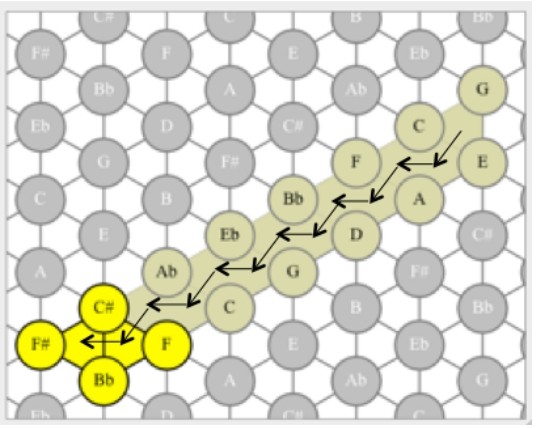
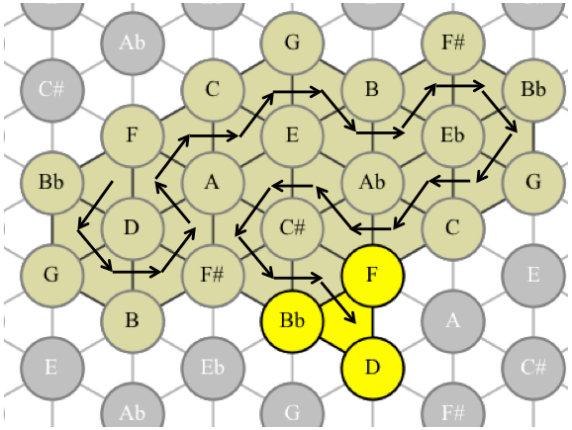
Sciences, mathématique et musique



Gala des étudiants,
14 novembre 2013



Moreno Andreatta
Equipe Représentations Musicales /
IRCAM/CNRS UMR 9912

La recherche à l'Ircam / CNRS / UPMC

www.ircam.fr

UMR STMS

L'Unité mixte de recherche UMR9912 associe le CNRS, l'UPMC, le ministère de la Culture et l'Ircam autour d'une thématique de recherche interdisciplinaire sur les **sciences et technologies de la musique et du son**.

Elle est rattachée à titre principal au CNRS à l'Institut des sciences informatiques et de leurs interactions (INS2I) et à titre secondaire aux Instituts des Sciences de l'ingénierie et des systèmes (INSIS), des Sciences humaines et sociales (INSHS) et des Sciences biologiques (INSB). Elle est intégrée à l'UPMC à la Faculté d'ingénierie - UFR 919 relevant du Pôle de recherche Modélisation et ingénierie.

Directeur : Gérard Assayag (Ircam)
Directeur adjoint : Hugues Vinet (Ircam)

Au 1er janvier 2012, le laboratoire est constitué des équipes suivantes :

- Acoustique instrumentale
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- Représentations musicales
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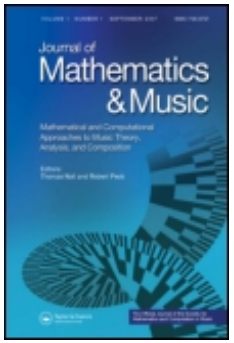
http://omax.ircam.fr/

Boulez conducts zappa
ajouter à mes albums

Frank Zappa
Pop, Rock
Oct 1984
Universal / Universal Music Division
Classics Jazz
7 titres

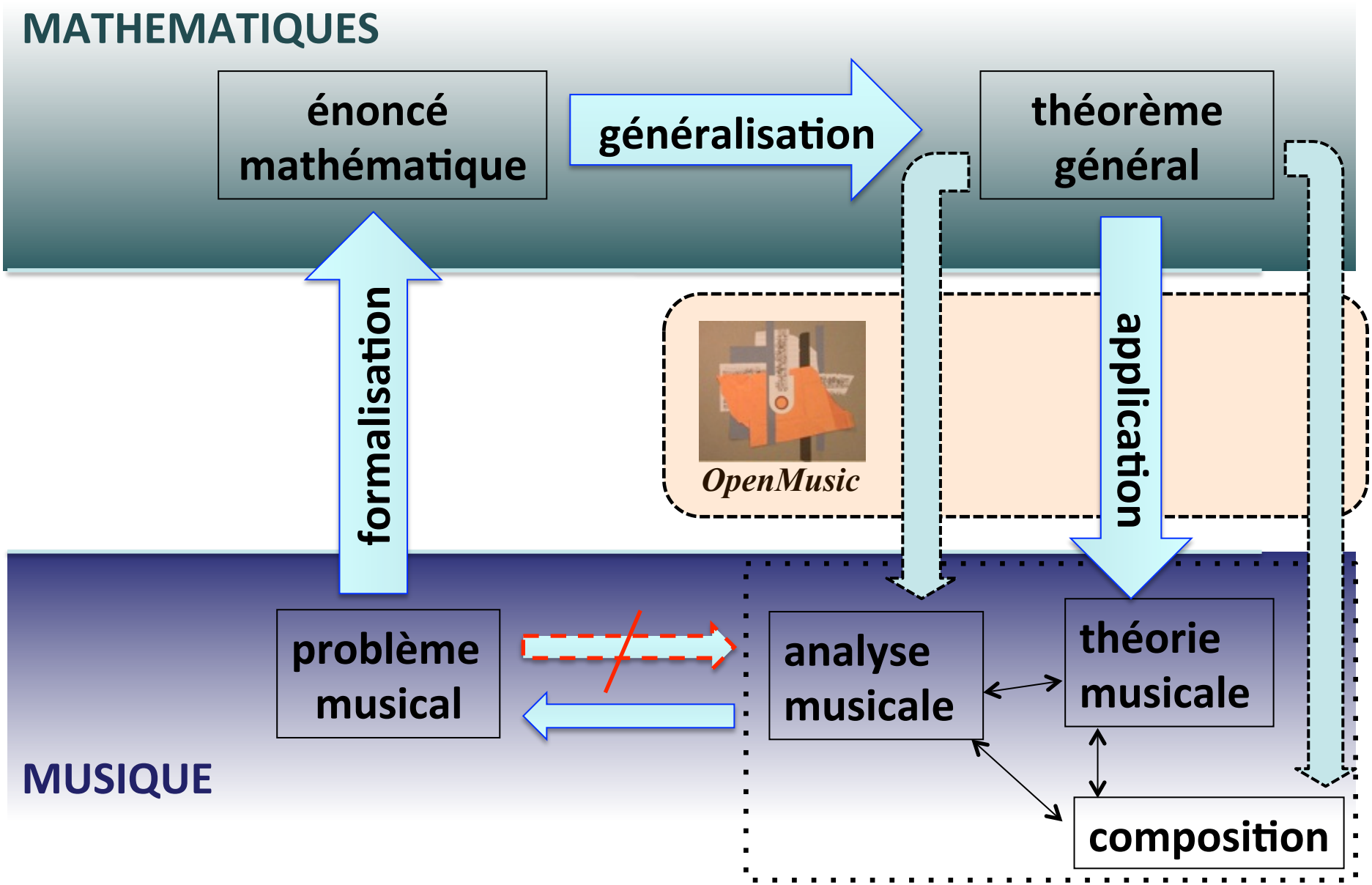
Mathematiques/Musique…une histoire récente !

- 1999: 4e Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (G. Assayag, H.G. Feichtinger, J.F. Rodrigues, Springer, 2001)
- 2000-2001: *MaMuPhi Seminar, Penser la musique avec les mathématiques ?* (Assayag, Mazzola, Nicolas éds., Coll. 'Musique/Sciences', Ircam/Delatour, 2006)
- 2000-2003: International Seminar on *MaMuTh* (*Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Luis-Puebla eds, epOs, 2004)
- 2003: *The Topos of Music* (G. Mazzola et al.)
- 2001-…: *MaMuX* Seminar at Ircam
- 2004-…: *mamuphi* Seminar (Ens/Ircam)
- 2006: Collection 'Musique/Sciences' (Ircam/Delatour France)
- 2007: Journal of Mathematics and Music (Taylor & Francis) and SMCM
- 2007: First MCM 2007 (Berlin) and Proceedings by Springer
- 2007-…: AMS Special Session on Mathematical Techniques in Musical Analysis
- 2009: *Computational Music Science* (eds: G. Mazzola, M. Andreatta, Springer)
- 2009: MCM 2009 (Yale University) and Proceedings by Springer
- 2010: Mathematics Subject Classification : 00A65 Mathematics and music
- 2011: MCM 2011 (Ircam, 15-17 June 2011) and Proceedings LNCS Springer
- 2013: MCM 2013 (McGill University, Canada, 12-14 June 2013) - Springer



Société Mathématique de France

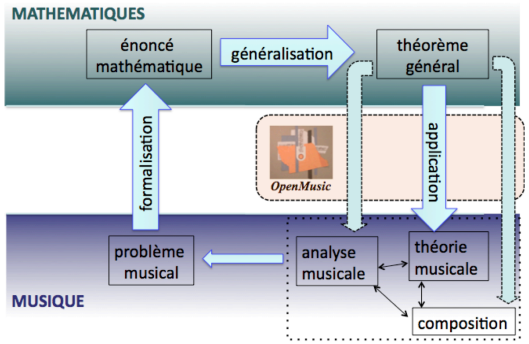
Double mouvement d'une dynamique mathémusicale

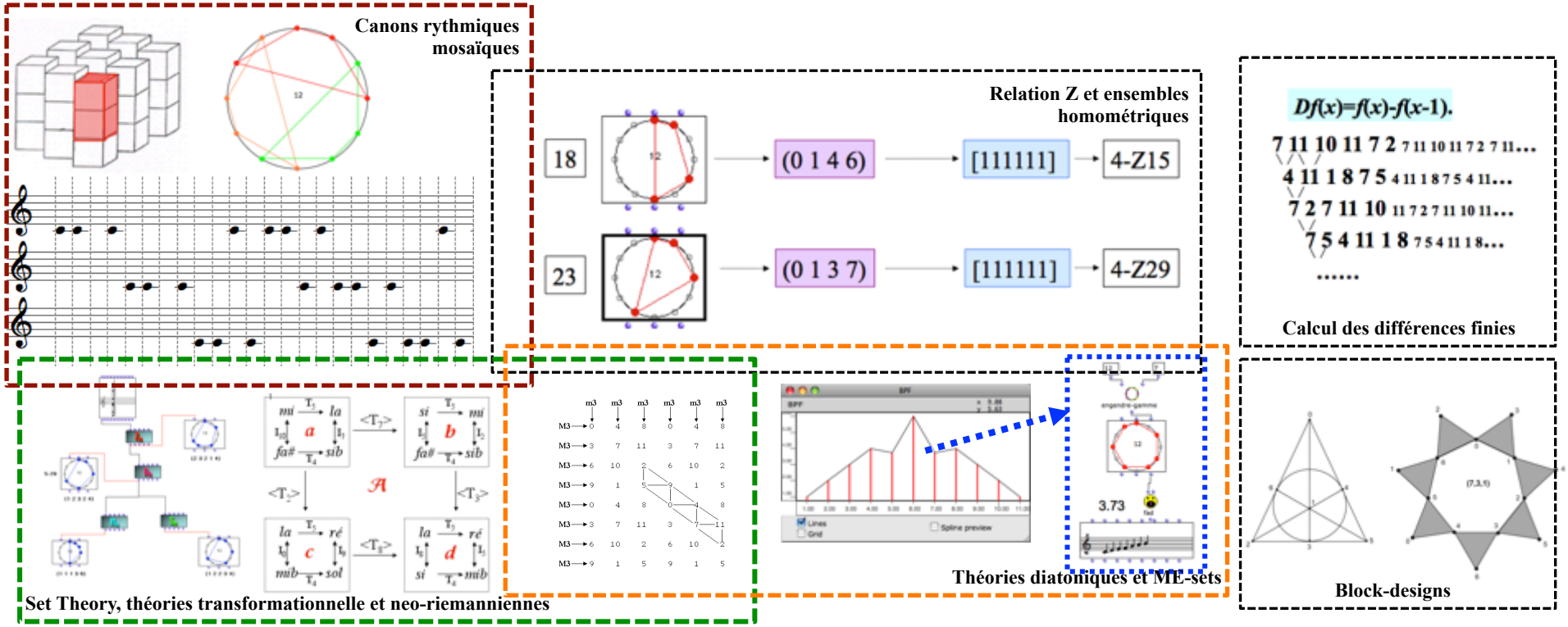


Petit catalogue d'objets mathémusicaux

[*Cf.* M. Andreatta : *Mathematica est exercitium musicae*, HDR, octobre 2010]

1. La construction des canons rythmiques mosaïques : de Minkowski à Fuglede
2. La relation *Z* et la théorie des ensembles homométriques
3. La *Set Theory* et la théorie transformationnelle
4. Les théories diatoniques et les *ME-sets*
5. Suites périodiques et calcul de différences finies
6. La théorie des block-designs en composition algorithmique
7. Modèles algébriques et catégoriels pour la cognition musicale



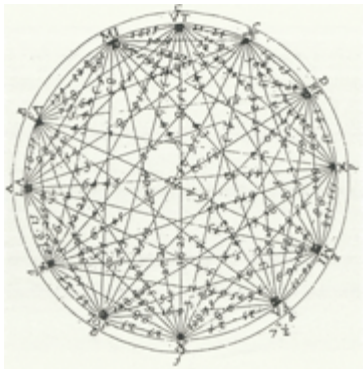


Musique et mathématiques : une longue histoire…

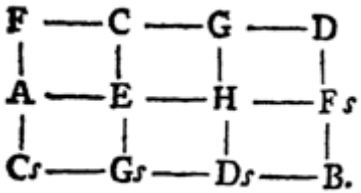
MUSIQUE	MATHS
500 av. J. C. Relation hauteur/longueur corde. La musique est source d'inspiration pour la théorie des nombres et la géométrie.	Nombres naturels et rationnels
300 a.J. Invention (théorique) de la gamme chromatique tempérée égale par Atistoxène de Tarente) et **prémonition de la théorie des groupes**. Isomorphismes entre les logarithmes (intervalles musicaux) et les exponentiels (longueur d'une corde)	*Aucune relation.*
1000 Invention de la représentation bidimensionnelle des hauteurs	*Aucune correspondance*
1500 *Aucune reprise des concepts précédents*	Nombres négatifs. Construction des rationnels
1600 *Aucune relation*	Nombres réels et les logarithmes
Marin Mersenne (1588-1648) : combinatoire musicale	Calcul des probabilités
1700 La fugue comme un automate abstrait. Manipulation inconsciente du groupe de Klein	Nombres complexes (Euler, Gauss), les quaternions (Hamilton), continuité (Cauchy), structure de groupe (Galois, Abel)
Leonhard Euler : *Speculum Musicum* (1773)	Théorie des graphes
1900 Libération de la prison de la tonalité (Loquin, Hauer, Schoenberg)	Nombres infinis et transfinis (Cantor). Axiomatique de Peano. Théorie de la mesure (Lebesgue, Borel)
1920 Formalisation radicale des macrostructures à travers le système sériel (Schoenberg)	*Aucun développement de la théorie des nombres.*
Ernst Krenek (1900-1991) : les axiomes dans le système dodécaphonique	*David Hilbert, Les fondements de la géométrie (1899)*

Iannis Xenakis, *Musique. Architecture*, Tournai, Casterman, 1971, (New, revised edition: Tournai, Casterman, 1976)

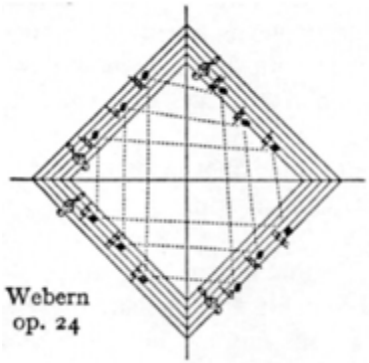
Pythagore et le monocorde, VIe-Ve siècle av. J. C.

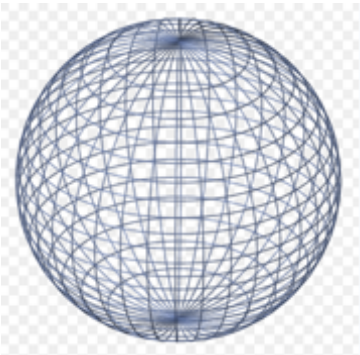


Mersenne, *Harmonicorum Libri* XII, 1648

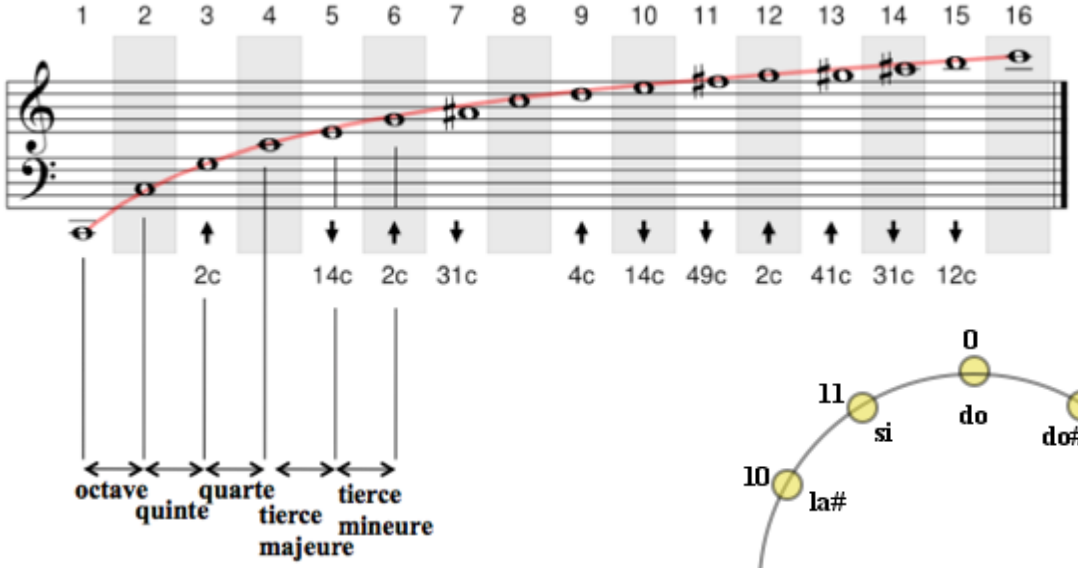


Euler : *Speculum musicum*, 1773



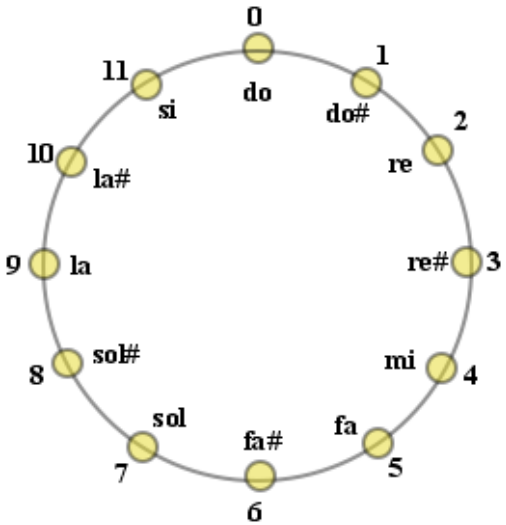


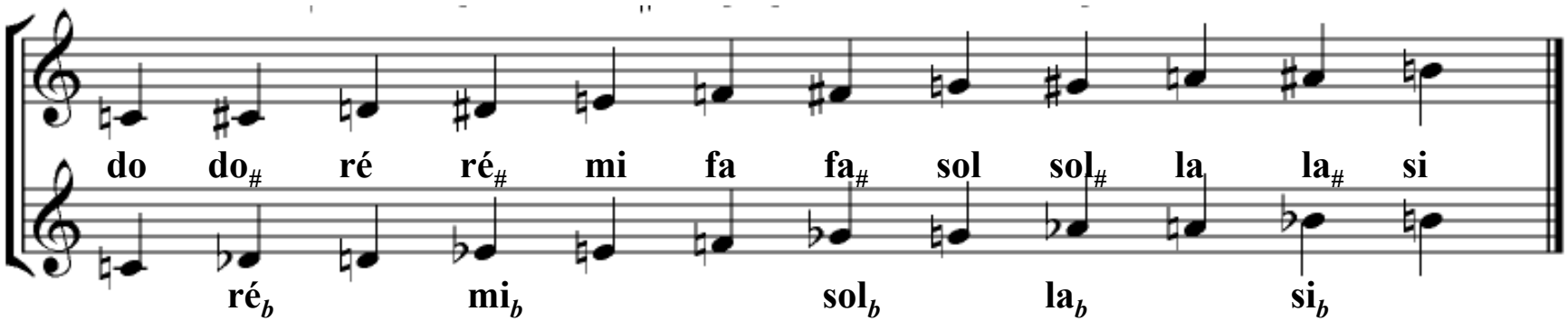
De Pythagore aux harmoniques au cercle chromatique



Physique

Mathématiques





Marin Mersenne et la naissance de la combinatoire musicale

114 **Marin Mersenne, *Harmonicorum Libri XII*, 1648**

LIBER SEPTIMVS.

DE CANTIBVS, SEV CANTILENIS, EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 12.

I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6127020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000

HARMONICORVM LIBRI XII

IN QVIBVS AGITVR

DE SONORVM NATVRA, CAVSIS, ET EFFECTIBVS: DE CONSONANTIIS, Dissonantiis, Rationibus, Generibus, Modis, Cantibus, Compositione, orbisque totius Harmonicis Instrumentis.

Authore F. M. MERSENNO Minimo.

Ad Illustr. V. HENRICVM LVDOVICVM HABERTVM DE MONTMOR.

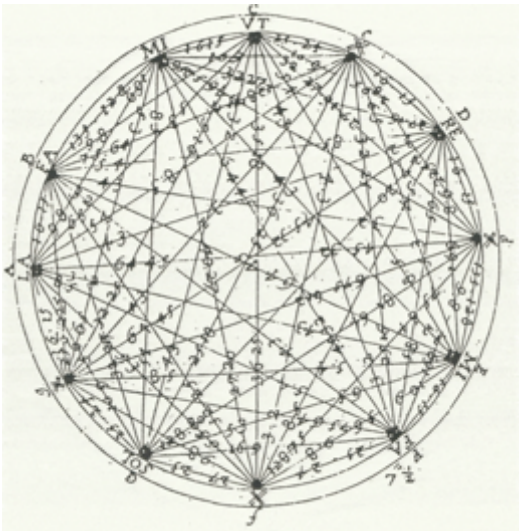
Laudate eum in cymbalis benesonantibus, laudate eum in cymbalis iubilationis: Omnis spiritus laudet Dominum. PSALM. 150.

EDITIO AVCTA.

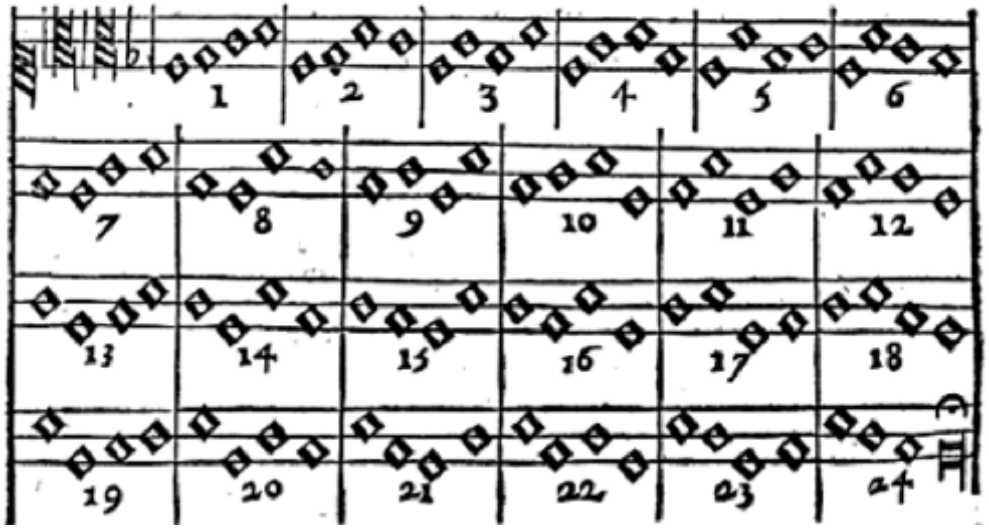
LVTETIÆ PARISIORVM.
Sumptibus GVILLELMI BAVDRY, viâ Iacobæâ, prope Collegium Plessæum.

M. DC. XLVIII.

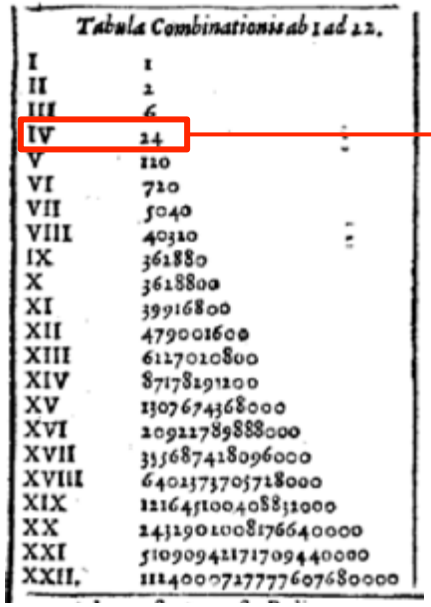
Cum Priuilegio Regis Christianiss. & Approbatione Superiorum.



Varietas, seu Combinatio quatuor notarum.

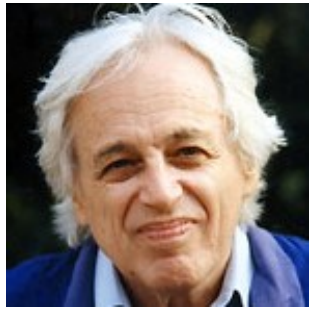


Quatre notes peuvent parfois suffire…

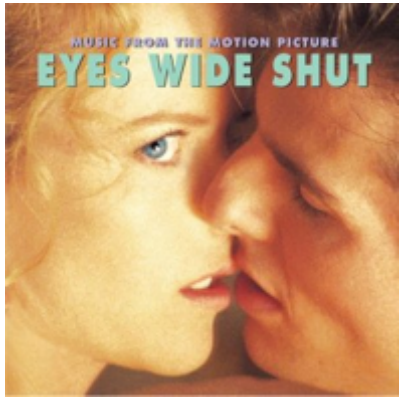


Tabula Combinationis ab 1 ad 22.

