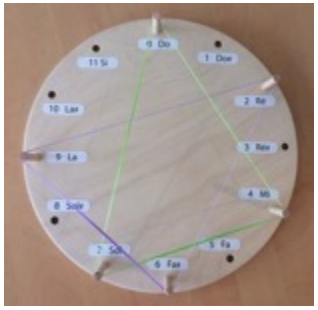
Centre de culture numérique Université de Strasbourg

Les maths dans la musique…
…la musique des maths !



CCN
Université de Strasbourg
15 novembre 2023

Moreno Andreatta

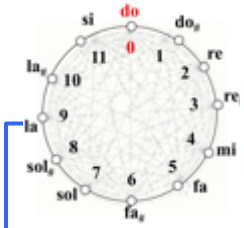
CNRS / IRMA / Université de Strasbourg
CNRS / IRCAM / Sorbonne Université
www.morenoandreatta.com

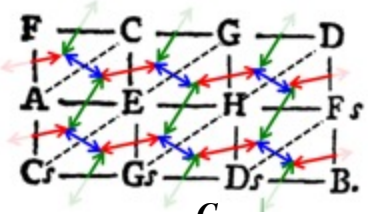
IRMA Institut de Recherche Mathématique Avancée
Université de Strasbourg
cnrs
ircam Centre Pompidou
SORBONNE UNIVERSITÉ

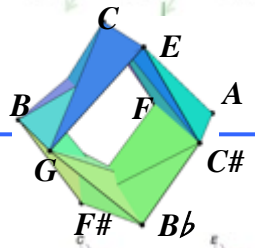
La recherche ‘mathémusicale’ : approches symboliques et signal

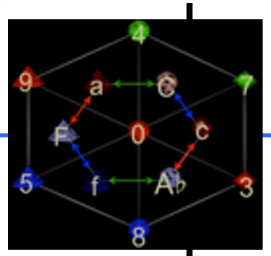
Approches *‘signal’*

Modèles algébriques

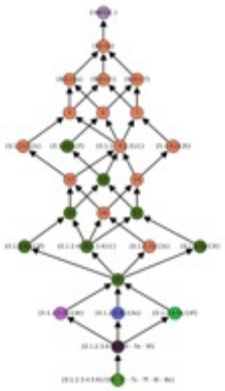


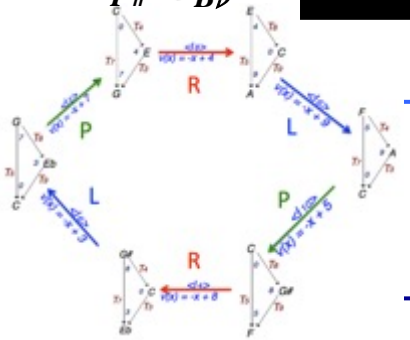


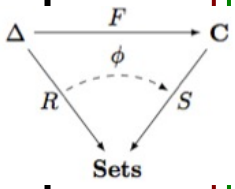




Modèles géométriques



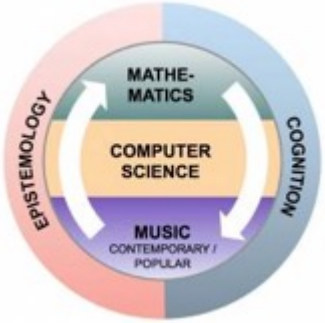




Modèles catégoriels

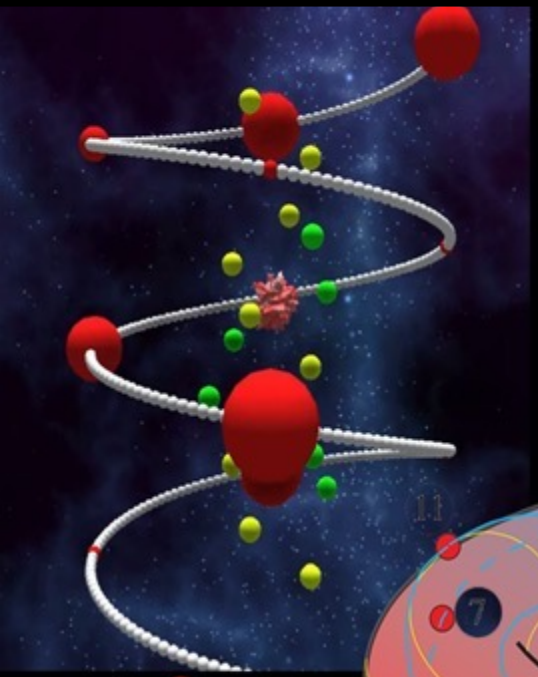
Modèles computationnels

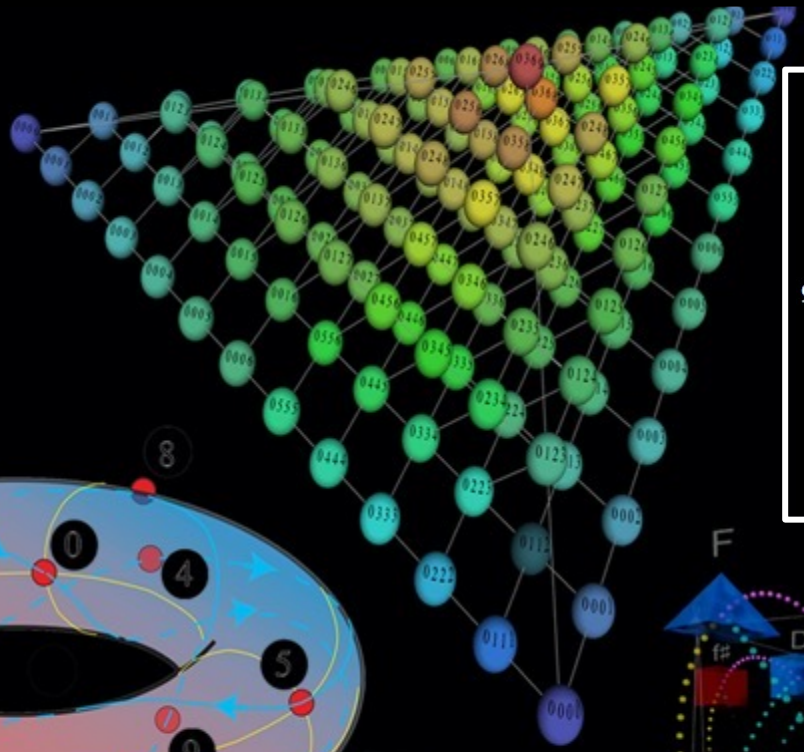
Modèles cognitifs

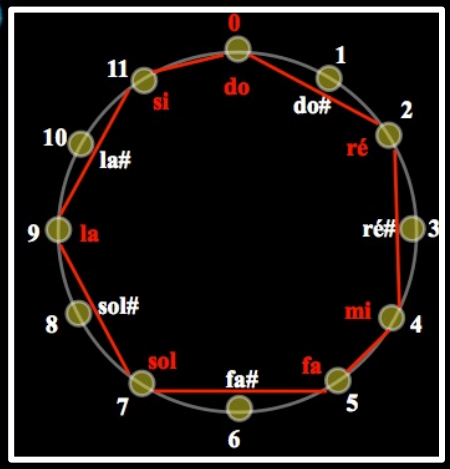


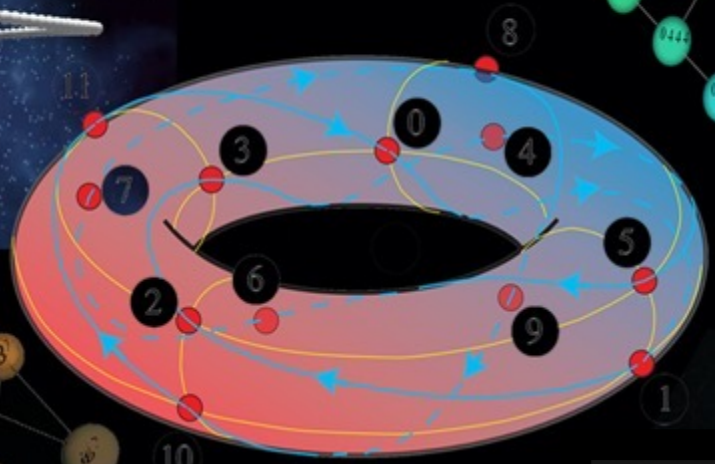
Approches *‘symboliques’*

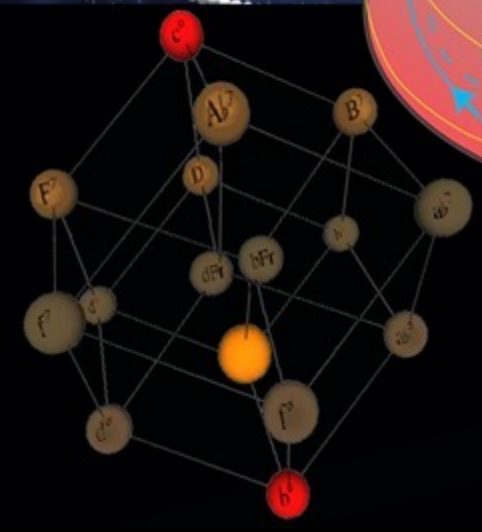
La galaxie des modèles géométriques en musique

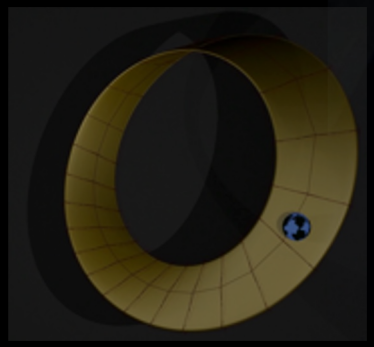


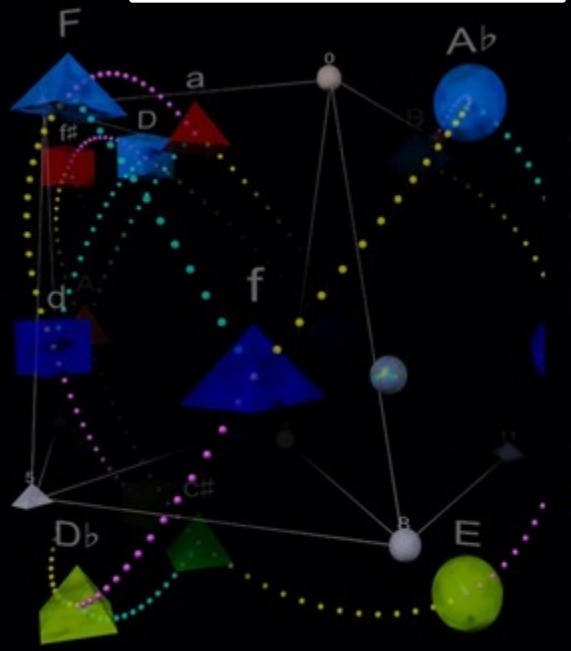




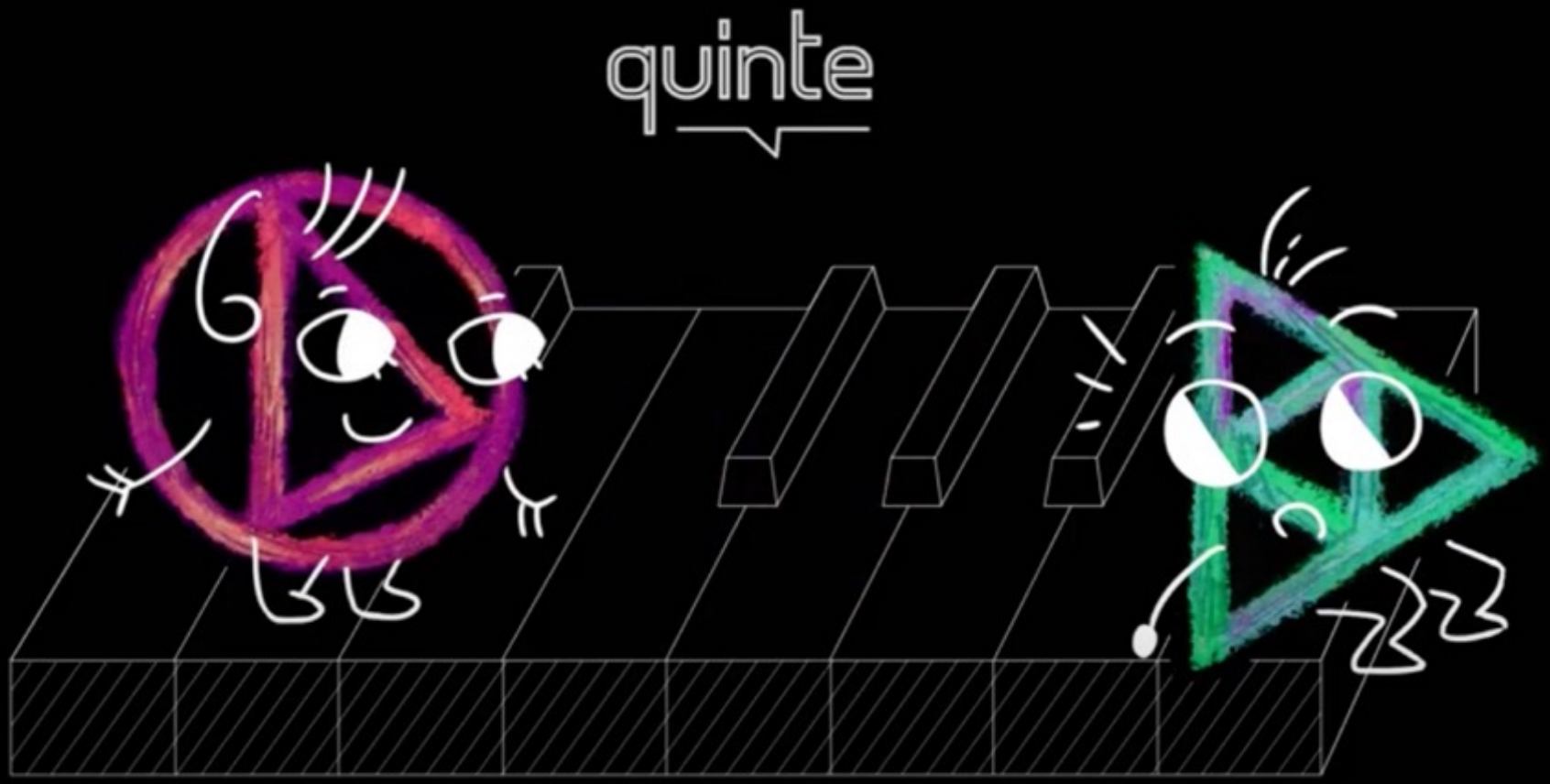








La rencontre entre Madame le Cercle et Monsieur le Triangle



Musique et mathématiques - Histoire d'une rencontre

➔ YouTube + « Musique et mathématiques : histoire d’une rencontre »

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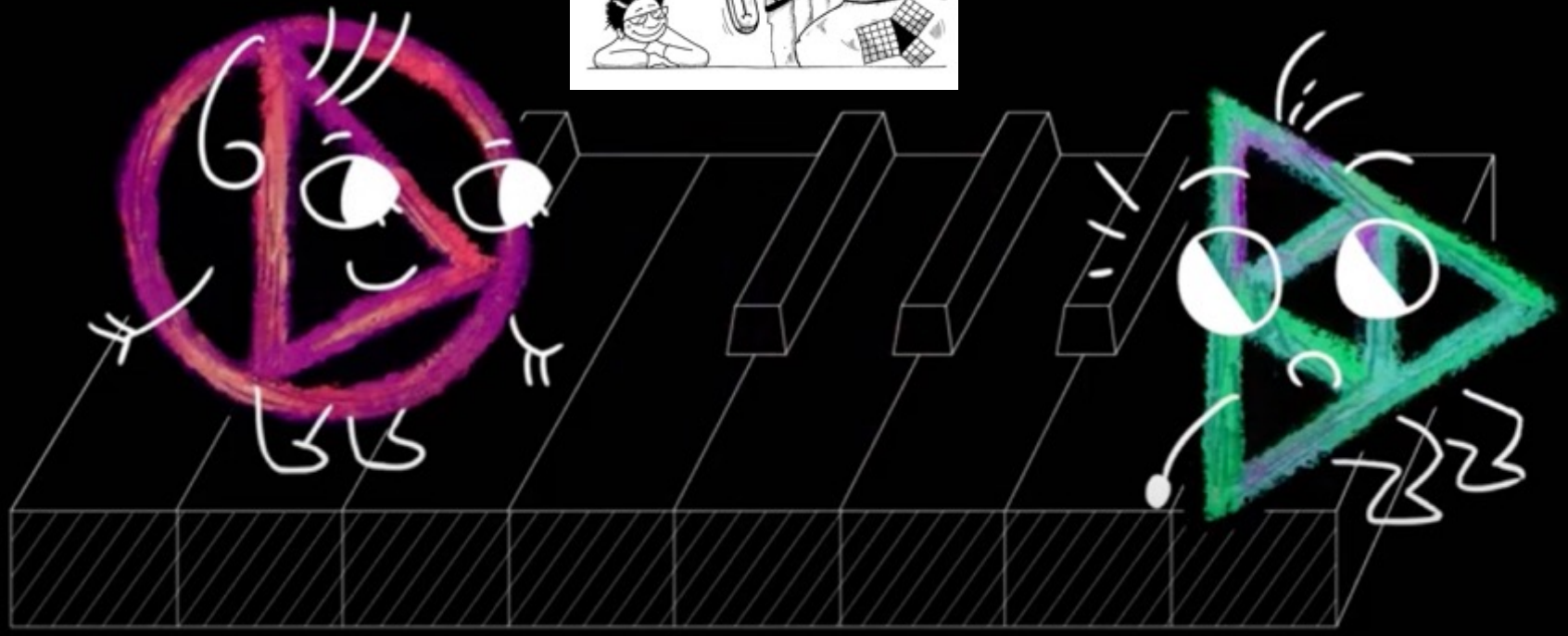

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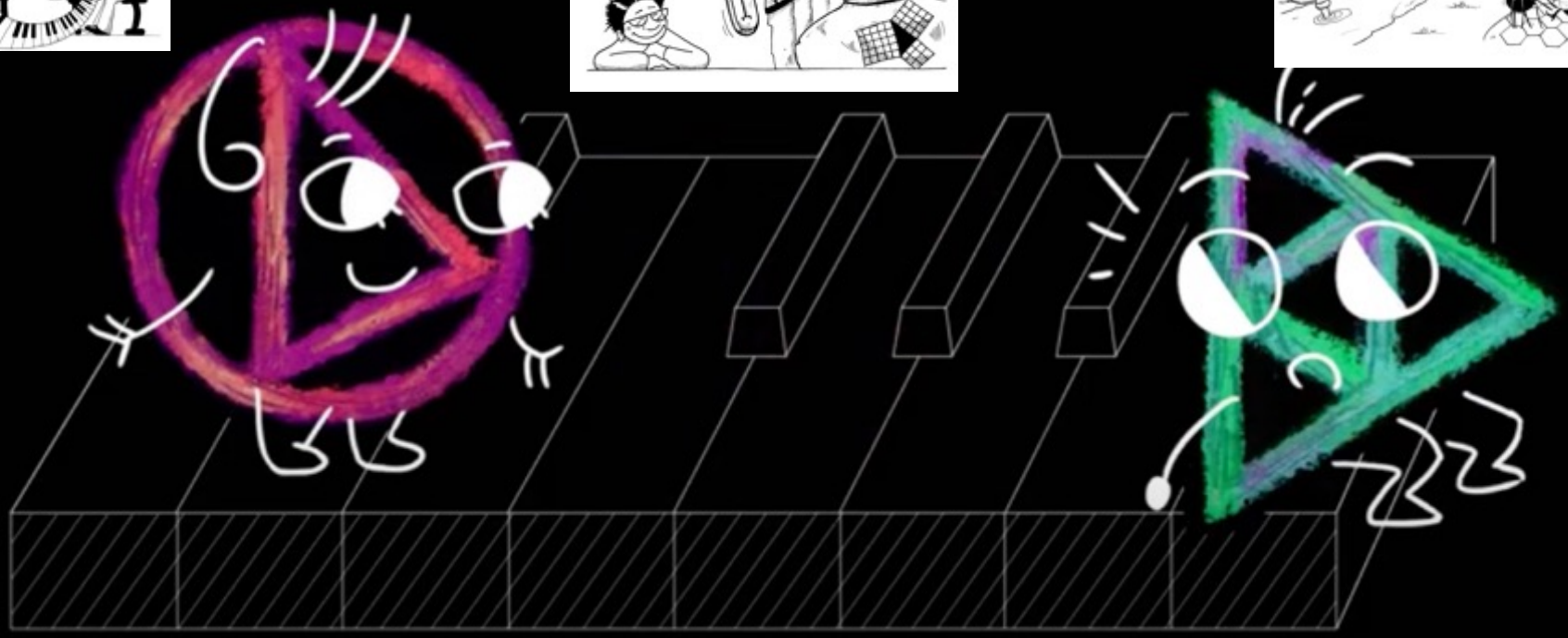

Musique et mathématiques - Histoire d'une rencontre

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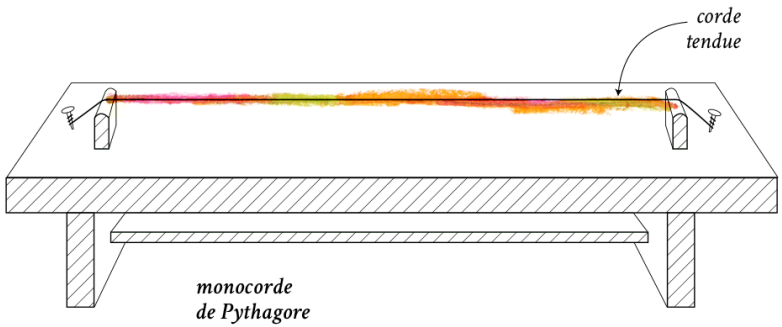
Pythagore et le monocorde

Les intervalles selon le monocorde



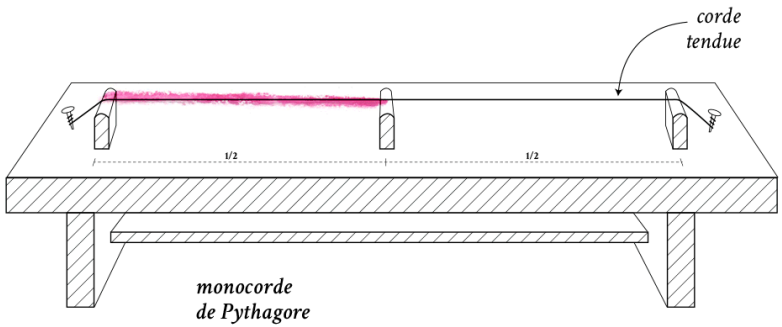
VOICI UNE NOTE

Le monocorde est un instrument contenant une unique corde tendue et une caisse de résonance.

Plus cette corde est courte, plus le son obtenu en la pinçant est aigu, car la **fréquence du son est inversement proportionnelle à la longueur de la corde**.

Ainsi, si le rapport de longueur de la corde est égal à 1 (comme ici), son rapport de fréquence sera égal à 1/1 donc 1 !

Les intervalles selon le monocorde



Longueur de la corde	Rapport de fréquence	Intervalle correspondant
1/2	2	OCTAVE

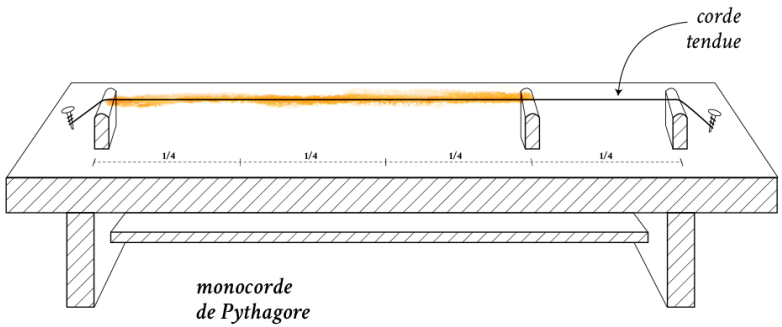
Le monocorde est un instrument contenant une unique corde tendue et une caisse de résonance.

Plus cette corde est courte, plus le son obtenu en la pinçant est aigu, car la **fréquence du son est inversement proportionnelle à la longueur de la corde.**

Ainsi, si le rapport de longueur de la corde est égal à 1 (comme ici), son rapport de fréquence sera égal à 1/1 donc 1 !

Si le rapport de longueur de la corde est égal à 1/2, son rapport de fréquence sera égal à 1/(1/2) donc 2, ce qu'on appelle l'intervalle d'**octave**

Les intervalles selon le monocorde



Longueur de la corde	Rapport de fréquence	Intervalle correspondant
3/4	4/3	QUARTE

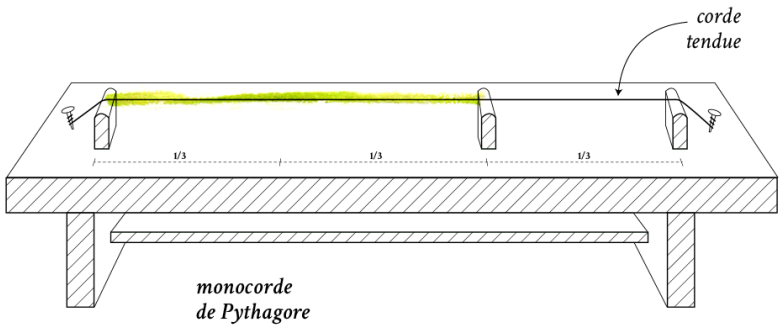
Le monocorde est un instrument contenant une unique corde tendue et une caisse de résonance.

Plus cette corde est courte, plus le son obtenu en la pinçant est aigu, car la **fréquence du son est inversement proportionnelle à la longueur de la corde.**

Ainsi, si le rapport de longueur de la corde est égal à 1 (comme ici), son rapport de fréquence sera égal à 1/1 donc 1 !

Si le rapport de longueur de la corde est égal à 3/4, son rapport de fréquence sera égal à 1/(3/4) donc 4/3, ce qu'on appelle l'intervalle de **quarte.**

Les intervalles selon le monocorde



Longueur de la corde	Rapport de fréquence	Intervalle correspondant
2/3	3/2	QUINTE

Le monocorde est un instrument contenant une unique corde tendue et une caisse de résonance.

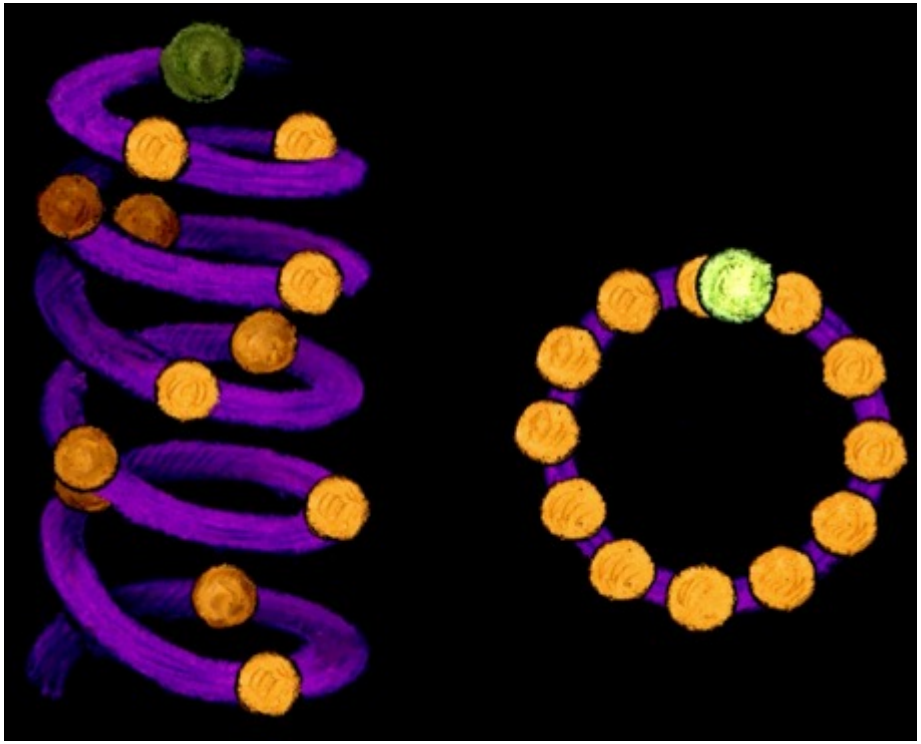
Plus cette corde est courte, plus le son obtenu en la pinçant est aigu, car la **fréquence du son est inversement proportionnelle à la longueur de la corde.**

Ainsi, si le rapport de longueur de la corde est égal à 1 (comme ici), son rapport de fréquence sera égal à 1/1 donc 1 !

Si le rapport de longueur de la corde est égal à 2/3, son rapport de fréquence sera égal à 1/(2/3) donc 3/2, ce qu'on appelle l'intervalle de **quinte.**

Comment construire une gamme à partir de la quinte (juste) ?

De quinte en quinte… à l'infini !



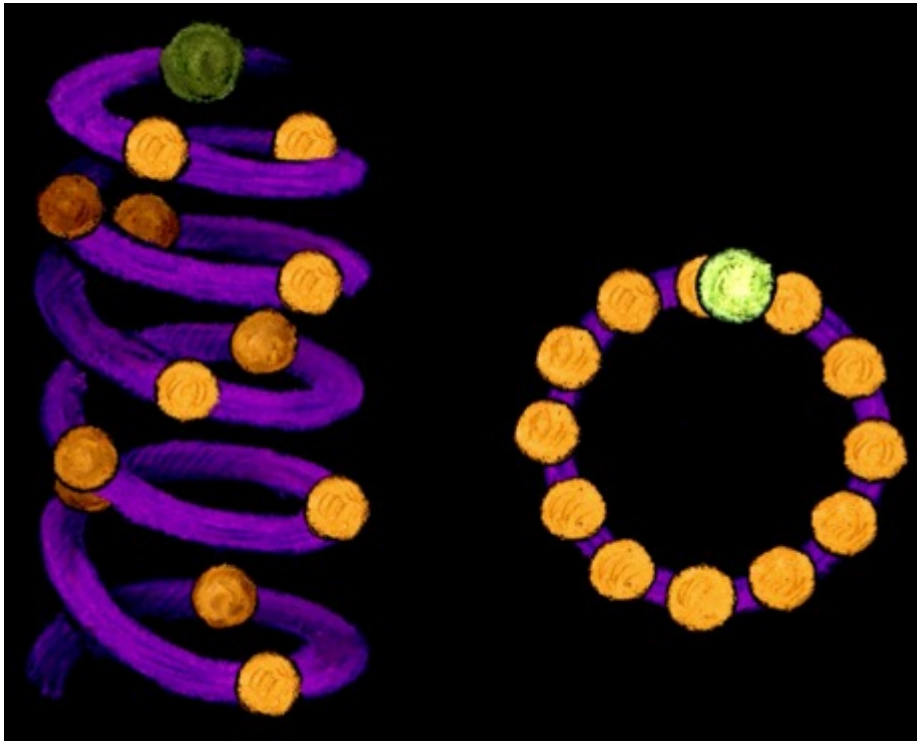
Calculer des puissances et des quotients en lien avec le **cycle des quintes**.

do (grave)	1
ré	$\frac{9}{8}$
mi	$\frac{81}{64}$
fa	

sol	$\frac{3}{2}$
la	$\frac{27}{16}$
si	$\frac{243}{128}$

do (aïgu)	2
ré	
mi	
fa	

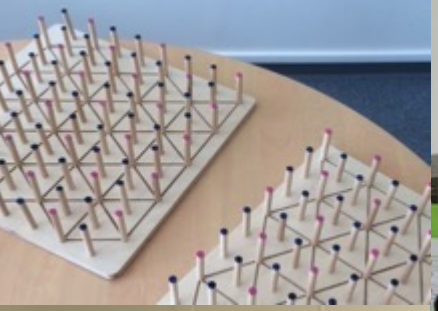
De quinte en quinte… à l'infini !



