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Formalizing musical structure: from Information to Group Theory
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<u>Summary:</u>

O. Introduction

"Les identifications musique-message, musique-communication, musique-langage
sont des schématisations qui entraînent vers des absurdités et des dessèchements.
Trop de flou en musique ne peut se plier à trop de précision théorique.
Plus tard, avec l'affinement et l'invention de nouvelles théories, peut être."[1]

The history of Information Theory begins with a technical engineering problem about the efficiency of a communication channel, such as a telephone or a telegraph, once it can be abstracted and formalized in terms of a probability system. Although the basic ideas of modern Information Theory are contained in a paper which Sir R.V.L. Hartley published in 1928 in the Bell System Technical Journal[2] the first coherent presentation of a Theory of Communication in mathematical form can be considered Shannon and Weaver's report of twenty years later from the Bell Laboratories.[3] For the first time general concepts like "information", "uncertainty", "noise", "redundancy" were defined rigorously, with consequences which we will discuss trying to follow the different shapes that an informational approach to music has taken over almost 50 years. When music is regarded as a communicative art, Information Theory becomes not only a technical source for applications to music at different levels, from an analytic to a synthetic one, but also a "mental habitus" (Cohen, 1962 - p.161) for more sophisticated compositional ideas, as in the case of Iannis Xenakis.

Regarding music as a probability system, as suggested by the first aesthetically oriented discussion of Information Theory,[4] has many technical and theoretical consequences. Perhaps the most evident one concerns the emphasis which has been given to statistics as a method for grasping the informational content of musical scores. At this first level statistical speculations apply specifically to what is called the "symbolic representation of music" (Knopoff and Hutchinson, 1981 - p. 17). The basic assumption is that musical style can be described mathematically (read statistically) once the word "information" is understood as "the freedom of choice which a composer has in working with his materials or [...] the degree of uncertainty which a listener feels in responding to the results of a composer's tonal choices" (Youngblood, 1958 - p. 25).

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[1] Iannis Xenakis: Vers un métamusique, in "La Nef", 29, 1967.
[2] See Hiller and Bean (1965) p. 99.
[3] Shannon, C.E., Weaver, W. (1949).
[4] See Meyer (1967).

In one of the first attempts to apply Information Theory to musical composition[5] information is simply identified with an amount of meaning of a particular piece of music, in contrast to Shannon and Weaver's previous assumption that the mathematical theory applies only to the 'technical' level of transmission, not to other levels like the 'semantic' or the 'esthetic' one.[6] For "the word information relates not so much to what you do say, as to what you could say" (Weaver, 1949 - p.100). In other words "mathematical formulations of information content are not descriptive of conveyed meaning, but rather describe distributional aspects of the symbolic characters used in the transmission of that meaning" (Knopoff and Hutchinson, 1981 - p. 17).

Beneath *entropy*, for which we shall give a formal and an intuitive definition later, there is another parameter - the *redundancy* - which is related to the former in a way that will be clear when we will consider Moles' approach to Information Theory and aesthetic perception. Intuitively the redundancy of a (musical) sequence of objects (like a melodic or a rhythmic line) corresponds to the degree of regularity contained in the given message. Redundancy, claims Moles, creates an internal organisation in the message, assuming that a composition is a sequence of random musical events according with well-established rules of selection. "Because such a selection makes some sequences or classes of sequences more probable than others, composition implies, in term of information theory, imposing redundancy on a sequence" (Cohen, 1962 - p. 147). Attempts to use redundancy as a parameter of style, as has been done by Youngblood (1958) and, subsequently, by Hiller and Bean (1966), have been criticised in various way by Cohen (1962), Knopoff and Hutchinson (1981, 1983) and, more recently, Snyder (1990). For not only are redundancy values 'redundant' in respect to the "information [...] already imparted by the entropy values" (Knopoff and Hutchinson, 1983 - p. 81) but they are taken as meaningful under the assumption that "information theory may be applied to small samples, perhaps even individual pieces" (Cohen, 1962 - p. 156). A small sample of music, like any other small collection of objects, is rarely suitable for a pertinent analysis under statistical methods.

Beside these well known critiques it could be suggested that sometimes the quality of redundancy is much more important that its quantity. As we will see in detail in the third chapter of this study,

[5] See Pinkerton (1956).

[6] The distinction between levels will be developed with particular attention in discussing A. Moles' extension of Shannon's original model.

when discussing Iannis Xenakis' compositional universe, the basic idea of a "stochastic music" is not that of minimalising its redundancy-content but is an attempt to create a new way of listening. It will be clear that in elaborating his idea of "formalized music" Xenakis has been largely influenced by informational-theoretical assumptions derived from the works of the Nobel-prize winning physicist Dennis Gabor and from the research conducted by Abrahm Moles at the GRM (Groupe de Research Musicale) in Paris. Information Theory also represents the scientific justification of a critique, which the composer directed against serial music in his famous article that appeared in the "Gravesaner Blätter",[7] more than a collection of analytical tools which seem to be so fascinating to his contemporaries. Returning to these approaches, their common difficulty (which is the same for statistical studies in general) "is that of defining the problem correctly"[8], which means considering "the assumptions made prior to collecting and analysing the data" (Snyder, 1990 - p. 121). If we try to follow the different assumptions historically, we can have an idea of the limited validity of many results obtained in informational-oriented analysis. These were critically pointed in the Sixties by Cohen[9] and are summarised in a claim which can be considered the underlying principle of this dissertation, when he says that "[...] the main value in the application of information theory to music lies not in the collection of specific results obtained, but in the mental habits developed when dealing with an aesthetic object or process such music" (Cohen, 1962 - p.161). He also seems to suggest that almost all the mathematical assumptions of these analytical works are "difficult if not impossible to establish" (Ibid. - p. 157). If we consider, for example, one of the first attempts to apply probabilistic methods in the formalization of style[10] we find some problematic assumptions which it may be useful to discuss here briefly. The most general one is that both music and language are communication systems and therefore (?) "a technique which has proved useful in describing the latter should be applicable to the former" (Youngblood, 1958 -

[7] when he pointed out, for example, that "linear [serial] polyphony is destroying itself by its present complexity. What one hears is, in reality, only piles of notes in various registers. The vast complexity prevents the listener from following the entanglement of lines and has as its major effect an irrational and fortuitous dispersal of sound over the whole range of the sound spectrum" (I. Xenakis: La crise de la musique serielle, in "Gravesaner Blätter", 1955, n.1. Also quoted by M. Fleuret in Fleuret (1972, p. 22).