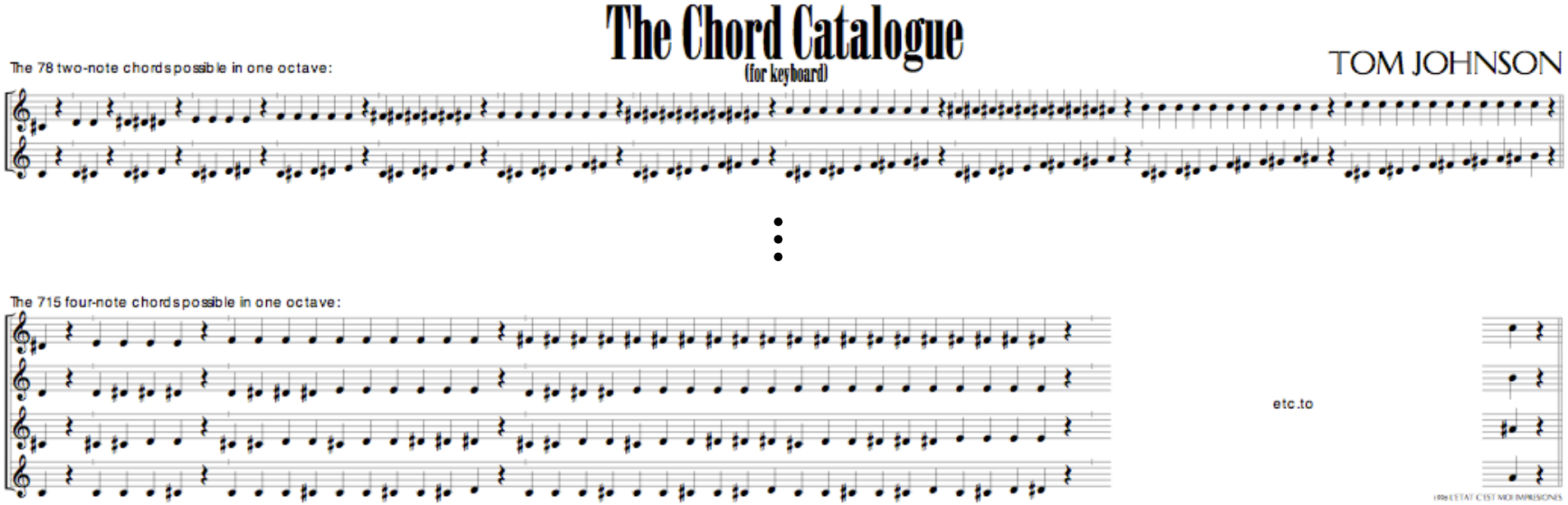
I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6227020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000



Six Bagatelles
(G. Ligeti, 1953)

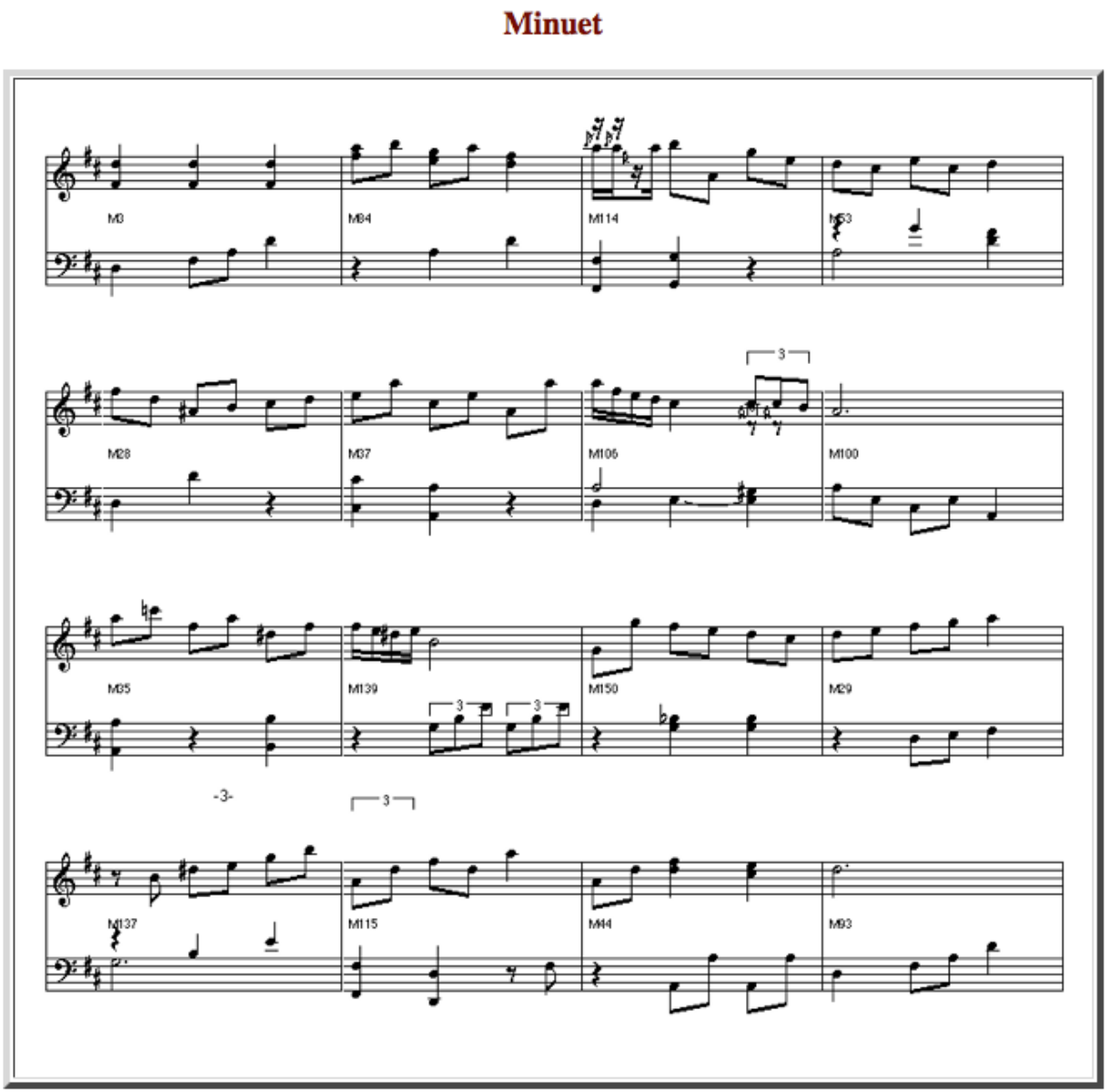


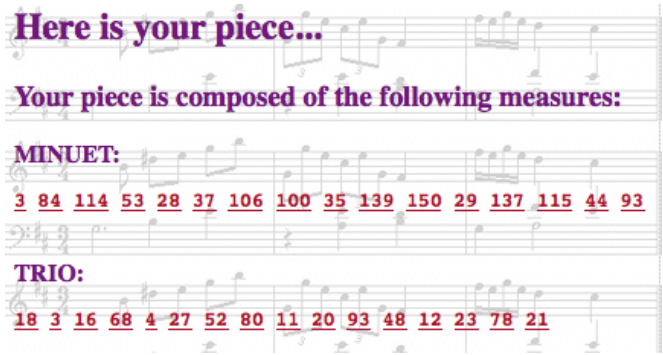
The Four Note Opera (T. Johnson, 1972)



Totale: 8178 accordi

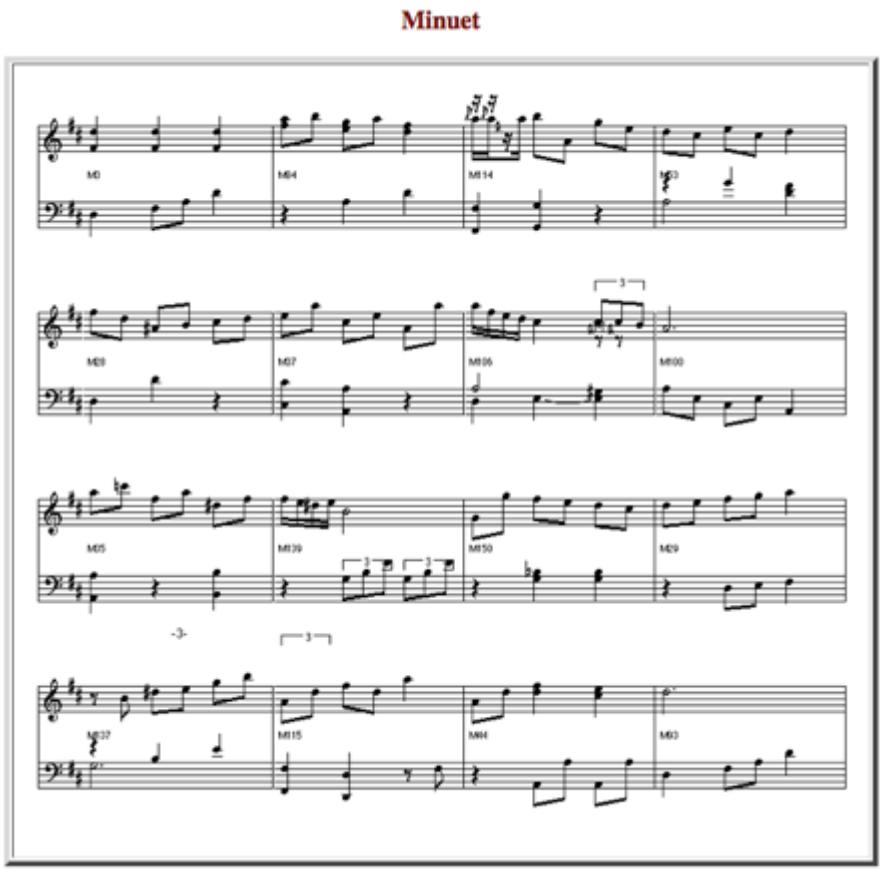
« Procédés algorithmiques » versus « musique algorithmique »

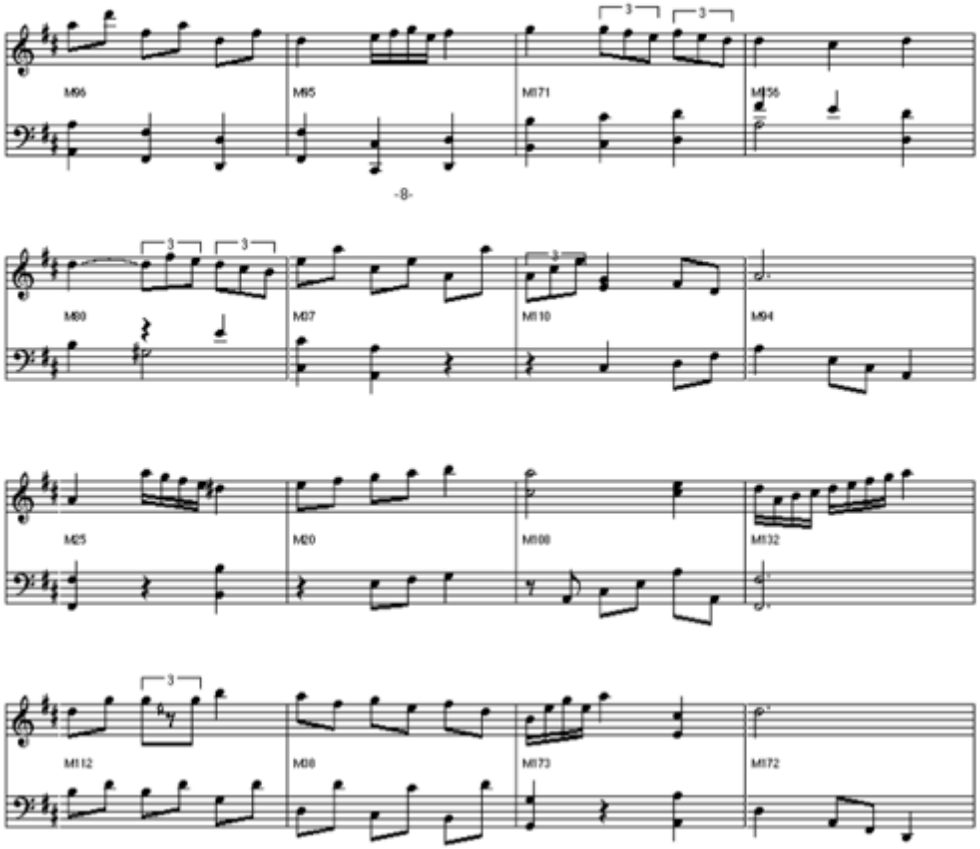




(http://sunsite.univie.ac.at/Mozart/dice/)

« Procédés algorithmiques » versus « musique algorithmique »





Here is your piece...

Your piece is composed of the following measures:

MINUET:

3 84 114 53 28 37 106 100 35 139 150 29 137 115 44 93

TRIO:

18 3 16 68 4 27 52 80 11 20 93 48 12 23 78 21

Here is your piece...

Your piece is composed of the following measures:

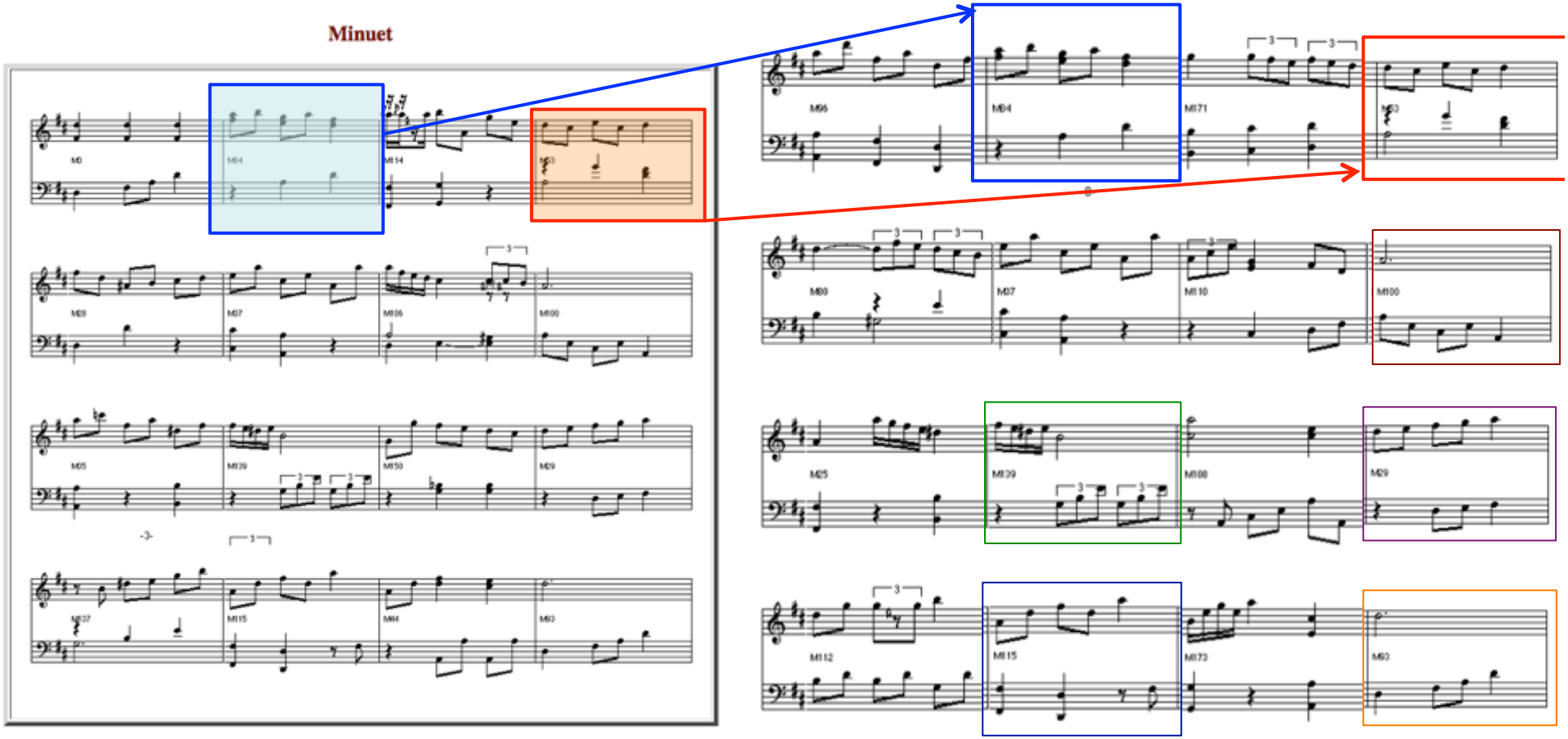
MINUET:

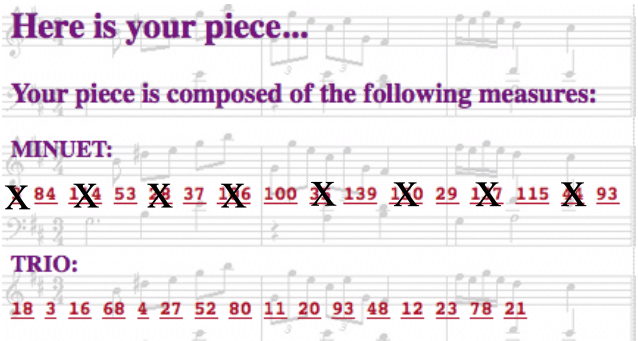
96 95 171 156 80 37 110 94 25 20 108 132 112 38 173 172

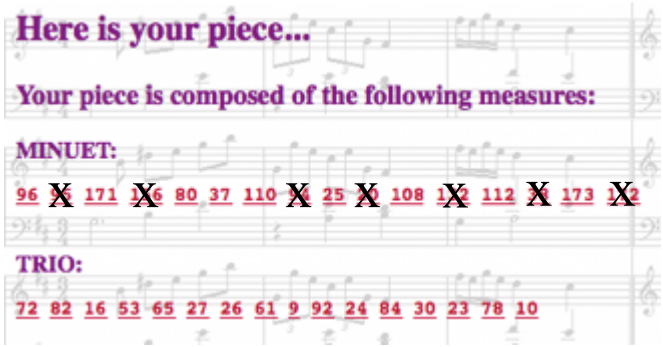
TRIO:

72 82 16 53 65 27 26 61 9 92 24 84 30 23 78 10

La substitution (ou copier-coller…)

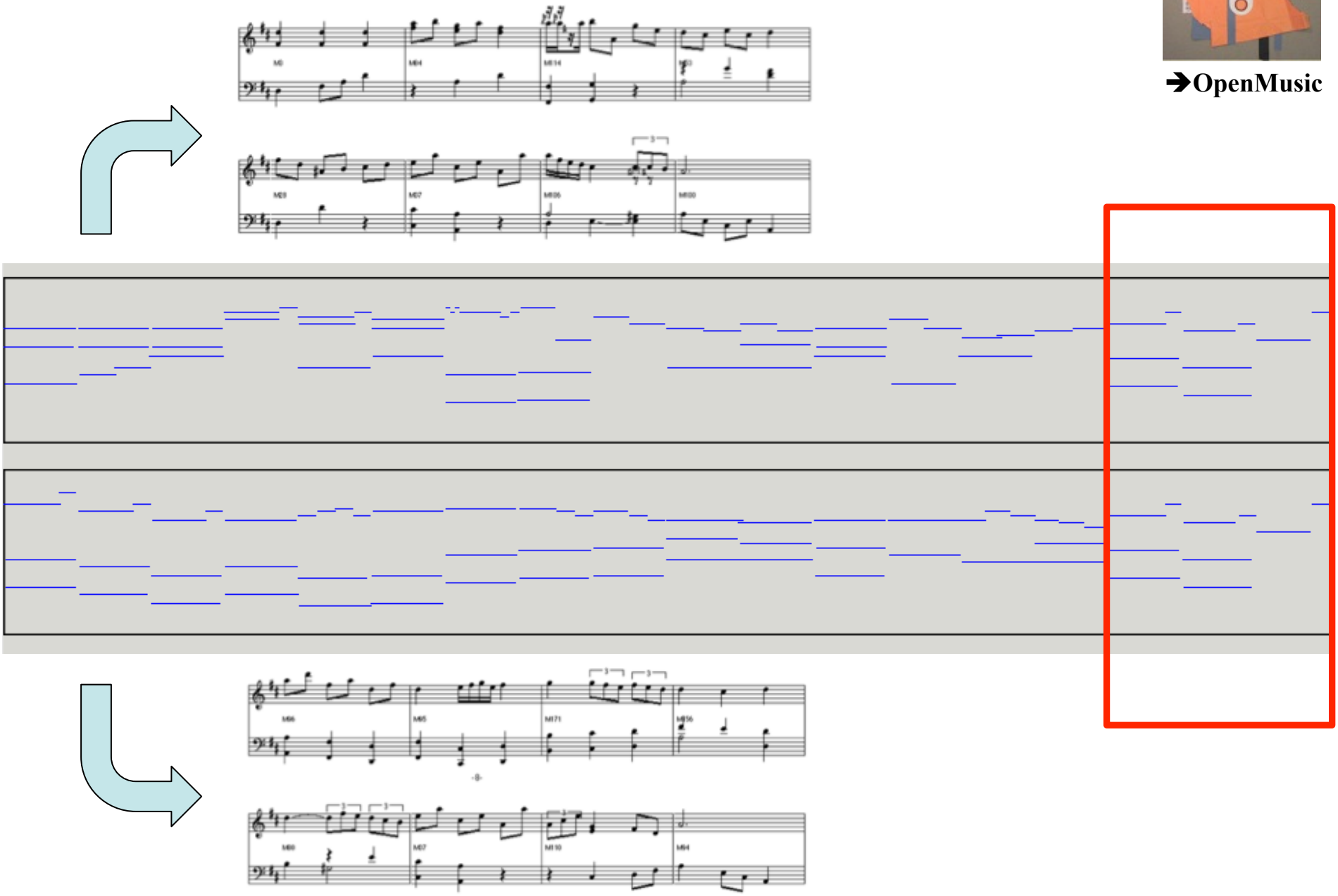






La superposition « non-cacophonique »

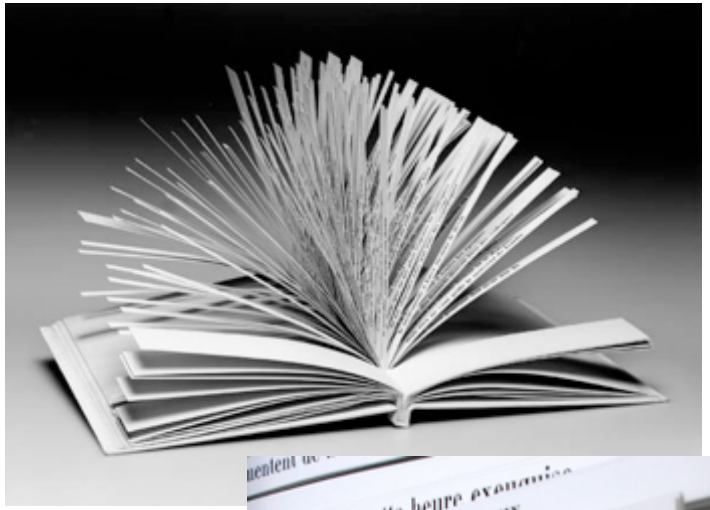
➔OpenMusic



Caractère combinatoire de la production artistique

L'Oulipo : « L'ouvroir de littérature potentielle »

« Ce petit ouvrage permet à tout un chacun de composer à volonté cent mille milliards de sonnets, tous réguliers bien entendu. C'est somme toute une sorte de machine à fabriquer des poèmes, mais en nombre limité ; il est vrai que ce nombre, quoique limité, fournit de la lecture pour près de deux cents millions d'années (en lisant vingt-quatre heures sur vingt-quatre) »
(Raymond Queneau, *Cent mille milliards de poèmes*, 1961)
➔ **`http://x42.com/active/queneau.html`**



Il Castello dei destini incrodiati (1969)

Raymond Queneau

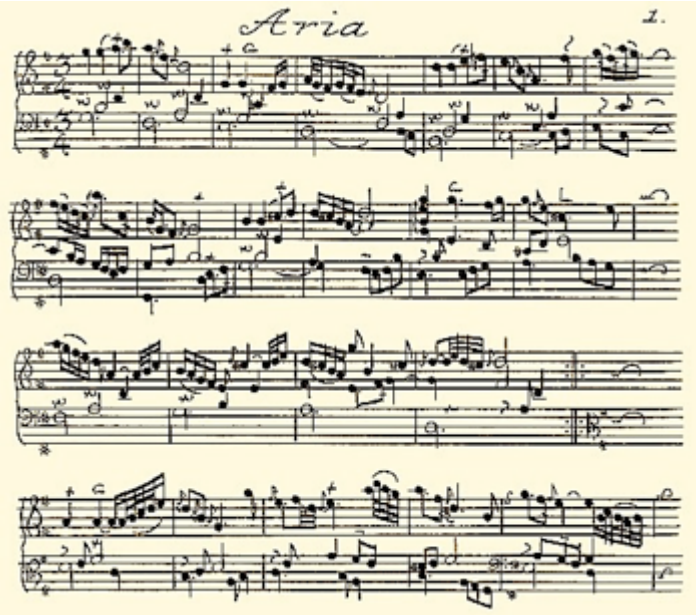
Georges Perrec

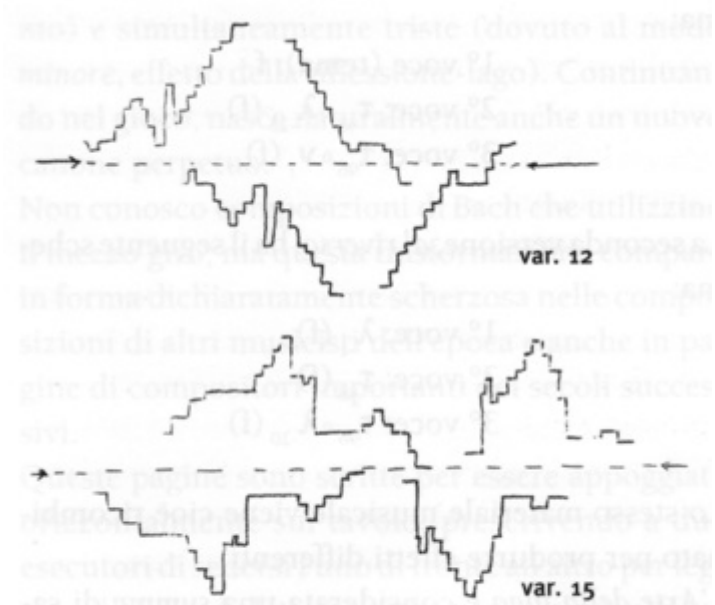
Italo Calvino

Contrepoint et transformations géométriques

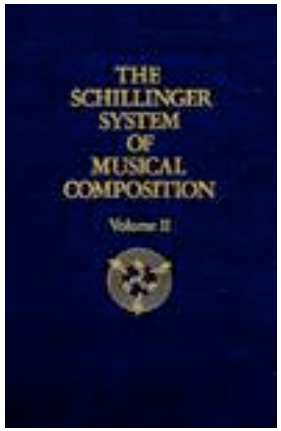
Benedetto Scimemi, « Contrappunto e trasformazioni geometriche », *Matematica e Cultura*, Springer, 1996