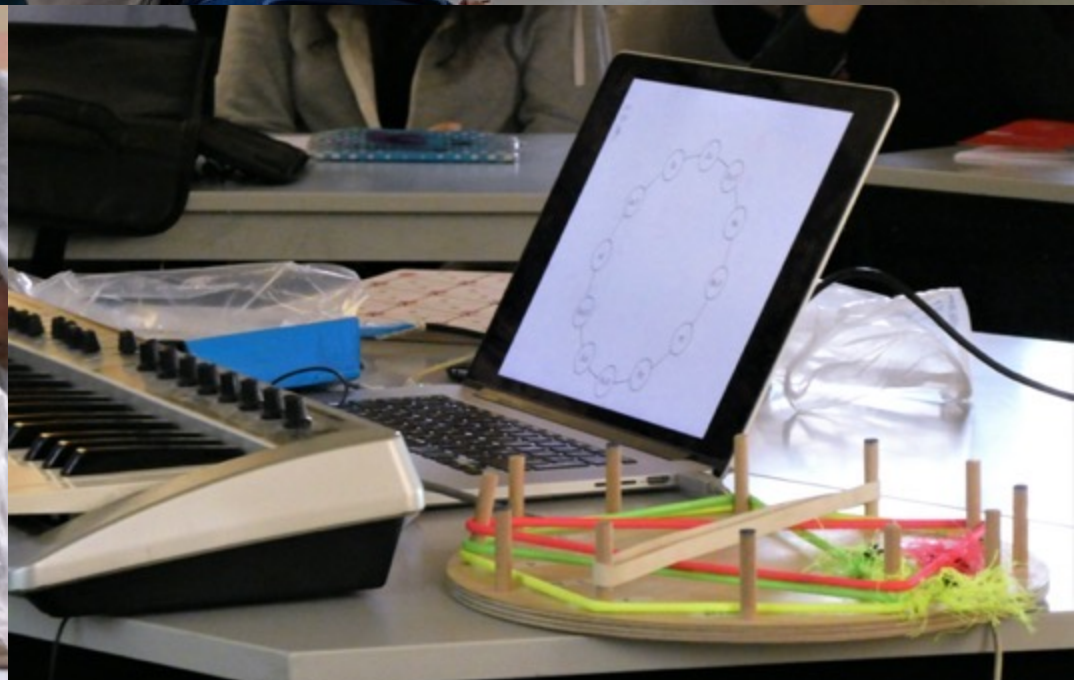
Calculer des puissances et des quotients en lien avec le **cycle des quintes**.
Mettre en place un raisonnement mathématique pour **prouver que le cycle des quintes est infini.**

La **preuve par l'absurde** : supposons qu'un multiple de la quinte est égal à un multiple de l'octave et montrons que cela conduit à une contradiction.


MATHÉMATIQUES,
DESSINEZ-MOI
LA MUSIQUE

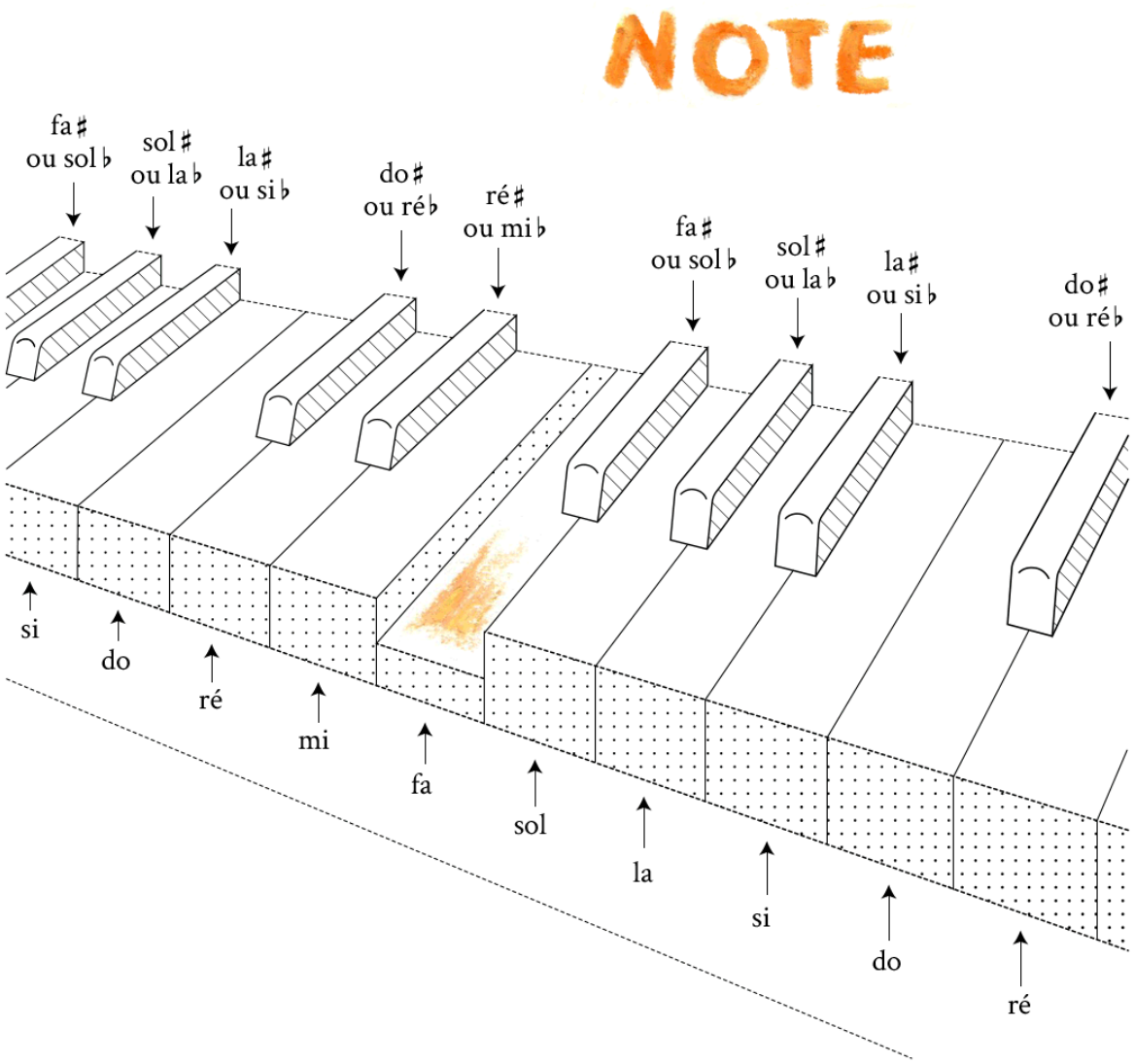



Rakuten
M-AUDIO

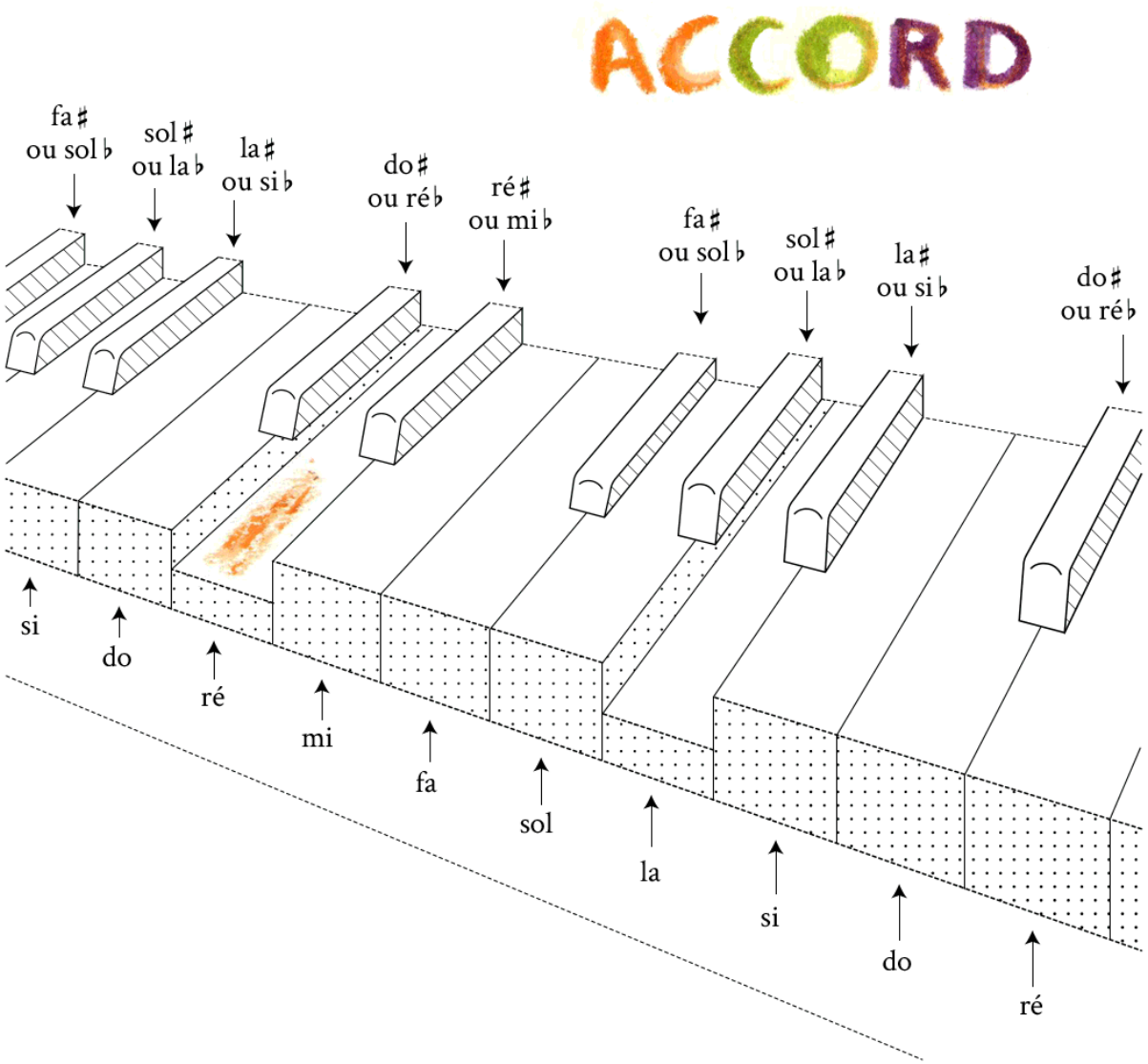


Que connaissez-vous du clavier de piano ?

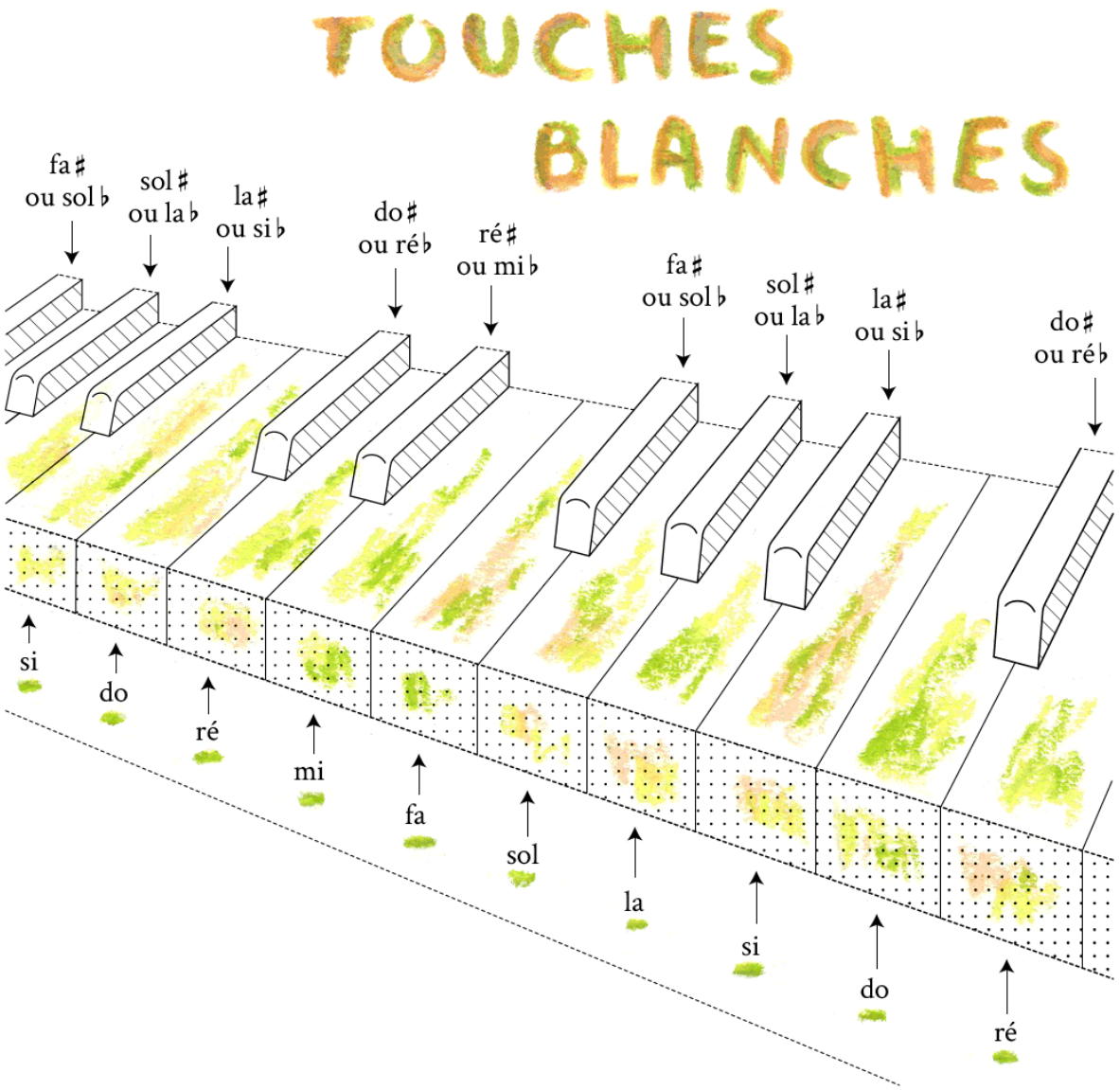
Petit rappel des principaux concepts musicaux



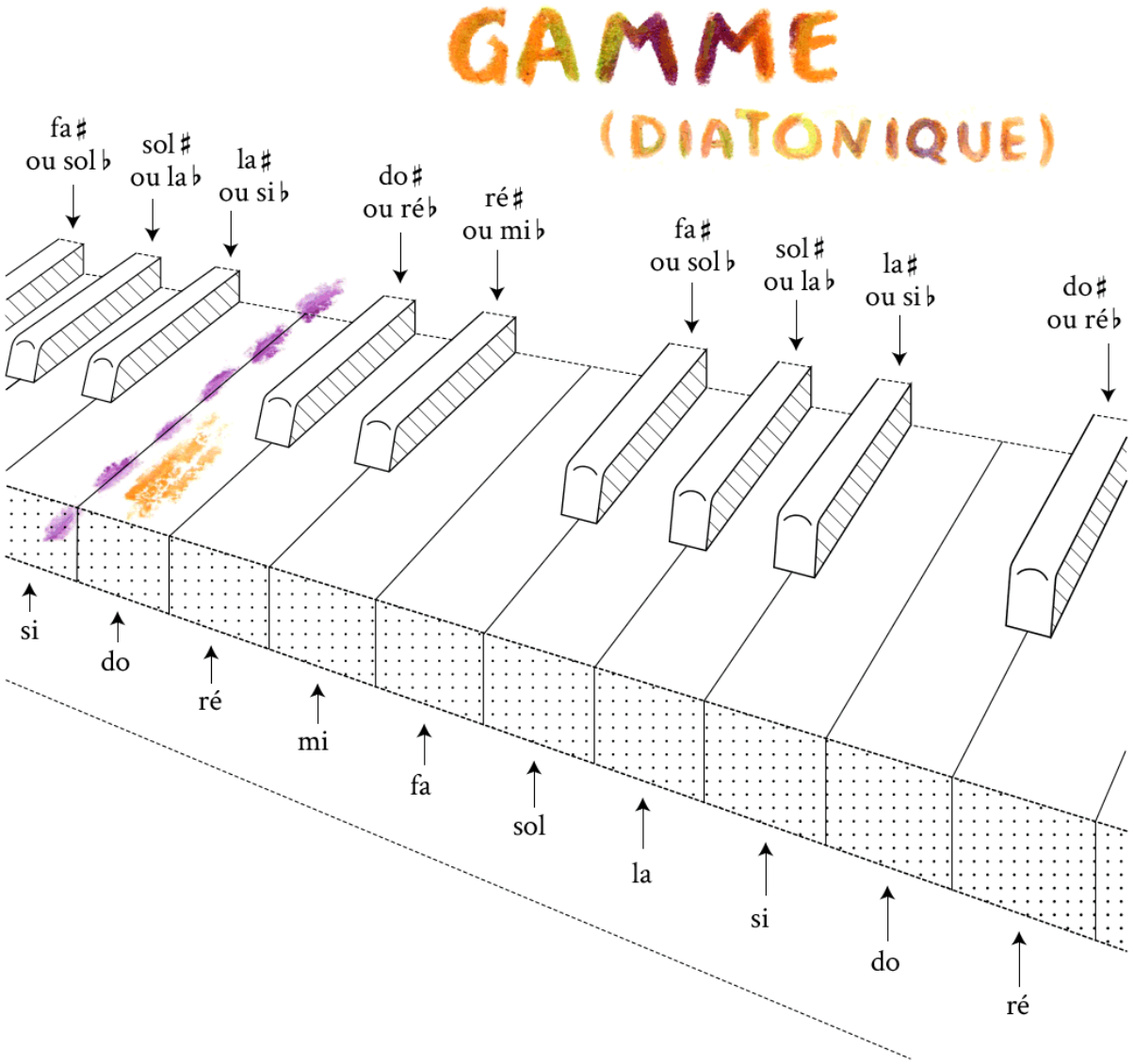
Petit rappel des principaux concepts musicaux



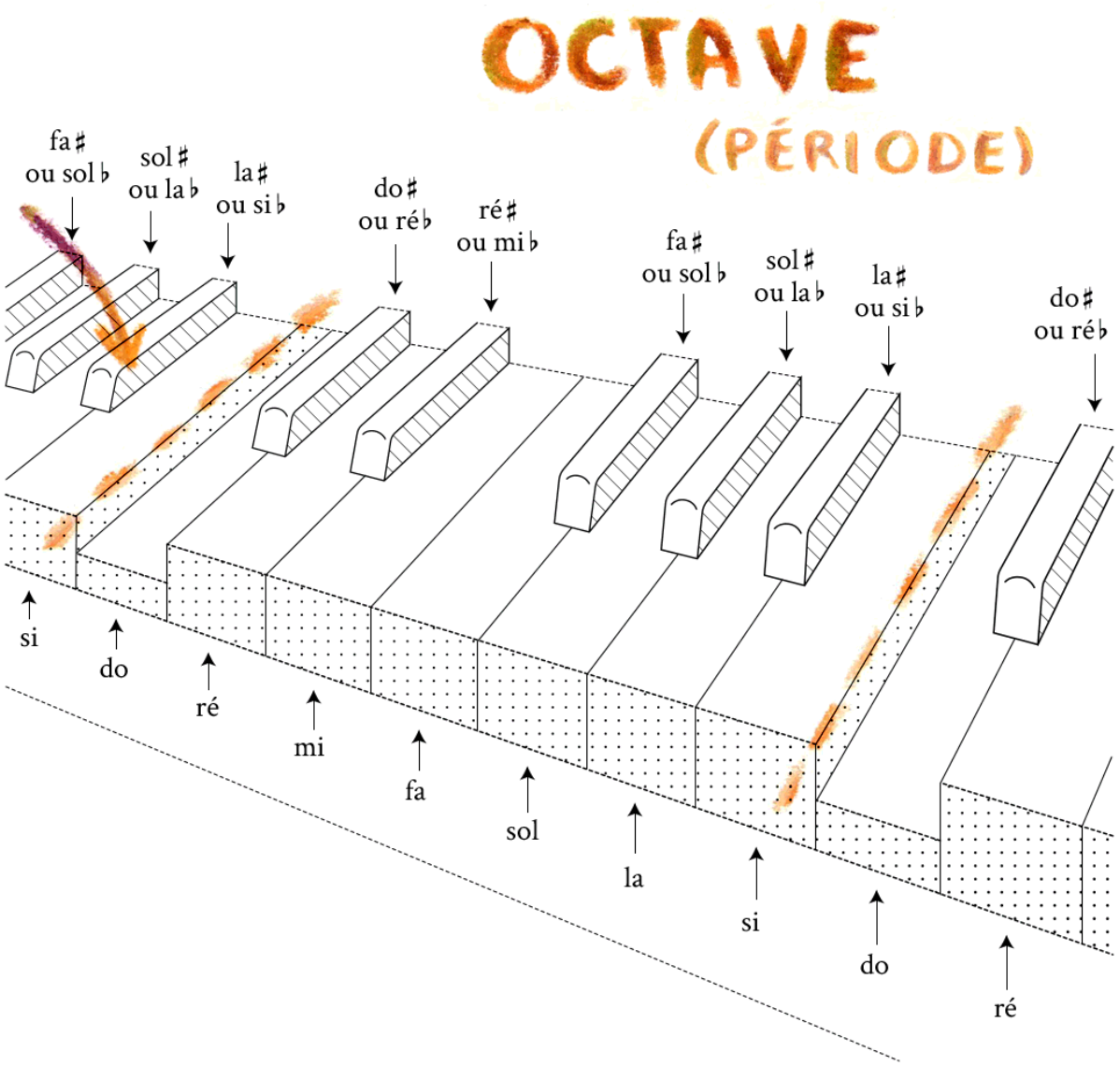
Petit rappel des principaux concepts musicaux



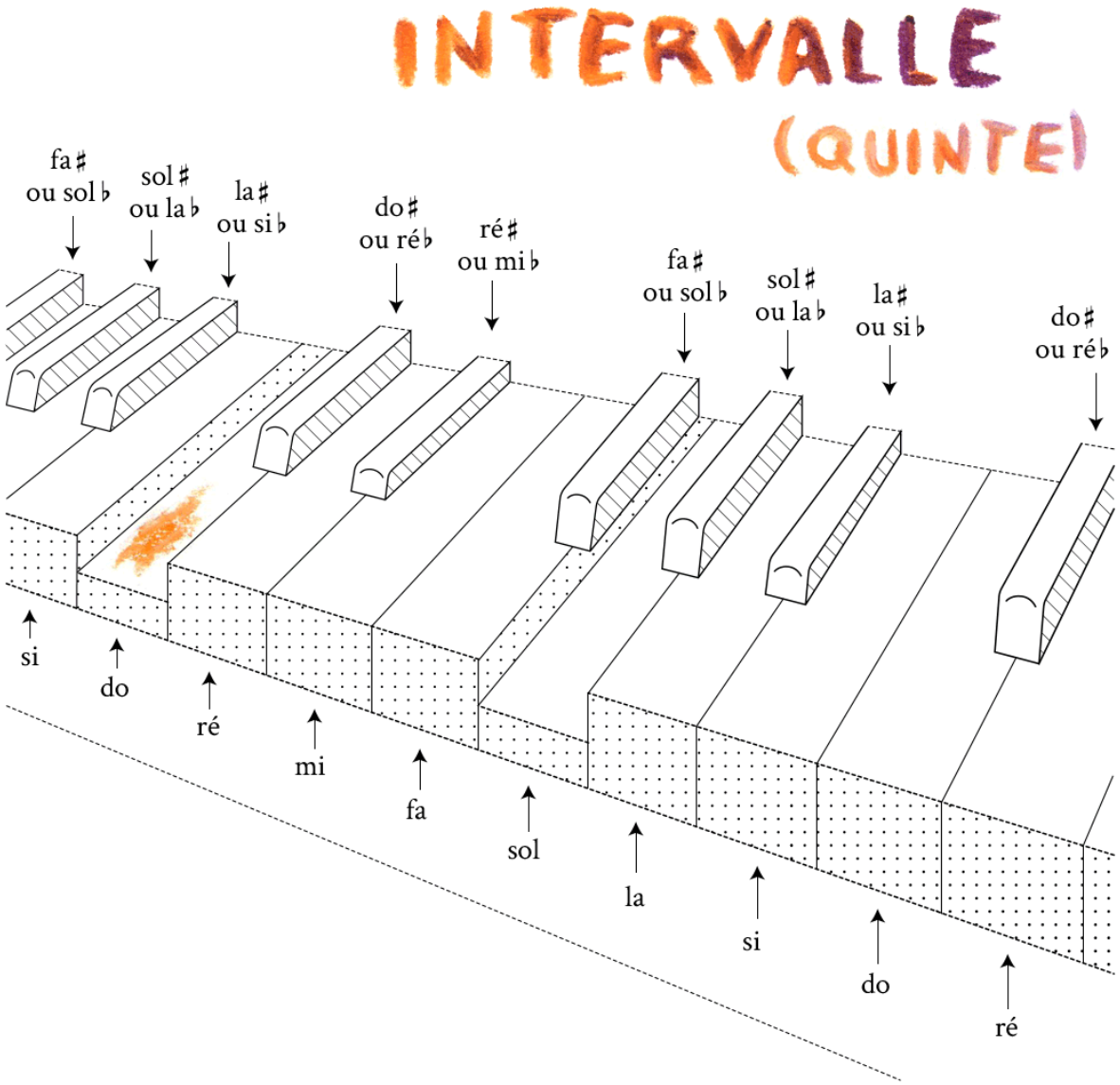
Petit rappel des principaux concepts musicaux



Petit rappel des principaux concepts musicaux

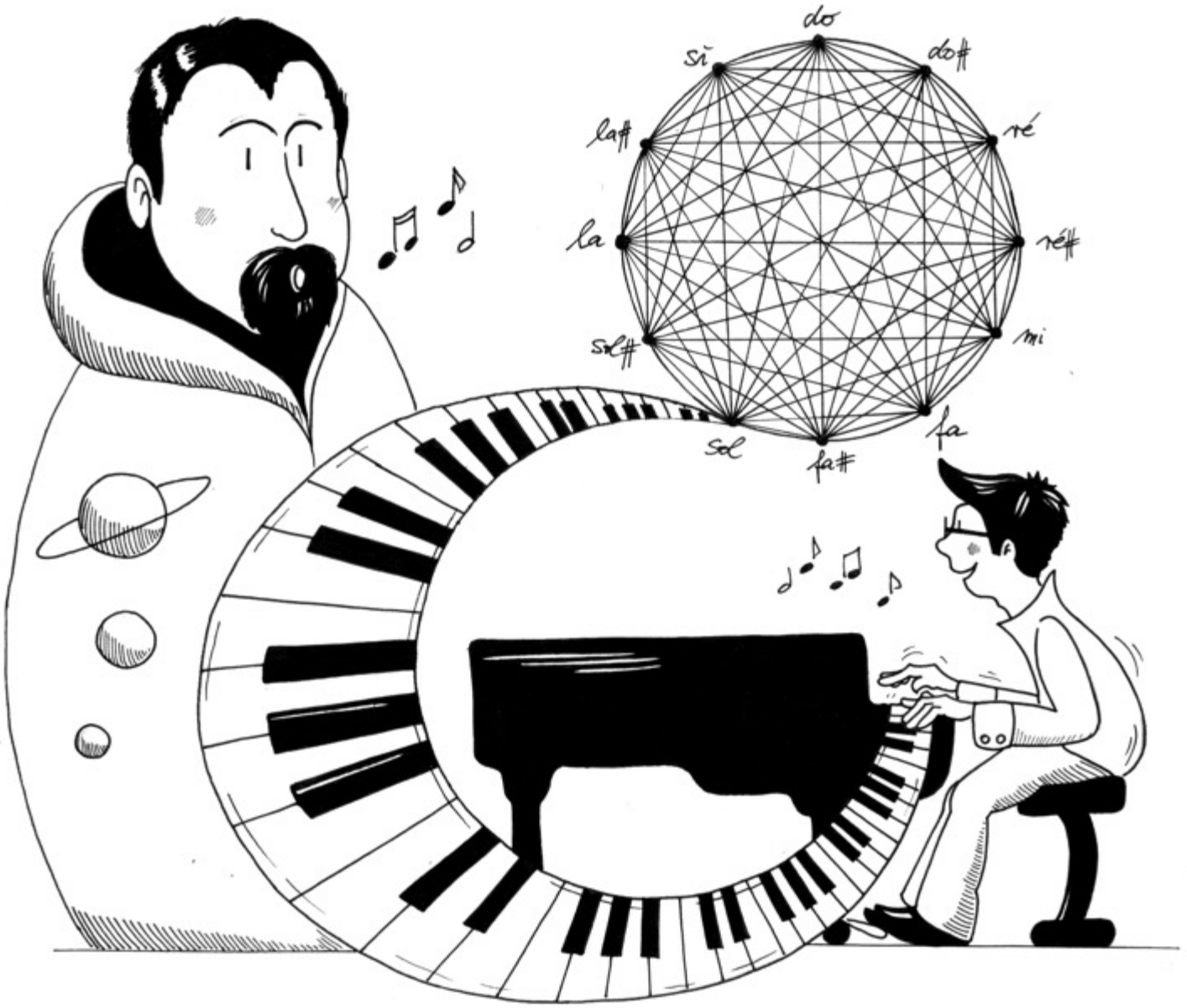


Petit rappel des principaux concepts musicaux



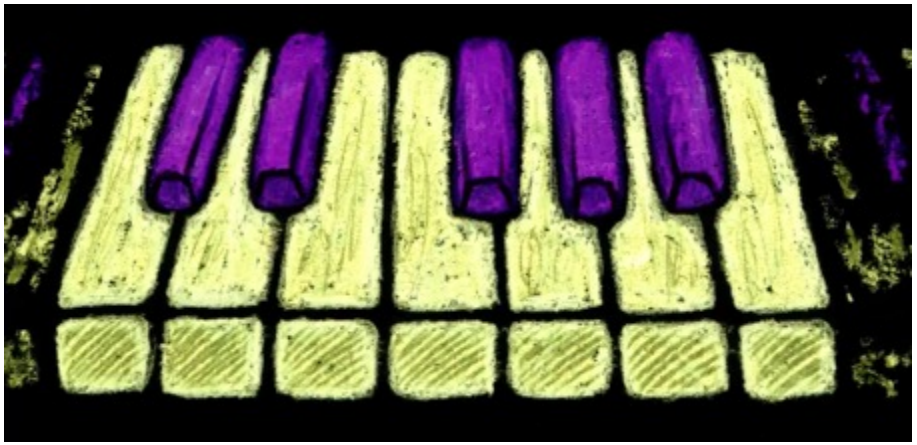
Du piano au cercle

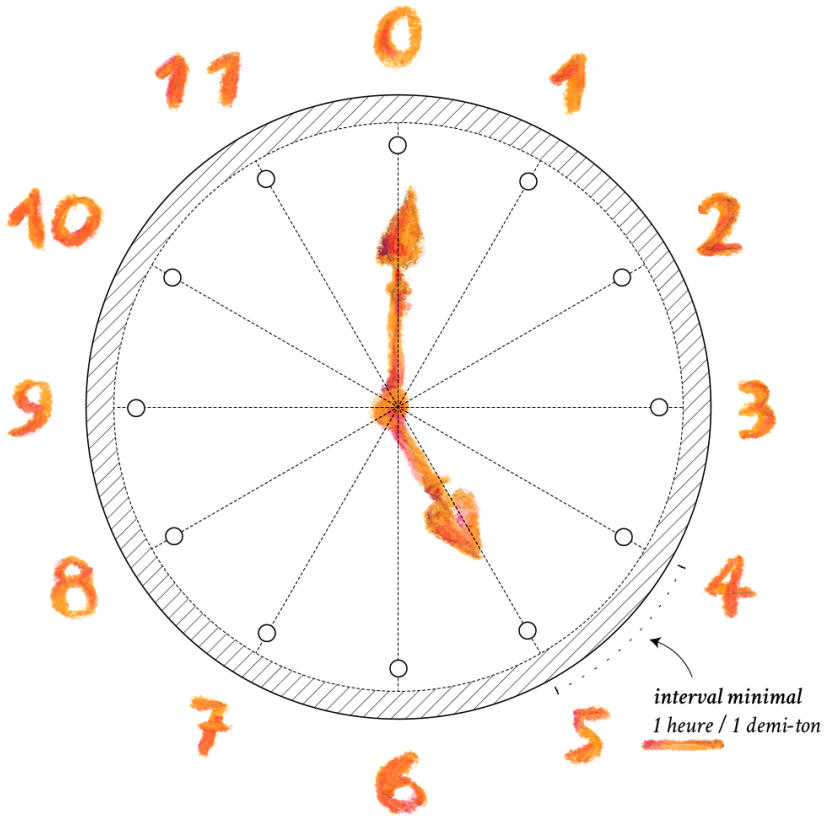
Mersenne et l'invention de la combinatoire



Que ce que cela vous évoque ?

