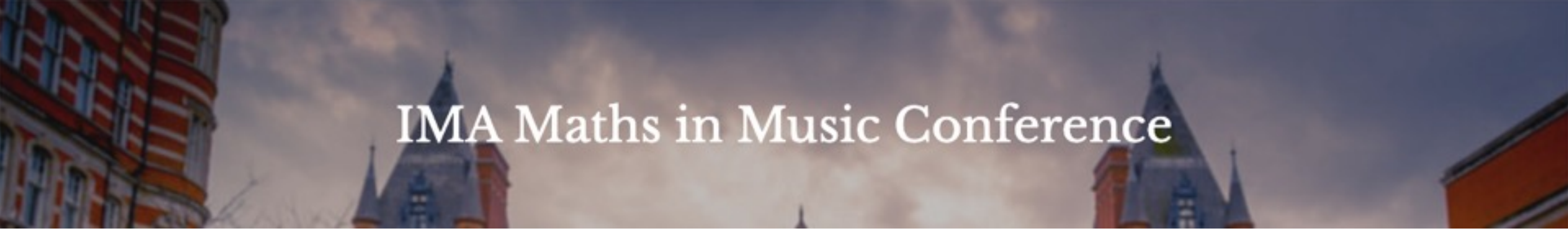


The Music of Maths: a ‘mathemusical’ journey

July 13-15, 2022

Moreno Andreatta
CNRS / IRMA / Université de Strasbourg
CNRS / IRCAM / Sorbonne Université
www.morenoandreatta.com

IRMA Institut de Recherche Mathématique Avancée · Université de Strasbourg · cnrs · ircam Centre Pompidou · Sorbonne Université

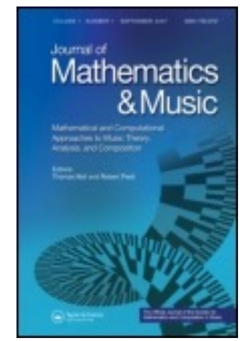
Maths/Music in academic research

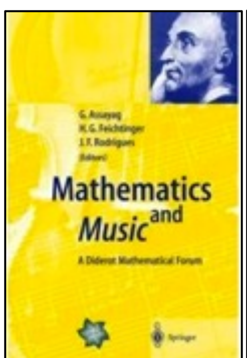
Conferences of the SMCM (with Springer LNAI Proceedings):

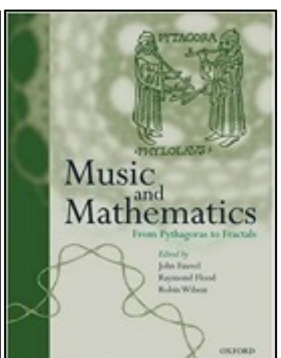
- 2007 Technische Universität (Berlin, Allemagne)
- 2009 Yale University (New Haven, USA)
- 2011 IRCAM (Paris, France)
- 2013 McGill University (Canada)
- 2015 Queen Mary University (Londres)
- 2017 UNAM (Mexico City)
- 2019 Universidad Complutense de Madrid (Spain)
- 2022 Georgia State University (Atlanta, USA)

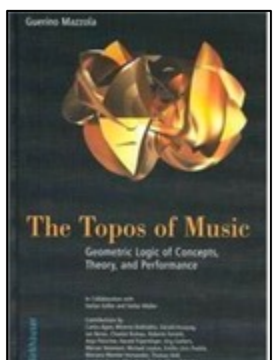
Official Journal and MC code (00A65: Mathematics and Music)

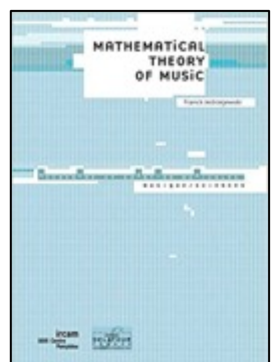
- *Journal of Mathematics and Music*, Taylor & Francis
  (Editors: E. Amiot, J. Yust | Associate ed.: D. Conklin)

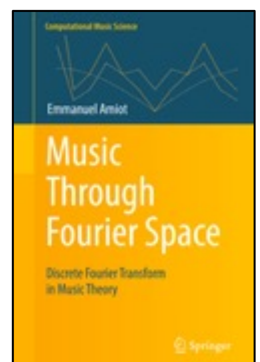


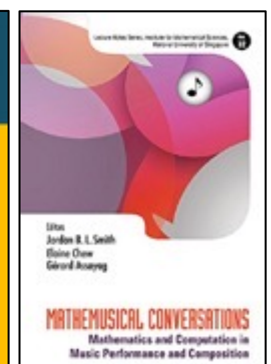


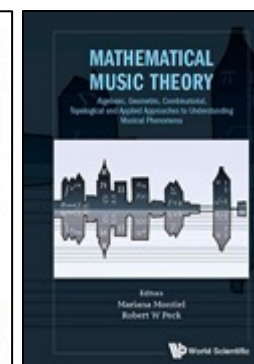






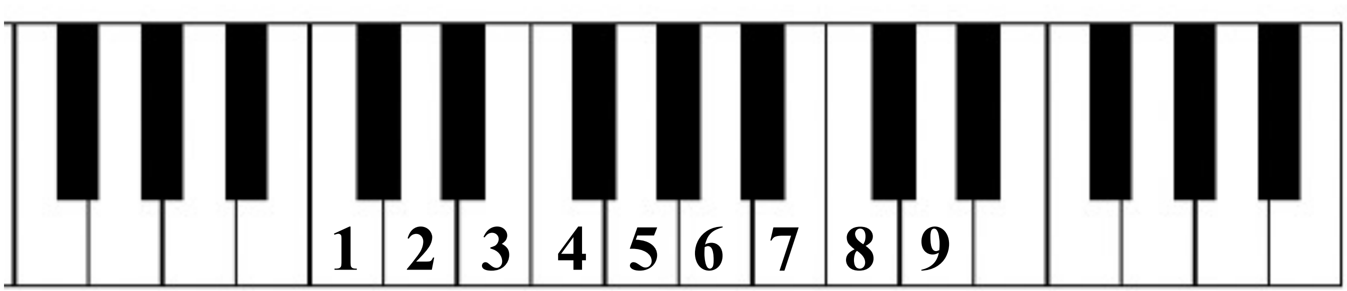




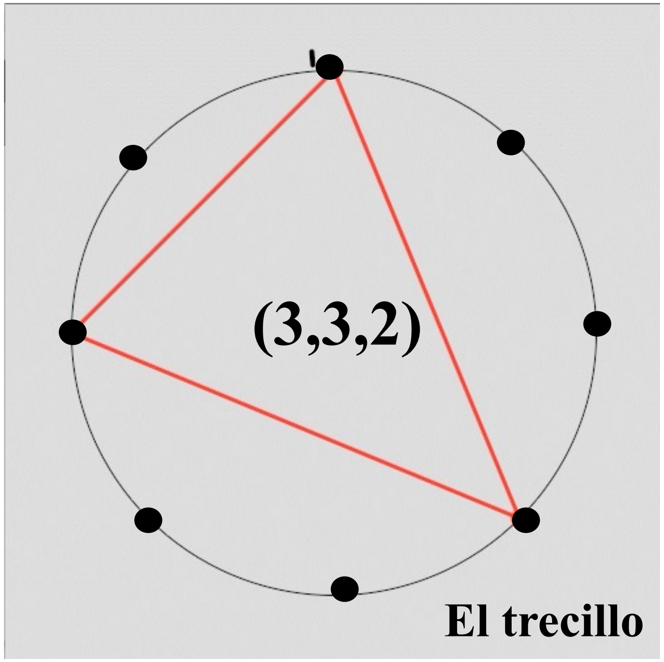


(non-exhaustive list…)

A Song for π

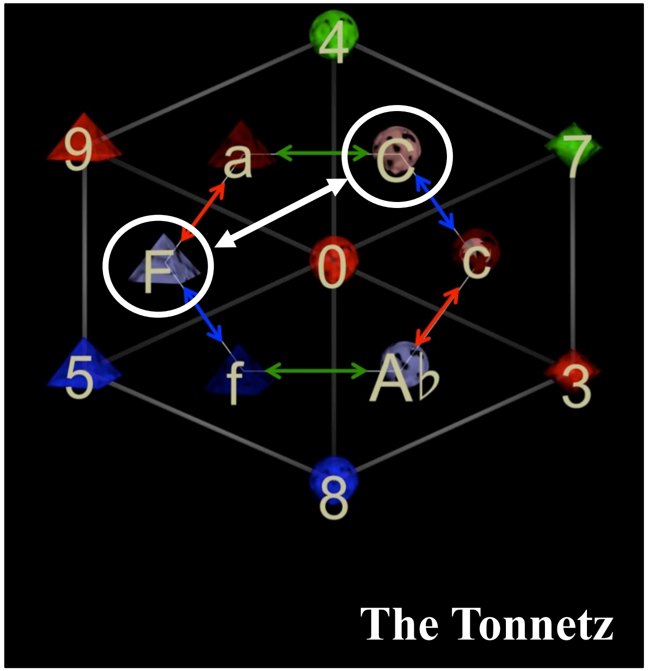


3,1415926535897932384626433832795028841971693993751... PACE



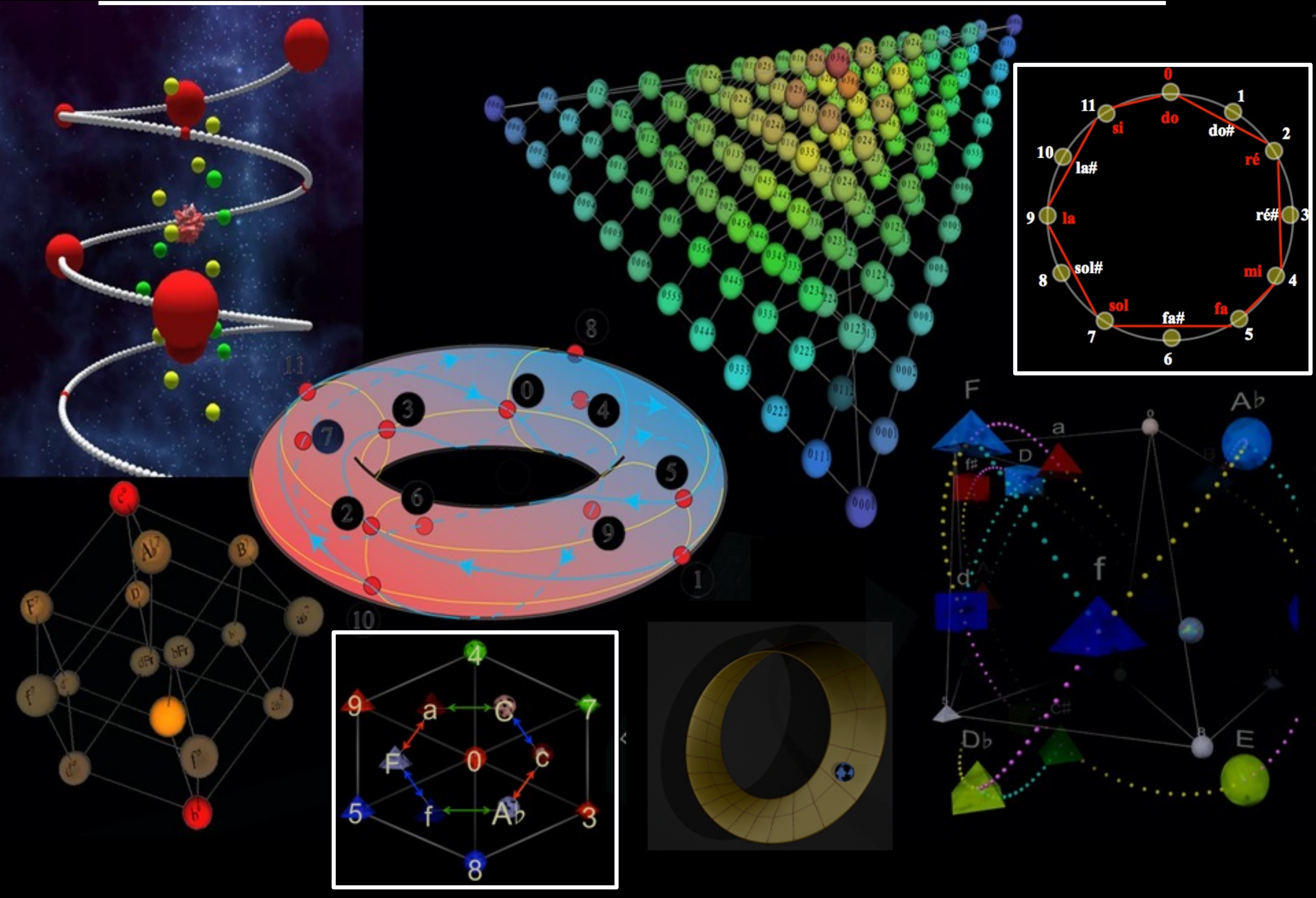
El trecillo

INTERNATIONAL DAY OF MATHEMATICS MARCH 14

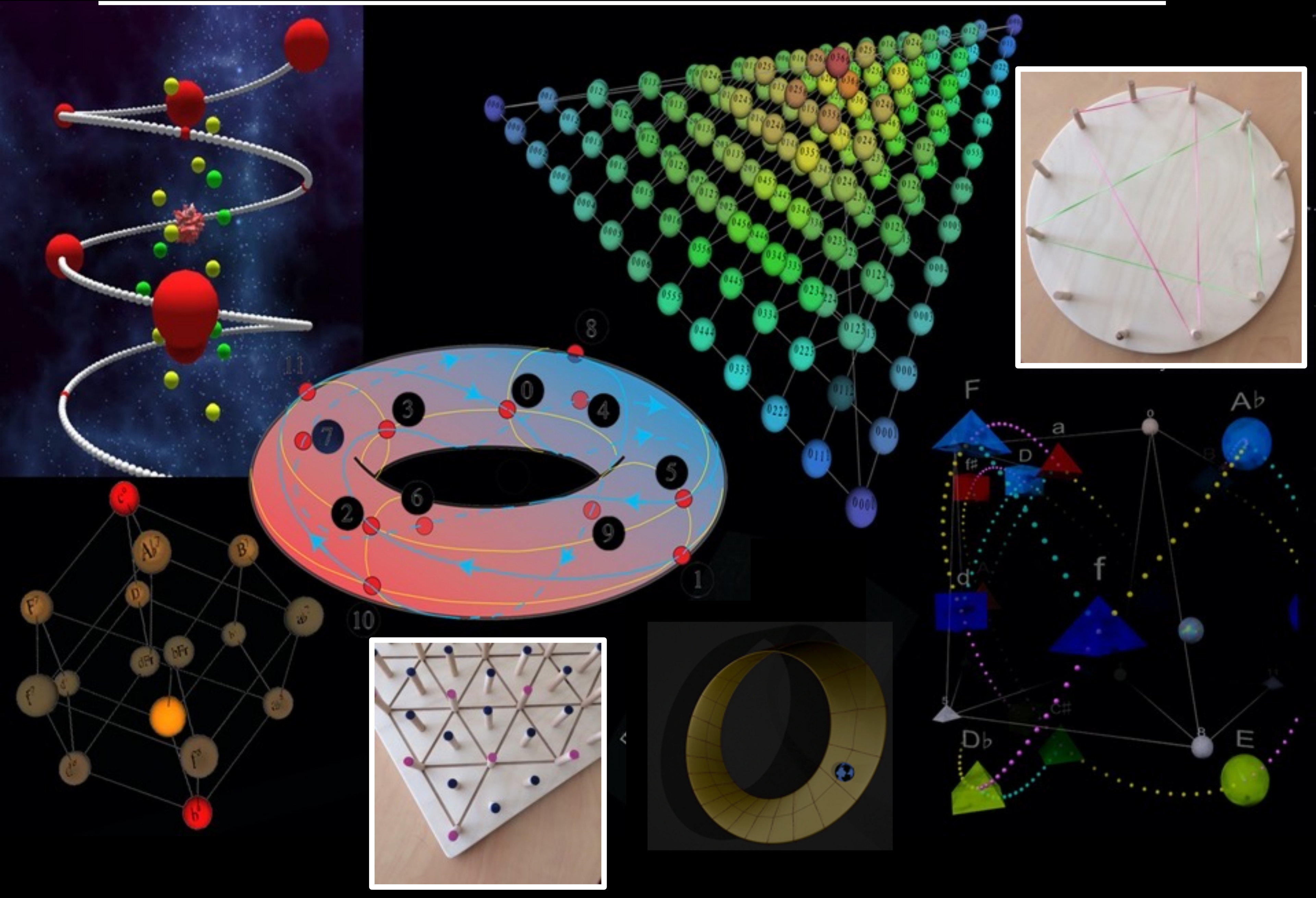


The Tonnetz

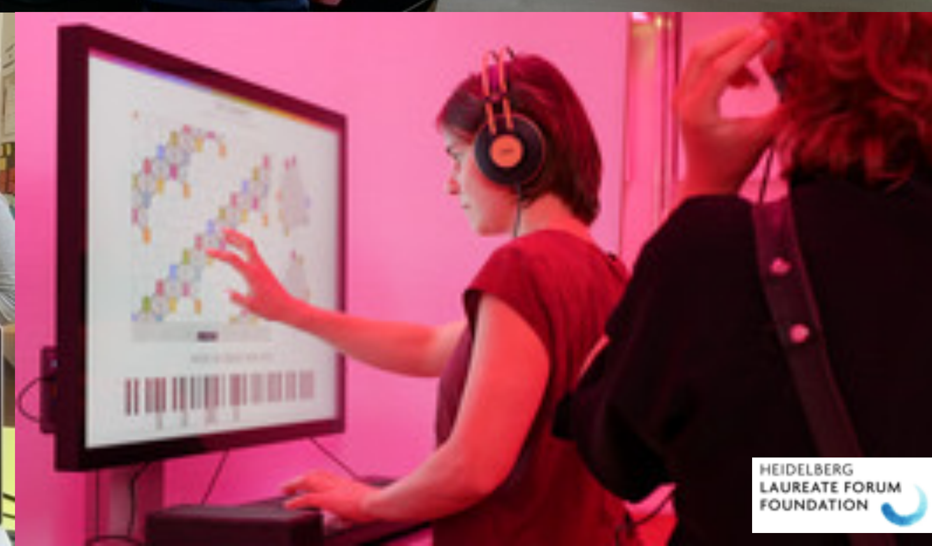
The galaxy of geometrical models at the service of music



The galaxy of geometrical models at the service of music




Rakuten
M-AUDIO
EVOLUTION


HEIDELBERG
LAUREATE FORUM
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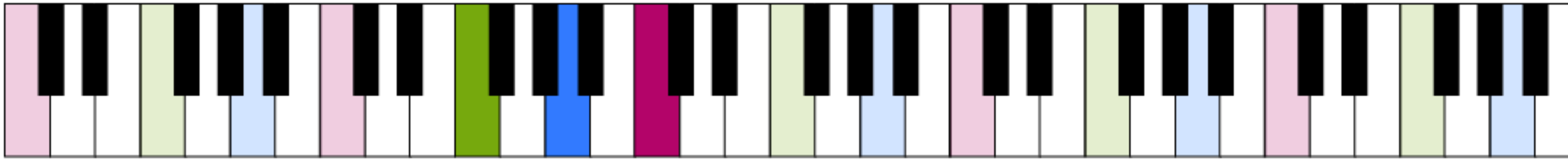
la.la.lab
THE MATHS OF MUSIC

May. 17, 2019 to Dec. 2020

IMAGINARY
open mathematics

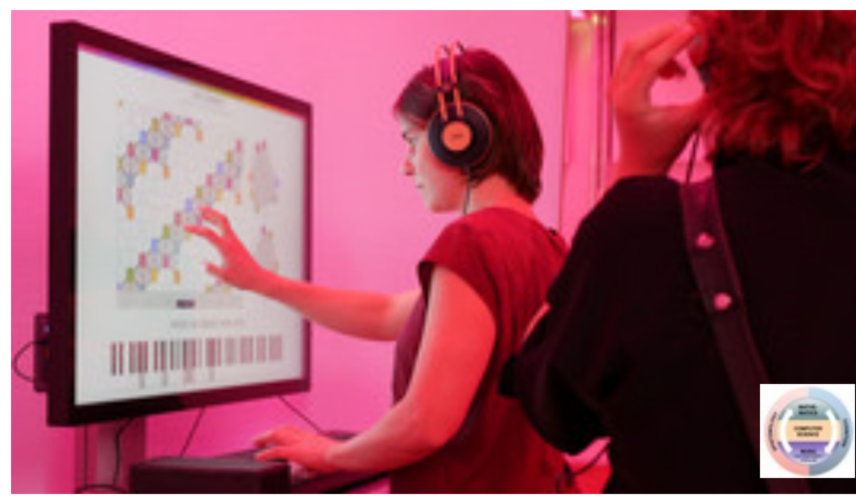
HEIDELBERG
LAUREATE FORUM
FOUNDATION

https://imaginary.org/



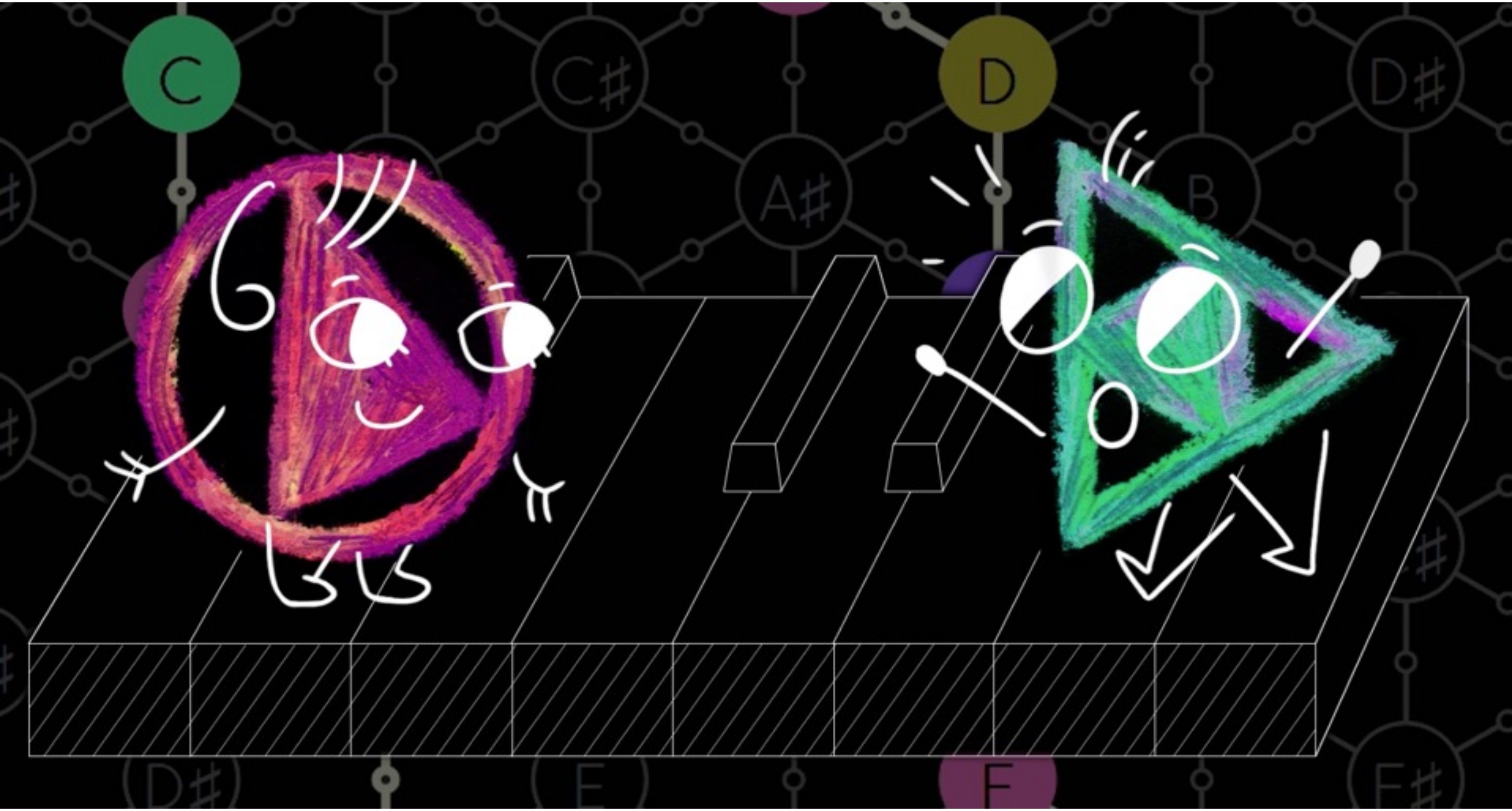
La.La.Lab brings the visitor to an interactive exploration and discovery of music from a mathematical perspective. The exhibition pivots over three axis:

