

FestivalScienza
C A G L I A R I



MATEMATICA E MUSICA

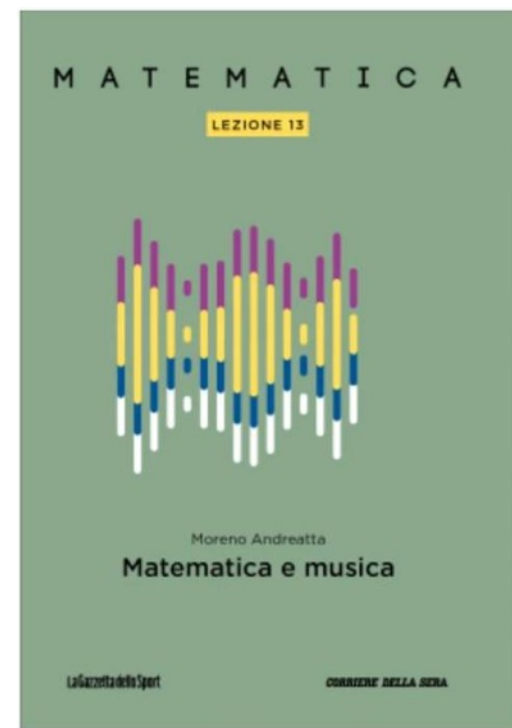
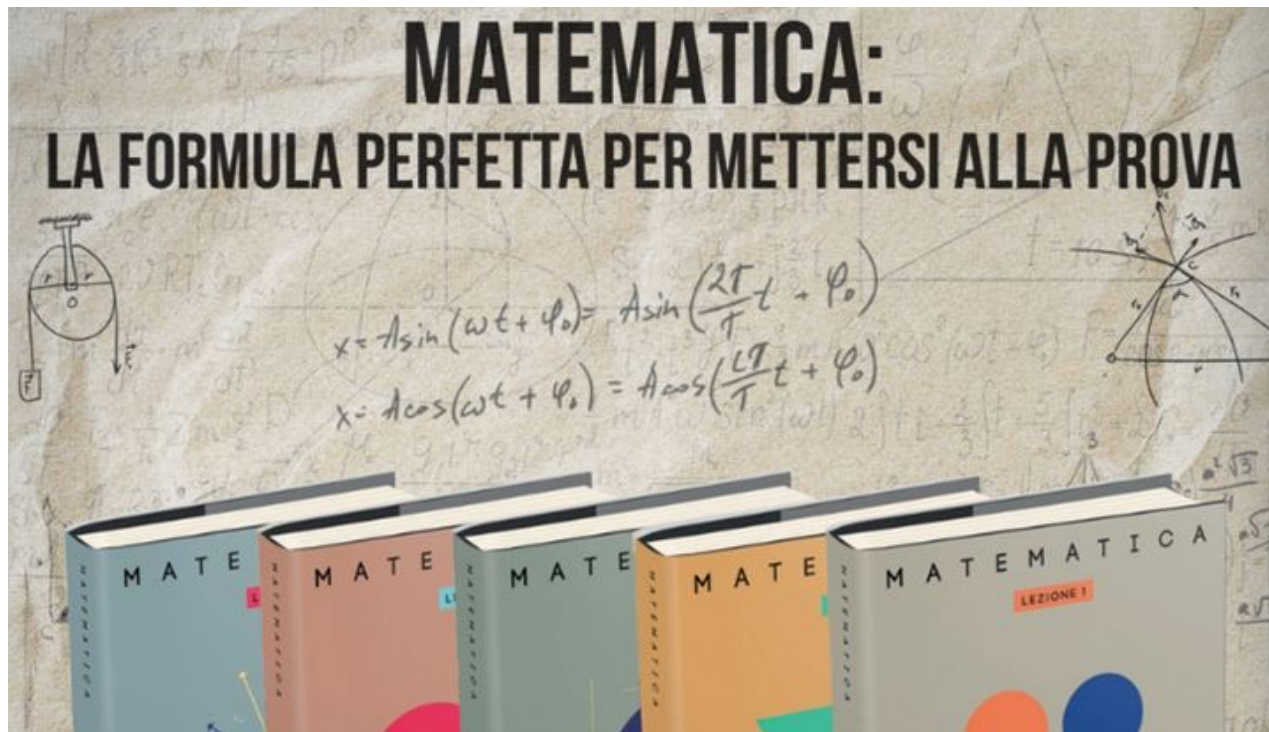
**Presentazione, dimostrazione interattiva
e performance musicale**

Moreno Andreatta

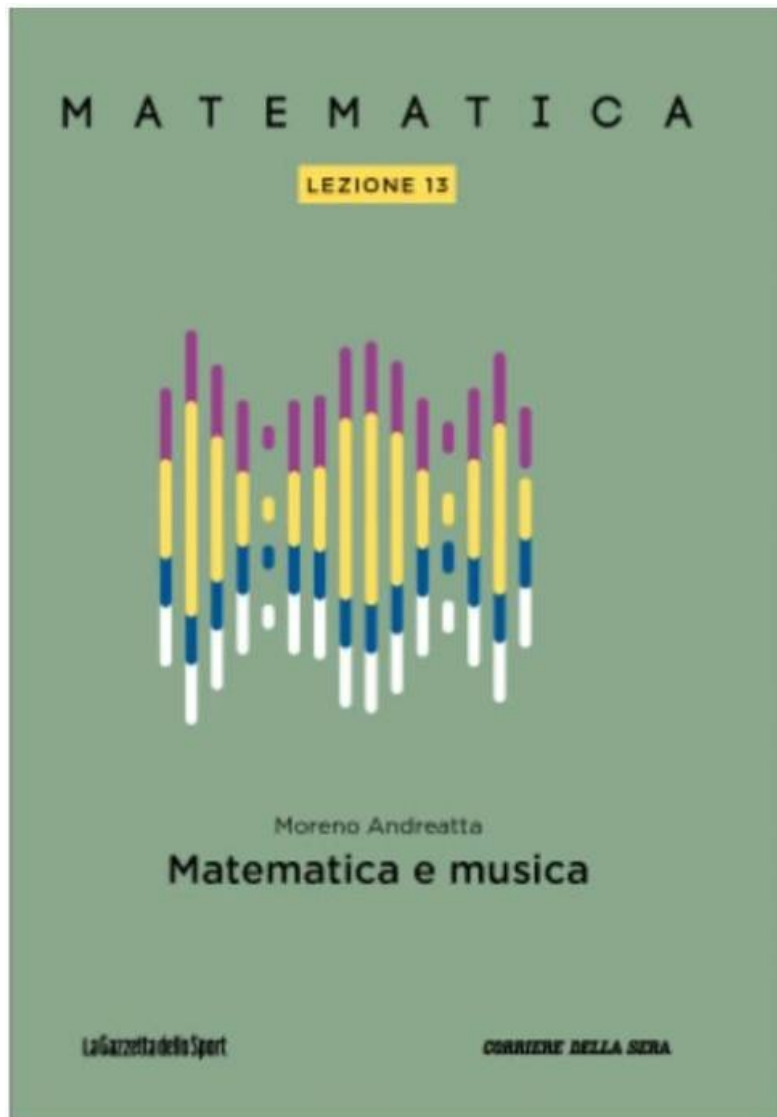
CNRS / IRMA / Université de Strasbourg

CNRS / IRCAM / Sorbonne Université

www.morenoandreatta.com

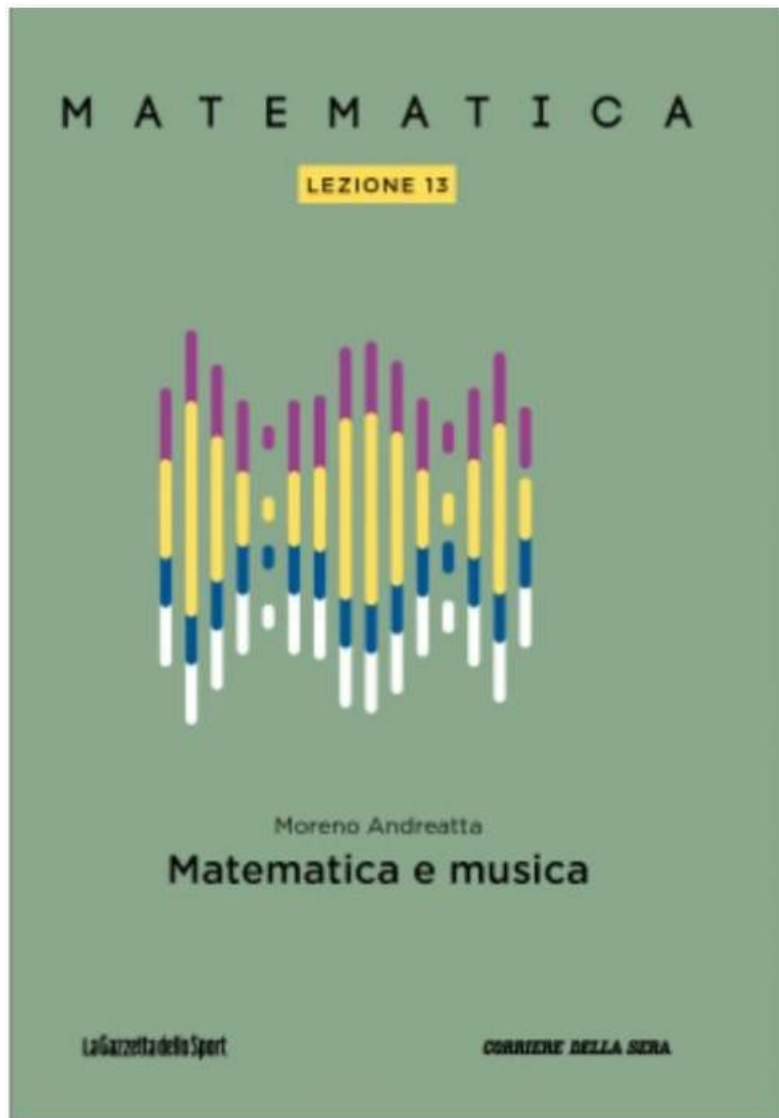


Che legame c'è fra la matematica e la musica? La riflessione sui rapporti fra queste due discipline, che possiamo affermare cominci con **Pitagora** (almeno per quanto riguarda la storia del pensiero filosofico in Occidente), ha di fatto attraversato i secoli per giungere sino a noi incorporando, di volta in volta, dei campi della matematica dove il legame ha potuto trovare un fondamento teorico e nuove possibilità di applicazioni in ambito non solo musicale ma nella matematica stessa. La musica ha in effetti contribuito attraverso le riflessioni di **teorici della musica, analisti e compositori** ad aprire nuovi orizzonti di ricerca all'interno della matematica, come storicamente è avvenuto per il **calcolo combinatorio** e la **teoria dei grafi** rispettivamente nel XVII e XVIII secolo e come continua a fare oggi nei laboratori di **ricerca matematica** di tutto il mondo.



Sommario

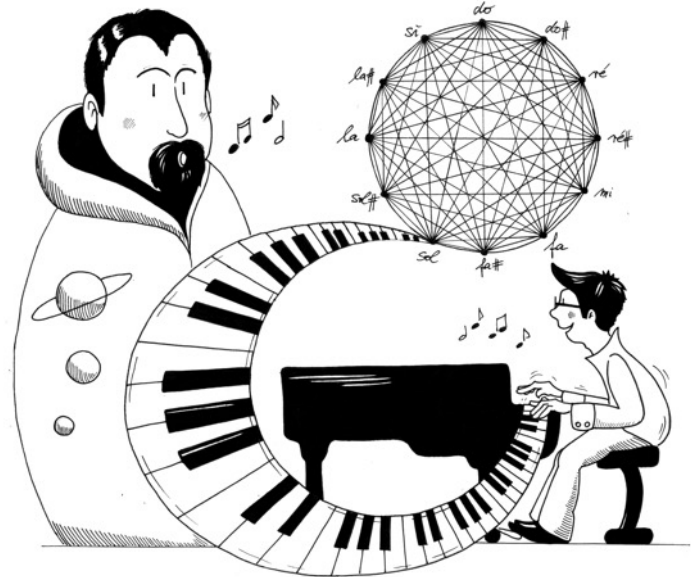
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 - Babbage e Lovelace
- **GIOCHI MATEMATICI** (M. Codogno)
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 - Soluzioni
- **ESERCIZI**
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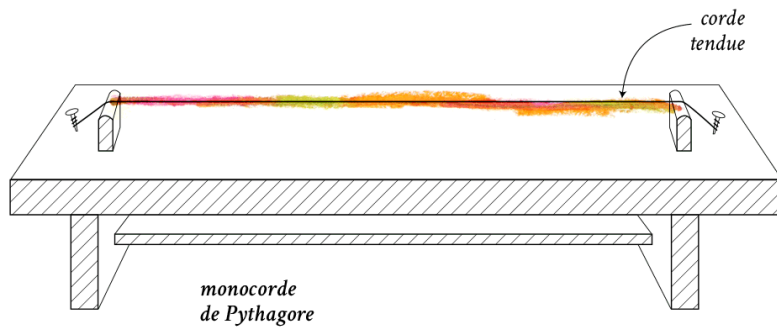
Quando la musica precede la matematica



Pitagora, il monocordo e le frazioni

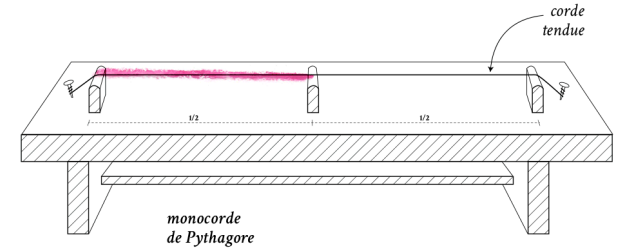
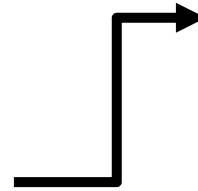
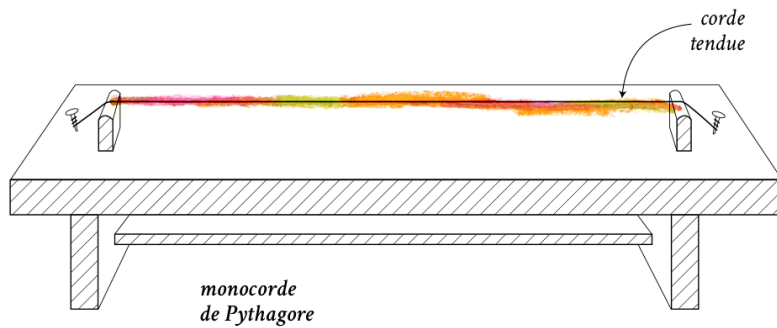