[8] i.e., quoted by Meyer, the "mere collection and counting of phenomena do not lead to significant concepts. Behind any statistical investigation must be hypotheses that determine which facts shall be collected and counted" (Meyer, 1967- p.18).

[9] See Cohen, 1962.

[10] See Youngblood, 1958.

p.25). In particular a stochastic process such as a Markov chain,[11] because of its analogy with some properties of English language, can be assumed to be a faithful representation for the formal description of musical style. Almost contemporary with Xenakis' critique on Serial Music, Chomsky, a linguistic with mathematical background, showed that Markov processes are inadequate for language because they cannot generate the "embedded structures" (Chomsky, 1956) i.e. those "necessitating reference to a higher order of grammatical organisation" (NGD., p.394). Applied to music this implies the inadequacy of Markov processes to give an interesting description of music which is less aleatoric , like a classical fugue, a serial composition, or even standard jazz.

One of the most remarkable consequences of Chomsky's *new paradigm* was that methods deriving from classical linguistics have been largely preferred to stochastic processes in describing musical structures. We will not analyse the ever suggestive connections between music and language. We only mention that, beside the well-known approach of Lerdahl and Jackendoff[12] Chomsky's original idea of "generative grammars" (Chomsky, 1956) has been applied, differently, by H. Christopher Longuet-Higgins, one of the founding figures of cognitive science[13].

The common basic assumption is that "the set of all possible melodies in a particular musical style[14] "constitutes a language which can be described in finite terms by a generative grammar " (Longuet-Higgins, 1994 - p. 105). Longuet-Higgins' generative grammar has the advantage of allowing representations of "mental processes" in computational form and therefore is "closer (...) both to the Chomskian paradigm and to the ways of thinking of the active musician" (Longuet-Higgins, 1989 - p.17). Note that the term "generative" is related to music in two distinct ways: rhythmically and tonally. In fact "metrical rhythms resemble syntactic structures in being generated by phrase-structure grammars; as for the pitch relations between notes, the tonal intervals of Western music form a mathematical group *generated* [italics mine] by the octave, the fifth and the third" (Longuet-Higgins, 1994 - p.103).We refer to more technical works on algebraic methods applied to music[15] for all the concepts that cannot be considered, for a lack of time and space, in this study. We would

[11] Note that we require besides the "stochasticity" and the "Markov consistency" other properties such "ergodicity" and "stationarity" which we will discuss later because of their relevance in Xenakis' compositions. See Cohen (1962 - p.155 - 157) for a brief summary of these assumptions).

[12] Lerdahl, F and R. Jackendoff: A generative Theory of Tonal Music, Cambridge, 1983.

[13] In particular the second part of his Mental Processes (1987) is dedicated to his essays in music that were published in periodicals from the 1960s.

[14] Note that we restrict the field of applicability of a generative grammatical approach to the Western classical tradition.

[15] See Andreatta (1996, 1997, 1997 Diss.)

like to point out, at the conclusion of this introduction, that the only emphasis that we will give to group-theoretical methods applied to music is motivated, also, by the way in which mathematical objects such as Transformational Graphs and Networks[16] have been applied, recently, to model non tonal music analytically[17]. This seems to be a result that could be very difficult to obtain from a strict grammatical approach as well from an information theoretical perspective of the type described above.

To return to a more aesthetic perspective on Information Theory, one of the most historically relevant interests of this theory is concerned with an attempt to explain the listener's affective (or emotional) response to music. We will analyse two theories which are almost contemporary and, although with some specific differences, "are in no way antithetical" (Cohen, 1962 - p.160): Meyer's theory of musical meaning based on the frustration of expectations[18] and Moles' ideas about the psychology of the listener as a relevant for the transmission of the musical meaning itself[19]. Finally, from a compositional point of view, the case of Xenakis is intriguing. Even if, in fact, Information Theory failed in applying Markov processes to produce stochastic melodies, as pointed out in various ways by Cohen (1962), Xenakis constructed his sophisticated compositional theory starting from the same theoretical background. It will be interesting to observe how the work of three such informationally oriented theorists as Gabor, Meyer-Eppler and Moles, became a starting point in Xenakis' elaboration of a 'formalized music', of which the stochastic type is most closely related to our study.

[16] See Lewin, (1987).
[17] See Lewin (1993) and Rahn (1995).
[18] See Meyer (1956).
[19] See Moles (1968).

I. Abraham Moles: principles and methods of Information Theory in Music.

As suggested by the title of this dissertation, we are interested in seeing how the concept of musical *structure* has been at the centre of interest of different scientific approaches to music.

In analysing A. Moles' Information Theory and Esthetic Perception (1968) we have the first confirmation of the relevance of such a concept. The assumption of this "study of the perception of musical structures", as it is described by the author, (Moles, 1968 - p.165), is that the term *Information Theory* is interchangeable with that of *Structuralist Theory*, since "the basic hypotheses are in fact the ones of a philosophical attitude now called *Structuralism*"   (Ibid. - p. v).

Besides this first philosophical approach we note that Moles is also fascinated by a so called "phenomenologic study of music" (Ibid. - p. 5) which is the key to interpreting his interest in the "microstructure" of the sonic message (or *sonic object*, as he calls it using a well known expression of P. Schaeffer). It will be interesting to see how this phenomenological attitude is not only close to Xenakis' "corpuscular" (or atomistic) theory of sound, but also to a music-theoretical approach (the dogmatic foundation of which makes it inadequate for a "scientific esthetics" (p.122) confirmed by psychological experiments) such as that developed by the American Music Theorist D. Lewin, for example, (Lewin, 1986).

