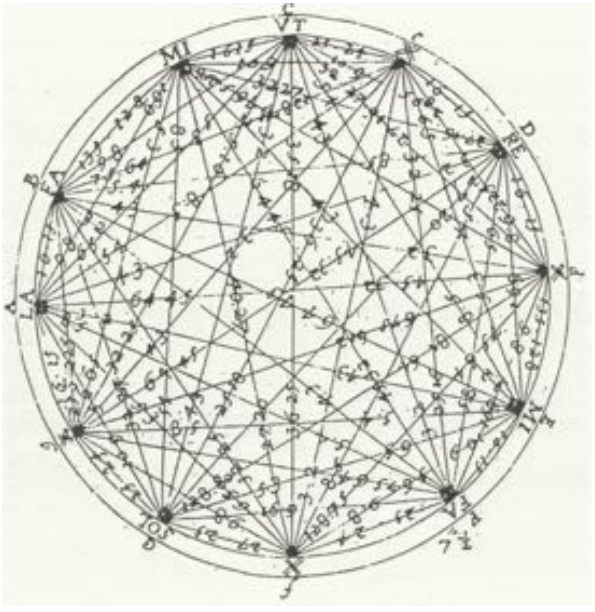
## LIBER SEPTIMVS.

DE CANTIBVS, SEV CANTILENIS,

EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

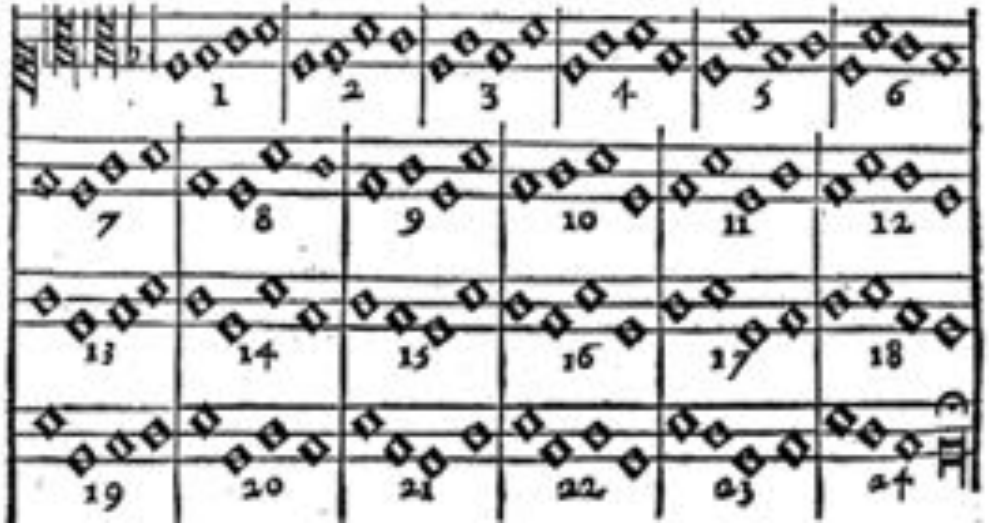
*Tabula Combinationis ab 1 ad 22.*

| | |
|---|---|
| I | 1 |
| II | 2 |
| III | 6 |
| IV | 24 |
| V | 120 |
| VI | 720 |
| VII | 5040 |
| VIII | 40320 |
| IX | 362880 |
| X | 3628800 |
| XI | 39916800 |
| XII | 479001600 |
| XIII | 6227020800 |
| XIV | 87178291200 |
| XV | 1307674368000 |
| XVI | 20922789888000 |
| XVII | 355687428096000 |
| XVIII | 6402373705728000 |
| XIX | 121645100408832000 |
| XX | 2432902008176640000 |
| XXI | 51090942171709440000 |
| XXII. | 1124000727777607680000 |



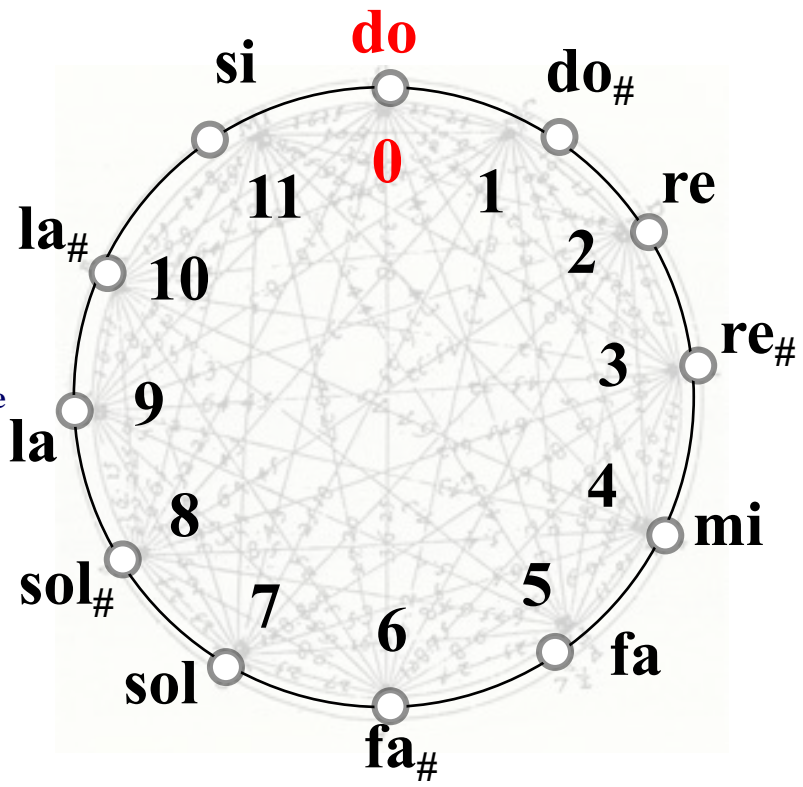
Marin Mersenne

*Varietas, seu Combinatio quatuor notarum.*



# The circular representation of the pitch space

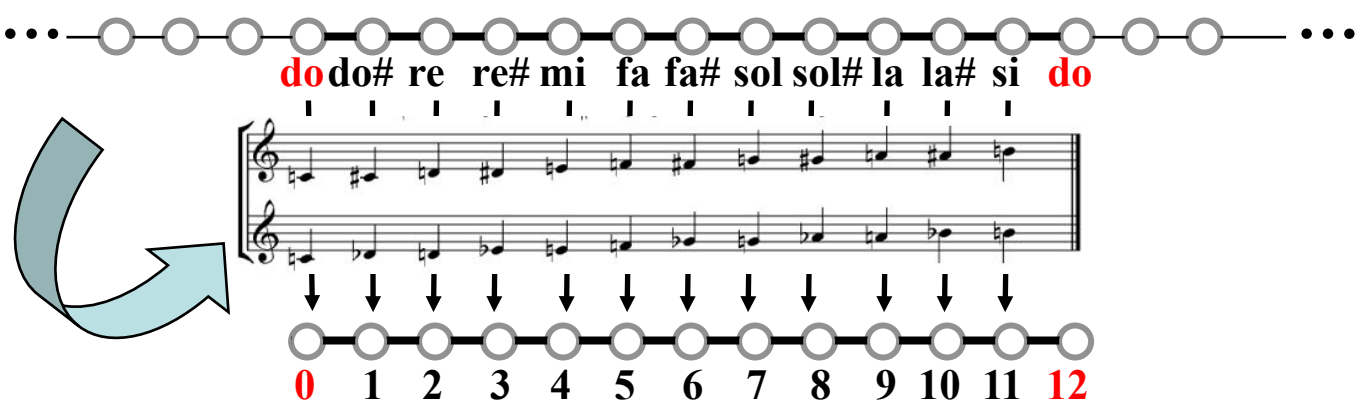
Marin Mersenne




*Harmonicorum Libri XII*, 1648

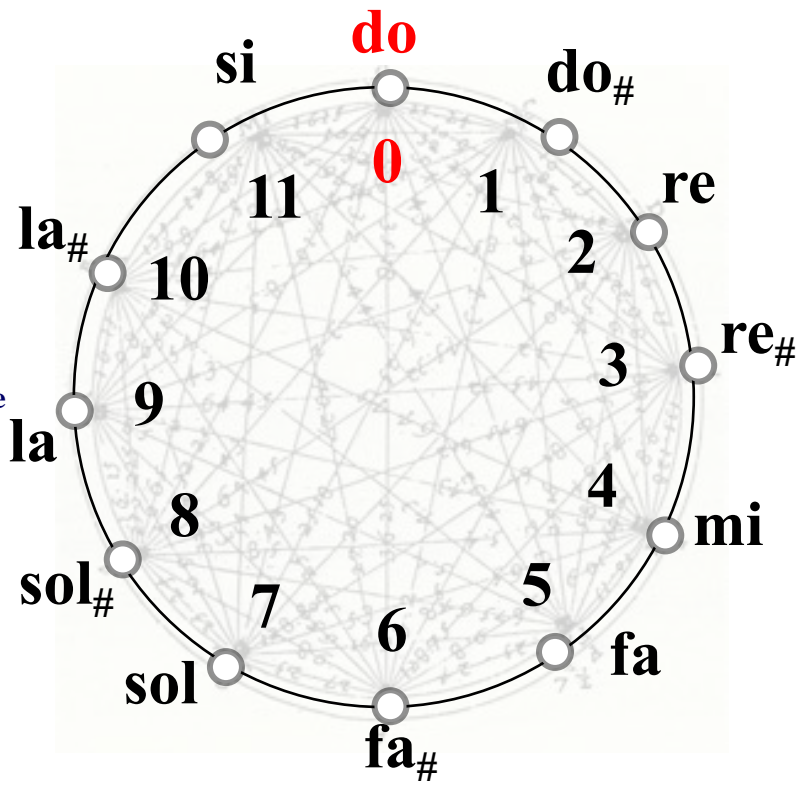




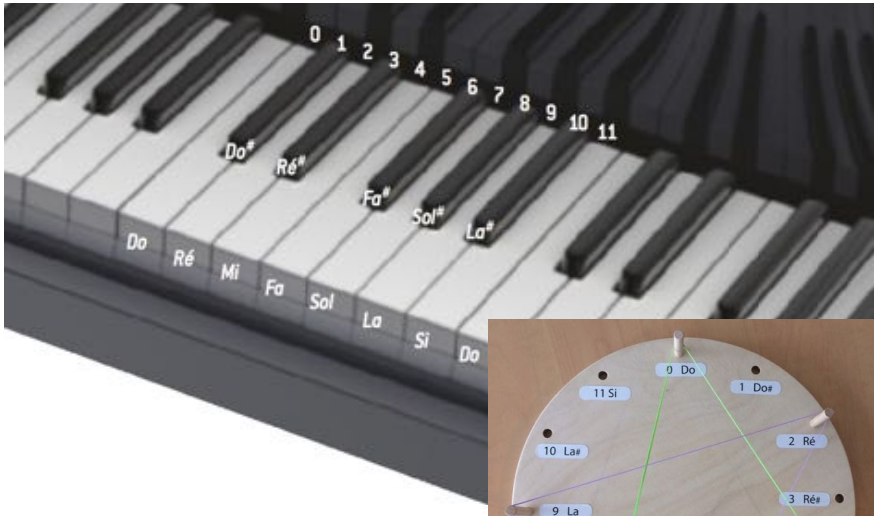


# The circular representation of the pitch space

Marin Mersenne

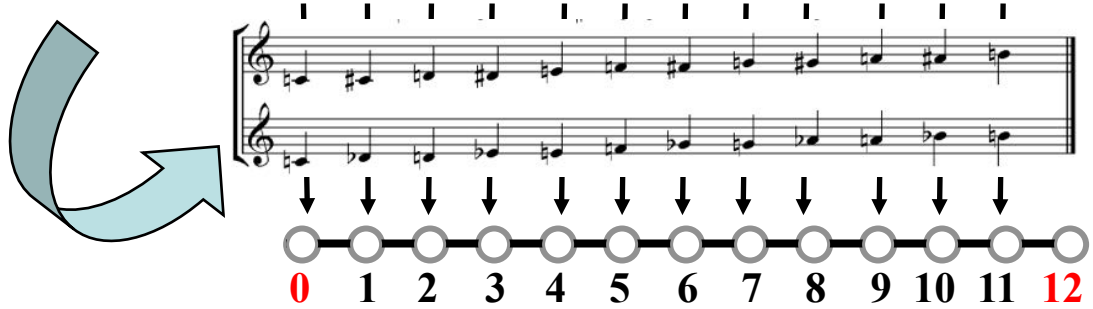



*Harmonicorum Libri XII*, 1648





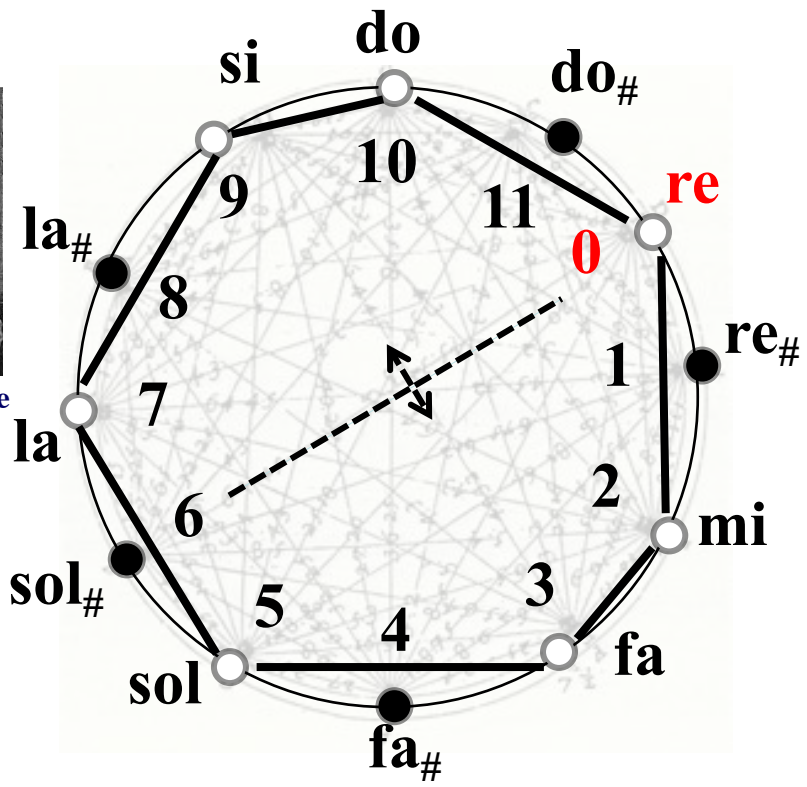




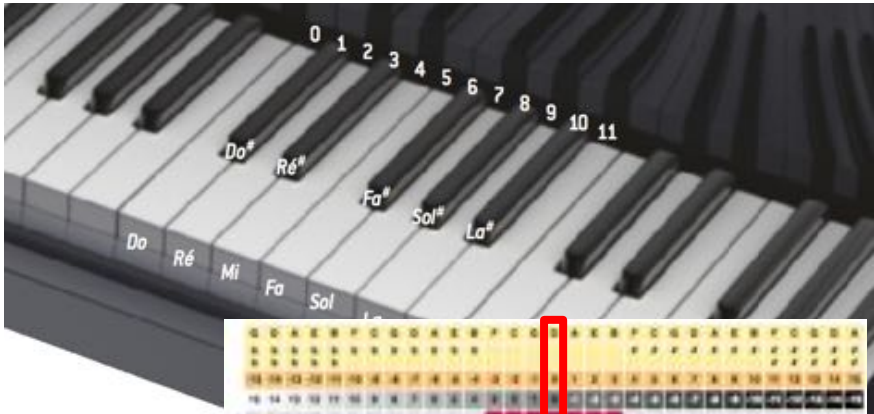

➔ DEMO

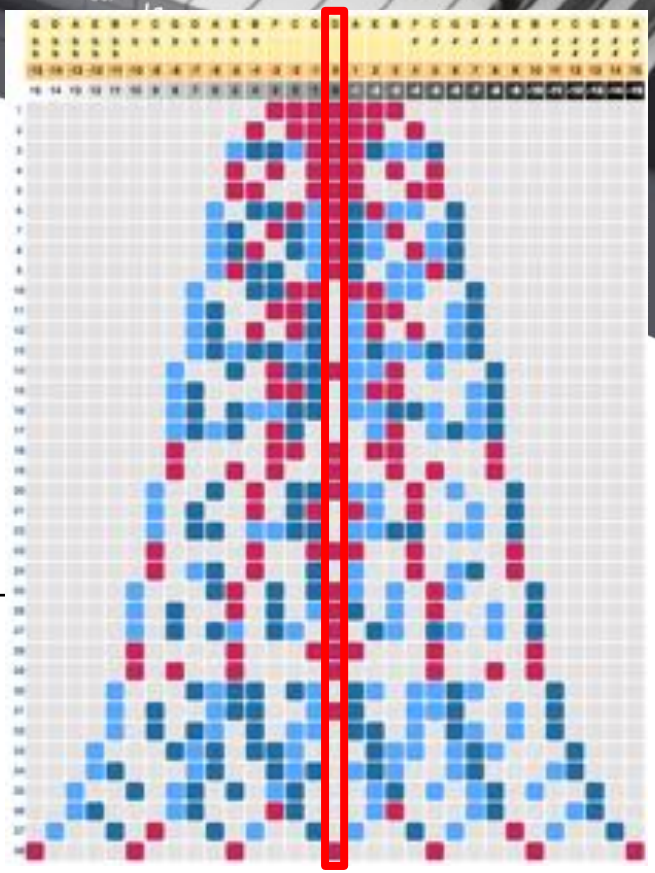
# The circular representation of the pitch space

Marin Mersenne

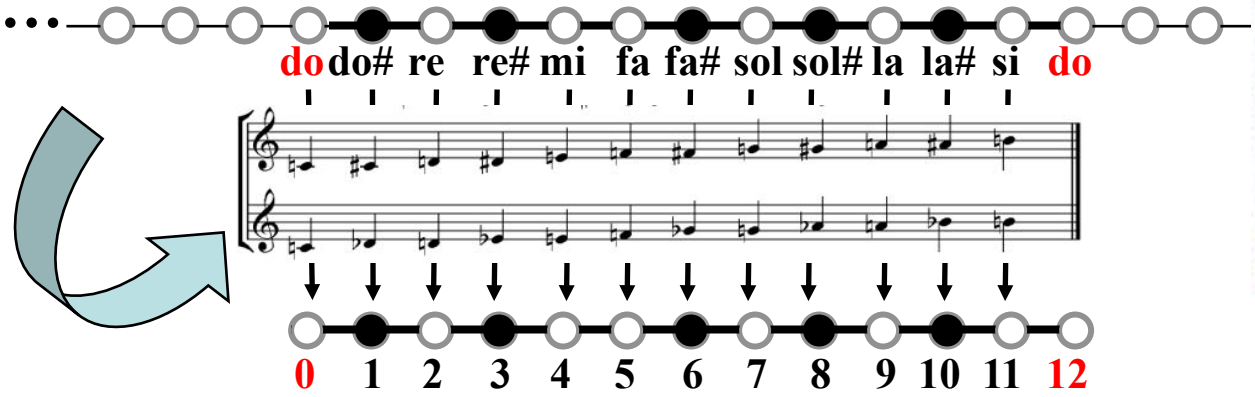



*Harmonicorum Libri XII*, 1648



http://www.cloche-diatonique.ch/



# Permutational melodies in contemporary (art) music

114 **Marin Mersenne, *Harmonicorum Libri XII*, 1648**

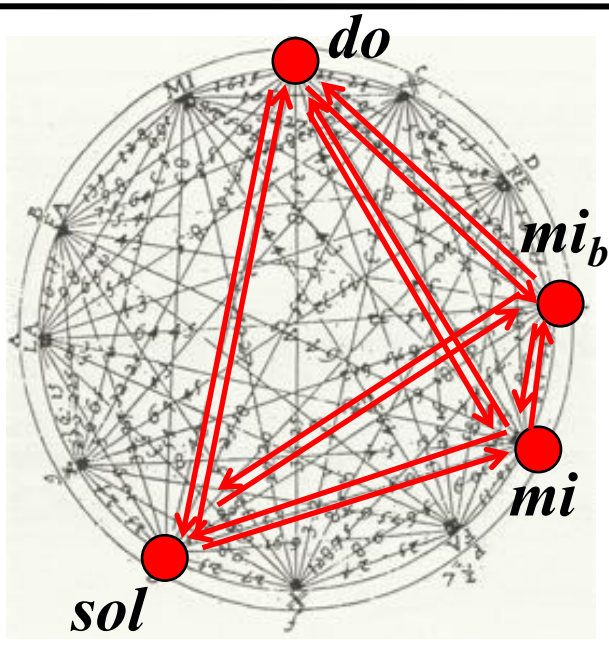
LIBER SEPTIMVS.

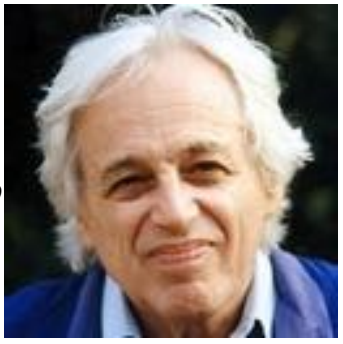
DE CANTIBVS, SEV CANTILENIS,

EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

*Tabula Combinationis ab 1 ad 22.*

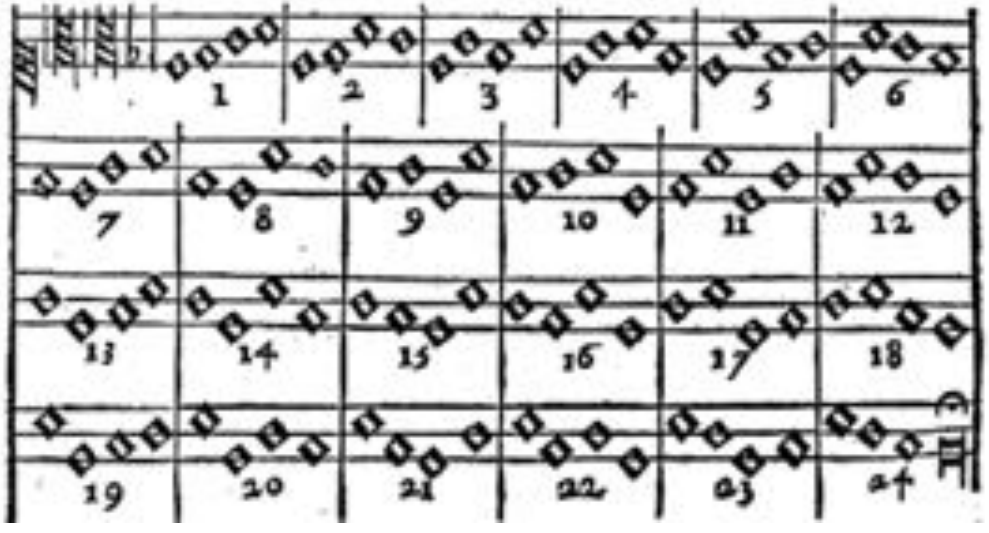
| | |
|---|---|
| I | 1 |
| II | 2 |
| III | 6 |
| IV | 24 |
| V | 120 |
| VI | 720 |
| VII | 5040 |
| VIII | 40320 |
| IX | 362880 |
| X | 3628800 |
| XI | 39916800 |
| XII | 479001600 |
| XIII | 6227020800 |
| XIV | 87178291200 |
| XV | 1307674368000 |
| XVI | 20922789888000 |
| XVII | 355687428096000 |
| XVIII | 6402373705728000 |
| XIX | 121645100408832000 |
| XX | 2432902008176640000 |
| XXI | 51090942171709440000 |
| XXII. | 1124000727777607680000 |



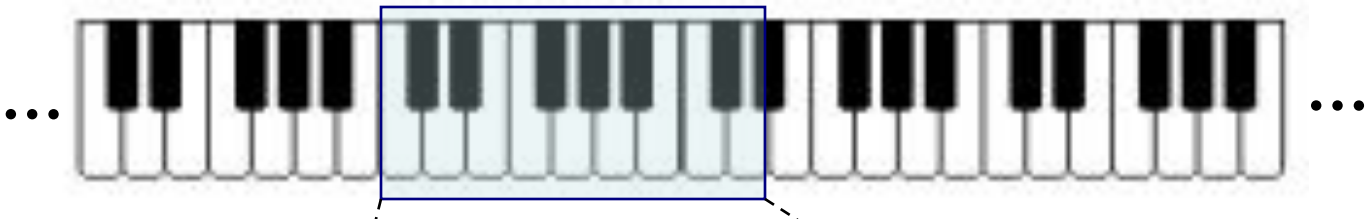



***Six Bagatelles***
**(G. Ligeti, 1953)**

*Varietas, seu Combinatio quatuor notarum.*

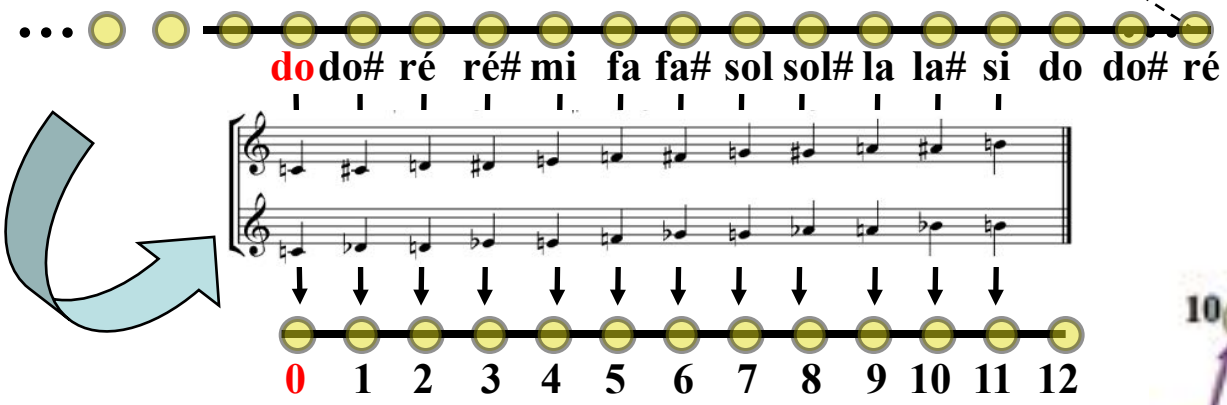


# The equal tempered space is a cyclic group

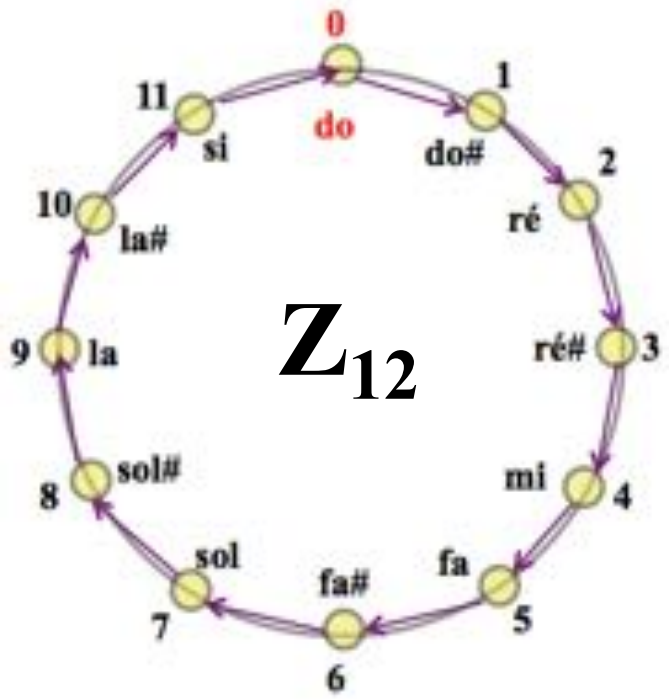


The generators of the cyclic group of order 12 are the transpositions $T_1$, $T_5$, $T_7$ et $T_{11}$ where

$T_k: x \rightarrow x+k \bmod 12$




$Z_{12} = < T_1 \mid (T_1)^{12} = T_0 >$



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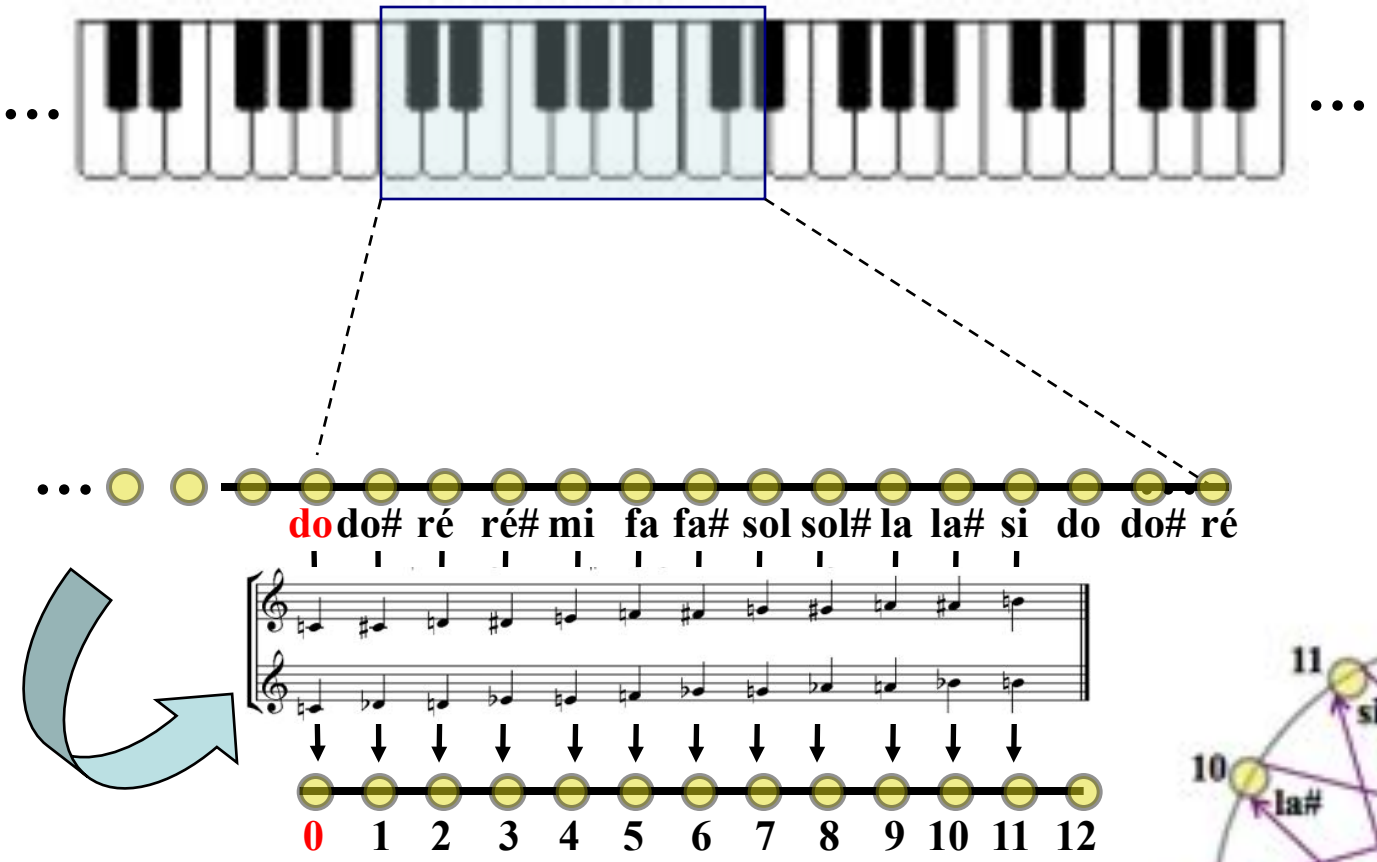
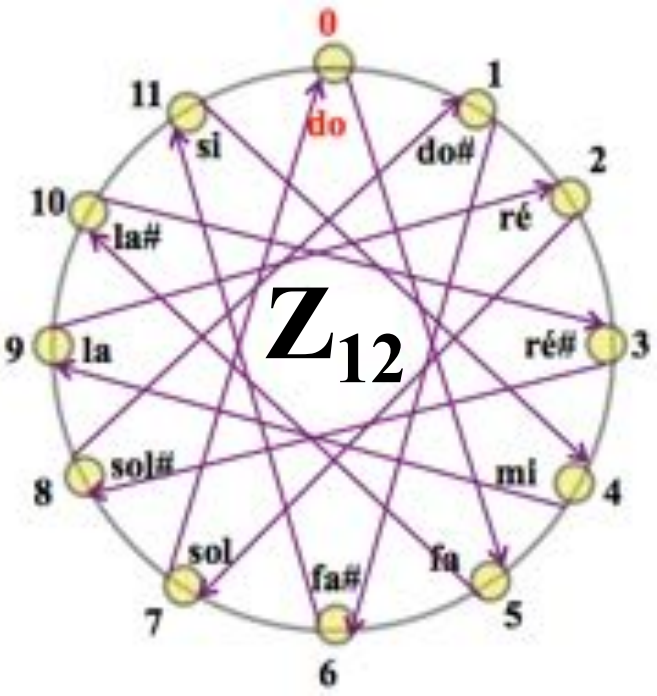


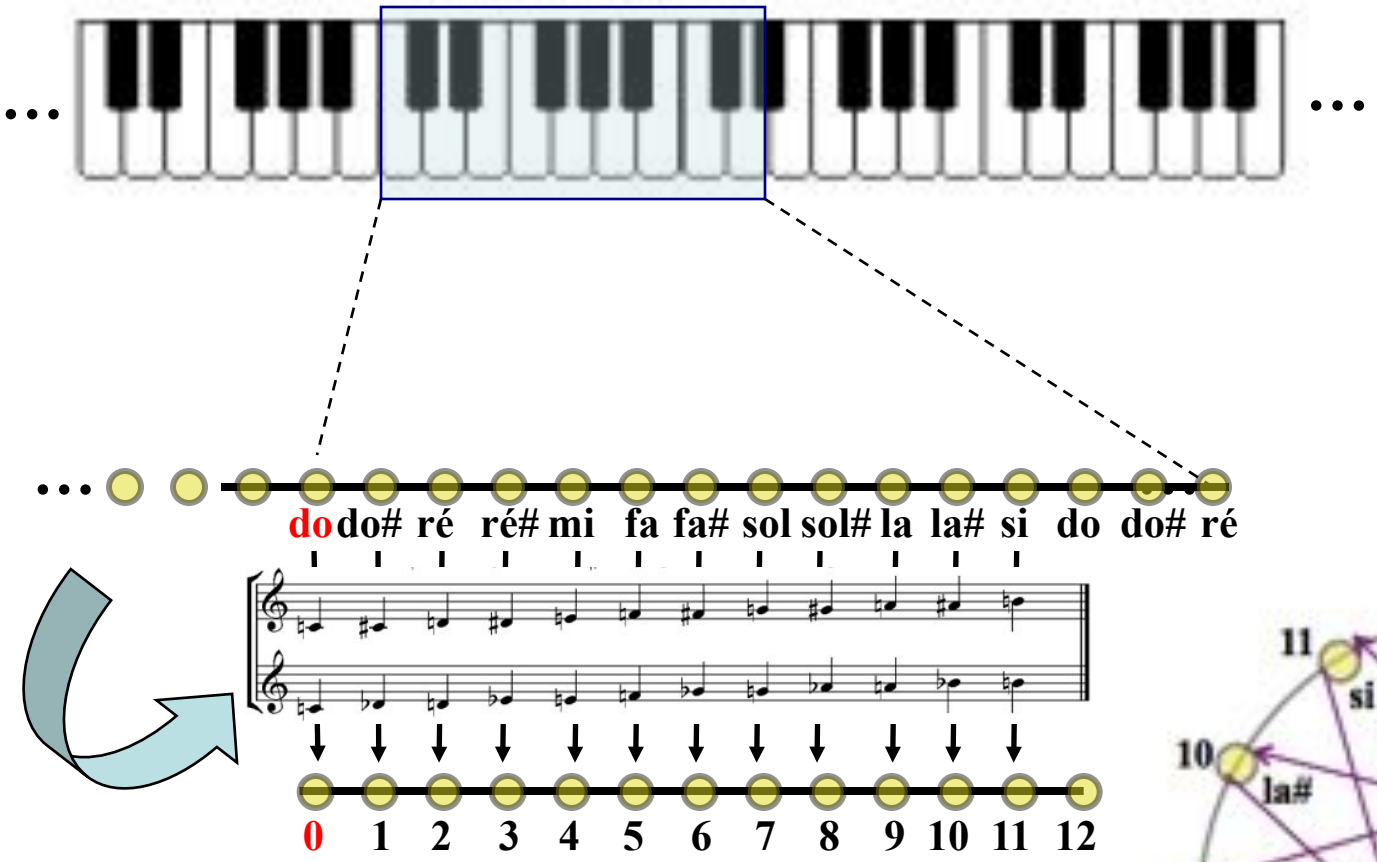

The generators of the cyclic group of order 12 are the transpositions $T_1$, $T_5$, $T_7$ et $T_{11}$ where

$T_k: x \rightarrow x+k \bmod 12$

$$Z_{12} = < T_1 \mid (T_1)^{12} = T_0 > =$$
$$= < T_5 \mid (T_5)^{12} = T_0 >$$



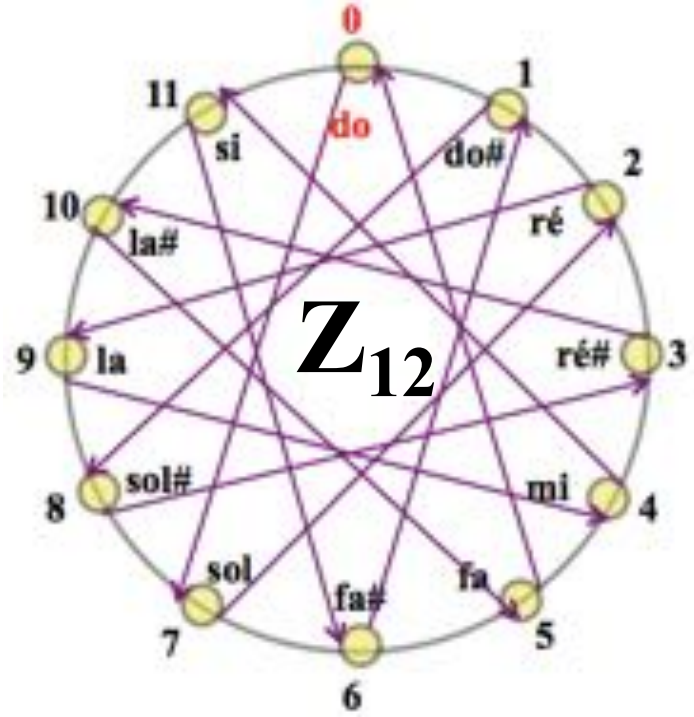
# The equal tempered space is a cyclic group



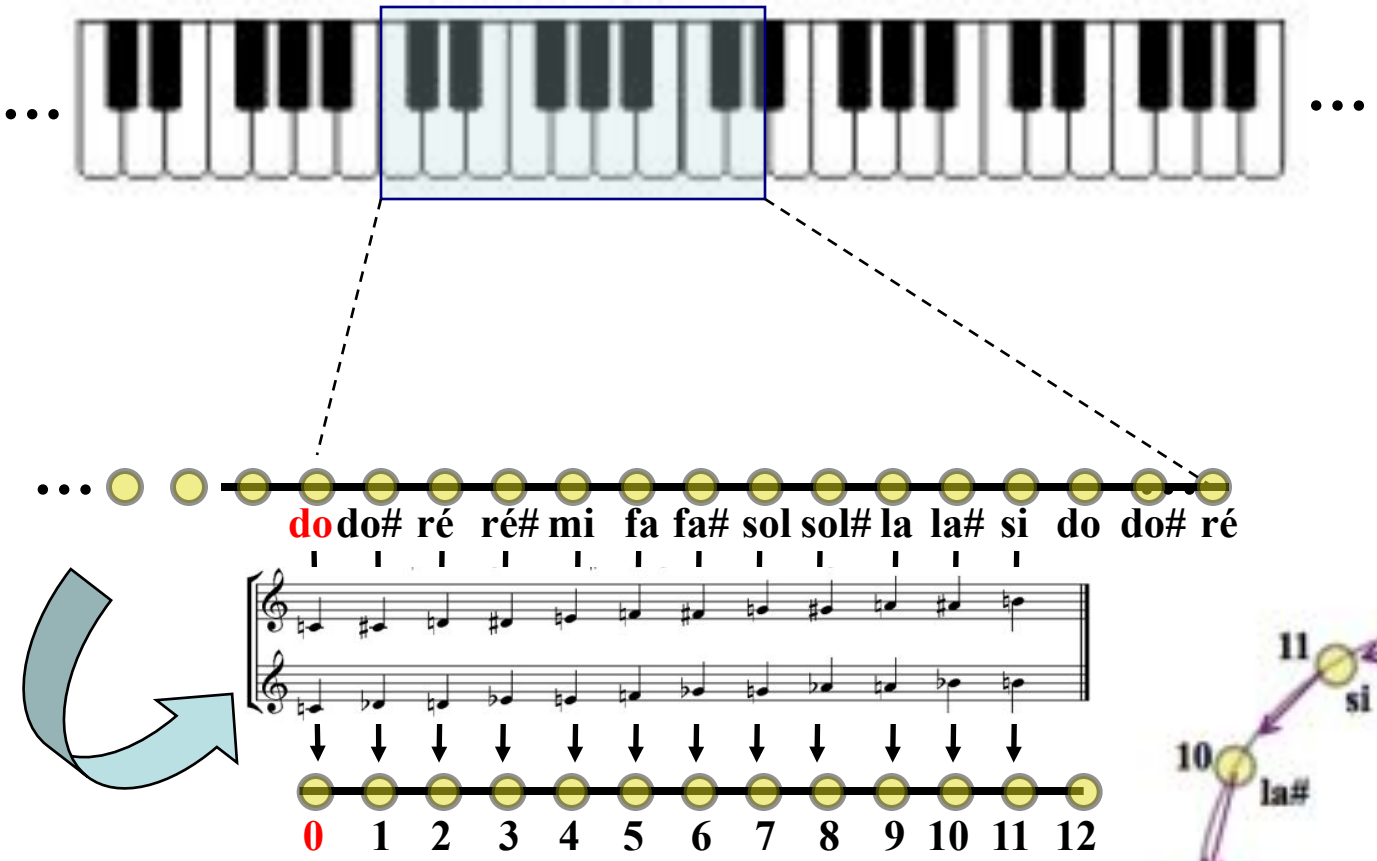

The generators of the cyclic group of order 12 are the transpositions $T_1$, $T_5$, $T_7$ et $T_{11}$ where

$T_k: x \rightarrow x+k \bmod 12$

$$Z_{12} = < T_1 \mid (T_1)^{12} = T_0 > = < T_5 \mid (T_5)^{12} = T_0 > = < T_7 \mid (T_7)^{12} = T_0 >$$



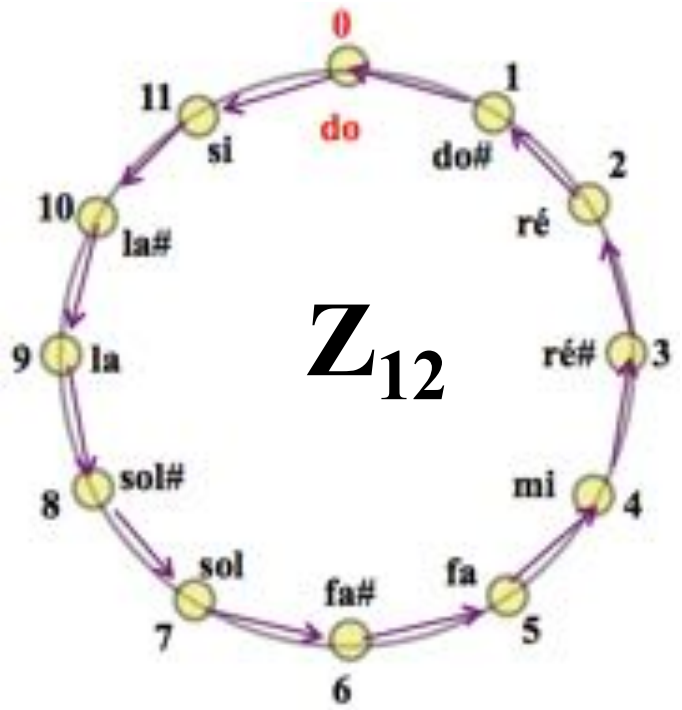
# The equal tempered space is a cyclic group




The generators of the cyclic group of order 12 are the transpositions $T_1$, $T_5$, $T_7$ et $T_{11}$ where

$T_k: x \rightarrow x+k \bmod 12$

$$Z_{12} = < T_1 \mid (T_1)^{12} = T_0 > =$$
$$= < T_5 \mid (T_5)^{12} = T_0 > =$$
$$= < T_7 \mid (T_7)^{12} = T_0 > =$$
$$= < T_{11} \mid (T_{11})^{12} = T_0 >$$

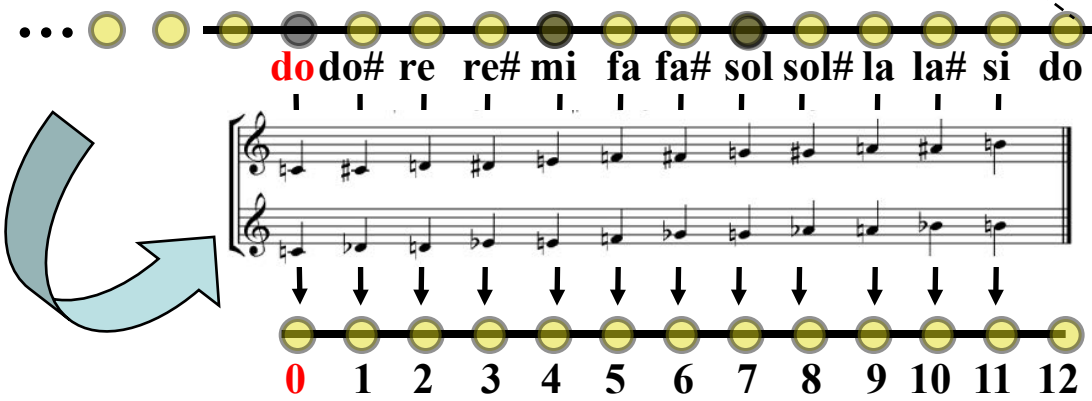


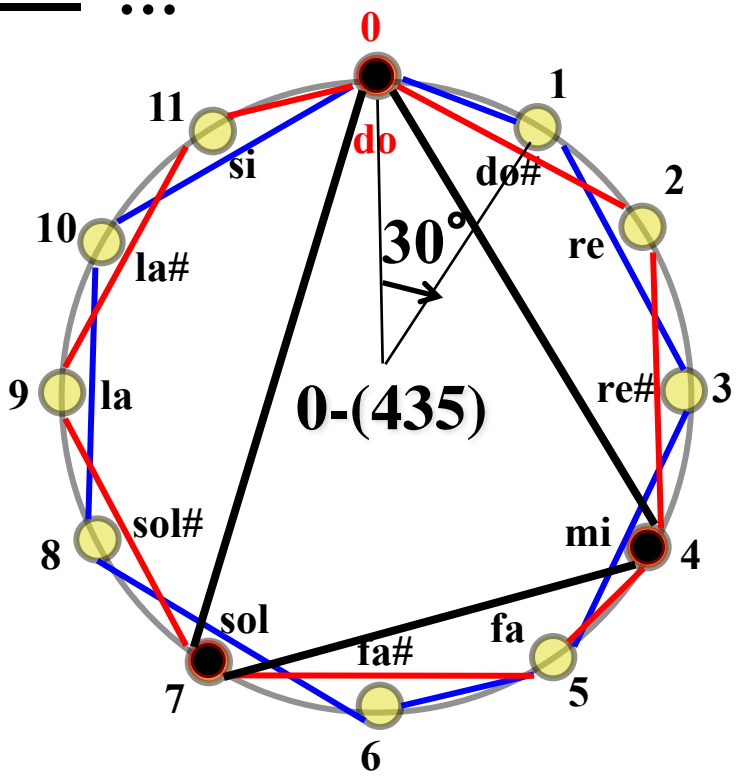
# Musical transpositions are additions...

$T_k : x \rightarrow x+k \bmod 12$

***Do* maj** = {0,2,4,5,7,9,11} +1

***Do#* maj** = {1,3,5,6,8,10,0}



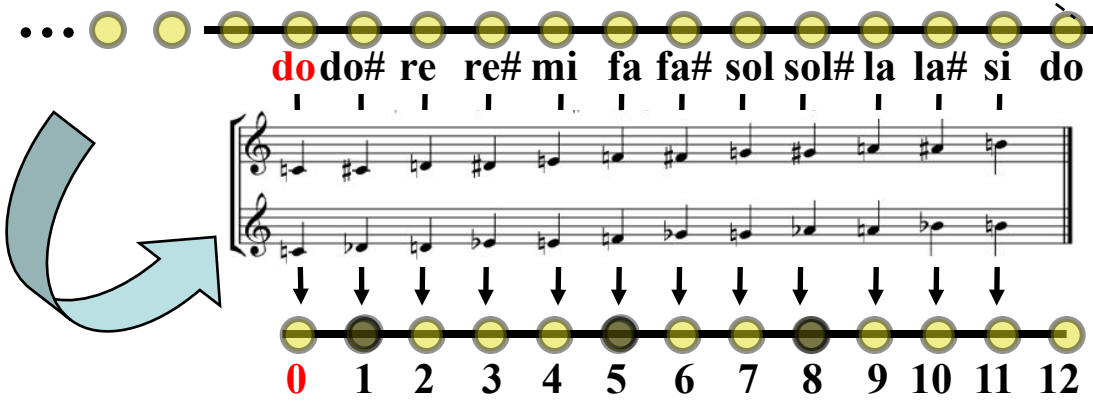

**...or rotations !**

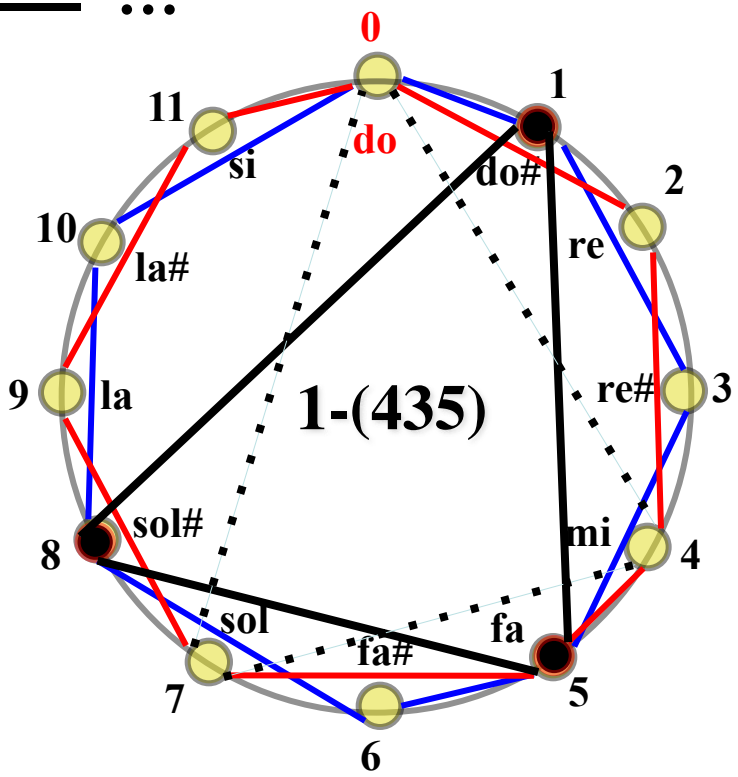
# Musical transpositions are additions...

