ATIAM

PARCOURS DU MASTER SCIENCES ET TECHNOLOGIES
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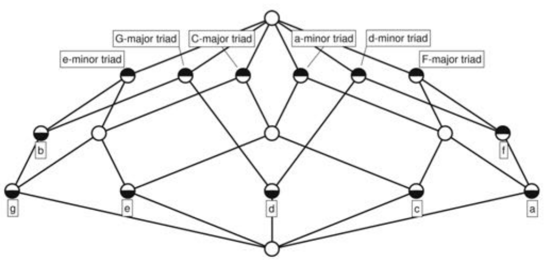
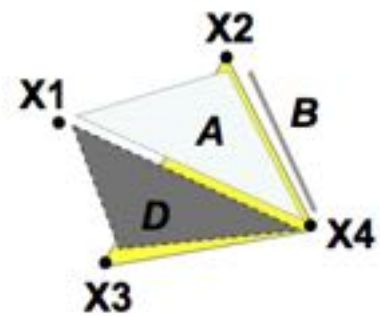
The interplay between algebra and geometry in computational musicology

– Part I –

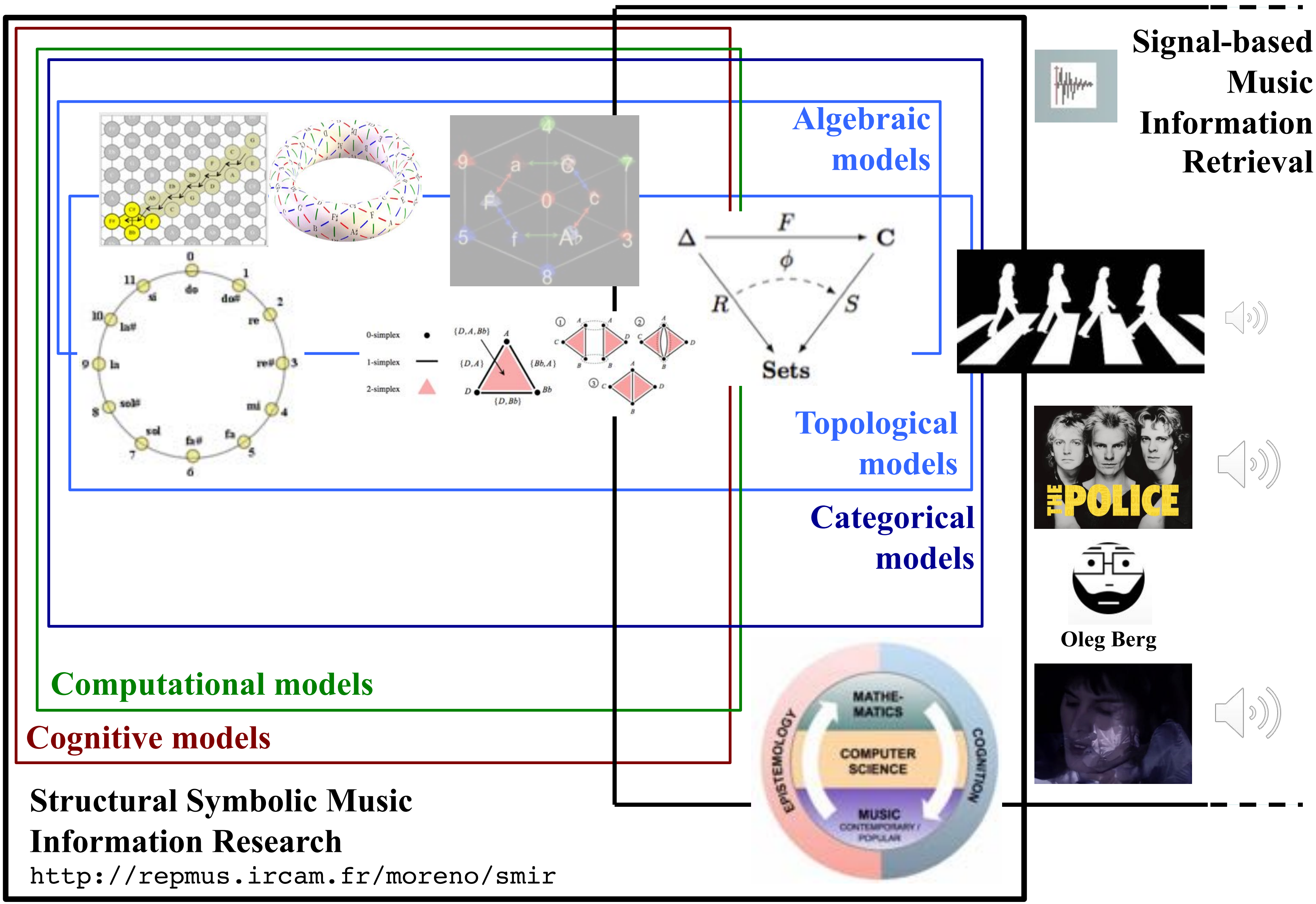




Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS/UPMC
http://www.ircam.fr/repmus.html

ircam Centre Pompidou · cnrs · SORBONNE UNIVERSITÉ · IRMA Institut de Recherche Mathématique Avancée · GREAM · USIAS Université de Strasbourg Institut d'Études Avancées

The SMIR Project: Structural Music Information Research



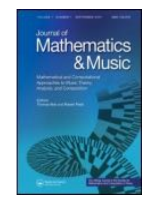
Computational Musicology in academic research

Conferences of the SMCM:

- **2007 Technische Universität (Berlin, Allemagne)**
- **2009 Yale University (New Haven, USA)**
- **2011 IRCAM (Paris, France)**
- **2013 McGill University (Canada)**
- **2015 Queen Mary University (Londres)**
- **2017 UNAM (Mexico City)**
- **2019 Universidad Complutense de Madrid (Spain)**

Official Journal and MC code (00A65: Mathematics and Music)

- ***Journal of Mathematics and Music*, Taylor & Francis**
  (Editors: Th. Fiore, C. Callender | Associate eds.: E. Amiot, J. Yust)



Books Series:

- ***Computational Music Sciences* Series, Springer (G. Mazzola & M. Andreatta eds. – 12 books published (since 2009)**
- **Collection *Musique/Sciences*, Ircam-Delatour France (J.-M. Bardez & M. Andreatta dir. – 16 books published (since 2006)**

Some examples of PhD on maths & music

- **Alessandro Ratoci**, Vers l'hybridation stylistique assistée par ordinateur, PhD in music **composition & research**, Sorbonne University / IRCAM (cosupervised with Laurent Cugny)
- **Sonia Cannas**, *Représentations géométriques et formalisations algébriques en musicologie computationnelle*, PhD in **maths** in cotutelle agreement, **University of Pavia** (L. Pernazza) / **Université de Strasbourg** (A. Papadopoulos & M. Andreatta). To be defended in 2019.
- **Grégoire Genuys**, *Théorie de l'homométrie et musique*, PhD in **maths**, **Sorbonne University** / IRCAM (cosupervised with Jean-Paul Allouche), 2017.
- **Hélianthe Caure**, *Pavages en musique et conjectures ouvertes en mathématiques*, PhD in **computer science**, **Sorbonne University** (cosupervised with Jean-Paul Allouche), 2016.
- **Mattia Bergomi**, *Dynamical and topological tools for (modern) music analysis,* PhD in **maths** in a cotutelle agreement Sorbonne University / University of Milan (cosupervised with Goffredo Haus, 2015).
- **Charles De Paiva**, *Systèmes complexes et informatique musicale*, thèse de doctorat, Programme Doctoral International « Modélisation des Systèmes Complexes », PhD in **musicology** in a cotutelle agreement, **Sorbonne University** / **UNICAMP**, Brésil, 2016.
- **John Mandereau**, *Des systèmes d'Intervalles Généralisés aux Systèmes Evolutifs à Mémoire : aspects théoriques et computationnels*, thèse de doctorat en mathématiques, PhD in cotutelle agreement **University of Pisa** / **Sorbonne University** (cosupervised with F. Acquistapace). PhD in **maths** (aborted).
- **Louis Bigo**, *Représentation symboliques musicales et calcul spatial*, PhD in **computer science**, **University of Paris Est Créteil** / **IRCAM**, 2013 (cosupervised with Olivier Michel and Antoine Spicher)
- **Emmanuel Amiot**, *Modèles algébriques et algorithmiques pour la formalisation mathématique de structures musicales*, PhD in, **Sorbonne University** / **IRCAM**, 2010 (cosupervised with Carlos Agon) **computer science**
- **Yun-Kang Ahn**, *L'analyse musicale computationnelle*, PhD in **computer science**, **Sorbonne University** / **IRCAM**, 2009 (cosupervised with Carlos Agon)

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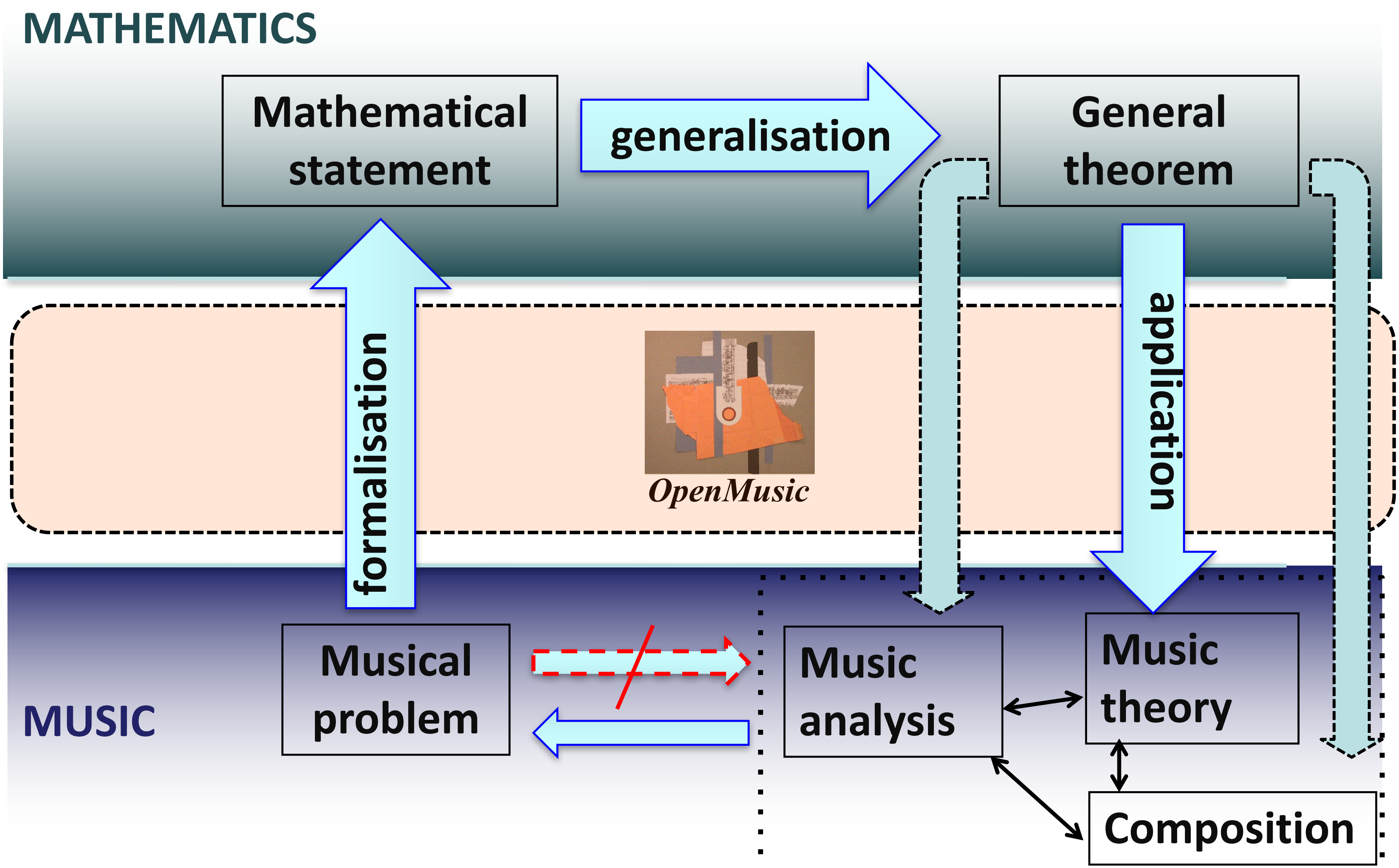
The double movement of a 'mathemusical' activity



Periodic sequences and finite difference calculus

$$Df(x)=f(x)-f(x-1)$$

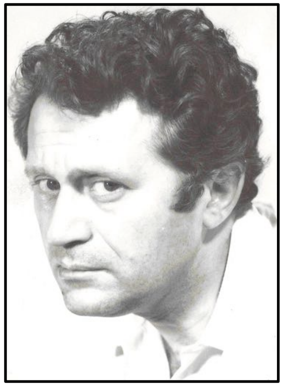
f = **7 11 10 11 7 2** 7 11 10 11 7 2 7 11…

Df = **4 11 1 8 7 5** 4 11 1 8 7 5 4 11…

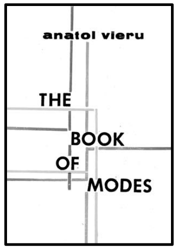
$D^2 f$ = **7 2 7 11 10** 11 7 2 7 11 10 11…

$D^3 f$ = **7 5 4 11 1 8** 7 5 4 11 1 8…

$D^k f$ = ……



Anatol Vieru



V	0	3	8	7	11	0	11	10	6	9	0	9	1	2	9	8	4	3	6
VIII	0	0	0	0	3	3	7	2	0	0	0	6	3	3	3	4	8	0	0
IV	3	3	4	4	1	11	11	8	3	3	9	4	1	7	11	8	11	3	9
IX	0	0	0	0	0	3	6	[1]	3	3	3	3	9	0	3	6	[10]	6	6
IV	0	10	3	9	10	0	9	7	0	6	7	9	6	4	9	3	4	6	3

Anatol Vieru: *Zone d'oubli* pour alto (1973)

Reducible and reproducible sequences

f = 11 6 7 2 3 10 11 6 …

Df = 7 1 7 1 7 1 7 1…

D^2f = 6 6 6 6 6…

D^4f = 0 0 0

Reducible sequences:

$\exists\ k \geq 1$ such that

$D^k f = 0$

---

f = 7 11 10 11 7 2 7 11 …

Df = 4 11 1 8 7 5 4 11 1 …

D^2f = 7 2 7 11 10 11 7 2 7 …

D^3f = 7 5 4 11 1 8 7 5 4 11 1 8…

D^4f = 10 11 7 2 7 11 10 11 …

D^5f = 1 8 7 5 4 11 1 8 …

D^6f = 7 11 10 11 7 2 7 11 …

Reproducible sequences:

$\exists\ k \geq 1$ such that

$D^k f = f$

A decomposition property of any periodic sequence

$Df(x)=f(x)-f(x-1).$

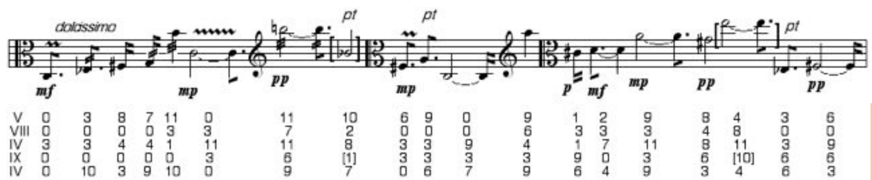
7 11 10 11 7 2 7 11 10 11 7 2 7 11...

4 11 1 8 7 5 4 11 1 8 7 5 4 11...

7 2 7 11 10 11 7 2 7 11 10 11...

7 5 4 11 1 8 7 5 4 11 1 8...

......



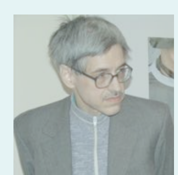
Anatol Vieru: *Zone d'oubli* for viola (1973)

Reducible sequences:
$\exists\ k\geq 1$ **such that** $D^k f = 0$

Reproducible sequences:
$\exists\ k\geq 1$ **such that** $D^k f = f$

- **Decomposition theorem**

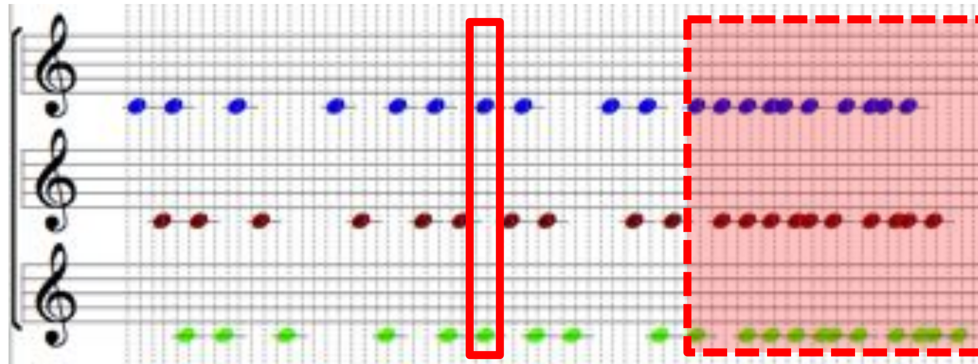
(Vuza & Andreatta, *Tatra M.*, 2001)
Every periodic sequence f can be decomposed in a unique way as a sum $f_1 + f_2$ of a reducible sequence f_1 and a reproducible sequence f_2



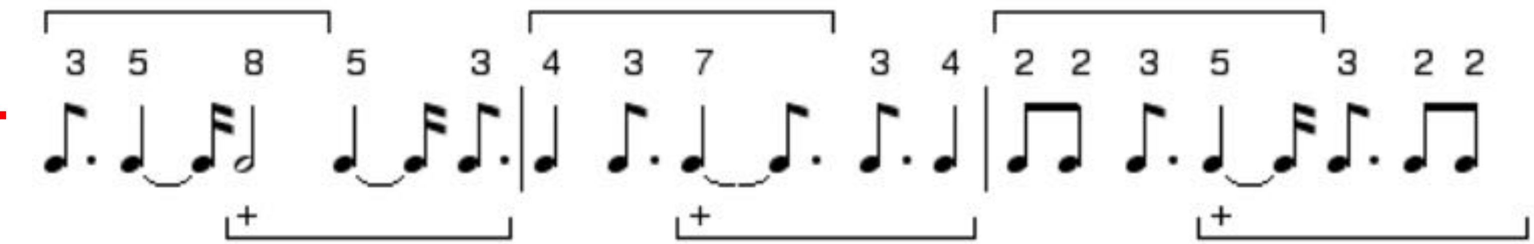
D. Vuza, M. Andreatta (2001), « On some properties of periodic sequences in Anatol Vieru's modal theory », Tatra Mountains Mathematical Publications, Vol. 23, p. 1-15

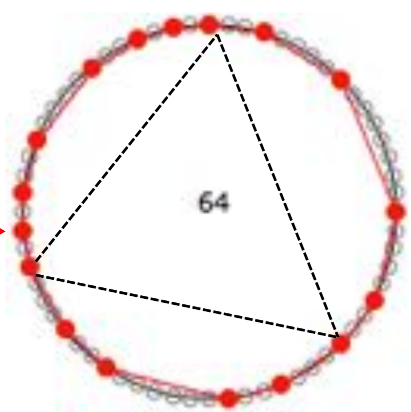
Periodic rhythmic sequences and tiling canons

Harawi (1945)



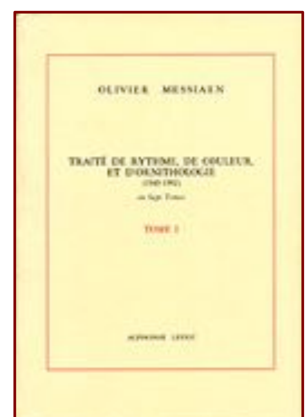
Harawi: rhythmic reduction

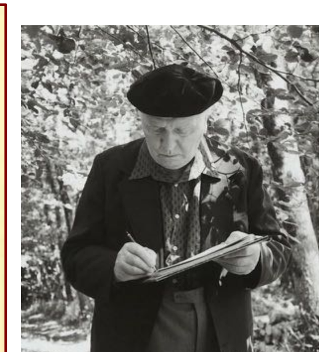




« …il résulte de tout cela que les différentes sonorités se mélangent ou s’opposent de manières très diverses, jamais au même moment ni au même endroit […]. C’est du désordre organisé »

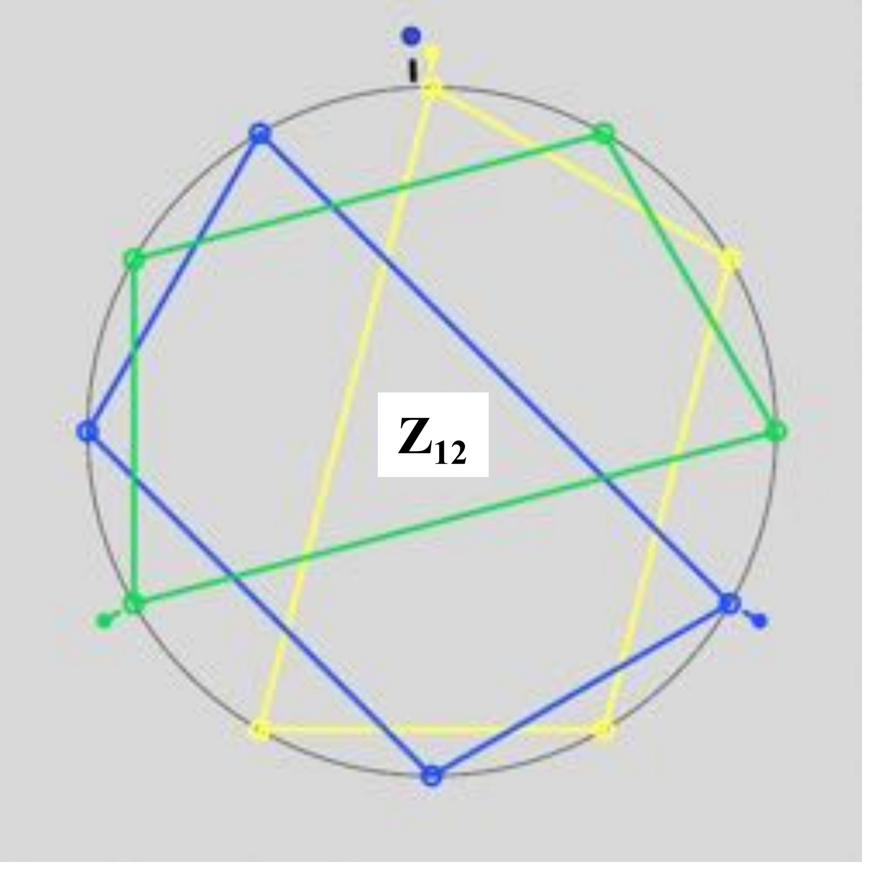
O. Messiaen: *Traité de Rythme, de Couleur et d’Ornithologie,* tome 2, Alphonse Leduc, 1992.

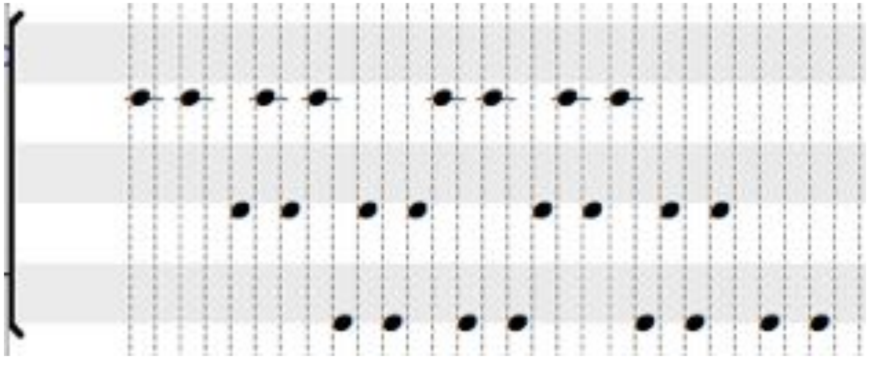




Olivier Messiaen

Formalizing the tiling process as a direct sum of subsets





$A_1 = \{0, 2, 5, 7\}$

T_4

$A_2 = \{4, 6, 9, 11\}$

T_4

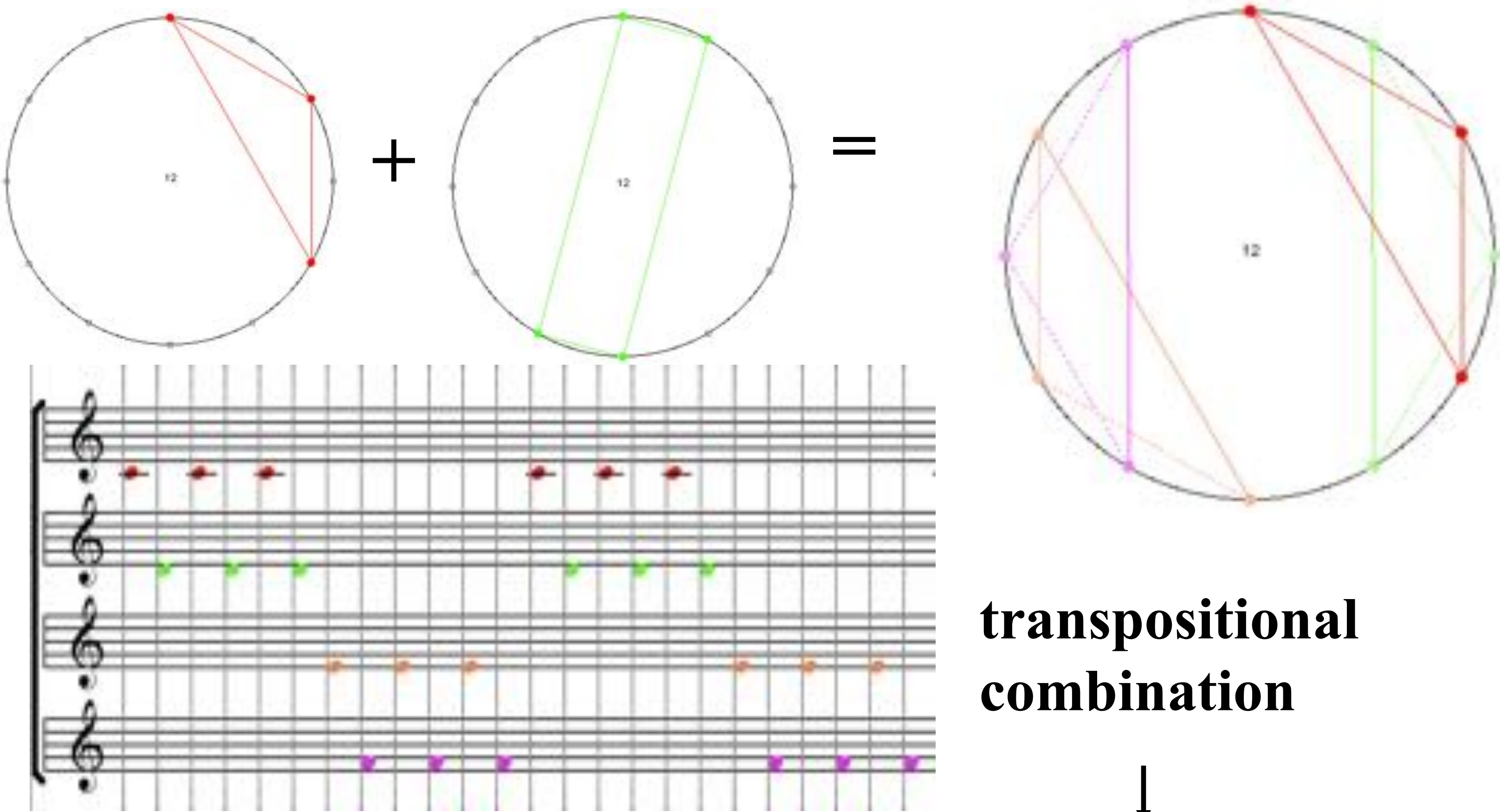
$A_3 = \{8, 10, 1, 3\}$

$Z_{12} = A_1 \cup A_2 \cup A_3$

$Z_{12} = A \oplus B$

$A = \{0,2,5,7\}$
$B = \{0,4,8\}$

Rhythmic tiling canons with no regular entries

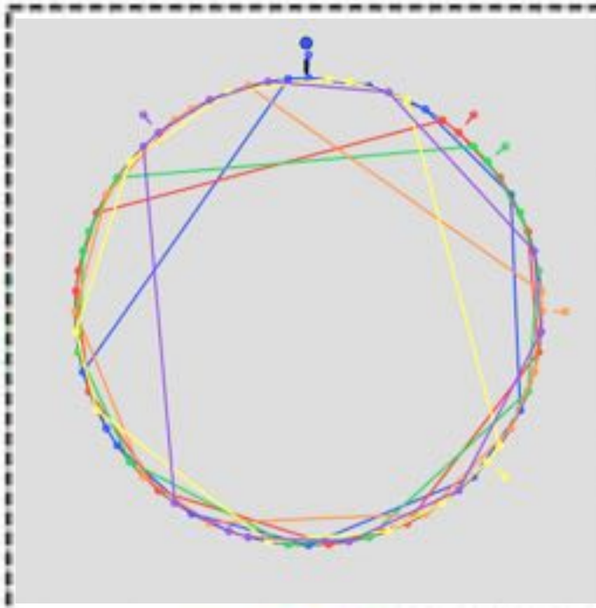


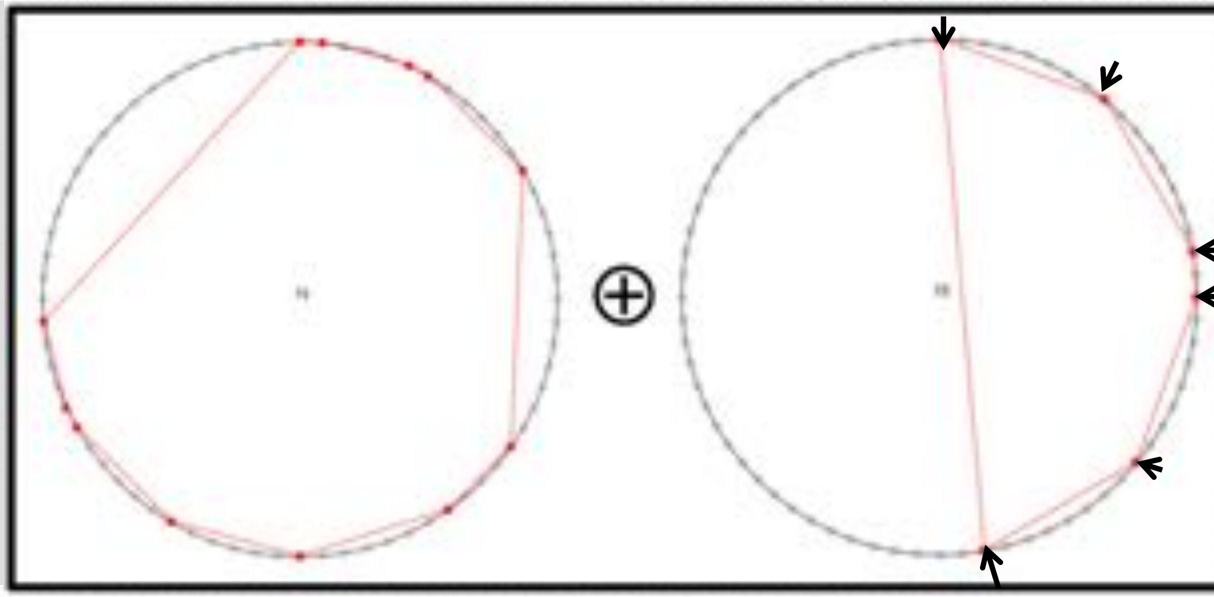
transpositional combination

$$\{0,2,4\} \oplus \{0,1,6,7\} = Z_{12} = (2\ 2\ 8) \bullet (1\ 5\ 1\ 5)$$

One of the two factors is a Messiaen's mode of limited transposition

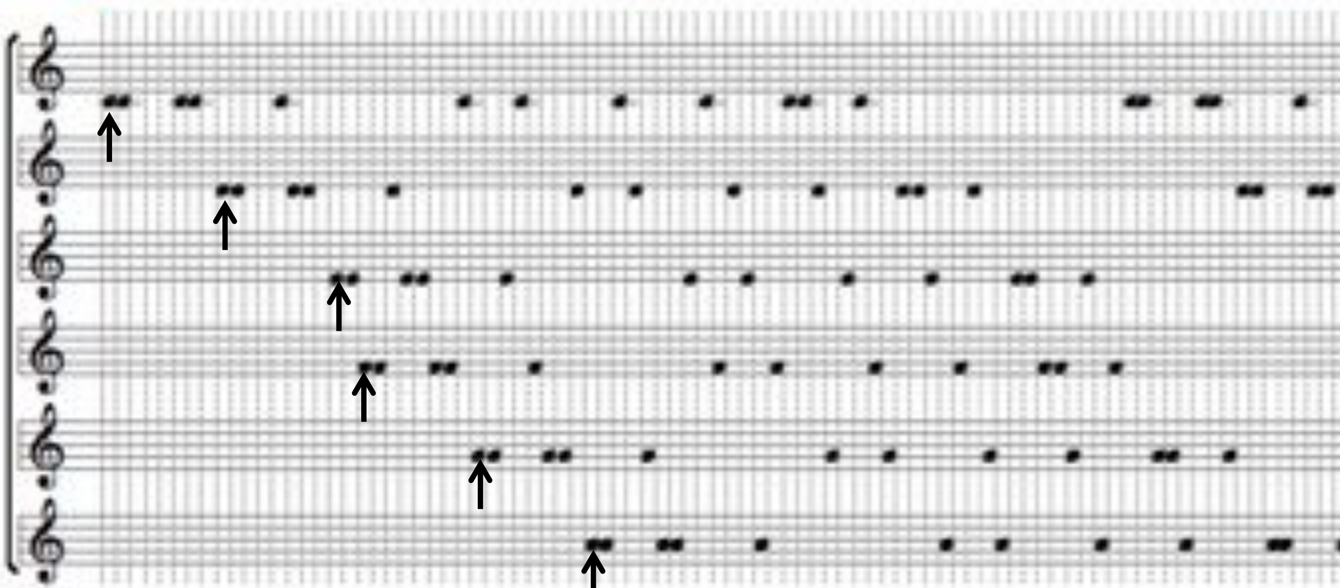
Aperiodic Rhythmic Tiling Canons (Vuza Canons)