However the word "information" has to be taken in the technical sense derived from Shannon's original "Communication Theory". Music, like speech, is in the first analysis a message, i.e. "a finite, ordered set of elements of perception drawn from a repertoire and assembled in a structure" (Moles, 1968 - p. 9). In general, the problem of communication can be formalized in terms of a channel of transmission between a *transmitter* and a *receiver*. The "quantity" transmitted is, by definition, the information (or information content) of the message and it is a "measurable quantity", as Moles points out in the first introductory chapter of his book, which is a "General Outline of Physical Information Theory". The idea of definition of the information content of a message implies the characterisation of this quantity as not directly related to the length of the message itself. What is needed is a kind of "weighting on the intrinsic value of the message" (Ibid. - p.19) where the term *value* must not be confused with that of *meaning* (or *significance*). "Value", explains Moles, "is bound up with the unexpected, the unforseeable, the original"(p.19). This fact has at least two remarkable consequences: the first one is that as a measure of unforseeability the information can be described, accurately, by means of statistical and, more generally, probabilistic methods, a conclusion that he shares with Xenakis, although his considerations are different. The second main consequence is that *information* and *originality* become synonymous. If one takes, for example, a melodic message composed of a sequence of elements (notes) which are extracted from a family of objects (repertoire) it is possible, in a probabilistic way, to describe the information content H of the given sequence as a "function of the improbability of the received message" (p. 22). Starting with Fechner's law which describes the *sensation* as proportional to the logarithm of the received *physical excitation* given by the message and applying the discovered result (which is analogous to the classical Boltzmann formula of entropy in statistical thermodynamics) to the case of a message of a length $N \cdot t$ and which repertoire consists of n symbols it follows Shannon's original formulae for discrete channels:

$$H = H(n,p_i,N,t) = -N \cdot t \cdot \sum_i (p_i \cdot \log_2 p_i) \qquad i=1,...,n$$

where p_i are the probabilities of occurrence of the symbols for the given message. Expressed in the bracket is the dependence of the information to the time t, the density N of elements, the number of elements n and the statistical distribution p_i of the number of each element of the repertoire. Of course in the case in which there are no repetitions $n=N$.

Note that the originality of a message consisting of M elements can be expressed "independently of the time t during which the message lasts" (p.37). In the hypothesis in which the sequence "yields uniformly M/t elements per second" we have the following formula; in this, the dependence on time is eliminated[20]:

$$H = H(n,p_i,M) = -M \cdot \sum_i (p_i \cdot \log_2 p_i) \qquad i=1,...,n$$

[20]This assumption, which will be discussed at the end of the next chapter, seems to be particularly relevant in an aesthetic-phenomenological discussion of the musical message. Loosing the time direction is something like loosing the memory and this point will be the necessary condition, as we'll see, for Xenakis' foundation of a symbolic (or 'outside of time') music.

For an intuitive explanation of the reason for which, historically, the scale of logarithms (base 2) has been preferred to, for example, the natural one (base *e*) see (Fichet, 1996, pp. 172-174).

The dream of all artistically inclined mathematicians, that of reducing aesthetic concepts like the creativity or the originality to a mathematical expression, is far from definitively solved. As it has been pointed out before, this concept of information must be carefully separated from the concept of "signification". It can also happen (see Fichet, 1996, p.185) that in applying the previous result to different melodic patterns, they actually have the same value of originality, although every listener could find a different degree of obviousness between them. This is the reason why such a result cannot be used crudely in analysis of musical pieces as a "stylistic parameter", as has been done very frequently (see Introduction). Before going into the problem of more or less obviousness in a given message we observe that, in the case that all n symbols of the message have equal probabilities $p_i=p=1/n$ we have a maximal value H_{max} of originality, which is:

$$H_{max} = -log_2(1/n)$$

This corresponds to the information content that would be transmitted if all the symbols of the message were equiprobable. It follows that, if we indicate by H_t the information transmitted by a given message the ratio H_t/H_{max} is a positive quantity that is less than (or almost equal to) one. Following Shannon's terminology Moles calls this quantity the "relative information" of the message (p. 42) and uses it to define a new parameter *R* which is:

$$R = 1 - H_t/H_{max} .$$

This is called the *redundancy* of the given message and it expresses not only the "superfluous" content of the message (in respect to the ideal or optimal communication system) but also the necessary condition which allows the comprehensibility of the message itself. As he explained in his previous work, "l'intelligibilité, et donc la redondance, mesurent ainsi le degré d'ordre que l'esprit recherche dans le monde extérieur, et qui est une condition nécessaire du plasir esthétique" (Les musique expérimentales, 1960, p.95). As we will see in the third chapter, this kind of "postulate of intelligibility" is completely rejected by Xenakis in the name of a new compositional style that has, as a starting point, the same information-theoretical background but in which Moles' concept of "redundancy" simply disappears. Xenakis' "Stochastic Music" is qualitatively different from Moles' "most difficult to transmit" message (Moles, 1968-p.62) as representing the minimum amount of redundancy. Nevertheless Moles' suggestions concerning the perceptual capacity of the listener are quite important not only for being the basis for the "atomistic theory of sound" that Xenakis developed in accordance with D. Gabor's ideas, but also because they imply the attempt to go further in the discussion of information. Besides the semantic aspect of information a new dimension is needed: the aesthetic. We will spend the last part of this chapter discussing the important division between semantic and esthetic information, leaving the description of the macrostructure of the musical space, together with the microstructure of the sonic object, until the third chapter.

As we pointed out in the Introduction of this study, the *Theory of Communication* was initially only concerned with the technical level of the transmission of information. The theory has firstly been generalised to the "semantic level" by Weaver, who does not, really, explain how to do it.

