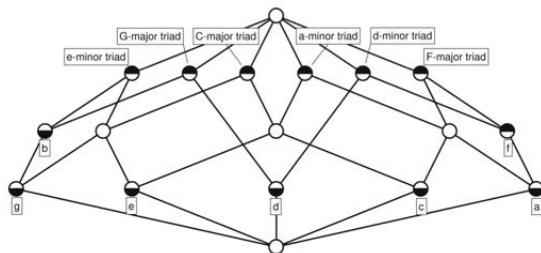


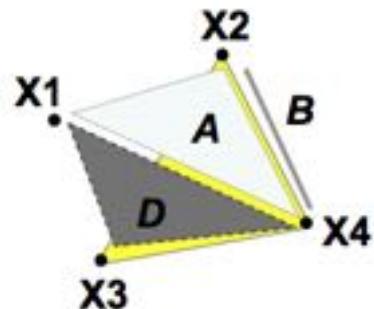
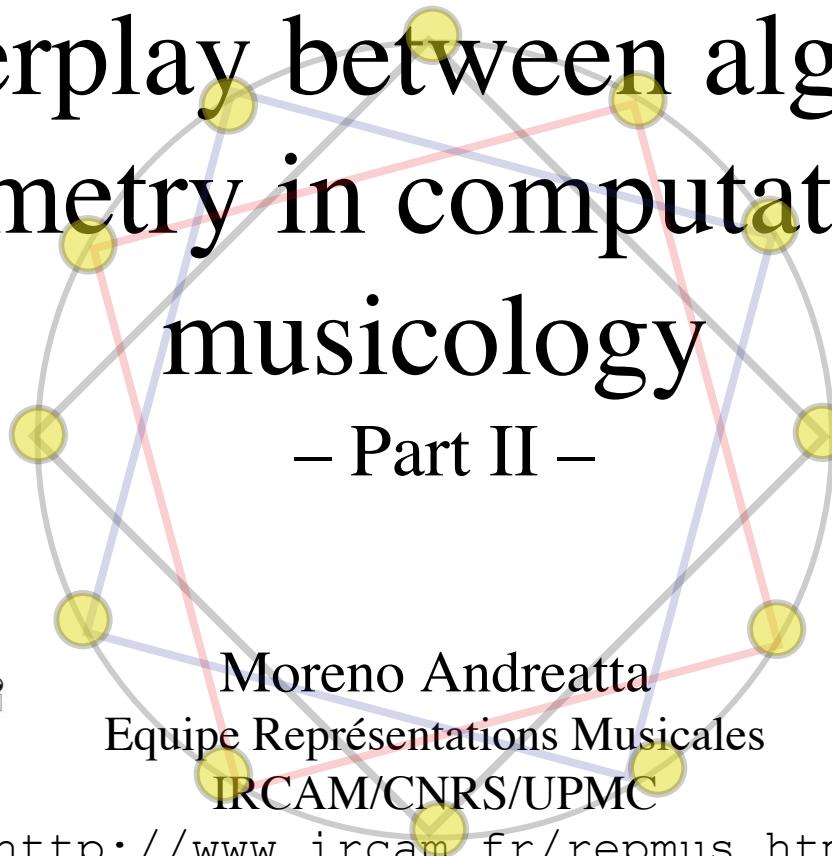


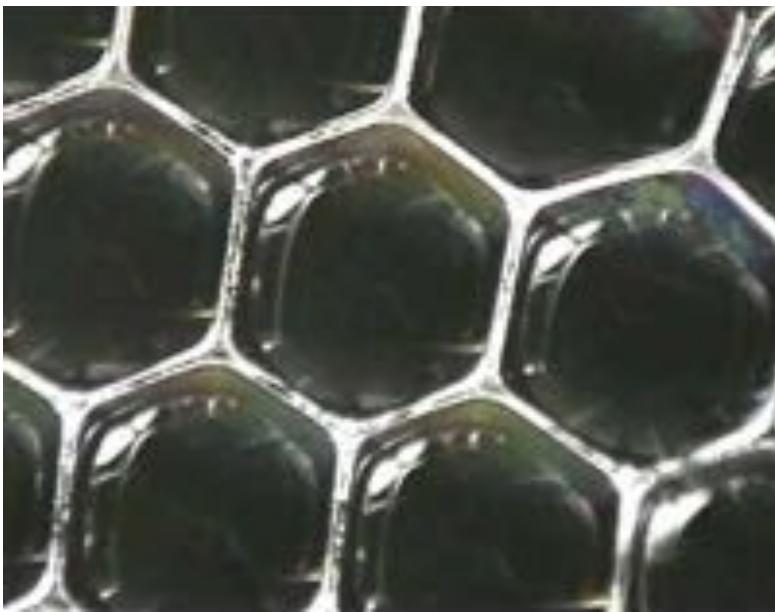
The interplay between algebra and geometry in computational musicology

– Part II –

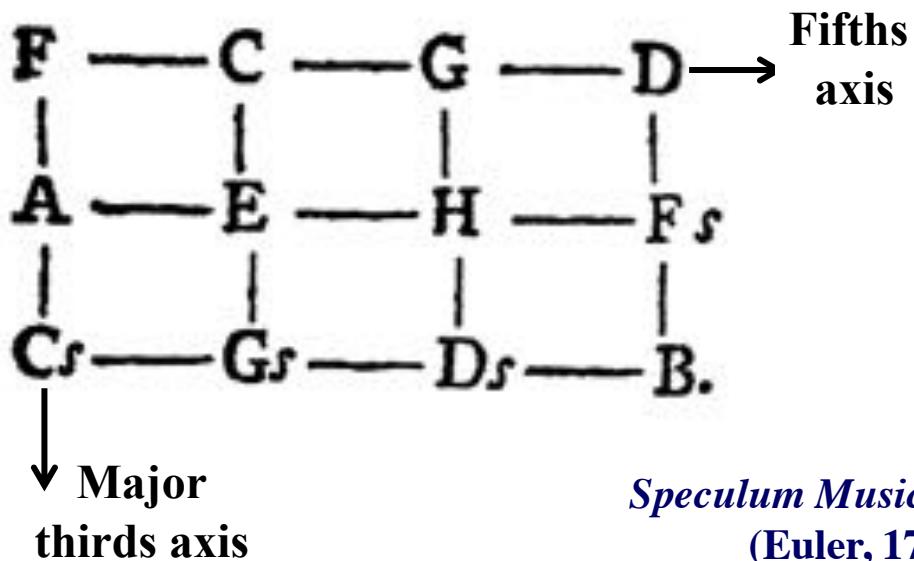


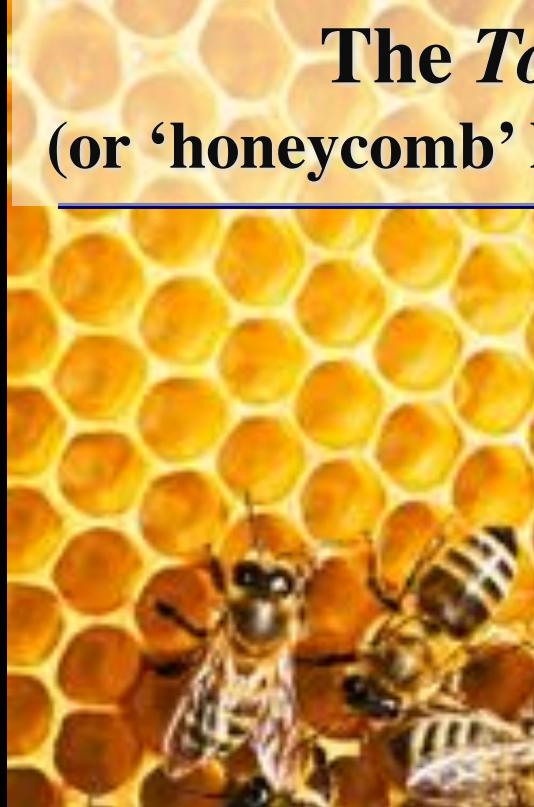
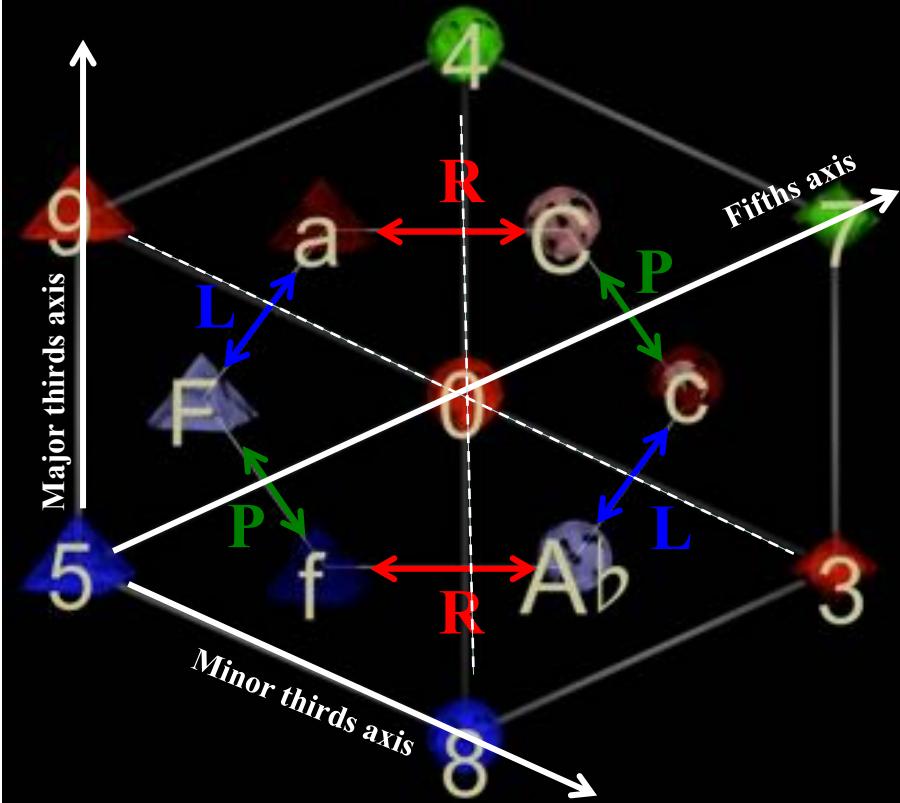
Moreno Andreatta
Equipe Représentaions Musicales
IRCAM/CNRS/UPMC
<http://www.ircam.fr/repmus.html>





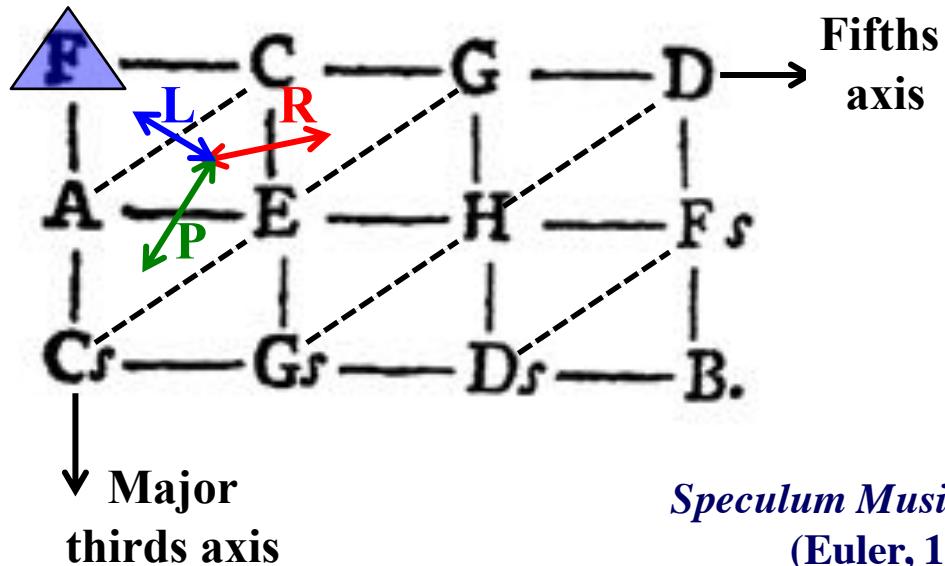
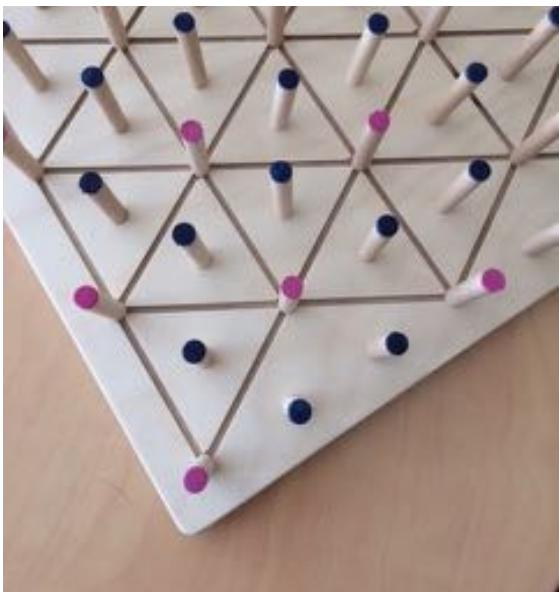
The Tonnetz (or ‘honeycomb’ hexagonal tiling)





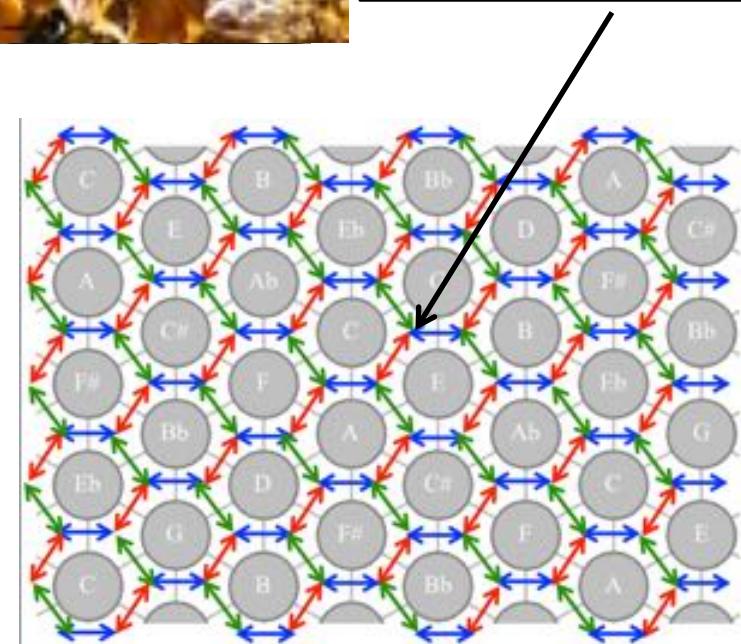
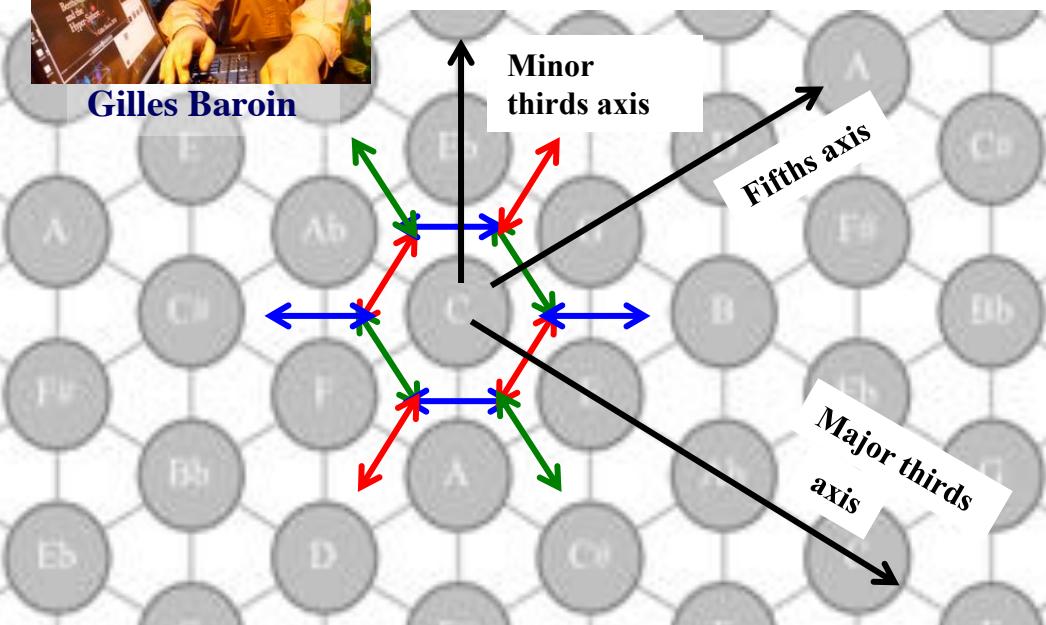
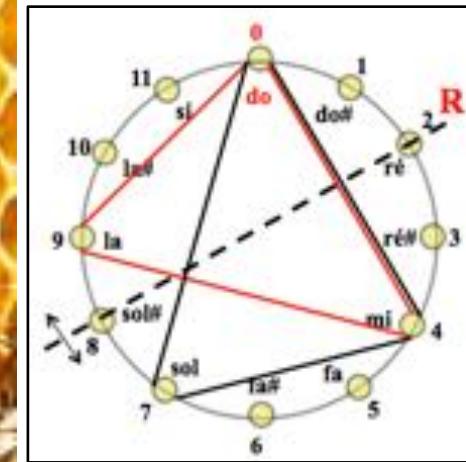
The Tonnetz

(or ‘honeycomb’ hexagonal tiling)

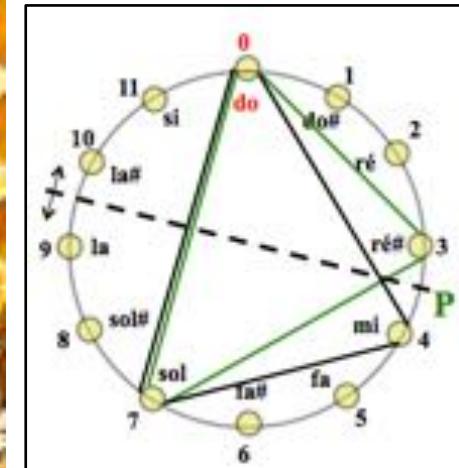


Speculum Musicum
(Euler, 1773)

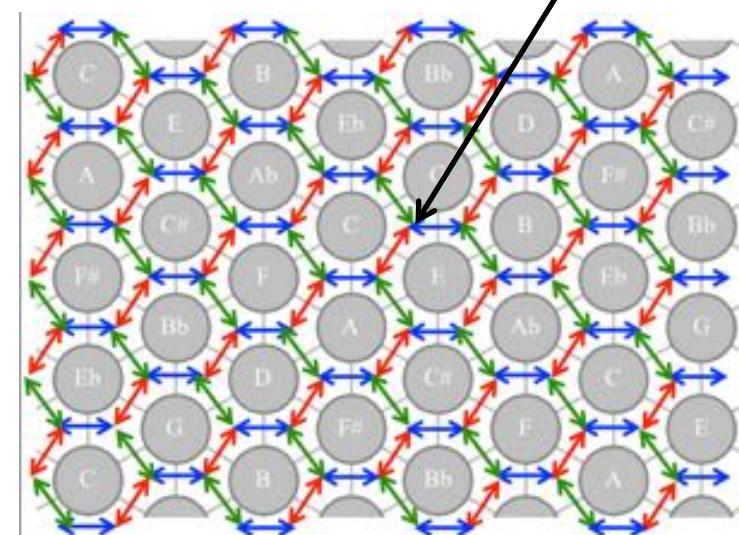
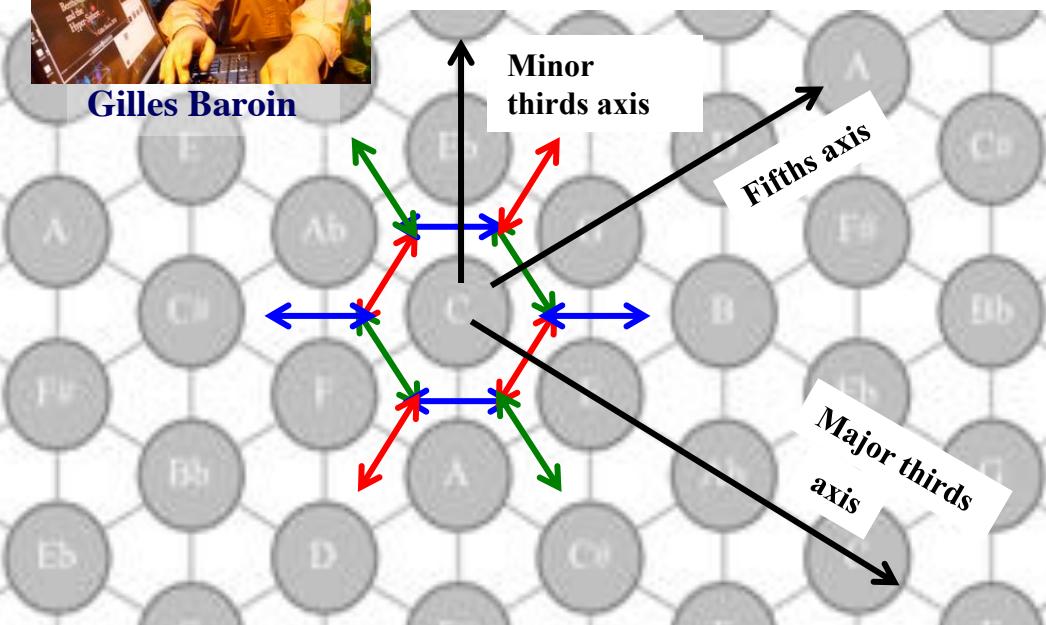
The Tonnetz (or hexagonal tiling honeycomb)



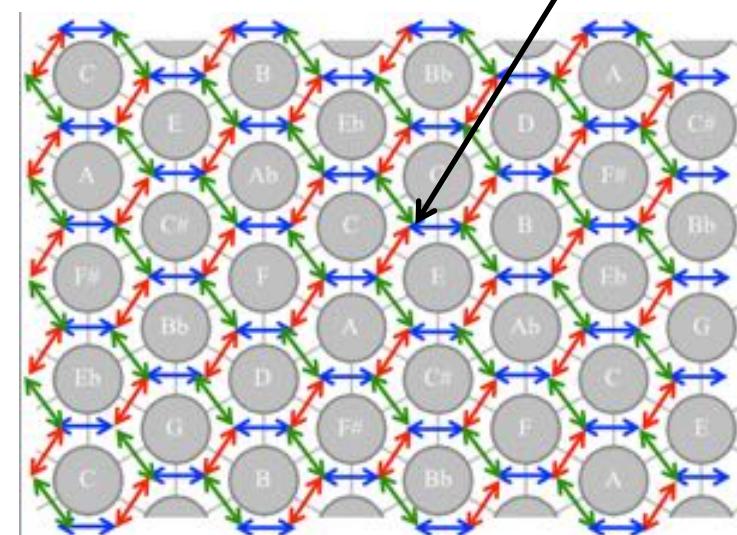
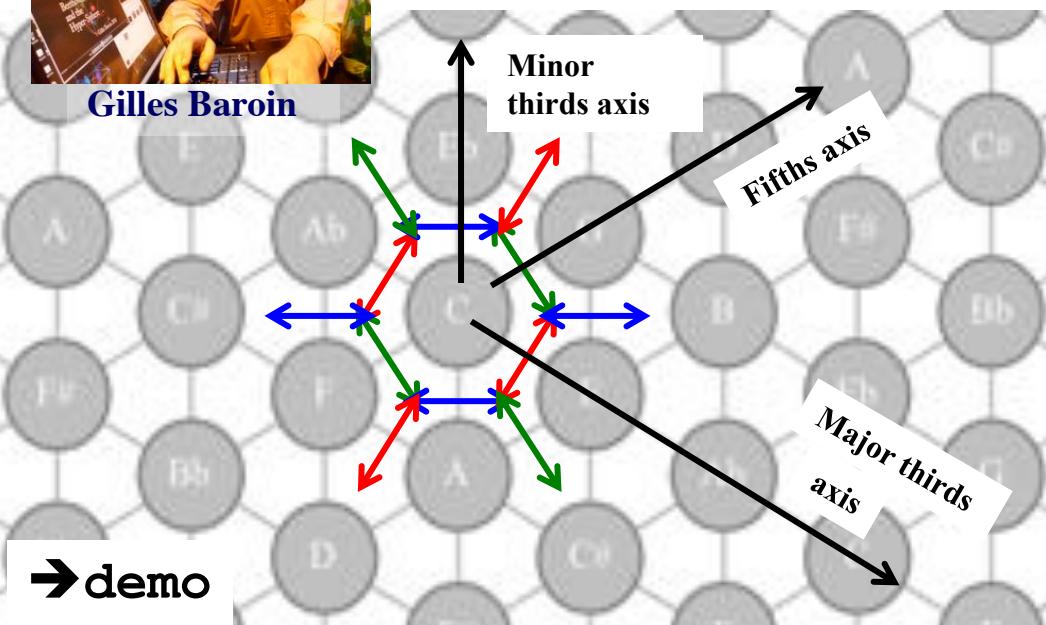
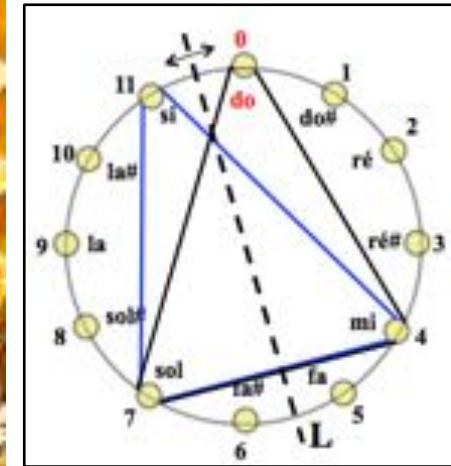
The Tonnetz (or hexagonal tiling honeycomb)