Calcolo degli intervalli col monocordo



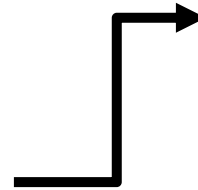
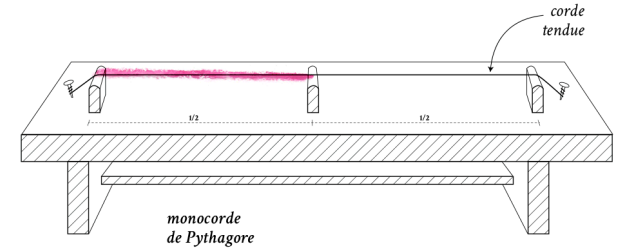
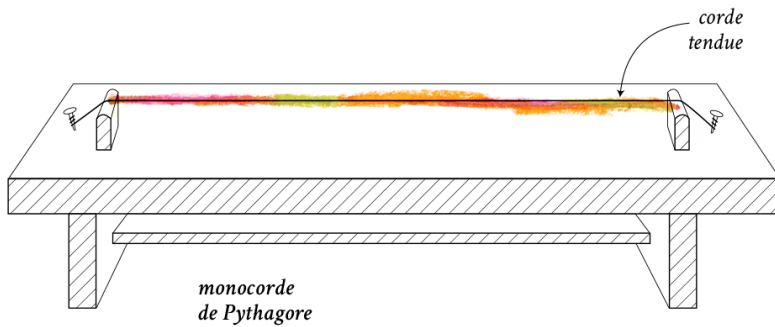
VOICI UNE NOTE

Calcolo degli intervalli col monocordo



VOICI UNE NOTE

Calcolo degli intervalli col monocordo



Longueur
de la corde

Rapport
de fréquence

Intervalle
correspondant

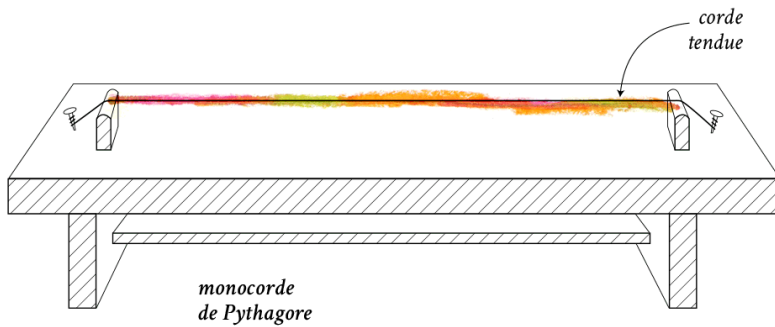
$1/2$

2

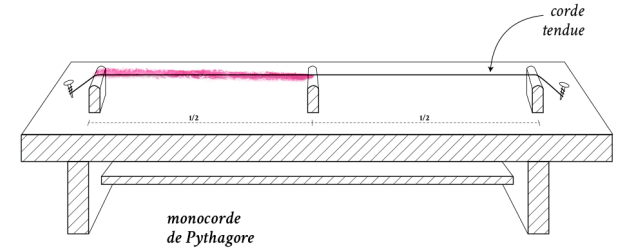
OCTAVE

VOICI UNE NOTE

Calcolo degli intervalli col monocordo



VOICI UNE NOTE



Longueur
de la corde

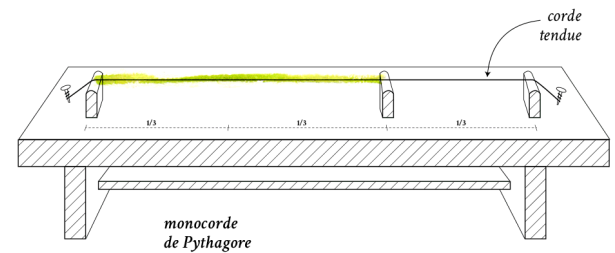
1/2

Rapport
de fréquence

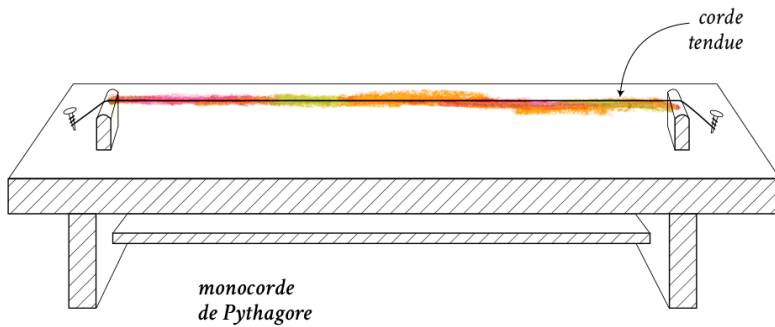
2

Intervalle
correspondant

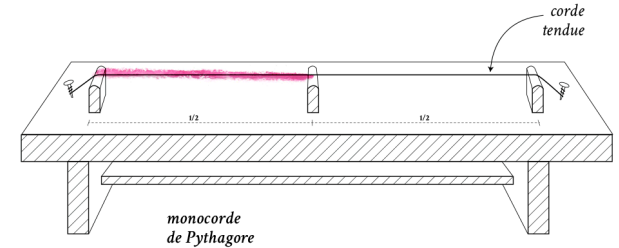
OCTAVE



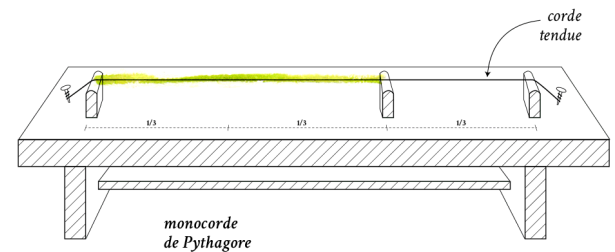
Calcolo degli intervalli col monocordo



VOICI UNE NOTE

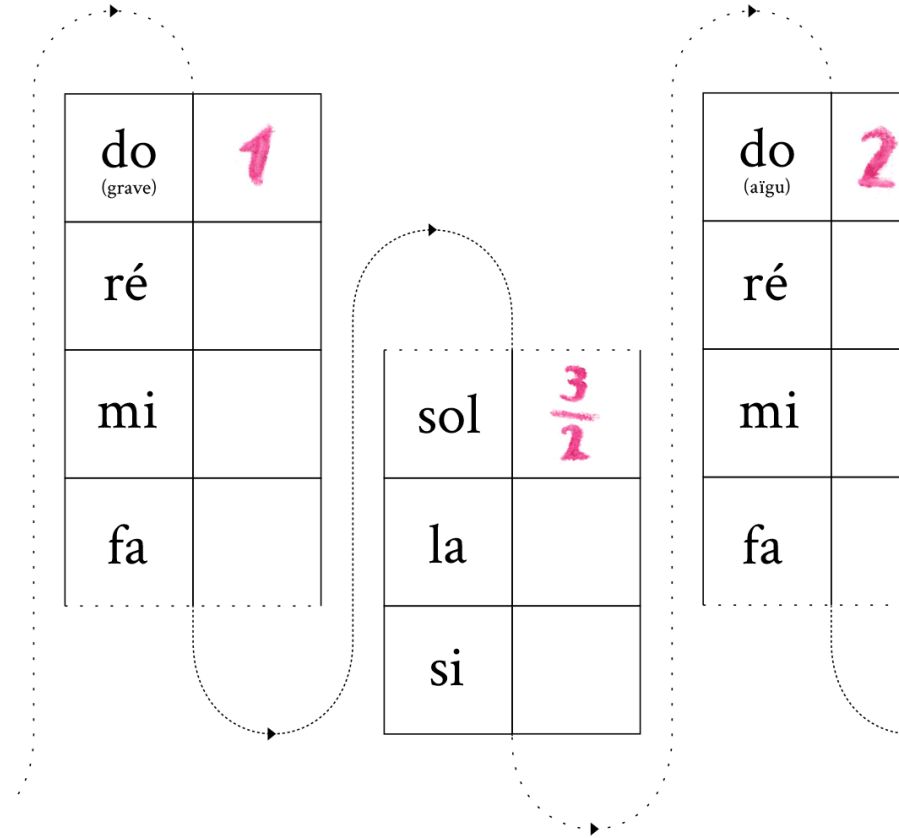
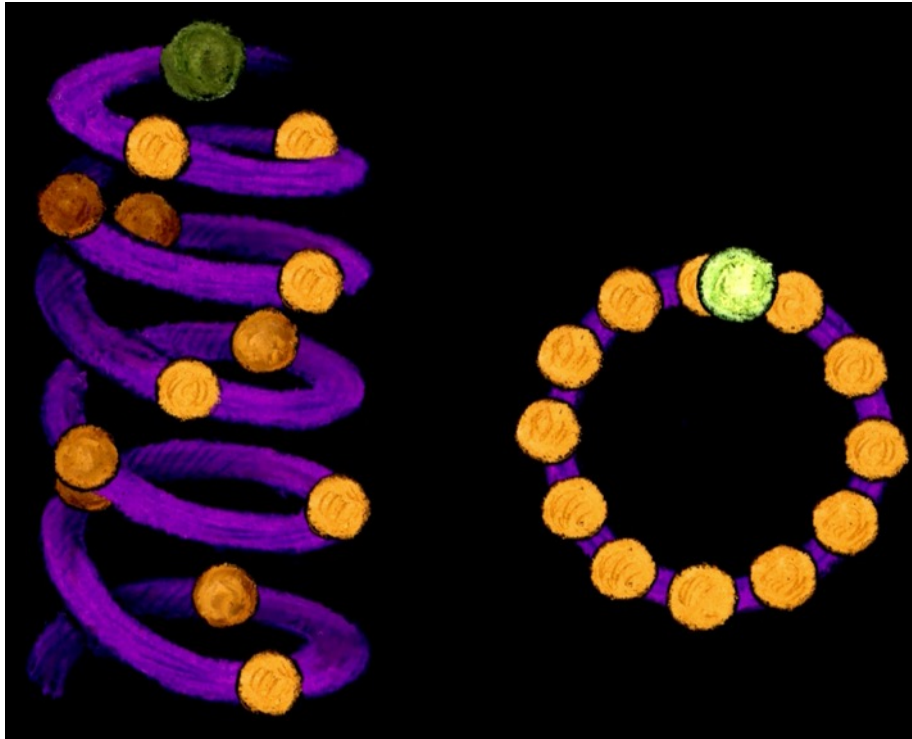


Longueur de la corde	Rapport de fréquence	Intervalle correspondant
$1/2$	2	OCTAVE



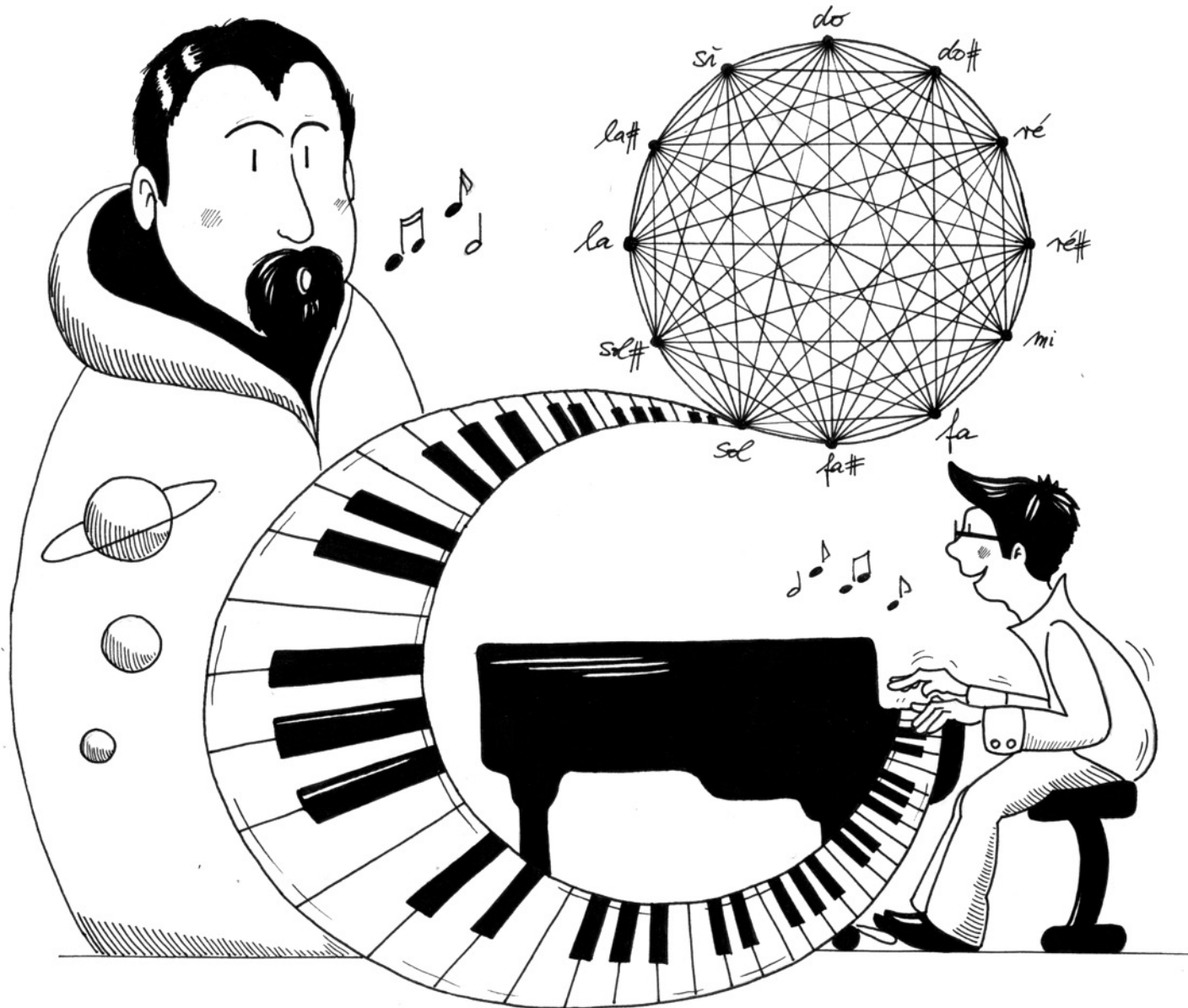
Longueur de la corde	Rapport de fréquence	Intervalle correspondant
$2/3$	$3/2$	QUINTE

Di quinta in quinta...all'infinito!