$T_k : x \rightarrow x+k \bmod 12$

***Do* maj** = {0,2,4,5,7,9,11} +1

***Do#* maj** = {1,3,5,6,8,10,0}



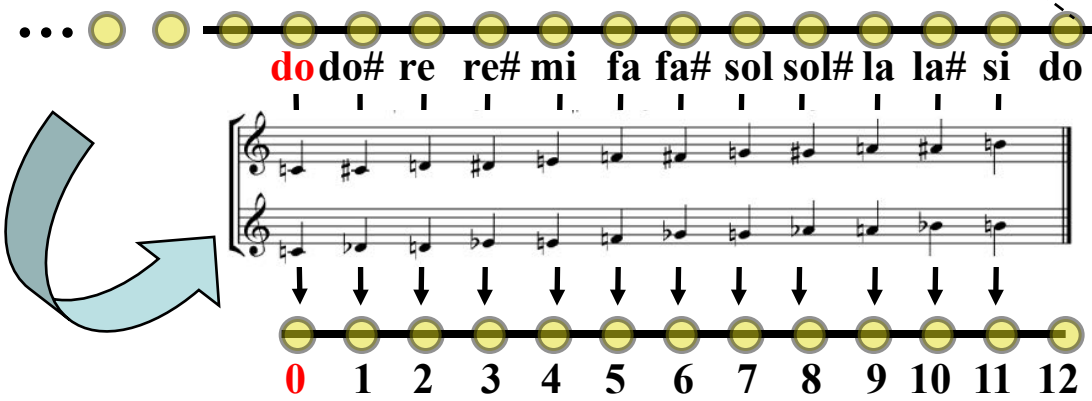

**...or rotations !**

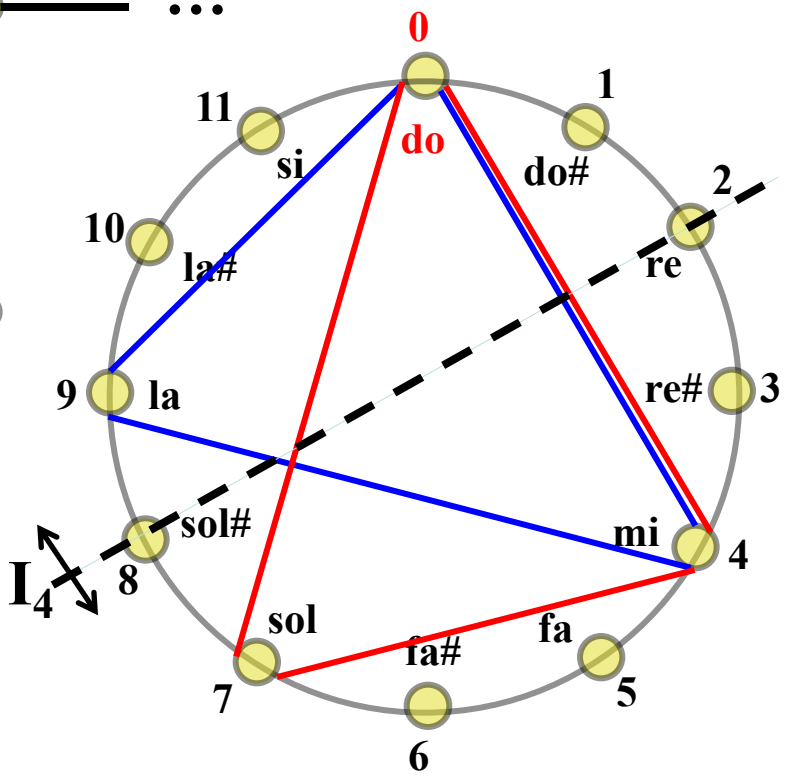
# Musical inversions are differences...

I : $x \rightarrow -x$ mod 12

*Do* maj = {0,4,7}
*La* min = {0,4,9}

$I_4(x)=4-x$



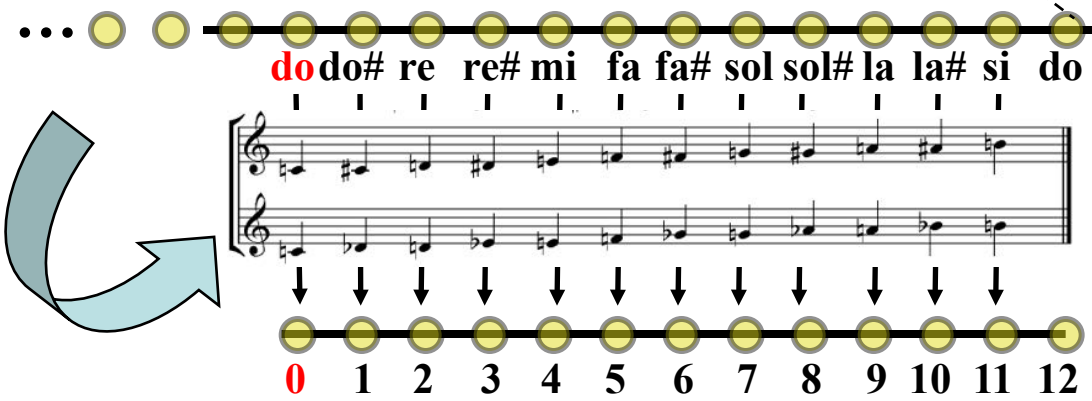

... or axial symmetries!

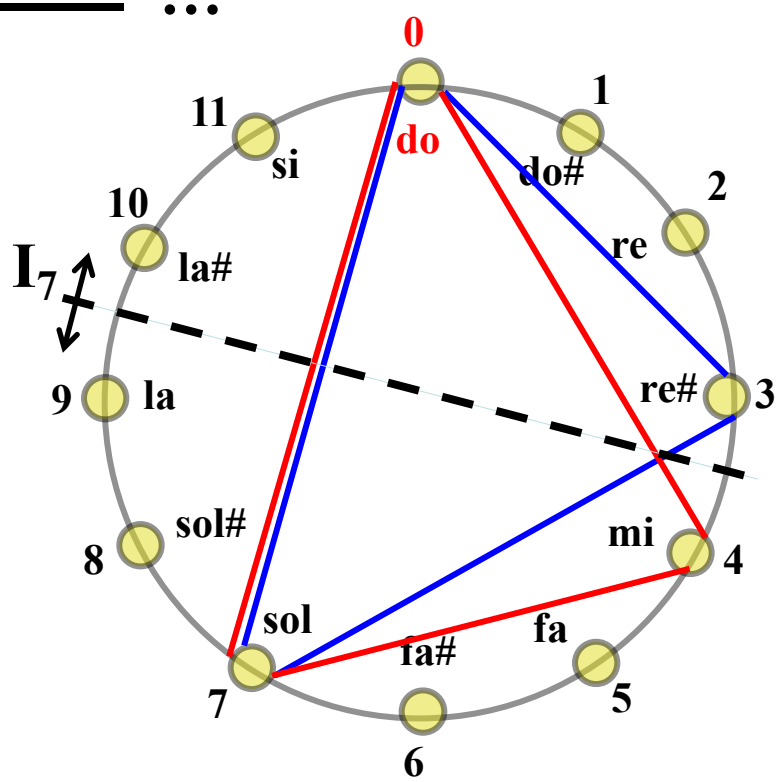
# Musical inversions are differences...

$I : x \rightarrow -x \bmod 12$

***Do* maj** $= \{0,4,7\}$
***Do* min** $= \{0,3,7\}$

$I_7(x)=7-x$



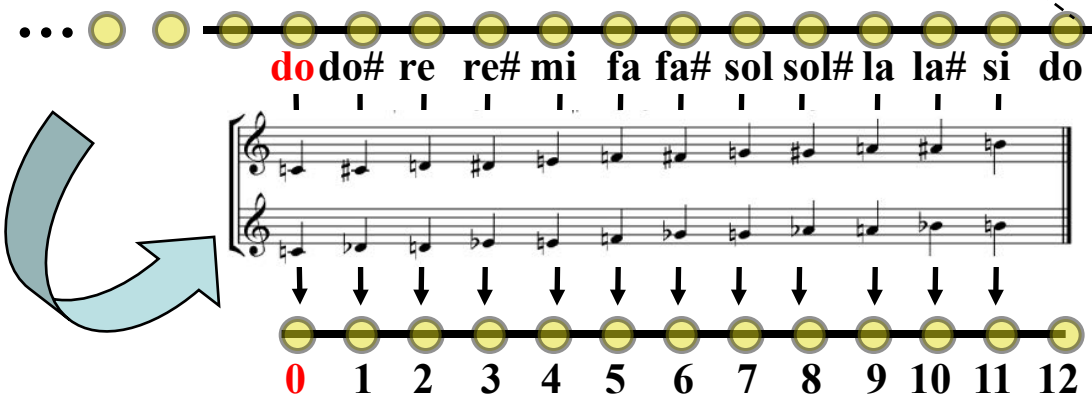

## ... or axial symmetries!

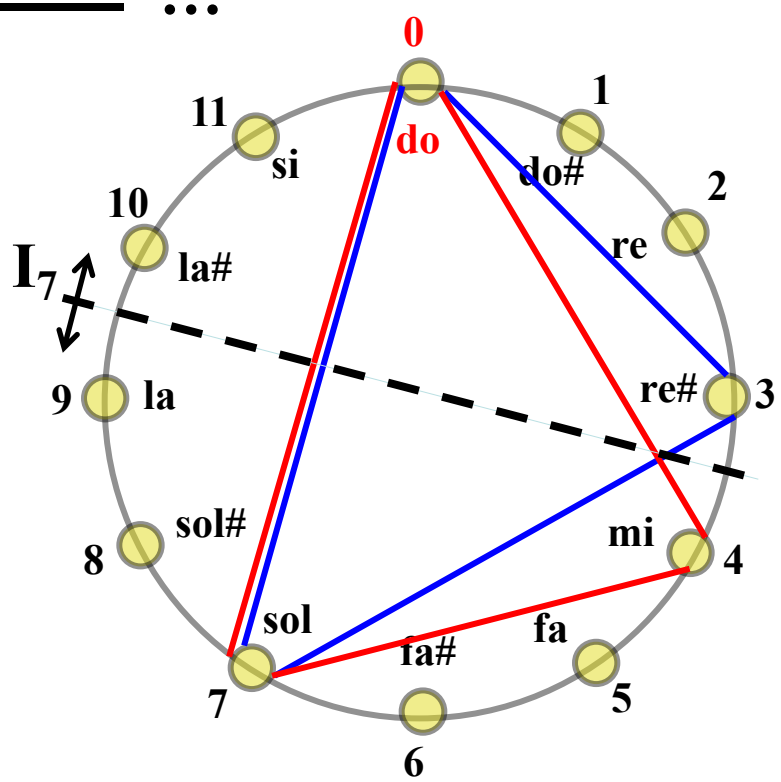
# Musical inversions are differences...

$I : x \rightarrow -x \bmod 12$

***Do* maj** $= \{0,4,7\}$
***Do* min** $= \{0,3,7\}$

$I_7(x)=7-x$



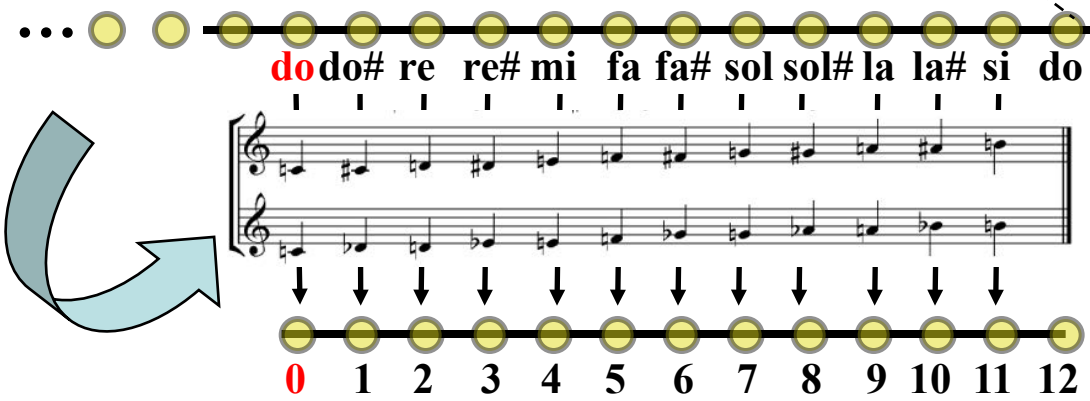

## ... or axial symmetries!

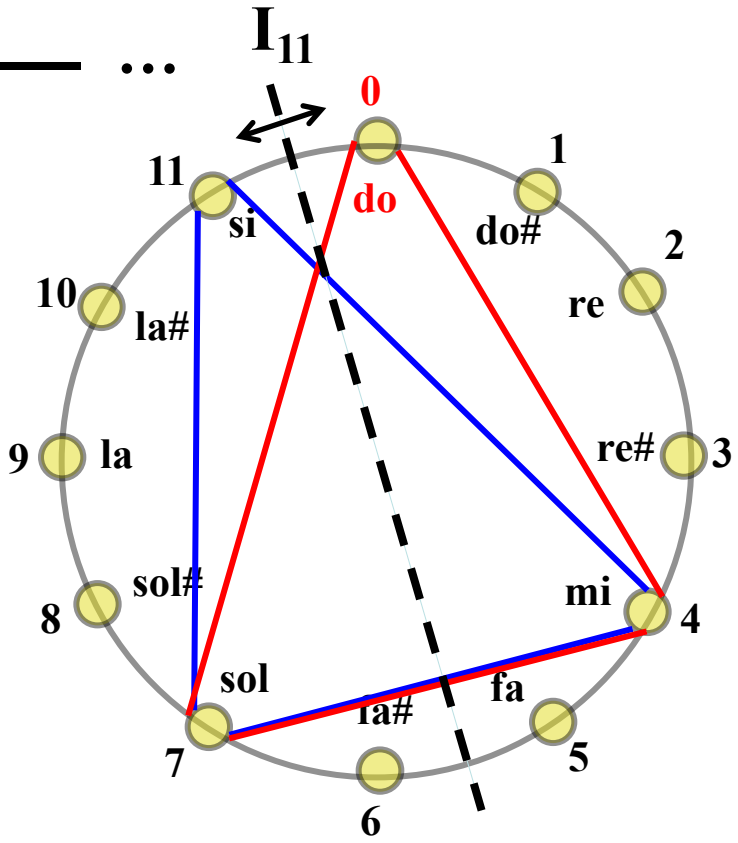
# Musical inversions are differences...

$I : x \rightarrow -x \bmod 12$

***Do* maj** $= \{0,4,7\}$
***Mi* min** $= \{4,7,11\}$

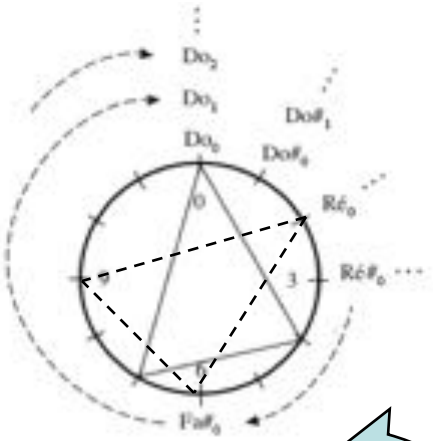
$I_{11}(x)=11-x$




## ... or axial symmetries!

# Circular representation and intervallic structure

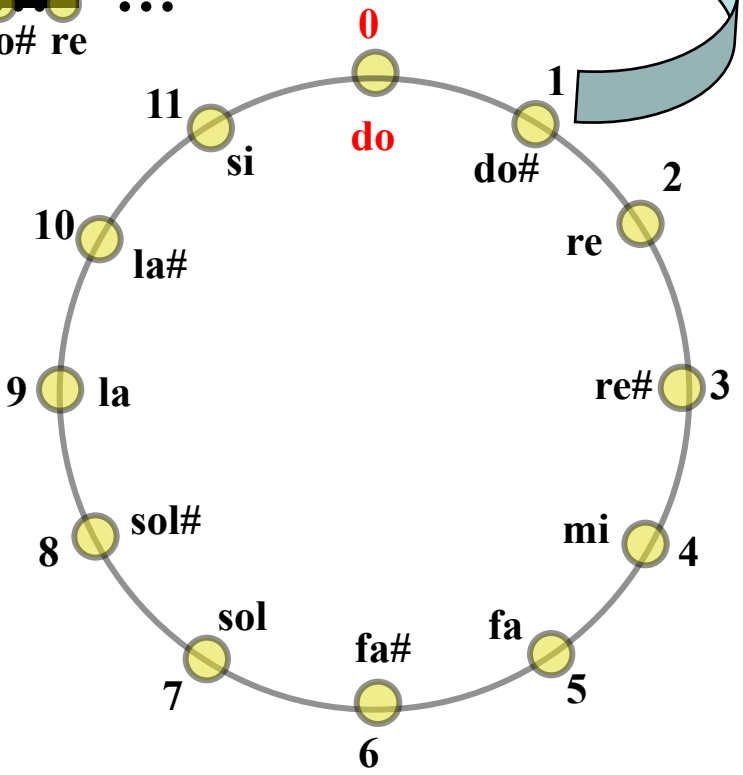


The intervallic structure is invariant

(4,3,5)

do do# re re# mi fa fa# sol sol# la la# si do do# re

0 1 2 3 4 5 6 7 8 9 10 11 12

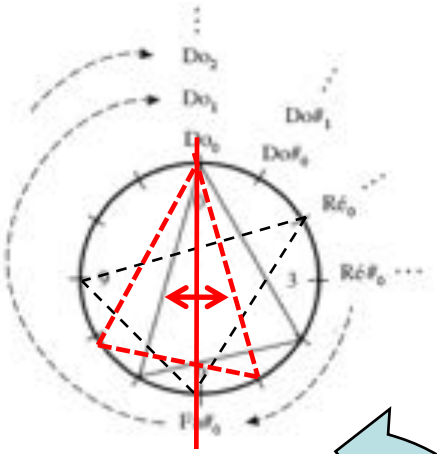



**The two basic musical applications**

$T_k : x \rightarrow x+k \bmod 12$ (transposition)

$I : x \rightarrow -x \bmod 12$ (inversion)

# Circular representation and intervallic structure

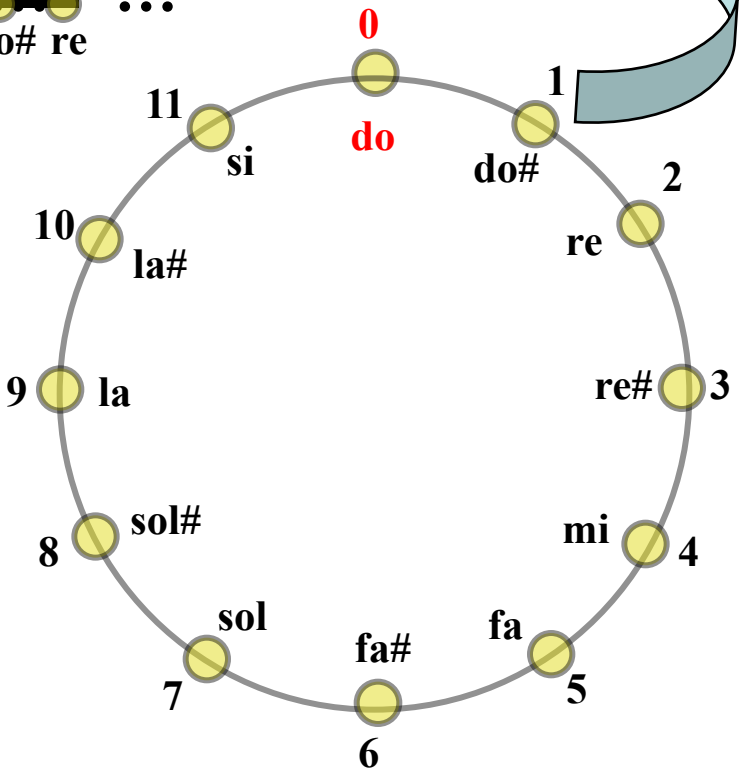


The intervallic structure is reversed

(5,3,4)

do do# re re# mi fa fa# sol sol# la la# si do do# re

0 1 2 3 4 5 6 7 8 9 10 11 12




**The two basic musical applications**

$T_k : x \rightarrow x+k \bmod 12$ (transposition)

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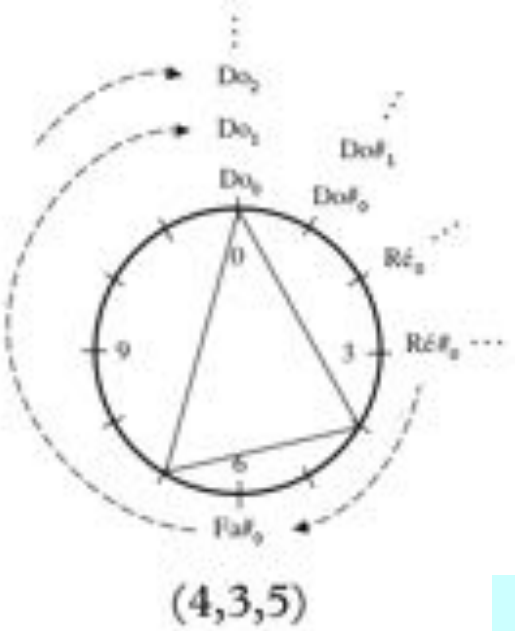
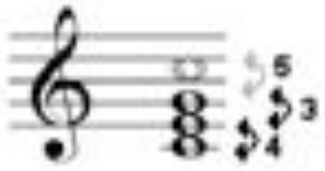
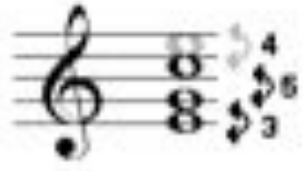
# Circular representation and intervallic structure



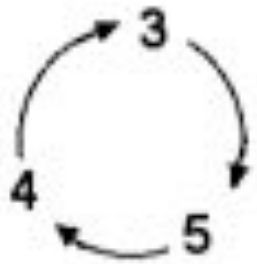

(4,3,5)

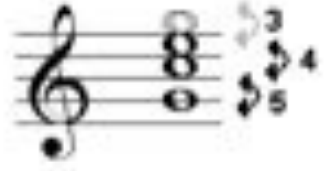


(4 3 5)

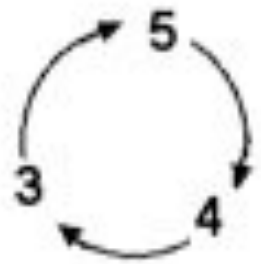



(3 5 4)

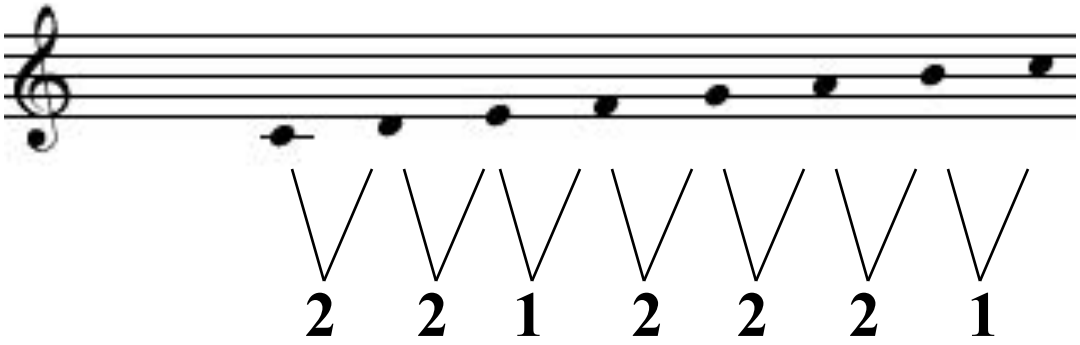


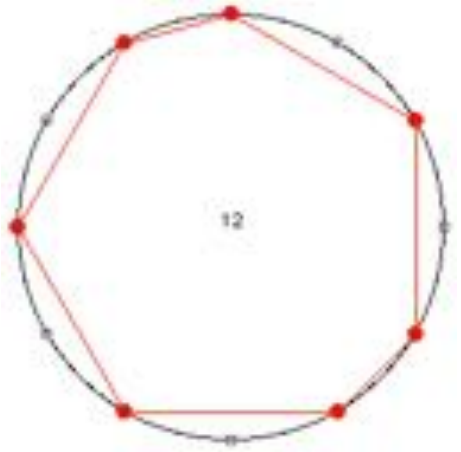



(5 4 3)

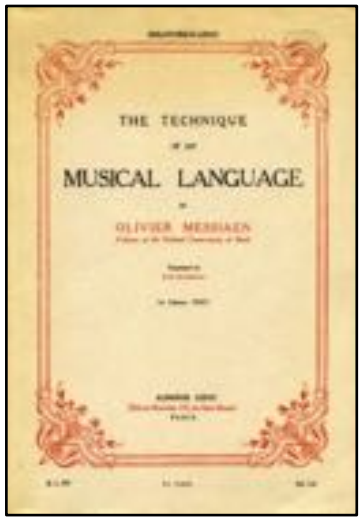



***The « inversions » of a chord are all circular permutations on an intervallic structure***



# The pitch-rhythm *mystic* isomorphic correspondence



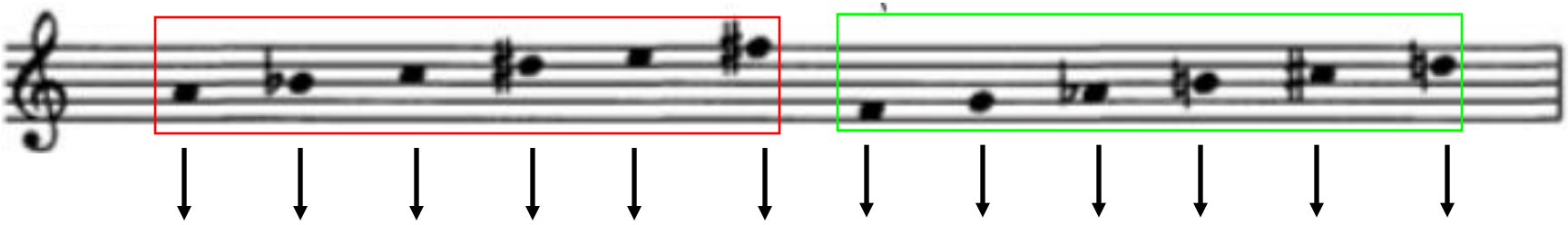


"**These modes** [of Limited Transpositions] **realize in the vertical direction** (transposition) **what non-retrogradable rhythms realize in the horizontal direction** (retrogradation). In fact, these modes *cannot be transposed* beyond a certain number of transpositions without falling again into the same notes, enharmonically speaking; likewise, these rhythms *cannot be read in a retrograde sense* without one's finding again exactly the same order of values as in the right sense. These modes cannot be transposed because they are—without polytonality—in the modal atmosphere of several keys at once and contain in themselves small transpositions; these rhythms cannot be retrograded because they contain in themselves small retrogradations. These modes are divisible into symmetrical groups; these rhythms, also, with this difference: the symmetry of the rhythmic groups is a retrograde symmetry. Finally, the last note of each group of these modes is always *common* with the first of the following group; and the groups of these rhythms frame a central value *common* to each group. **The analogy is now complete**".

- **O. Messiaen, *Technique de mon langage musical*, Alphonce Leduc, 1944**
- **O. Messiaen, *Traité de rythme, de couleurs et d'ornithologie*, Alphonce Leduc, 1949-1992**

# Messiaen's Modes of Limited Transpositions

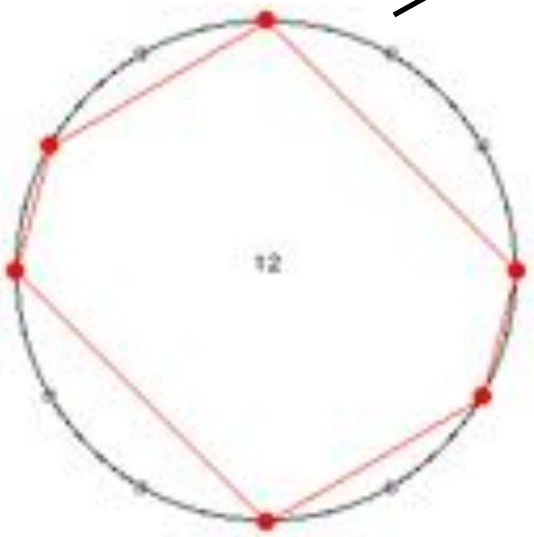
Schoenberg: Serenade Op.24, Mouvement 5



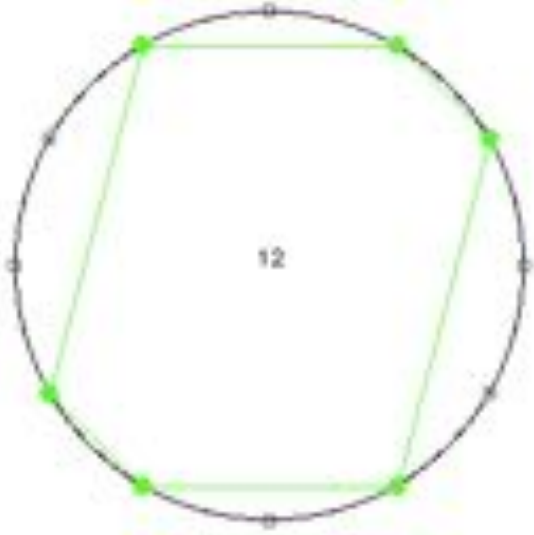
A = {9 , 10 , 0 , 3, 4 , 6} {5 , 7, 8, 11, 1, 2}

T6{9,10,0,3,4,6}=
={6+9,6+10,6,6+3,6+4,6+6}=
={3,4,6,9,10,0}

T6(A)=A

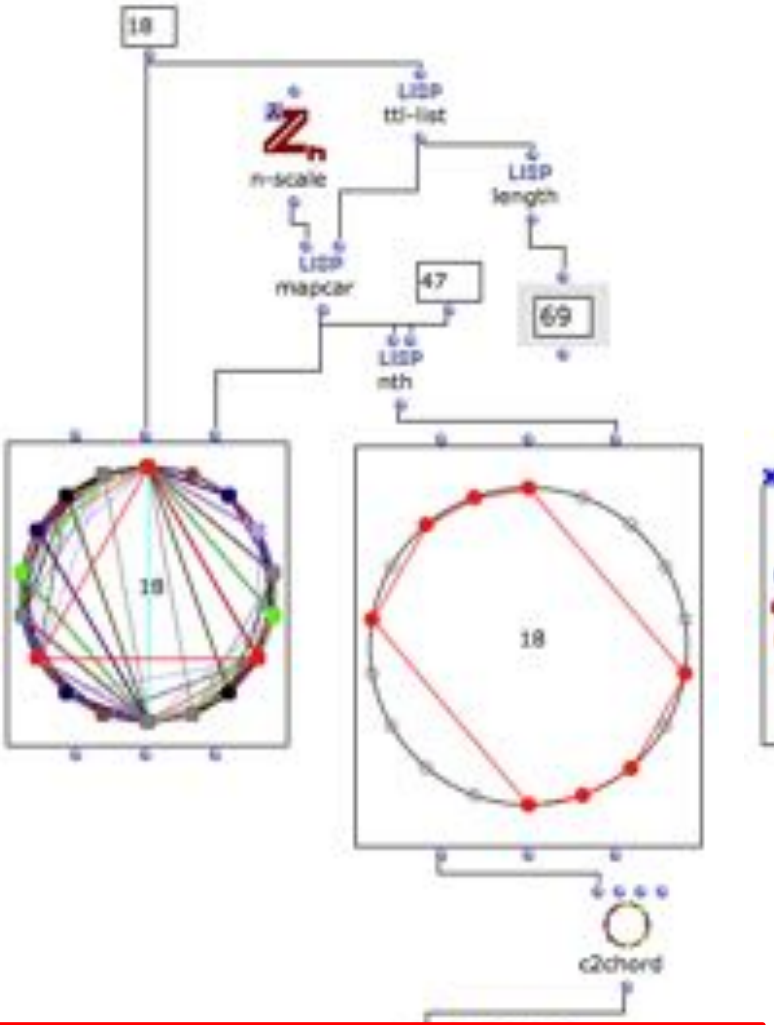


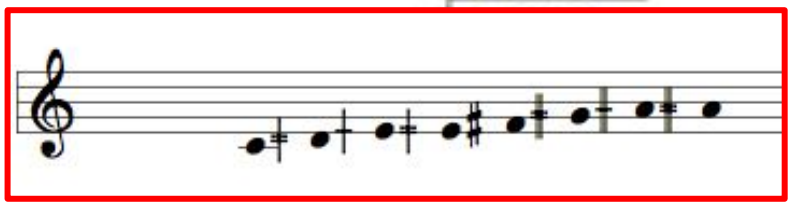

(3, 1, 2, 3, 1, 2)

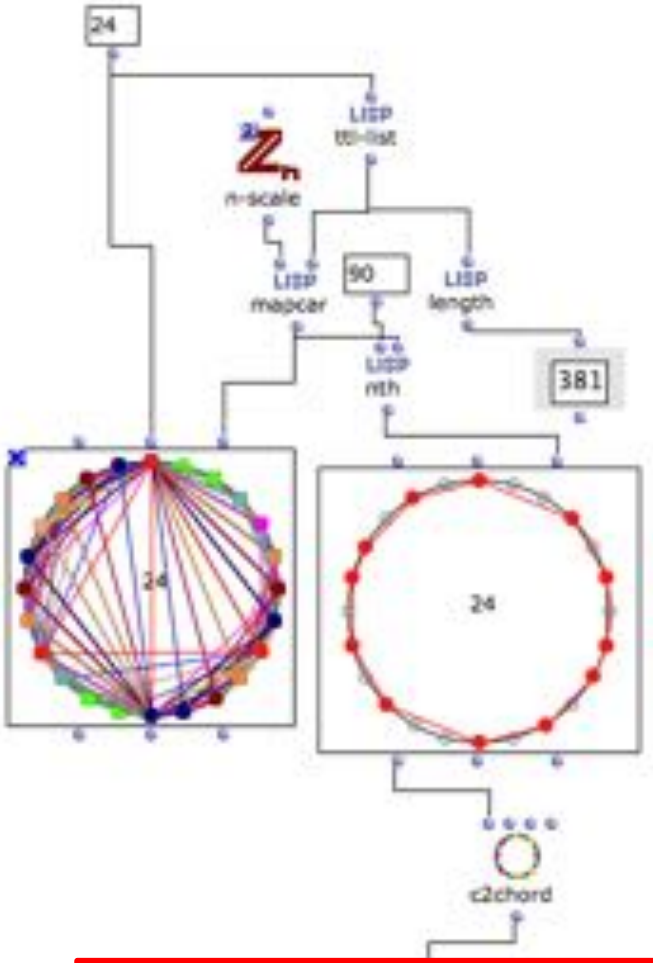


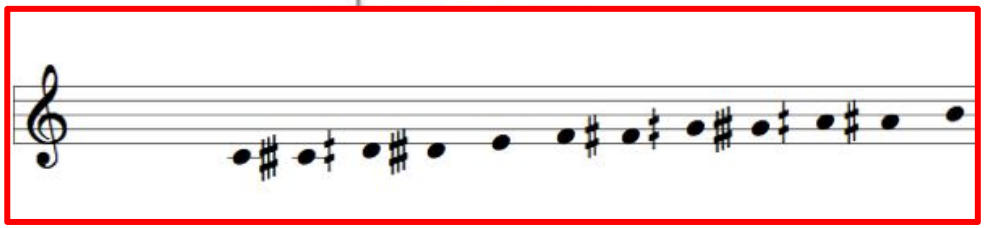

(2, 1, 3, 2, 1, 3)

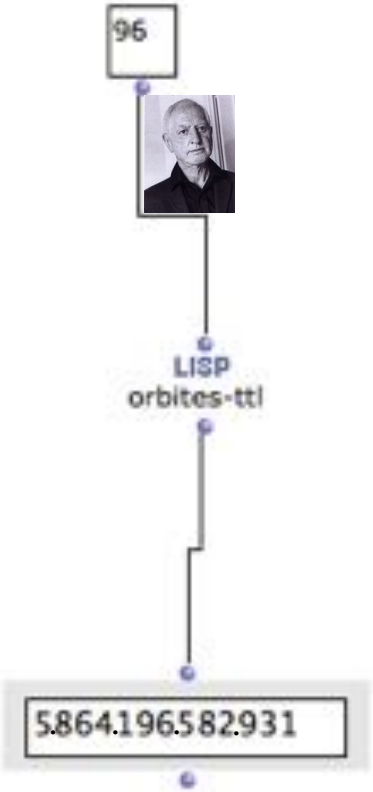
# Messiaen's Modes and their microtonal extensions



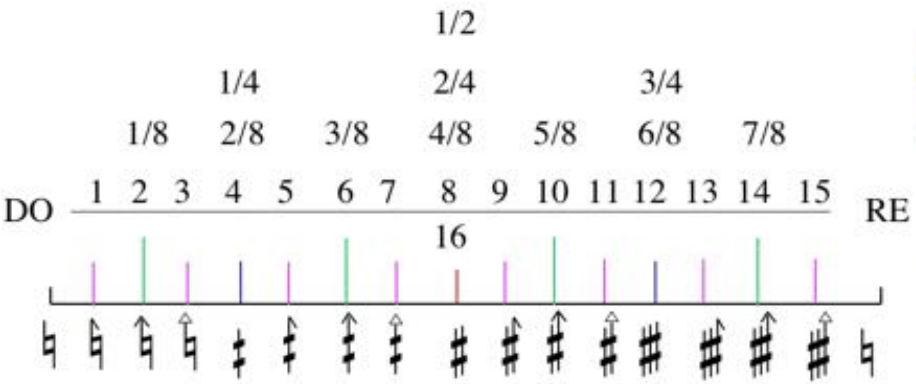



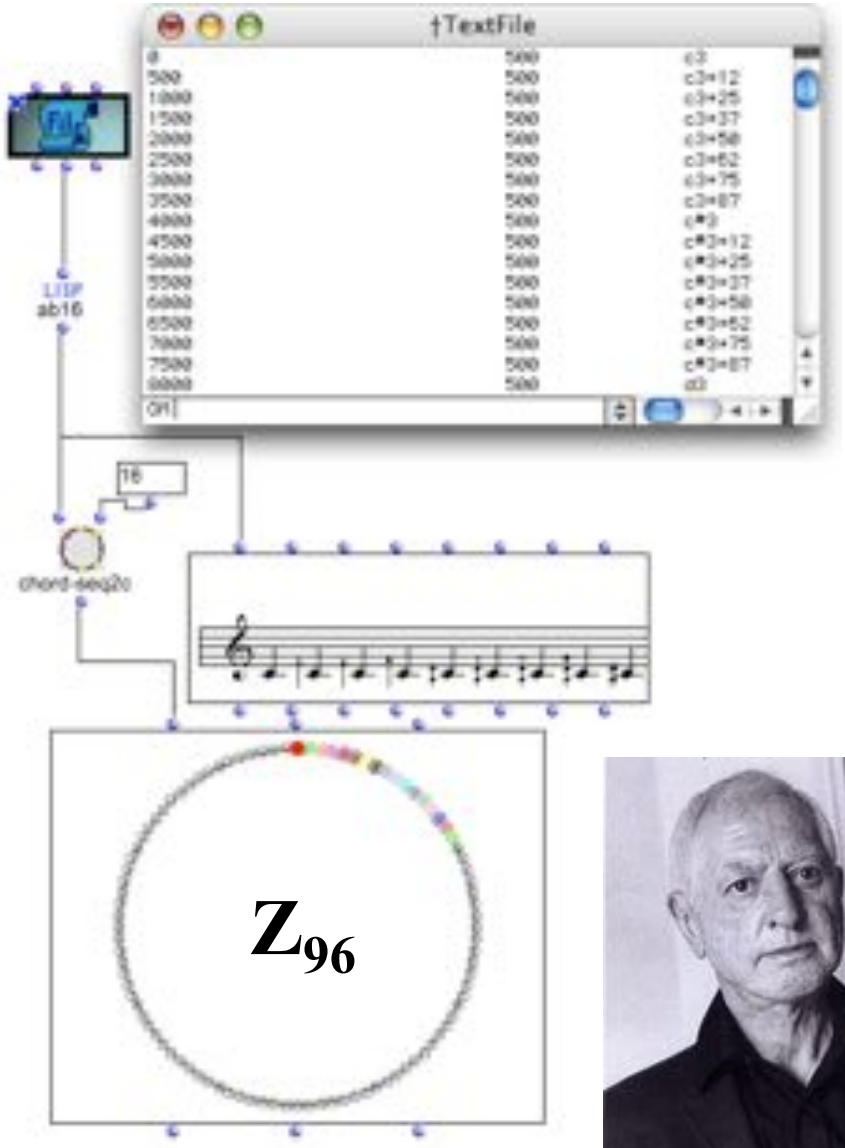




# Microtonal Composition







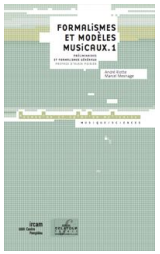



Alain Bancquart

➔ 

Microtonality

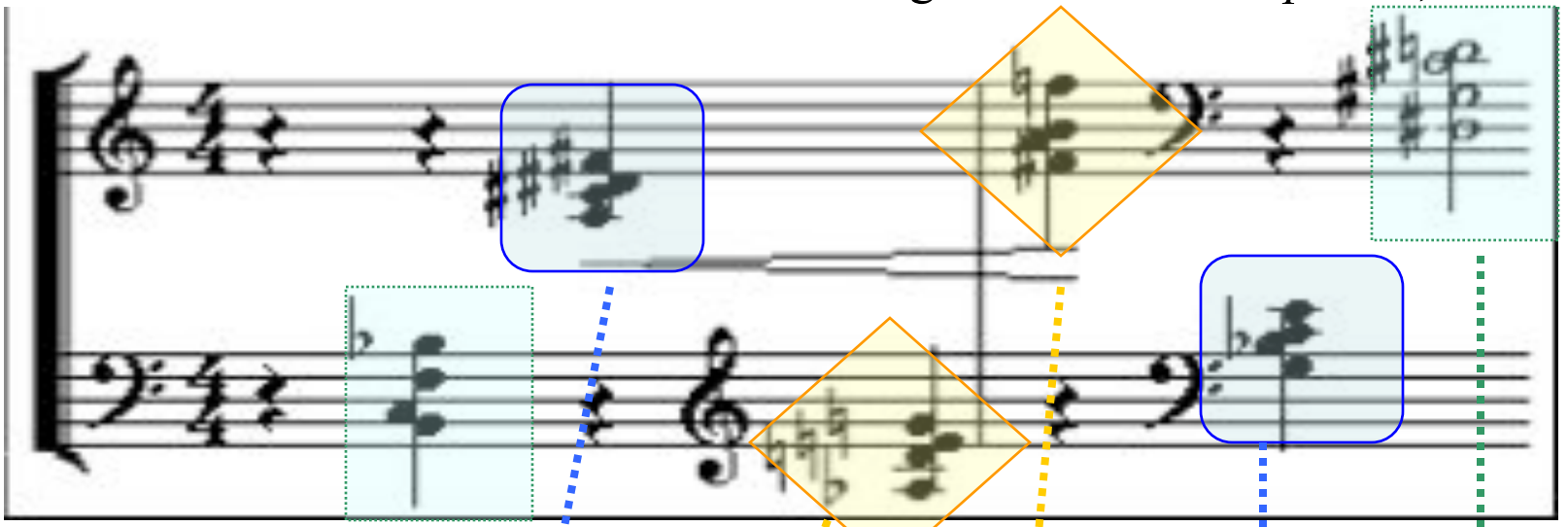
A. Bancquart, M. Andreatta, et C. Agon, « Microtonal Composition », The OM Composer's Book 2, éd. Jean Bresson, Carlos Agon, Gérard Assayag (Ircam/Delatour France, Sampzon), 2008, p. 279-302.