Le cadran d'horloge musical et les accords





La représentation circulaire

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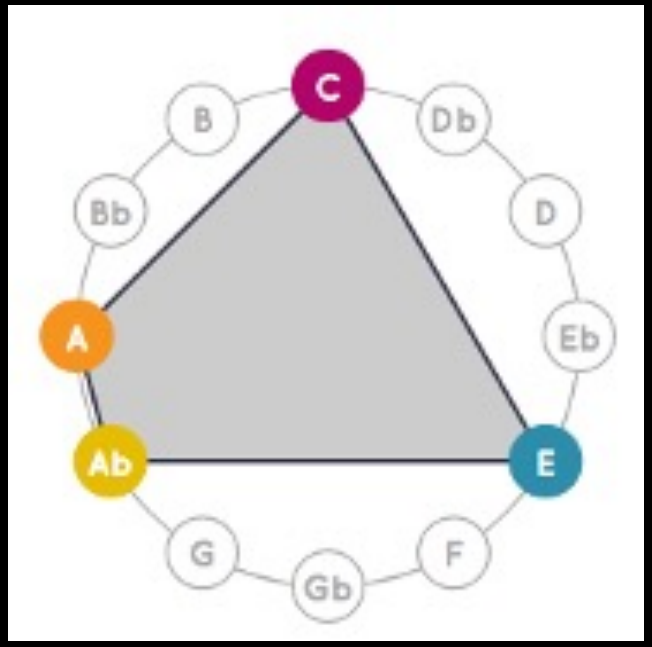
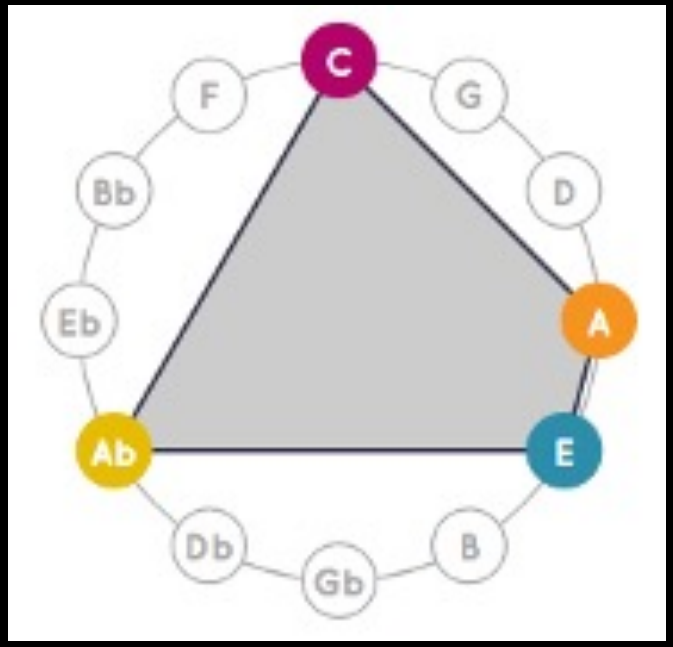
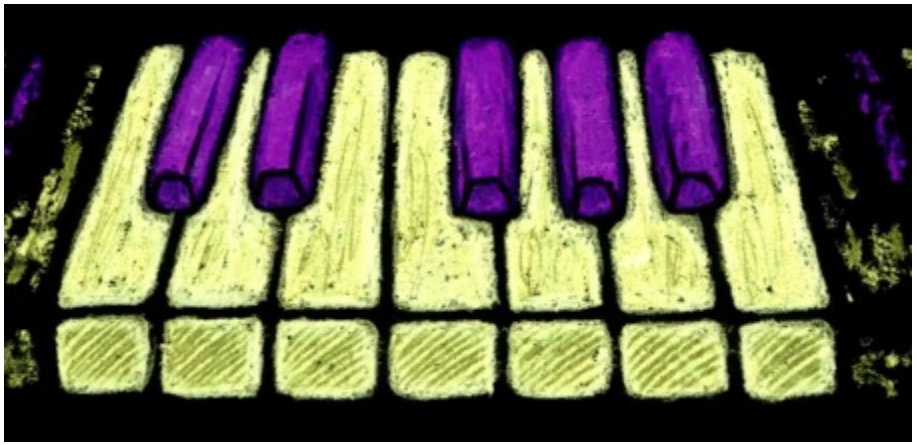
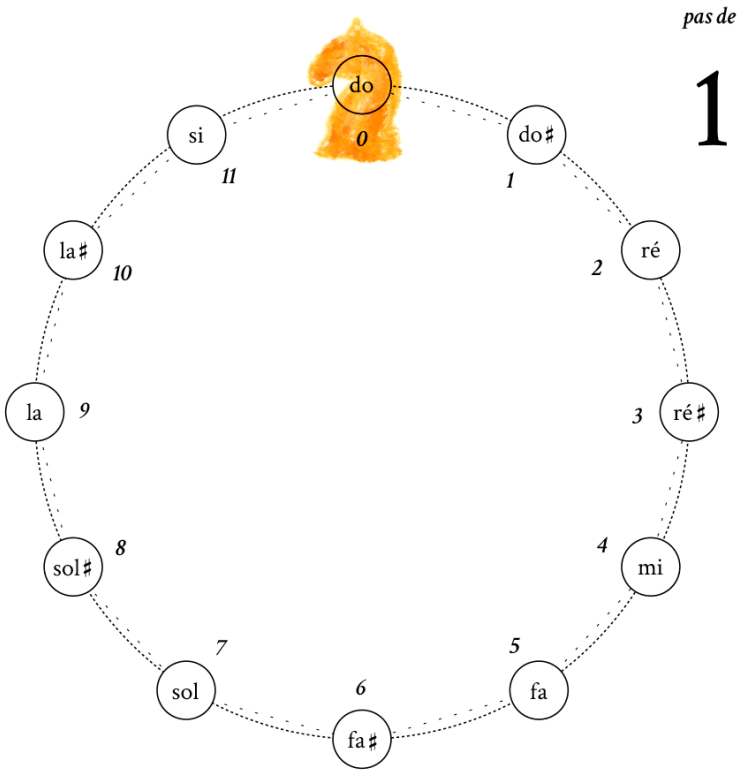
DEMO



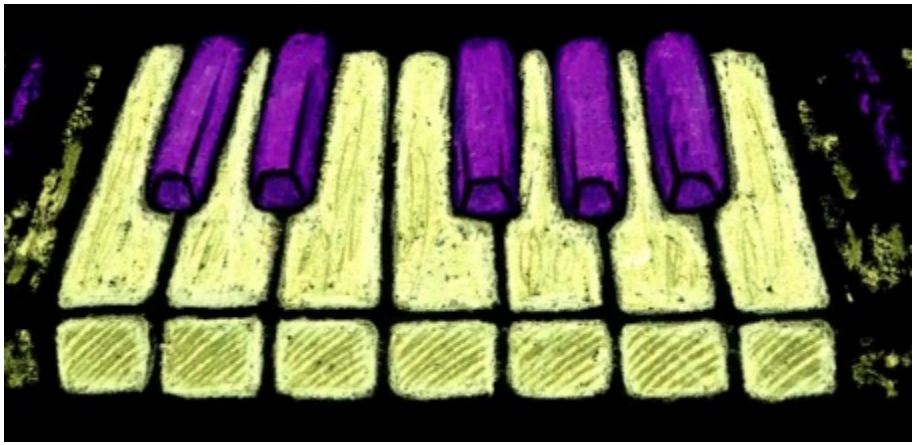


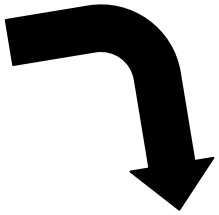
Intervalles générateurs du tempérament égal

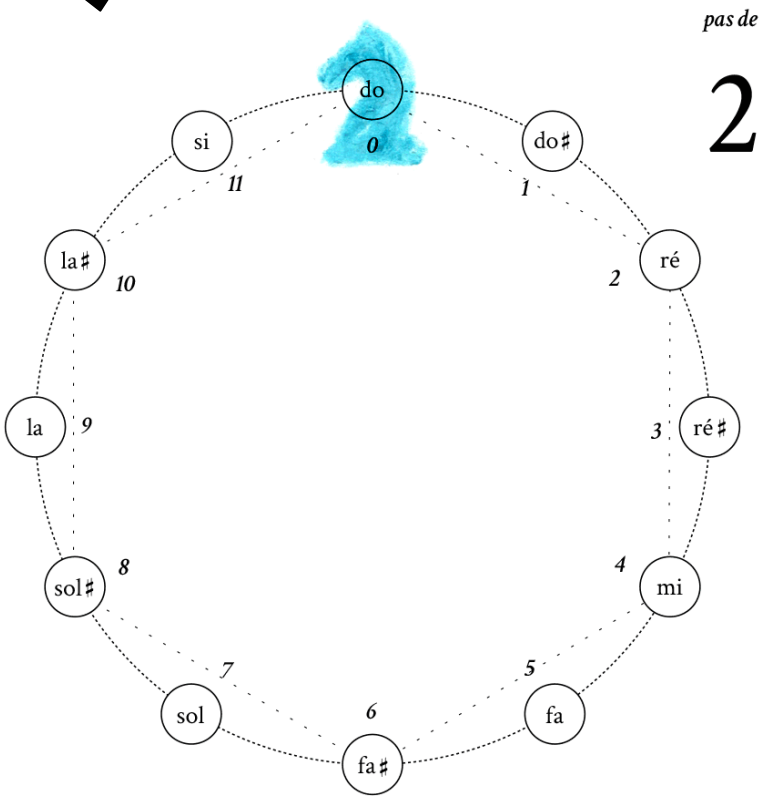




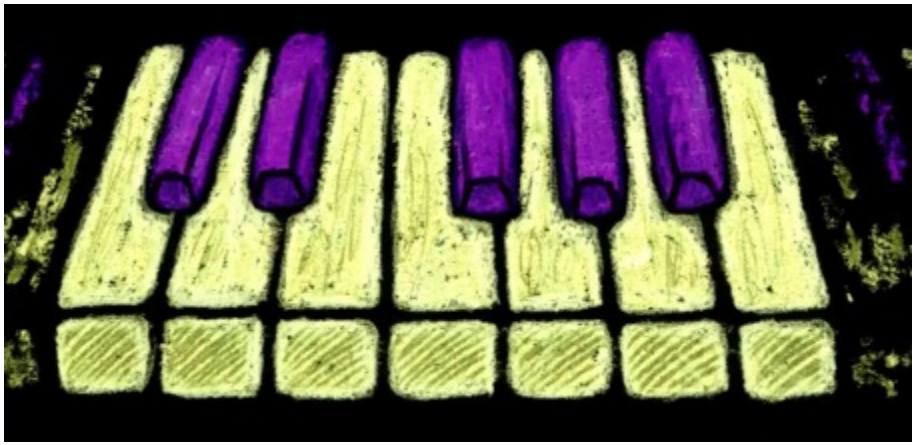
Intervalles générateurs du tempérament égal

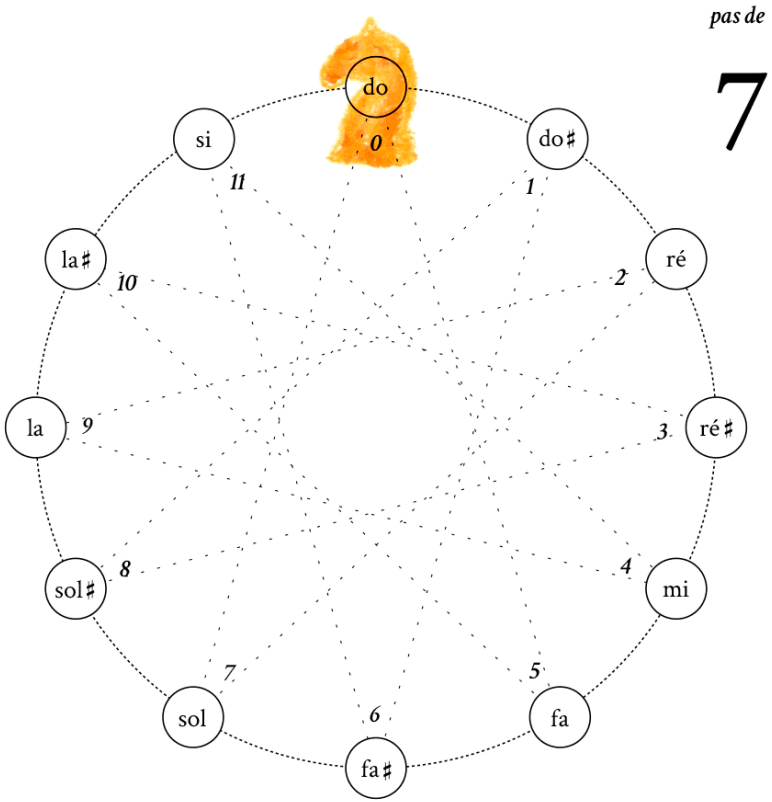




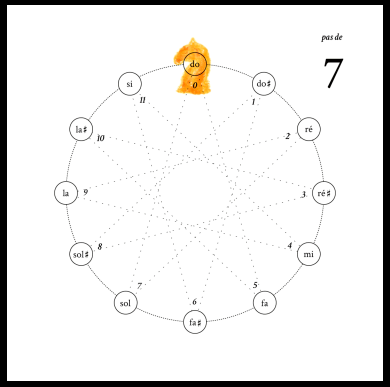


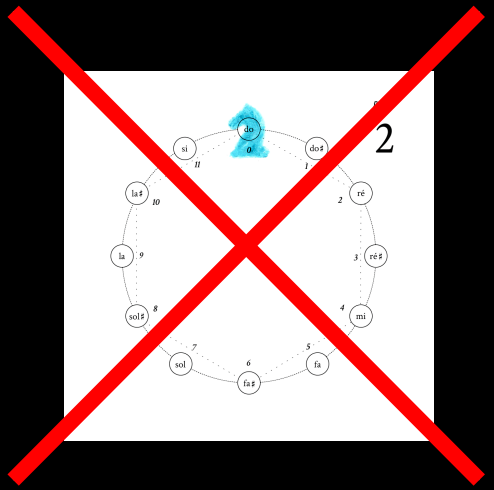
Intervalles générateurs du tempérament égal





Quels sont les intervalles qui marchent?





Intervalles générateurs du tempérament égal

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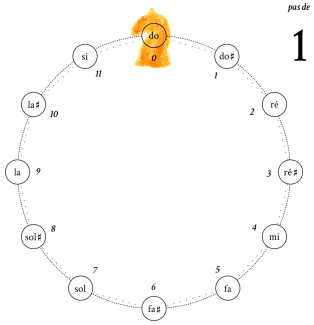

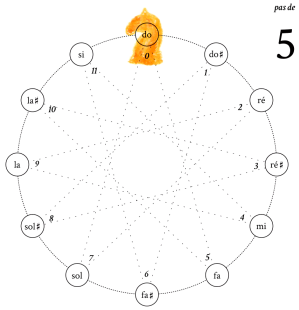

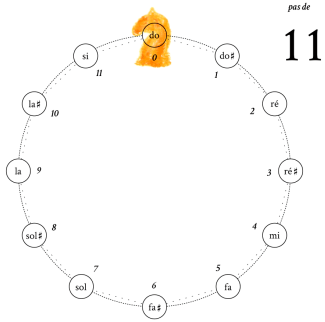

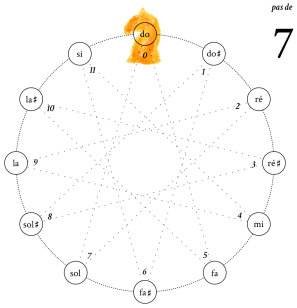

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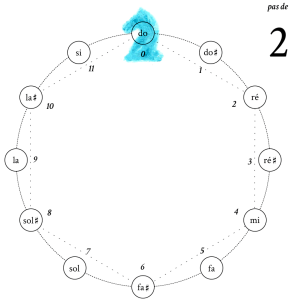
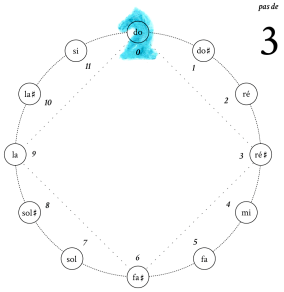
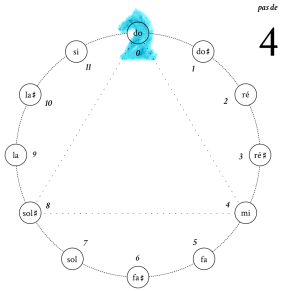
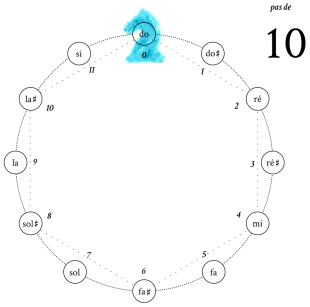
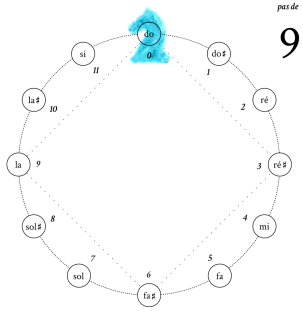
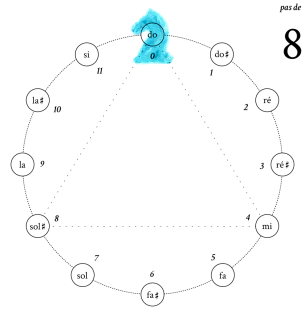




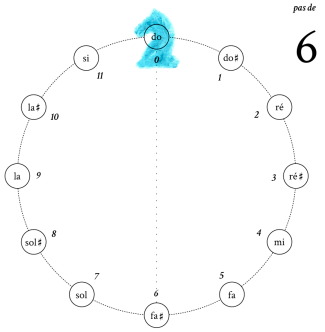


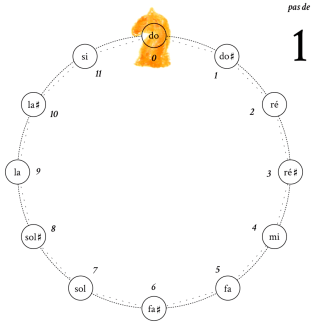


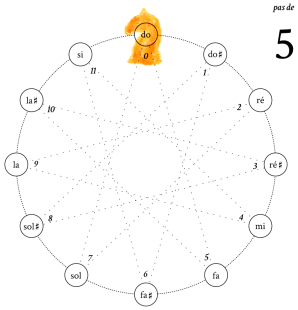


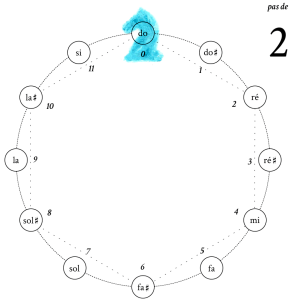


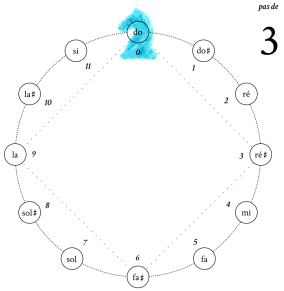
Intervalles générateurs du tempérament égal

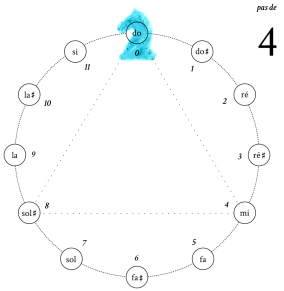


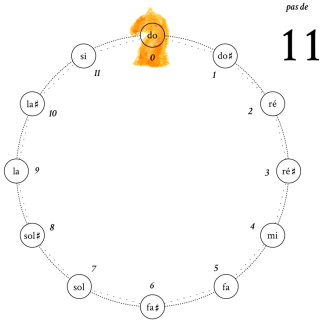


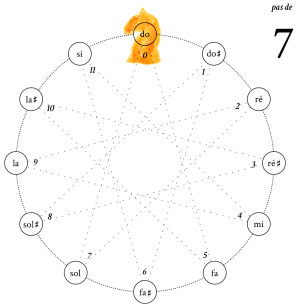


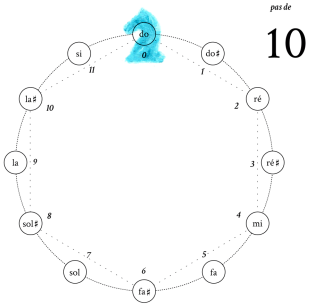


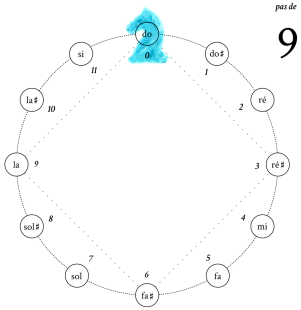


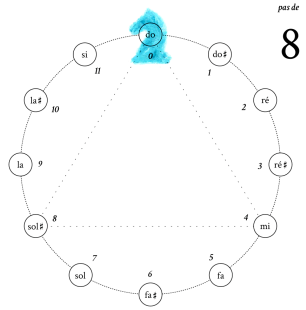






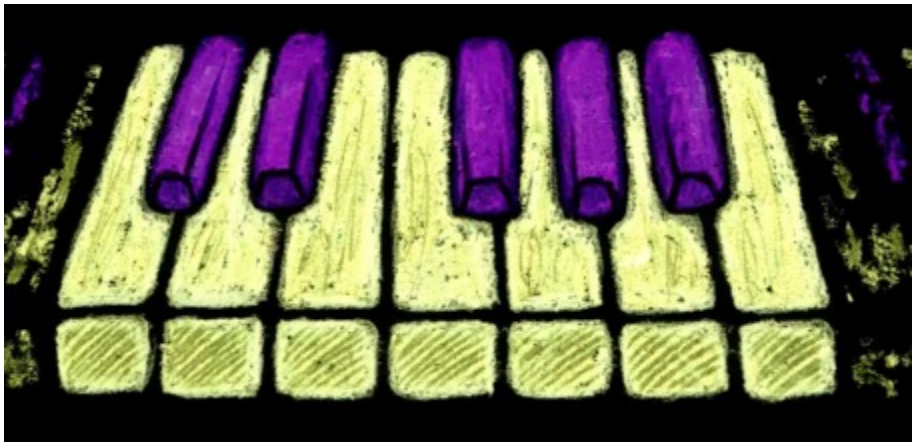


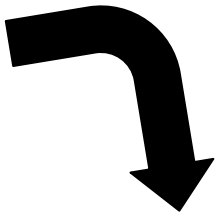


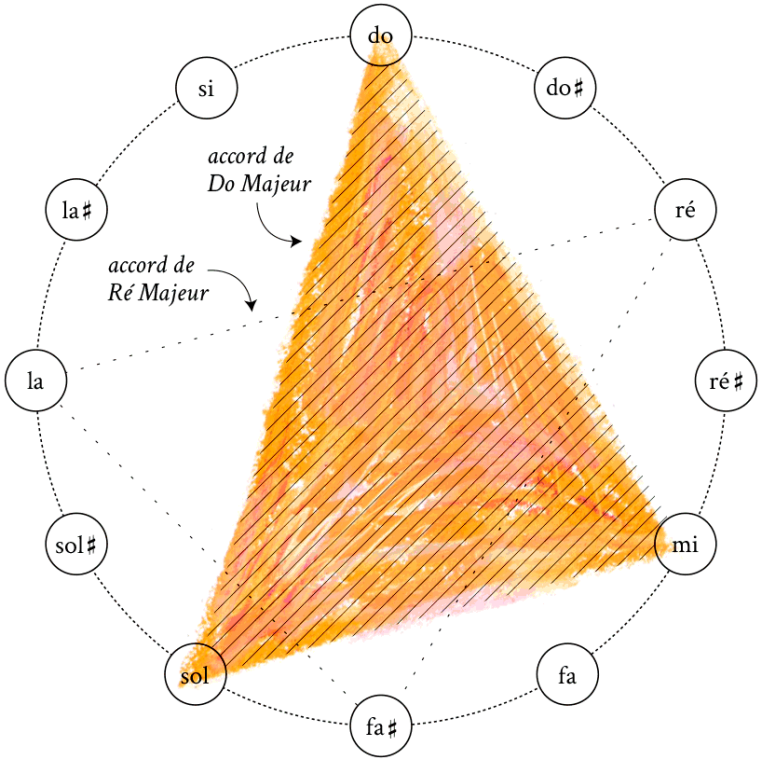


Comment transformer les accords dans le cercle ?

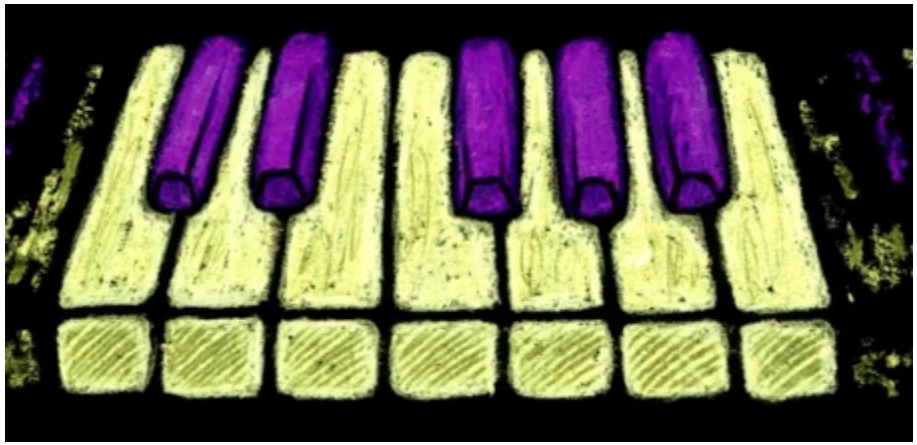
Les transpositions musicales sont des rotations

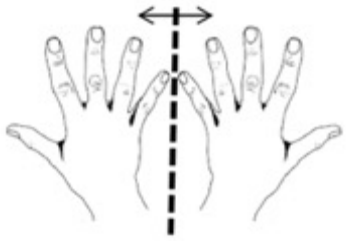


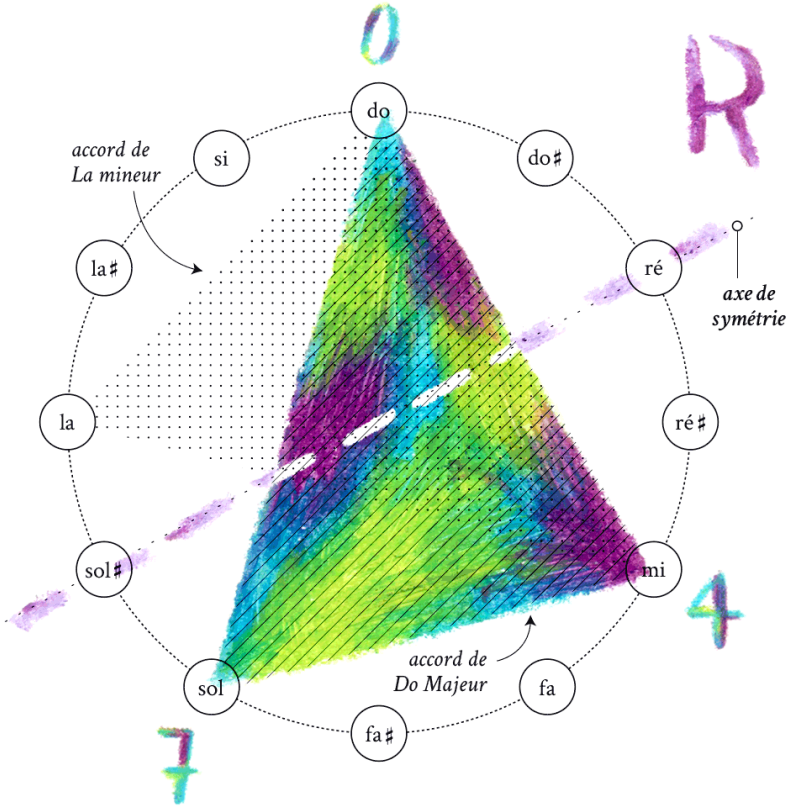




Quelle est la relation entre accord majeur et accord mineur?