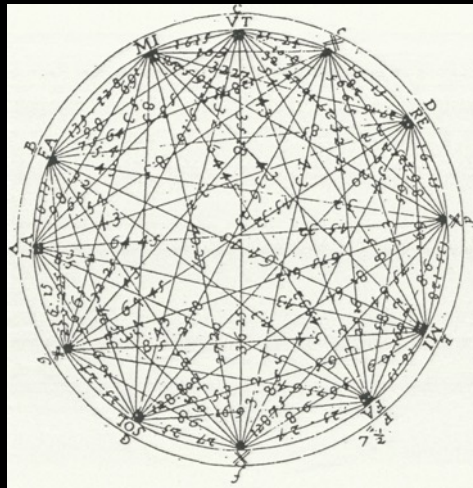


**Dal pianoforte alla
rappresentazione circolare**

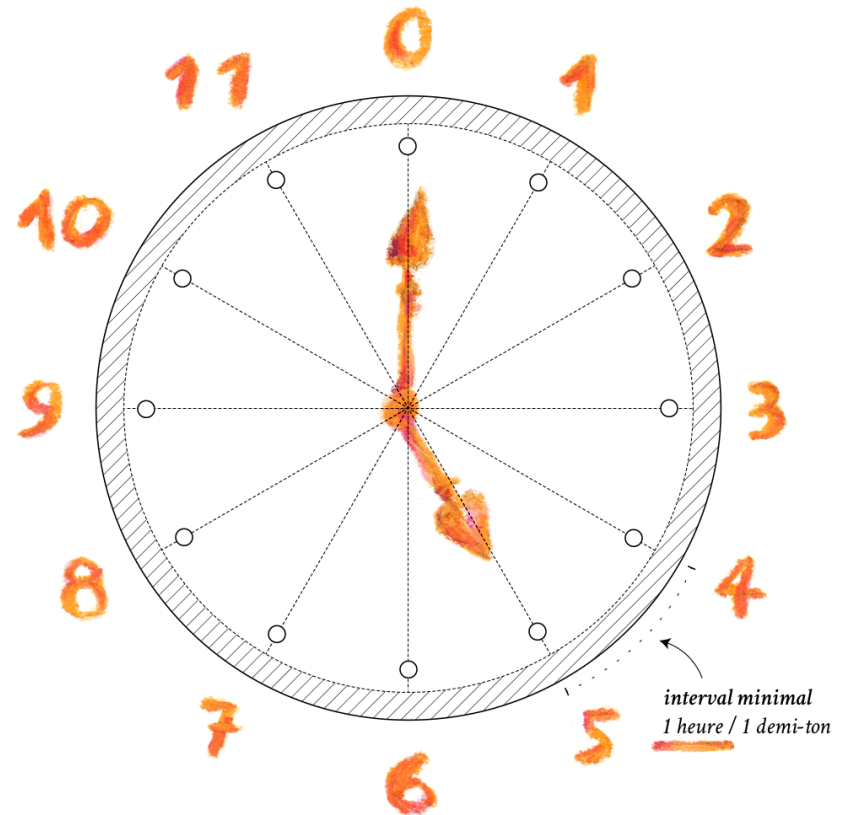
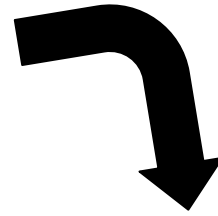
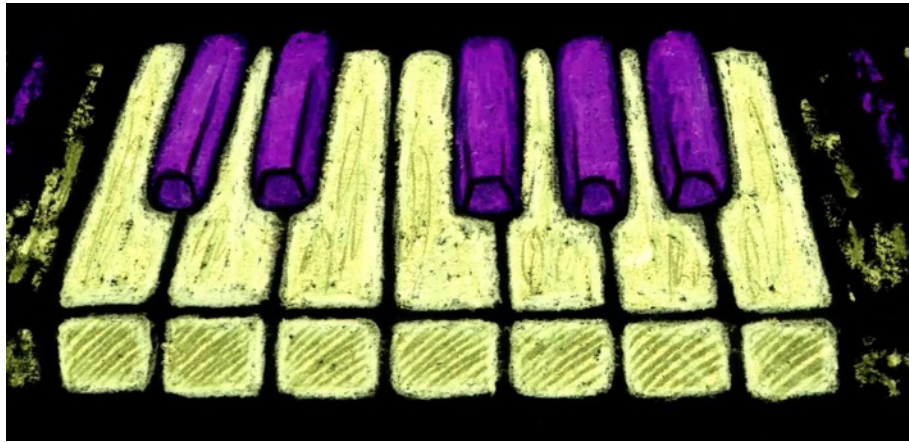
Mersenne e l'invenzione della combinatoria

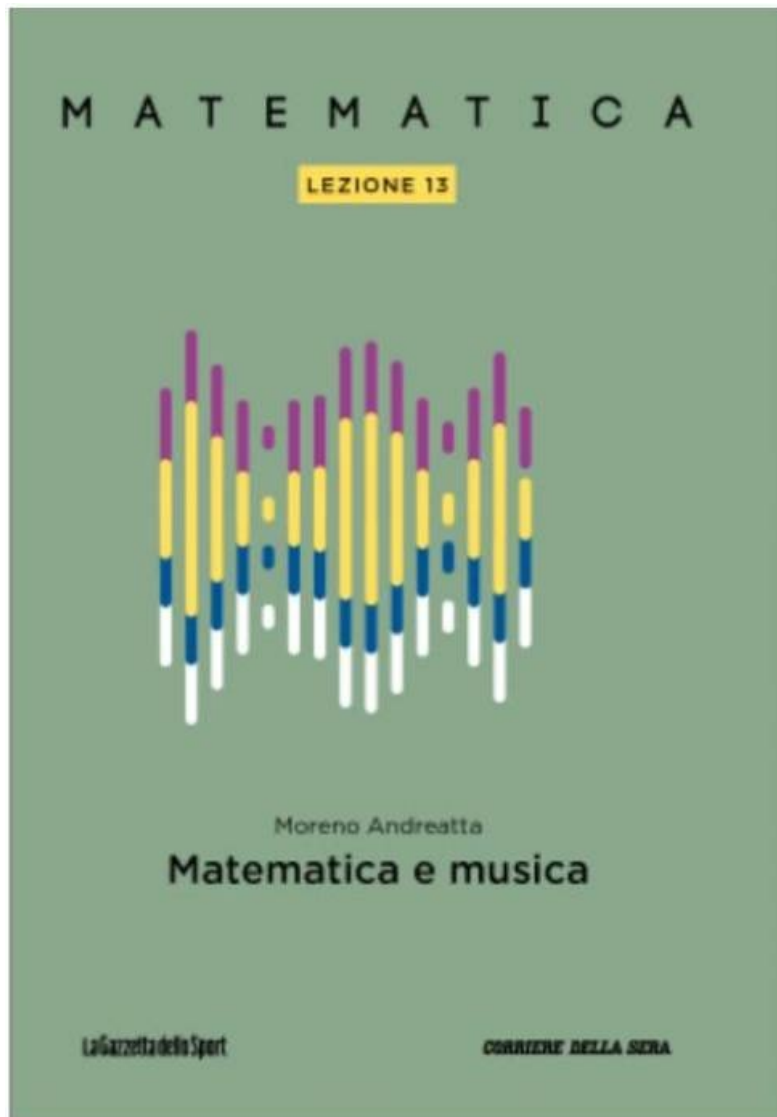


Vi ricorda qualcosa?



L'orologio musicale e gli accordi

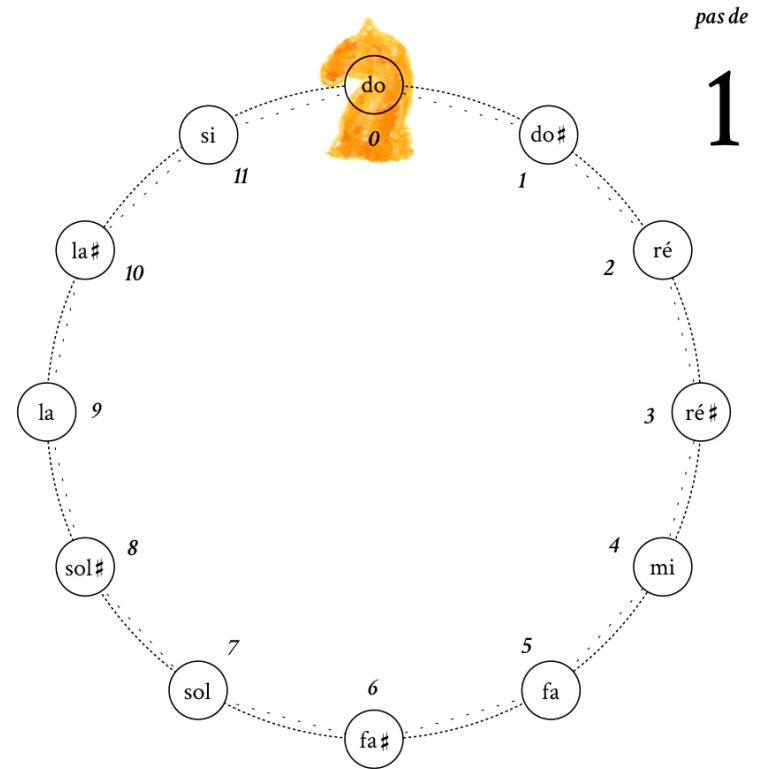
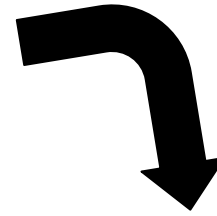
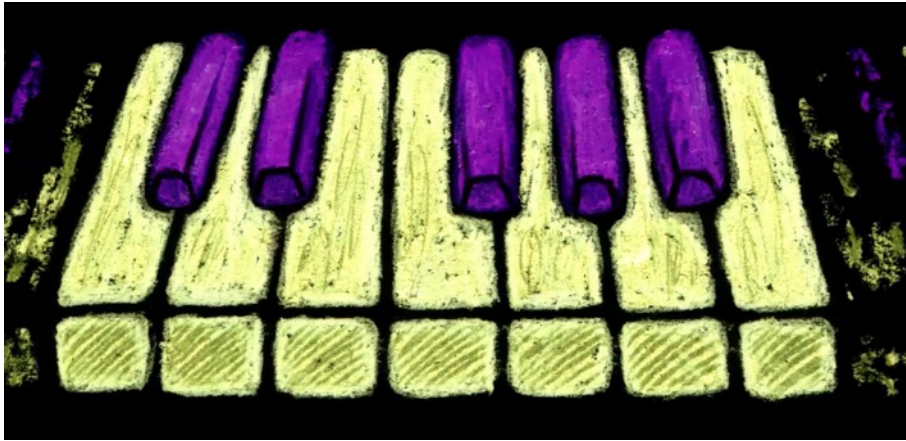




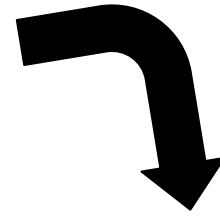
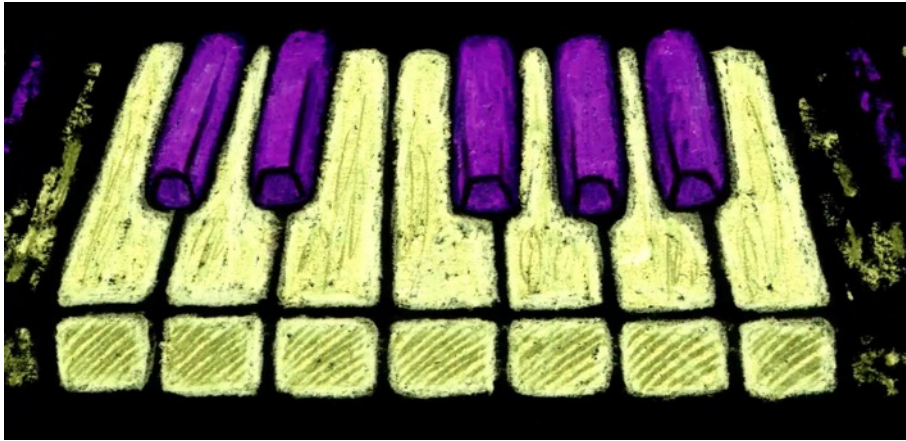
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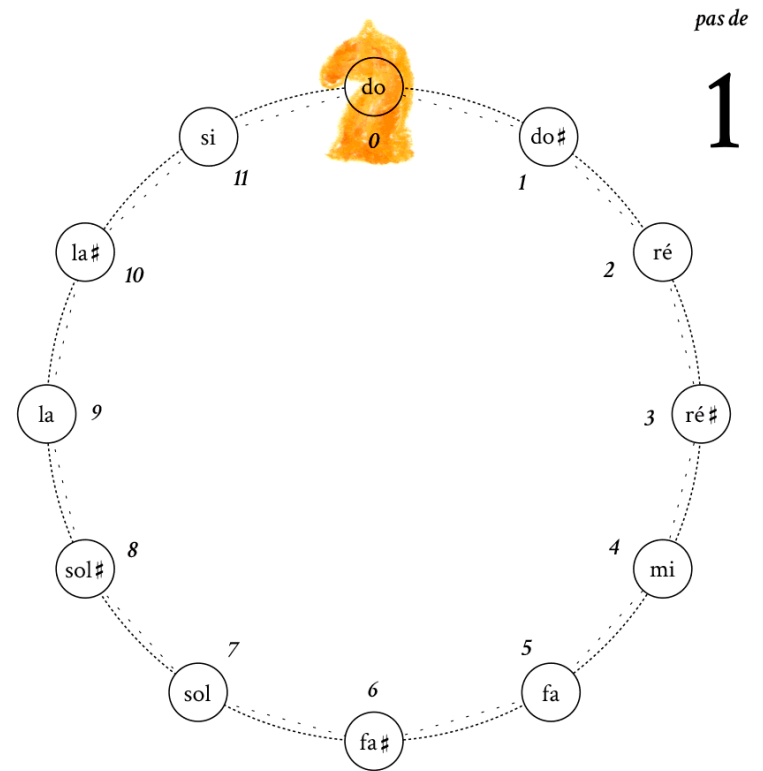
Cerchiamo gli intervalli generatori