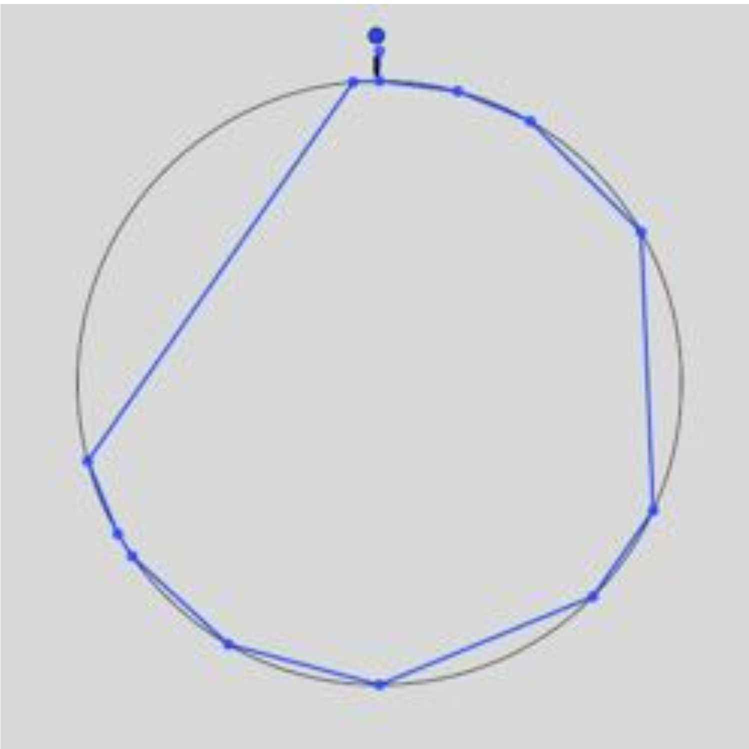


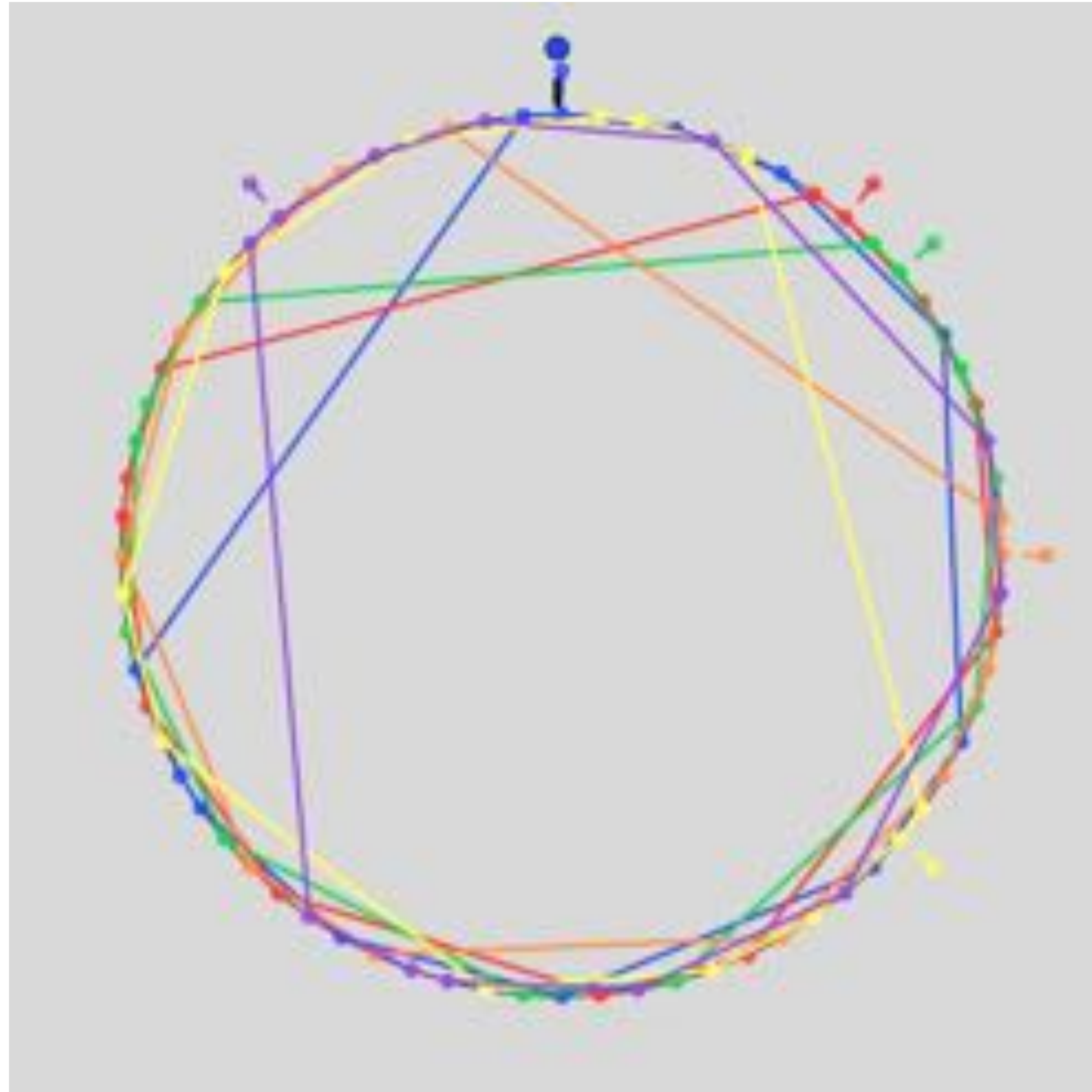
Dan Vuza

Anatol Vieru

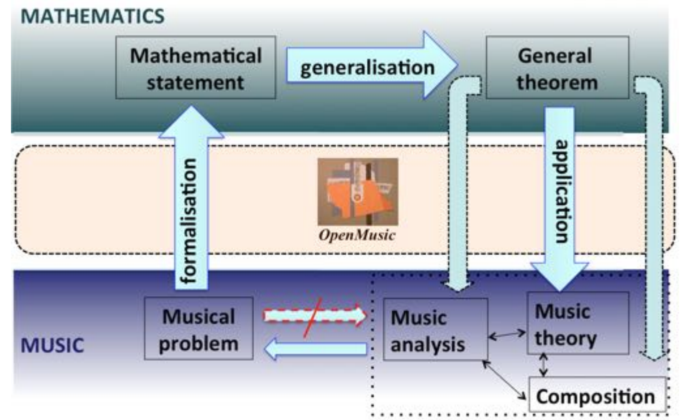


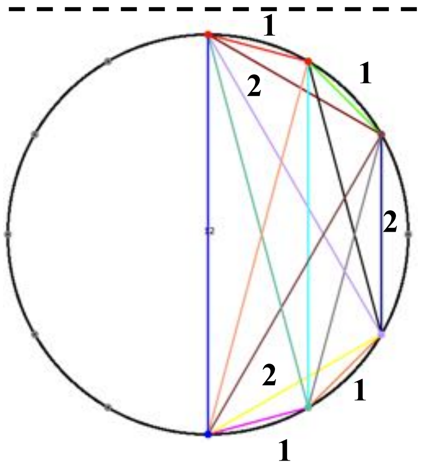
The melody of a Vuza Canon





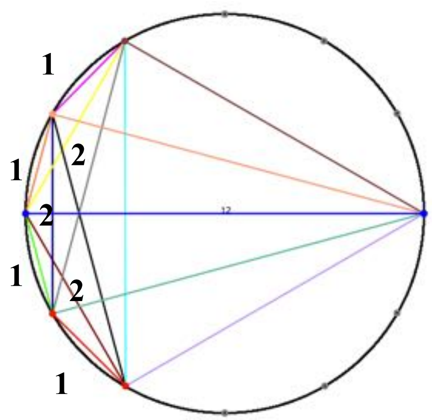
A historical example of “mathemusical” problem

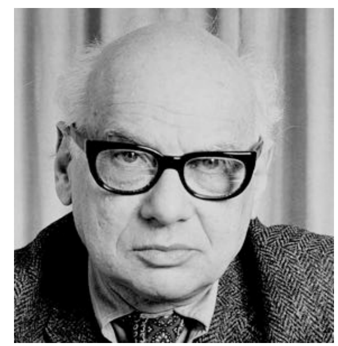




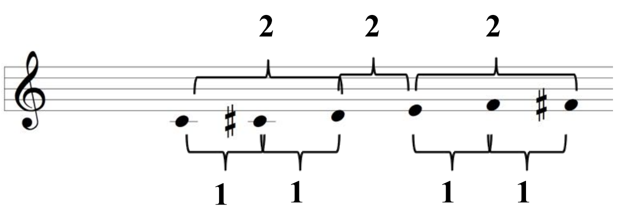
HOMOMETRY ⟷

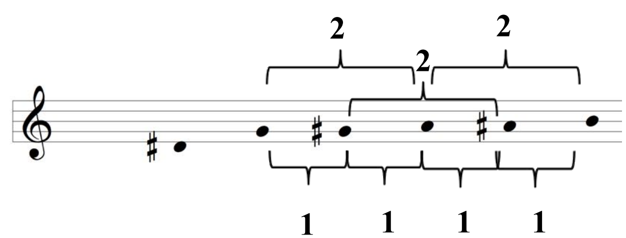
Hexachord Theorem



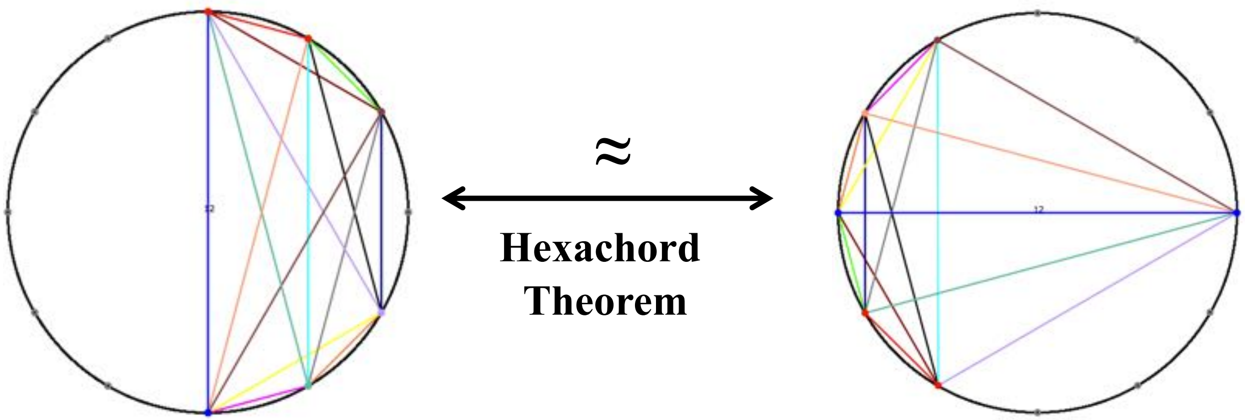


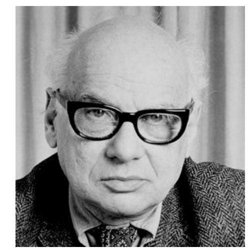
M. Babbitt





The shortest proof of Babbitt's Theorem?





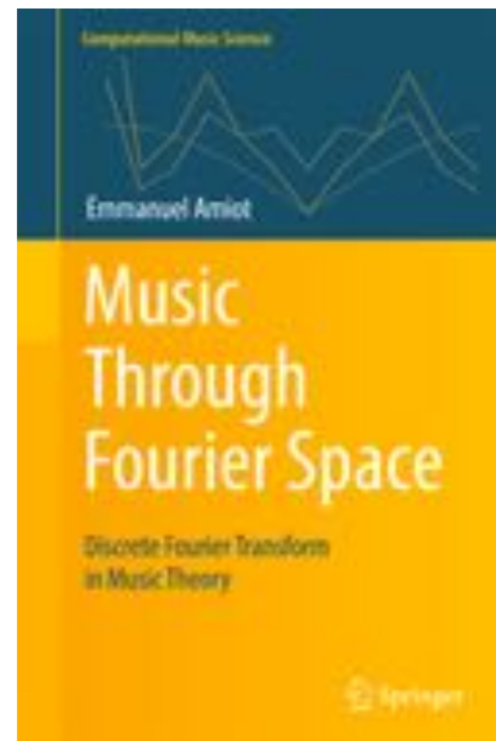
M. Babbitt

$IC_A = [4, 3, 2, 3, 2, 1\ ] = [4, 3, 2, 3, 2, 1\ ] = IC_{A'}$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$\forall \mathrm{k}\ \mathcal{F}(\mathrm{IC}_{\mathbb{Z}_c \setminus A})(\mathrm{k}) = \mathcal{F}(\mathrm{IC}_A)(\mathrm{k})$$



E. Amiot : « Une preuve élégante du théorème de Babbitt par transformée de Fourier discrète », Quadrature, 61, 2006.

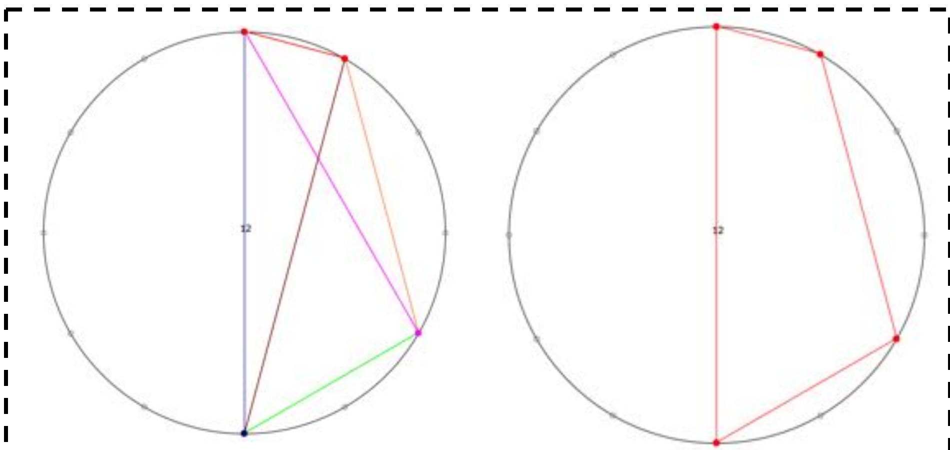
Z-relation, homometry and phase retrieval problem

- Two sets are Z-related if they have the same module of the DFT

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

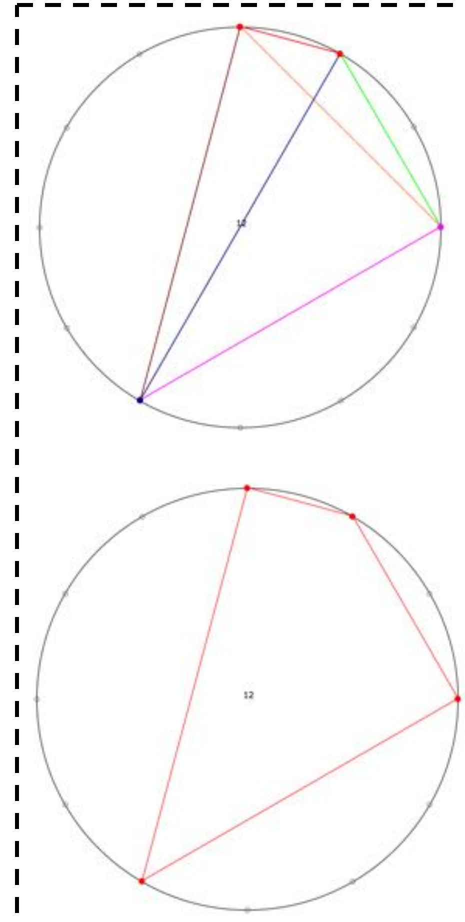
$$\mathcal{F}(IC_A) = \mathcal{F}_A \times \mathcal{F}_{-A} = |\mathcal{F}_A|^2$$



Z-relation

⟷

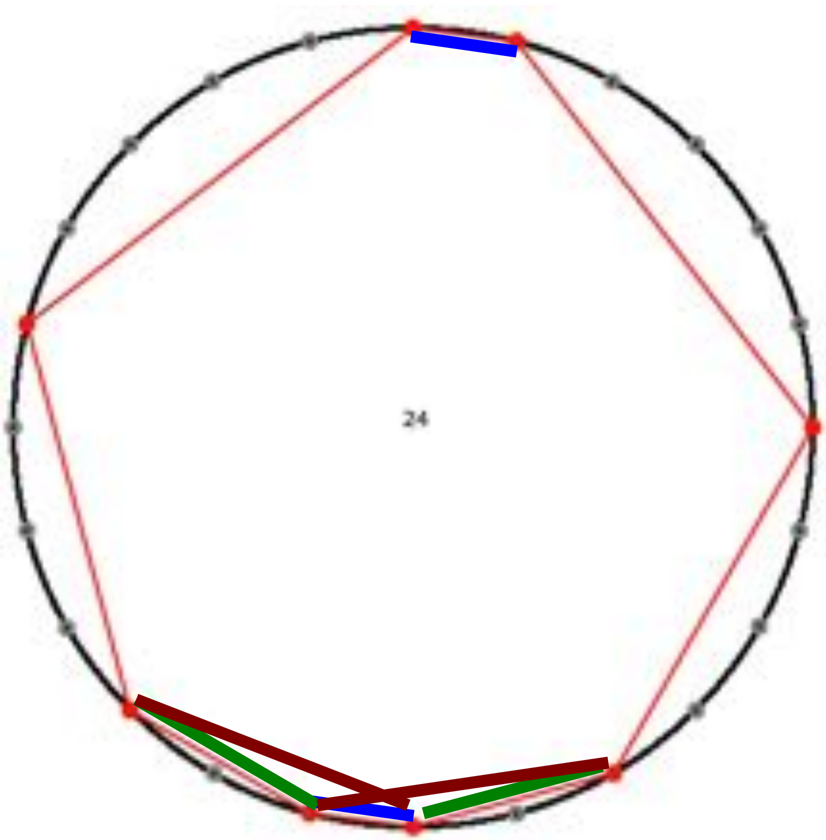
homometry



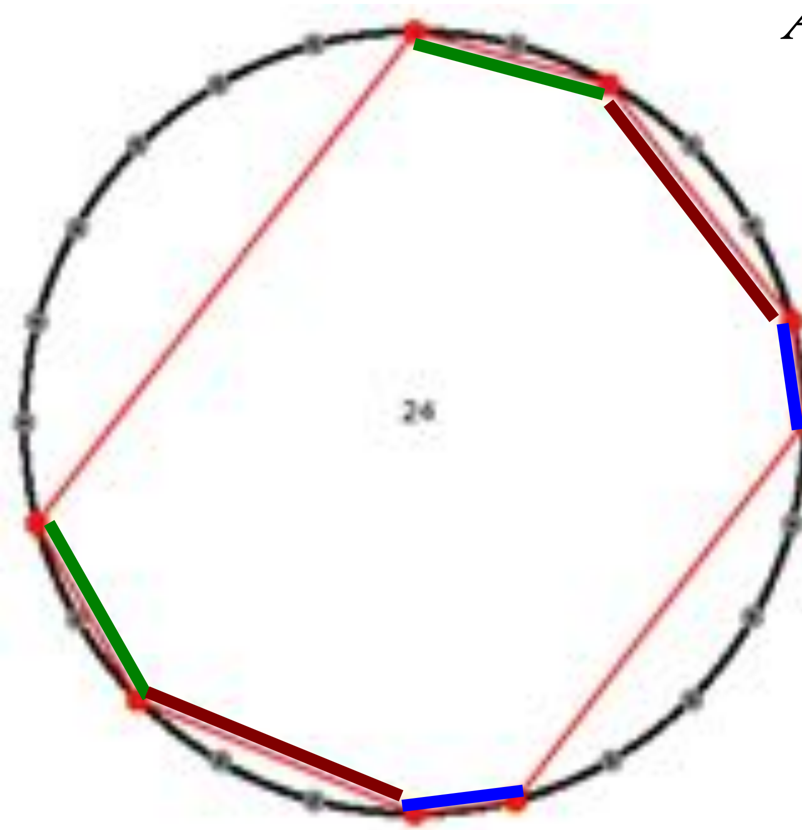
• **Mandereau J., D. Ghisi, E. Amiot, M. Andreatta, C. Agon, (2011), « Z-relation and homometry in musical distributions »,** ***Journal of Mathematics and Music*****, vol. 5, n° 2, p. 83-98.**
• **Mandereau J, D. Ghisi, E. Amiot, M. Andreatta, C. Agon (2011), « Discrete phase retrieval in musical structures »,** ***Journal of Mathematics and Music*****, vol. 5, n° 2, p. 99-116.**

Z-relation (music) and homometry (cristallography)

A



A′

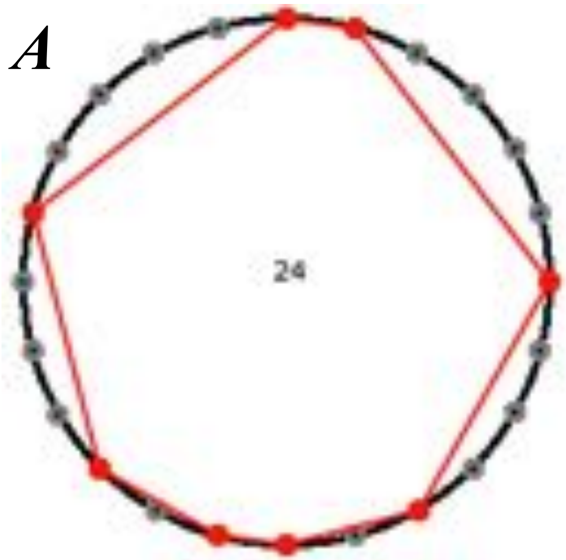


$$IC_A(k) = (1_A \star 1_{-A})(k)$$

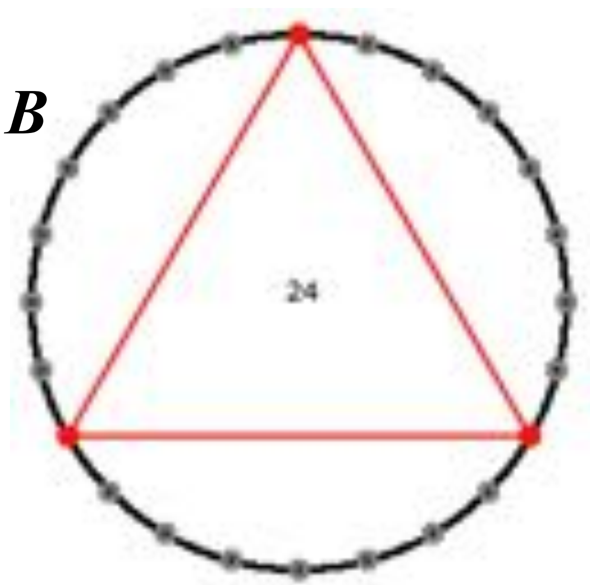
$$A \sim A' \quad \overset{\text{Z-relation}}{\underset{\text{Homometry}}{\Longleftrightarrow}} \quad IC_A(k) = IC_{A'}(k) \quad \Longleftrightarrow \quad |F_A|^2 = |F_{A'}|^2$$

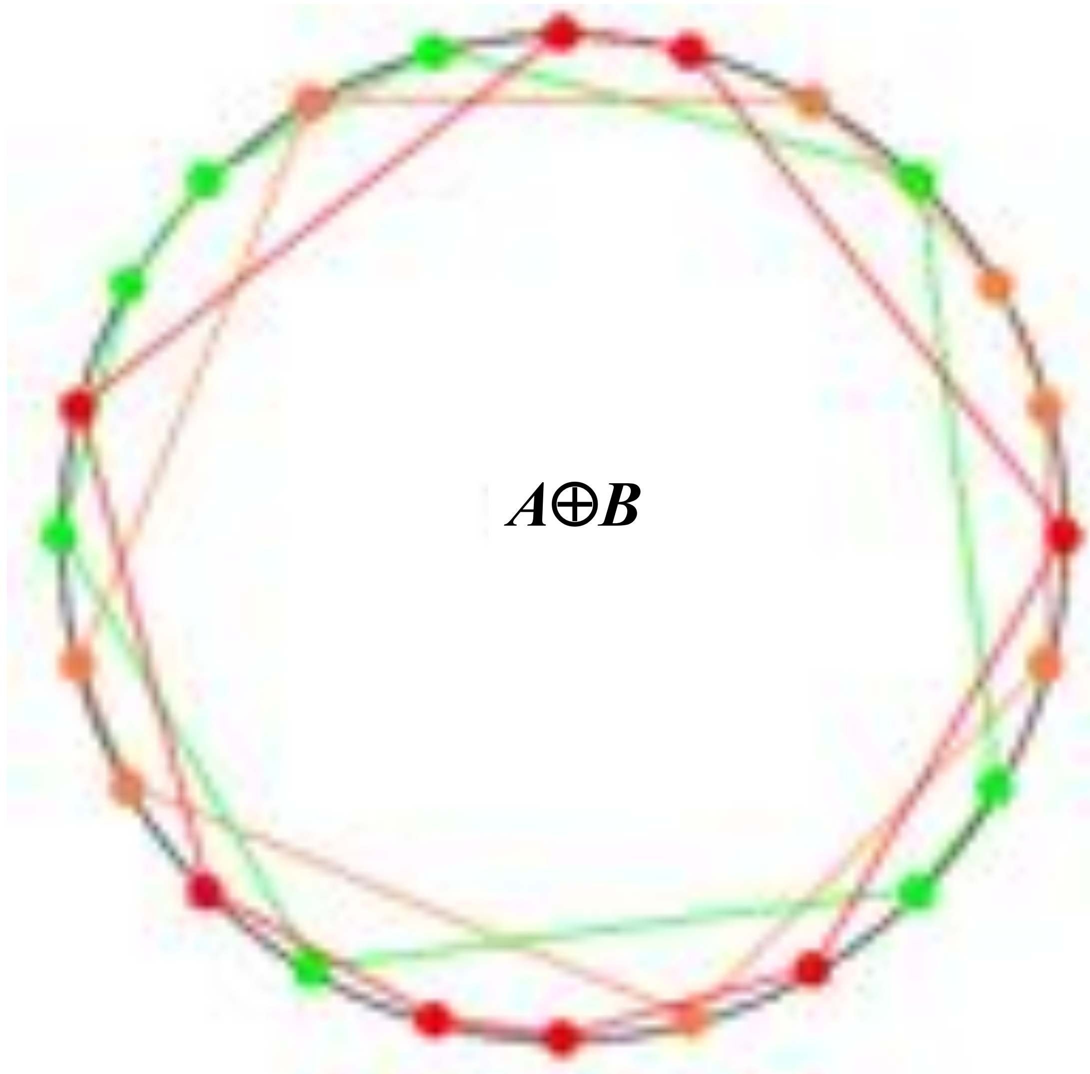
Tiling Rhythmic Canons and Homometry

A



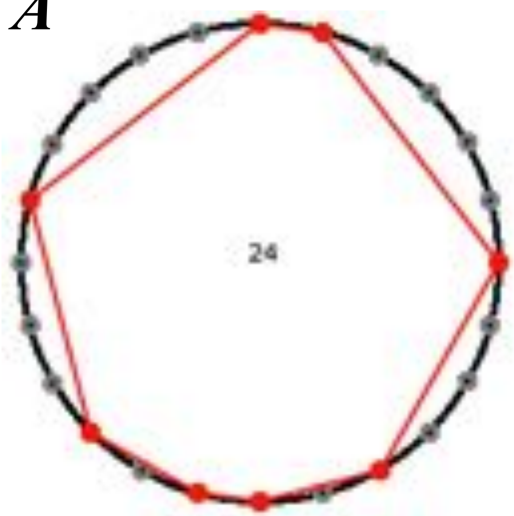
B



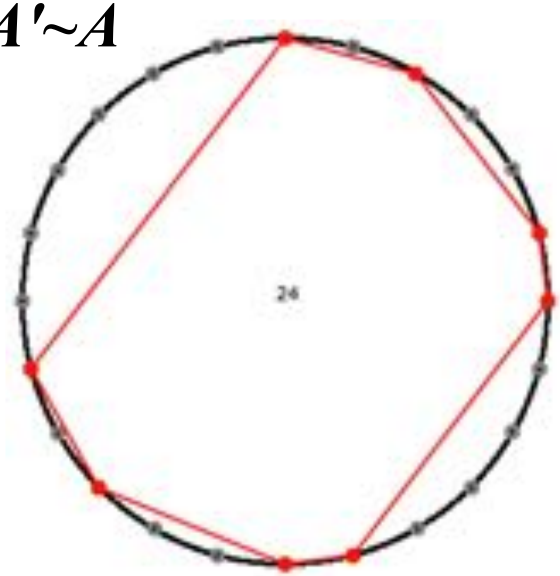


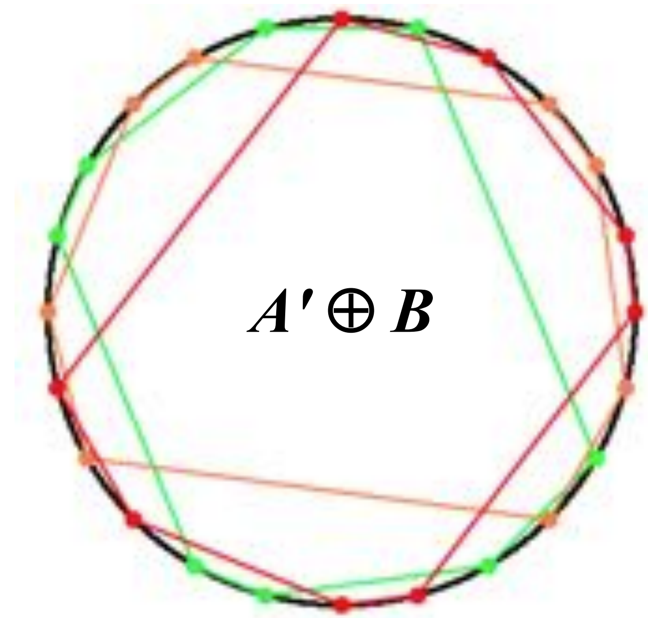
Tiling Rhythmic Canons and Homometry

A

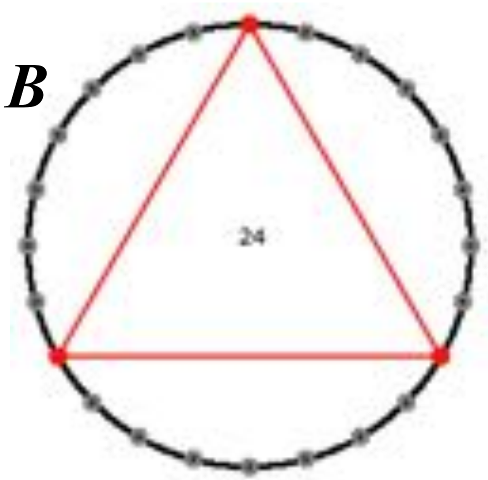


A'~A





B

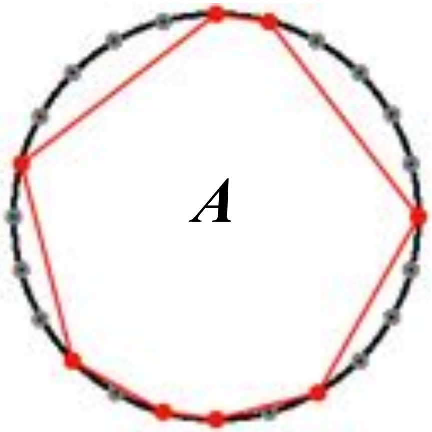


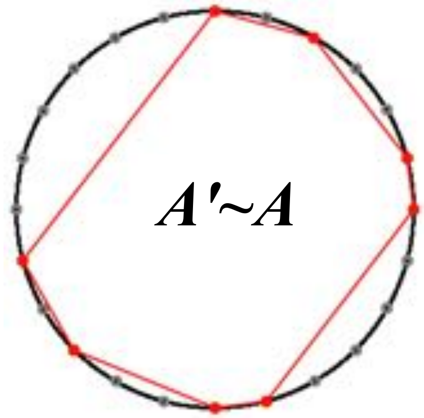
A musical offering:

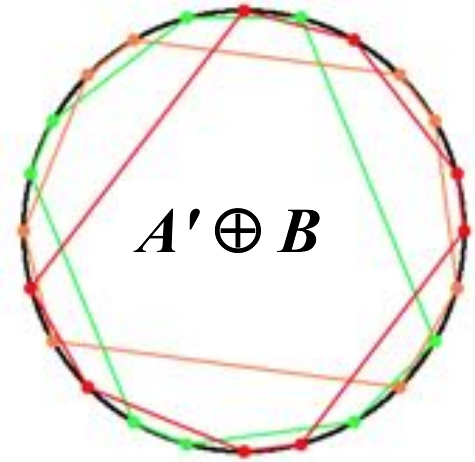
- ***Theorem***:
 If A tiles with B and A' has the same IC, then A' tiles with B, too.