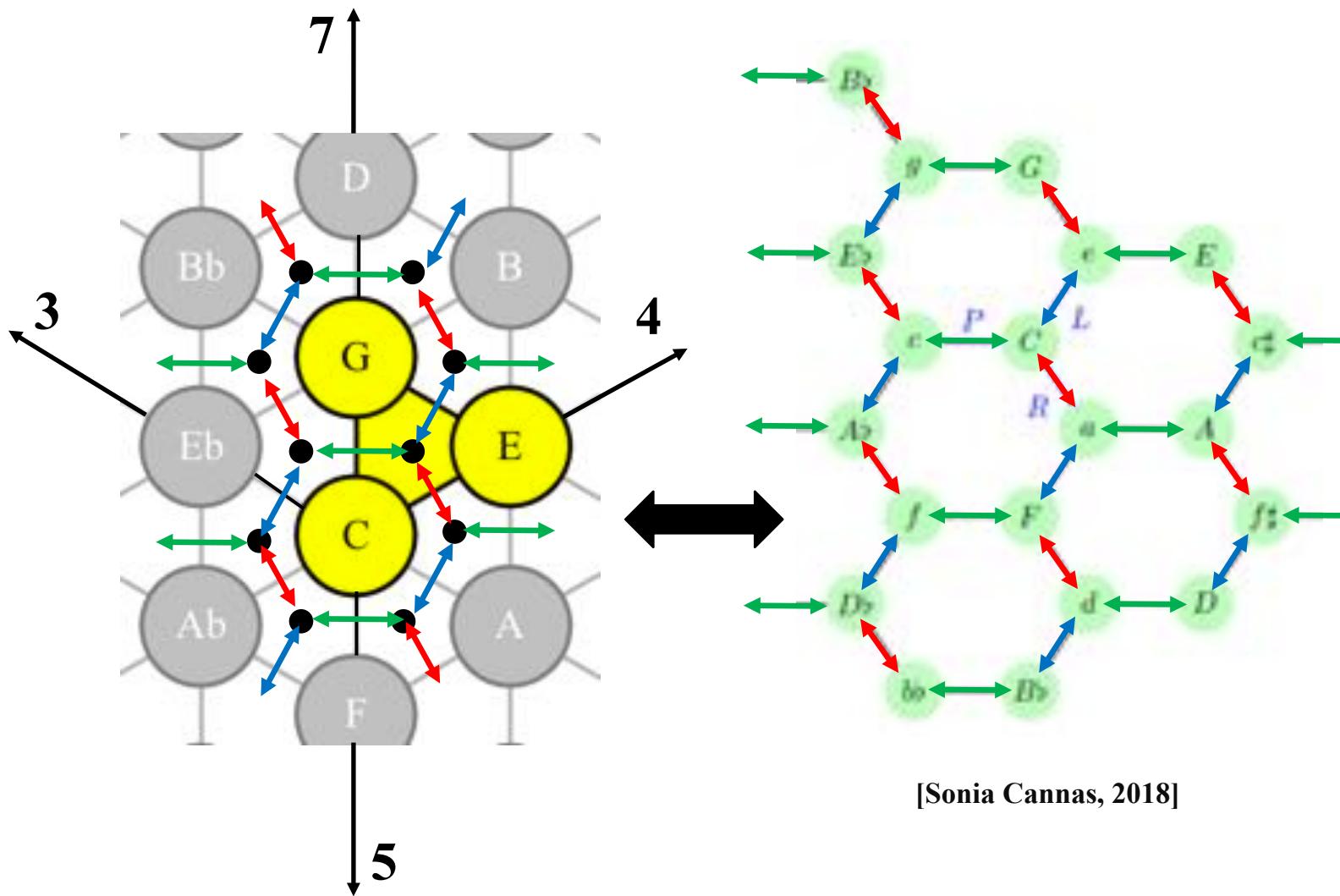
Gilles Baroin



The Tonnetz (or hexagonal tiling honeycomb)

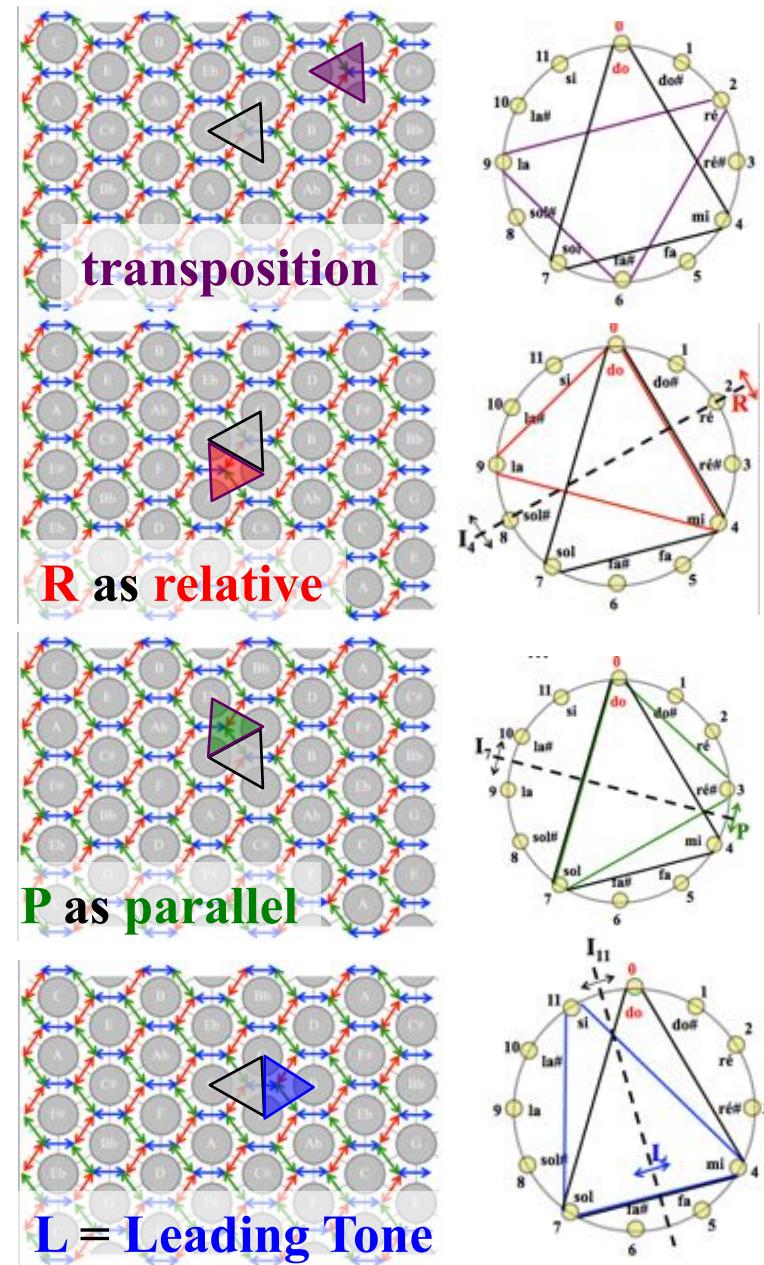
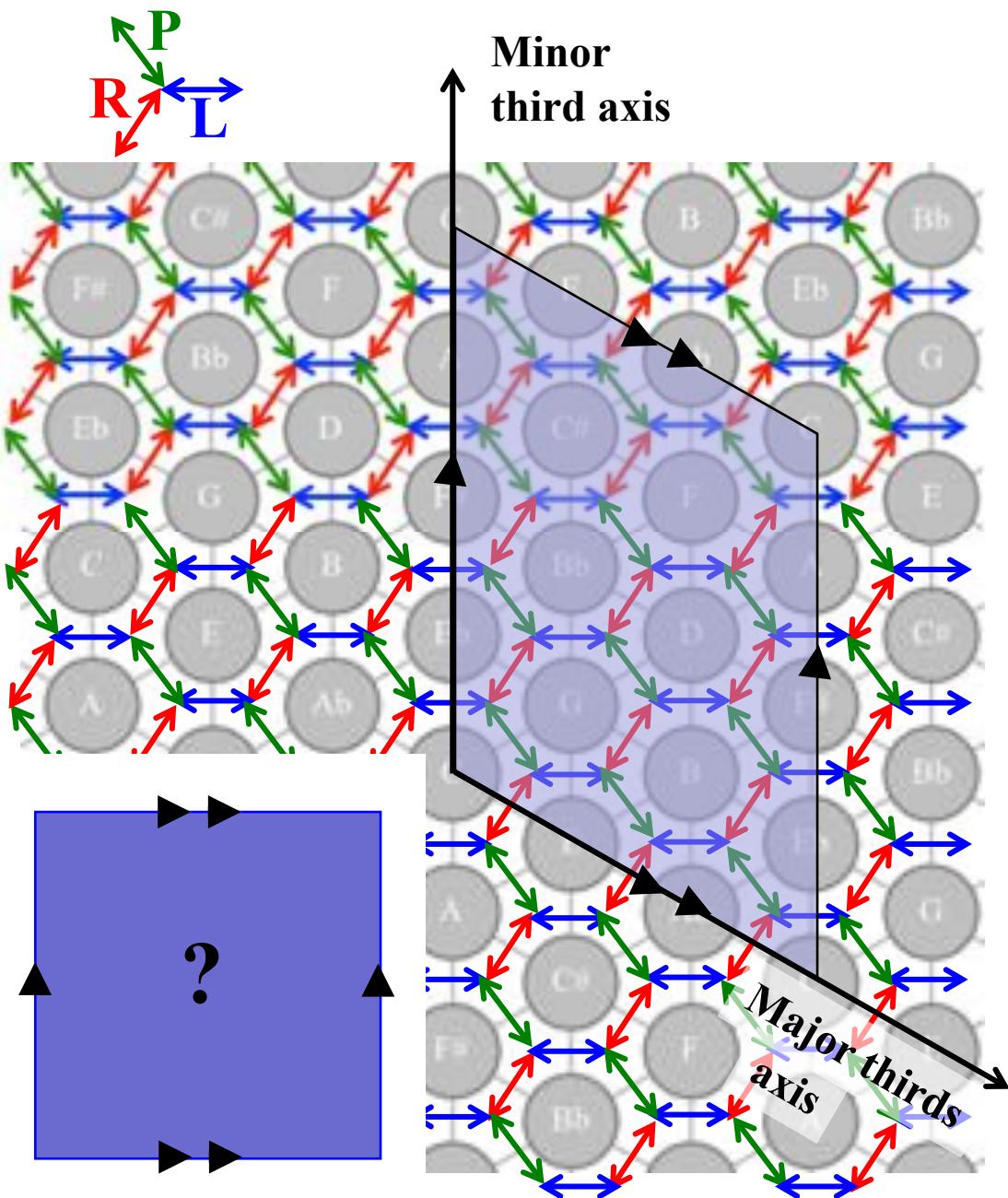


From the Tonnetz to the dual one (and vice-versa)

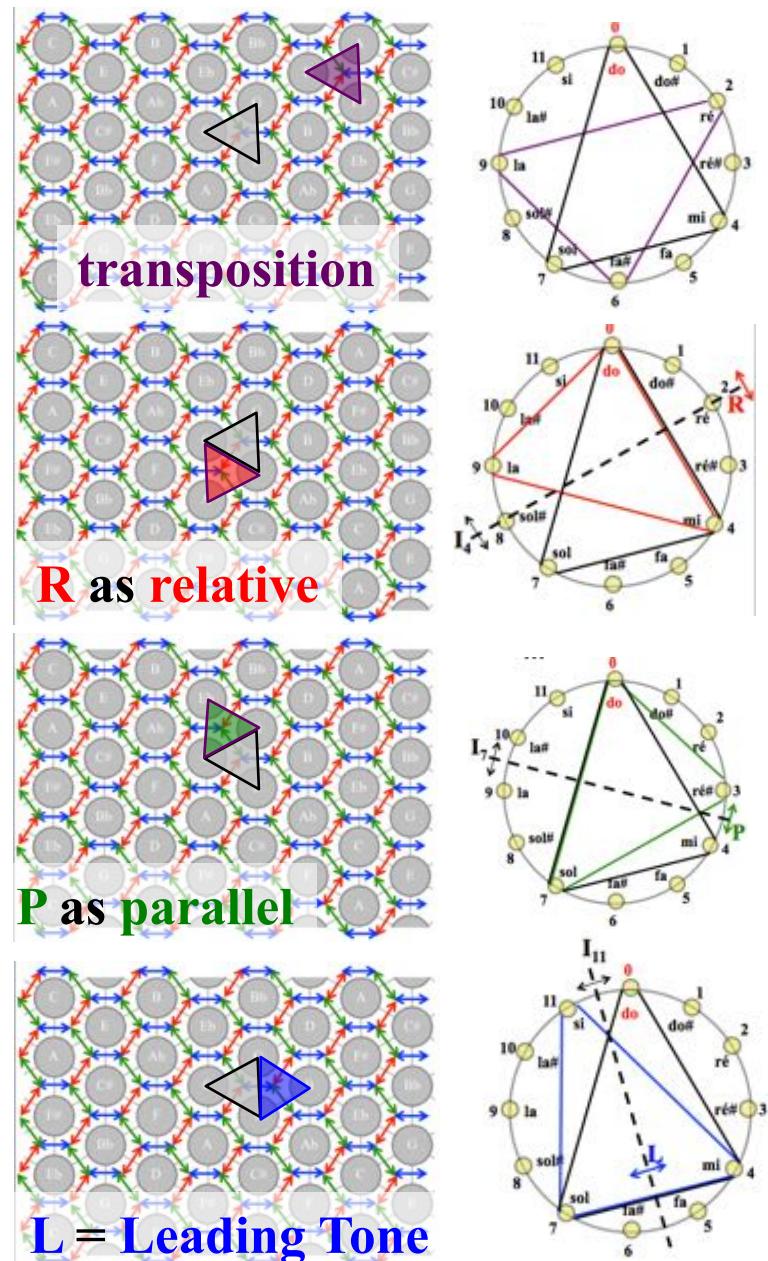
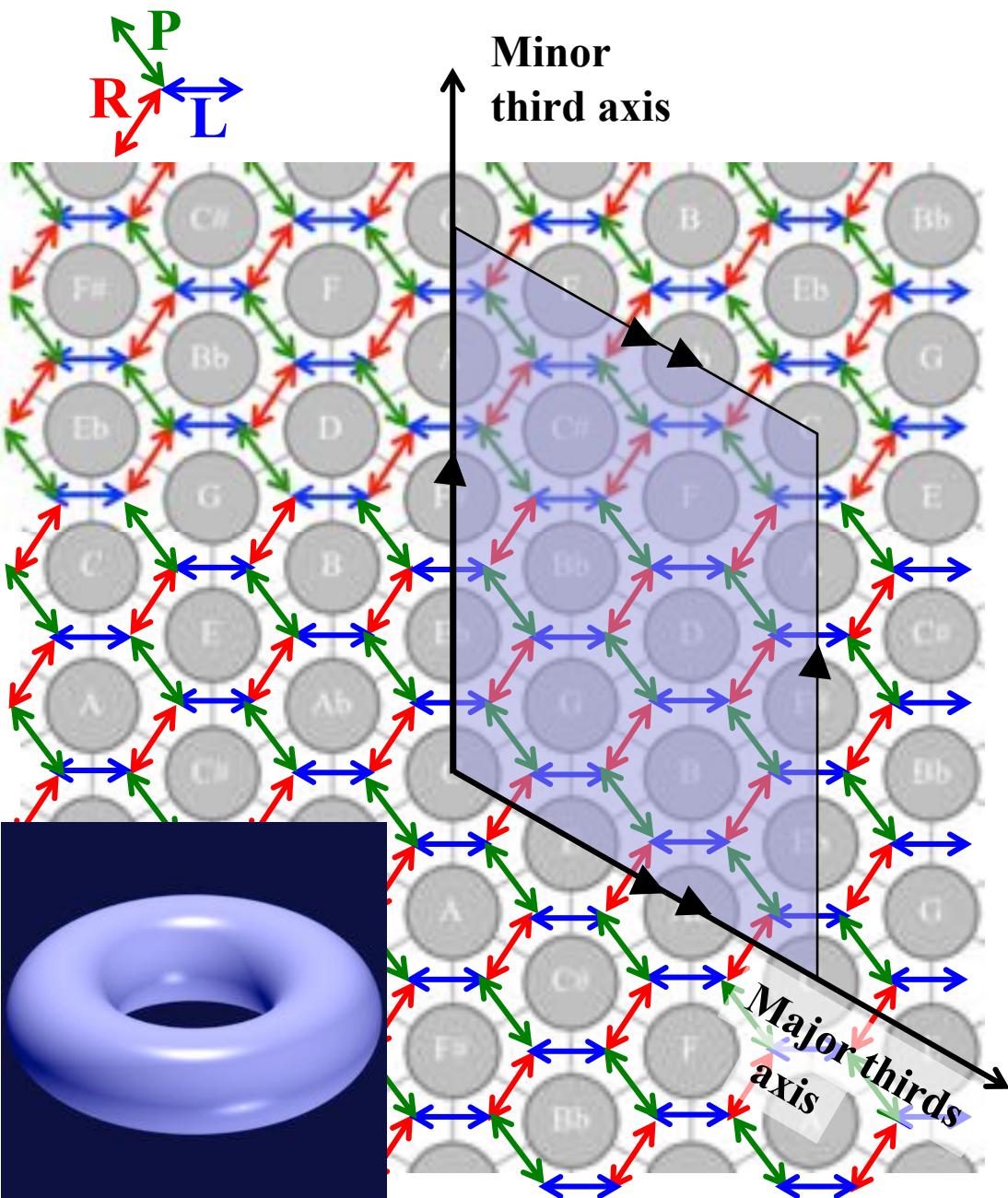


[Sonia Cannas, 2018]

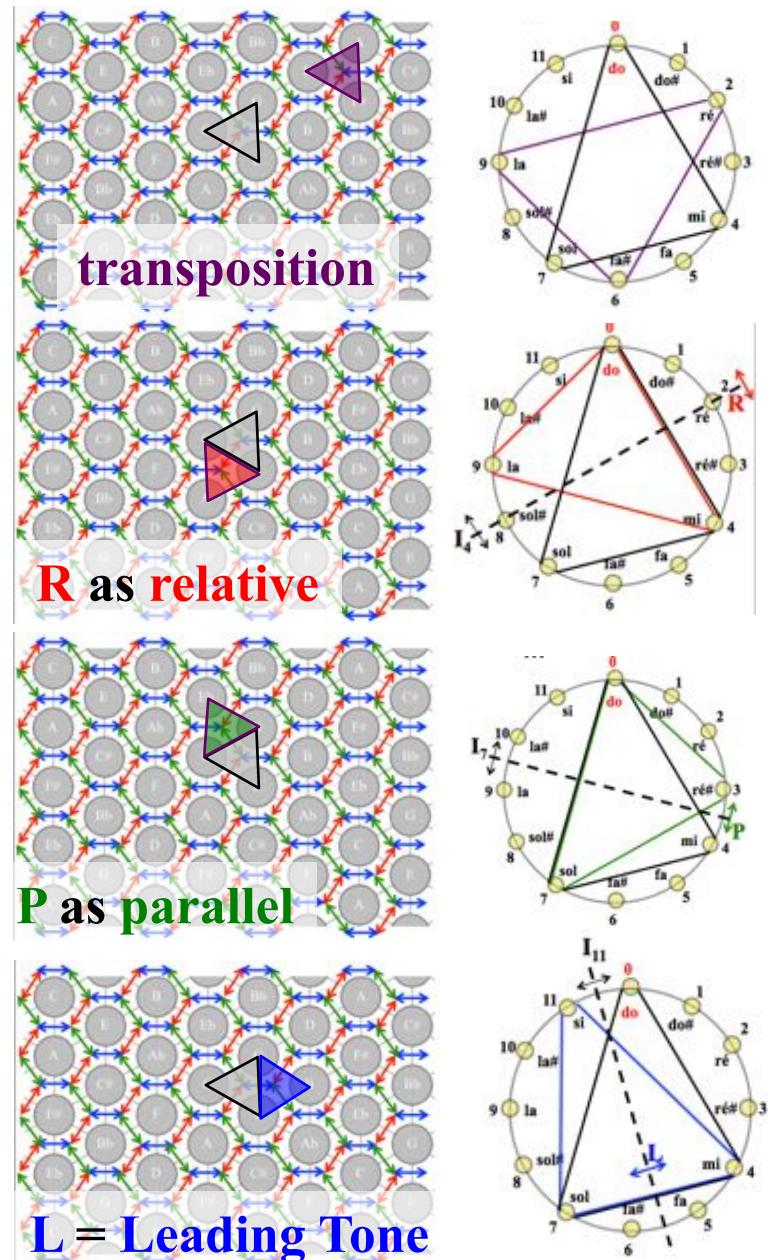
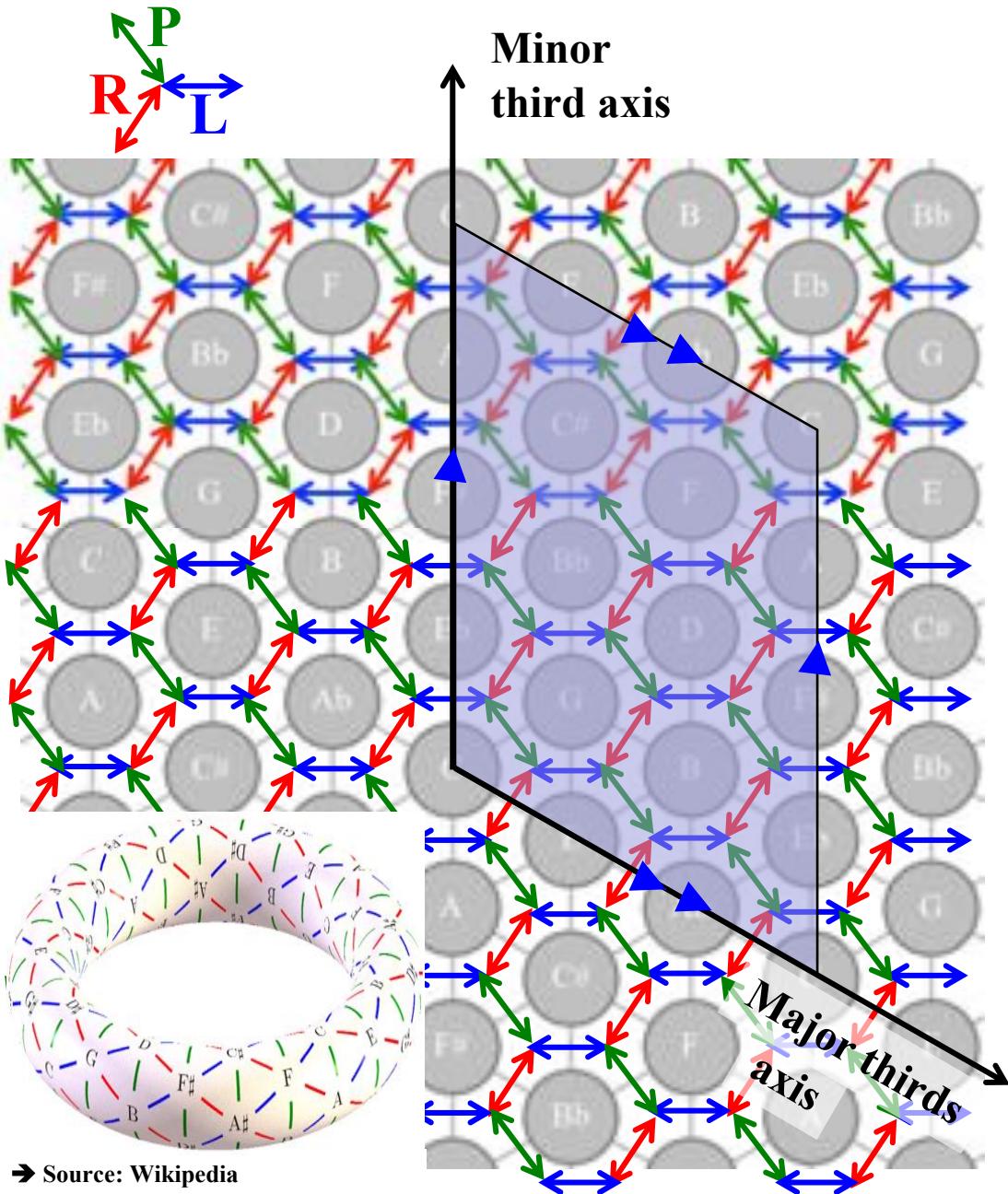
The Tonnetz, its symmetries and its topological structure



The Tonnetz, its symmetries and its topological structure



The Tonnetz, its symmetries and its topological structure



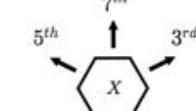
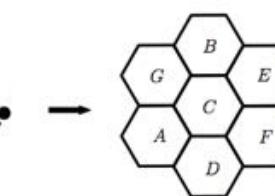
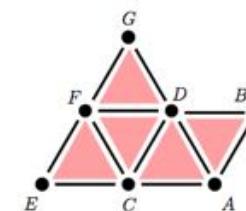
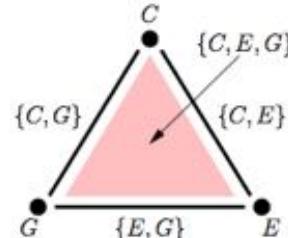
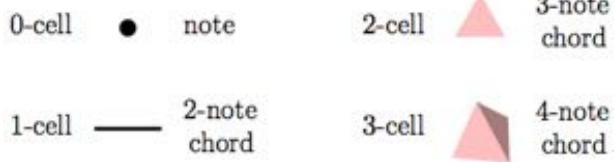
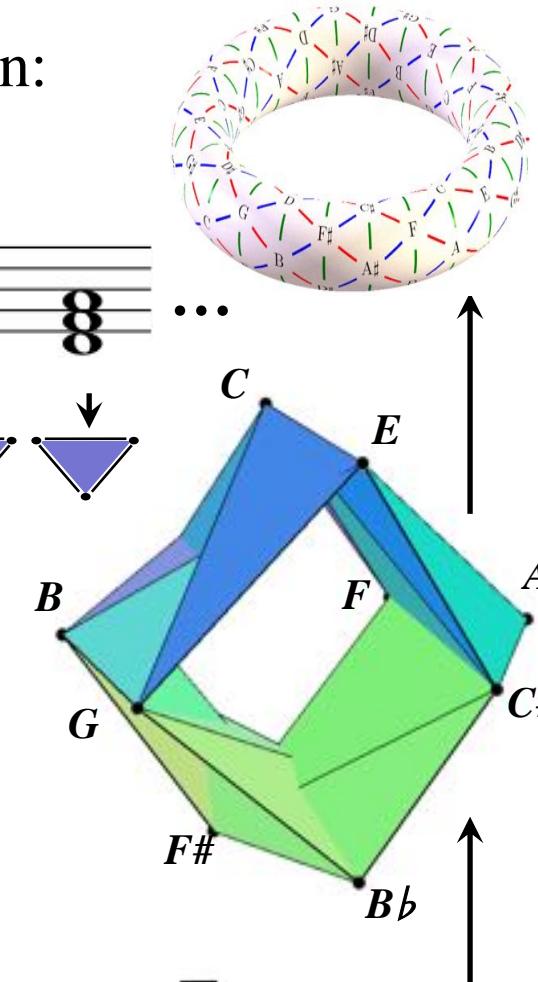
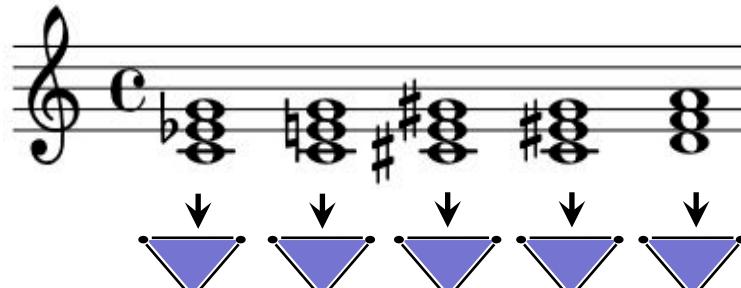
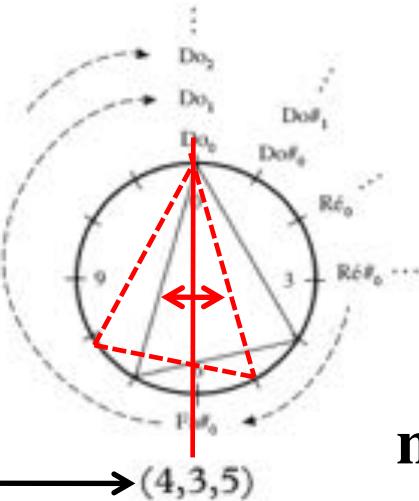


The Tonnetz as a simplicial complex

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

Louis Bigo

- Assembling chords related by some equivalence relation
 - Equivalence up to transposition/inversion:

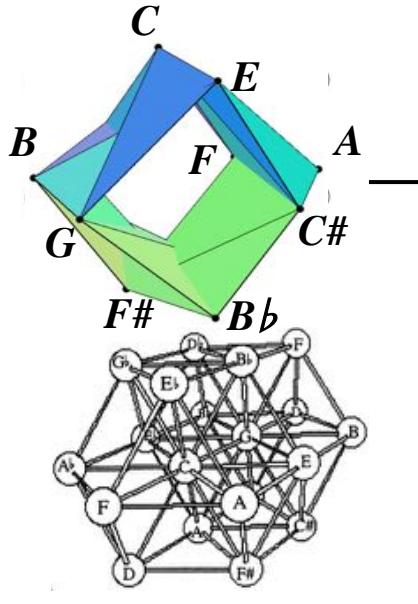


Classifying Chord Complexes

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

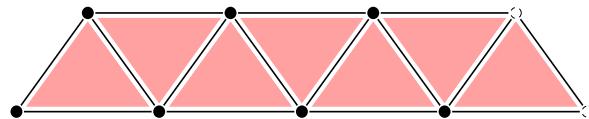
- Complexes enumeration in the chromatic system