- **Music theory.** Learning what tools build music, and how these tools are used to create art. Basic concepts and historical comments.
- **Current research.** The latest trends of research in the connection of maths and music. Artificial Intelligence, theoretical and new instruments, classification and composition tools.
- **Art and entertainment.** A joyful display of artworks from artists and mathematicians in the field. Talks/concerts at scheduled events



The Tonnetz web environment (developer: C. Guichaoua)

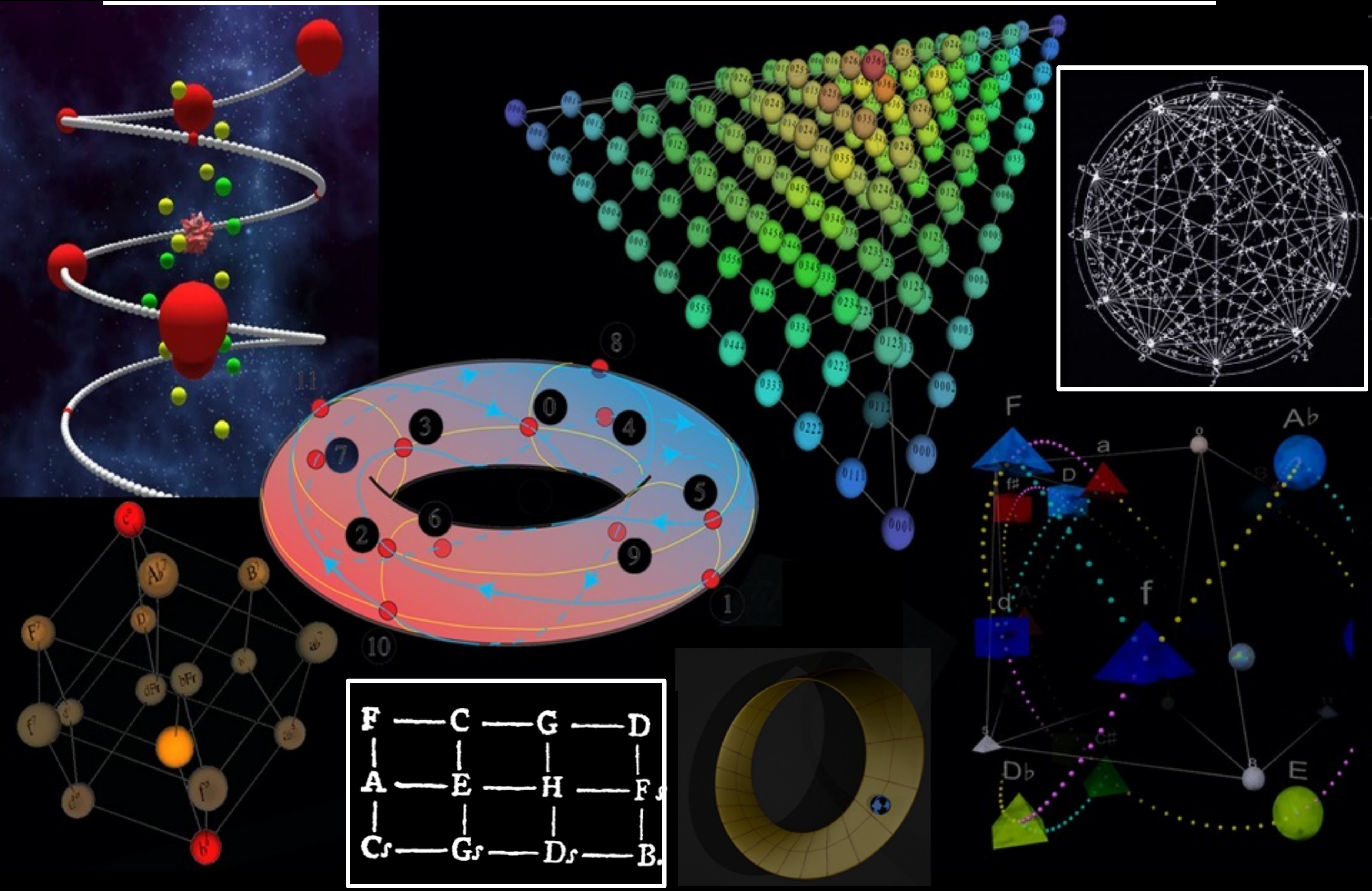
« Musique et mathématiques »: a pedagogical film



➔ www.morenoandreatta.com

AuDiMATH
AUTOUR DE LA DIFFUSION
DES MATHÉMATIQUES

The galaxy of geometrical models at the service of music

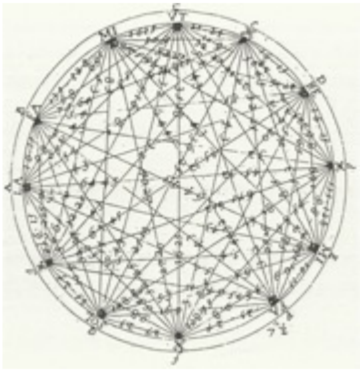


Music&maths: parallel destiny or mutual influences?

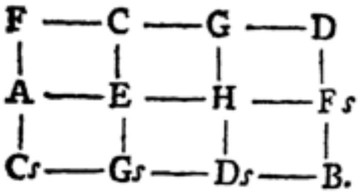
I. Xenakis (1922-2001)

MUSIC	MATHS
500 B.C. Pitches and lengths of strings are related. Here **music** gives a marvelous thrust to **number theory** and **geometry**.	Discovery of the fundamental importance of **natural numbers** and the invention of **fractions**.
No correspondence in music.	Positive irrational numbers [...]
300 B.C. [...] Music theory highlights the discovery of the **isomorphism between the logarithms** (musical intervals) and **exponentials** (string lengths) more than 15 centuries before their discovery in mathematics; also a **premonition of group theory** is suggested by Aristoxenos.	No reaction in mathematics. [...]
1000 A.D. Invention of the **two-dimensional spatial representation of pitches** linked with time by means of staves and points [...] seven centuries (1635-37) before the magnificent **analytical geometry** of Fermat and Descartes.	*No parallel in mathematics.*
1500 *No response or development of the preceding concepts.*	Zero and negative numbers are adopted. Construction of the set of rationals.
1600 *No equivalence, no reaction.*	The sets of real numbers and of logarithms are invented.
1648 Invention of **musical combinatorics** by Marin Mersenne (*Harmonicorum Libri*)	Probability theory by Bernoulli (*Ars Conjectandi*, 1713)
1700 [...] The **fugue**, for example, is an **abstract automaton** used two centuries before the birth of the science of automata. Also, there is an **unconscious manipulation of finite groups** (Klein group) in the four variations of a melodic line used in counterpoint.	Number theory is ahead of but has no equivalent yet in temporal structures. [...]
1773 A first **geometric** and **graph-theoretic** representation of pitches (*Speculum Musicum*)	Invention of graph theory
1900 Liberation from the tonal yoke. First acceptance of the neutrality of chromatic totality (Loquin [1895], Hauer, Schoenberg).	The infinite and transfinite numbers (Cantor). Peano axiomatics. [...] The beautiful measure theory (Lebesque, ...)
1920 First radical formalization of macrostructures through the serial system of Schoenberg.	No new development of the number theory.
1929 and **1937-1939** Susanne K. Langer and Ernst Krenek on the role of axioms in music	David Hilbert, *Die Grundlage der Geometrie* (1899)
1946 Milton Babbitt on group theory and integral serialism	Rudolf Carnap, *The Logical Syntax of Language* (1937)

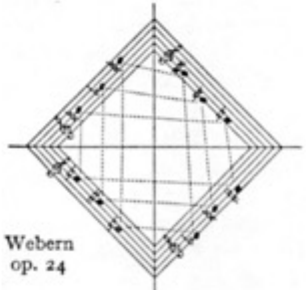
Pythagoras and the monochord, VI^th^-V^th^ Century B.C.

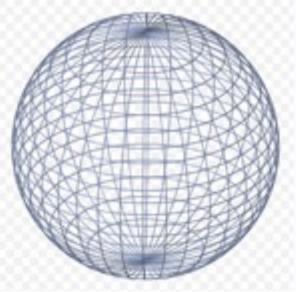


Mersenne and the 'musical clock', 1648



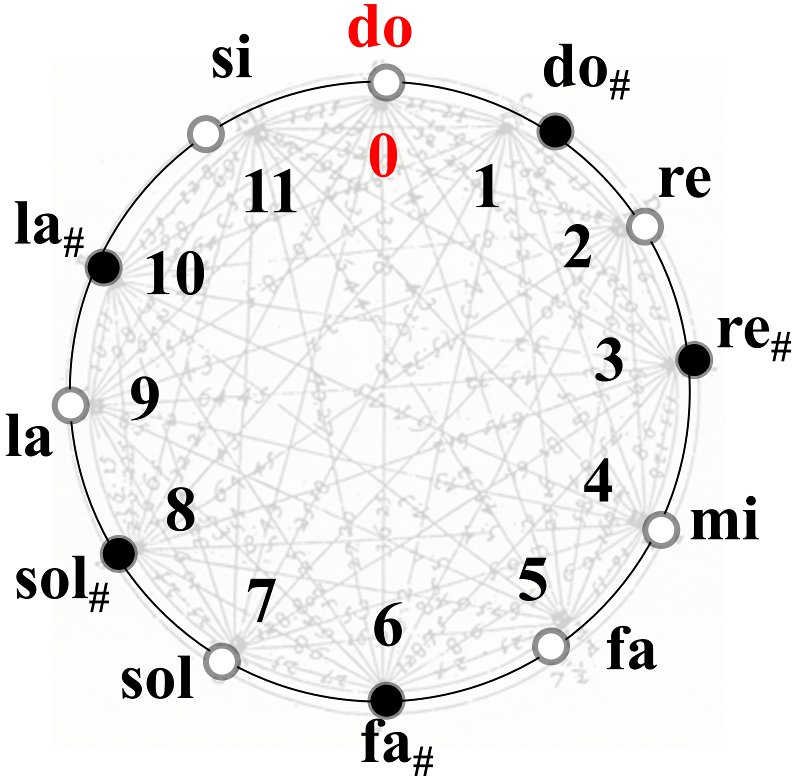
Euler and the *Speculum musicum*, 1773



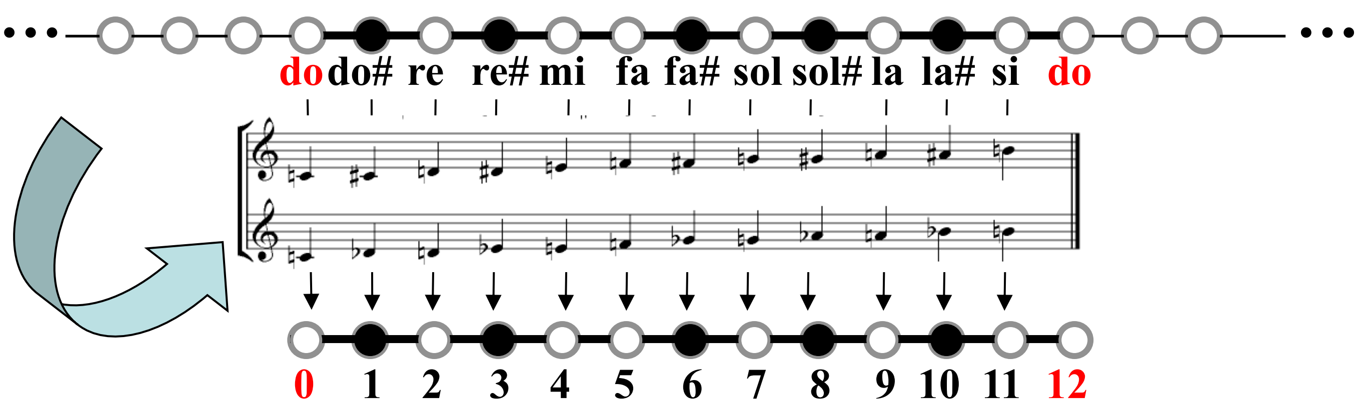


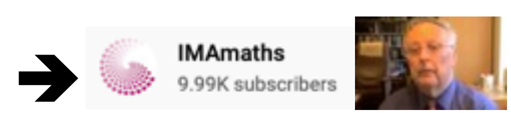
The circular representation of the pitch space

M. Mersenne



Harmonicorum Libri XII, 1648





Permutational melodies in contemporary music

114 Marin Mersenne, *Harmonicorum Libri XII*, 1648

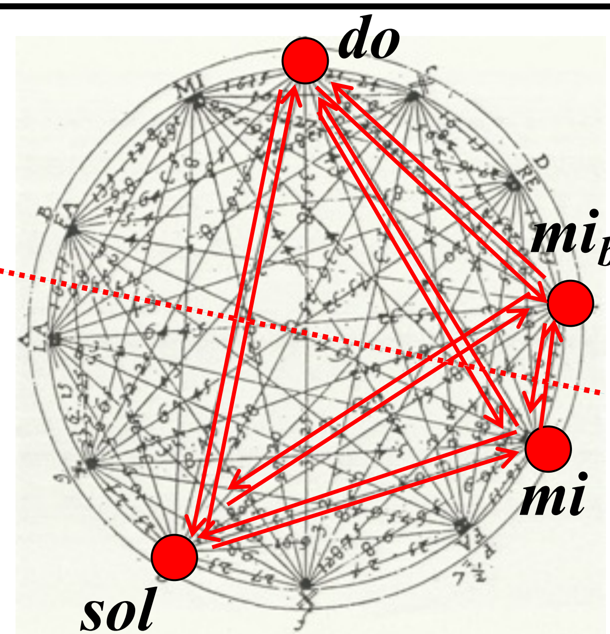
LIBER SEPTIMVS.

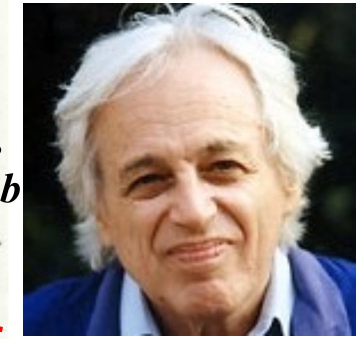
DE CANTIBVS, SEV CANTILENIS,

EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 22.

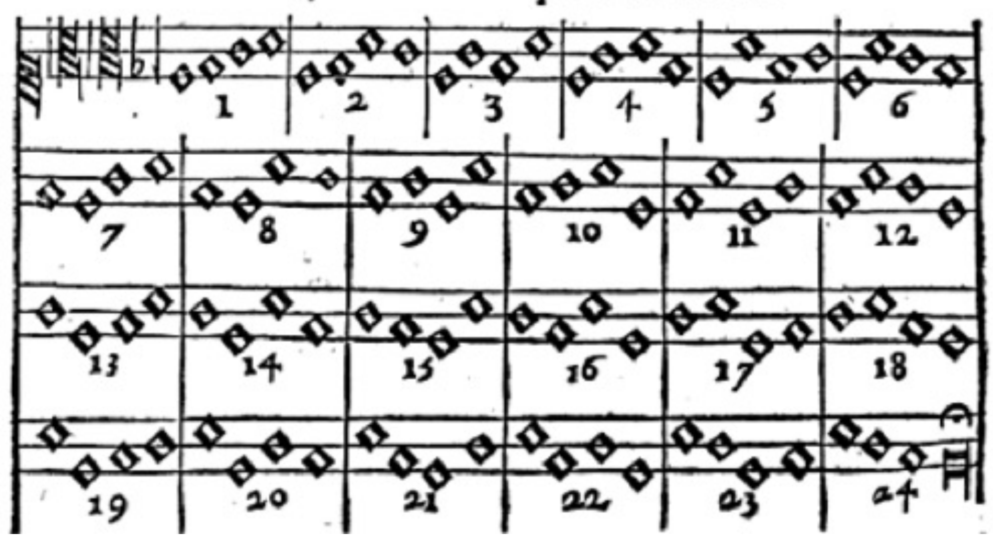
I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6227020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000





Six Bagatelles
(G. Ligeti, 1953)

Varietas, seu Combinatio quatuor notarum.



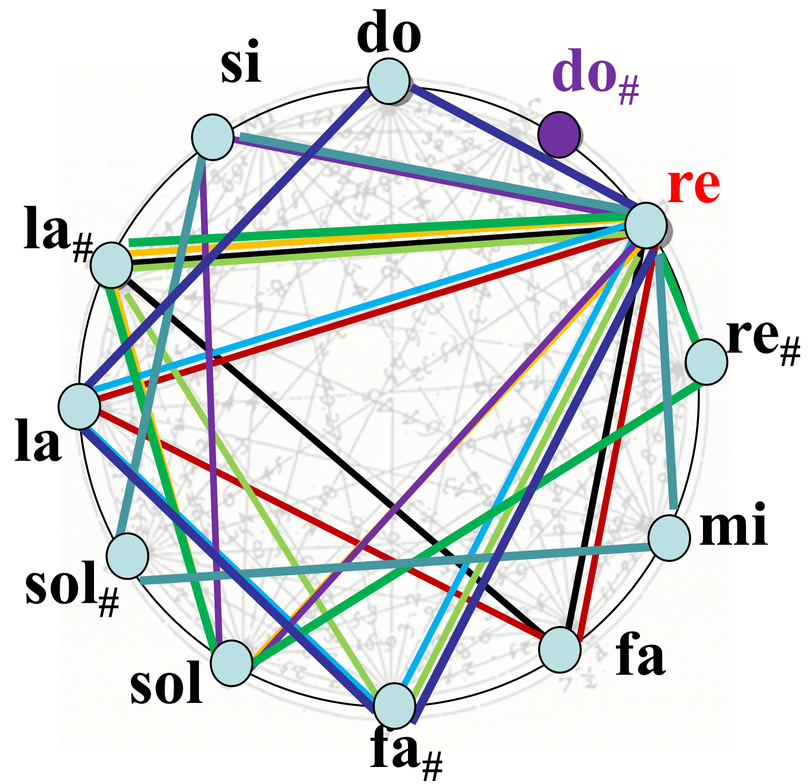
A permutational song: one sentence, one note (one note left!)

Una volta soltanto una storia d'amore finisce (M. Andreatta)

Una volta una storia d'amore
soltanto una storia
Una storia d'amore
soltanto una storia d'amore
Una storia
soltanto una storia
Una storia d'amore
soltanto
Una volta soltanto
una storia d'amore soltanto

Un amore soltanto una volta
soltanto una storia d'amore soltanto
Una storia d'amore
soltanto una volta una storia
Un amore
una volta soltanto
Un amore finisce
soltanto
Una volta una storia d'amore finisce
Un amore soltanto

Una volta soltanto una storia d'amore finisce



➔ http://repmus.ircam.fr/_media/moreno/11_una_storia_moreno_andreatta.mp3

Antes, despues

Julio Cortazar

Como los juegos al llanto
como la sombra a la columna
el perfume dibuja el jazmín
el amante precede al amor
como la caricia a la mano
el amor sobrevive al amante
pero inevitablemente
aunque no haya huella ni presagio

aunque no haya huella ni presagio
como la caricia a la mano
el perfume dibuja el jazmín
el amante precede al amor
pero inevitablemente
el amor sobrevive al amante
como los juegos al llanto
como la sombra a la columna

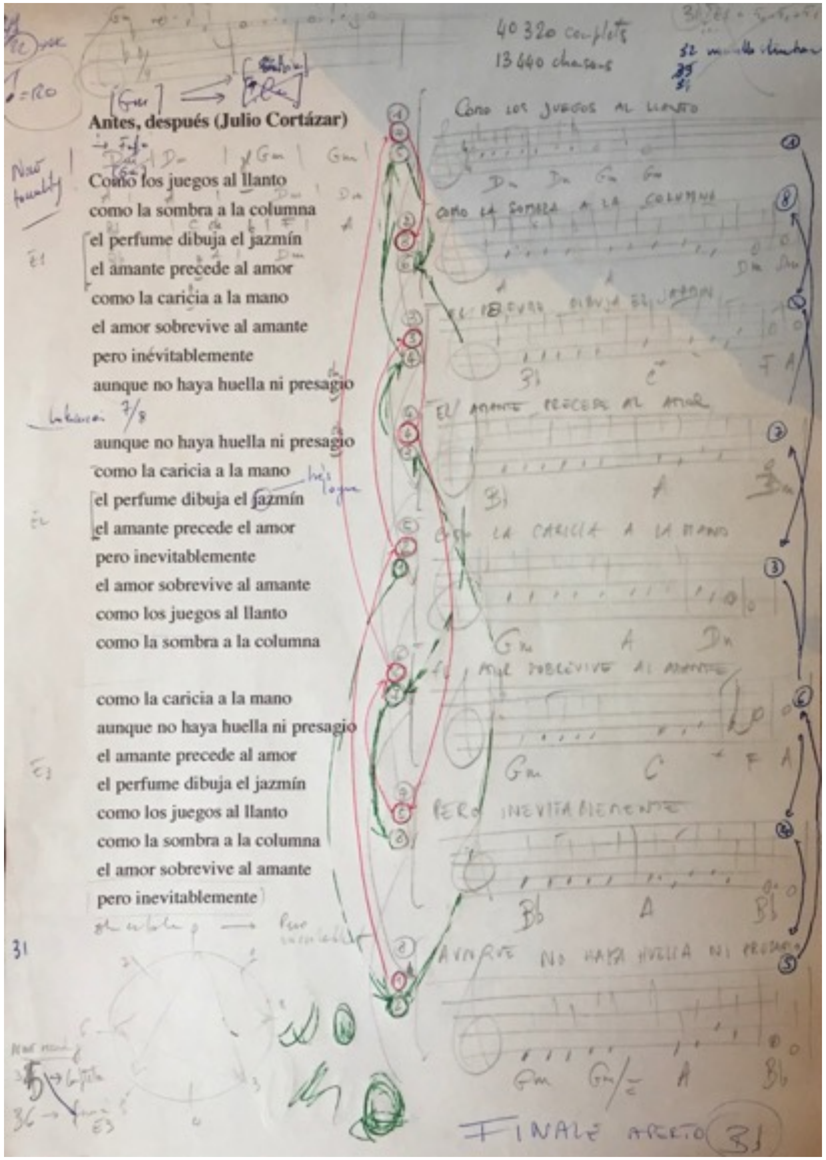
como la caricia a la mano
aunque no haya huella ni presagio
el amante precede al amor
el perfume dibuja el jazmín
como los juegos al llanto
como la sombra a la columna
el amor sobrevive al amante
pero inevitablemente

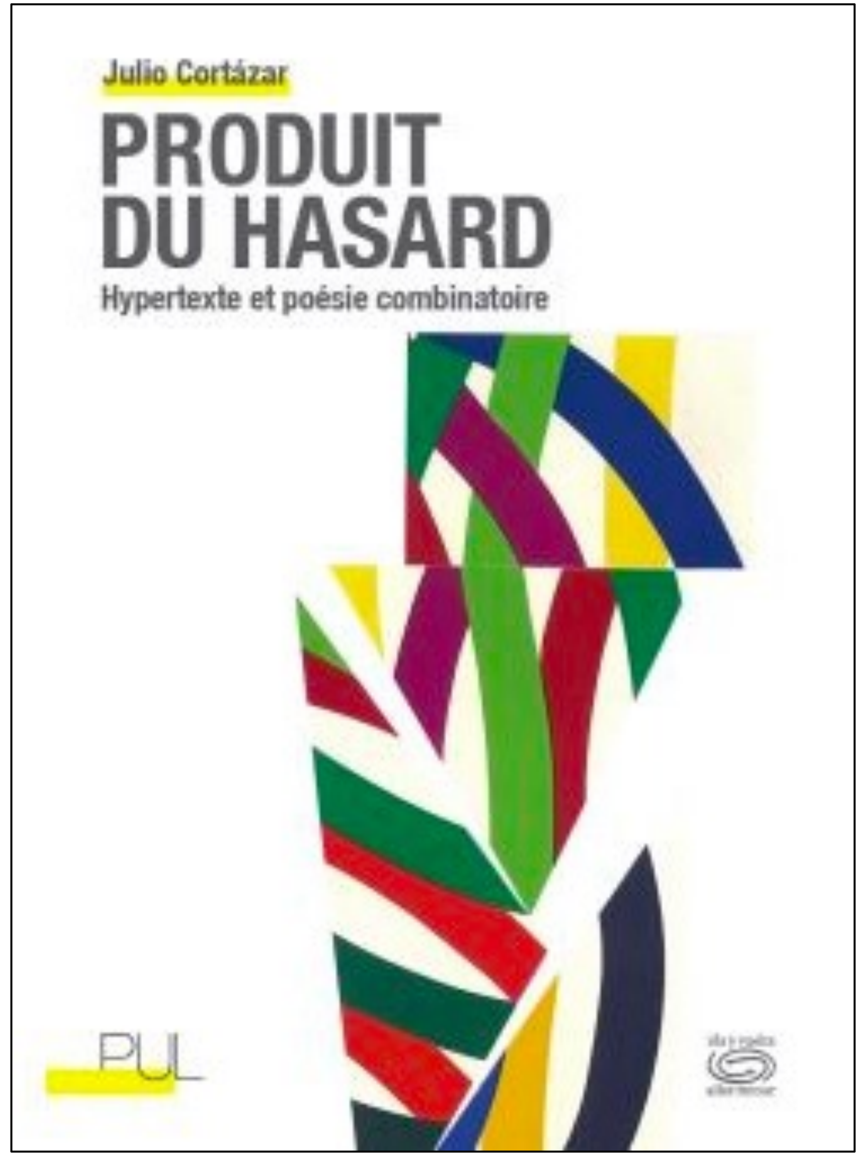
Come i giochi le lacrime
come l'ombra la colonna
il profumo disegna il gelsomino
l'amante precede l’amore
come la carezza la mano
l’amore fa durare l'amante
ma allora inevitabilmente
Anche in assenza di traccia o di presagio