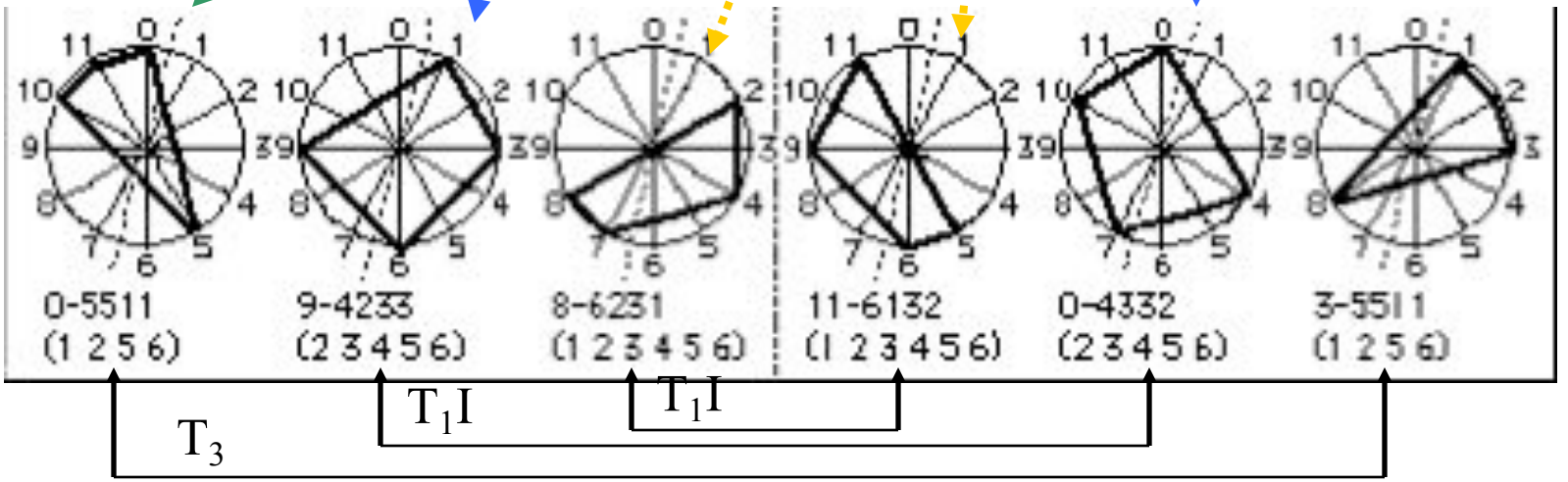


# « Entités formelles pour l'analyse musicale »

*Marcel Mesnage (1998)*

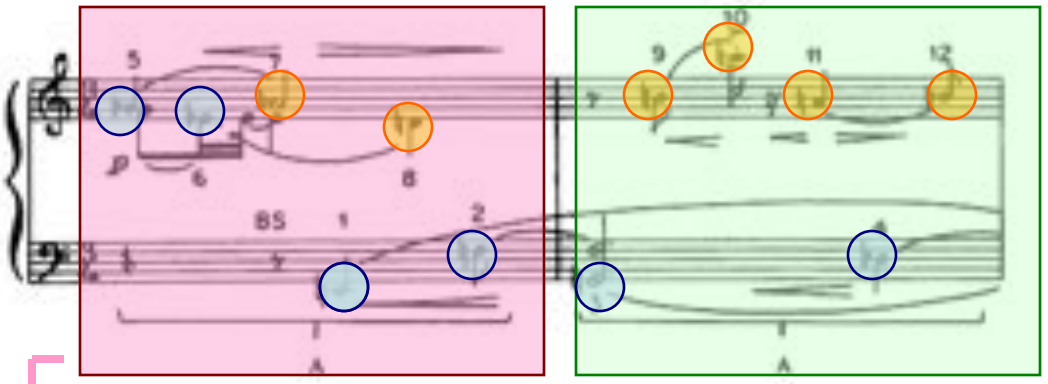
A. Schoenberg : *Klavierstück Op. 33a*, 1929



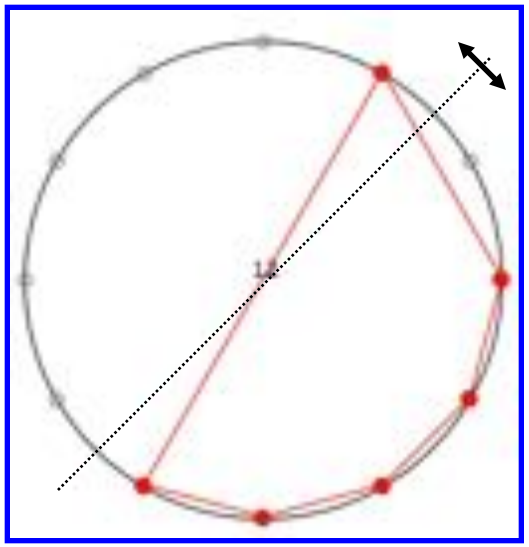


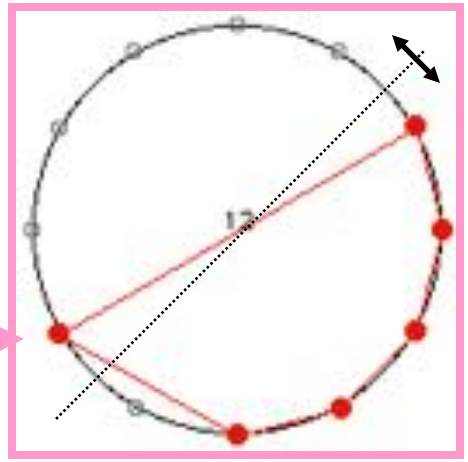
# Serialism and hexachordal combinatoriality

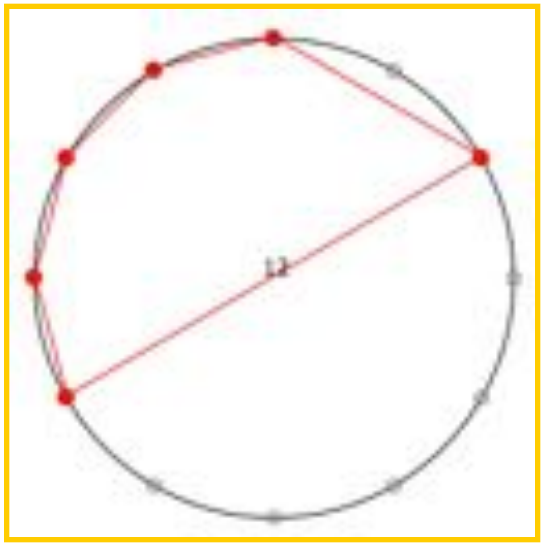
Schoenberg: Suite Op.25, Minuetto



**Double combinatoriality**

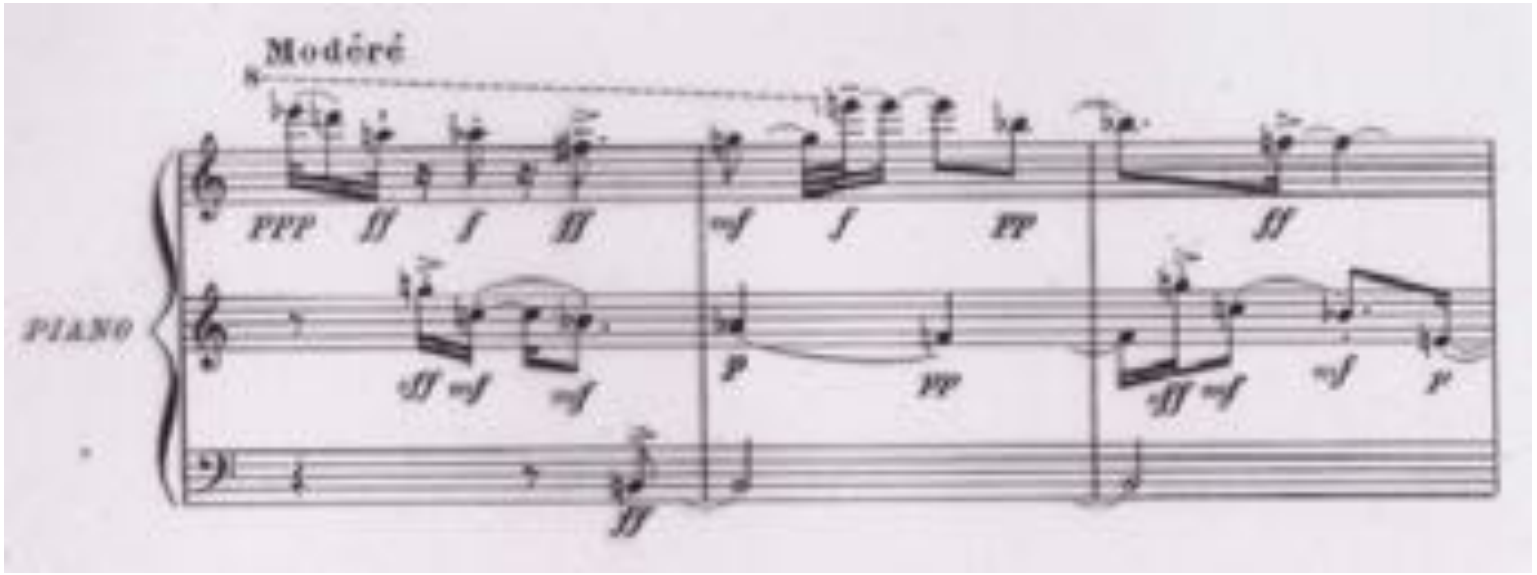


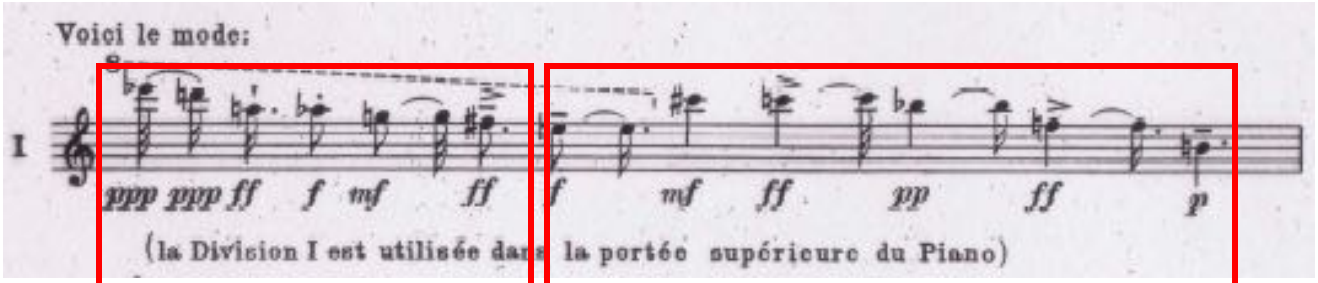


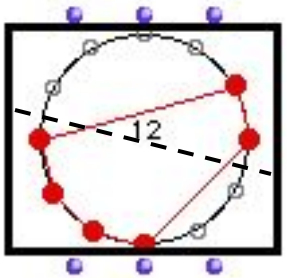


# Hexachordal Combinatoriality in Messiaen

- Mode de valeurs et d'intensités (1950)

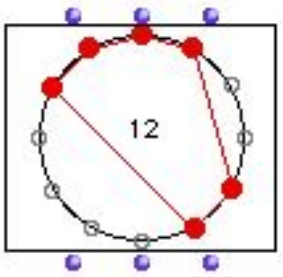


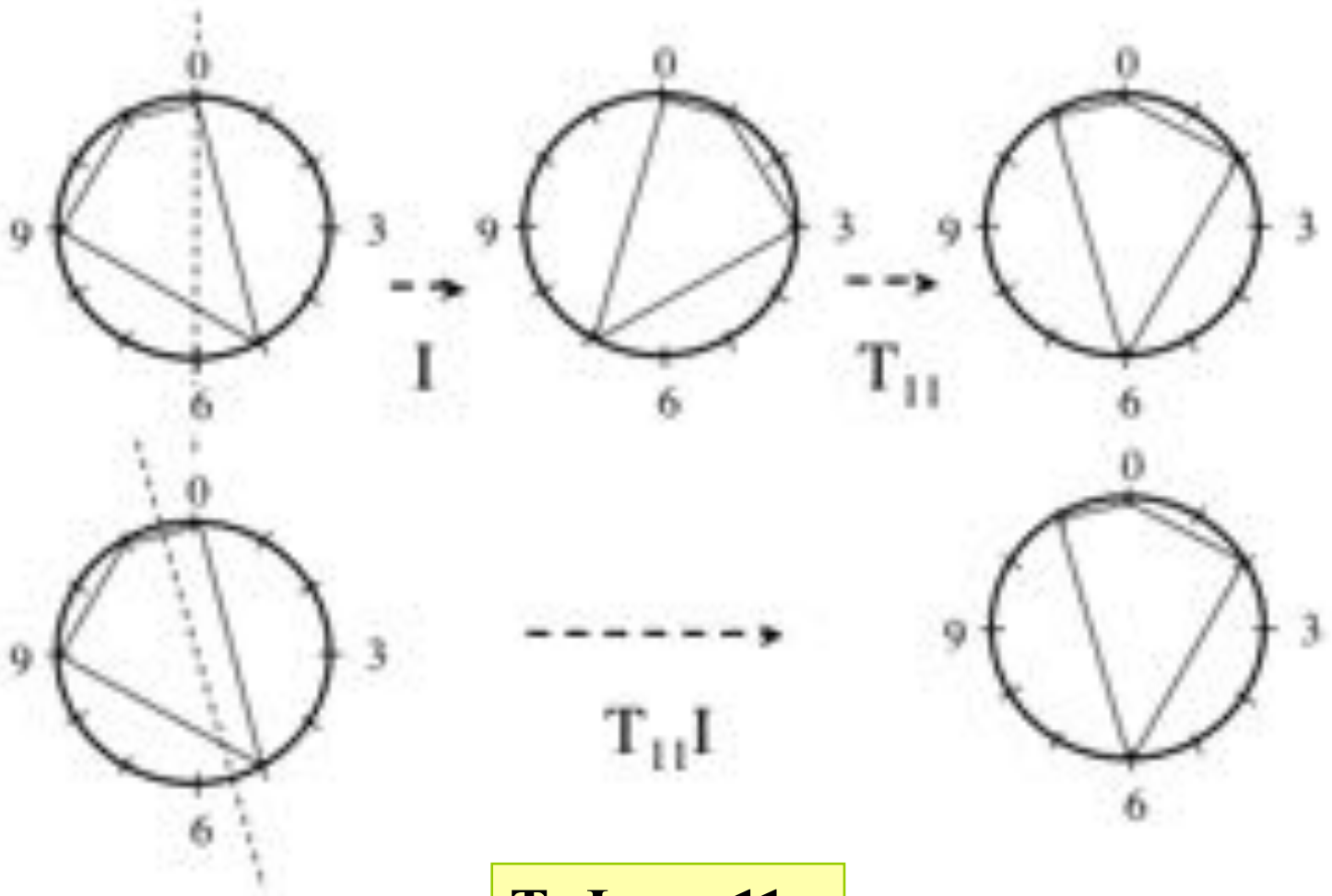

$\{3,2,9,8,7,6\} \longrightarrow \{4,5,10,11,0,1\}$

$T_7I : x \rightarrow 7\text{-}x$



# Transposition and Inversion

**I: $x \rightarrow 12-x$** **$T_k$: $x \rightarrow k+x$**



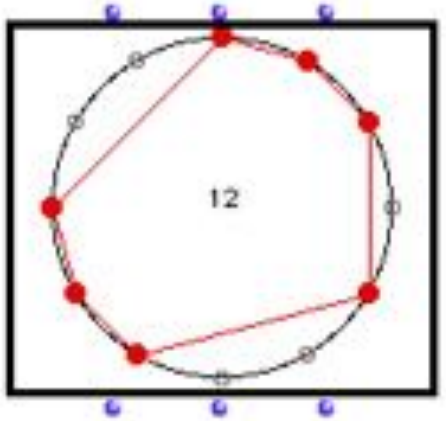

**$T_{11}I$: $x \rightarrow 11-x$**

**{0, 5, 9, 11}** ⟶ **{11, 6, 3, 0}**

# Combinatorics and modal compositional processes

*On a combinatorial problem posed by G. Benjamin*


**The scale :**

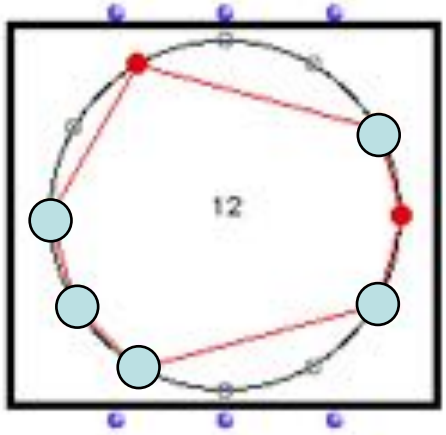



$T_7$

**Is it unique ?**

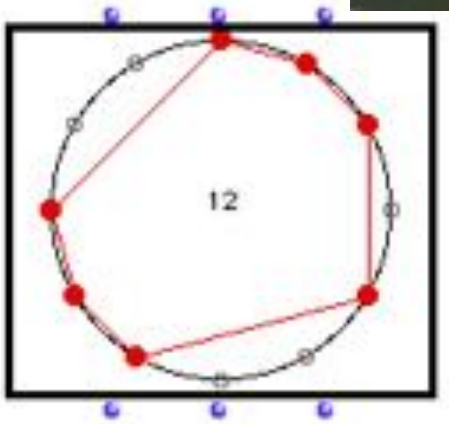
**How to find 7-notes scales preserving :**

1) **4 notes by inversion**
2) **5 notes by fifth transposition**
3) **5 notes once the inversion is transposed by a fourth**



# Combinatorics and modal compositional processes

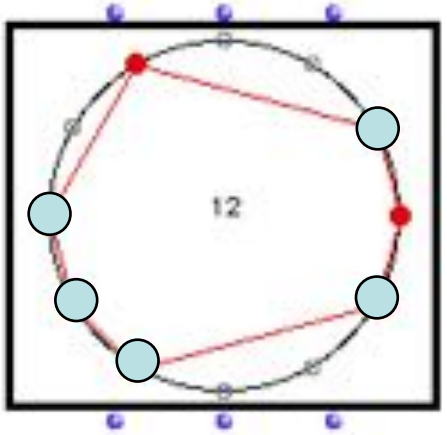
*On a combinatorial problem posed by G. Benjamin*


**How to find 7-notes scales preserving :**

1) **4 notes by inversion**
2) **5 notes by fifth transposition**
3) **5 notes once the inversion is transposed by a fourth**

$T_7$




*pcset* — *Interval Content*

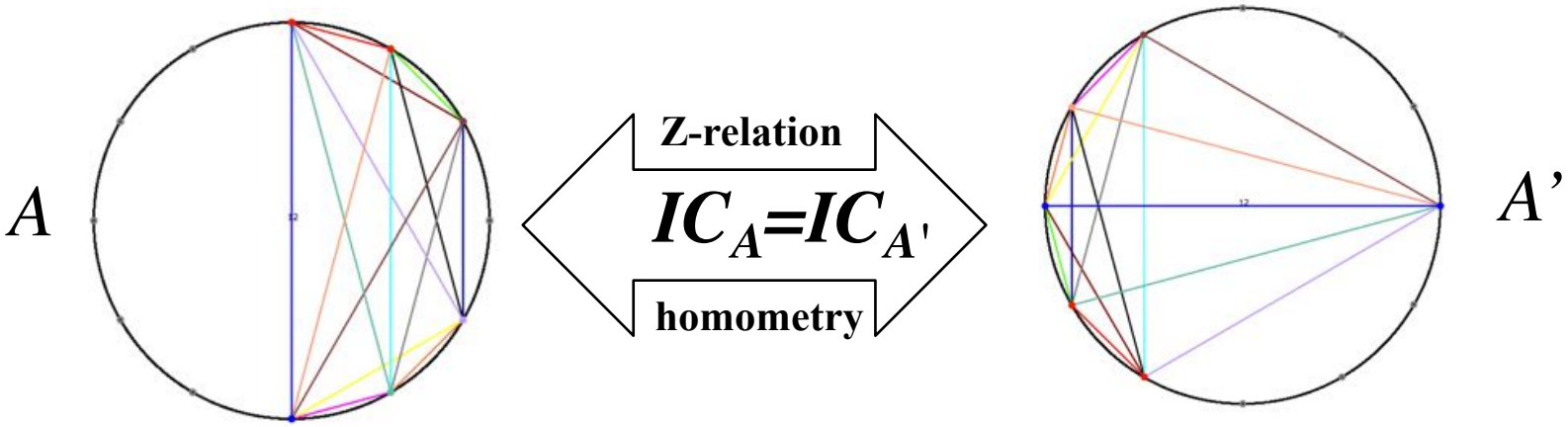
**(0 1 3 4 6 8 10)** ⇢ **[7 4 3 3 4 5 4 5 4 3 3 4]**

$T_0$ ... $T_7$

**Interval content**

# Z-relation and Babbitt's Hexachord Theorem

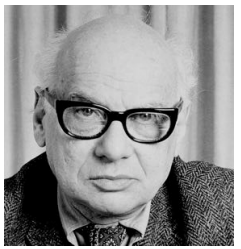
**The *interval content* $IC_A$ of a chord provides the multiplicity of occurrences of its intervals**




$$IC_A = [4, 3, 2, 3, 2, 1\ ] \quad = \quad [4, 3, 2, 3, 2, 1\ ] = IC_{A'}$$

**Babbitt's Hexachord Theorem:**

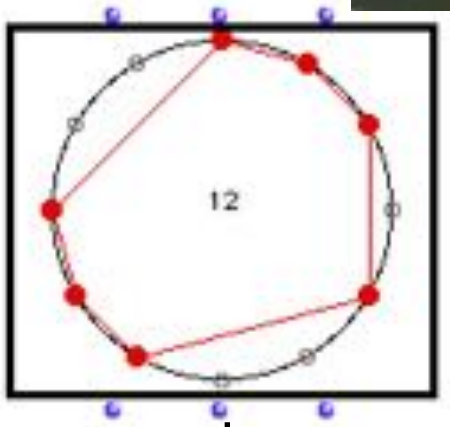
*A hexacord and its complement have the same interval content*



Milton Babbitt

# Combinatorics and modal compositional processes

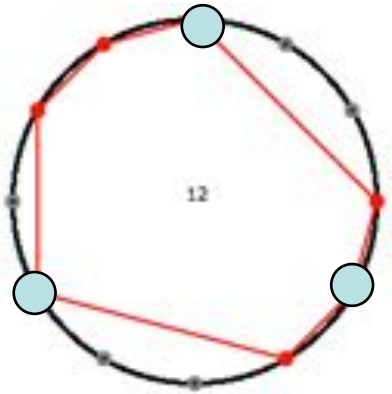
*On a combinatorial problem posed by G. Benjamin*


**How to find 7-notes scales preserving :**

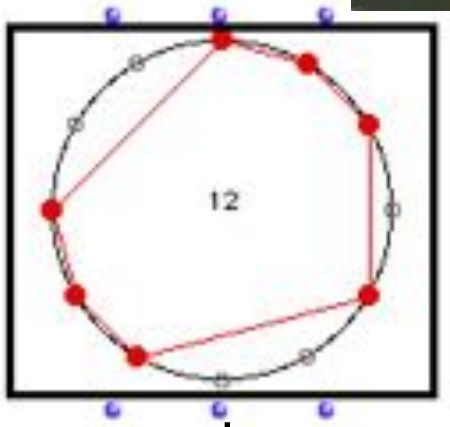
1) **4 notes by inversion**
2) **5 notes by fifth transposition**
3) **5 notes once the inversion is transposed by a fourth**

**I**



# Combinatorics and modal compositional processes

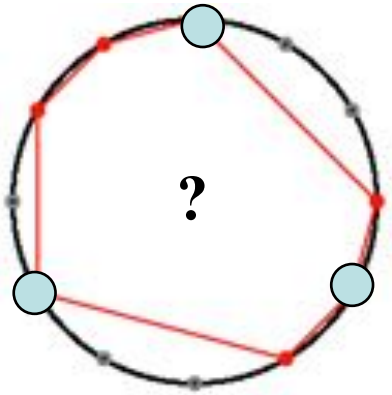
*On a combinatorial problem posed by G. Benjamin*


**How to find 7-notes scales preserving :**

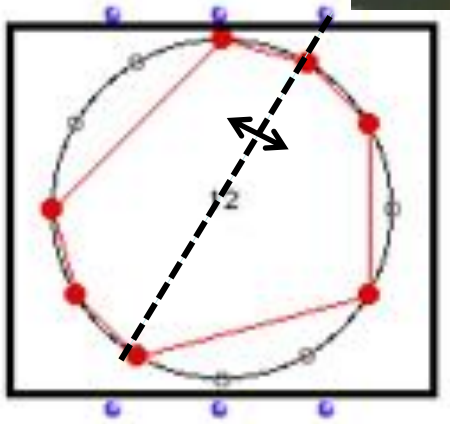
1) **4 notes by inversion**
2) **5 notes by fifth transposition**
3) **5 notes once the inversion is transposed by a fourth**

**I**



# Combinatorics and modal compositional processes

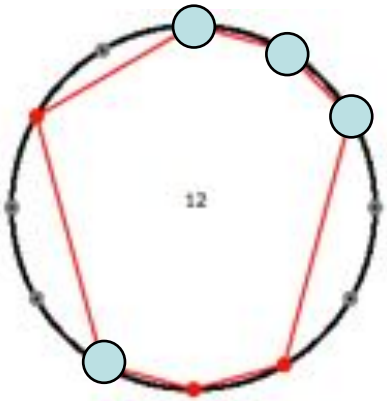
*On a combinatorial problem posed by G. Benjamin*


**How to find 7-notes scales preserving :**

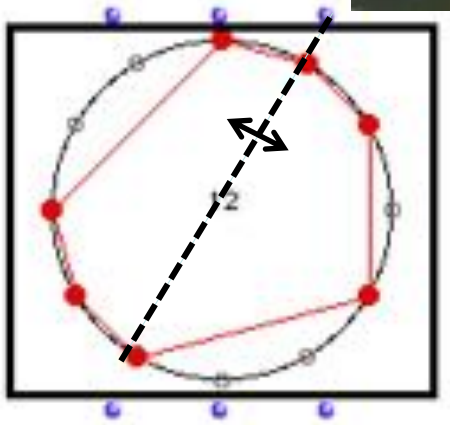
1) **4 notes by inversion**
2) **5 notes by fifth transposition**
3) **5 notes once the inversion is transposed by a fourth**

$I_2$



# Combinatorics and modal compositional processes

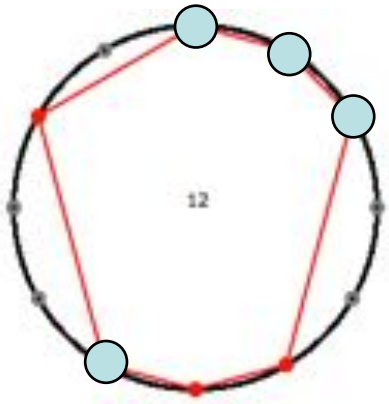
*On a combinatorial problem posed by G. Benjamin*

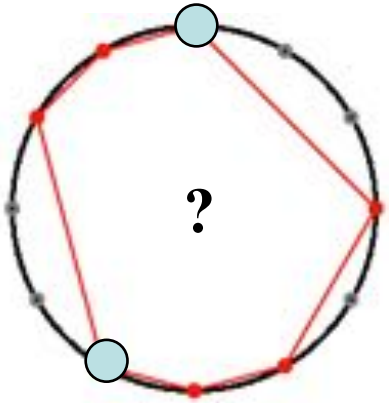

**How to find 7-notes scales preserving :**

1) **4 notes by inversion**
2) **5 notes by fifth transposition**
3) **5 notes once the inversion is transposed by a fourth**

$I_2$

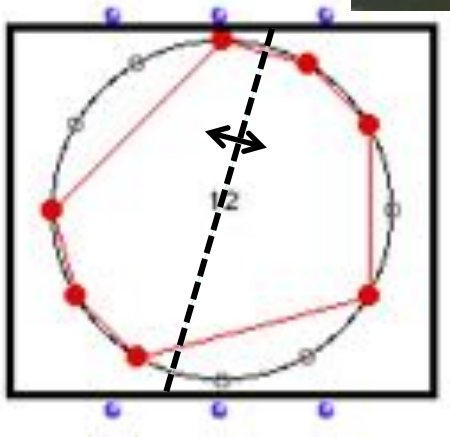



$T_5$



# Combinatorics and modal compositional processes

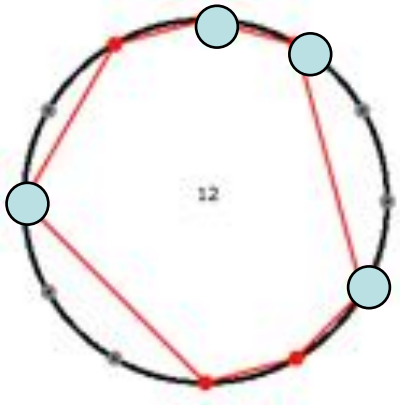
*On a combinatorial problem posed by G. Benjamin*

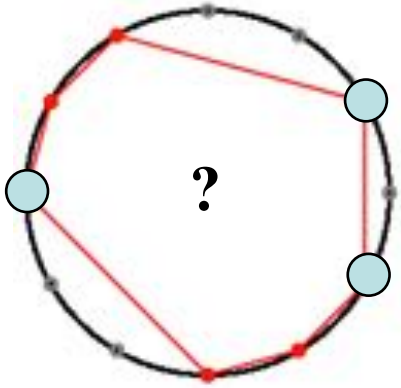

**How to find 7-notes scales preserving :**

1) **4 notes by inversion**
2) **5 notes by fifth transposition**
3) **5 notes once the inversion is transposed by a fourth**

$I_1$

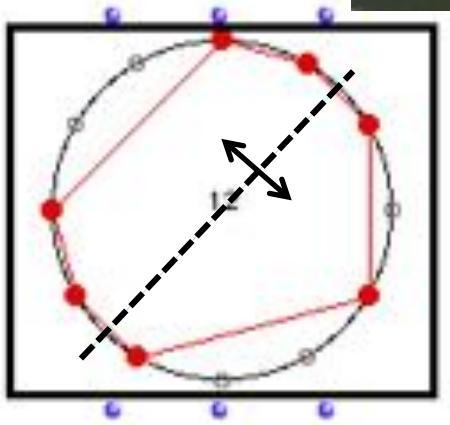



$T_5$



# Combinatorics and modal compositional processes

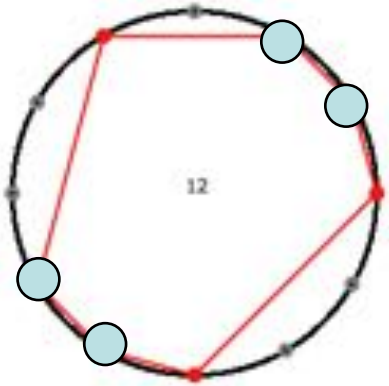
*On a combinatorial problem posed by G. Benjamin*

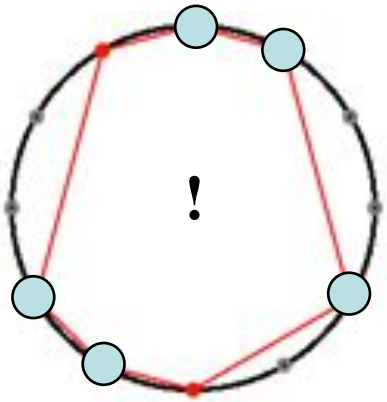

**How to find 7-notes scales preserving :**

1) **4 notes by inversion**
2) **5 notes by fifth transposition**
3) **5 notes once the inversion is transposed by a fourth**

$I_3$

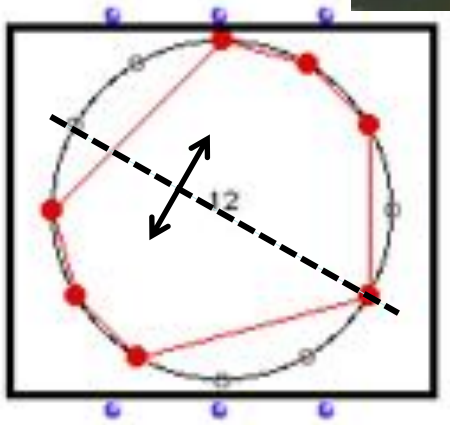



$T_5$



# Combinatorics and modal compositional processes

*On a combinatorial problem posed by G. Benjamin*

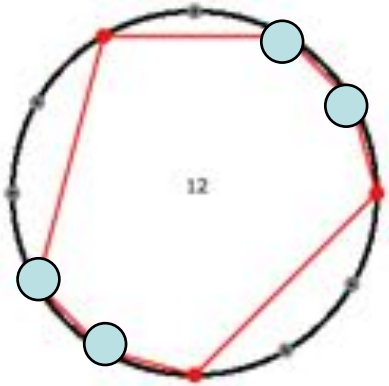




**How to find 7-notes scales preserving :**

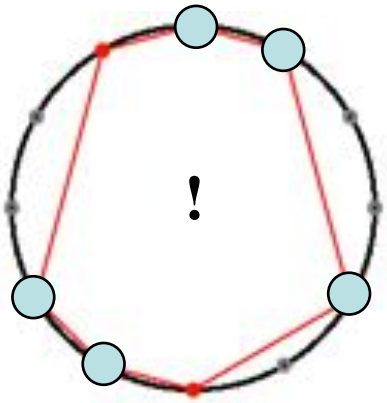
1) **4 notes by inversion**
2) **5 notes by fifth transposition**
3) **5 notes once the inversion is transposed by a fourth**

$I_3$

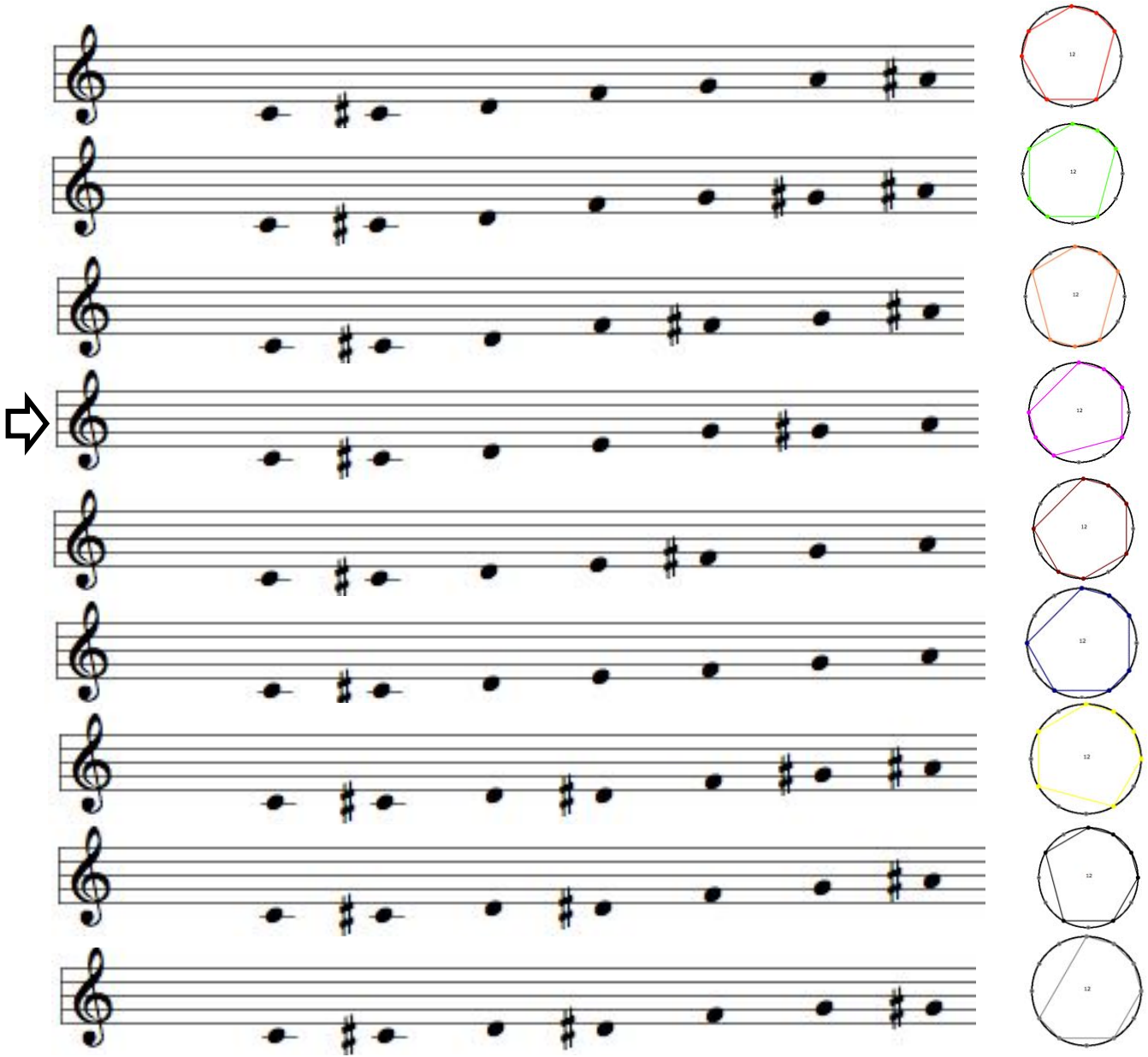
$I_8$



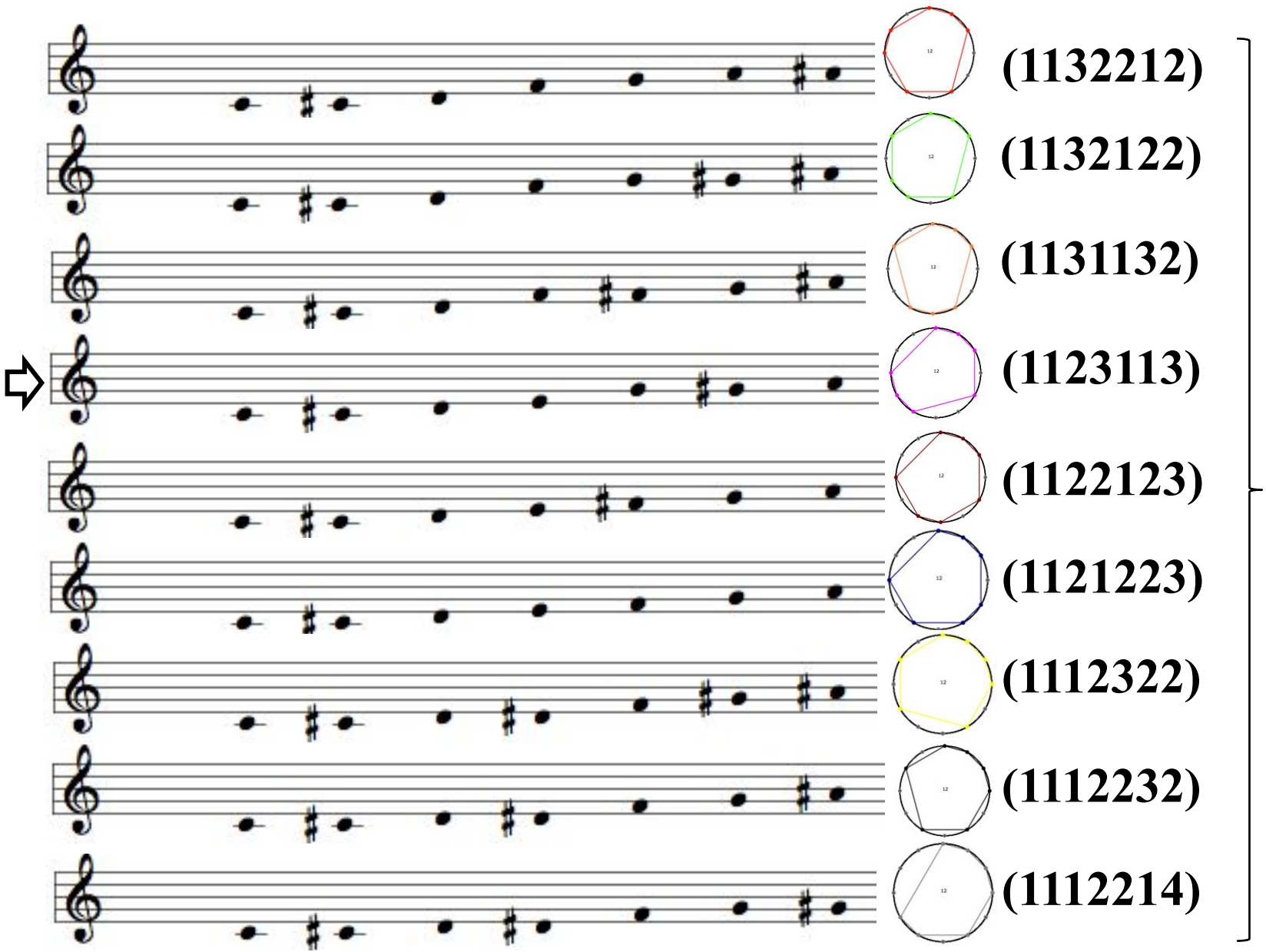

$T_5$



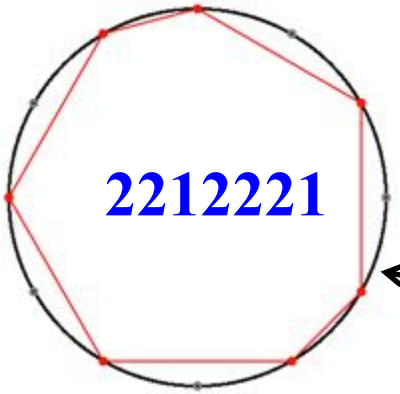
# The family of solutions of Benjamin's problem

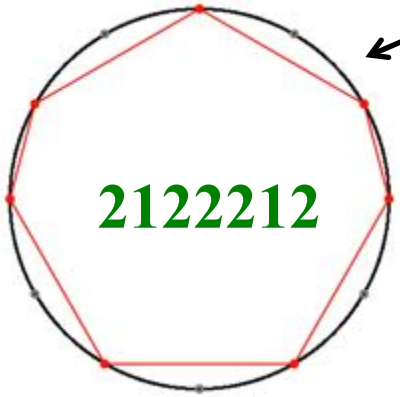


# The catalogue of all solutions (up to transposition)



# The « cognitive » pitch/rhythm isomorphisms




...

cyclic permutations

TABLE I

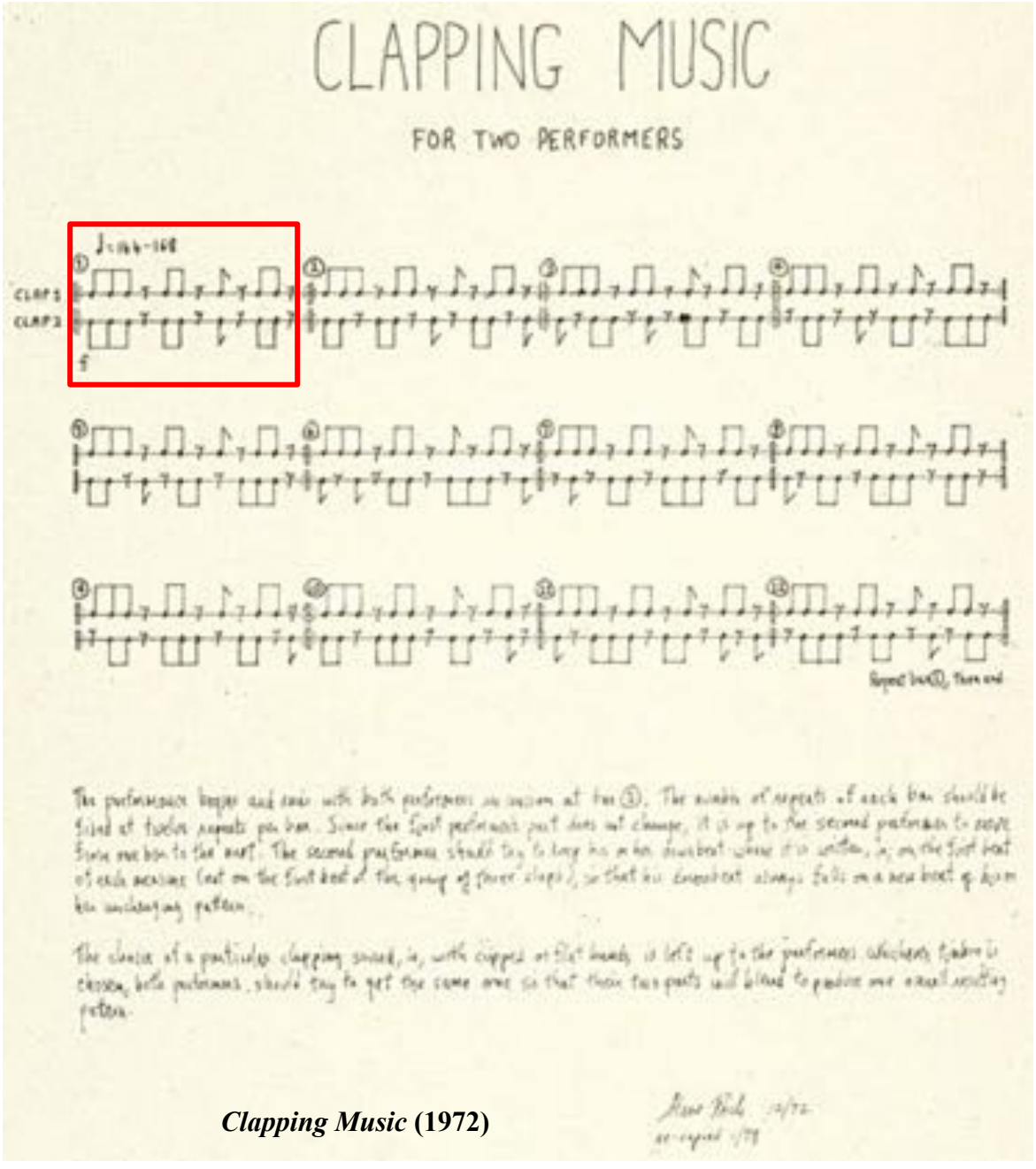
Comparison of M = 7, L = 12 patterns for pitch (scales) and rhythm (time-lines)

| | pattern | pitch domain name and notation (in C) | rhythm domain notation | examples from West Africa | references |
|---|---|---|---|---|---|
| 1. | 2212221 | major scale (Ionian) CDEFGAB | | Ewe (Atsiabek , Sogba, Atsia) also Yoruba | Jones (1959), C. K. Ladzekpo, S. K. Ladzekpo and Pantaleoni, Locke |
| 2 | 2122212 | Dorian CDE♭FGAB♭ | | Bemba—Northern Rhodesia | Jones (1965), (Ekwueme) |
| 3. | 1222122 | Phrygian C D♭ E♭ F G A♭ B♭ | | ——— | ——— |
| 4. | 2221221 | Lydian CDEF♯GAB | | Ga-Adangme (common) also common Haitian pattern, Akan (Ab fo ) | C. K. Ladzekpo, Combs (1974), R. Hill, Asiama |
| 5. | 2212212 | Mixolydian CDEFGAB♭ | | Yoruba sacred music from Ekiti | King |
| 6. | 2122122 | Aeolian CDE♭FGA♭B♭ | | Ashanti (Ab fo , Mpre) | Koetting |
| 7. | 1221222 | Locrian C D♭ E♭ F G♭ A♭ B♭ | | Ghana* | Nketia (1963a) |
| 8. | 2121222 | (#2 Locrian) CDE♭FG♭A♭B♭ | | Ashanti (Asedua) | C. K. Ladzekpo |
| 9. | 2112123 | ——— CDD♯EF♯GA | | Akan (juvenile song) | Nketia (1963b) |

* clap pattern
† mute stroke on bell

**J. Pressing, "Cognitive isomorphisms between pitch and rhythm in world musics: West Africa, the Balkans and Western tonality", *Studies in Music*, 17, p. 38-61, 1983**

# Cyclic permutations in minimalistic music


CLAPPING MUSIC

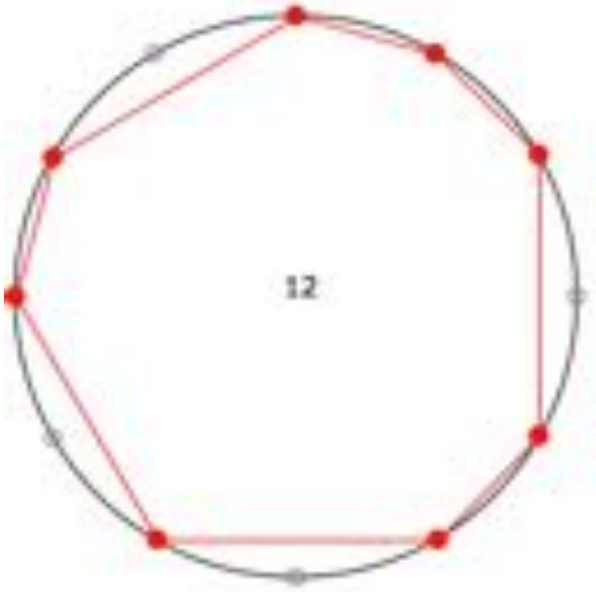
FOR TWO PERFORMERS

CLAP 1
CLAP 2

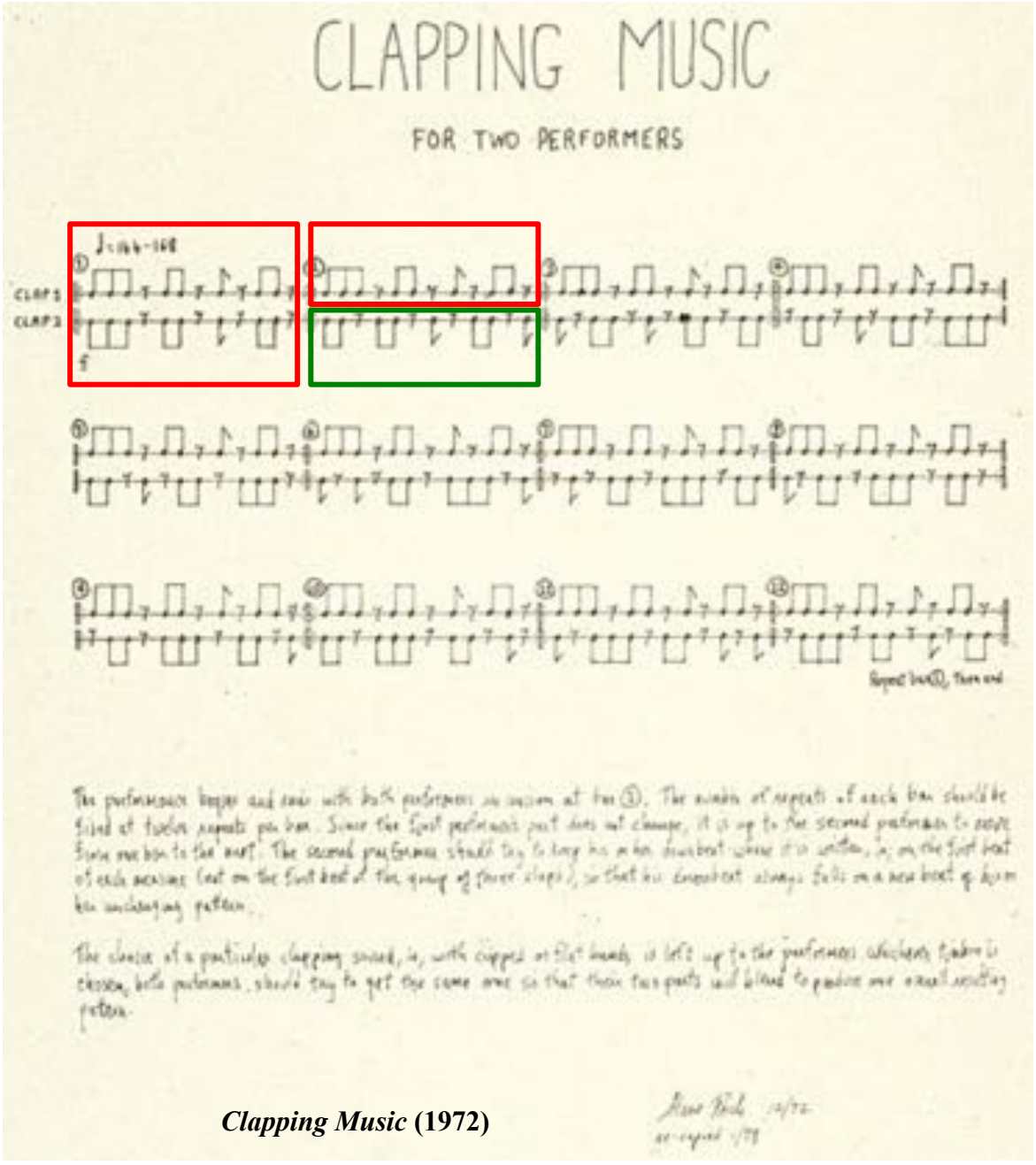
The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer's part does not change, it is up to the second performer to cue from one bar to the next. The second performer should try to stay his or her slowbeat where it is written, i.e. on the first beat of each sections (not on the first beat of the group of seven claps), so that his counterpart always falls on a new beat of their oscillating pattern.

The choice of a particular clapping sound, i.e. with clapped or flat hands is left up to the performers whichever pattern is chosen, both performers should try to get the same one so that these two parts will blend to produce one overall rhythmic pattern.

***Clapping Music*** **(1972)**



# Cyclic permutations in minimalistic music

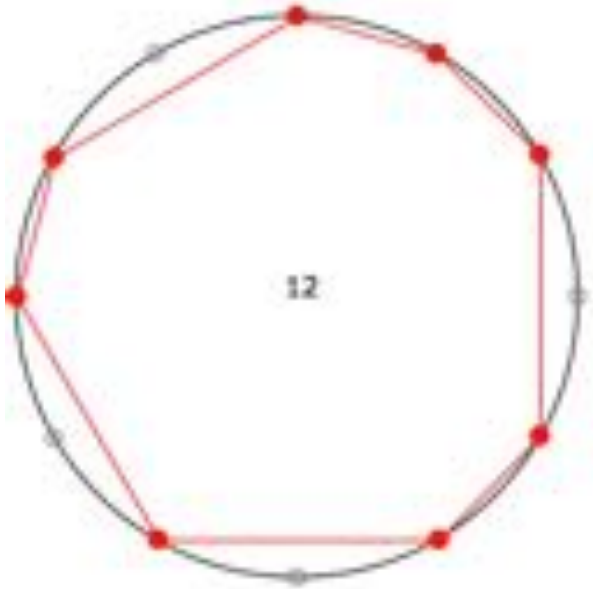

CLAPPING MUSIC

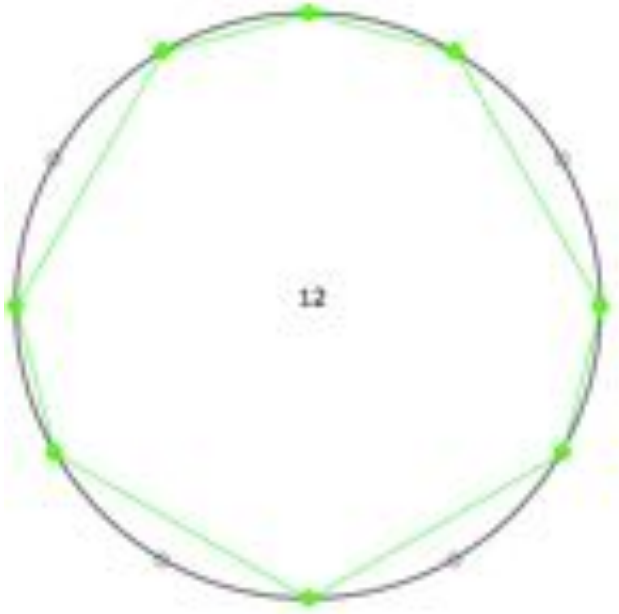
FOR TWO PERFORMERS

The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer's part does not change, it is up to the second performer to cue from one bar to the next. The second performer should try to stay his or her shortest space as written, i.e. on the first beat of each sections (not on the first beat of the group of seven claps), so that his counterpart always falls on a new beat of their oscillating pattern.

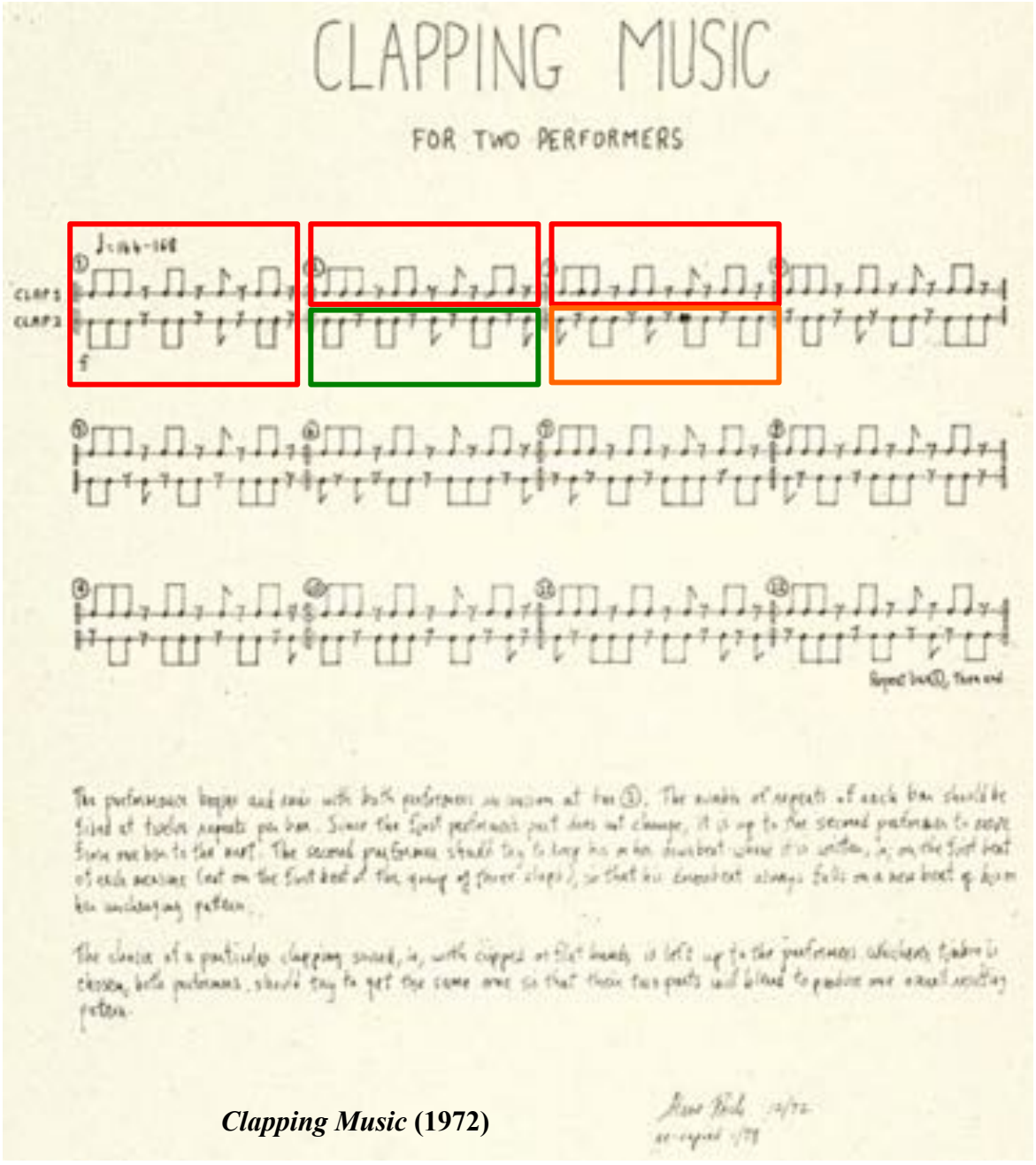
The choice of a particular clapping sound, i.e. with clapped or flat hands, is left up to the performers. Whichever system is chosen, both performers should try to get the same one so that these two parts will blend to produce one overall resulting pattern.