$K_{TI}[3,4,5]$
[Cohn – 1997]



$K_{TI}[2,3,3,4]$
[Gollin - 1998]

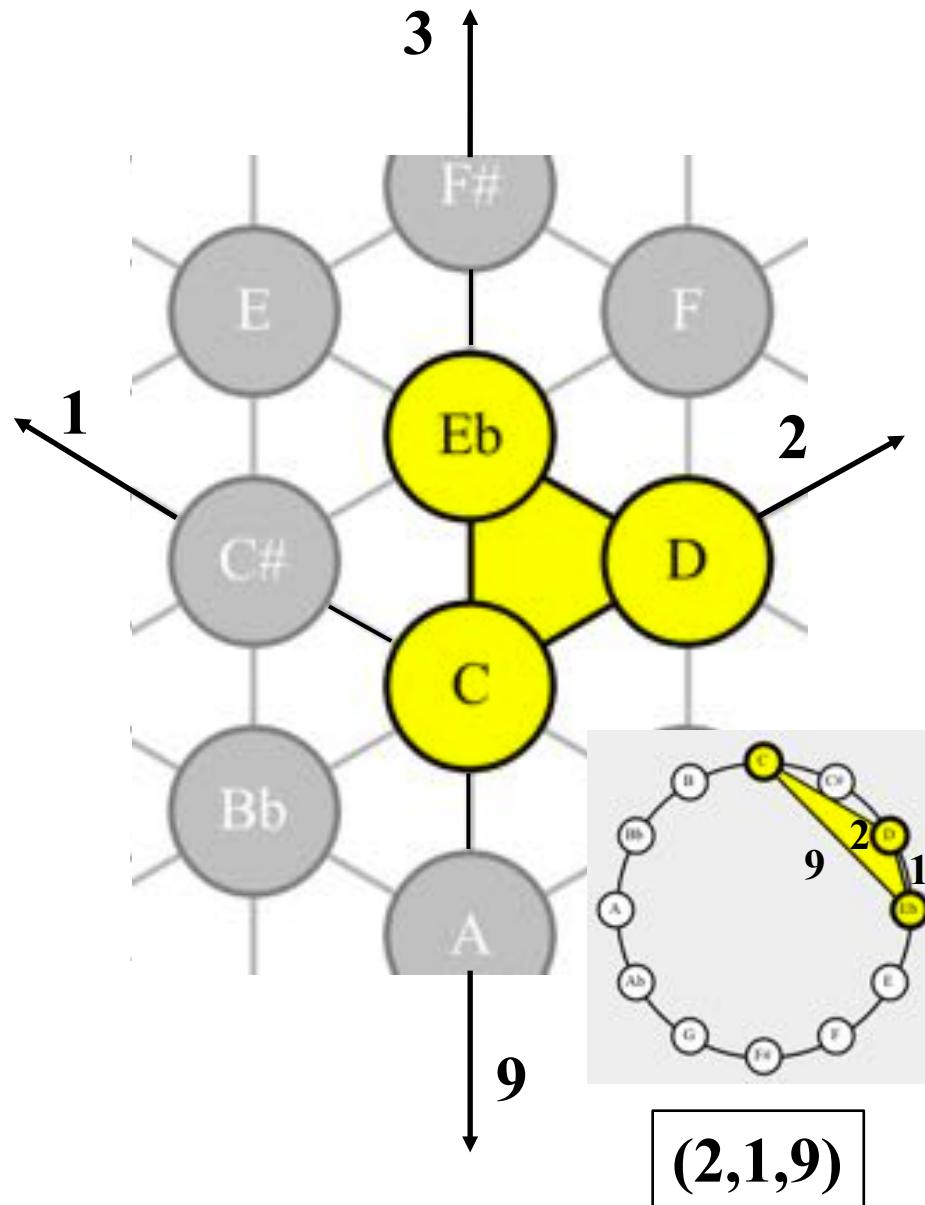
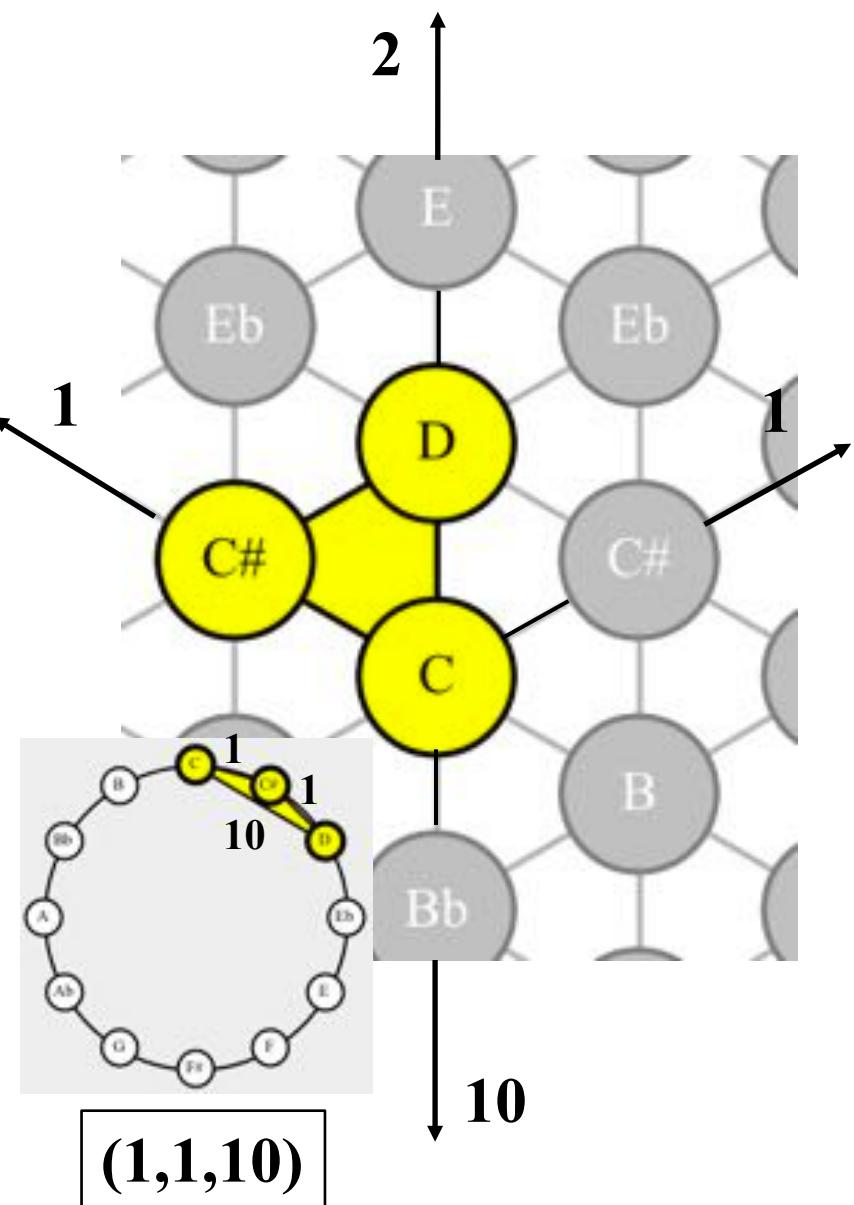
$K_T[2,2,3]$
[Mazzola – 2002]



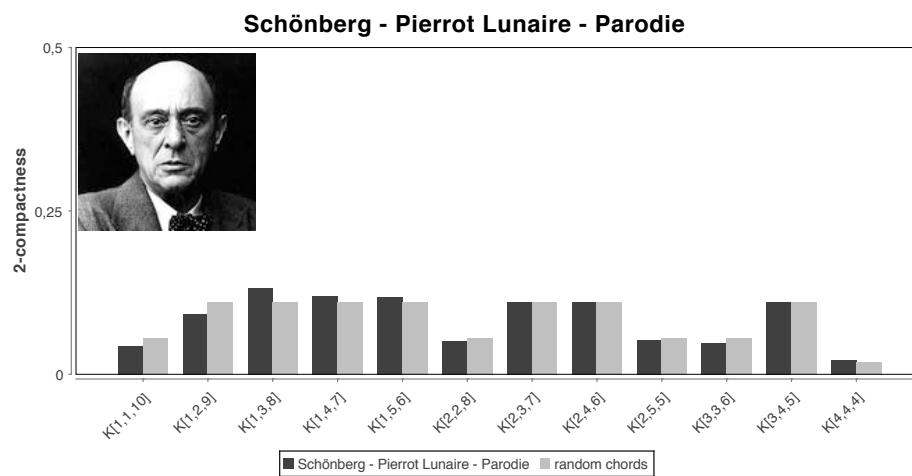
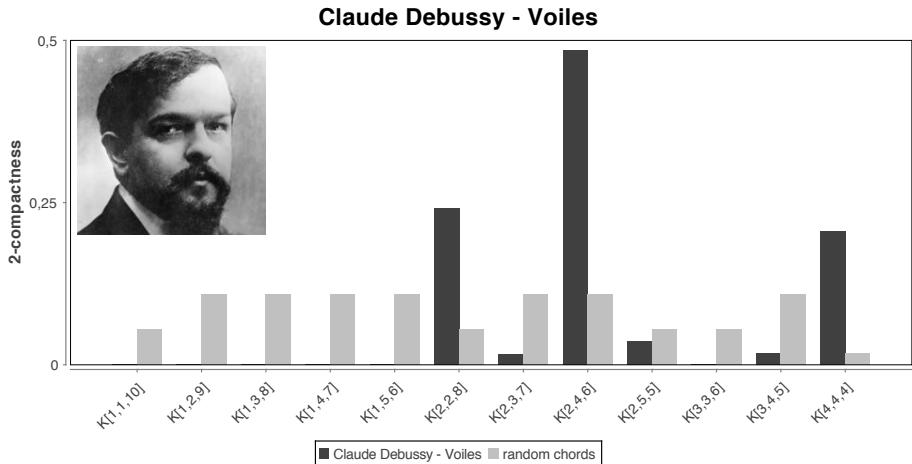
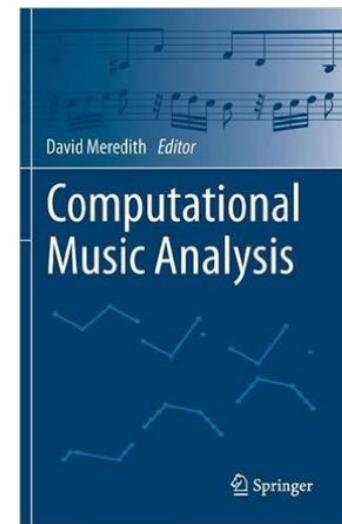
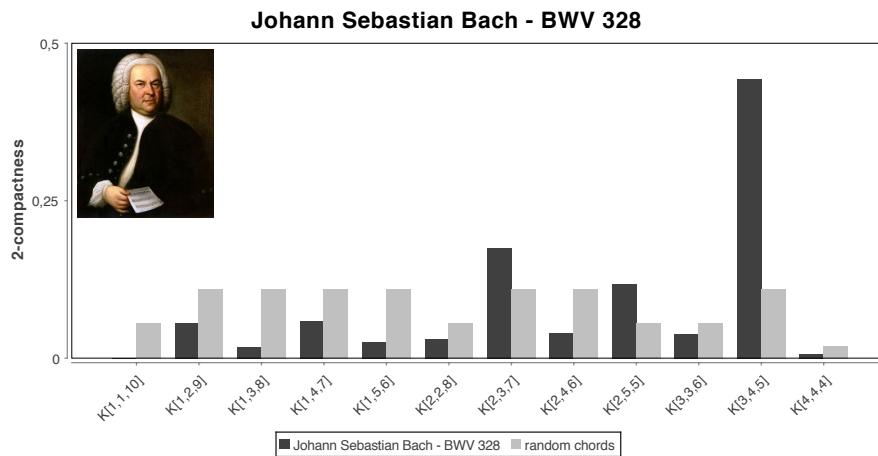
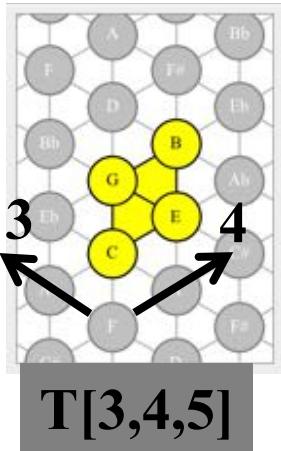
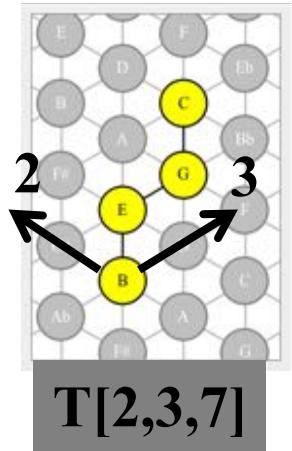
...

d	complexe	taille	b_n	p-v	χ
-	K_\emptyset	0	[0]		0
0	$K_{TI}[0]$	0	[0]		0
1	$K_{TI}[1, 11]$	12	[1, 1]	x	0
	$K_{TI}[2, 10]$	12	[2, 2]		0
	$K_{TI}[3, 9]$	12	[3, 3]		0
	$K_{TI}[4, 8]$	12	[4, 4]		0
	$K_{TI}[5, 7]$	12	[1, 1]	x	0
	$K_{TI}[6, 6]$	6	[6, 0]		6
2	$K_{TI}[1, 1, 10]$	12	[1, 1, 0]	x	0
	$K_{TI}[1, 2, 9]$	24	[1, 2, 1]	x	0
	$K_{TI}[1, 3, 8]$	24	[1, 2, 1]	x	0
	$K_{TI}[1, 4, 7]$	24	[1, 2, 1]	x	0
	$K_{TI}[1, 5, 6]$	24	[1, 1, 6]		6
	$K_{TI}[2, 2, 8]$	12	[2, 2, 0]		0
	$K_{TI}[2, 3, 7]$	24	[1, 2, 1]	x	0
	$K_{TI}[2, 4, 6]$	24	[2, 2, 6]		6
	$K_{TI}[2, 5, 5]$	12	[1, 1, 0]	x	0
	$K_{TI}[3, 3, 6]$	12	[3, 0, 3]		6
	$K_{TI}[3, 4, 5]$	24	[1, 2, 1]	x	0
	$K_{TI}[4, 4, 4]$	4	[4, 0, 0]		4
3	$K_{TI}[1, 1, 1, 9]$	12	[1, 1, 0, 0]	x	0
	$K_{TI}[1, 1, 2, 8]$	24	[1, 1, 12, 0]		12
	$K_{TI}[1, 1, 3, 7]$	24	[1, 2, 13, 0]		12
	$K_{TI}[1, 1, 4, 6]$	24	[1, 1, 18, 0]		18
	$K_{TI}[1, 1, 5, 5]$	12	[1, 1, 6, 0]		6

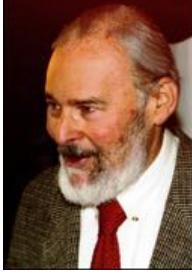
The panoply of *Tonnetze* at the service of the analyst



The geometric character of musical logic



Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015



D. Lewin

Simply
transitive
action

Système d'Intervalles Généralisés - Système Généralisé d'Intervalles David Lewin's *Generalized Interval System* [GMIT, 1987]

$$\text{GIS} = (S, G, \text{int})$$

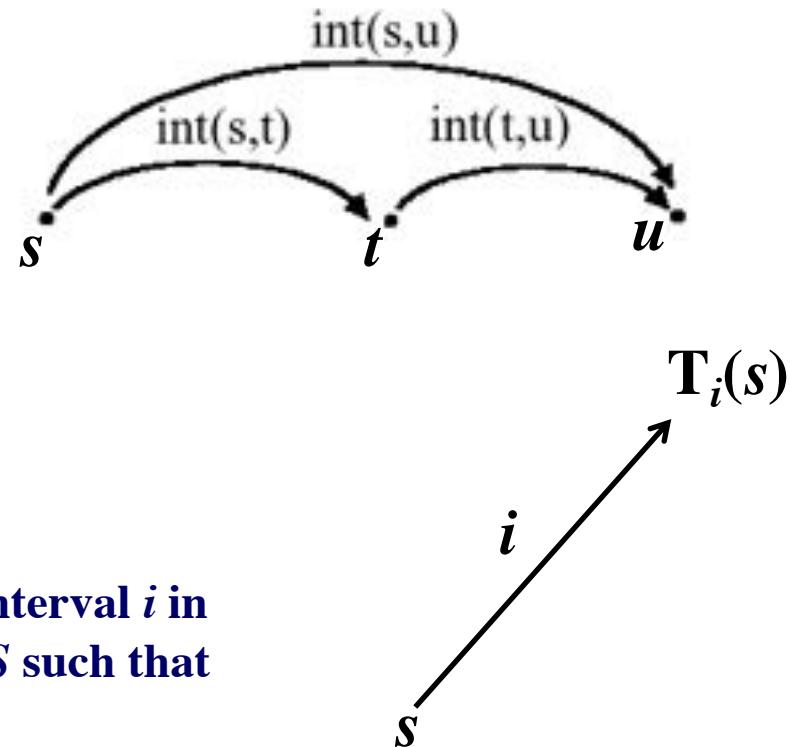
S = set

(G, \bullet) = group of intervals

int = intervallic function

$$S \times S \xrightarrow{\text{int}} G$$

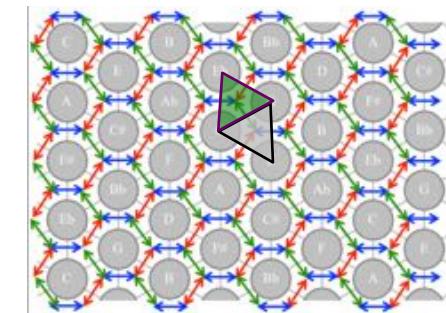
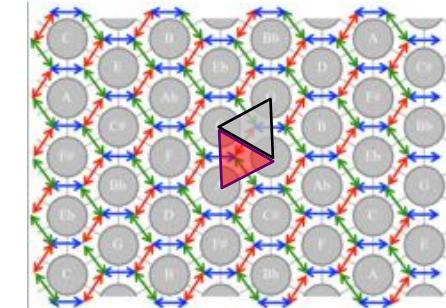
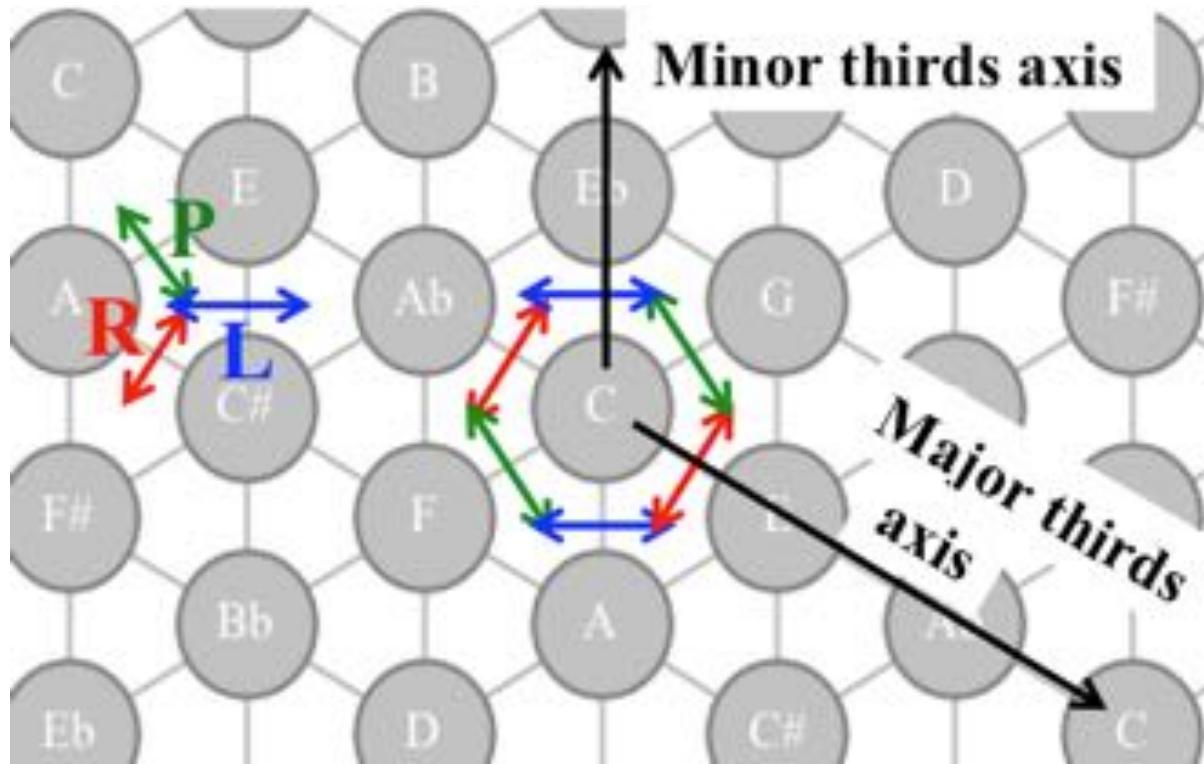
- 1. For all objects s, t, u in S :
 $\text{int}(s, t) \bullet \text{int}(t, u) = \text{int}(s, u)$
- 2. For all object s in S and for all interval i in G there exists a unique object t in S such that
 $\text{int}(s, t) = i$



Let $\tau = \{T_i ; i \in G\}$ be the group of transpositions

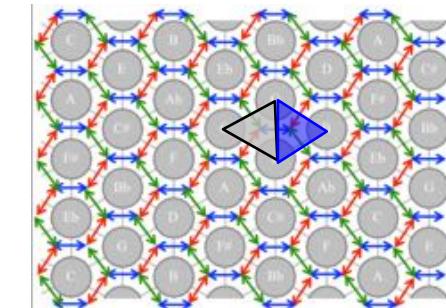
$\text{GIS} = (S, G, \text{int}) \Leftrightarrow \tau \times S \rightarrow S$ such that $(T_i, s) \rightarrow T_i(s)$ where $\text{int}(s, T_i(s)) = i$

The Tonnetz as “Generalized Interval System”

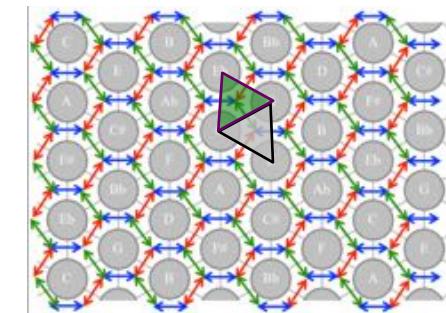
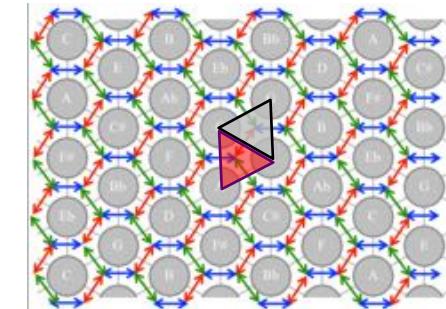
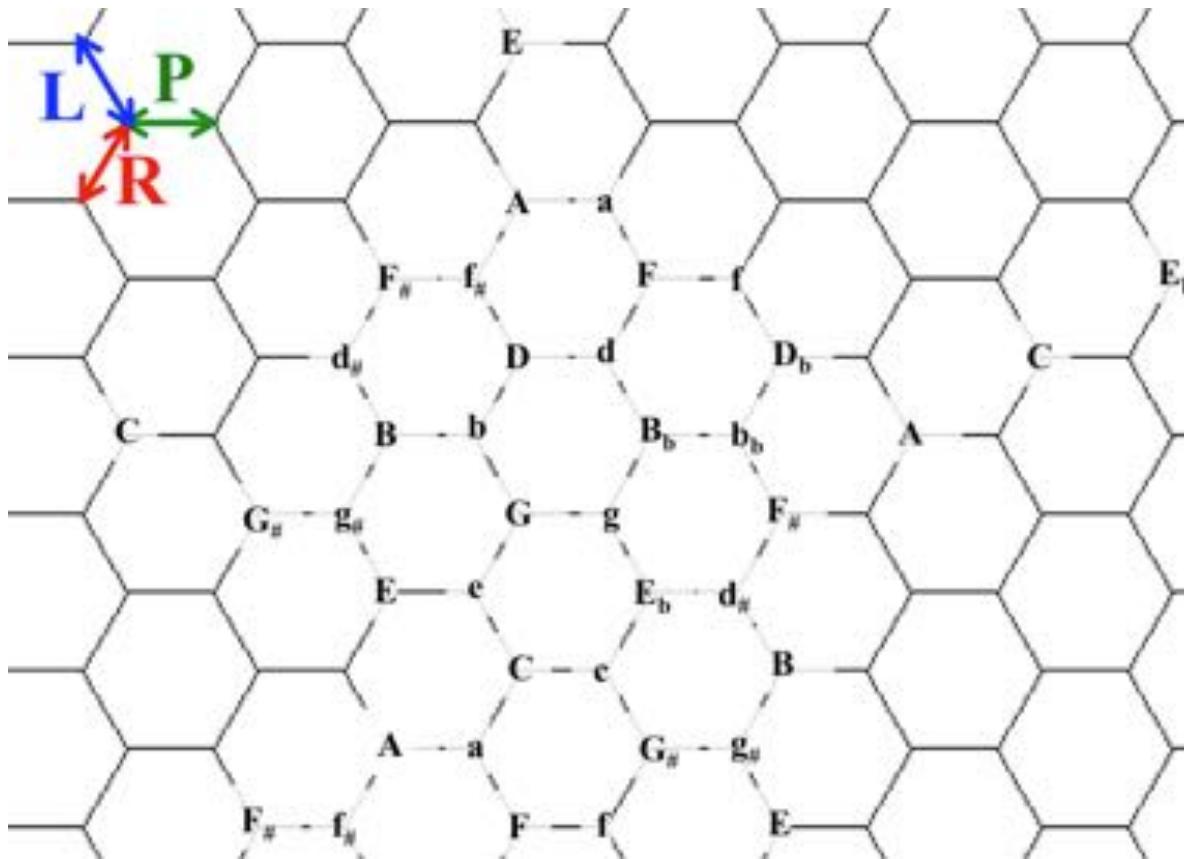


$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 ; LRL = L(LR)^{-1} \rangle$$

ρ acts in a simply transitive way on the set S of the 24 consonant triads

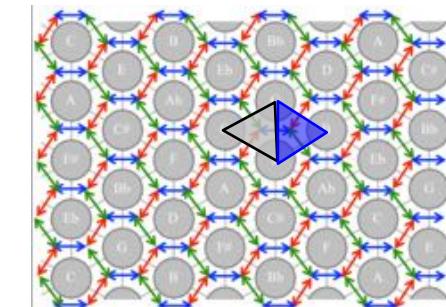


The Tonnetz as “Generalized Interval System”