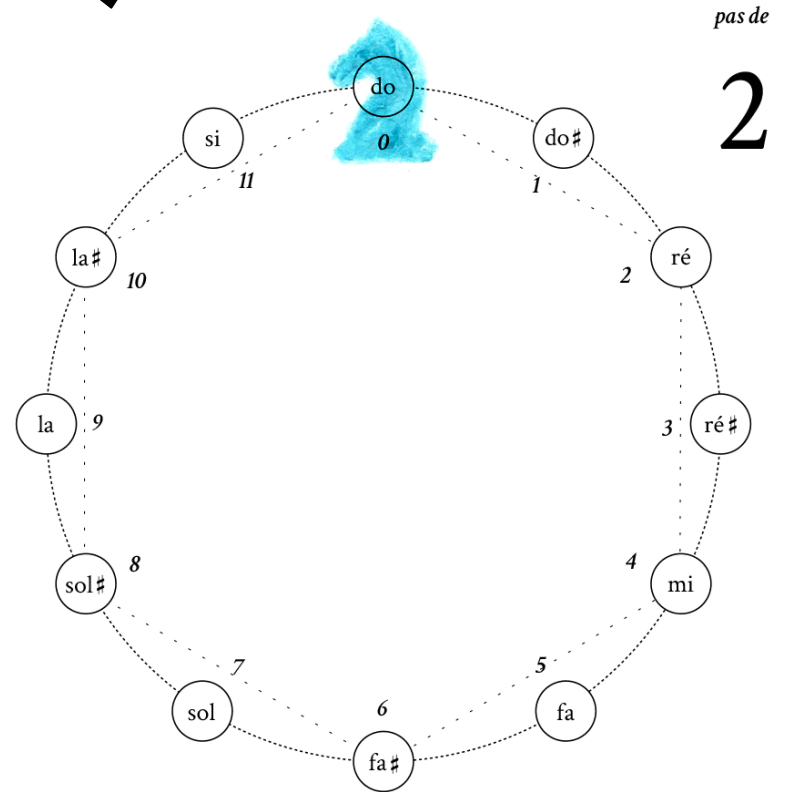
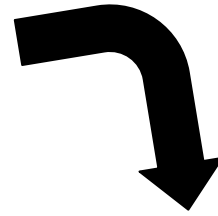
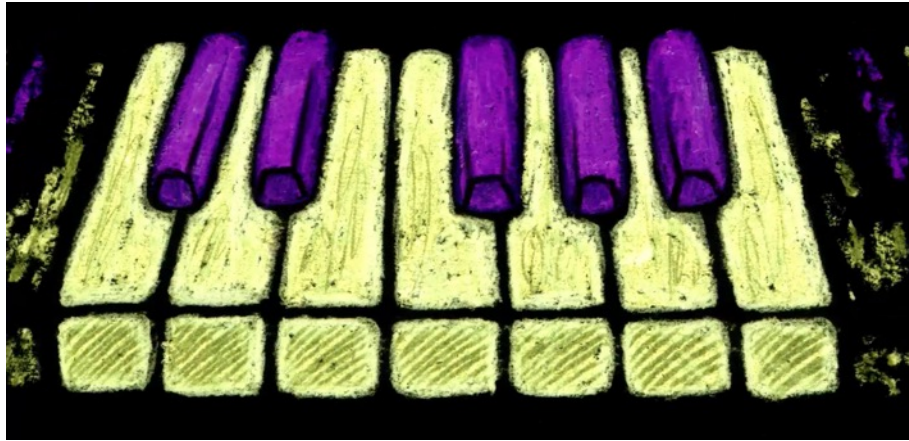
Cerchiamo gli intervalli generatori



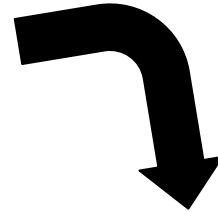
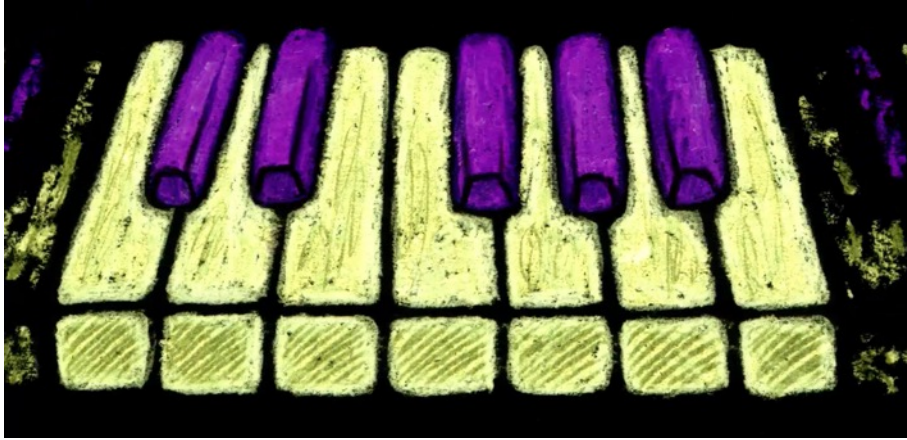
YES!



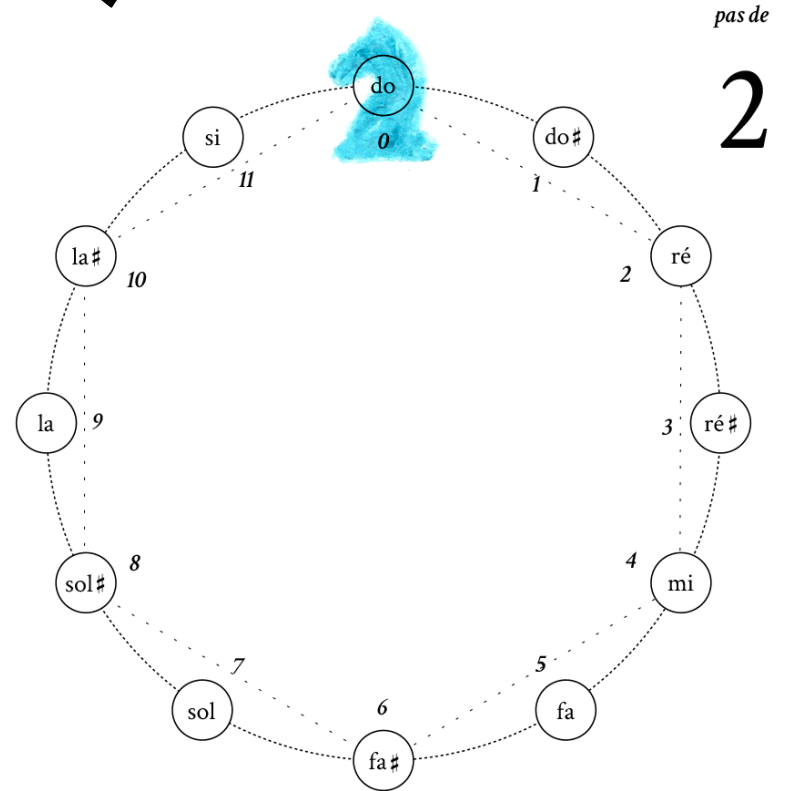
Cerchiamo gli intervalli generatori



Cerchiamo gli intervalli generatori

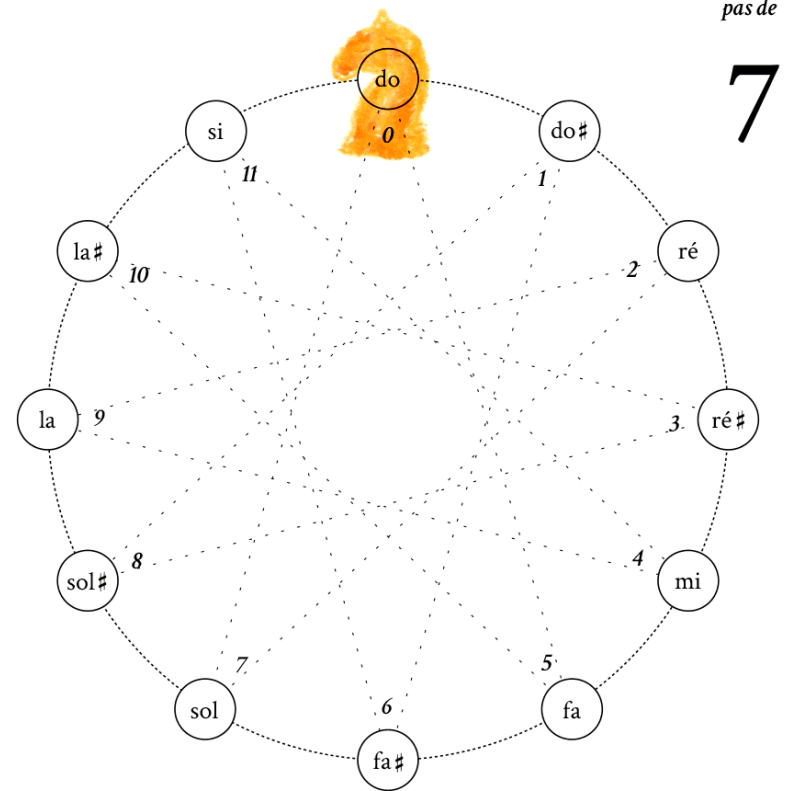
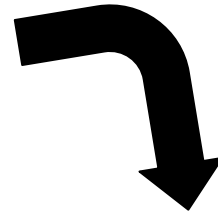
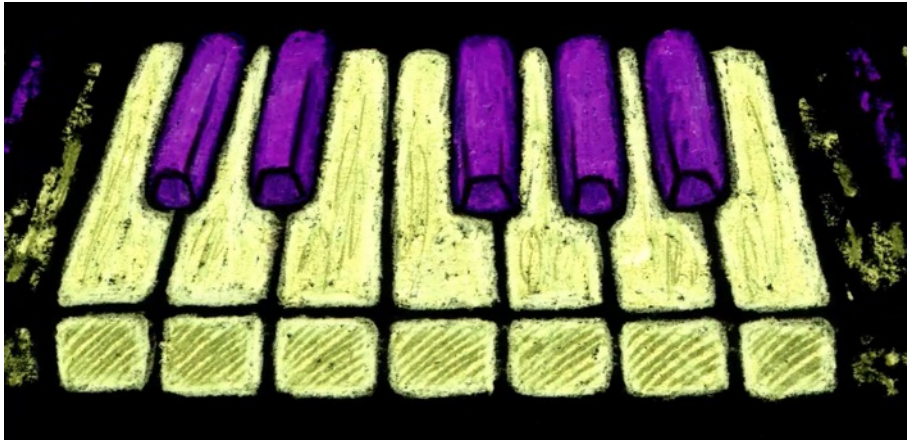


NO!

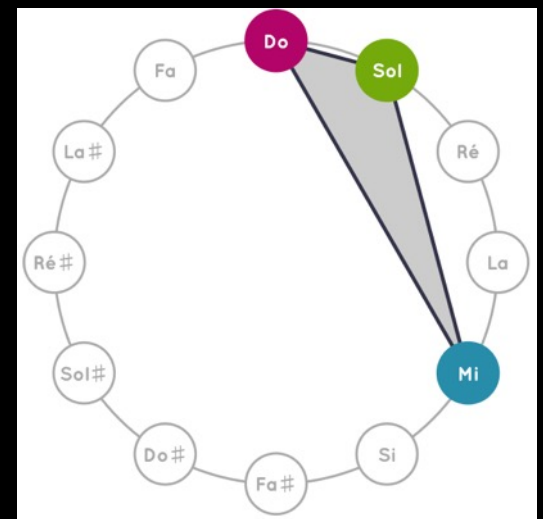
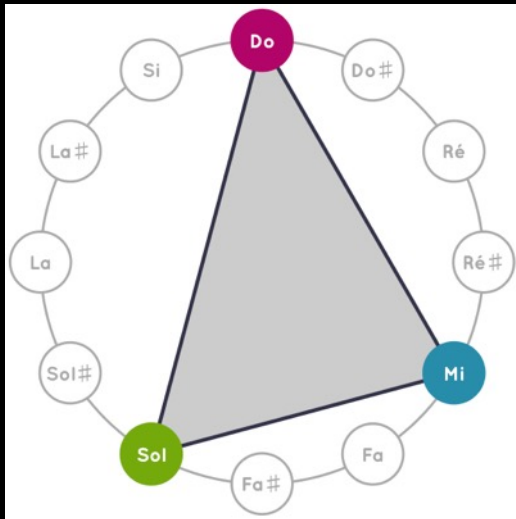


E se prendo l'intervallo di quinta?

Il ciclo delle quinte!