Moles' extension originates from the fact that "while the work's semantic information may be exhausted and eventually memorised, the peculiarity of the work of art is that its richness transcends the individual's perceptual capacity" (Ibid.-p. 166). The distinction between "semantic information", having a universal logic that is also commutable from one channel to another, and "esthetic information" that is, on the contrary, "specific to the channel which transmits it", being radically changed if "transferred from one channel to another" (Ibid.-p. 131), also gives an answer to the kind of paradox about *well known messages.* According to the theory of information-content as proportional to its unpredictability, it seems that if the message is already known, a transmission of the same message would be superfluous. Paradoxes such as this are - of course - only apparent and the previous distinction between different levels shows how in music "the esthetic message is infinitely richer in elements and carries more information that the semantic message" (Ibid.-p. 167). In fact, using a poetic image which is also very close to what Xenakis would say about the nature of musical composition, "the *realization* [italics mine] of the musical work differs each time and constitutes the *field of freedom* [italics mine] of the work" (Ibid.-p. 166). Moles renounces to a scientifically or, more specifically, mathematical definition of "esthetic information" of the type we can find, for example, in Birkhoff (1933). He also limits his speculation by the *simplest* and *most valid* assumption that "esthetic information obeys the same general laws that govern all informative messages" (Moles, 1968-p. 132) and can be generally characterised , as we said, as *not universal* and *not translatable.* This new level of information, he claims, "is specific to the receptor, since it varies according to his repertoire of knowledge, symbols, and a priori structurings, which in turn relate to his sociological background" (p. 132). Moles postulates the existence of a "semantic-esthetic counterpoint" (p. 153) which is deduced by experimental strategies, like that of "infinite clipping of very complex orchestral messages" (p. 153). But this postulate has different consequences, particularly for what concerns the limitation rules in the perception of artistic message. Remembering that we can associate with a given message a value H_{max} which represents the maximum information content that can be perceived by the listener, it follows the simple *heuristic rule*[21]: "in order for the musical message to remain entirely intelligible to a listener of capacity H_{max}, the semantic plus the esthetic information in this interval[22] must be lower than H_{max}" (p.160).

The almost total lack of mathematical rigorousness in this final speculation about different levels of the artistic message must not be considered too negatively. Concepts like the "field of freedom" of the musical message compared with, for example, the score (which is, in his hypothesis, the *scheme*, i.e. "the essential part of the semantic message of music" - p.137) have a central position not only in Xenakis' compositional universe but also in an aesthetic study which is, as we pointed out in the Introduction, in no way antithetical to what is described above. The following chapter will accordingly be also dedicated to L. Meyer's contribution to the relation between the concept of musical Meaning and an information-theoretical approach to music.

[21]See also Joel E. Cohen's Preface on the book, p. x, for the *heuristic* role of Information Theory in Moles' description of the esthetic perception.
[22]i.e. in the "interval of maximum extent of presence" (p. 160), which he defines as varying between 5 and 10 seconds.

II. Leonard Meyer: style as a system of expectations.

Meyer's basic hypothesis about the relations between Information Theory and Music is that the "psycho-stylistic conditions which give rise to musical meaning, whether affective or intellectual, are the same as those which communicate information" (Meyer, 1967-p.5).

For a detailed description of the reasons why the problem of musical meaning and its communication is of particular interest, together with the distinction between *affective* and *intellectual* meaning, we must refer to Meyer's previous work <u>Emotion and Meaning in Music</u> (1956). In particular, his thesis about the musical arousal of emotion is essential in this context: "affect or emotion-felt is aroused when unconscious expectation - a tendency to respond - activated by the musical stimulus, is temporarily inhibited or permanently blocked" (Meyer, 1956-p.31).

Many critiques have been raised to Meyer's idea on the production of emotion. In particular the required inhibition of a tendency to respond seems to be, e.g., for Budd (1985) as well for Xenakis (although for different reasons), neither a necessary or sufficient condition for the arousal of emotion. Despite all these criticisms it is vital to point out that assuming "frustration or inhibition in the fulfillment of expectations usually lead to affective response" (Cohen, 1962-p.159) is for Meyer a necessary condition for what he will call later "the probabilistic nature of musical style" (Meyer, 1967-p.5). Therefore, because style "constitutes the universe of discourse within which musical meaning arises" (Meyer, 1967-p.7) it is quite important to distinguish the different ways in which a given *stimulus* can be meaningful. There are, in fact, stimuli or *processes* that refer to something which is different from themselves in kind and, on the contrary, stimuli that refer to something which is like themselves in kind. This main distinction leads to a contrapuntal relation between a *designative* and a *embodied* meaning respectively. Although "music give rise to both types of meaning" (Meyer, 1967-p.6), Meyer's study is concerned specifically "with those meanings which arise within the context of the work itself" (Meyer, 1967-p.7), i.e. embodied meaning.

According with W. Weaver's definition of information as "a measure of one's freedom of choice in selecting a message" (Weaver, 1949-p.273) it appears that both "[embodied] meaning and information are thus related through probability to uncertainty" (Meyer, 1967-p.11). In an information-theoretical context, such as this, Meyer's alternative meanings of the term *uncertainty* are interesting, particularly as concerns the relationship with Moles' original concept of redundancy. Firstly Meyer distinguishes between *systemic* and *designed* uncertainty. Using the assumption that "music, like information, is an instance of a Markov process" (Ibid.-p.15) it follows that there exists a kind of uncertainty which is "systemic in nature" (Ibid.-p.15), i.e. "built into" the special case of the stochastic process itself. The decreasing of such uncertainty, which implies the decreasing of meaning and information through the functioning of the stochastic process, is balanced by the designed uncertainty, i.e. the uncertainty introduced by the composer. Note that the so called Markov-consistency, which is (as we will discuss briefly in the end of this chapter) far from obvious in a musical context, is - on the contrary - deliberately assumed by Iannis Xenakis as a property of *his* particular instance of stochastic music. It is notable that this first distinction between systemic and designed uncertainty in Meyer is concerned with a composer's perspective. By changing the point of view in problems which are more related to the perception of the listener, Meyer distinguishes between *desirable* and *undesirable* uncertainty. By "desirable uncertainty" Meyer means that which "arises within and as a result of the structured probabilities of a style system in which a finite number of antecedents and consequents become mutually relevant through the habits, beliefs, and attitudes of a group of listener" (Ibid.-p.17). The concept of style as a "complex system of probabilities" (Ibid.-p.8) has been generalized here to that of *culture* as "learned probability system" (Ibid.-p.17). The point seems to be quite intriguing, particularly together with that of *behaviour* of a given musical work, but proceeding in such a direction would take us far from the purpose of this study.