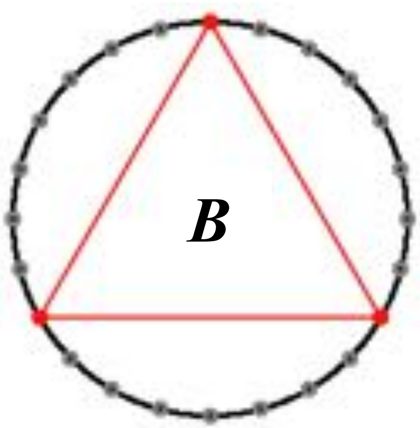
E. Amiot, « New Perspectives on Rhythmic Canons and the Spectral Conjecture », *JMM*, 2009.

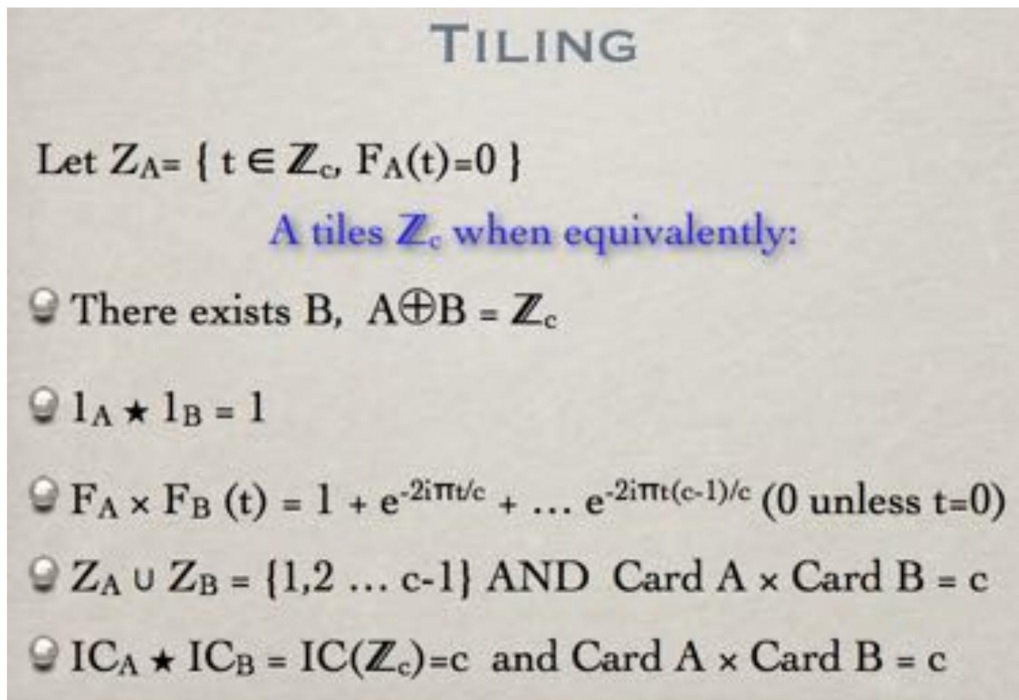
Tiling Rhythmic Canons and Homometry





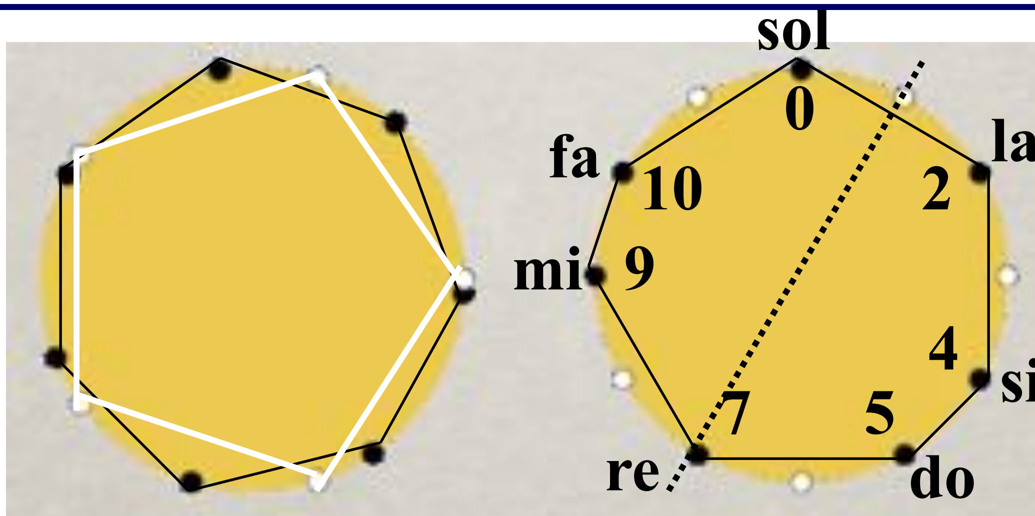






E. Amiot, « New Perspectives on Rhythmic Canons and the Spectral Conjecture », *JMM*, 2009.

Maximally Even Sets



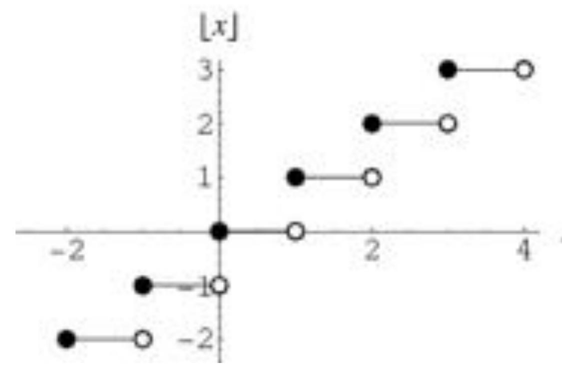
Diatonic scale:
{0, 2, 4, 5, 7, 9, 10}

Pentatonic scale:
{1, 3, 6, 8, 11}

Definition (Clough-Myerson-Douthett) A set *A* with cardinality *d* in a given equal tempered space $\mathbf{Z}_c$ is maximally even if $A = \{a_k\}$

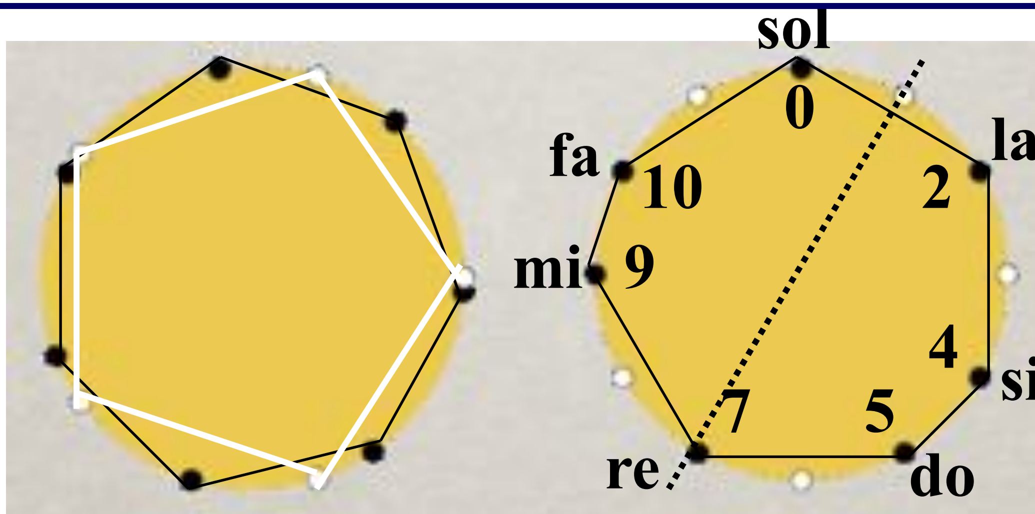
$$a_k = J_{c,d}^{\alpha}(k) = \lfloor \frac{kc + \alpha}{d} \rfloor$$

where $\alpha \in \mathbf{R}$
$\lfloor x \rfloor$ is the integer part of x



$$J_{12,7}^{5} = \left\{ \left\lfloor \frac{12k+5}{7} \right\rfloor \right\}_{k=0}^{6} = \{0,2,4,5,7,9,11\}$$

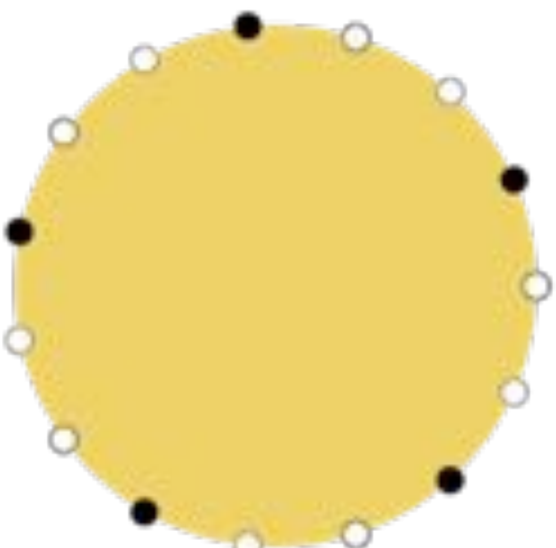
Maximally Even Sets



Diatonic scale:
{0, 2, 4, 5, 7, 9, 10}

Pentatonic scale:
{1, 3, 6, 8, 11}

Definition (Amiot, 2005) A set A with cardinality d given equal tempered space $\mathbf{Z}_c$ is maximally even if $|F_A(d)| \geq |F_B(d)|$ for all subsets B of cardinality d in $\mathbf{Z}_c$.

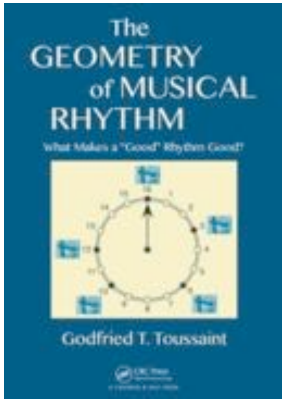


where $F_{set}(t) := \sum_{k \in set} e^{2i\pi kt/12}$

$$|F_A(5)| = 1+1+1+1+1 = 5$$

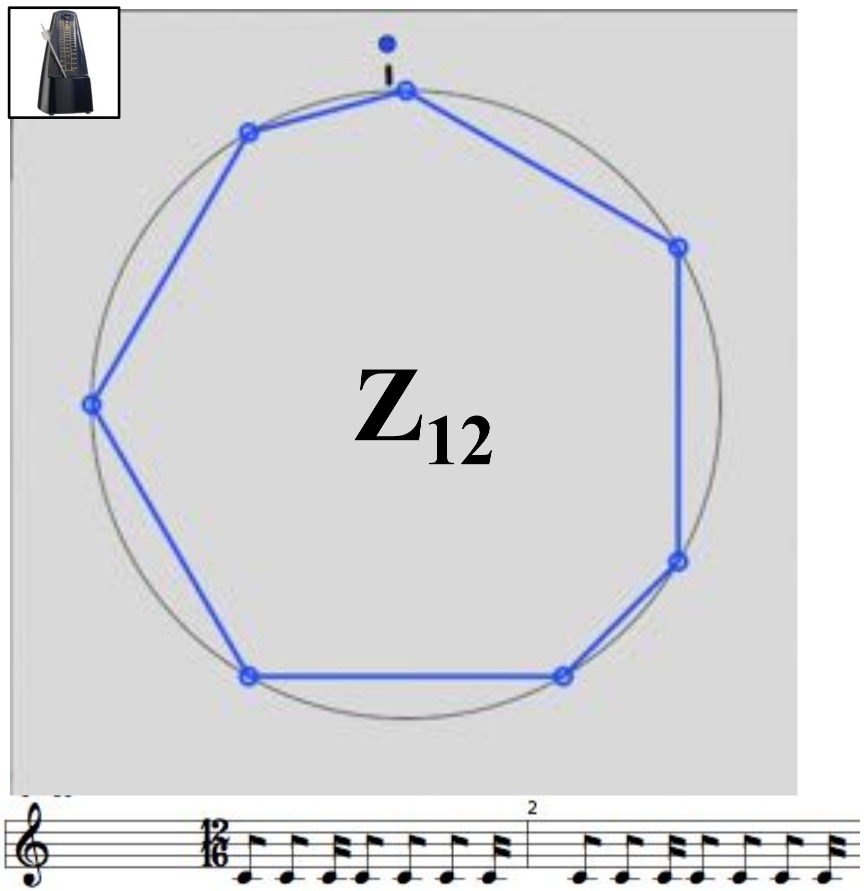
En général, $|F_A(t)| \leq \#A$

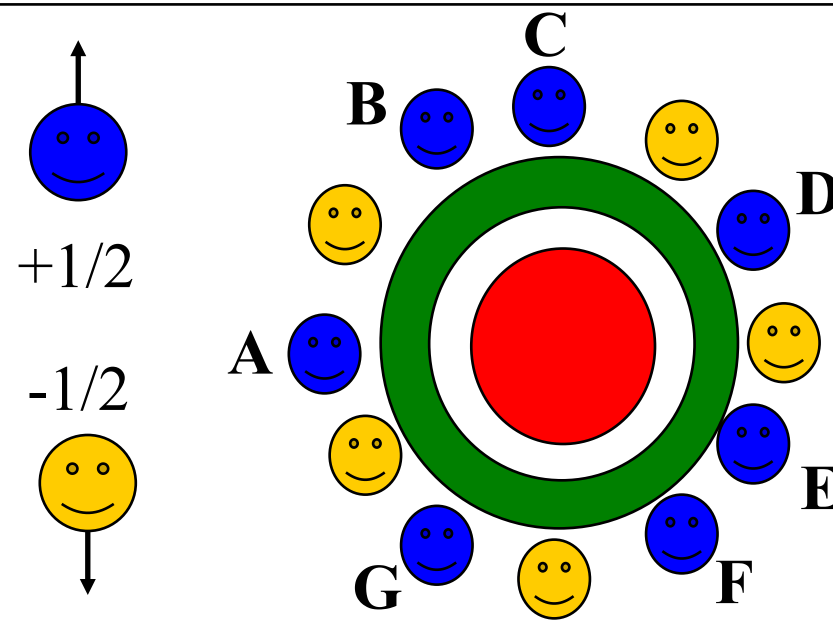
The pitch-rhythm isomorphic correspondence



"It is well known that there exists an isomorphic relation between pitch and rhythm, which several authors have pointed out from time to time."

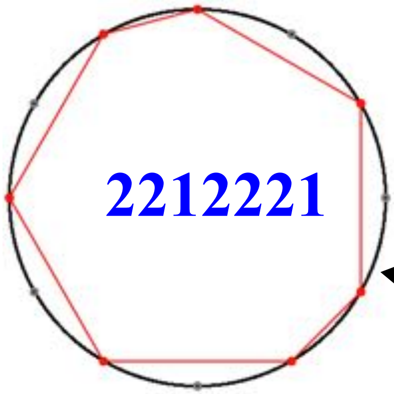
(G. Toussaint, *The geometry of musical rhythm. What makes a "Good" Rhythm Good?* CRC Press, 2013, p. xiii)

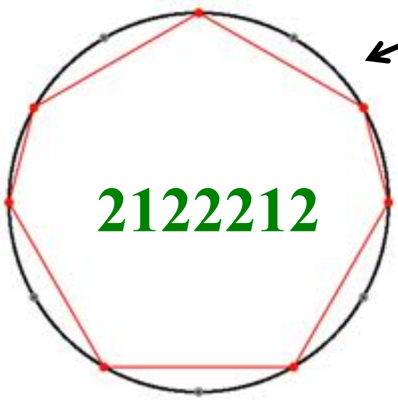




J. Douthett & R. Krantz, "Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction", *J. Math. Phys.* 37 (7), July 1996

The pitch-rhythm *cognitive* isomorphic correspondence





...

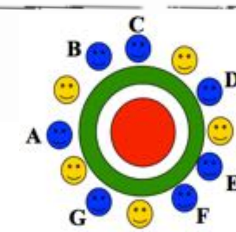
cyclic permutations

TABLE I

Comparison of M = 7, L = 12 patterns for pitch (scales) and rhythm (time-lines)

pattern	pitch domain name and notation (in C)	rhythm domain notation	examples from West Africa	references
1. 2212221	major scale (Ionian) CDEFGAB	♩ ♩ ♪♩ ♩ ♩ ♪	Ewe (Atsiabek , Sogba, Atsia) also Yoruba	Jones (1959), C. K. Ladzekpo, S. K. Ladzekpo and Pantaleoni, Locke
2. 2122212	Dorian CDE♭ FGAB♭	♩ ♪♩ ♩ ♩ ♪♩	Bemba—Northern Rhodesia	Jones (1965), (Ekwueme)
3. 1222122	Phrygian CD♭ E♭ FGA♭ B♭	♪♩ ♩ ♩ ♪♩ ♩	———	———
4. 2221221	Lydian CDEF#GAB	♩ ♩ ♩ ♪♩ ♩ ♪	Ga-Adangme (common) also common Haitian pattern, Akan (Ab fo)	C. K. Ladzekpo, Combs (1974), R. Hill, Asiama
5. 2212212	Mixolydian CDEFGAB♭	♩ ♩ ♪♩ ♩ ♪♩	Yoruba sacred music from Ekiti	King
6. 2122122	Aeolian CDE♭ FGA♭ B♭	♩ ♪♩ ♩ ♪♩ ♩	Ashanti (Ab fo , Mpre)	Koetting
7. 1221222	Locrian CD♭ E♭ FG♭ A♭ B♭	♪♩ ♩ ♪♩ ♩ ♩	Ghana*	Nketia (1963a)
8. 2121222	(#2 Locrian) CDE♭ FG♭ A♭ B♭	♩ ♪♩ ♪♩ ♩ ♩	Ashanti (Asedua)	C. K. Ladzekpo
9. 2112123	——— CDD#EF#GA	♩ ♫♩ ♪♩ ♩.	Akan (juvenile song)	Nketia (1963b)

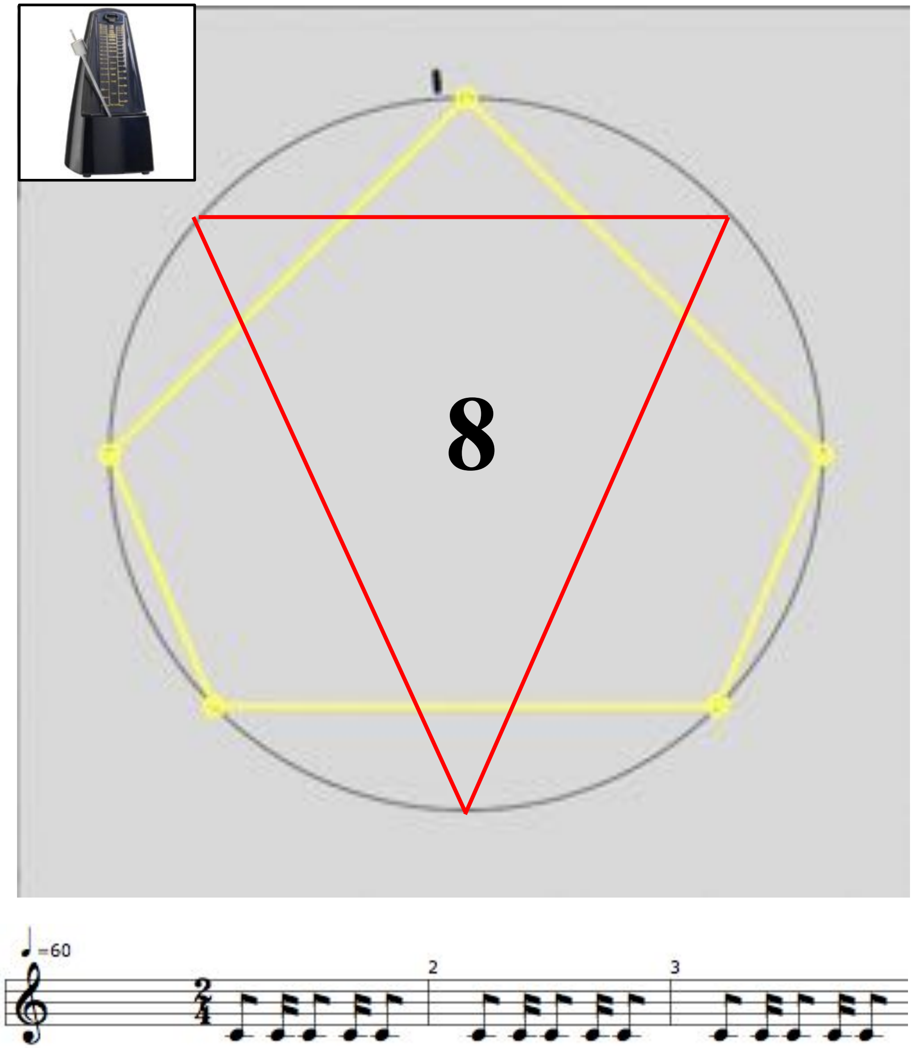
* clap pattern
† mute stroke on bell



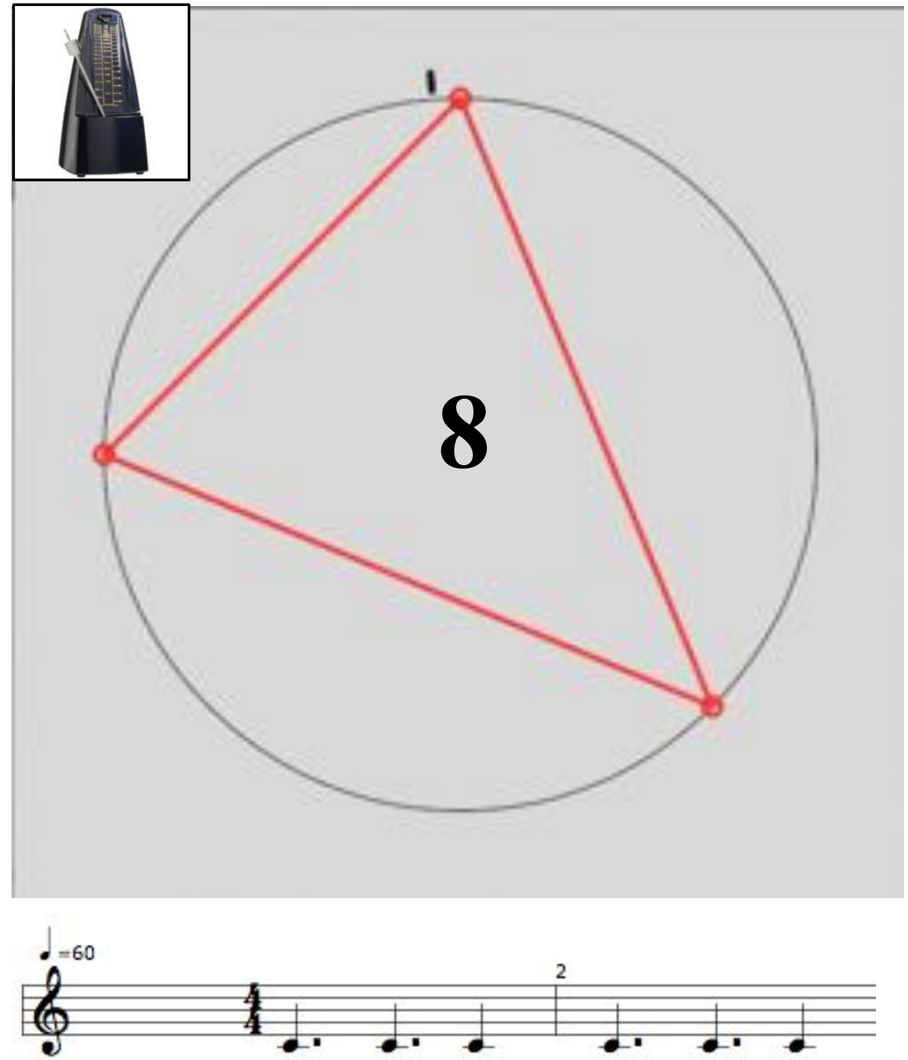
J. Pressing, "Cognitive isomorphisms between pitch and rhythm in world musics: West Africa, the Balkans and Western tonality", *Studies in Music*, 17, p. 38-61