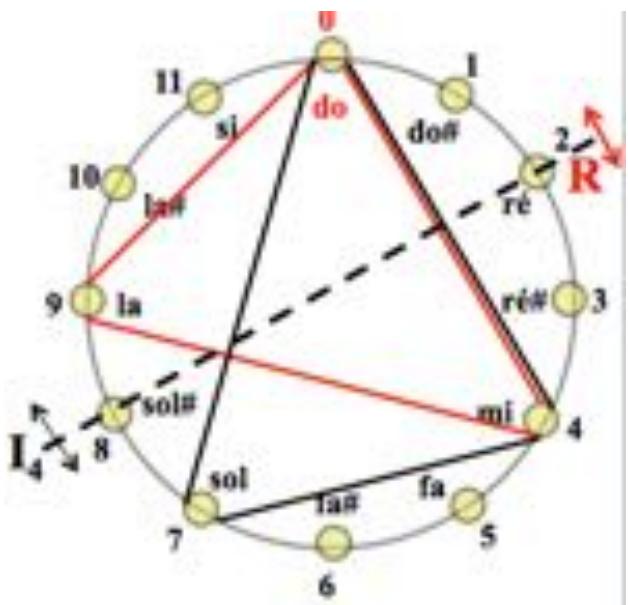


$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 ; LRL = L(LR)^{-1} \rangle$$

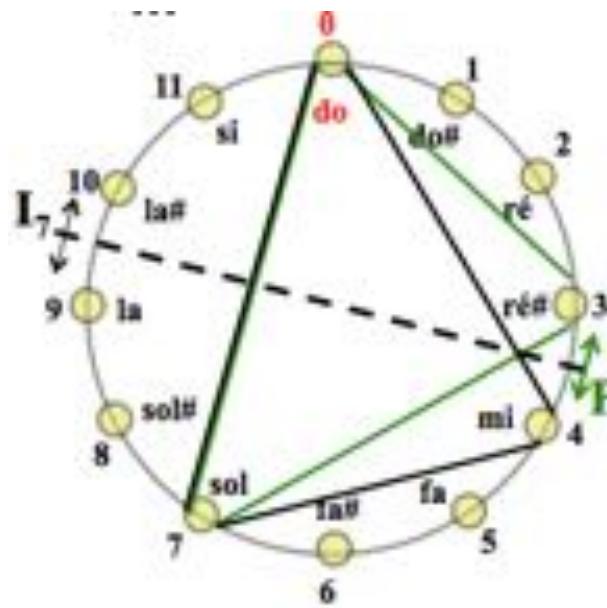
ρ acts in a simply transitive way on the set S of the 24 consonant triads



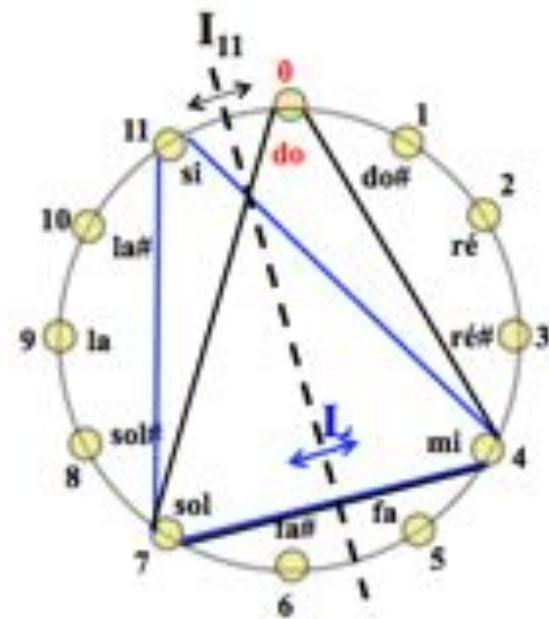
A different GIS structure on the same set S



$$I_4: x \rightarrow 4-x$$



$$I_7: x \rightarrow 7-x$$



$$I_{11}: x \rightarrow 11-x$$

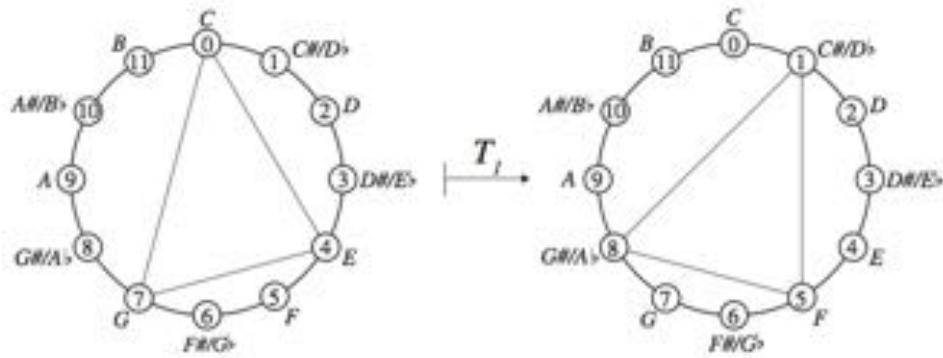
$$D_{12} = \langle I, T \mid I^2 = T^{12} = 1 ; ITI = I(TI)^{-1} \rangle$$

D_{12} acts in a simply transitive way on the set S of the 24 consonant triads

Two “dual” actions on the set of consonant triads

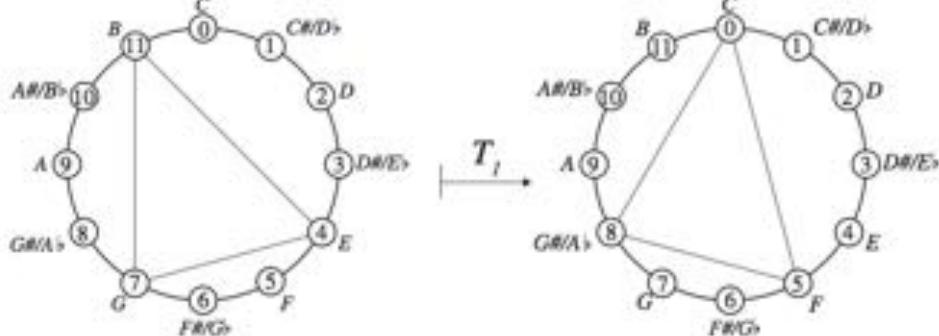
$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 ; LRL = L(LR)^{-1} \rangle$$

$$\leftrightarrow D_{12} = \langle I, T \mid I^2 = T^{12} = 1 ; ITI = I(IT)^{-1} \rangle$$

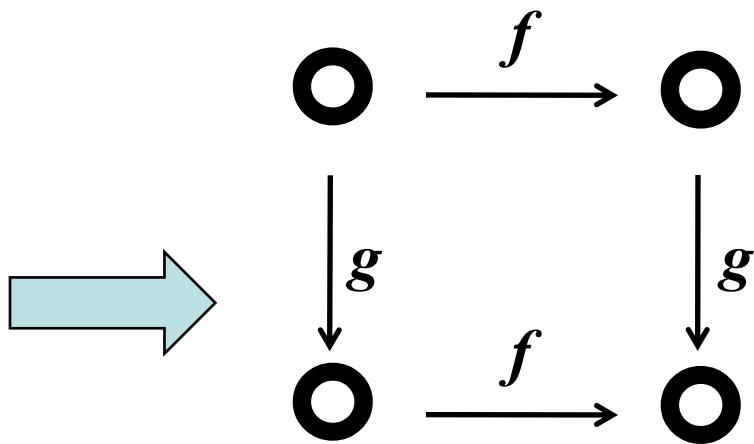


T_I

L



T_I

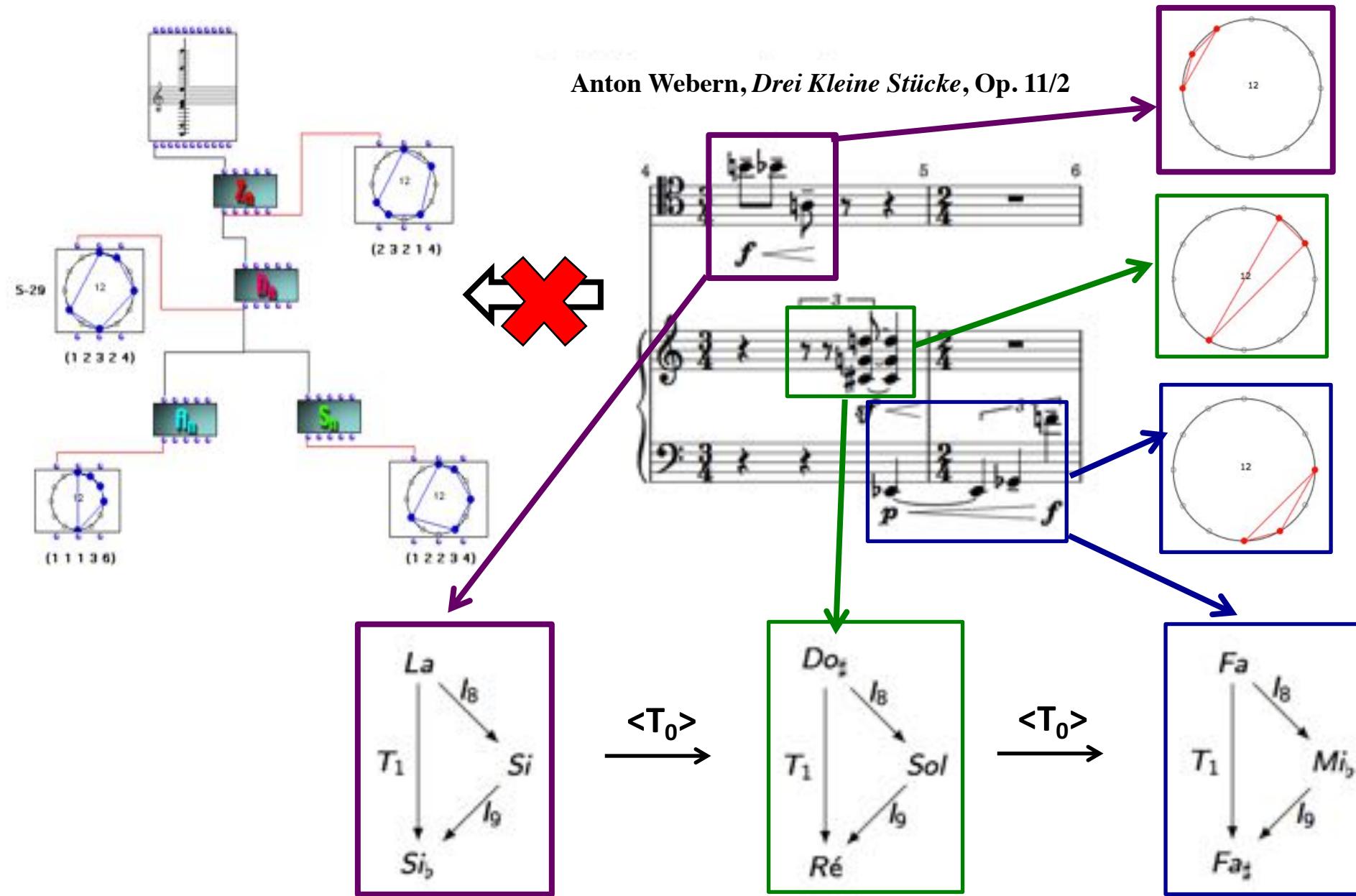


Every diagram commutes

$$\forall f \in D_{12}$$

$$\forall g \in \rho$$

K-Nets and the paradigmatic approach



K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

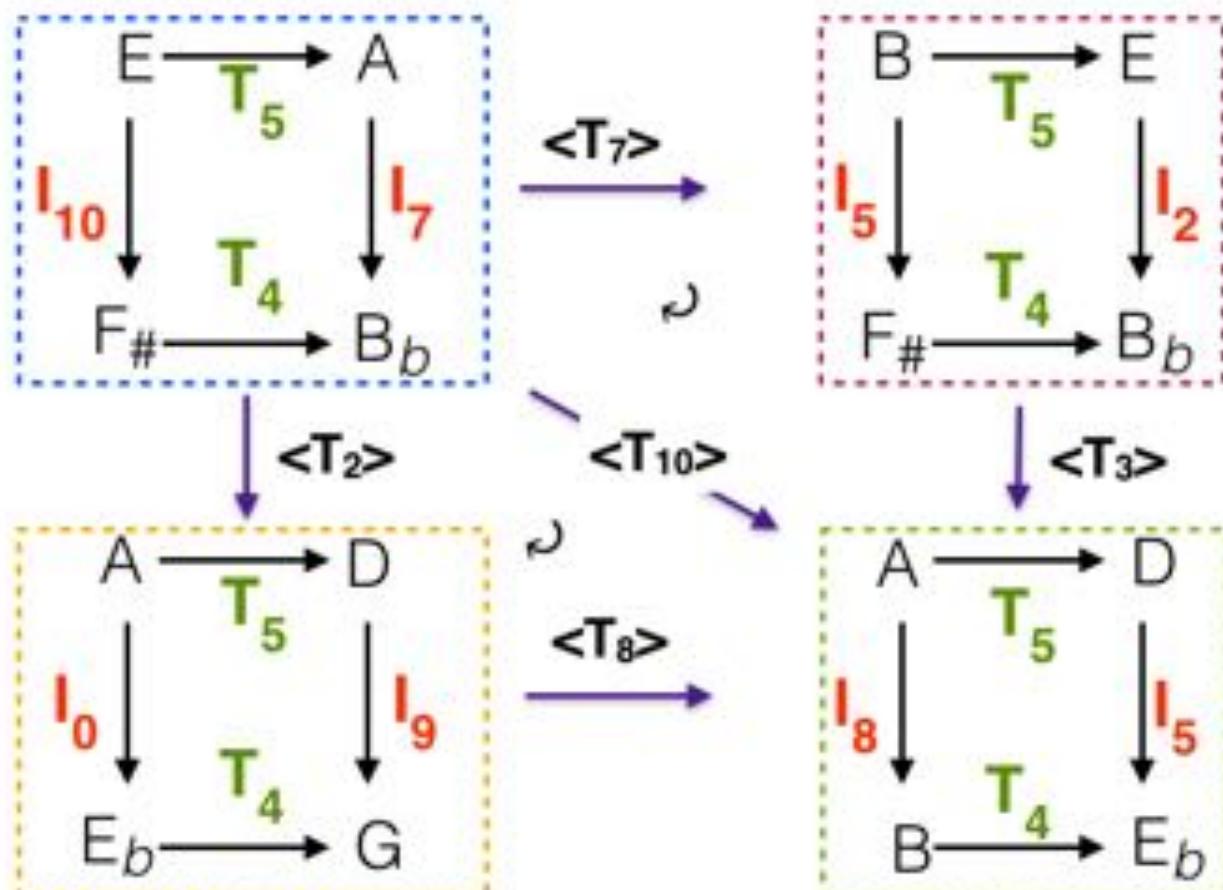


D. Lewin

H. Klumpenhouwer

A musical score excerpt on two staves. The top staff shows a melody with various notes and rests. The bottom staff shows harmonic changes indicated by Roman numerals (I, II, III, IV, V) and sharps or flats. Colored dashed boxes highlight specific notes: blue for E, green for A, yellow for D, and orange for G. These notes are part of a K-net diagram below.

$$\langle T_k \rangle : T_m \rightarrow T_m \\ I_m \rightarrow I_{k+m}$$



K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994



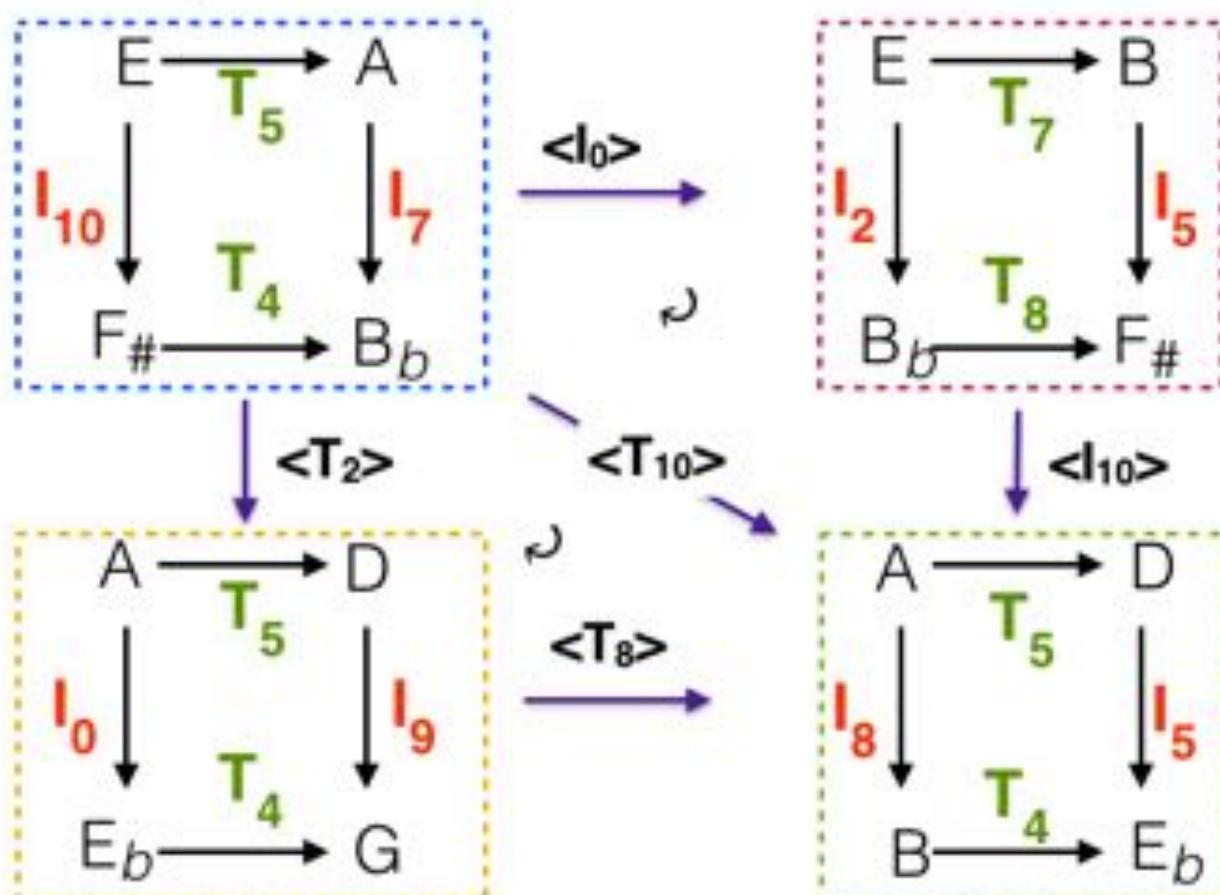
D. Lewin

H. Klumpenhouwer



$$\langle T_k \rangle : T_m \rightarrow T_m \\ I_m \rightarrow I_{k+m}$$

$$\langle I_k \rangle : T_m \rightarrow T_{-m} \\ I_m \rightarrow I_{k-m}$$



K-nets as a transformational construction

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

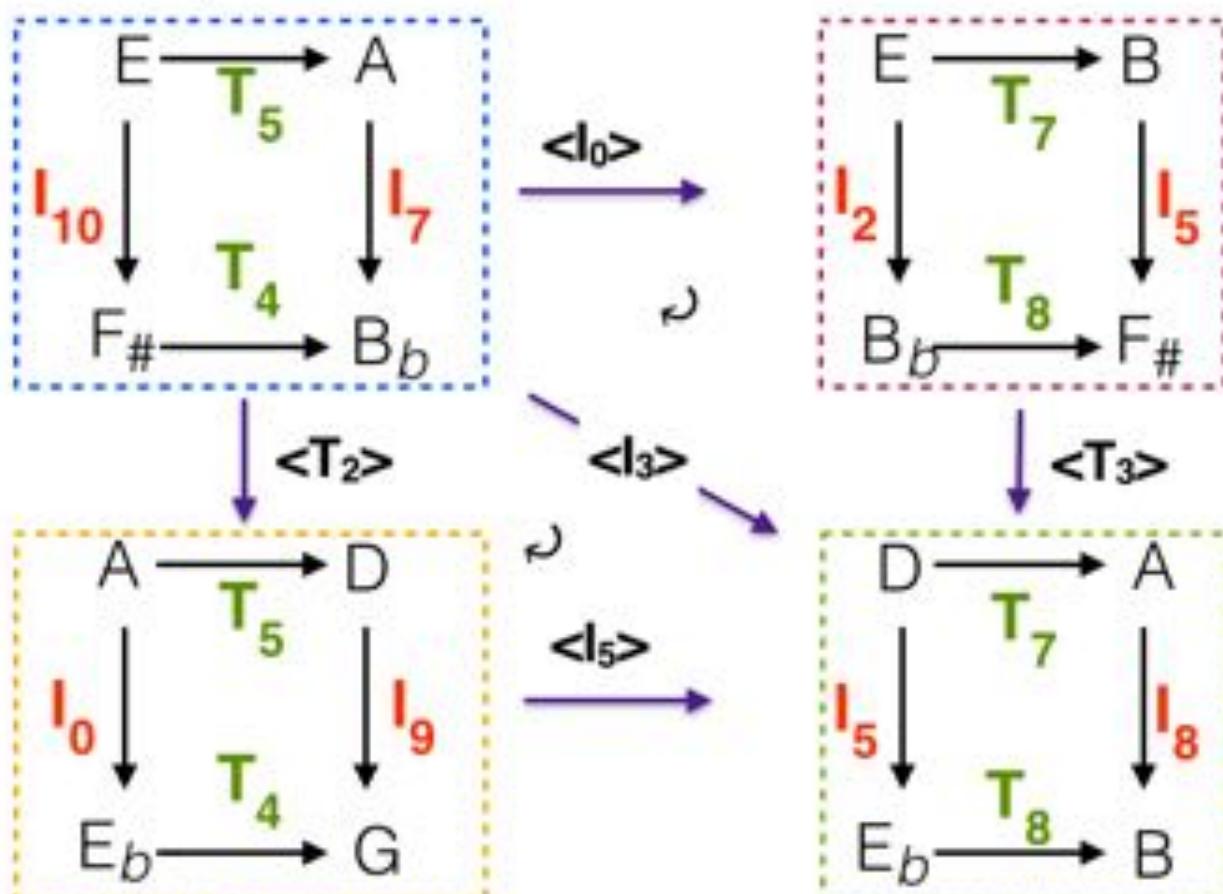


D. Lewin

H. Klumpenhouwer

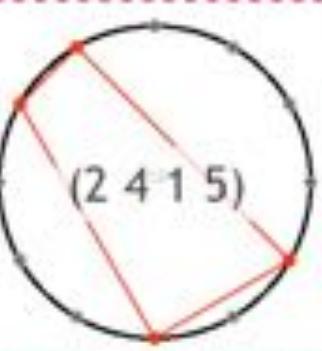
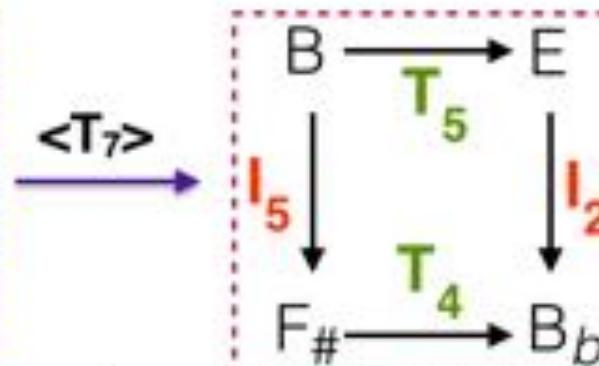
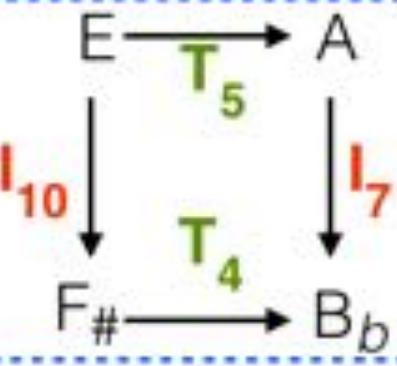
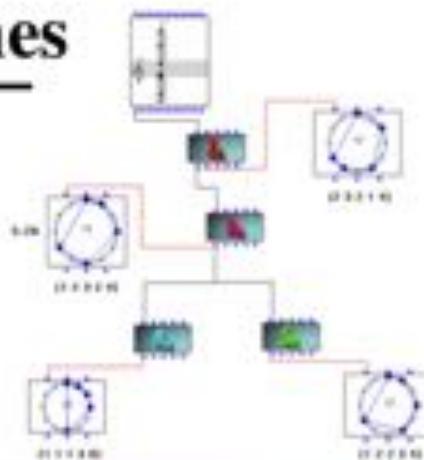
$$\langle T_k \rangle : T_m \rightarrow T_m \\ I_m \rightarrow I_{k+m}$$

$$\langle I_k \rangle : T_m \rightarrow T_{-m} \\ I_m \rightarrow I_{k-m}$$



Transformational vs set-theoretical approaches

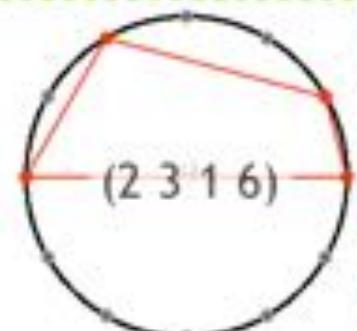
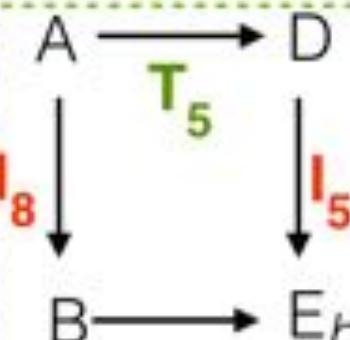
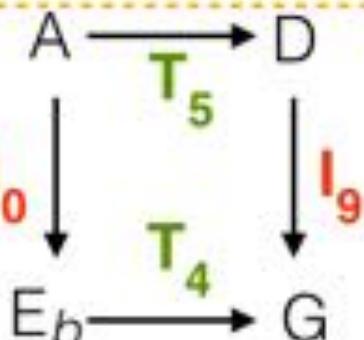
D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 ", JMT, 1994