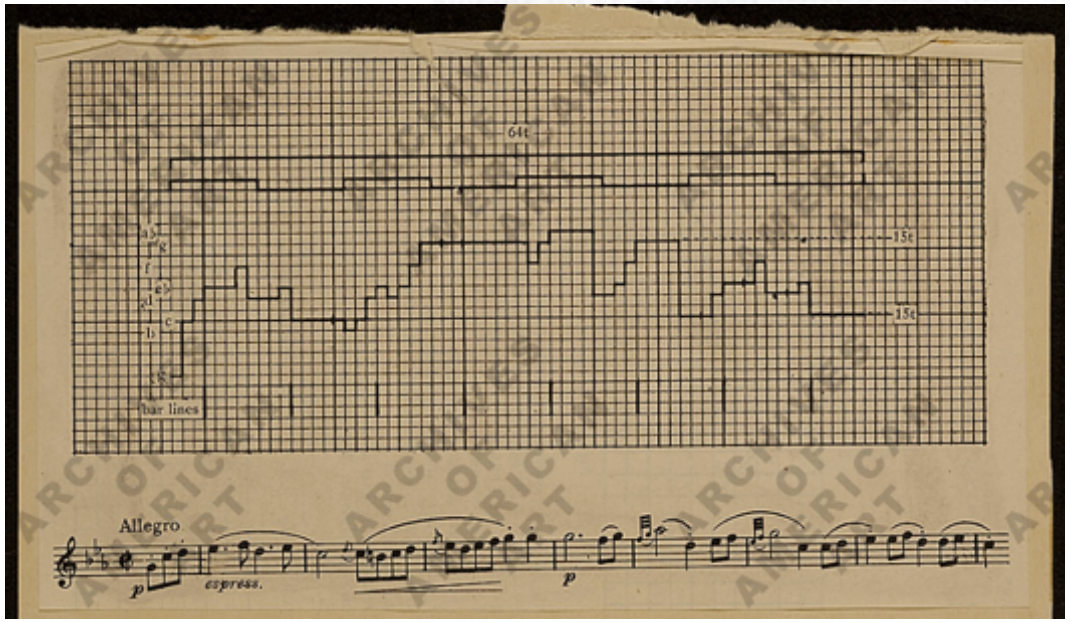
➔OpenMusic



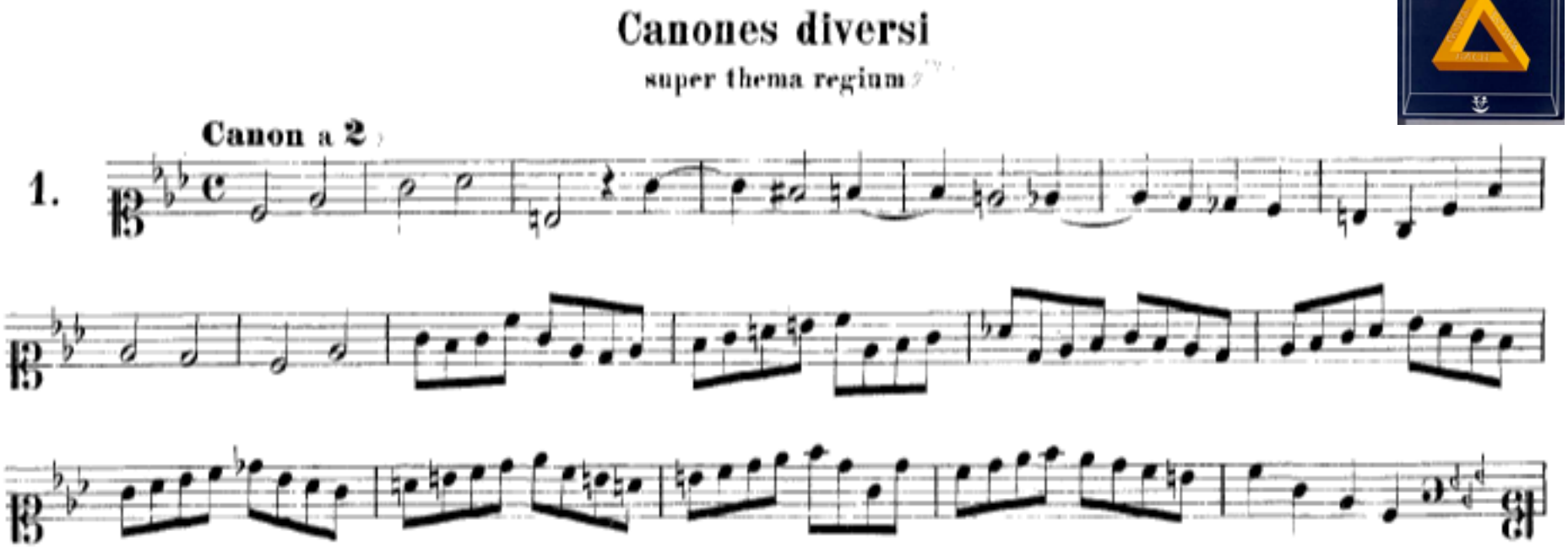


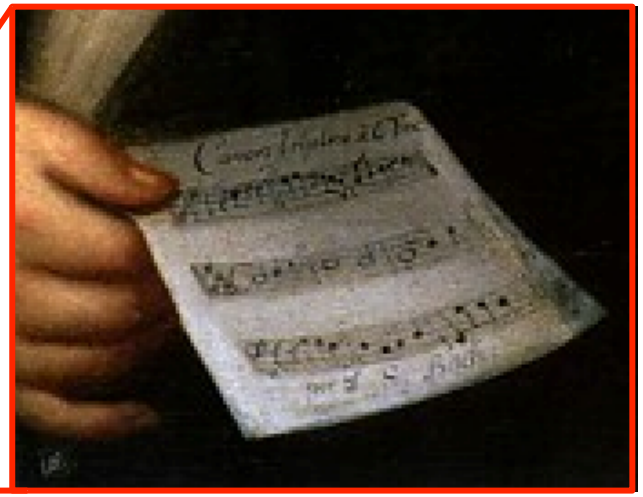
Joseph Schillinger

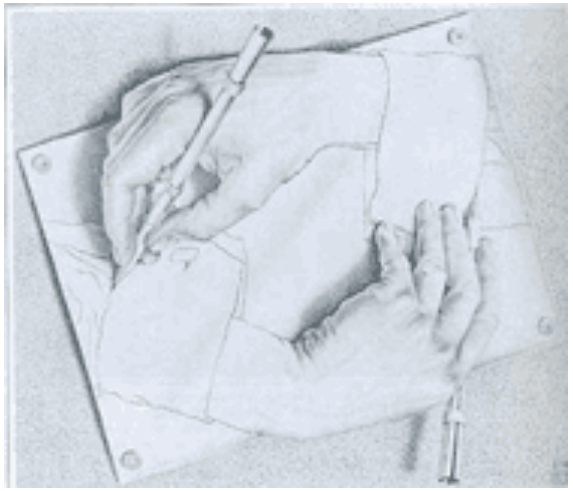




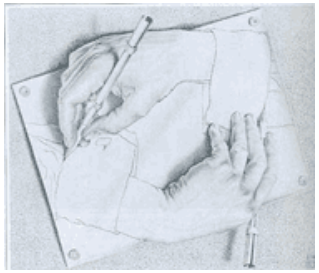
Canons énigmatiques chez Bach et géométrie

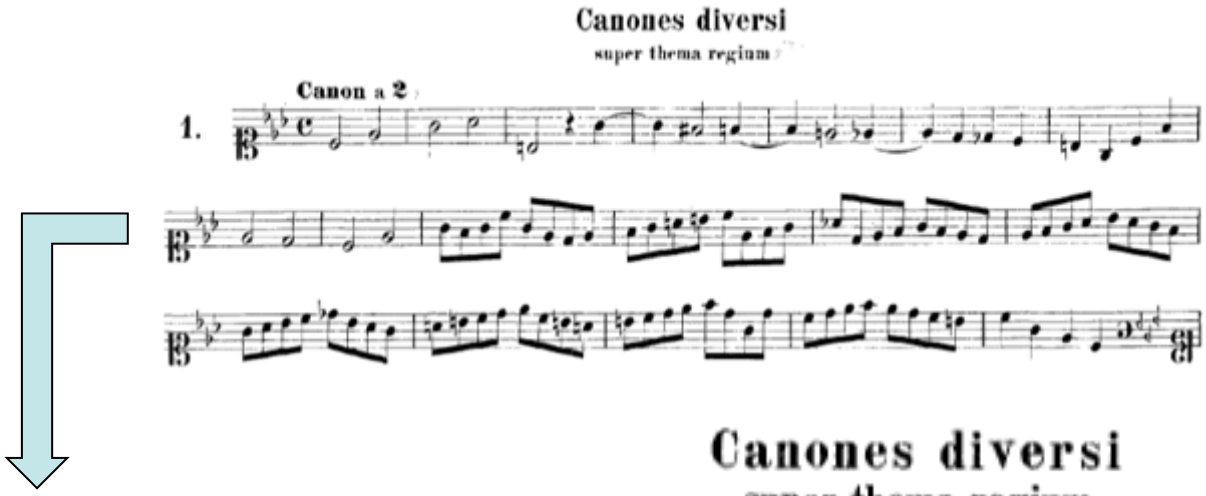


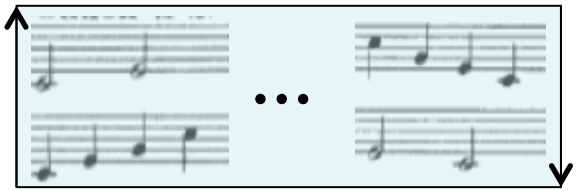


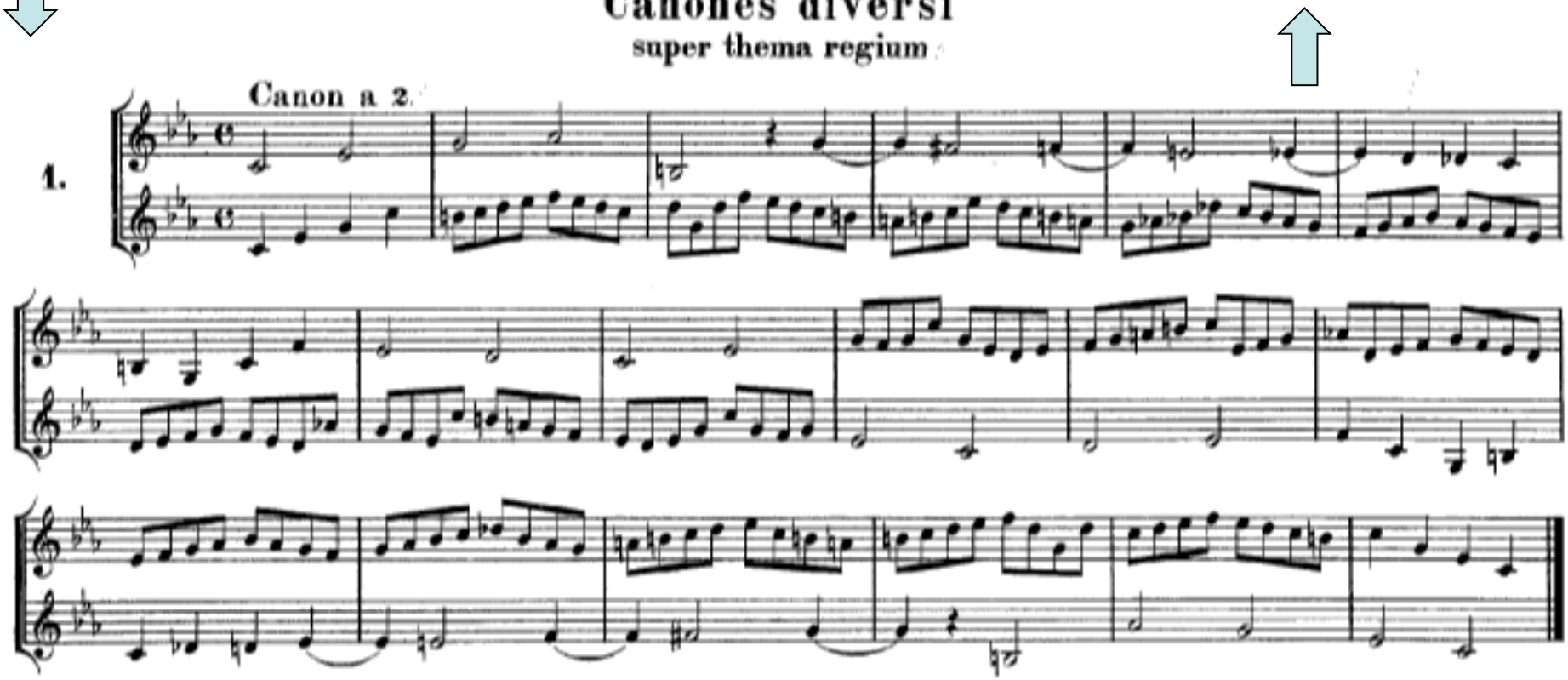


Ma fin est mon début (renversé !)

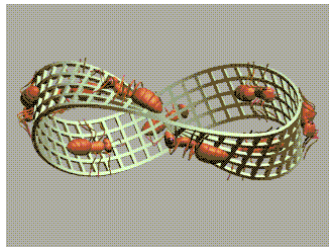


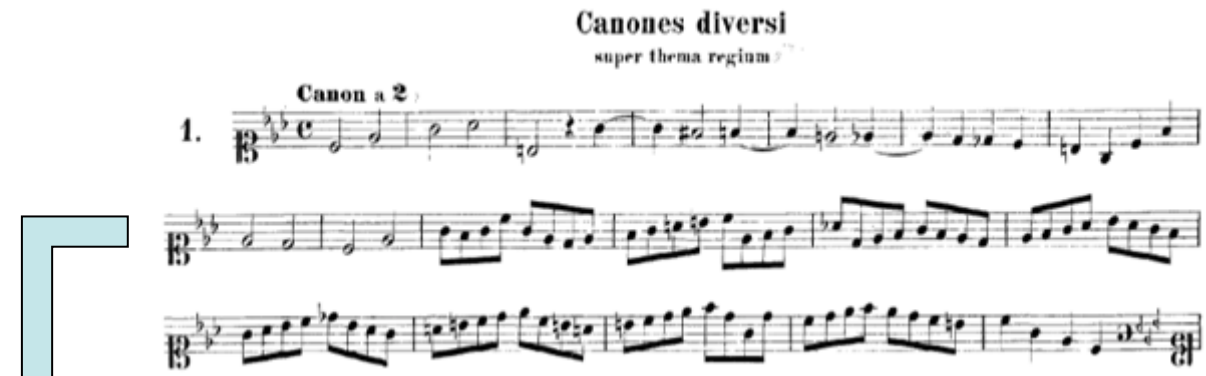


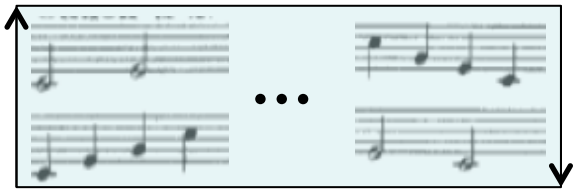


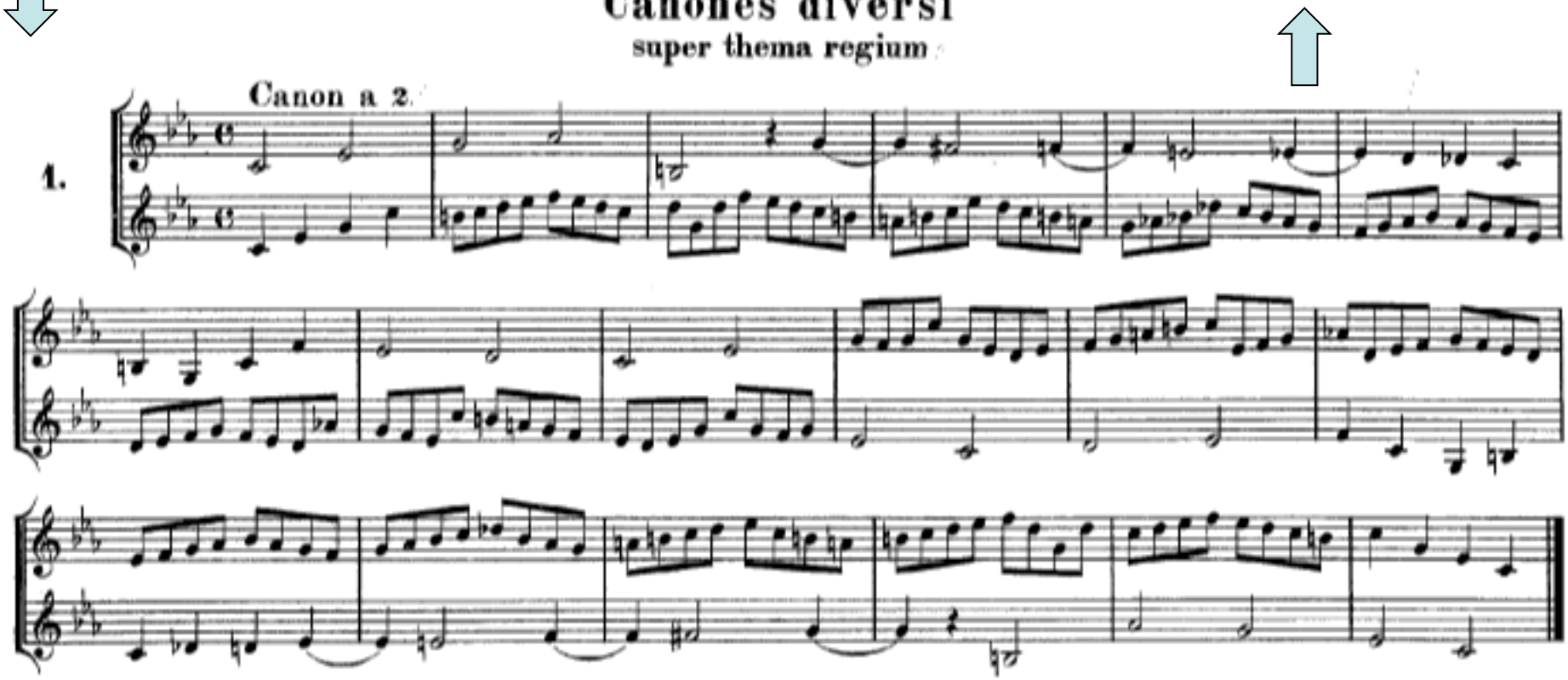


Le ruban de Moebius ou la fourmi musicienne









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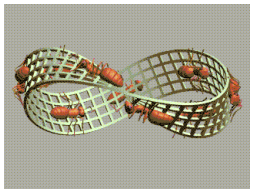
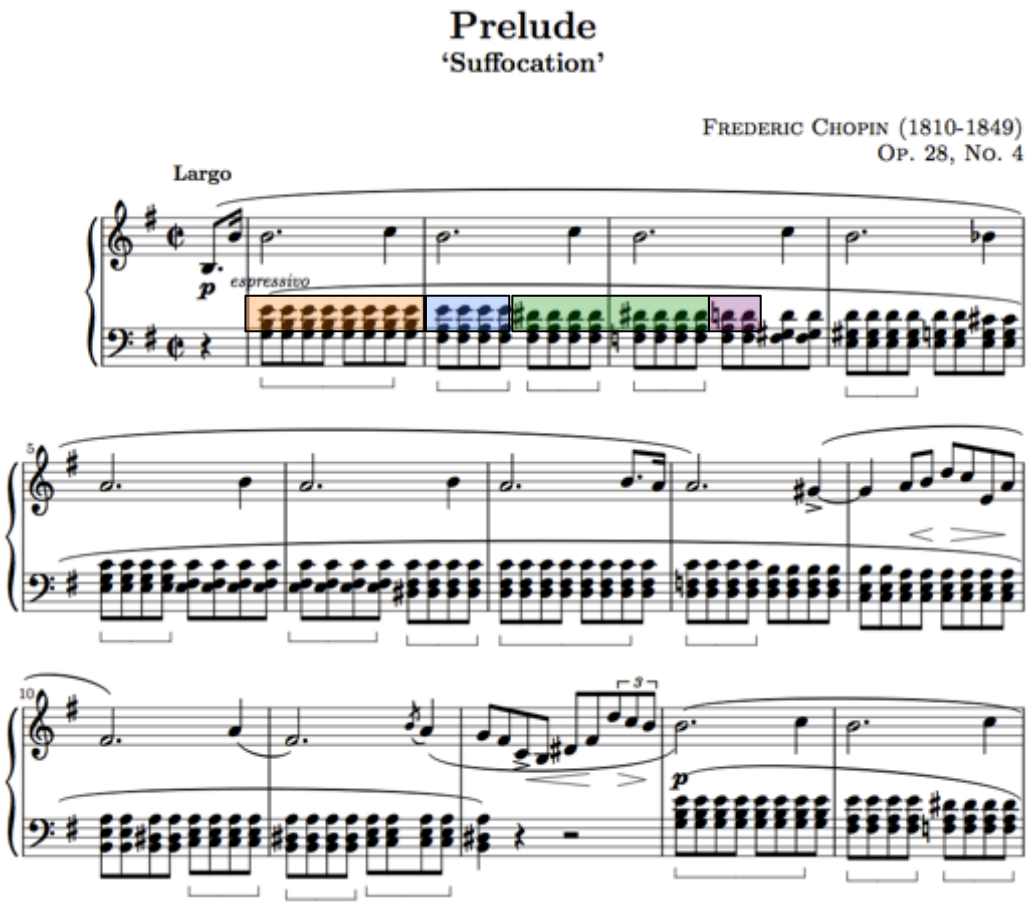
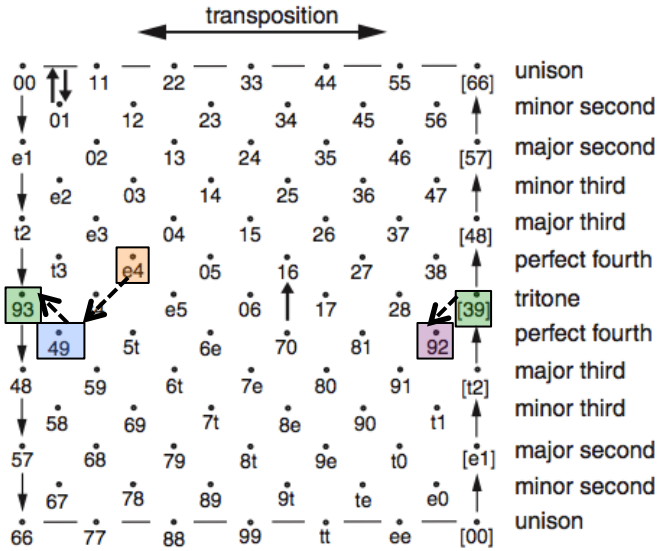
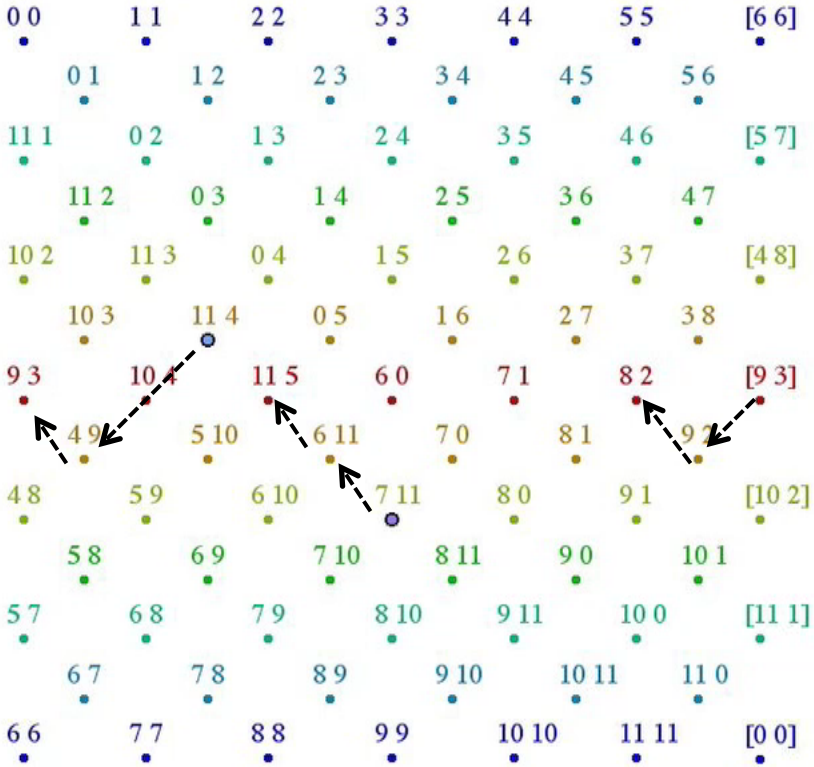


http://www.josleys.com/Canon/Canon.html

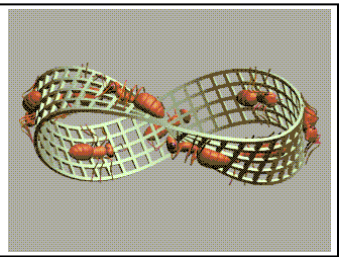
$$T^2 = \mathbf{R/12Z} \times \mathbf{R/12Z} \longrightarrow T^2/S_2$$



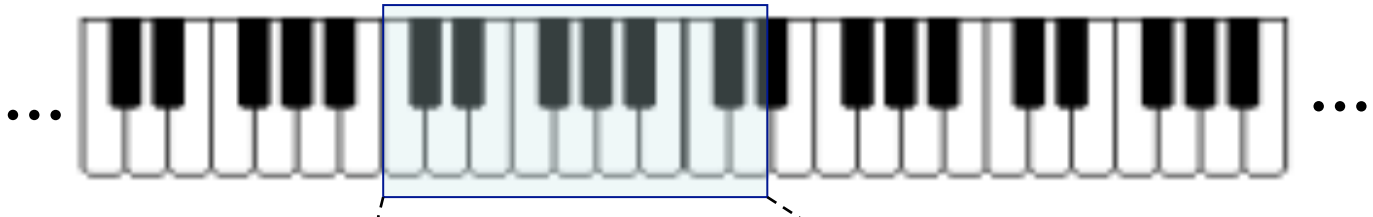


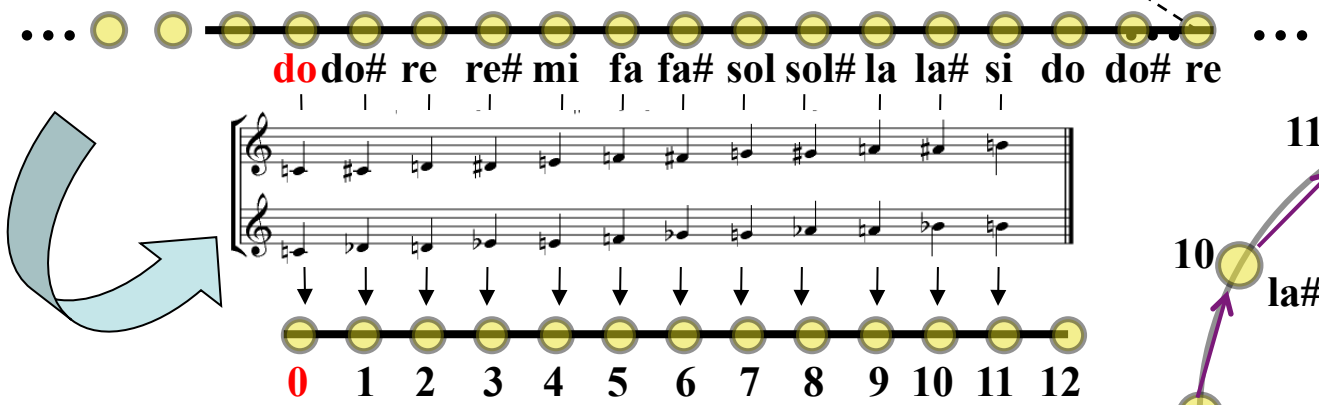


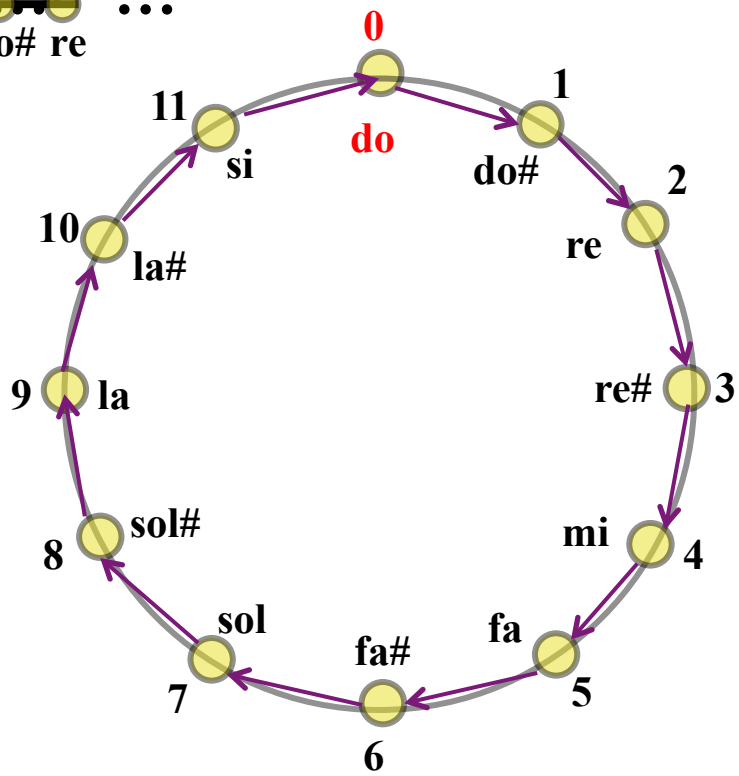
Dmitri Tymoczko : « The Geometry of Musical Chords », *Science*, 313, 2006



Reduction à l'octave et congruence modulo 12



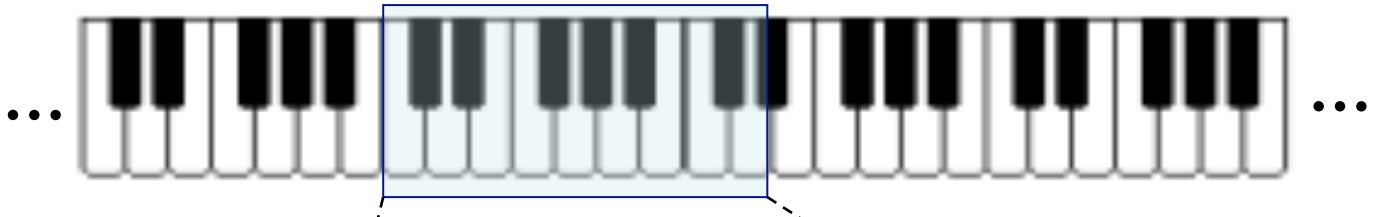




1

➔ Quels sont les autres entiers qui engendrent le cercle ?