Besides the desirable uncertainty, Meyer considers the case of unknown probabilities, "either because the listener's habit responses are not relevant to the style (...) or because *external interferences* (...) obscure the structure of the situation being considered" (Ibid.-p.17). In the first case we observe the existence of a "cultural noise" (Ibid.-p.17) that is so-called because it "refers to disparities which may exist between the habit responses required by the musical style and those which a given individual actually possesses" (Ibid.-p.16). *External interferences* are, on the contrary, an instance of "acoustical noise" (Ibid.-p.16). Both cultural and acoustical noises are for Meyer relevant in a discussion of the redundancy parameter in music. In particular, looking at the difficulties which the listener seems to have with contemporary music, Meyer suggests consideration of the fact that "the redundancy rate of this music is at times so low as to be unable to counteract the cultural noise which is aways present in a communication situation" (Ibid.-p.17). We will discuss in the following chapter Xenakis' personal defence of this kind of *overloading* of "the channel capacity of the auduence" (Ibid.-p.17) which is, also, intrinsically an answer to Moles' concept of "limiting information rate" (Moles 1968-p.200). Excluding, for the moment, Xenakis' particular compositional style, we will spend the last part of this chapter discussing some historical consequences of an informational definition of musical style, as described by Meyer. As we pointed out very briefly in the introduction of this study, there have been many attempts historically to use Information Theory for a statistical analysis of style. Meyer claims that there are some requirements these studies must assume if they want to have "anything more than a curiosity value" (Meyer, 1967-p.18). We attempt now to describe some of these hypotheses of validity together with other mathematical assumptions which have been discussed by Cohen (1962, pp.155-157).

1. Difference between *systemic* and *designed* uncertainty.

Assuming that the samples collected are stochastic, one has to consider, beside the "natural" tendency of uncertainty to diminish, the active contribution of the composer. With, perhaps, the single exception of Xenakis' *Stochastic Music*, where the musical system obeys the same laws as govern a physical system, this second contribution is not trivial and has to be taken into account.

2. The *ergodicity* of the source, i.e. "a sufficiently large sample from an infinite sequence has the same statistical structure [as that] of the infinite sequence" (Cohen, 1962-p.155).

It has been observed, for example, by Cohen that the ergodicity of the small sample is "doubtful" and therefore "the results based on the assumption [...] have no rigorous validity and must be considered grossly approximative" (Ibid., p.156). In addition to what was pointed out previously, we observe, with Meyer, that the search for probabilistic parameters involves not only "phrases and smaller parts of a musical structure" (Meyer, 1967-p.19) but also relations between them. This fact implies an architectonic analysis, i.e. an analysis where "different sets of probability must be discovered for different hierarchic levels" (Ibid.-p.19).

3. *Stationarity* and *Markov consistency*.

The first property assumes that "the statistical structure of the sequence is independent of the time at which observation of the sequence begins" (Cohen, 1962-p.157) i.e. "probability remains relatively constant throughout musical works" (Meyer, 1967-p19). It is not difficult to see that not only "within a given piece the stationarity assumption is not satisfied" (Cohen, 1962-p.156) but that there are also difficulties in assuming a stationarity property in "large corpora of stylistic homogeneus pieces" because the corpus, within which the structure of any large sequence is supposed to be the same, is in most cases "too small to provide several 'sufficiently large' sequences for comparison" (Meyer, 1967-p.157).

What extends the Markov consistency, i.e. the property of a "stochastic process in which the probabilities depend upon the previous events" (Weaver, 1949-p.267) is its inadequacy in describing musical as well as linguistic structures, and this has been pointed out already in the introduction. An exception to this is found in those compositions which are based explicitly on Markov properties. The first part of the next chapter will be devoted to the analysis of the stochastic universe of a composer who, adapting an expression of Meyer, has been able to construct "devices for composing music" on the basis of the same information theoretical problems that we have discussed in this and the previous chapter.

III. Iannis Xenakis: toward a group-theoretical *formalized* music.

The title of this chapter, which reminds us of a section of Xenakis' most famous work Formalized Music (1991), tries to limit the "field of possibility" in a description of the musical universe of the composer. The temptation to discuss the relevance of mathematics in his compositional and philosophical *Weltanschauung* would take us far from the purpose of this study. For, even by analysing one of Xenakis most humanistic-oriented papers, like his Thesis Defence Arts/Science:Alloys (1985) for the "doctorat d'État" at the Sorbonne, one would be tempted to claim that for him musical works answer questions, in a sort of "mosaic of hierarchical coherencies" at the top of which he placed philosophy. Questions of, for example, 'existentiality', 'causality', 'inference', 'connectedness', 'compacity', 'impure (or pure) determinism' though there is no bijunivocal correspondence between a particular problem and an answer (i.e. a musical work). "Several of these questions are interrelated and create intersections belonging to the same philosophical domain" (Xenakis, 1985-p. 6) so that "a work (answer) can, in itself, respond to a whole group of questions" (Ibid.-p.6). In such a context "mathematics plays an essential role [...] as a philosophical catalyst, but also as a spring board toward self-liberation" (Ibid.-p. 6). Strategies are needed for several questions, and this fact implies different mathematical theories, from probability (and statistics) to more algebraic approaches, like Sieve Theory and - more generally - Group Theory.

With regard to this kind of methodological attitude to answering questions, it is not surprising that, beside his books, one finds perhaps more enlightening several 'Conversations' with the composer which have been published throughout the years. These interviews, which Varga suggests as a necessary introduction to Xenakis' own crucial books Musique. Architecture and Formalized Music, appeared in many music periodicals finding, sometimes, their best collocation as (self)-portraits in form of books.[23] Specific help in following his theoretical and musical evolution since the first experiences in granular theory of sound toward a more algebraic compositional strategy is given by

[23] amongst all these conversation Bois (1967), Restagno (1988) and Varga (1996) seem to me to be the most intriguing. This latter suggests, also, that one should take all these sources as an introduction to the world of Iannis Xenakis.

a very recent interview which appeared in the *Computer Music Journal*.[24] The granular nature of acoustical phenomena was pointed out, originally, in the late 1940s by the Nobel-prize winning physicist Dennis Gabor (Gabor 1946, 1947, 1951). His idea of *Acoustical Quanta*, which have been largely described in Werner Meyer-Eppler's Grundlagen und Anwendungen der Informationstheorie (1959), originate in some difficulties of the standard (or Fourier) representation of musical signals. A brief discussion of the way in which the same kind of problems have been considered by Abrahm Moles could help us to understand the originality of Gabor-Xenakis' perspective.