African-cuban ME-rhythms

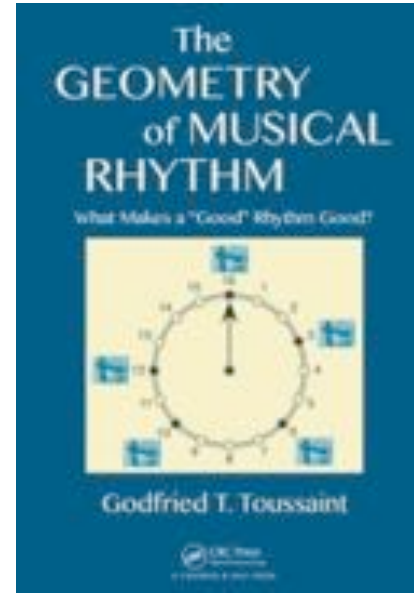
El cinquillo

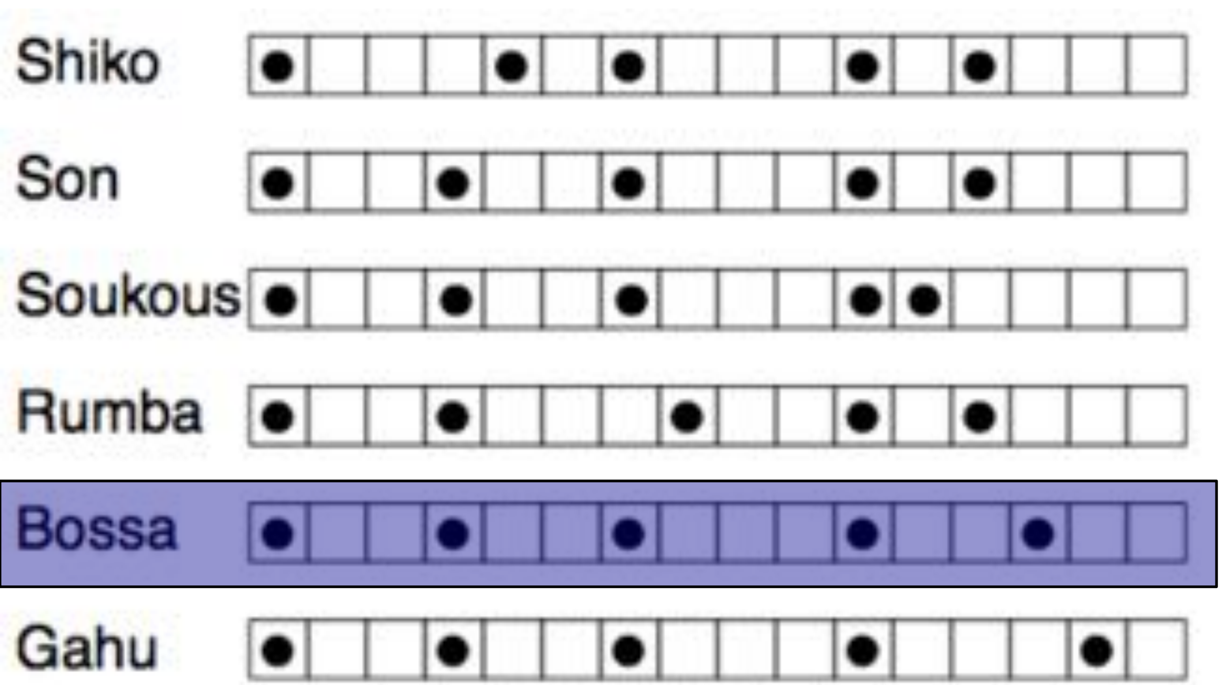


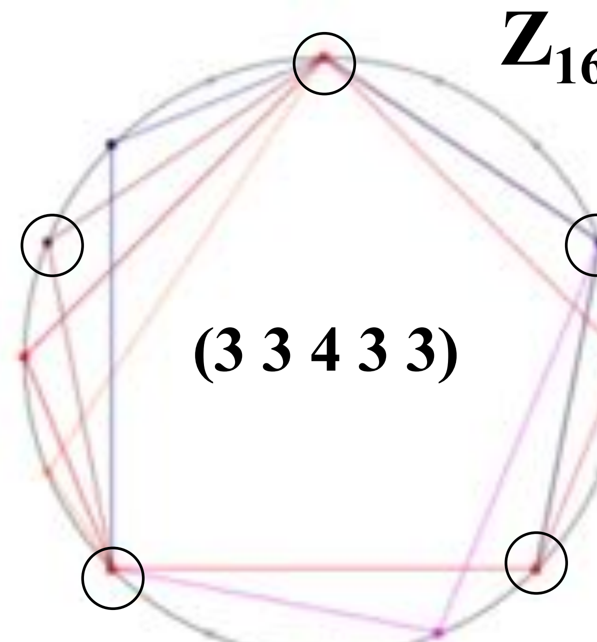
El trecillo



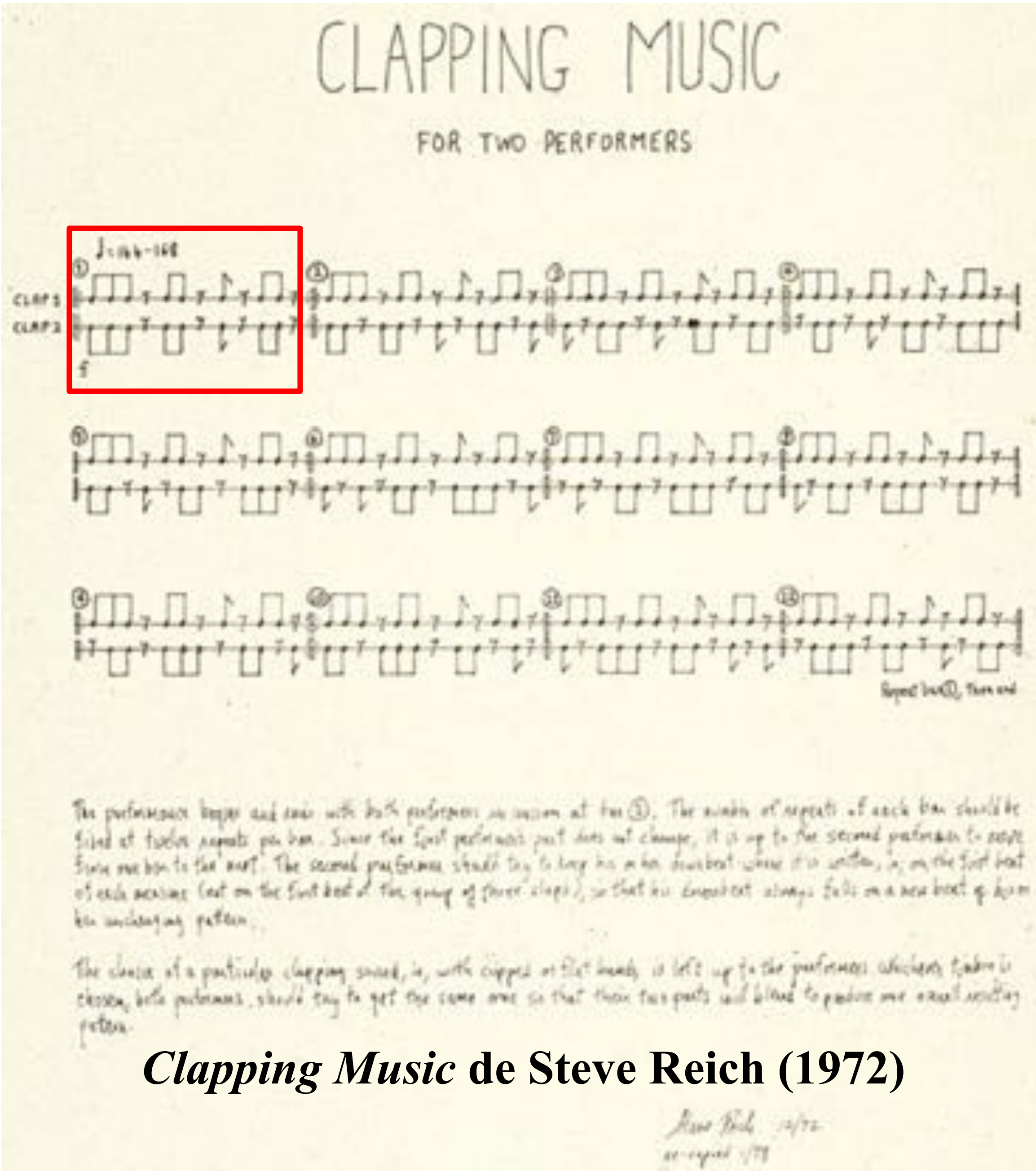
The geometry of African-Cuban rhythms



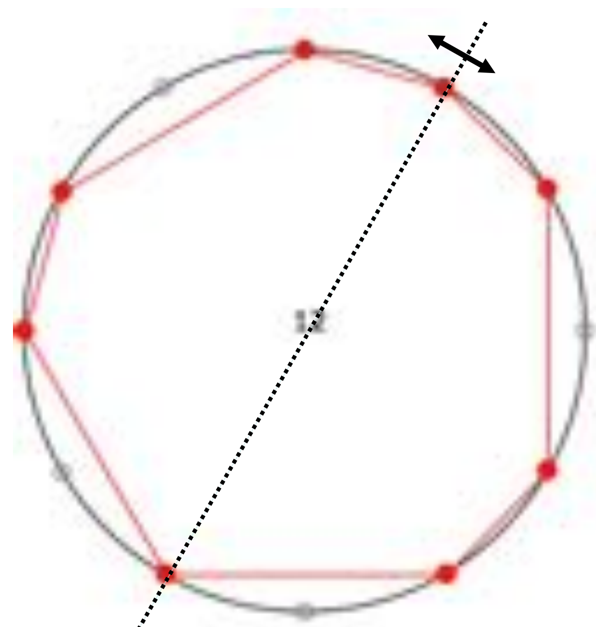




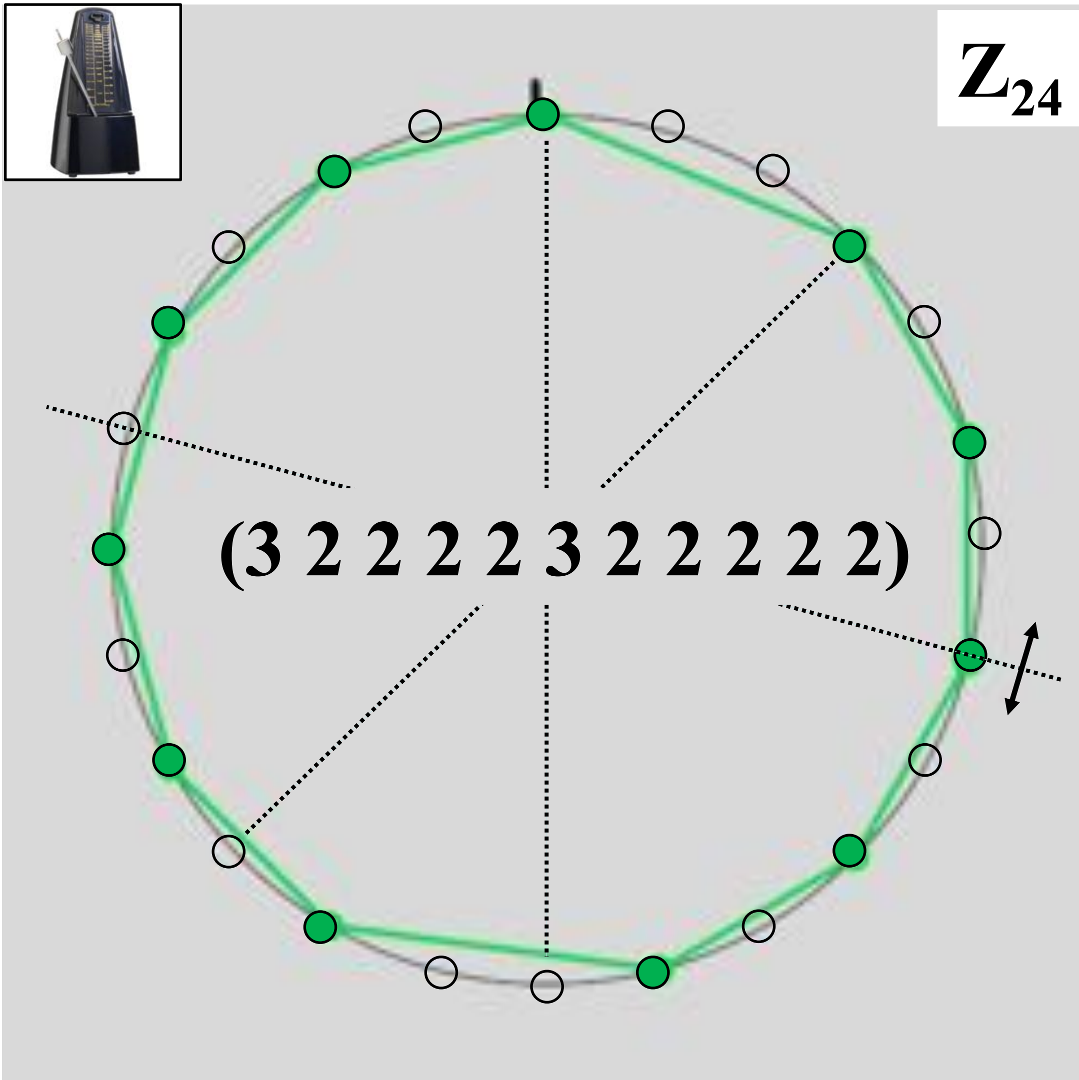
Palindromic structures in Steve Reich’s music



Clapping Music de Steve Reich (1972)

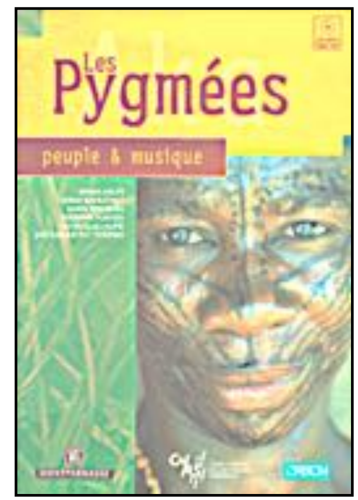


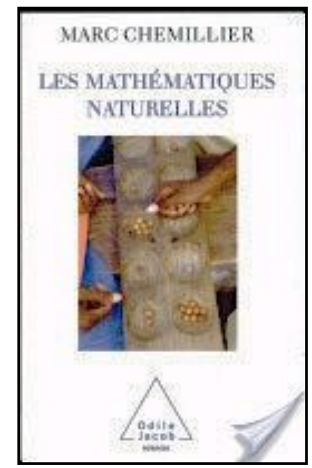
Odditive property of orally-trasmitted practices



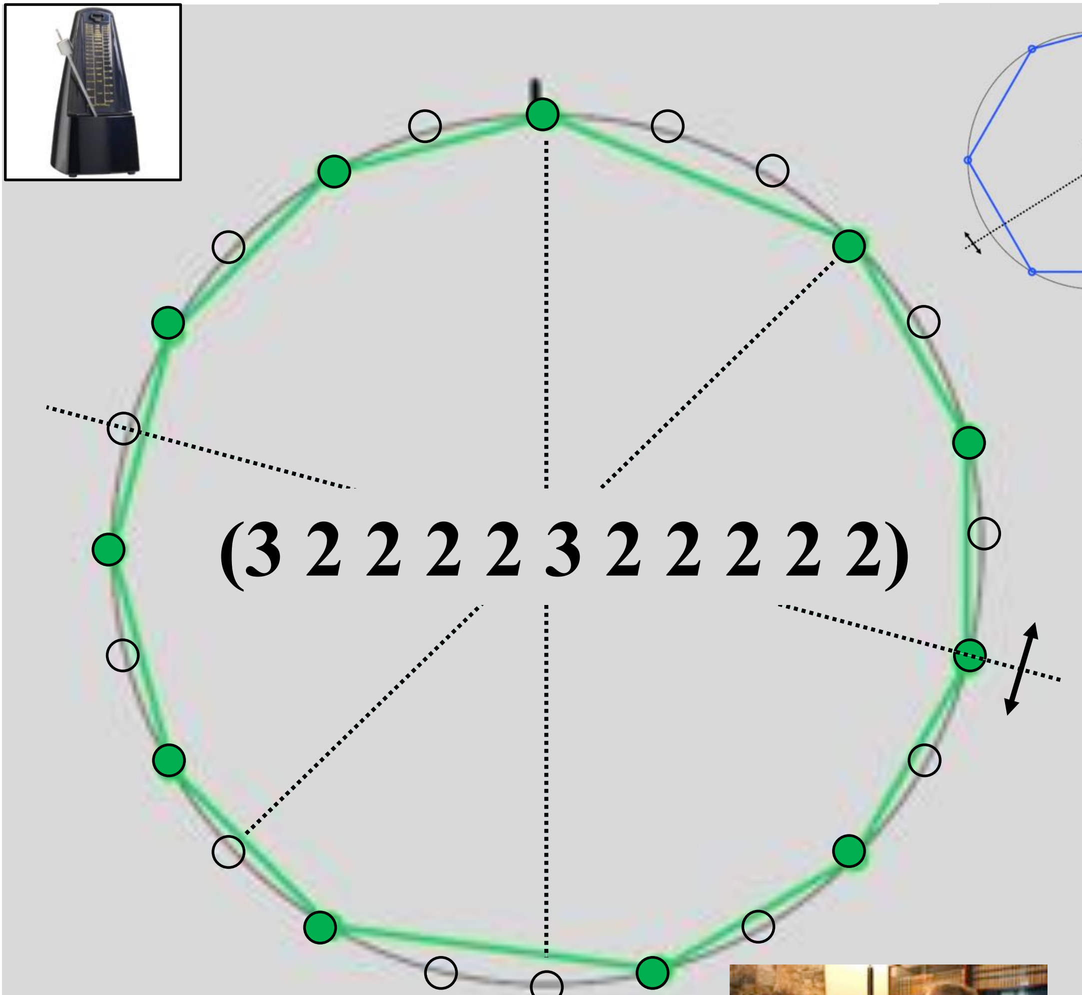
Simha Arom

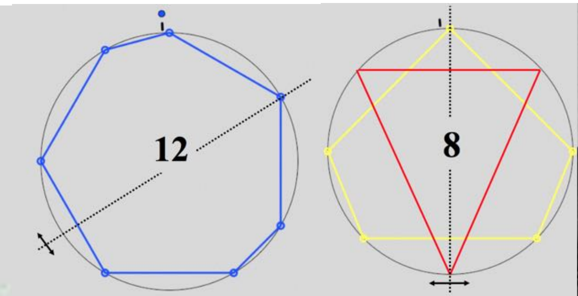
Marc Chemillier

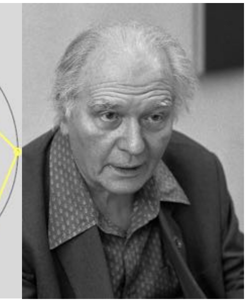




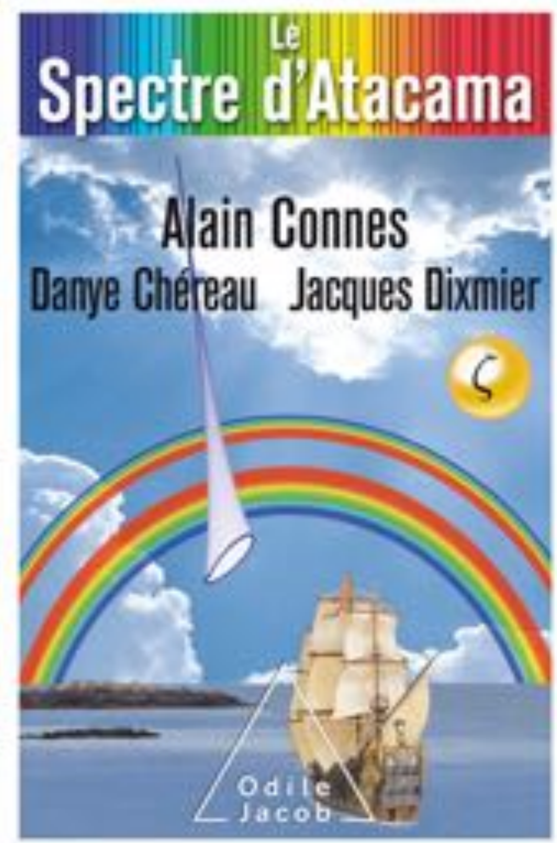
Olivier Messiaen’s non-invertible rhythms

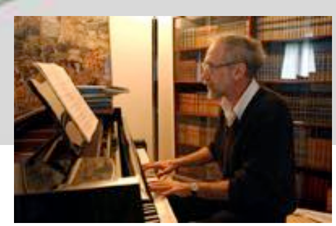






Olivier Messiaen

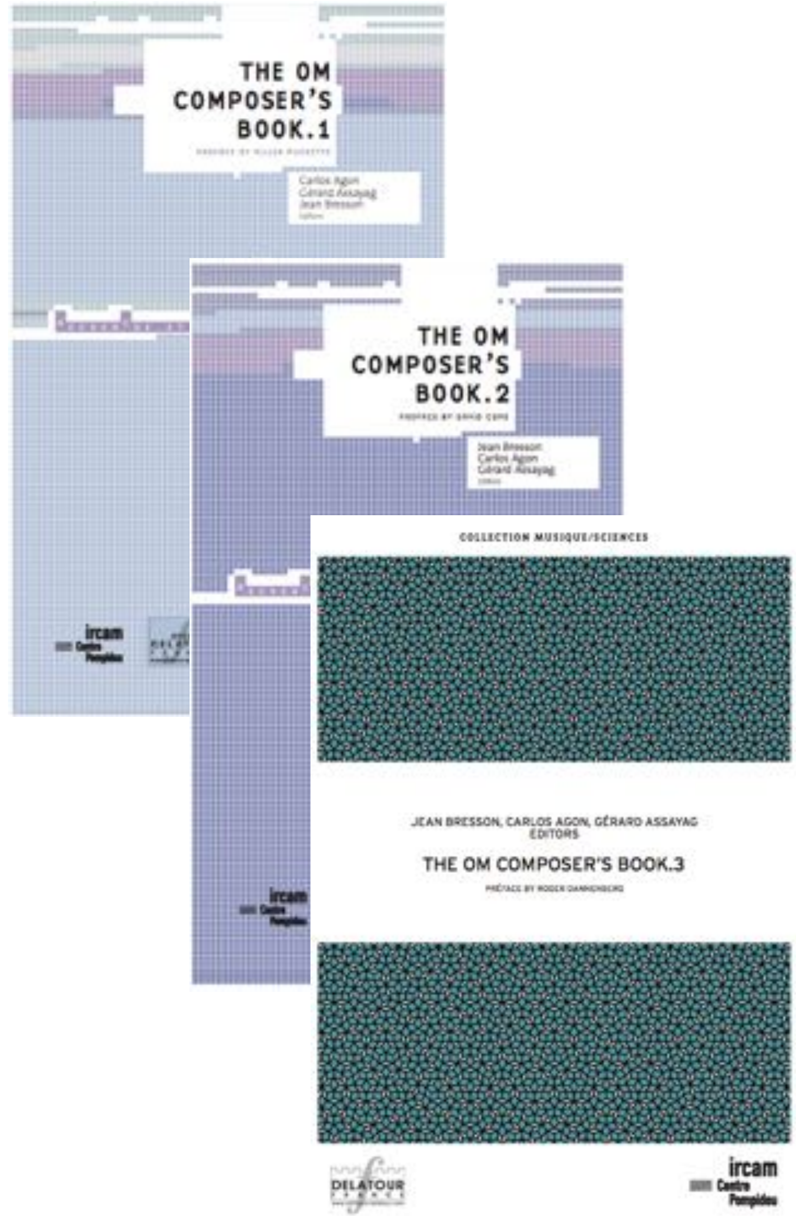




Alain Connes

OpenMusic, a Visual Programming Language for computer-aided composition

www.repmus.ircam.fr/openmusic/home

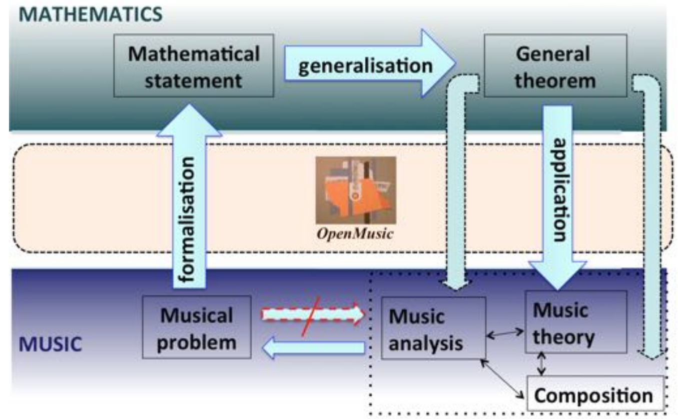


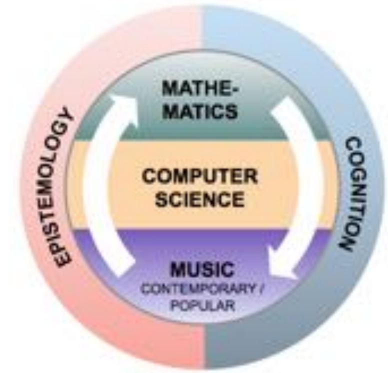
C. Agon, G. Assayag and J. Bresson, *The OM Composer's Book* (3 volumes) "Musique/Sciences" Series, Ircam/Delatour, 2006, 2007 and 2016

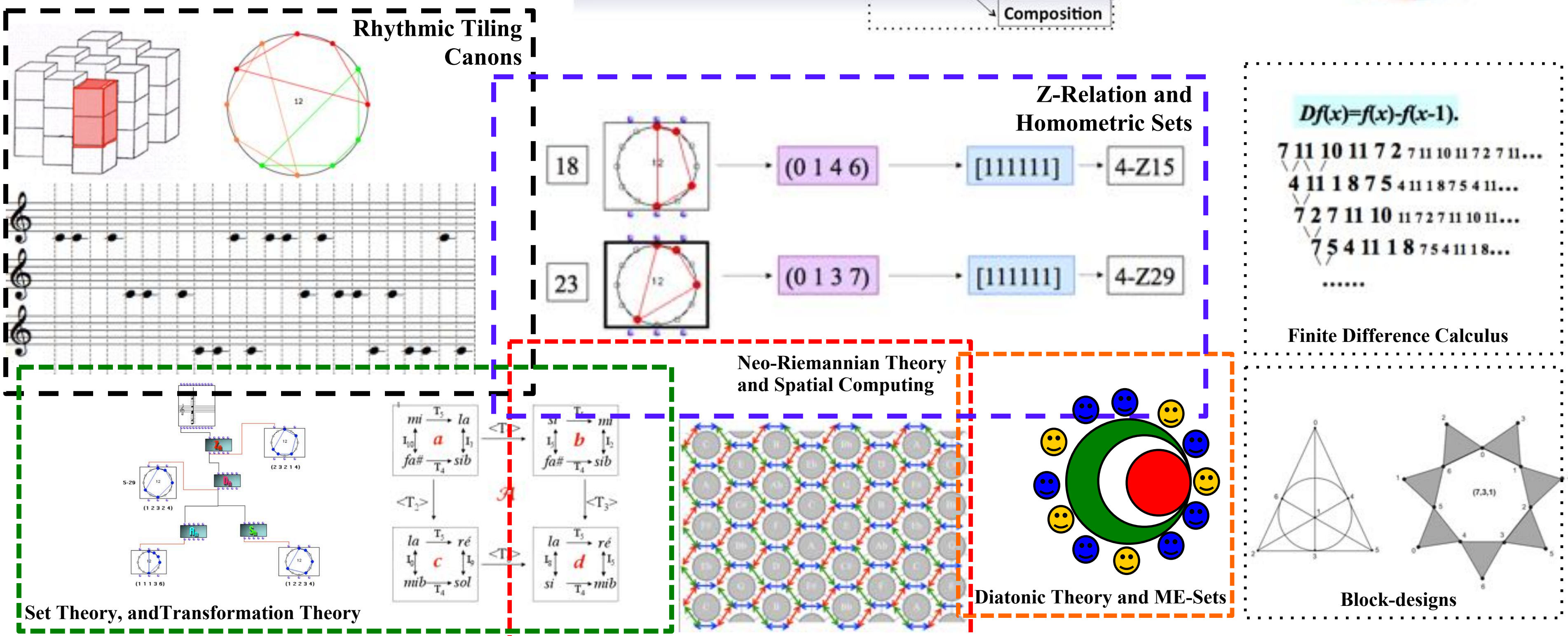
A short catalogue of mathemusical problems

M. Andreatta : *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

- **Tiling Rhythmic Canons**
- ***Z* relation and homometry**
- **Transformational Theory**
- **Music Analysis, SC and FCA**
- **Diatonic Theory and ME-Sets**
- **Periodic sequences and FDC**
- **Block-designs in composition**





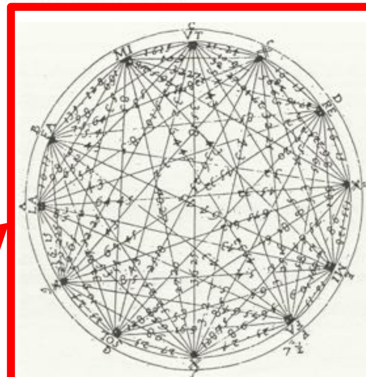


Music and mathematics: « prima la musica »!

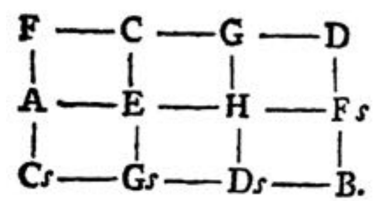
I. Xenakis

MUSIC	MATHS
500 B.C. Pitches and lengths of strings are related. Here **music** gives a marvelous thrust to **number theory** and **geometry**.	Discovery of the fundamental importance of **natural numbers** and the invention of **fractions**.
No correspondence in music.	Positive irrational numbers [...]
300 B.C. [...] Music theory highlights the discovery of the **isomorphism between the logarithms** (musical intervals) and **exponentials** (string lengths) more than 15 centuries before their discovery in mathematics; also a **premonition of group theory** is suggested by Aristoxenos.	No reaction in mathematics. [...]
1000 A.D. Invention of the **two-dimensional spatial representation of pitches** linked with time by means of staves and points [...] seven centuries (1635-37) before the magnificent **analytical geometry** of Fermat and Descartes.	*No parallel in mathematics.*
1500 *No response or development of the preceding concepts.*	Zero and negative numbers are adopted. Construction of the set of rationals.
1600 *No equivalence, no reaction.*	The sets of real numbers and of logarithms are invented.
1648 Invention of **musical combinatorics** by Marin Mersenne (*Harmonicorum Libri*)	Probability theory by Bernoulli (*Ars Conjectandi*, 1713)
1700 [...] The **fugue**, for example, is an **abstract automaton** used two centuries before the birth of the science of automata. Also, there is an **unconscious manipulation of finite groups** (Klein group) in the four variations of a melodic line used in counterpoint.	Number theory is ahead of but has no equivalent yet in temporal structures. [...]
1773 A first **geometric** and **graph-theoretic** representation of pitches (*Speculum Musicum*)	Invention of graph theory
1900 Liberation from the tonal yoke. First acceptance of the neutrality of chromatic totality (Loquin [1895], Hauer, Schoenberg).	The infinite and transfinite numbers (Cantor). Peano axiomatics. [...] The beautiful measure theory (Lebesque, ...)
1920 First radical formalization of macrostructures through the serial system of Schoenberg.	No new development of the number theory.
1929 and 1937-1939 Susanne K. Langer and Ernst Krenek on the role of axioms in music	David Hilbert, *Die Grundlage der Geometrie* (1899)
1946 Milton Babbitt on group theory and integral serialism	Rudolf Carnap, *The Logical Syntax of Language* (1937)

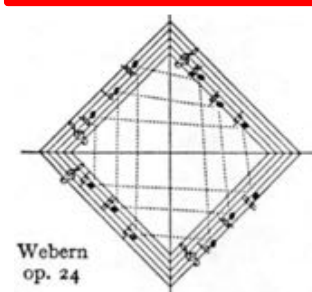
Pythagoras and the monochord, VIth-Vth Century B.C.



Mersenne and the ‘musical clock’, 1648

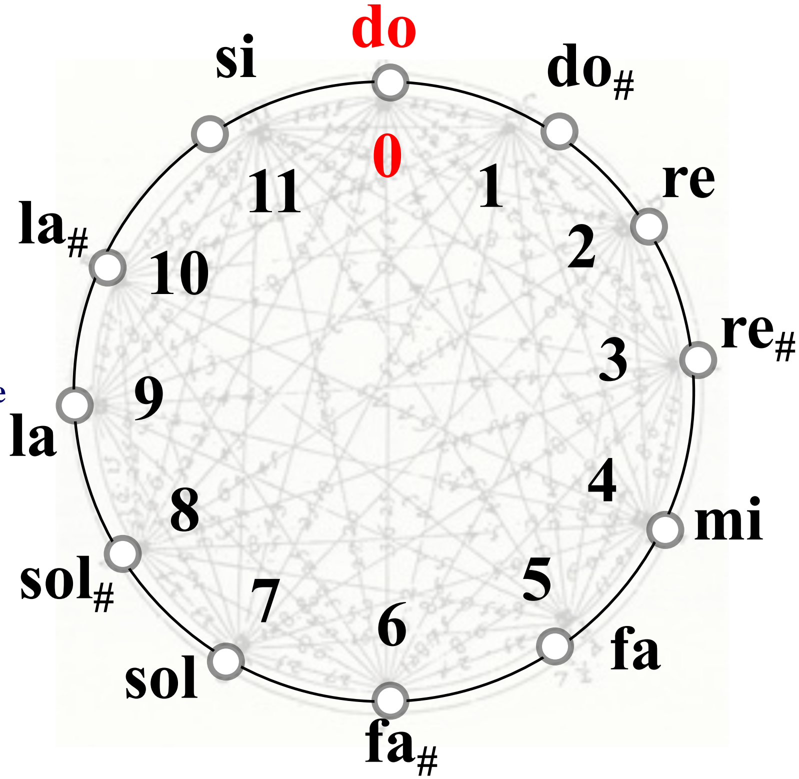


Euler and the *Speculum musicum*, 1773

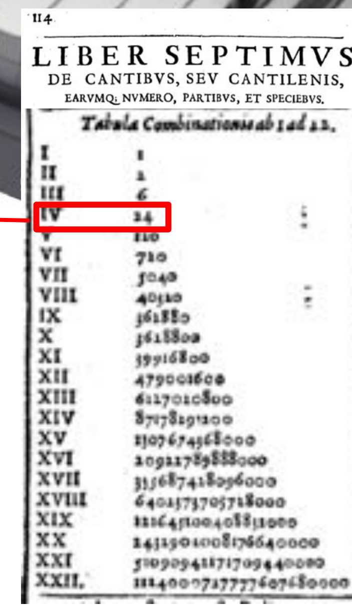


The circular representation of the pitch space

Marin Mersenne



Harmonicorum Libri XII, 1648


114

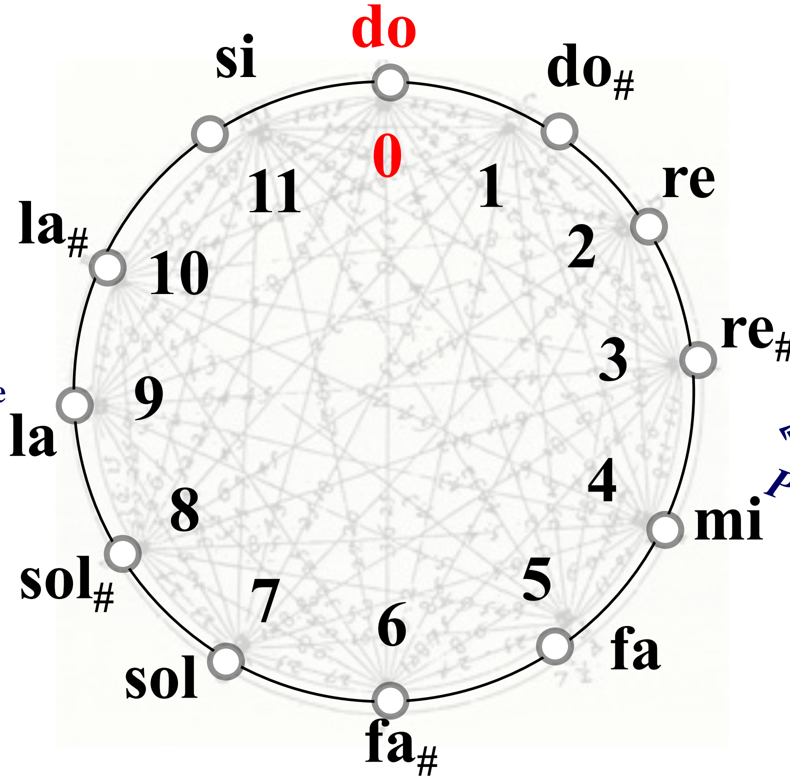
LIBER SEPTIMVS.
DE CANTIBVS, SEV CANTILENIS,
EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 22.