Reduction à l'octave et congruence modulo 12



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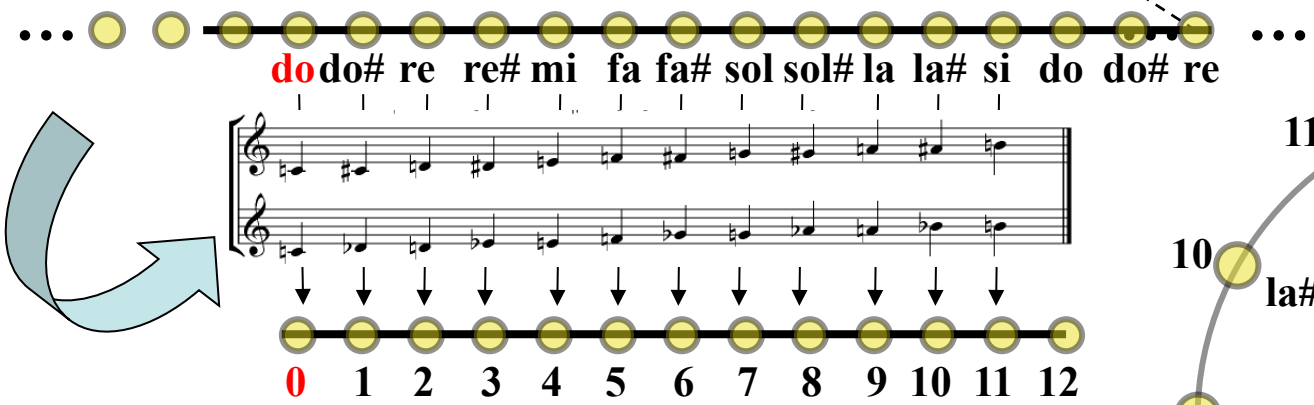

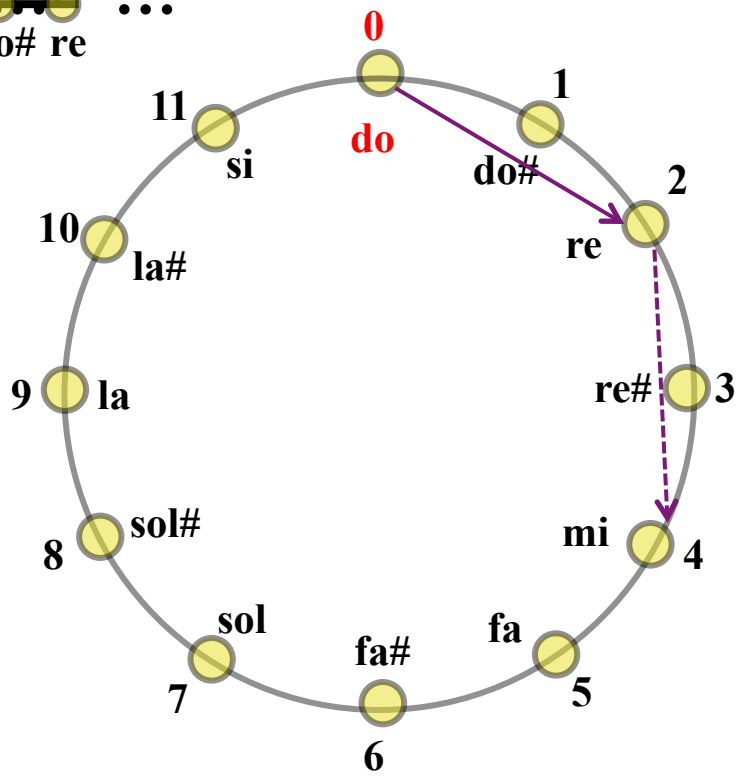

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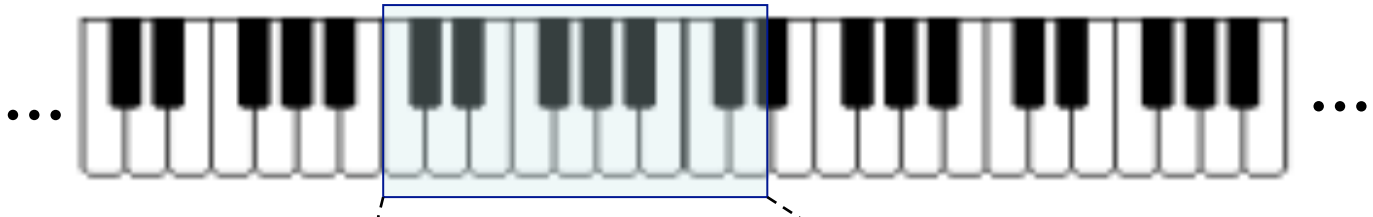
Reduction à l'octave et congruence modulo 12



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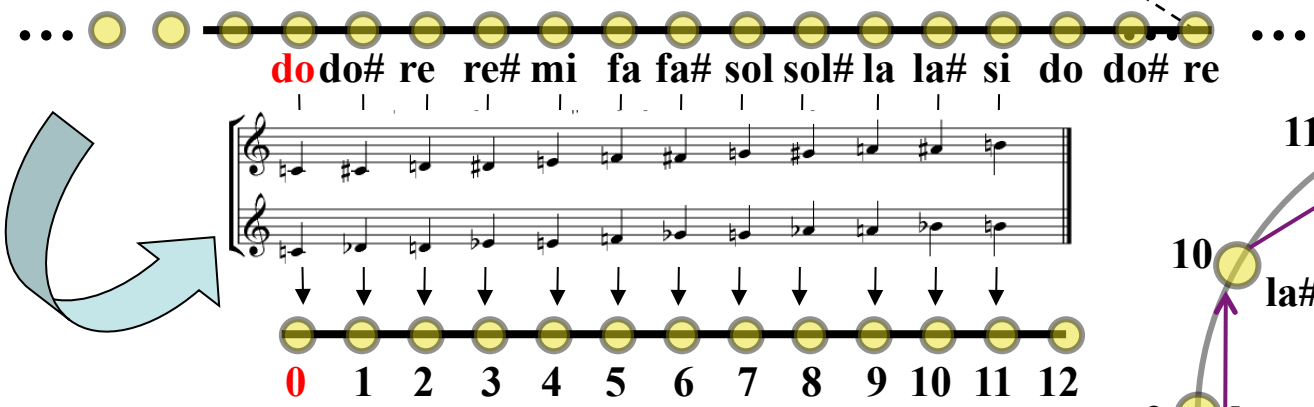

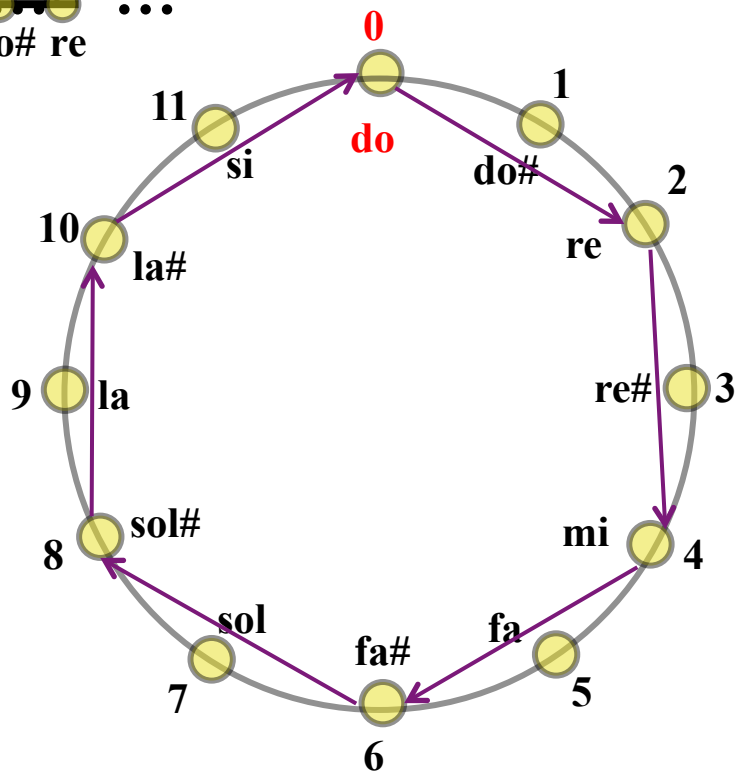

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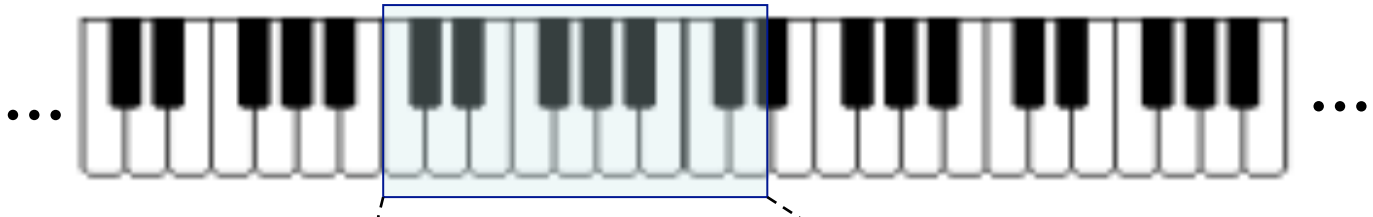

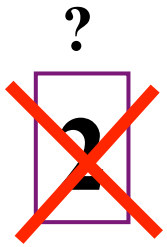

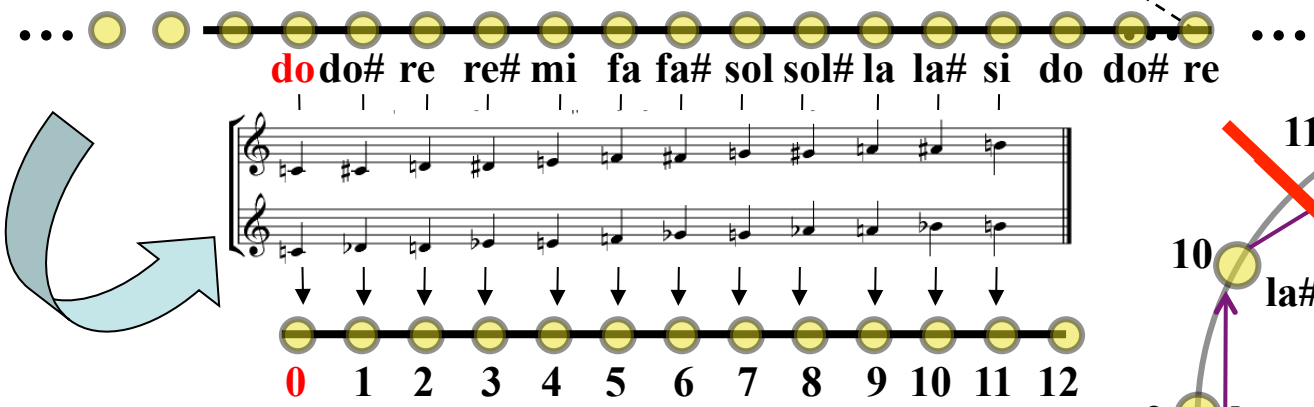

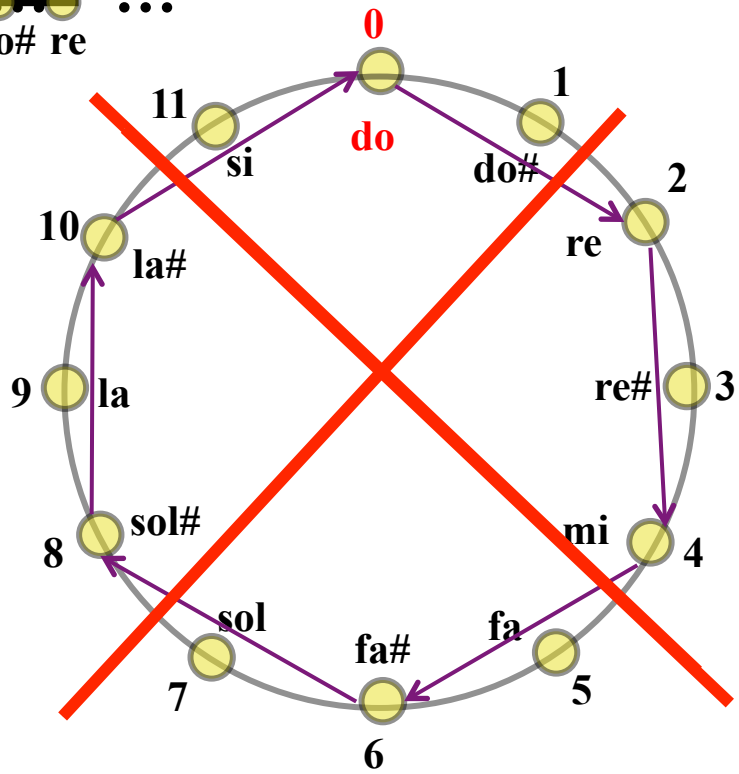

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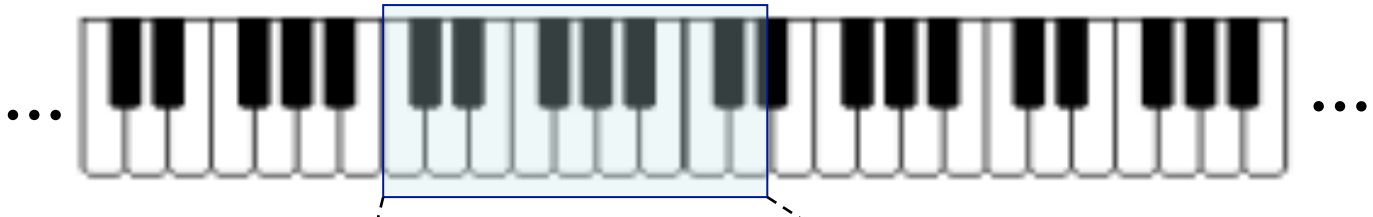

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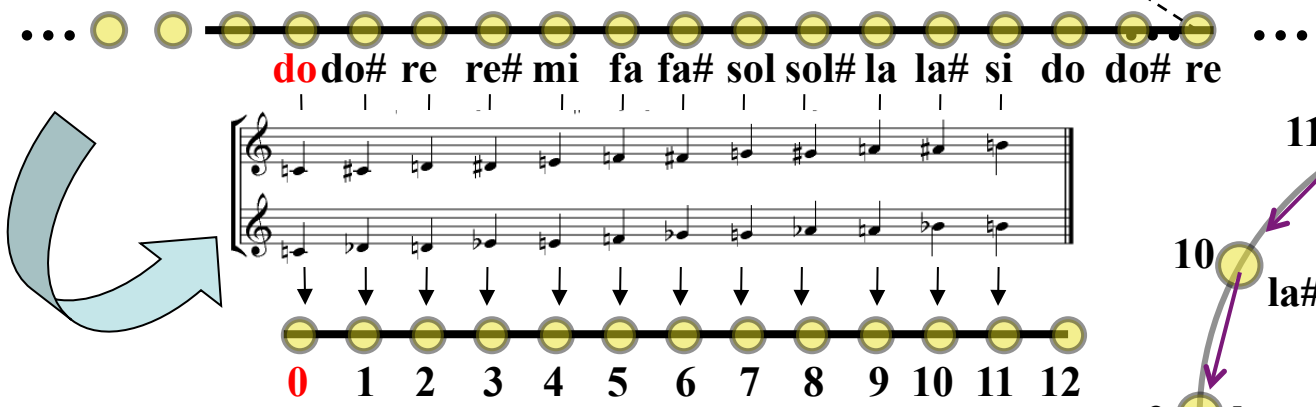

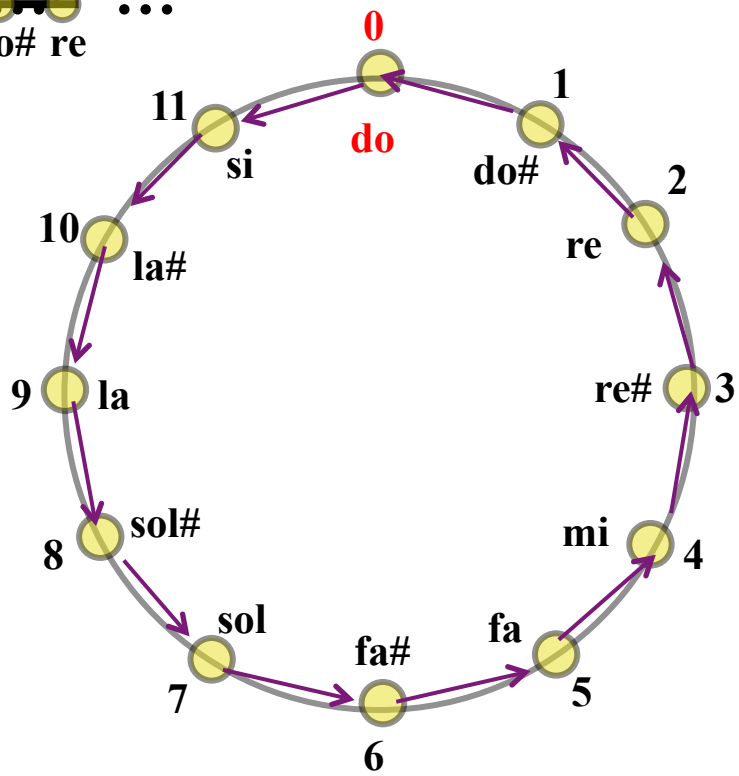

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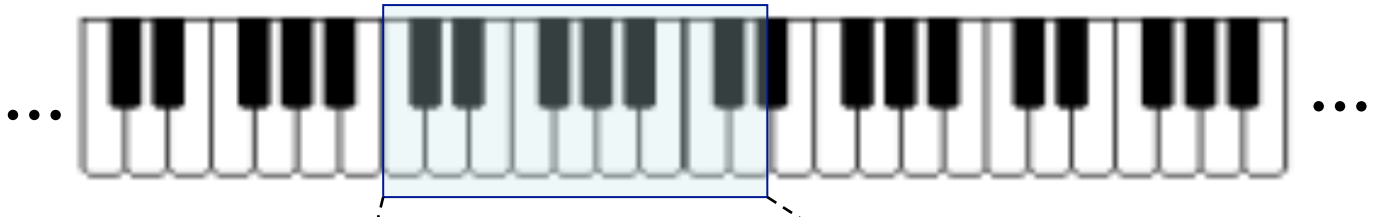

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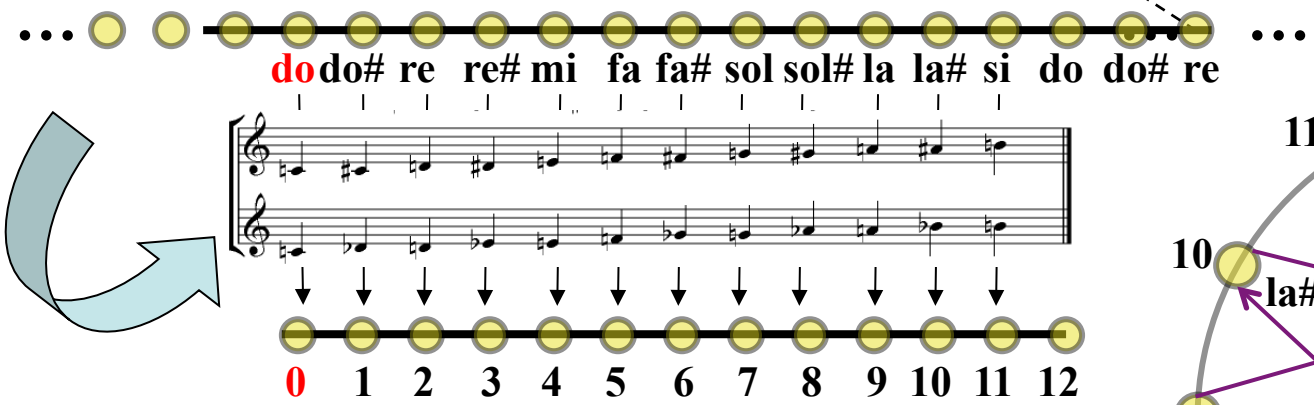

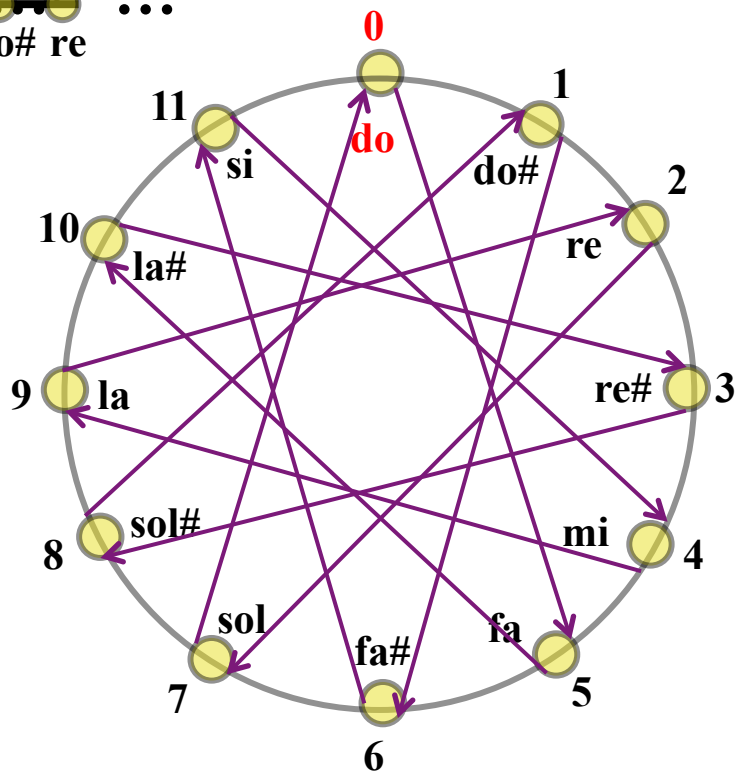

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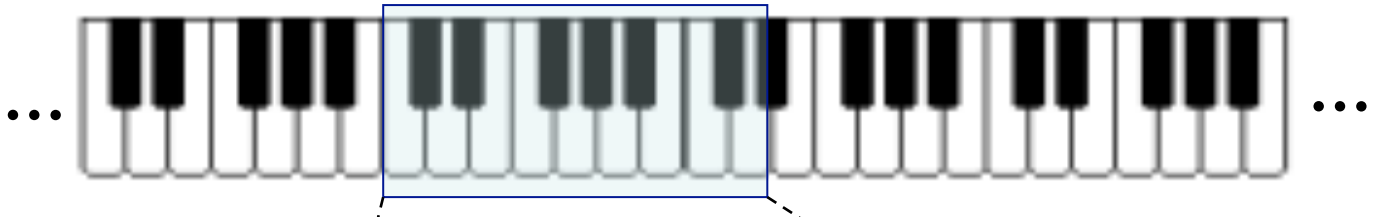

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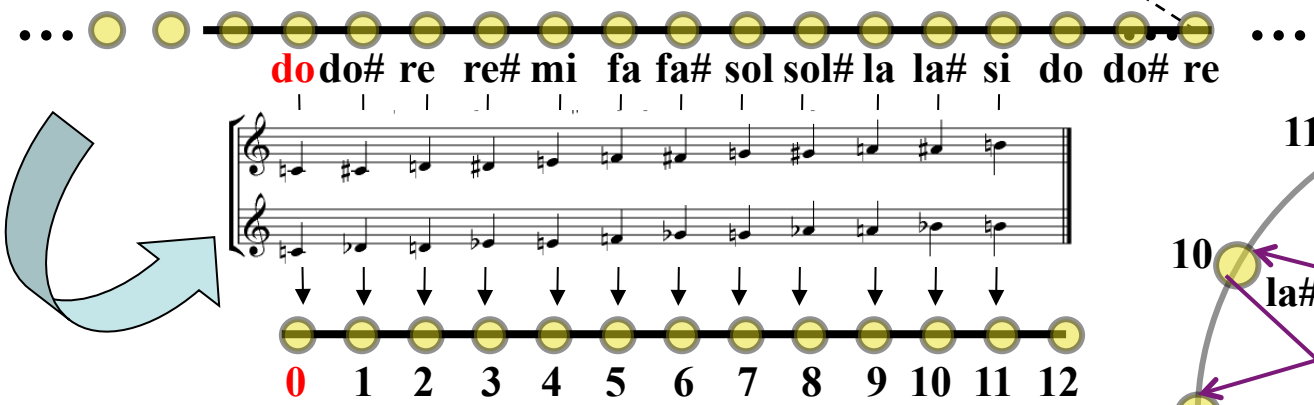