Anche in assenza di traccia o di presagio
come la carezza la mano
il profumo disegna il gelsomino
l'amante precede l’amore
ma allora inevitabilmente
l’amore fa durare l'amante
come i giochi le lacrime
come l'ombra la colonna

come la carezza la mano
anche in assenza di traccia o di presagio
l'amante precede l’amore
il profumo disegna il gelsomino
come i giochi le lacrime
come l'ombra la colonna
l’amore fa durare l'amante
ma allora inevitabilmente

A permutation song based on a permutational poem



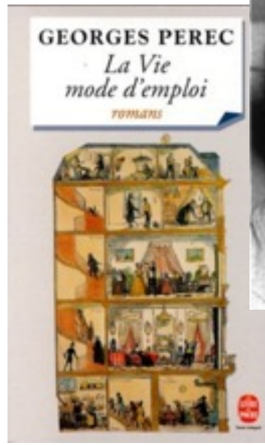


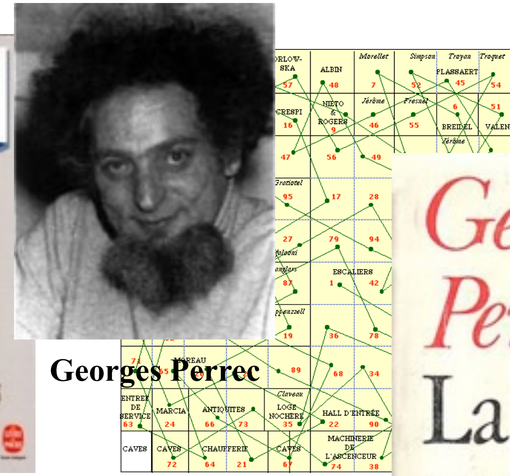
The song was premiered the 13 November 2021 at Ircam within the *mamuphi* Seminar
➔ **https://medias.ircam.fr/x61f8d4 (10:57-14:50)**

Rules and constraints in the artistic process

OuLiPo (Ouvroir de Littérature Potentielle)

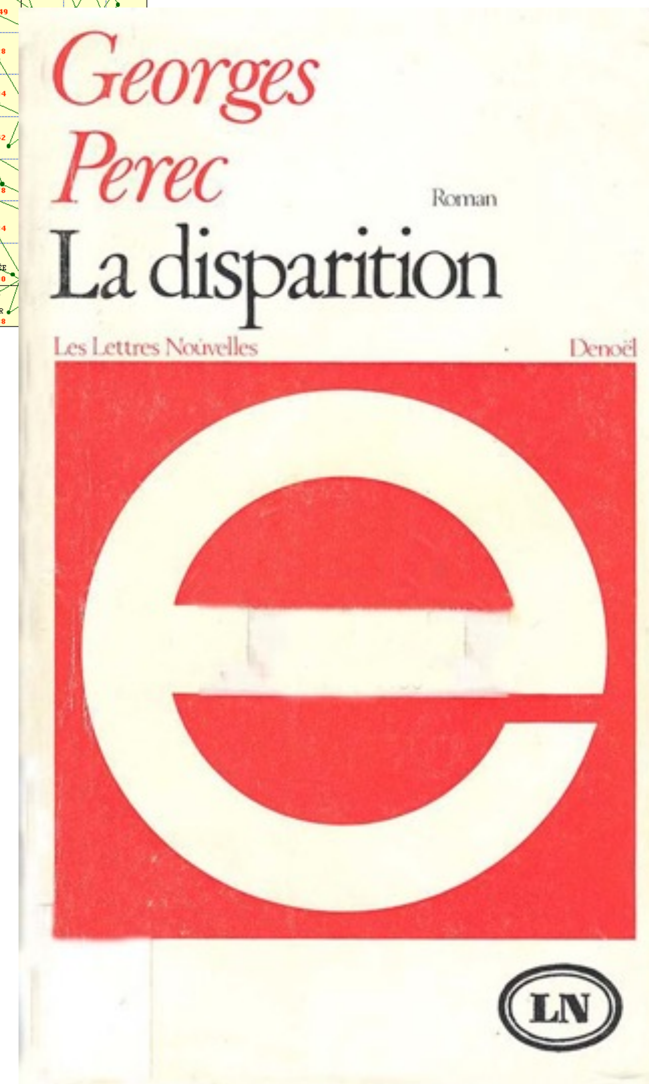
Cent mille milliards de poèmes, 1961





Georges Perrec

La vie mode d'emploi,



Raymond Queneau

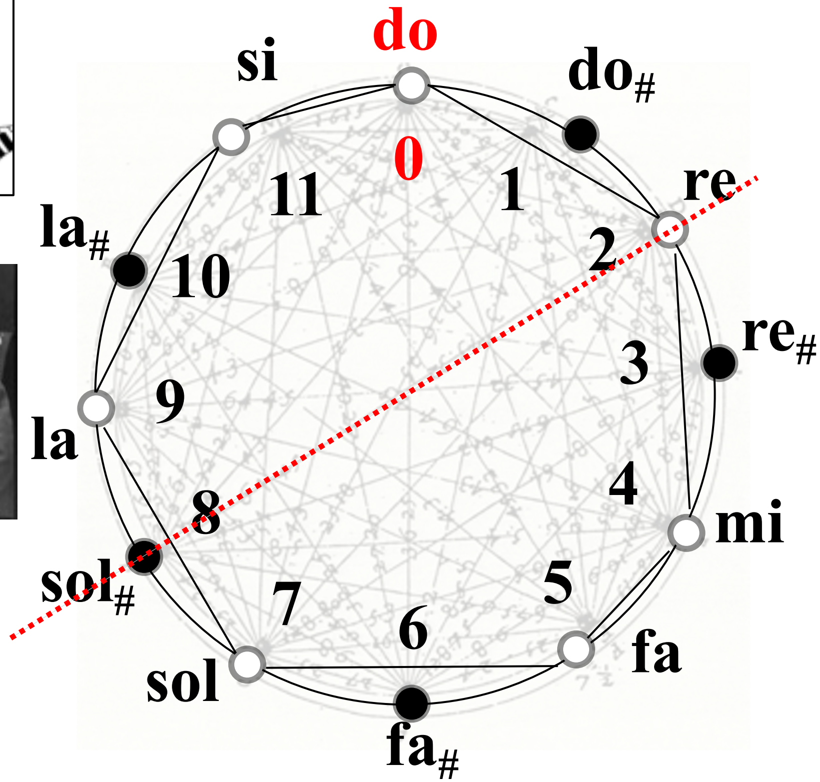
Italo Calvino

Il castello dei destini incrociati, 1969

The circular representation of the pitch space

M. Mersenne

C. Durutte



Harmonicorum Libri XII, 1648

ESTHÉTIQUE MUSICALE.

TECHNIE

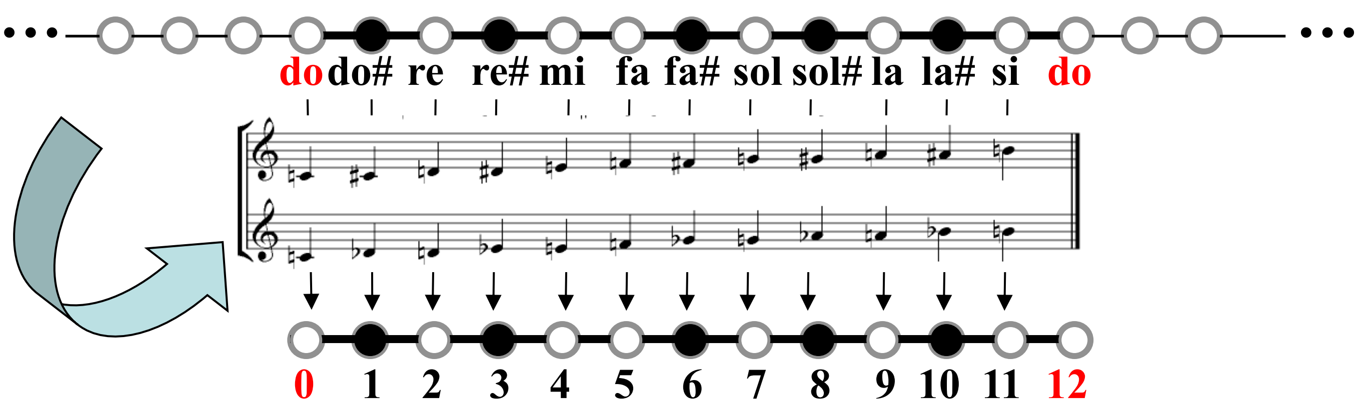
OU

LOIS GÉNÉRALES DU SYSTÈME HARMONIQUE,

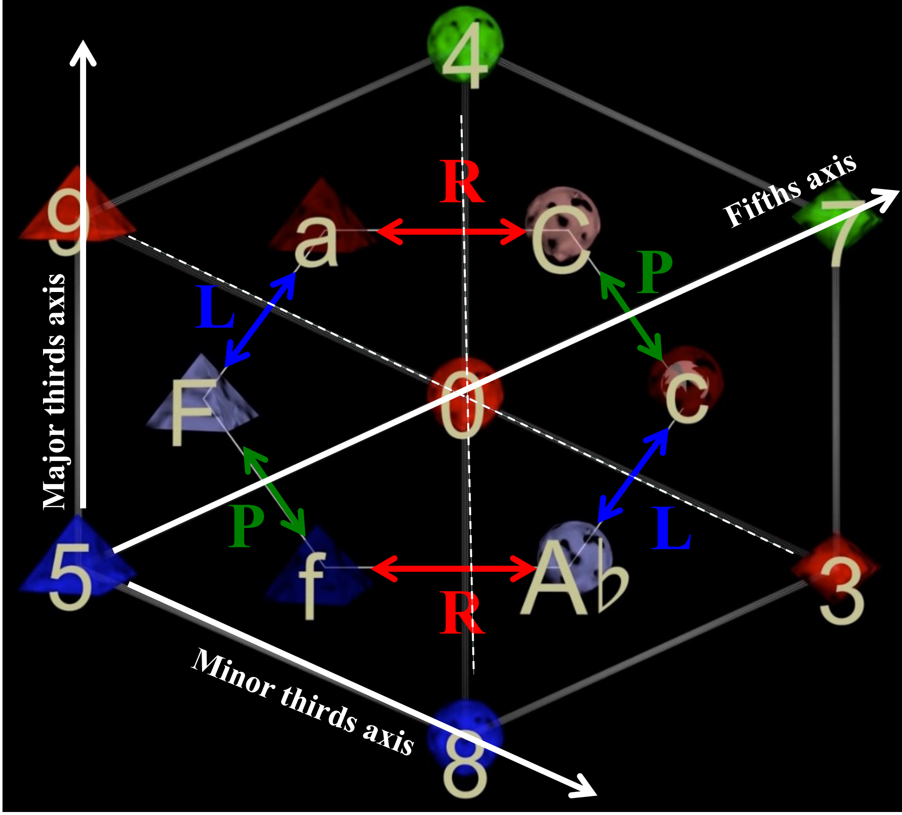
PARIS,

METZ,

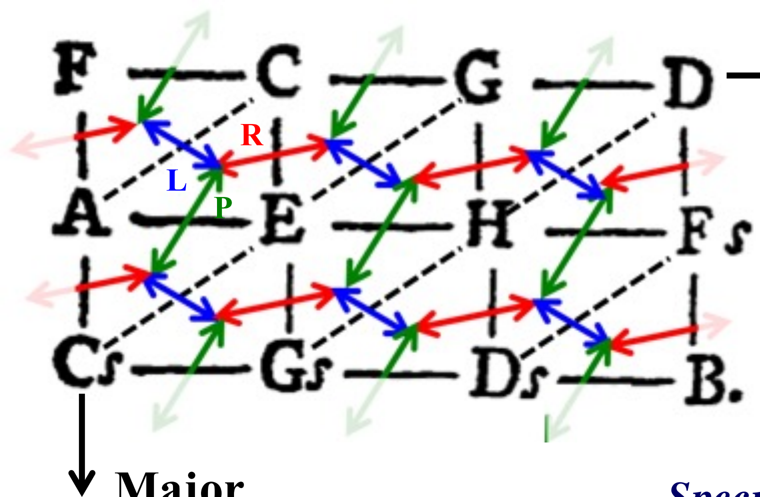
1855.



The *Tonnetz*

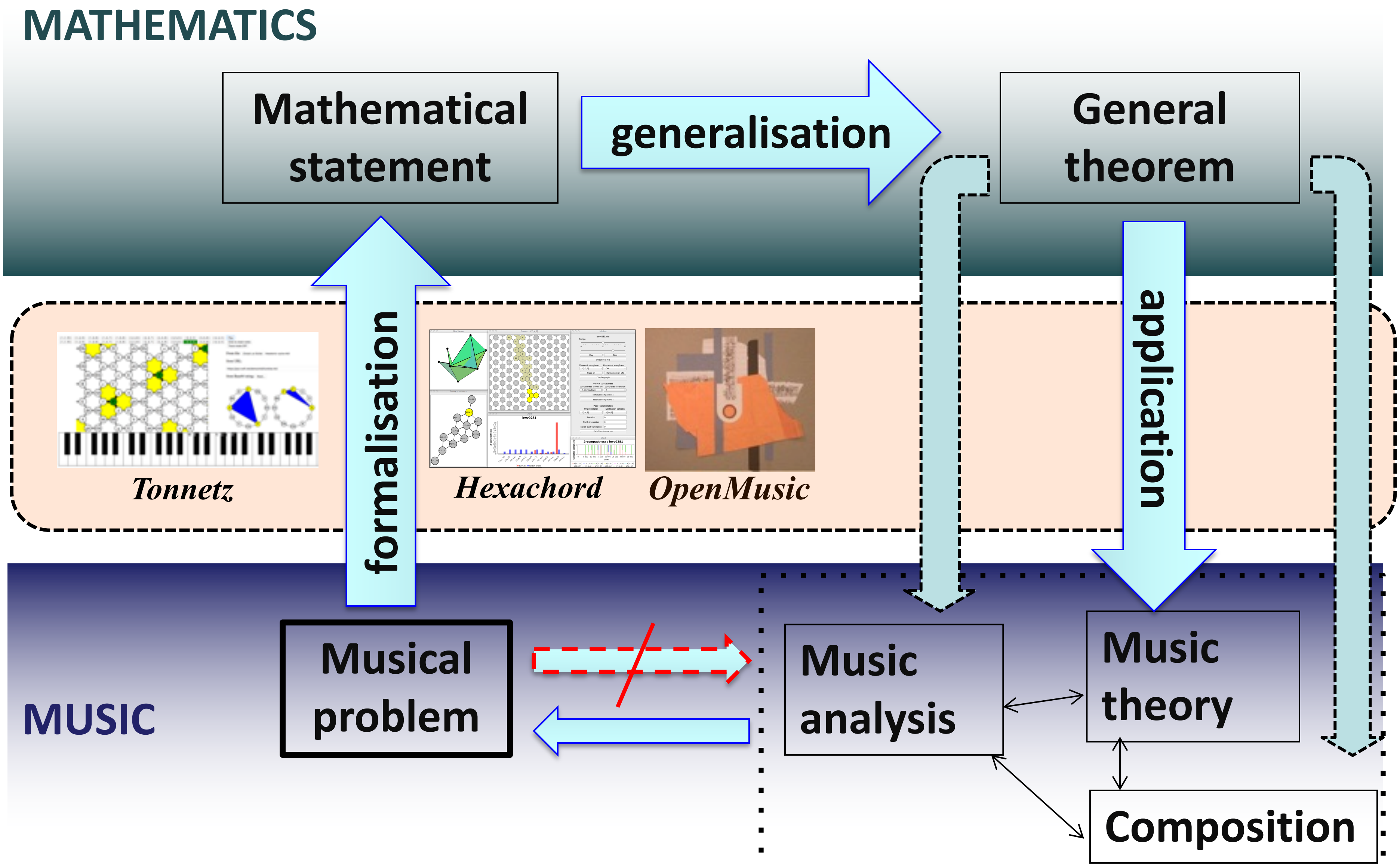


© Wiebke Drenckhan – Math'n Pop

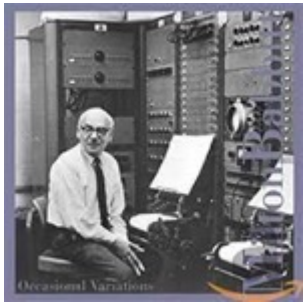


Speculum Musicum
(Euler, 1773)

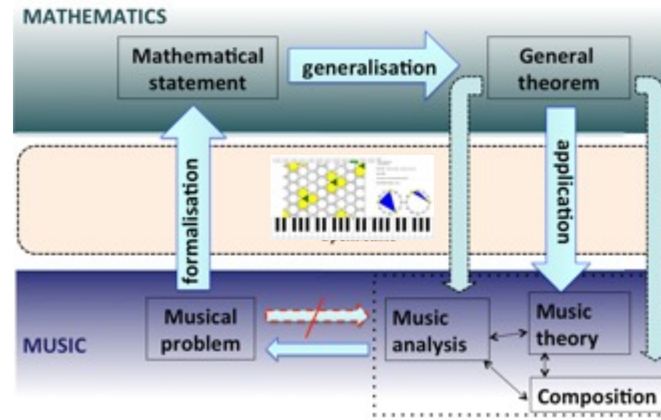
The double movement of a 'mathemusical' activity

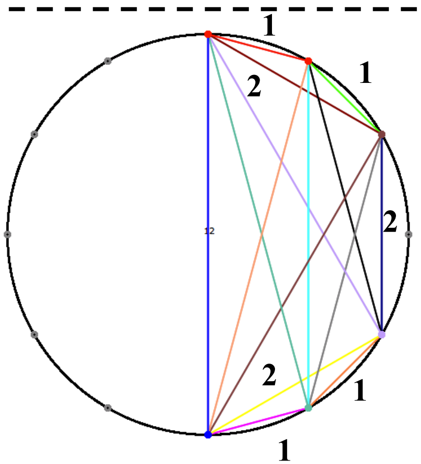


A historical example of “mathemusical” problem



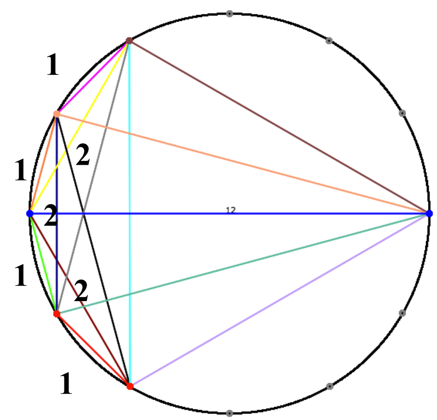
Milton Babbitt’s Hexachord Theorem

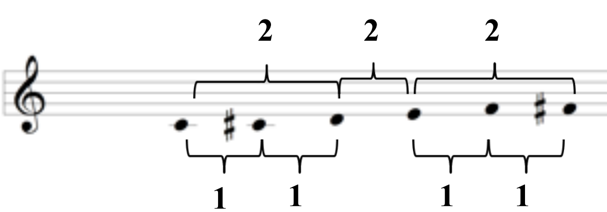


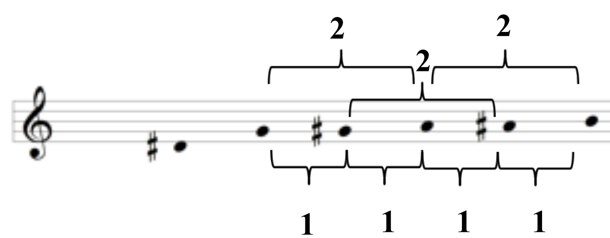


HOMOMETRY

Hexachord Theorem







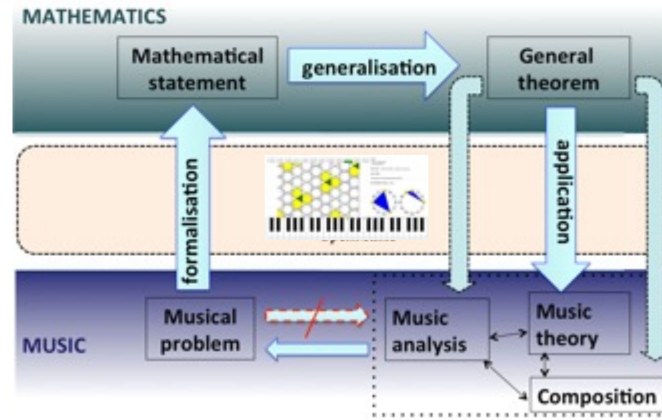
➔ **Check the interactive environment:** https://guichaoua.gitlab.io/web-hexachord/hexachordTheorem

A historical example of "mathemusical" problem

New hexachordal theorems in metric spaces with a probability measure

SUBMITTED

Abstract. The Hexachordal Theorem is a fancy combinatorial property of the sets in $\mathbf{Z}/12\mathbf{Z}$ discovered and popularized by the musicologist Milton Babbitt (1916-2011). Its has been given several explanations and partial generalizations. Here we complete the comprehension of the phenomenon giving both a geometrical and a probabilistic perspective.



Continuous Hexachordal Theorems

Theorem

Let $f : \mathbb{S}^k \mapsto [0,1]$ ($k = 1, 3$ or 7) be such that $\int_{\mathbb{S}^k} f = 1/2$ and $\bar{f} = 1 - f$. Then for every t one has

$$\int f(x)f(x \cdot t)\, dx = \int \bar{f}(x)\bar{f}(x \cdot t)\, dx.$$

Application to $f = \mathbf{1}_A$.

Probabilistic version

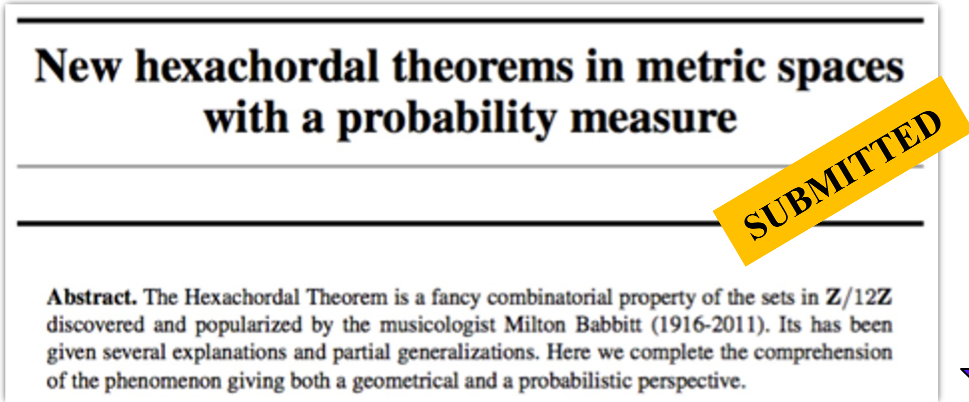
Let $(\mathfrak{X}, d, \mu)$ be a metric measure space of mass 1. We consider two $\mathcal{X}$-valued random variables, X and Y that are μ-uniform and independent.