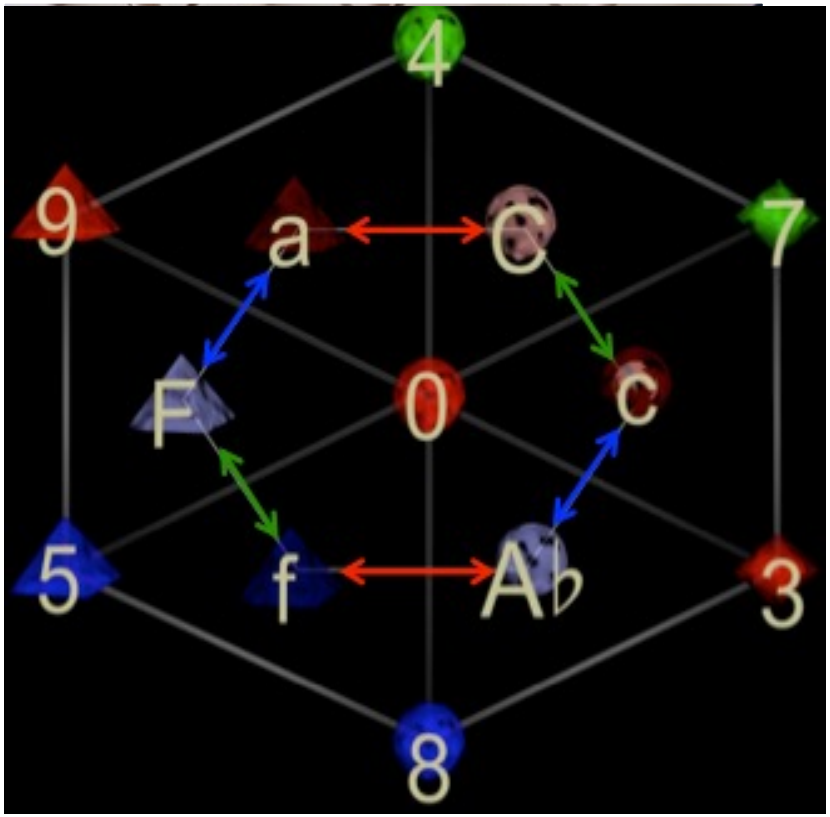




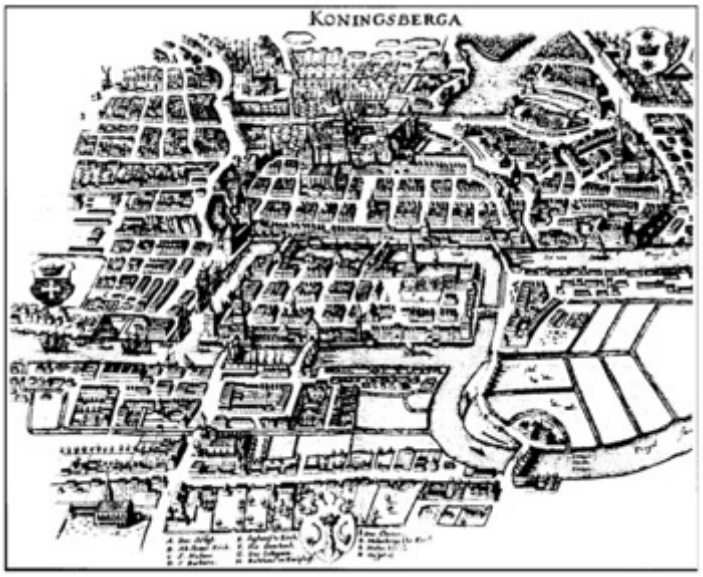


Comment visualiser les symétries dans le plan ?

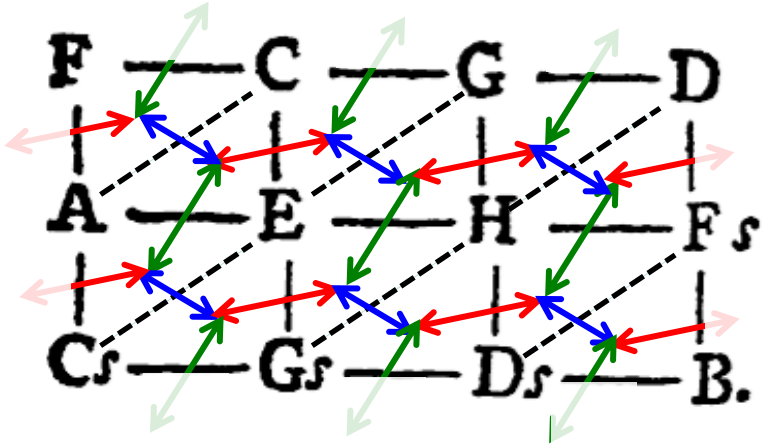
Euler et le problème du cavalier des échecs



Le *Tonnetz* (ou nid musical d'abeilles)

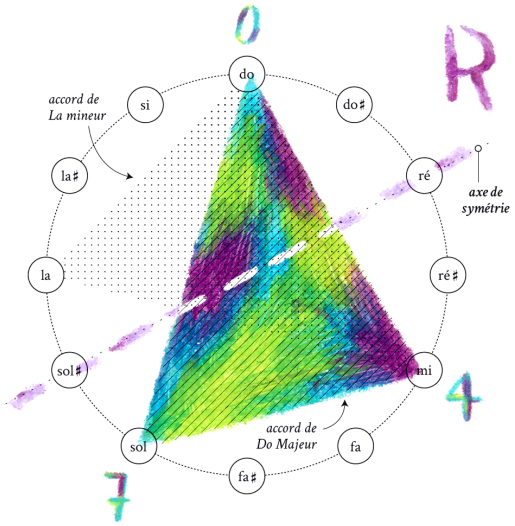


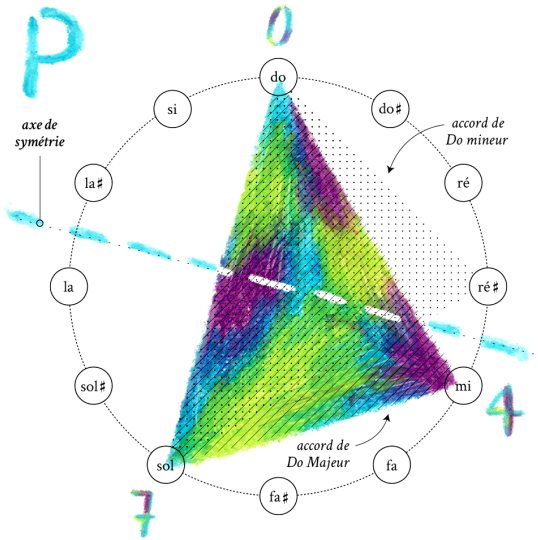
Leonhard Euler

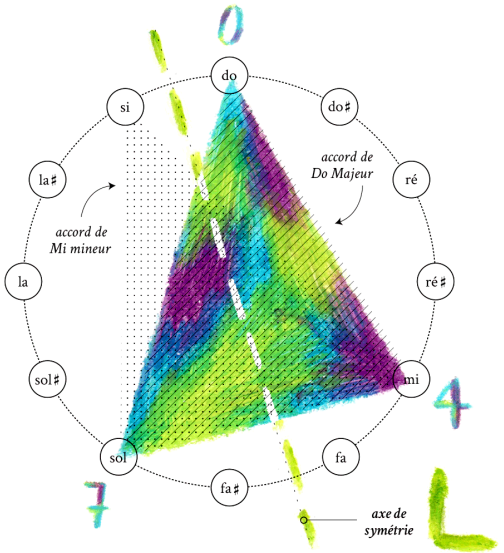


Speculum Musicum (1773)

Les trois symétries néo-riemanniennes







R = RELATIF

C majeur

↕

A mineur

P = PARALLÈLE

C majeur

↕

C mineur

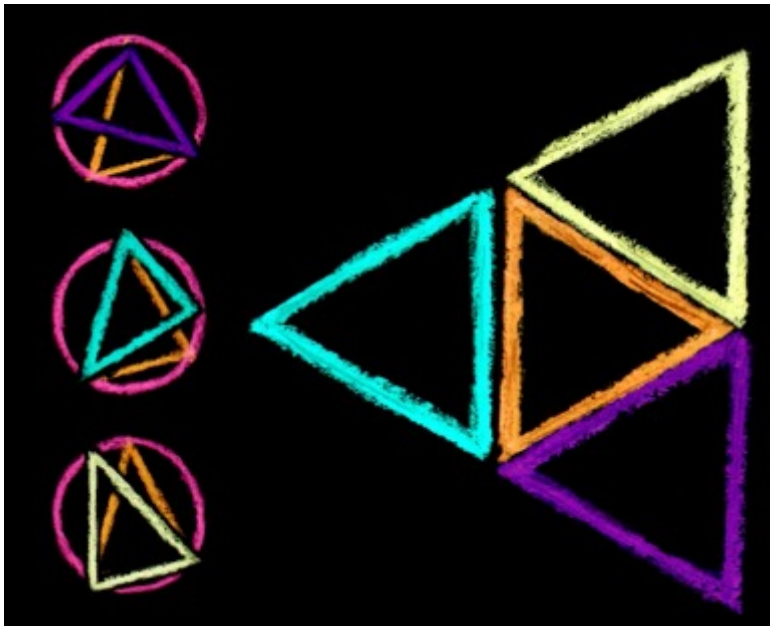
L = LEADING-TONE (EXCHANGE)

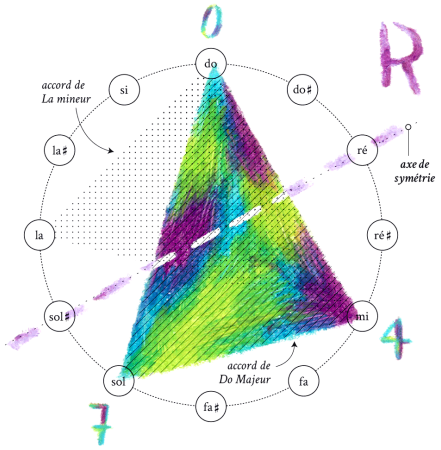
C majeur

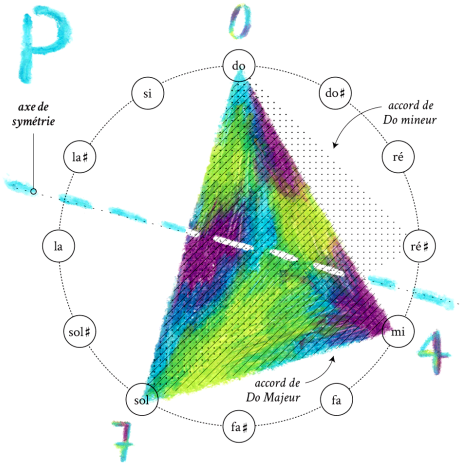
↕

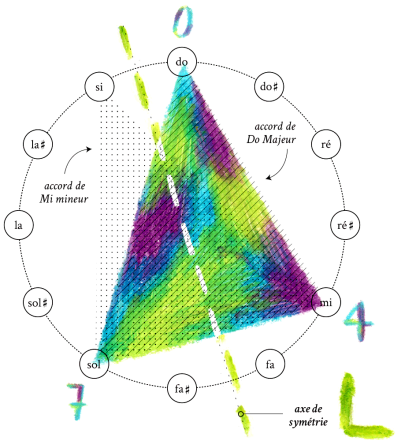
E mineur

Le système Tonnetz : trois symétries pour trois équations

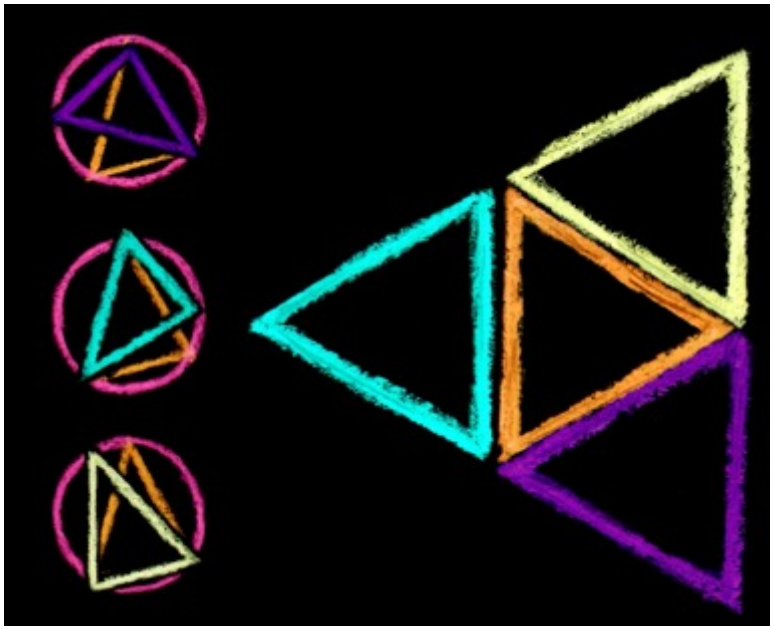


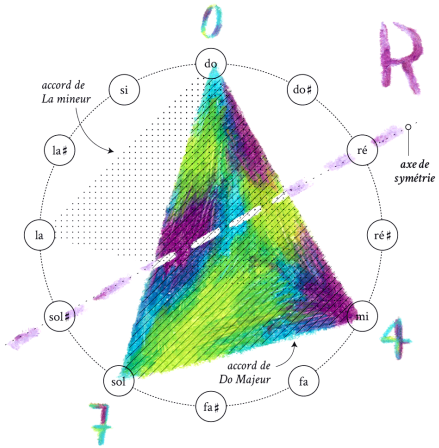


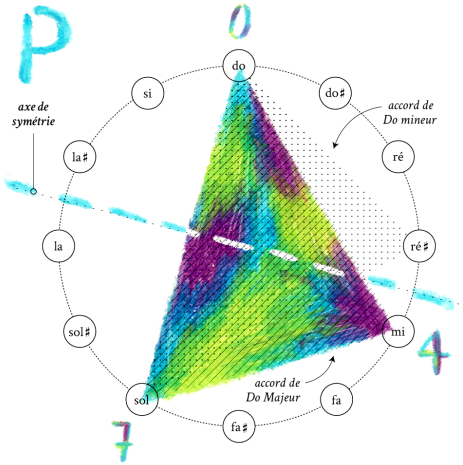


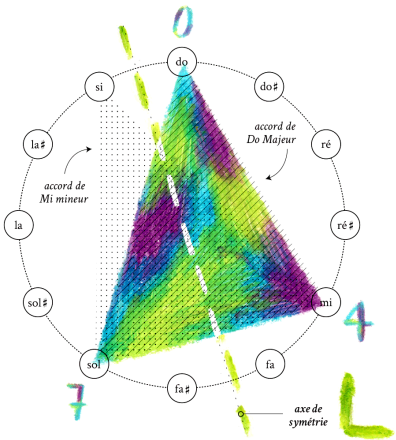


Le système Tonnetz : trois symétries pour trois équations





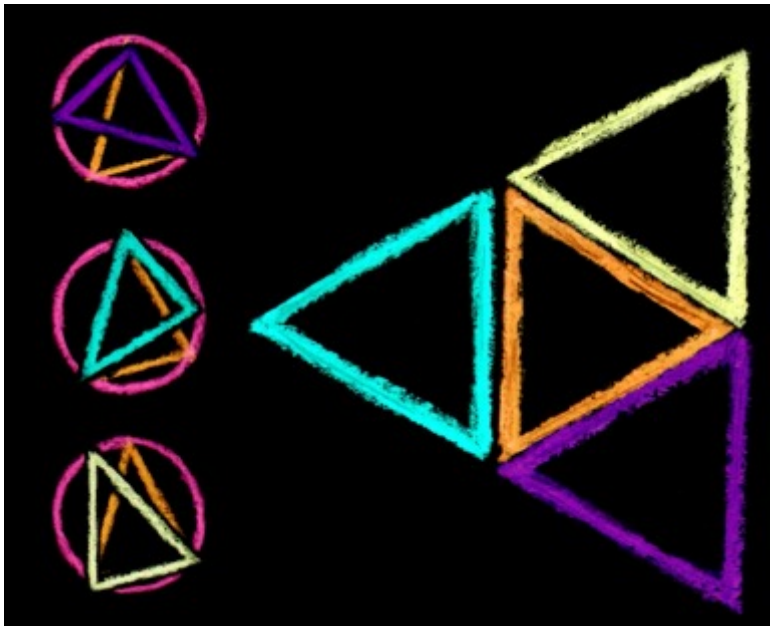


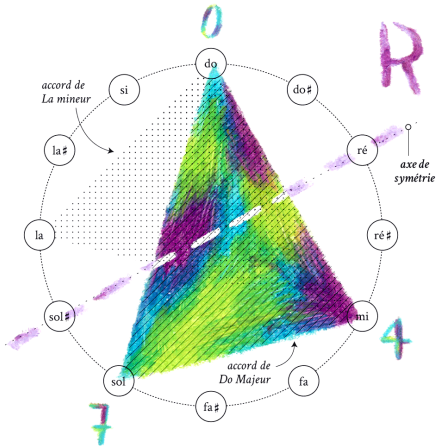


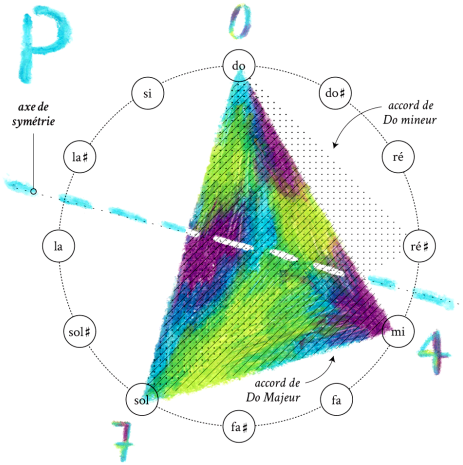
$x \longrightarrow 4\text{-}x$

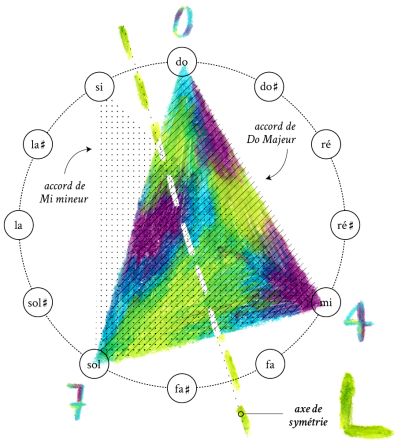
$\{0,4,7\} \longrightarrow \{\ \_\ ,\ \_\ ,\ \_\ \}$

Le système Tonnetz : trois symétries pour trois équations





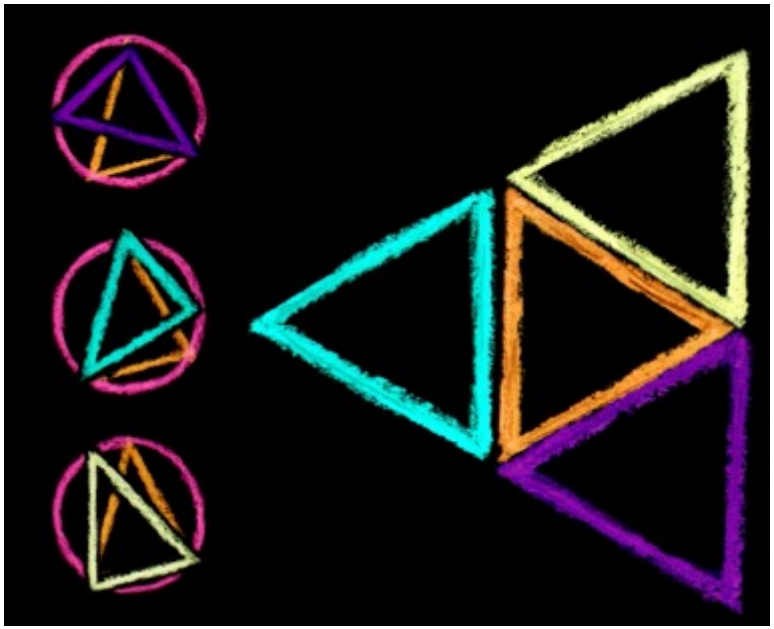


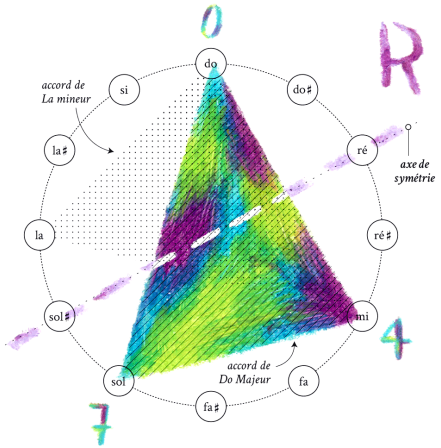


$x \longrightarrow 4\text{-}x$

$\{0,4,7\} \longrightarrow \{4,0,9\}$

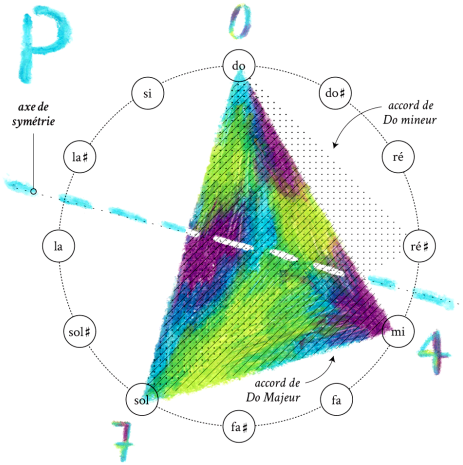
Le système Tonnetz : trois symétries pour trois équations





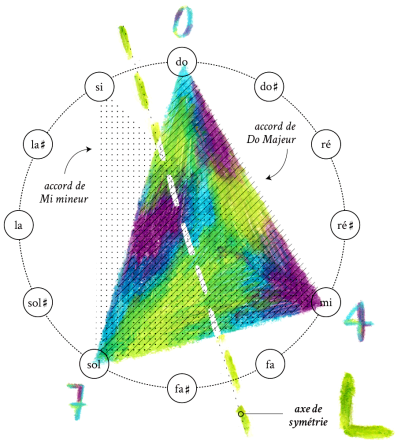
$x \longrightarrow 4\text{-}x$

$\{0,4,7\} \longrightarrow \{4,0,9\}$



$x \longrightarrow k\text{-}x$

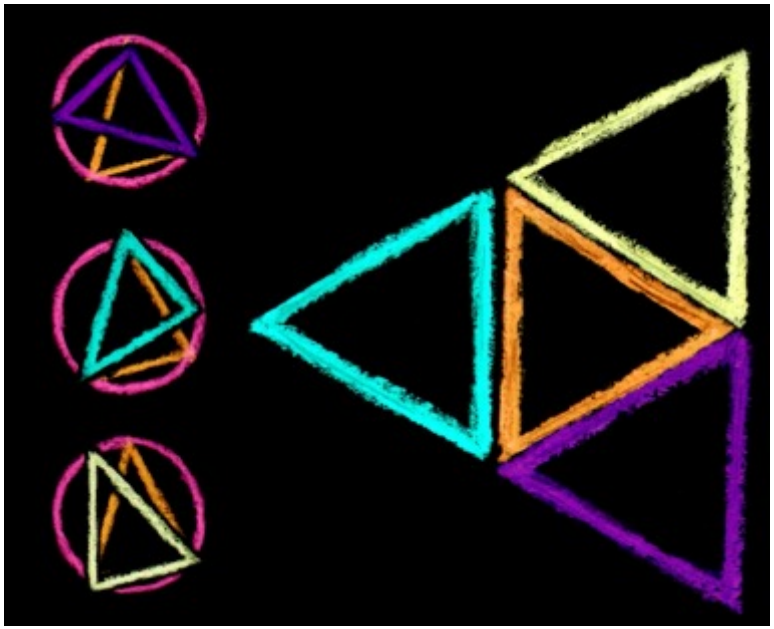
k=?

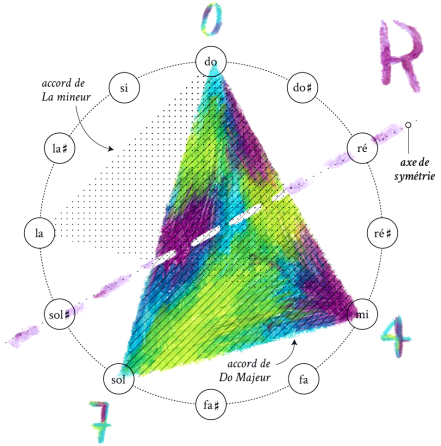


$x \longrightarrow k\text{-}x$

k=?

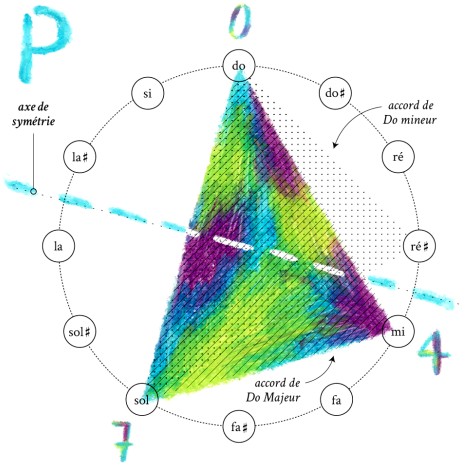
Le système Tonnetz : trois symétries pour trois équations





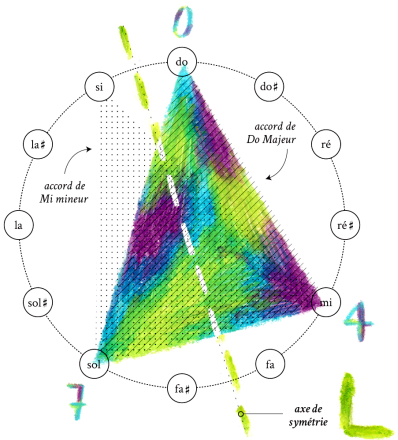
$x \longrightarrow 4\text{-}x$

$\{0,4,7\} \longrightarrow \{4,0,9\}$



$x \longrightarrow 7\text{-}x$

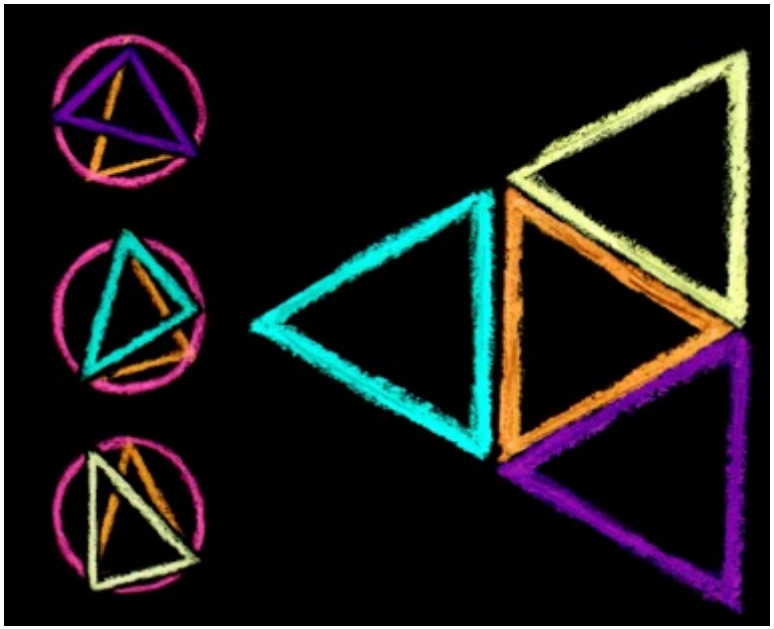
$\{0,4,7\} \longrightarrow \{7,3,0\}$

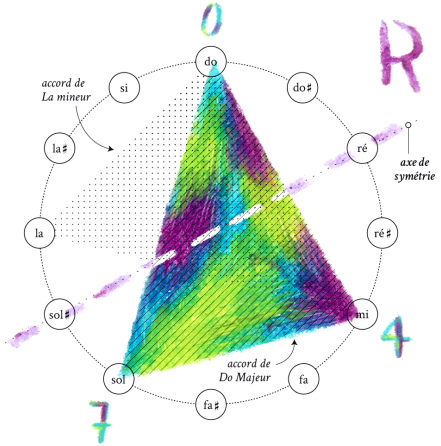


$x \longrightarrow k\text{-}x$

k=?