As pointed out in the first chapter of this study, Moles quoted from Pierre Schaeffer the idea of decomposing the musical material of sound into *sonic objects.* Each of these is represented in a three dimensional space which coordinates respectively:

1. The level of sonor pressure, i.e. the physical intensity of the object (expressed in decibels) with respect to the minimum amount of audibility.
2. The Pitch level, i.e. logarithm of physical frequency.
3. The level of time, expressed in seconds.

The underlying philosophy is that the "continuity of sound is subdivided *operationally* [italics mine] into sonic objects each possessing an autonomous center of interest" (Moles, 1968-p.122). As it has been pointed out in a recent study on the Phenomenology of Experimental Music, Xenakis, in his atomistic theory of sound, maintains the previous three dimensional representations but he completely changes the perspective.[25] The continuity of sound is only an illusion, for "all sound is an integration of grains, of elementary sonic particles, of sonic quanta.[26] Each of these elementary grains has a threefold nature: duration, frequency and intensity.[27] All sound (...) is *conceived* [italics mine] as an assemblage of a large number of elementary grains adequately disposed in time" (Xenakis, 1991).

In composition this idea of grains of sound as "elementary units almost indistinguishable to human aural perception" (CMJ, 1996-p.11) found its first application in *Analogique B*, an electroacoustic piece of the late 1950s. From a strict mathematical point of view this piece, together with

[24] We will refer to this paper with (CMJ, 1996).
[25] See Orcalli (1993, p. 81). For more recent applications of a granular theory of sounds see Roads (1978).
[26] This word, that we found also in Gabor, suggests some analogies with the basic assumptions orf quantum physics. In one of the quoted conversations Xenakis admitted that originally he was unaware of Gabor's theories and the idea of sonic quanta (or "phonon") came to him from Planck's hypothesis about the discontinuous nature of light.
[27] Note the perfect analogy with Moles.

Analogique A of the previous year, represents the first instance of "Markovian Music", i.e. music based on stochastic Markov processes. The essential point is that he overturns one of the first assumptions of Information Theory concerning the reliability of Markov chains in representing the formal properties of musical style as well as of a language.[28] In agreement with Noam Chomsky he suspects the validity of Markov processes in linguistics but, he claims, "what is impossible in the case of language can be realized in music. After all, music is not a language [...]. We can therefore use the Markov chain, provided the result is interesting" (Varga, 1996-p.82). Two points here seem to be very close to the aim of this study. Firstly, Xenakis' position is clear, on what extends possible relations between music and language. "Music", as he suggests again by means of a naturalistic metaphor, is not a language. Every musical piece is like a highly complex rock with ridges and designs engraved within, and without a single one being the best or the most true" (Xenakis, 1987-p.32). But beside this naturalistic image, that could recall Moles' concept of "field of freedom" in the aesthetic message, Xenakis' recourse to Markov chains and - more generally - to stochastic processes, is submitted to a necessary condition: it must "yield interesting results to the ear" (CMJ, 1996-p.14). In other words "in composition the primarly factor is the human capacity for discernment" (Ibid. p.12) but there is no contradiction in trying to overload "the channel capacity of the audience", as Moles would say. Clouds of sound, like that employed, for example, in the orchestral work *Terrêtektorh* (1965-66), require, from the human ear, a different kind of listening. The loss of the individual line of each instrument is, in Xenakis' new compositional universe, quite different from the "dispersion irraisonnée et fortuite des sons sur toute l'étendue du spectre sonore" (Xenakis, 1994-p.42) which he denounced in his famous critique on serial music[29]. What has been discussed above has some interesting consequences. The first one is Xenakis' predilection for mathematical theories such as statistics and probability that can help the composer to manage a great amount of data. The second main consequence is that, in this new way of listening, information theoretical concepts like "redundancy" or "predictability" play no role. "The interest in the music is not linked to unpredictability" (PNM, 1975-p.96) which can only be, he claims, "something very relative".

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[28] See the Introduction of this study for more references.

[29] See Xenakis (1955). For example, with regard to *Pithoprakta* (1957), a piece for orchestra of 50 instruments, Maurice Fleuret pointed out that "in a granulation of sound of variable density 'the individual noise loses its importance to the profit of the ensemble perceived as a block, in its totality' "(Fleuret, 1970 - p.23).

As far as I know, Xenakis has never spoken extensively in explaining the dichotomy 'predictability/unpredictability', or the role of parameters like 'redundancy' in his compositions. The hypothesis that I would like to suggest here is concerned with one of Xenakis' most recurrent ideas: the distinction between *in time* and *outside time* (or 'hors-temps'). With regard to predictability (or "forseeability", as Moles would say) as a phenomenological category, one inevitably has to involve a temporal dimension. In a informational perspective 'redundancy' is also a parameter that is strictly influenced by the time. We suggested[30] looking at it as a *mirror reflection* of information (i.e. originality) but we can go further in this metaphor by describing it as 'temporal mirror reflection of informational content'. Formula [4] on p. 8 explains, clearly, this temporal dependence. Practically, one can have an idea of the inadequacy of such an approach by using the previous formula to characterise the redundancy values of Xenakis' *Markovian Music.* Redundancy would be asymptotically zero with the aesthetic consequences that Meyer would be tempted to draw out[31]. Xenakis' category of *hors-temps* seems to be very helpful in understanding these kinds of paradoxes; at the same time it gives an aesthetic justification for our final discussion on Group Theory applied to Music. For "music" - he claims - " is basically outside time and time serves only for it to manifest itself" (Varga, 1996 - p.83).