Definition

Two sets A and B of mass $1/2$ are <u>(metrically) homometric</u> if

$$\mathbb{P}(d(X,Y) \in \cdot | X \in A, Y \in A) = \mathbb{P}(d(X,Y) \in \cdot | X \in B, Y \in B).$$

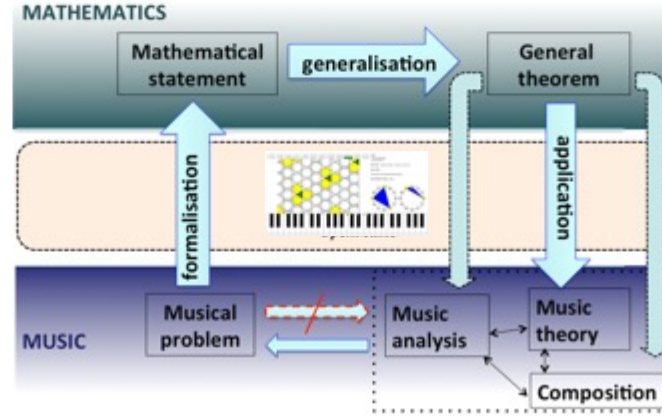
A historical example of “mathemusical” problem



New hexachordal theorems in metric spaces with a probability measure

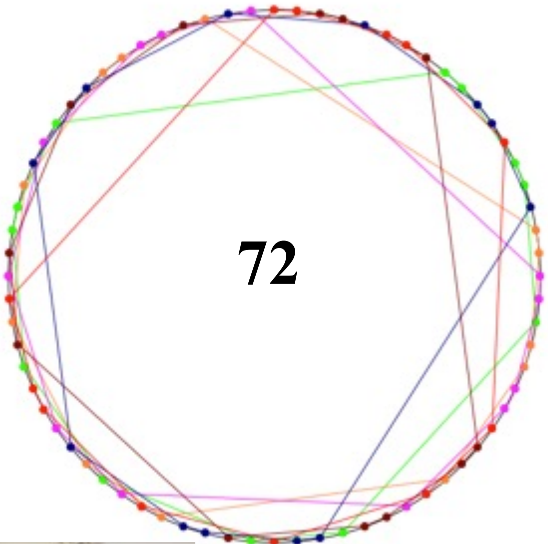
SUBMITTED

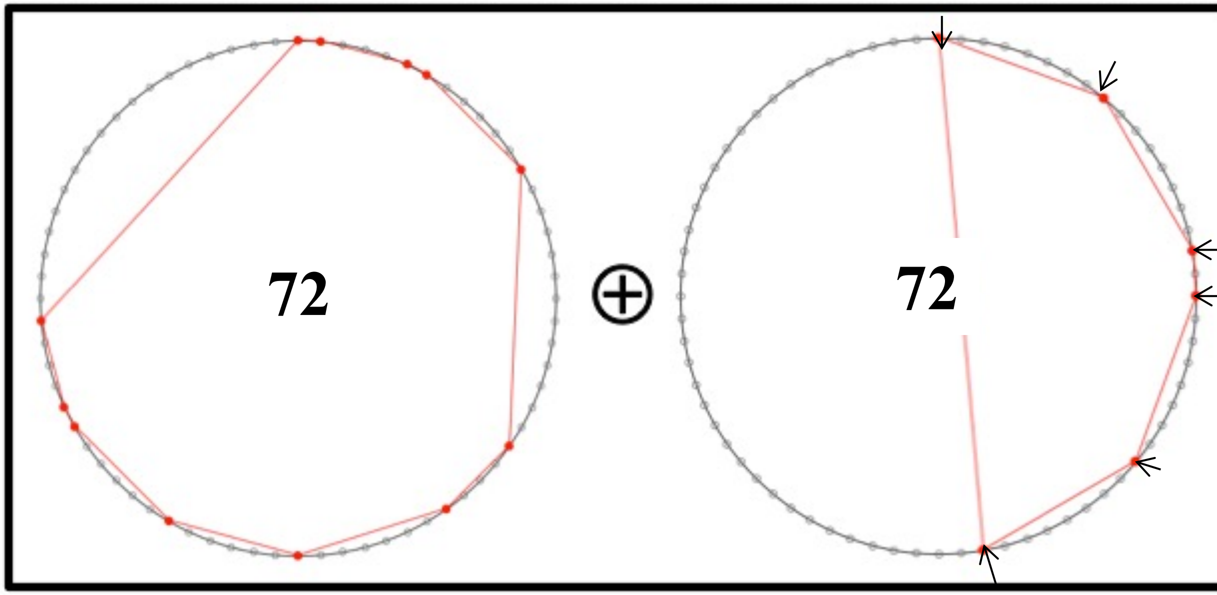
Abstract. The Hexachordal Theorem is a fancy combinatorial property of the sets in $\mathbf{Z}/12\mathbf{Z}$ discovered and popularized by the musicologist Milton Babbitt (1916-2011). Its has been given several explanations and partial generalizations. Here we complete the comprehension of the phenomenon giving both a geometrical and a probabilistic perspective.



- The finite/compact spaces on which a group is transitively acting (homogeneous spaces).
  - Compact Lie groups
  - Spheres, torii, etc.
  - Cayley graphs of finite groups with generators (Hypercubes, symmetric group, etc.)
  - Other homogeneous graphs (Petersen graph, truncated icosahedron, etc.)

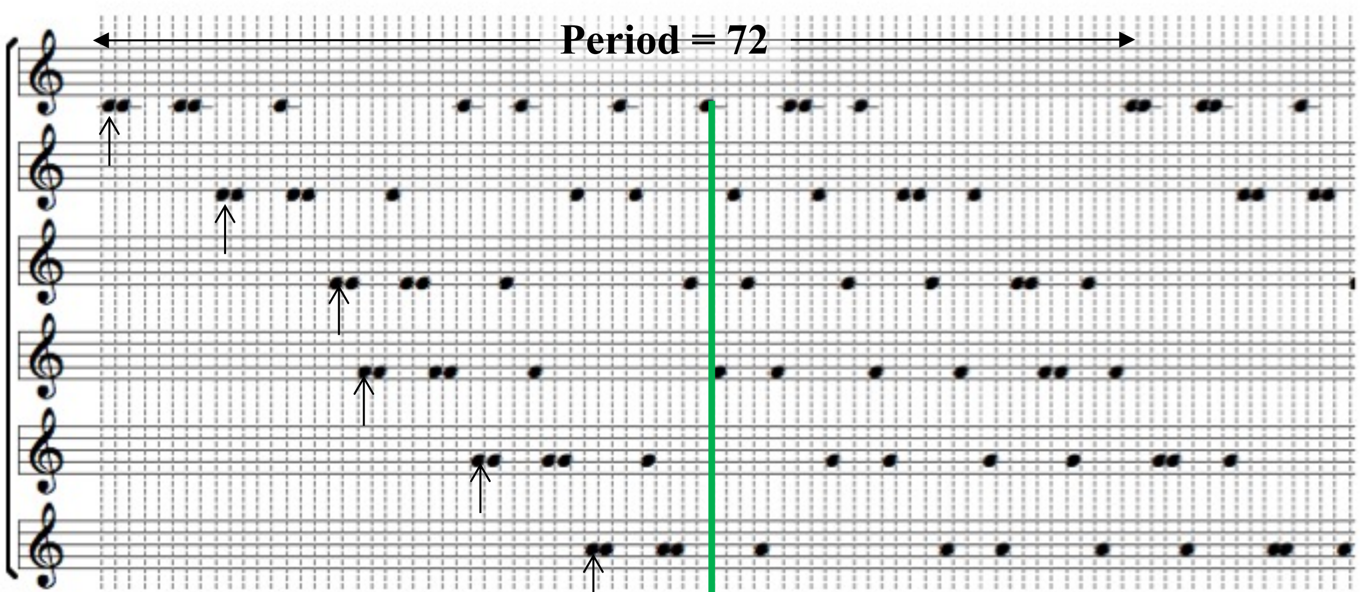
Aperiodic Rhythmic Tiling Canons (Vuza Canons)



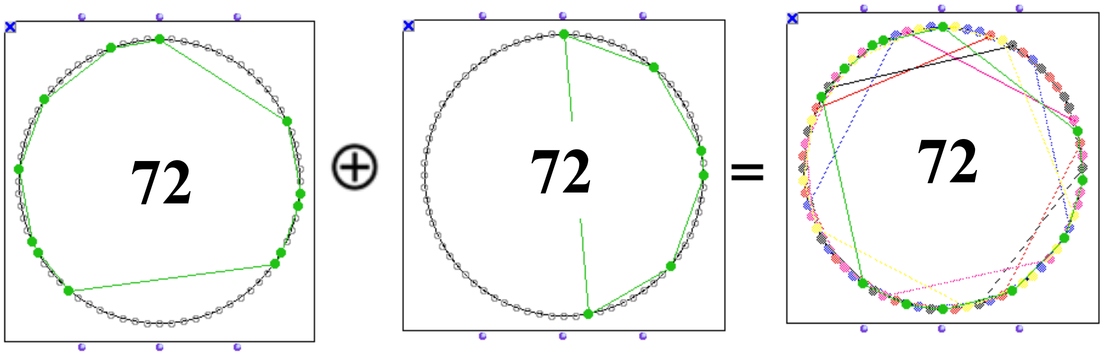


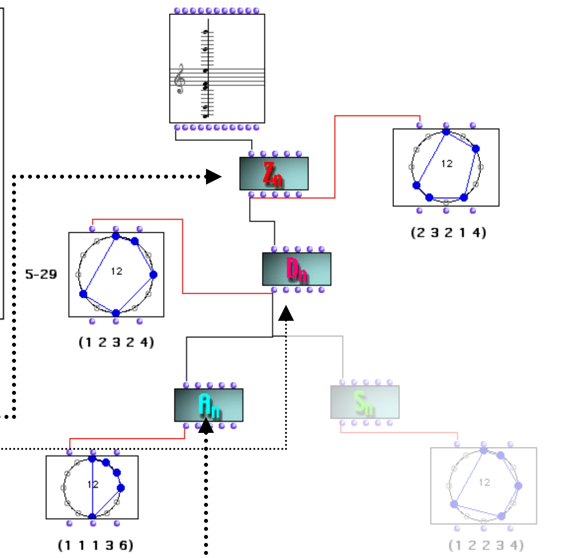
Dan Vuza

Anatol Vieru



Group-based paradigmatic classification of Vuza Canons





Z_n

(1 3 3 6 11 4 9 6 5 1 3 20)
(20 3 1 5 6 9 4 11 6 3 3 1)
(1 4 1 19 4 1 6 6 7 4 13 6)
(6 13 4 7 6 6 1 4 19 1 4 1)
(1 5 15 4 5 6 6 3 4 17 3 3)
(3 3 17 4 3 6 6 5 4 15 5 1)

(8 8 2 8 8 38)
(16 2 14 2 16 22)
(14 8 10 8 14 18)

D_n

(1 3 3 6 11 4 9 6 5 1 3 20)
(1 4 1 19 4 1 6 6 7 4 13 6)
(1 5 15 4 5 6 6 3 4 17 3 3)

(8 8 2 8 8 38)
(16 2 14 2 16 22)
(14 8 10 8 14 18)

A_n

(1 3 3 6 11 4 9 6 5 1 3 20)
(1 4 1 19 4 1 6 6 7 4 13 6)

(14 8 10 8 14 18)

Tijdeman's
'Fundamental Lemma' (1995)
***R* tiles Z_n => *aR* tiles Z_n <*a*,*n*>=1**

Vuza Canons and Fuglede Spectral Conjecture

A subset of the *n*-dimensional Euclidean space tiles by translation *iff* it is *spectral*.

(*J. Func. Anal.* 16, 1974)

➔False in dim. $n \geq 3$
(Tao, Kolountzakis, Matolcsi, Farkas and Mora)

➔ Open in dim. 1 et 2

DEFINITION 6 *A subset A of some vector space (say $\mathbb{R}^n$) is spectral iff it admits a Hilbert base of exponentials, i.e. if any map $f \in L^2(A)$ can be written*

$$f(x) = \sum f_k \exp(2i\pi\lambda_k.x)$$

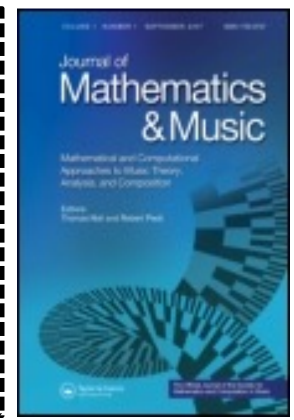
for some fixed family of vectors $(\lambda_k)_{k\in\mathbb{Z}}$ where the maps $e_k : x \mapsto \exp(2i\pi\lambda_k.x)$ are mutually orthogonal (i.e. $\int_A \overline{e_k} e_j = 0$ whenever $k \neq j$).

$\downarrow$ ***n*=1**

DEFINITION 8. *A subset $A \in \mathbb{Z}$ is* spectral *if there exists a spectrum $\Lambda \subset [0,1]$ (i.e., a subset with the same cardinality as A) such that $e^{2i\pi(\lambda_i - \lambda_j)}$ is a root of $A(X)$ for all distinct $\lambda_i, \lambda_j \in \Lambda$.*

Theorem (Amiot, 2009)

- *All non-Vuza canons are spectral.*
- *Fuglede Conjecture is true (or false) iff it is true (or false) for Vuza Canons*



Periods of Vuza Canons

72
108 120 144 168 180
200 216 240 252 264 270 280 288
300 312 324 336 360 378 392 396
400 408 432 440 450 456 468 480
500 504 520 528 540 552 560 576 588 594
600 612 616 624 648 672 675 680 684 696
700 702 720 728 744 750 756 760 784 792
800 810 816 828 864 880 882 888…

(Sloane's sequence A102562)

M. Andreatta & C. Agon (eds), "Tiling Problems in Music", Special Issue of the *Journal of Mathematics and Music*, Vol. 3, Number 2, July 2009 (with contributions by E. Amiot, F. Jedrzejewski, M. Kolountzakis and M. Matolcsi)

Towards a complete classification of non-periodic canons

Extended Vuza Canons

Greta Lanzarotto[1,2,3,4] and Ludovico Pernazza[1](✉)

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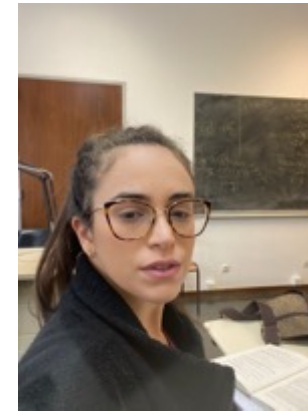
https://matematica.unipv.it/en/homepage-english/, https://www.matapp.unimib.it/en, https://www.altamatematica.it/en/, https://irma.math.unistra.fr/?lang=en



Greta Lanzarotto

Abstract. Starting from well-known constructions of aperiodic tiling rhythmic canons by G. Hajós, N.G. de Bruijn and D.T. Vuza, several generalisations are given. In this way, it is possible to find new aperiodic canons, that we call *extended Vuza canons*.

- **G. Lanzarotto, *Tiling problems in music and Fuglede's spectral conjecture*, ongoing PhD, University of Pavia / University of Strasbourg**
- **G. Lanzarotto, L. Pernazza, "Extended Vuza Canons", Proceedings MCM 2022, Springer.**

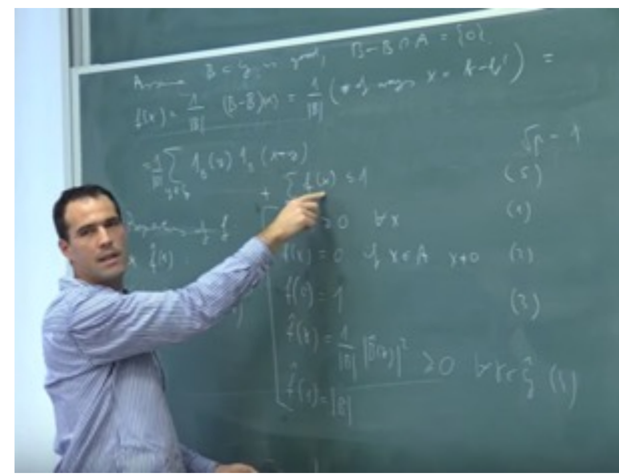
Fuglede Spectral conjecture for convex domains is true

(Submitted on 28 Apr 2019)

Let Ω be a convex body in $\mathbb{R}^d$. We say that Ω is spectral if the space $L^2(\Omega)$ has an orthogonal basis of exponential functions. There is a conjecture going back to Fuglede (1974) which states that Ω is spectral if and only if it can tile the space by translations. It has long been known that if a convex body Ω tiles then it must be a polytope, and it is also spectral. The converse, however, was proved only in dimensions $d \leq 3$ and under the a priori assumption that Ω is a polytope.
In this paper we prove that for every dimension d, if a convex body $\Omega \subset \mathbb{R}^d$ is spectral then it must be a polytope, and it can tile the space by translations. The result thus settles Fuglede's conjecture for convex bodies in the affirmative. Our approach involves a construction from crystallographic diffraction theory, that allows us to establish a geometric "weak tiling" condition necessary for the spectrality of Ω.

Subjects:	**Classical Analysis and ODEs (math.CA)**; Functional Analysis (math.FA); Metric Geometry (math.MG)
MSC classes:	42B10, 52B11, 52C07, 52C22
Cite as:	arXiv:1904.12262 **[math.CA]**
	(or arXiv:1904.12262v1 **[math.CA]** for this version)

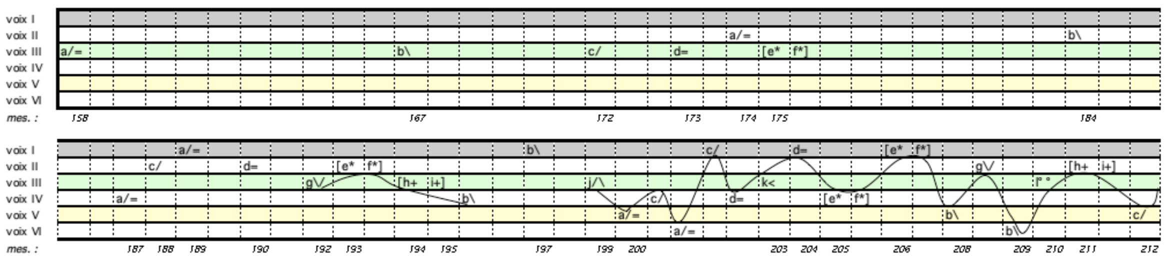
Nir Lev (Bar-Ilan University, Tel-Aviv)

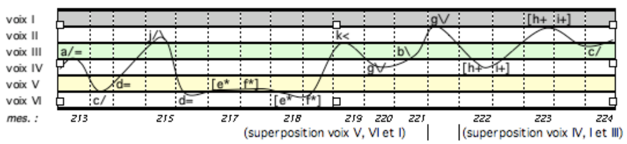


Mate Matolcsi (Rényi Institute, Budapest)

N. Lev & M. Matolcsi, "Fuglede conjecture for convex domains is true in all dimensions", *Acta Mathematica* 228, n. 2, p. 385-420, 2022.

Some compositional applications of the Vuza Canons model



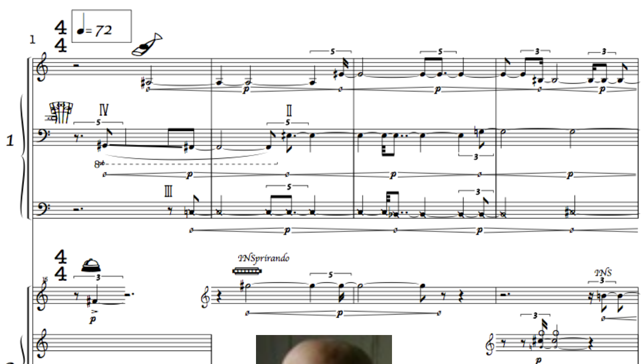


a/= : montée vers accord puis "mise en pulsation"
b\ : "mise en pulsation" superposé à un gliss. descendant
c/ : montée vers accord (tête de a/=)
d= "mise en pulsation" en diminuendo (fin de a/=)
[e*f*] : accord mis en "cross rythm I" (durée double)
g\/ : gliss. descendant puis ascendant
[i+j+] : accord mis en "cross rythm II" (durée double)
j\/ : gliss. ascendant puis descendant avec accent
k< : "son à l'envers"
l° ° : deux impacts brefs et piano

Coïncidences (1999)

Coïncidences - Fabien Levy : déroulement du canon (mes. 158 à 226)
(chaque impact fait 3 temps)

F. Lévy



M. Lanza

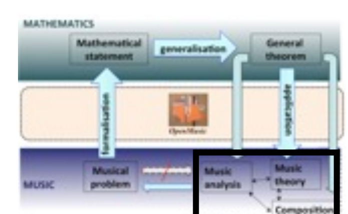
La bataille de caresme et de charnage
(pour violoncelle et accompagnement, 2012)

G. Bloch

A piece based on Monk **(2007)**
(« Well You Need'nt »)

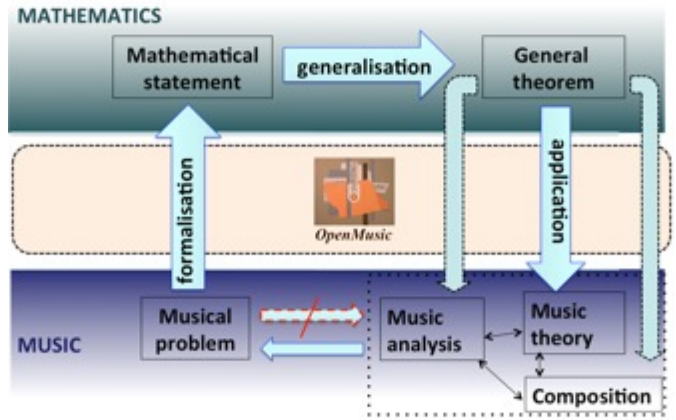
La notte poco prima della foresta
(opéra de chambre
pour acteur, mezzo-soprano, baryton, ensemble et électronique, 2009)

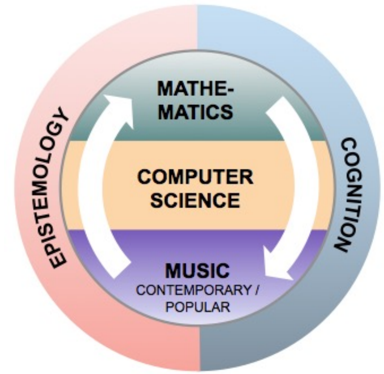
D. Ghisi

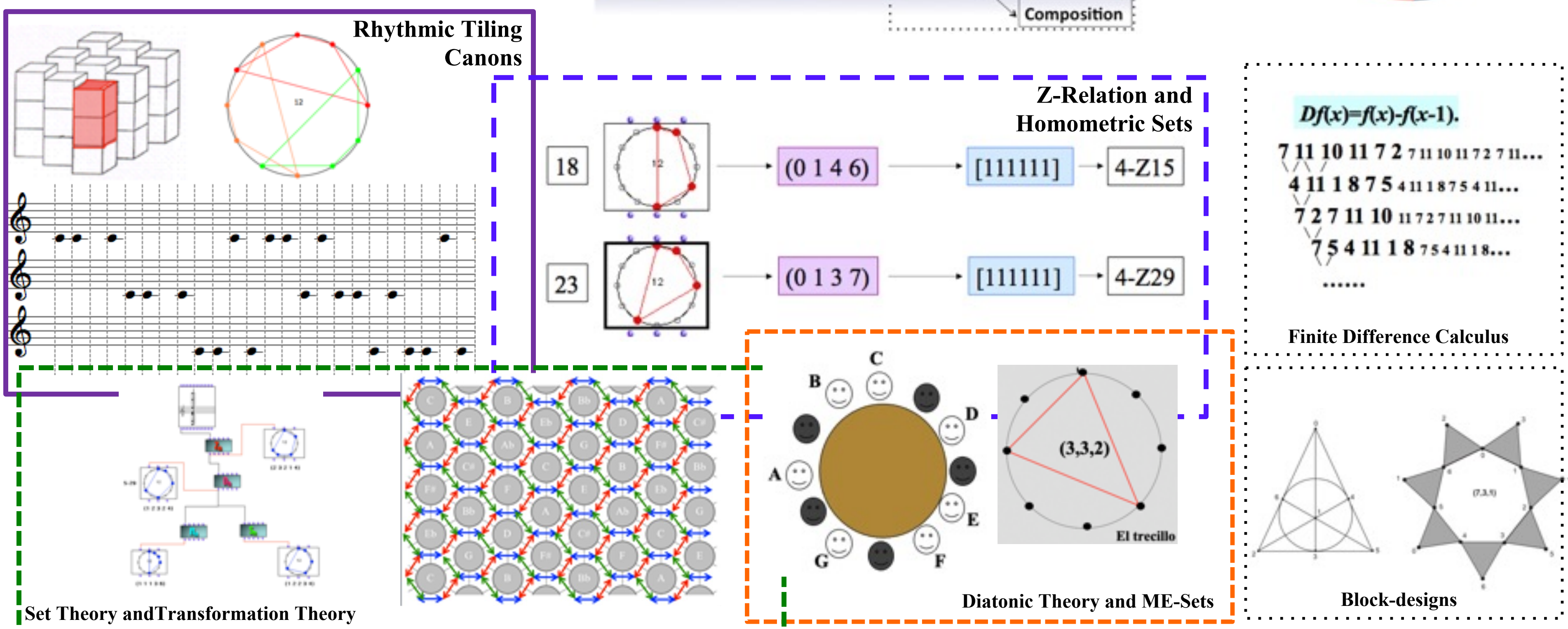


Some musically-driven mathematical problems

- Tiling Rhythmic Canons
- *Z* relation and homometry
- Set and Transformation Theory
- Diatonic Theory and ME-Sets
- Periodic sequences and FDC
- Block-designs in composition