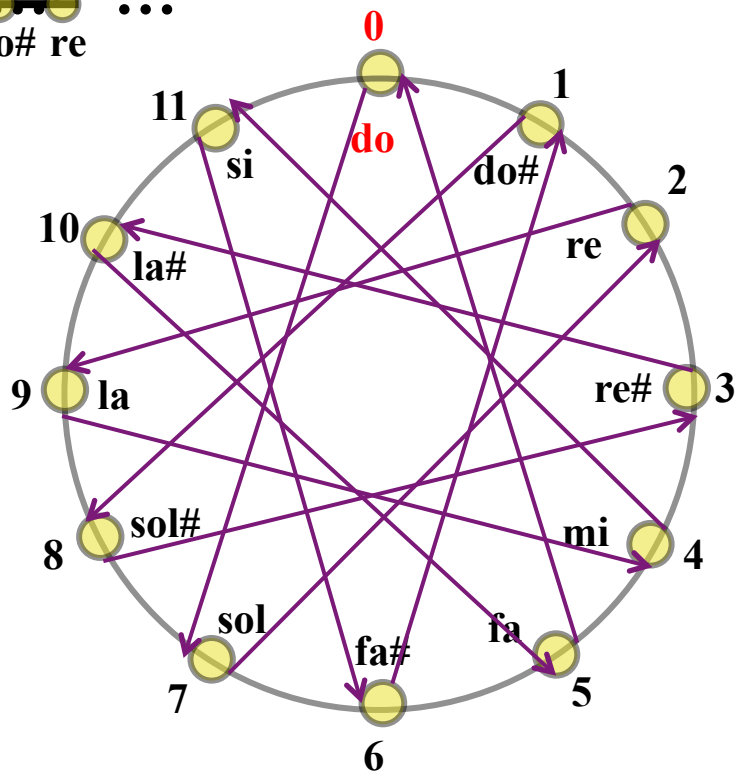

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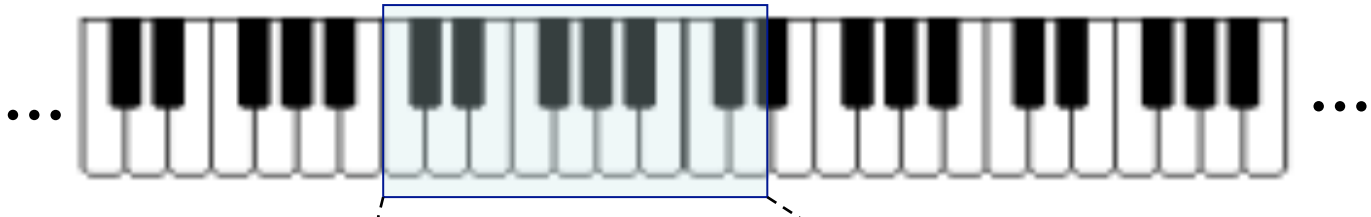
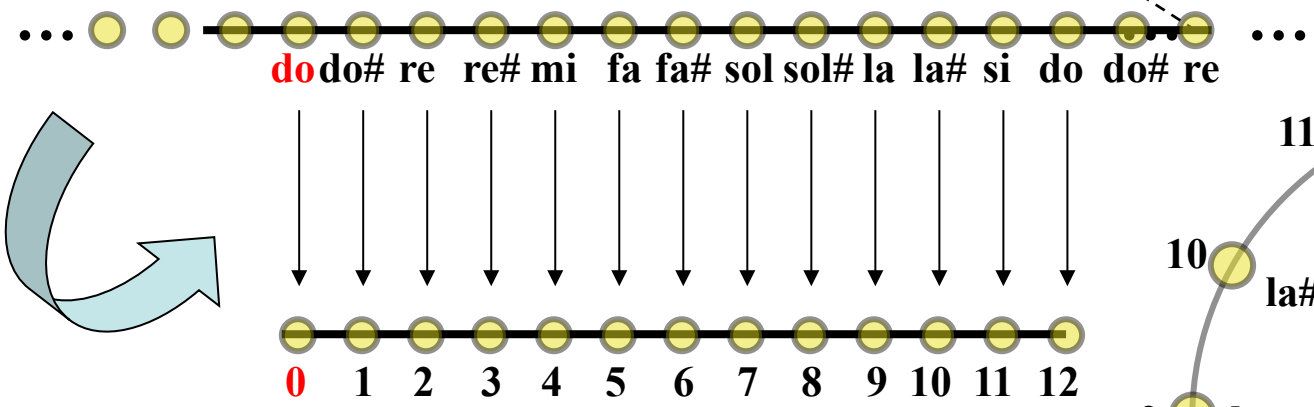
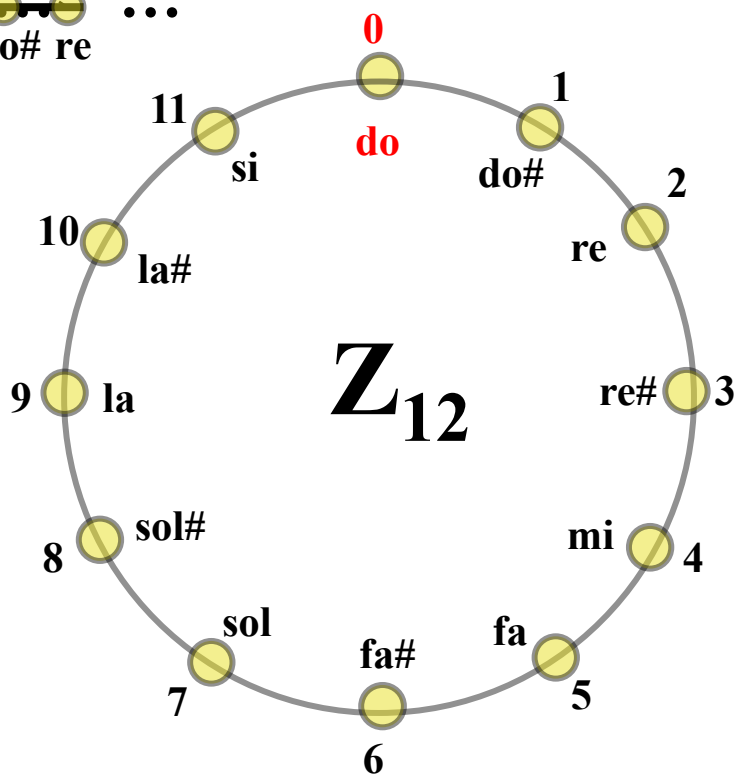


$$\mathbf{Z_{12}} = < T_k \mid (T_k)^{12} = T_0 >$$

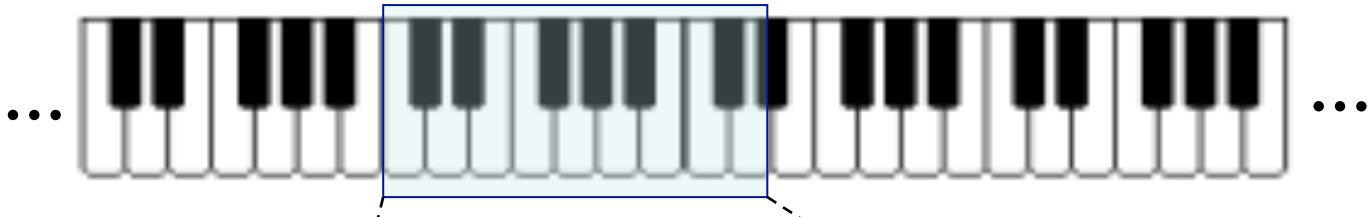
$T_k : x \rightarrow x+k \bmod 12$





➔ **Les générateurs du groupe cyclique d'ordre 12 sont les transpositions T_1, T_5, T_7 et T_{11}**

Reduction à l'octave et congruence modulo 12



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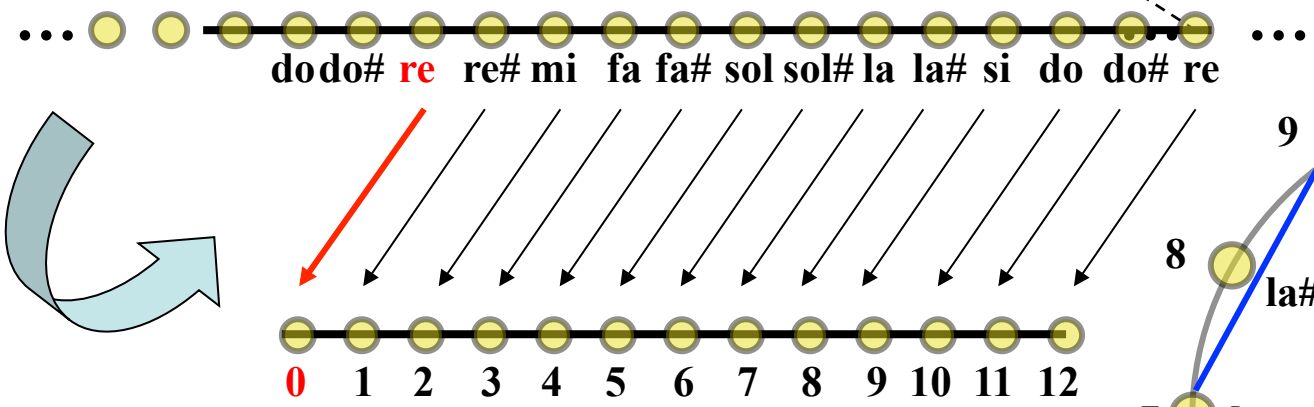

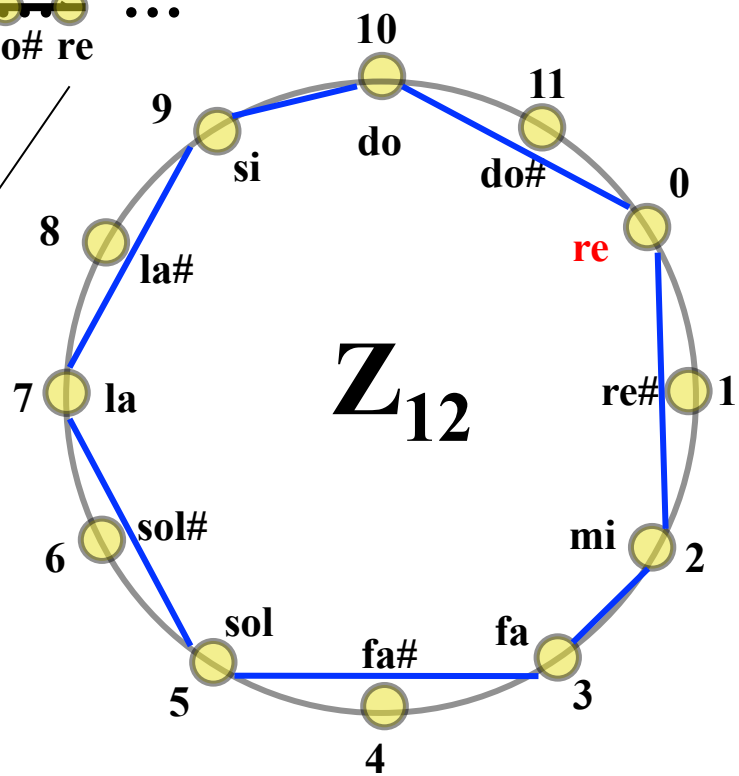

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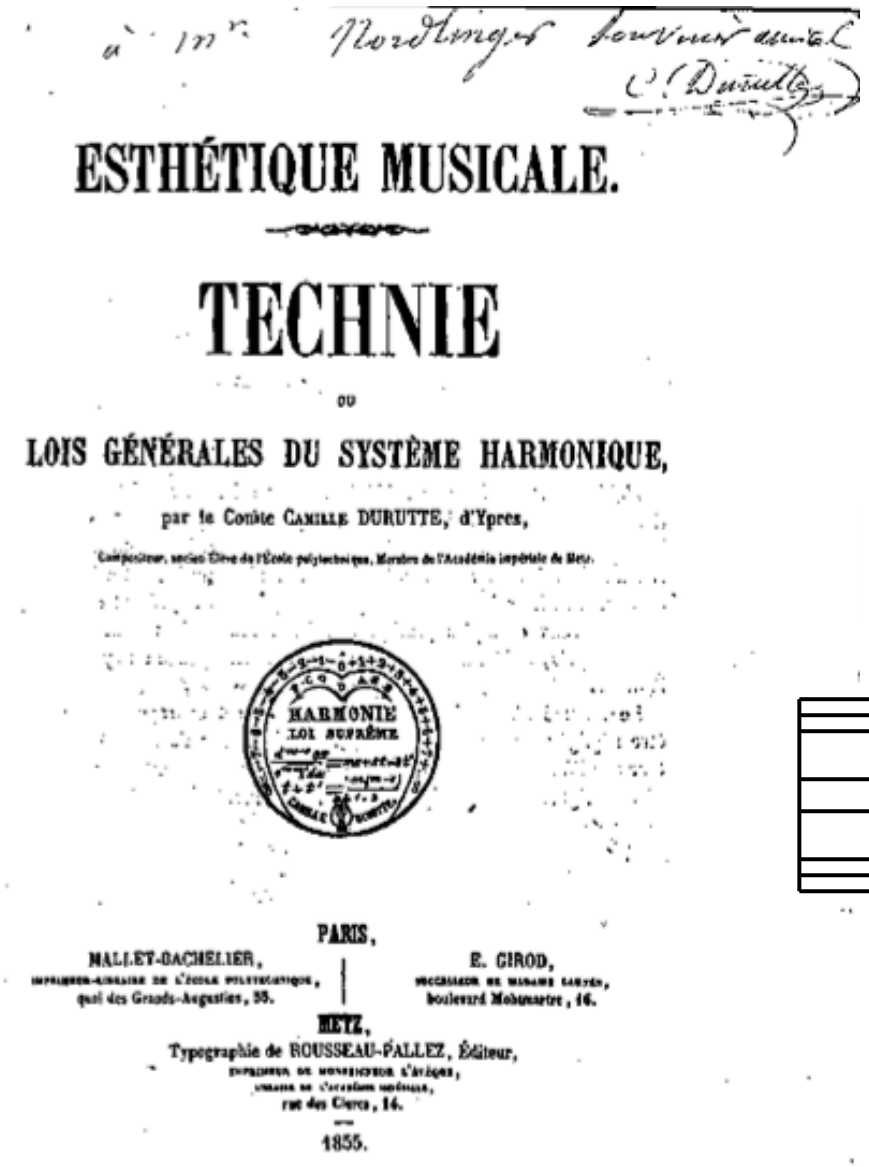
La symétrie du ré…



à Mr Nordinger Souvenir amical
C. Durutte

ESTHÉTIQUE MUSICALE.

TECHNIE

OU

LOIS GÉNÉRALES DU SYSTÈME HARMONIQUE,

par le Comte CAMILLE DURUTTE, d'Ypres,

Compositeur, ancien Élève de l'École polytechnique, Membre de l'Académie impériale de Metz.

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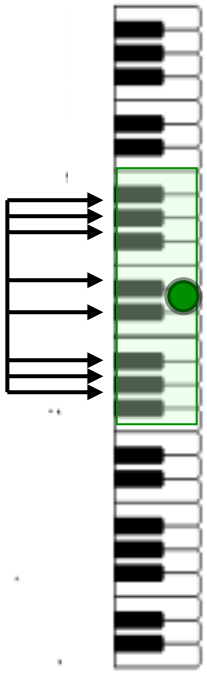
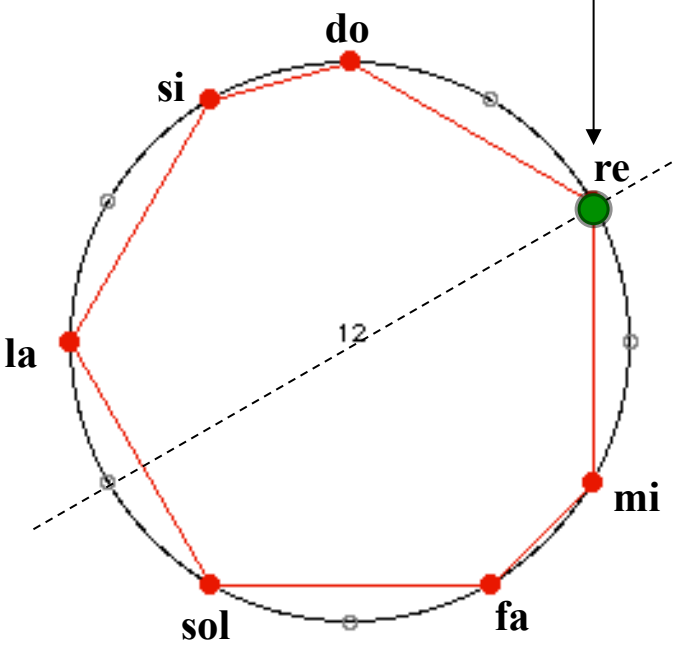
1855.

Camille Durutte:

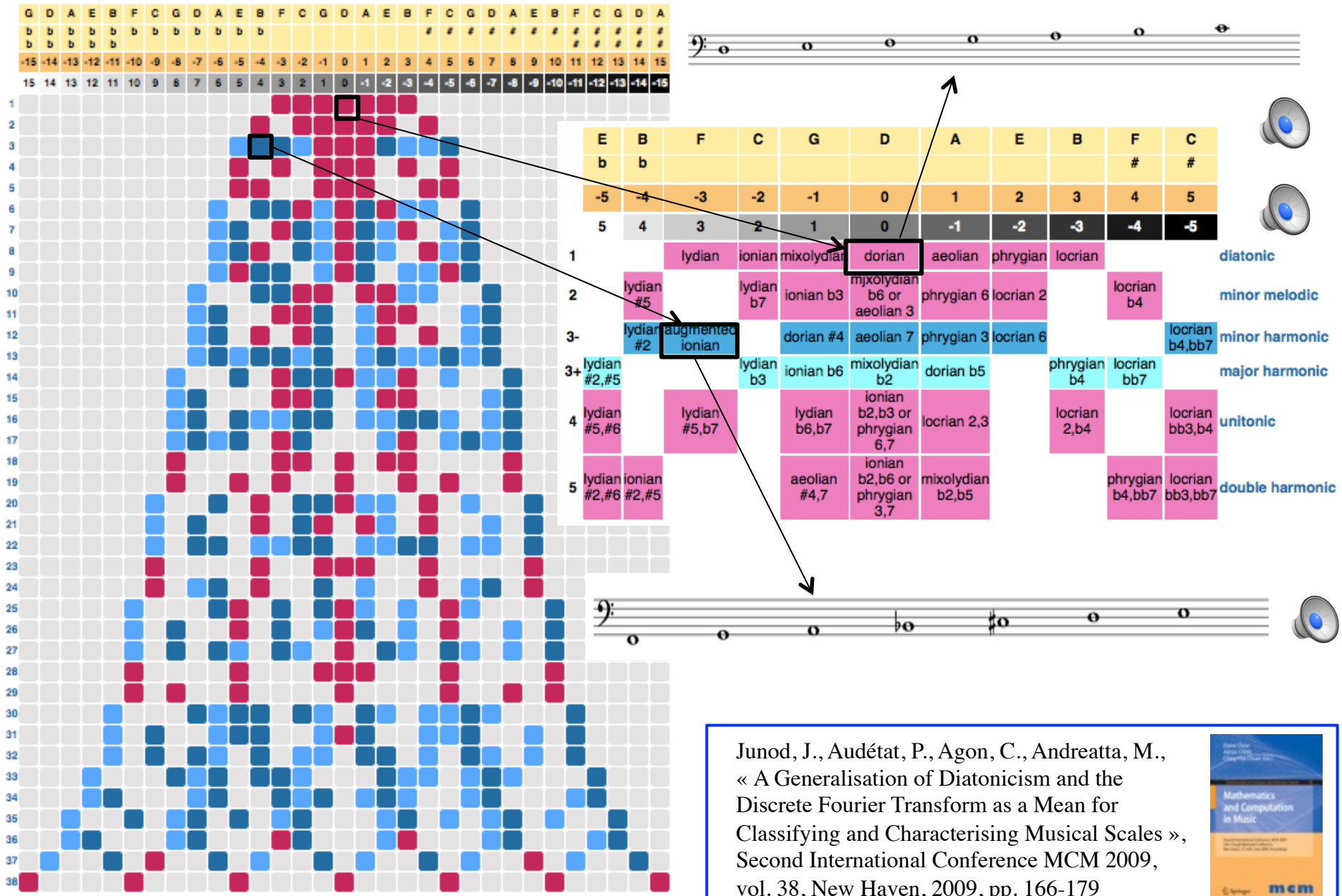
- *Technie, ou lois générales du système harmonique* (1855)
- *Résumé élémentaire de la Technie harmonique, et complément de cette Technie* (1876)

Solbb	Rébb	.....	Fa	Ut	Sol	Ré	La	Mi	Si	.....	Ré×	La×
−15	−14	.....	−3	−2	−1	0	+1	+2	+3	.....	+14	+15





La cloche diatonique (P. Audétat & co.)



http://www.cloche-diatonique.ch/

Junod, J., Audétat, P., Agon, C., Andreatta, M., « A Generalisation of Diatonicism and the Discrete Fourier Transform as a Mean for Classifying and Characterising Musical Scales », Second International Conference MCM 2009, vol. 38, New Haven, 2009, pp. 166-179

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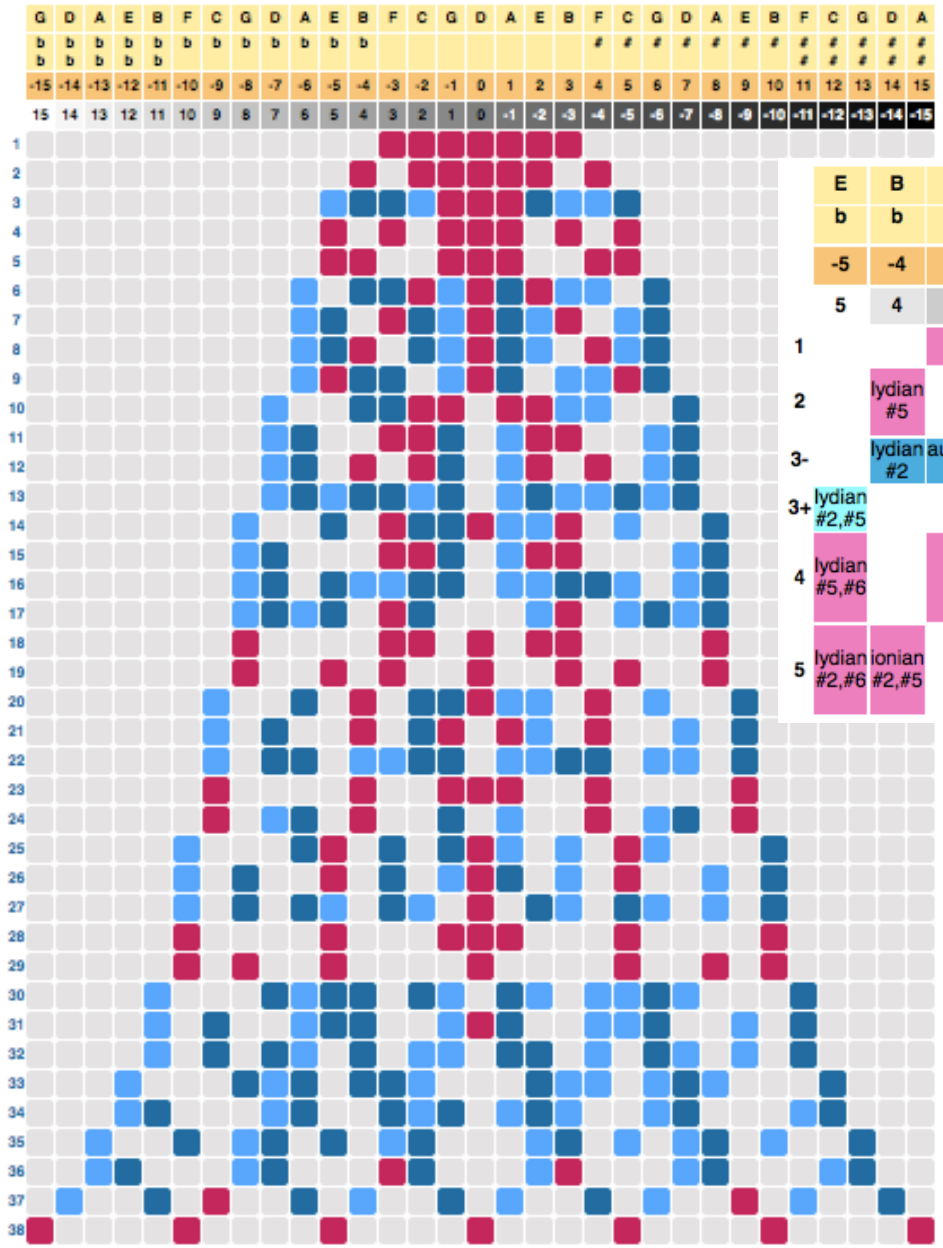

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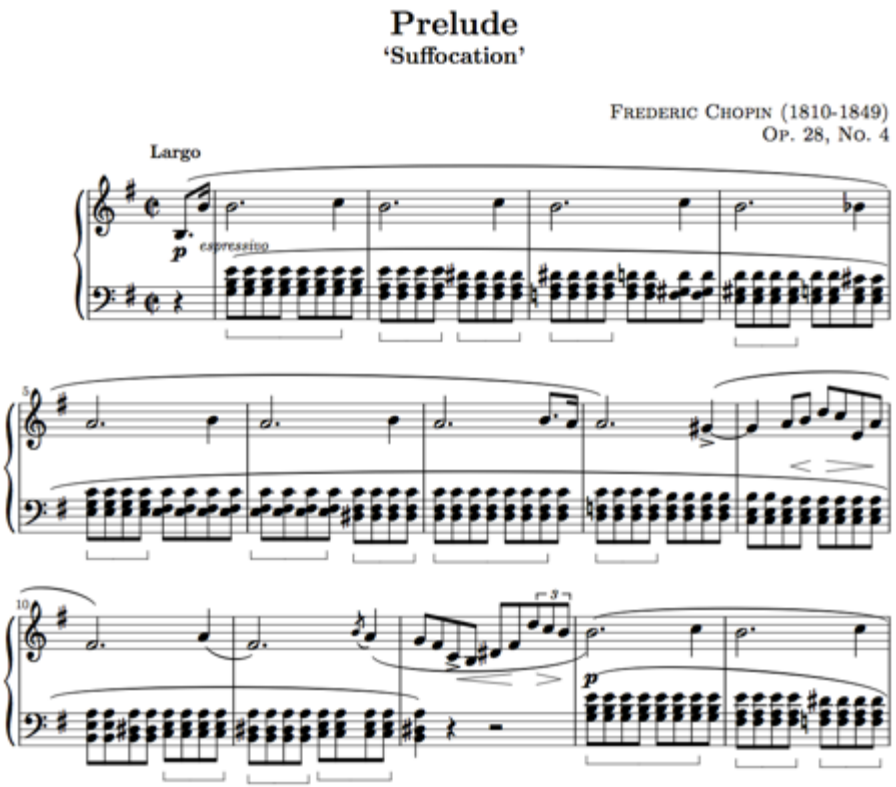
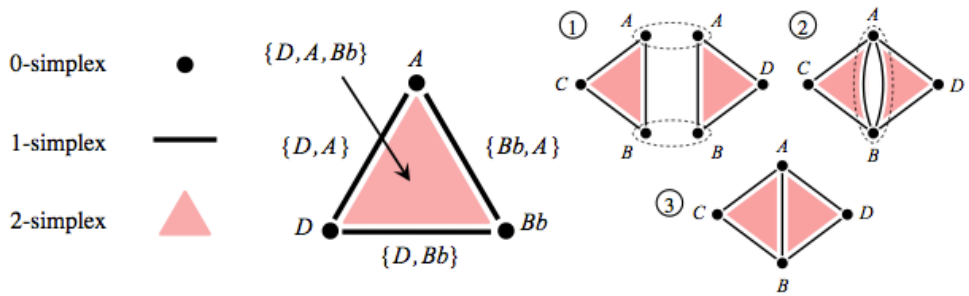
	E	B	F	C	G	D	A	E	B	F	C	
	b	b								#	#	
	-5	-4	-3	-2	-1	0	1	2	3	4	5	
	5	4	3	2	1	0	-1	-2	-3	-4	-5	
1			lydian	ionian	mixolydian	dorian	aeolian	phrygian	locrian			diatonic
2		lydian #5		lydian b7	ionian b3	mjxolydian b6 or aeolian 3	phrygian 6	locrian 2		locrian b4		minor melodic
3-		lydian #2	augmented ionian		dorian #4	aeolian 7	phrygian 3	locrian 6			locrian b4,bb7	minor harmonic
3+	lydian #2,#5			lydian b3	ionian b6	mixolydian b2	dorian b5		phrygian b4	locrian bb7		major harmonic
4	lydian #5,#6		lydian #5,b7		lydian b6,b7	ionian b2,b3 or phrygian 6,7	locrian 2,3		locrian 2,b4		locrian bb3,b4	unitonic
5	lydian #2,#6	ionian #2,#5			aeolian #4,7	ionian b2,b6 or phrygian 3,7	mixolydian b2,b5			phrygian b4,bb7	locrian bb3,bb7	double harmonic

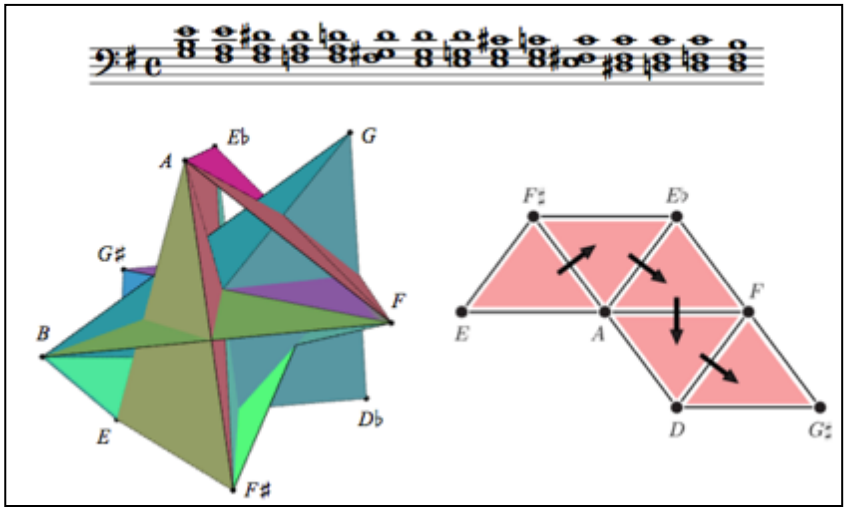
Les cinq échelles les plus utilisées dans la musique occidentale apparaissent toutes au sommet de la cloche diatonique, et ce, dans un ordre proche de celui de leur apparition historique. Une exception toutefois : à l'heure actuelle l'**échelle unitonique sensible** n'est pas considérée comme une échelle susceptible de proposer une harmonie fonctionnelle, contrairement aux cinq autres. C'est pourtant le cas dans la cloche diatonique, où cette échelle est considérée comme fonctionnelle car occupant la quatrième ligne.