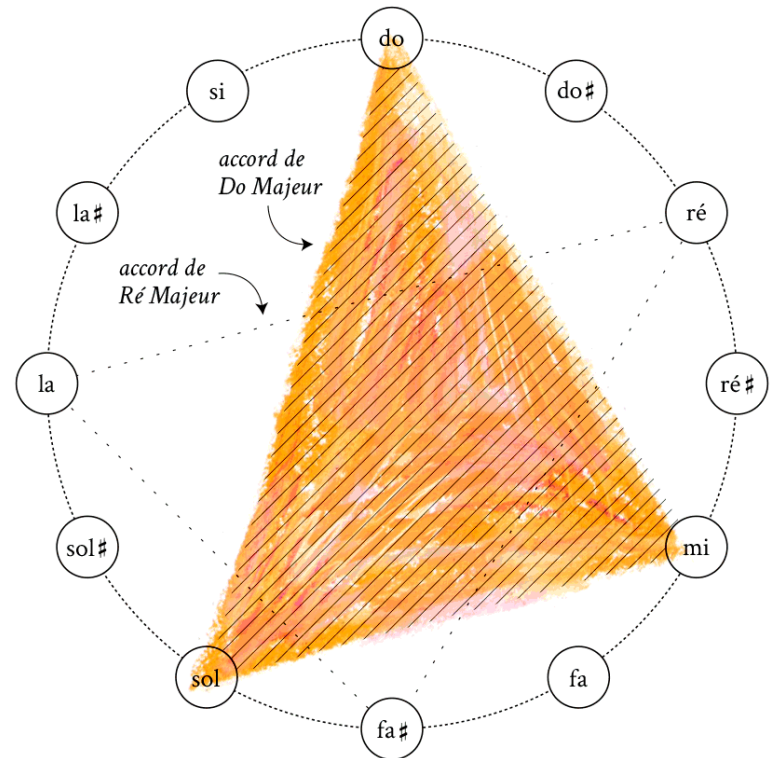
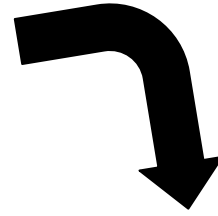
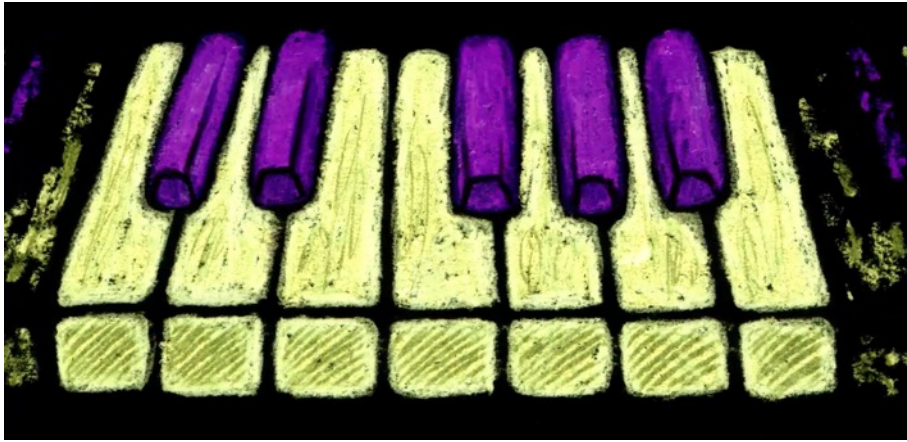


Giochiamo con la rappresentazione circolare

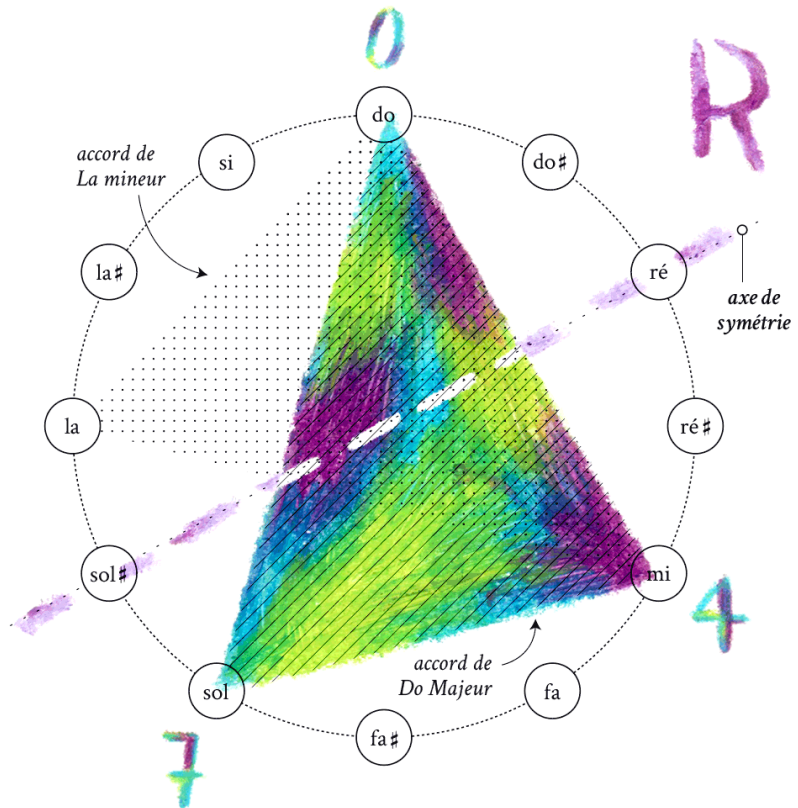
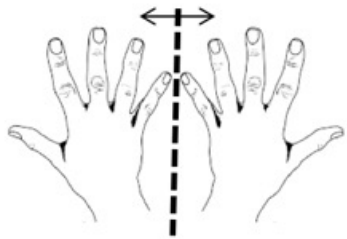
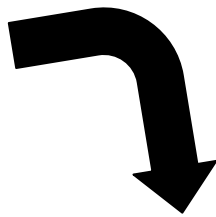
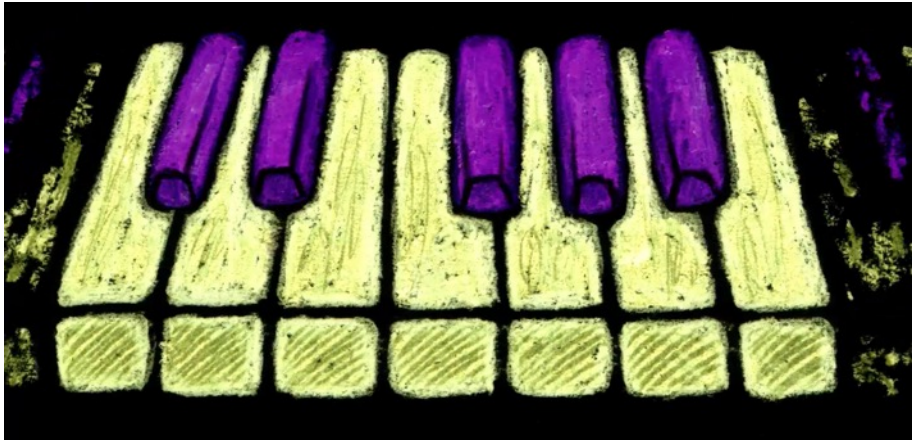


Riepilogando...

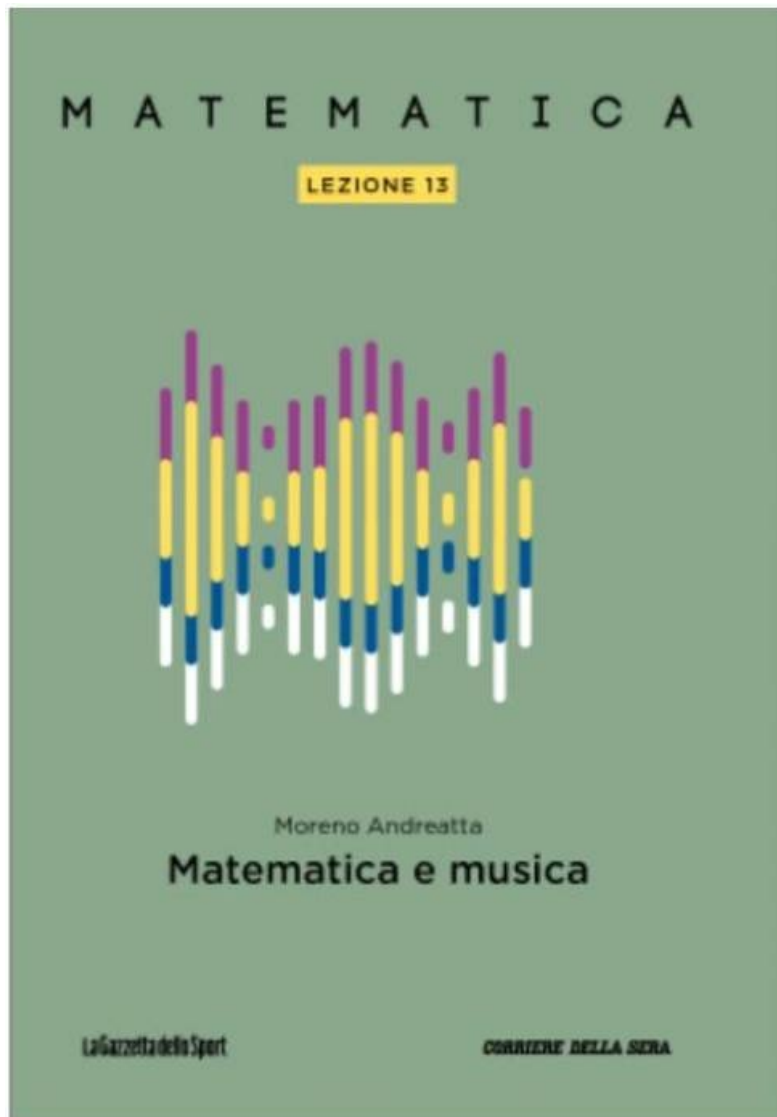
Le trasposizioni musicali sono delle rotazioni



Le inversioni sono delle simmetrie assiali



Come visualizzare le simmetrie nel piano (= spazio a due dimensioni)?



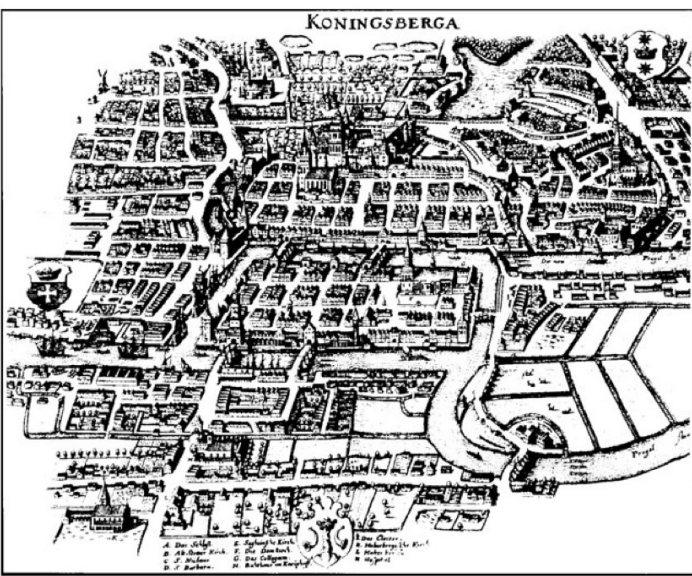
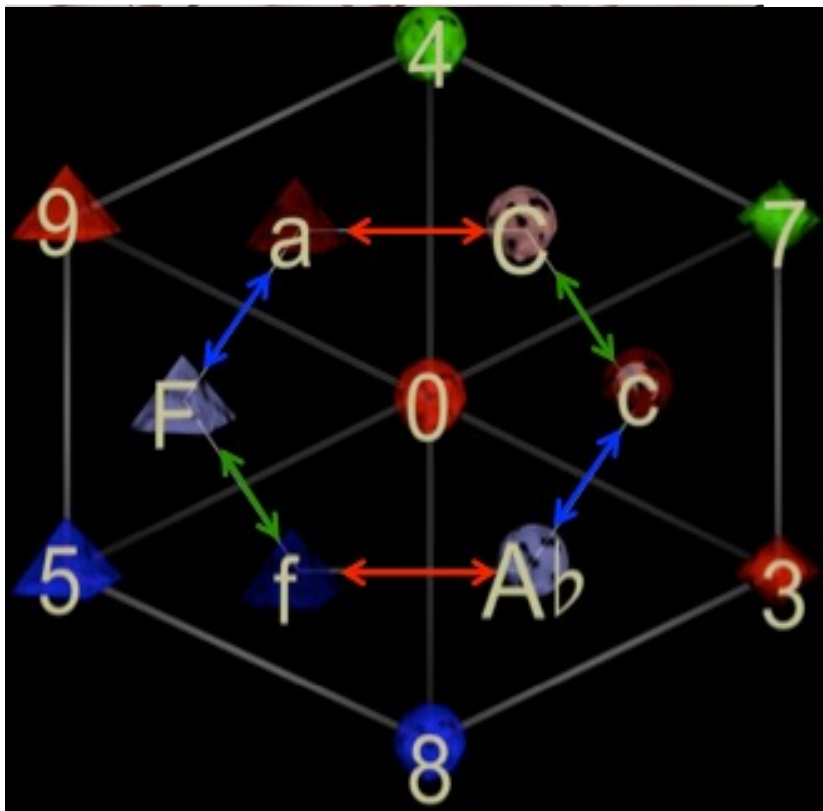
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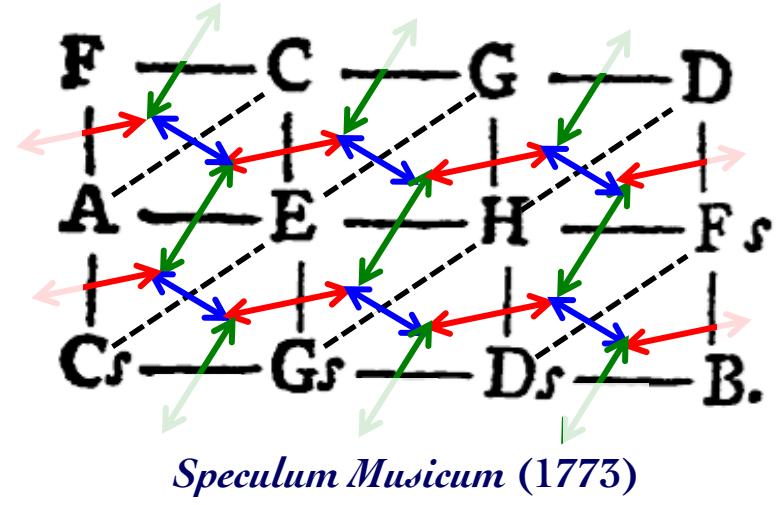
Eulero e la teoria die grafi