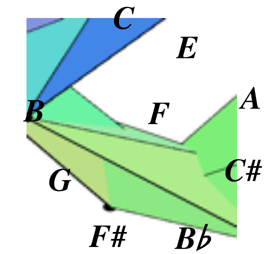




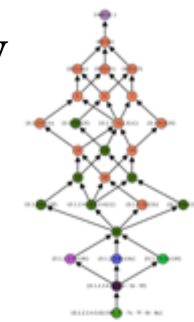
➔ M. Andreatta, *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

The SMIR Project: Structural Music Information Research

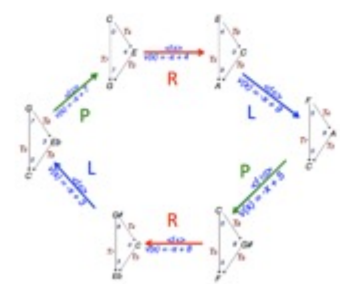
- **Generalized Tonnetze and Persistent Homology**
  - The Tonnetz as a simplicial complex
  - Algebraic classification of the twelve possible Tonnetze
  - Isotropic and anisotropic Tonnetze
  - Application to automatic stylistic classification



- **Formal Concept Analysis and Mathematical Morphology**
  - Lattice structure of formal concepts
  - Derivation operators (in FCA) and dilation/erosion in MM
  - Application to pattern recognition and extraction



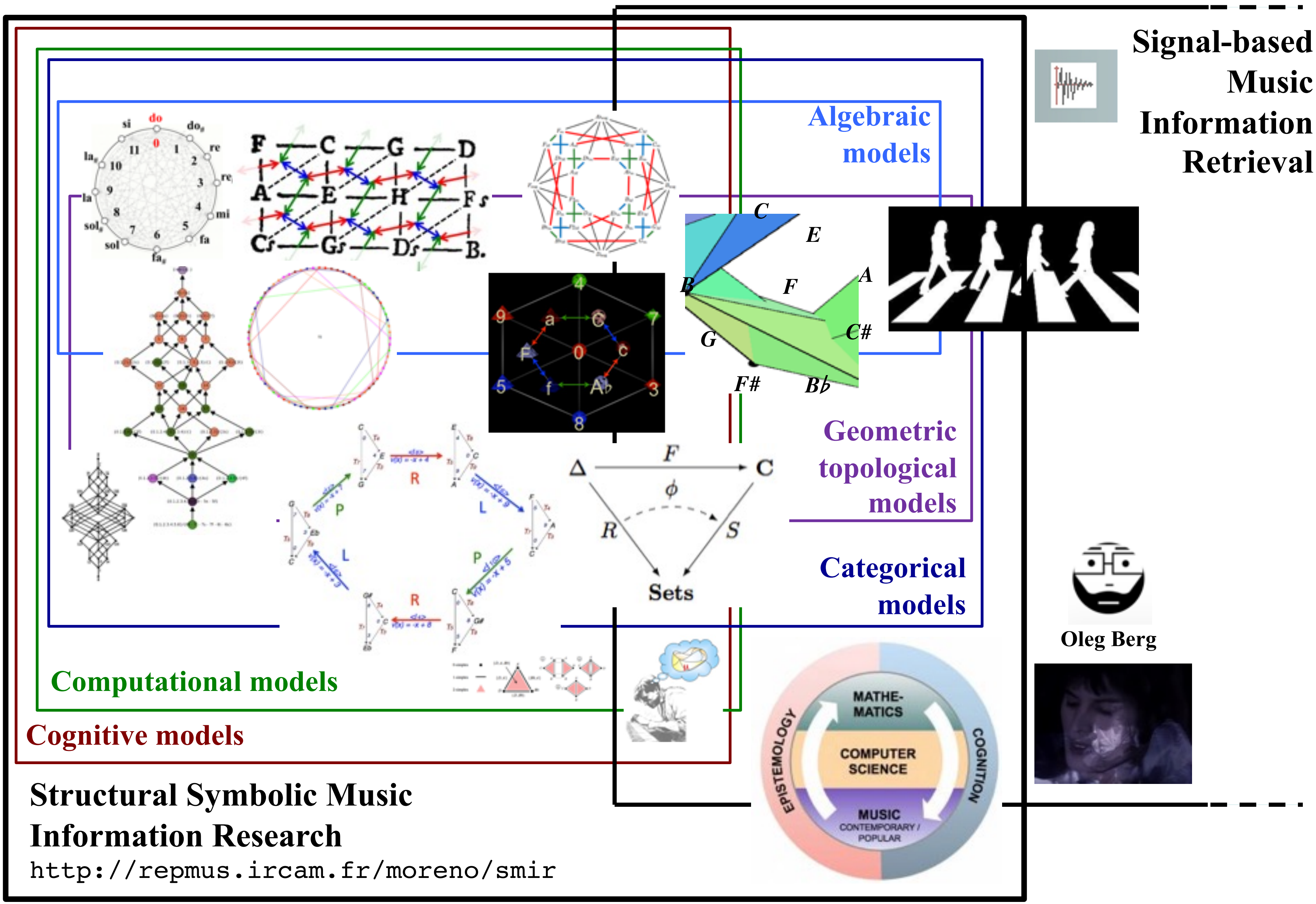
- **Category theory and Transformational Music Analysis**
  - From K-nets to PK-nets
  - Categorical formalisation of generalized Tonnetze



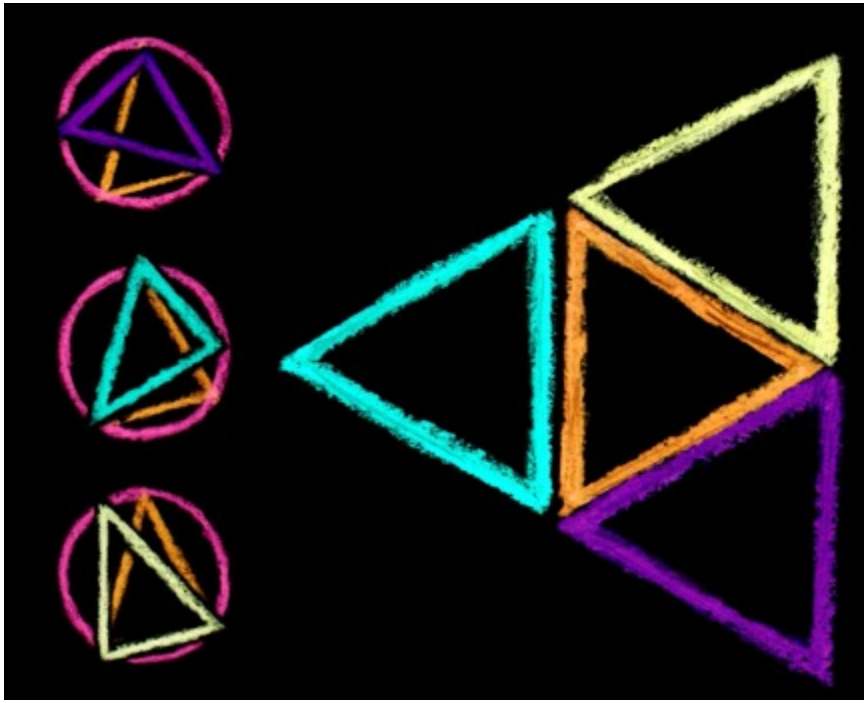
- **'Mathemusical' problems and open conjectures**
  - Tiling rhythmic canons and Fuglede Spectral Conjecture
  - Homometric musical structures and the phase retrieval problem
- **Philosophy, Epistemology and Cognitive Science**
  - Geometry-based Neo-structuralism in music analysis
  - Processes and techniques of mathemusical learning

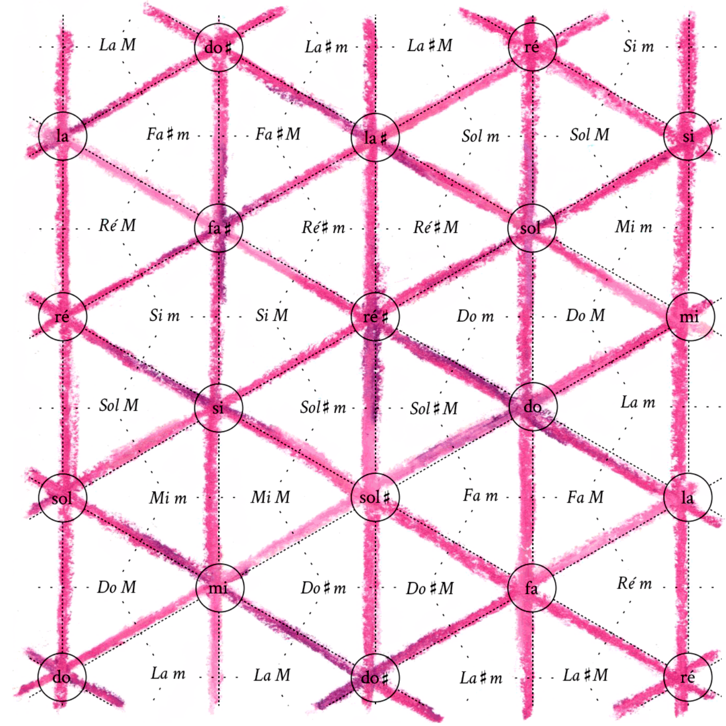
Andreatta, M. (2018), "From music to mathematics and backwards: introducing algebra, topology and category theory into computational musicology", in M. Emmer and M. Abate (eds.), *Imagine Math 6 - Mathematics and Culture*, Springer, pp. 77-88

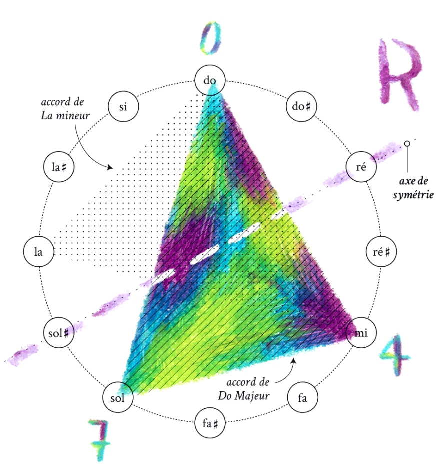
The SMIR Project: advanced maths for the working musicologist

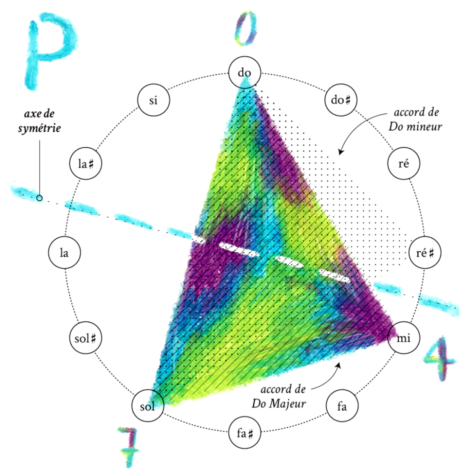


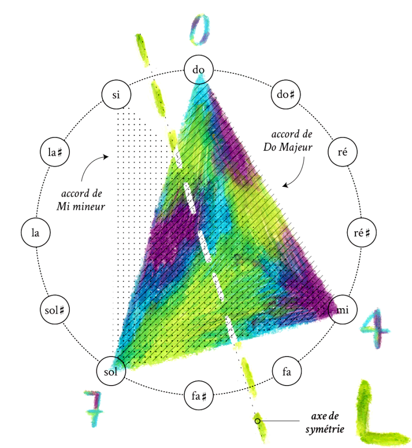
Tonnetz and duality

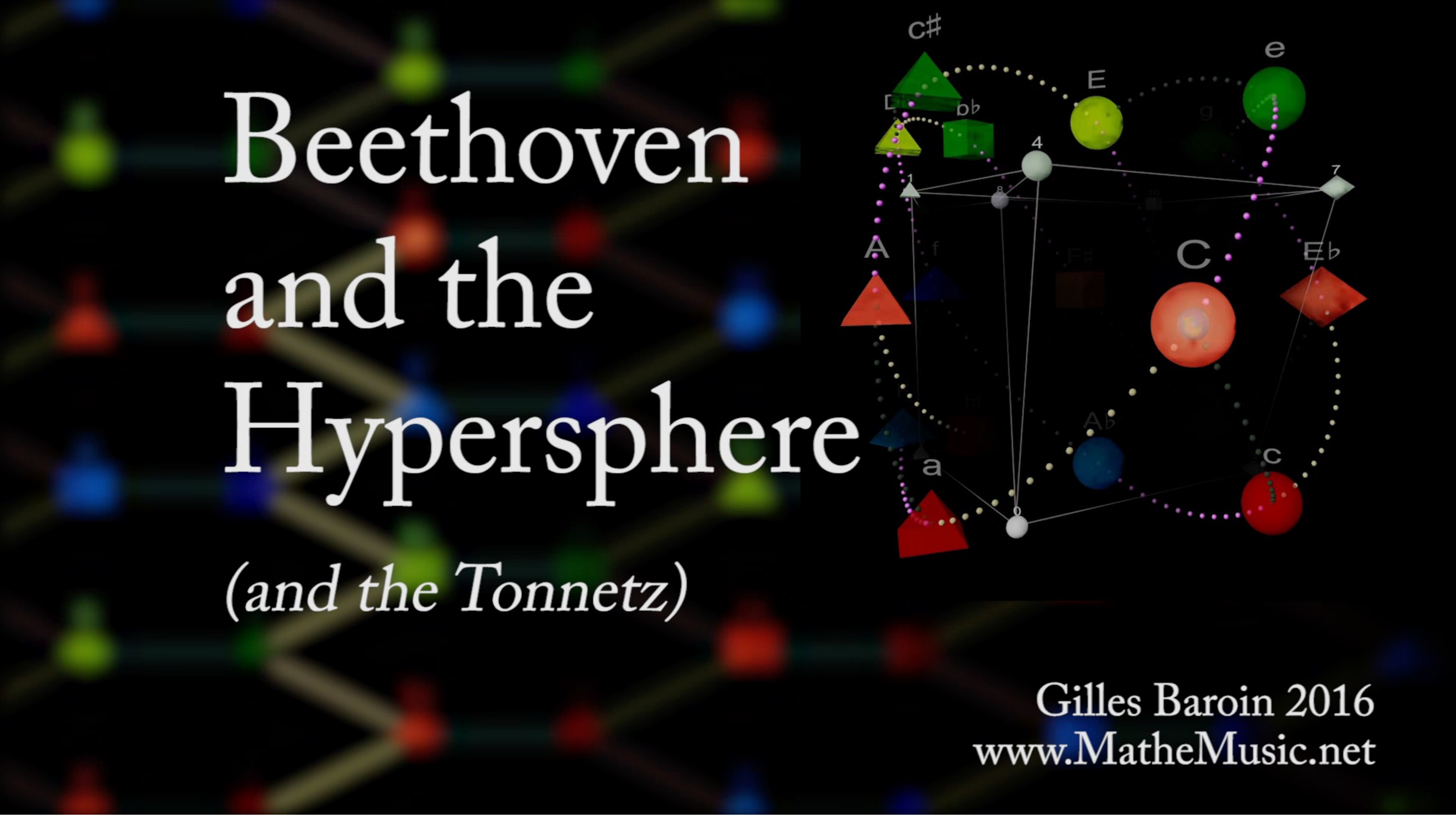








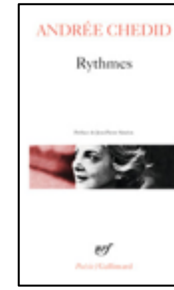




Gilles Baroin

➔ www.mathemusic.net

From poetry to song writing: hamiltonian compositional strategies



A part (Andrée Chedid, poème tiré du recueil *Rhymes* Collection Poésie/Gallimard (n. 527), Gallimard, 2018)

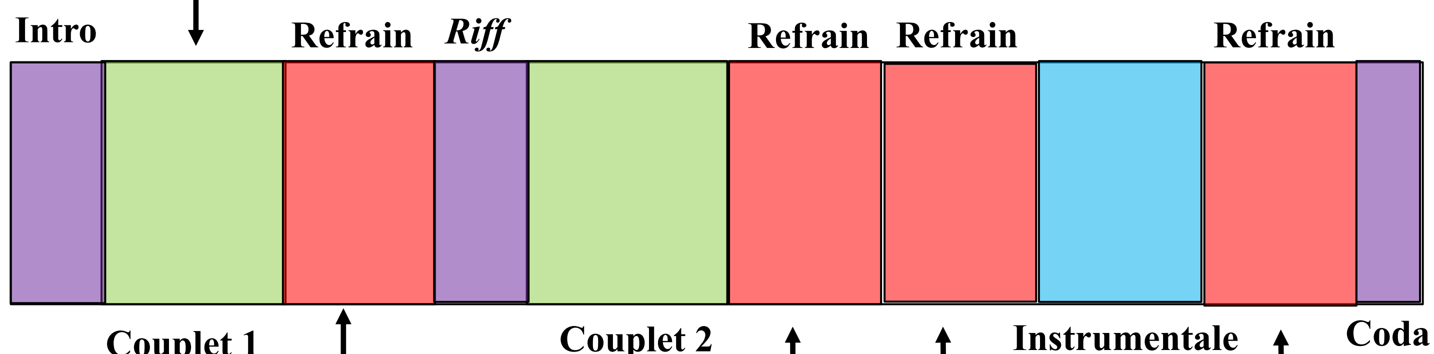
➔ http://repmus.ircam.fr/_media/moreno/prix_chedid_2018_moreno.mp3

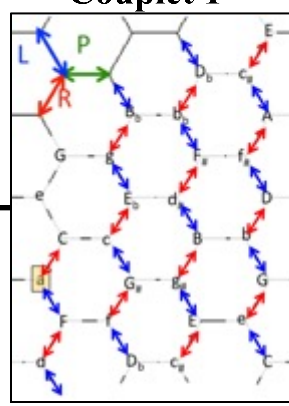
À part le temps
Et ses rouages
À part la terre
En éruptions
À part le ciel
Pétrisseur de nuages
À part l'ennemi
Qui génère l'ennemi

À part le désamour
Qui ronge l'illusion
À part la durée
Qui moisit nos visages

À part les fléaux
À part la tyrannie
À part l'ombre et le crime
Nos batailles nos outrages

Je te célèbre ô Vie
Entre cavités et songes
Intervalle convoité
Entre le vide et le rien



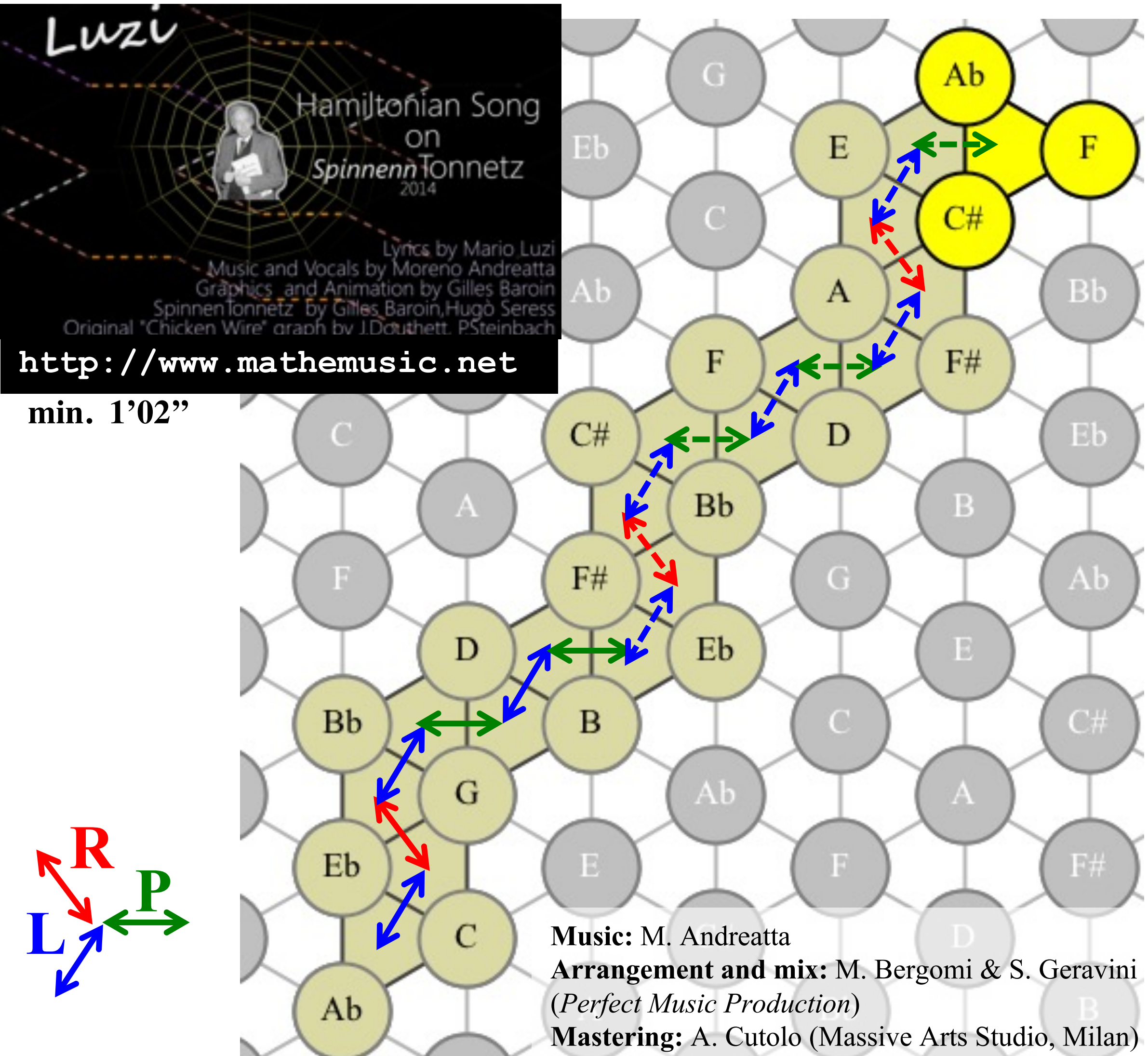


Hamiltonian Cycles with inner periodicities

8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR

→

L P L P L R …
P L P L R L …
L P L R L P …
P L R L P L …
L R L P L P …
R L P L P L …



min. 1'02"

Music: M. Andreatta
Arrangement and mix: M. Bergomi & S. Geravini (*Perfect Music Production*)
Mastering: A. Cutolo (Massive Arts Studio, Milan)

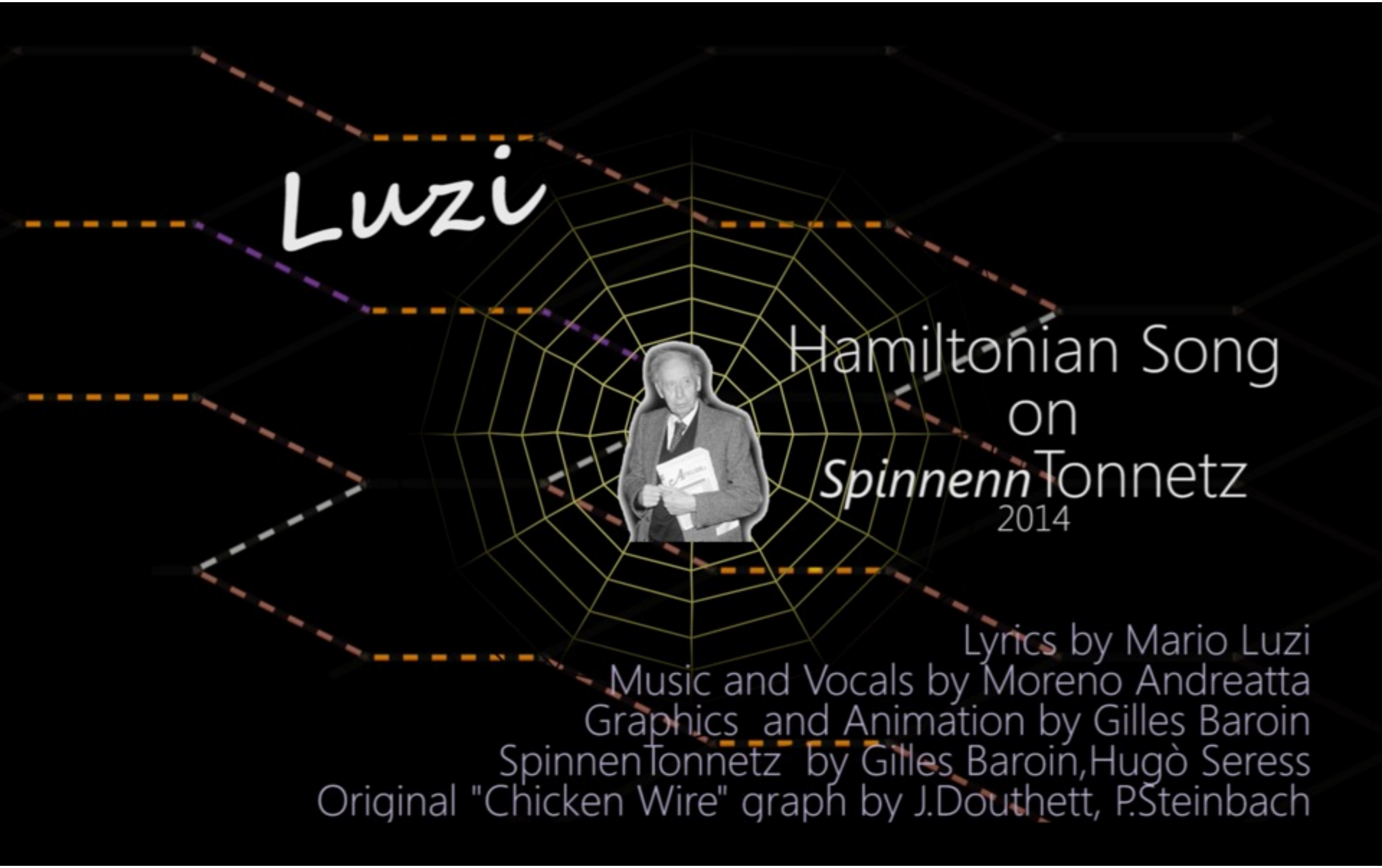
La sera non è più la tua canzone
(Mario Luzi, 1945, in *Poesie sparse*)

La sera non è più la tua canzone,
è questa roccia d'ombra traforata
dai lumi e dalle voci senza fine,
la quiete d'una cosa già pensata.