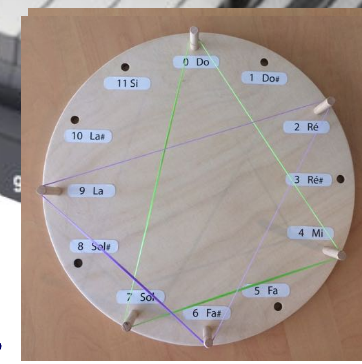
I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6227020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000

The circular representation of the pitch space

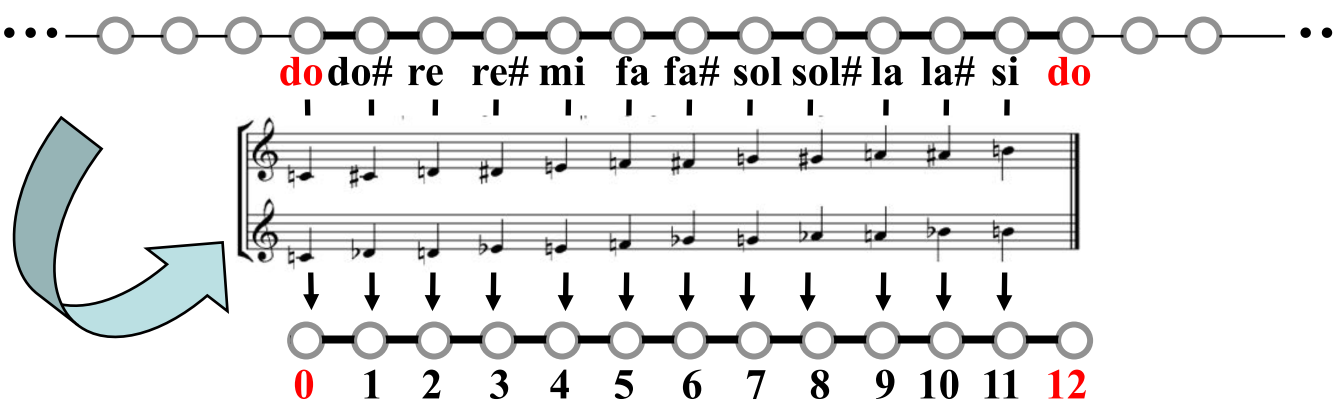

Marin Mersenne



Harmonicorum Libri XII, 1648

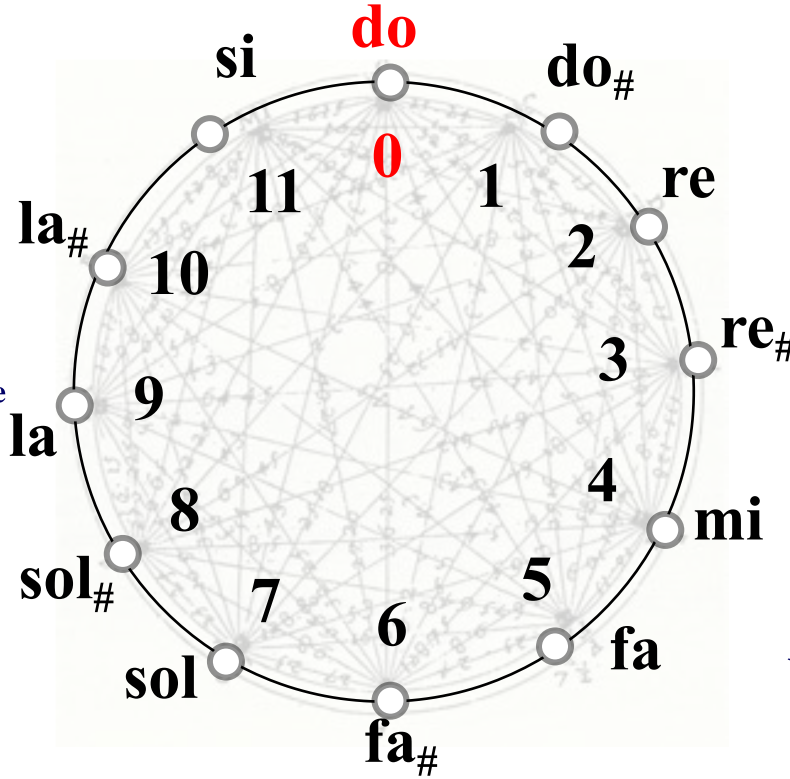


M. Andreatta, C. Agon, «La musique mise en algèbre», *Pour la Science*, 2008

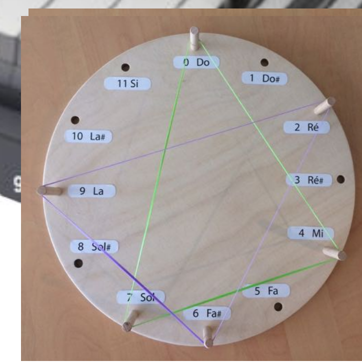


The circular representation of the pitch space

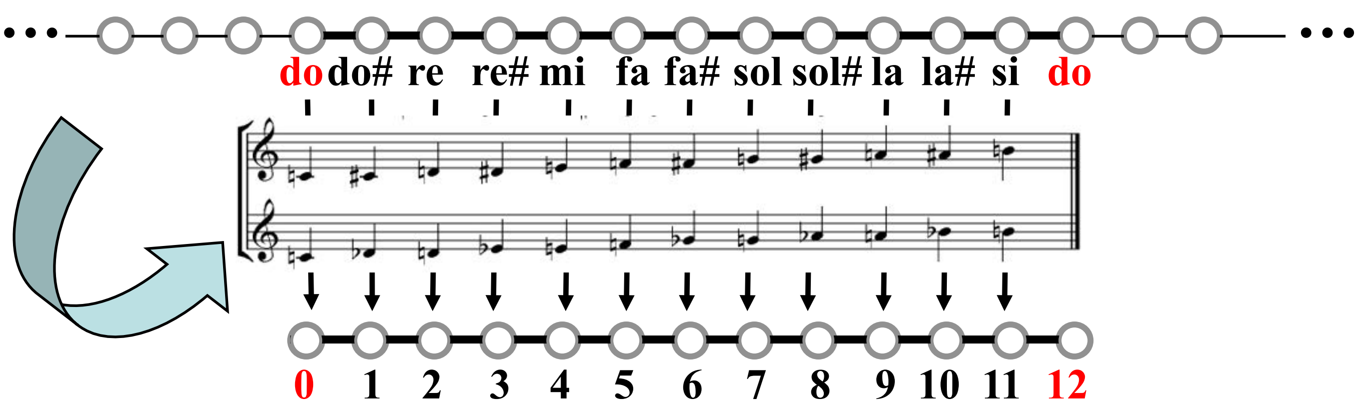
Marin Mersenne



Harmonicorum Libri XII, 1648

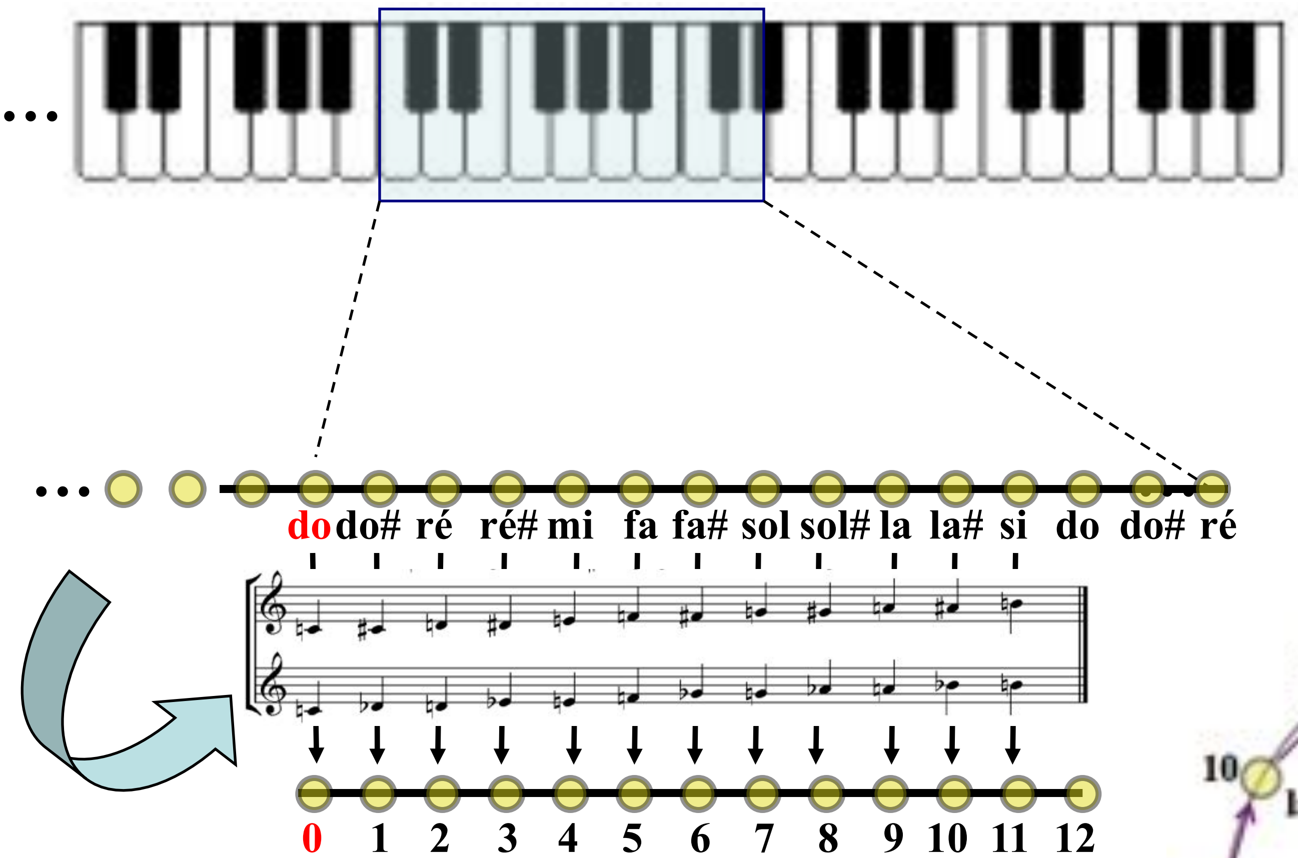


M. Andreatta, C. Agon, « Algèbre et géométrie : sont-elles inscrites dans le cerveau ? », ***Pour la Science*****, 2018**



➔ DEMO

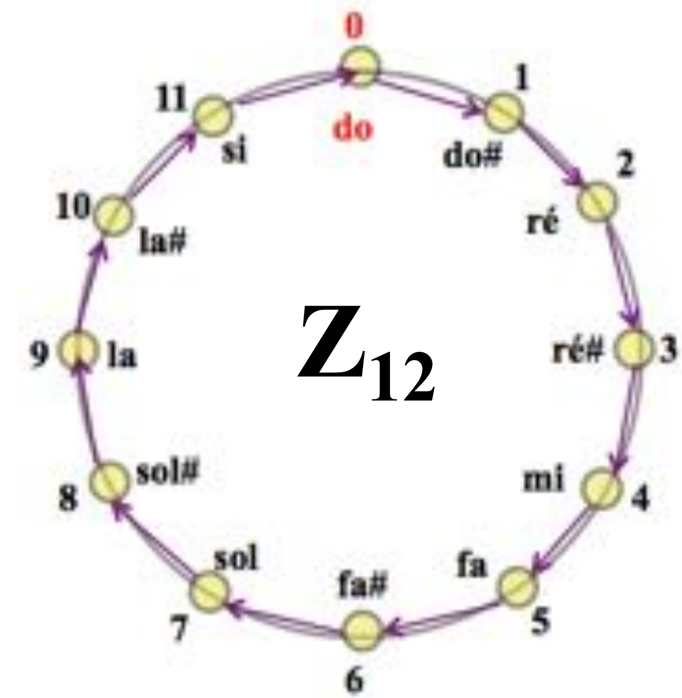
The equal tempered space is a cyclic group



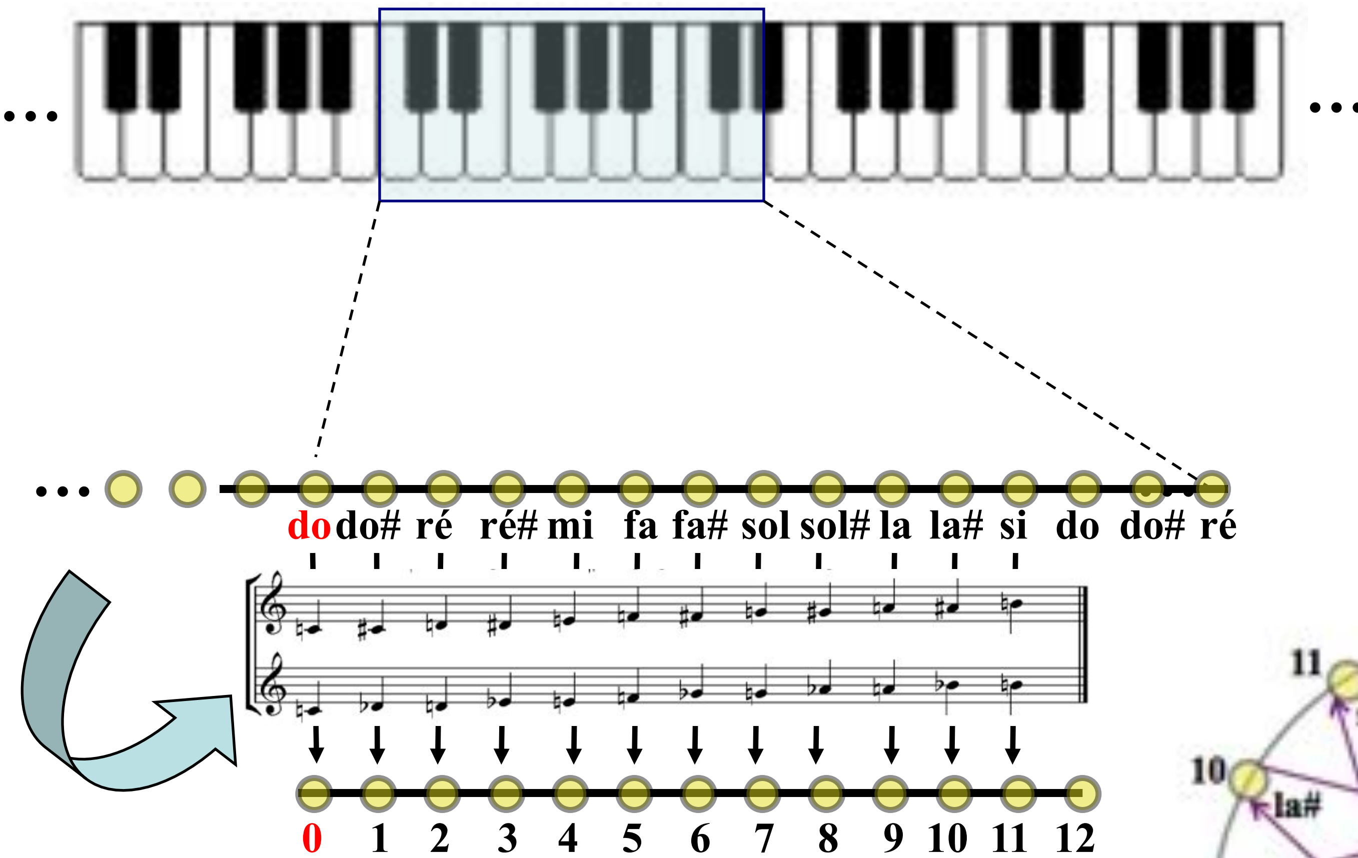
The generators of the cyclic group of order 12 are the transpositions T_1, T_5, T_7 et T_{11} where

$T_k: x \rightarrow x+k \bmod 12$

$$Z_{12} = \langle T_1 \mid (T_1)^{12} = T_0 \rangle$$



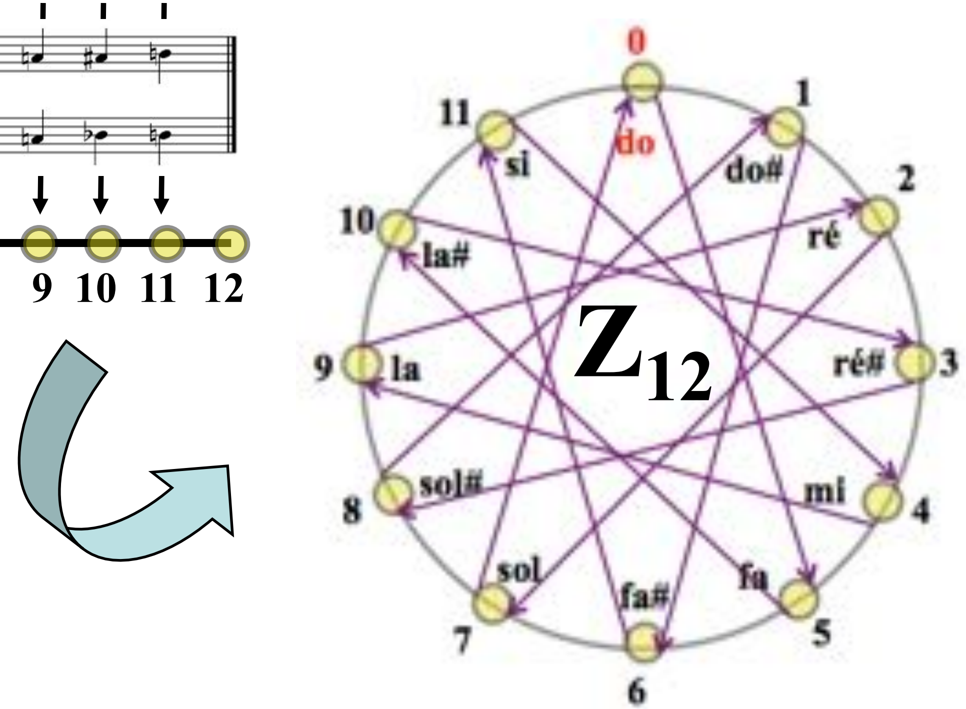
The equal tempered space is a cyclic group



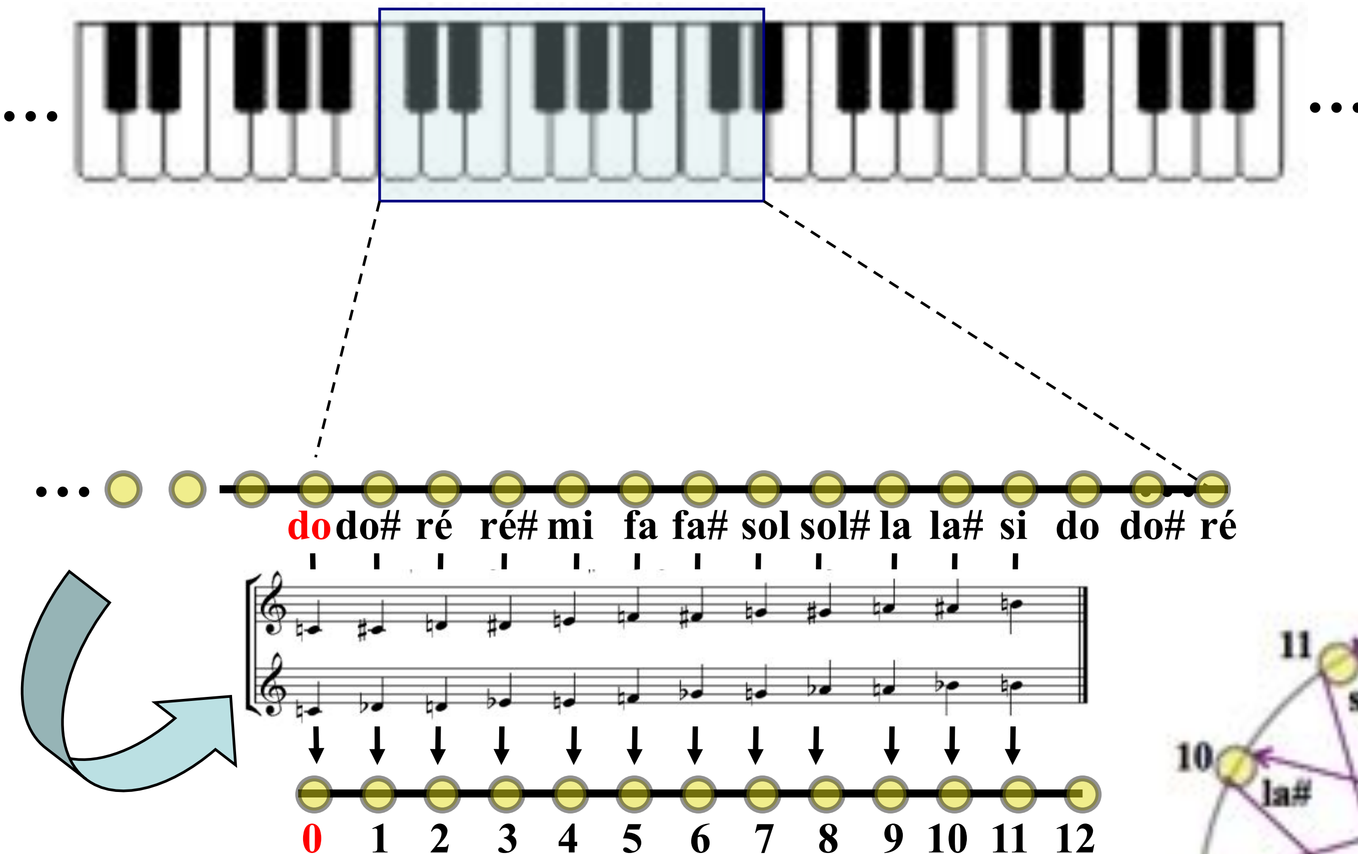
The generators of the cyclic group of order 12 are the transpositions T_1, T_5, T_7 et T_{11} where

$T_k: x \rightarrow x+k \bmod 12$

$$Z_{12} = < T_1 \mid (T_1)^{12} = T_0 > =$$
$$= < T_5 \mid (T_5)^{12} = T_0 >$$



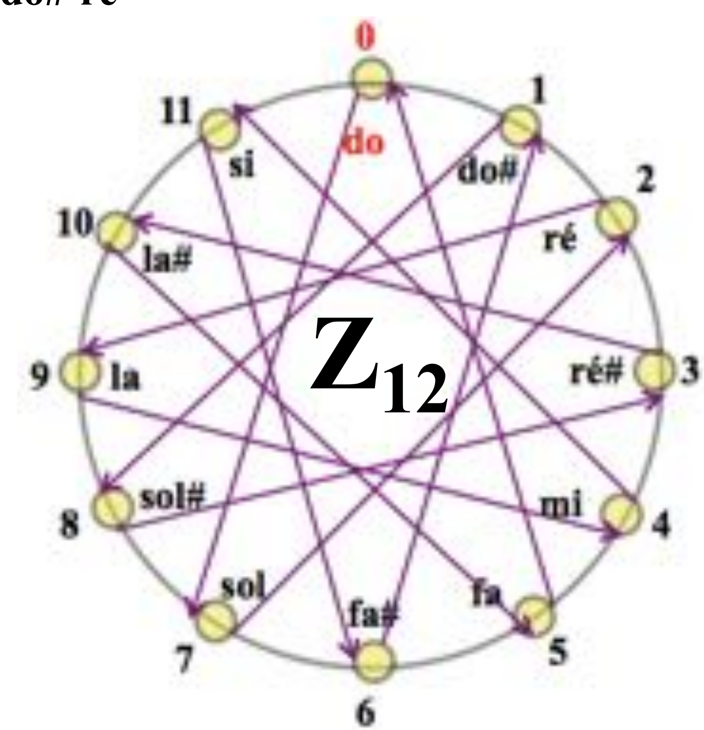
The equal tempered space is a cyclic group



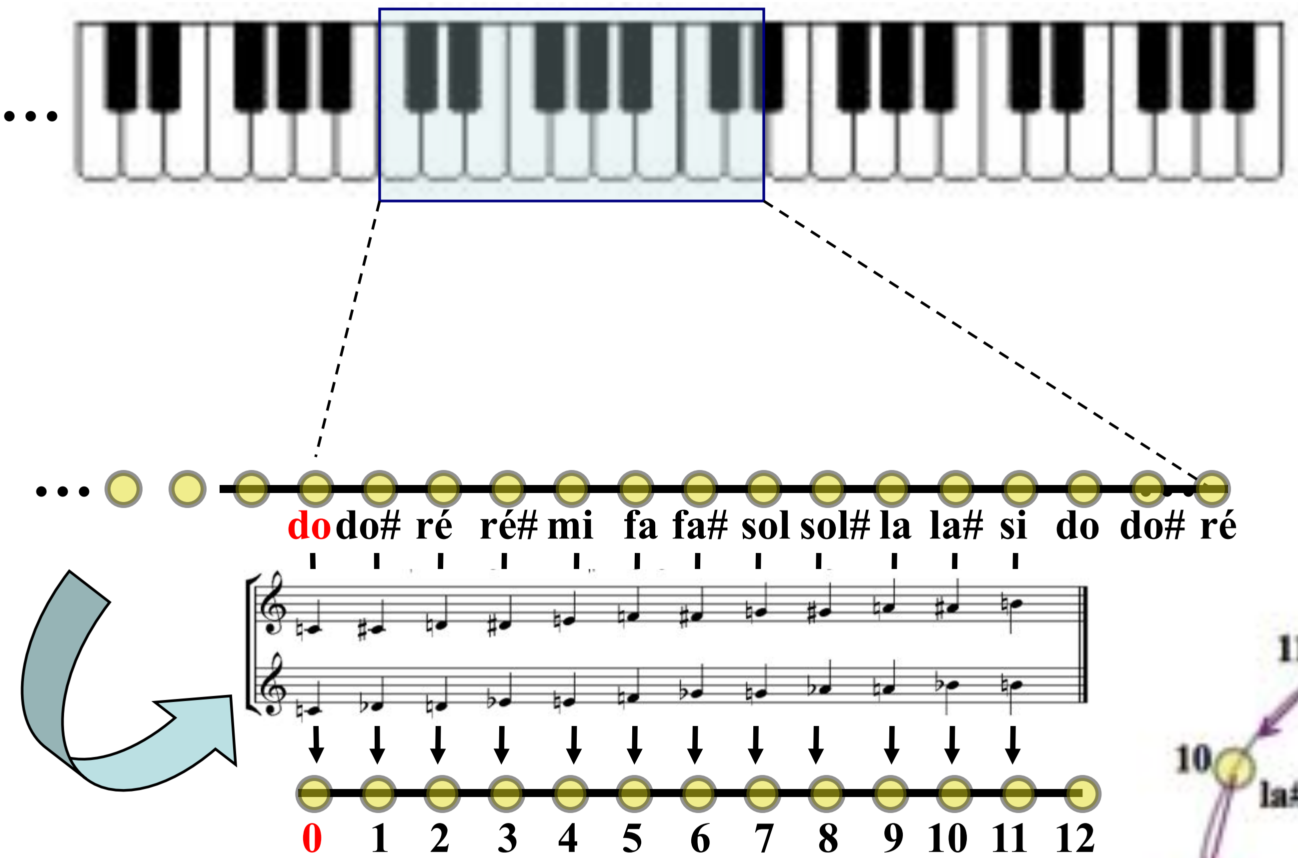
The generators of the cyclic group of order 12 are the transpositions T_1, T_5, T_7 et T_{11} where

$T_k: x \rightarrow x+k \bmod 12$

$$\mathbf{Z_{12}} = \langle T_1 \mid (T_1)^{12} = T_0 \rangle =$$
$$= \langle T_5 \mid (T_5)^{12} = T_0 \rangle =$$
$$\mathbf{= \langle T_7 \mid (T_7)^{12} = T_0 \rangle}$$

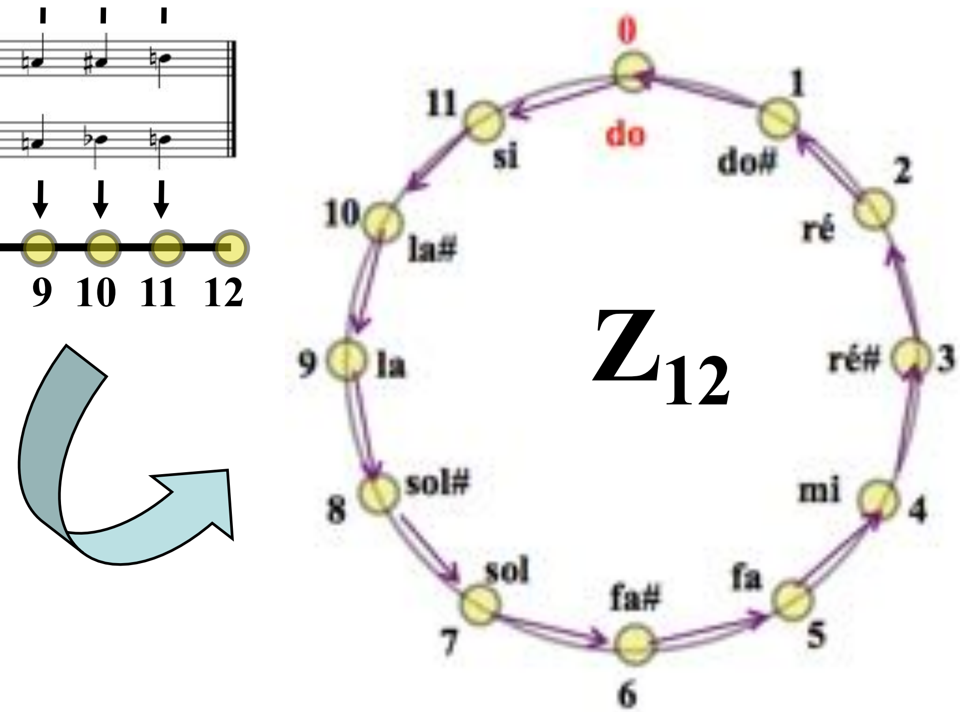


The equal tempered space is a cyclic group



The generators of the cyclic group of order 12 are the transpositions T_1, T_5, T_7 et T_{11} where

T_k: $x \to x+k \bmod 12$

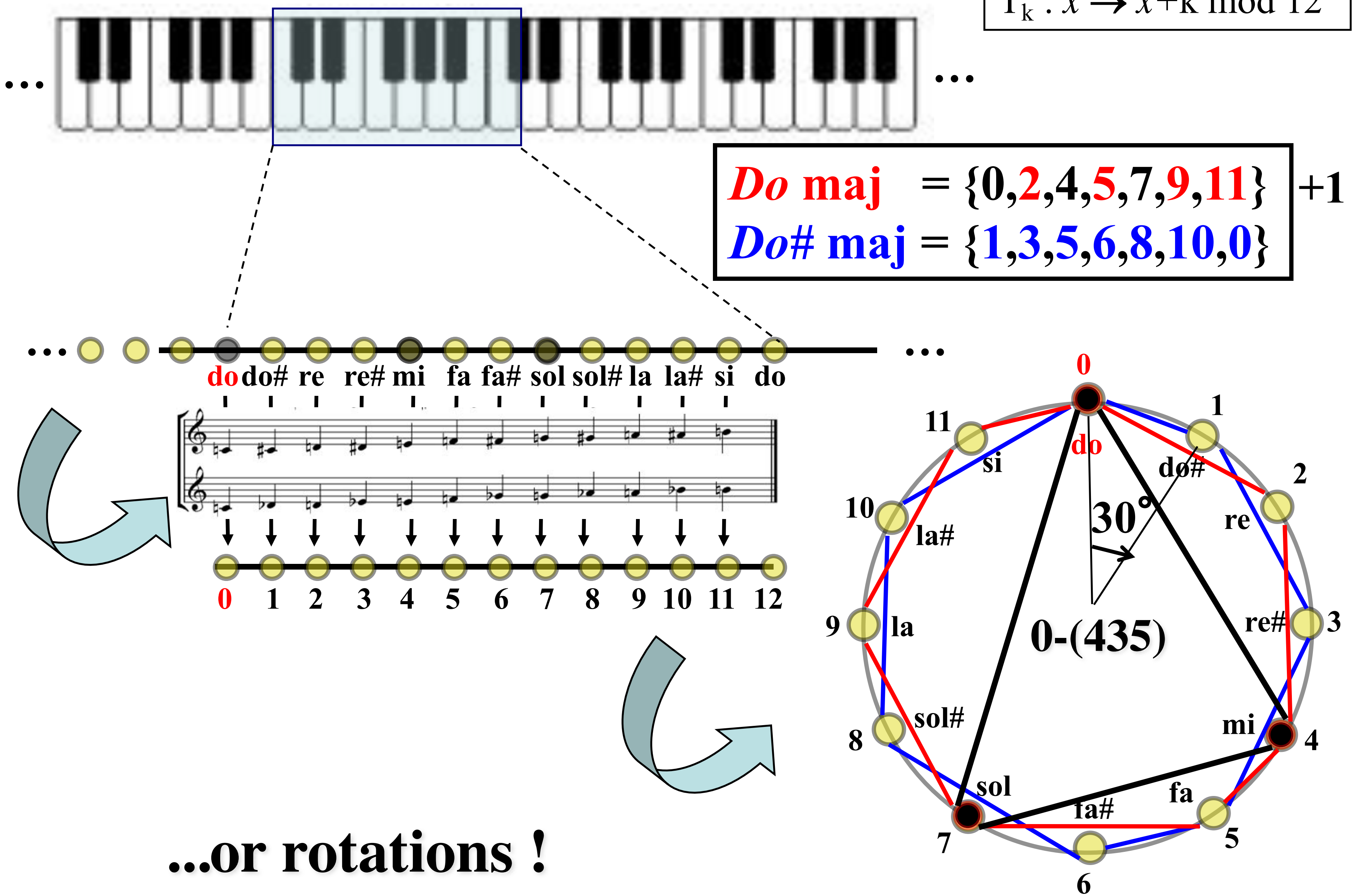


$$\mathbf{Z}_{12} = < T_1 \mid (T_1)^{12} = T_0 > =$$
$$= < T_5 \mid (T_5)^{12} = T_0 > =$$
$$= < T_7 \mid (T_7)^{12} = T_0 > =$$
$$= < T_{11} \mid (T_{11})^{12} = T_0 >$$

Musical transpositions are additions...