Il Tonnetz (o alveare musicale)

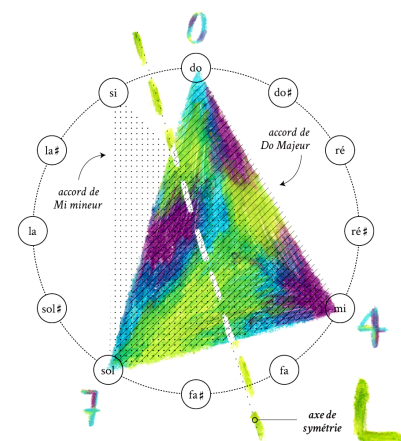
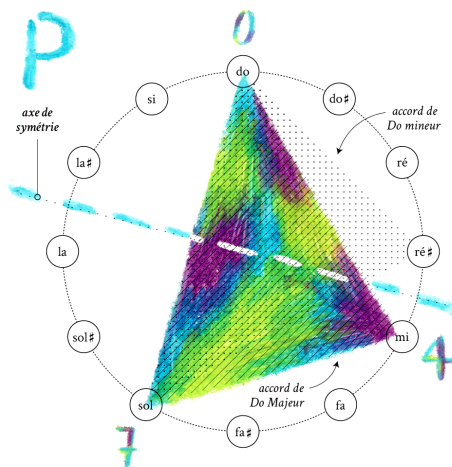
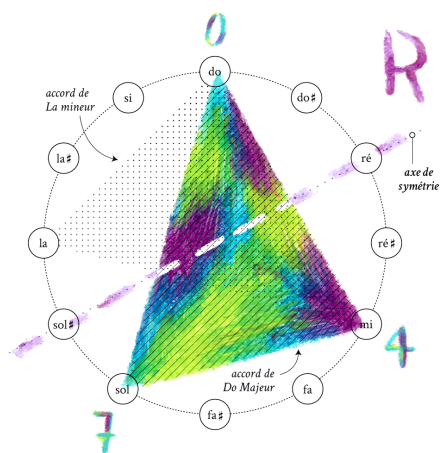
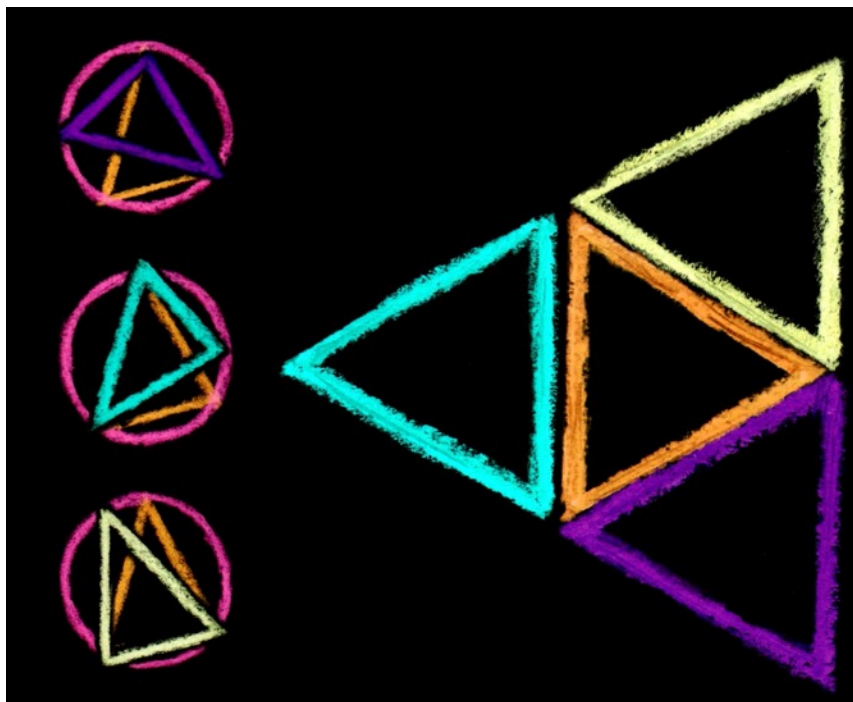


Leonardo Eulero

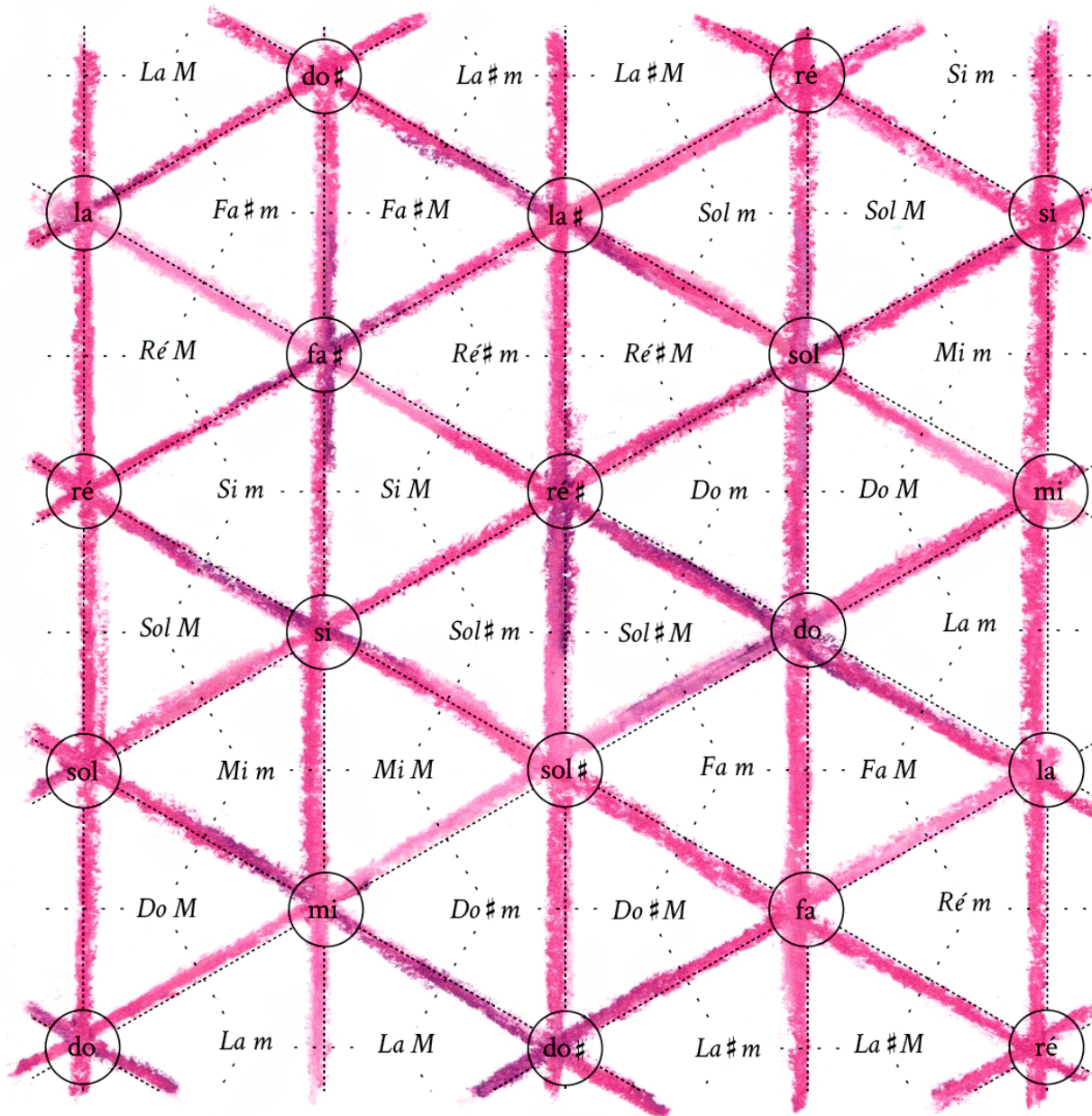


Speculum Musicum (1773)

Il Tonnetz e le tre simmetrie planari



Le due facce del Tonnetz



Harmonic Progressions

In Paolo Conte

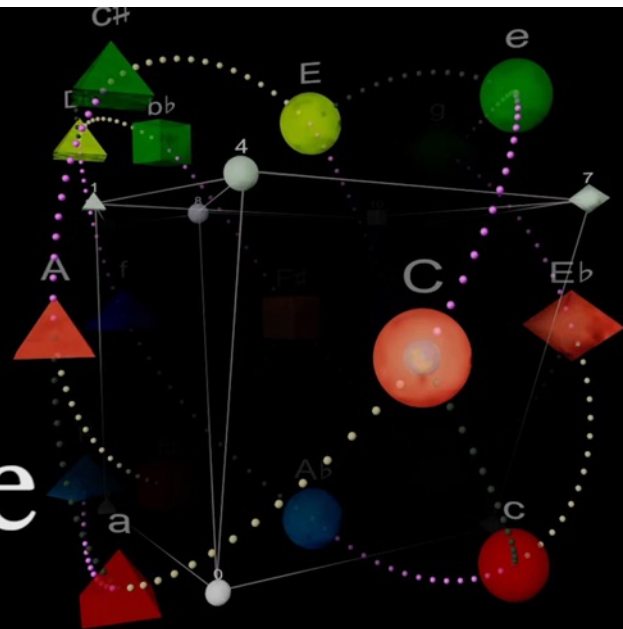
Sotto le Stelle del Jazz



Supervision Moreno Andreatta
Modelisation Gilles Baroin 2016

Beethoven and the Hypersphere

(and the Tonnetz)



Gilles Baroin 2016
www.MatheMusic.net

➔ www.mathemusic.net



Gilles Baroin

Hamiltonian Song
www.MatheMusic.net

Le Blé en Herbe

Lyrics, music and performance by Polo
Mathematical supervision by Moreno Andreatta
Hyperspheres and animation by Gilles Baroin, 2016