Ah questa luce viva e chiara viene
solo da te, sei tu così vicina
al vero d'una cosa conosciuta,
per nome hai una parola ch'è passata
nell'intimo del cuore e s'è perduta.

Caduto è più che un segno della vita,
riposi, dal viaggio sei tornata
dentro di te, sei scesa in questa pura
sostanza così tua, così romita
nel silenzio dell'essere, (compiuta).

L'aria tace ed il tempo dietro a te
si leva come un'arida montagna
dove vaga il tuo spirito e si perde,
un vento raro scivola e ristagna.



➔https://www.youtube.com/watch?v=1TlwFLYCYhY&ab_channel=MatheMusic4D

Exploring Hamiltonian cycles in music composition

1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPRPR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPRLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
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23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
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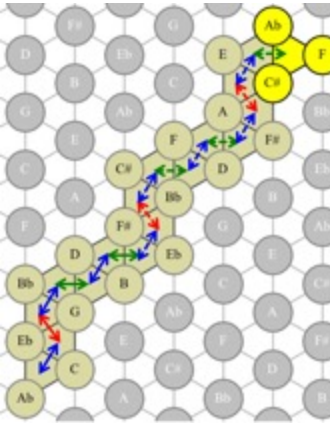
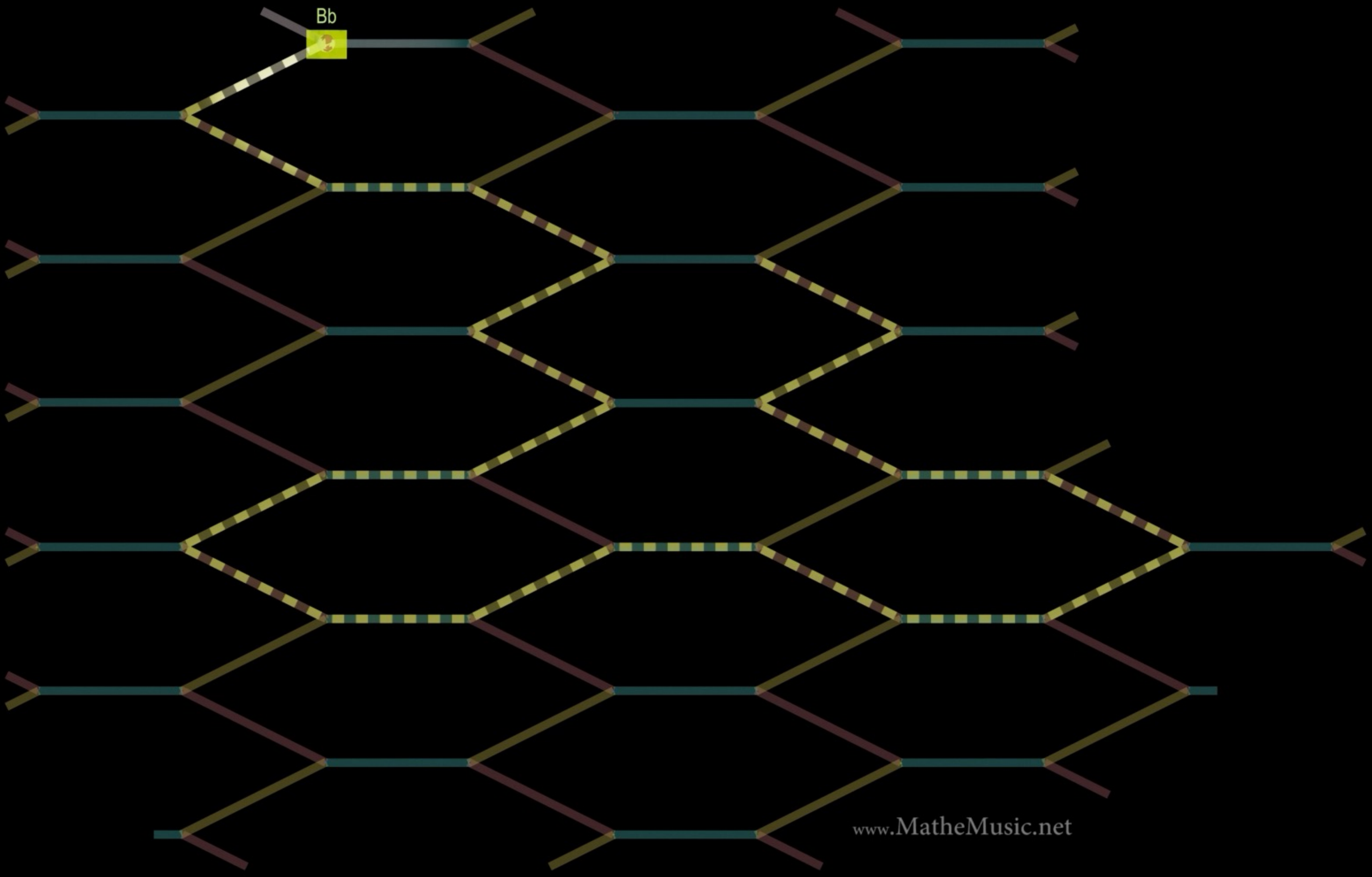
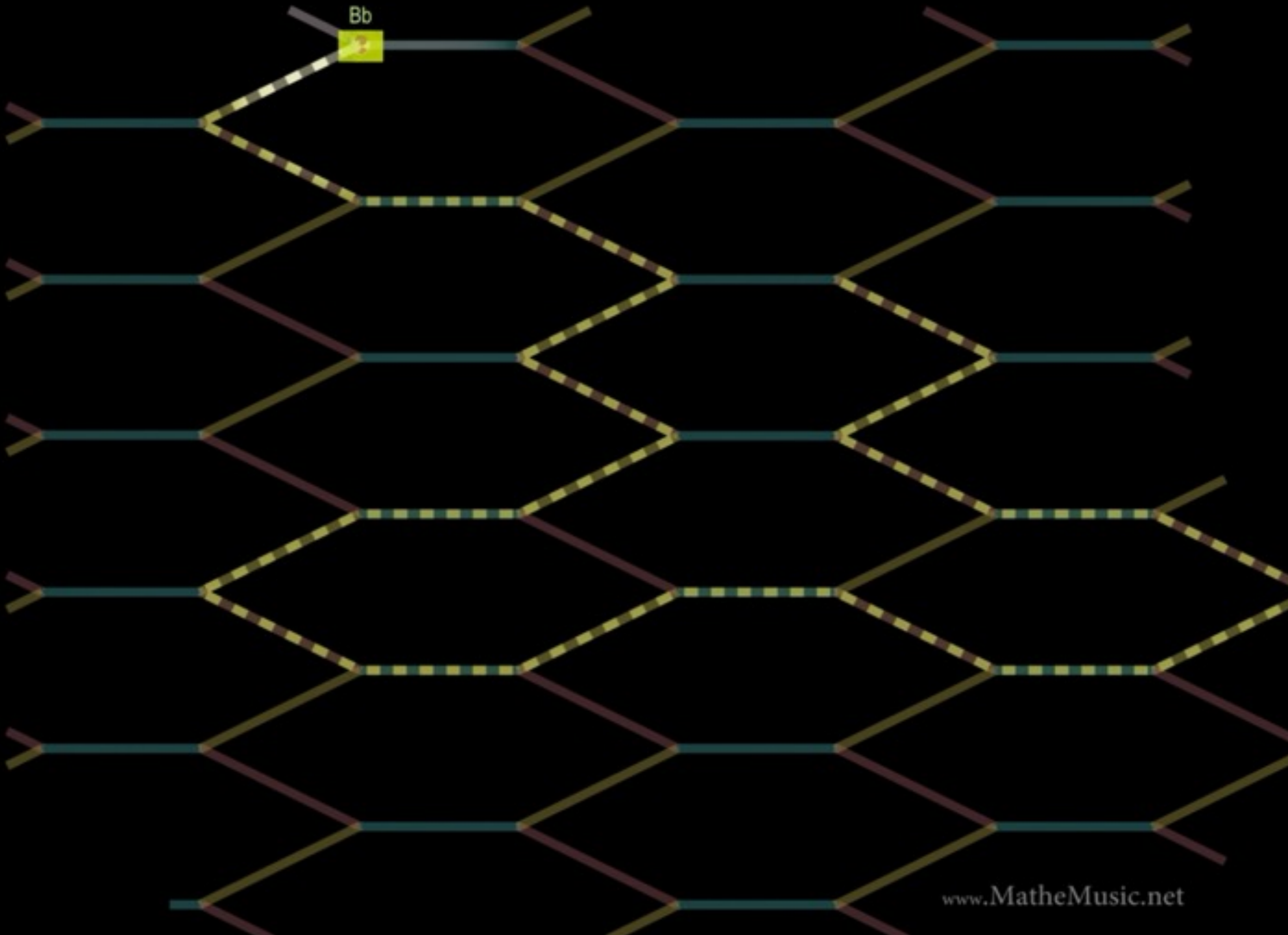
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLRP
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm--RLRLRPRP
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL

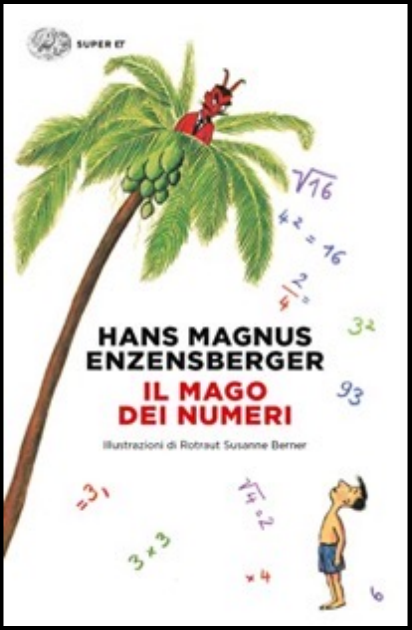
LPLPLR…
PLPLRL…
LPLRLP…
PLRLPL…
LRLPLP…



Hamiltonial Cycles without inner periodicity

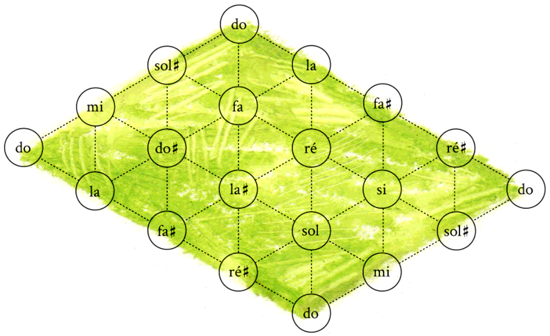


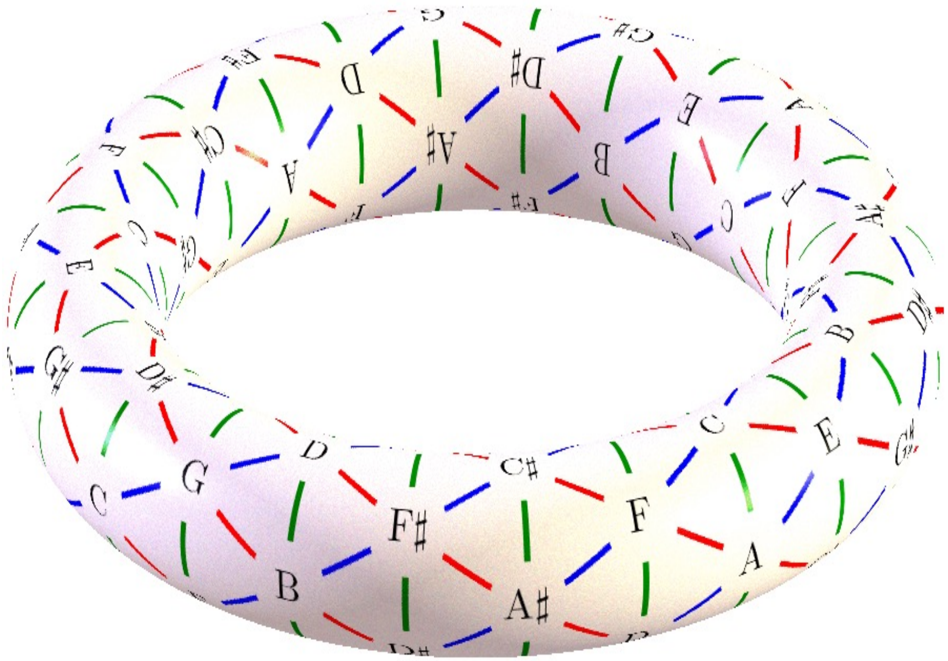




Renzo Sicco
(Assemblea Teatro)

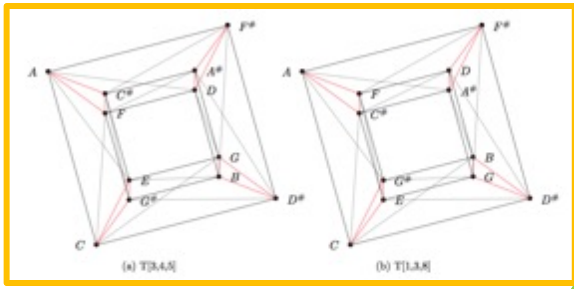
The topological structure of the *Tonnetz*

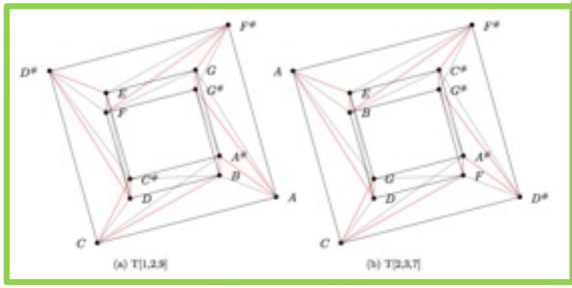




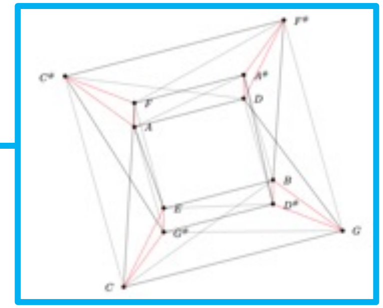
(Source: www.wikimedia.org/)

Classifying Tonnetze as Simplicial Chord Complexes



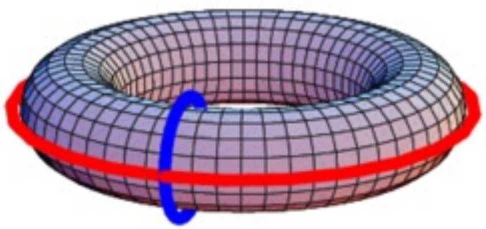


Tonnetz	Nombres de Betti		
	β_0	β_1	β_2
$T[1,2,9]$	1	2	1
$T[1,3,8]$	1	2	1
$T[1,4,7]$	1	2	1
$T[2,3,7]$	1	2	1
$T[3,4,5]$	1	2	1
$T[1,1,10]$	1	1	0
$T[2,5,5]$	1	1	0
$T[2,2,8]$	2	2	0
$T[1,5,6]$	1	1	6
$T[2,4,6]$	2	2	6
$T[3,3,6]$	3	0	3
$T[4,4,4]$	4	0	0



Louis Bigo

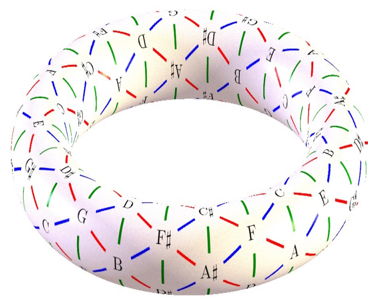
Paul Lascabettes



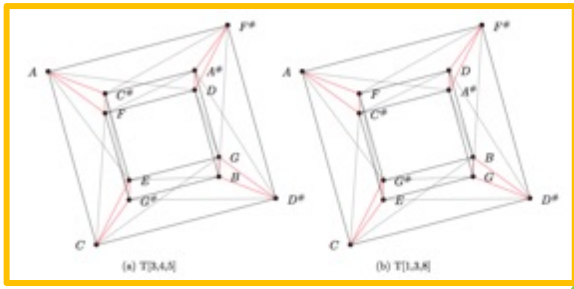
$\beta_0=1$

$\beta_1=2$

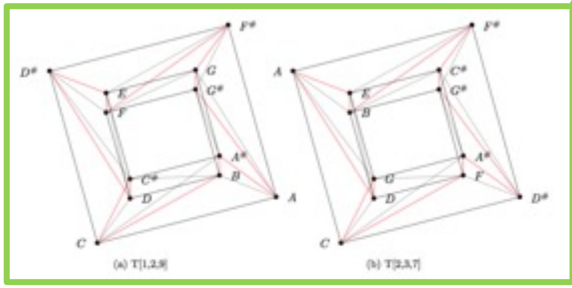
$\beta_2=1$



Classifying Tonnetze as Simplicial Chord Complexes

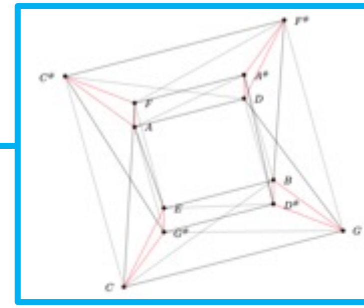


(a) T[3,4,5] (b) T[1,3,8]



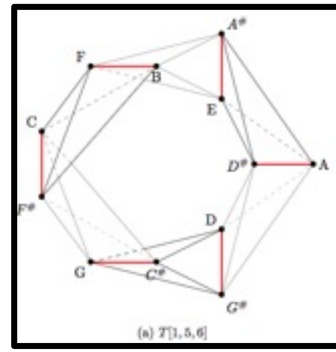
(a) T[1,2,9] (b) T[2,3,7]

Tonnetz	Nombres de Betti		
	β_0	β_1	β_2
$T[1,2,9]$	1	2	1
$T[1,3,8]$	1	2	1
$T[1,4,7]$	1	2	1
$T[2,3,7]$	1	2	1
$T[3,4,5]$	1	2	1
$T[1,1,10]$	1	1	0
$T[2,5,5]$	1	1	0
$T[2,2,8]$	2	2	0
$T[1,5,6]$	1	1	6
$T[2,4,6]$	2	2	6
$T[3,3,6]$	3	0	3
$T[4,4,4]$	4	0	0

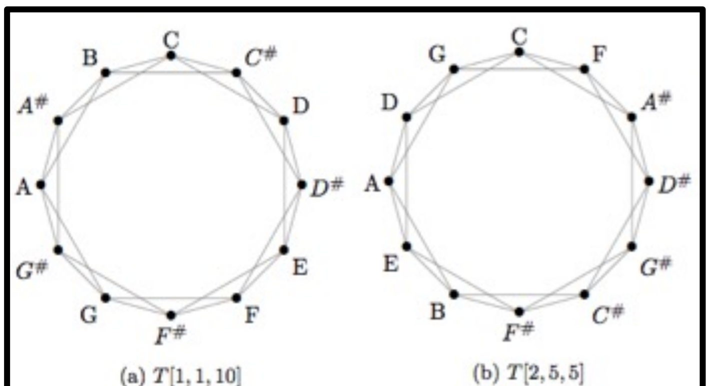


Louis Bigo

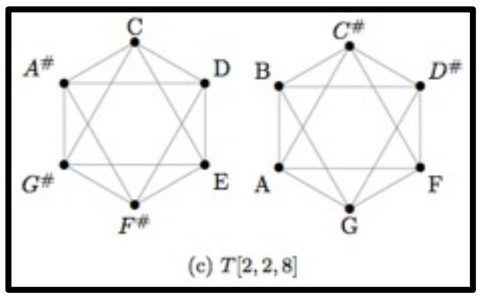
Paul Lascabettes



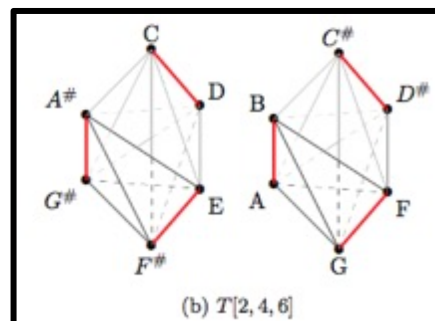
(a) $T[1,5,6]$



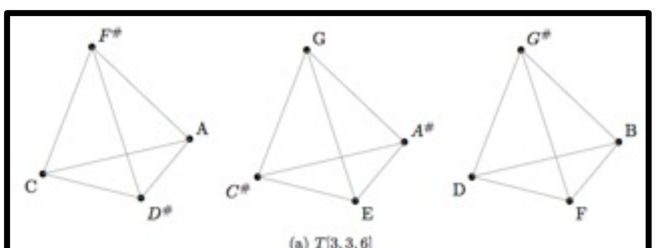
(a) $T[1,1,10]$ (b) $T[2,5,5]$



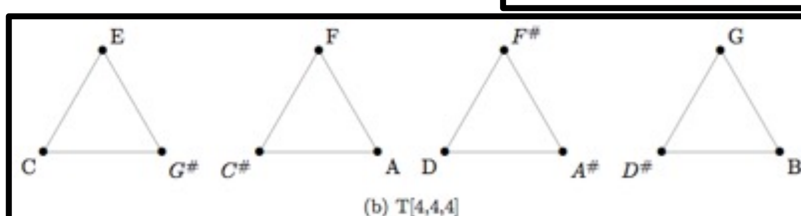
(c) $T[2,2,8]$



(b) $T[2,4,6]$

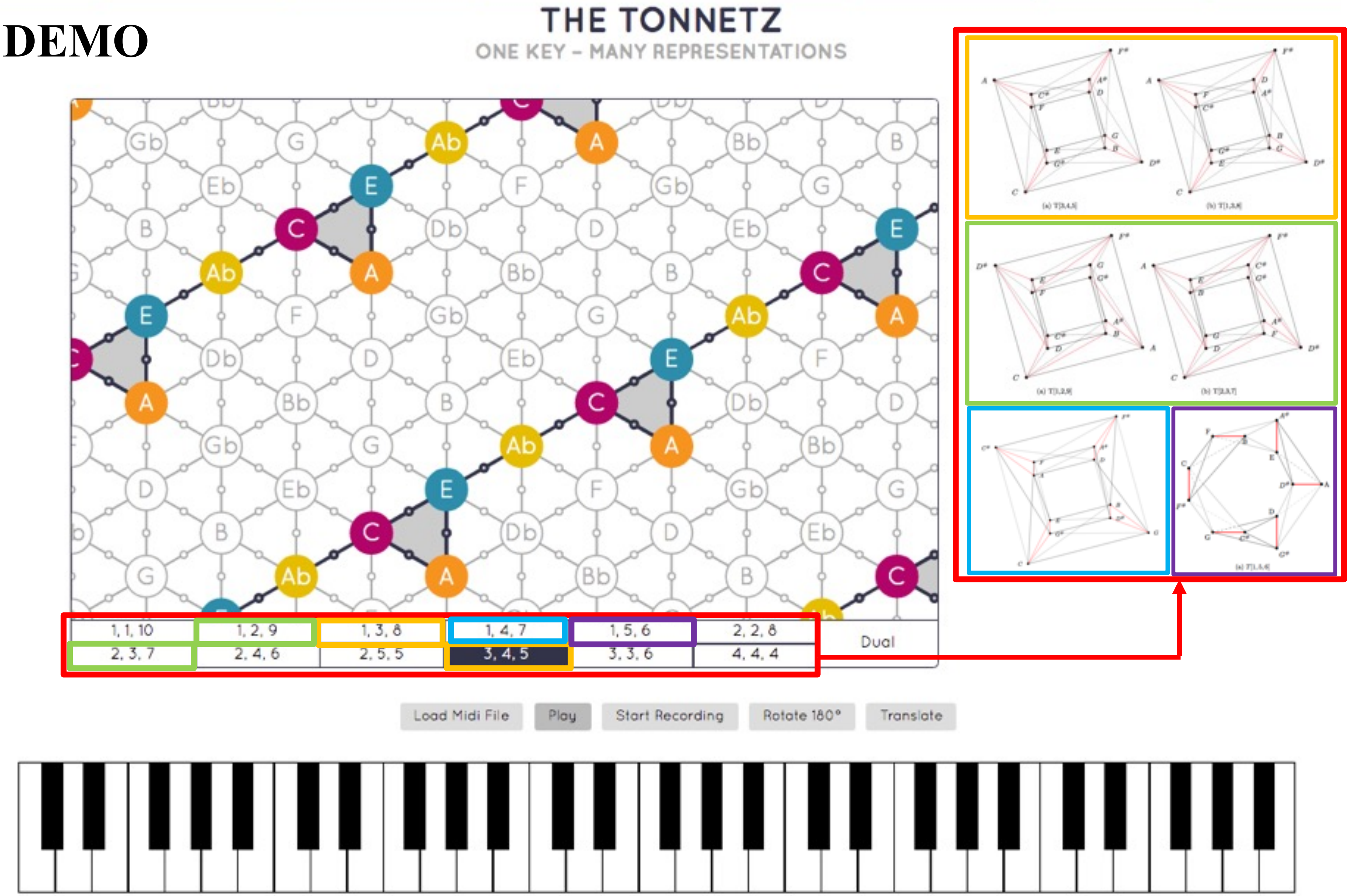


(a) $T[3,3,6]$



(b) T[4,4,4]