$\downarrow < T_2 >$

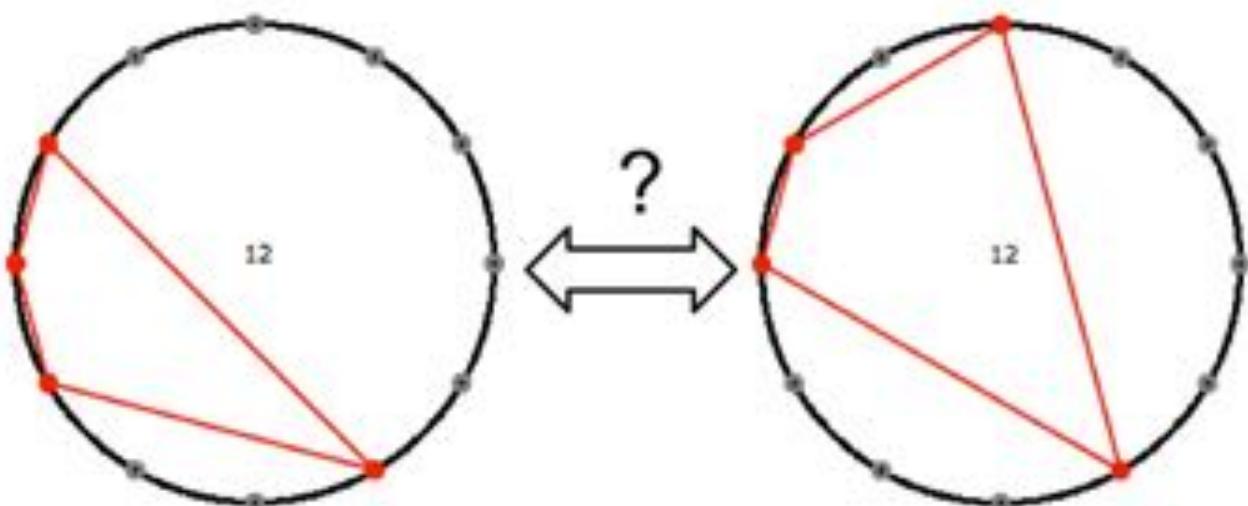
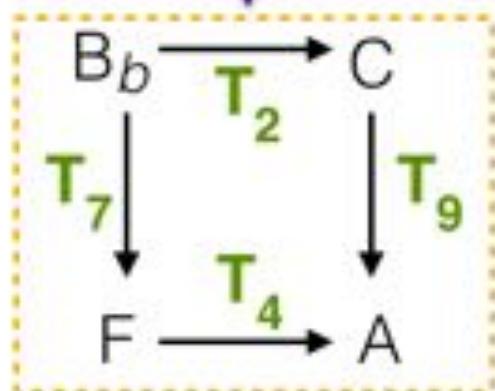
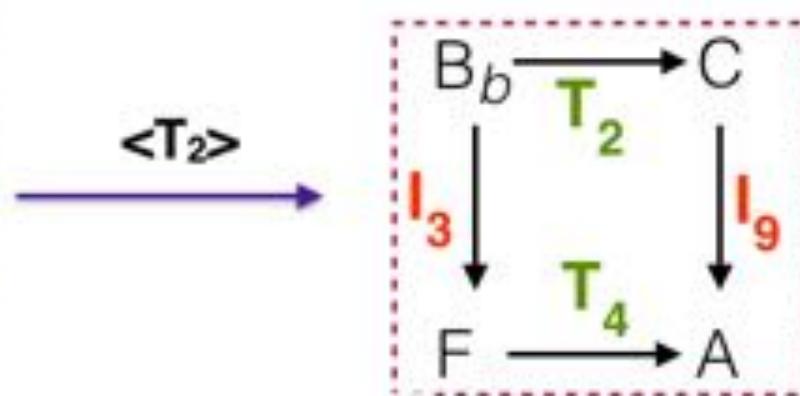
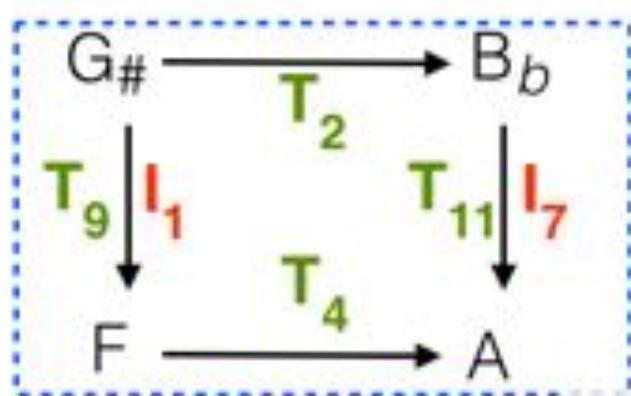
\curvearrowleft

$\downarrow < T_3 >$



$\downarrow < T_8 >$

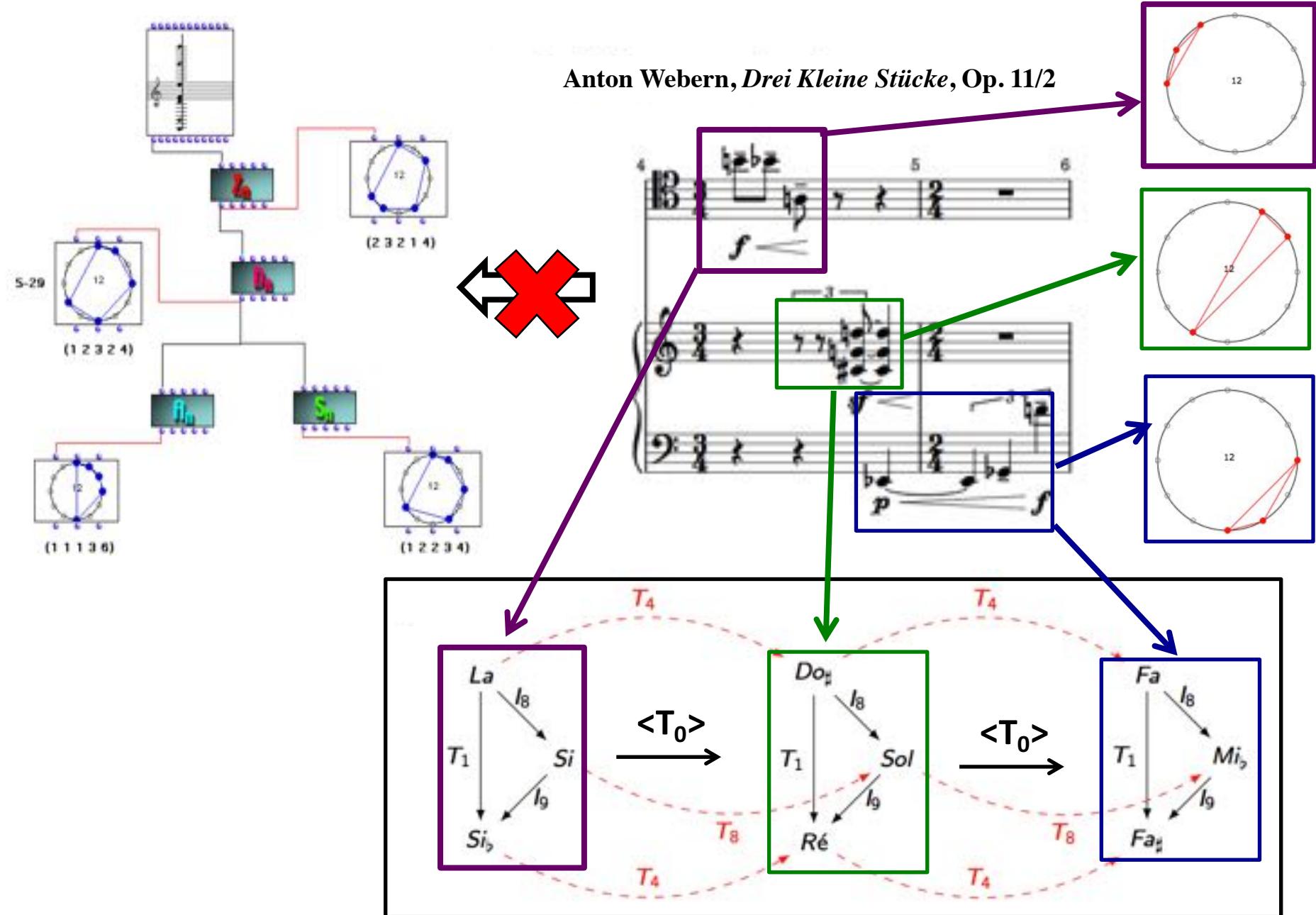
Some theoretical difficulties with the isographic relations



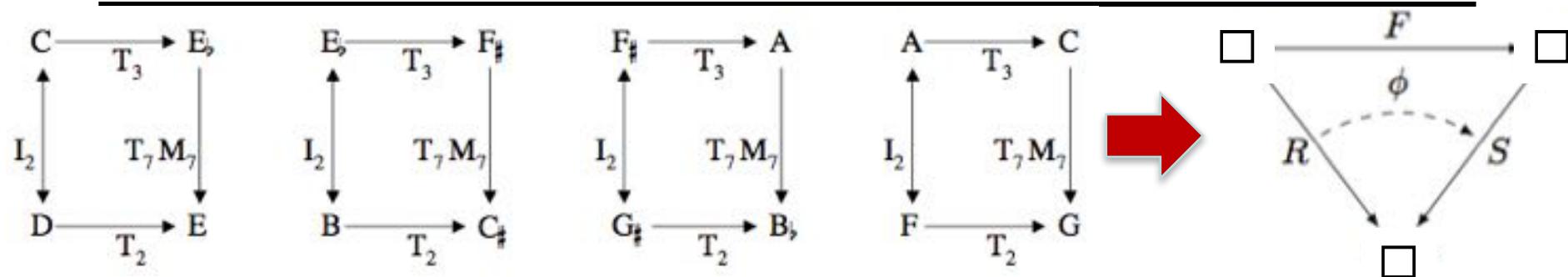
CONCLUSION

There are K-Nets which are not always isographic to a given one, i.e. the isographic relations are highly sensitive to the transformations used to label the arrows.
Is it possible to overstep this theoretical limitation? Which new definition of K-nets allows one to do that?

The categorical vs paradigmatic action-based approach



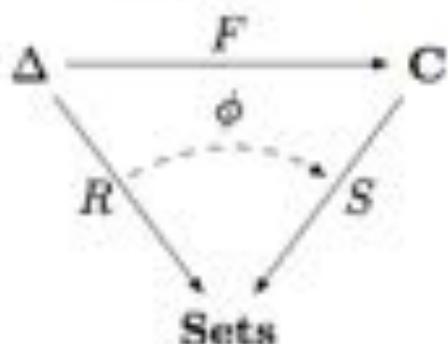
From K-Nets to category-based PK-Nets



Definition 1 Let \mathbf{C} be a category, and S a functor from \mathbf{C} to the category Sets. Let Δ be a small category and R a functor from Δ to Sets. A PK-net of form R and of support S is a 4-tuple (R, S, F, ϕ) , in which

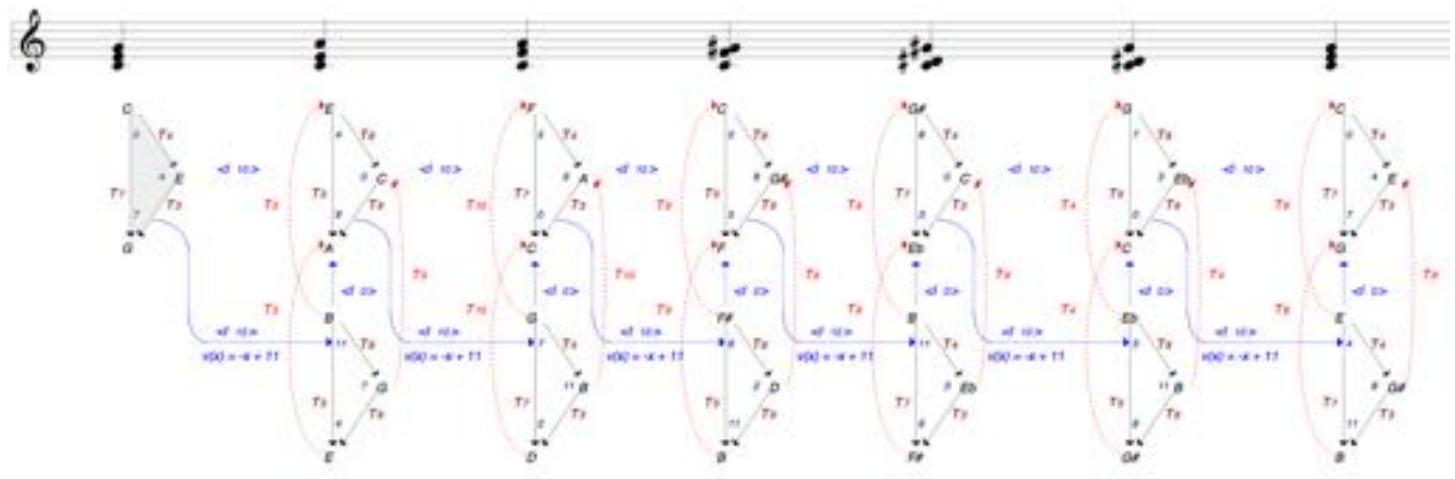
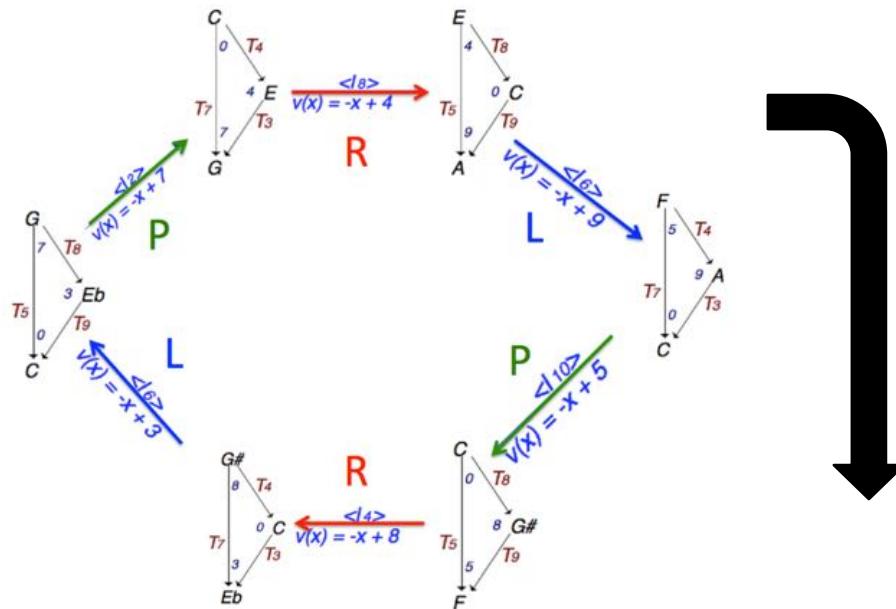
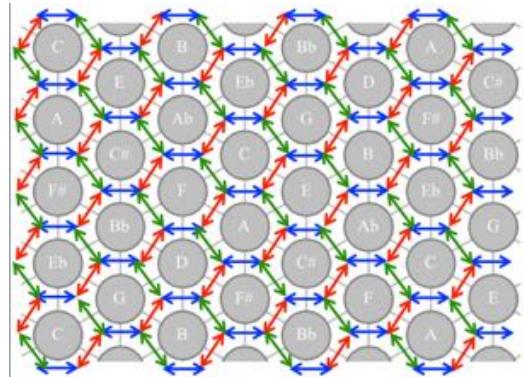
- F is a functor from Δ to \mathbf{C} ,
- and ϕ is a natural transformation from R to SF .

The definition of a PK-net is summed up by the following diagram:



Popoff A., M. Andreatta, A. Ehresmann,
« A Categorical Generalization of
Klumpenhouwer Networks », MCM 2015,
Queen Mary University, Springer, p. 303-314

Topological vs categorical construction of the Tonnetz

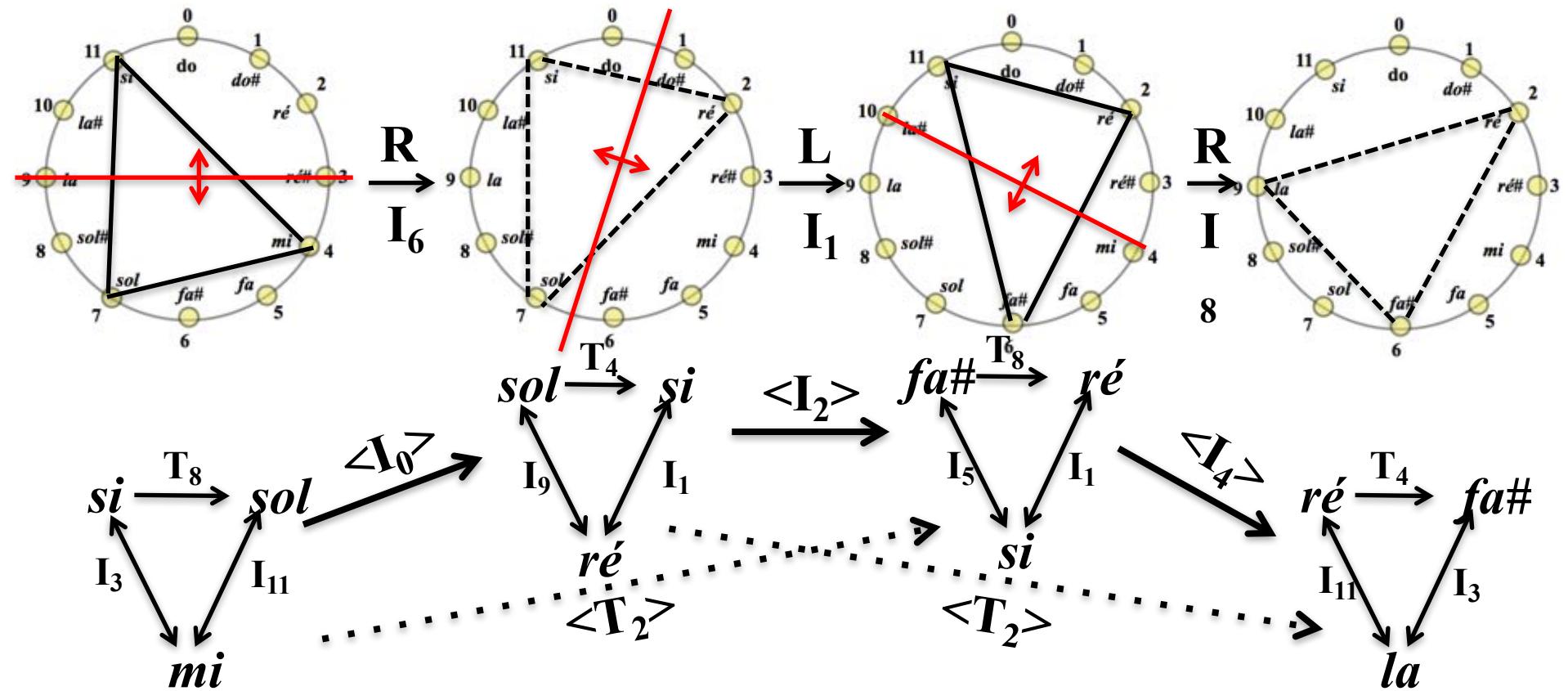


- Popoff A., C. Agon, M. Andreatta, A. Ehresmann (2016), « From K-Nets to PK-Nets: A Categorical Approach », PNM, 54(1)
- Popoff A., M. Andreatta, A. Ehresmann, « Relational PK-Nets for Transformational Music Analysis » (forthcoming in the JMM)

Em**G****Bm****D**

Musical score showing four measures of a piece in Em, G, Bm, and D. Measures are enclosed in dashed boxes labeled Em, G, Bm, and D. Below each measure is a transformation arrow:

- Em:** $R = I_6 = T_6 I$
- G:** $L = I_1 = T_1 I$
- Bm:** $R = I_8 = T_8 I$





Two Dimensions

Traditional Chordal Space

© Gilles Baroin 2011



Gilles Baroin

→ www.mathemusic.net

Harmonic Progressions

In Paolo Conte

Sotto le Stelle del Jazz

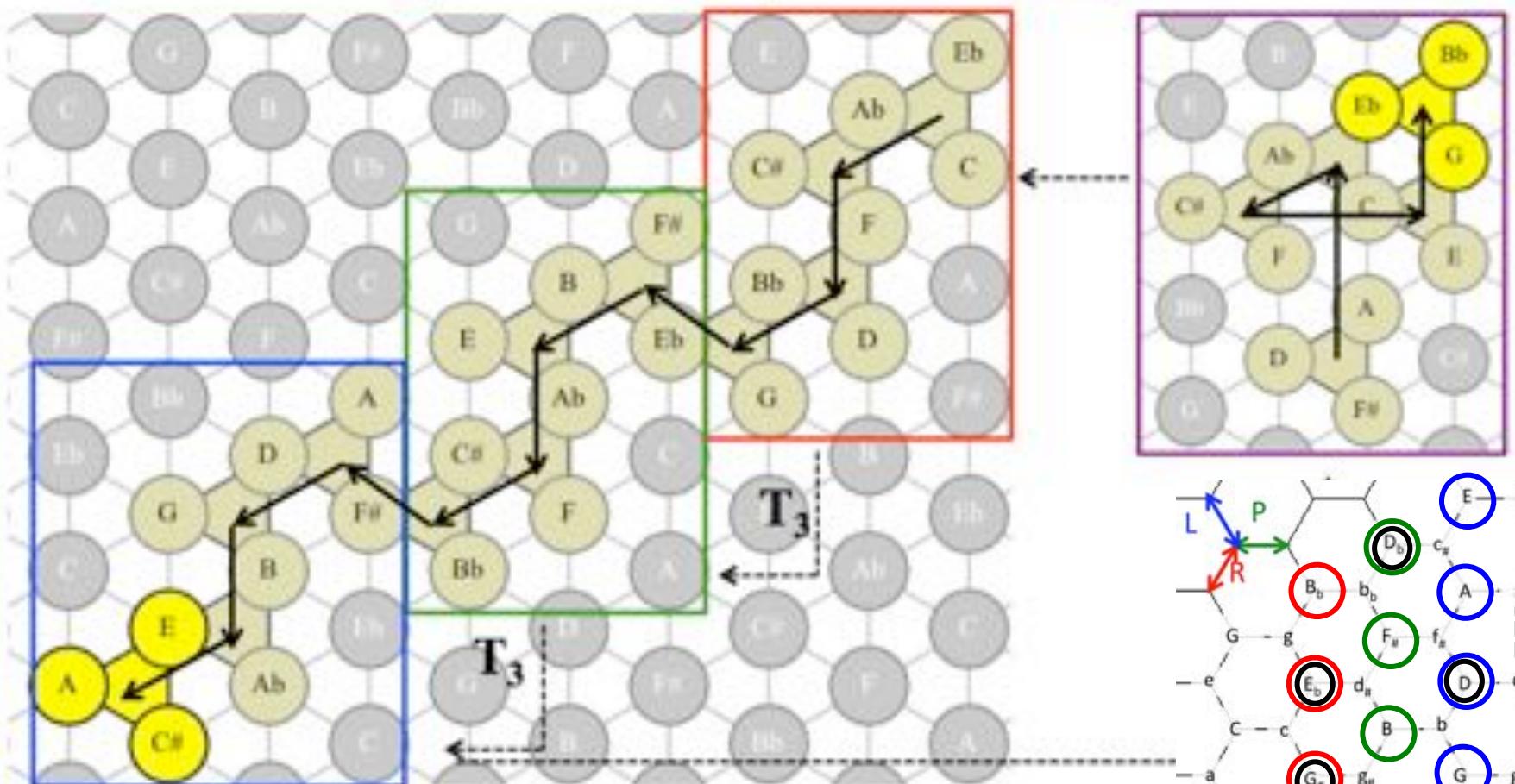
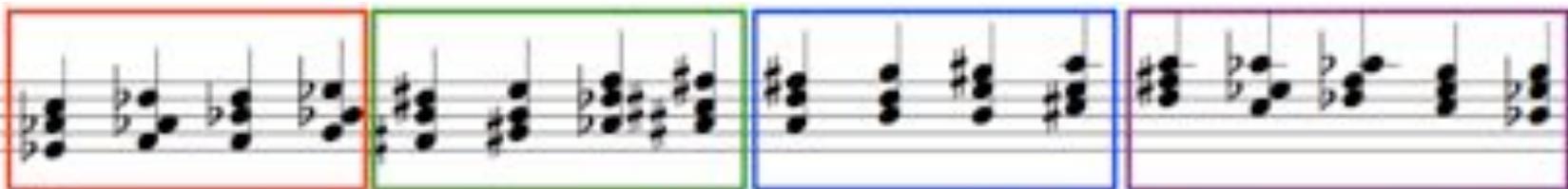


*Supervision Moreno Andreatta
Modelisation Gilles Baroin 2016*

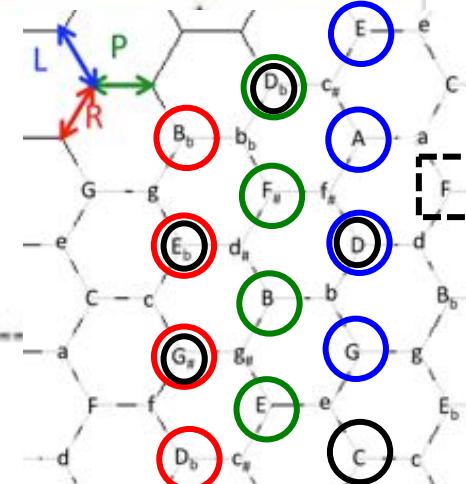


Symmetries in Paolo Conte's *Madeleine*

La_b Re_b Si_b Mi_b Si Mi Re_b Fa_# Re Sol Mi La Re La_b Re_b Do Mi_b



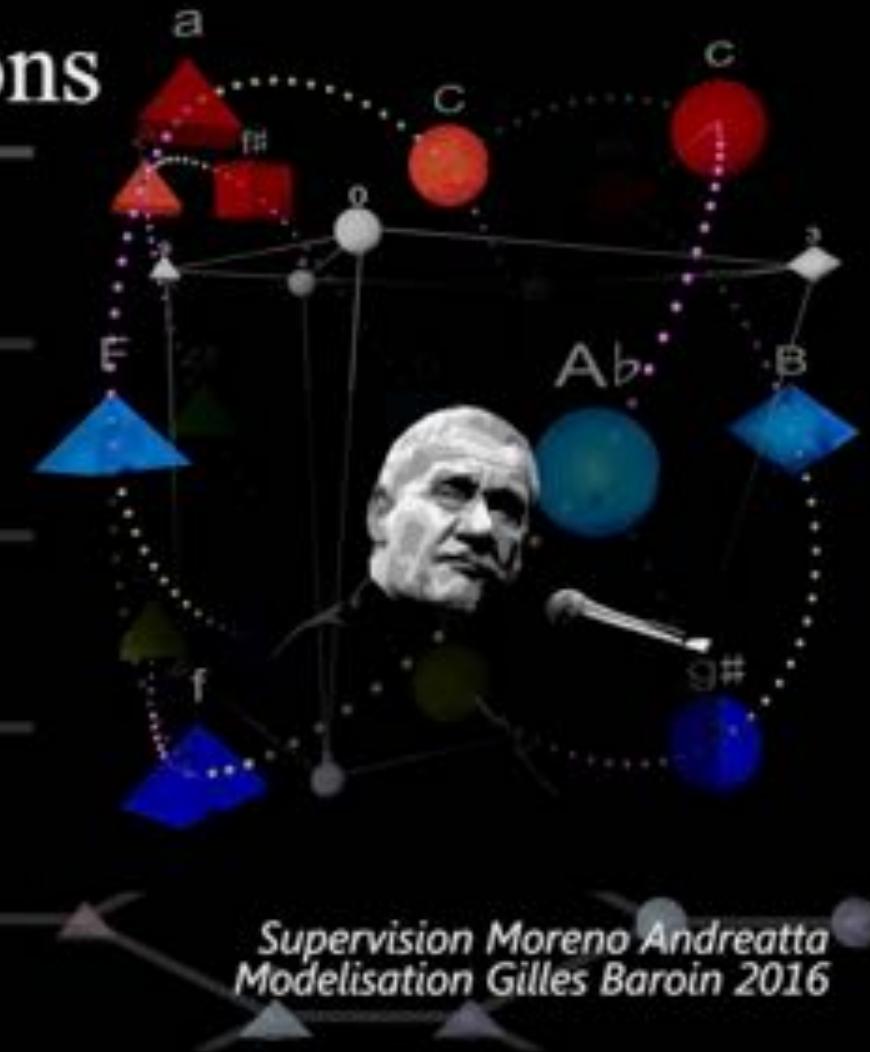
Almost total covering of the major-chords space