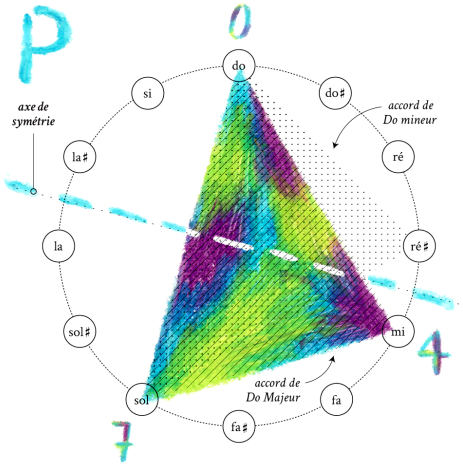
Le système Tonnetz : trois symétries pour trois équations





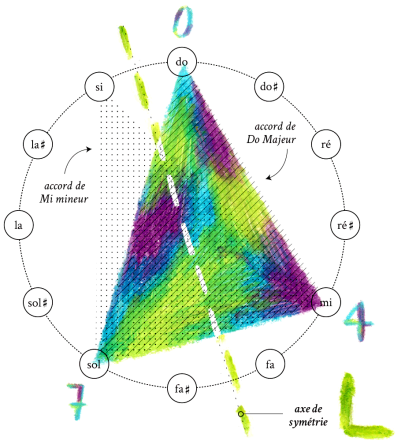
$x \longrightarrow 4\text{-}x$

$\{0,4,7\} \longrightarrow \{4,0,9\}$



$x \longrightarrow 7\text{-}x$

$\{0,4,7\} \longrightarrow \{7,3,0\}$

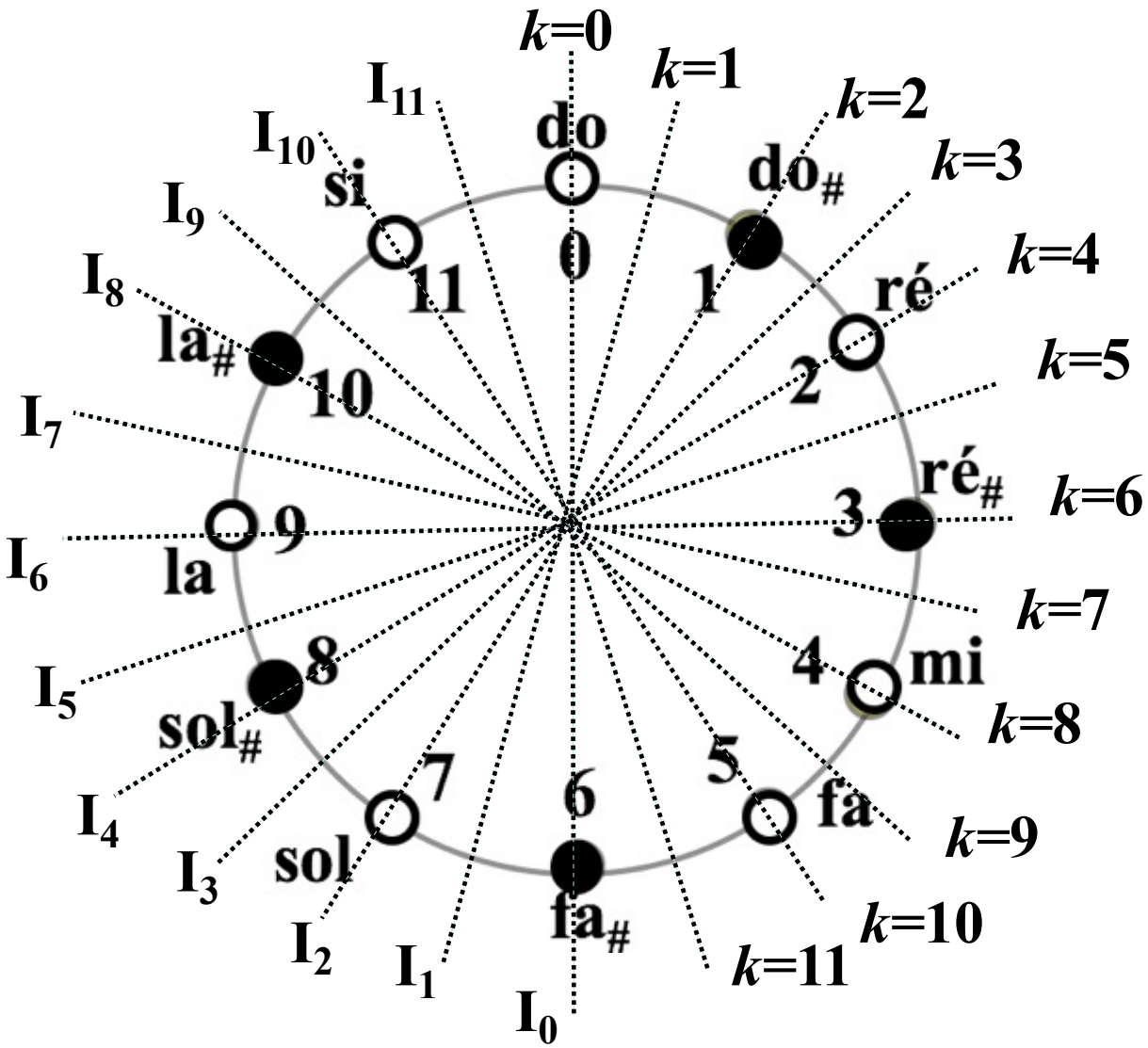


$x \longrightarrow 11\text{-}x$

$\{0,4,7\} \longrightarrow \{11,7,4\}$

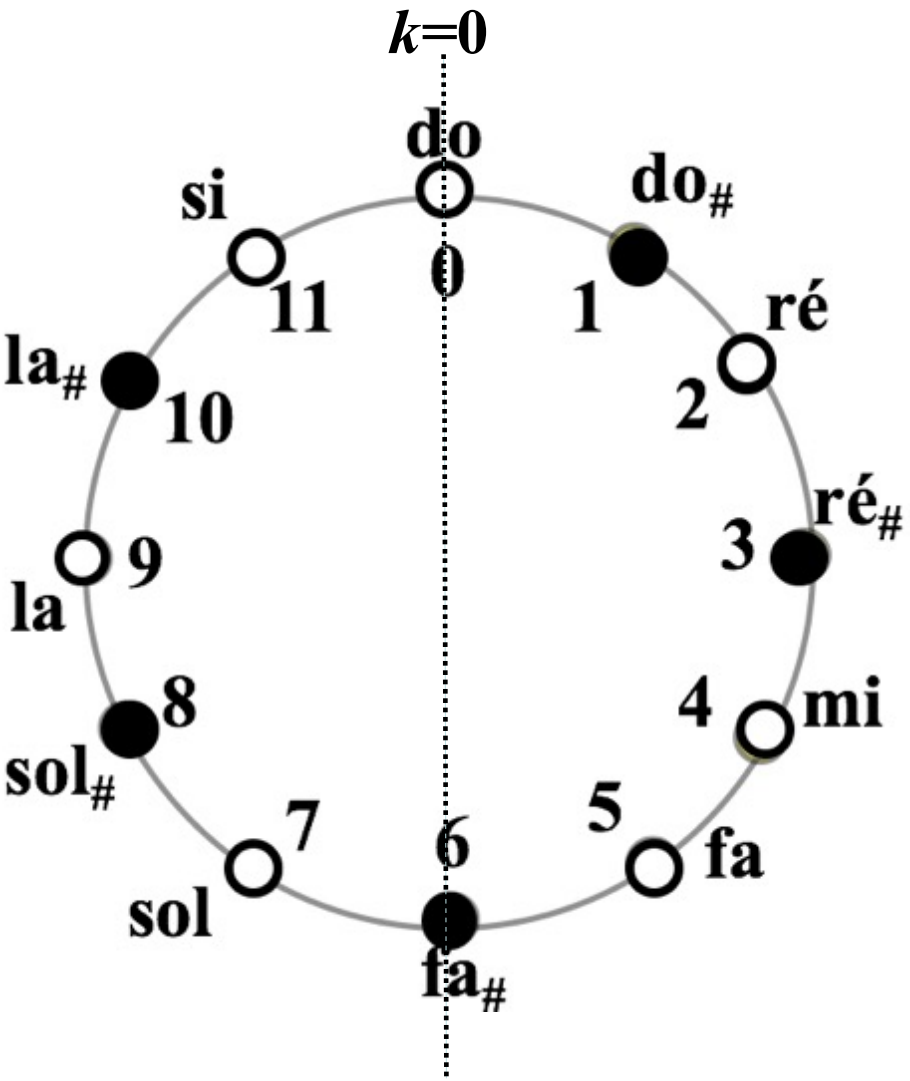
Les inversions sont des symétries axiales généralisées

$x \xrightarrow{I_k}$ k-x modulo 12



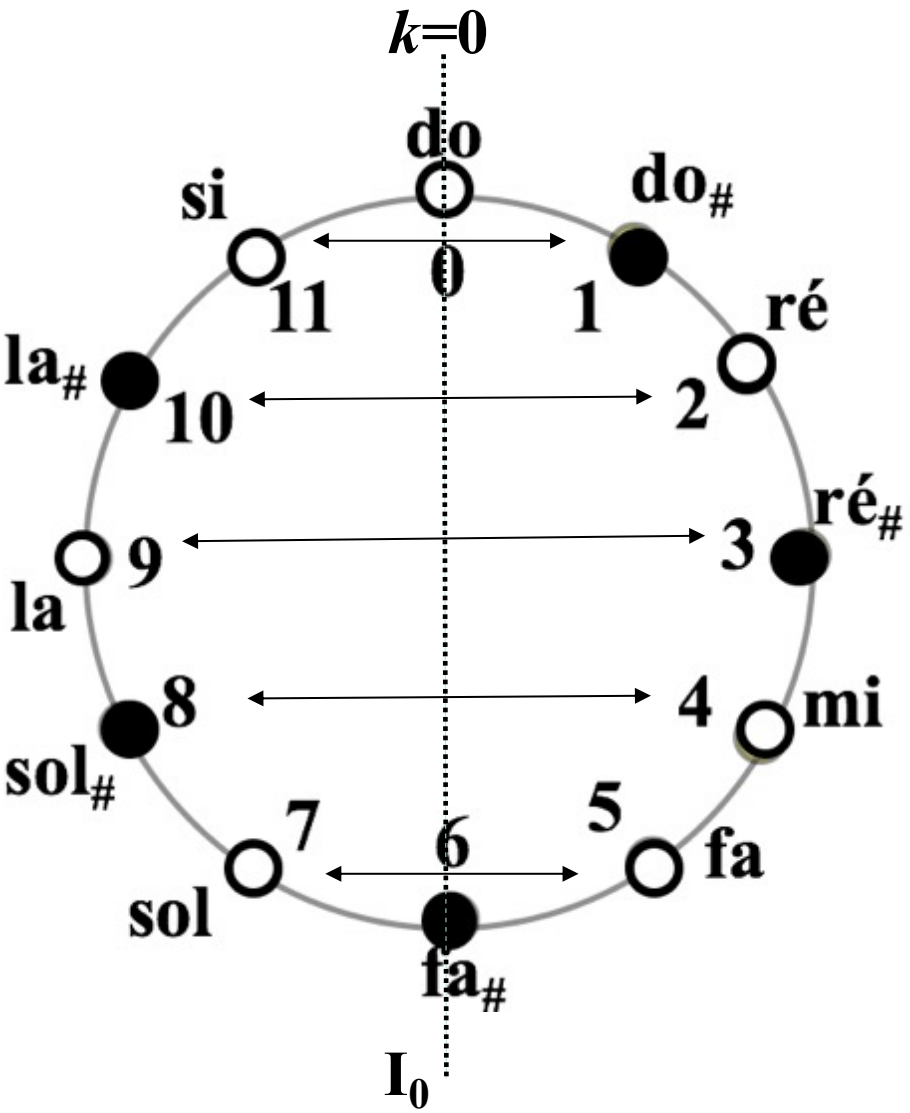
Quelques exemples

$x \xrightarrow{I_0} -x$ modulo 12



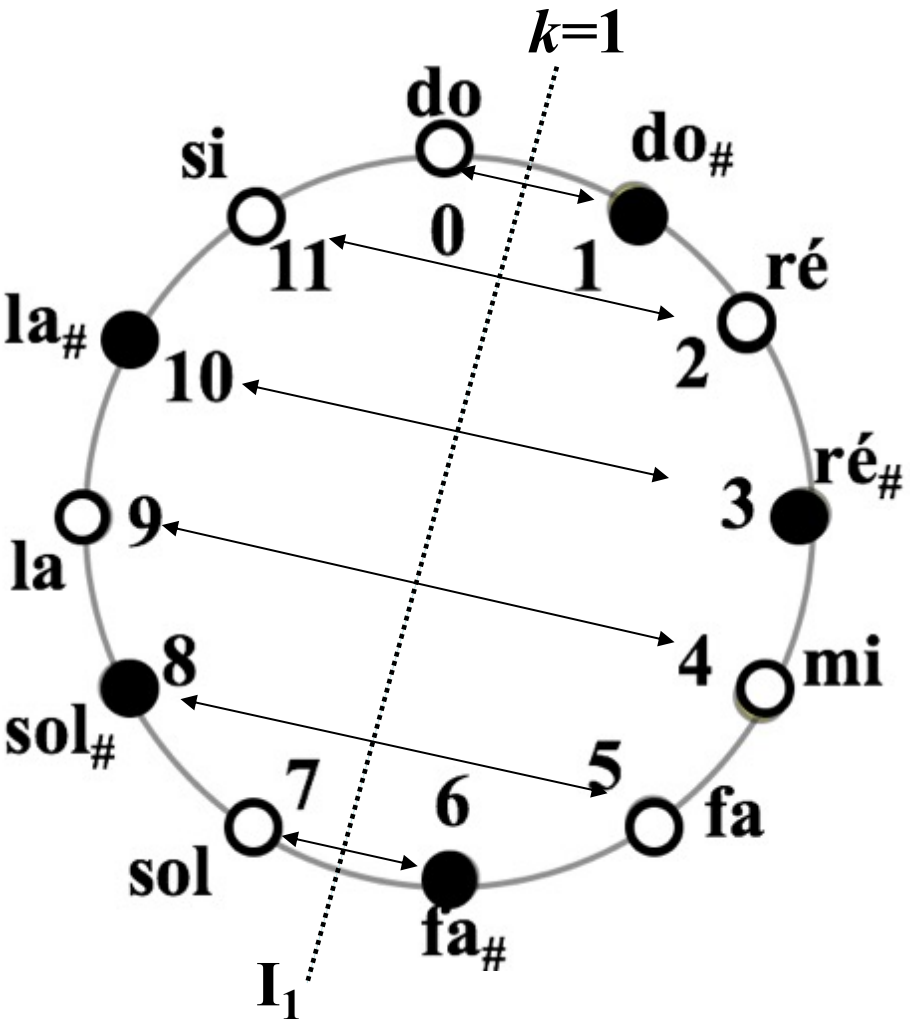
Quelques exemples

$x \xrightarrow{I_0} -x$ modulo 12



Les inversions sont des symétries axiales généralisées

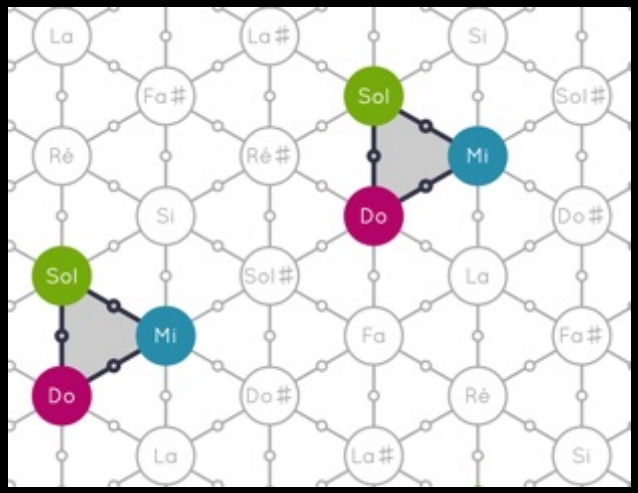
$x \xrightarrow{I_1}$ 1-x modulo 12

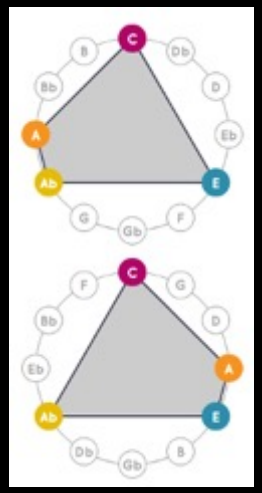


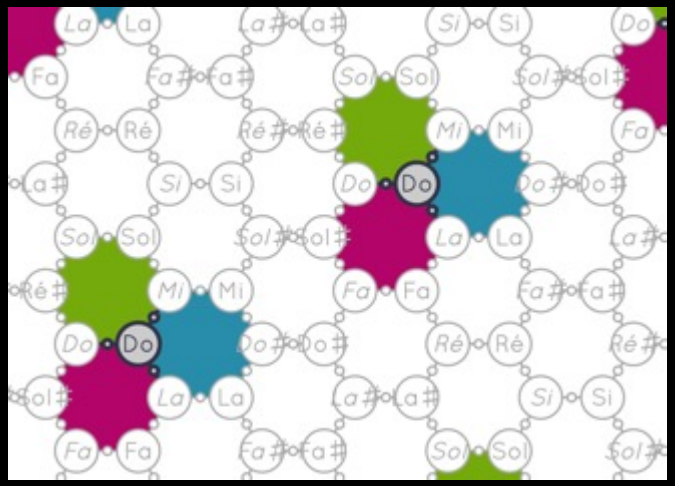
Le Tonnetz et les accords

www.morenoandreatta.com/software/

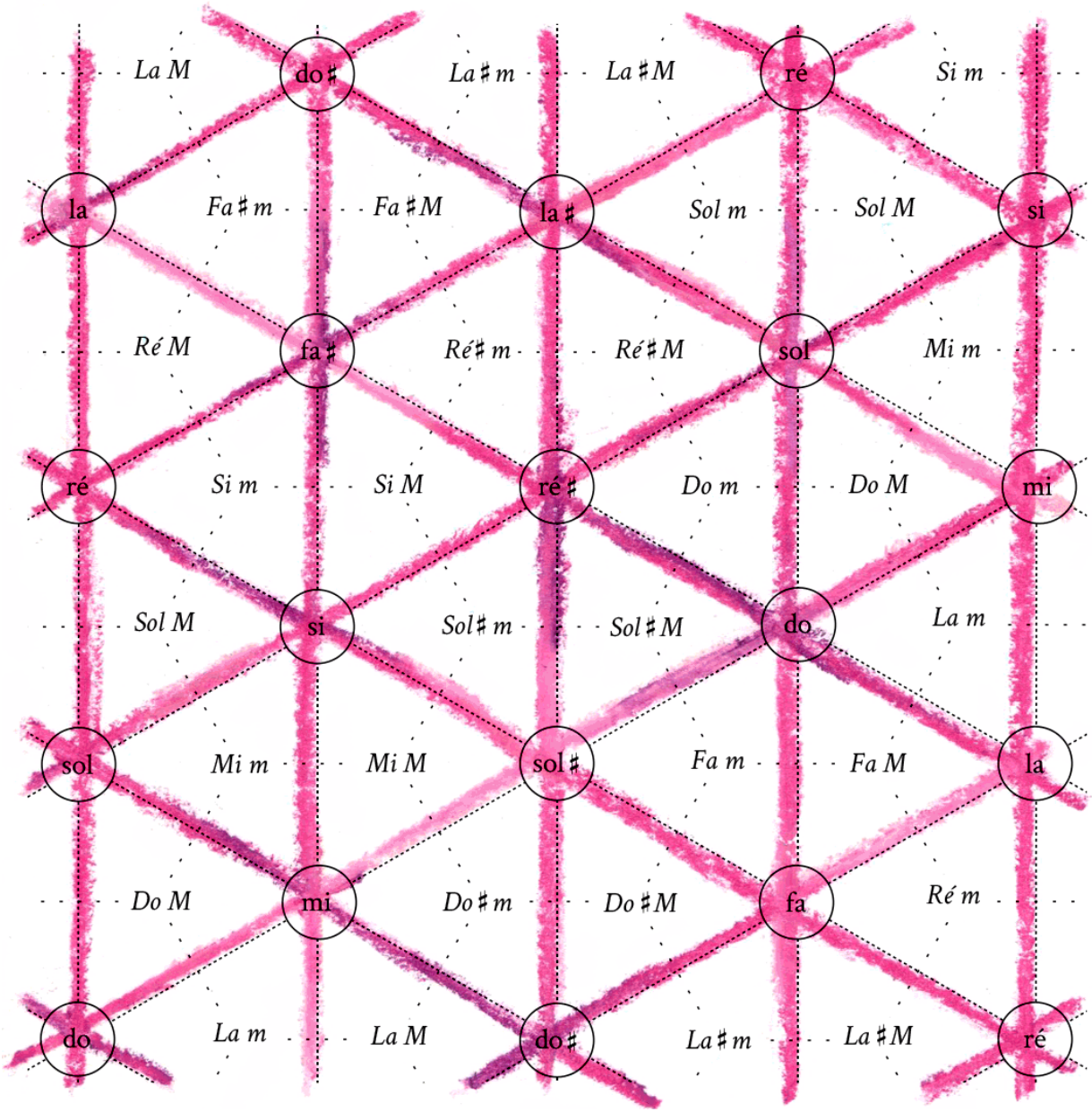
DEMO







Les deux facettes du système Tonnetz



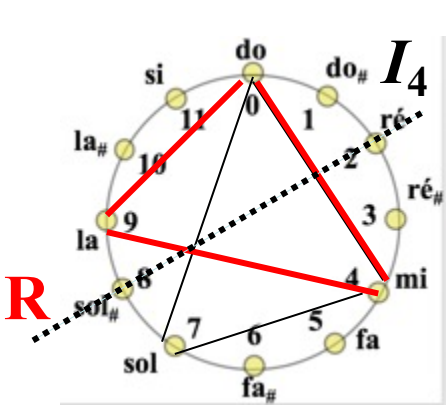
Harmonic Progressions

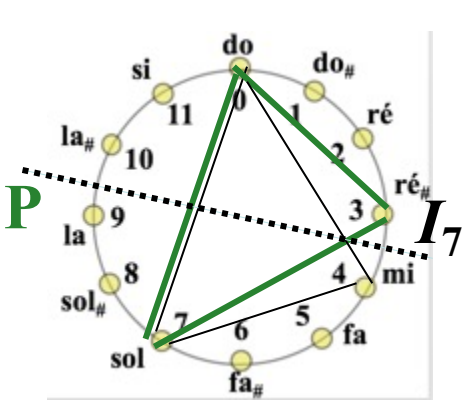
In Paolo Conte

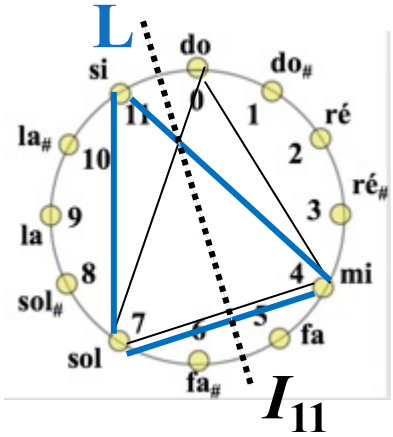
Sotto le Stelle del Jazz

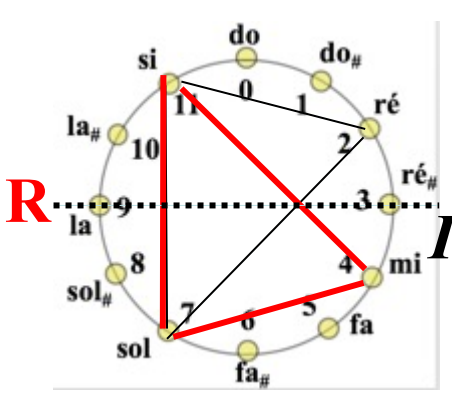
Supervision Moreno Andreatta
Modelisation Gilles Baroin 2016

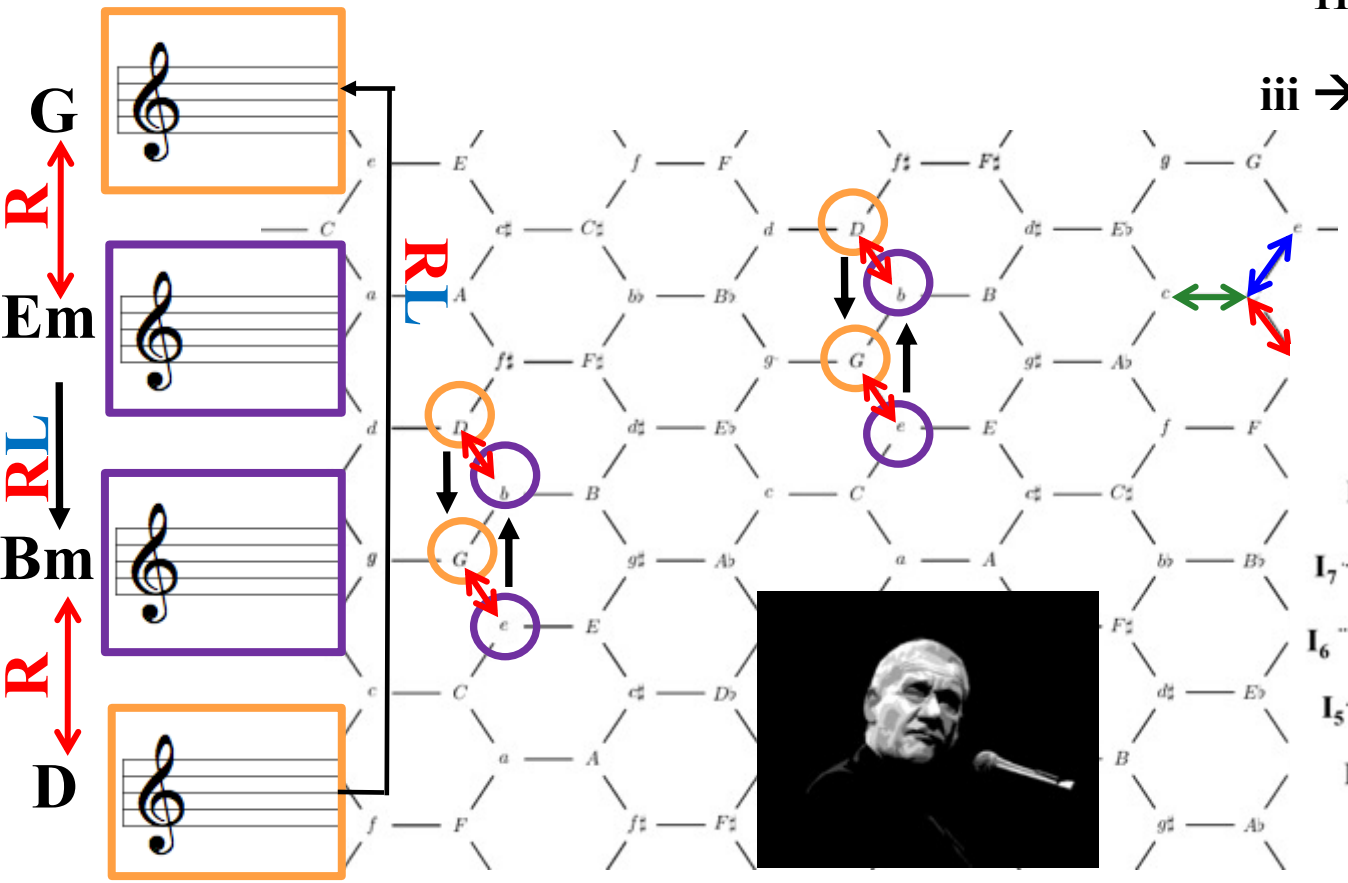
Quelques exemples analytiques : …chez Paolo Conte









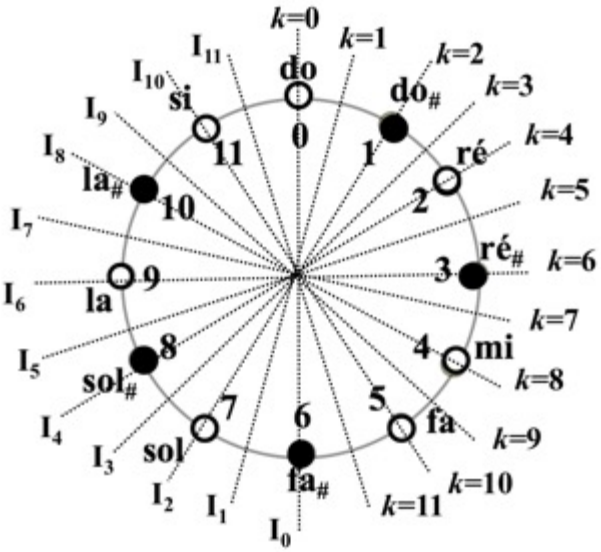


iii → V → I → vi

V → I → vi → iii

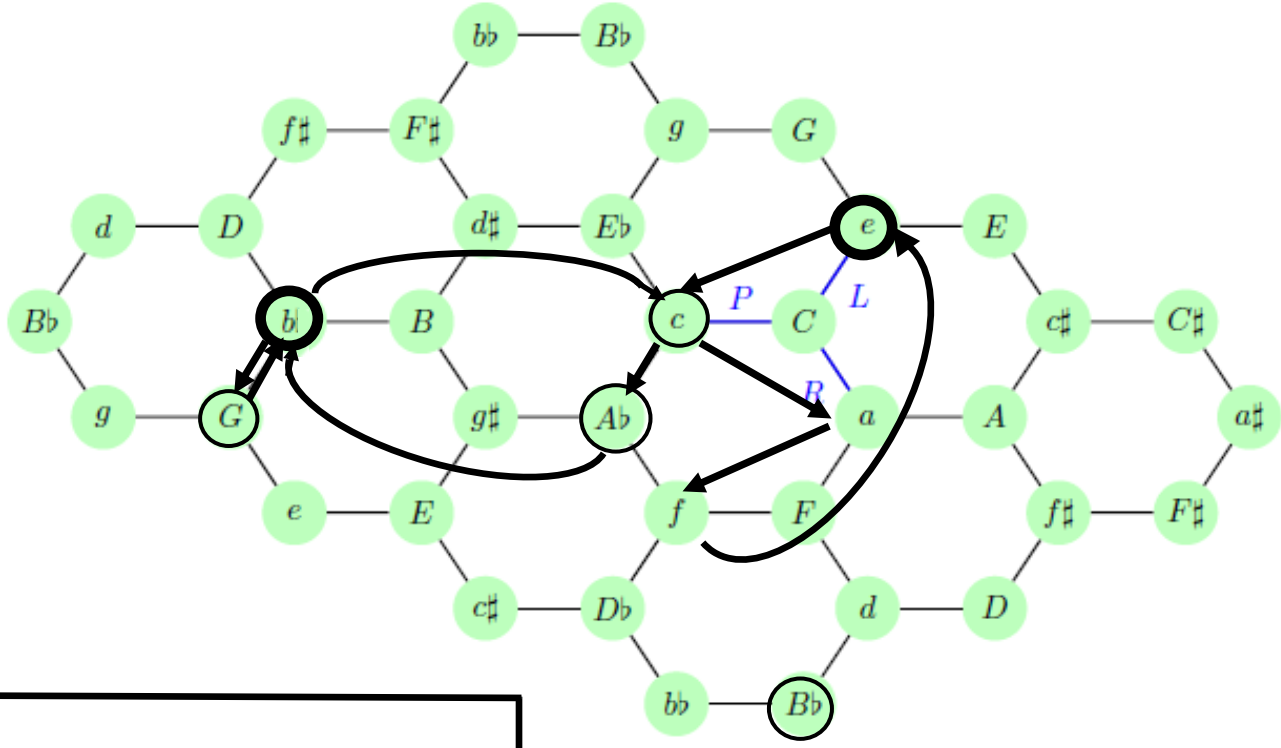
I → vi → iii → V

vi → iii → V → I



Parcours harmoniques chez Arthur H

Le Baron noir
(album *Piano solo*, 2002)