*Clapping Music* **(1972)**



# Cyclic permutations in minimalistic music



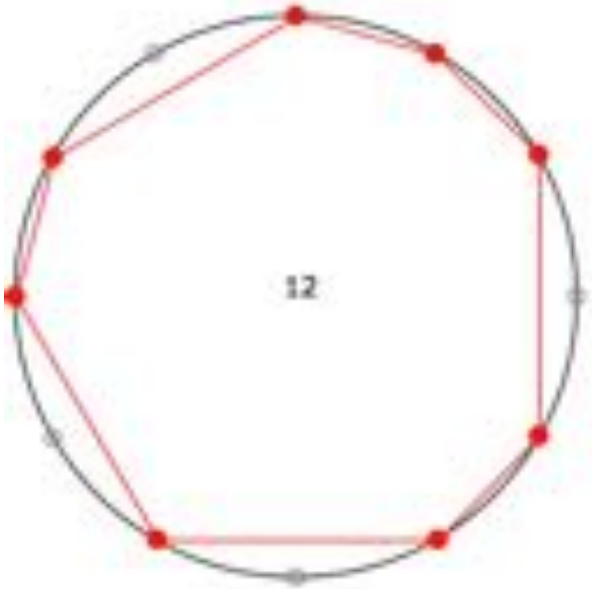
CLAPPING MUSIC

FOR TWO PERFORMERS

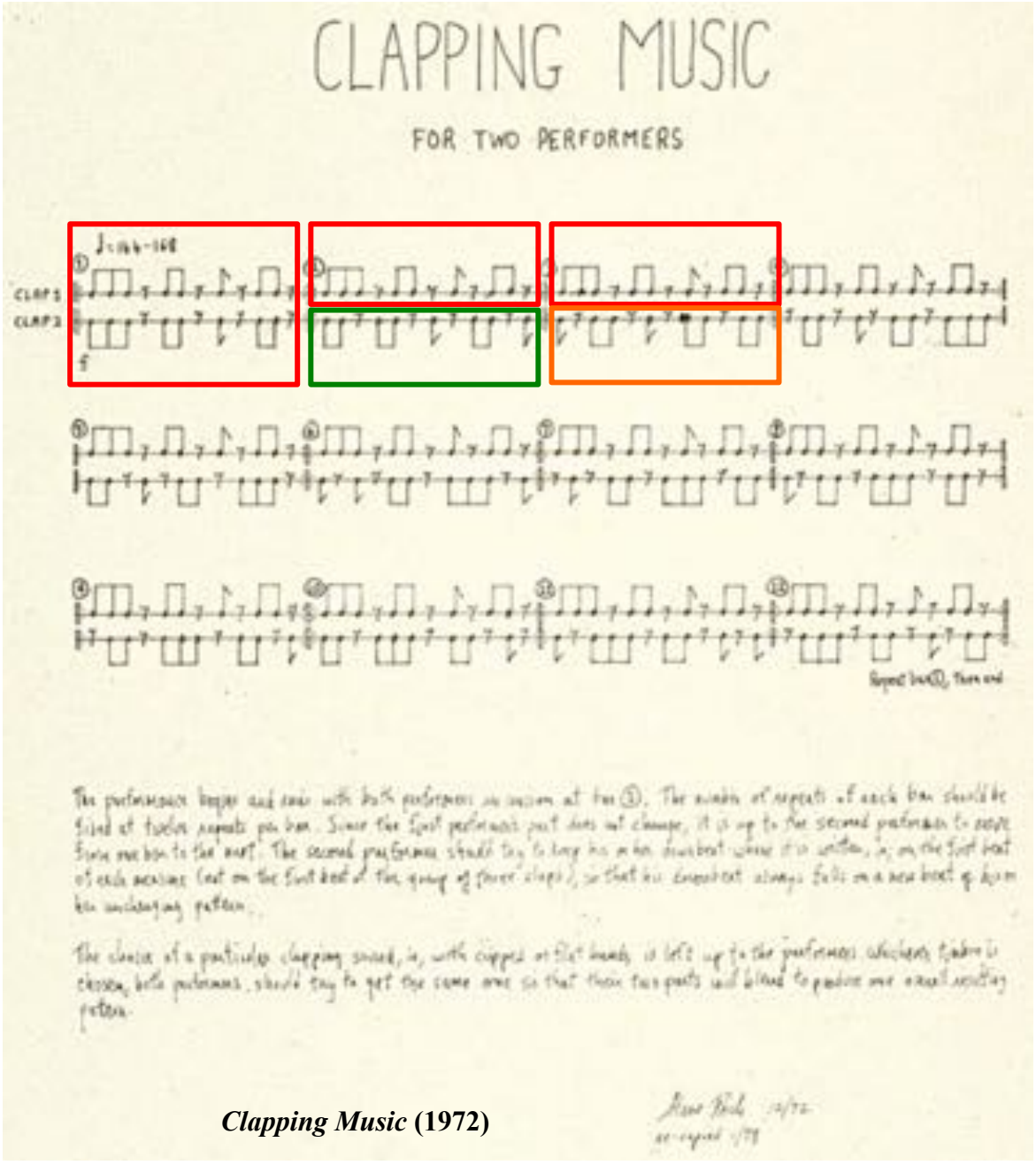
The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer's part does not change, it is up to the second performer to cue from one bar to the next. The second performer should try to stay in an inaudible unison with the first at the beginning of each section (not on the first beat of the group of eleven claps), so that his counterpart always falls on a new beat of the accompanying pattern.

The choice of a particular clapping section, i.e., with claps or flat hands, is left up to the performers. Whichever pattern is chosen, both performers should try to get the same one so that these two parts will blend to produce one overall rhythmic pattern.

*Clapping Music* **(1972)**




# Cyclic permutations in minimalistic music



CLAPPING MUSIC

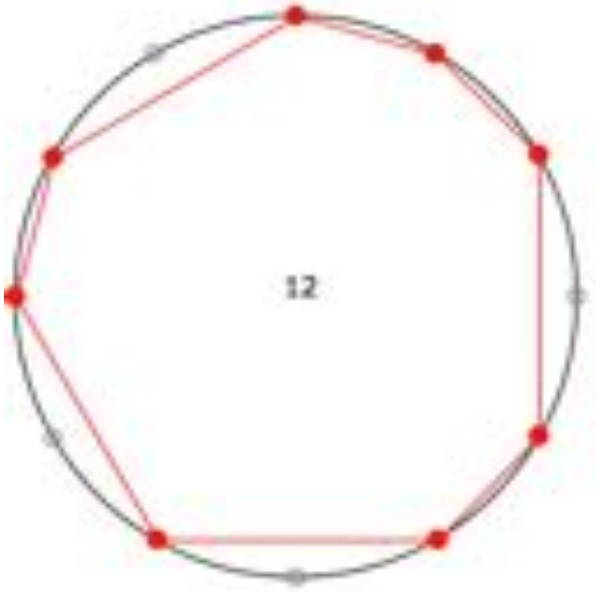
FOR TWO PERFORMERS

CLAP 1
CLAP 2

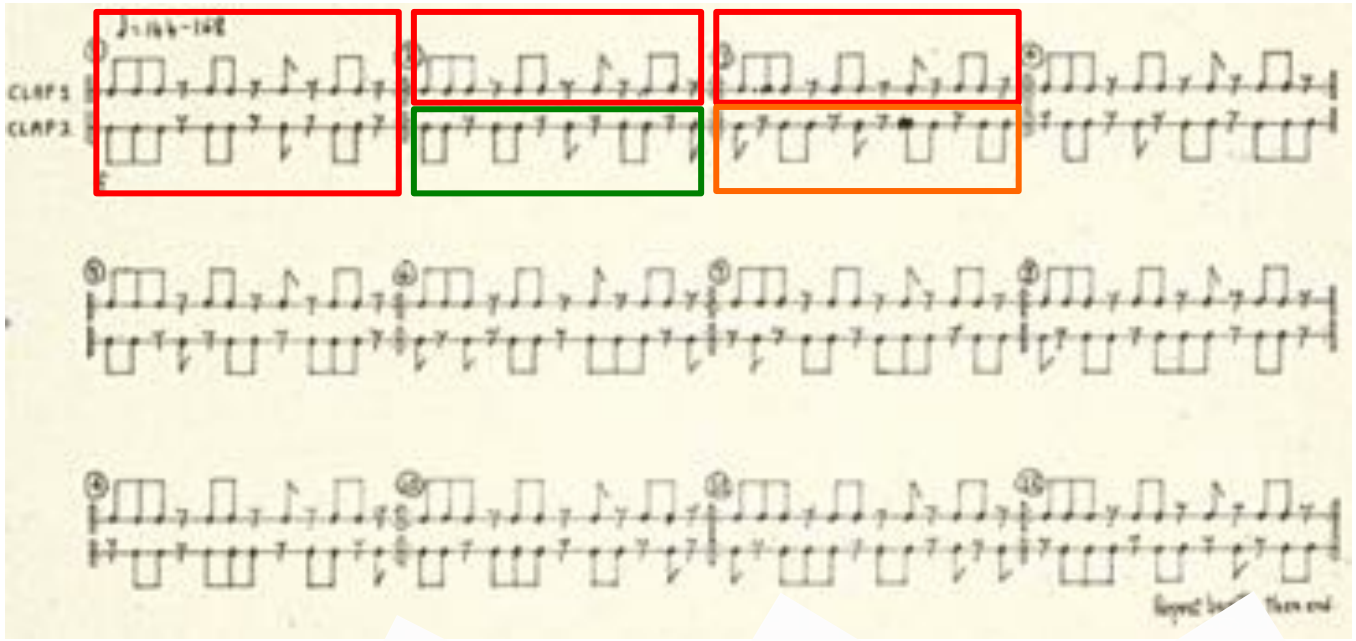
The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer's part does not change, it is up to the second performer to cue from one bar to the next. The second performer should try to stay his or her slowbeat where it is written, i.e. on the first beat of each sections (not on the first beat of the group of seven claps), so that his counterpart always falls on a new beat of their oscillating pattern.

The choice of a particular clapping sound, i.e. with clapped or flat hands, is left up to the performers. Whichever system is chosen, both performers should try to get the same one so that these two parts will blend to produce one overall rhythmic pattern.

*Clapping Music* **(1972)**

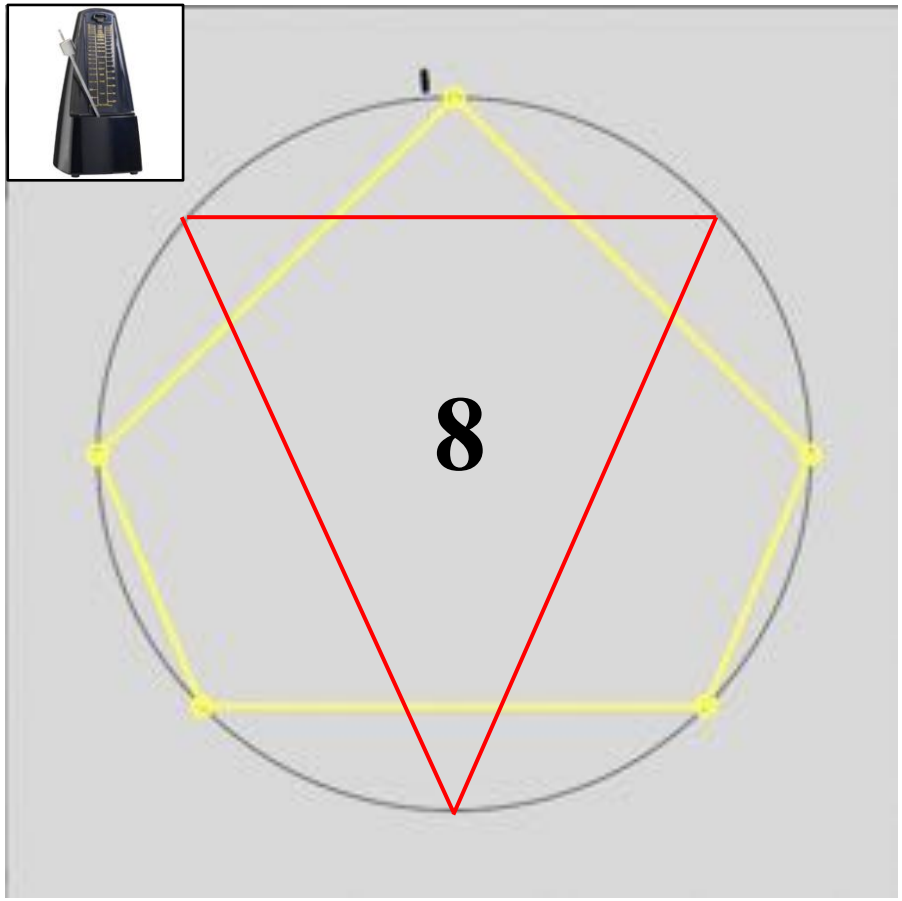


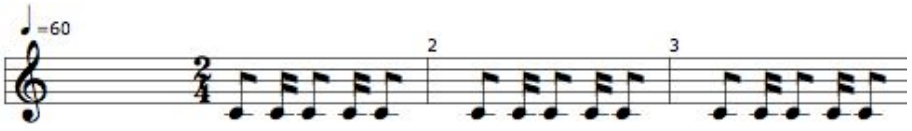
# Cyclic permutations in minimalistic music



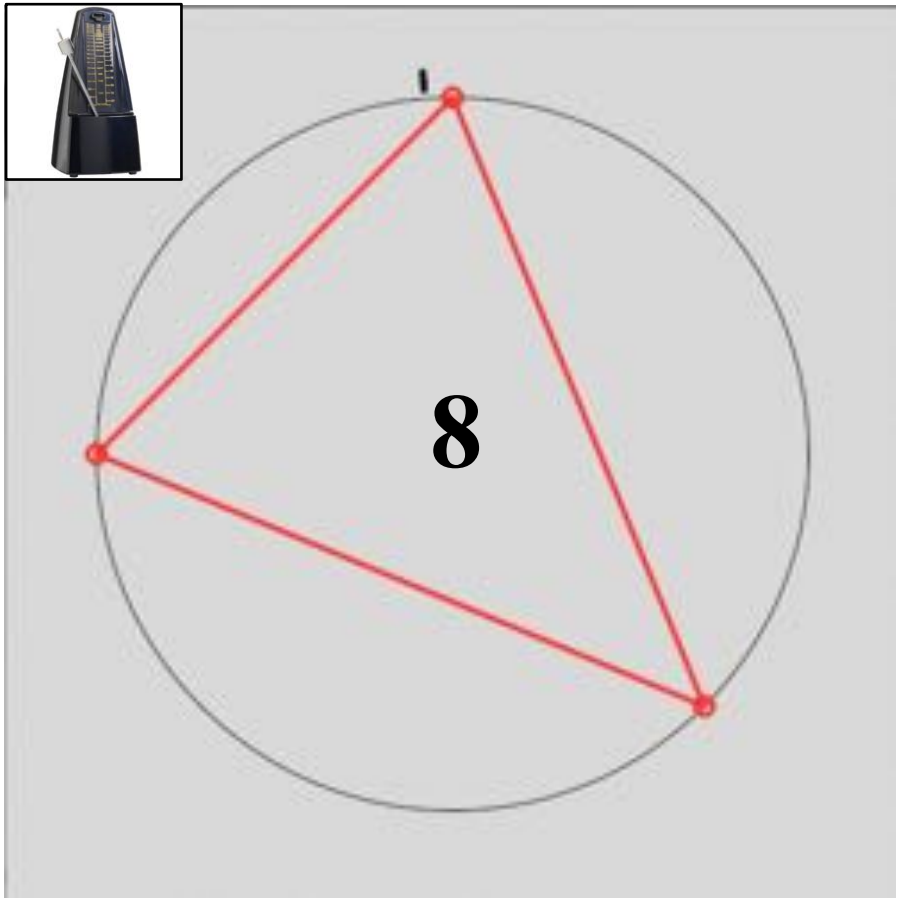
# Clapping African-cuban ME-rhythms?

## *El cinquillo*



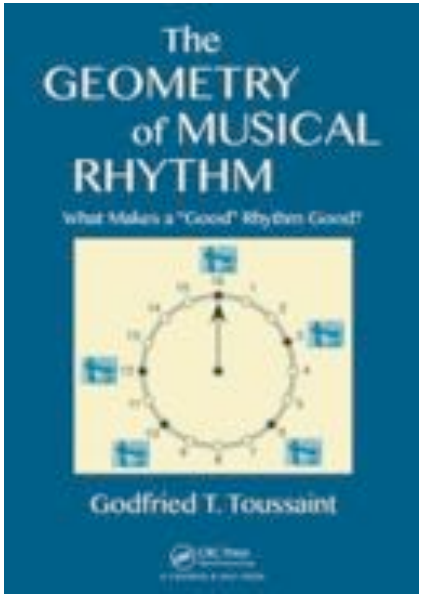



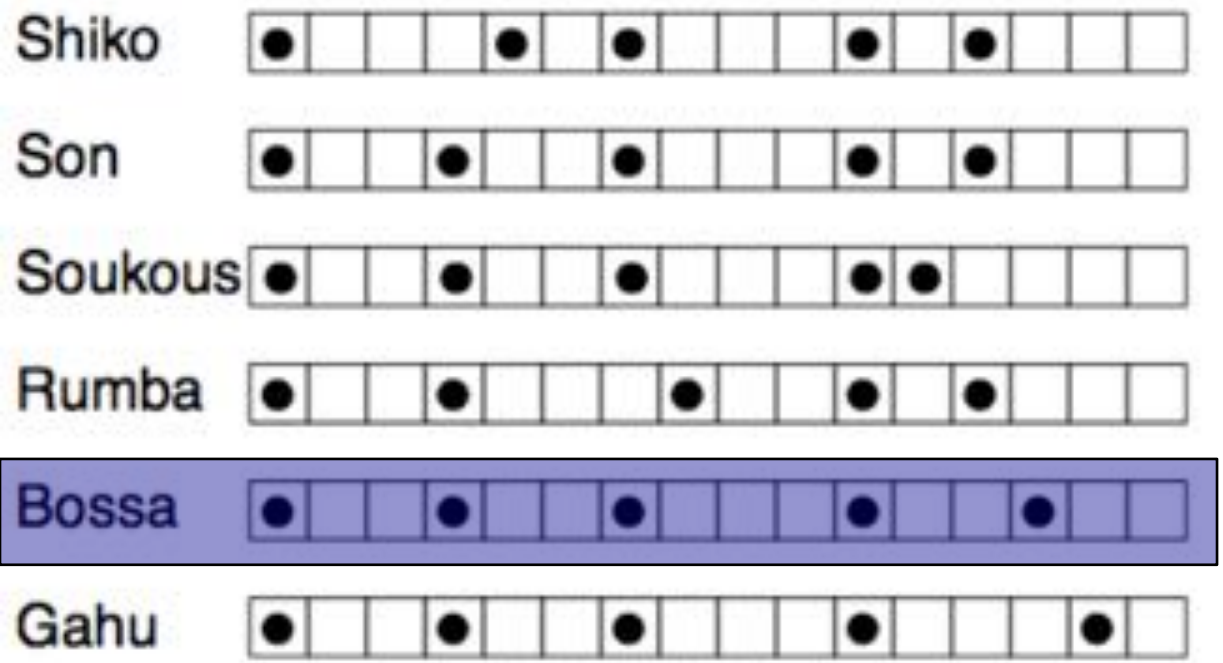

## *El trecillo*

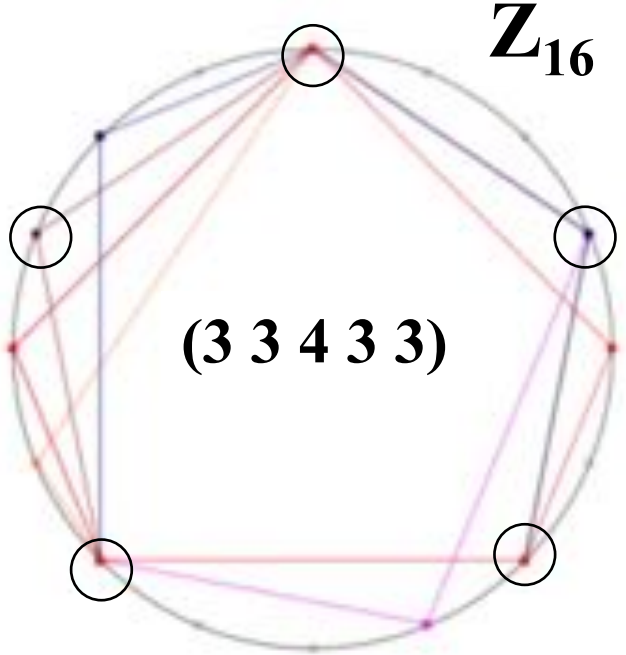



# Clapping African-cuban (ME-)rhythms

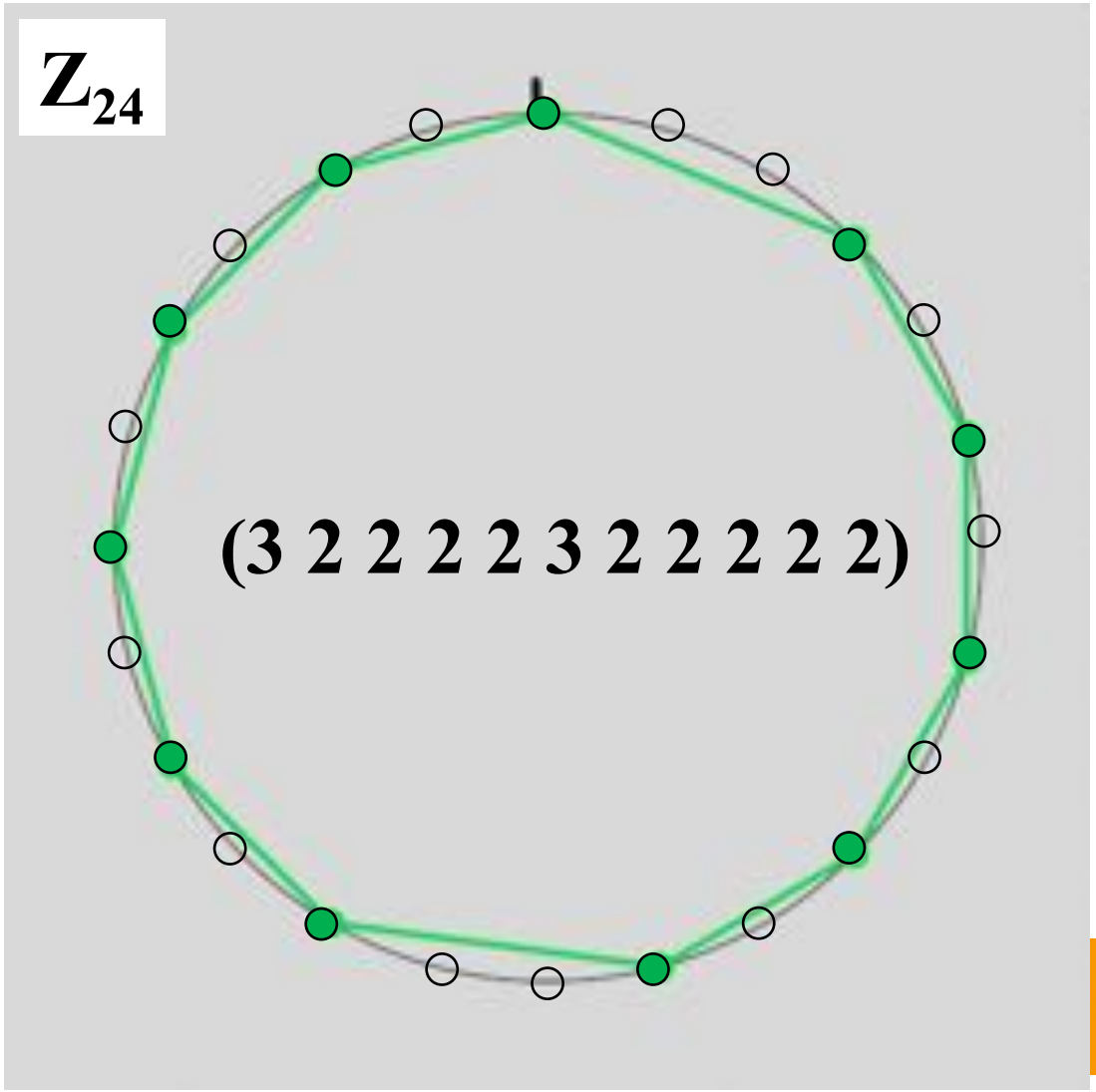




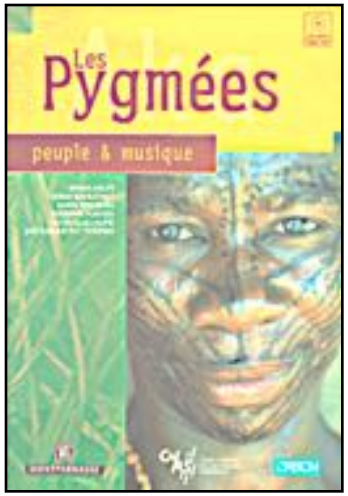



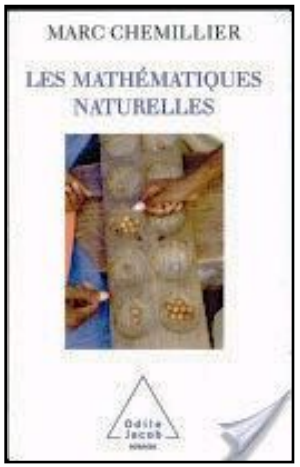
# Clapping odditive rhythms?




Simha Arom

Marc Chemillier

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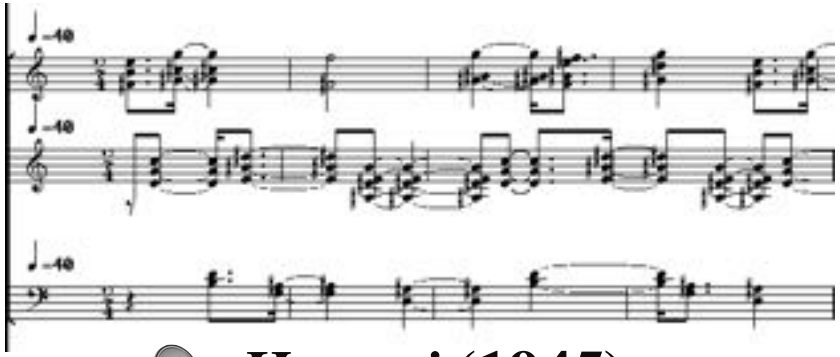
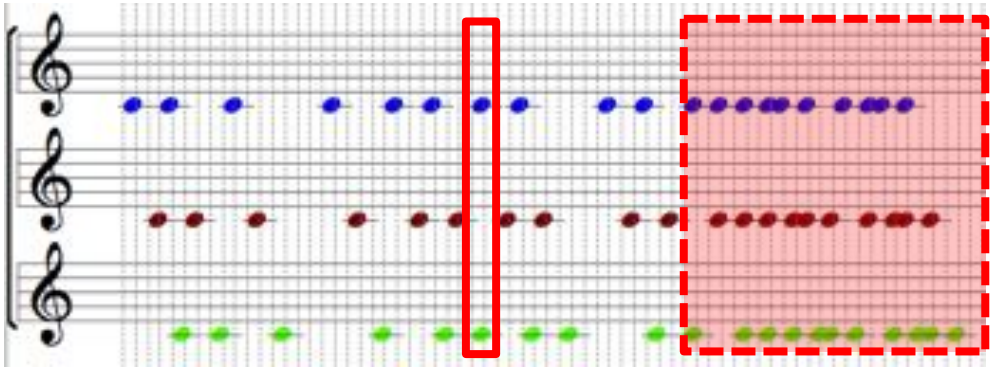
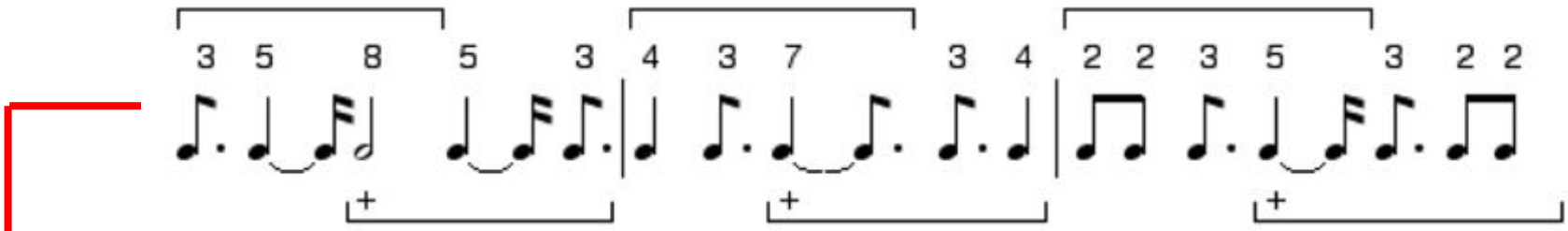
# Periodic rhythmic sequences and tiling canons

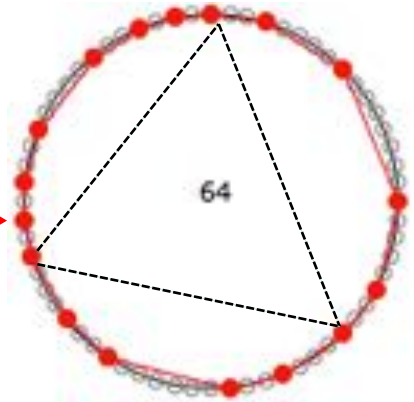


*Harawi* (1945)



***Harawi*: rhythmic reduction**

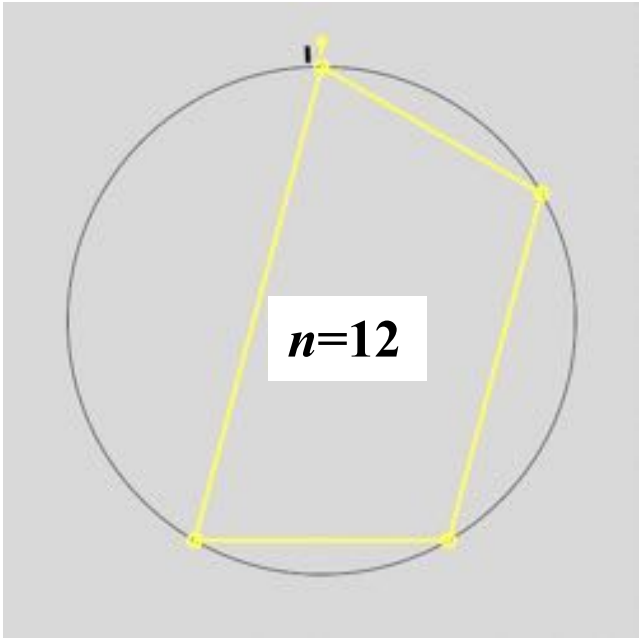


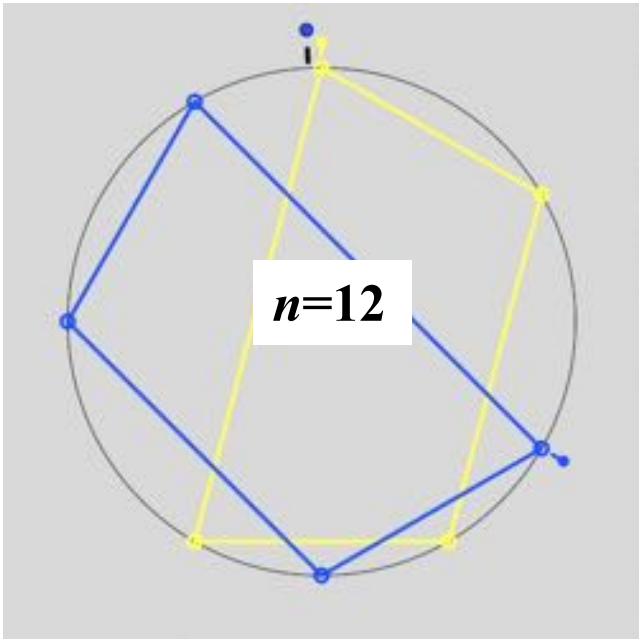



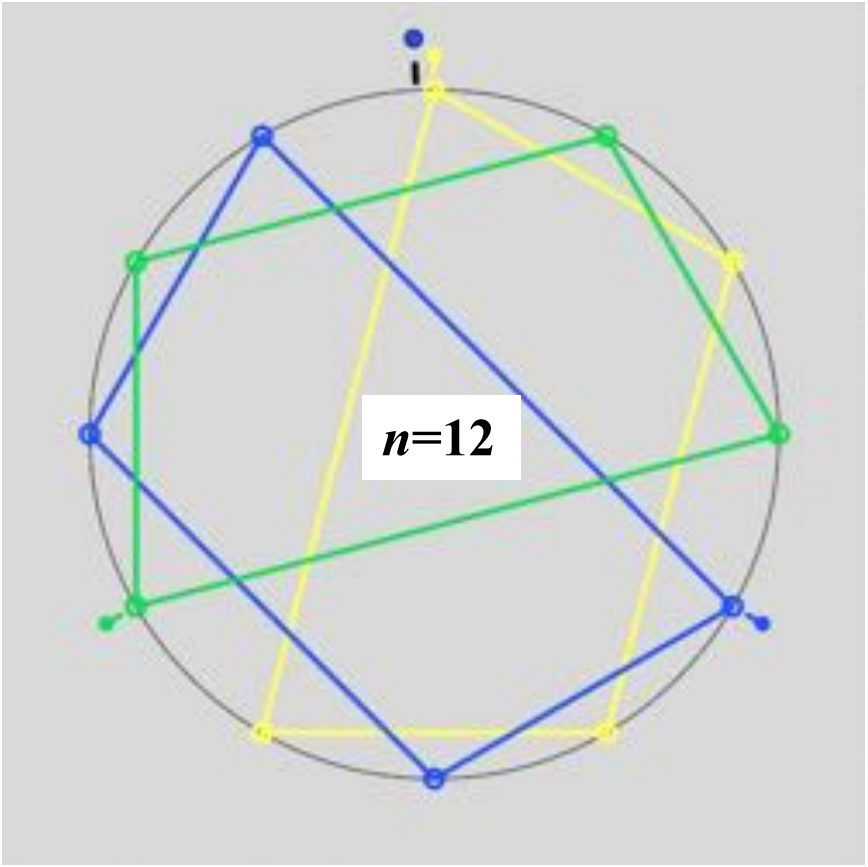

**« …il résulte de tout cela que les différentes sonorités se mélangent ou s'opposent de manières très diverses, jamais au même moment ni au même endroit […]. C'est du désordre organisé »**

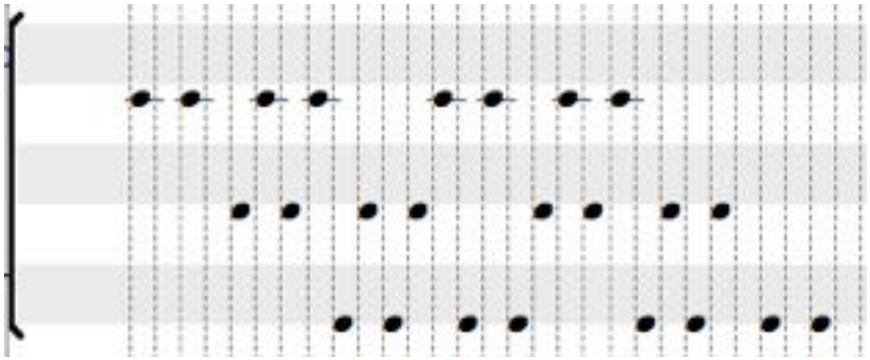
**O. Messiaen: *Traité de Rythme, de Couleur et d'Ornithologie,* tome 2, Alphonse Leduc, 1992.**

# Tiling the time axis with translates of one tile

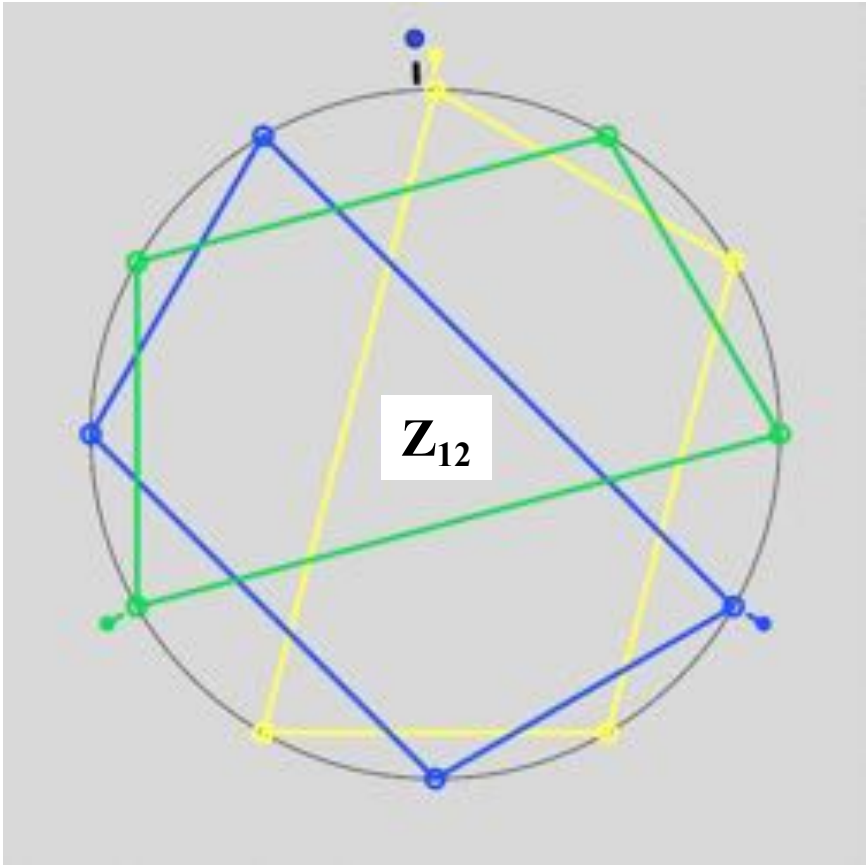


# Formalizing the tiling process as set-theoretical operations




$A_1 = \{0, 2, 5, 7\}$

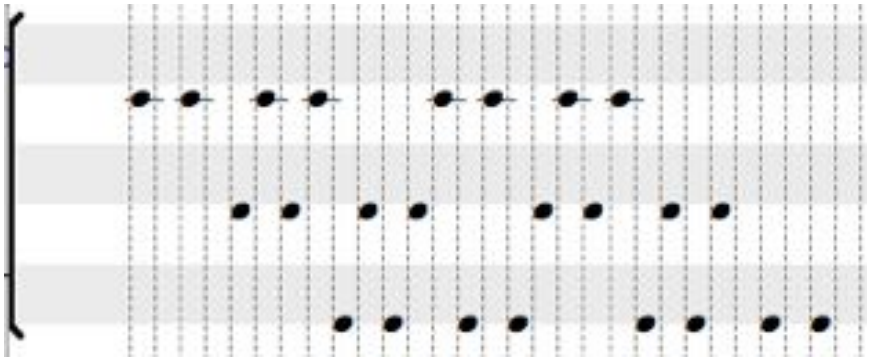
$T_4$ ↓

$A_2 = \{4, 6, 9, 11\}$

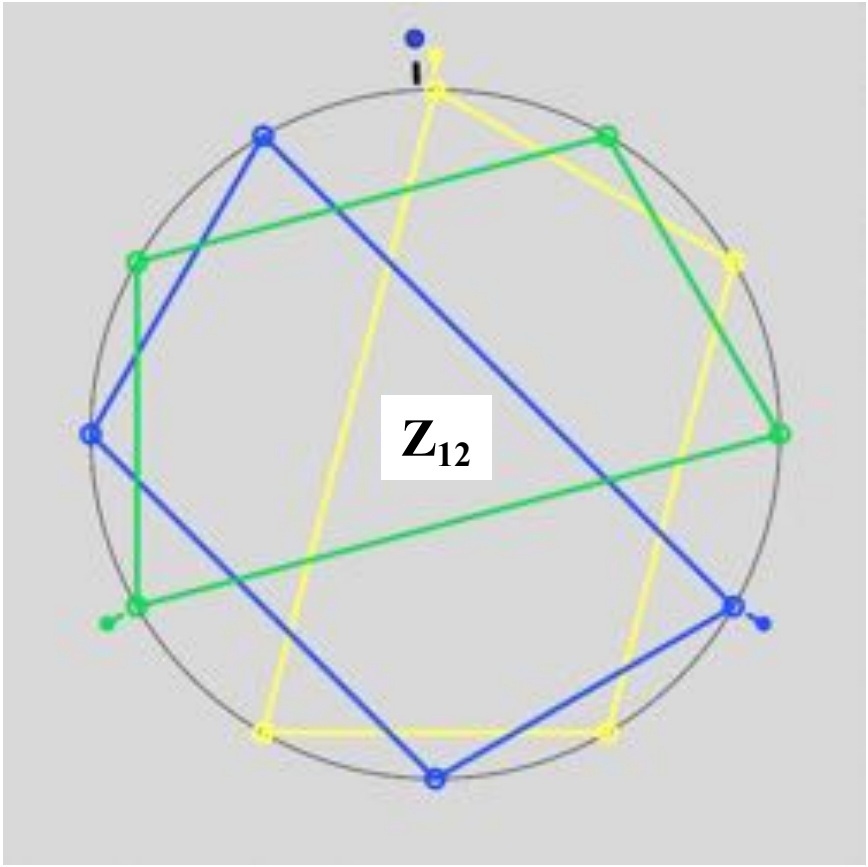
$T_4$ ↓

$A_3 = \{8, 10, 1, 3\}$

$Z_{12} = A_1 \cup A_2 \cup A_3$



# Formalizing the tiling process as a direct sum of subsets



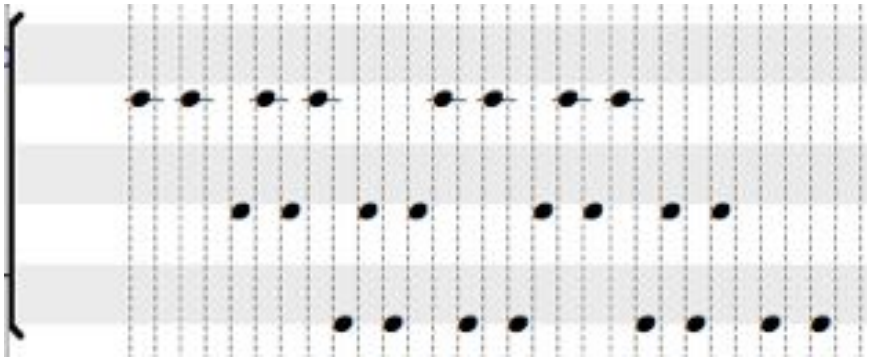

$A_1 = \{0, 2, 5, 7\}$

$T_4$ ↓

$A_2 = \{4, 6, 9, 11\}$

$T_4$ ↓

$A_3 = \{8, 10, 1, 3\}$



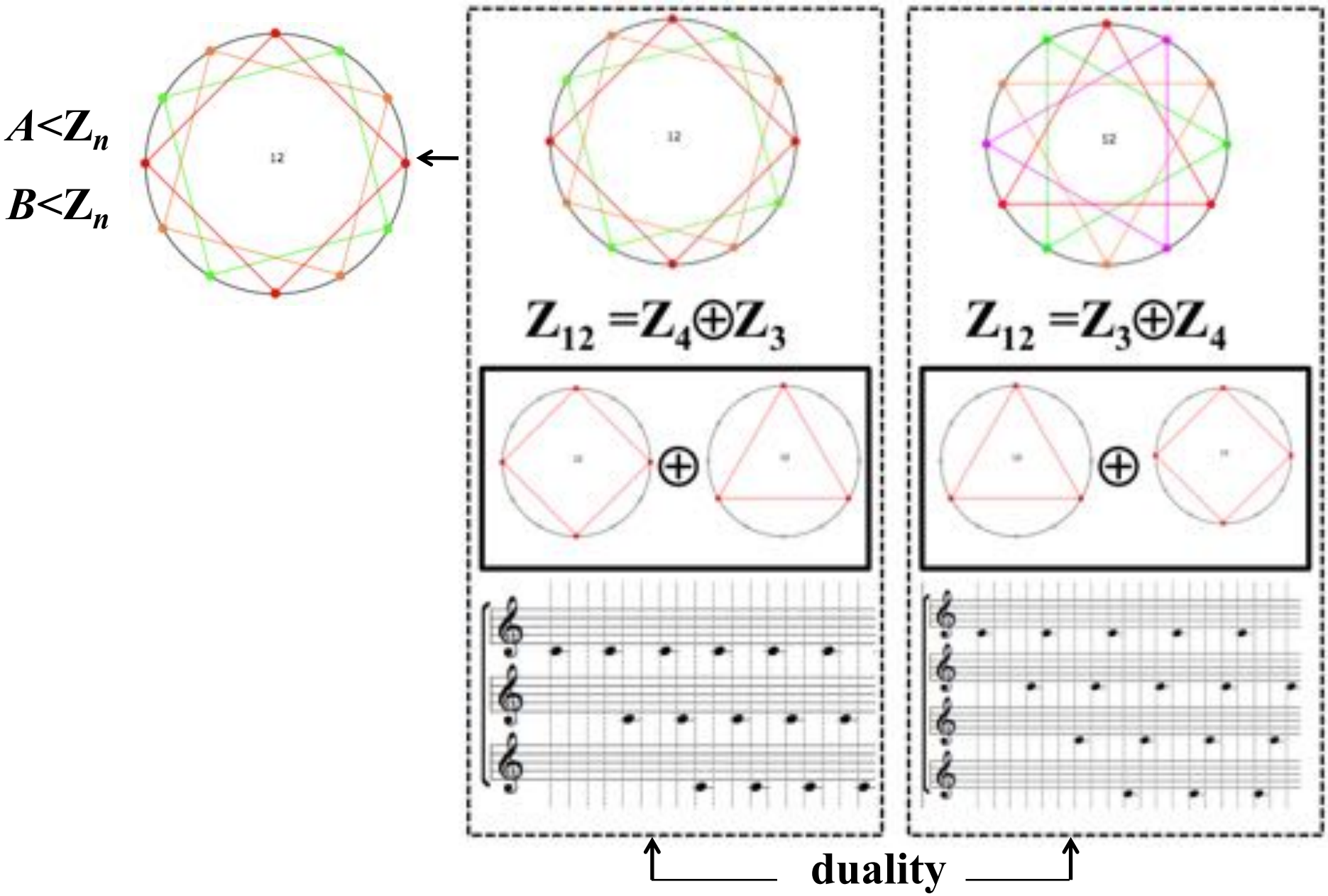
$Z_{12} = A_1 \cup A_2 \cup A_3$

$Z_{12} = A \oplus B$

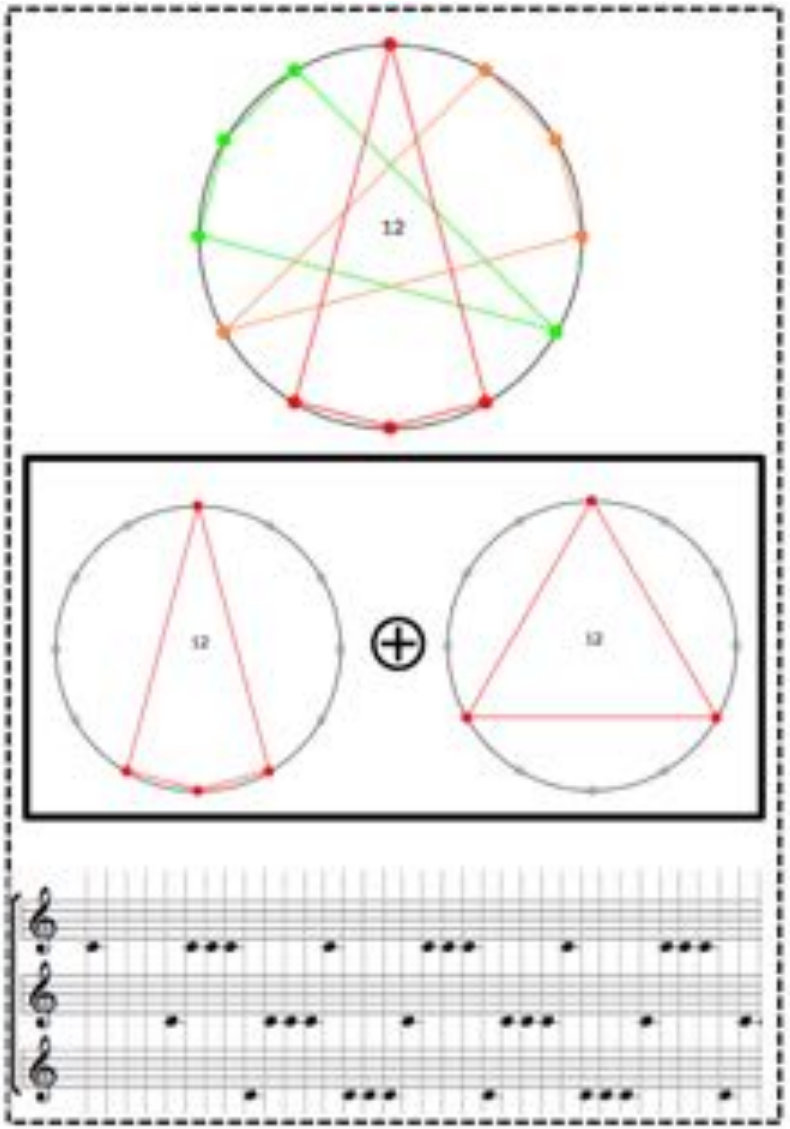
$A = \{0,2,5,7\}$

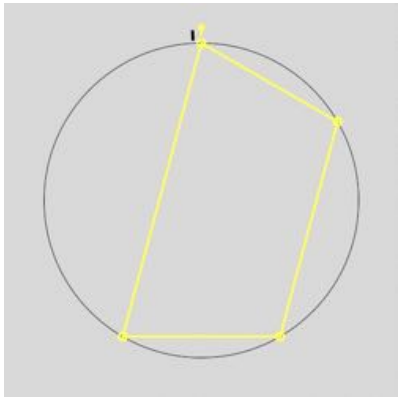
$B = \{0,4,8\}$

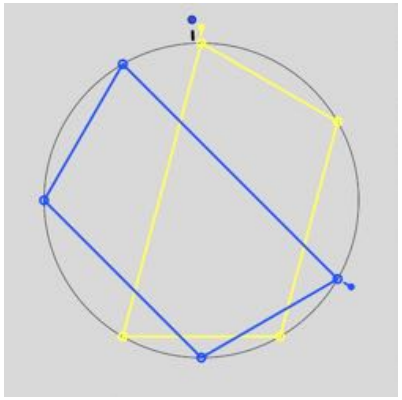
# Type 1: tiling rhythmic canons with max regularity