Pierre Audétat et Julien Junod, 2006-2009.

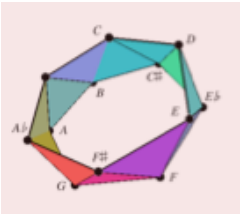
L'espace géométrique d'un *Prélude* de Chopin



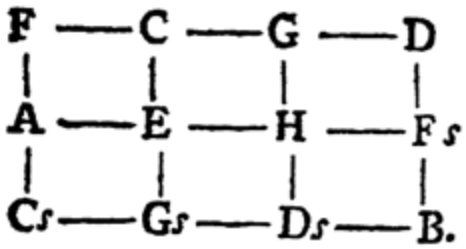




Louis Bigo, *Représentations symboliques musicales et calcul spatial*,
Thèse LAC/Ircam

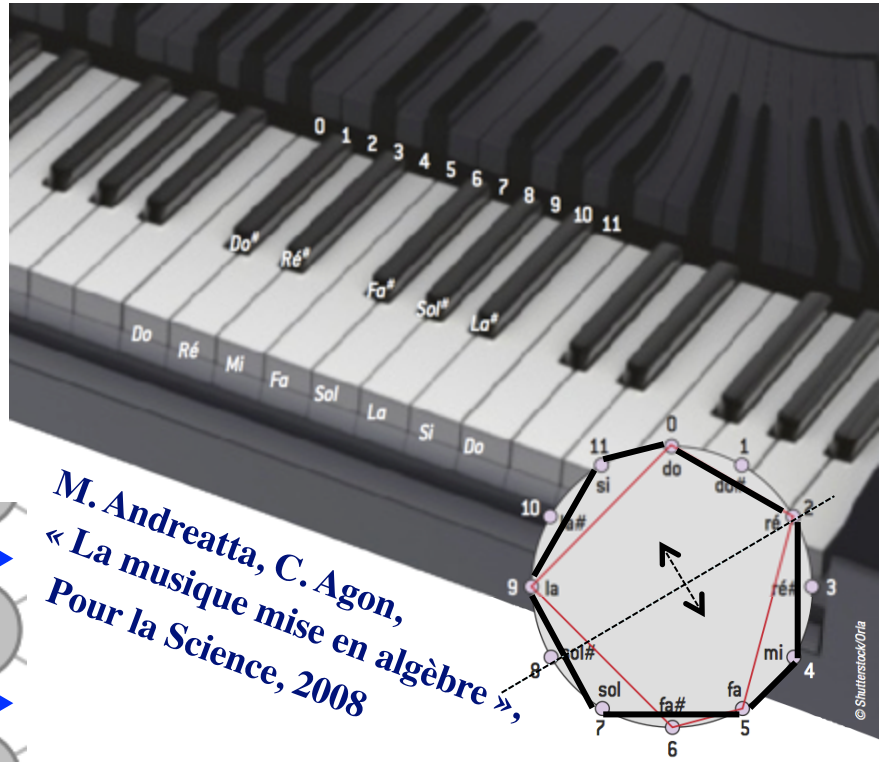


Deux représentations équivalentes : le cercle et le tore



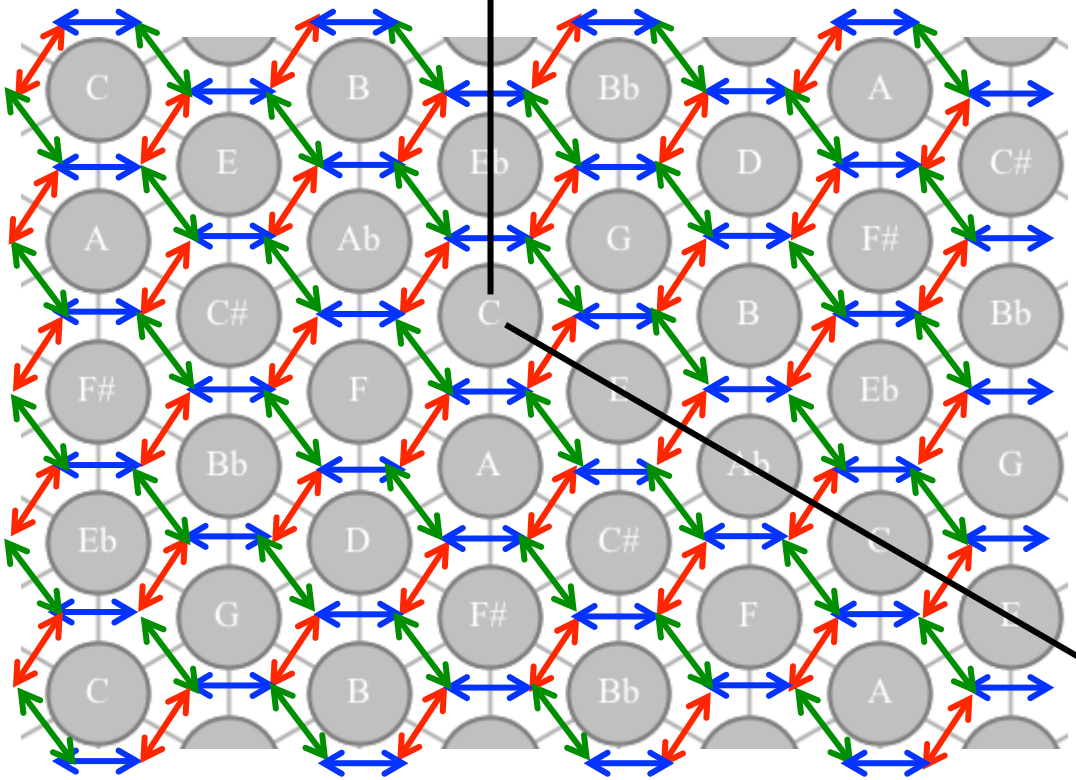
Euler (1773)

Mersenne (1648)

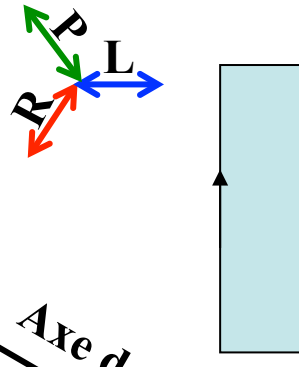


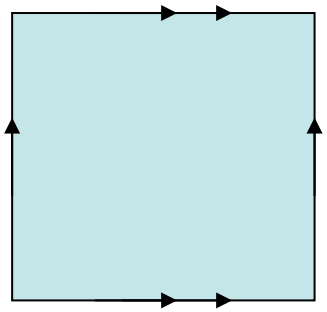
M. Andreatta, C. Agon, « La musique mise en algèbre », Pour la Science, 2008

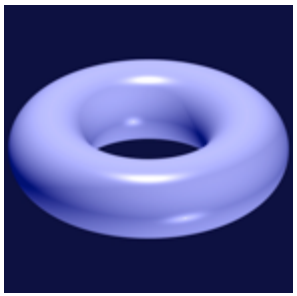
Axe de tierces mineures



Hexachord (Louis Bigo, 2013)

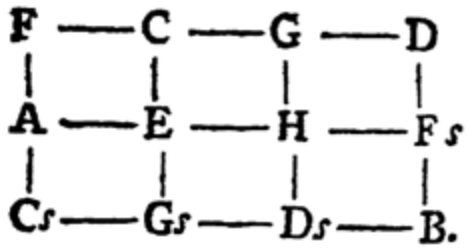






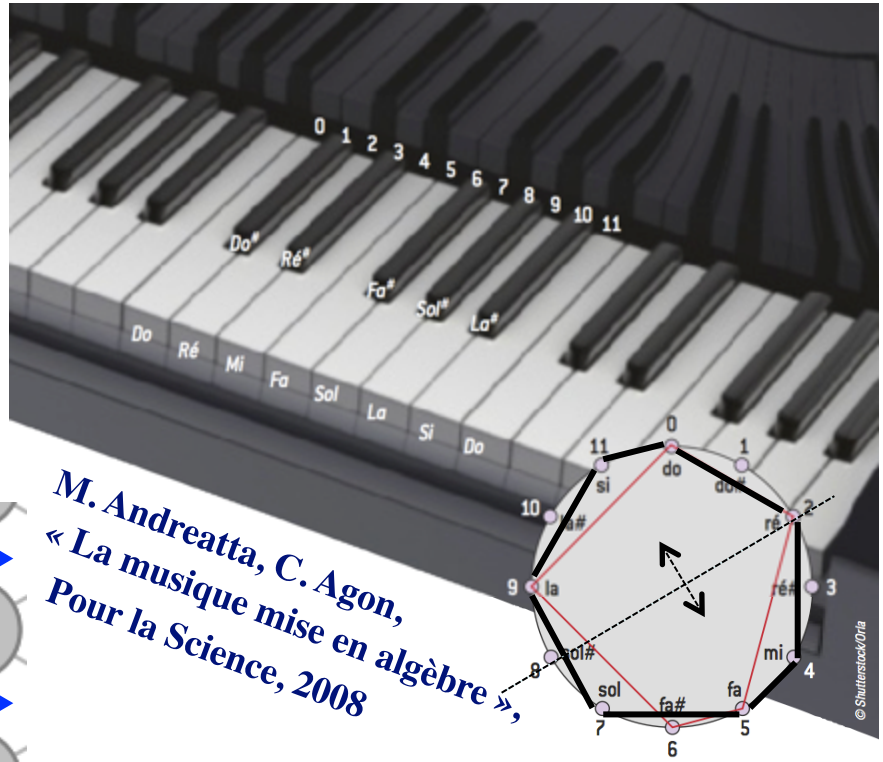
Axe des tierces majeures

Deux représentations équivalentes : le cercle et le tore



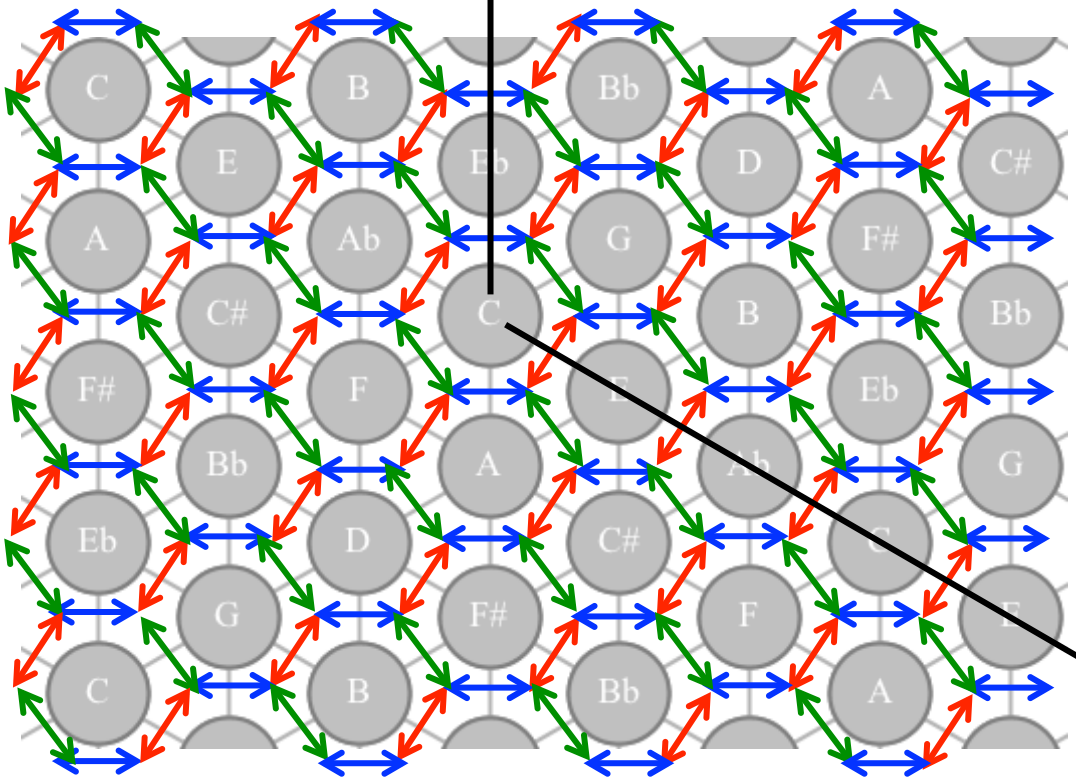
Euler (1773)

Mersenne (1648)

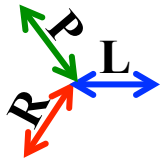


M. Andreatta, C. Agon, « La musique mise en algèbre », Pour la Science, 2008

Axe de tierces mineures



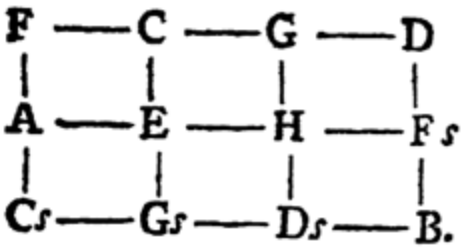
Hexachord (Louis Bigo, 2013)



Axe des tierces majeures

Gilles Baroin

Trajectoires hamiltoniennes dans le « bon » *Tonnetz*



Euler (1773)

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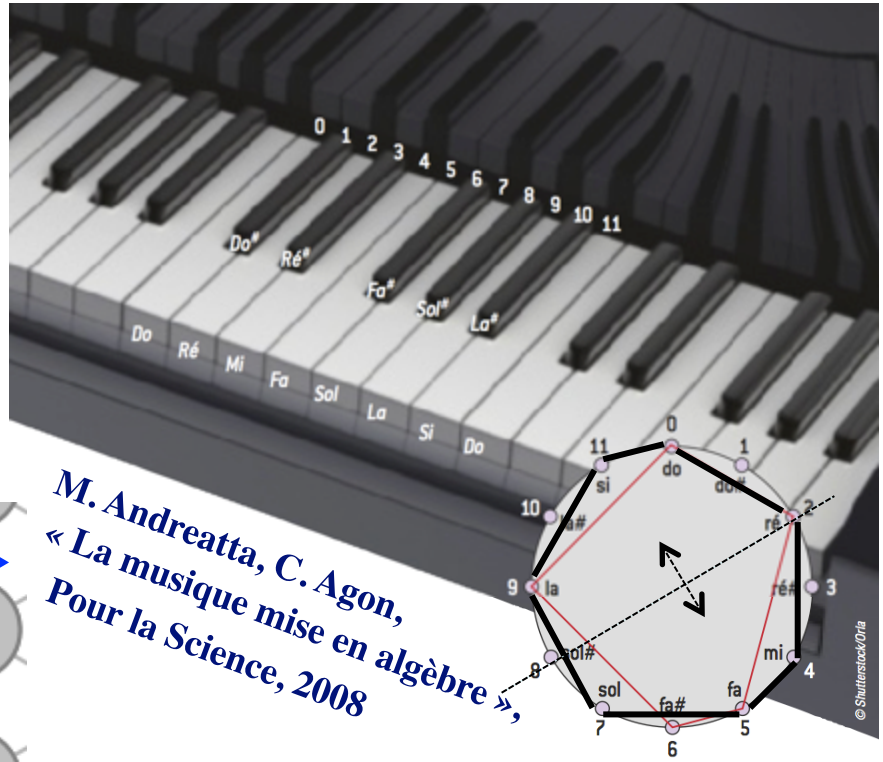
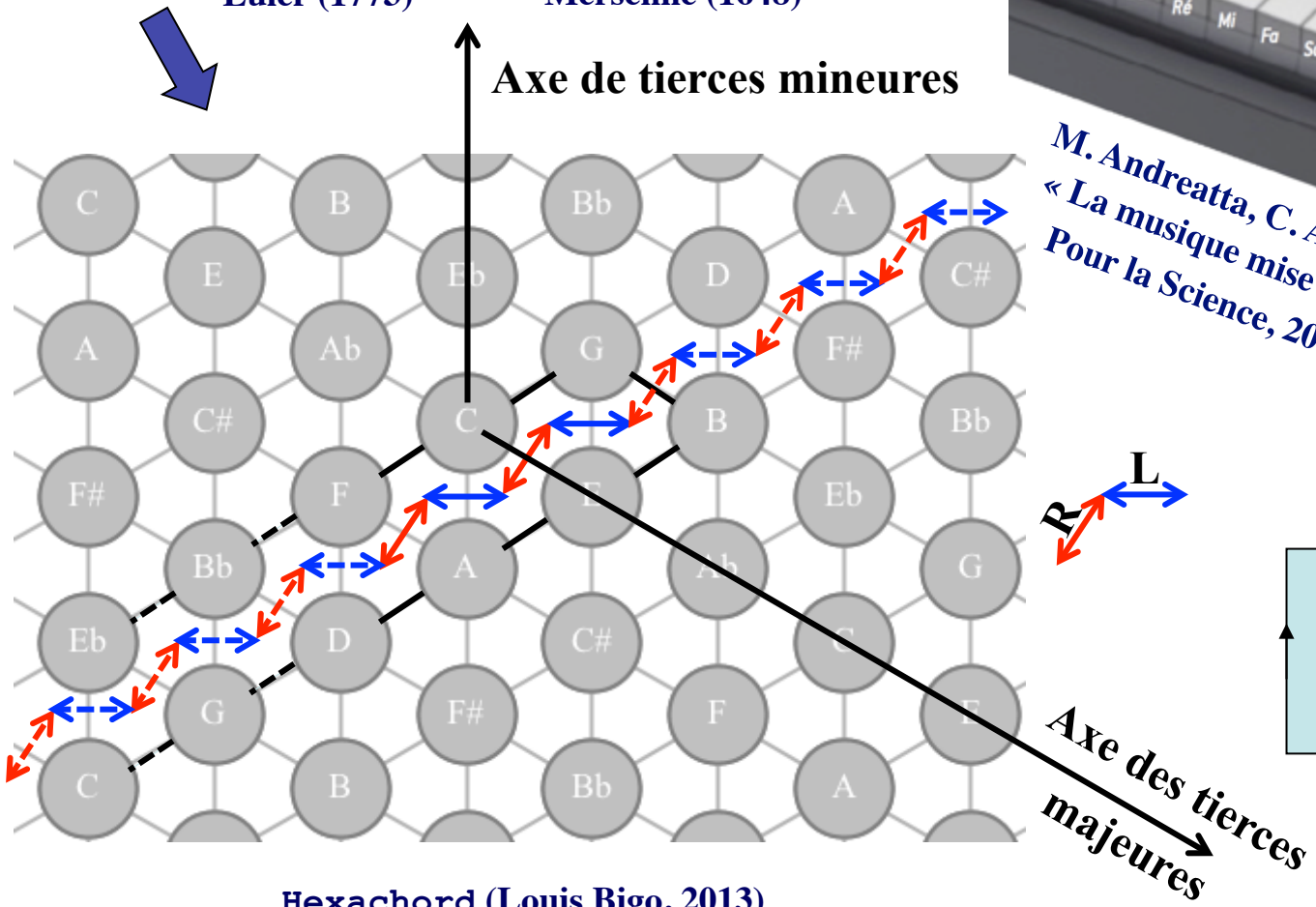
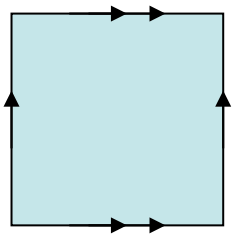
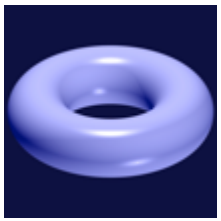


M. Andreatta, C. Agon, « La musique mise en algèbre », Pour la Science, 2008

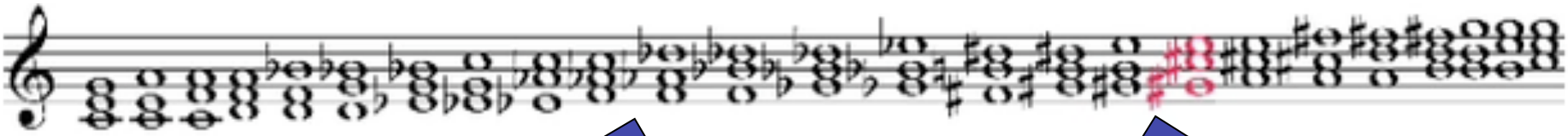


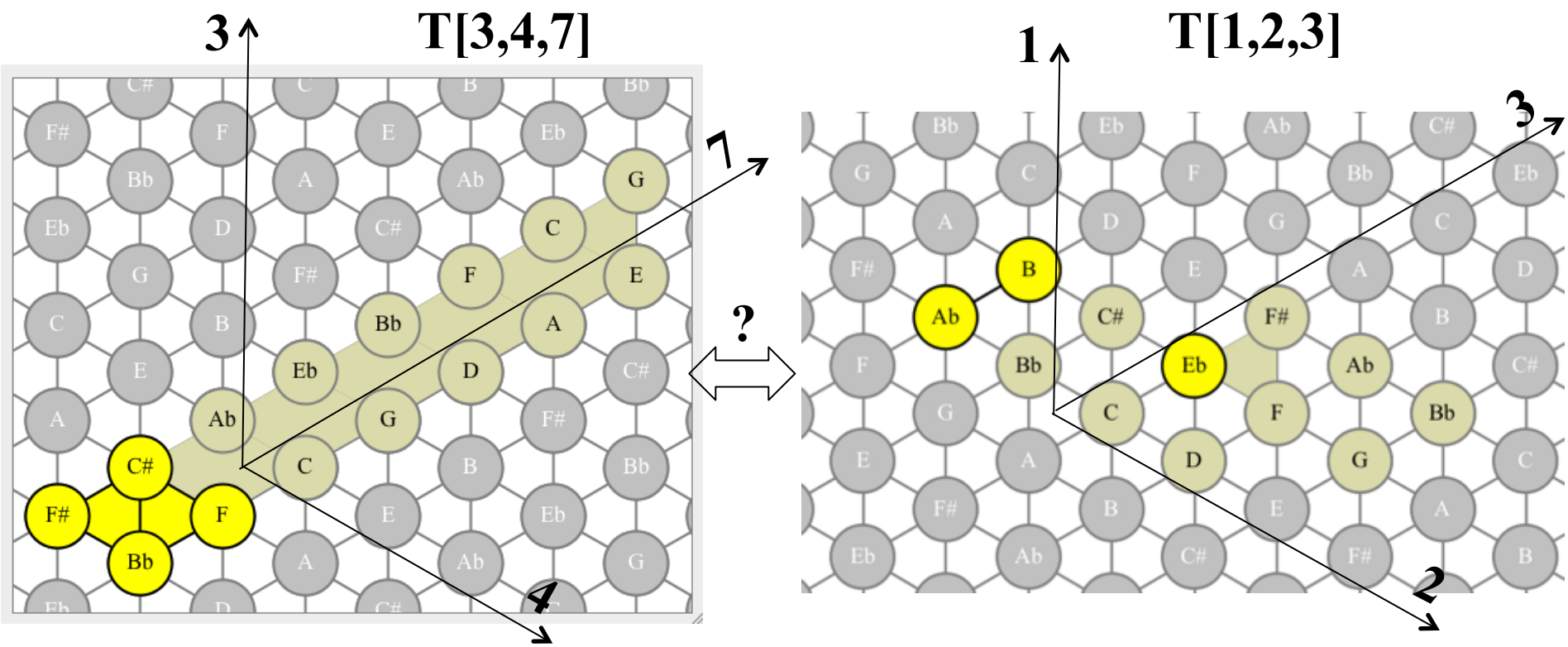
Hexachord (Louis Bigo, 2013)





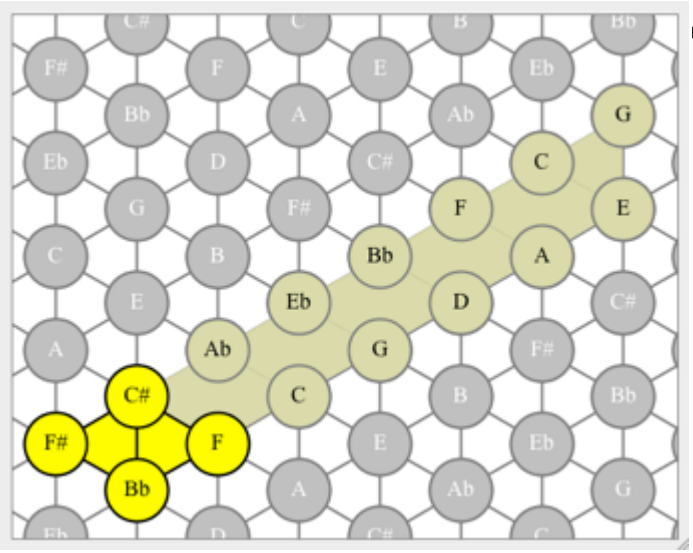
Le caractère spatial de la « logique musicale »





L. Bigo, M. Andreatta, J.-L. Giavitto, O. Michel, et A. Spicher, « Computation and Visualization of Musical Structures in Chord-based Simplicial Complexes », Proceedings Mathematics and Computation in Music Conference 2013 – Montreal, Springer

Le caractère spatial du « style musical »



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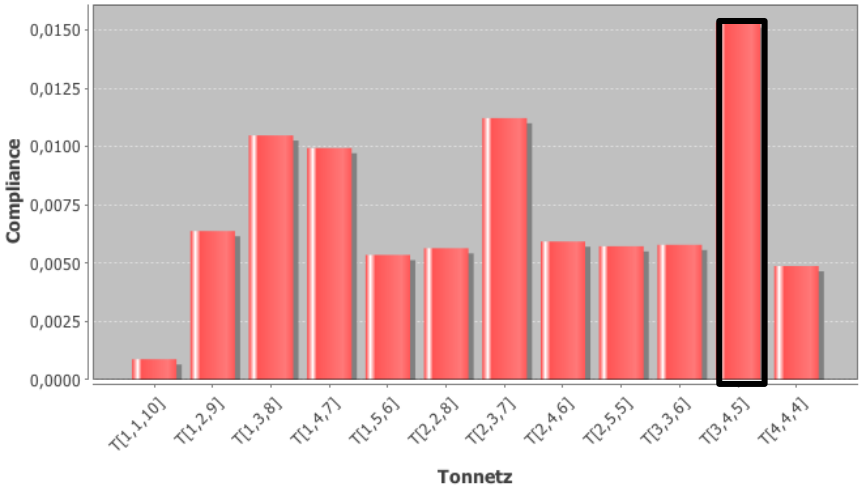
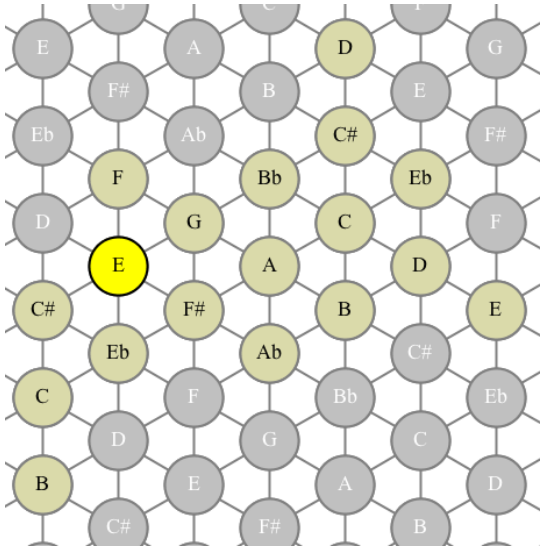