$T_k : x \rightarrow x + k \bmod 12$

$\textit{Do}$ maj = {0,2,4,5,7,9,11} +1
$\textit{Do\#}$ maj = {1,3,5,6,8,10,0}



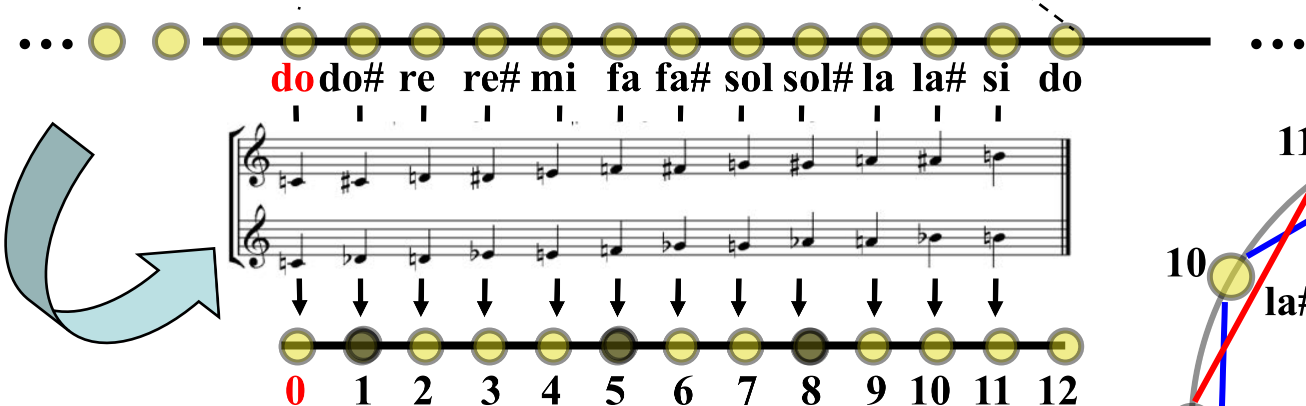
...or rotations !

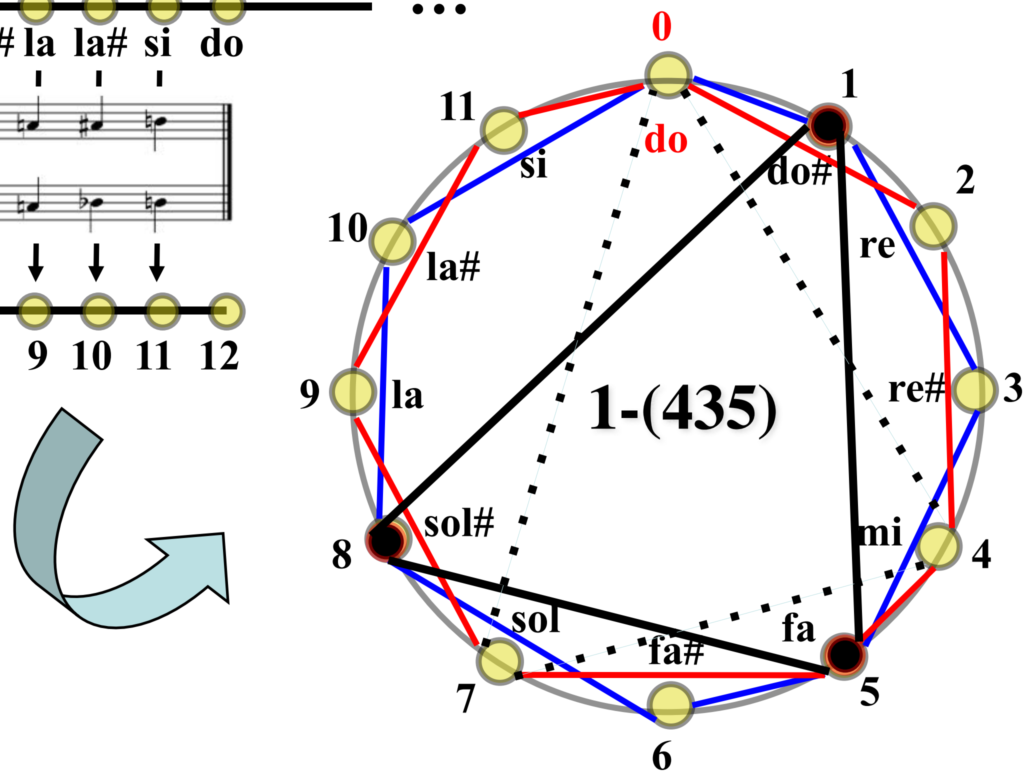
Musical transpositions are additions...

$T_k : x \rightarrow x+k \bmod 12$

Do **maj** = {0,2,4,5,7,9,11} +1

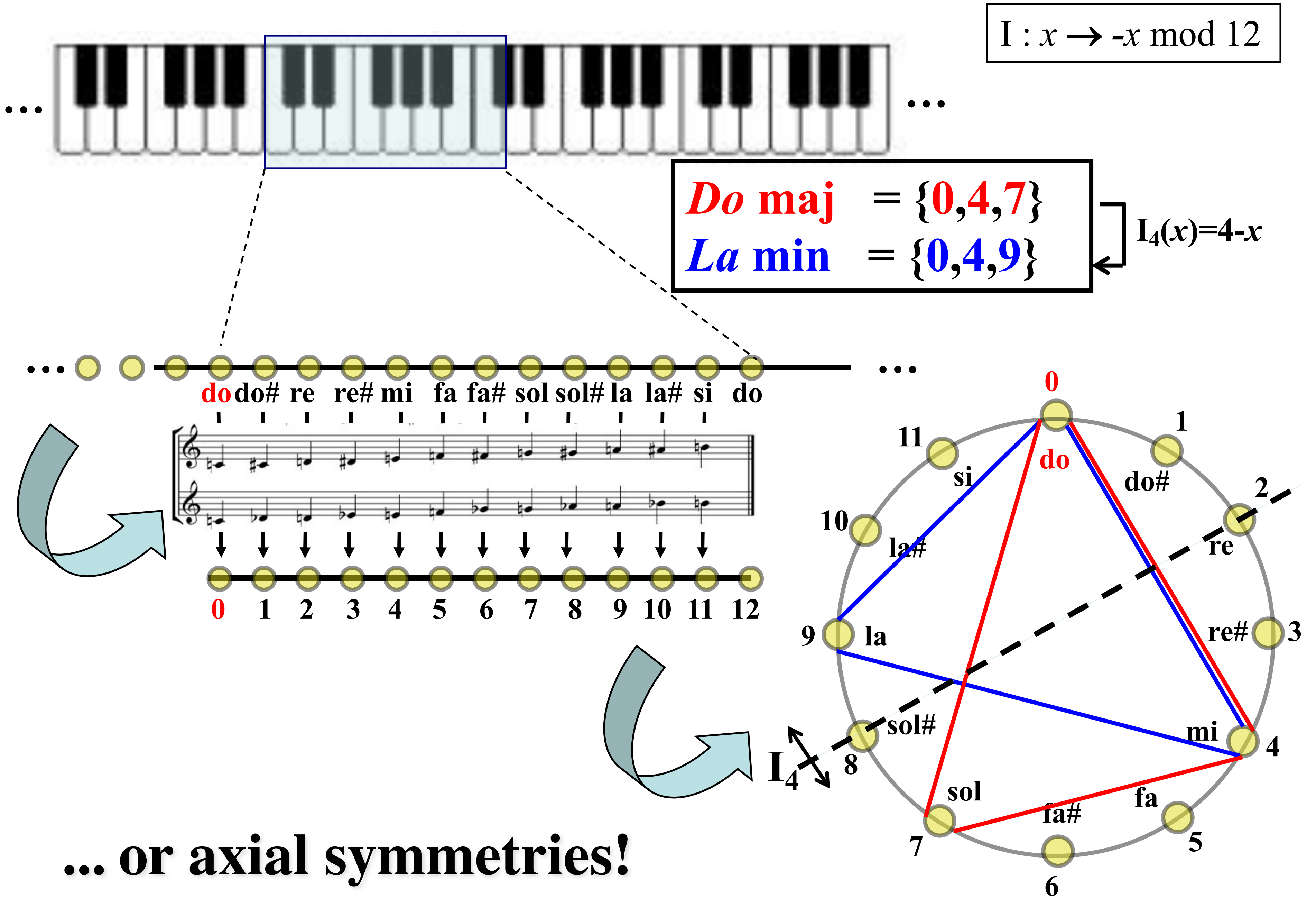
Do# **maj** = {1,3,5,6,8,10,0}





...or rotations !

Musical inversions are differences...



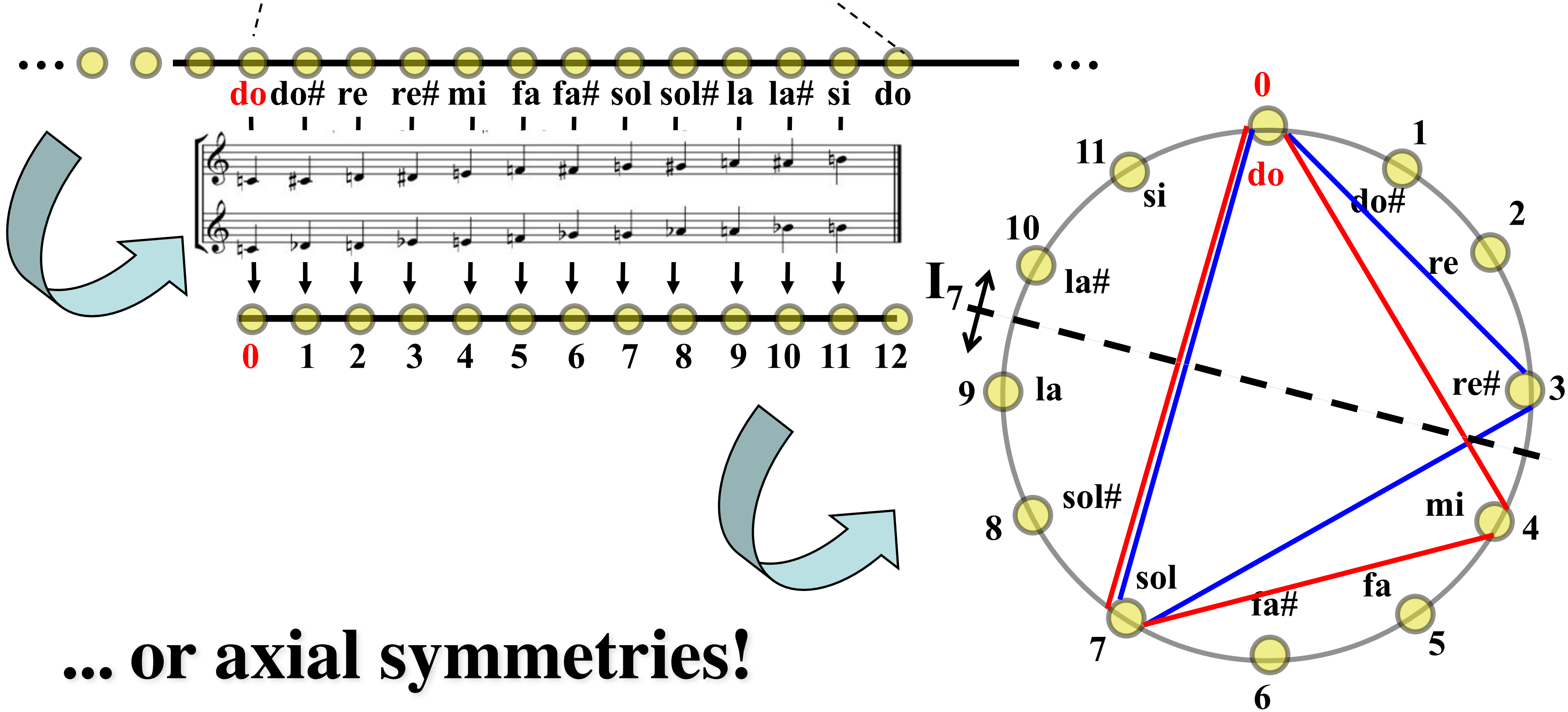
... or axial symmetries!

Musical inversions are differences...

$I : x \rightarrow -x \mod 12$

***Do* maj**	$= \{0,4,7\}$
***Do* min**	$= \{0,3,7\}$

$I_7(x)=7-x$



... or axial symmetries!

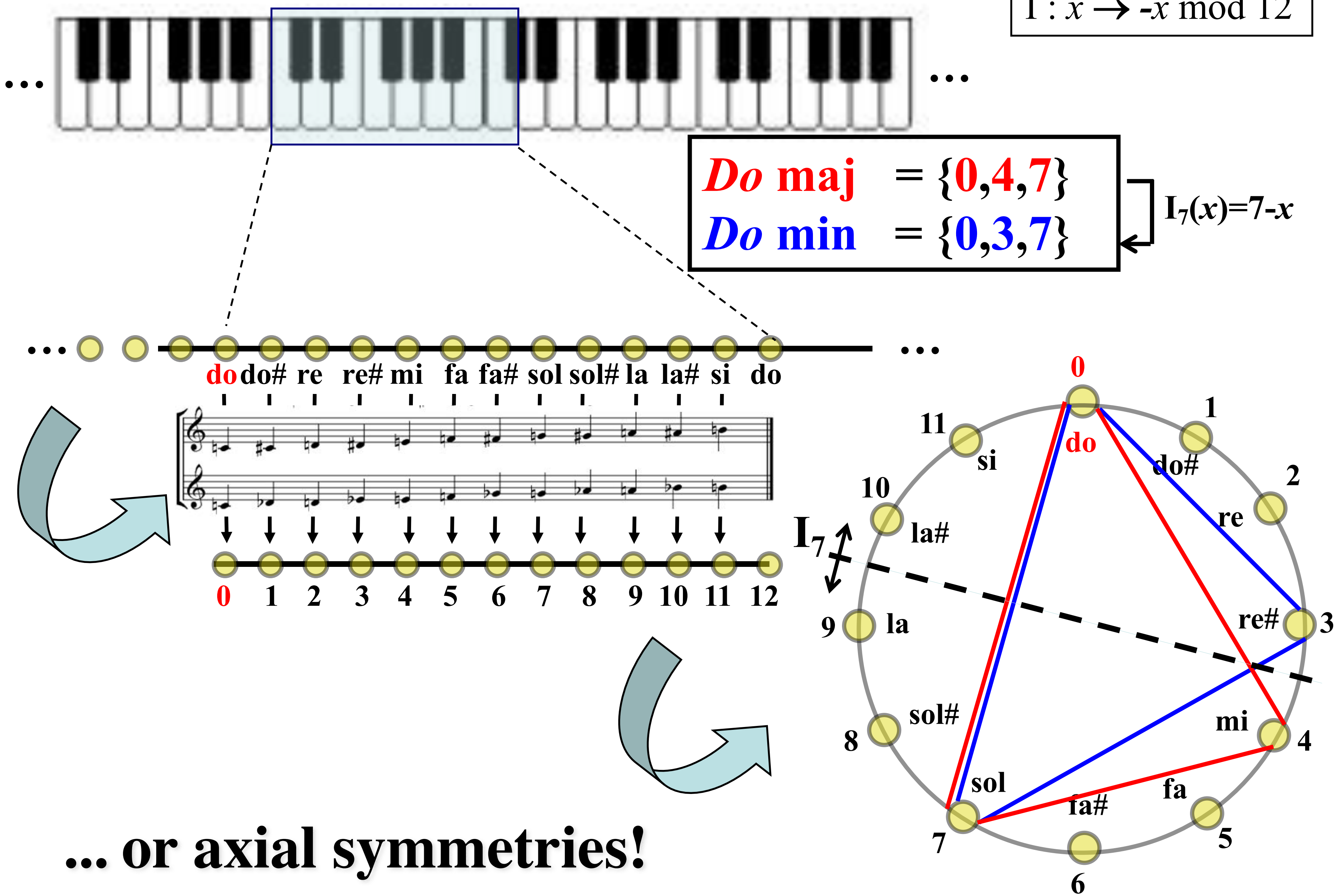
Musical inversions are differences...

$I : x \rightarrow -x \bmod 12$

Do maj $= \{0,4,7\}$

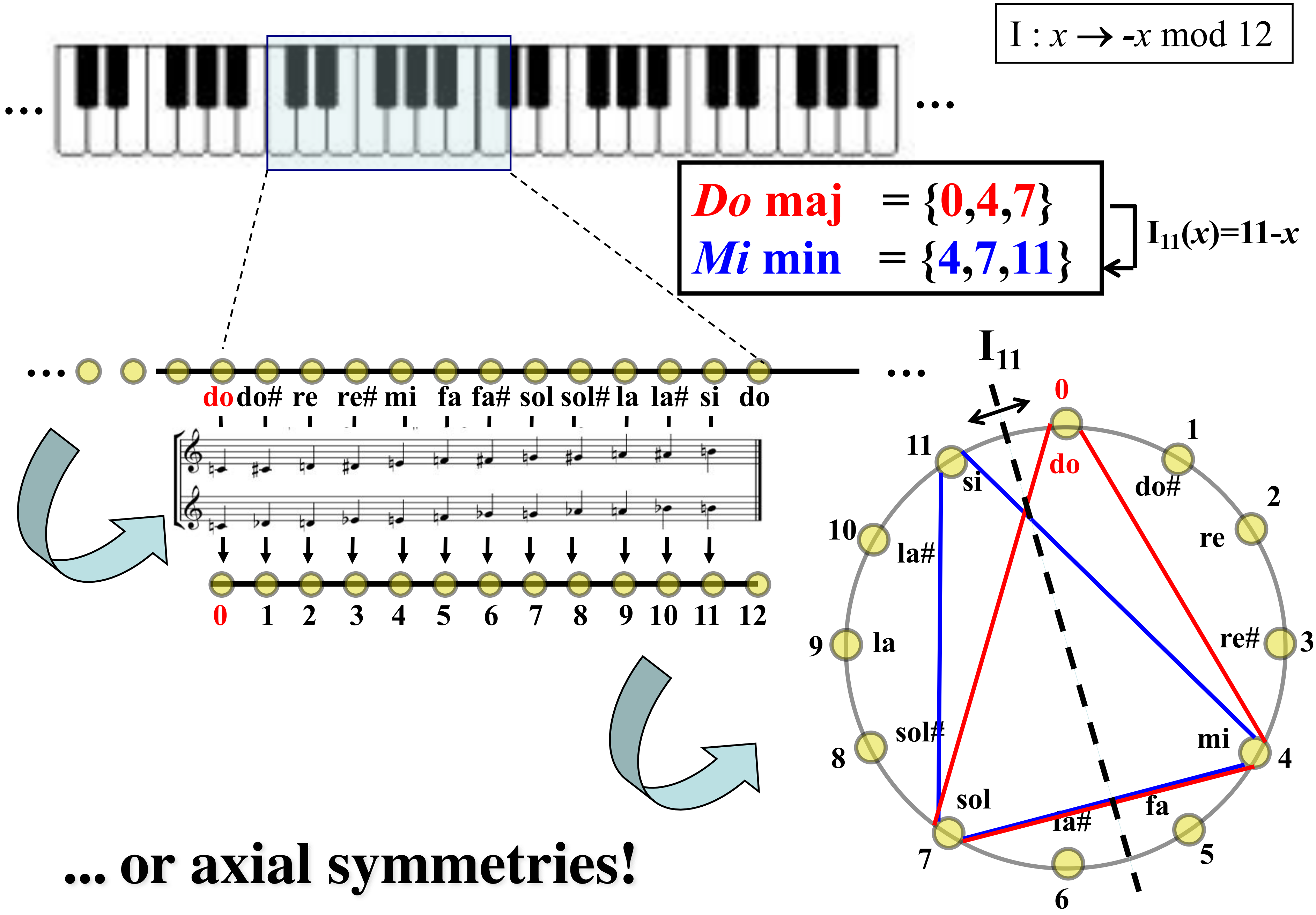
Do min $= \{0,3,7\}$

$I_7(x) = 7 - x$



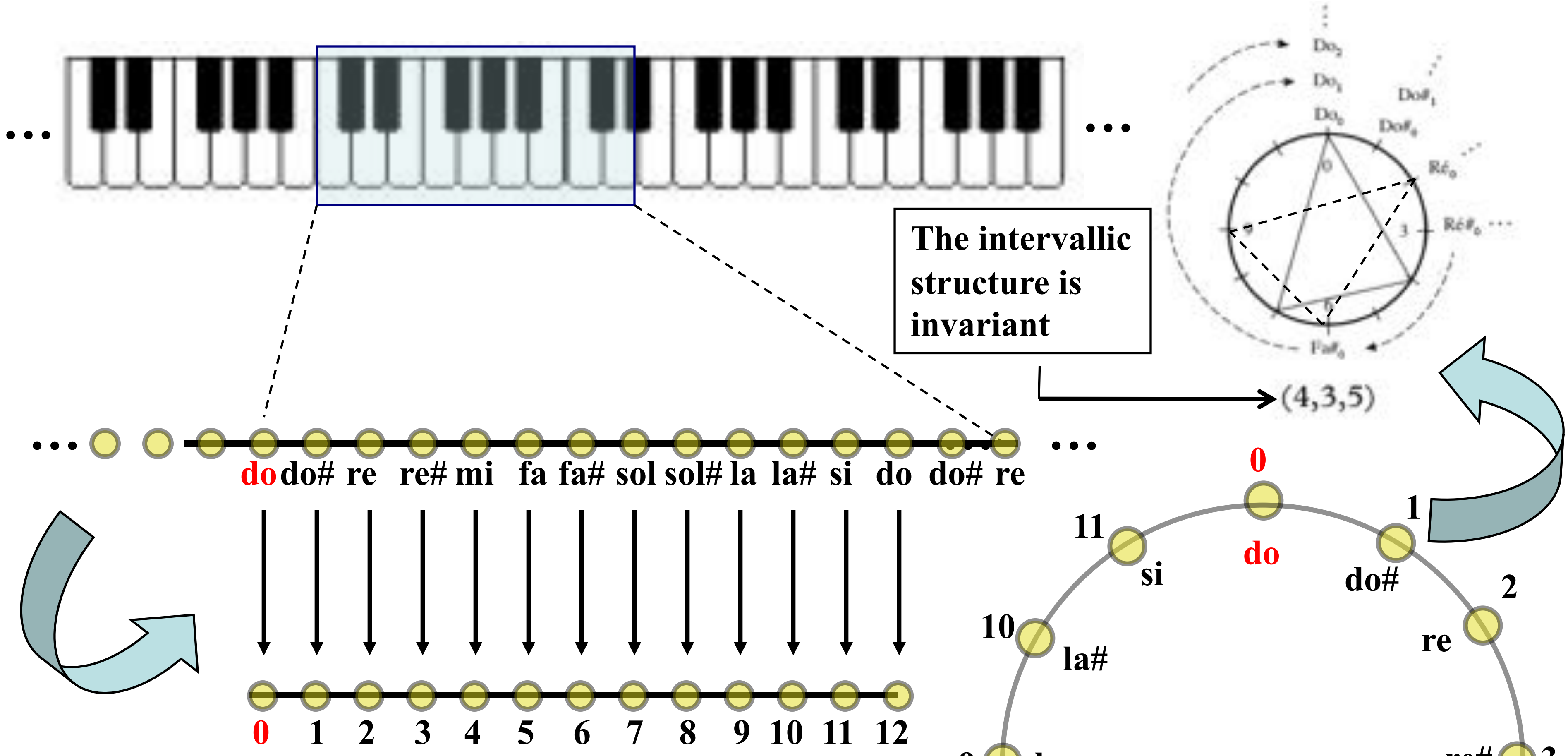
... or axial symmetries!

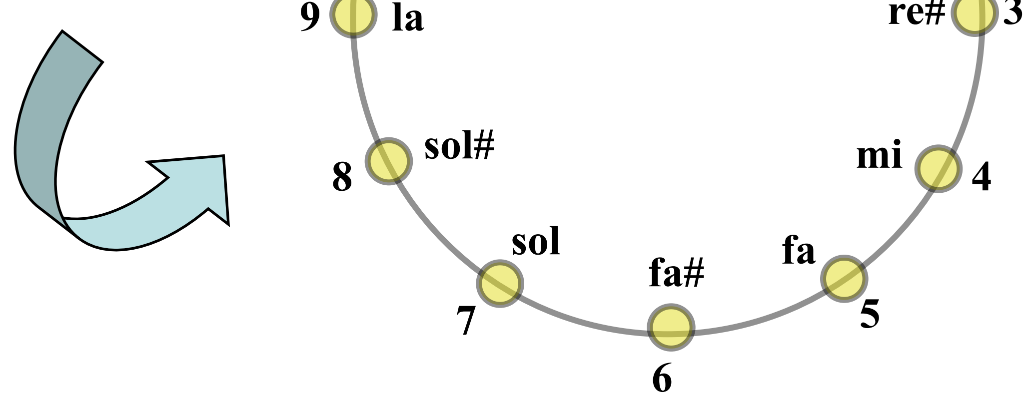
Musical inversions are differences...



... or axial symmetries!

Circular representation and intervallic structure



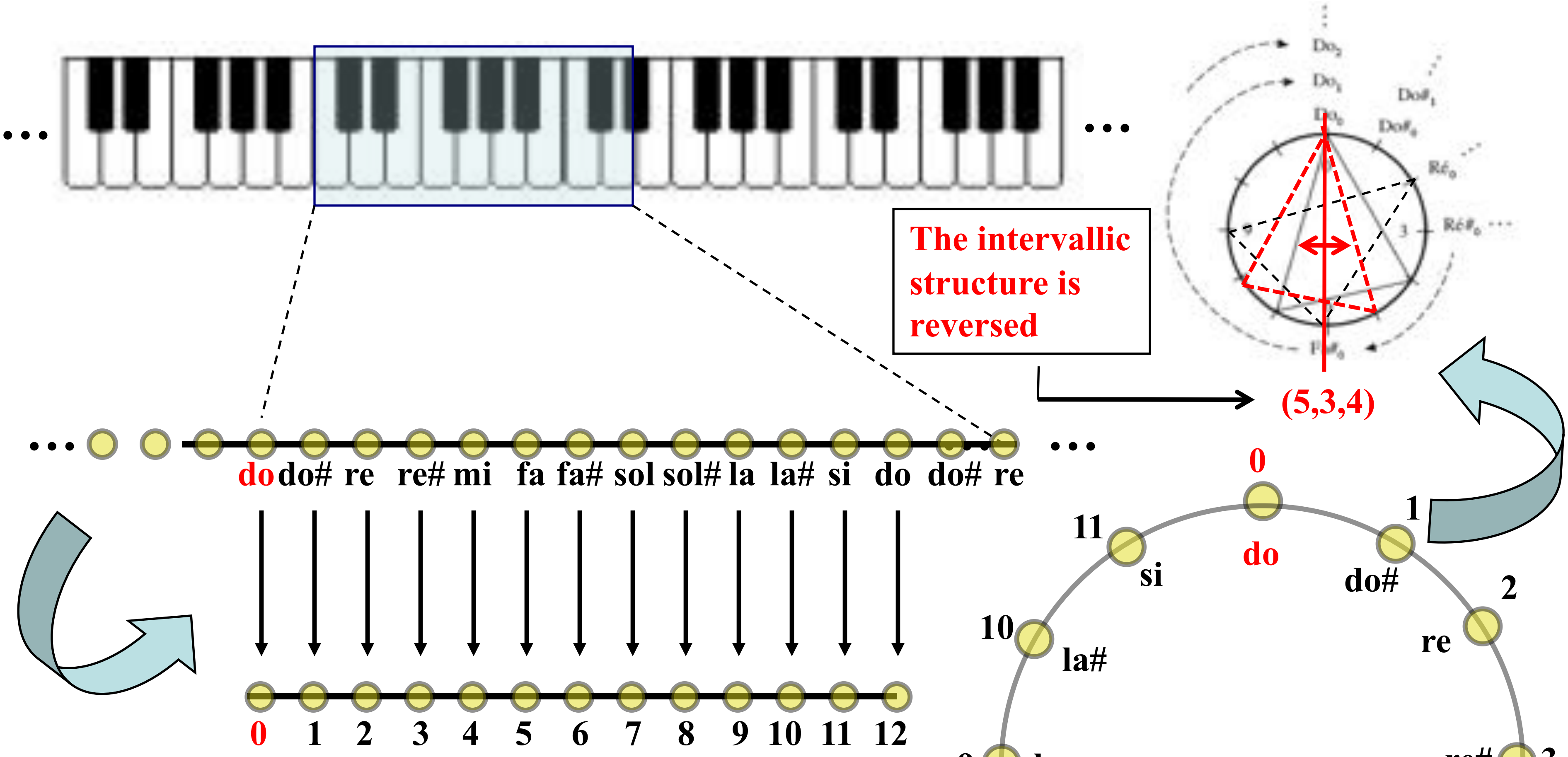


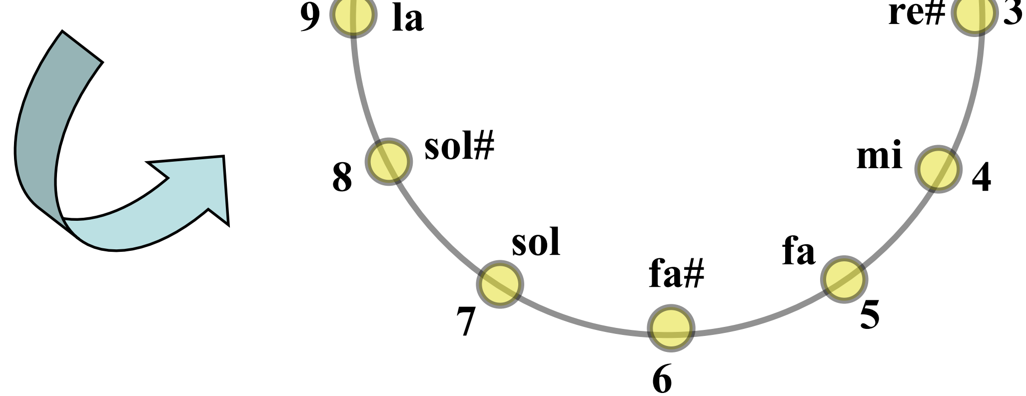
The two basic musical applications

$T_k : x \rightarrow x+k \bmod 12$ (transposition)

$I : x \rightarrow -x \bmod 12$ (inversion)

Circular representation and intervallic structure



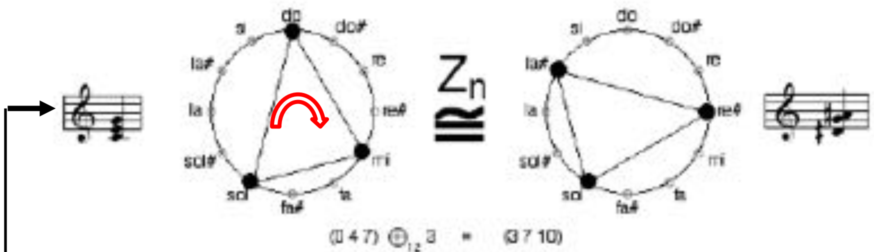


The two basic musical applications

$T_k : x \rightarrow x+k \bmod 12$ (transposition)

$I : x \rightarrow -x \bmod 12$ (inversion)

Equivalence classes of musical structures
(up to a group action)