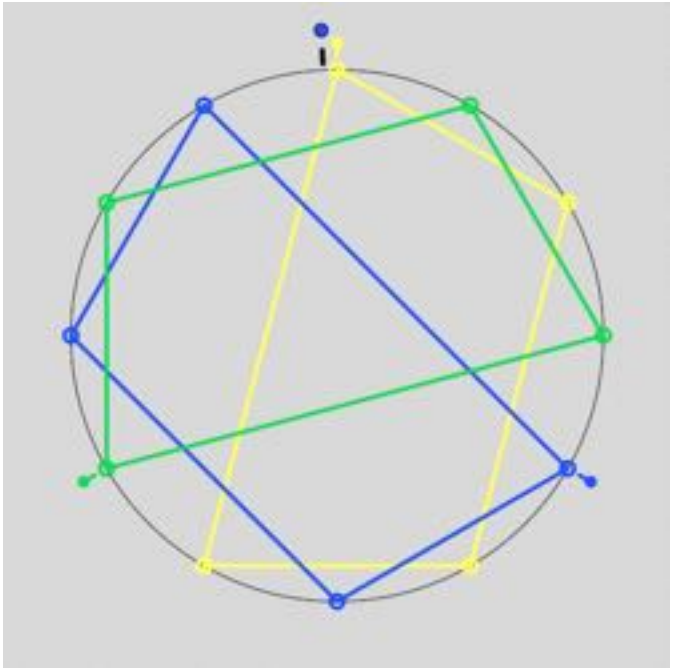


# Type 2: Rhythmic Tiling Canons with less regularity



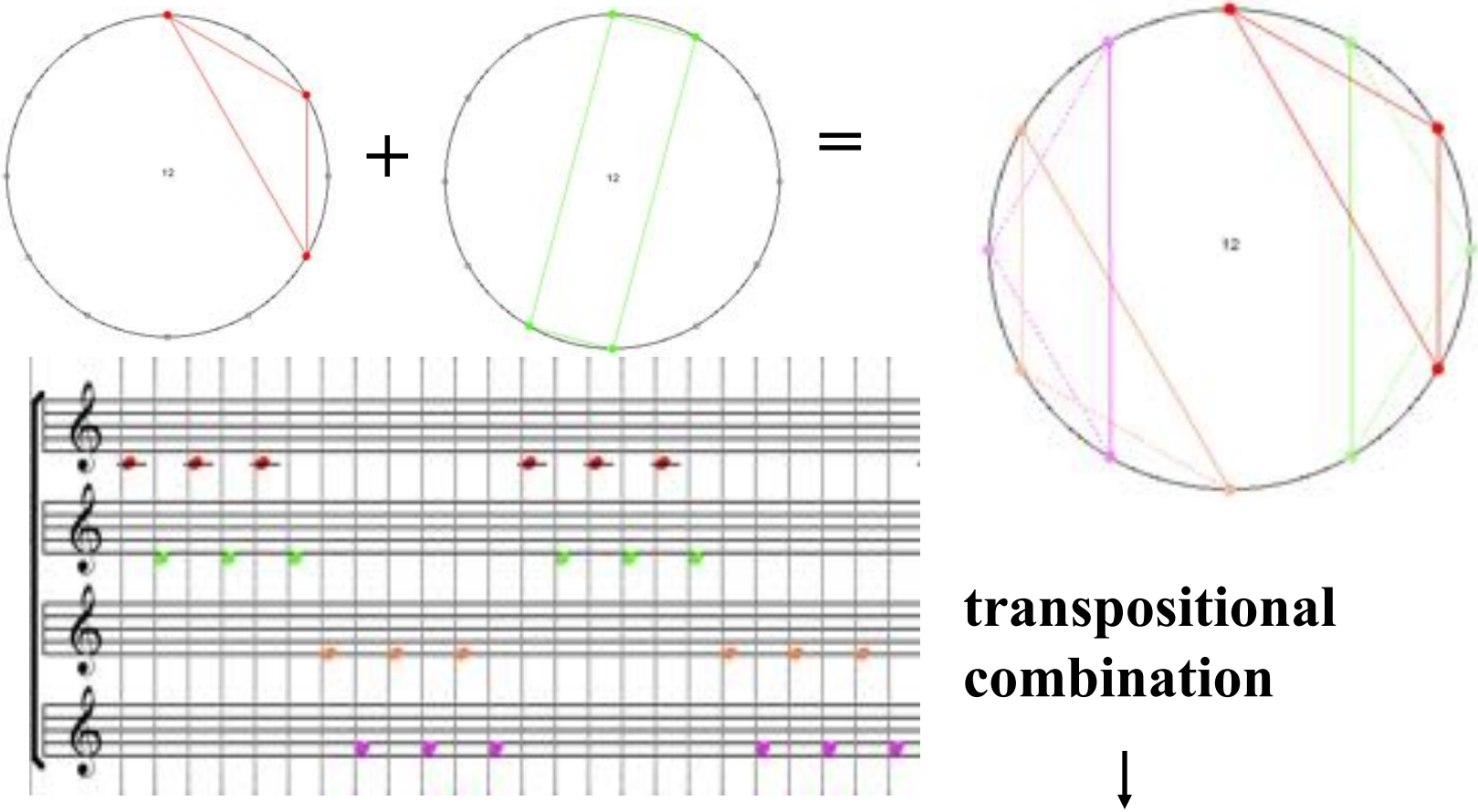




{0,5,6,7}⊕{0,4,8}

{0,2,5,7}⊕{0,4,8}

# Type 3: rhythmic tiling canons with no regular entries

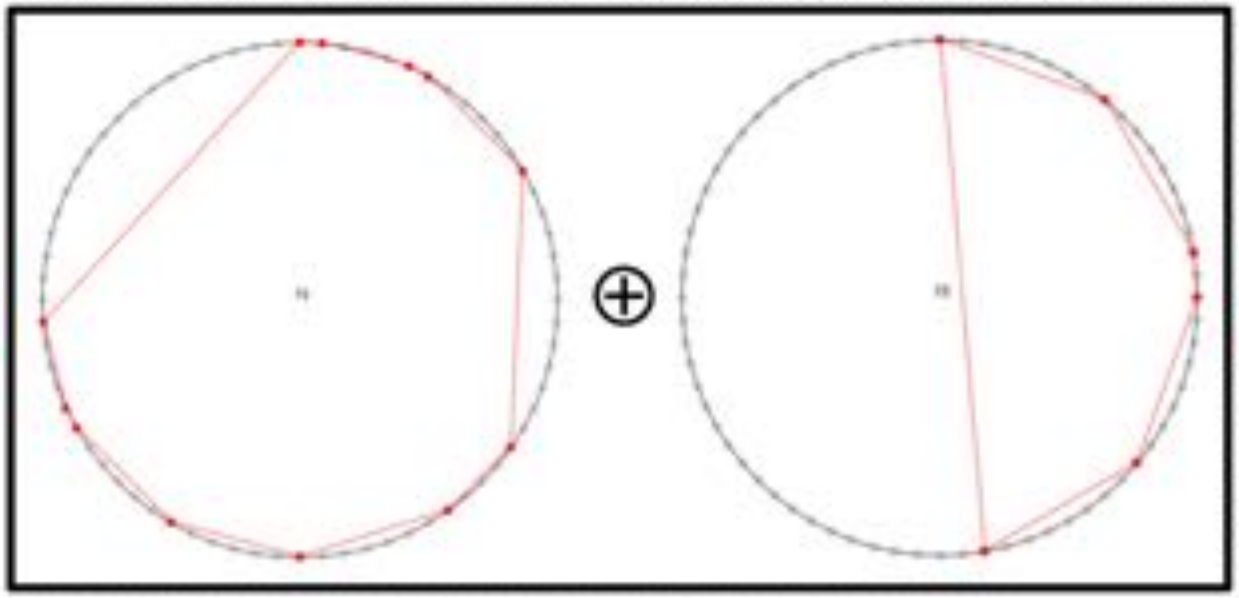


transpositional combination

$$\{0,2,4\} \oplus \{0,1,6,7\} = Z_{12} = (2\ 2\ 8) \bullet (1\ 5\ 1\ 5)$$

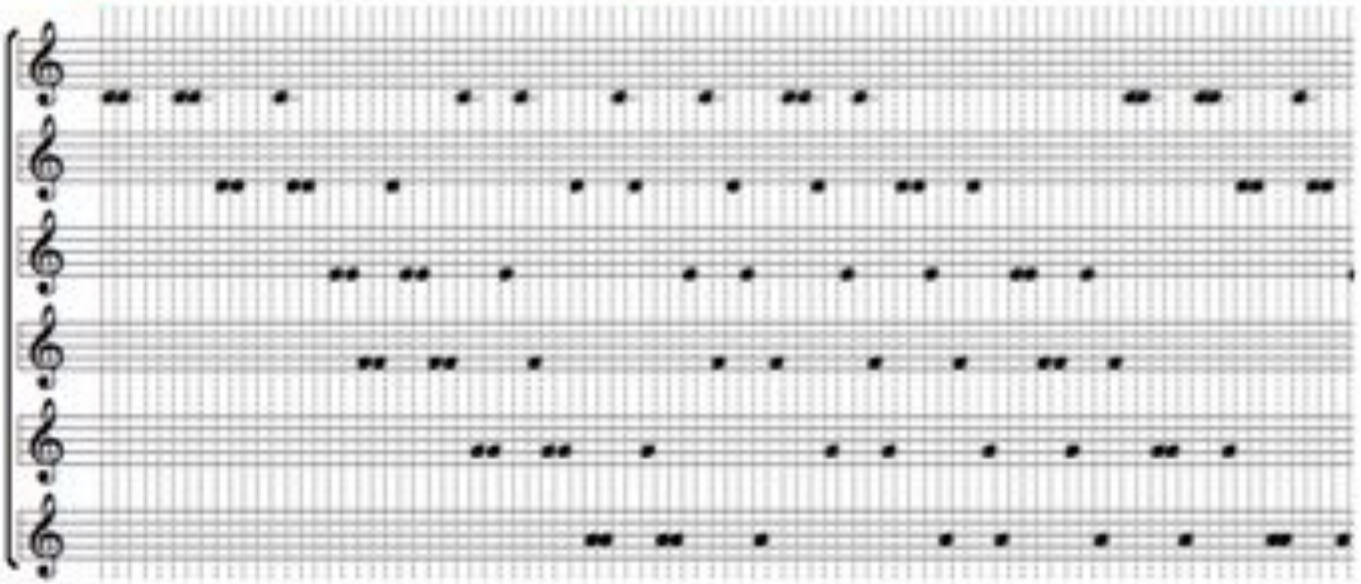
One of the two factors is a Messiaen's mode of limited transposition

# Aperiodic Rhythmic Tiling Canons (Vuza Canons)

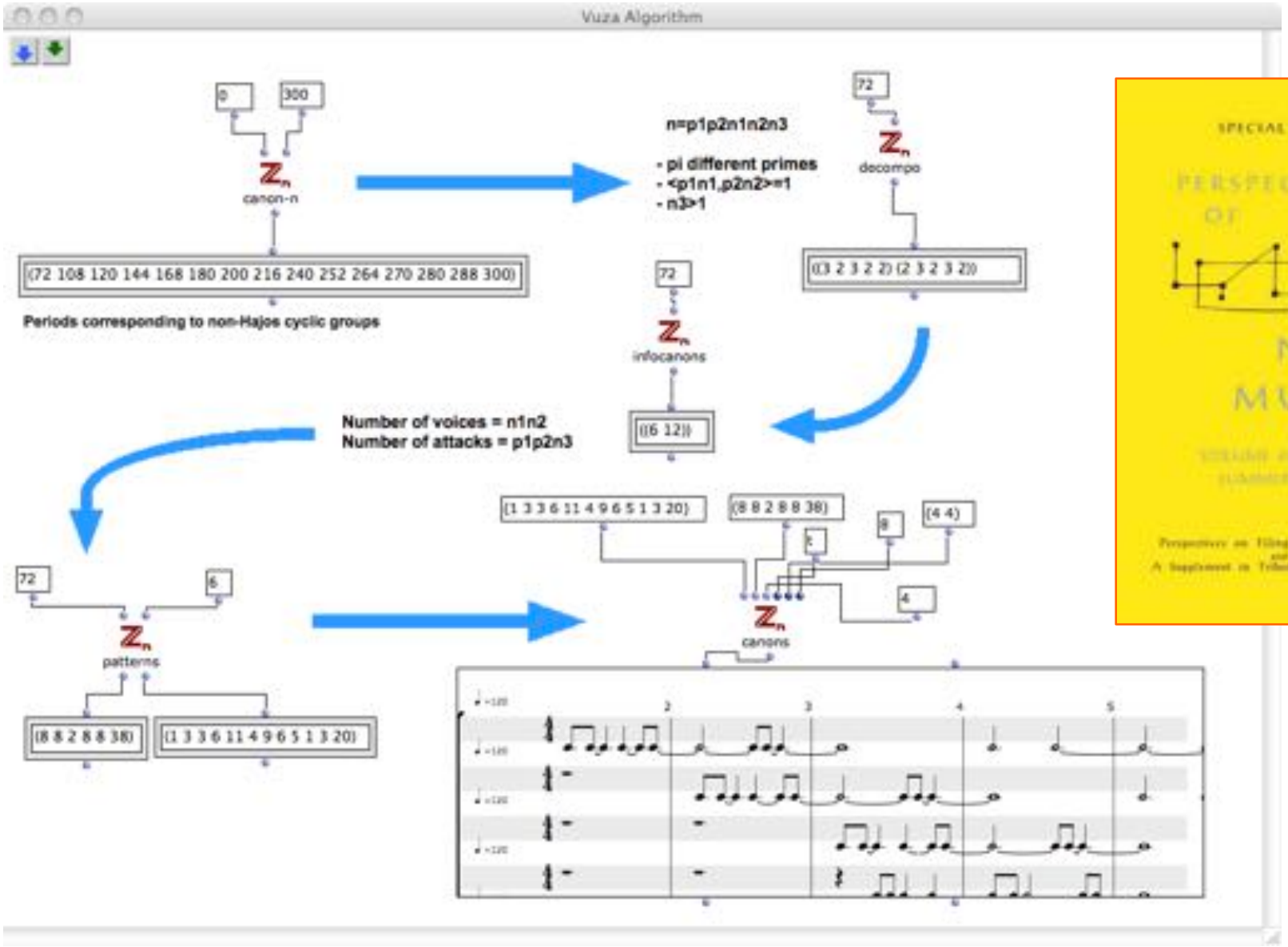


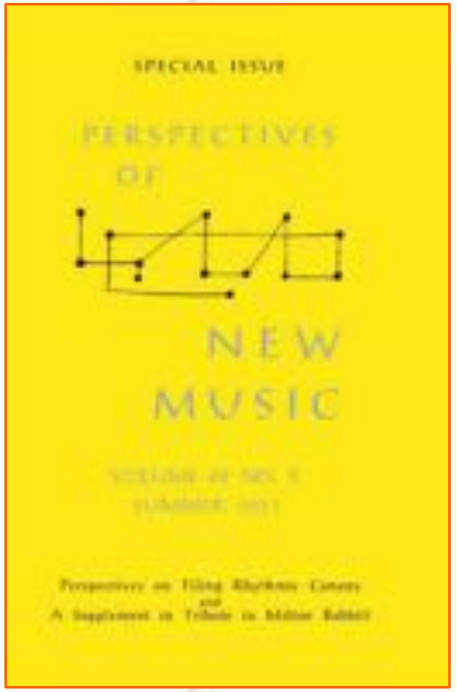
Dan Vuza

Anatol Vieru



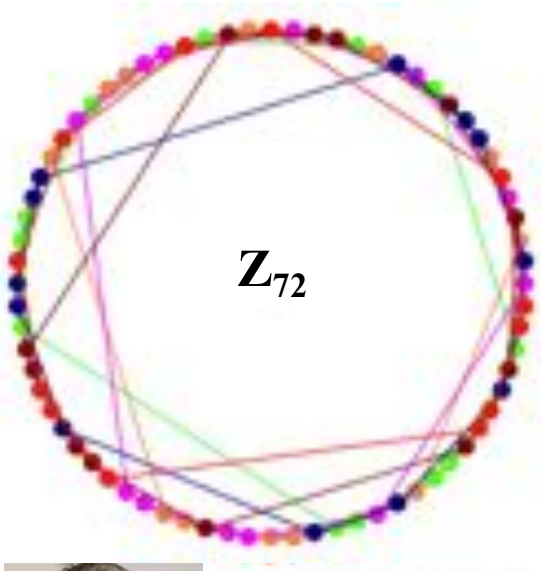
# Vuza Canons in OpenMusic 'MathTool' environment

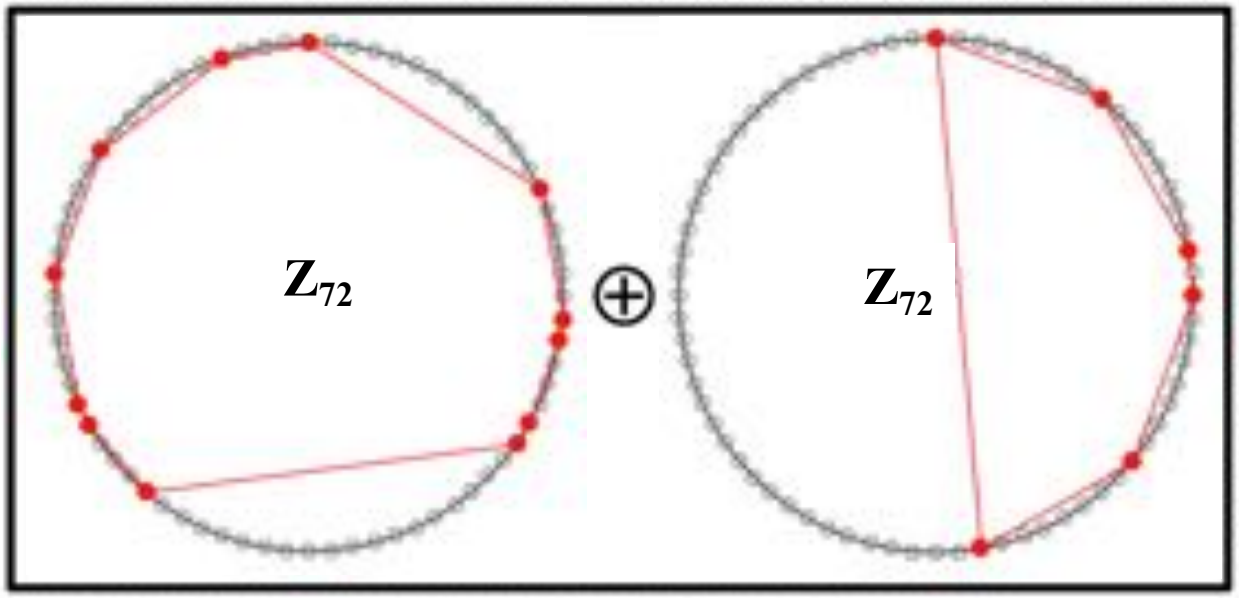



Vuza Algorithm

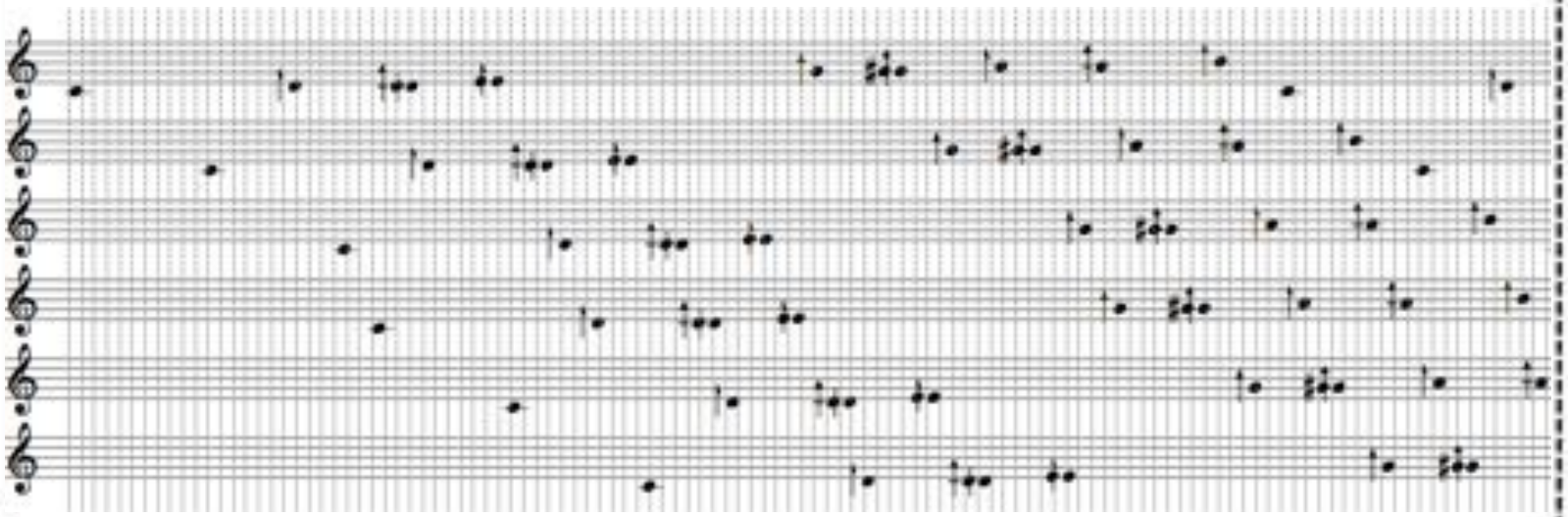
# Microtonal Vuza Canons



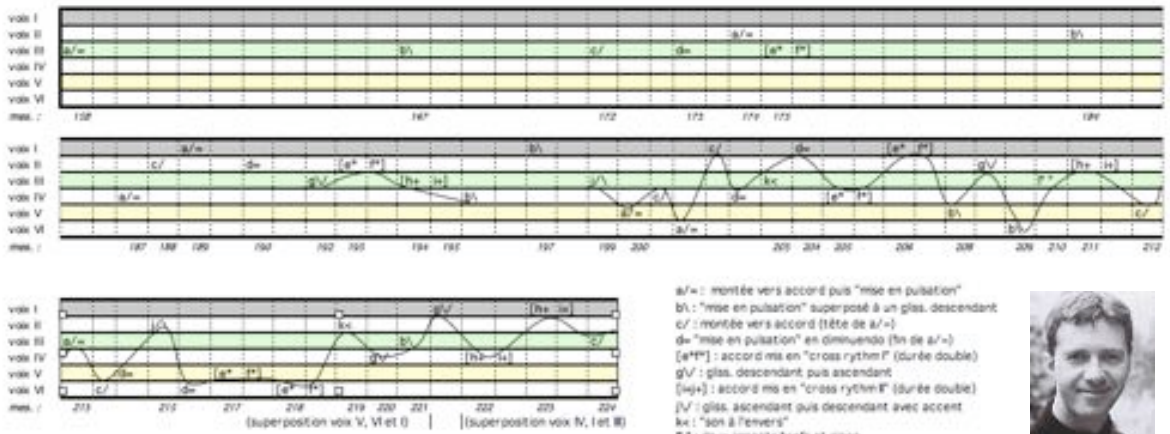




Dan Vuza

Anatol Vieru



# Some compositional applications of the Vuza Canons model

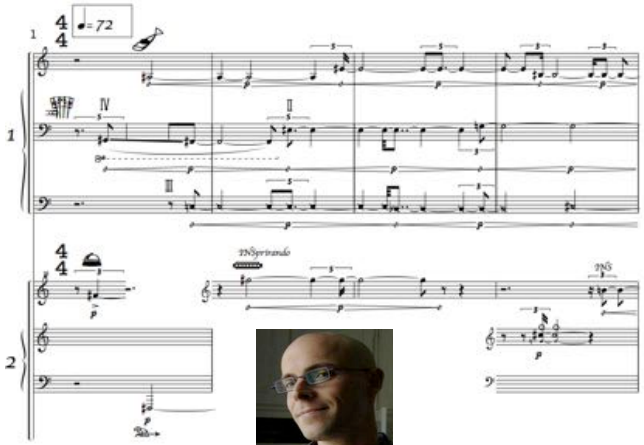



***Coïncidences* (1999)**

Coïncidences - Fabien Levy : déroulement du canon (mes. 158 à 226)
(chaque impact fait 3 temps)



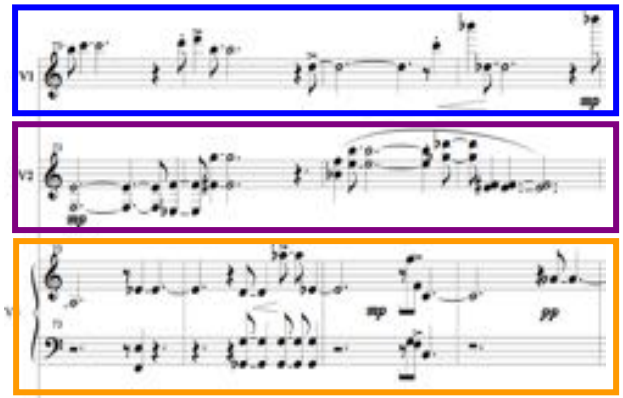
**F. Lévy**





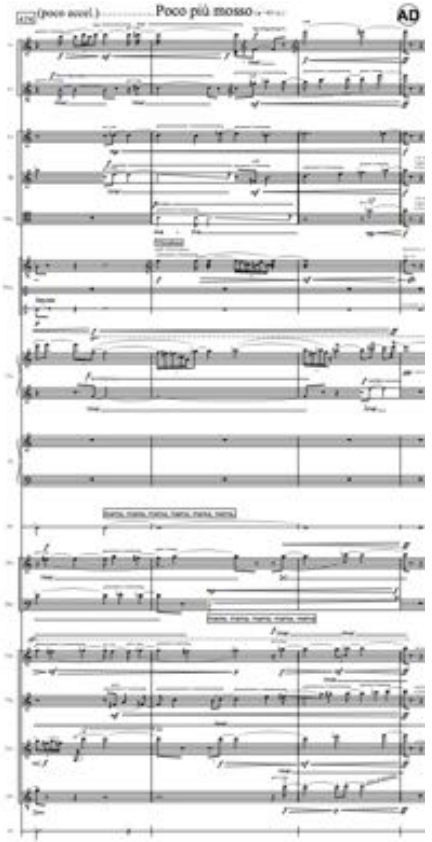
***La bataille de caresme et de charnage***
(pour violoncelle et accompagnement, 2012)

**M. Lanza**




***A piece based on Monk* (2007)**
**(« Well You Need'nt »)**

**G. Bloch**



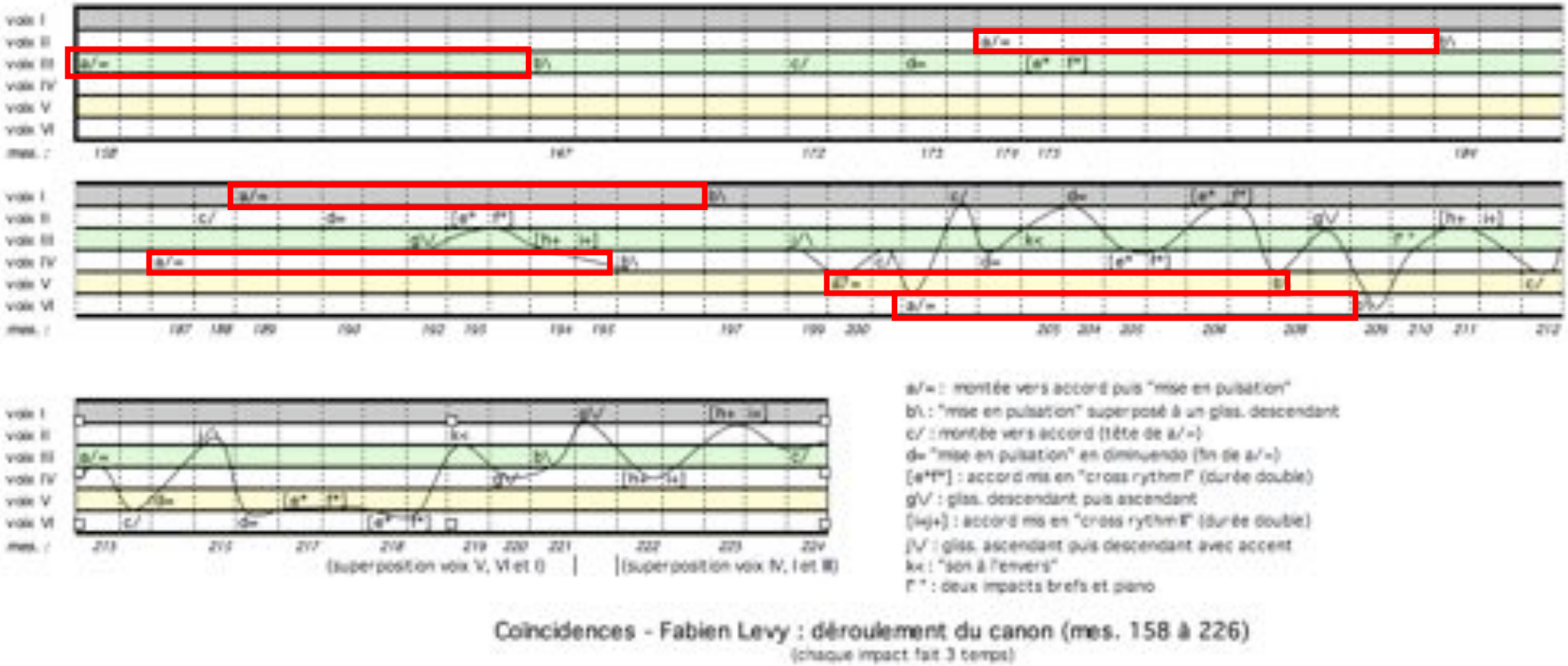

***La notte poco prima della foresta***
(opéra de chambre
pour acteur, mezzo-soprano, baryton, ensemble et électronique, 2009)

**D. Ghisi**

# Fabien Lévy

## Morphological Tiling Canons


- *Coïncidences* (pour 33 musiciens, 1999-2007)




Coïncidences - Fabien Levy : déroulement du canon (mes. 158 à 226)
(chaque impact fait 3 temps)

Tokyo Symphony Orchestra, Dir.: Kazuyoshi Akiyama, 05/09/2007, Suntory Hall, Tokyo, Japon

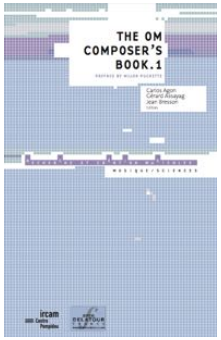
F. Lévy, « Three uses of Vuza Canons », *Perspectives of New Music*, 49(2), 2011, p. 23-31.

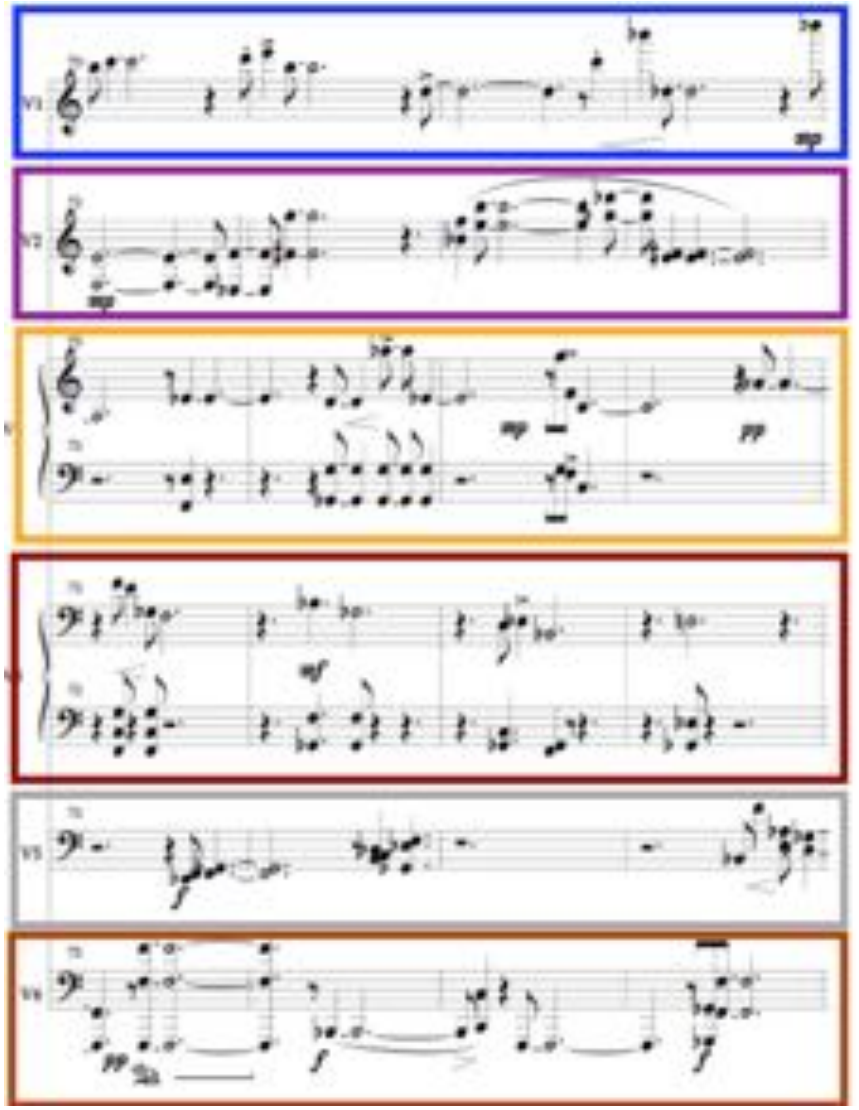
# Georges Bloch

## Several compositional strategies

- **Metrical organization of a tiling canon**
- **Self-similarity processes**
- **Metrical modulations between canons**

- ***Projet Beyeler*** **(2001)**
- *Projet Hitchcock*
- *Visite des tours de la cathédrale de Reims*
- *Noël des Chasseurs*
- *Canons à marcher*
- *Canon à eau*
- *Harawun* (2004)
- *L'Homme du champ* (2005)
- ***A piece based on Monk*** **(2007)**
- *Peking Duck Soup* (2008)



• ***A piece based on Monk*****, 2007 (« Well You Need'nt »)**

**G. Bloch, « Vuza Canons into the museum », *The OM Composers' Book*, 2006**

# Mauro Lanza

## Vuza Canons and local periodicities

- ***La descrizione del diluvio*** (Ricordi, 2007-2008)


*"6 voices are live and 8 are in the electronic part. The choice of the notes and the durations is made in order to stress some* ***quasi-periodicities*** *of the underlying Vuza canon and this gives to each voice a much more "redundant" character".*

# Mauro Lanza

## Vuza Canons and local periodicities

- ***La descrizione del diluvio*** (Ricordi, 2007-2008)

*"The choice of the notes and the durations is made in order to stress some* ***quasi-periodicities*** *of the underlying Vuza canon* [...]*"*

**(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)**

**(*1 3 25 27 1 3* 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 *1 3 25 27 1 3* 52)**

**(*1 3 25 27* 1 3 11 14 27 *1 3 25 27* 4 25 27 1 3 25 14 13 *1 3 25 27* 1 3 52)**

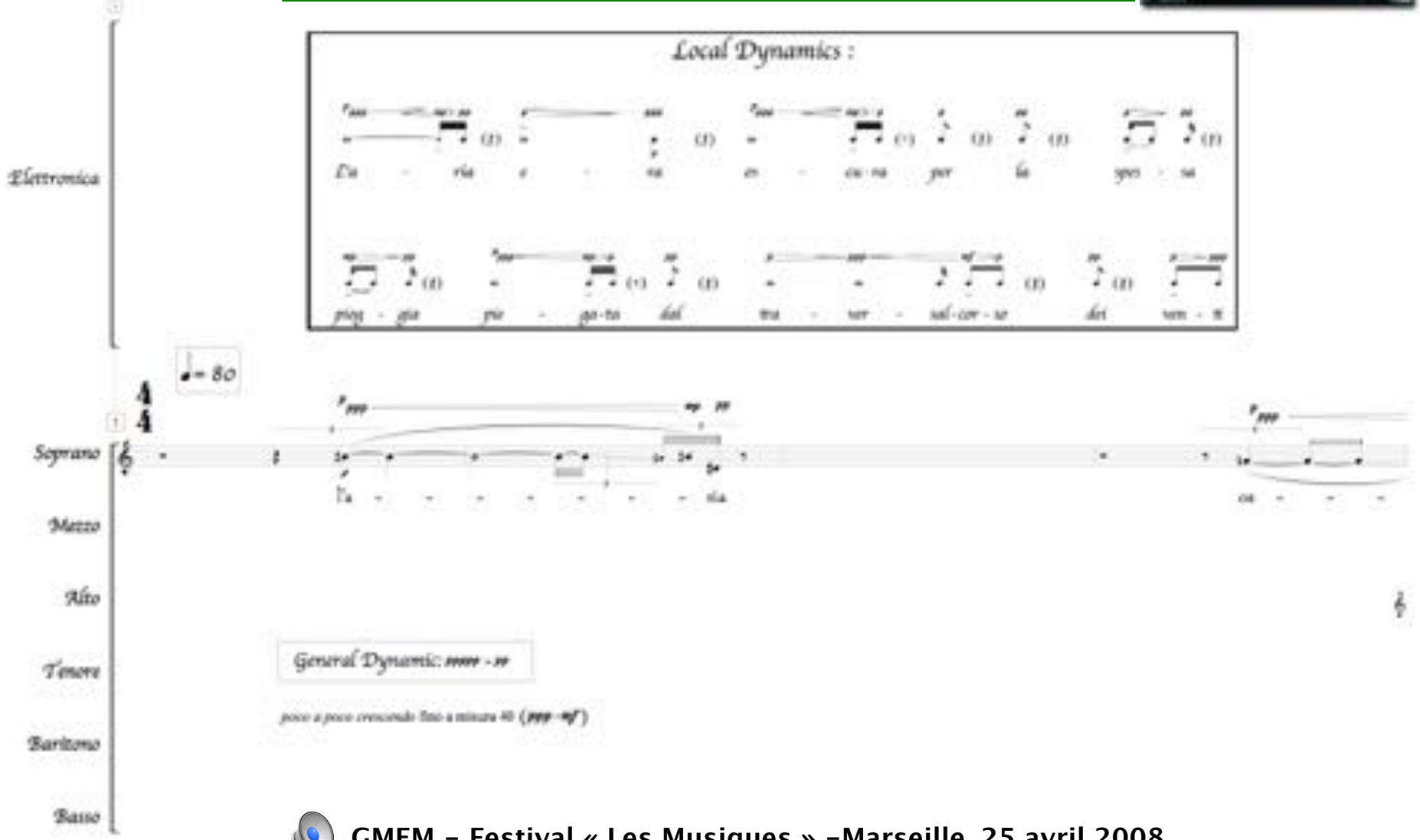
**(*1 3 25* 27 1 3 11 14 27 *1 3 25* 27 4 25 27 *1 3 25* 14 13 *1 3 25* 27 1 3 52)**

**(*1 3* 25 27 *1 3* 11 14 27 *1 3* 25 27 4 25 27 *1 3* 25 14 13 *1 3* 25 27 *1 3* 52)**

# Mauro Lanza

## Vuza Canons and local periodicities

• ***La descrizione del diluvio*** (Ricordi, 2007-2008)



Mauro Lanza

# LA BATAILLE DE CARESME ET DE CHARNAGE

per violoncello e accompagnamento (2012)

**(min. 10’15”)**

190

*Canone sulla II corda*

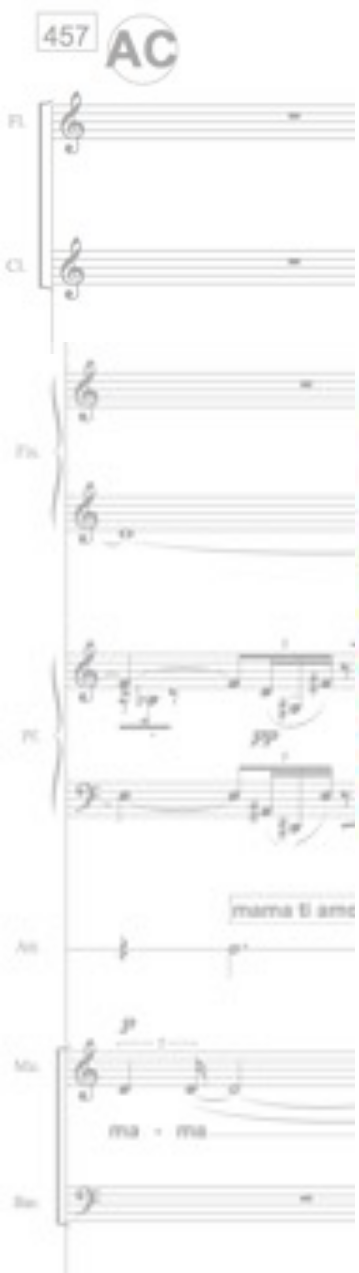
pp

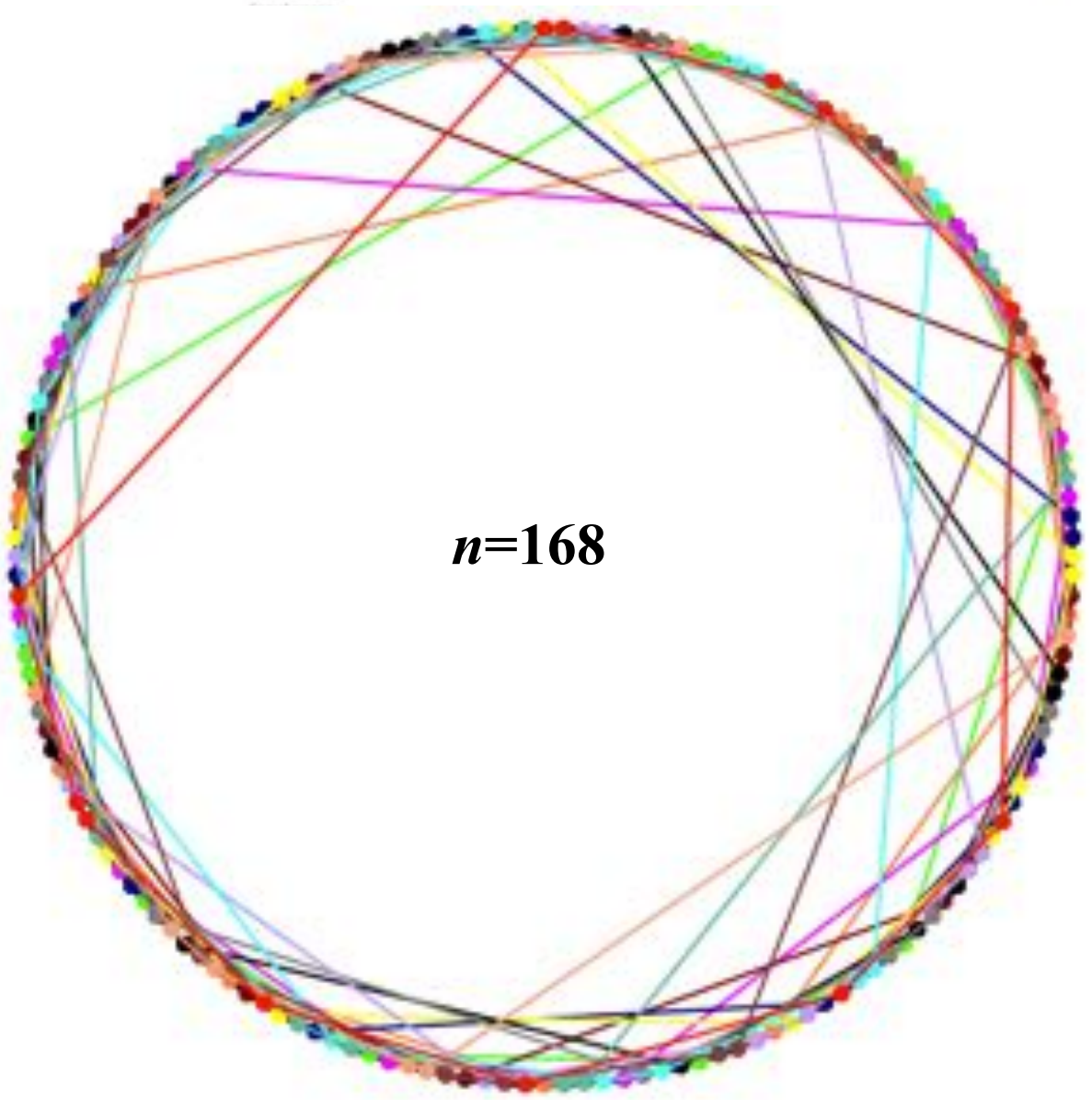
194

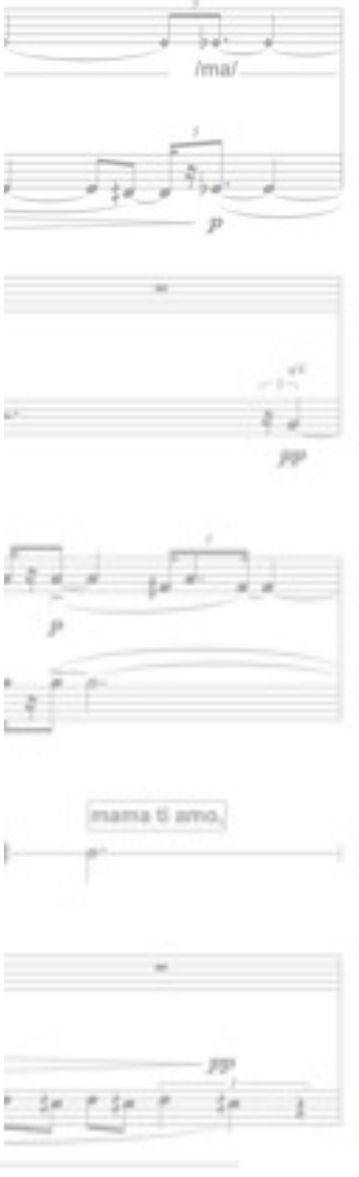
# *La notte poco prima della foresta* (2009)

(opéra de chambre pour acteur, mezzo-soprano, baryton, ensemble et électronique)

**D. Ghisi**



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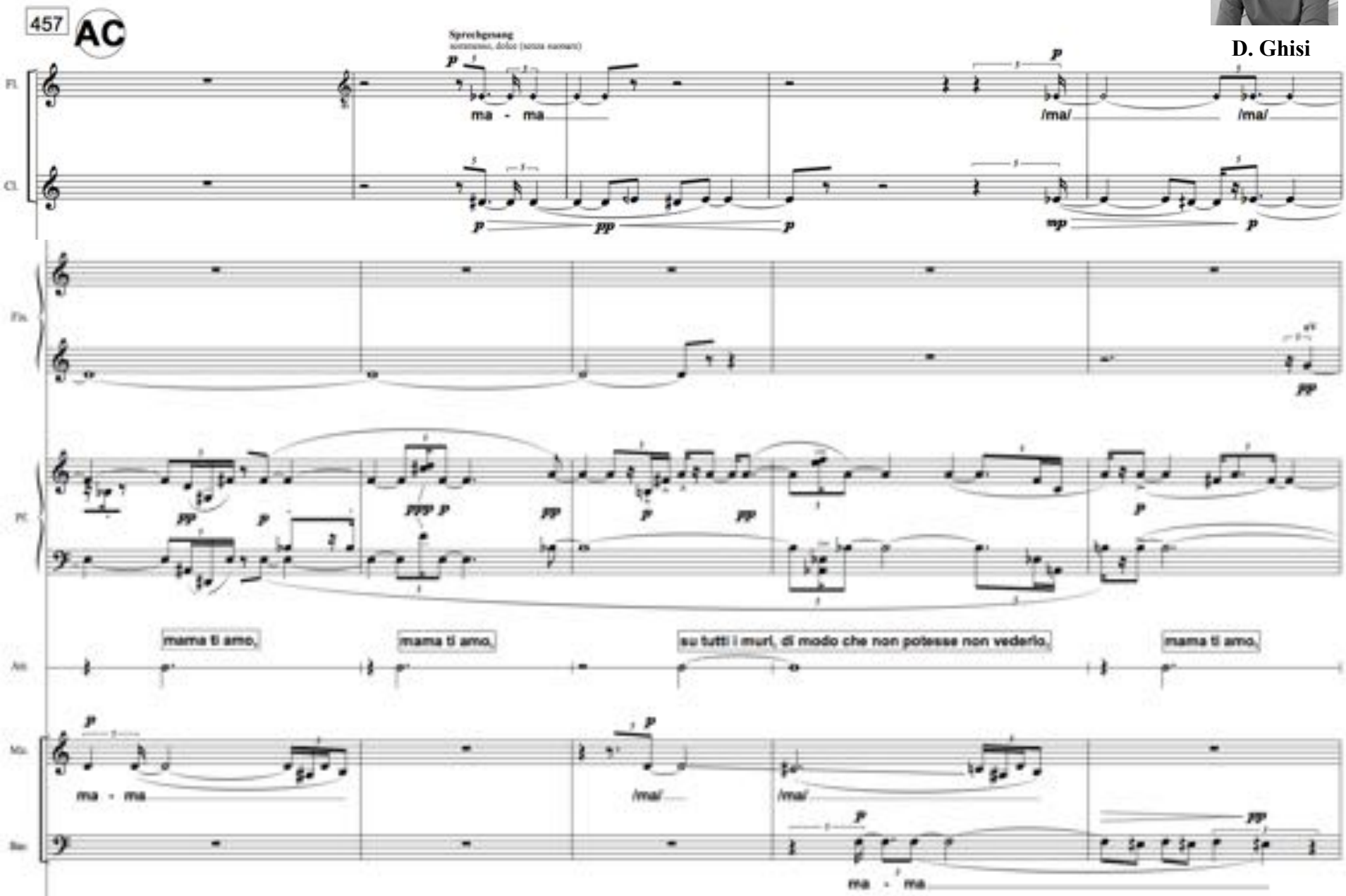


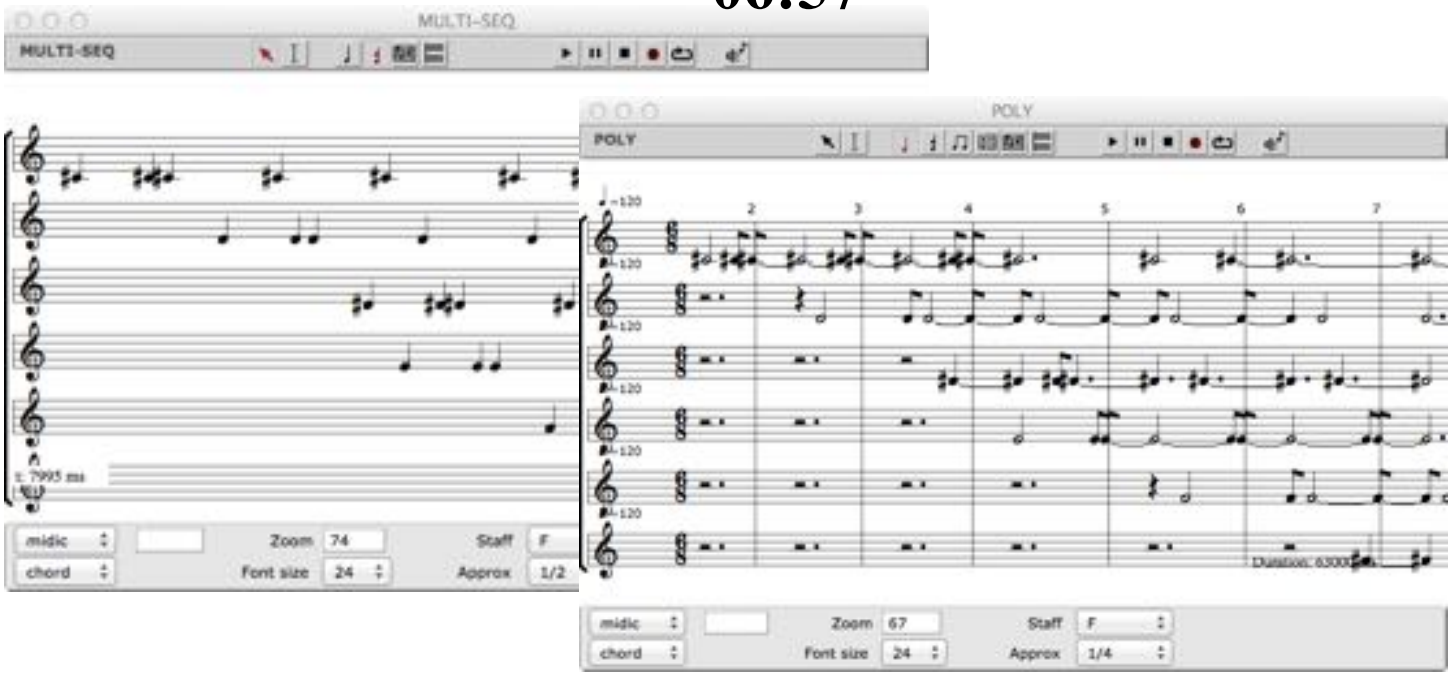
# Vuza Canons and electronic music


« Cinq canons de Vuza » de Sébastien Roux (Diffusion intégrale et portrait)

le dimanche 10 avril 2016

**06:57**

**Sébastien Roux**
**(ATIAM 2001-2002)**




➔**https://podcloud.fr/podcast/alla-breve-lintegrale/episode/cinq-canons-de-vuza-de-sebastien-roux**

# Recent pieces using Vuza Canons

## Als Gregor und Griselda

Kanon für 6 Stimmen (auch Amateure)
(deutsche Fassung)

Fabien Lévy

*Auftrag des Rümlingen Festivals*
*à Melba Fenouil*

Dieser Kanon folgt das mathematische Prinzip des "Vuza-Kanons". Ich bedanke Moreno Andreatta, von Ircam, für seine Hilfe, um diesen Vuza-Kanon zu rechnen, und Melba Fenouil für die deutsche Fassung des Textes.