The easiest way to understand the previous statement is probably given by the composer himself, when he describes the path from *stochastic* pieces like *Achorripsis* (1958) and the *ST* works[32] to the idea of a *symbolic* music. As he admitted, these were three basic questions to answer: "What is time, what is pitch, what is the relationship between the two" (Varga, 1996 - p.82). A question such as the latter can be approached in different ways. Physically the Theory of Relativity - to which he also refers in his Concerning Time, Space and Music (Xenakis, 1991 - Ch. X) - describes time as intrinsically related to space: a non zero temporal interval separates distinct points in space.

[30] See chapters I and II.

[31] The complexity of the problem that concerns the relationships between the nature of information and the nature of redundancy cannot be examined in this study. Again, with regard to formula [4] on p. 8, one could discuss the complementary relationship between the two *quantities* in a more general way, as a qualitative description of abstract categories, which could be different in kind (and, therefore, not suitable for any comparison). To all these 'secondary' questions probably refers Xenakis, when he claims that "les technocrates actuels et leurs suiveurs assimilent la musique à un message que le compositeur (source) transmet à un auditeur (rècepteur). De cette façon, ils croient résoudre en formules [...] la nature de la musique et des arts en général. [...]Pourtant, en dehors d'une cuisine statistique élémentaire, cette théorie, valable pour les transmission technologiques, s'est révélée incapable de donner les caractéristiques de valeur esthétique" (Xenakis, 1967).

[32] i.e. *ST/4, ST/10, ST/48.*

Restricted to music, the question of time, as Xenakis suggests, is related to "the notion of separability" (i.e. non-synchronisation) which is a "prerequisite to the notion of anteriority" (Ibid.- p. 262). By means of the 'principle of anteriority', he concludes, "the flux of time is locally equipped with a structure of total order in a mathematical[33] sense" (Ibid. - p.264) and can also "be placed in a one-to-one correspondence with the points of a line" (Ibid. - p.264). The path toward a 'symbolic music' expresses Xenakis' predilection for concepts like 'formalization' and 'axiomatization' which "constitute a procedural guide, better suited to modern thought " (Xenakis, 1991 - p.178). In the same way that real numbers can be shown as points on a straight line (i.e. by means of axioms), the tempered chromatic scale can be formalized in terms of logical functions. This attitude, the composer demonstrates in the construction of his *Sieve Theory*[34], leads, from a compositional perspective, to pieces like *Herma* (1962), *Eonta* (1964), *Akrata* (1966) and *Nomos Alpha* (1966). In the context of a 'Symbolic Music' the word *symbol* is used "in its former meaning, of sign: the idea come from symbolic logic and symbolic mathematics" (Varga, 1996 - p.84). Therefore it seems that, from the previous list of compositions, only *Herma* could be called 'symbolic'. Its "outside time" architecture is based, in fact, on the abstract (i.e. logical) combinations of sets (i.e. collections of pitches of the keyboard). Examples of logical operations are the intersection ($\cap$), the union ($\cup$) and the negation (-) or complement. One could say that he is a precursor of well known analytical approaches of American music Theorists, e.g., Allen Forte's *Pitch Class Set Theory*. In fact, as he suggests, "nous comprenons la place de choix qui revient à la théorie des ensembles, non seulement pour la construction d'oevres nouvelles, mais aussi pour l'analyse et la meilleure compréhension des oeuvres du passé"[35]. What happens, he asks, if sets are not amorphous, as in *Herma*, but incidentally have an internal structure or symmetry? According to a famous *cliché* that "wherever symmetry occurs *Groups* describe it"[36] one is conducted back to a new field of modern mathematics, the

[33] As Xenakis explains in one of the quoted conversations, a *ordered structure* means that "given three elements of one set, you are able to put one of them in between the other two" (PNM, 1975-77, p.97).

[34] By looking at the earlier writings by Xenakis, as suggested by S. Kanach in Xenakis (1991, p. 380) one can discover that the first formalization of a 'Sieve Theory' appeared in his article La voie de la recherche et de la question (Preuves 177, Nov. 1965). In a definition given by the composer "it will be said that Sieve Theory is the study of the internal symmetries of a series of points either constructed intuitively, given by observation, or invented completely from moduli of repetition" (Xenakis, 1991 - p.276). For a technical mathematical discussion on Sieve Theory in relationship with, e.g., Messiaen's 'Modes of limited transposition' we refer to Andreatta (1997: Dissertation).

[35] See Iannis Xenakis, Revue d'esth., XIV, 3-4, 1961.

[36] See, for example, F.J. Budden: *The Fascination of Groups*, Cambridge University Press, 1972.

Abstract Algebra and, specifically, the *Group Theory*.[37] Note that with regard to the characteristics of the sonic object, as described at the begin of this chapter, all the sets H (pitch), G (intensity) and U (durations) have an Abelian (or commutative) group structure. Another particular musical group is the group of symmetries of the rectangle in the plane (or Klein four group). This is the theoretical basis of A. Schoenberg's 'Dodecaphonic System', as suggested by Xenakis (1991 - p.169) and discussed in Andreatta (1997: Diss.). More generally it can be shown that the collection of symmetries of a regular polyhedron has a group structure. This fact has been applied by Xenakis in compositions like *Nomos Alpha* and *Akrata* which are based, respectively, on the group of symmetries of cube and tetrahedron in space. In relation to what was discussed above, it seems reasonable to speak of these compositions as *Algebraic* rather than *Symbolic Music*. The final Appendix will show how an intelligent[38] combination of symbolic methods (i.e. Pitch-Class-Set Theory) and algebraic structures (i.e. Lewin's Generalized Interval System) have been applied, recently, in music analysis.[39] For the lack of time (and, intrinsically, of space) we will concentrate our discussion on the *methodology*, more than the results.

[37] For a formal definition see, e.g., Andreatta (1997: Dissertation).

[38] in the etymological sense, of *inter* (or *intus*) - *legere*, i.e. reading among and inside (the score, in this case).

[39] in particular such a new analytical approach seems to realise Information Theory's old dream of grasping the 'content' of a non tonal musical piece (K. Stockhausen's *Klavierstück III has never been analised by grammatical or informational-oriented theorists).*

Appendix.

David Lewin: the "art of *making* a network analysis" (Lewin, 1993 - p.53).