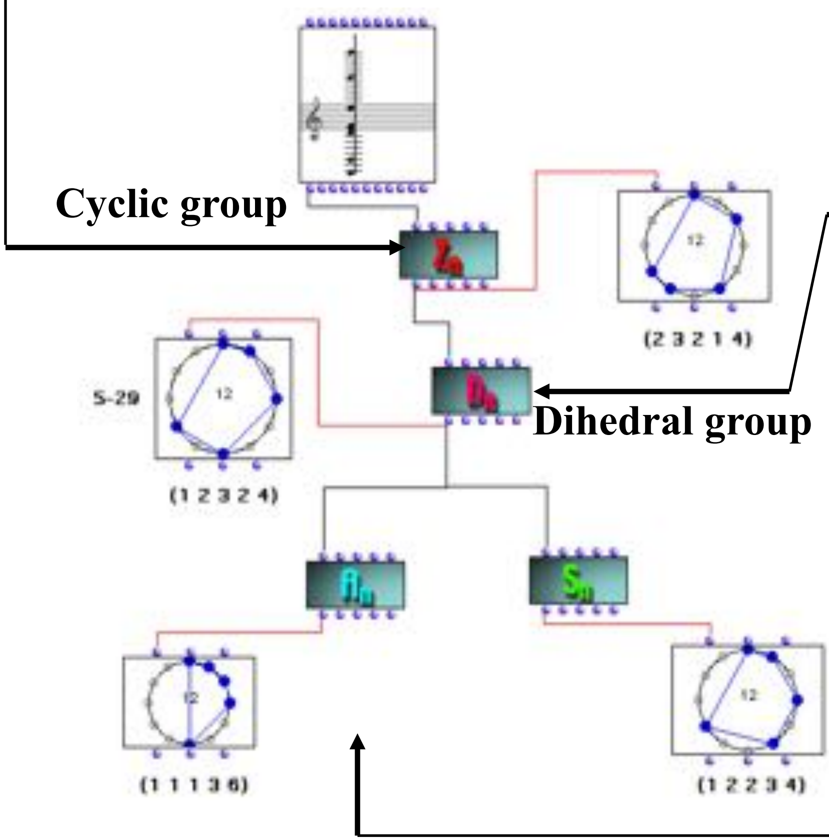


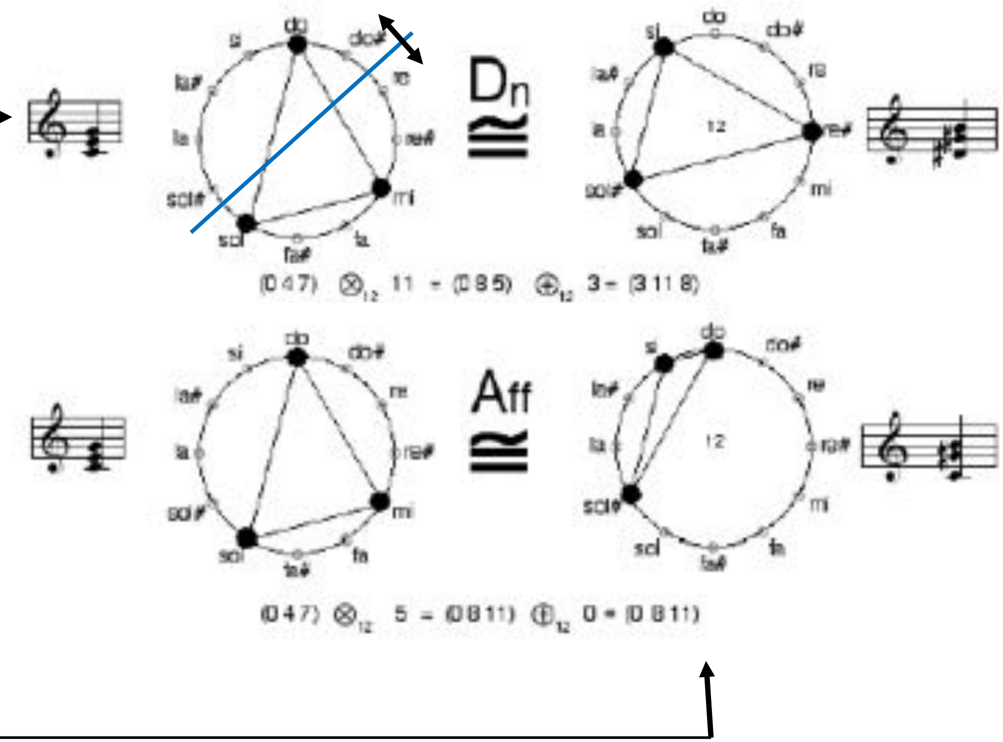
$\mathbf{Z_{12}} = < T_k \mid (T_k)^{12} = T_0 >$ where $T_k(x)=x+1$

$\mathbf{D_{12}} = < T_k , I \mid (T_k)^{12} = I^2 = T_0 , ITI=I(IT)^{-1} >$

where $I(x)=-x$

$\mathbf{Aff} = \{ f \mid f(x)=ax+b,\ a \in (\mathbf{Z_{12}})^*,\ b \in \mathbf{Z_{12}} \}$

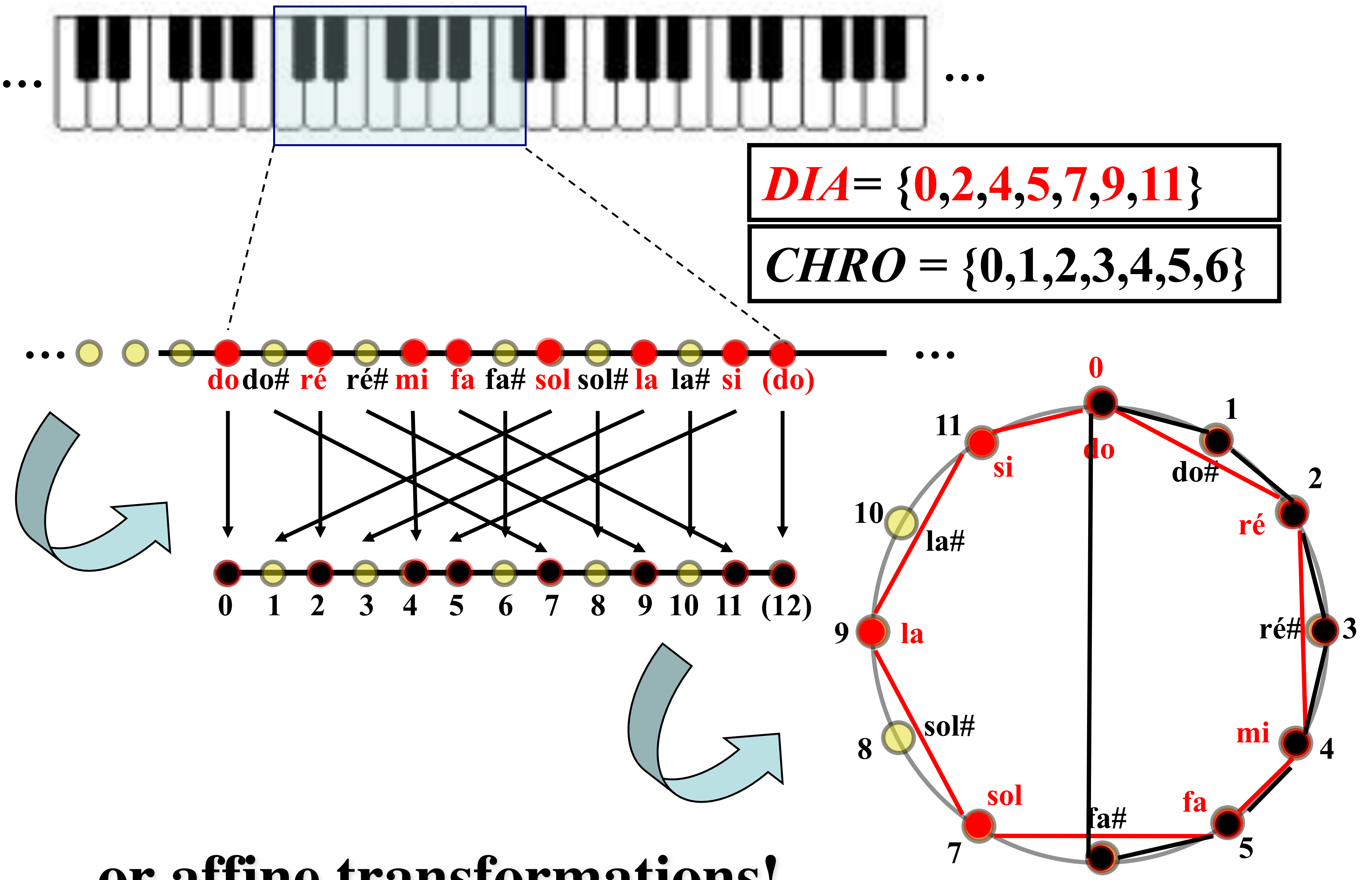




Affine group

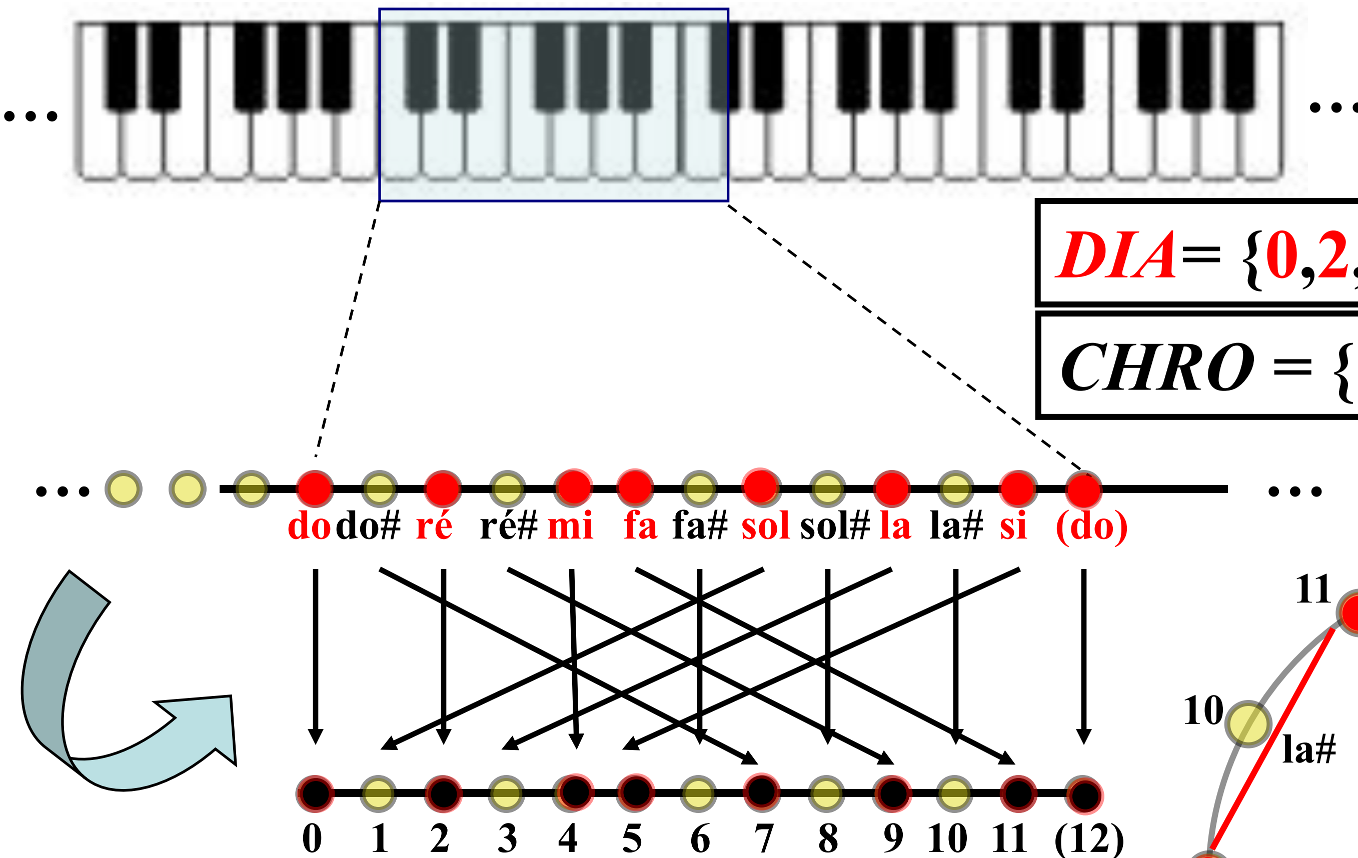
Paradigmatic architecture

Augmentations are multiplications...



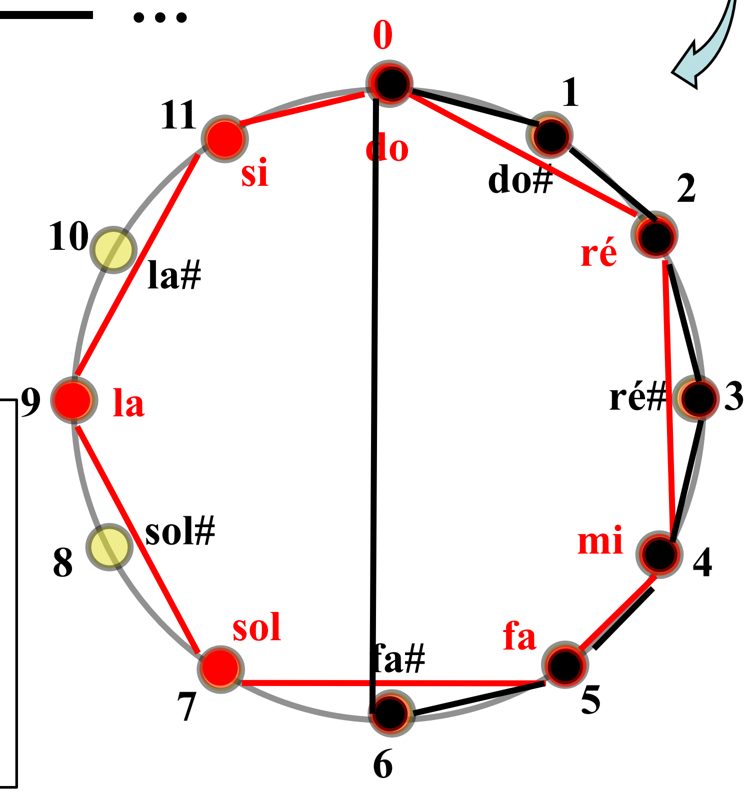
... or affine transformations!

Affine transformations and DIA/CHRO duality



DIA= {0,2,4,5,7,9,11}

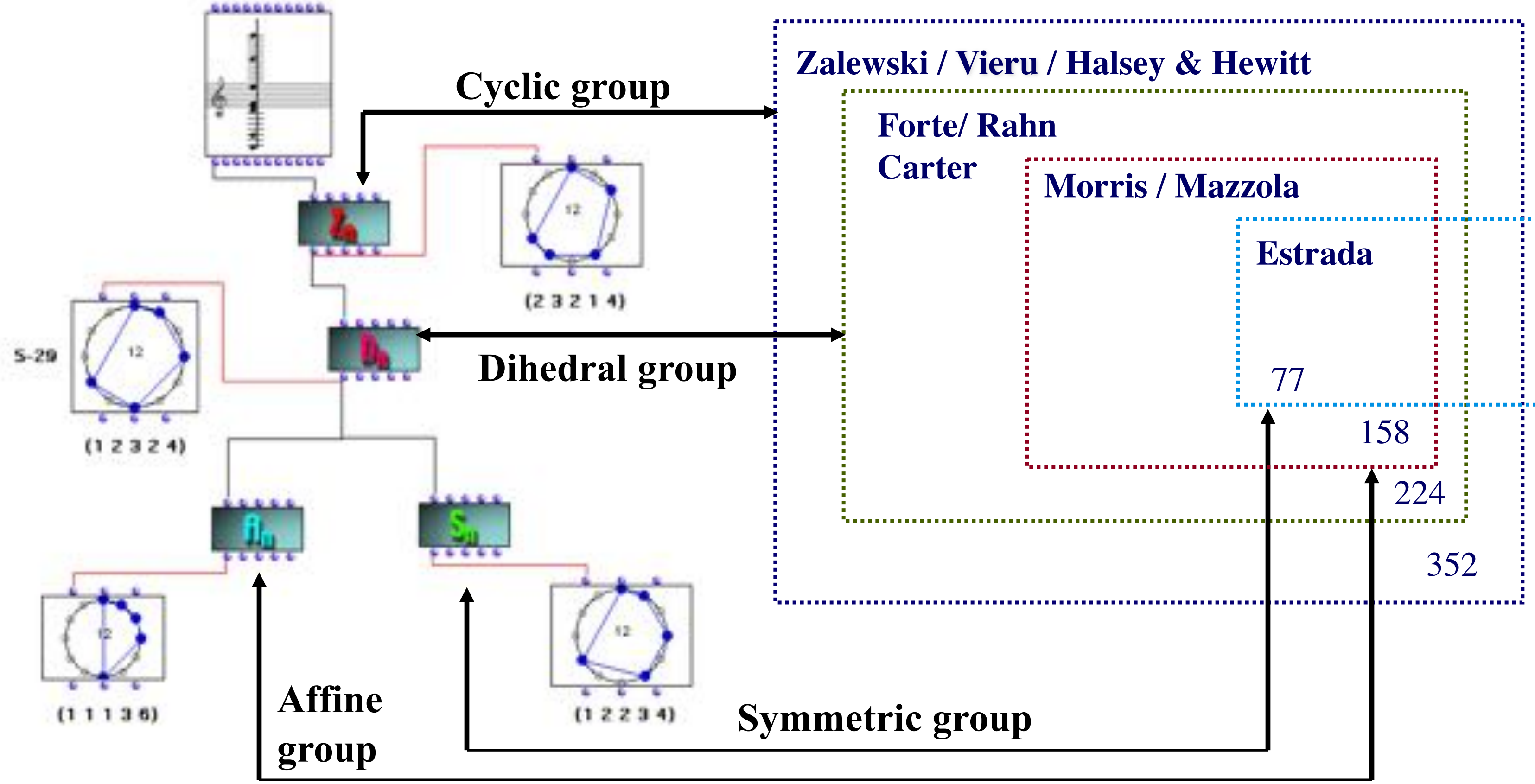
CHRO = {0,1,2,3,4,5,6}



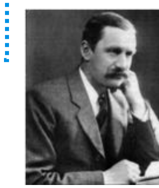
« Le **diatonisme** et le **chromatisme** ne peuvent pas être envisagés en termes de simplicité ou de complexité, comme on le pensait jadis. Il s'agit plutôt d'une question **d'unité des contraires** dans le groupe **Z/12Z** »

(A. Vieru, « The Musical Signification of Multiplication by 7. Diatonicity and Chromaticity », *Muzica,* 1995)

Group actions and the classification of musical structures




F. Klein


W. Burnside


G. Polya

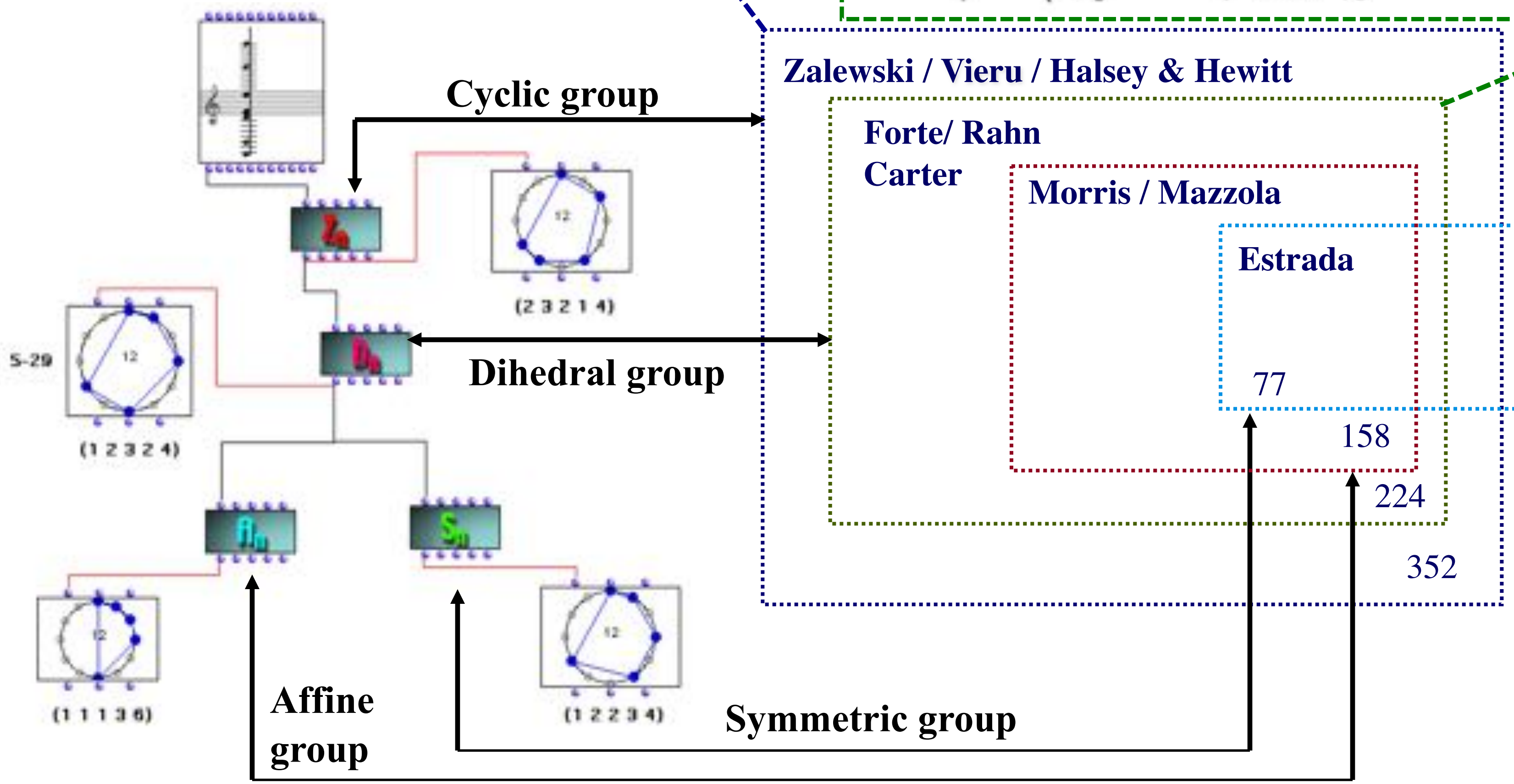
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Z_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1
S_n	***1***	***6***	***12***	***15***	***13***	***11***	***7***	***5***	***3***	***2***	***1***	***1***

A group action based classification of musical structures

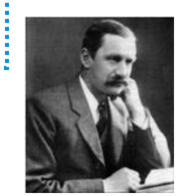
$$\# \text{ of } k\text{-chords} = \frac{1}{n}\sum_{j|(n,k)} \phi(j)\binom{n/j}{k/j} = \frac{1}{n}\Phi_n(k)$$

$$\# \text{ of } k\text{-chords} = \begin{cases} \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n-1)/2}{[k/2]}\right], & \text{if } n \text{ is odd,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{n/2}{k/2}\right], & \text{if } n \text{ is even and } k \text{ is even,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n/2)-1}{[k/2]}\right], & \text{if } n \text{ is even and } k \text{ is odd.} \end{cases}$$

Paradigmatic architecture



F. Klein

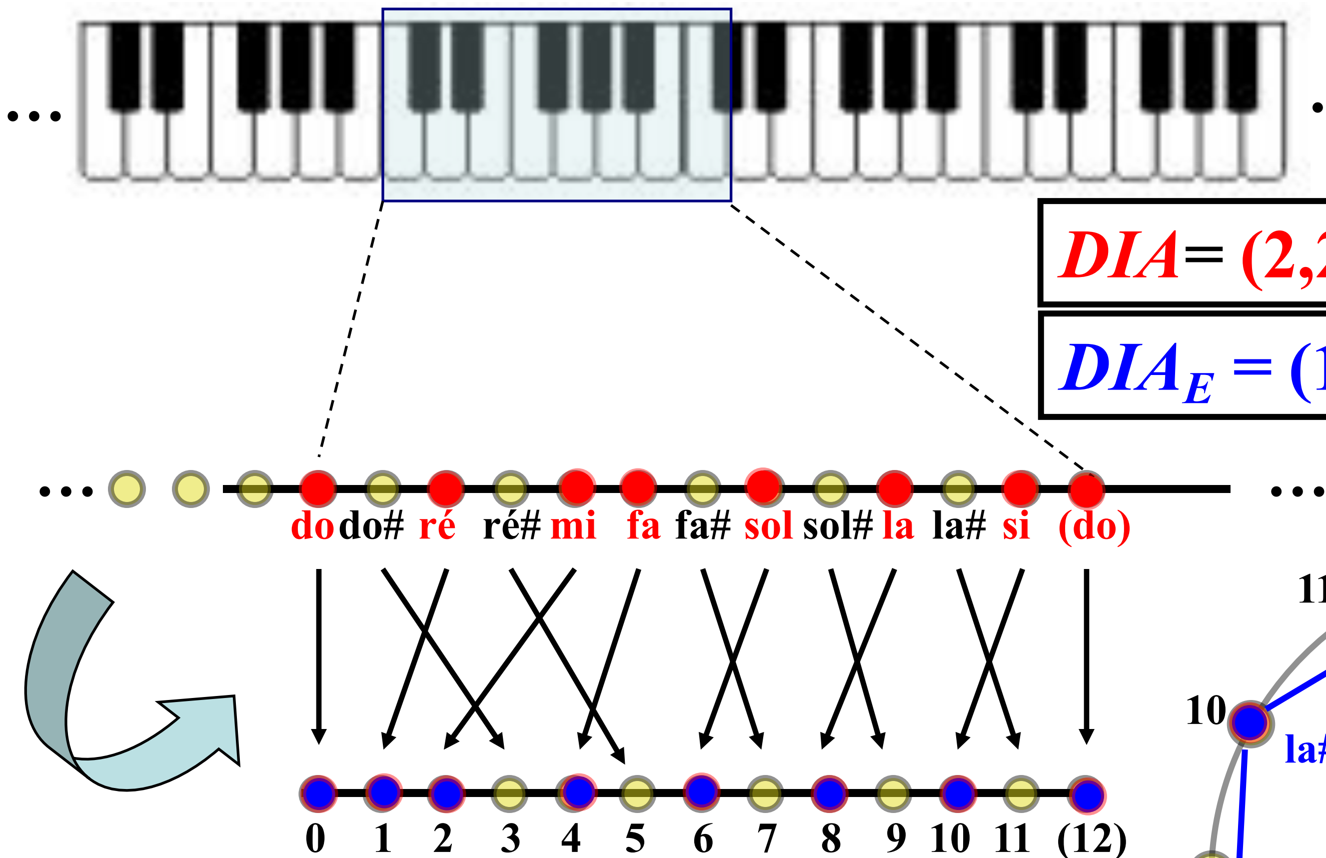


W. Burnside

G. Polya

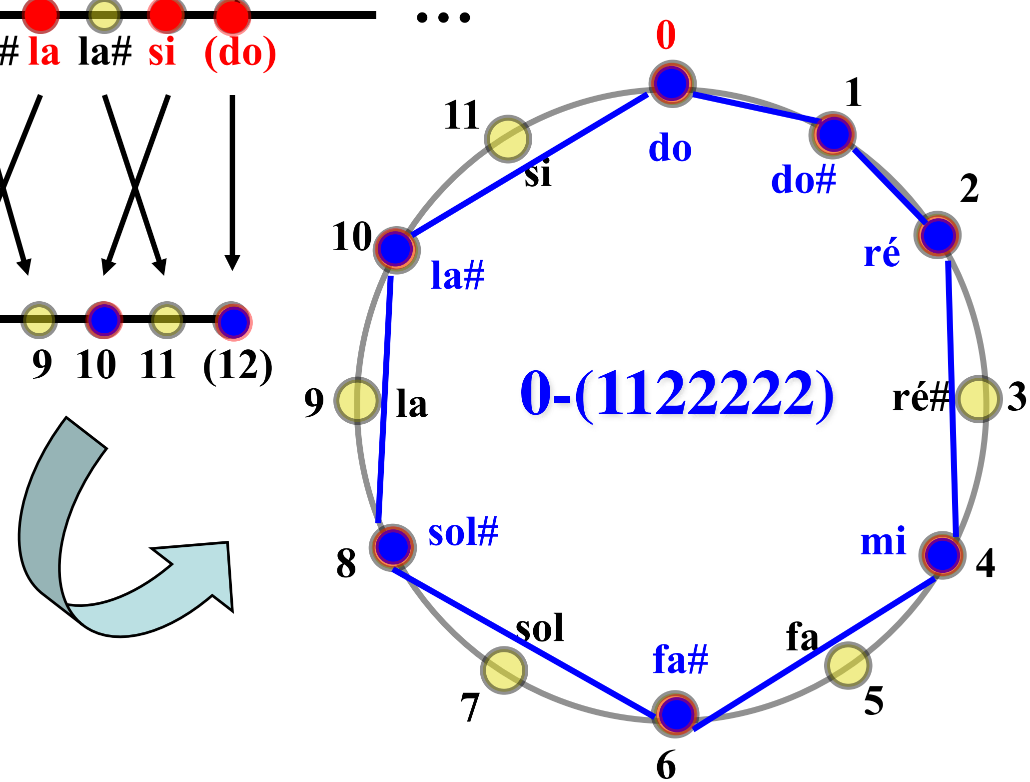
- D. Halsey & E. Hewitt, “Eine gruppentheoretische Methode in der Musik-theorie”, *Jahr. der Dt. Math.-Vereinigung*, 80, 1978
- D. Reiner, “Enumeration in Music Theory”, *Amer. Math. Month*. 92:51-54, 1985
- H. Fripertinger, “Enumeration in Musical Theory”, *Beiträge zur Elektr. Musik*,1,1992
- R.C. Read, “Combinatorial problems in the theory of music”, *Discrete Mathematics* 1997
- H. Fripertinger, “Enumeration of mosaics”, *Discrete Mathematics*, 1999
- H. Fripertinger, “Enumeration of non-isomorphic canons”, *Tatra Mt. Math. Publ.*, 2001

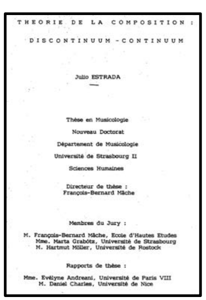
Classifying chords up to permutations of intervals



DIA= (2,2,1,2,2,2,1)

DIA_E = (1,1,2,2,2,2,2)




THEORIE DE LA COMPOSITION :

DISCONTINUUM - CONTINUUM

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Thèse en Musicologie

Nouveau Doctorat

Département de Musicologie

Université de Strasbourg II

Sciences Humaines

Directeur de thèse :
François-Bernard Mâche

Membres du Jury :

M. François-Bernard Mâche, Ecole d'Hautes Etudes
Mme. Marta Grabótz, Université de Strasbourg
M. Hartmut Möller, Université de Rostock

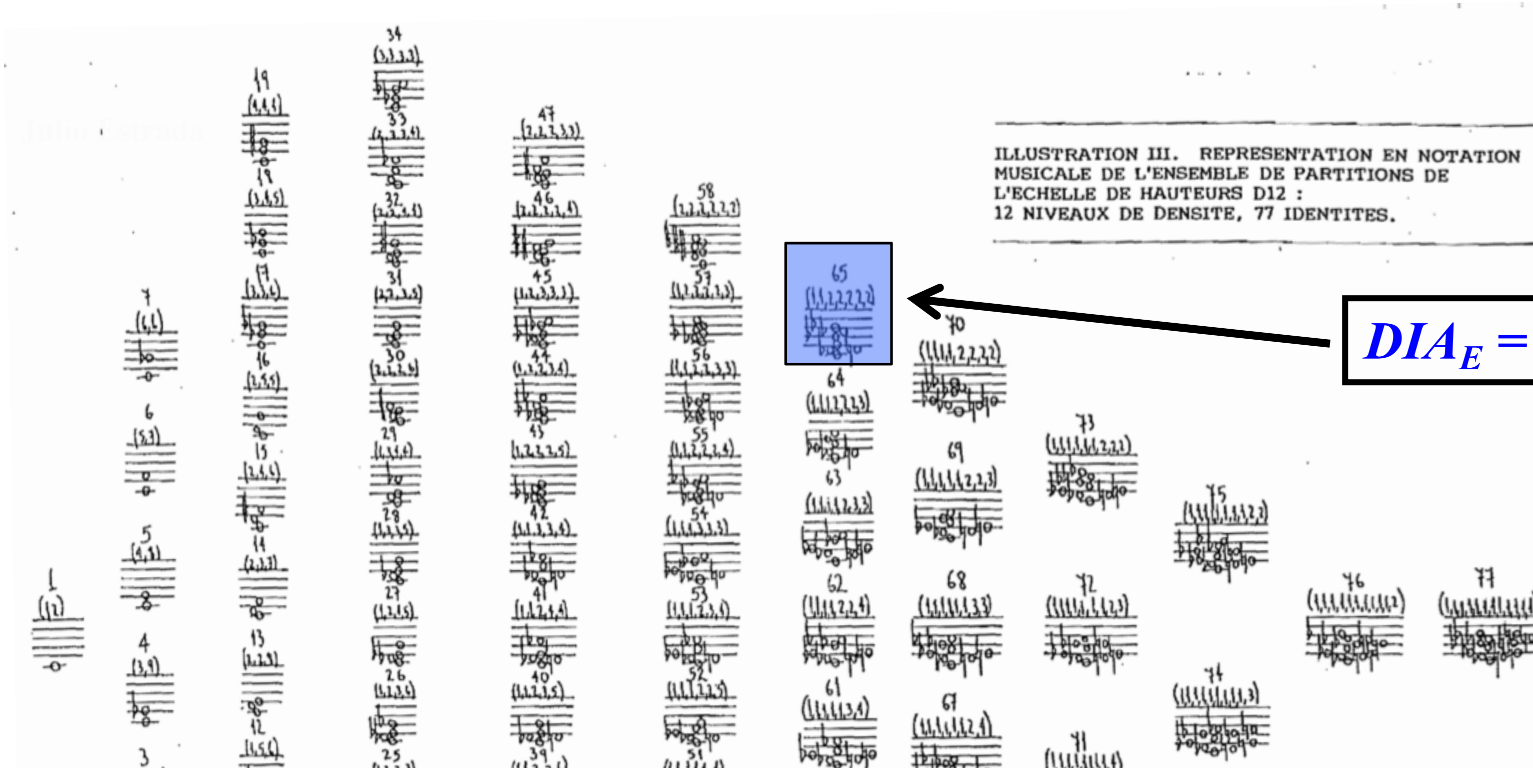
Rapports de thèse :