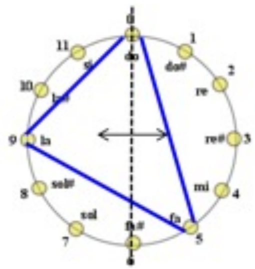
DEMO



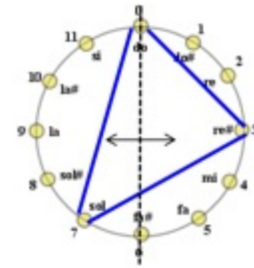
➔ **https://morenoandreatta.com/software/**

Negative Harmony as a trajectory rotation

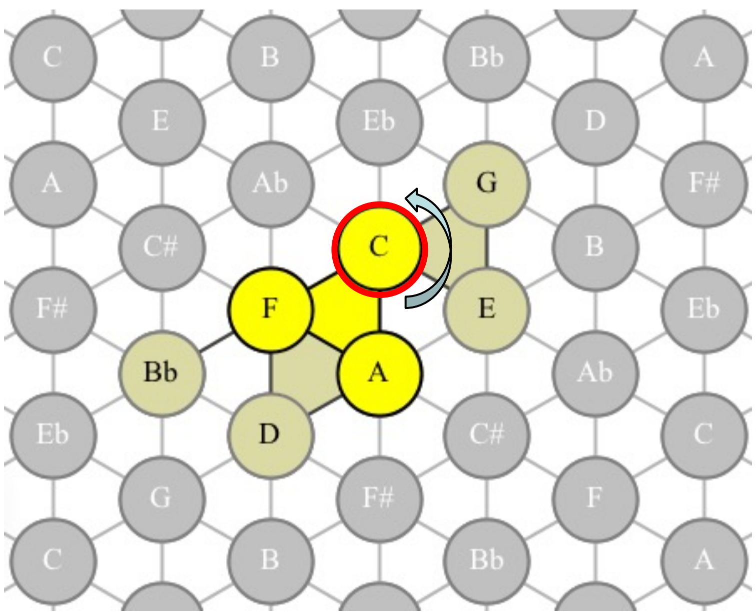
F major chord



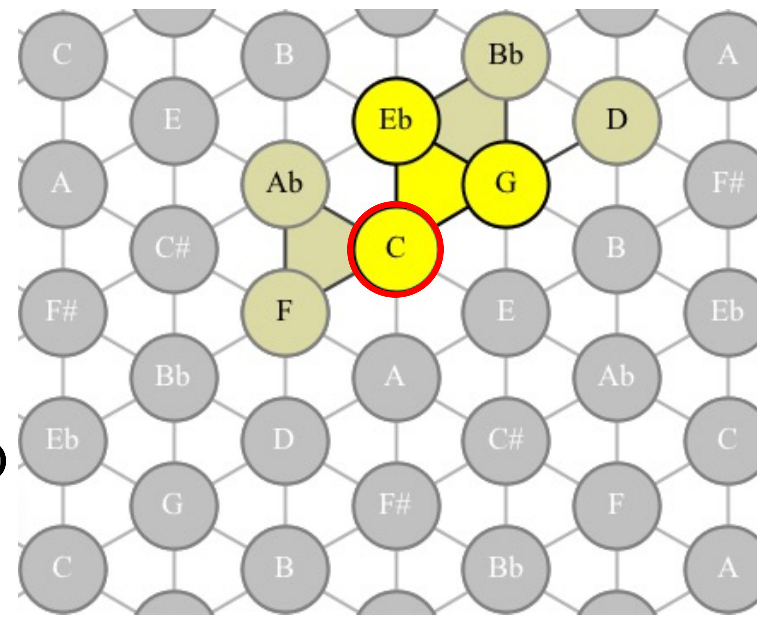
inversion



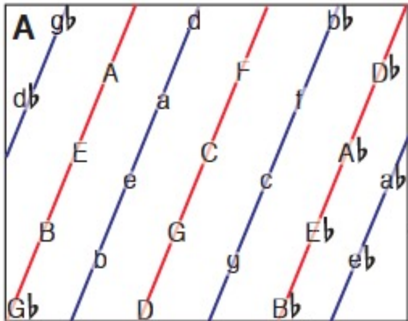
C minor chord



Rotation
(around de C)



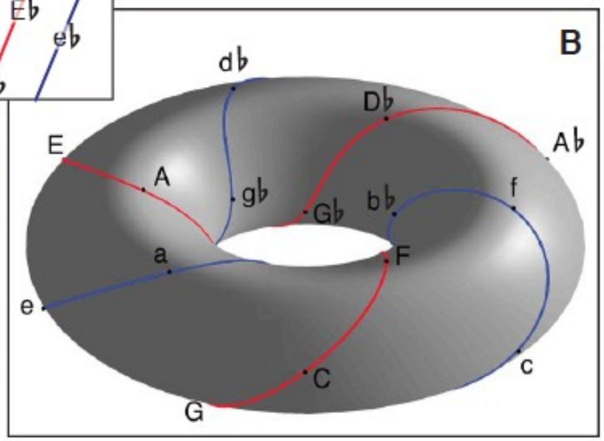
Neurosciences and *Mathemusical* Learning



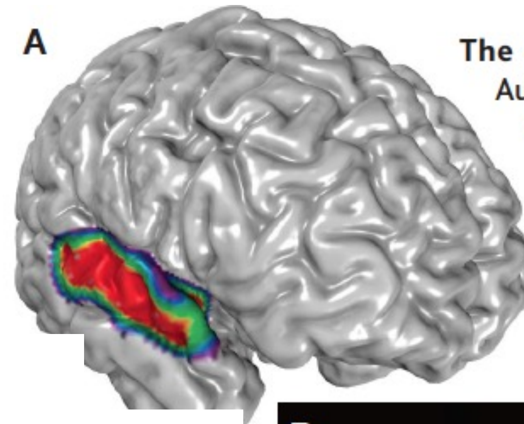
PERSPECTIVES: NEUROSCIENCE

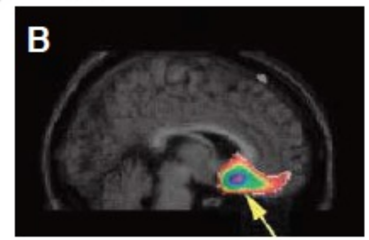
Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl



Mental key maps. (A) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). **(B)** Musical keys as points on the surface of a torus.





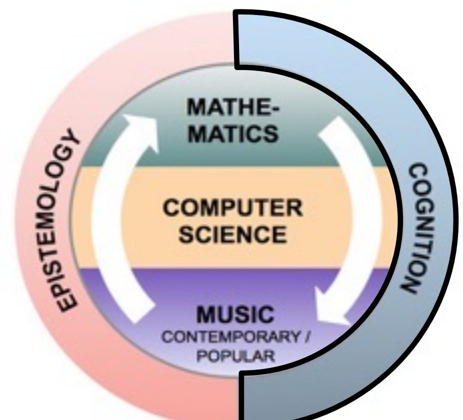
The sensation of music. (A) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. **(B)** Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.

E. Bisesi

M. Andreatta

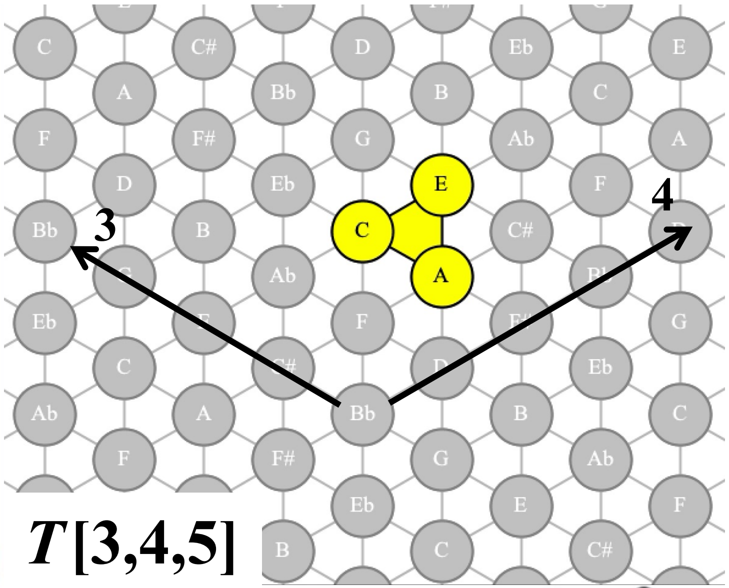
J. L. Besada

C. Guichaoua

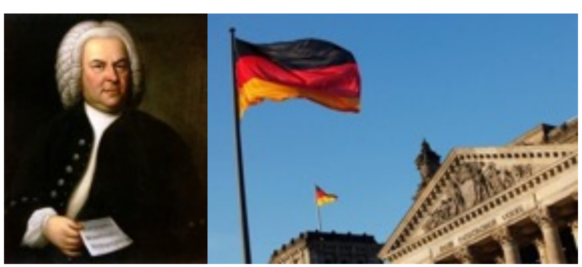


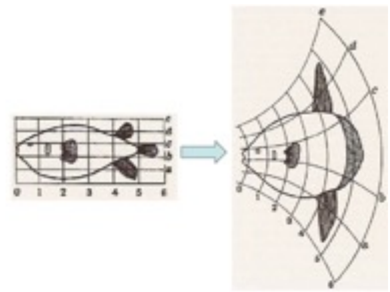
E. Bisesi, J. L. Besada, C. Guichaoua, M. Andreatta, "Conceptualizing chord relationships via spatial visualization within the Tonnetz", *International Conference on Spatial Cognition* (Rome, 13-17 September 2021).

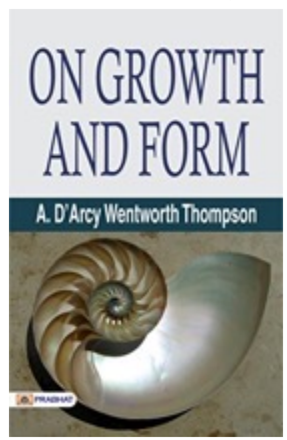
The musical style...is the space!

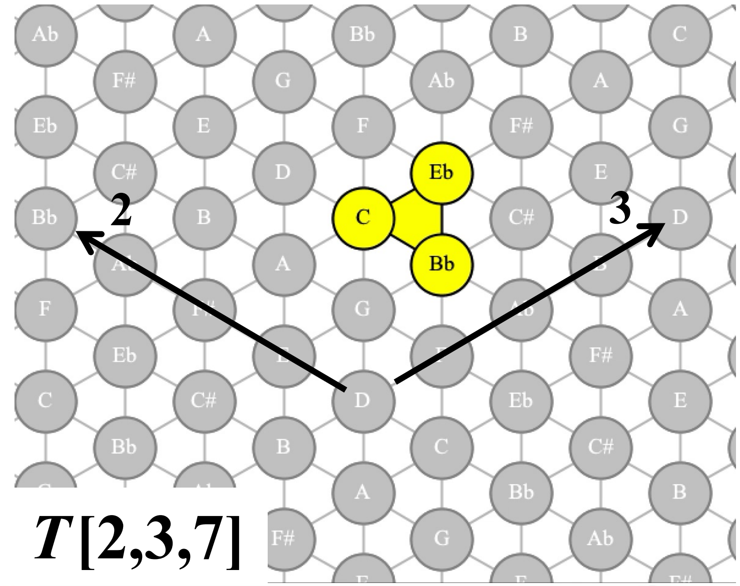


$T[3,4,5]$



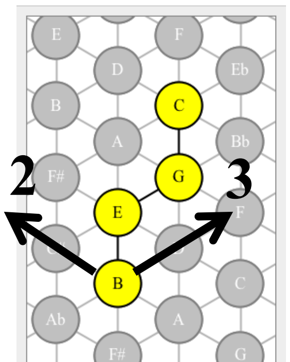




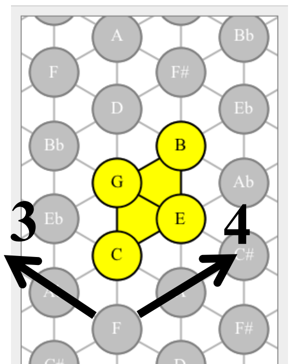


$T[2,3,7]$

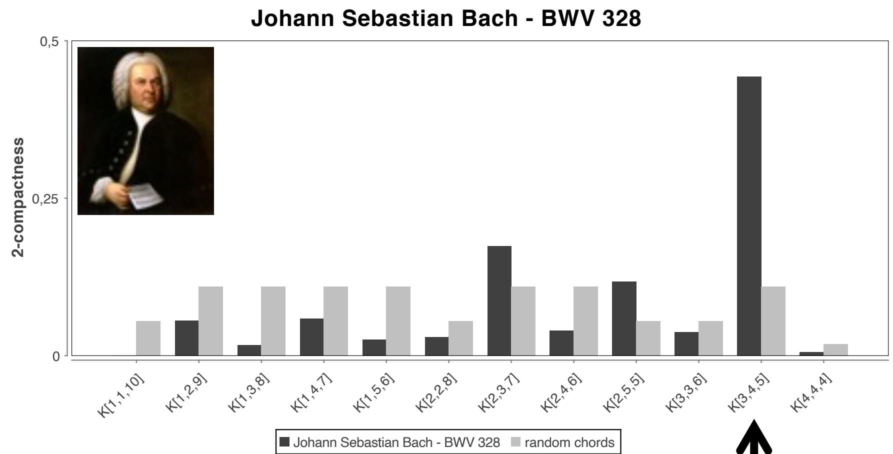
The geometric and topological character of musical style



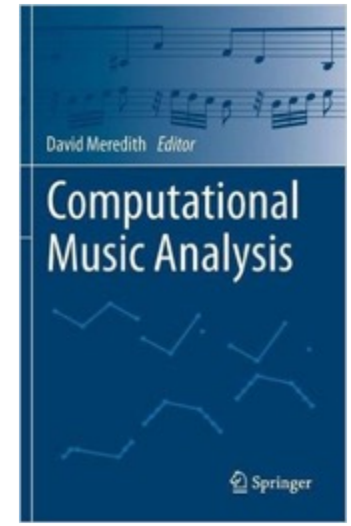
T[2,3,7]

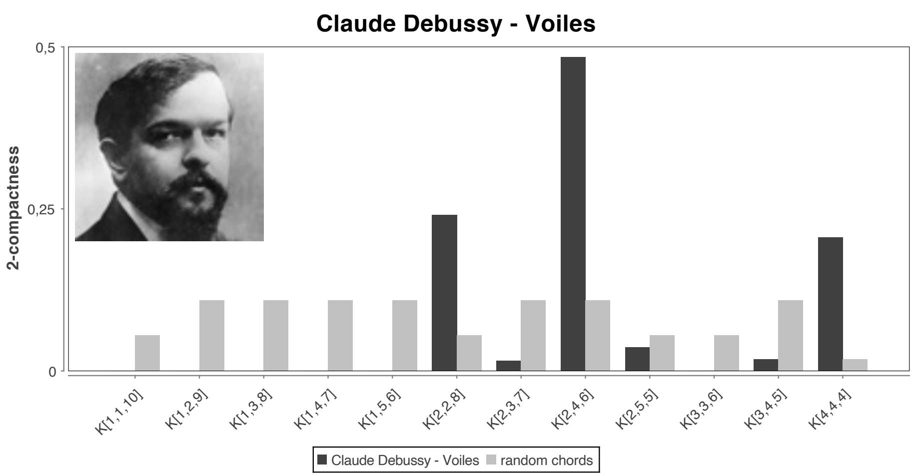


T[3,4,5]

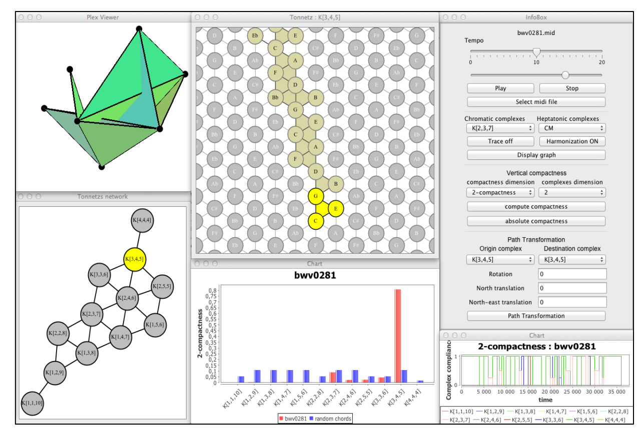


T[3,4,5]





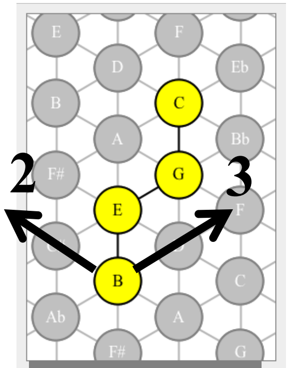
T[2,4,6]



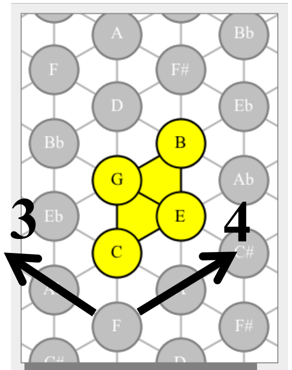
Hexachord **software (©Louis Bigo)**

- **Bigo L., M. Andreatta, "Musical analysis with simplicial chord spaces", in D. Meredith (ed.),** ***Computational Music Analysis*****, Springer, 2015**

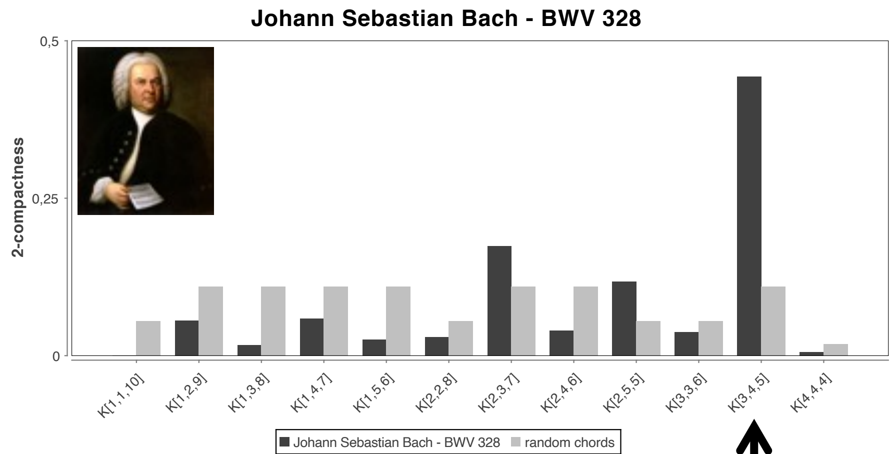
The geometric and topological character of musical style



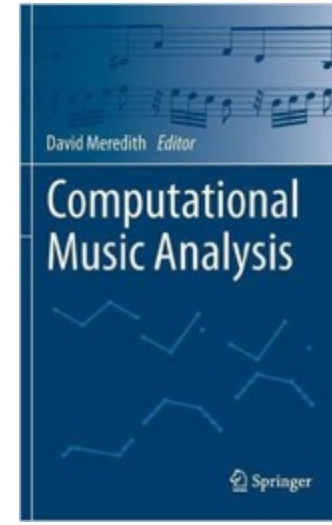
T[2,3,7]

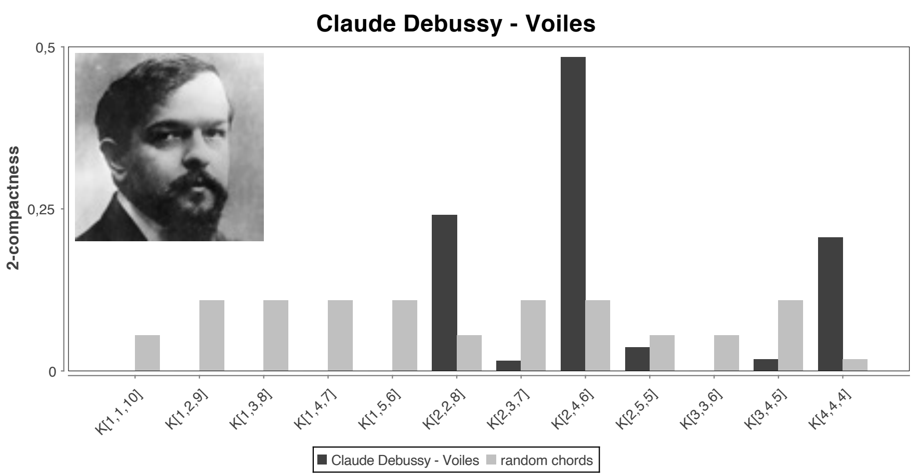


T[3,4,5]