➔ www.mathemusic.net



Polo Lamy



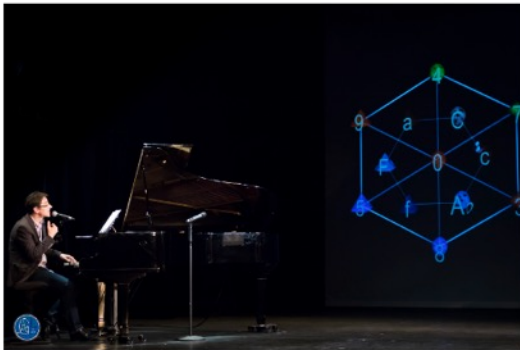
Gilles Baroin

www.morenoandreatta.com

- Music and Poetry
- Music and Maths
- Improvisation
- Software

Music at the interface with mathematics and poetry

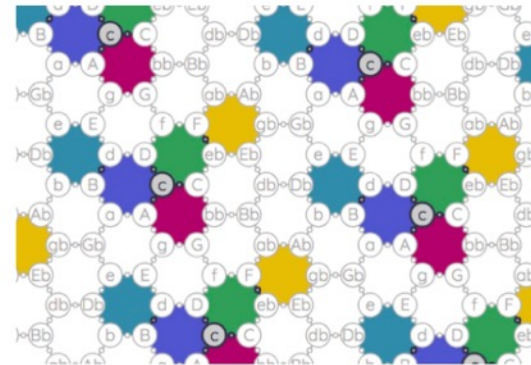
Music and maths

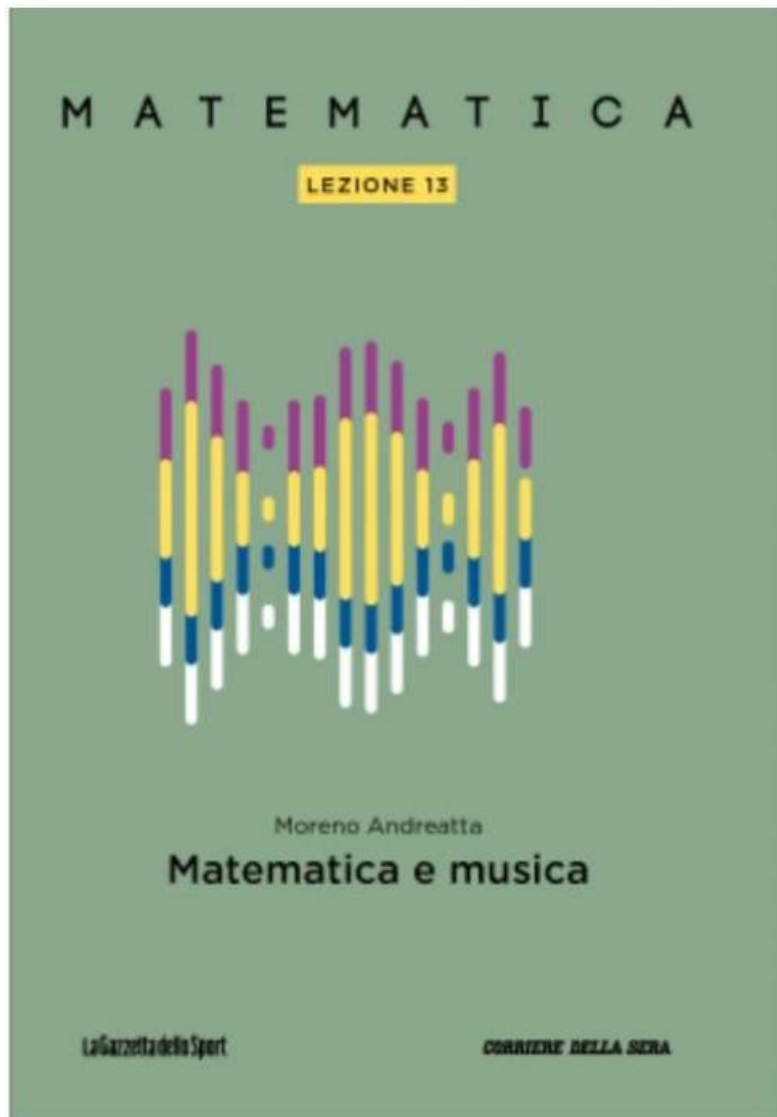


Improvisation



Software

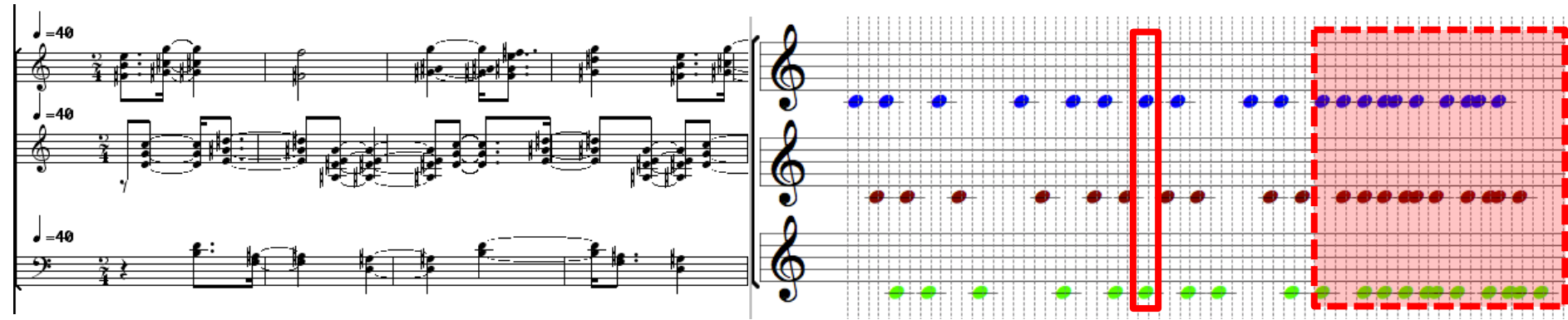




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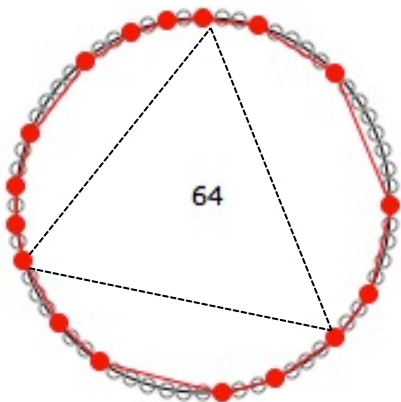
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Canoni ritmici a mosaico



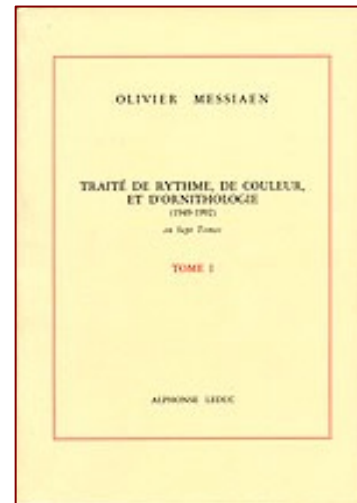
Harawi (1945)

riduzione ritmica

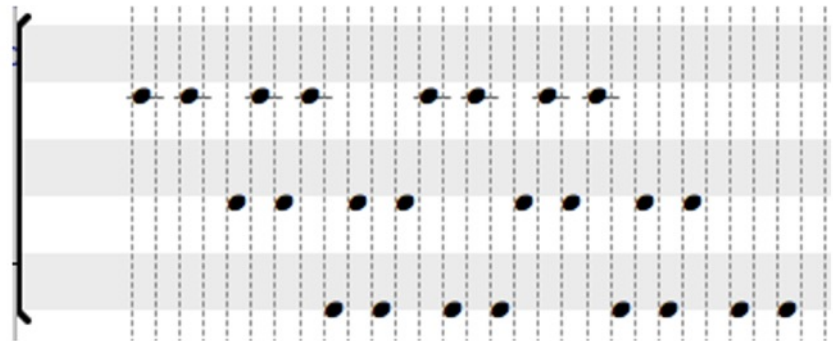
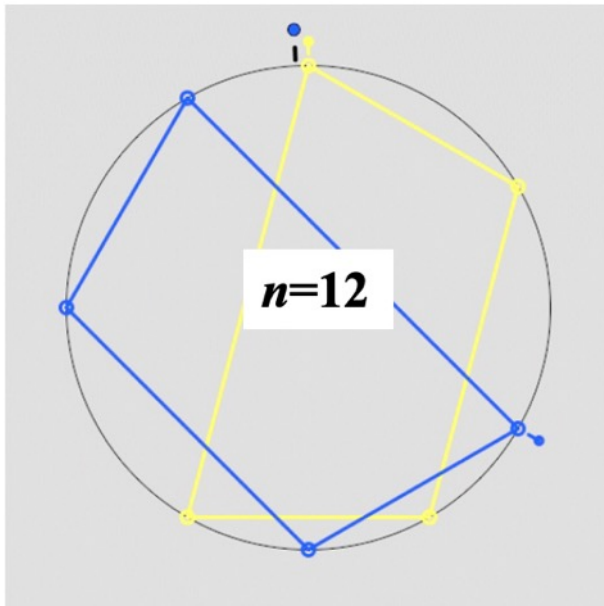
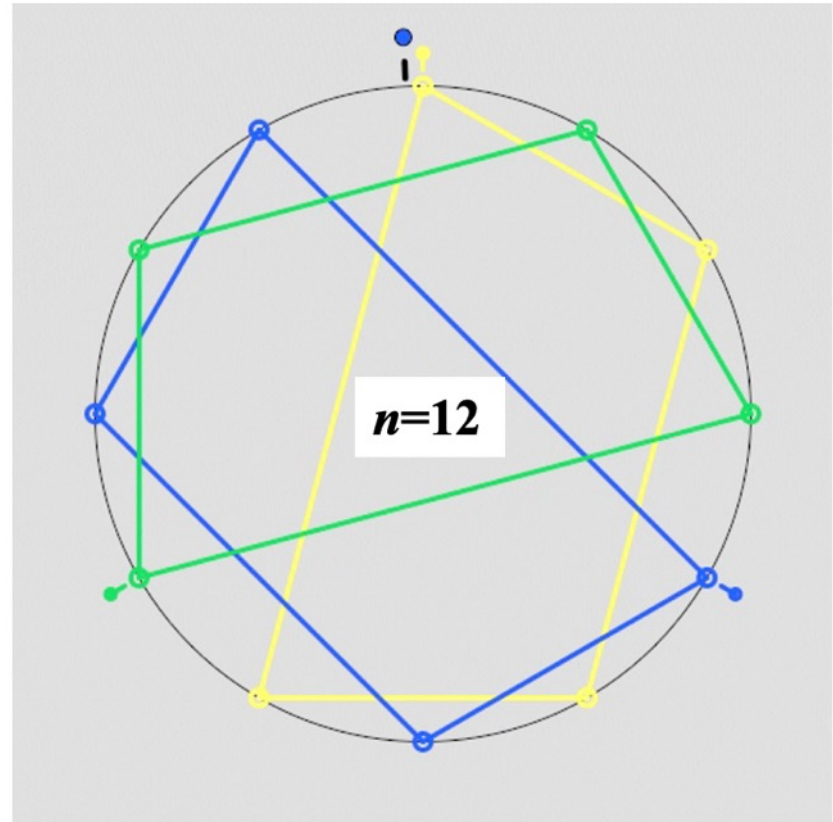
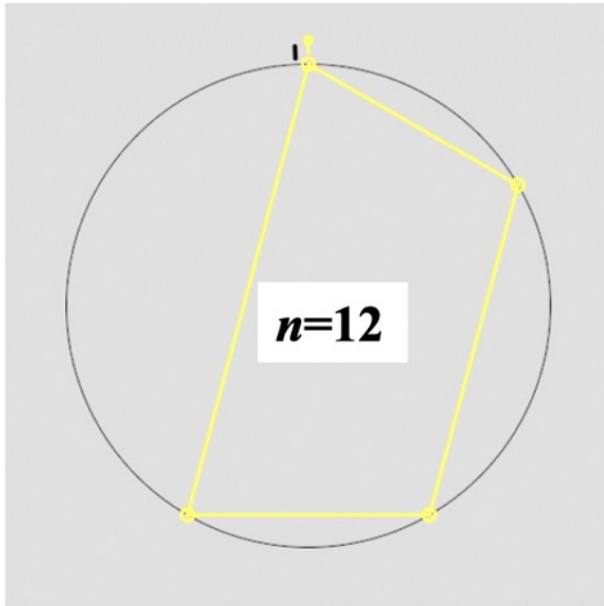


«...risulta da tutto ciò che le diverse sonorità si mescolano o si oppongono in maniera diversa, mai nello stesso momento né nello stesso istante temporale. Si tratta di un disordine organizzato »

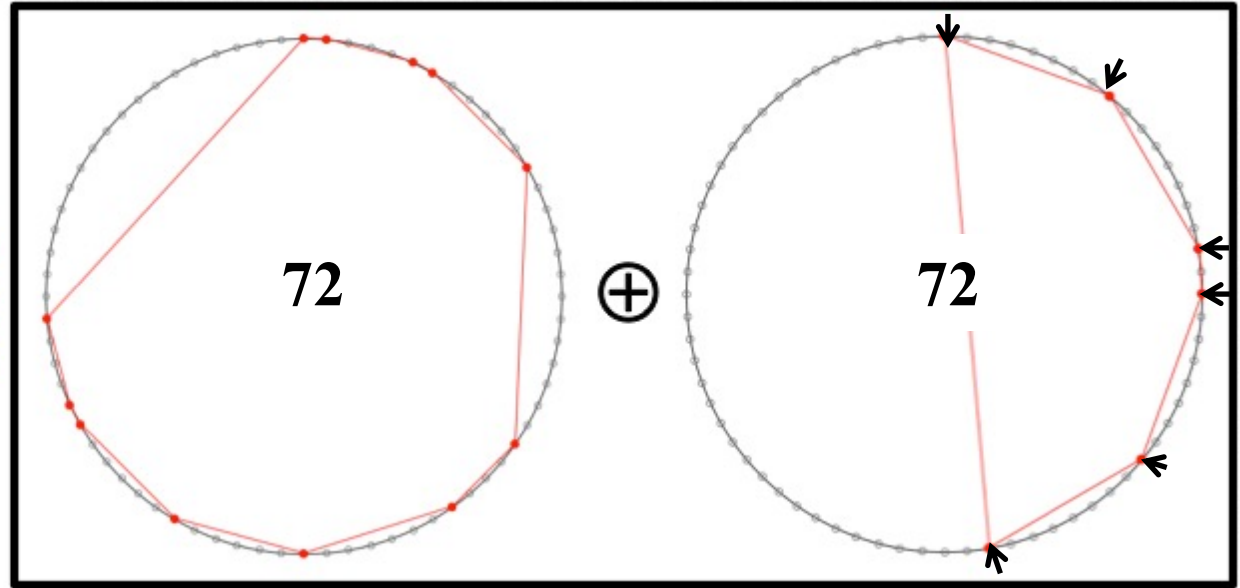
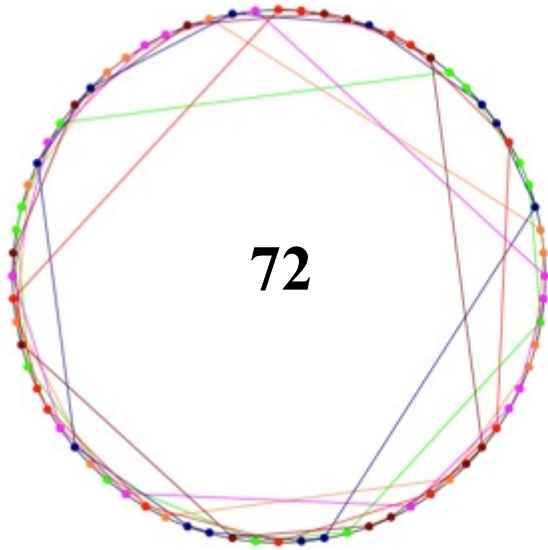
O. Messiaen: *Traité de Rythme, de Couleur et d'Ornithologie*, tome 2, Alphonse Leduc, 1992.



Un esempio di canone ritmico a mosaico



Canoni ritmici a mosaico aperiodici



Anatol Vieru



Dan Vuza

Musical score with five staves and rhythmic annotations.

Il cerchio ritmico e le sue rotazioni

CLAPPING MUSIC

FOR TWO PERFORMERS

$\text{♩} = 144-168$

CLAP 1
CLAP 2

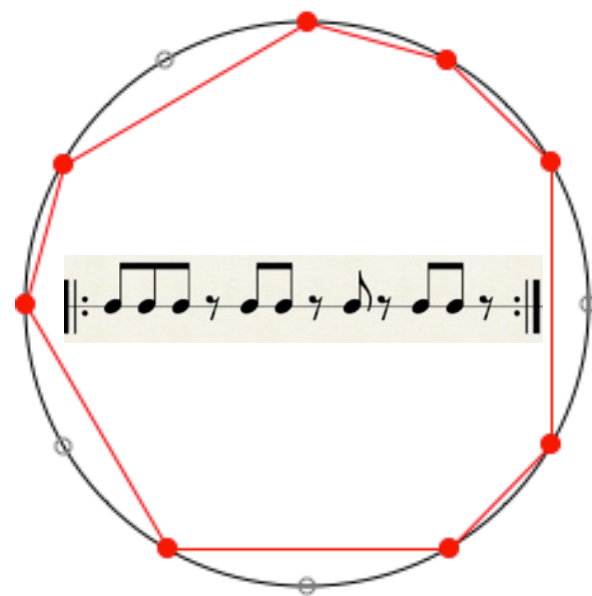
Repeat bar 1, then end.