### Phonetisches Alphabet

**Konsonanten**

b : Bad
s : Sellerie
k : Käse
d : Datum
f : Fön
g : Sagen
ʒ : jolie
l : Lärm
m : Mutti
n : Nein
ɲ : agneau
p : Punkt
ʁ : Relevant
t : Tanken
v : Wagon, Witwe
z : Süsse, sagen
ʃ : Kirsche

**Halbe-Konsonanten (oder halbe-Vokalen)**

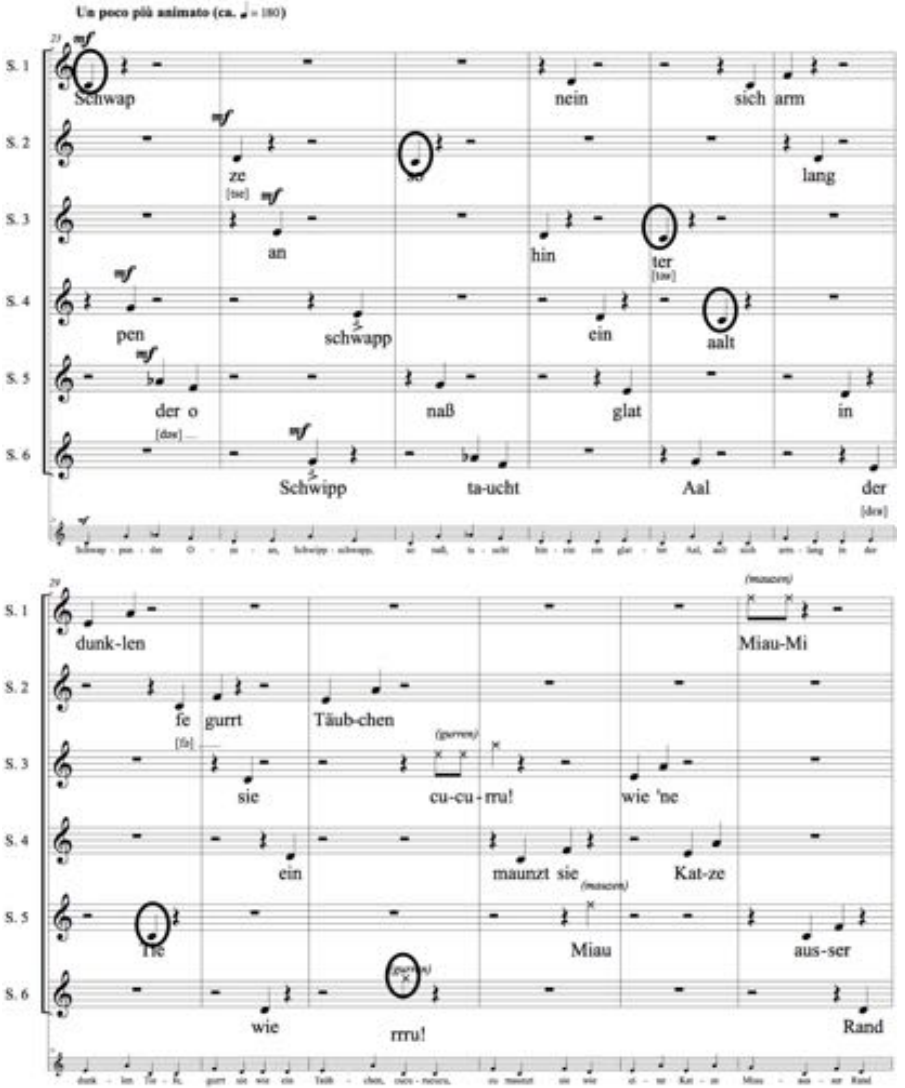
j : jodeln
w : fouet (/fwɛ/), voir (/vwaʁ/)
ɥ : Suite (/sɥita/)

**Vokalen**

a : Papa
ɑ : Kartoffeln
ə : wieder
ø : Fön
œ : Fleur
e : Merit
ɛ : fern
o : Oktopus
ɔ : Ort
i : wieder
u : du, Suppe
y : lügen

**Nasale Vokalen**

ɑ̃ : rang, avant
ɛ̃ : rein, brin, pain
ɔ̃ : bon, ton
œ̃ : brun, un



# From permutational songs to the Tonnetz

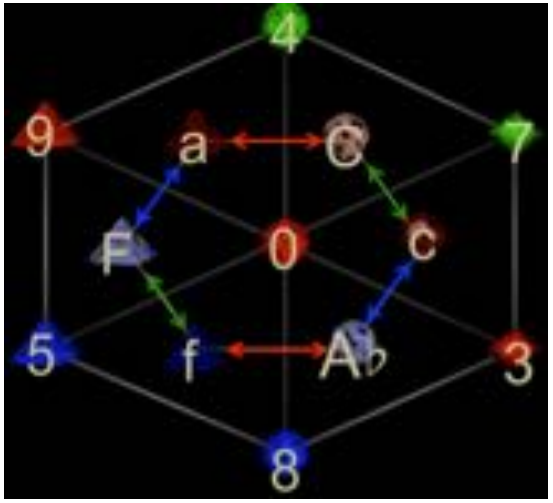
***Se telefonando*, 1966 (Maurizio Costanzo/Ennio Morricone) / Mina**


**(min. 0'53")**


***Je changerais d'avis*, 1966 (Françoise Hardy)**


**Ennio Morricone**

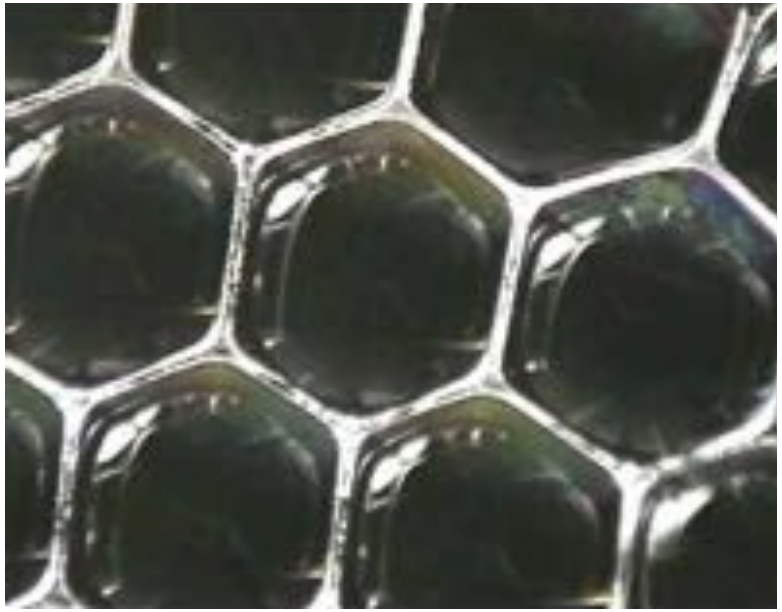
## The harmonic space

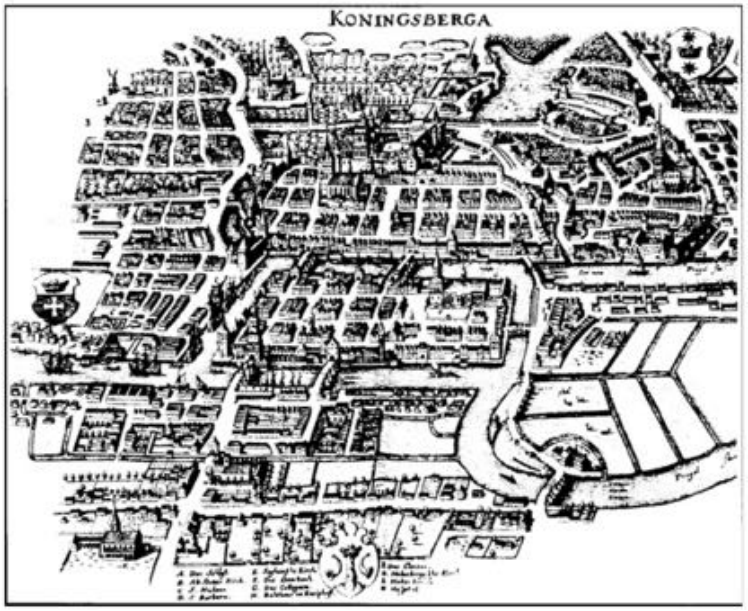


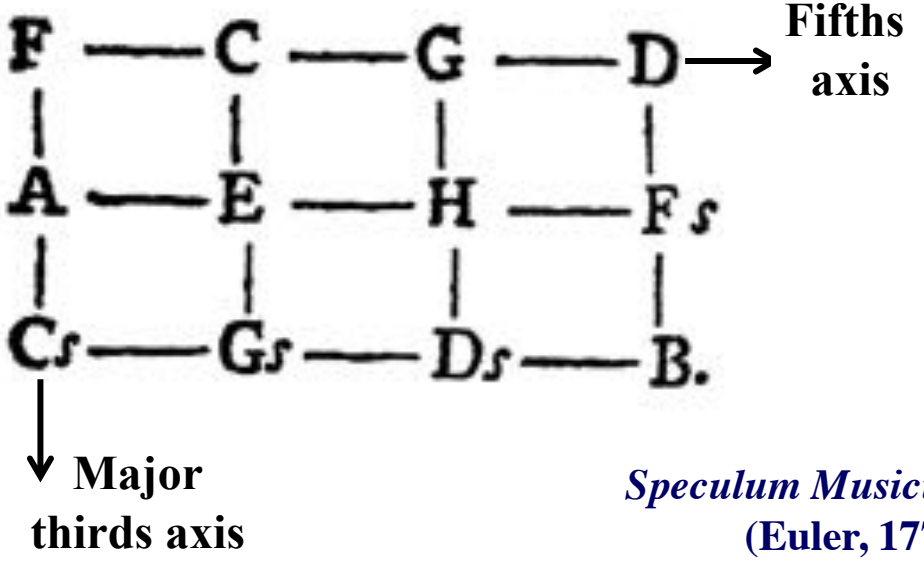

| | | | | | |
|---|---|---|---|---|---|
| C | c | $C_{\#}$ | $c_{\#}$ | D | **d** |
| $E_b$ | $e_b$ | E | e | **F** | f |
| $F_{\#}$ | $f_{\#}$ | G | **g** | $G_{\#}$ | $g_{\#}$ |
| A | a | $B_b$ | $b_b$ | **B** | b |

**Chord enumeration**

# The *Tonnetz*
## (or 'honeycomb' hexagonal tiling)

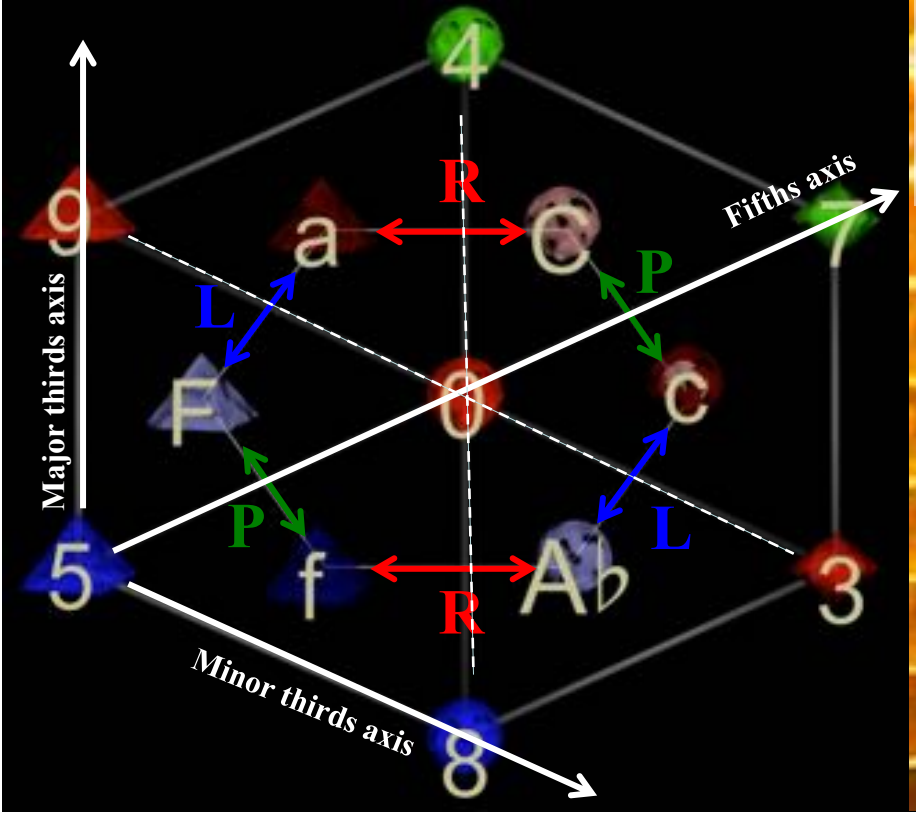


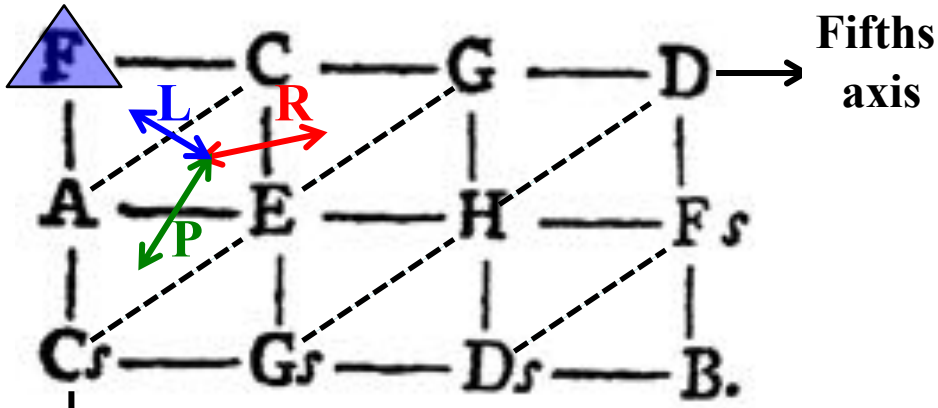



*Speculum Musicum*
(Euler, 1773)

# The *Tonnetz*
## (or 'honeycomb' hexagonal tiling)



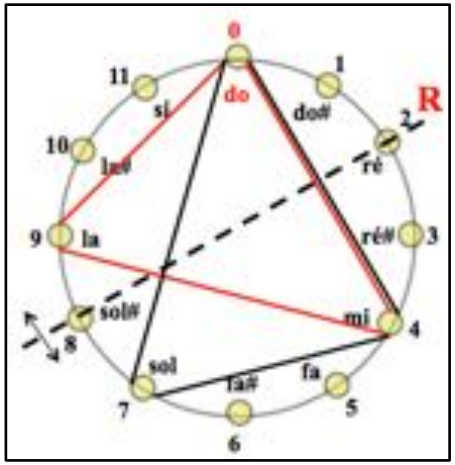



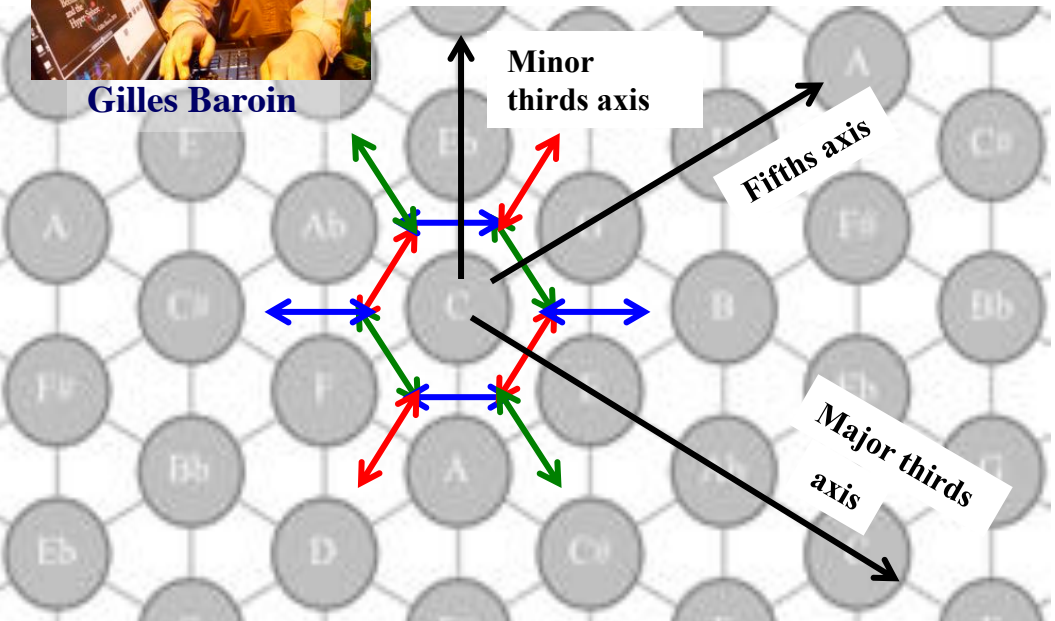

*Speculum Musicum*
(Euler, 1773)

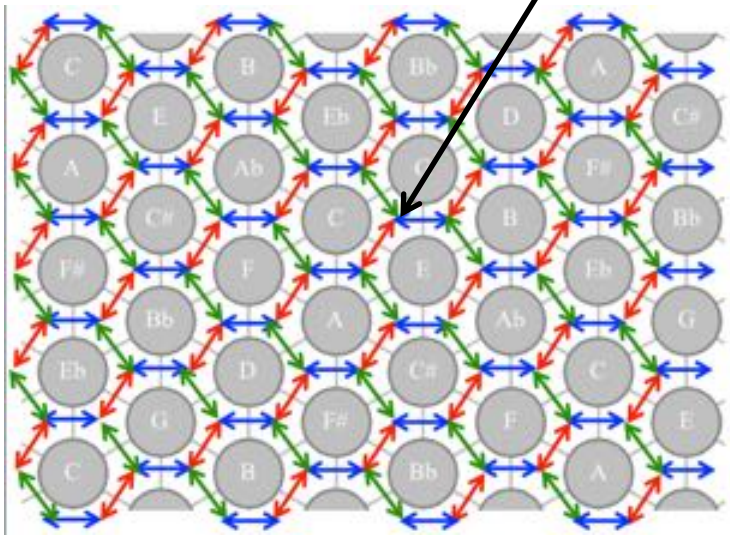
# The *Tonnetz*
## (or hexagonal tiling honeycomb)




Gilles Baroin



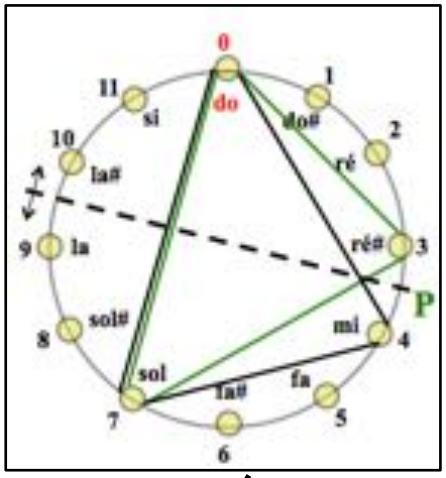



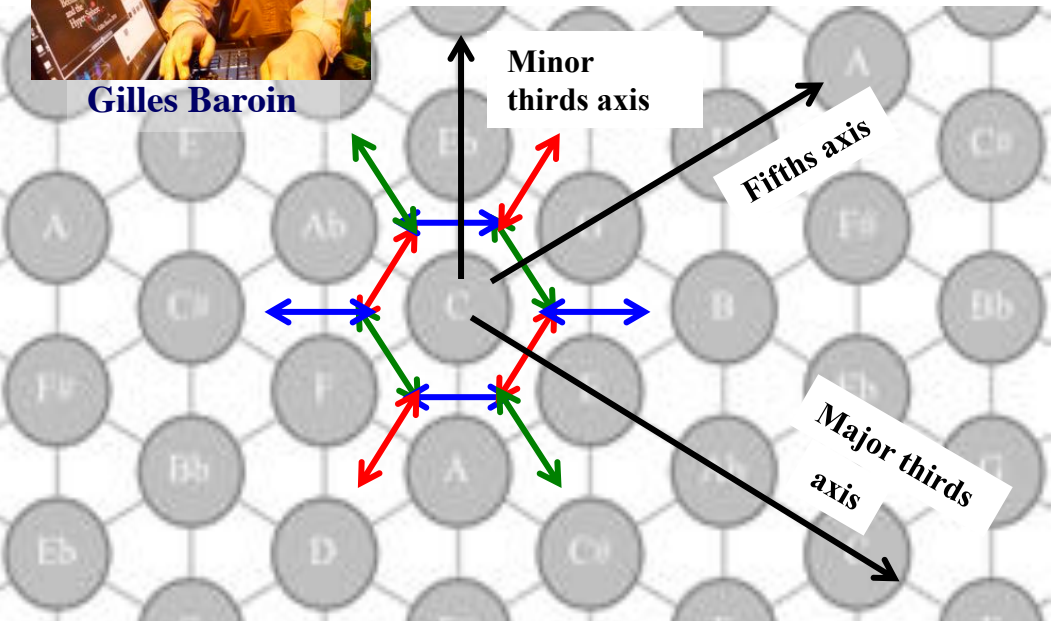



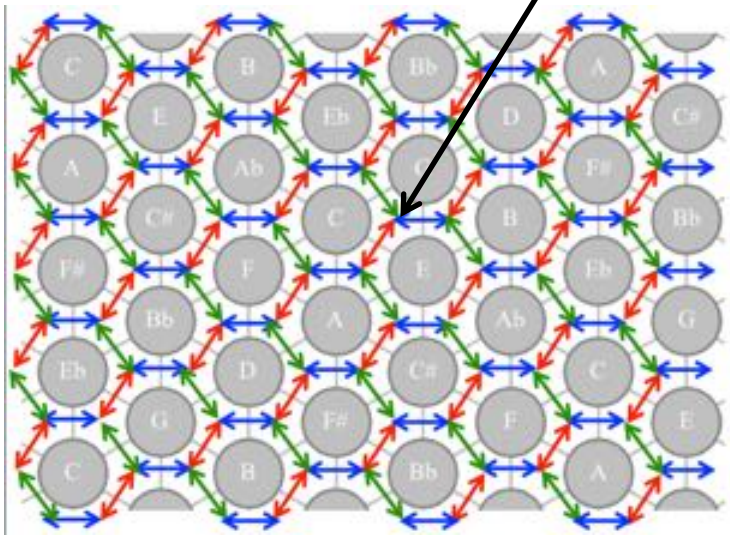
# The *Tonnetz*
## (or hexagonal tiling honeycomb)



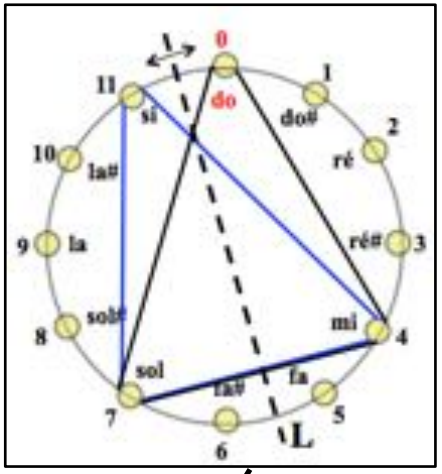

Gilles Baroin



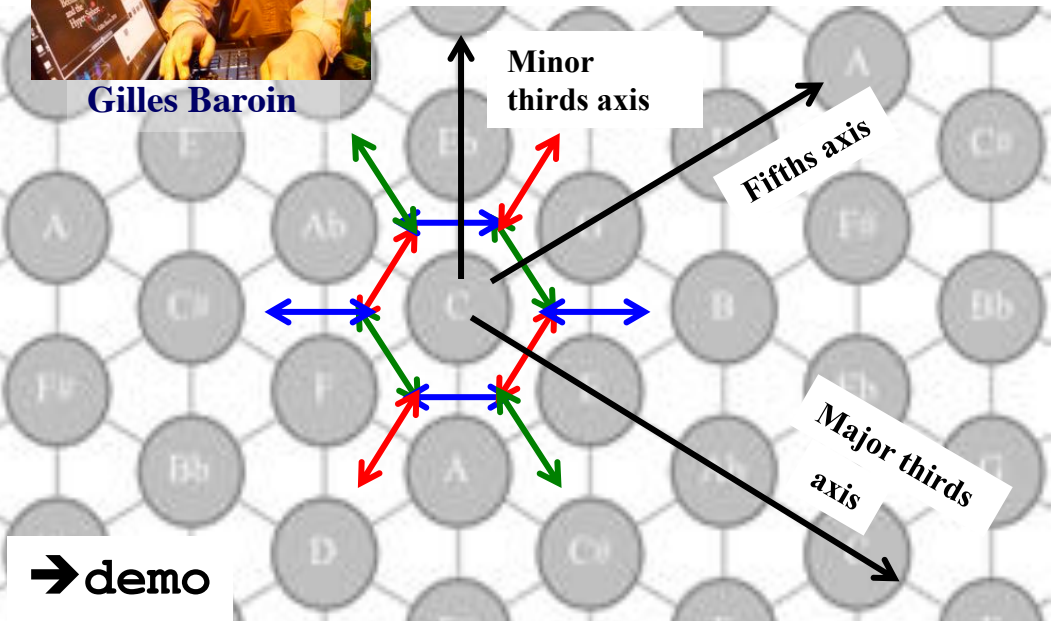



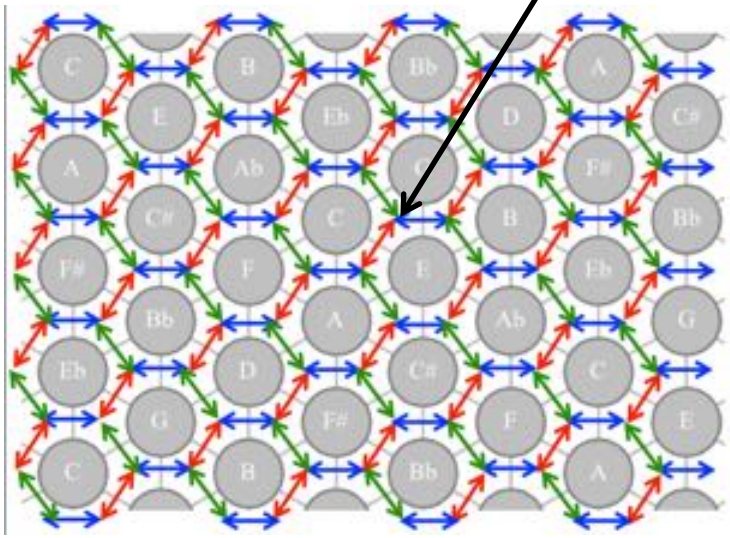
# The *Tonnetz*
## (or hexagonal tiling honeycomb)




Gilles Baroin



From Circles to Hypersheres Chapter 6

# Two Dimensions

## Traditional Chordal Space



Gilles Baroin

# Harmonic Progressions

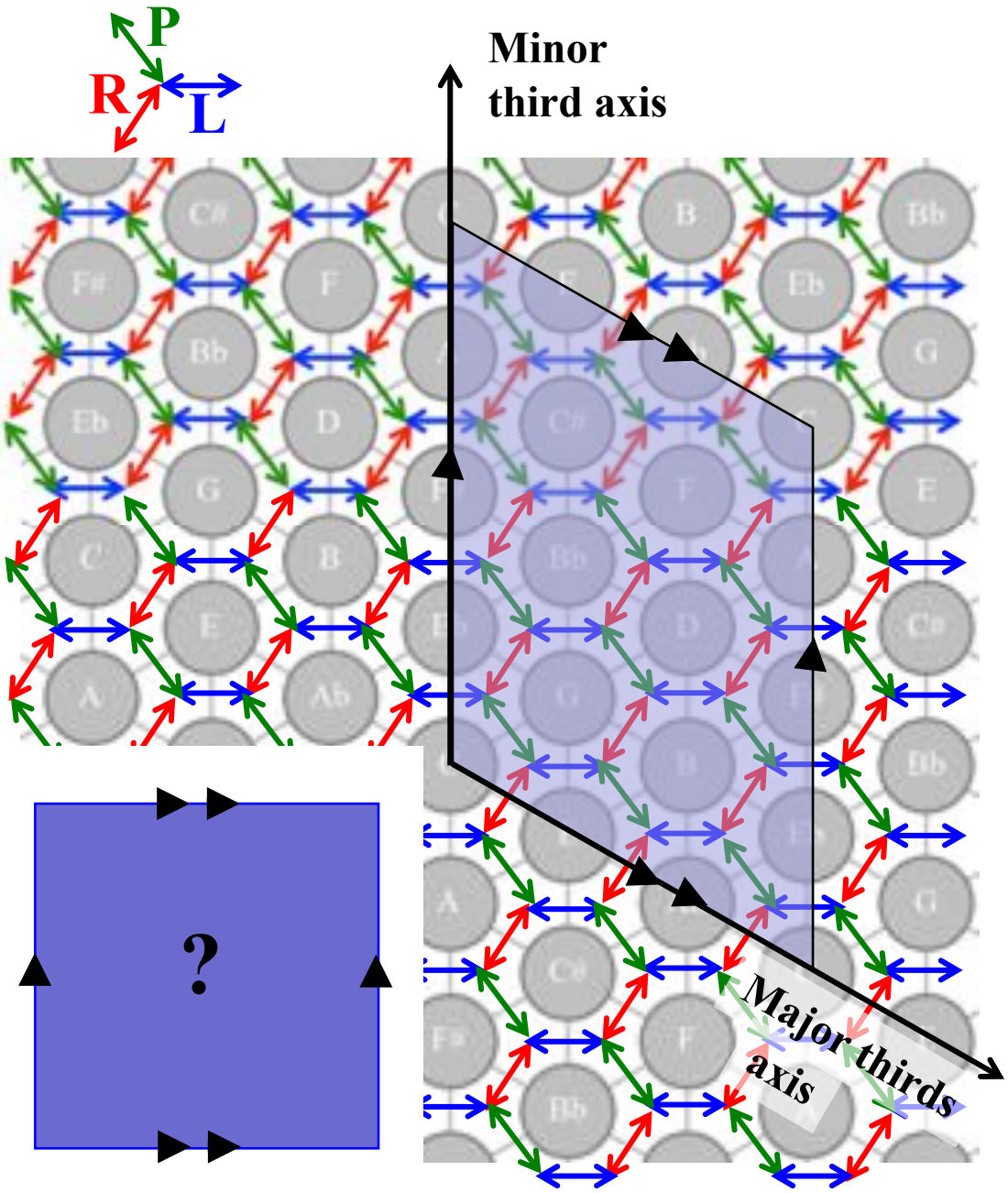
## In Paolo Conte

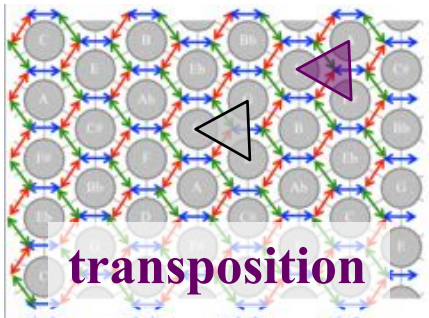
## *Sotto le Stelle del Jazz*

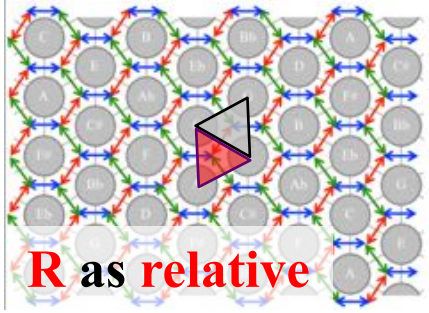

*Supervision Moreno Andreatta*
*Modelisation Gilles Baroin 2016*

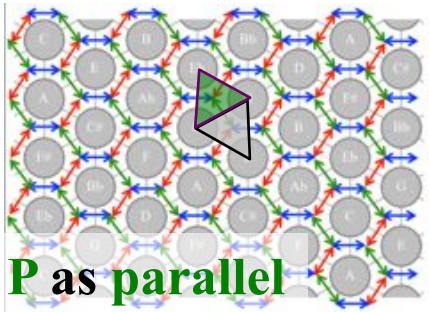
➔ **www.mathemusic.net**

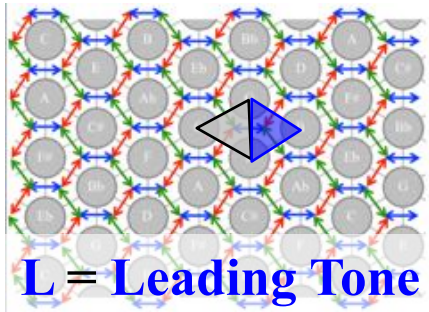
# The *Tonnetz*, its symmetries and its topological structure

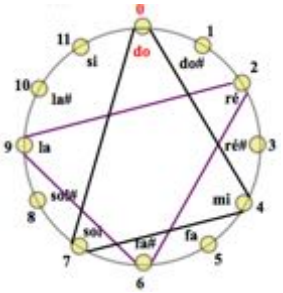


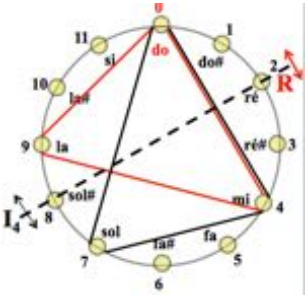



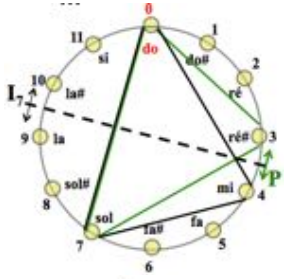



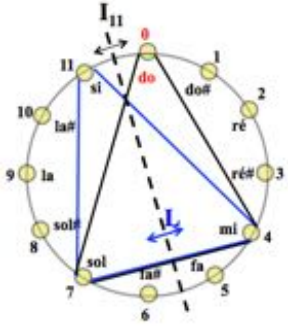



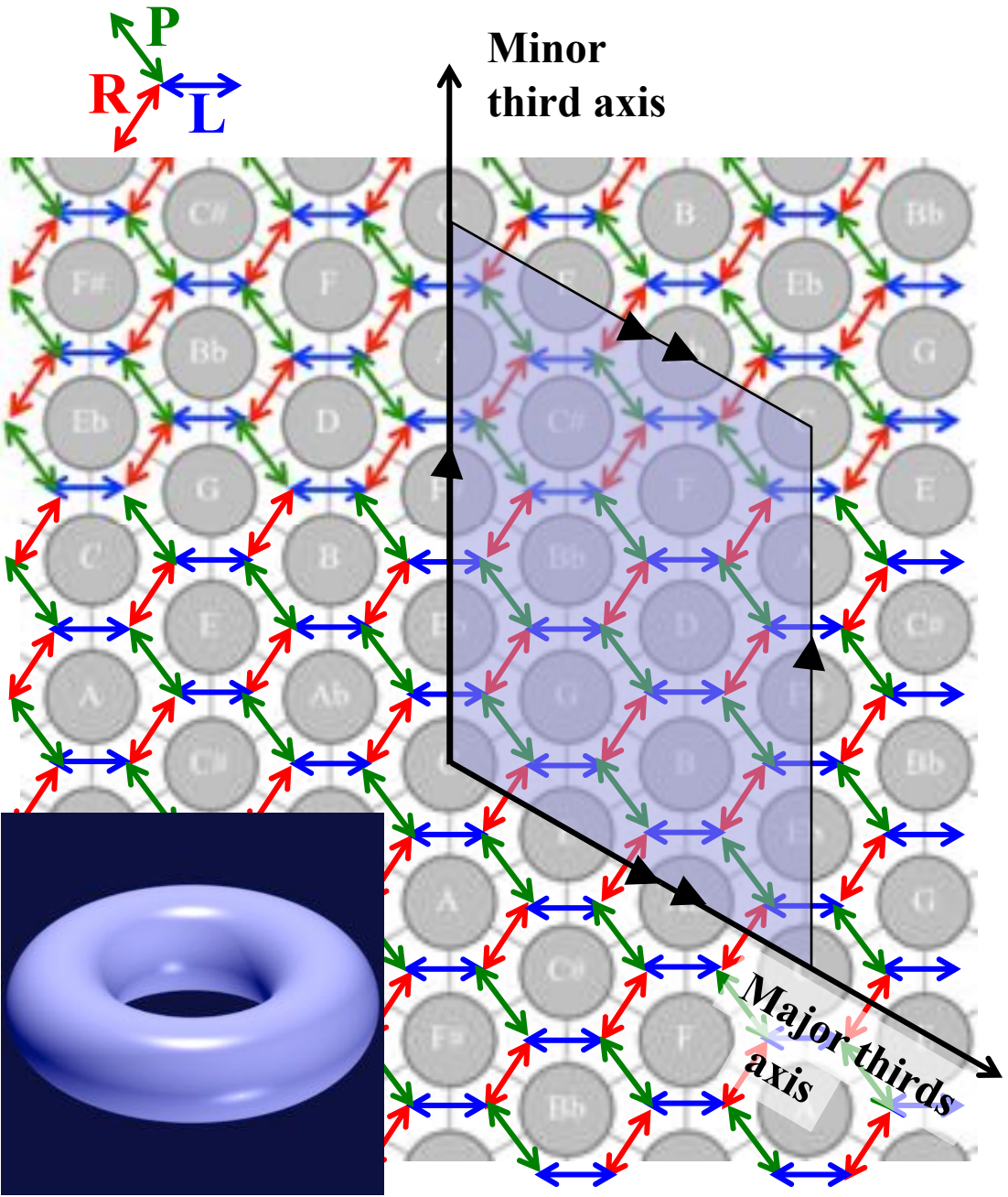



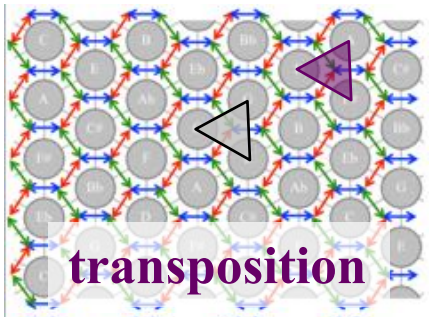


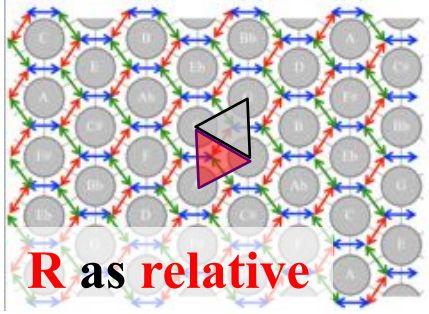


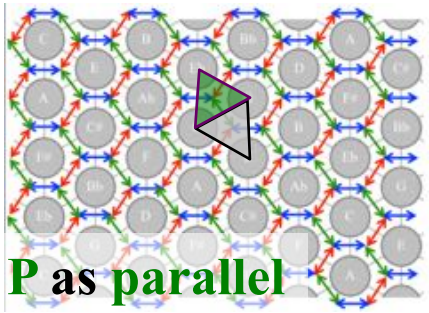


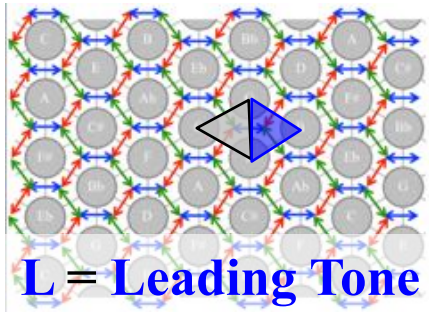
# The *Tonnetz*, its symmetries and its topological structure

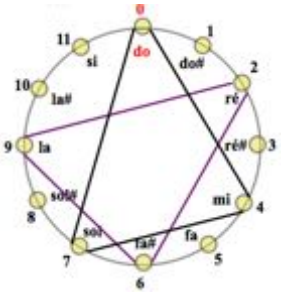


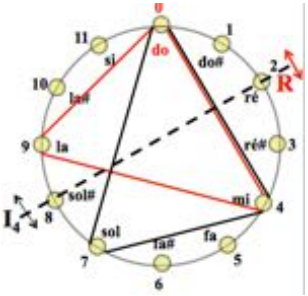



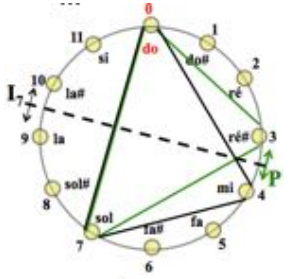



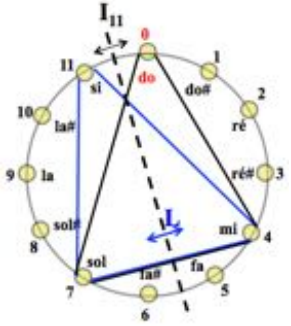



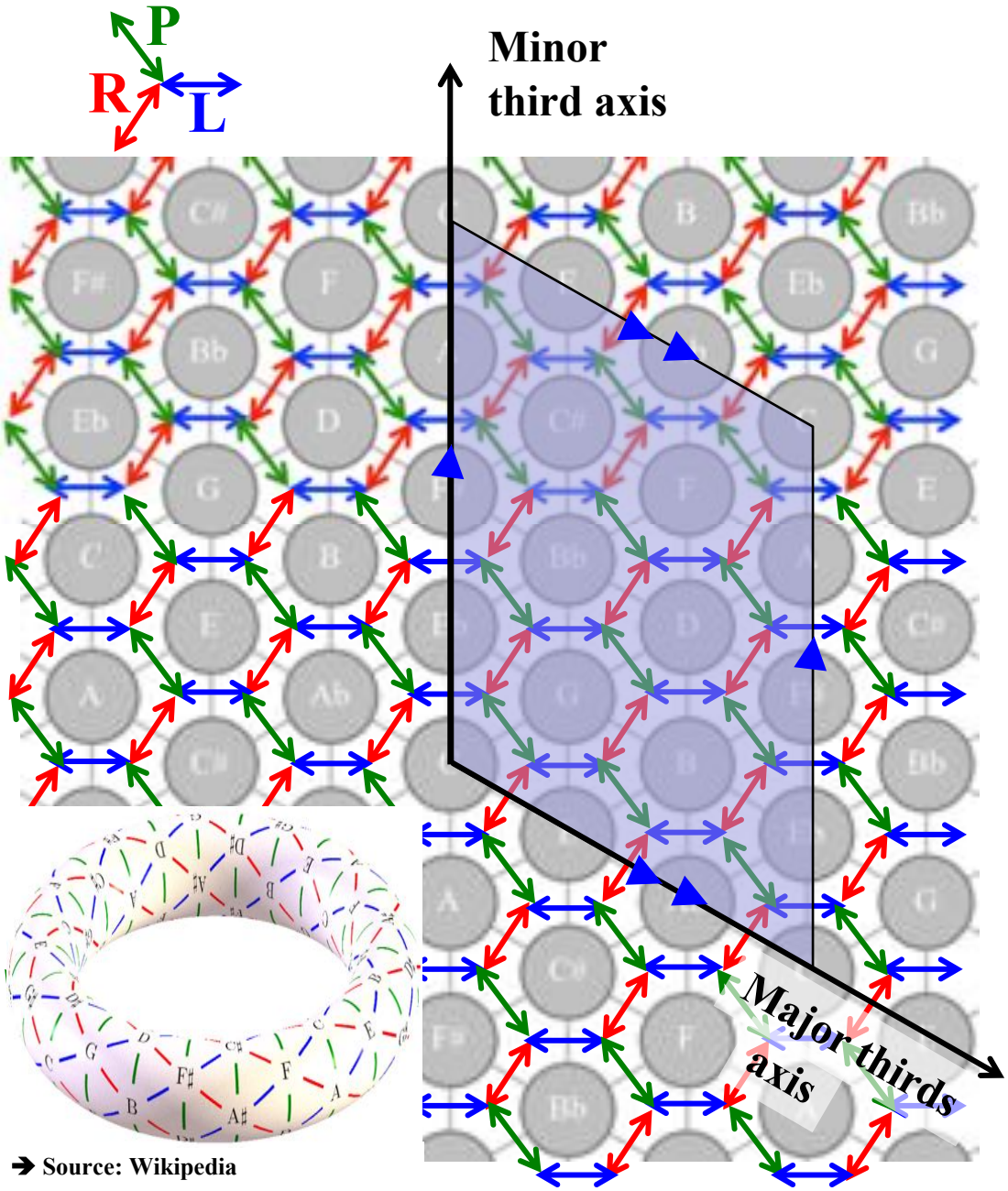



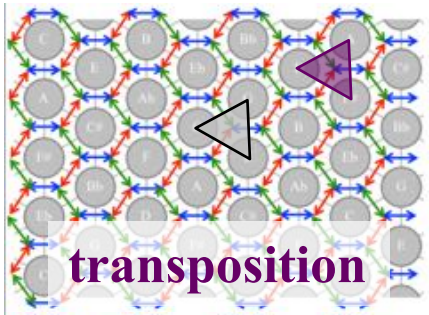


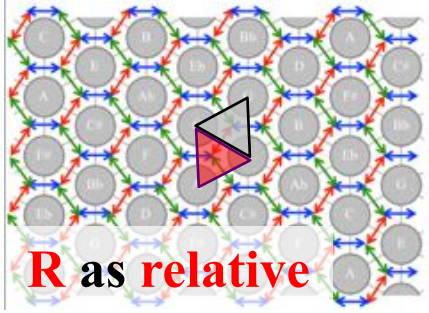


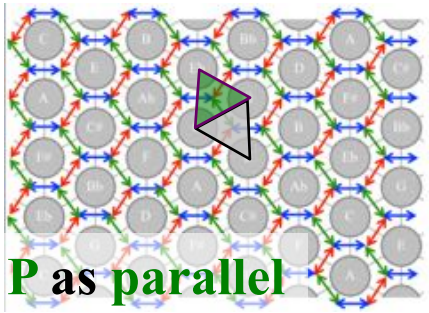


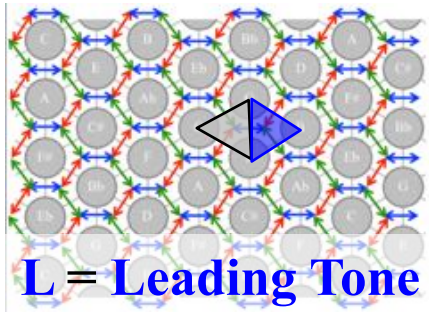
# The *Tonnetz*, its symmetries and its topological structure