Beethoven, 2e mouvement de la *9e Symphonie*



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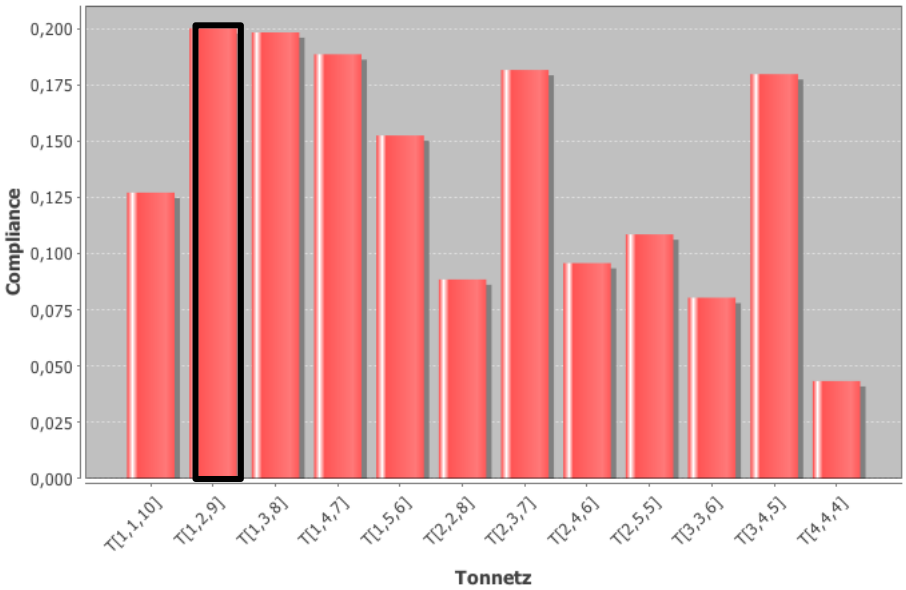
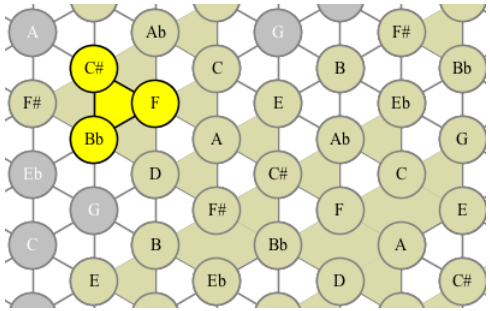
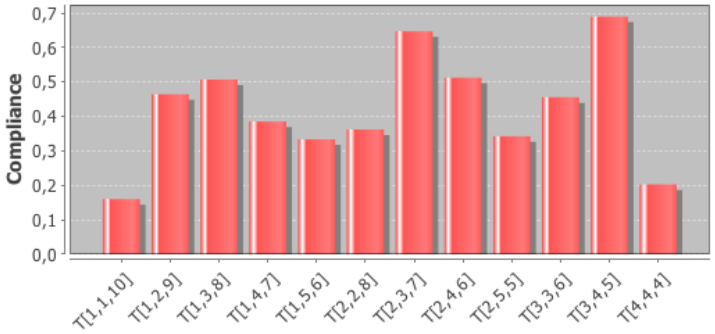


Babbitt, *Semi-Simple Variations*

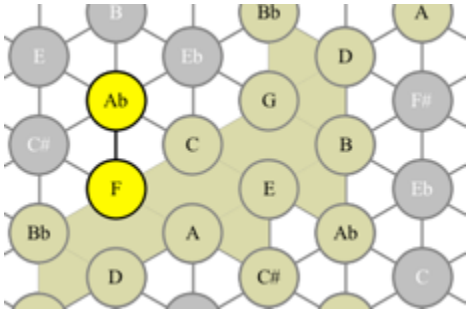
L'espace géométrique comme paramètre de style

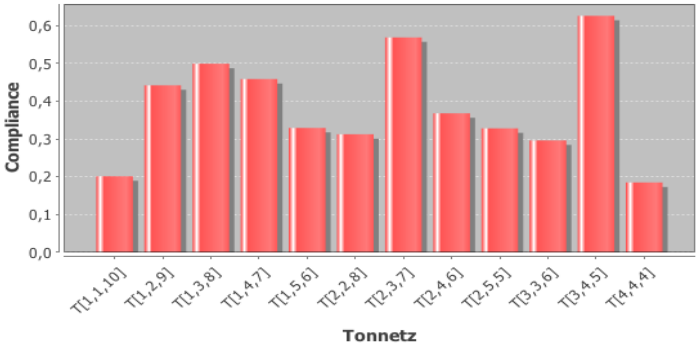
Thelonious Monk, Brilliant Corners



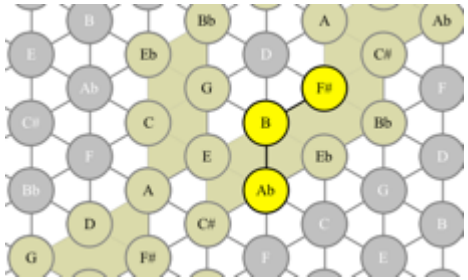


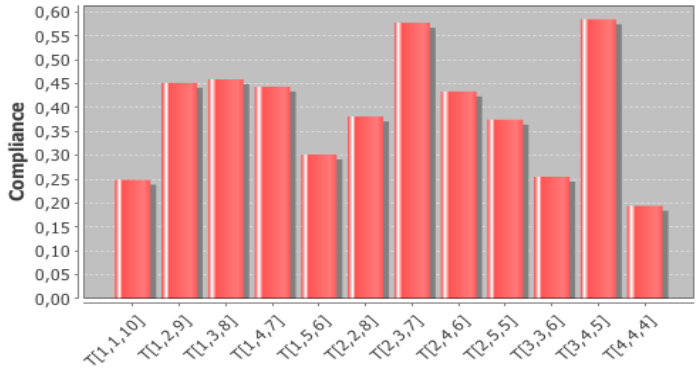
Chick Corea, Eternal Child





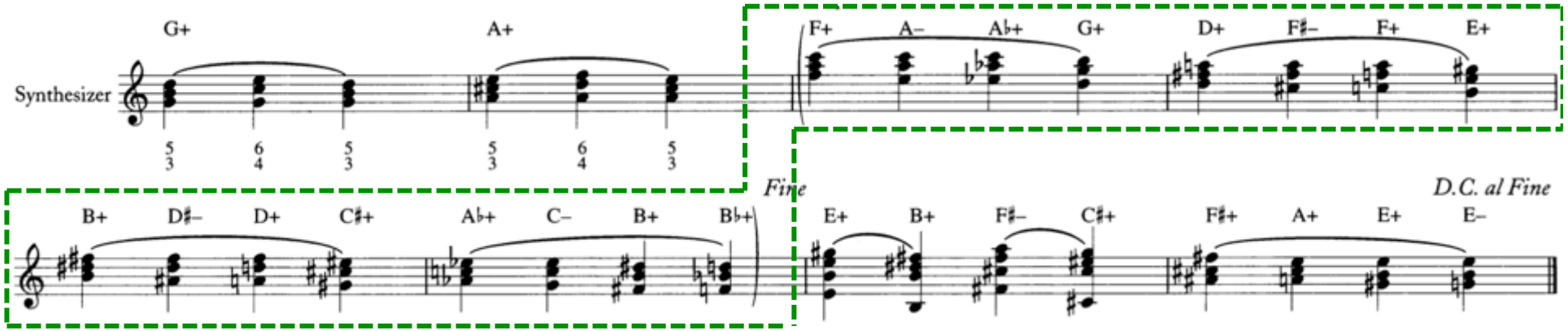
Bill Evans, Turn Out the Stars





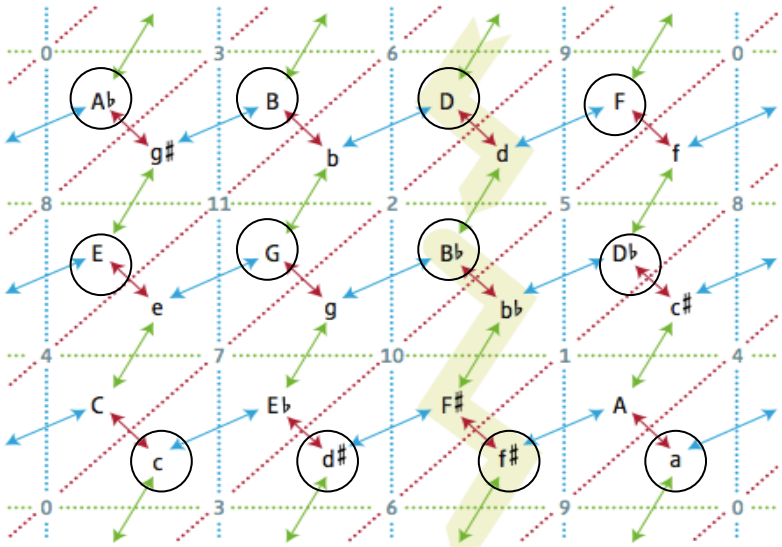
Progressions harmoniques chez Zappa: où sont les symétries ?

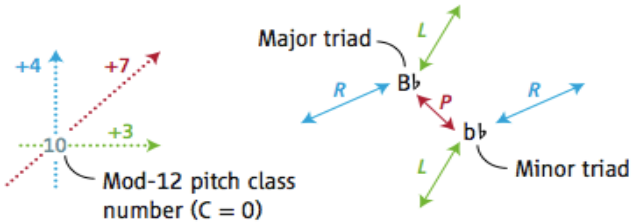
- Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), p. 177-199, 2004



« Easy Meat » - 1981 (Frank Zappa)

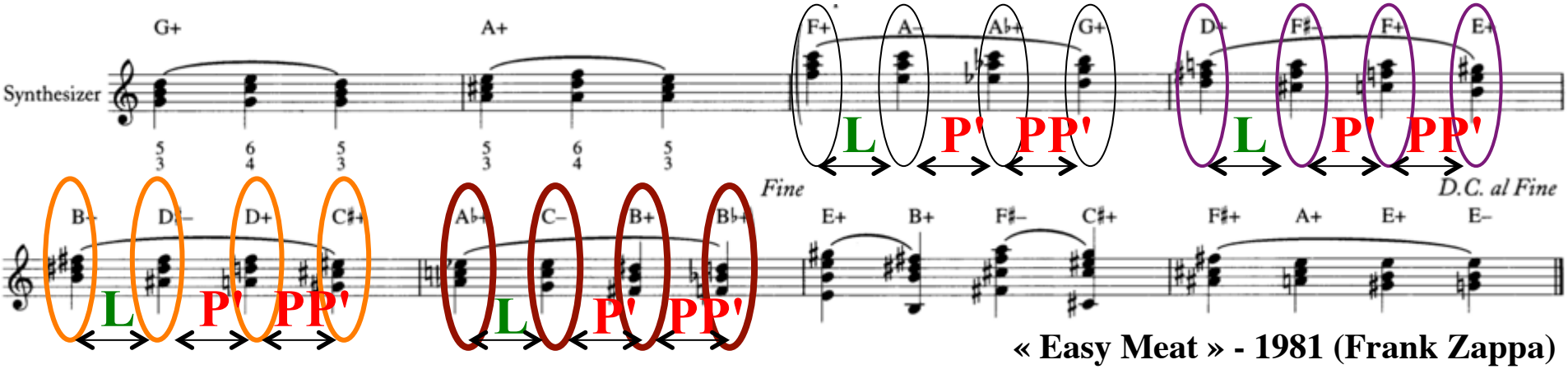
(min. 2'29")



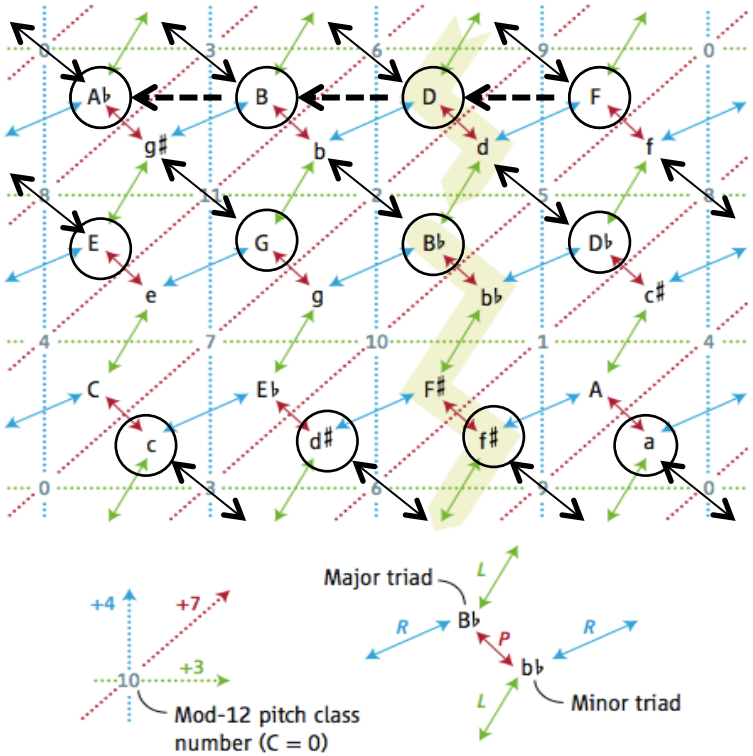


La cellule génératrice et ses translations spatiales

- Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), p. 177-199, 2004



« Easy Meat » - 1981 (Frank Zappa)



$$F+ \xrightarrow{L} A- \xrightarrow{P'} A\flat+ \xrightarrow{PP'} G+$$

T_{-3}

$$D+ \xrightarrow{L} F\sharp- \xrightarrow{P'} F+ \xrightarrow{PP'} E+$$

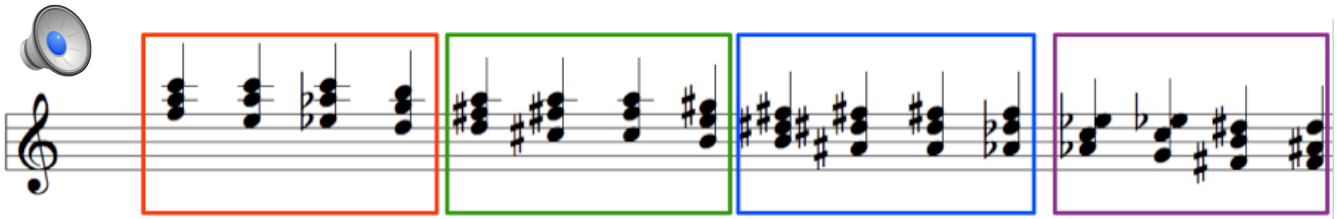
T_{-3}

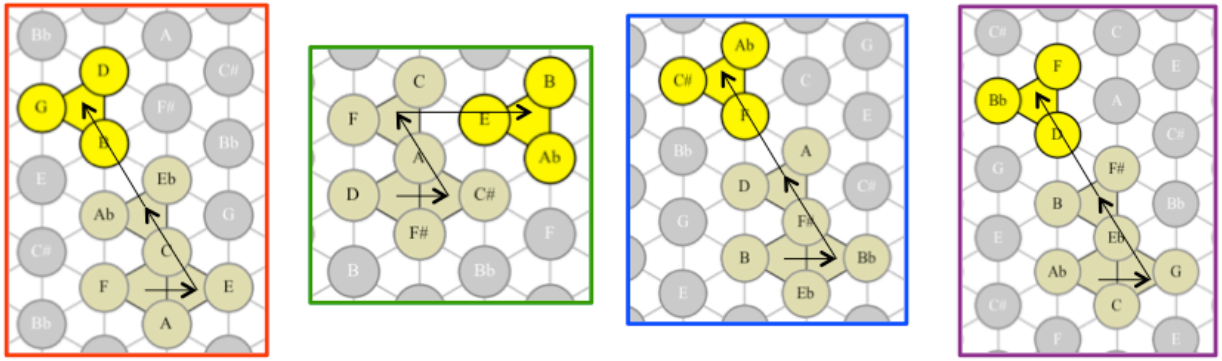
$$B+ \xrightarrow{L} D\sharp- \xrightarrow{P'} D+ \xrightarrow{PP'} C\sharp+$$

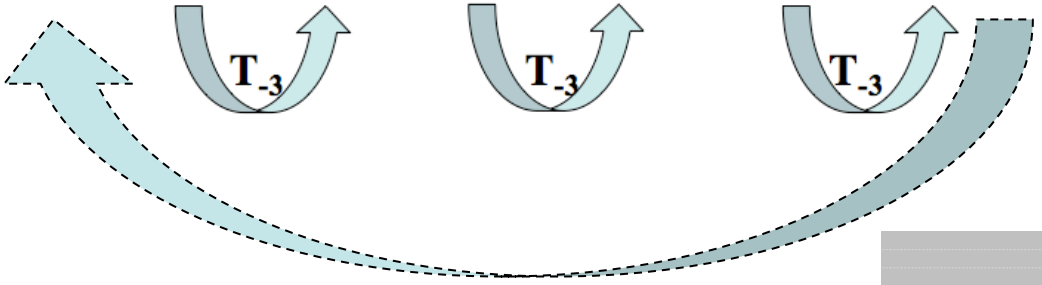
T_{-3}

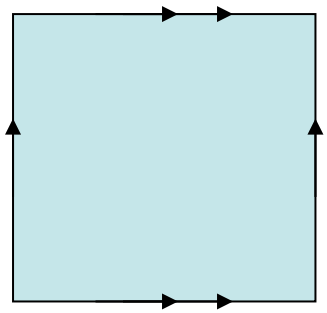
$$A\flat+ \xrightarrow{L} C- \xrightarrow{P'} B+ \xrightarrow{PP'} B\flat+$$

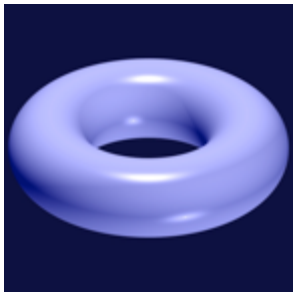
L’espace (de la progression harmonique)

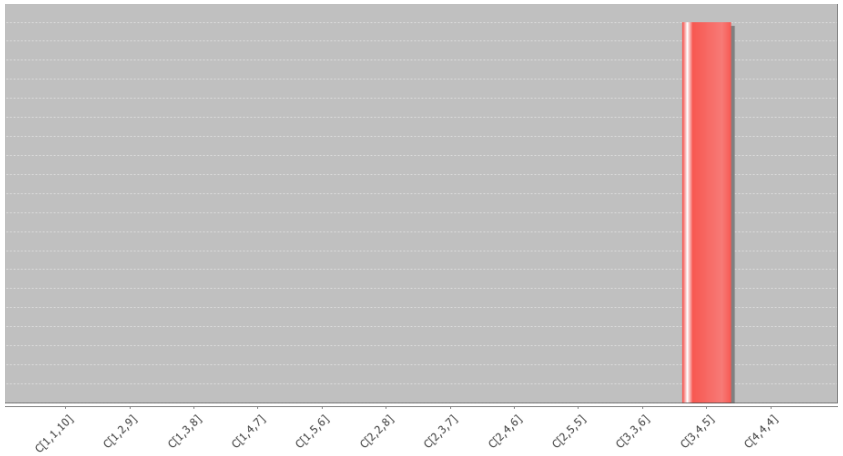




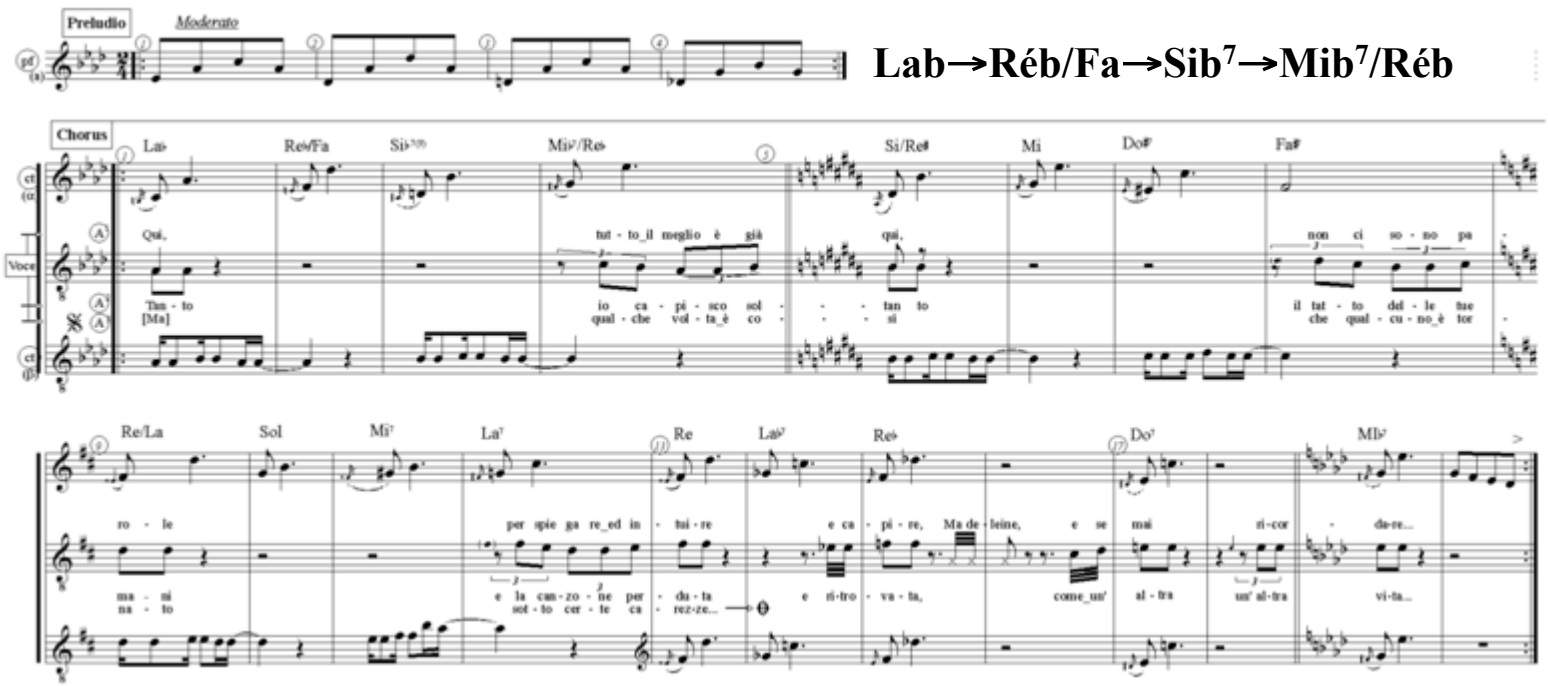








Progressions harmoniques dans « Madeleine » (Paolo Conte)



Lab→Réb/Fa→Sib^7→Mib^7/Réb

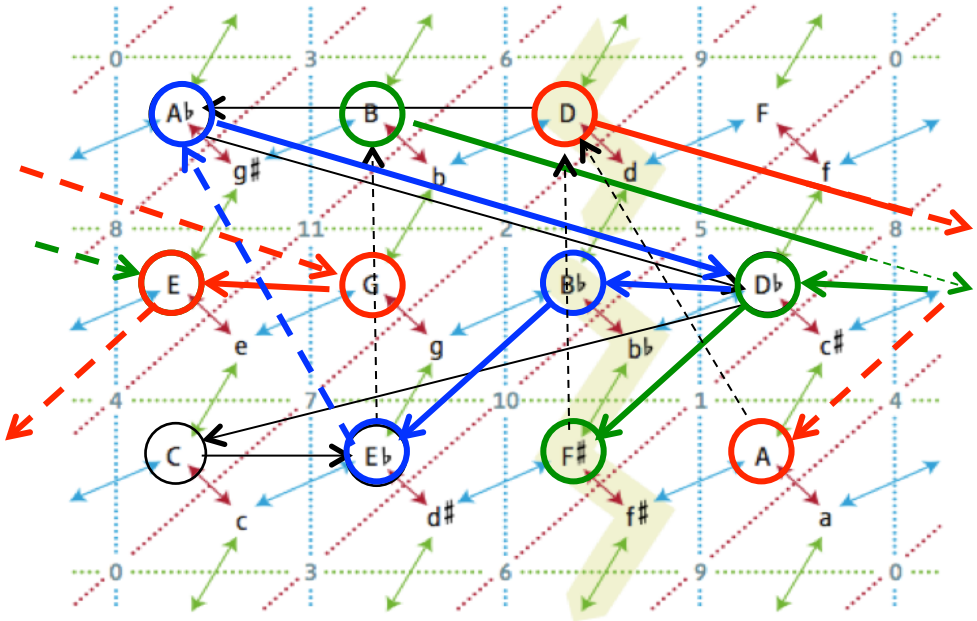
S. La Via, *Poesia per musica e musica per poesia. Dai trovatori a Paolo Conte*, Carocci, 2006

Lab→Réb/Fa→Sib^7→Mib^7/Réb

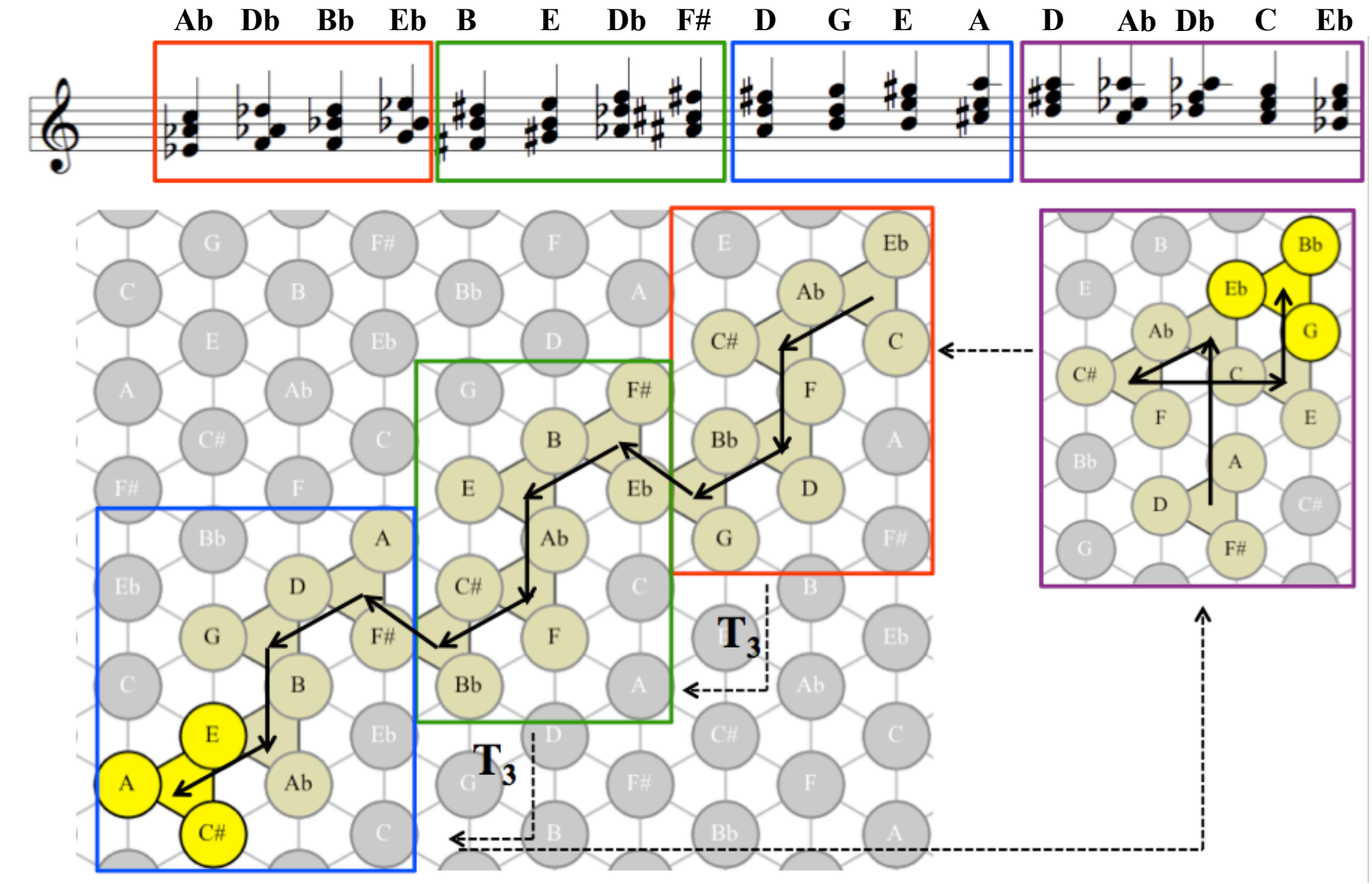
Si/Ré#→Mi→Do#→Fa#

Ré/La→Sol→Mi^7→La^7

Ré→Lab^7→Réb→Do^7→Mib



La trajectoire spatiale de « Madeleine » et son caractère ‘hamiltonien’