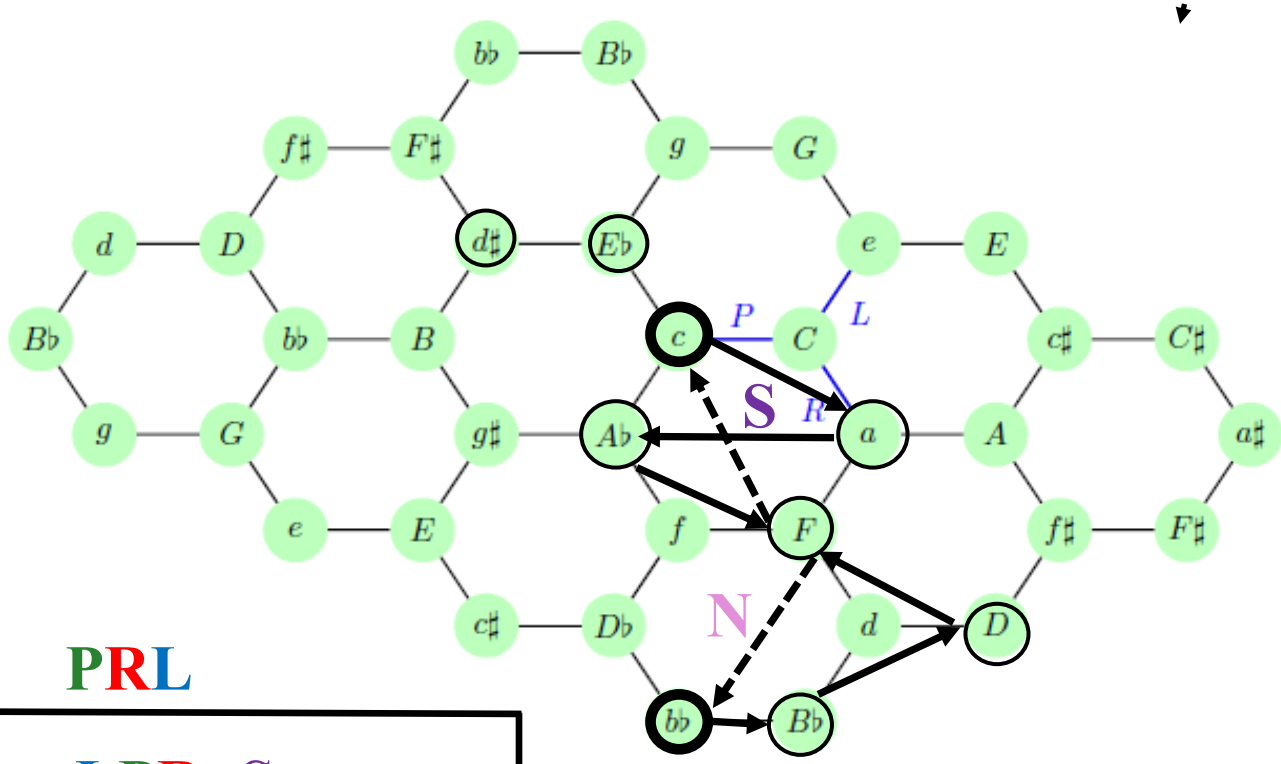


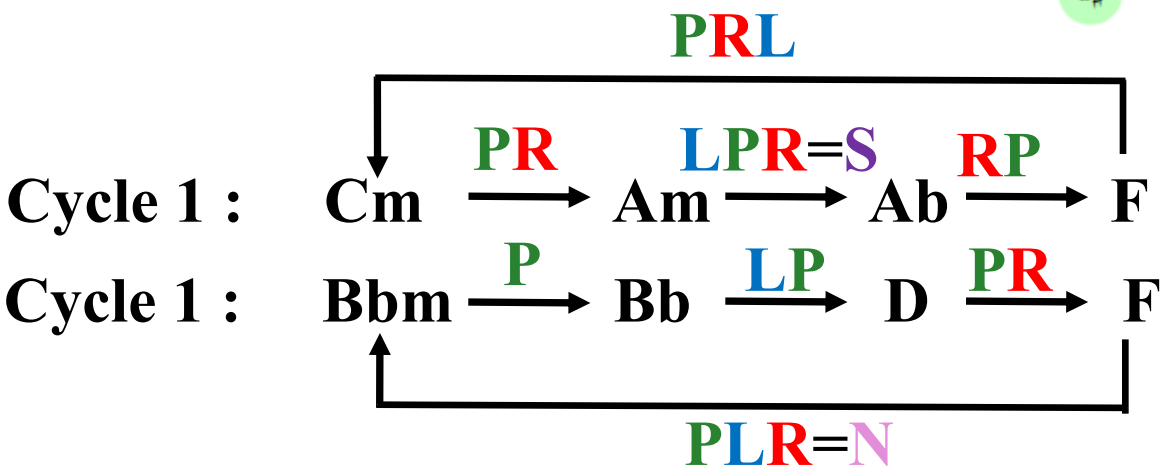
Cycle 1 : Bm —PLPR→ Cm —L→ Ab —PRP→ Bm —L→ G —L→ Bm —L→ G —L→ Bm

Cycle 2 : Em —LP→ Cm —PL→ Em —LP→ Cm —PR→ Am —LP→ Fm —PLRL→ Em

Parcours harmoniques chez Arthur H

Les Parures Secrètes
(album *Pour Madame X*, 2000)

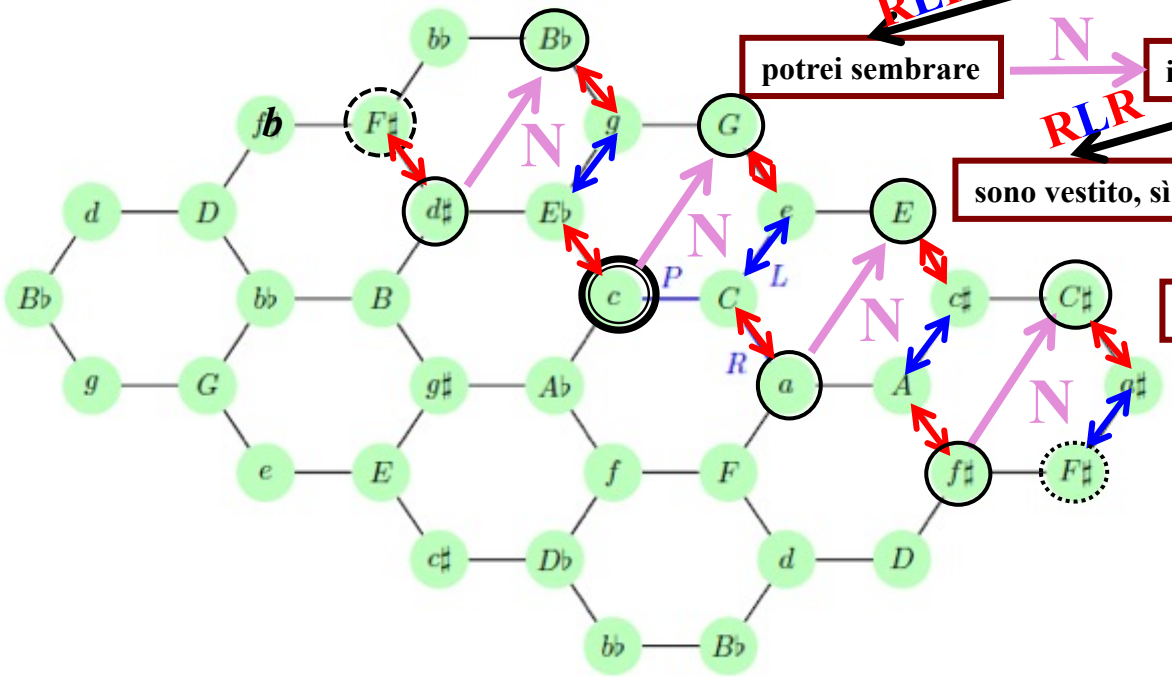




Les zig-zag chez Paolo Conte

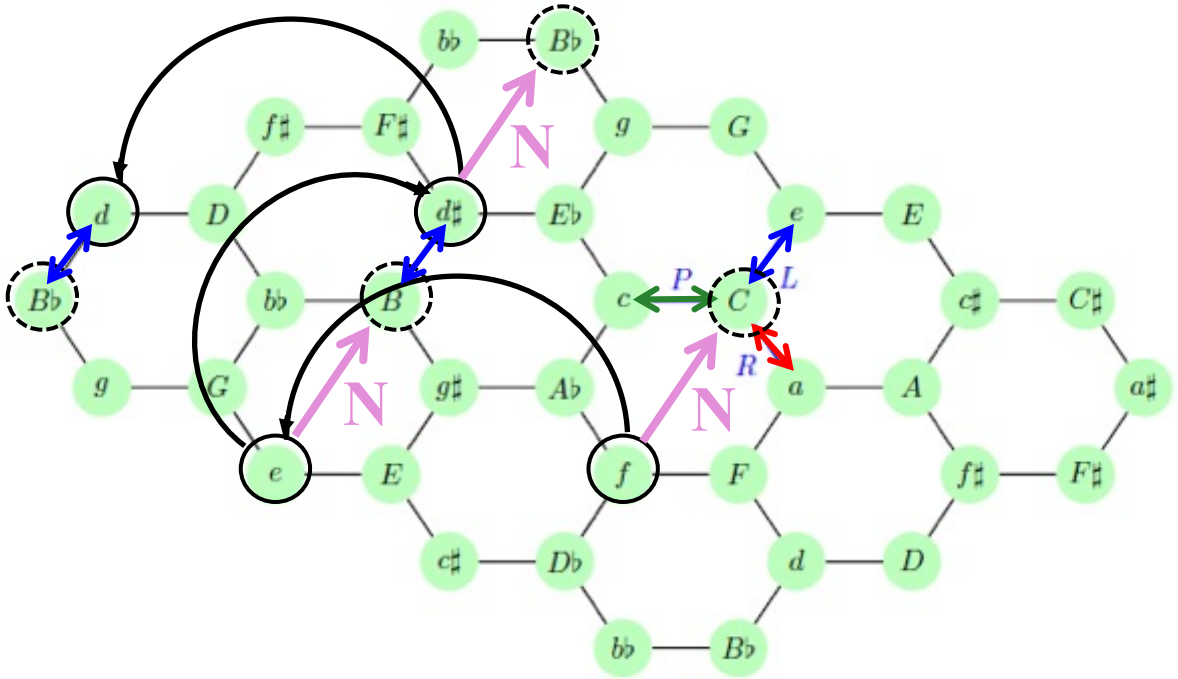
IL REGNO DEL TANGO (Paolo Conte)

Non son neanche del —N→ paese
—RLR→ ho una valigia di —N→ carton
—RLR→ sono vestito, sì —N→ in borghese,
—RLR→ ma dentro c'è —N→ il bandoneon...
—RLR→ potrei sembrare —N→ in borghese,
—RLR→ sono vestito, sì —N→ un ragioniere,
—RLR→ anche un geometra —N→ potrei,
—RLR→ ma un tango sento io —N→ gridare
—RLR→ in fondo ai sentimenti —N→ miei



Modulations et transformations géométriques

01:29



FRANGIFLUTTI
(L. Mello / M. Andreatta)

Nel rifrangere i nostri cuori,
– gente nata che esige cura –
sono stato tra i meno bravi
nella tua favola insicura.

NL

Ora è tardi lo so oramai
venti freddi come una fiaba
però tengo a cantare il canto
Eva sola che resta maga.

NL

Eva cenno di ribellione
Eva fascio pronto all'addio
Eva donna che sa parlare
Al silenzio fatto d'oblio.

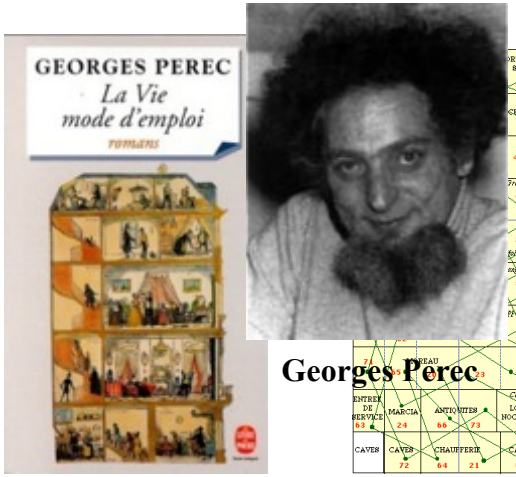
NL

Sai parole da incatenare
sai sorridere per piacere
sai scappare senza tornare
sai ferire e sai far l'amore.

f — N — *C* — L — *e* — N — *B* — L — *d#* — N — *Bb* — L — *d*

Les contraintes dans l'art : l'OuLiPo (Ouvroir de Littérature Potentielle)

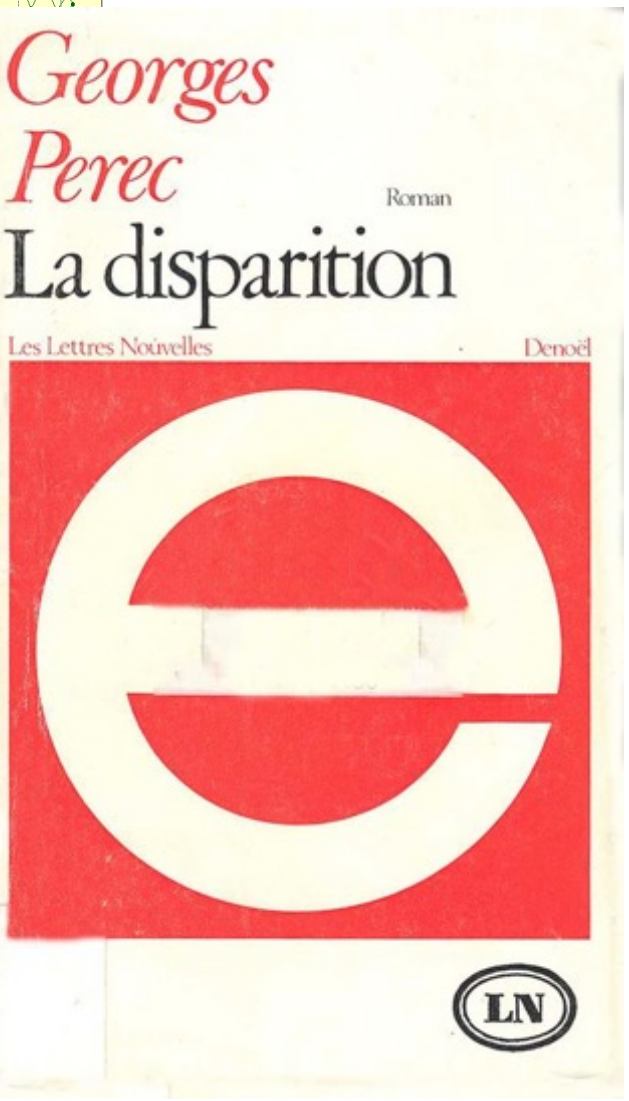
Cent mille milliards de poèmes, 1961



Georges Perec

La vie mode d'emploi,

oulipo

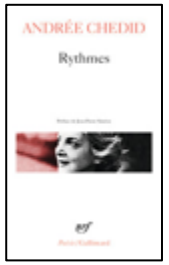


Raymond Queneau

Italo Calvino

Le Château des destins croisés, 1969

A nouveau le zig-zag de Beethoven (mais en modalité mineure)



A part (Andrée Chedid, poème tiré du recueil *Rythmes* Collection Poésie/Gallimard (n. 527), Gallimard, 2018)

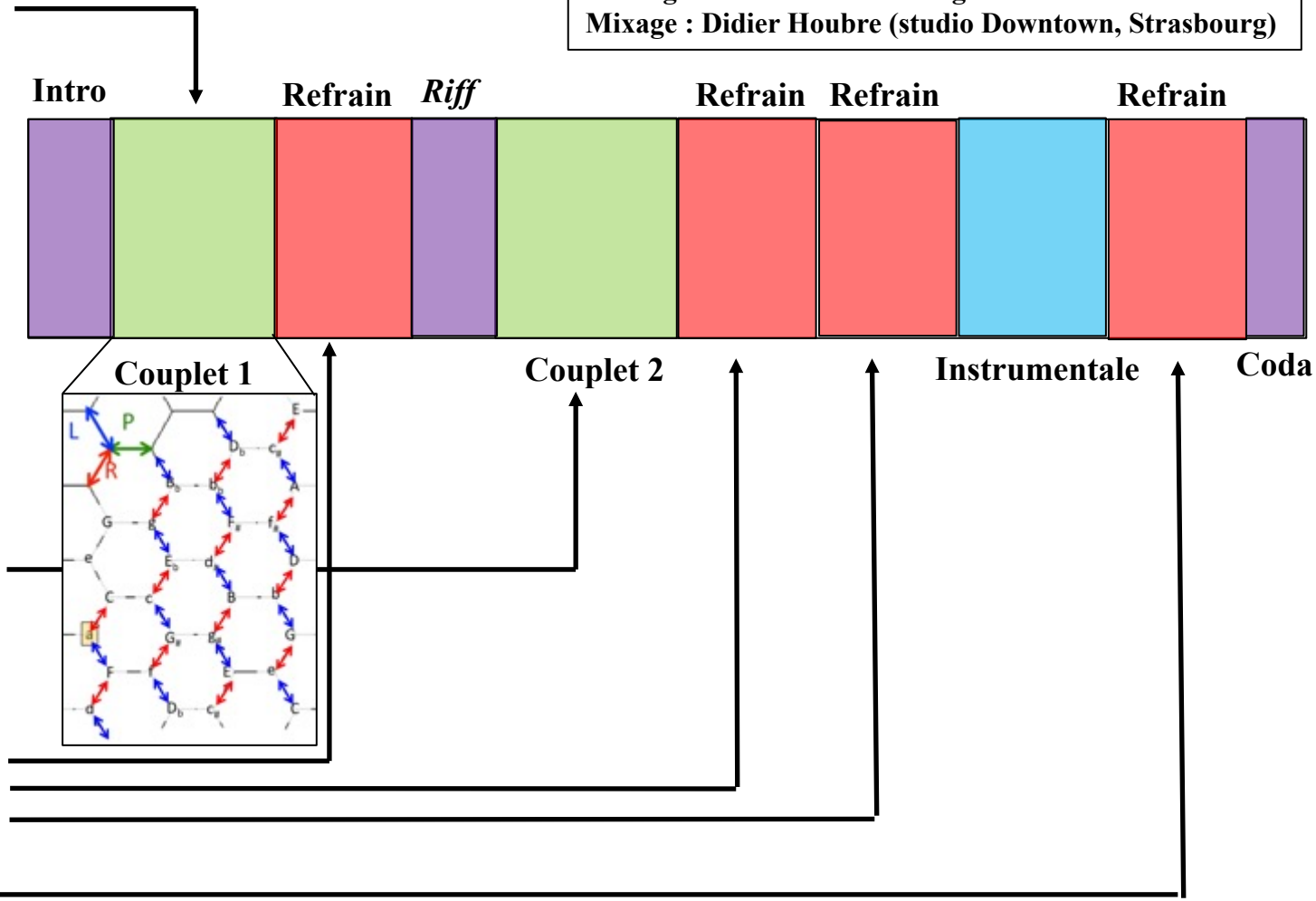
Composition : Moreno Andreatta
Arrangement : Benoît Messinger
Mixage : Didier Houbre (studio Downtown, Strasbourg)

À part le temps
Et ses rouages
À part la terre
En éruptions
À part le ciel
Pétrisseur de nuages
À part l'ennemi
Qui génère l'ennemi

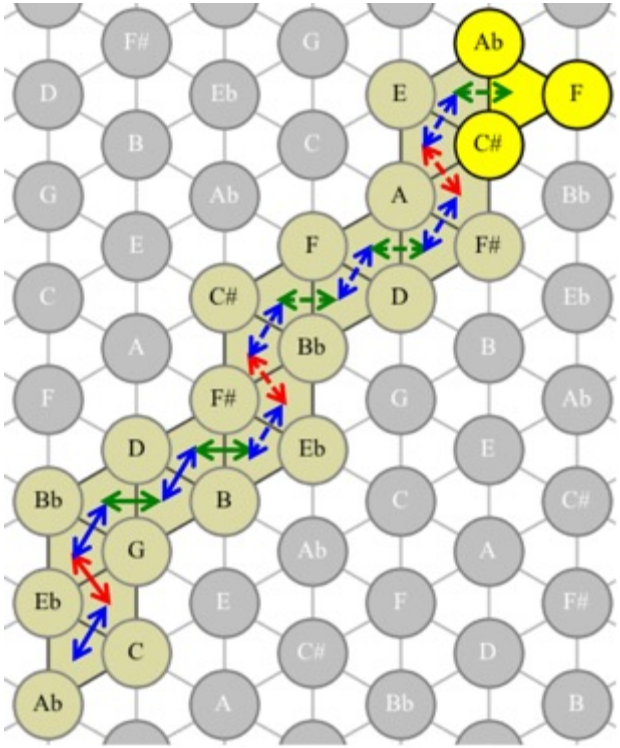
À part le désamour
Qui ronge l'illusion
À part la durée
Qui moisit nos visages

À part les fléaux
À part la tyrannie
À part l'ombre et le crime
Nos batailles nos outrages

Je te célèbre ô Vie
Entre cavités et songes
Intervalle convoité
Entre le vide et le rien



La sera non è più la tua canzone (sur un texte de Mario Luzi)

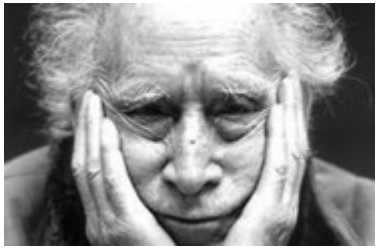


La sera non è più la tua canzone,
è questa roccia d'ombra traforata
dai lumi e dalle voci senza fine,
la quiete d'una cosa già pensata.

Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d'una cosa conosciuta,
per nome hai una parola ch'è passata
nell'intimo del cuore e s'è perduta.

Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell'essere, (compiuta).

L'aria tace ed il tempo dietro a te
si leva come un'arida montagna
dove vaga il tuo spirito e si perde,
un vento raro scivola e ristagna.



M. Luzi (1914-2005)

Le soir n'est plus ta chanson,
c'est ce rochet d'ombre transpercé
par les lumières et les voix sans fin,
la paix d'une chose déjà pensée.

Ah, cette lumière vive et claire vient
uniquement de toi, tu es si proche
du vrai d'une chose connue,
tu as pour nom une parole qui est passée
dans l'intimité du cœur où elle s'est
perdue .

Tombé est plus qu'un signe de la vie,
tu te reposes, du voyage tu es revenue
à l'intérieur de toi même, tu es
descendue dans cette
pure substance qui est si tienne,
si éloignée
dans le silence de l'être, achevée.

L'air se tait et le temps derrière toi
se lève tel une montagne aride
où plane ton esprit et se perd,
un vent rare glisse et stagne.

(tr. Antonia Soulez, philosophe et poète)

Musique : M. Andreatta
Arrangements et mixage : M. Bergomi & S. Geravini
(*Perfect Music Production*)
Mastering : A. Cutolo (Massive Arts Studio, Milan)

Utilisation inconsciente (?) des symétries chez *Muse*

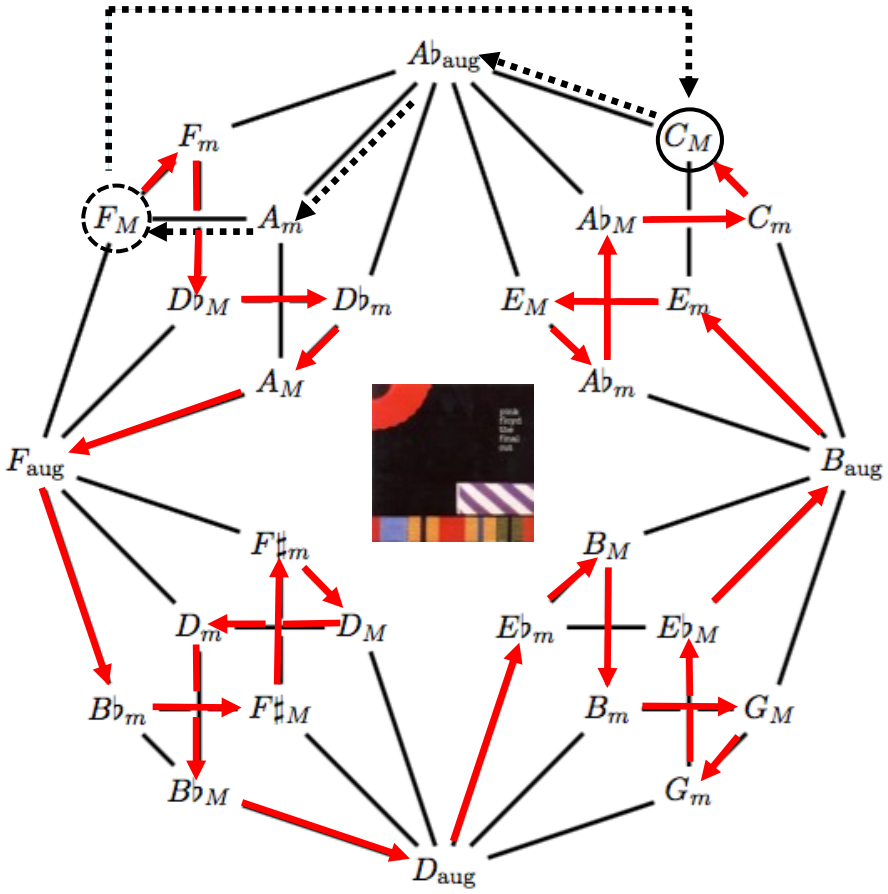
“Take a bow” (*Black Holes and Revelations*, 2006)

R
L
P
M

Hexachord (Louis Bigo, 2013)

axe temporel

The Gunner's Hamiltonian Dream (une expérience *oumupienne* autour de Pink-Floyd)



The Gunner's dream (R. Waters, 1983 / M. Andreatta, 2018)

C C+
Floating down through the clouds
Am F
Memories come rushing up to meet me now.
Fm
In the space between the heavens
C# C#m
and in the corner of some foreign field
A F+ Bbm
I had a dream.
F# F#m D Dm
I had a dream.
Bb
Good-bye Max.
D+
Good-bye Ma.
Ebm B
After the service when you're walking slowly to the car
Bm G
And the silver in her hair shines in the cold November air

Gm
You hear the tolling bell
Eb
And touch the silk in your lapel
G+ Em E G#m
And as the tear drops rise to meet the comfort of the band
G# Cm
You take her frail hand
C
And hold on to the dream.

The three Hamiltonian Cycles (C_M = C, C_m = Cm, C_{aug} = C+)

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->Bbm-->F#-->F#m-->D-->Dm-->Bb-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->F#-->Bbm-->Bb-->Dm-->D-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

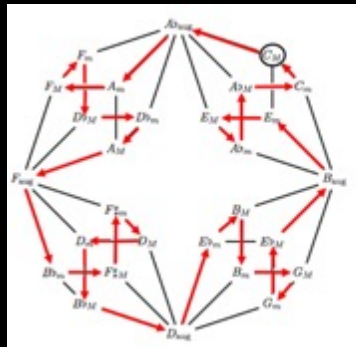
C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->D-->Dm-->Bb-->Bbm-->F#-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Cm-->G#-->G#m-->E-->Em-->C

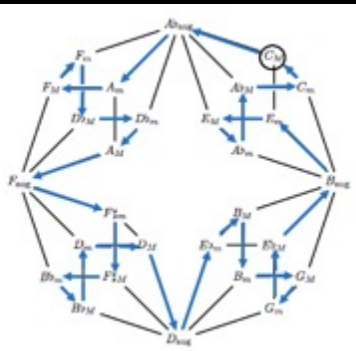
HamilFloyd

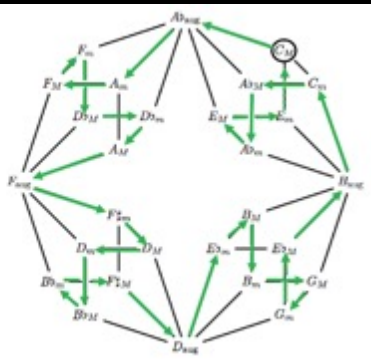
The Gunner's Hamiltonan Dream

Moreno Andreatta
Gilles Baroin 2022

➔**https://www.youtube.com/watch?v=nz5TYob02B4&ab_channel=MatheMusic4D**

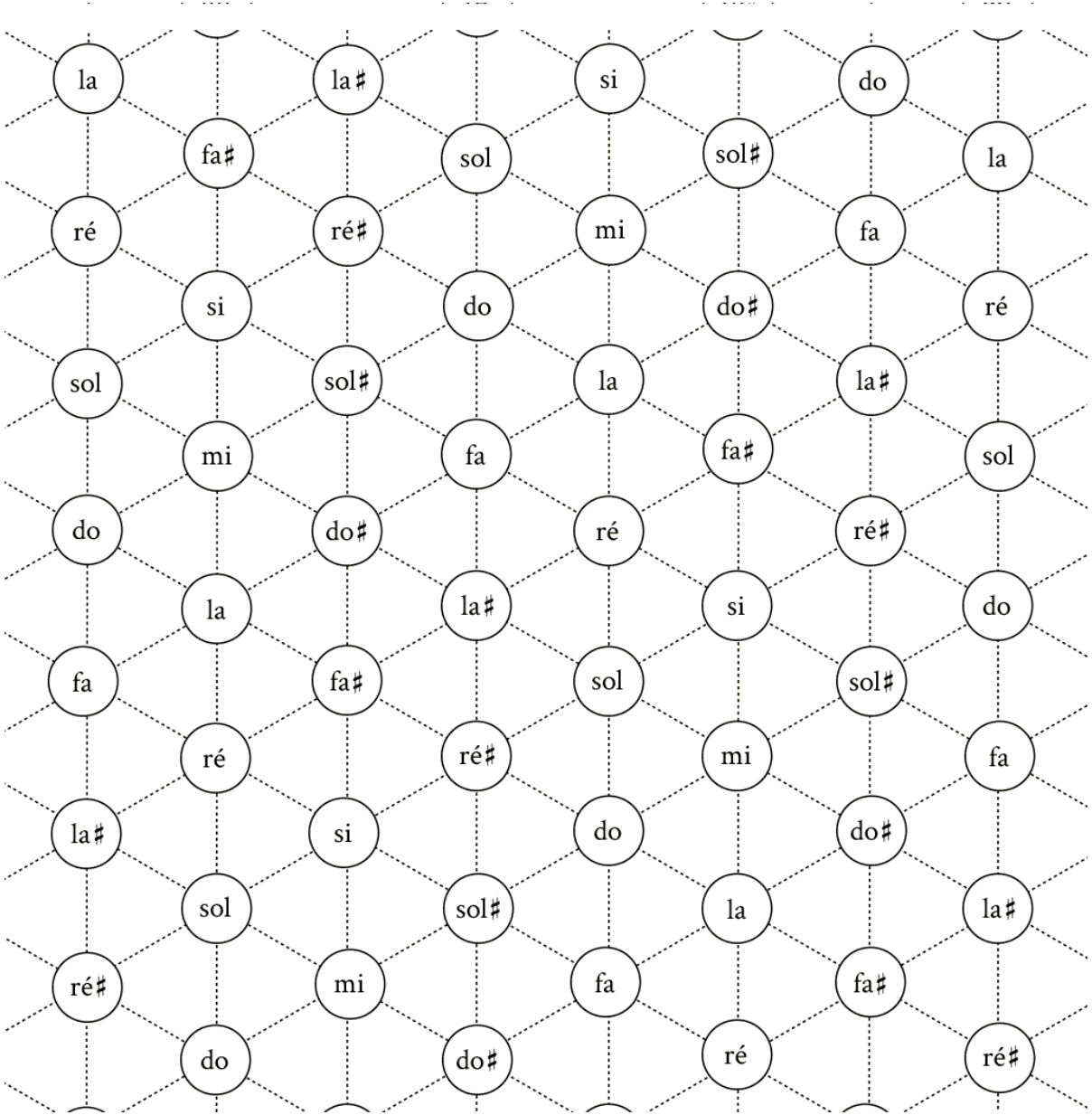






Retour sur le Tonnetz et sa topologie

Quelle est la forme géométrique du Tonnetz ?