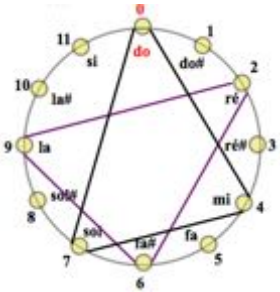


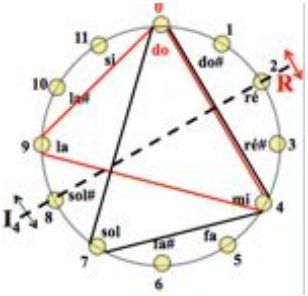



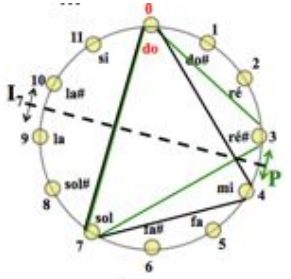



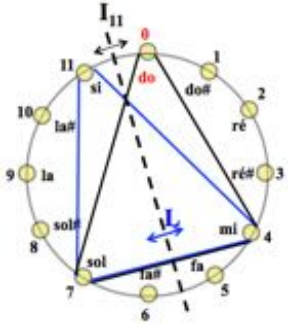







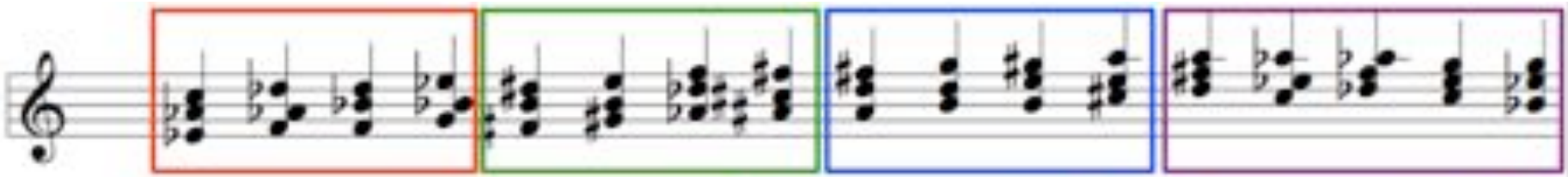


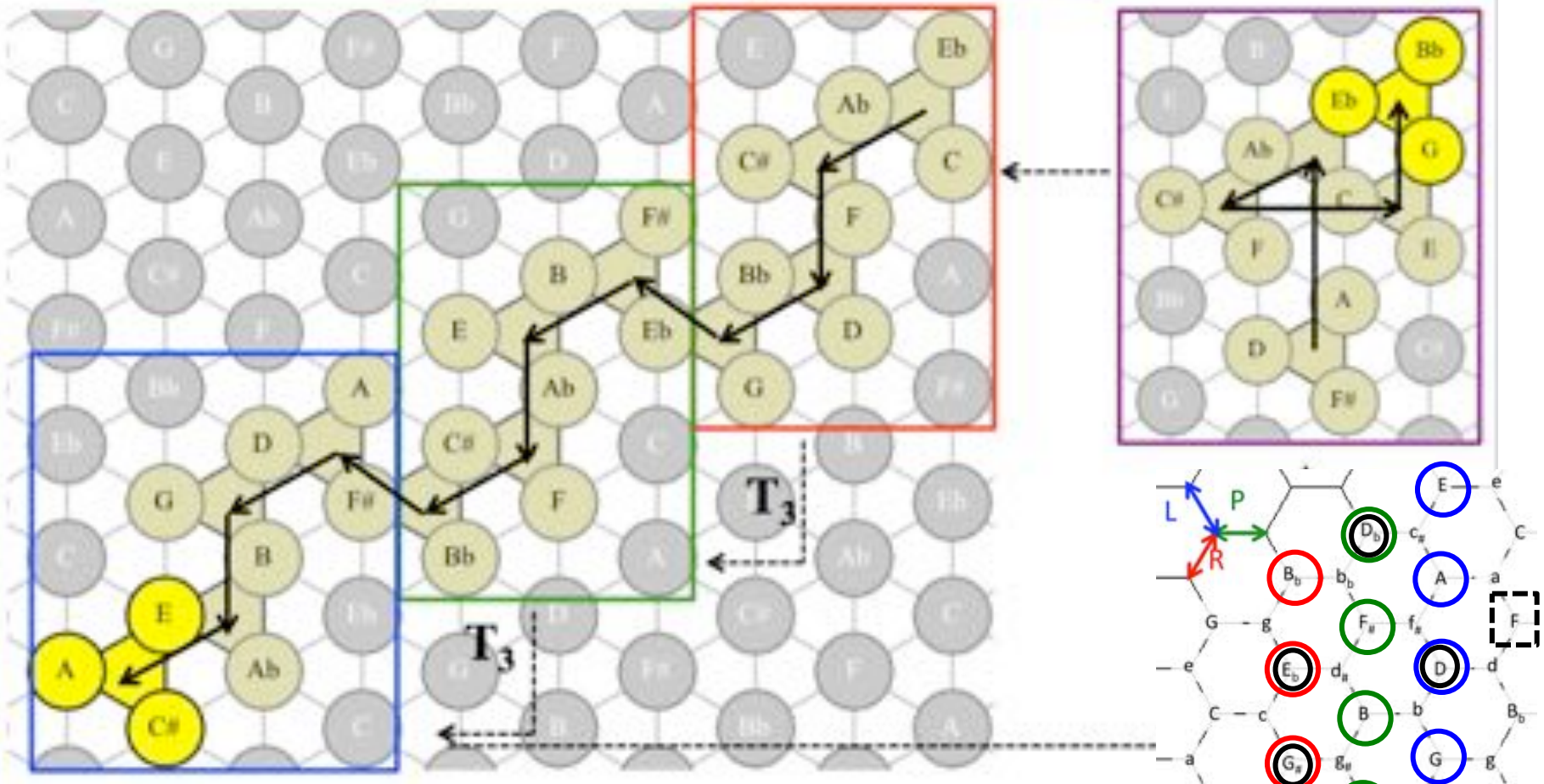


➔ Source: Wikipedia

# Symmetries in Paolo Conte's *Madeleine*

$La_b$ $Re_b$ $Si_b$ $Mi_b$ Si Mi $Re_b$ $Fa_\#$ Re Sol Mi La Re $La_b$ $Re_b$ Do $Mi_b$



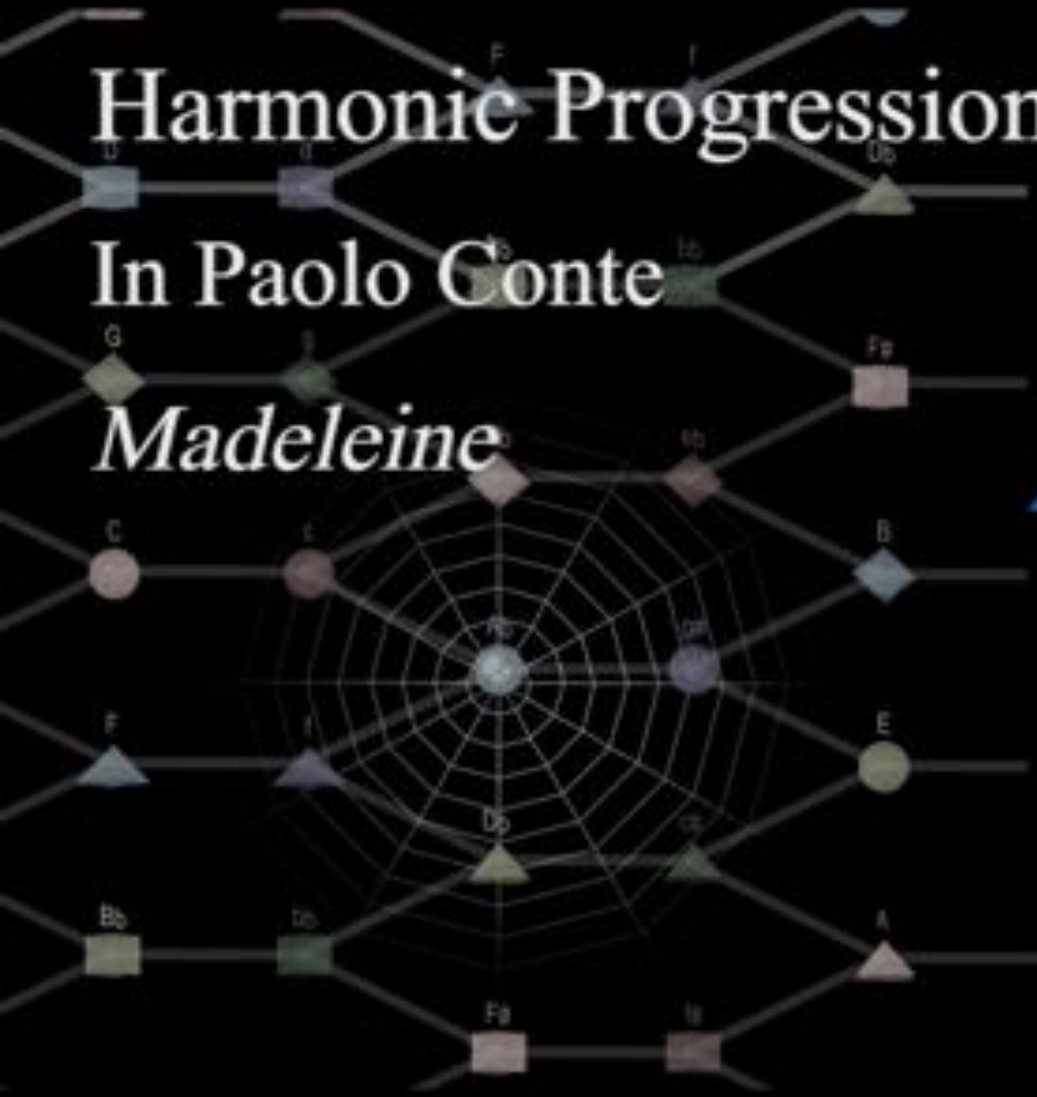


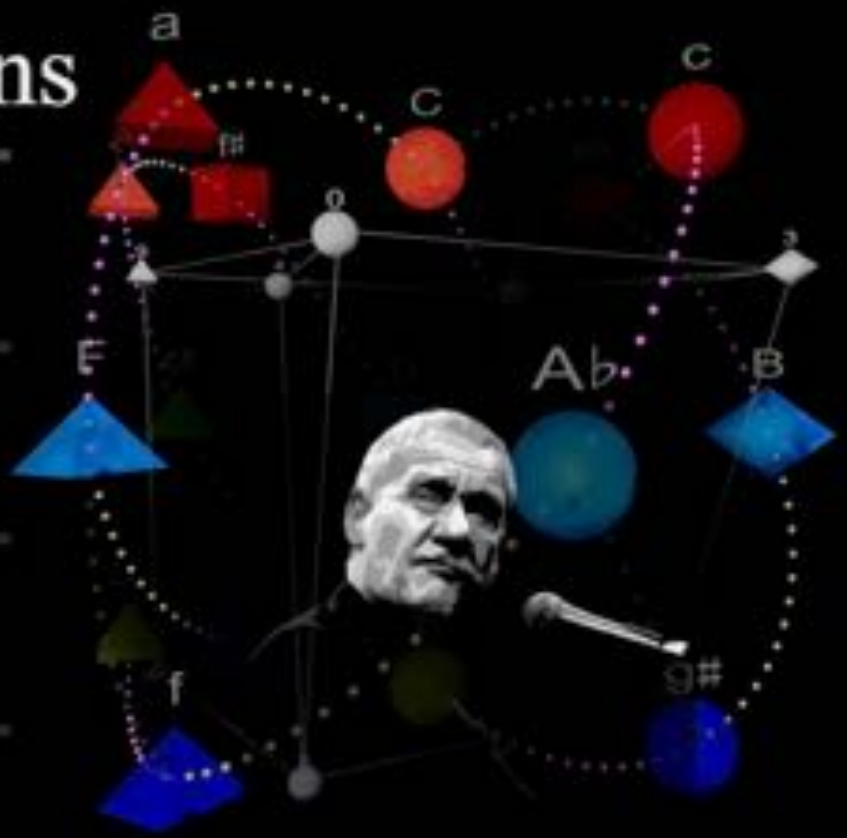

**Almost total covering of the major-chords space**

# Harmonic Progressions

## In Paolo Conte

## *Madeleine*





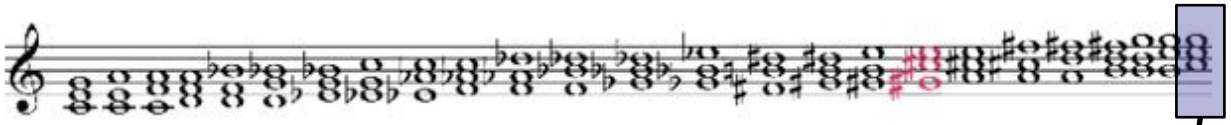

*Supervision Moreno Andreatta*
*Modelisation Gilles Baroin 2016*

# Beethoven and the Hypersphere

*(and the Tonnetz)*


Gilles Baroin 2016
www.MatheMusic.net

# Reading Beethoven backwards



time

## Le Blé en Herbe

(Polo/Moreno/Dieu)

Plonger comme un enfant, cheveux au vent
Sous l'océan du blé en herbe
Marée d'épis couleur d'amande
Qui tendent à caresser le ciel

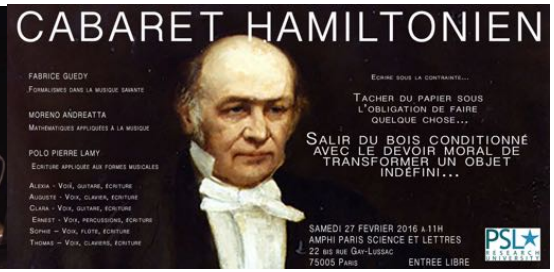
Algues tendres de mille plages
Frôlant le ventre des nuages
Cheveux de pluie, dos de poissons
Qui frissonnent à l'unisson

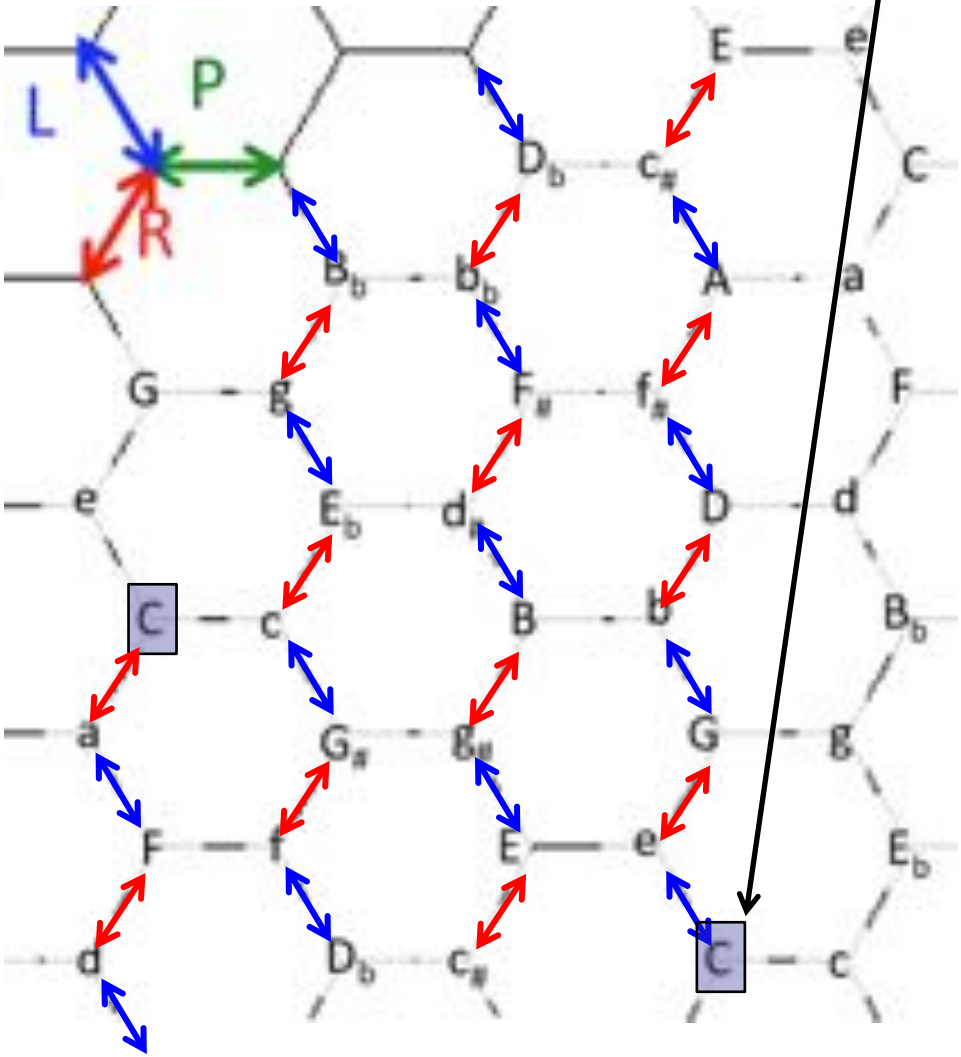
Suivre le bord des continents
Dans l'océan du blé en herbe
Pêcher le corail du pavot
Dans le sang des coquelicots

Croiser matin dans l'herbe folle
Deux tourterelles qui s'envolent
Suivre les jeux des hirondelles
Sur le paysage éternel
Nager comme un enfant, cheveux au vent
Sous l'océan
Du blé en herbe

Marée de fruits au goût amer
Acide et salée comme la mer

Vers l'îlôt d'un petit village
Vers un château d'eau sur la plage
Quand tout s'éteint avant l'orage
Quand se lève le vent du large
Sur le blé vert



# *Aprile*, a Hamiltonian « decadent » song

Do←$do_m$←Sol#←$fa_m$←Fa←$la_m$←La←$fa\#_m$←Fa#←$sib_m$←Do#←$do\#_m$

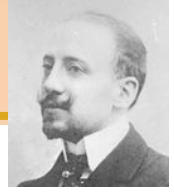
↘ La ↗ ↗

$mi_m$→Sol→$si_m$→Ré→$ré_m$→Sib→$sol_m$→Mib→$mib_m$→Si→$sol\#_m$→Mi

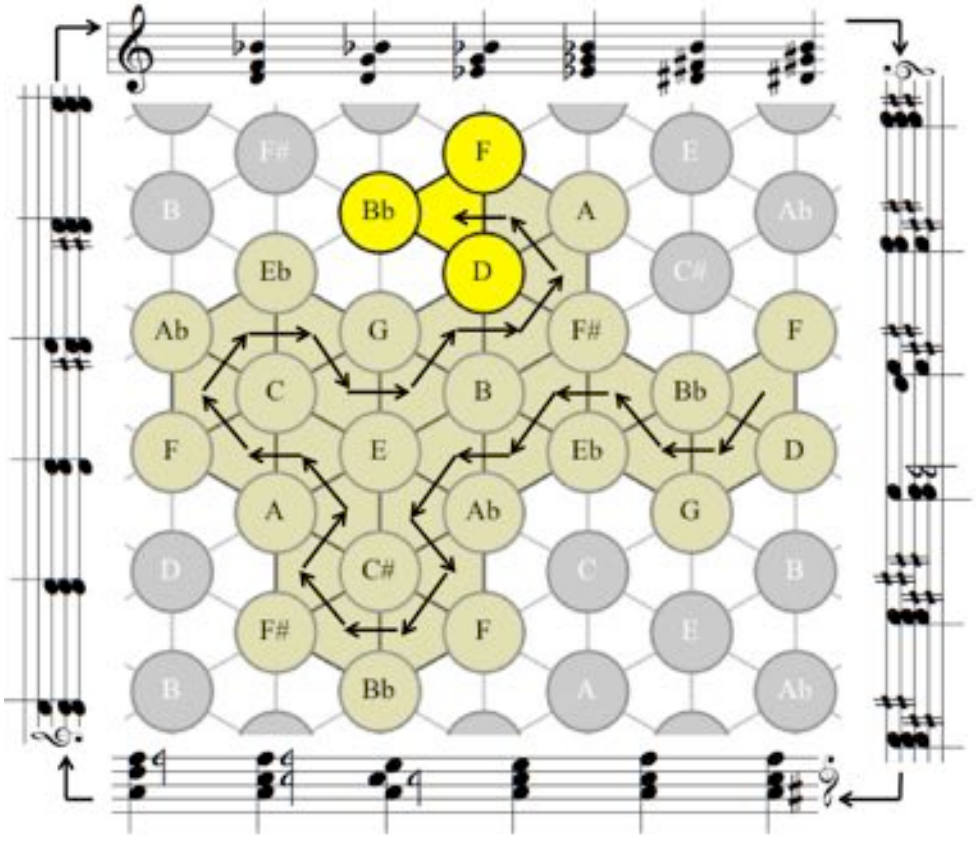
*Socchiusa è la finestra, sul giardino.*
*Un'ora passa lenta, sonnolenta.*
*Ed ella, ch'era attenta, s'addormenta*
*A quella voce che già si lamenta,*
*Che si lamenta in fondo a quel giardino.*

*Non è che voce d'acque su la pietra:*
*E quante volte, quante volte udita!*
*Quell'amore e quell'ora in quella vita*
*S'affondan come ne l'onda infinita*
*Stretti insieme il cadavere e la pietra.*

*Ella stende l'angoscia sua nel sonno.*
*L'angoscia è forte, e il sonno è così lieve!*
*(Par la luce d'april quasi una neve*
*che sia tiepida.)*
*Ed ella certo deve soffrire,*
*Vagamente, anche nel sonno.*



G. D'Annunzio (1863-1938)

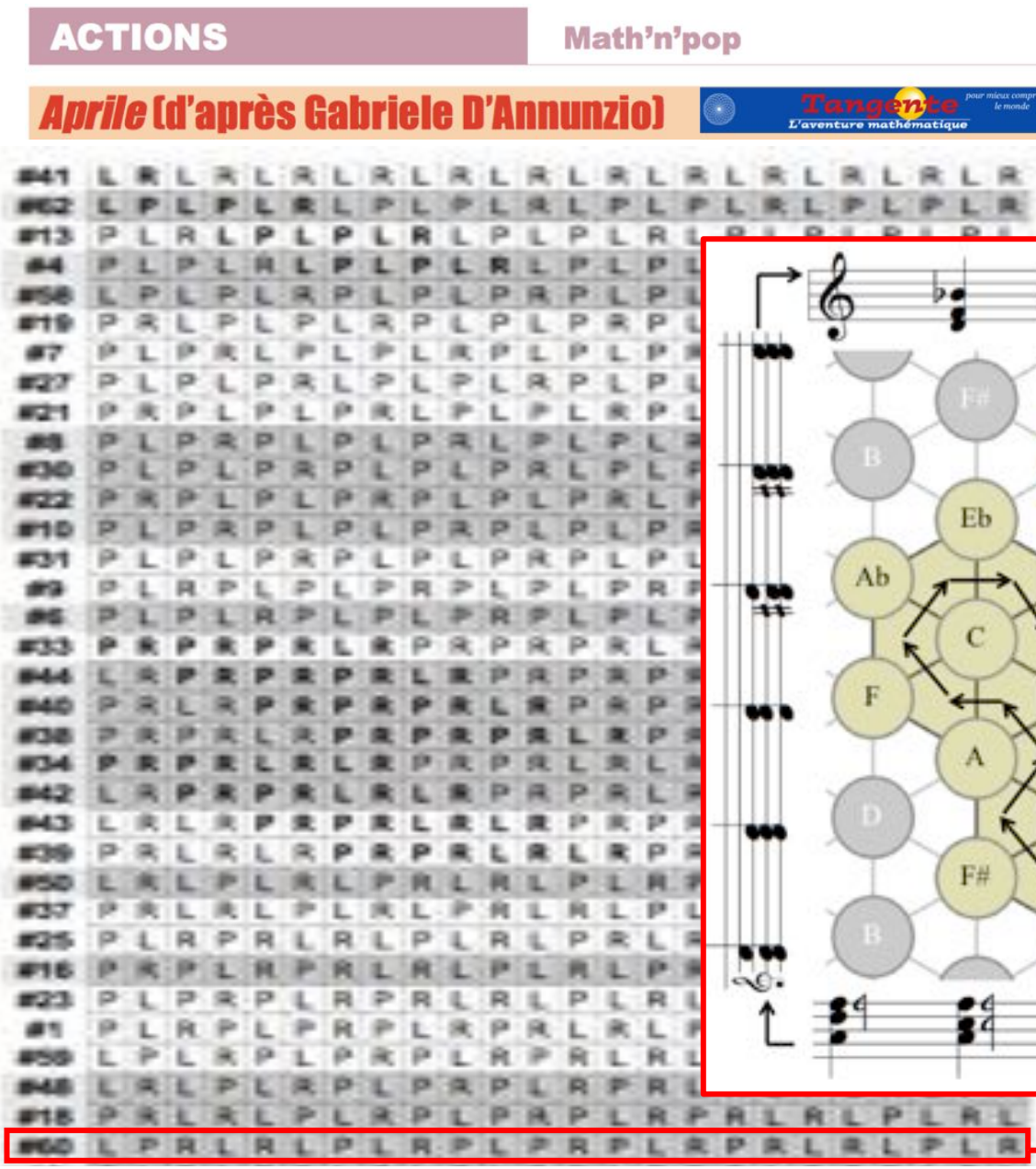


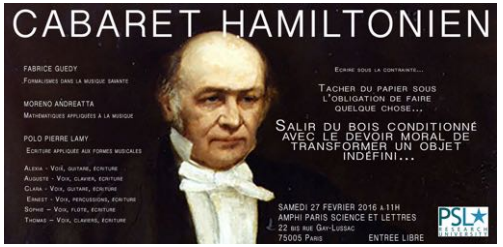

ACTIONS Math'n'pop

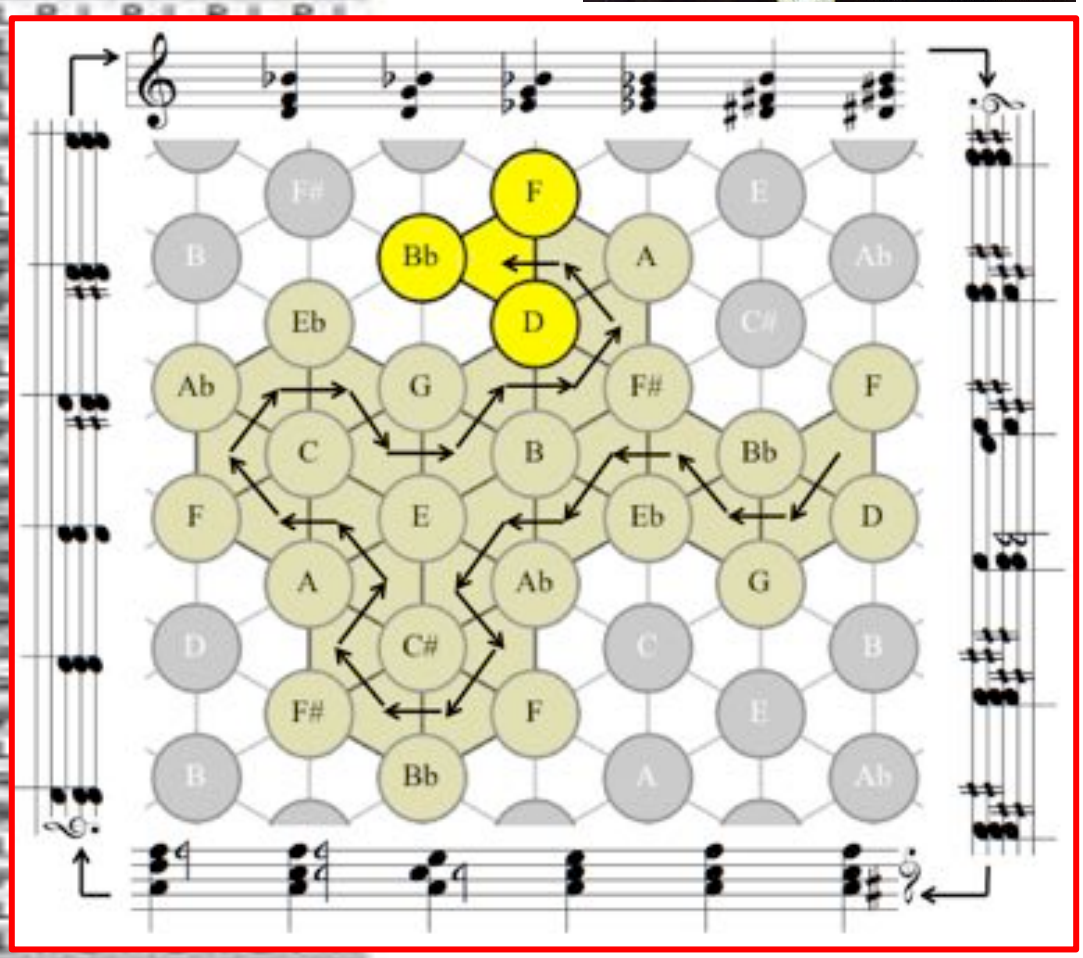
*Aprile* (d'après Gabriele D'Annunzio)

Tangente pour mieux comprendre le monde L'aventure mathématique

# The collection of 124 Hamiltonian Cycles




G. Albini & S. Antonini, University of Pavia, 2008

# *Aprile* (d'après Gabriele D'Annunzio)

Tangente pour mieux comprendre le monde
L'aventure mathématique

**M. Andreatta, « Math'n pop : symétries et cycles hamiltoniens en chanson », *Tangente***

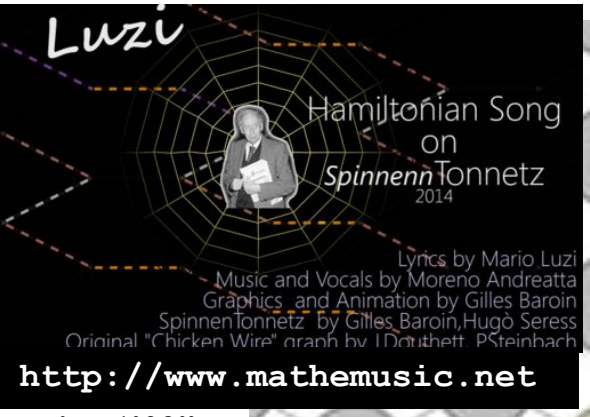
# The collection of 28 « redundant » Hamiltonian Cycles

1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPRPR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPRLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRLRPRP
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPRL
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRLRLRP
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm--RLRPRPRP
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em--RLRPRPRL
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLRP
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm--RLRLRPRP
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL

# Hamiltonian Cycles with inner periodicities

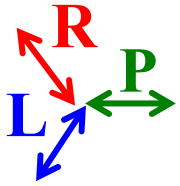
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR

L P L P L R …
P L P L R L …
L P L R L P …
P L R L P L …
**L R L P L P …**
R L P L P L …




**min. 1’02”**




**Music:** M. Andreatta
**Arrangement and mix:** M. Bergomi & S. Geravini (*Perfect Music Production*)
**Mastering:** A. Cutolo (Massive Arts Studio, Milan)

**La sera non è più la tua canzone**
**(Mario Luzi, 1945, in *Poesie sparse*)**

**La sera non è più la tua canzone,**
**è questa roccia d’ombra traforata**
**dai lumi e dalle voci senza fine,**
**la quiete d’una cosa già pensata.**

**Ah questa luce viva e chiara viene**
**solo da te, sei tu così vicina**
**al vero d’una cosa conosciuta,**
**per nome hai una parola ch’è passata**
**nell’intimo del cuore e s’è perduta.**

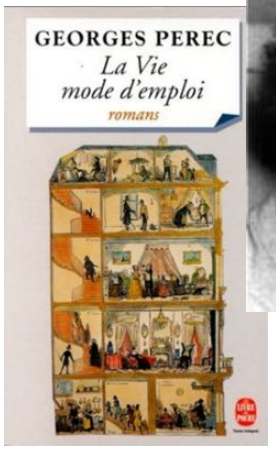
**Caduto è più che un segno della vita,**
**riposi, dal viaggio sei tornata**
**dentro di te, sei scesa in questa pura**
**sostanza così tua, così romita**
**nel silenzio dell’essere, (compiuta).**

**L’aria tace ed il tempo dietro a te**
**si leva come un’arida montagna**
**dove vaga il tuo spirito e si perde,**
**un vento raro scivola e ristagna.**


Luzi
Hamiltonian Song
on
SpinnennTonnetz
2014
Lyrics by Mario Luzi
Music and Vocals by Moreno Andreatta
Graphics and Animation by Gilles Baroin
SpinnenTonnetz by Gilles Baroin,Hugò Seress
Original "Chicken Wire" graph by J.Douthett, P.Steinbach

# The use of constraints in arts

*Cent mille milliards de poèmes*, 1961

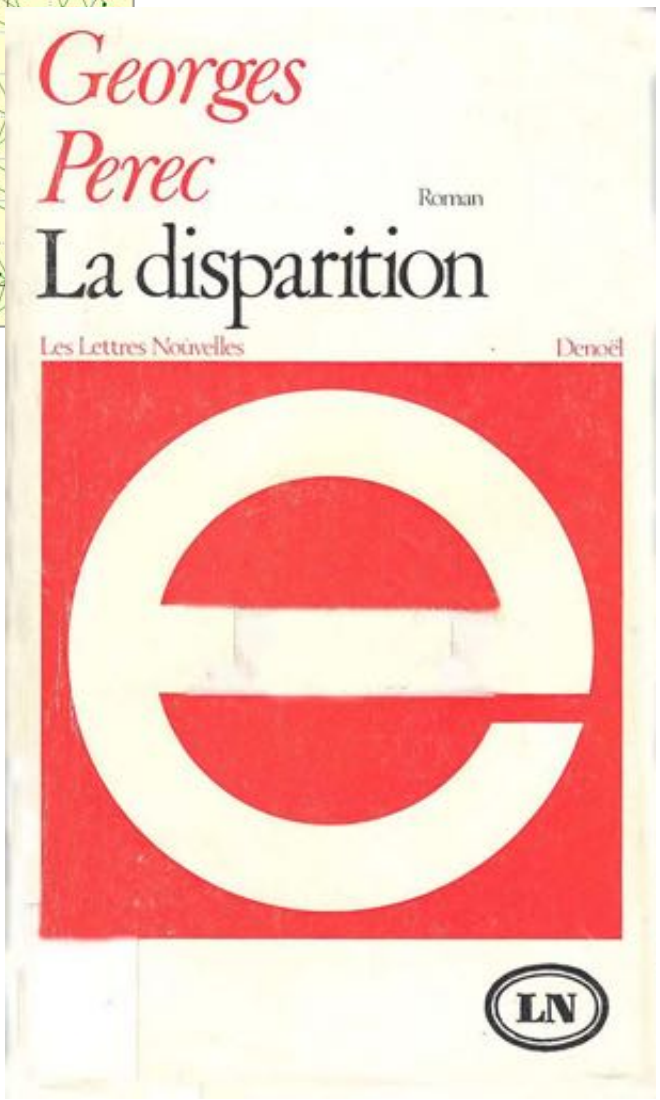





Georges Perrec

*La vie mode d'emploi*,

**OuLiPo (Ouvroir de Littérature Potentielle)**





**Raymond Queneau**

**Italo Calvino**

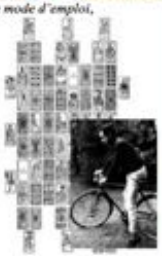
*Il castello dei destini incrociati,* 1969

# From the OuLiPo to the OuMuPo (ouvroir de musique potentielle)

Cent mille milliards de poèmes, 1961

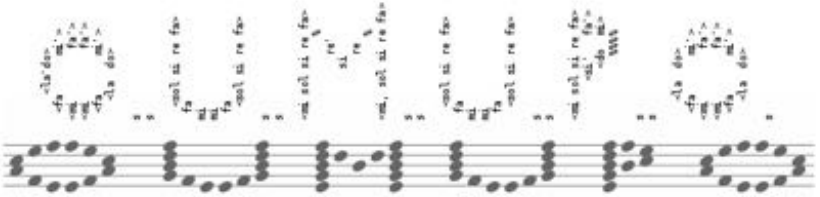

La vie mode d'emploi,

Raymond Queneau

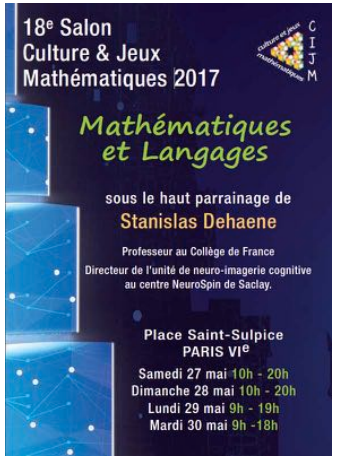


Italo Calvino

*Le Château des destins croisés*, 1969

http://oumupo.org/




{BnF

**9 novembre 2017**

**Valentin Villenave**

**Mike Solomon**

**Jean-François Piette**

**Martin Granger**

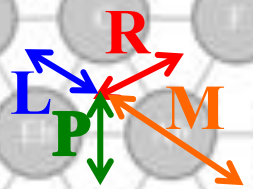
**Joseph Boisseau**

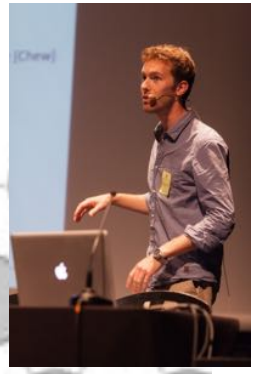
**Moreno Andreatta**

**Tom Johnson**

# Symmetries and algorithmic processes in *Muse*


"Take a bow" (*Black Holes and Revelations*, 2006)



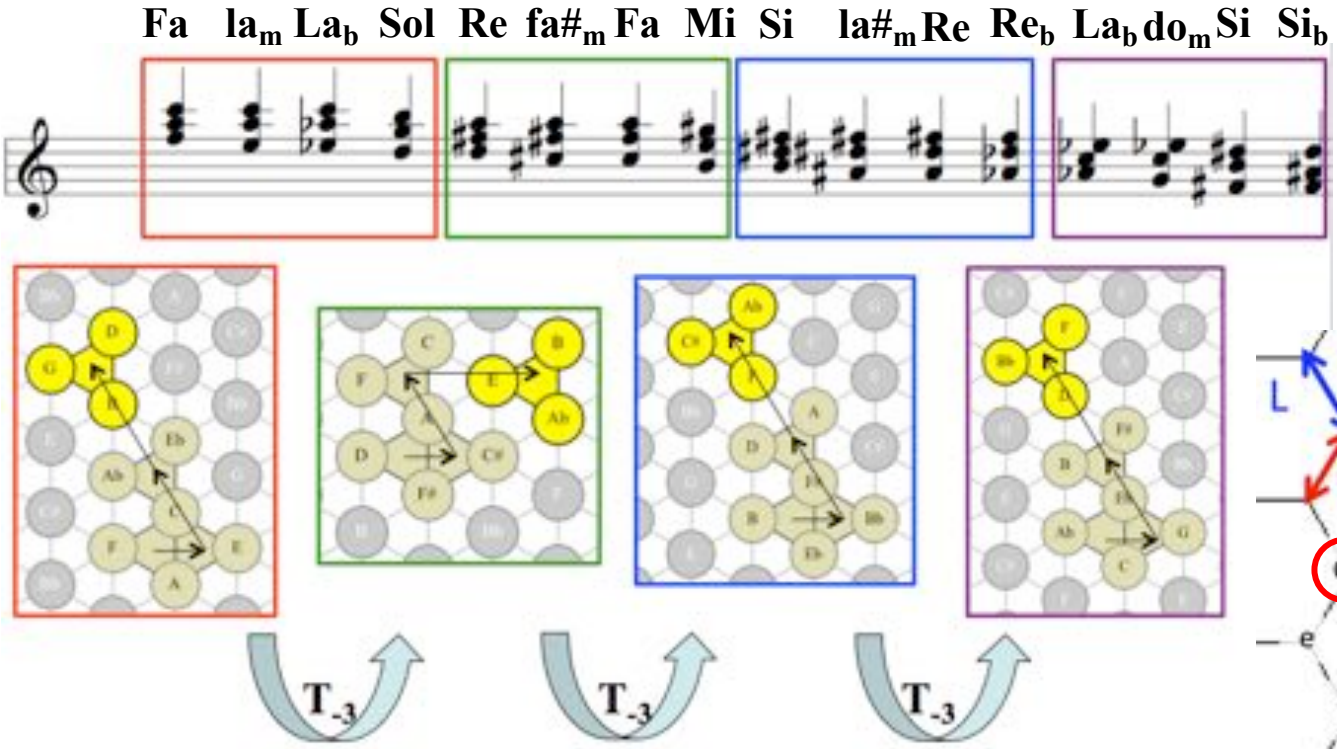
Louis Bigo

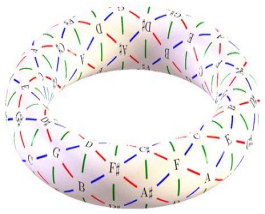



Hexachord (Louis Bigo, 2013)

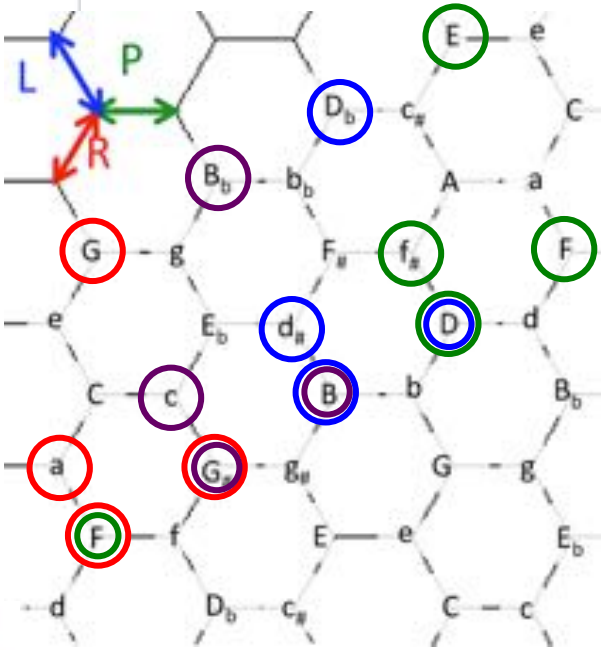
Temporal axis

# Symétries dans la musique de Frank Zappa



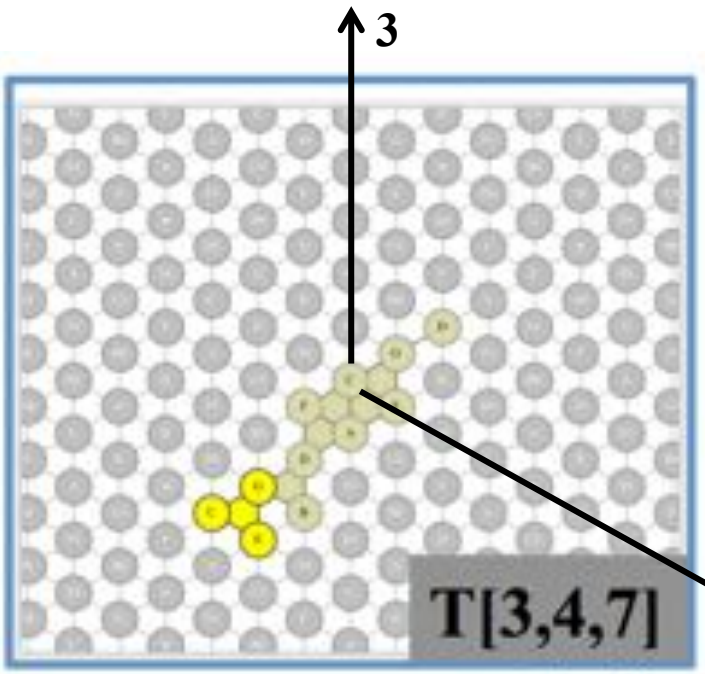



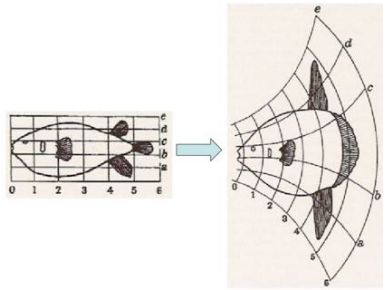
➔ Source: Wikipedia

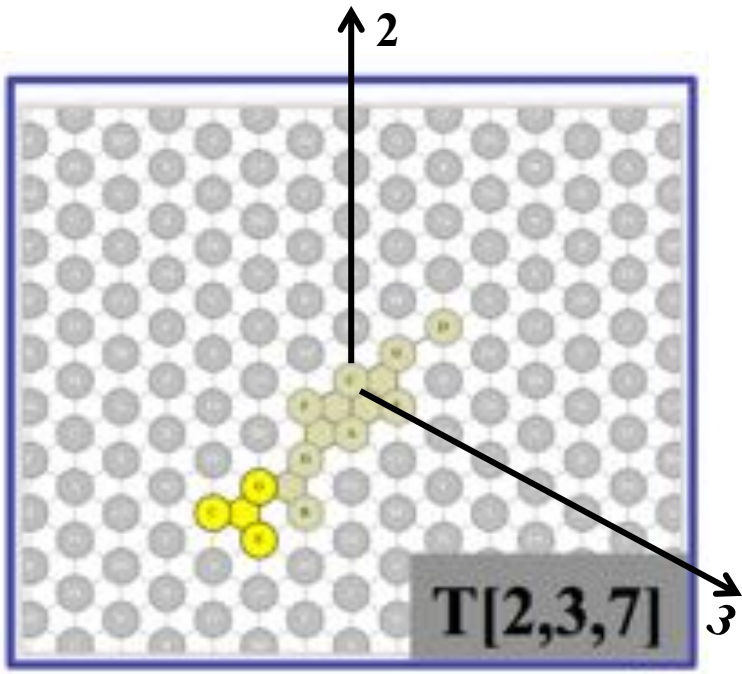


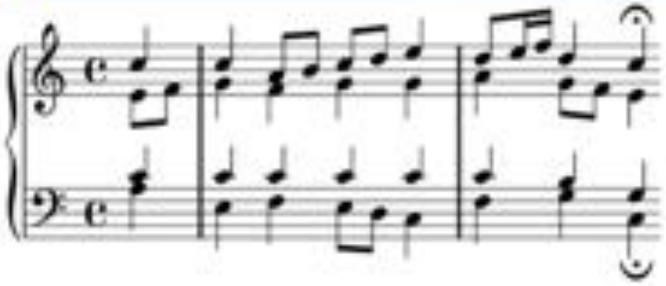

« Easy Meat » - 1981 (Frank Zappa)
min. 1'44" – 2'39"

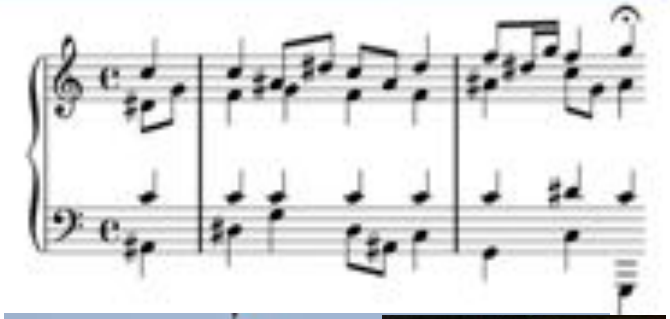
# The musical style...is the space!



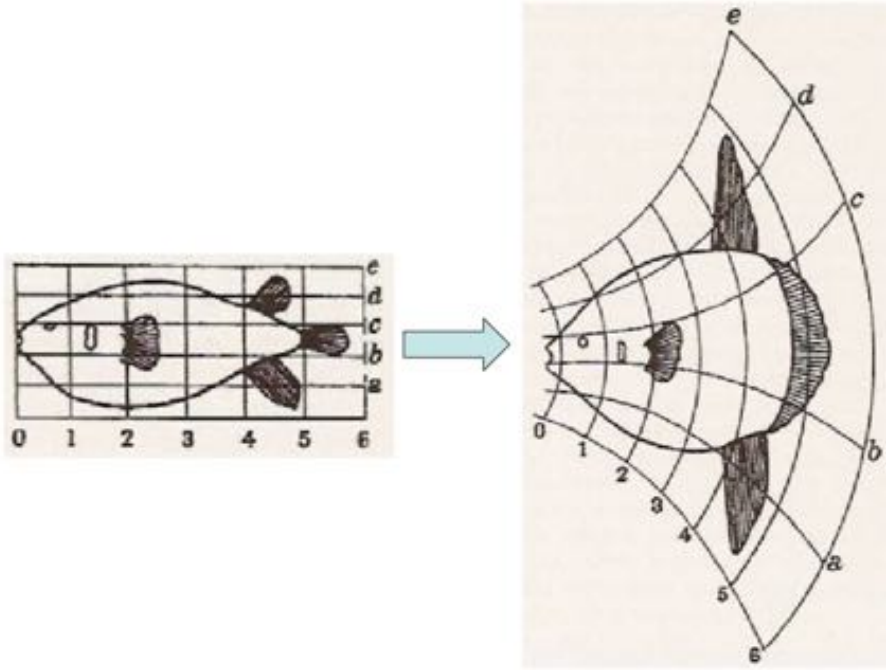



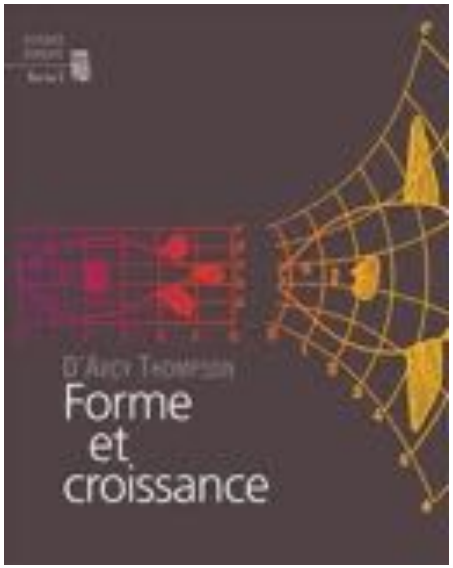




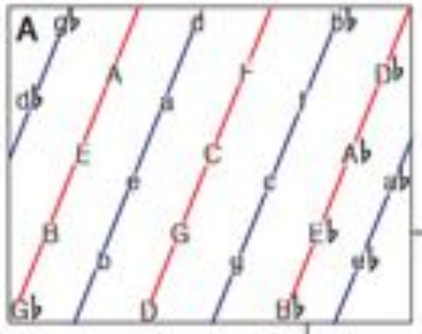
# The morphological genealogy of structuralism




« [La notion de **transformation**] me vient d'un ouvrage qui a joué pour moi un rôle décisif et que j'ai lu pendant la guerre aux États Unis: *On Growth and Form*, en deux volumes, de D'Arcy Wentworth Thompson, paru pour la première fois en 1917. L'auteur, naturaliste écossais, (...) interprétait comme des transformations les différences visibles entre les espèces ou organes animaux ou végétaux au sein d'un même genre. Ce fut une illumination, d'autant que j'allais vite m'apercevoir que cette façon de voir s'inscrivait dans une longue tradition: derrière Thompson, il y avait la botanique de Goethe, et derrière Goethe, Albert Dürer avec son Traité de la proportion du corps humain » (Lévi-Strauss et Eribon, 1988).

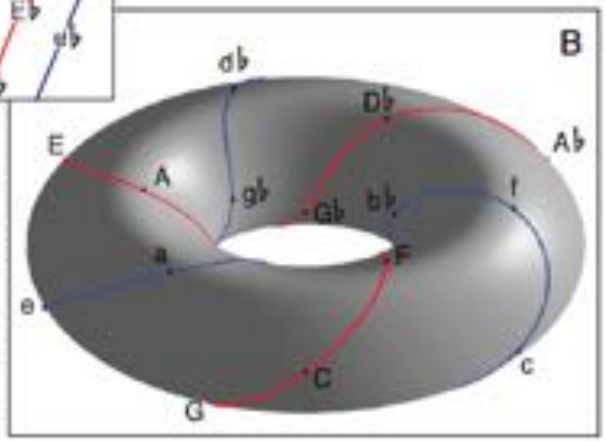
# *Tonnetz* and cognitive neurosciences



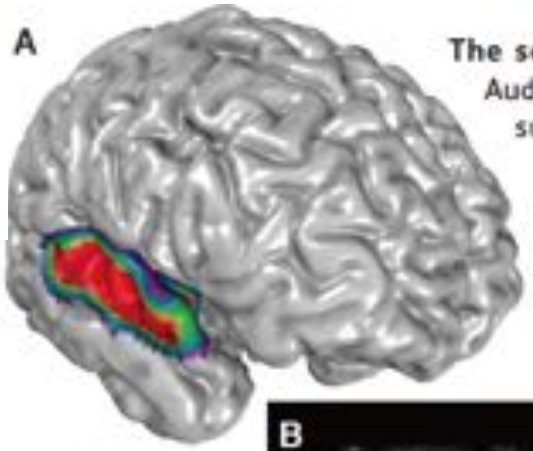

PERSPECTIVES: NEUROSCIENCE

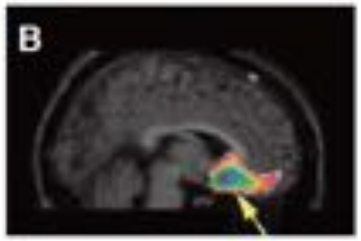
## Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl

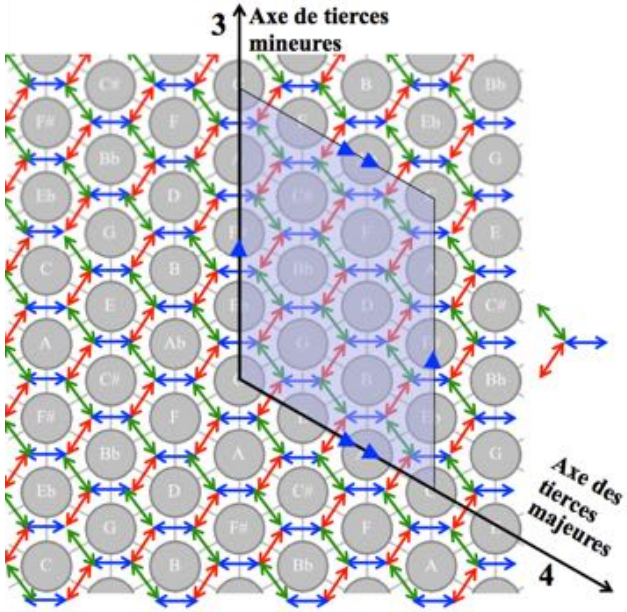



**Mental key maps.** (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (B) Musical keys as points on the surface of a torus.