Enumération des cycles hamiltoniens dans le *Tonnetz*

C	e	G	b	D	f#	A	c#	E	g#	B	d#	F#	a#	C#	f	G#	c	D#	g	A#	d	F	a	#41
C	e	E	g#	G#	c	D#	g	G	b	B	d#	F#	a#	A#	d	D	f#	A	c#	C#	f	F	a	#62
C	c	G#	f	C#	c#	A	a	F	d	A#	a#	F#	f#	D	b	G	g	D#	d#	B	g#	E	e	#13
C	c	G#	g#	E	c#	A	a	F	f	C#	a#	F#	f#	D	d	A#	g	D#	d#	B	b	G	e	#4
C	e	E	g#	G#	c	D#	d#	B	b	G	g	A#	a#	F#	f#	D	d	F	f	C#	c#	A	a	#58
C	c	D#	g	G	b	B	d#	F#	f#	D	d	A#	a#	C#	c#	A	a	F	f	G#	g#	E	e	#19
C	c	G#	g#	B	d#	D#	g	G	b	D	d	A#	a#	F#	f#	A	a	F	f	C#	c#	E	e	#7
C	c	G#	g#	E	e	G	b	B	d#	D#	g	A#	a#	F#	f#	D	d	F	f	C#	c#	A	a	#27
C	c	D#	d#	B	b	G	g	A#	d	D	f#	F#	a#	C#	c#	A	a	F	f	G#	g#	E	e	#21
C	c	G#	g#	B	b	G	g	D#	d#	F#	a#	A#	d	D	f#	A	a	F	f	C#	c#	E	e	#8
C	c	G#	g#	E	e	G	g	D#	d#	B	b	D	f#	F#	a#	A#	d	F	f	C#	c#	A	a	#30
C	c	D#	d#	B	b	G	g	A#	a#	F#	f#	D	d	F	a	A	c#	C#	f	G#	g#	E	e	#22
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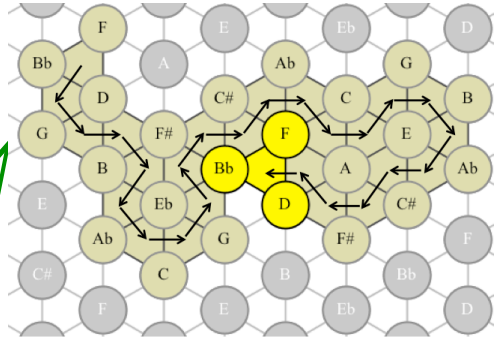
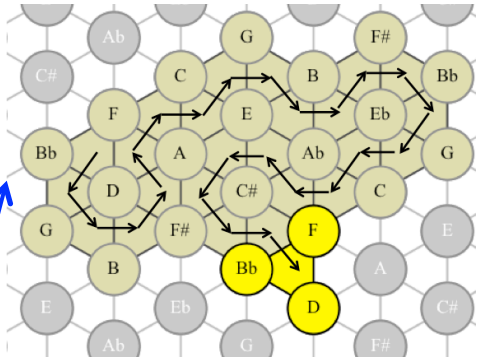
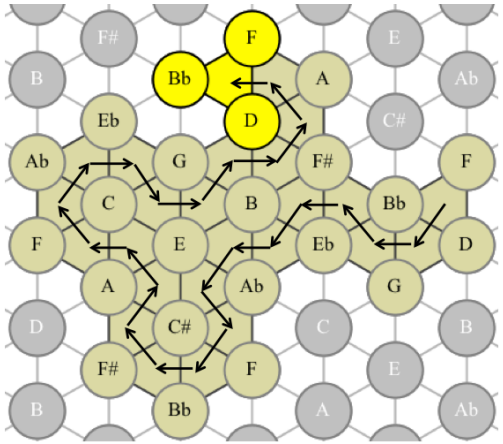
C	c	G#	g#	E	e	G	g	D#	d#	B	b	D	d	A#	a#	F#	f#	A	c#	C#	f	F	a	#31
C	c	G#	f	F	a	A	c#	C#	a#	A#	d	D	f#	F#	d#	D#	g	G	b	B	g#	E	e	#9
C	c	G#	g#	E	c#	C#	f	F	a	A	f#	F#	a#	A#	d	D	b	B	d#	D#	g	G	e	#6
C	c	D#	d#	F#	f#	A	c#	E	e	G	g	A#	a#	C#	f	G#	g#	B	b	D	d	F	a	#33
C	e	G	g	A#	a#	C#	c#	E	g#	B	b	D	d	F	f	G#	c	D#	d#	F#	f#	A	a	#44
C	c	D#	g	A#	a#	C#	c#	E	e	G	b	D	d	F	f	G#	g#	B	d#	F#	f#	A	a	#40
C	c	D#	d#	F#	a#	C#	c#	E	e	G	g	A#	d	F	f	G#	g#	B	b	D	f#	A	a	#38
C	c	D#	d#	F#	a#	C#	f	G#	g#	B	b	D	f#	A	c#	E	e	G	g	A#	d	F	a	#34
C	e	G	g	A#	a#	C#	f	G#	c	D#	d#	F#	f#	A	c#	E	g#	B	b	D	d	F	a	#42
C	e	G	b	D	d	F	f	G#	c	D#	g	A#	a#	C#	c#	E	g#	B	d#	F#	f#	A	a	#43
C	c	D#	g	A#	d	F	f	G#	g#	B	d#	F#	a#	C#	c#	E	e	G	b	D	f#	A	a	#39
C	e	G	b	B	d#	F#	a#	A#	g	D#	c	G#	g#	E	c#	C#	f	F	d	D	f#	A	a	#50
C	c	D#	g	A#	d	D	f#	A	c#	C#	a#	F#	d#	B	b	G	e	E	g#	G#	f	F	a	#37
C	c	G#	f	F	d	A#	g	D#	d#	B	g#	E	e	G	b	D	f#	F#	a#	C#	c#	A	a	#25
C	c	D#	d#	B	g#	G#	f	C#	a#	F#	f#	D	b	G	g	A#	d	F	a	A	c#	E	e	#16
C	c	G#	g#	B	b	G	e	E	c#	A	f#	D	d	A#	g	D#	d#	F#	a#	C#	f	F	a	#23
C	c	G#	f	F	a	A	f#	F#	a#	C#	c#	E	g#	B	d#	D#	g	A#	d	D	b	G	e	#1
C	e	E	g#	B	b	G	g	A#	a#	F#	d#	D#	c	G#	f	C#	c#	A	f#	D	d	F	a	#59
C	e	G	b	B	d#	F#	f#	D	d	F	f	C#	a#	A#	g	D#	c	G#	g#	E	c#	A	a	#48
C	c	D#	g	A#	d	D	f#	A	a	F	f	G#	g#	E	c#	C#	a#	F#	d#	B	b	G	e	#18
C	e	E	c#	A	f#	D	d	A#	g	G	b	B	g#	G#	c	D#	d#	F#	a#	C#	f	F	a	#60
C	c	G#	f	C#	c#	E	g#	B	d#	D#	g	A#	a#	F#	f#	A	a	F	d	D	b	G	e	#2



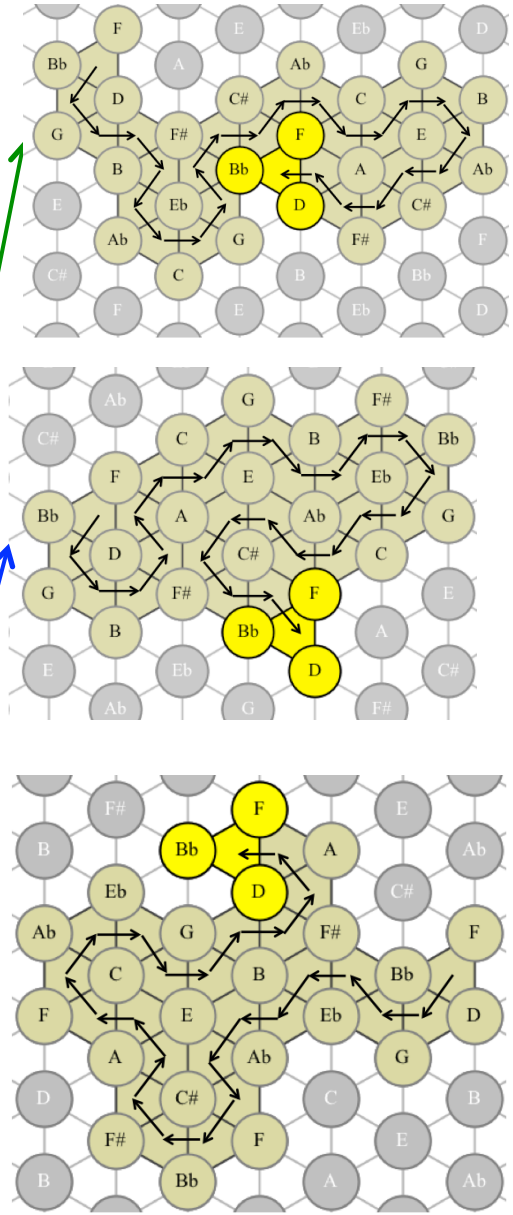




G. Albini et S. Antonini, « Hamiltonian Cycles in the Topological Dual of the Tonnetz », MCM 2009, LNCS Springer

Enumération des cycles hamiltoniens dans le *Tonnetz*

#41	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R
#62	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R
#13	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L
#4	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L
#58	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R
#19	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L
#7	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L
#27	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R
#21	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L
#8	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L
#30	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R
#22	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L
#10	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L
#31	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R
#9	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L
#6	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L
#33	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R
#44	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R
#40	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R
#38	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R
#34	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R
#42	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R
#43	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R
#39	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R
#50	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R
#37	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R
#25	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R
#16	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L
#23	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R
#1	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L
#59	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R
#48	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R
#18	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L
#60	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R
#2	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L



G. Albini et S. Antonini, « Hamiltonian Cycles in the Topological Dual of the Tonnetz », MCM 2009, LNCS Springer

Aprile, chanson hamiltonienne « décadente »

Do←do$_m$←Sol#←fa$_m$←Fa←la$_m$←La←fa#$_m$←Fa#←sib$_m$←Do#←do#$_m$

La

mi$_m$→Sol→si$_m$→Ré→ré$_m$→Sib→sol$_m$→Mib→mib$_m$→Si→sol#$_m$→Mi

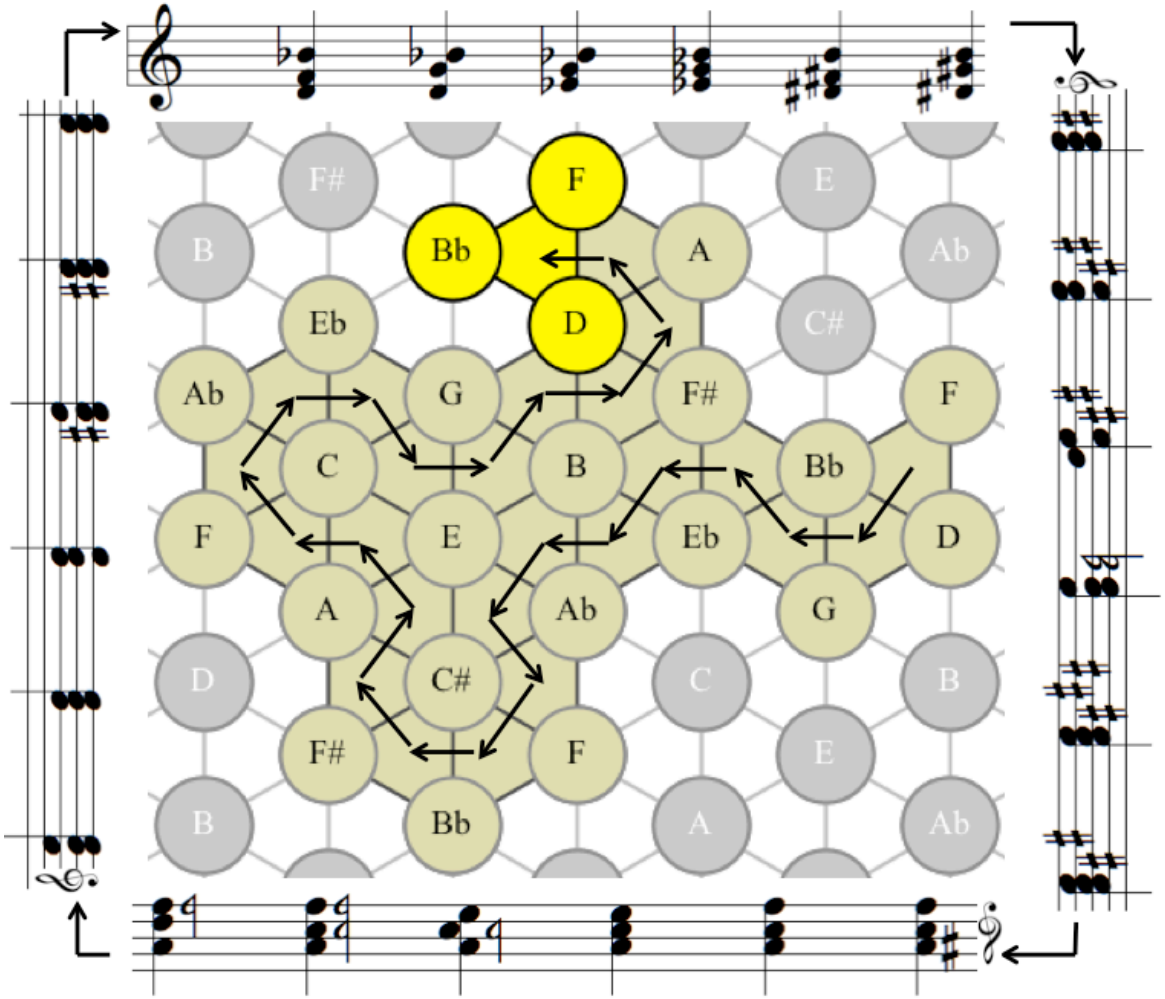
(Gabriele d'Annunzio)

Socchiusa è la finestra, sul giardino.
Un'ora passa lenta, sonnolenta.
Ed ella, ch'era attenta, s'addormenta
a quella voce che già si lamenta,
– che si lamenta in fondo a quel giardino.

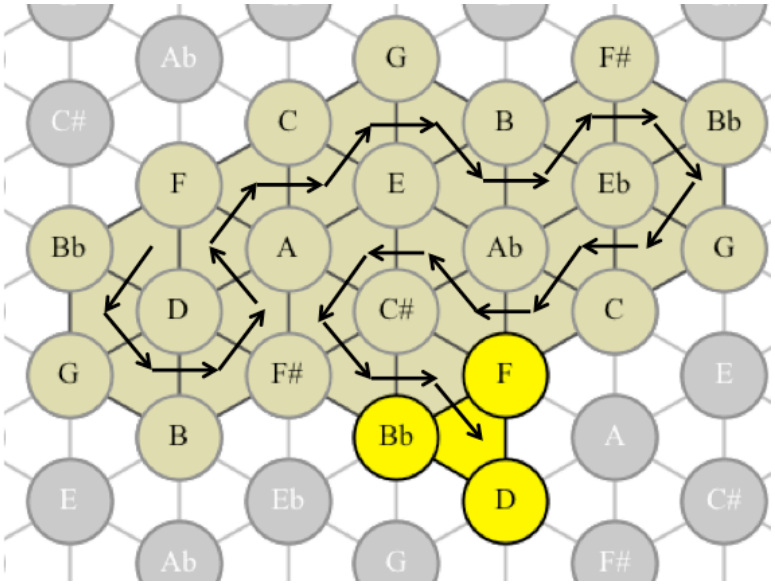
Non è che voce d'acque su la pietra:
e quante volte, quante volte udita!
Quell'amore e quell'ora in quella vita
s'affondan come ne l'onda infinita
stretti insieme il cadavere e la pietra.

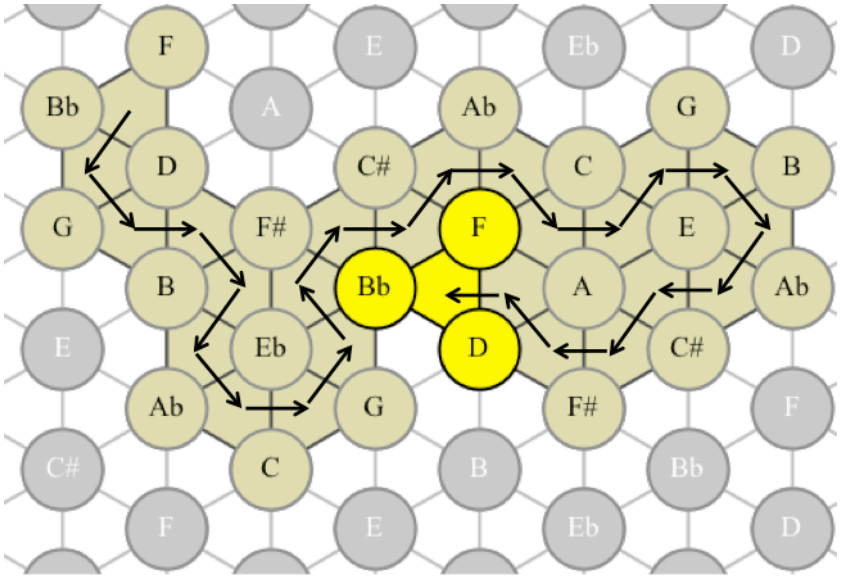
Ella stende l'angoscia sua nel sonno.
L'angoscia è forte, e il sonno è così lieve!
(Par la luce d'april quasi una neve
che sia tiepida.) Ed ella certo deve
soffrire, vagamente, anche nel sonno.

Gabriele D'Annunzio (1863-1938)



Deuxième et troisième cycle hamiltonien





Do→mi_m→Mi→$sol\#_m$→Si→$ré\#_m$→Re#→do_m→Lab→fa_m→Do#→$do\#_m$

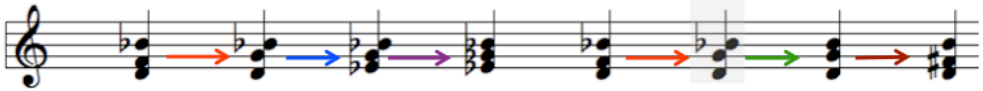
la_m←Fa←$ré_m$←Ré←si_m←Sol←sol_m←Sib←sib_m←Fa#←$fa\#_m$←La

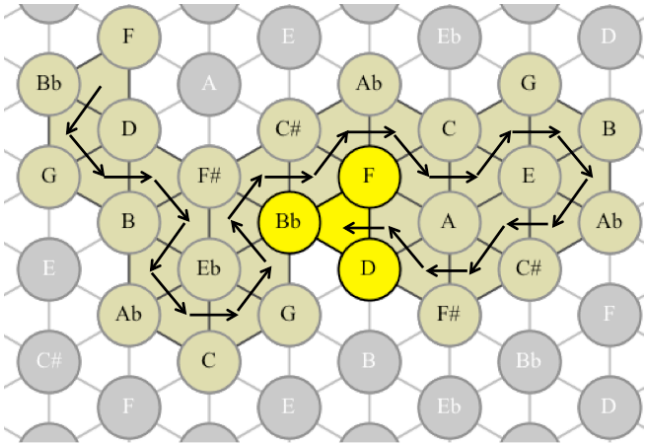
Mi←mi_m←Do←la_m←Fa←fa_m←Reb←sib_m←Fa#←mib_m←Mib←do_m

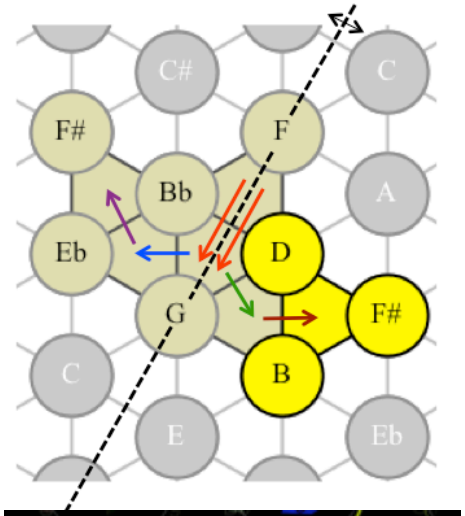
La

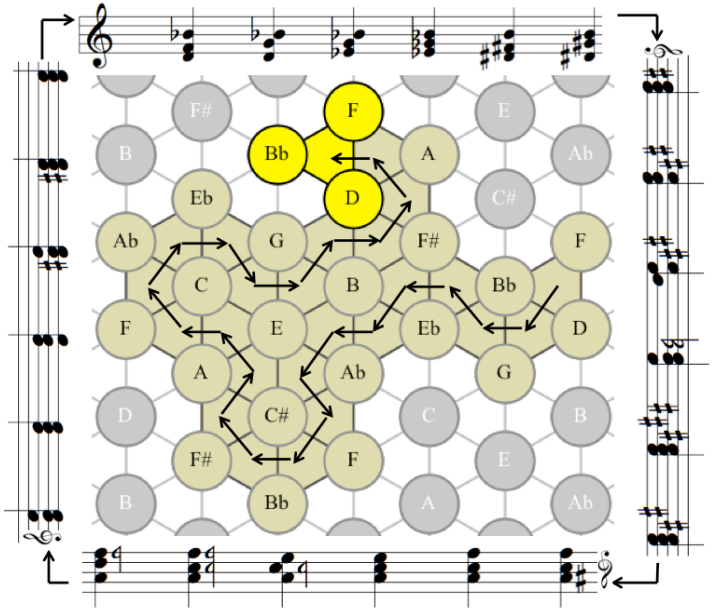
$do\#_m$→La→$fa\#_m$→Ré→$ré_m$→Sib→sol_m→Sol→si_m→Si→$sol\#_m$→Sol#

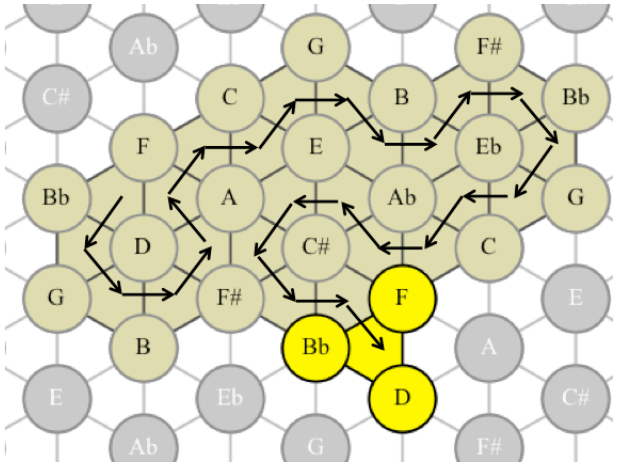
Maths et expériences oumu(po)piennes

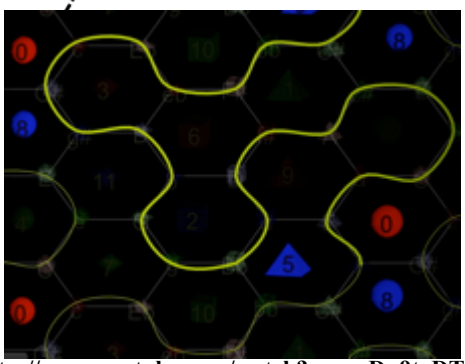








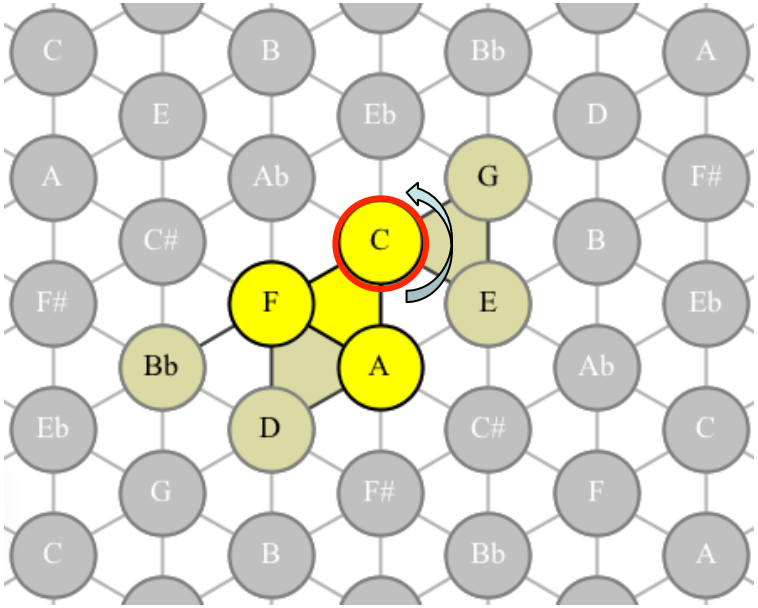




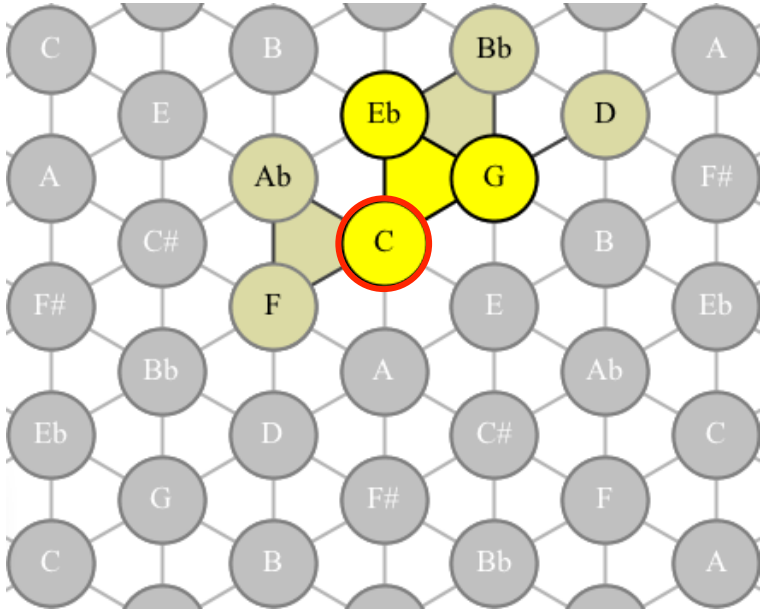
http://www.youtube.com/watch?v=oazDu9t_DTk

M. Andreatta, « Math'n pop : symétries et cycles hamiltoniens en chanson », *Tangente*, (à paraître)

Maths et expériences oumu(po)piennes sur les Beatles

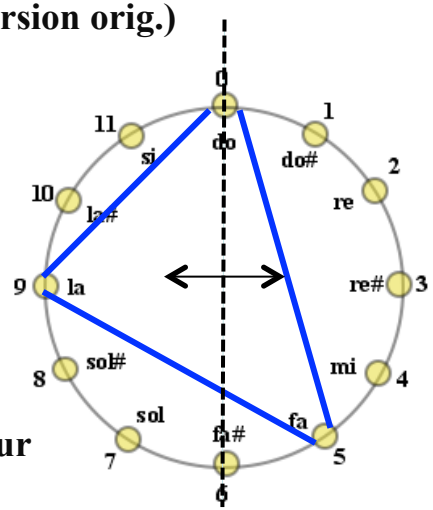


Rotation
(autour du *do*)

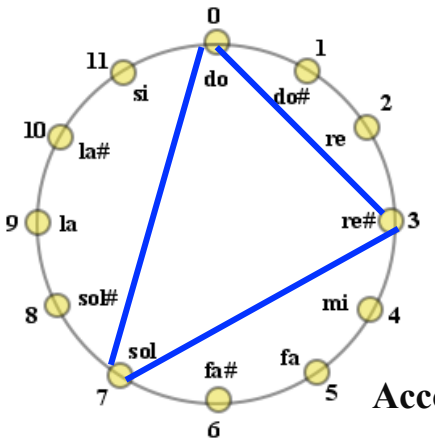


Beatles, Hey Jude
(version orig.)

Beatles, Hey Jude
(version transformée)



inversion



Accord majeur

Accord mineur

➔ `http://www.lacl.fr/~lbigo/scw13`

MERCI DE VOTRE ATTENTION !

Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS UMR 9912