The performance begins and ends with both performers in unison at bar 1. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer's part does not change, it is up to the second performer to rotate from one bar to the next. The second performer should try to keep his or her downbeat where it is written, i.e. on the first beat of each measure (not on the first beat of the group of three claps), so that his downbeat always falls on a new beat of his or her changing pattern.

The choice of a particular clapping sound, i.e. with cupped or flat hands is left up to the performers. Whichever timbre is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall resulting pattern.

Clapping Music (1972)

*New York 12/72
re-copied 1/78*



Il cerchio ritmico e le sue rotazioni

CLAPPING MUSIC

FOR TWO PERFORMERS

$\text{♩} = 144-168$

CLAP 1
CLAP 2

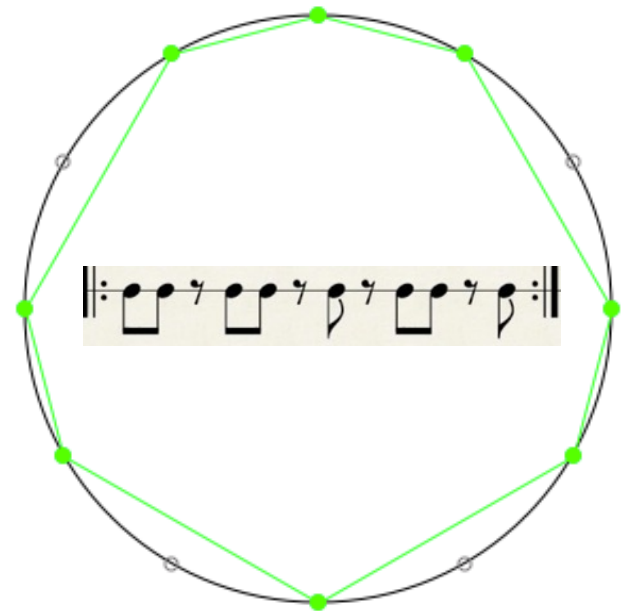
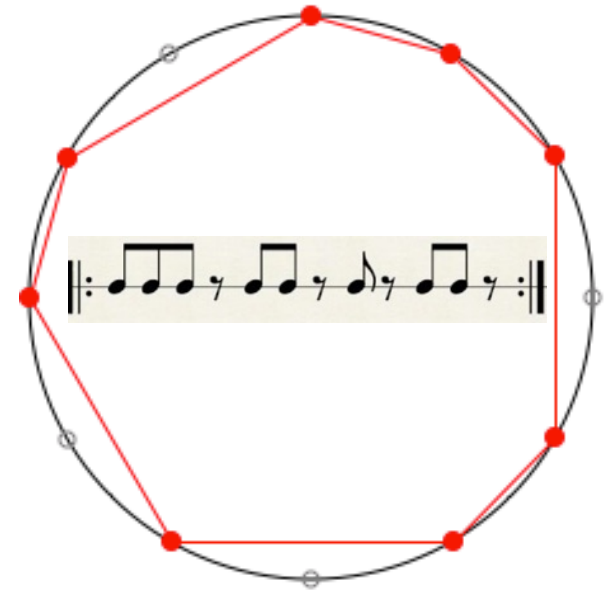
Repeat bar ①, then end.

The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer's part does not change, it is up to the second performer to move from one bar to the next. The second performer should try to keep his or her downbeat where it is written, i.e. on the first beat of each measure (not on the first beat of the group of three claps), so that his downbeat always falls on a new beat of his or her changing pattern.

The choice of a particular clapping sound, i.e. with cupped or flat hands is left up to the performers. Whichever timbre is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall resulting pattern.

Clapping Music (1972)

New York 12/72
re-copied 1/78



L'application web RHYTHM CIRCLE

<https://rhythm-circle.com>

en
fr

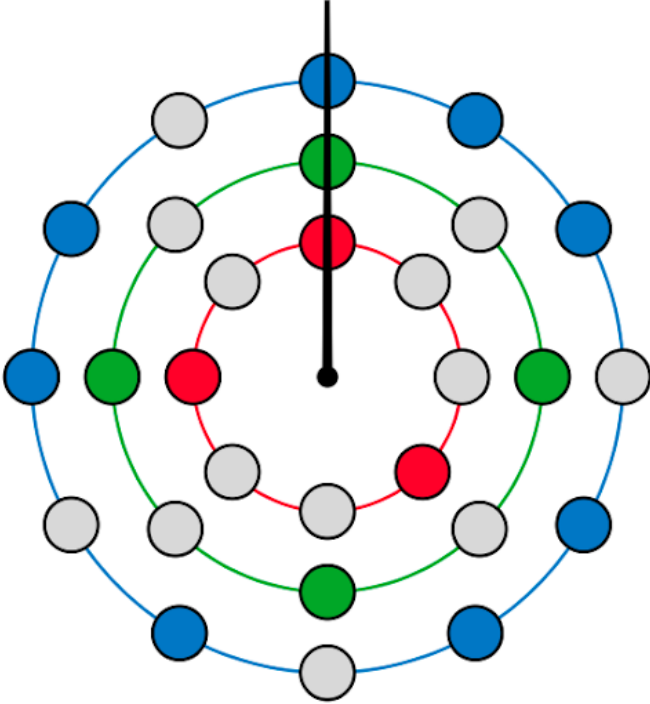
THE RHYTHM CIRCLE

Play

Tempo (mpm): 30


Reset rhythms

DEMO




Tresillo	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Son	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Shiko	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Soukous	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Rumba	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Bossa Nova	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Gahu	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Samba	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Fume-fume	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Bembé	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Steve Reich	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Basic 1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Basic 2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Basic 3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Basic 4	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>

Export

Snare Drum 


Subdivisions: 8

Rotation: - +

Kick Drum 

Subdivisions: 8

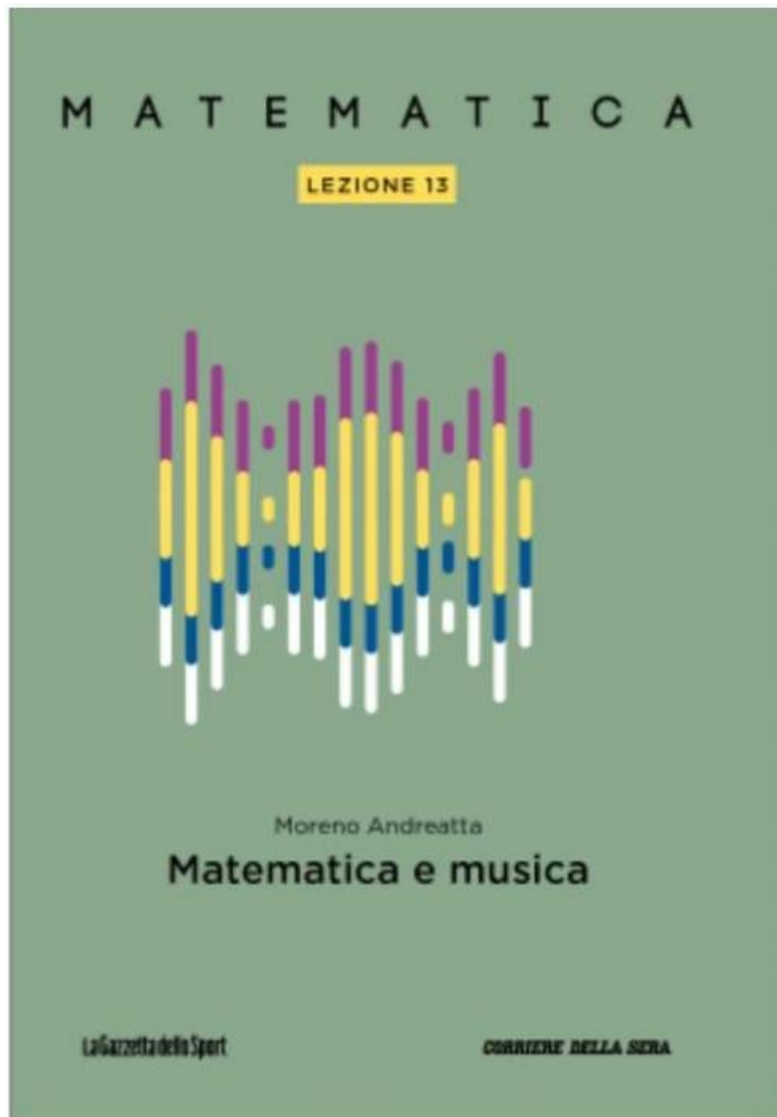
Rotation: - +

Hi Hat 

Subdivisions: 12

Rotation: - +

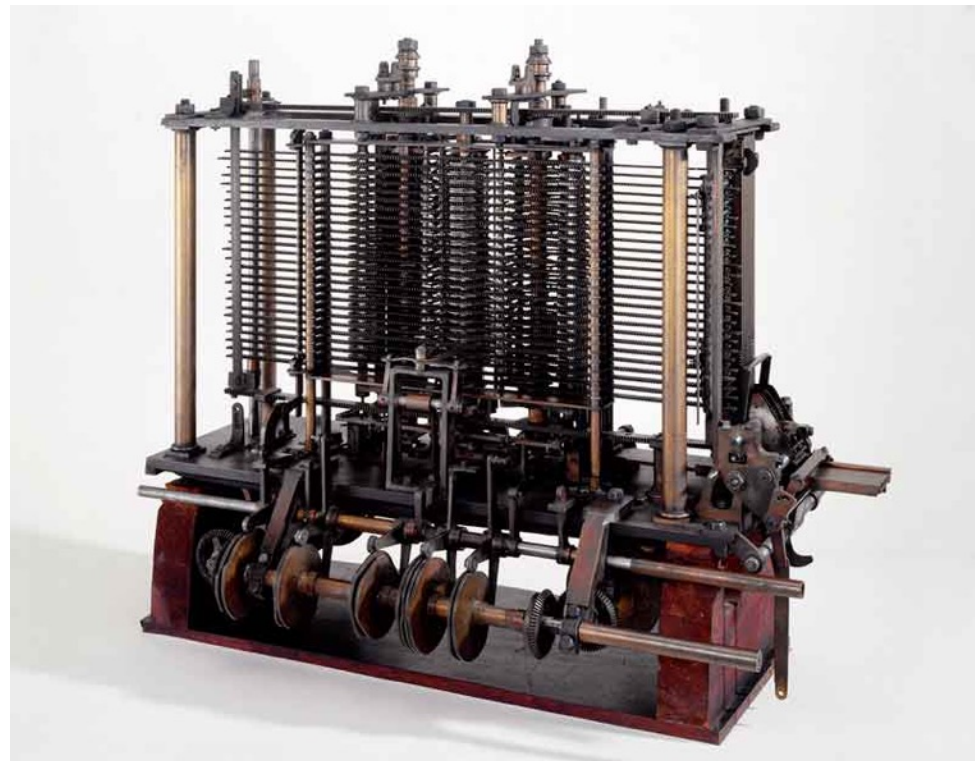
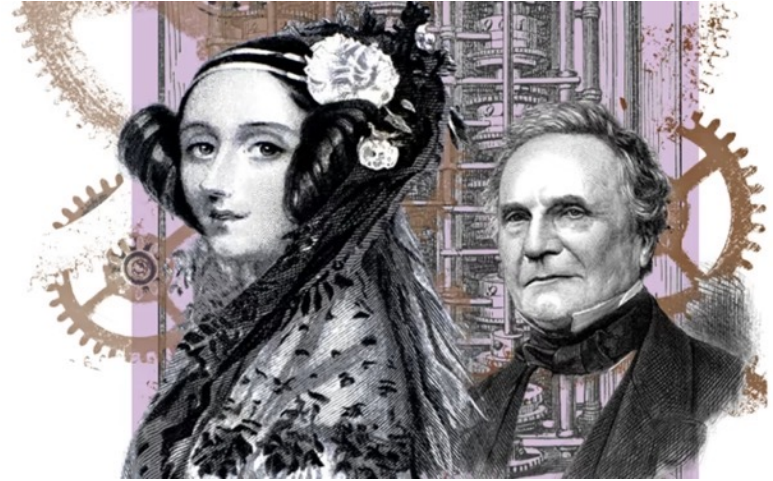
Credits



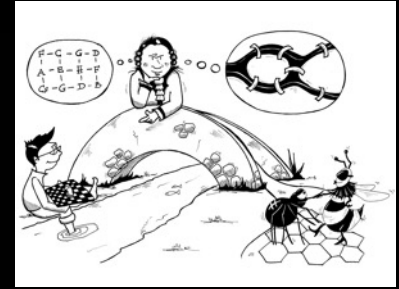
Sommario

- Introduzione
- **MATEMATICA E MUSICA**
 - Qualche parola per cominciare
 - Quando la musica precede la matematica...
 - Formalizzazione algebrica delle strutture musicali
 - Il Tonnetz e le sue generalizzazioni
 - Da Minkowski a Fuglede: canoni ritmici a mosaico e congetture aperte
- **I MAESTRI DELLA STORIA DELLA MATEMATICA**
 - **Babbage e Lovelace**
- **GIOCHI MATEMATICI**
 - Matemusica
 - Soluzioni
- **ESERCIZI**
 - Soluzioni

Babbage, Lovelace e la machina analitica



Un film pedagogico (in francese)



Musique et mathématiques, histoire d'une rencontre
L'incontro fra Mme la Circonferenza e Monsieur il Triangolo



Grazie per la vostra attenzione