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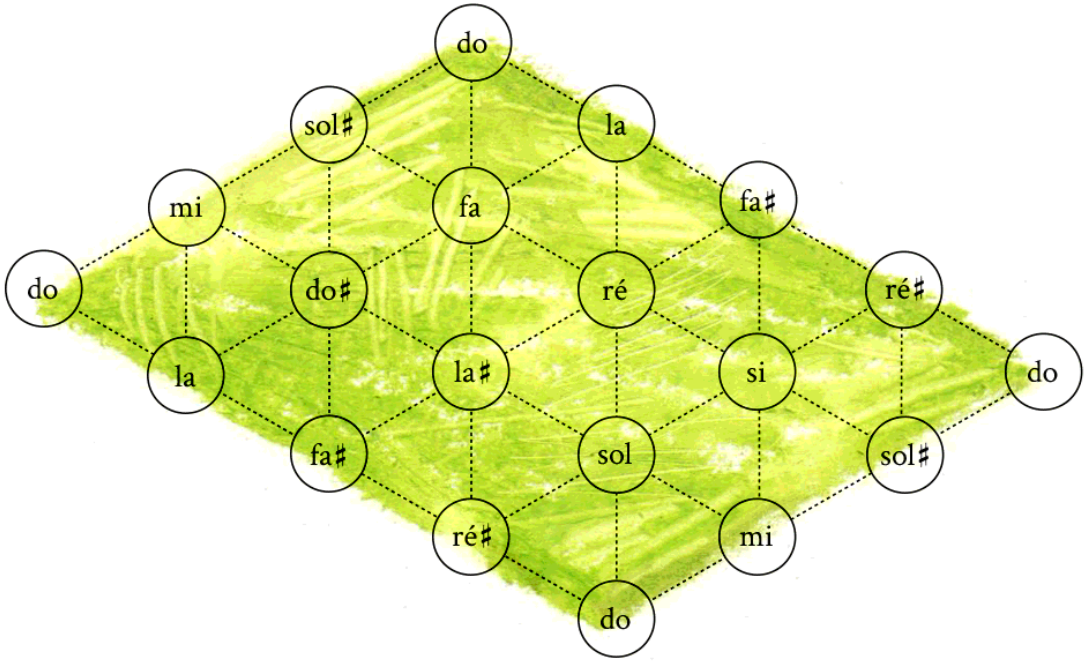

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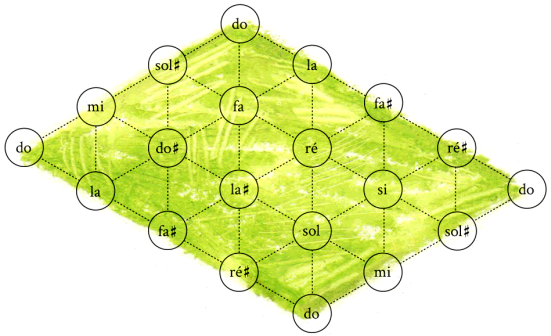

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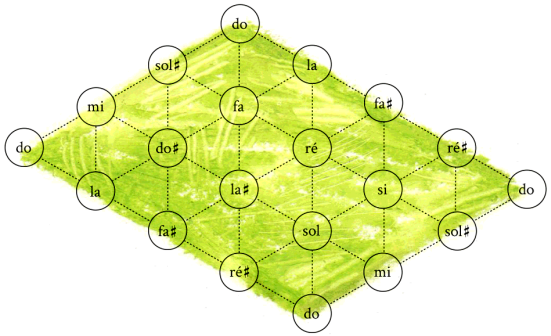
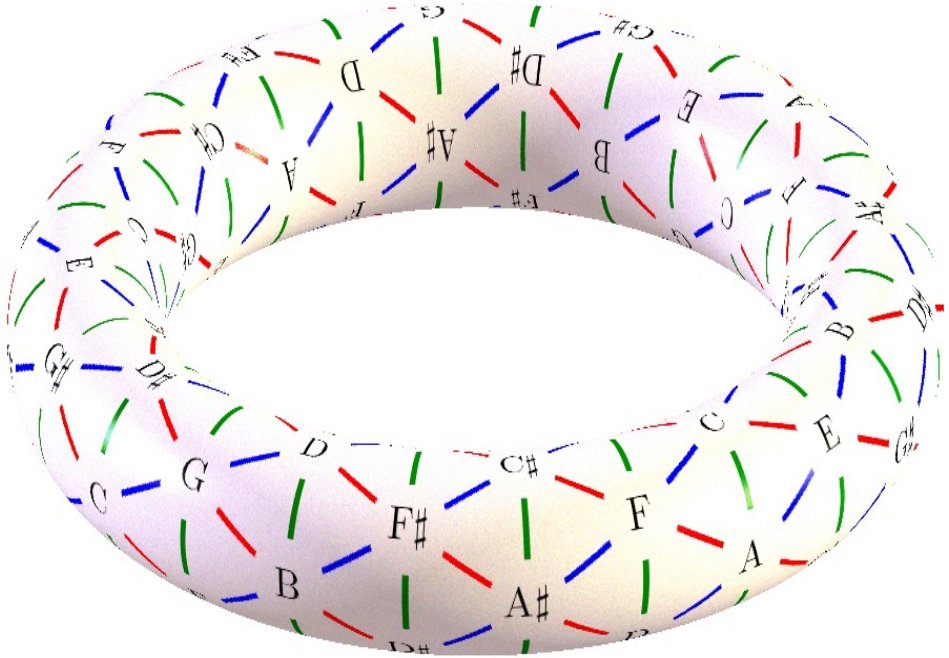
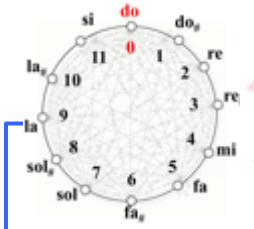
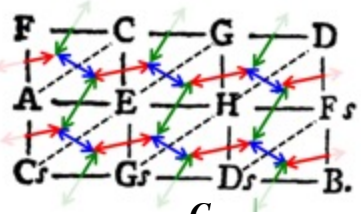


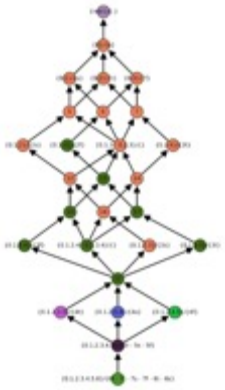


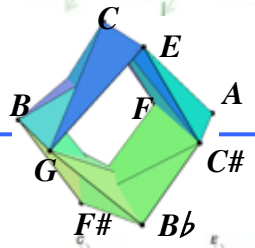
Retour sur la dualité majeur/mineur : l'harmonie négative

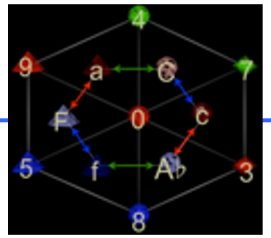




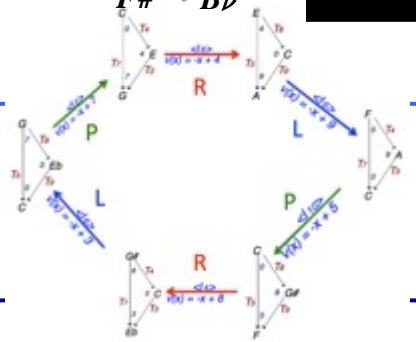
Modèles algébriques

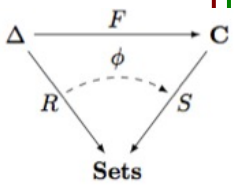






Modèles géométriques

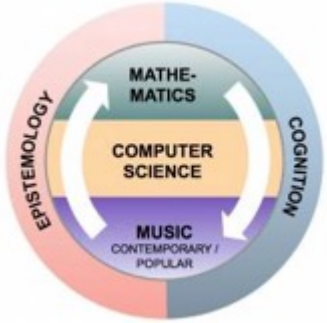




Modèles catégoriels

Modèles computationnels

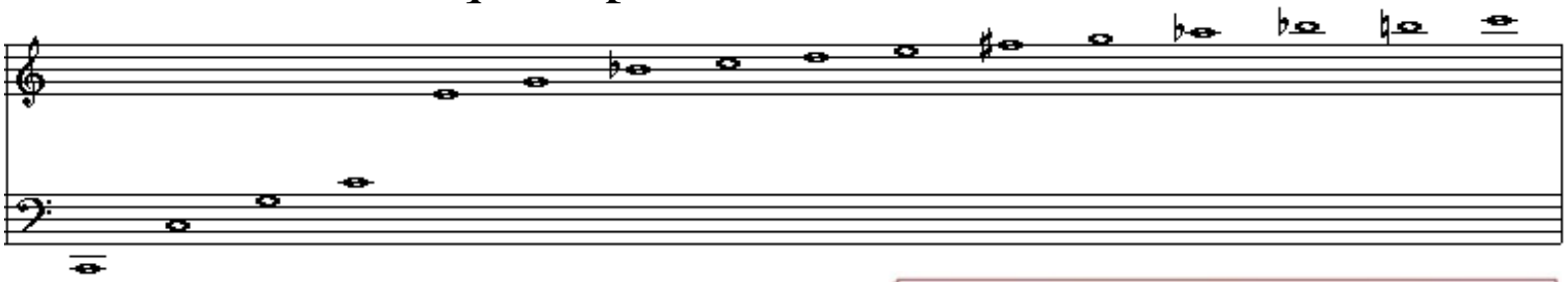
Modèles cognitifs

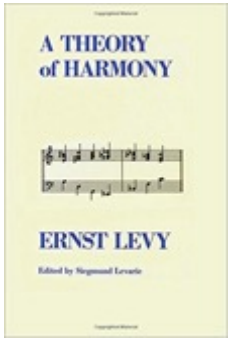


Approches '*symboliques*'

La « Negative/Symmetric Harmony » (de Jacob Collier/Steve Colemann)

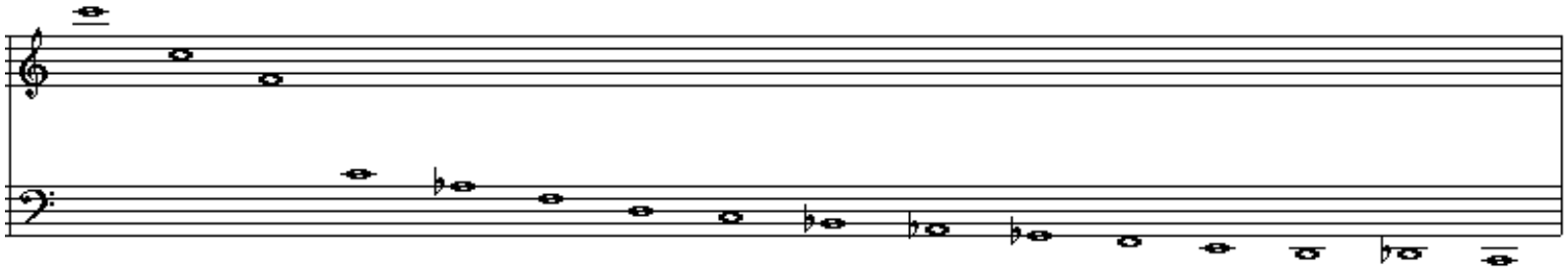
La série des harmoniques supérieurs





"The raw material of theories are facts. The raw materials of musical theory is music. Music is not, as some contemporary acousticians would like us to believe, 'something that happens in the air.' It is something that, first and last, happens in the soul."

Ernst Lévy
(1895-1981)



Steve Coleman

La série (imaginaire) des harmoniques inférieurs

C. Corea, J. Collier & H. Hancock

- Ernst Lévy, *A Theory of Harmony*, Albany, New Yortk, 1985
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La « Negative/Symmetric Harmony » (de Jacob Collier/Steve Colemann)

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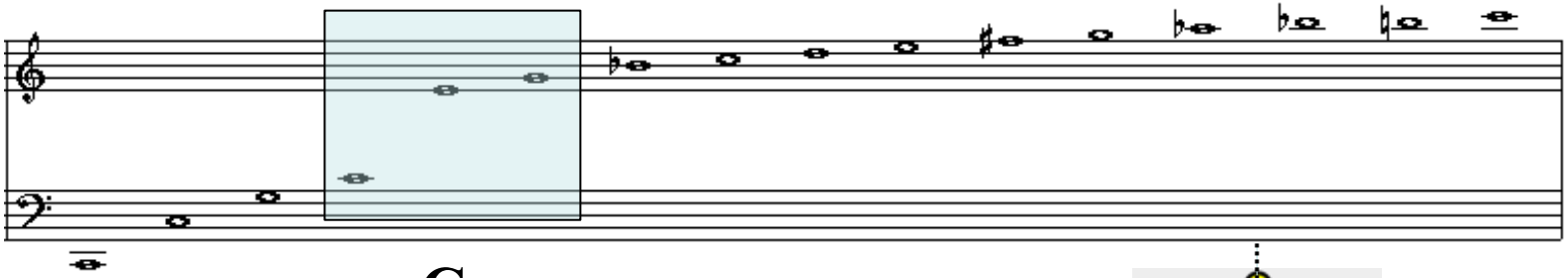

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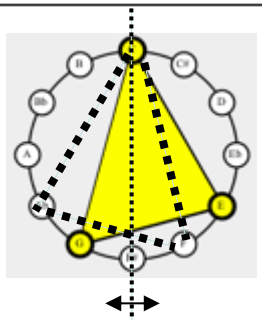

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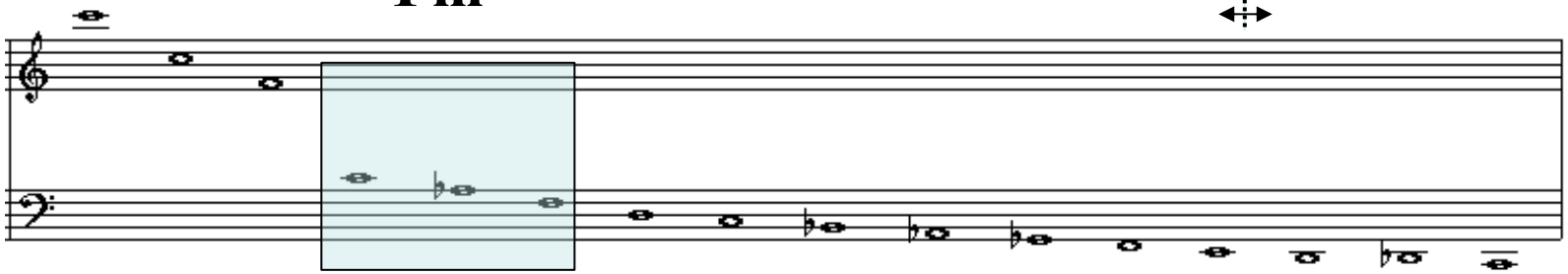

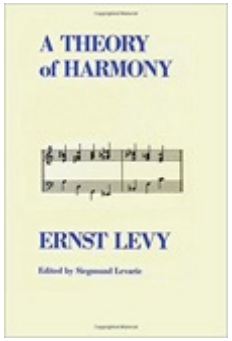

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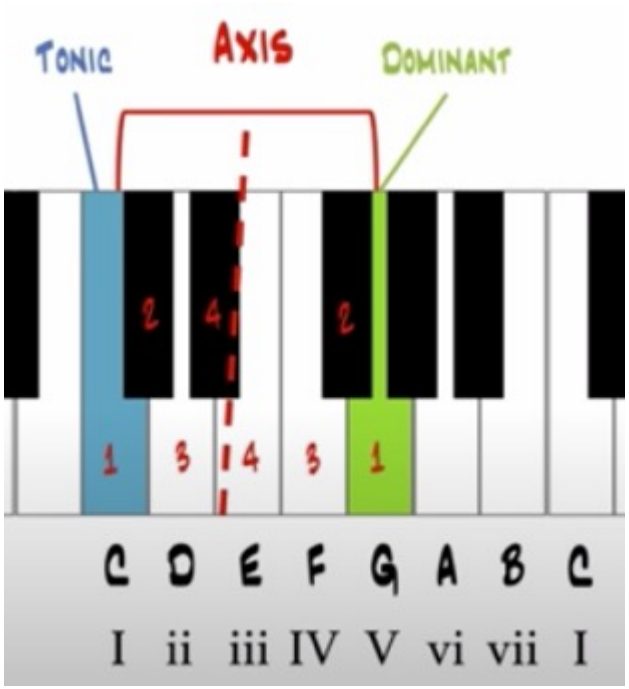
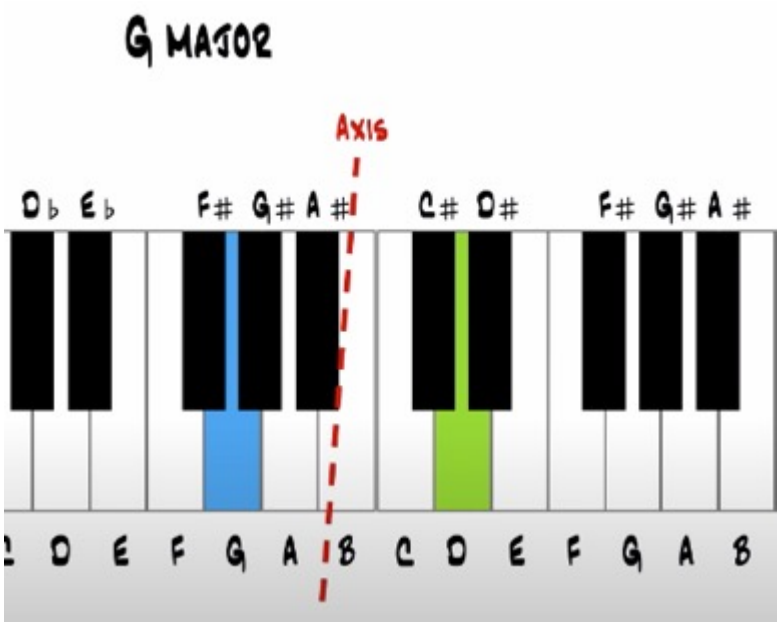
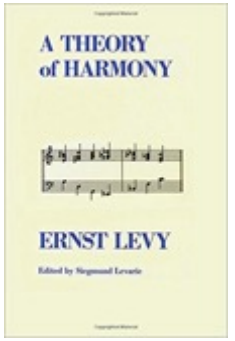


L'axe de symétrie dépend de la tonalité !





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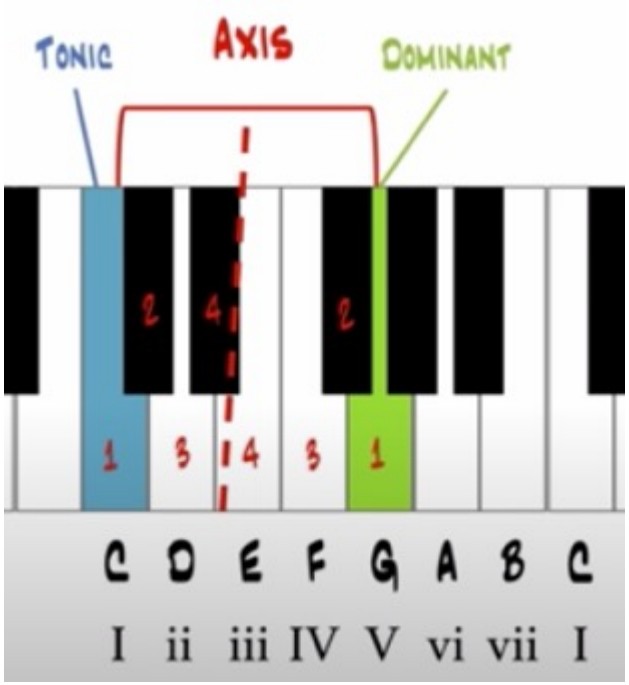

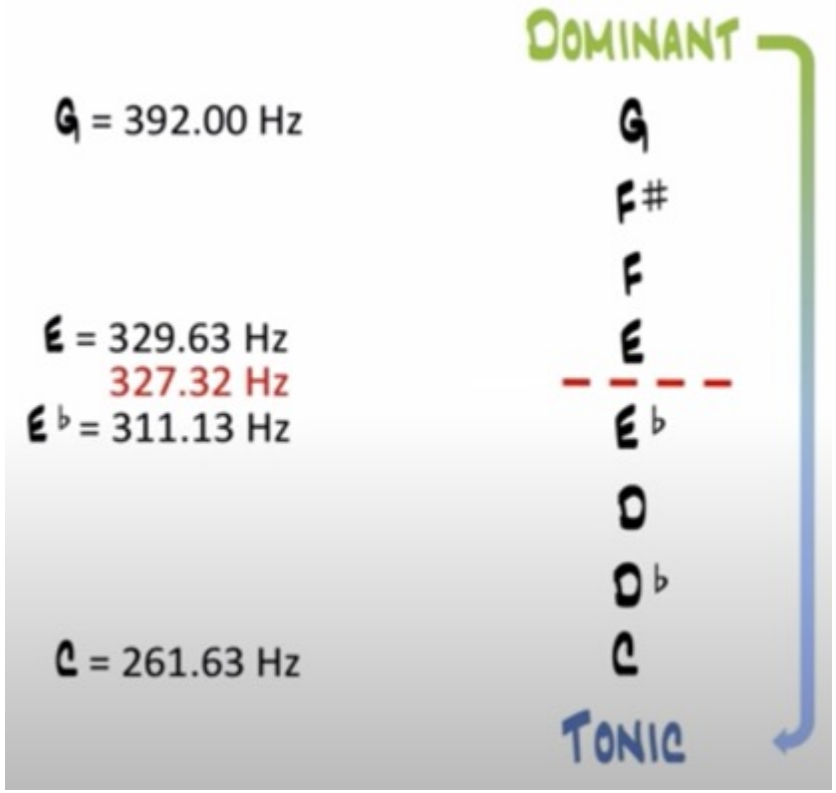

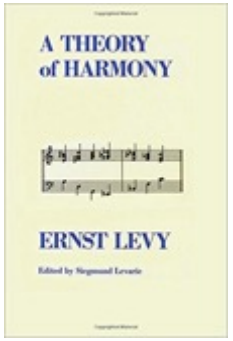

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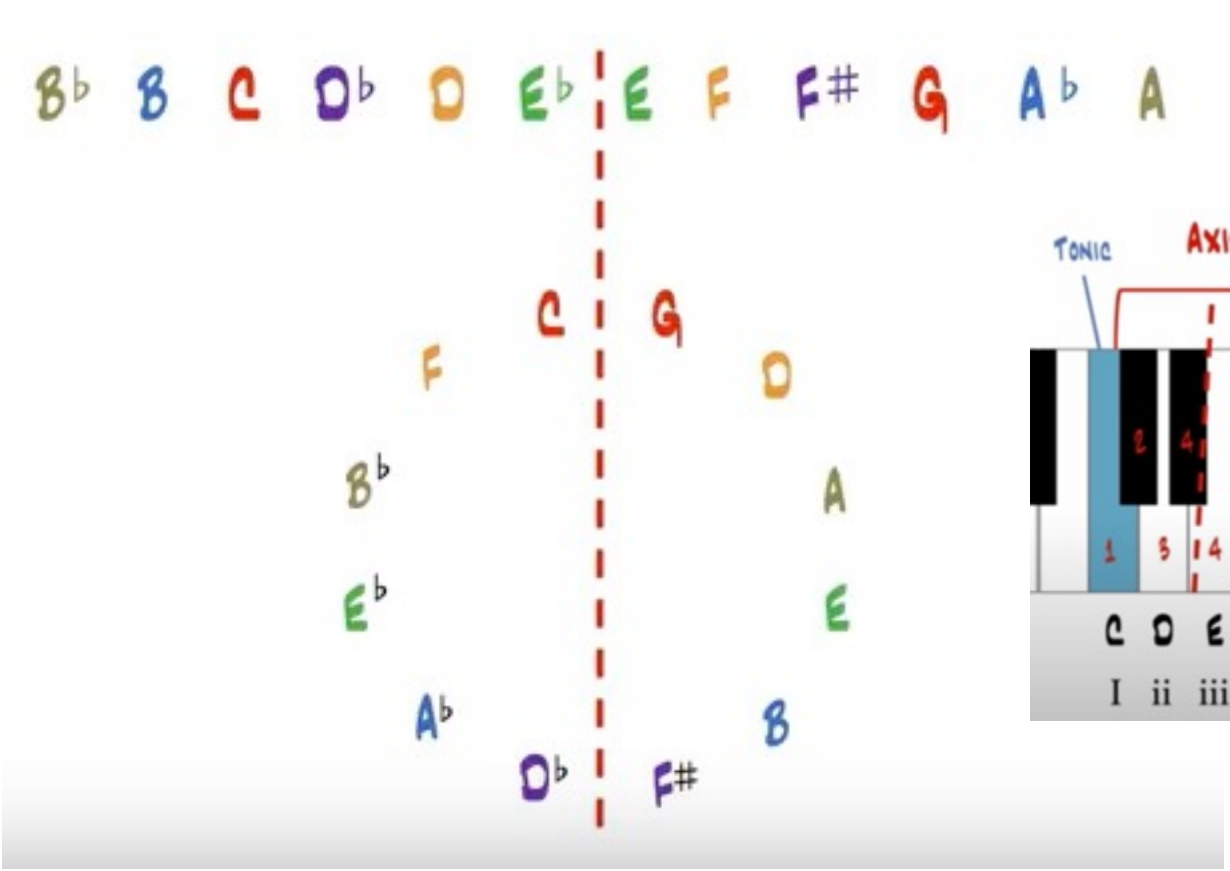

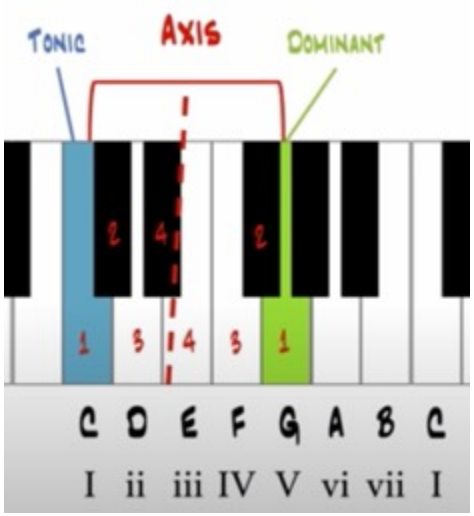

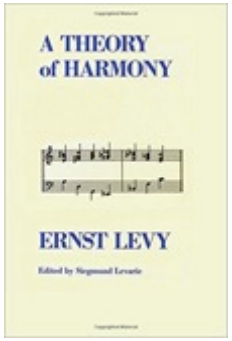

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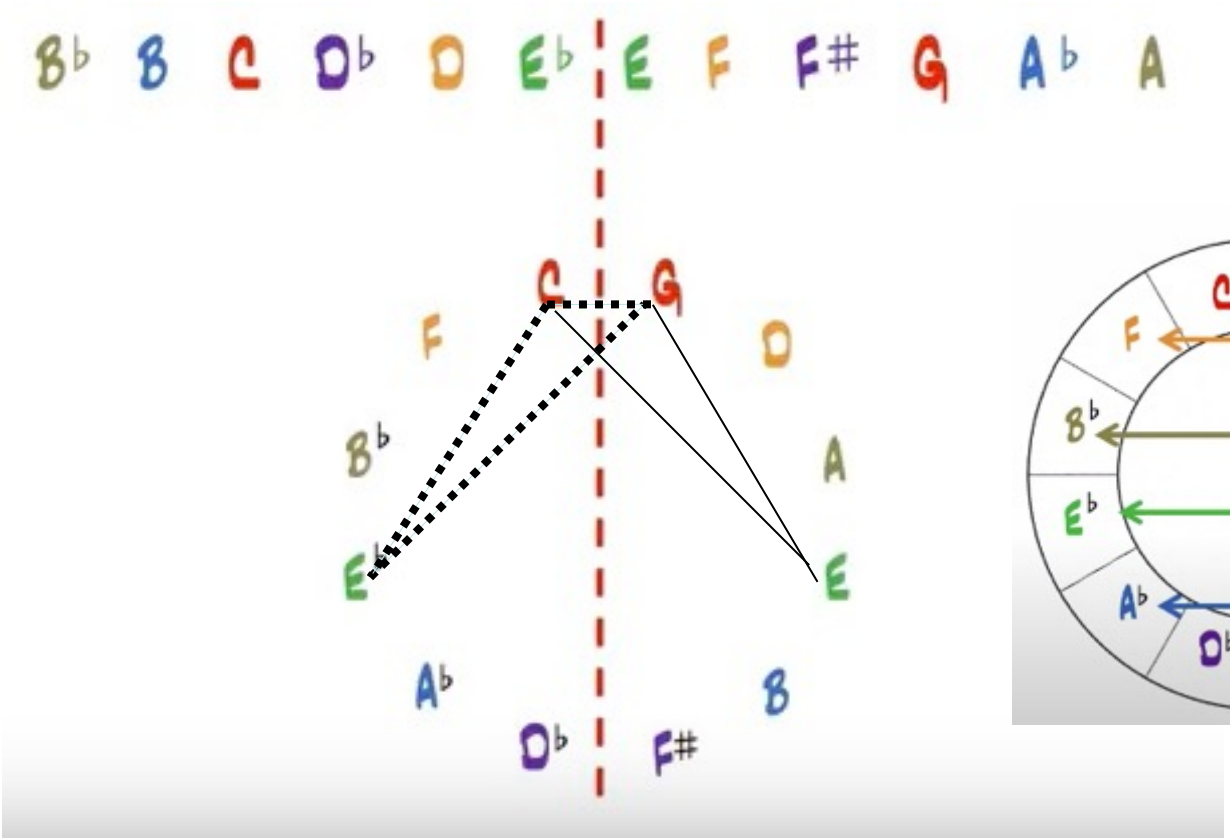