At the conclusion of a study dedicated to the problem of formalizing musical structures in a ever more algebraic perspective, the figure of the American Music Theorist D. Lewin is emblematic. By reading through the impressive list of articles written over almost forty years one could be surprised in discovering that the first contributions were very close to a probabilistic (even informational[40]) approach to music. Take, for example, his concept of *Interval Function* IFUNC, as introduced originally in Lewin (1959) with regard to sets (i.e. collection of notes). It represents not only a generalisation of Allen Forte's *Interval Vector* (Lewin, 1977) but, with the underlying structure of a *GIS*[41], it "can be given an interesting interpretation as a probability distribution" and therefore "it can be used to portray a statistical texture" (Lewin, 1987- p.101). To understand this point we have to refer to the notion of *Generalized Interval System* as largely explored in Lewin's first book Generalized Musical Intervals and Transformations (Yale: 1987).[42] This is the only technical definition that we will give, because the intent of this chapter is that of discussing the underlying methodology in a network analysis, more than the particular results.

A *GIS* can be defined as a triple (*S, IVLS, int*) where:

- *S* is a set (i.e. a collection of objects)
- *IVLS* is a mathematical group (the group of generalized intervals)
- *int* is a function which, given two objects *s, t* of *S* it assigns a value *int(s,t)* in *IVLS* such that the following properties are satisfied:

  -Given the objects *r, s, t* of *S*

$$int(r,s)\cdot int(s,t) = int(r,t)$$

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[40] See, e.g., his paper Some Applications of Communication Theory to the Study of Twelve-Tone Music (in *Journal of Music Theory* 12(1), 1968, pp. 50-84.)

[41] or *G*eneralized *I*nterval *S*ystem, for which we shall give a formal and an intuitive definition later. Note that probabilistic ideas were also discussed in Lewin (1979-80).

[42] In this "major treatise", as John Clough suggested in his Review on Lewin's book, "David Lewin has assembled thematic ideas from nearly 30 years of his published work, incorporated significant new material, constructed a formal mathematical theory encompassing it all [...]" (Clough, 1989 - p.226). No surprise, therefore, if the definition of *GIS* coincides with that of *FIS* (i.e. Formal Interval System) as described in a earlier work (Lewin, 1984).

- For every element *s* of *S* and for every "interval" *i* of *IVLS* there is a unique element *t* in *S* such that *int (s,t) = i.*

As explained by the author in a recent paper, "this notion generalized certain intuitions we have concerning traditional sorts of intervals that are directed from one pitch (or pitch class) to another. Generalized intervals are similarly directed, from one object of a *GIS* to another. These objects need not to be pitches or pitch classes; they may have a rhythmic, timbral, or other sort of character" (Lewin, 1995 - p.81). With regard to Xenakis' rhythmic model[43] one can take *S* as the set of points in (real number) time, as *IVLS* the group of numbers under addition and by *int(s,t)* the number of time units that *t* is later than *s.* It is easy to verify that for such an intervallic function as *int* the properties required in the formal definition of a *GIS* are verified. As pointed out before, we are more interested in the methodological problems involved by these ideas more than the practical results. This fact is expressed by Lewin, when he suggests to "conceive the formal space of a *GIS* as a space of *theoretical potentialities* [italics mine] rather than a compendium of musical practicalities" (Lewin, 1987 - p.27). In the same way in a network analysis, such as we will discuss later, *how* one arrives at his conclusions is more relevant that *what* those conclusions are. In other words, "one cannot be methodologically thorough in reporting an intellectual exercise without reporting the conditions under which the exercise was carried out" (Lewin, 1993 - p.20).[44] Before describing some aspects of the *intellectual exercise* necessary for a network analysis of Stockhausen's *Klavierstück III* we need to make the terminology clear. By using an image that sounds perfectly in line with an Information theoretical perspective, "music analysis is ultimately a problem of description. Description [...] must be viewed as an act of communication, and any study of such an act must address the problem that the agent doing the describing and the agent "receiving" the description must either share or negotiate a common language" (Smoliar, 1994). The new language is that of *Transformation Graphs* and *Networks*, which were introduced in the second part of Lewin (1987). The formal definitions are based on a earlier paper (Lewin, 1982-83) but we prefer to discuss them intuitively.[45]

[43] See Chapter III of this study.
[44] See Chapter II of this study for an equivalent perspective in Information Theory.
[45] See Rahn (1987) and (1995) for further aspects of this topic.

A *graph* is simply a system of points (called nodes) that are connected by arrows. By labelling opportunely the arrows and, according to these, the nodes, one obtains a transformation graph and a transformation network respectively. These concepts are musically relevant because they model "very naturally any situation in which music *moves* in some sense, perhaps temporally, but more generally independently of temporal order" (Rahn, 1987 - p.313). This last remark clearly explains D. Lewin's art of making a pcset network for Stockhausen *Klavierstück III*, "a piece that has resisted cogent explanation with previously available methods" (Morris, 1995 - p. 351)[46].

With his network Lewin reduces the work to a single pentachord that generates *musical forms* and *transformations*, "despite surface configurations which tend to camouflage the projection of the network to the ear" (Morris, 1995 - p. 351). Questions about the relationships between network-structures and the 'human mind' are perhaps the most interesting to pose today. Perceptual investigations, from a phenomenological[47] to a cognitive[48] or Artificial Intellicence-oriented perspective[49] could give, probably, more suggestions about the tension between formal (or formalized) musical structures and the mental processes involved in the act of listening.

[46] See, for example, Jonathan Harvey's analysis in The Music of Stockausen (Berkeley: University of California Press, 1975) or Nicholas Cook's approach, as described in his book A Guide to Musical Analysis (New York, 1987) and discussed in the Appendix of the second chapter in Lewin (1993- pp.53-67). One can easily comprehend the difficulties of grasping the informational-content of such a piece as well as giving a description of it in terms of generative grammars or Schenkerian techniques. It seems, therefore, reasonable to say that "Lewin provides the potential for new thoughts, not only about the music being analyzed, but also about the nature of music itself" (Smoliar, 1994 - p.77).

[47] See Lewin (1986).

[48] See, for example, Balzano (1982) or Longuet-Higgins (1987).

[49] As in Minsky (1982)

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