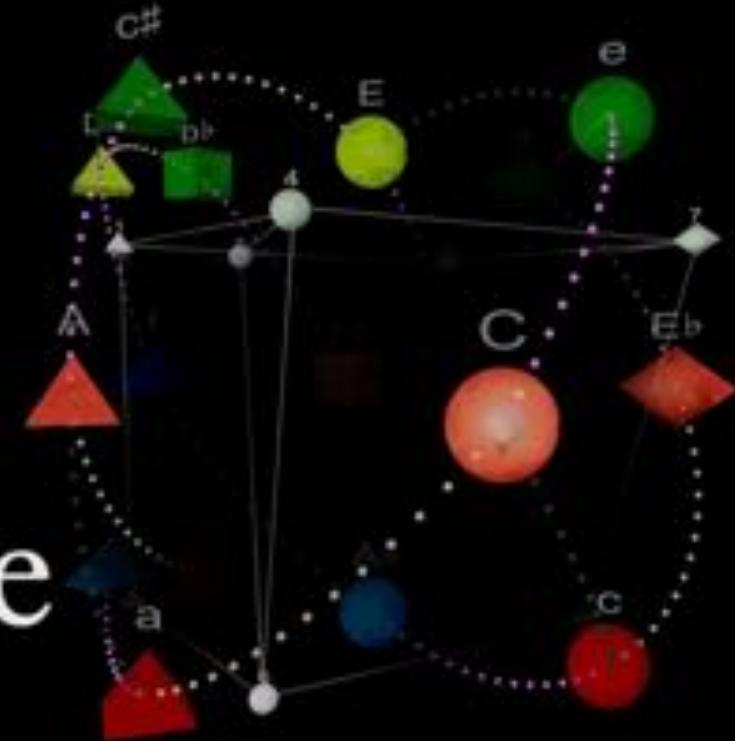
Harmonic Progressions

In Paolo Conte

Madeleine



Beethoven and the Hypersphere *(and the Tonnetz)*



Gilles Baroin 2016
www.MatheMusic.net

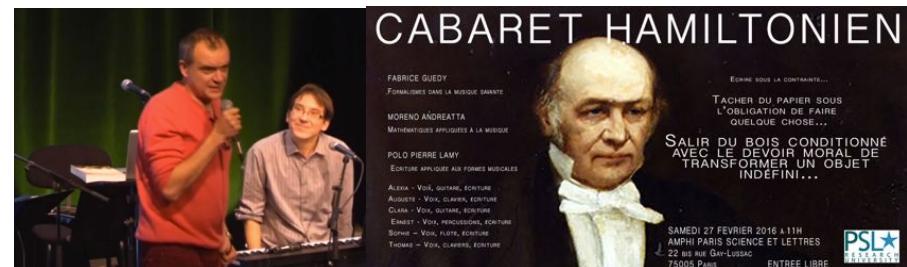
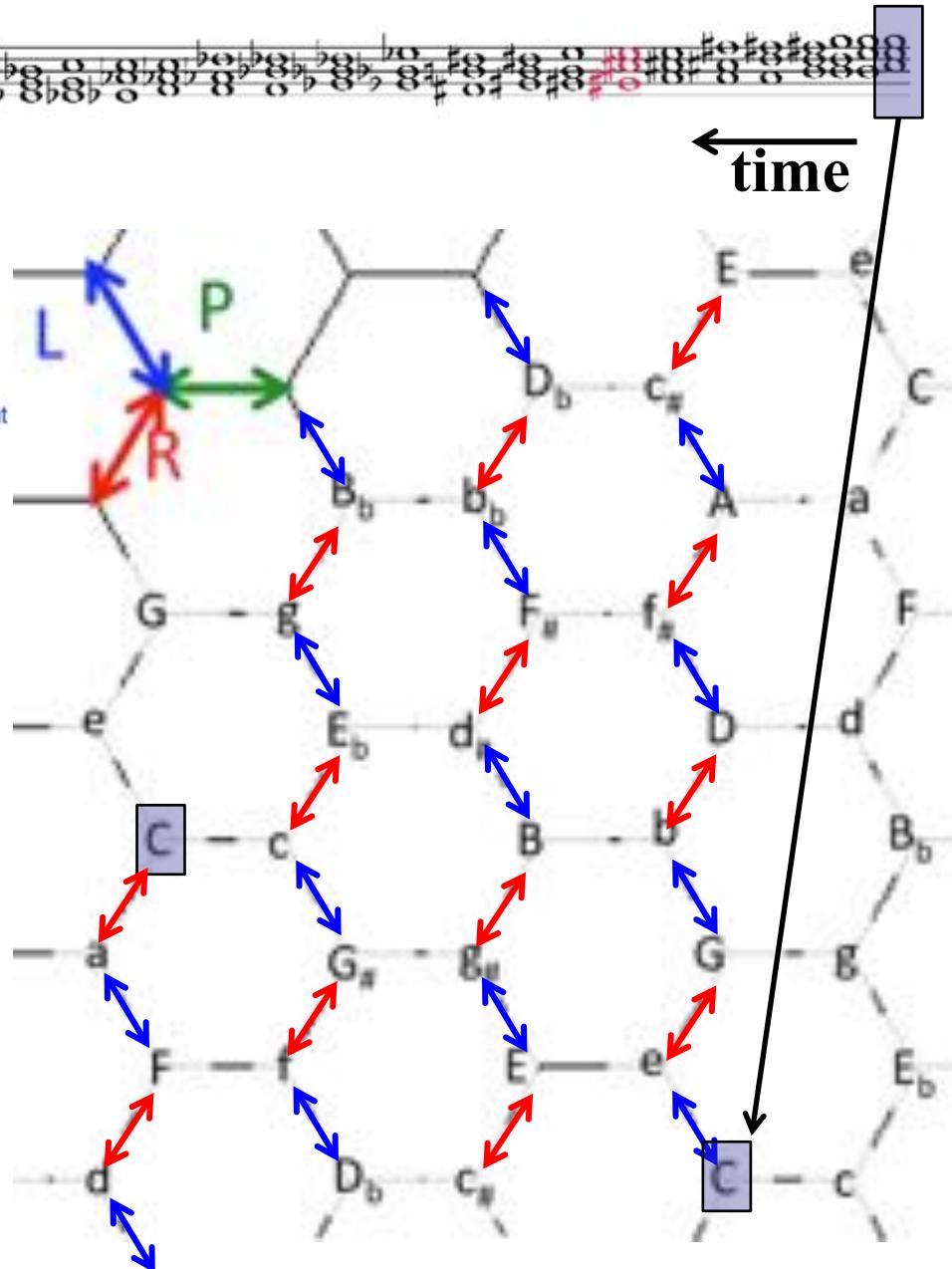
Reading Beethoven backwards



Le Blé en Herbe

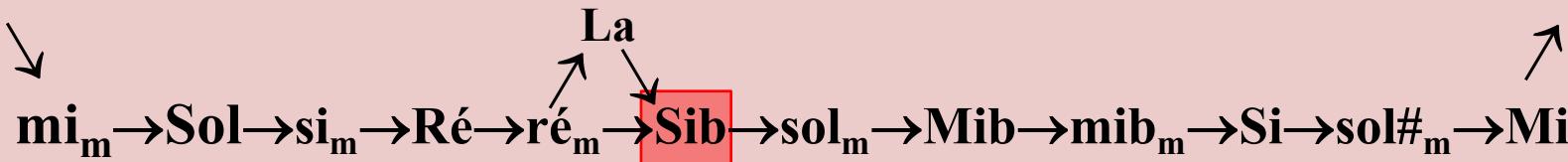
(Polo/Moreno/Dieu)

- | | |
|--|--|
| Plonger comme un enfant, cheveux au vent | Croiser matin dans l'herbe folle |
| Sous l'océan du blé en herbe | Deux tourterelles qui s'envolent |
| Marée d'épis couleur d'amande | Suivre les jeux des hirondelles |
| Qui tendent à caresser le ciel | Sur le paysage éternel |
| | Nager comme un enfant, cheveux au vent |
| Algues tendres de mille plages | Sous l'océan |
| Frôlant le ventre des nuages | Du blé en herbe |
| Cheveux de pluie, dos de poissons | |
| Qui frissonnent à l'unisson | Marée de fruits au goût amer |
| | Acide et salée comme la mer |
| Suivre le bord des continents | |
| Dans l'océan du blé en herbe | Vers l'ilôt d'un petit village |
| Pêcher le corail du pavot | Vers un château d'eau sur la plage |
| Dans le sang des coquelicots | Quand tout s'éteint avant l'orage |
| | Quand se lève le vent du large |
| | Sur le blé vert |



Aprile, a Hamiltonian « decadent » song

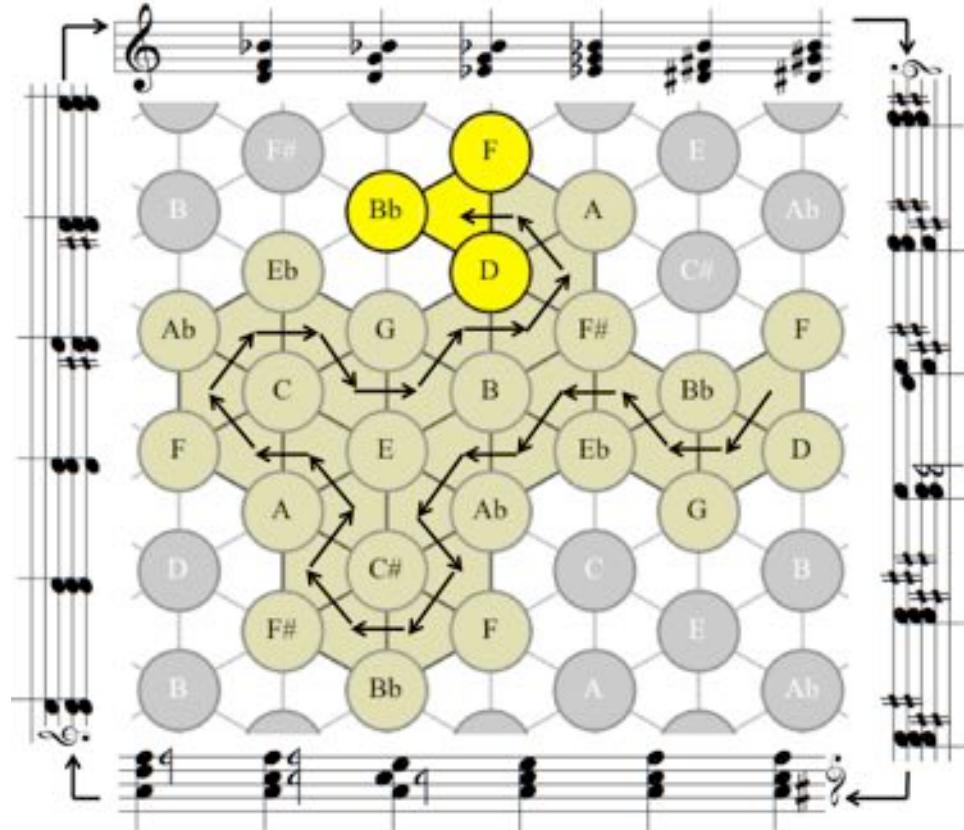
Do←**do**_m←**Sol#**←**fa**_m←**Fa**←**la**_m←**La**←**fa#**_m←**Fa#**←**sib**_m←**Do#**←**do#**_m



*Socchiusa è la finestra, sul giardino.
Un'ora passa lenta, sonnolenta.
Ed ella, ch'era attenta, s'addormenta
A quella voce che già si lamenta,
Che si lamenta in fondo a quel giardino.*

*Non è che voce d'acque su la pietra:
E quante volte, quante volte udita!
Quell'amore e quell'ora in quella vita
S'affondan come ne l'onda infinita
Stretti insieme il cadavere e la pietra.*

*Ella stende l'angoscia sua nel sonno.
L'angoscia è forte, e il sonno è così lieve!
(Par i' luce d'aprile quasi una neve
che sia tiepida.)
Ed ella certo deve soffrire,
Vagamente, anche nel sonno.*



ACTIONS

Math'n'pop

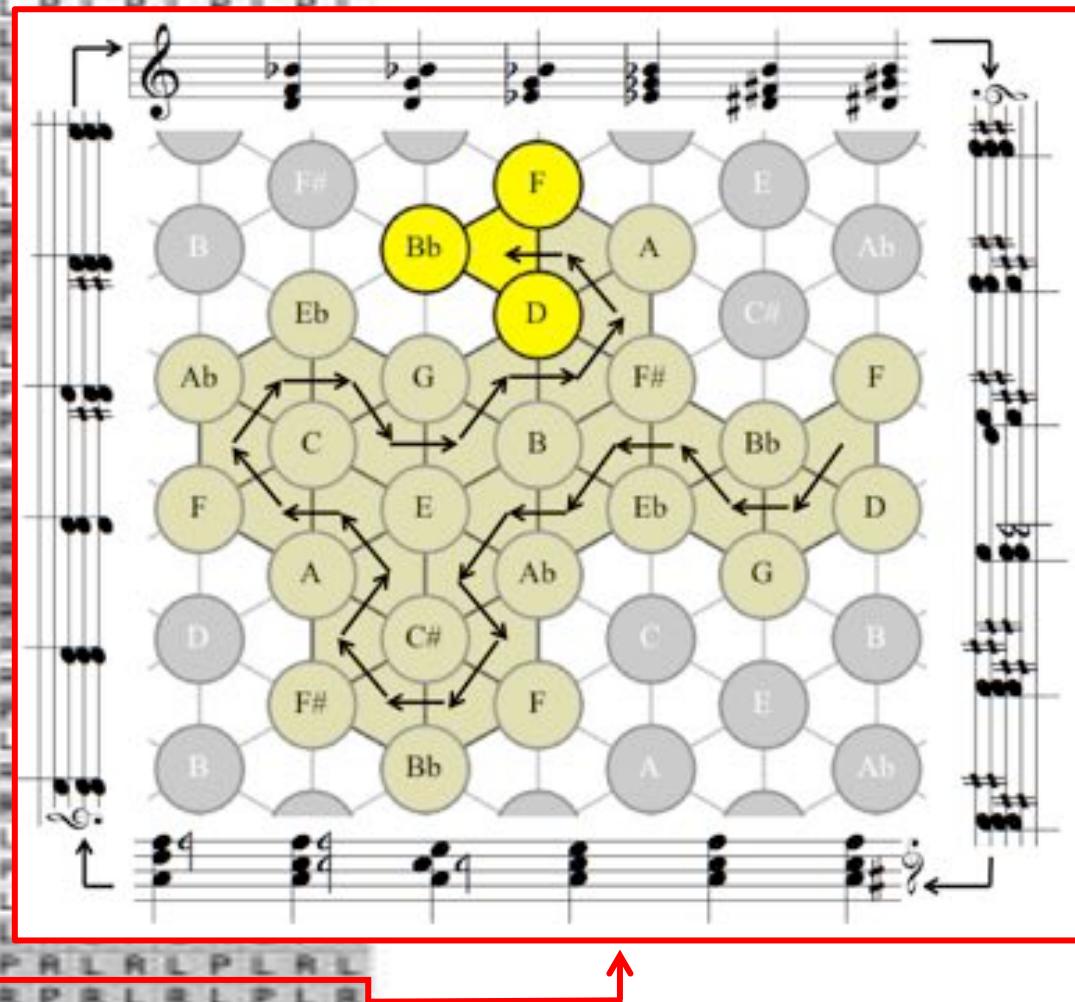
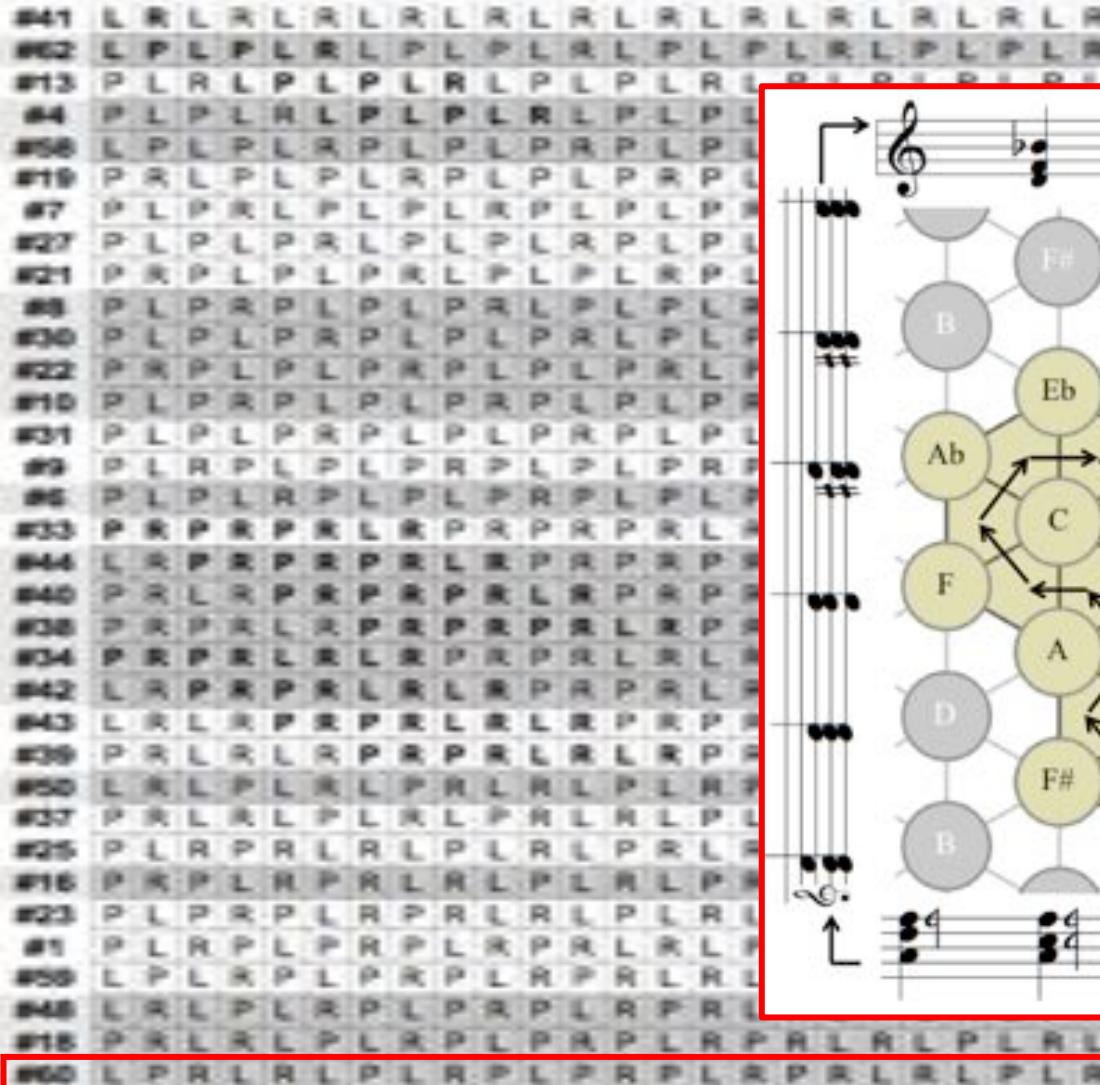
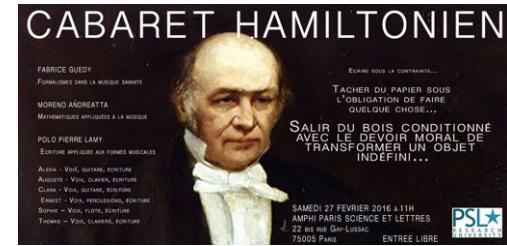
Aprile (d'après Gabriele D'Annunzio)

The collection of 124 Hamiltonian Cycles

ACTIONS

Math'n'pop

Aprile (d'après Gabriele D'Annunzio)



Aprile (d'après Gabriele D'Annunzio)



Tangente
L'aventure mathématique

pour mieux comprendre
le monde

M. Andreatta, « Math'n pop : symétries et cycles hamiltoniens en chanson », *Tangente*

Aprile

4D & 2D Visualizations
Hamiltonian Cycles
M.Andreatta, G.Baroin 2013

Lyrics: Gabriele d'Annunzio

Music and Vocals: Moreno Andreatta

Hypersphere and Ideogramms: Gilles Baroin

Original "Chicken Wire" graph: J.Douthett, P.Steinbach



<http://www.mathemusic.net>

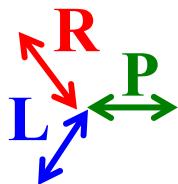
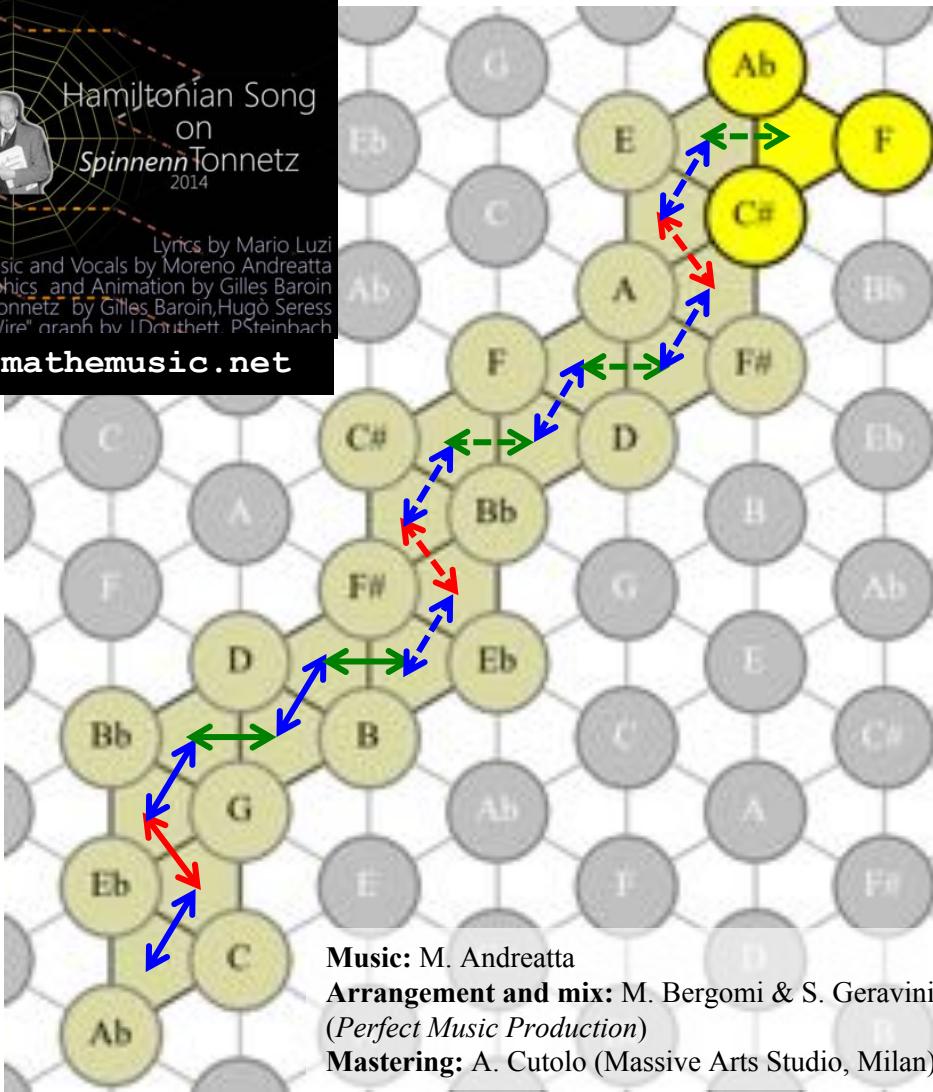
Hamiltonian Cycles with inner periodicities

8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR



L P L P L R ...
 P L P L R L ...
 L P L R L P ...
 PL R L P L ...
L R L P L P ...
 R L P L P L ...

Luzi



La sera non è più la tua canzone
(Mario Luzi, 1945, in *Poesie sparse*)

La sera non è più la tua canzone,
è questa roccia d'ombra traforata
dai lumi e dalle voci senza fine,
la quiete d'una cosa già pensata.

Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d'una cosa conosciuta,
per nome hai una parola ch'è passata
nell'intimo del cuore e s'è perduta.

Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell'essere, (compiuta).

L'aria tace ed il tempo dietro a te
si leva come un'arida montagna
dove vaga il tuo spirito e si perde,
un vento raro scivola e ristagna.