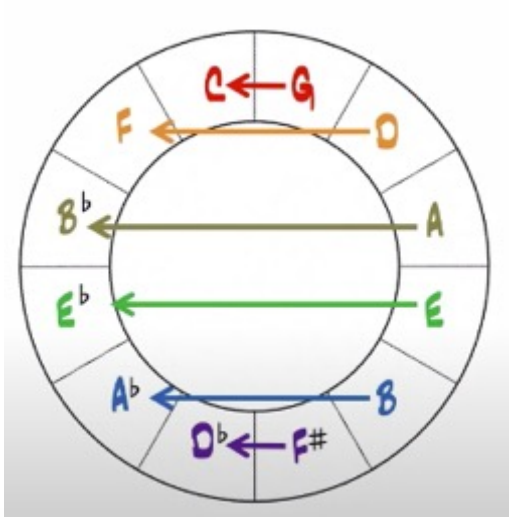

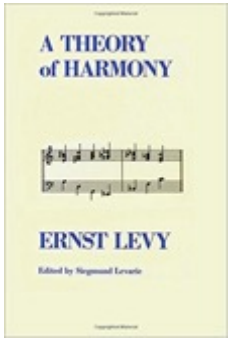

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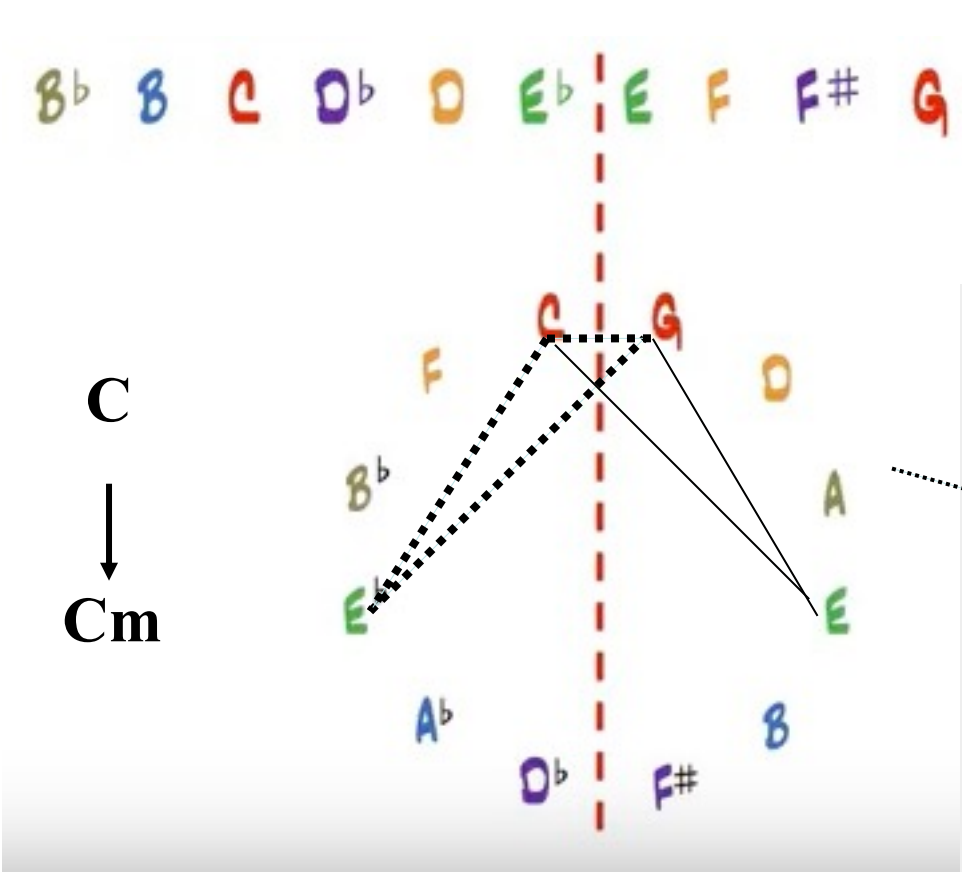

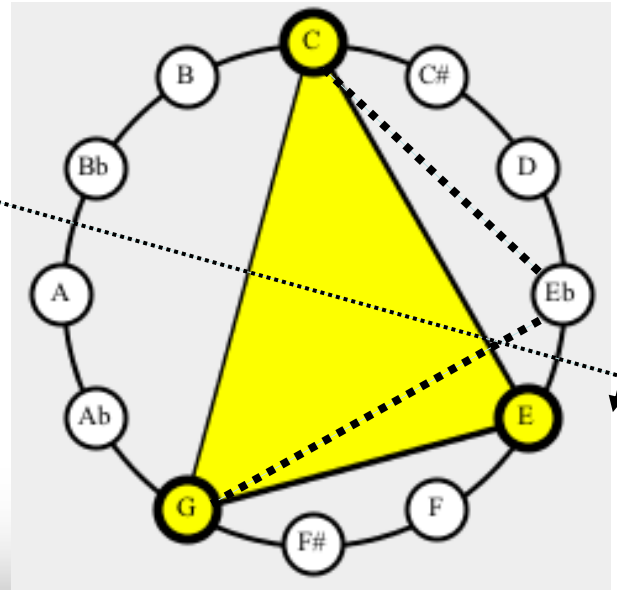

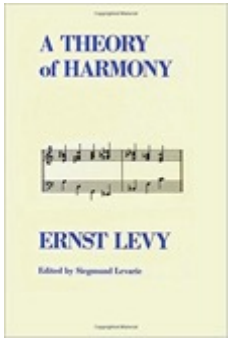

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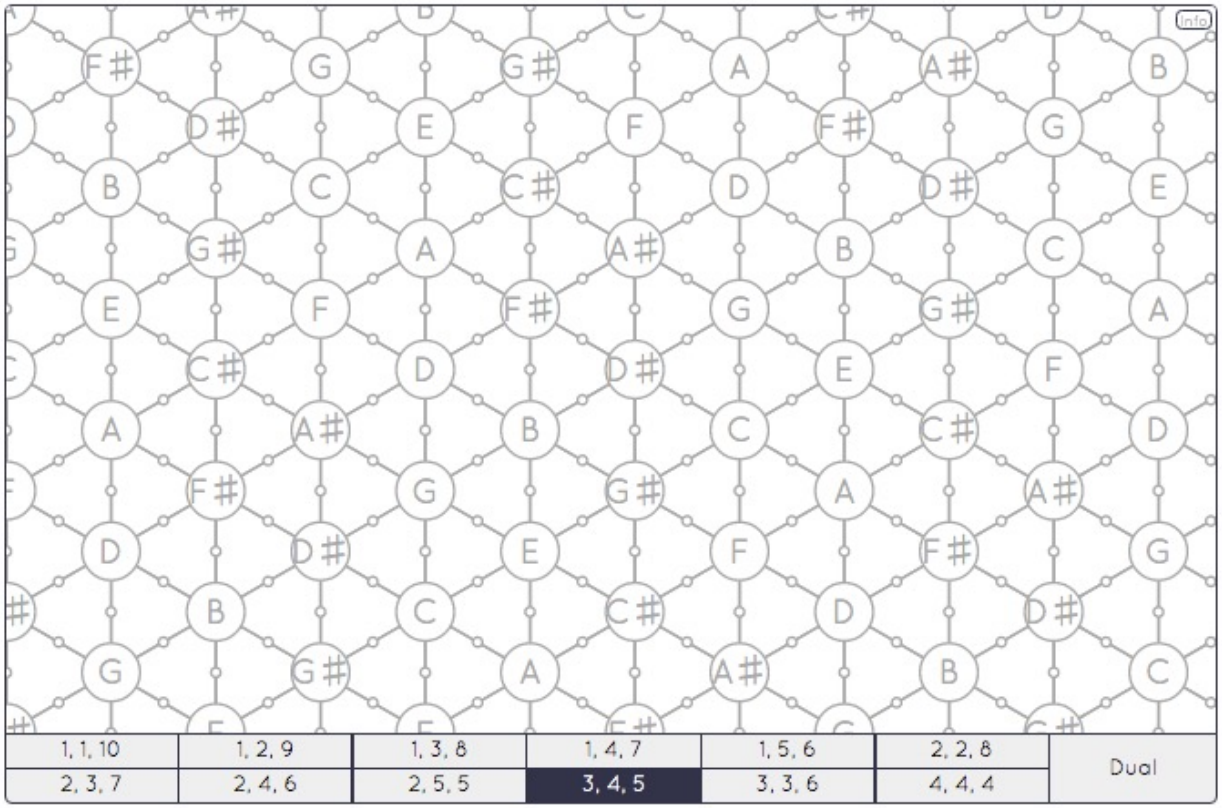
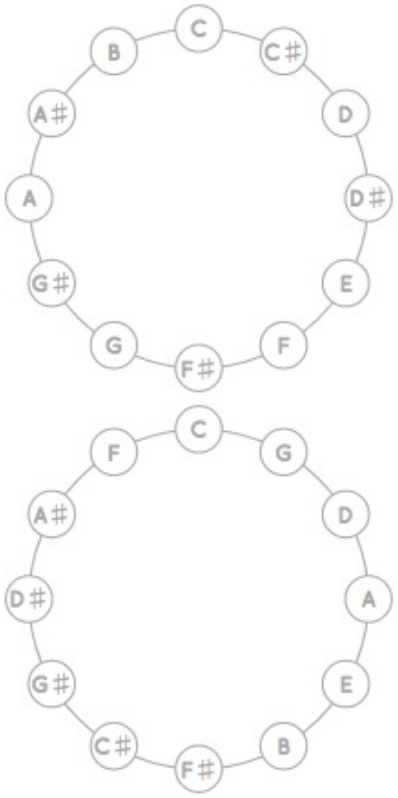
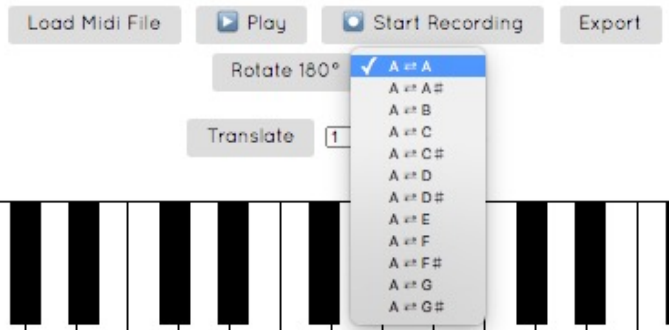
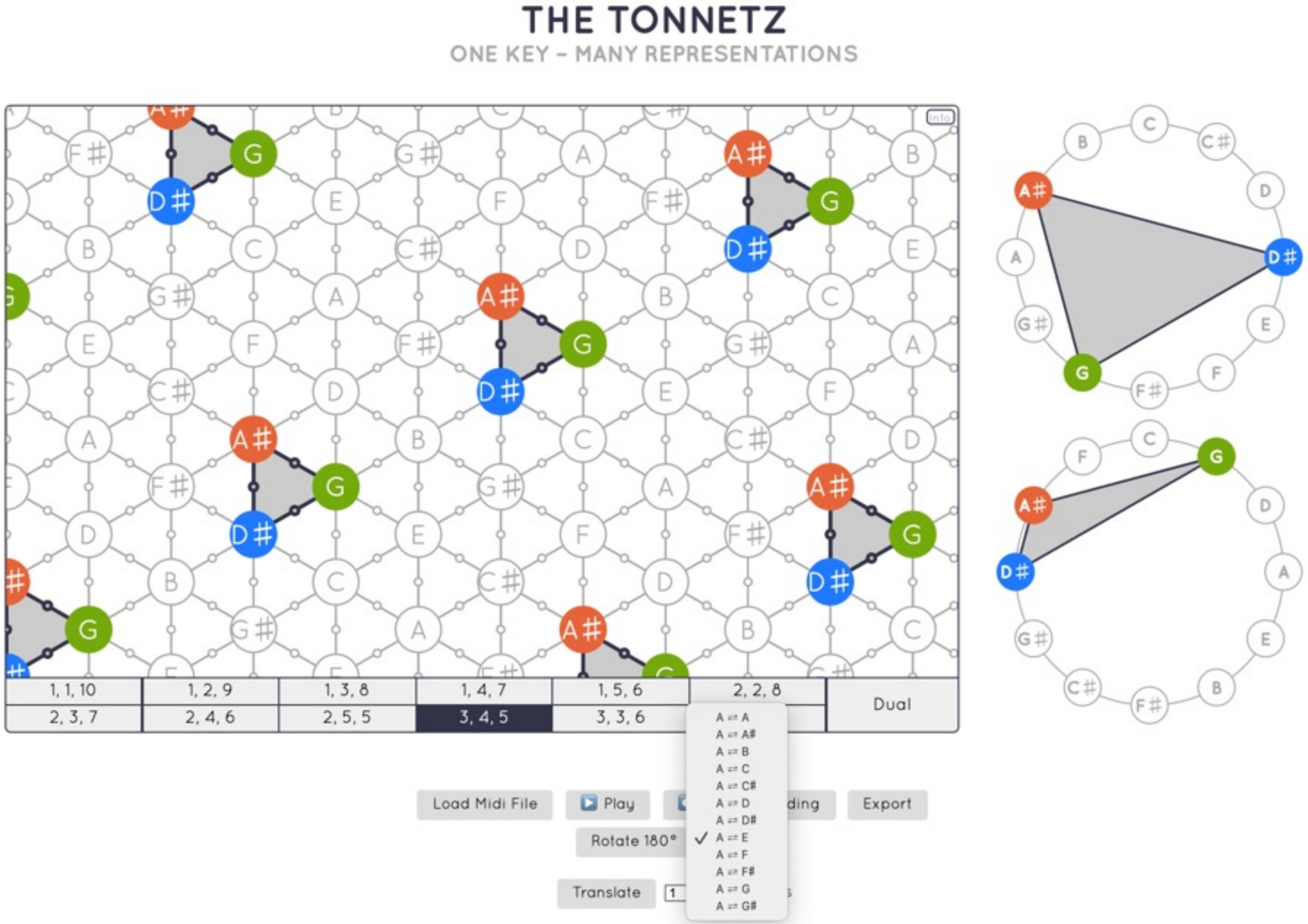
Harmonie négative dans l'environnement *Tonnetz*





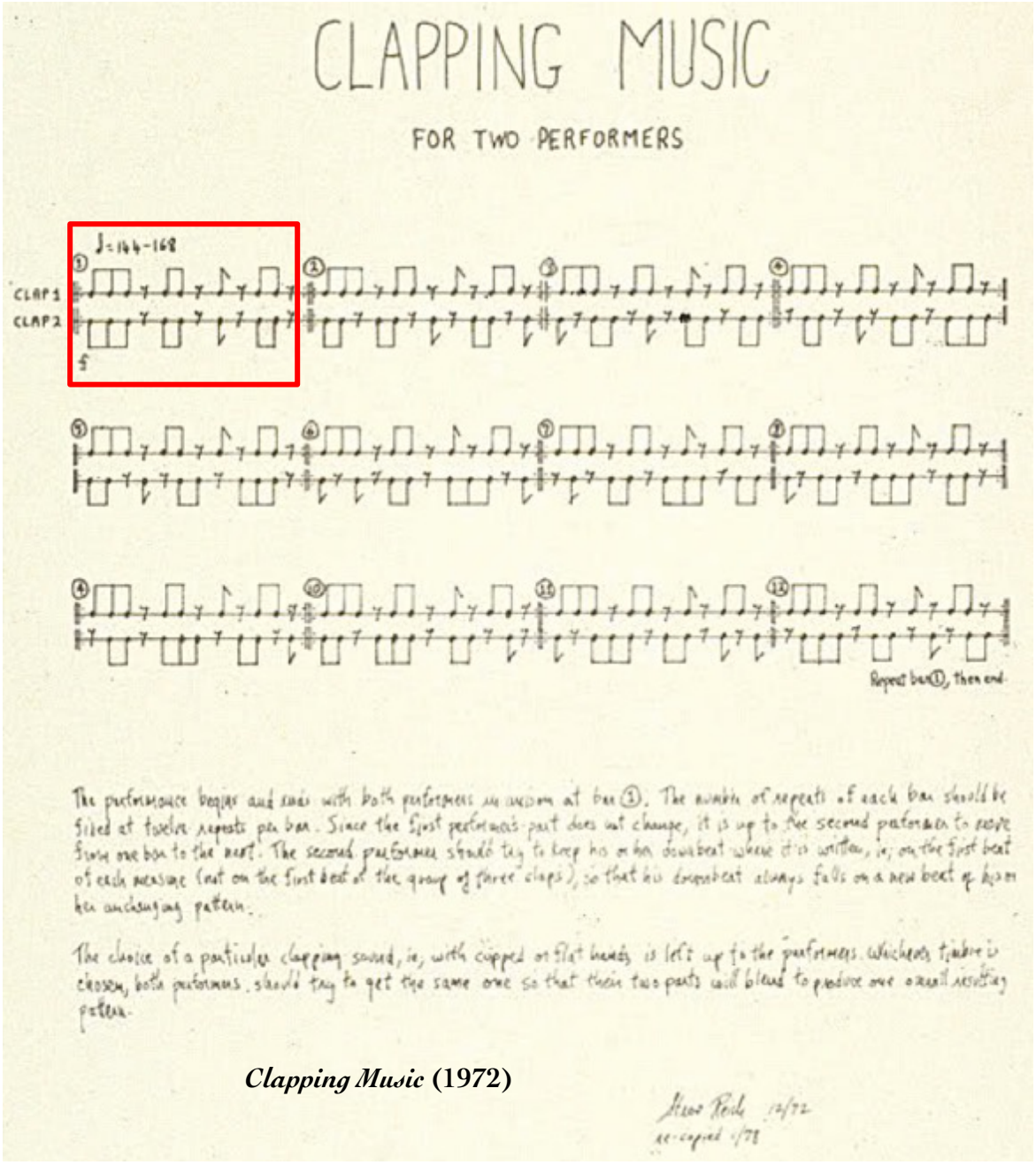


Exemple : harmonie négative de *Your Song* (E. John)



Et le rythme ?

Le « cercle rythmique » et ses rotations


CLAPPING MUSIC

FOR TWO PERFORMERS

♩=144-168

CLAP 1

CLAP 2

Repeat bar ①, then end

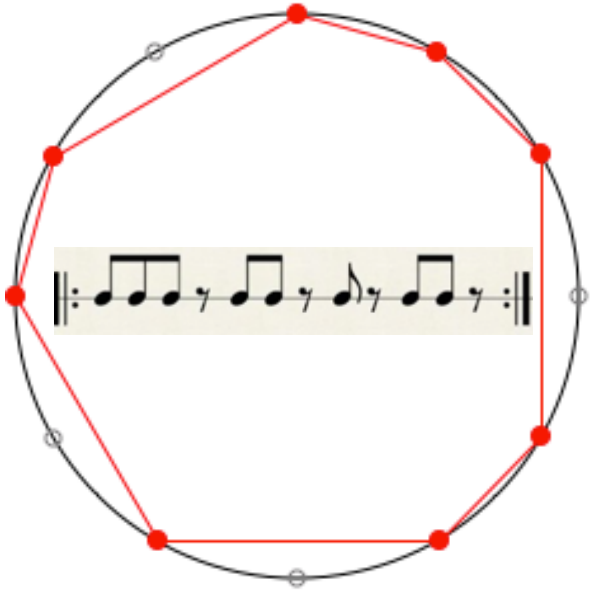
The performance begins and ends with both performers in unison at bar ①. The number of repeats of each bar should be fixed at twelve repeats per bar. Since the first performer's part does not change, it is up to the second performer to move from one bar to the next. The second performer should try to keep his or her downbeat where it is written, i.e. on the first beat of each measure (not on the first beat of the group of three claps), so that his downbeat always falls on a new beat as he or she unchanging pattern.

The choice of a particular clapping sound, i.e. with cupped or flat hands, is left up to the performers. Whichever timbre is chosen, both performers should try to get the same one so that their two parts will blend to produce one overall resulting pattern.

Steve Reich 12/72

re-copied 1/73

Clapping Music (1972)



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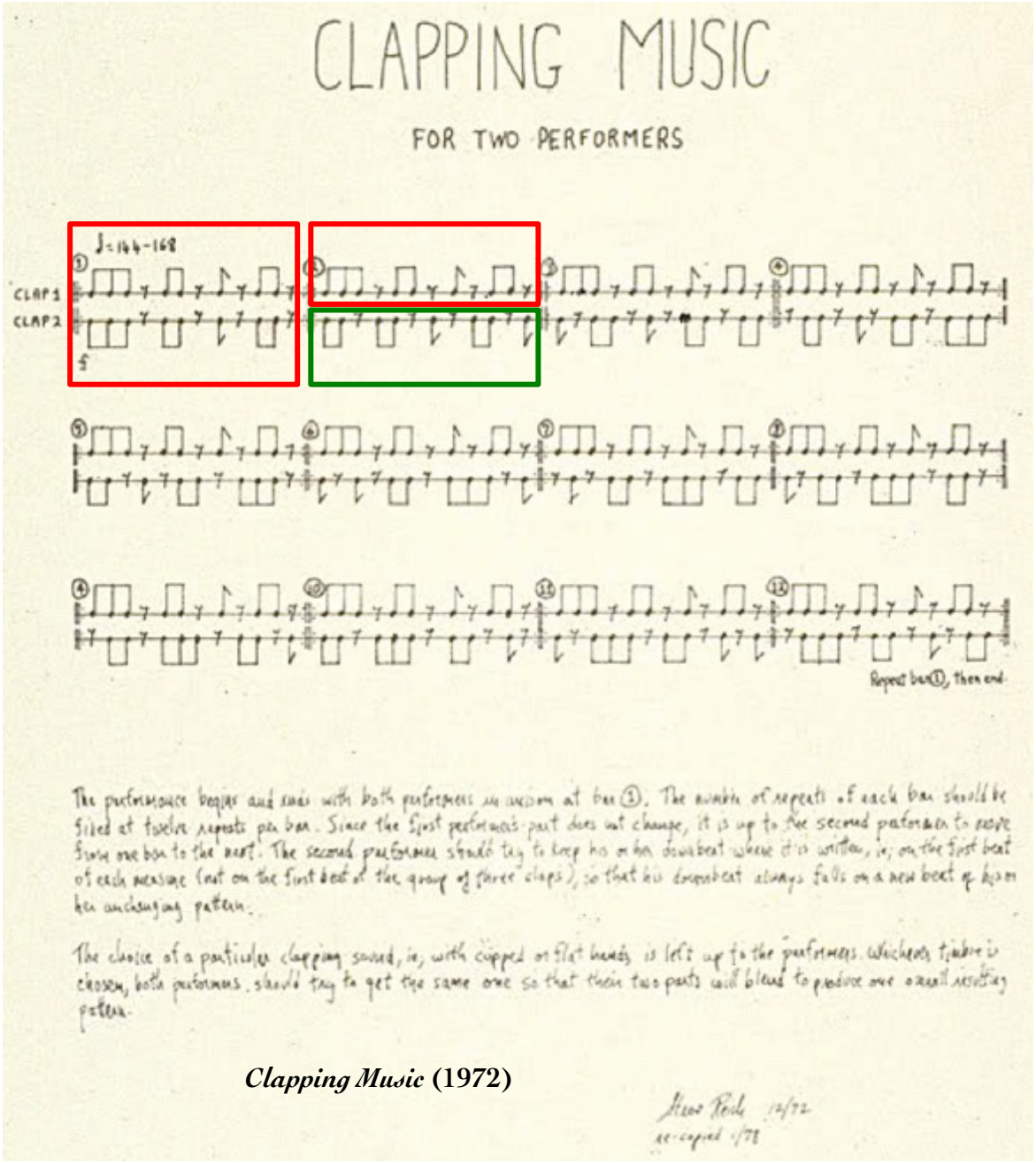

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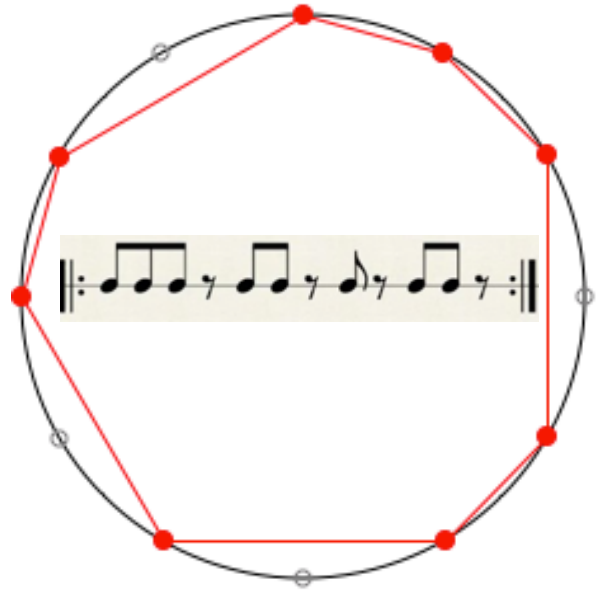
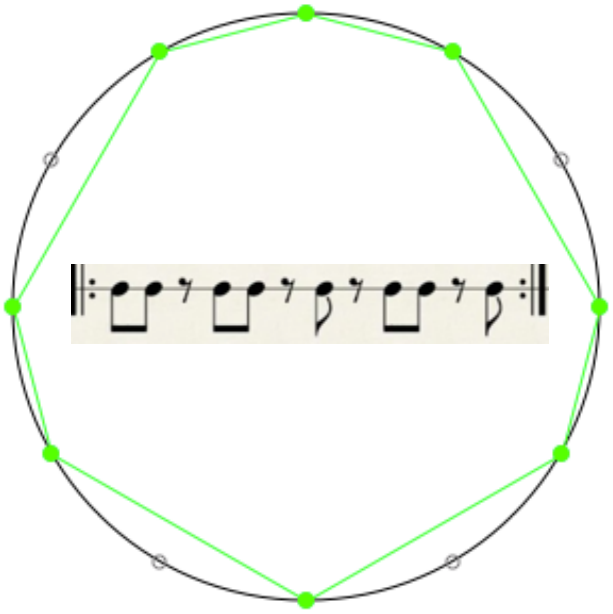
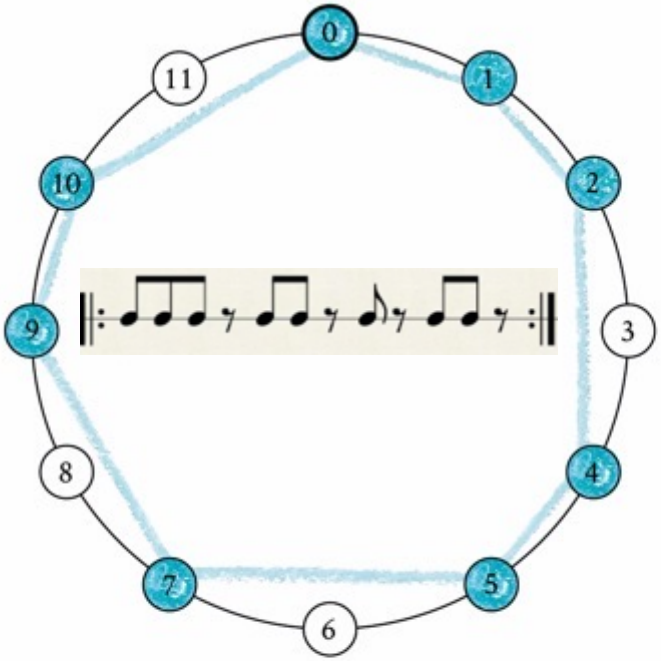
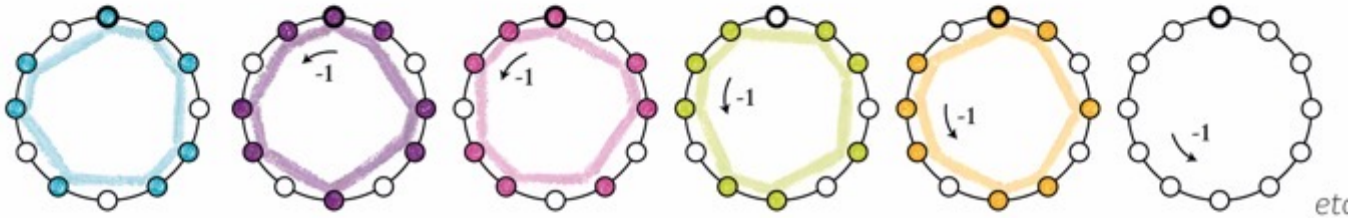


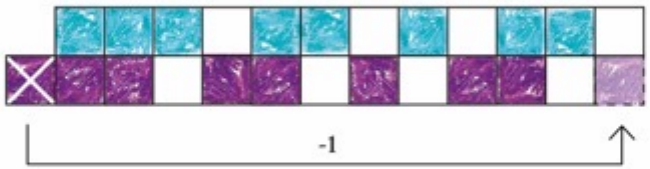


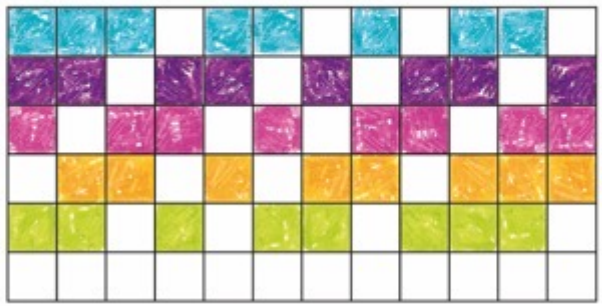
Pour se quitter en musique…



Gerubach's Scrolling Score Project
`http://www.gerubach.com`







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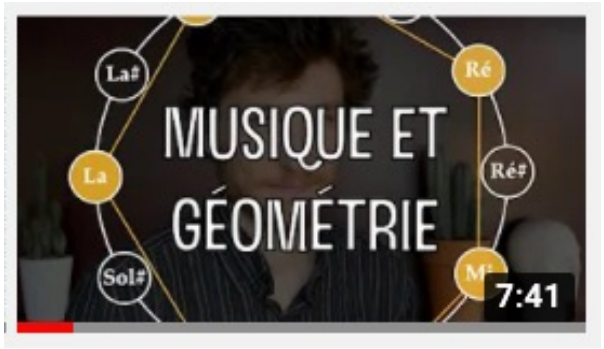
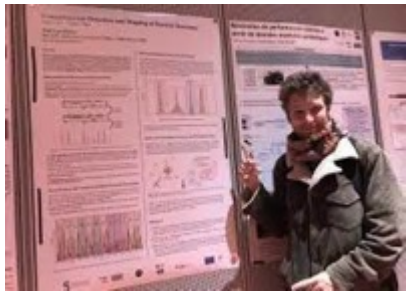
Mathémusique : chaîne youtube de Paul Lascabettes





P. Lascabettes (Equipe RepMus, IRCAM)

- **Musique et géométrie** : une introduction aux représentations géométriques des structures musicales (rythmes, gammes, accords)
- **Rythmes Euclidiens** ou les ensembles bien repartis (Maximally Even Sets)
- **Arbres rythmiques** : comment représenter les structures rythmiques complexes dans la pop (et dans les Beatles en particulier)
- Analyse rythmique de la chanson "Here Comes The Sun" des Beatles

➔ https://www.youtube.com/@mathemusique/

Math'n pop

Conférence Concert

www.mathnpop.com/