**The sensation of music.** (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

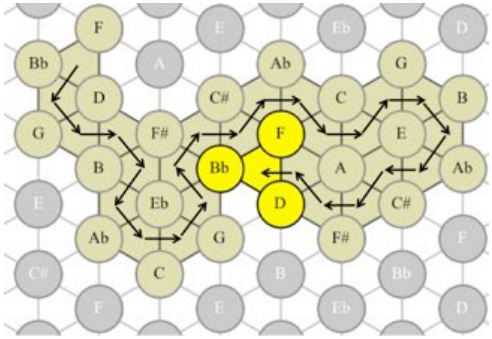


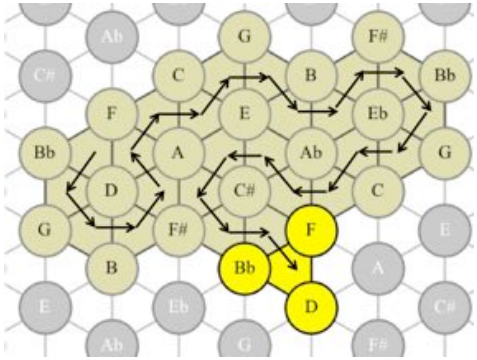


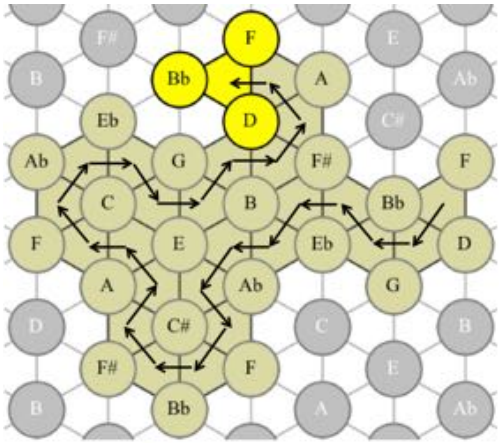
**Acotto E. et M. Andreatta (2012), « Between Mind and Mathematics. Different Kinds of Computational Representations of Music », *Mathematics and Social Sciences*, n° 199, 2012(3), p. 9-26.**

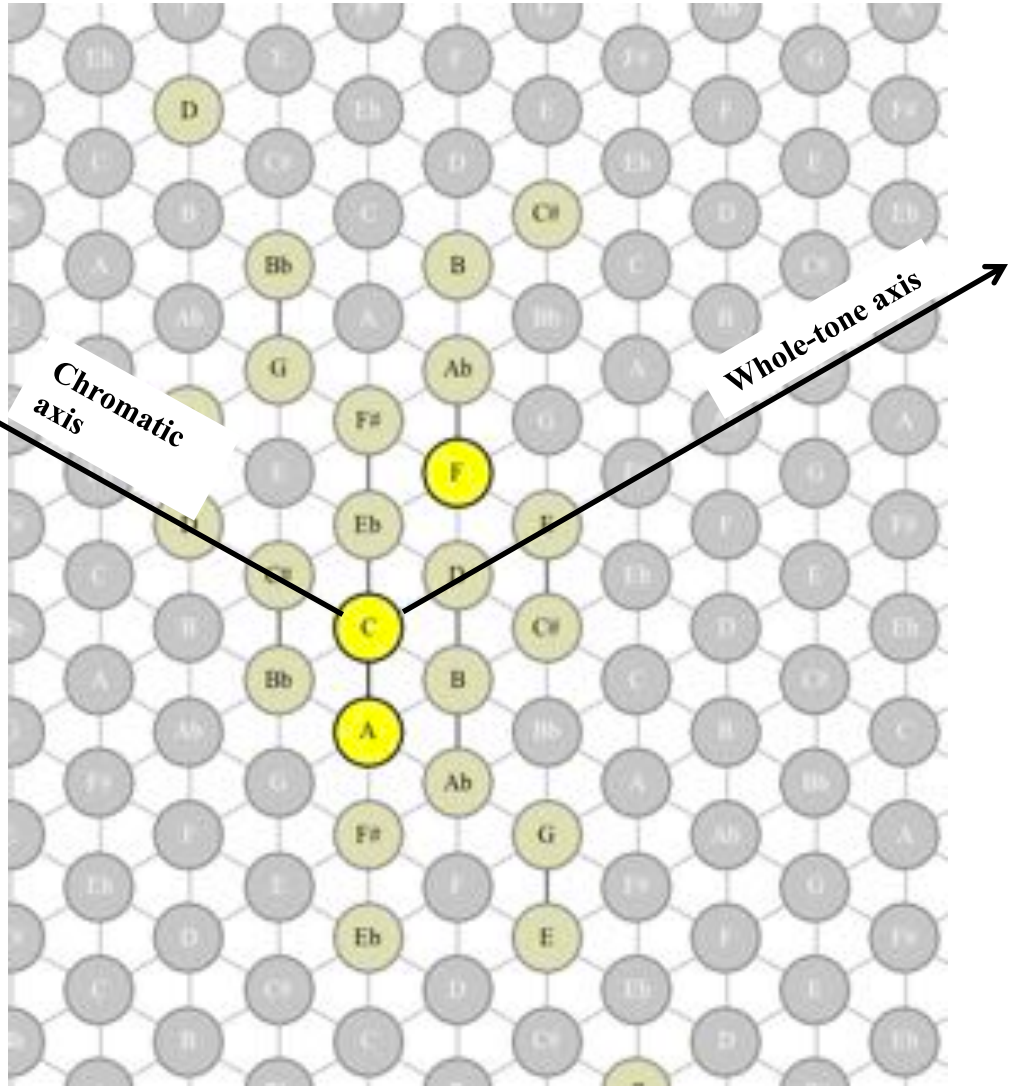

EMPG European Mathematical Psychology Group

# Different embeddings of Hamiltonian cycles

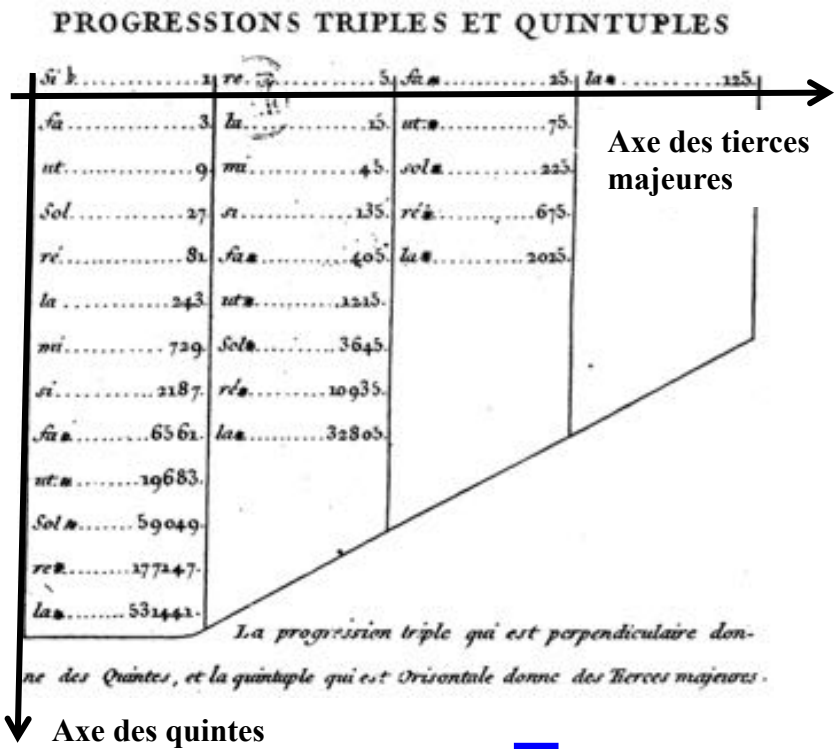


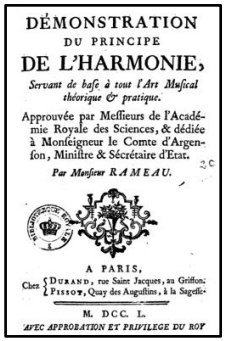



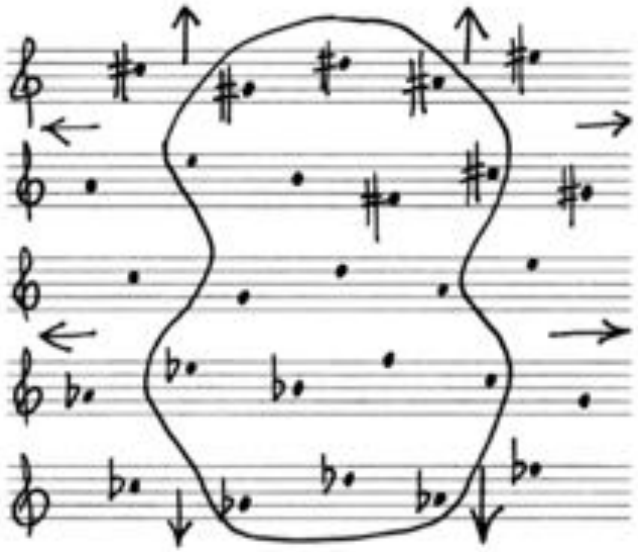



# Henri Pousseur's 'Network Theory' and Rameau's heritage





DÉMONSTRATION
DU PRINCIPE
DE L'HARMONIE,
*Servant de base à tout l'Art Musical théorique & pratique.*
Approuvée par Messieurs de l'Académie Royale des Sciences, & dédiée à Monseigneur le Comte d'Argenson, Ministre & Sécrétaire d'Etat.
*Par Monsieur RAMEAU.*
A PARIS,
Chez { DURAND, rue Saint Jacques, au Griffon; PISSOT, Quay des Augustins, à la Sagesse.
M. DCC. L.
*AVEC APPROBATION ET PRIVILEGE DU ROY*

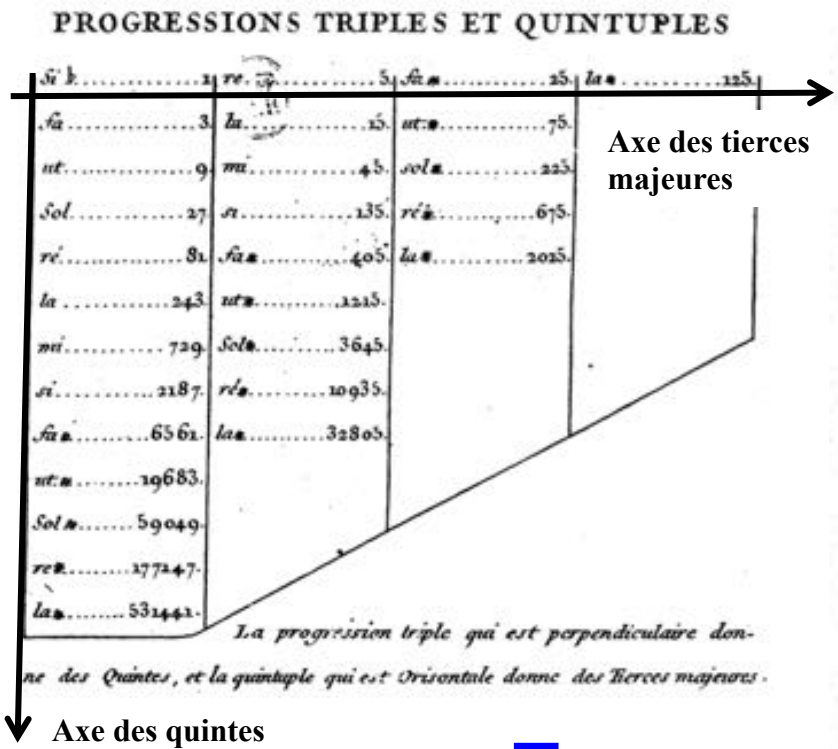


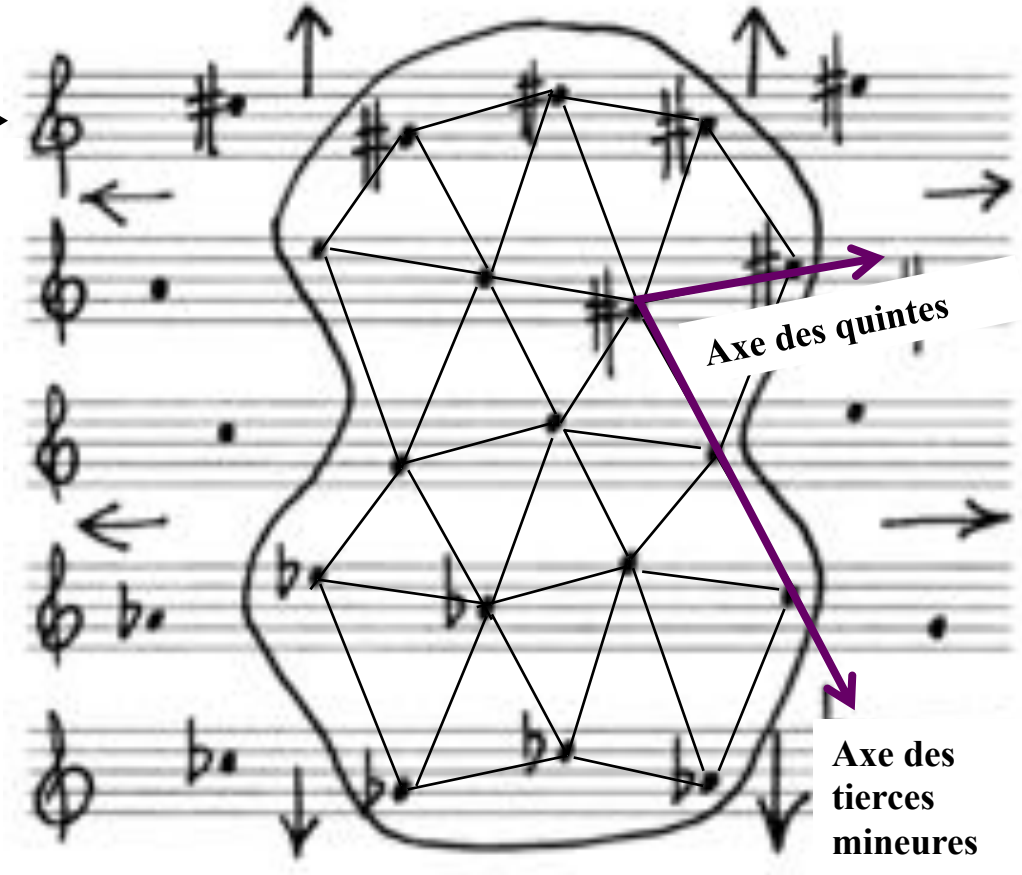
« Un réseau, au sens entendu ici, est une distribution de note [...] selon plusieurs (pour commencer deux) axes qui se caractérisent chacun comme une chaîne d'un seul et même intervalle »

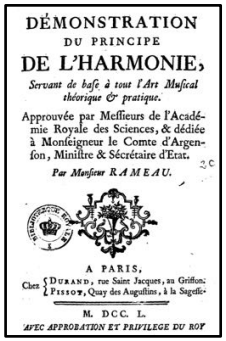
**« Applications Analytiques de la 'technique des réseaux' »,** ***Revue belge de Musicologie*****, Vol. 52, pp. 247-298, 1998**

- **J.-Ph. Rameau,** ***Démonstration du principe de l'harmonie,*** **1750**

# Henri Pousseur's 'Network Theory' and Rameau's heritage

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- « L'apothéose de Rameau. Essai sur la question harmonique, *Musiques Nouvelles. Revue d'esthétique,* 21, 105-172, 1968
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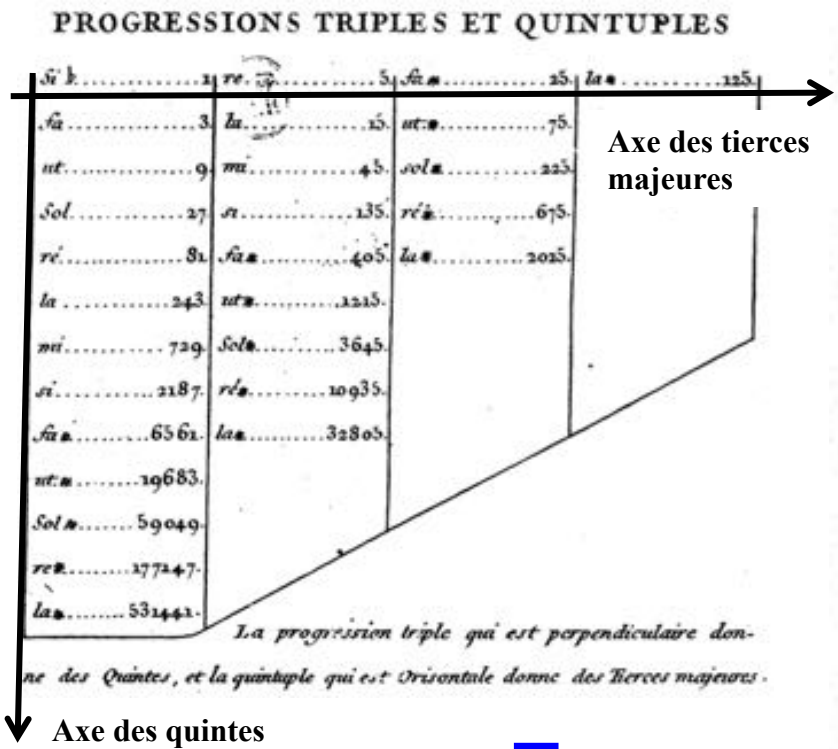


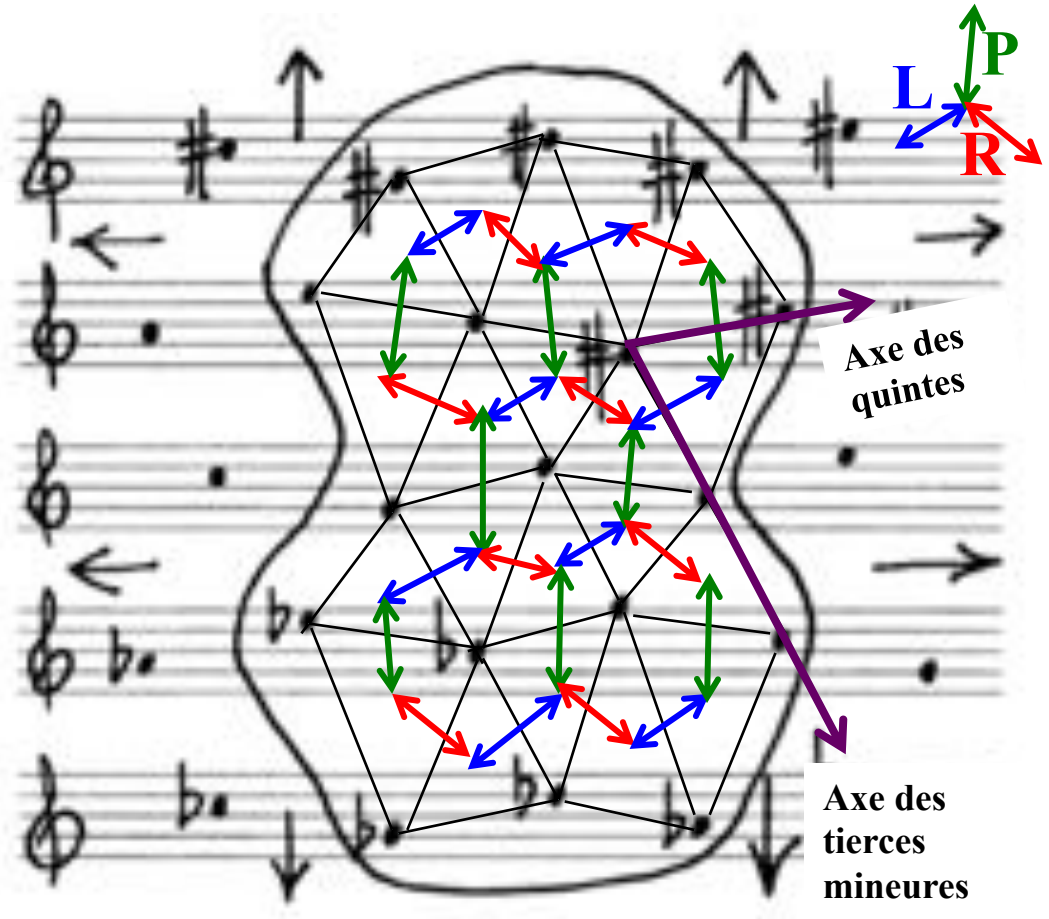


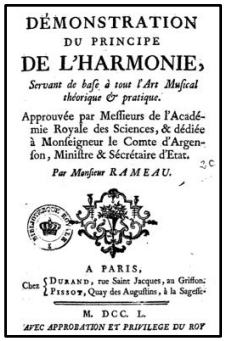
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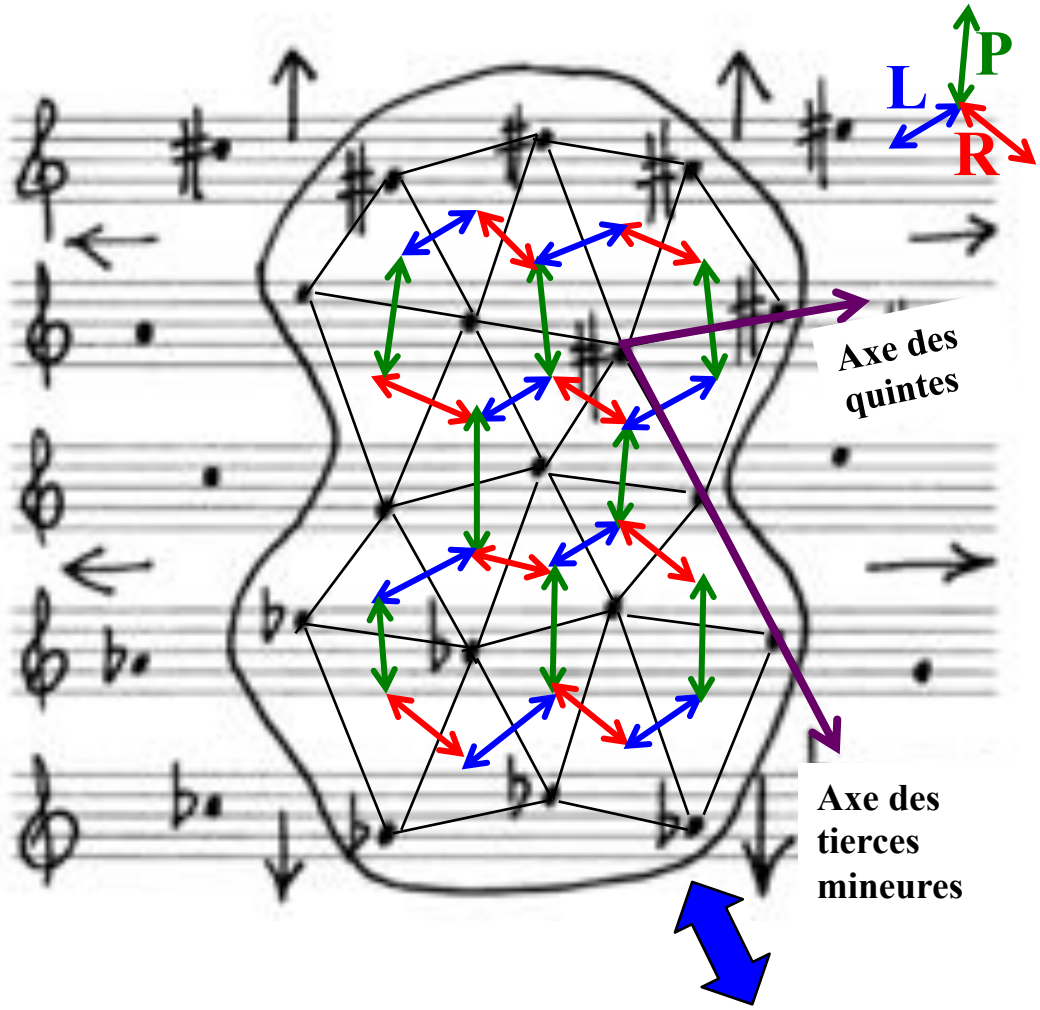
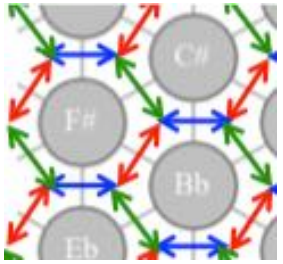
# Henri Pousseur's 'Network Theory' and Rameau's heritage

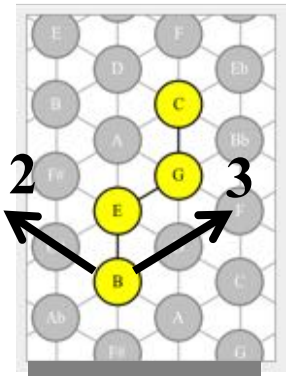
« Il ne faut toutefois pas oublier que le principe même de la méthode réside dans la volonté de construire le lacis de telle sorte que les relations musicales élémentaires effectives, donc 'en-temps', (analysées ou composées, mélodiques ou accordiques) soient les plus **serrées** possibles, s'expriment principalement entre notes **voisines** du réseau, dans un sens ou dans l'autre.
Ajoutons encore que l'on peut passer de certains réseaux à certains autres en faisant simplement 'basculer' les axes [...] ce qui modifie les rapports de proximité structurelle entre les notes et donc la hiérarchie de leurs intervalles ».



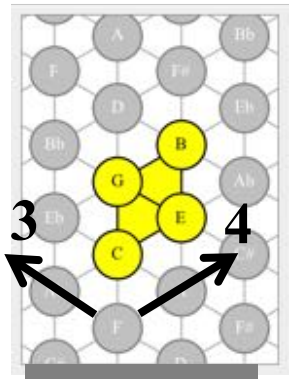




- **« Applications Analytiques de la 'technique des réseaux' », *Revue belge de Musicologie*, Vol. 52, pp. 247-298, 1998**

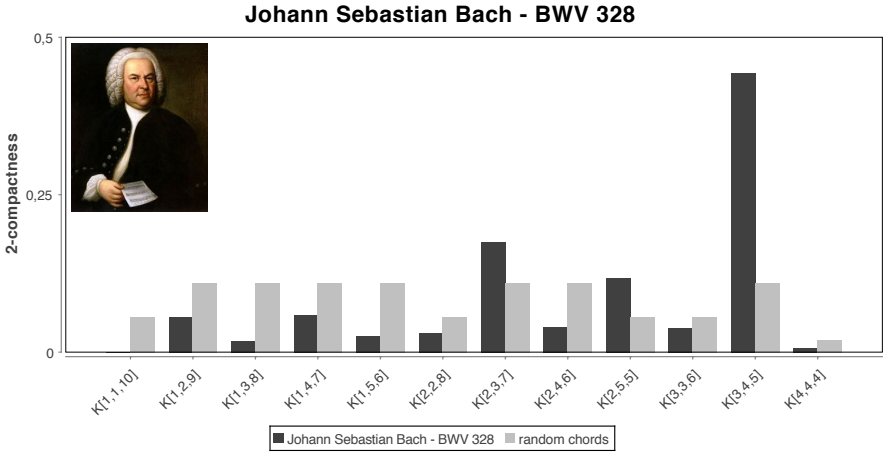
# The spatial character of the « musical style »

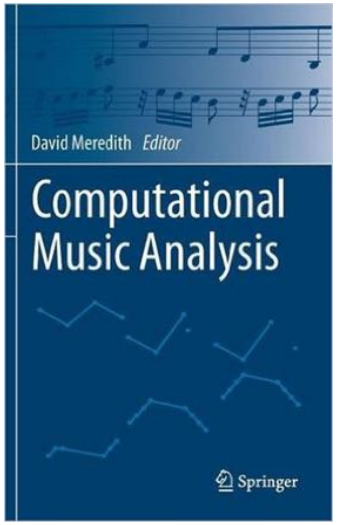


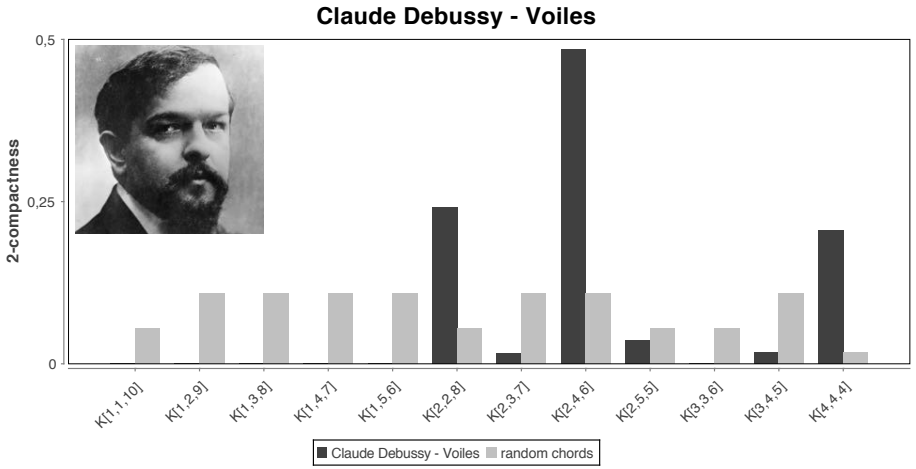

T[2,3,7]

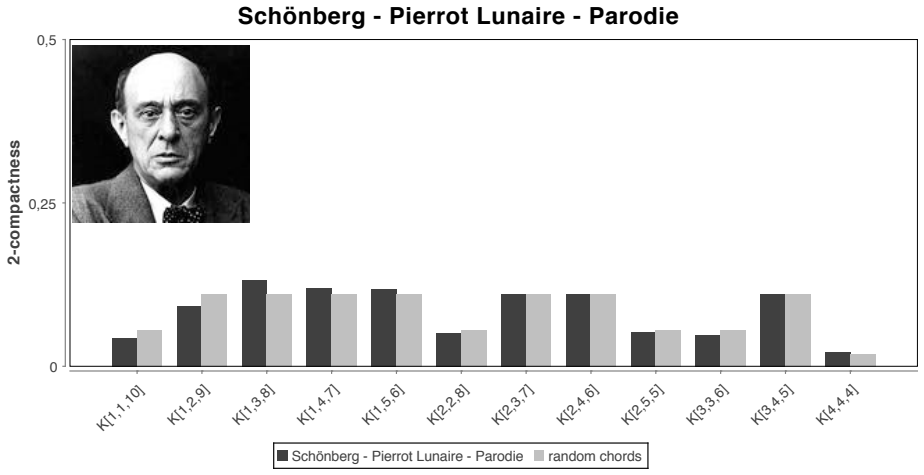



T[3,4,5]



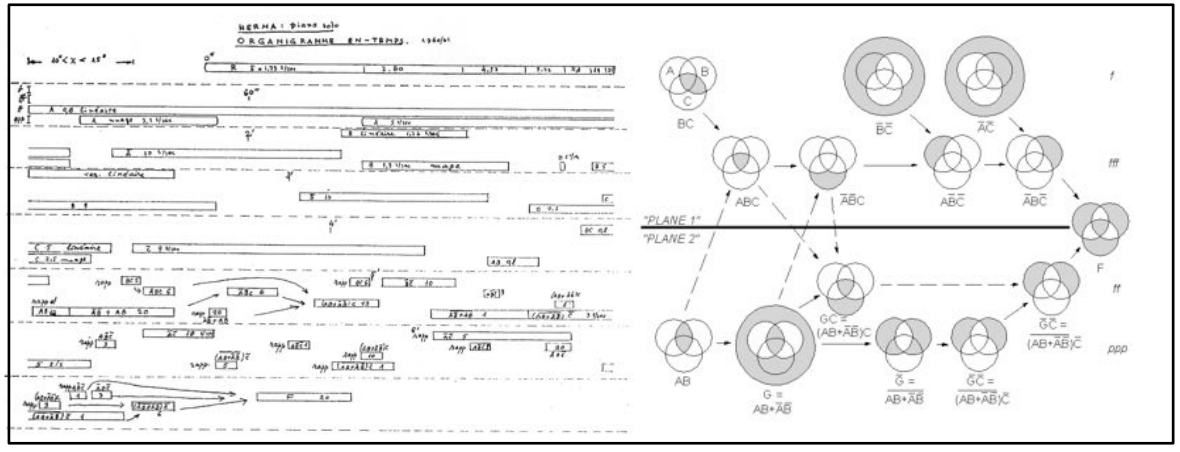



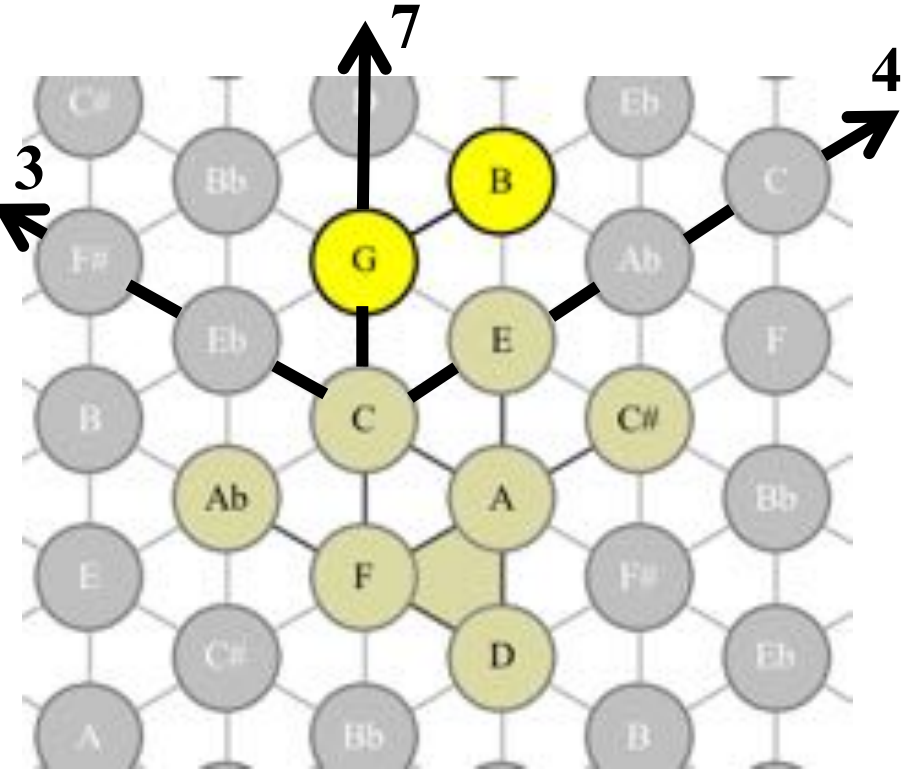



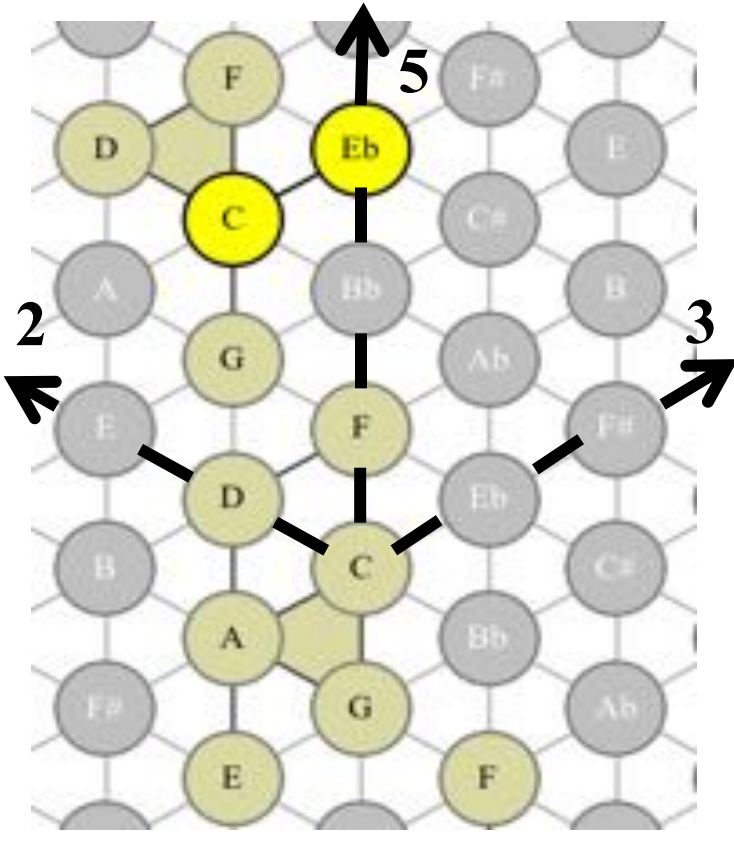




**Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2016**

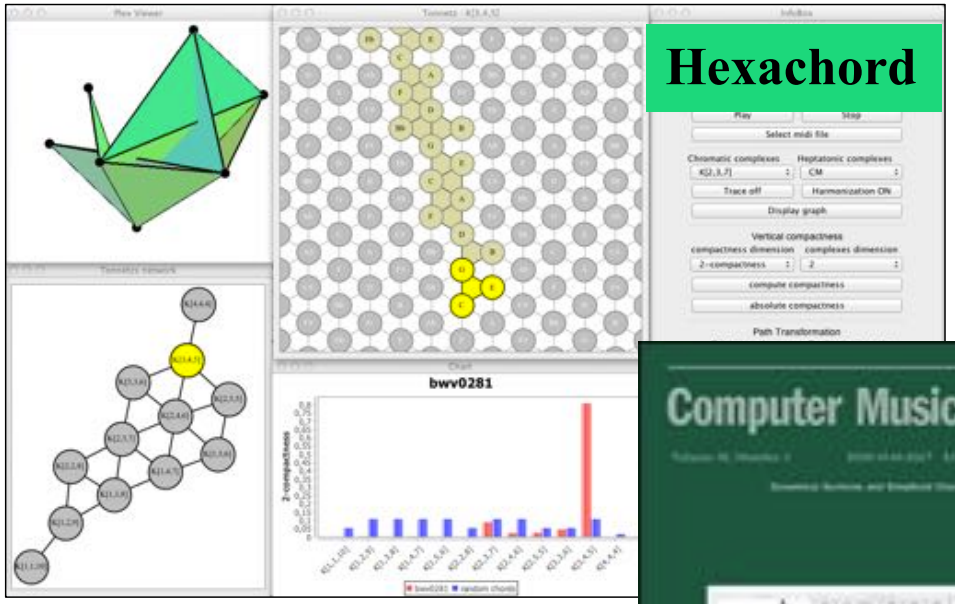
# A Tonnetz-based analysis of *Herma* by Iannis Xenakis



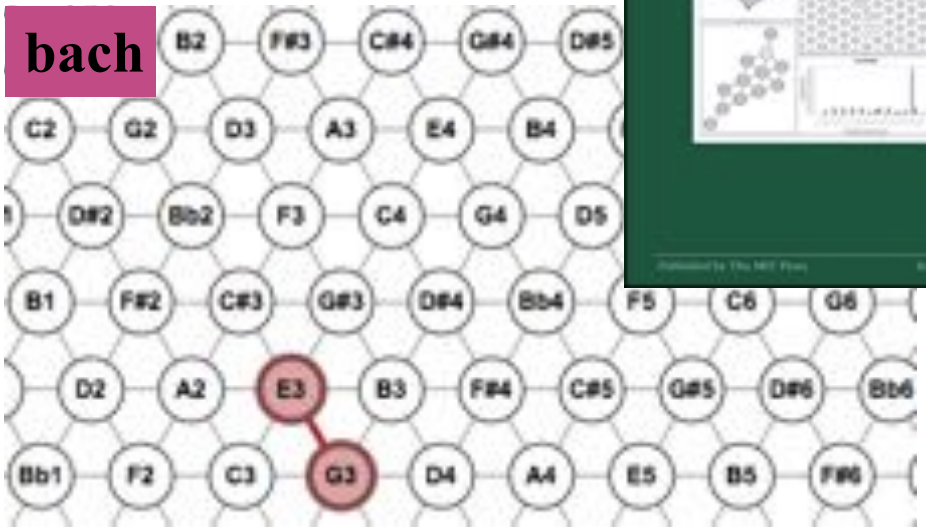



# Three environments for a Tonnetz-based Analysis



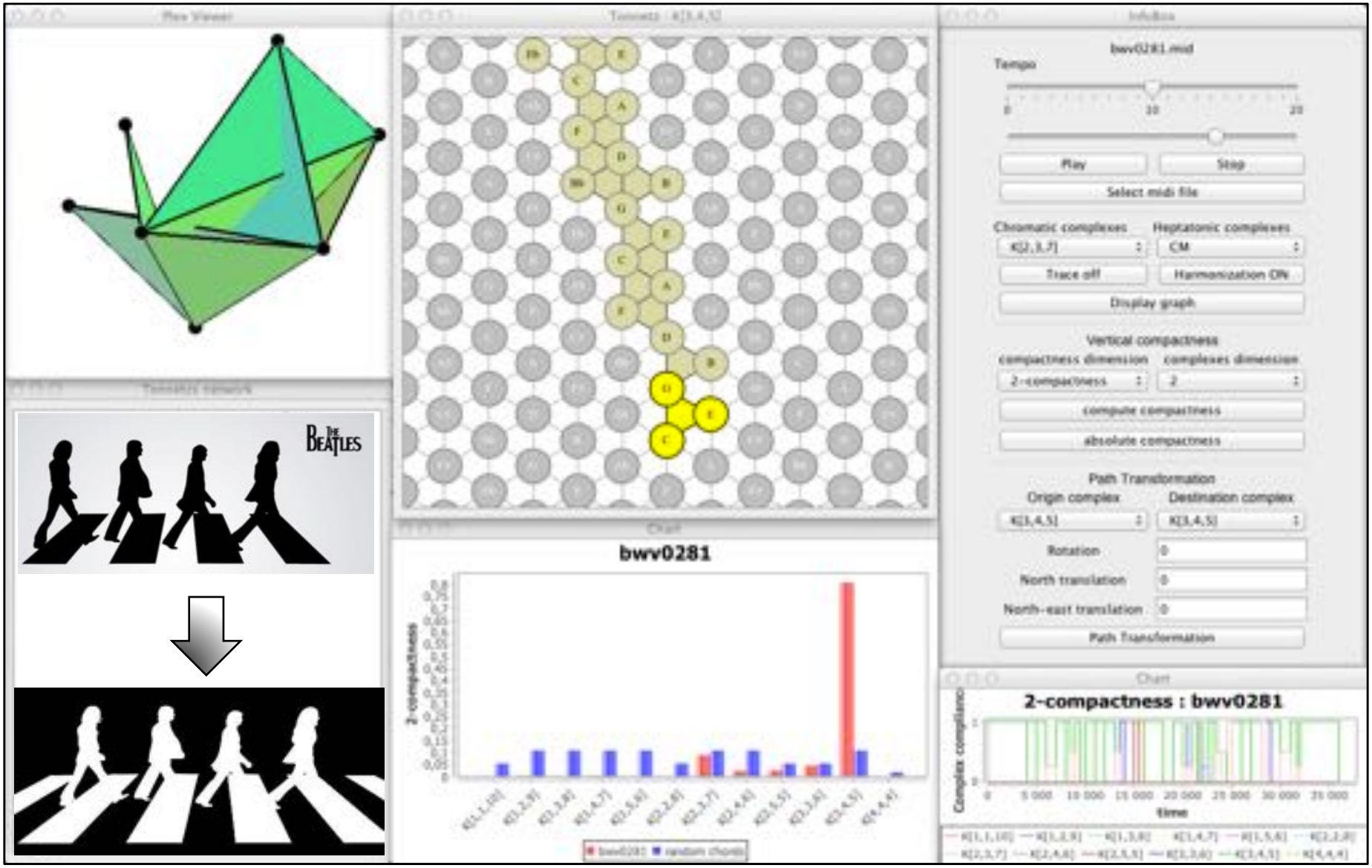






➔ demo

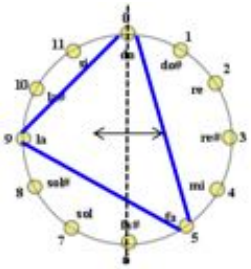
# Keeping the space...but changing the trajectory!



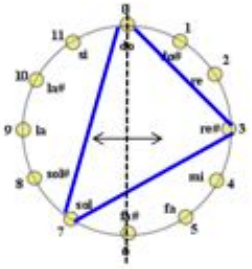

➔http://www.lacl.fr/~lbigo/hexachord

# Keeping the space...but changing the trajectory!

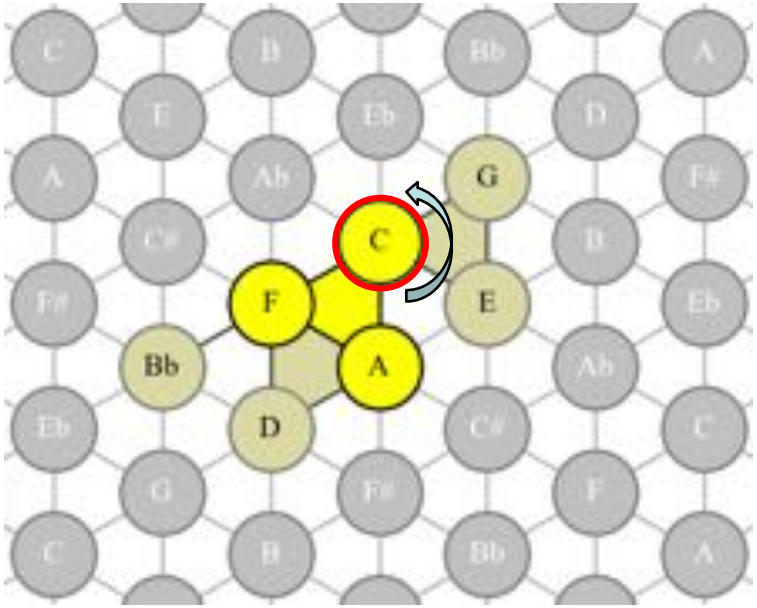

*F* major chord



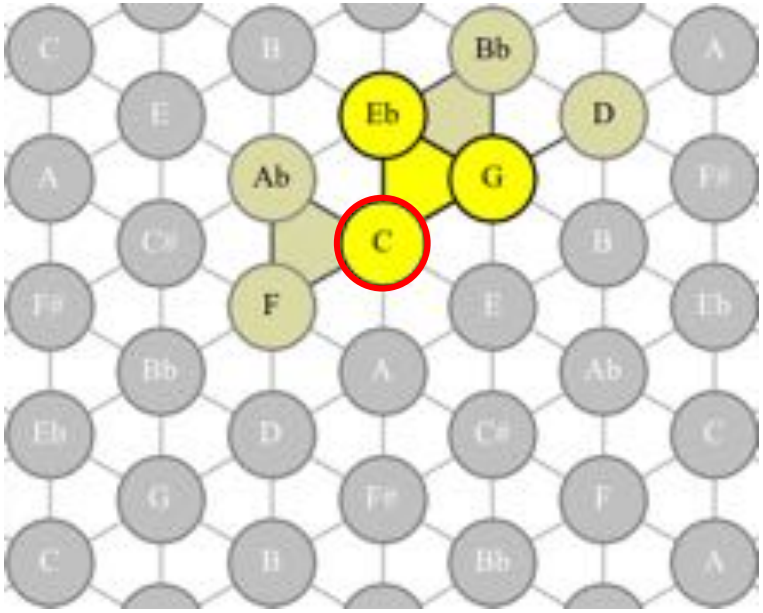
inversion



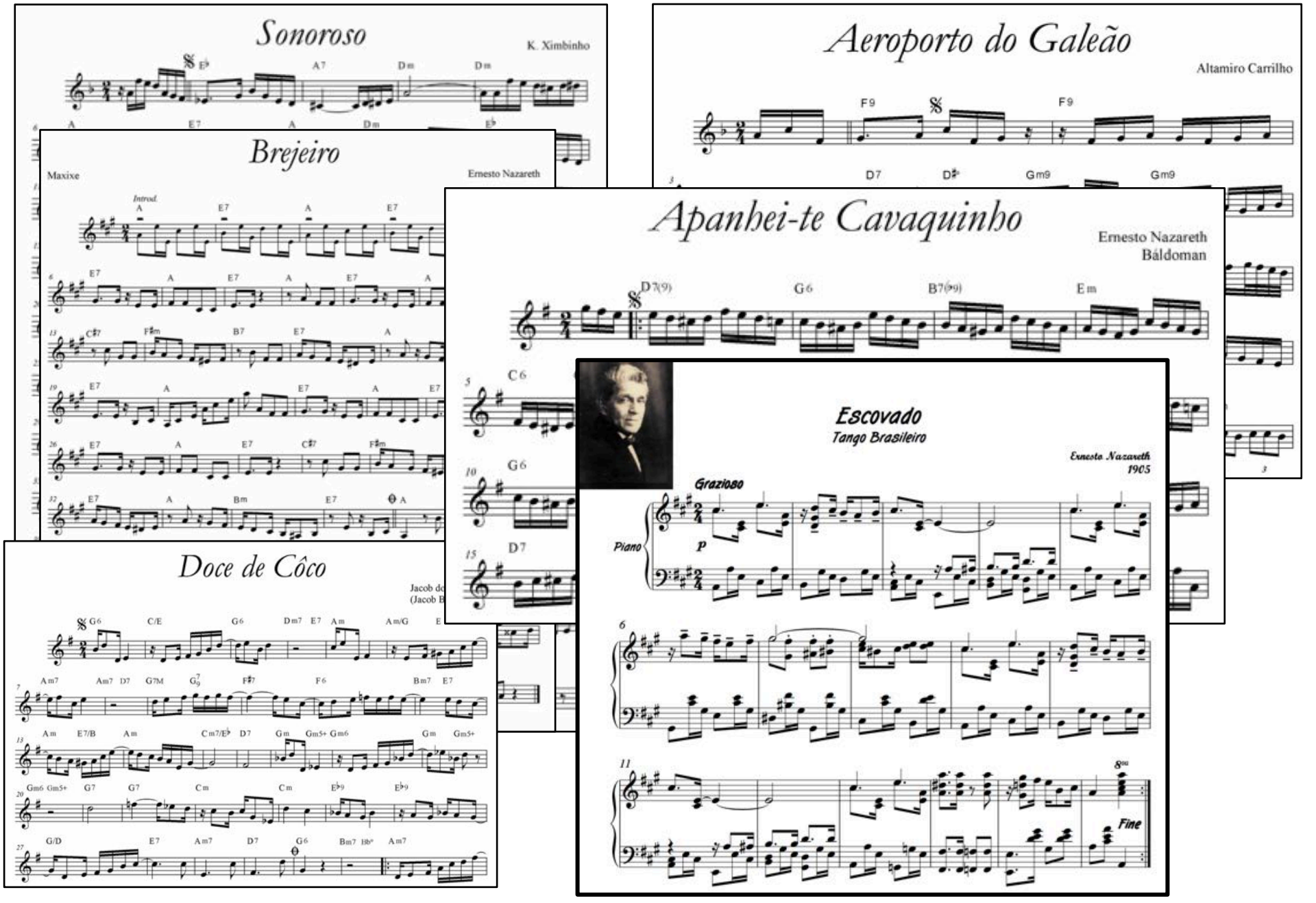
*C* minor chord




**Rotation**
**(autour du *do*)**



# Rotational symmetry applied to traditional Brazilian music



Thank you for your attention!