Luzi



Hamiltonian Song
on
SpinnenTonnetz
2014

Lyrics by Mario Luzi

Music and Vocals by Moreno Andreatta

Graphics and Animation by Gilles Baroin

SpinnenTonnetz by Gilles Baroin, Hugò Seress

Original "Chicken Wire" graph by J.Douthett, P.Steinbach



From poetry to song writing:

hamiltonian compositional strategies

A part (Andrée Chedid, poème tiré du recueil *Rhymes Collection Poésie/Gallimard* (n. 527), Gallimard, 2018)

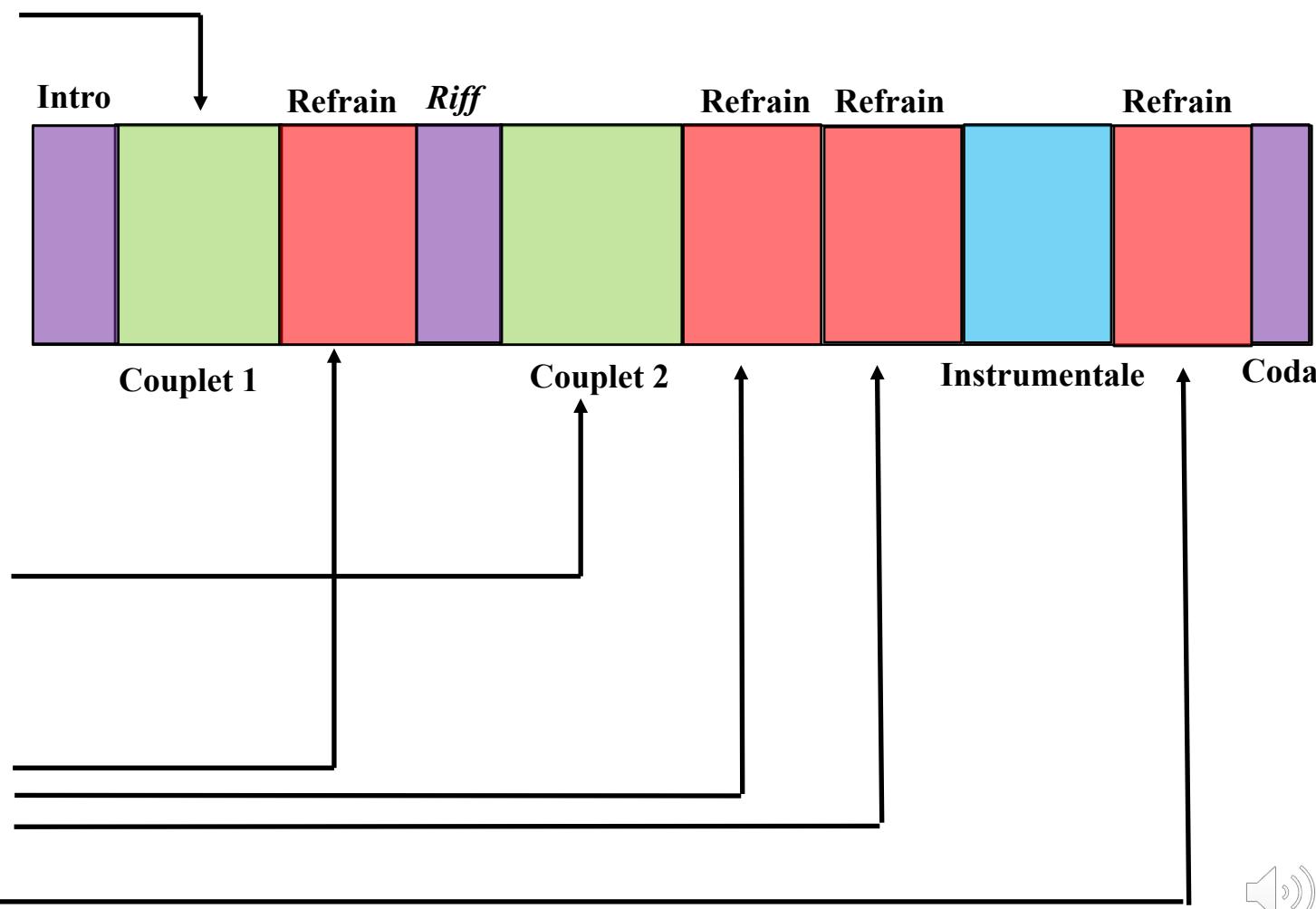


À part le temps
Et ses rouages
À part la terre
En éruptions
À part le ciel
Pétrisseur de nuages
À part l'ennemi
Qui génère l'ennemi

À part le désamour
Qui ronge l'illusion
À part la durée
Qui moisit nos visages

À part les fléaux
À part la tyrannie
À part l'ombre et le crime
Nos batailles nos outrages

Je te célèbre ô Vie
Entre cavités et songes
Intervalle convoité
Entre le vide et le rien



The catalogue of 28 hamiltonian cycles (with inner symmetry)

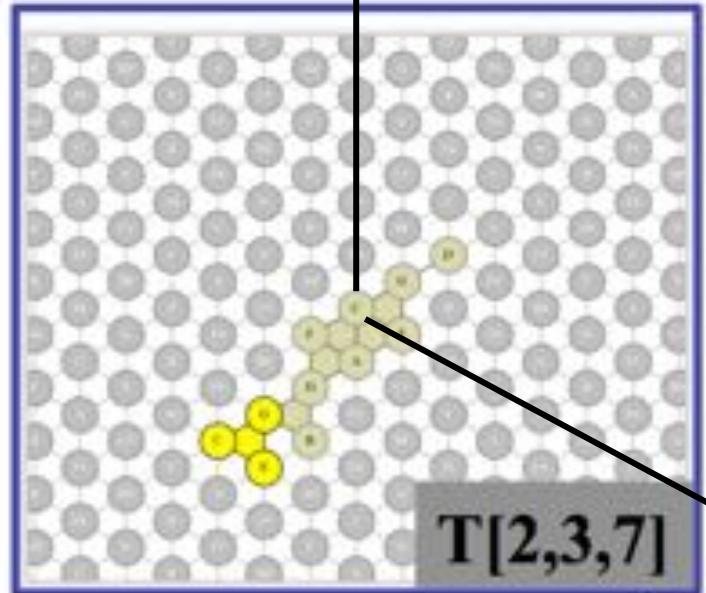
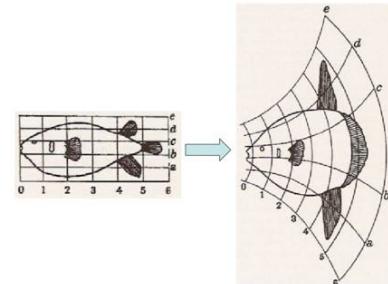
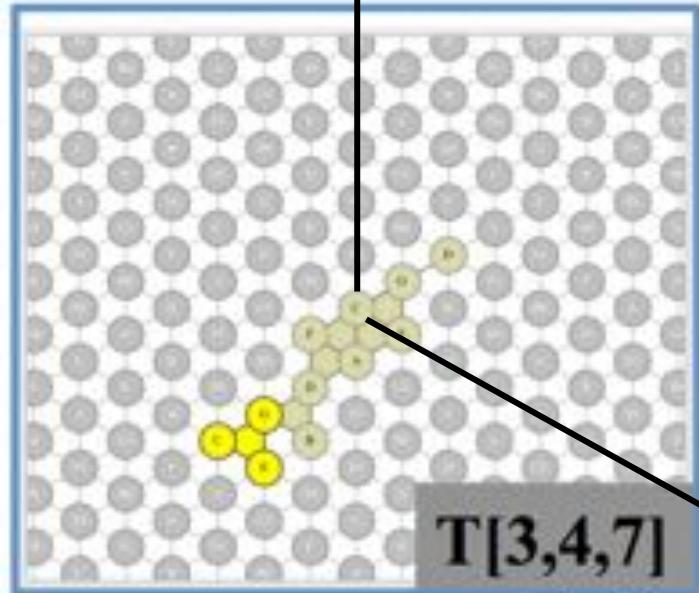
1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPRPR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPRLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRLRPRP
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPRPL
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRLRLRP
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm--RLRPRPRP
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em--RLRPRPRL
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLRP
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm--RLRLRPRP
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL



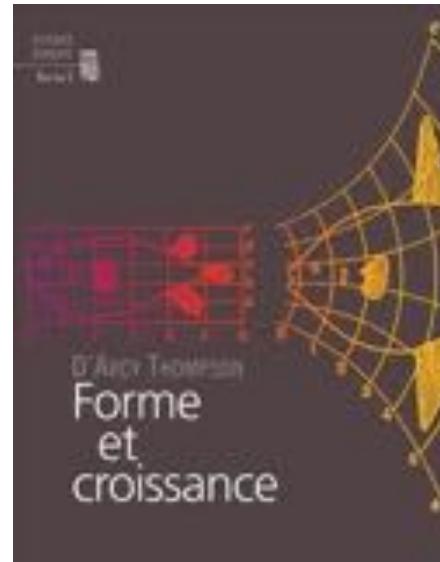
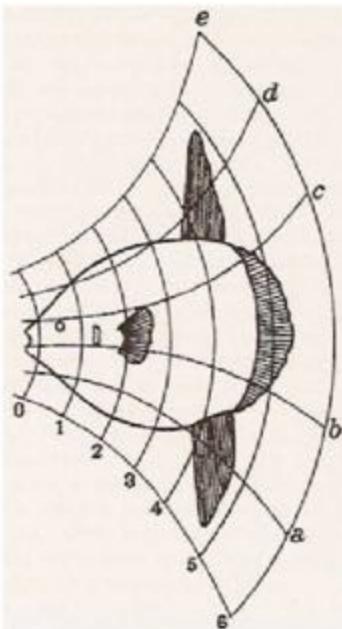
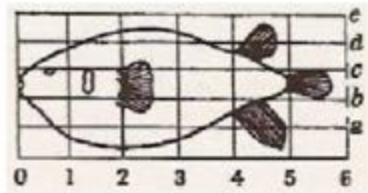
Le Blé en Herbe



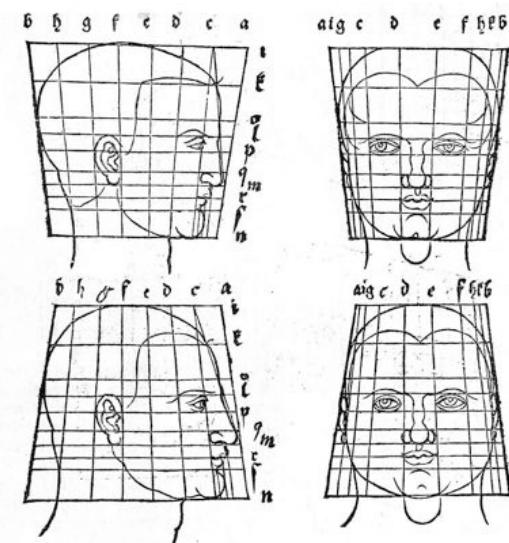
The musical style...is the space!



The morphological vs the mathematical genealogy of the structuralism



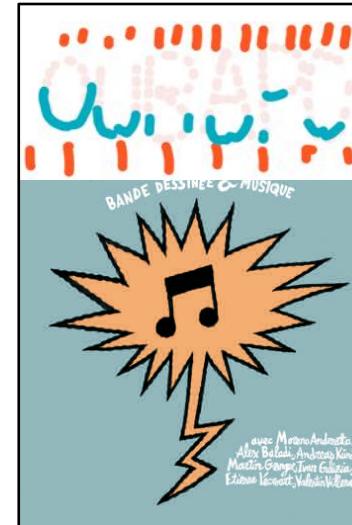
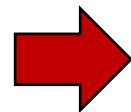
“[The notion of **transformation**] comes from a work which played for me a very important role and which I have read during the war in the United States : *On Growth and Form*, in two volumes, by **D'Arcy Wentworth Thompson**, originally published in 1917. The author (...) proposes an interpretation of the visible transformations between the species (animals and vegetables) within a same gender. This was fascinating, in particular because I was quickly realizing that this perspective had a long tradition: behind Thompson, there was **Goethe's** botany and behind Goethe, **Albert Dürer** with his *Treatise of human proportions*” (Lévi-Strauss, conversation with Eribon, 1988).



From the OuLiPo to the OuMuPo (ouvroir de musique potentielle)



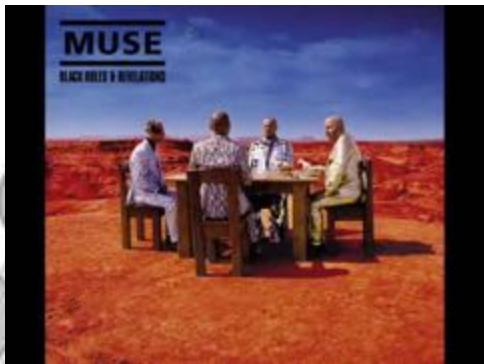
<http://oumupo.org/>



M. Andreatta et al., « Music, mathematics and language: chronicles from the Oumupo sandbox », in Kapoula, Z., Volle, E., Renault, J., Andreatta, M. (Eds.), *Exploring Transdisciplinarity in Art and Sciences*, Springer, 2018



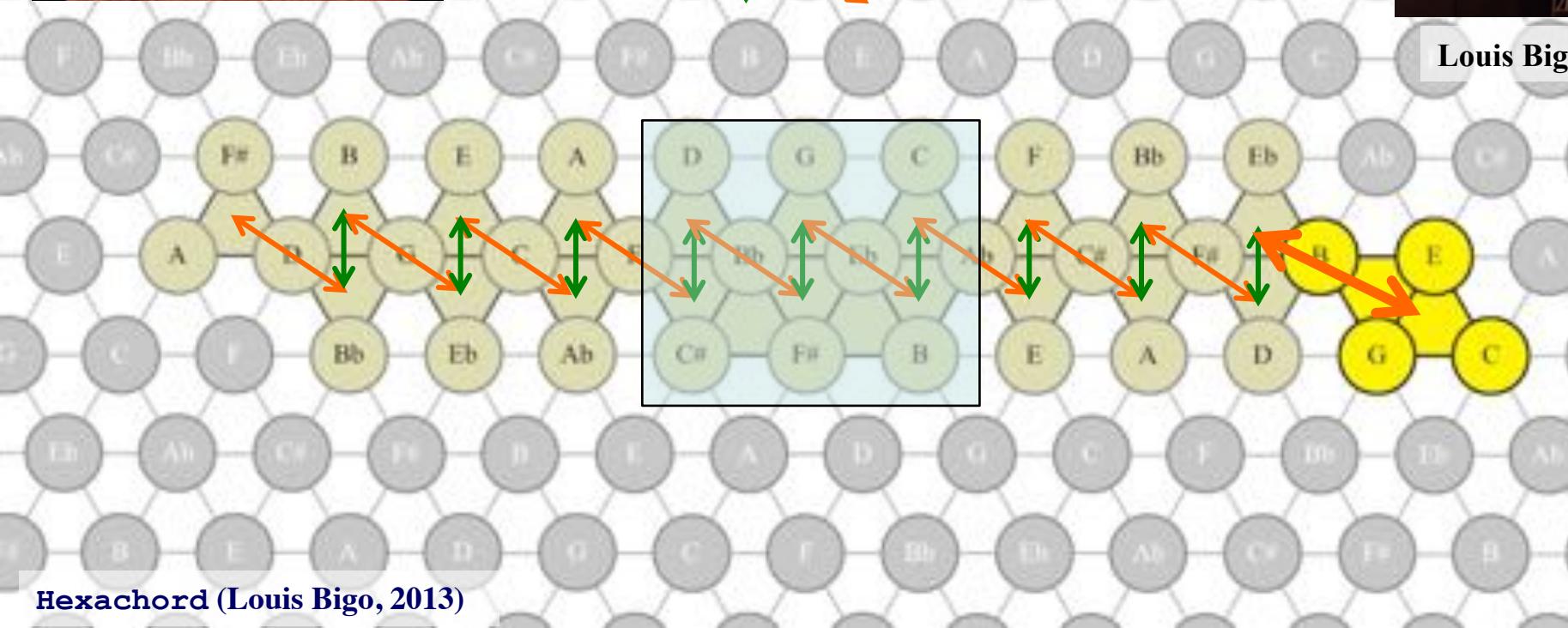
Symmetries and algorithmic processes in *Muse*



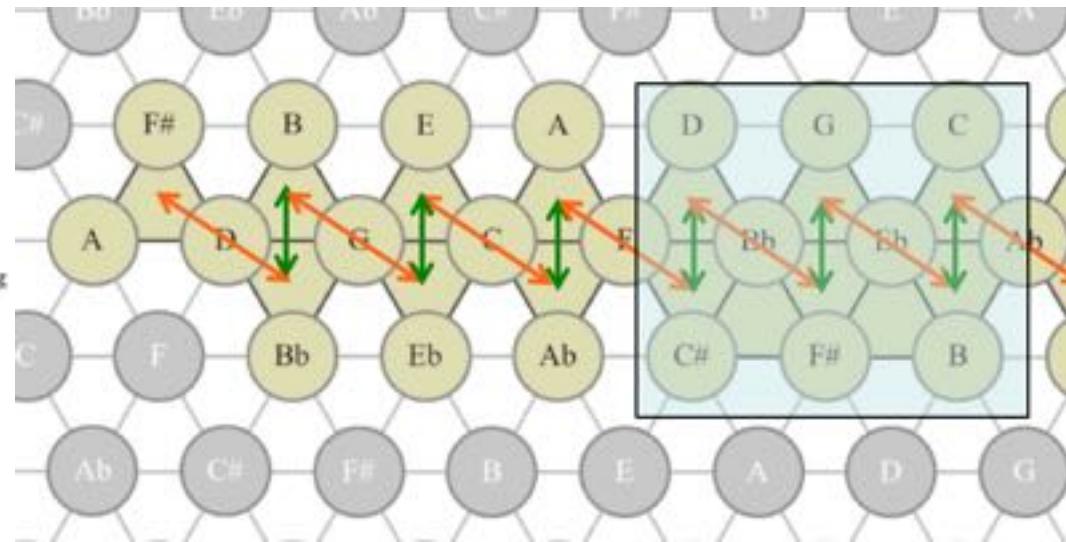
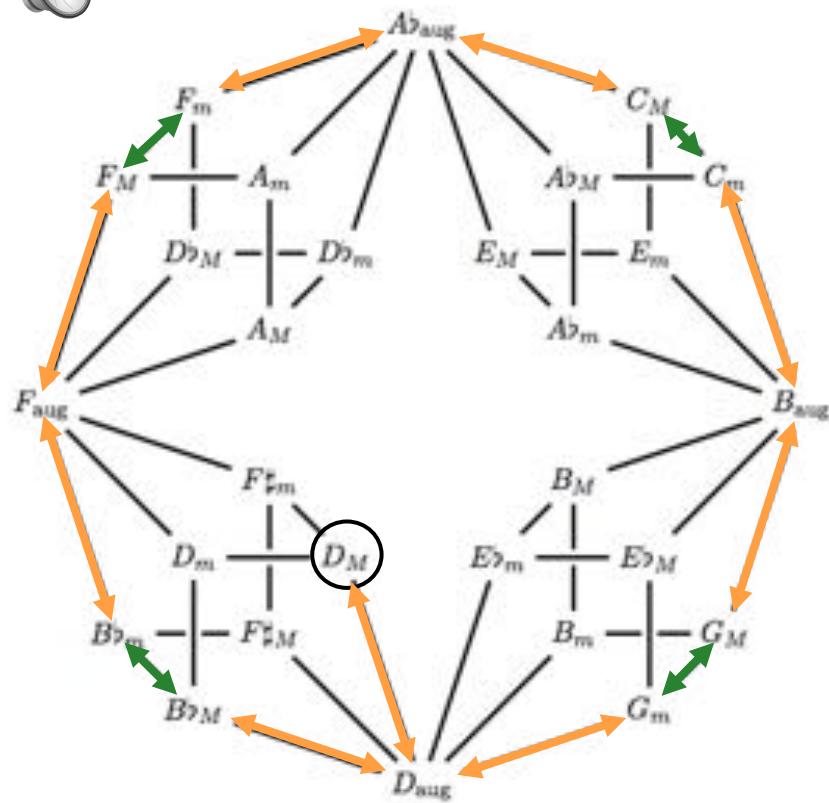
“Take a bow” (*Black Holes and Revelations*, 2006)



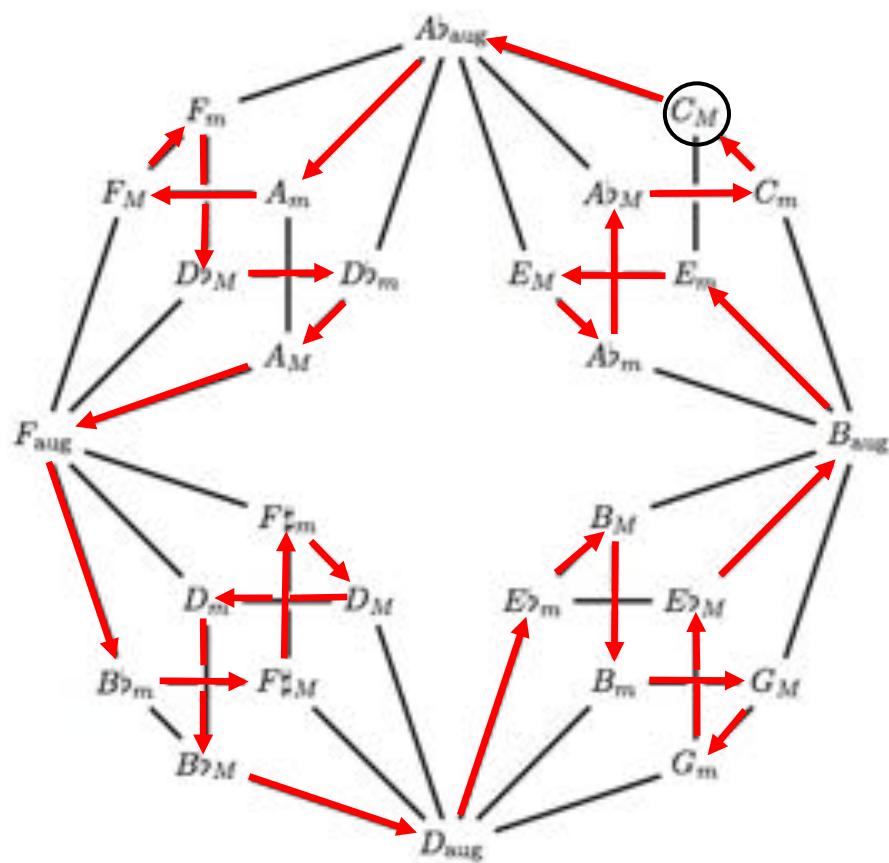
Louis Bigo



Representing Muse's progression in the Cube Dance



The Gunner's Hamiltonian Dream (an *oumouopian* experiment on a song by Pink-Floyd)



The three main hamiltonian cycles ($C_M = C$, $C_m = Cm$, $C_{aug} = C+$)

$C \rightarrow C+ \rightarrow Am \rightarrow F \rightarrow Fm \rightarrow C\#m \rightarrow C+ \rightarrow A \rightarrow F+ \rightarrow Bbm \rightarrow F\# \rightarrow F\#m \rightarrow D \rightarrow Dm \rightarrow Bb \rightarrow D+ \rightarrow Ebm \rightarrow B \rightarrow Bm \rightarrow G \rightarrow Gm \rightarrow Eb \rightarrow G+ \rightarrow Em \rightarrow E \rightarrow G\#m \rightarrow G\# \rightarrow Cm \rightarrow C$

$C \rightarrow C+ \rightarrow Am \rightarrow F \rightarrow Fm \rightarrow C\#m \rightarrow C+ \rightarrow A \rightarrow F+ \rightarrow F\#m \rightarrow F\# \rightarrow Bbm \rightarrow Bb \rightarrow Dm \rightarrow D \rightarrow D+ \rightarrow Ebm \rightarrow B \rightarrow Bm \rightarrow G \rightarrow Gm \rightarrow Eb \rightarrow G+ \rightarrow Em \rightarrow E \rightarrow G\#m \rightarrow G\# \rightarrow Cm \rightarrow C$

$C \rightarrow C+ \rightarrow Am \rightarrow F \rightarrow Fm \rightarrow C\#m \rightarrow C+ \rightarrow A \rightarrow F+ \rightarrow F\#m \rightarrow D \rightarrow Dm \rightarrow Bb \rightarrow Bbm \rightarrow F\# \rightarrow D+ \rightarrow Ebm \rightarrow B \rightarrow Bm \rightarrow G \rightarrow Gm \rightarrow Eb \rightarrow G+ \rightarrow Cm \rightarrow G\#m \rightarrow G\# \rightarrow E \rightarrow Em \rightarrow C$

The Gunner's dream (R. Waters, 1983 / M. Andreatta, 2018)

(C)

C+
Floating down through the clouds

Am

F

Memories come rushing up to meet me now.

Fm

In the space between the heavens

C#

and in the corner of some foreign field

A

F+

Bbm

I had a dream.

C#m

F#

F#m

D

Dm

I had a dream.

Bb

Good-bye Max.

D+

Good-bye Ma.

Ebm

B

After the service when you're walking slowly to the car

Bm

G

And the silver in her hair shines in the cold November air

Gm

You hear the tolling bell

Eb

And touch the silk in your lapel

G+

And as the tear drops rise to meet the comfort of the band

G#

Cm

You take her frail hand

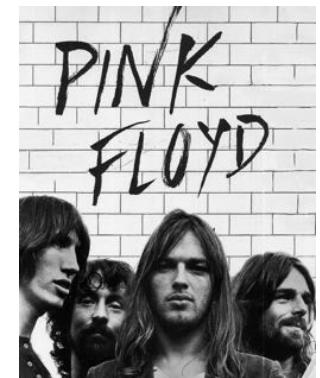
Em

E

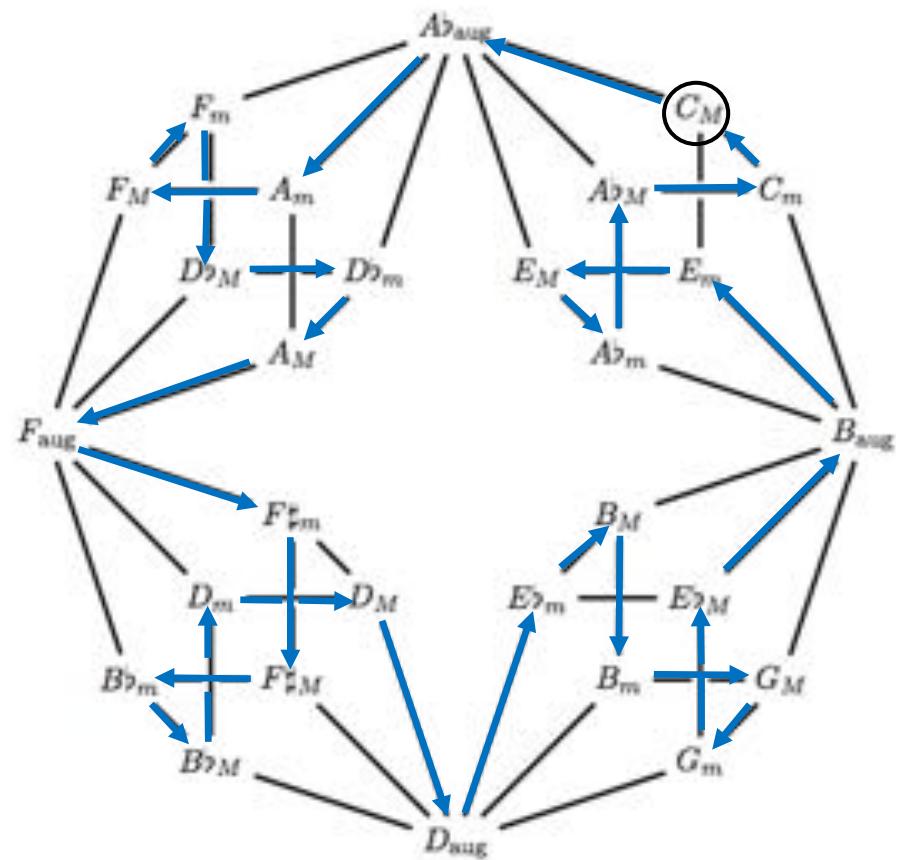
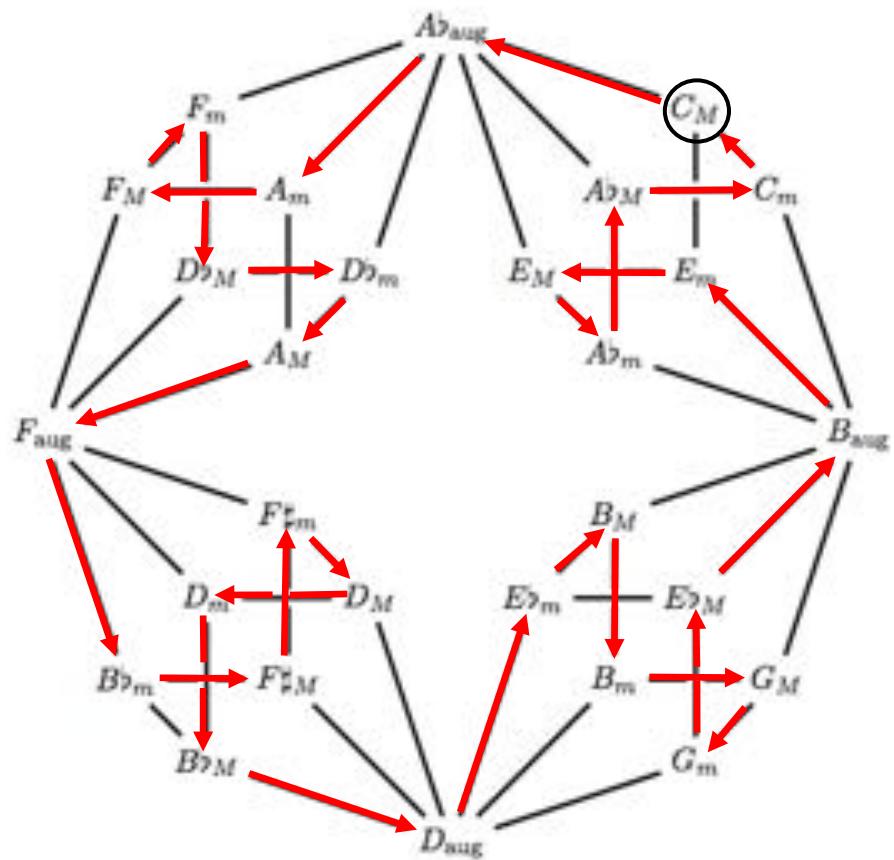
G#m

C

And hold on to the dream.