Mme. Evélyne Andreani, Université de Paris VIII
M. Daniel Charles, Université de Nice

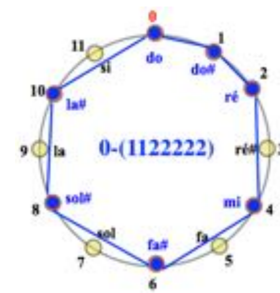
Julio Estrada, ***Théorie de la composition : discontinuum – continuum,*** **université de Strasbourg II, 1994**

The permutohedron as a combinatorial space

Julio Estrada, *Théorie de la composition : discontinuum – continuum,* université de Strasbourg II, 1994



$DIA_E = (1,1,2,2,2,2,2)$

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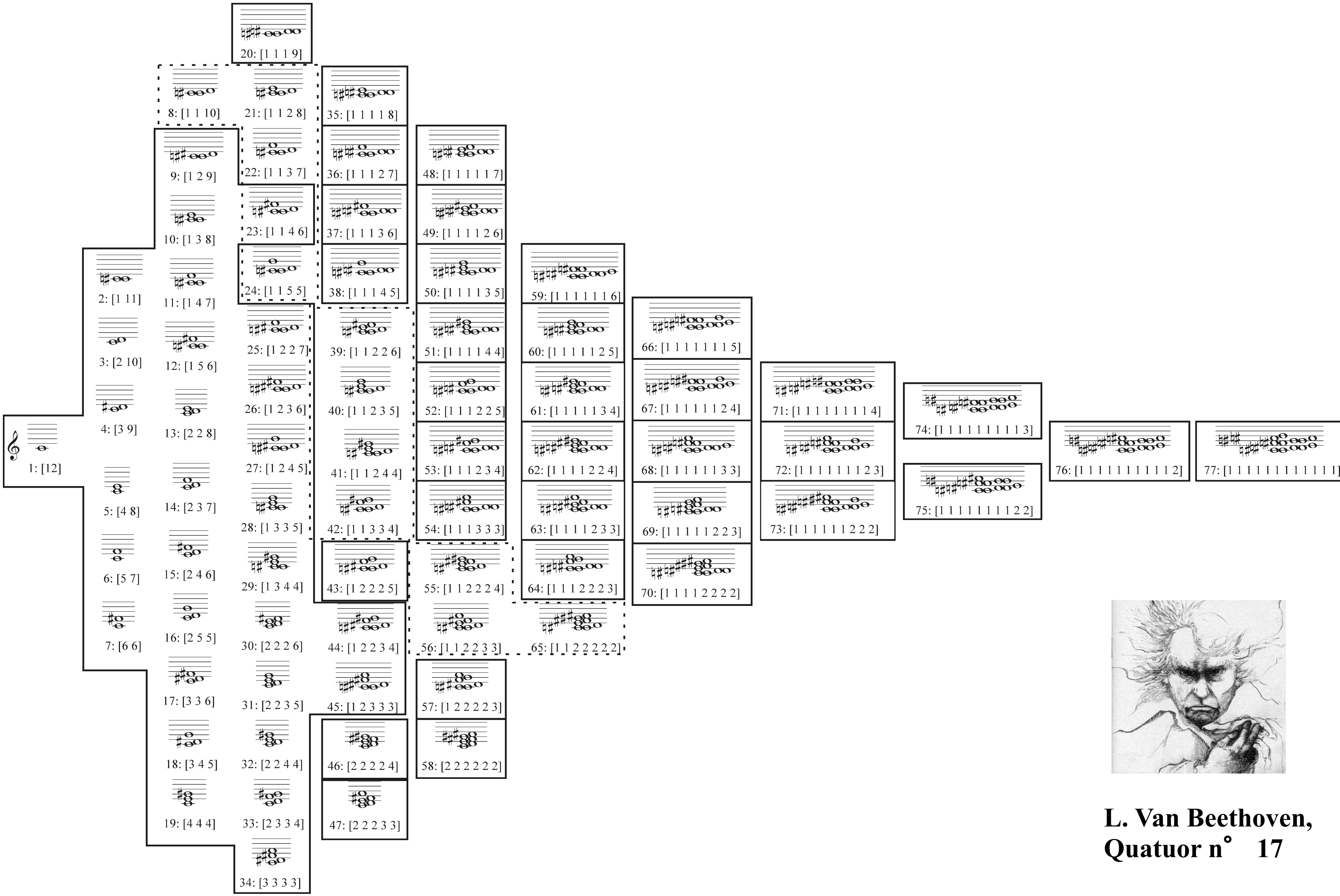
J. Estrada

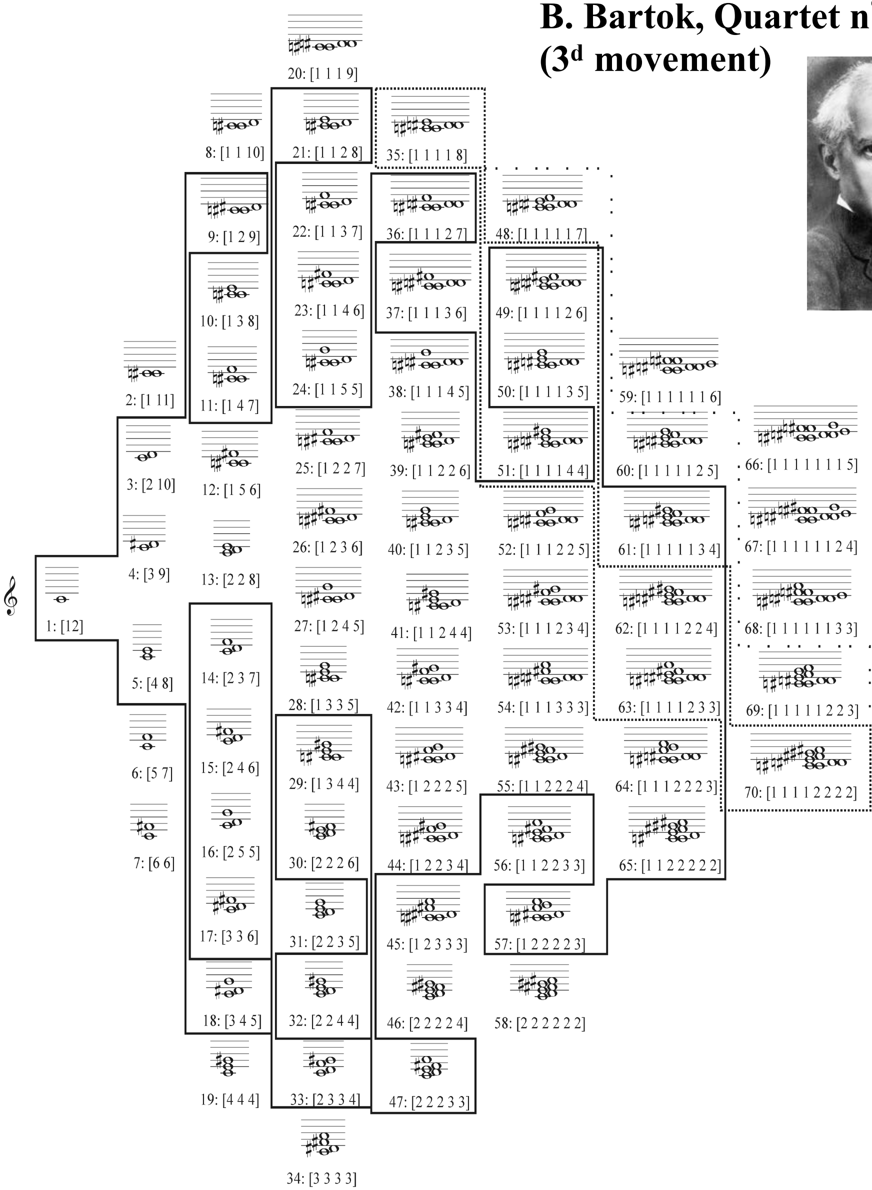
The permutohedron as a musical conceptual space



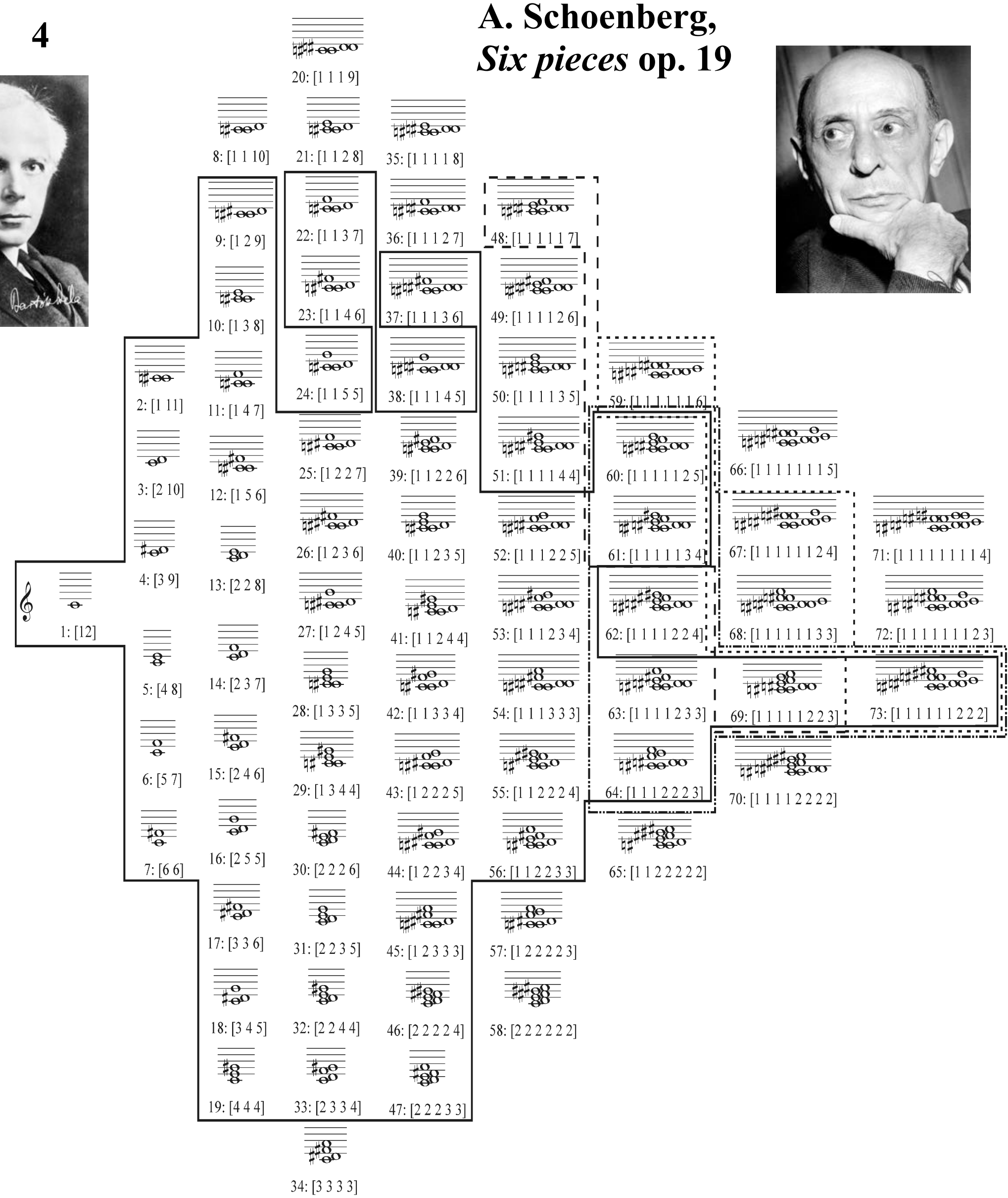
L. Van Beethoven,
Quatuor n° 17

The permutohedron as a musical conceptual space

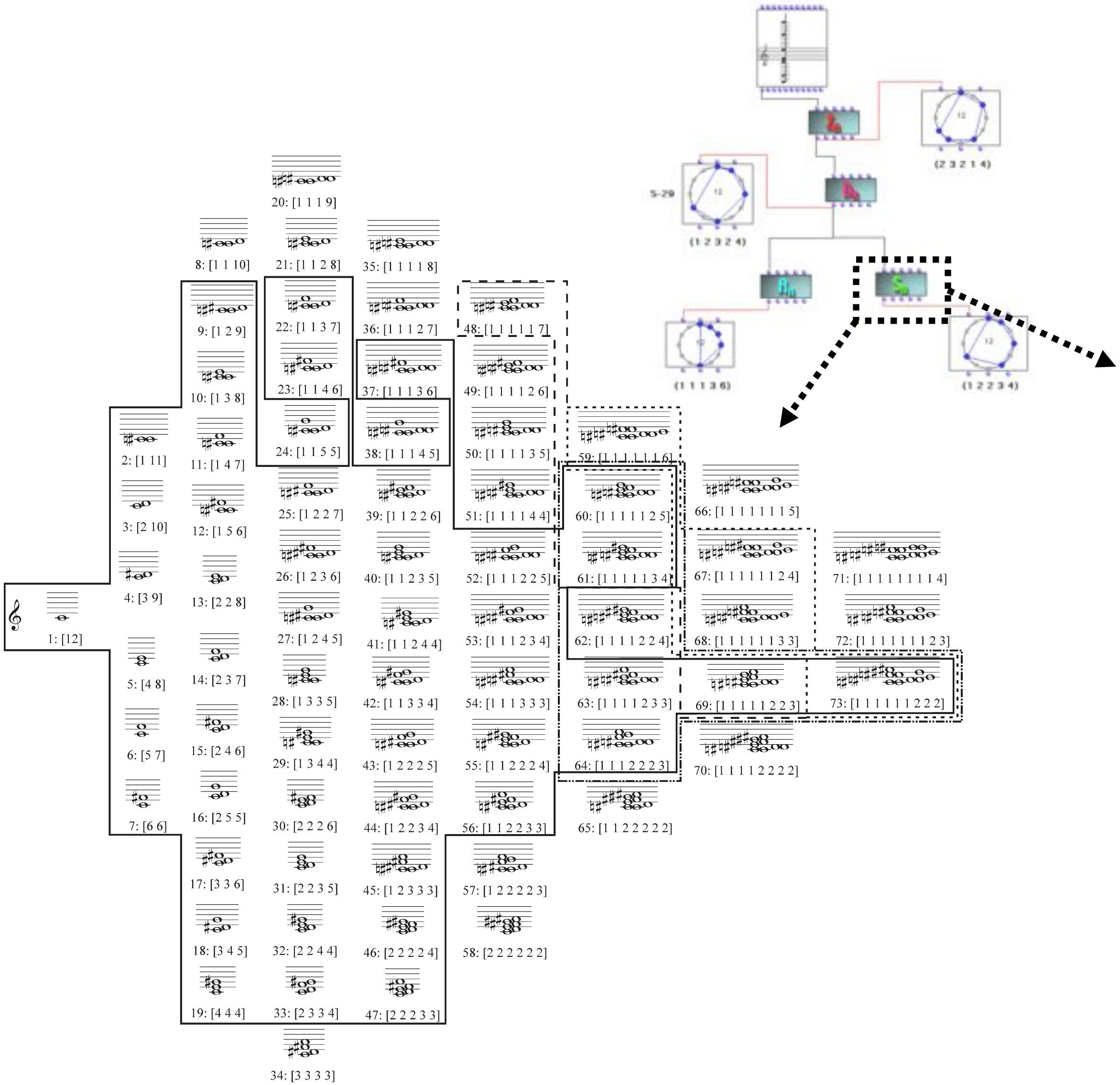
B. Bartok, Quartet n° 4 (3[d] movement)

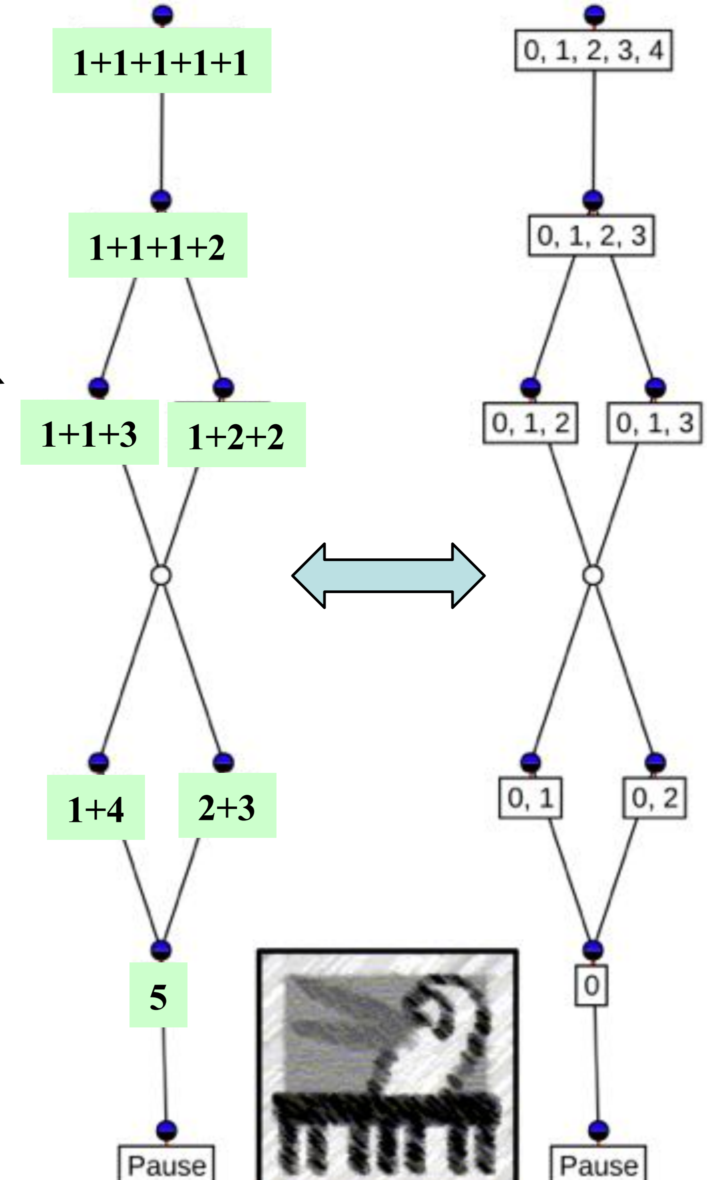


A. Schoenberg, ***Six pieces*** **op. 19**



The permutohedron as a lattice of formal concepts

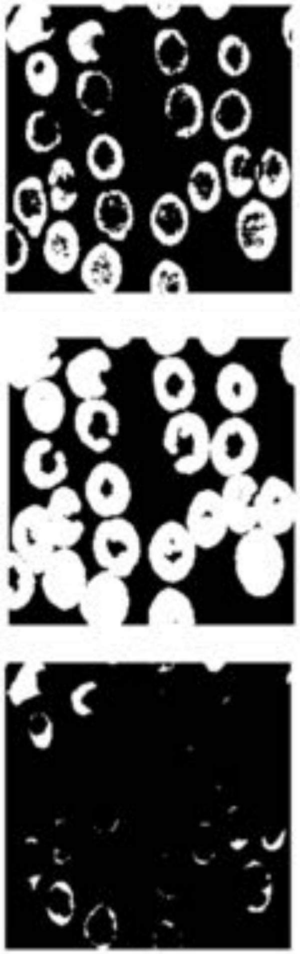


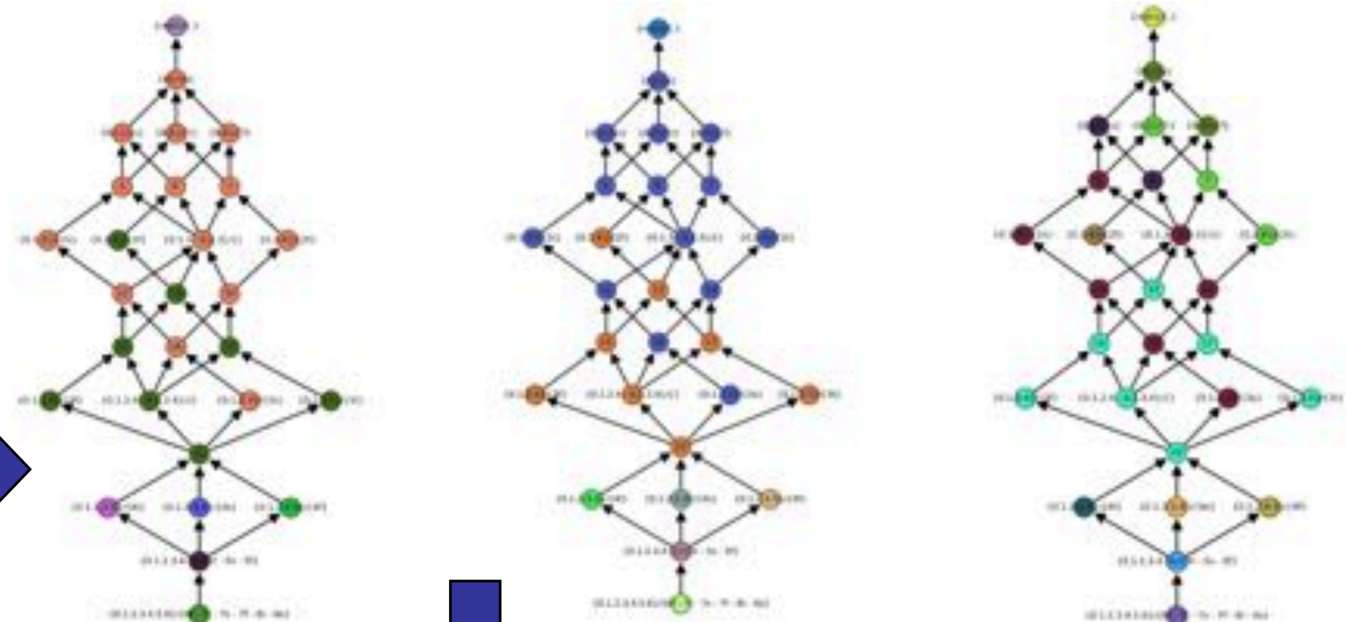


TECHNISCHE UNIVERSITÄT DRESDEN

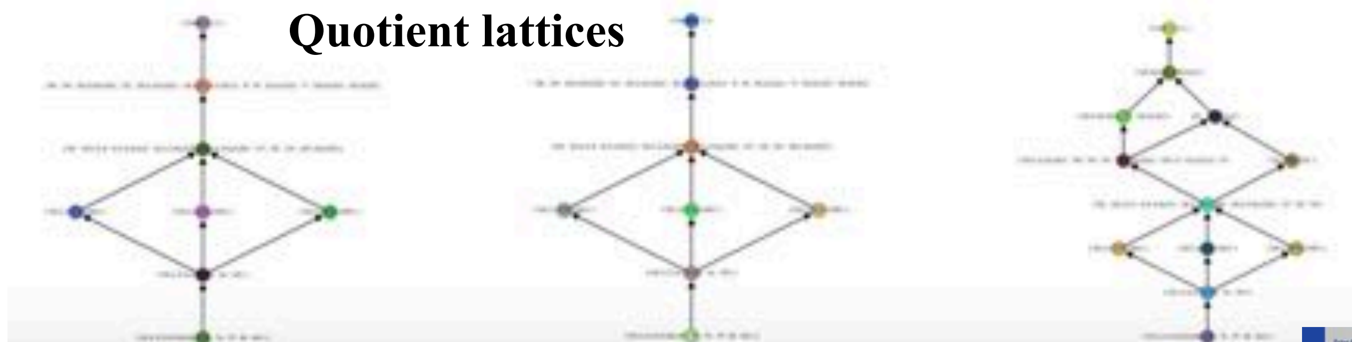
- T. Schlemmer, M. Andreatta, « Using Formal Concept Analysis to represent Chroma Systems », MCM 2013, McGill Univ., Springer, LNCS.

Concept lattices & mathematical morphology



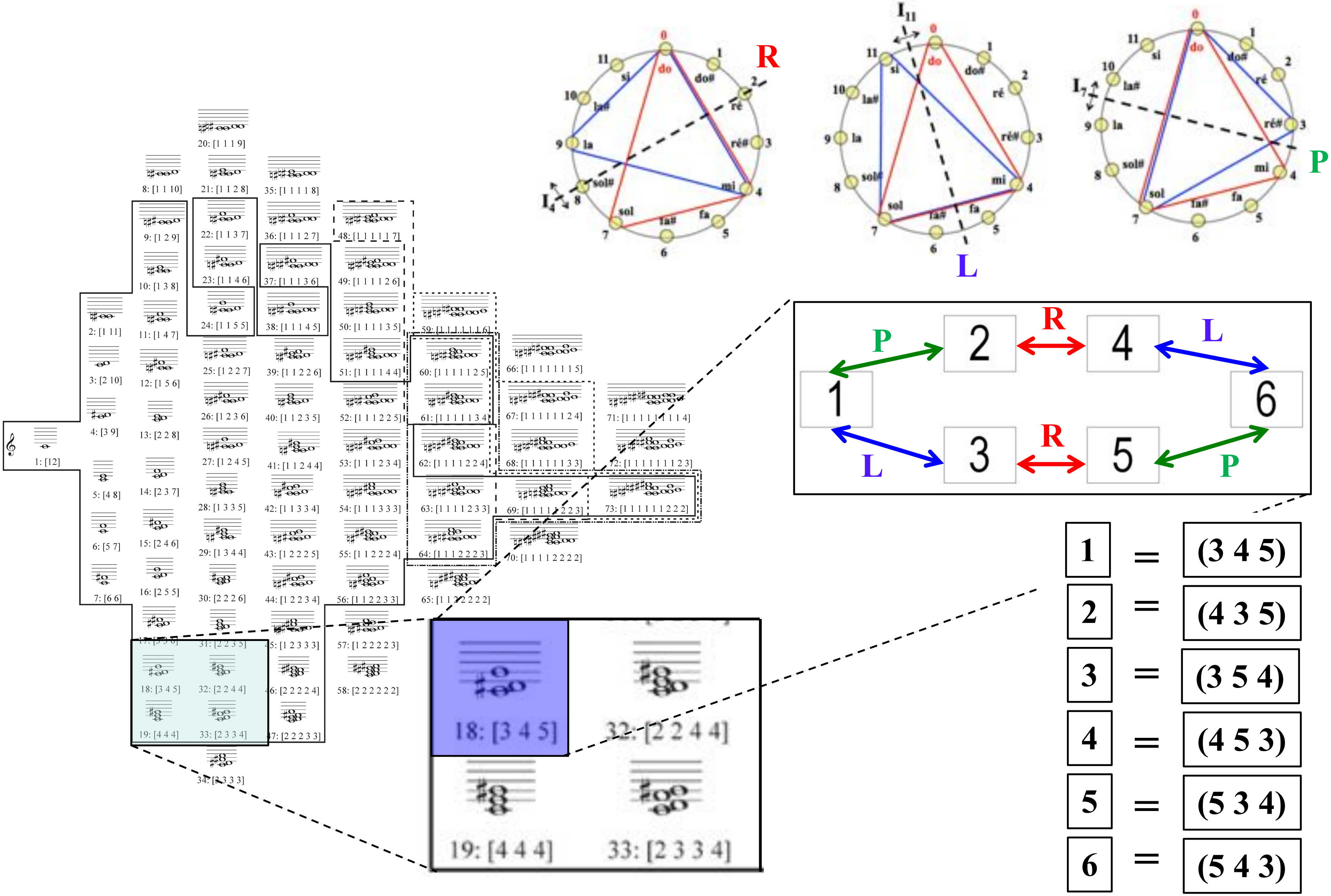


Quotient lattices



- **P. Relaño, *Morphologie mathématique, FCA et musicologie computationnelle*, Master 2, 2017 (codir. with I. Bloch (LTCI/Télécom ParisTech), J. Atif (LAMSADE, Université Paris Dauphine), C. Agon (IRCAM/Sorbonne)**
- **C. Agon, M. Andreatta, J. Atif, I. Bloch, and P. Relaño, "Musical Descriptions based on Formal Concept Analysis and Mathematical Morphology", Proceedings of the International Conference on Conceptual Structures, Springer, 2018**

Permutohedron and *Tonnetz*: a structural inclusion



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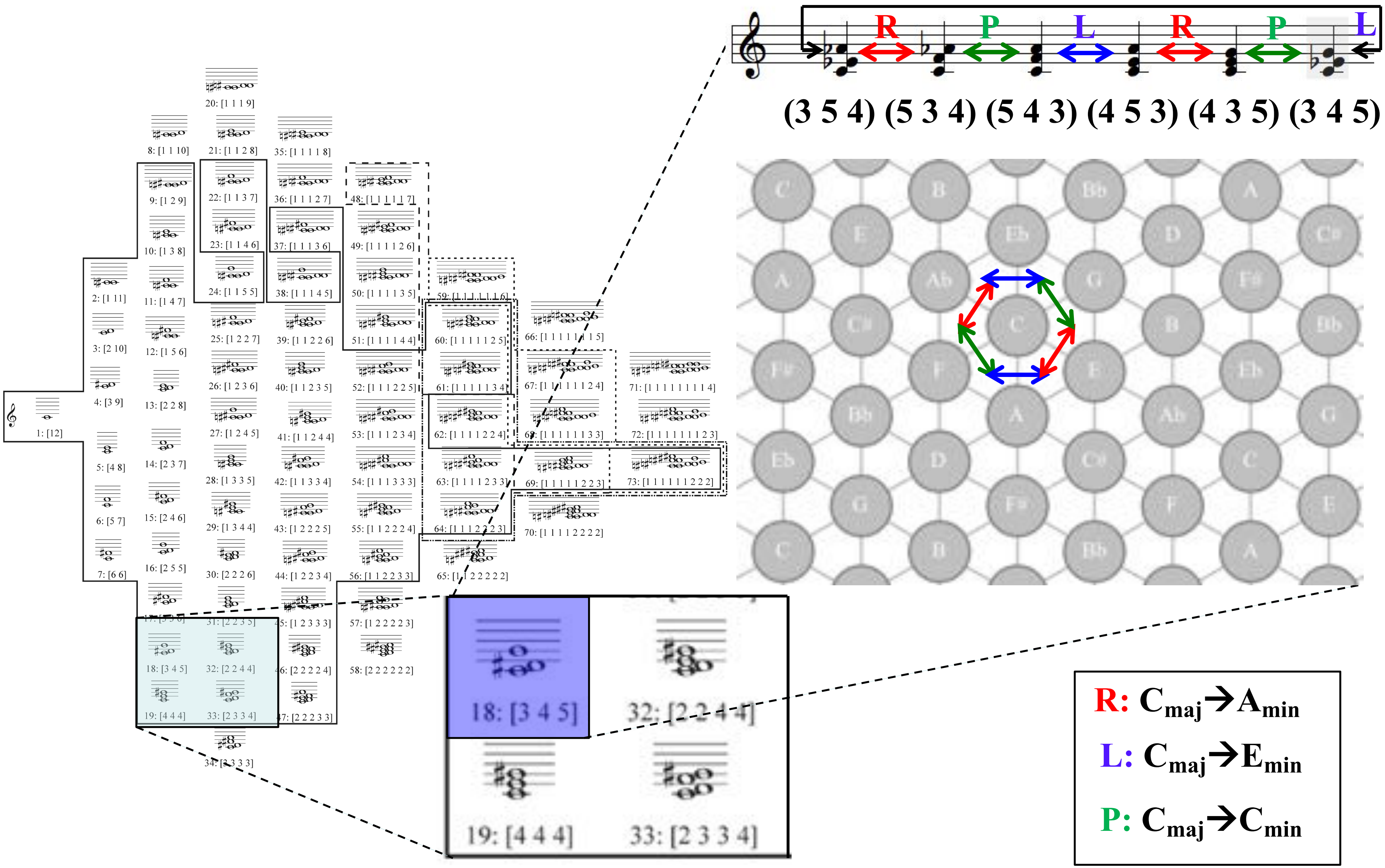

Permutohedron and *Tonnetz*: a structural inclusion