The Gunner's Hamiltonian Dream (an *oumouopian* experiment on a song by Pink-Floyd)

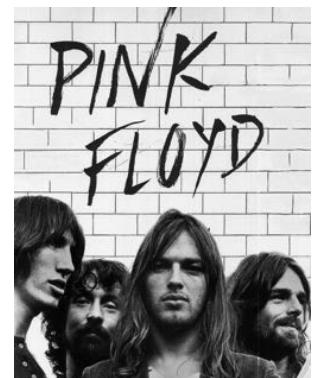


The three main hamiltonian cycles ($C_M = C$, $C_m = Cm$, $C_{aug} = C+$)

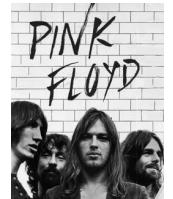
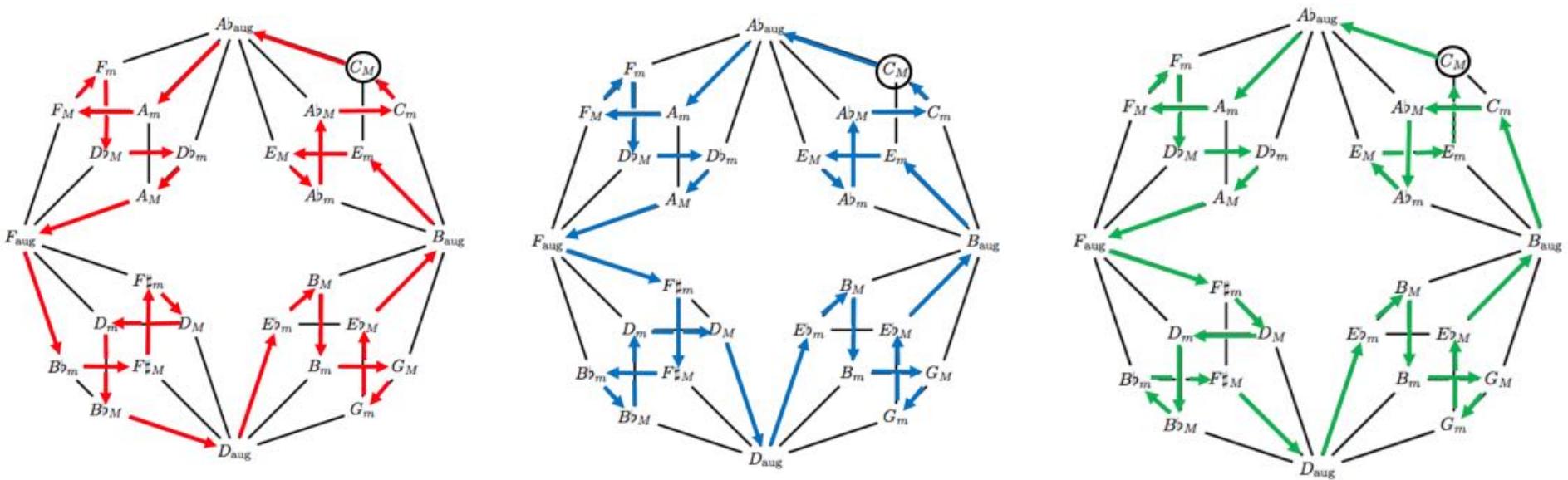
$C \rightarrow C \rightarrow Am \rightarrow F \rightarrow Fm \rightarrow C\# \rightarrow C\#m \rightarrow A \rightarrow F \rightarrow Bbm \rightarrow F\# \rightarrow F\#m \rightarrow D \rightarrow Dm \rightarrow Bb \rightarrow D \rightarrow Ebm \rightarrow B \rightarrow Bm \rightarrow G \rightarrow Gm \rightarrow Eb \rightarrow G \rightarrow Em \rightarrow E \rightarrow G\#m \rightarrow G\# \rightarrow Cm \rightarrow C$

$C \rightarrow C \rightarrow Am \rightarrow F \rightarrow Fm \rightarrow C\# \rightarrow C\#m \rightarrow A \rightarrow F \rightarrow F\#m \rightarrow F\# \rightarrow Bbm \rightarrow Bb \rightarrow Dm \rightarrow D \rightarrow D \rightarrow Ebm \rightarrow B \rightarrow Bm \rightarrow G \rightarrow Gm \rightarrow Eb \rightarrow G \rightarrow Em \rightarrow E \rightarrow G\#m \rightarrow G\# \rightarrow Cm \rightarrow C$

$C \rightarrow C \rightarrow Am \rightarrow F \rightarrow Fm \rightarrow C\# \rightarrow C\#m \rightarrow A \rightarrow F \rightarrow F\#m \rightarrow D \rightarrow Dm \rightarrow Bb \rightarrow Bbm \rightarrow F\# \rightarrow D \rightarrow Ebm \rightarrow B \rightarrow Bm \rightarrow G \rightarrow Gm \rightarrow Eb \rightarrow G \rightarrow Cm \rightarrow G\# \rightarrow G\#m \rightarrow E \rightarrow Em \rightarrow C$



The Gunner's Hamiltonian Dream (an *oumouopian* experiment on a song by Pink-Floyd)



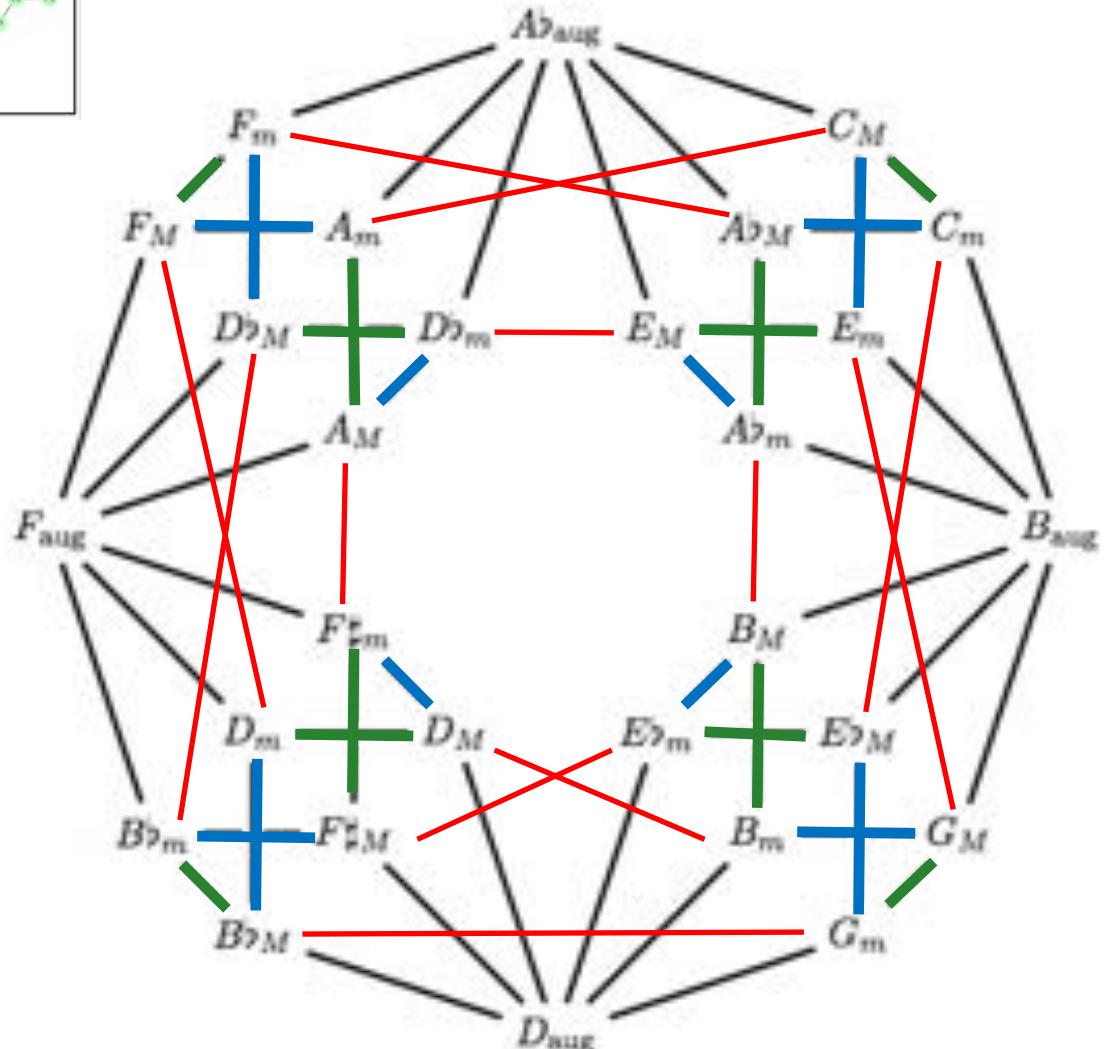
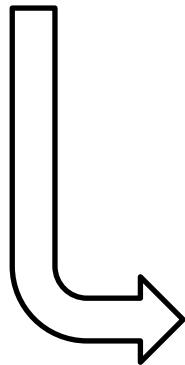
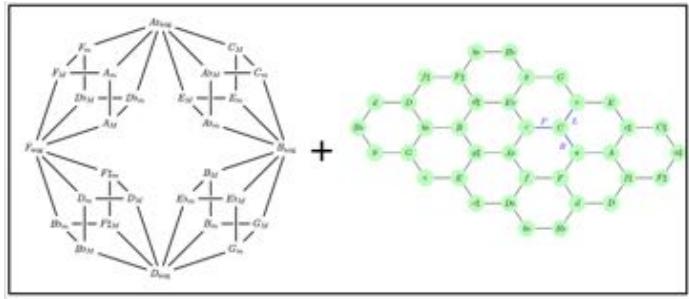
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Embedding the Cube Dance into the Tonnetz



Spatial music analysis via *Hexachord*

The image shows a composite screenshot of the Hexachord software interface and a copy of the 'Computer Music Journal'.

Hexachord Software Components:

- Hex Viewer:** A 3D visualization of a geometric polyhedron, likely representing a complex or set class.
- Tessellation:** A hexagonal grid representation of musical data, with specific hexagons highlighted in yellow and labeled with letters (A through H) and numbers (1 through 4).
- InfoBox:** A control panel for a MIDI file named "bwv0281.mid". It includes:
 - Tempo slider (set to 10).
 - Play and Stop buttons.
 - Select midi file input field.
 - Chromatic complexes and Heptatonic complexes dropdown menus.
 - Trace off and Harmonization ON buttons.
 - Display graph button.
 - Vertical compactness section with compactness dimension (2), 2-compactness, compute compactness, and absolute compactness buttons.
 - Path Transformation section with Origin complex (K[3,4,5]) and Destination complex (K[3,4,5]), Rotation (0), North translation (0), and North-east translation (0) inputs, and a Path Transformation button.
 - Chart section titled "2-compactness : bwv0281" showing a bar chart of 2-compactness values over time.

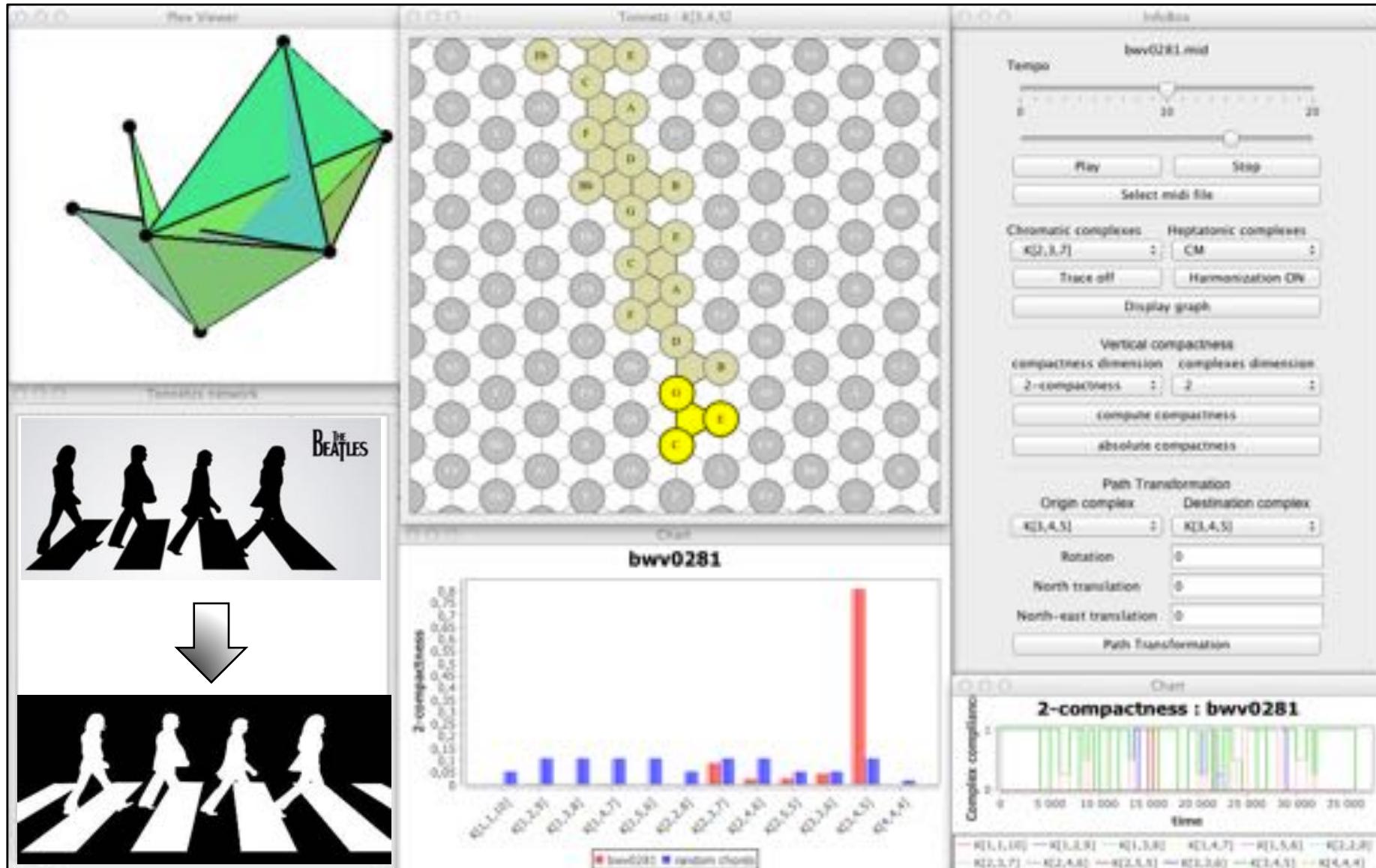
Computer Music Journal Cover:

Volume 30 Number 1, Spring 2006, ISSN 0899-2213
Digital Harmonies and Discreet Sound Sources

The journal cover features a green background with a white header and a dark green footer. It includes a small graphic of a geometric shape and some text in the footer.

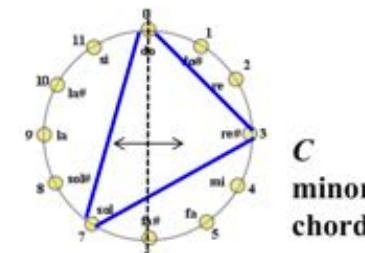
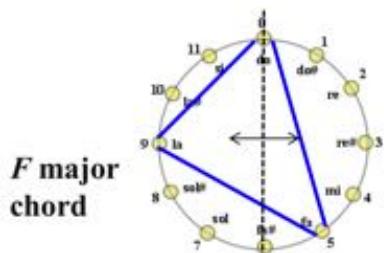
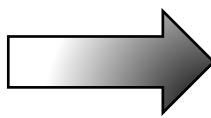
→ <http://www.lacl.fr/~lbigo/hexachord>

Keeping the space...but changing the trajectory!

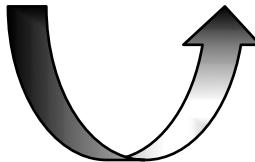
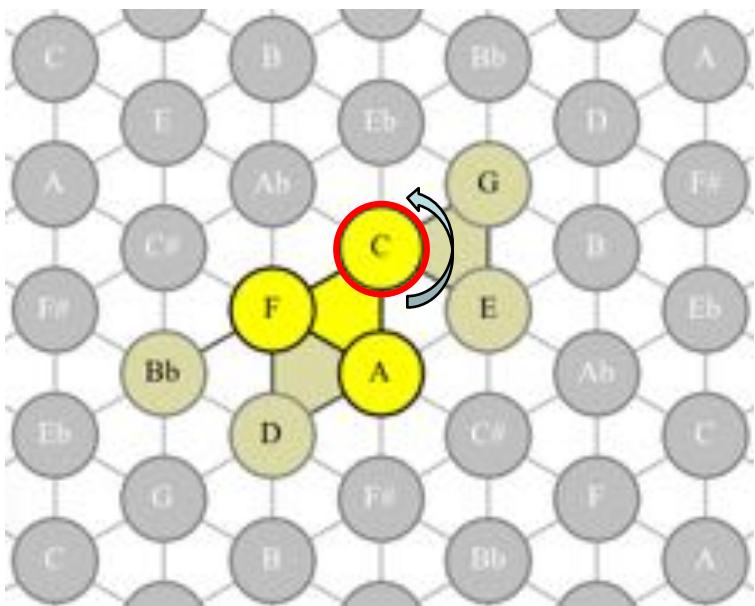


→ <http://www.lacl.fr/~lbigo/hexachord>

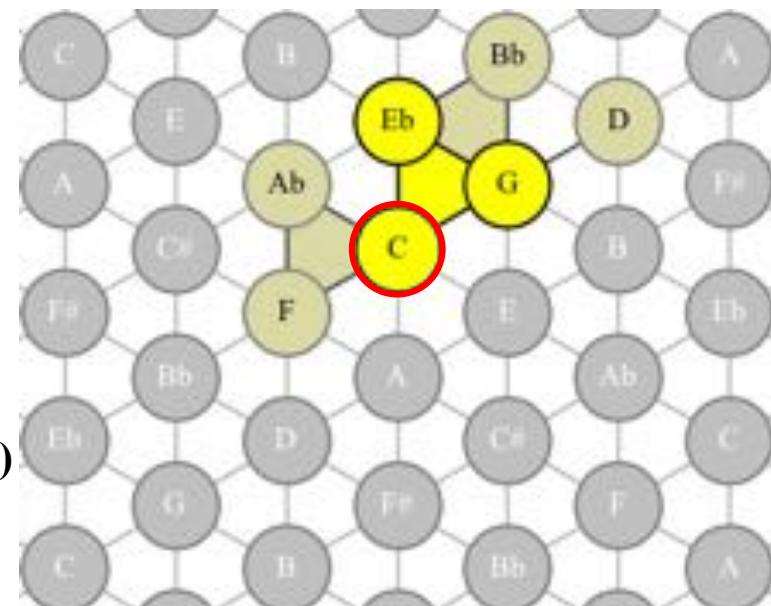
Keeping the space...but changing the trajectory!



C
minor
chord



Rotation
(autour du do)



Rotational symmetry applied to traditional Brazilian music

Sonoroso

K. Ximbinho

Brejeiro

Maxixe

Introd.

A

E7

A

Dm

Dm

E \flat

A

E7

A

E7



Doce de Côco

Jacob do
(Jacob E)

$\frac{8}{8}$ G6 C/E G6 Dm7 E7 Am Am/G E

A m7 Am7 D7 G7M G \flat 7 F \sharp 7 F6 Bm7 E7

A m E7/B A m Cm7/E \flat D7 G m Gm5+ Gm6 Gm Gm5+

Gm6 Gm5+ G7 G7 Cm Cm E \flat 9 E \flat 9

G/D E7 A m7 D7 G6 Bm7 B \flat 9 A m7

Aeroporto do Galeão

Altamiro Carrilho

F9 $\frac{8}{8}$ F9

D7 D \sharp Gm9 Gm9

Apanhei-te Cavaquinho

Ernesto Nazareth
Baldoman

D7(9) $\frac{8}{8}$ G6 B7(9) E m

C6 G6 D7

Escovado
Tango Brasileiro

Ernesto Nazareth 1905

Grazioso

Piano

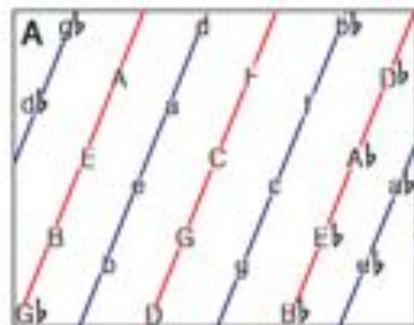
6

II

8^{va}

Fine

Some cognitive implications of mathemusical research

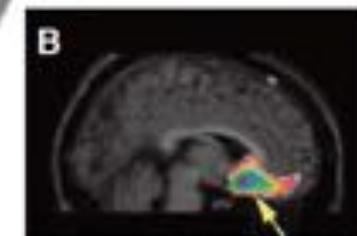
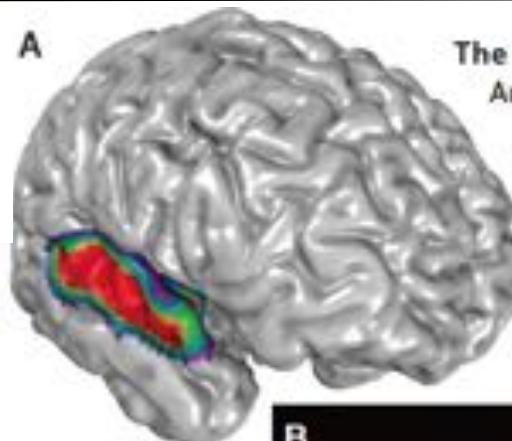
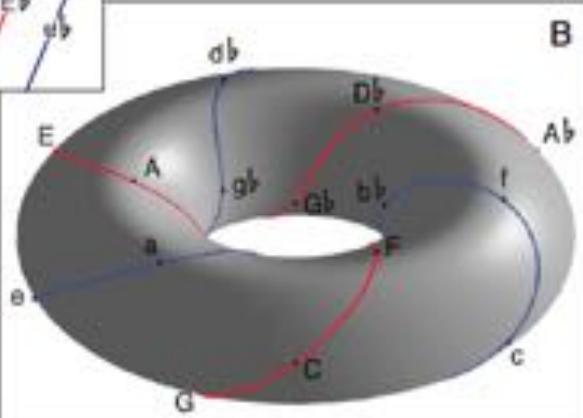


Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (B) Musical keys as points on the surface of a torus.

PERSPECTIVES: NEUROSCIENCE

Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl

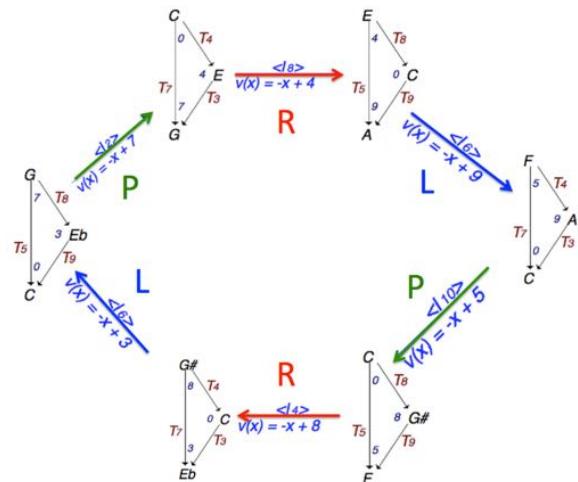


The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (B) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

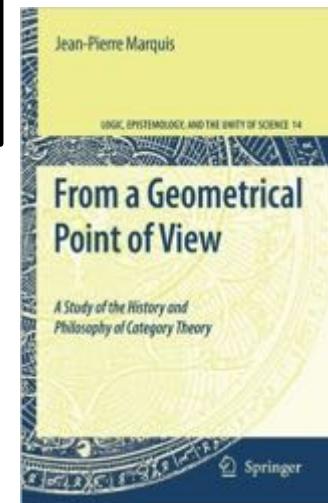
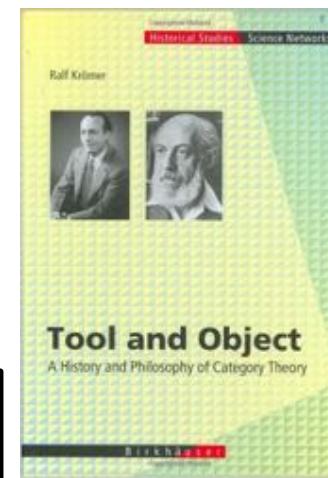
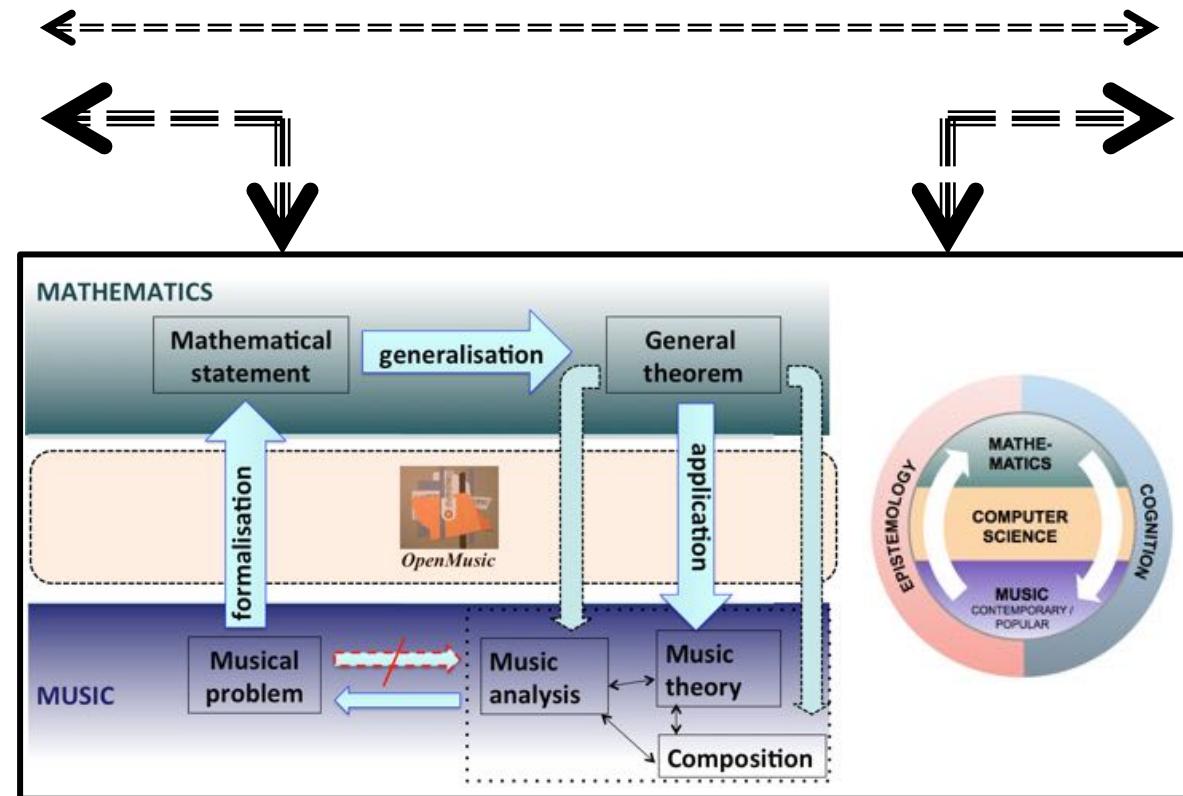
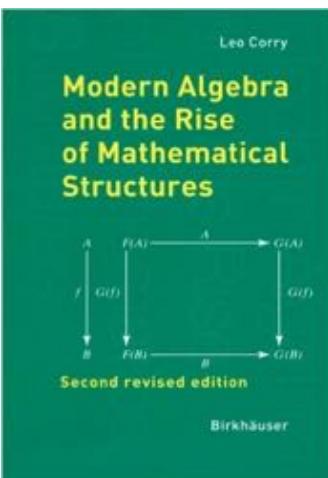
« La théorie des catégories est une théorie des constructions mathématiques, qui est macroscopique, et procède d'étage en étage. Elle est un bel exemple d'abstraction réfléchissante, cette dernière reprenant elle-même un principe constructeur présent dès le stade sensorimoteur. Le style catégoriel qui est ainsi à l'image d'un aspect important de la genèse des facultés cognitives, est un style adéquat à la description de cette genèse »



J. Piaget



Which type of philosophy for the *mathemusical* research?



A synthetic vision allows us to link together apparently distant strata of mathematics and culture, helping us to break down many artificial barriers. Not only can today's mathematics be appreciated through epistemic, ontic, phenomenological and aesthetic modes, but in turn, it should help to transform philosophy.