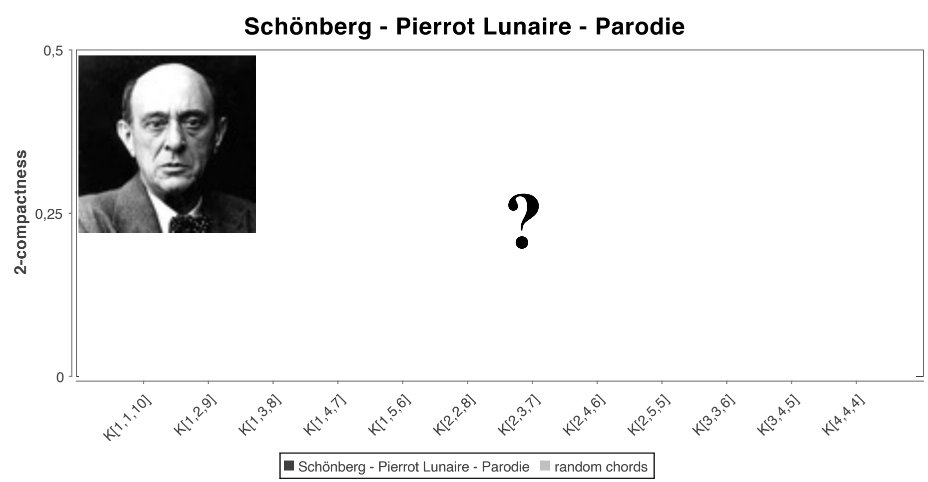


T[3,4,5]



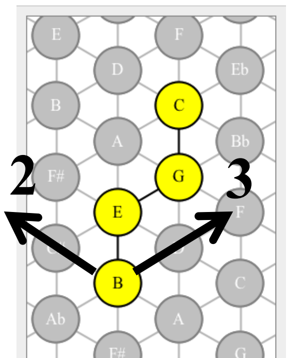


T[2,4,6]

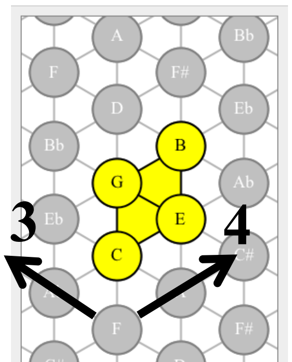


- **Bigo L., M. Andreatta, "Musical analysis with simplicial chord spaces", in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015**

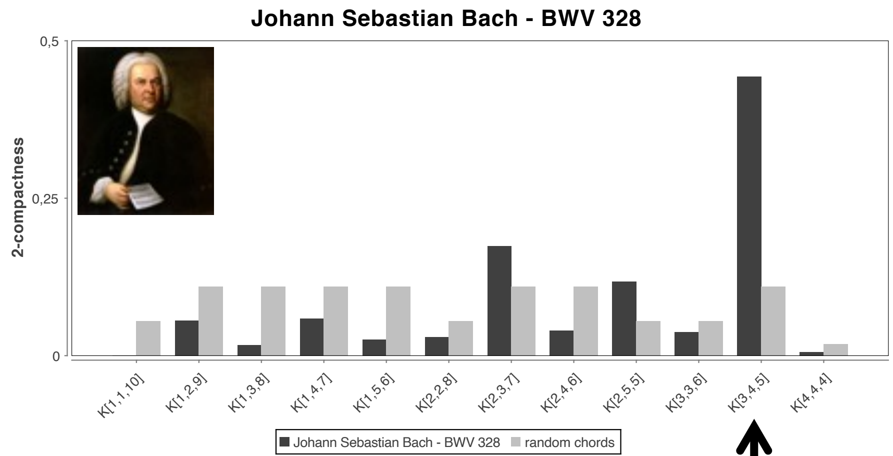
The geometric and topological character of musical style



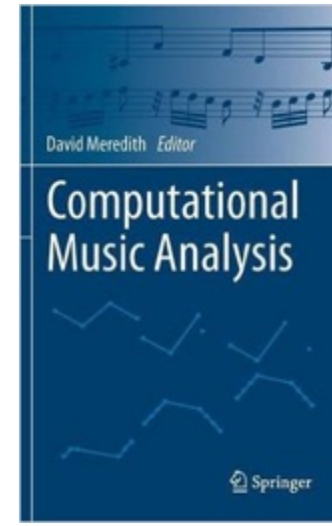
T[2,3,7]

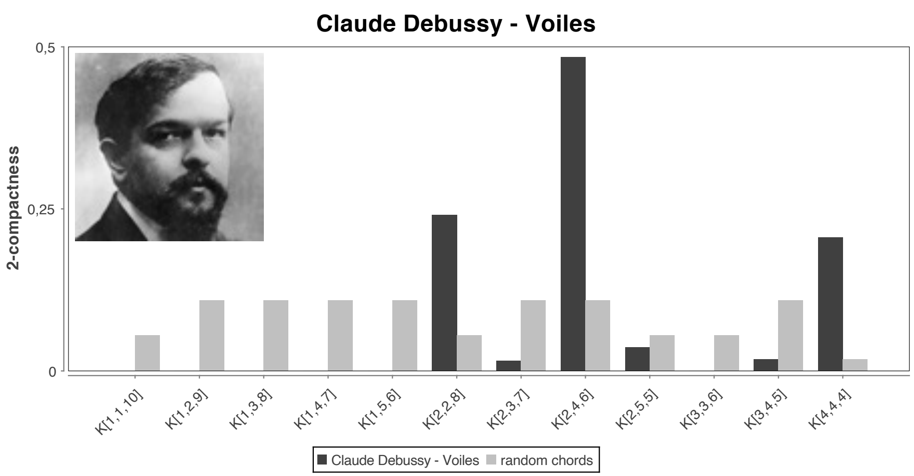


T[3,4,5]

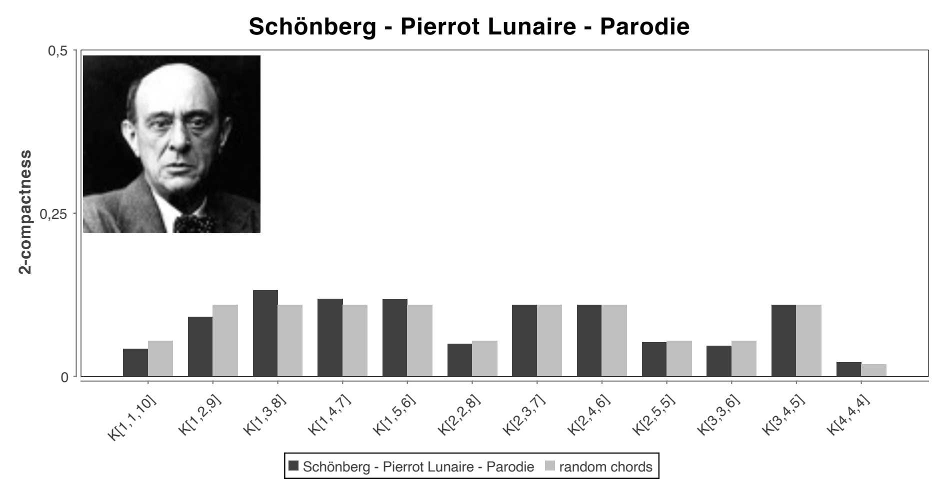


T[3,4,5]



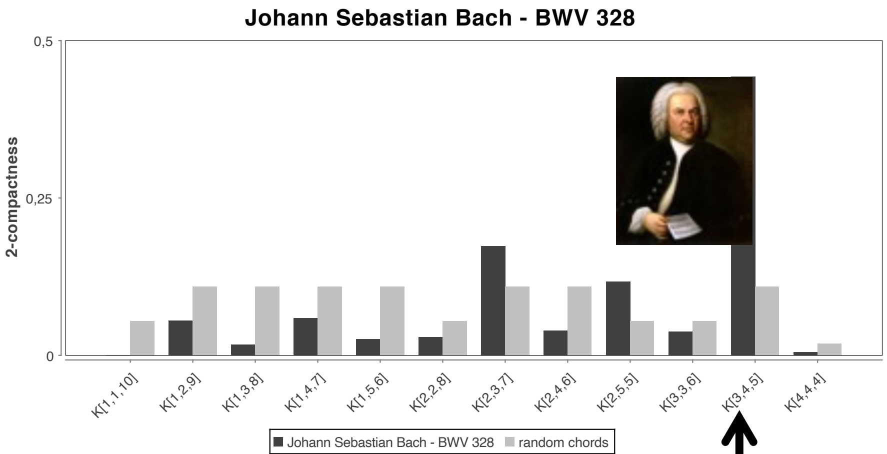


T[2,4,6]

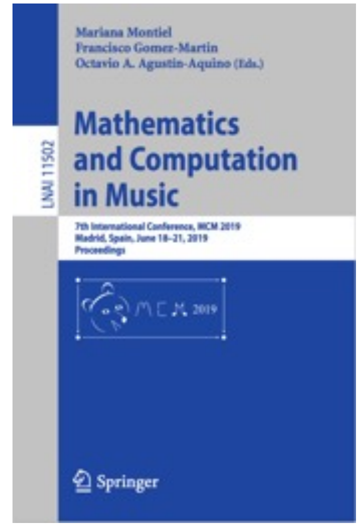


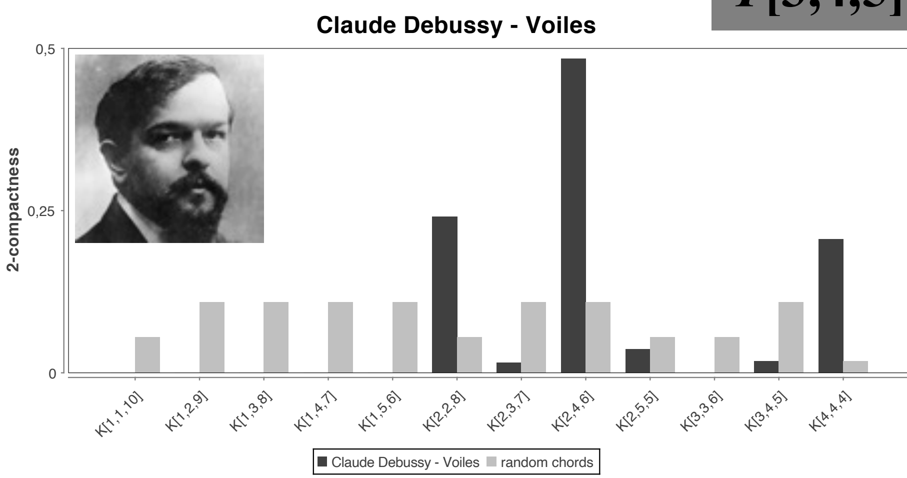
- **Bigo L., M. Andreatta, "Musical analysis with simplicial chord spaces", in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015**

The geometric and topological character of musical style

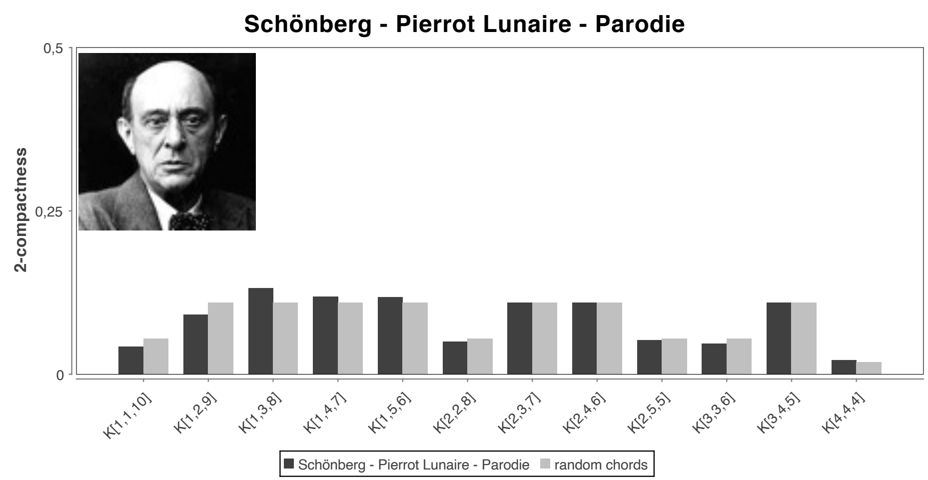


$T[3,4,5]$



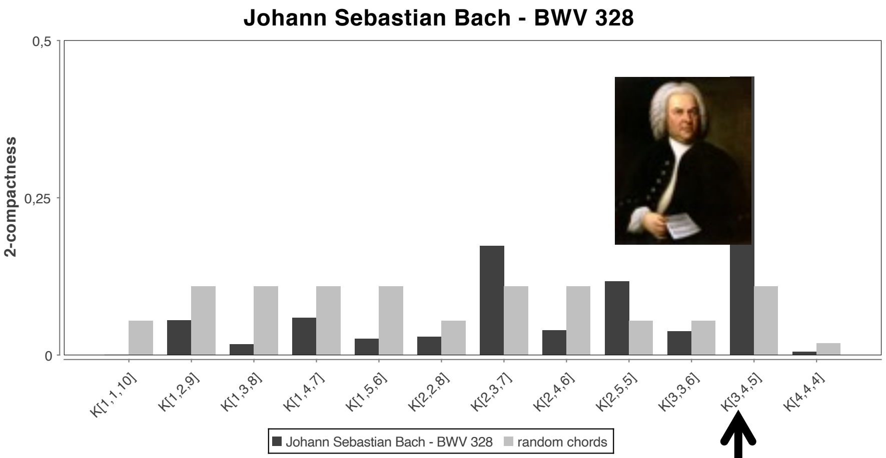


$T[2,4,6]$

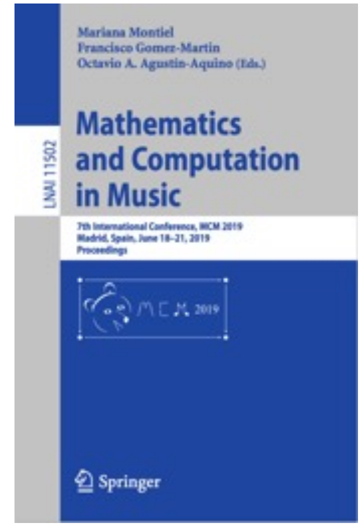


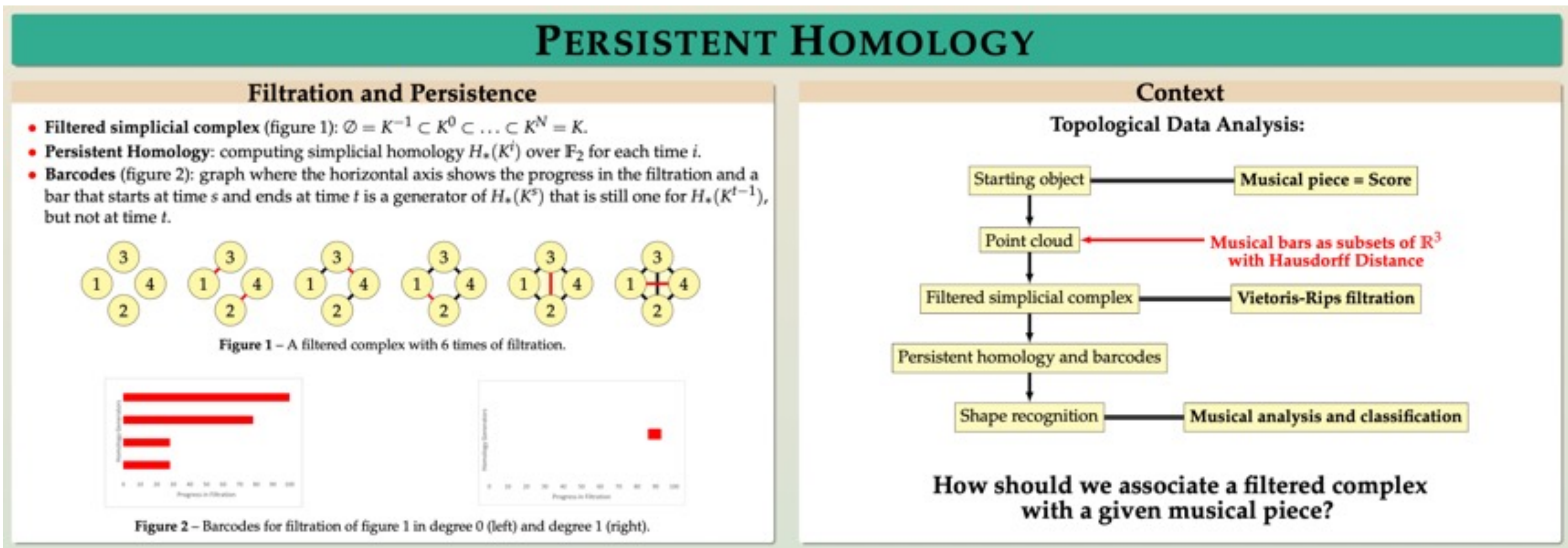
- **Bigo L., M. Andreatta, "Musical analysis with simplicial chord spaces", in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015**
- **Bigo L., M. Andreatta, "Filtration of Pitch-Class Sets Complexes", in M. Montiel et al. (eds.), Proceedings of MCM 2019, Madrid.**

The geometric and topological character of musical style



$T[3,4,5]$





Persistent Homology

Filtration and Persistence

- **Filtered simplicial complex** (figure 1): $\emptyset = K^{-1} \subset K^0 \subset \ldots \subset K^N = K$.
- **Persistent Homology**: computing simplicial homology $H_*(K^i)$ over $\mathbb{F}_2$ for each time i.
- **Barcodes** (figure 2): graph where the horizontal axis shows the progress in the filtration and a bar that starts at time s and ends at time t is a generator of $H_*(K^s)$ that is still one for $H_*(K^{t-1})$, but not at time t.

Figure 1 – A filtered complex with 6 times of filtration.

Figure 2 – Barcodes for filtration of figure 1 in degree 0 (left) and degree 1 (right).

Context

Topological Data Analysis:

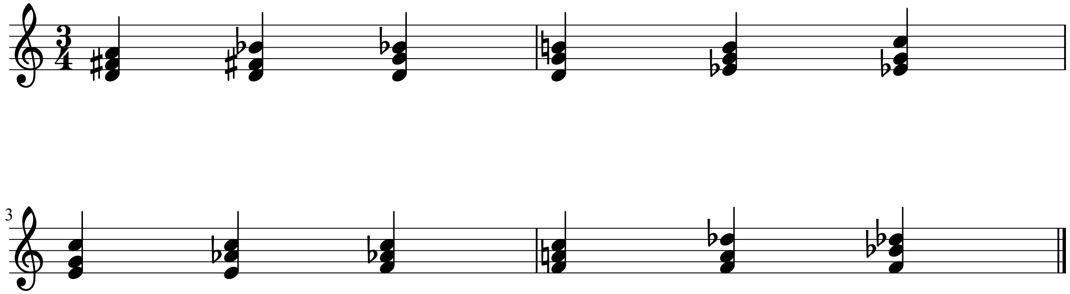
Starting object — Musical piece = Score
Point cloud ← Musical bars as subsets of $\mathbb{R}^3$ with Hausdorff Distance
Filtered simplicial complex — Vietoris-Rips filtration
Persistent homology and barcodes
Shape recognition — Musical analysis and classification

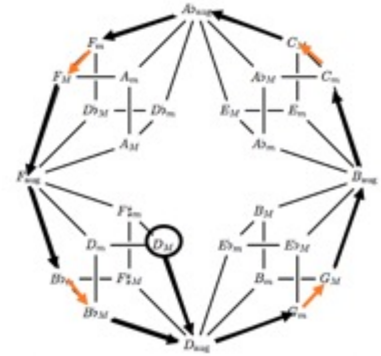
How should we associate a filtered complex with a given musical piece?

- Bigo L., M. Andreatta, "Musical analysis with simplicial chord spaces", in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015
- Bigo L., M. Andreatta, "Filtration of Pitch-Class Sets Complexes", in M. Montiel et al. (eds.), Proceedings of MCM 2019, Madrid.
- Callet V., "Persistent Homology on Musical Bars", in M. Montiel et al. (eds), Proceedings of MCM 2022, Atlanta

Two analytical approaches to Muse's *Take a bow*

Muse's "Take a bow" (*Black Holes and Revelations*, 2006)





- First analytical approach: $\mathcal{U}$-$\mathcal{U}$-$\mathcal{P}$ cycle

$$D_M \xrightarrow{\mathcal{U}} D_{\text{aug}} \xrightarrow{\mathcal{U}} G_m \xrightarrow{\mathcal{P}} G_M \xrightarrow{\mathcal{U}} B_{\text{aug}} \xrightarrow{\mathcal{U}} C_m \xrightarrow{\mathcal{P}} C_M \xrightarrow{\mathcal{U}} A\flat_{\text{aug}} \xrightarrow{\mathcal{U}} F_m \xrightarrow{\mathcal{P}} F_M \xrightarrow{\mathcal{U}} F_{\text{aug}} \xrightarrow{\mathcal{U}} B\flat_m$$

$\mathcal{U}$ —— (black)
$\mathcal{P}$ —— (orange)

- Second analytical approach: transpositions by fourth of the first cell

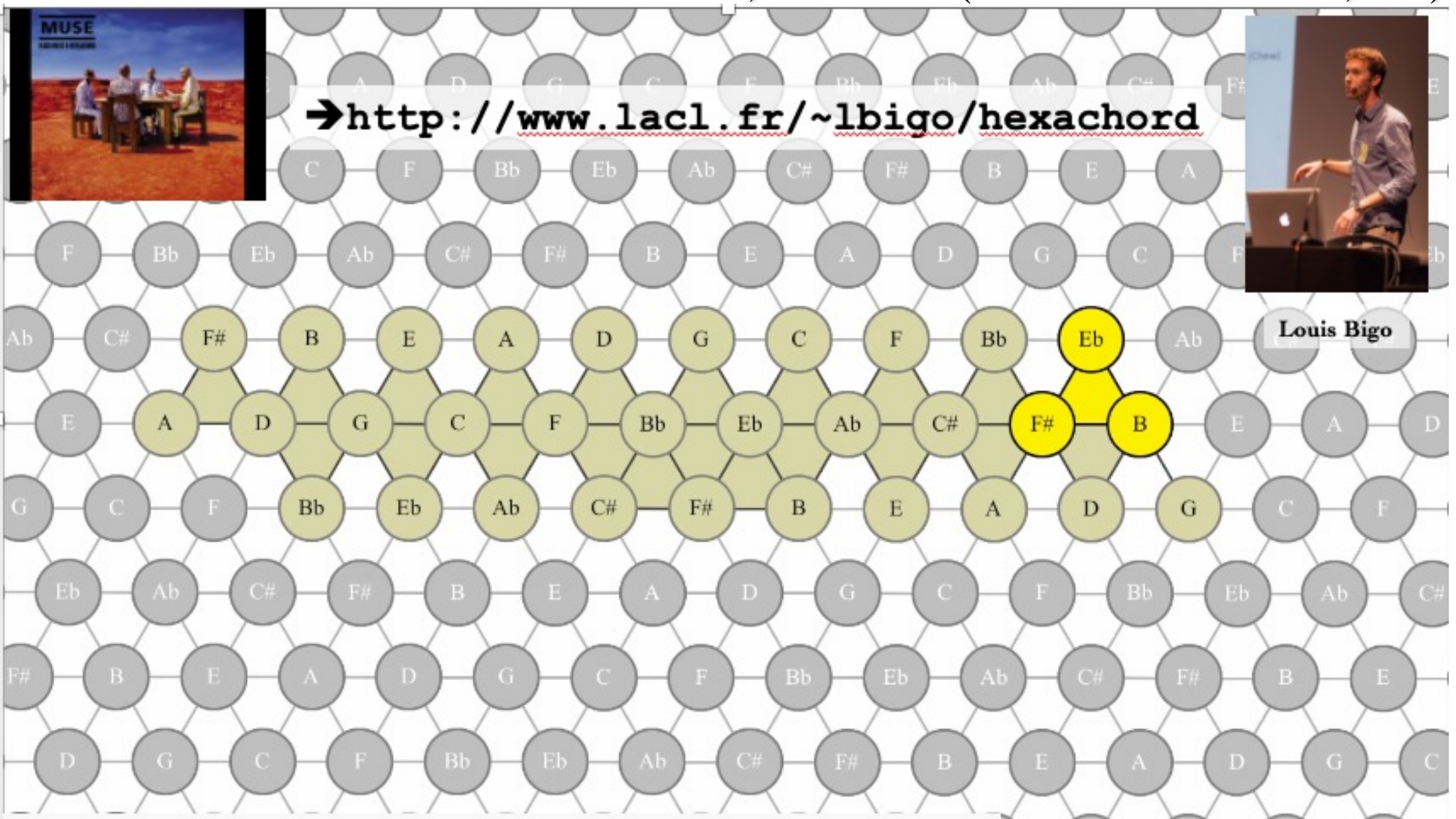
$$\begin{tikzcd}
D_M \arrow[dd, "\mathcal{U}^2"'] \arrow[dr, "\mathcal{U}"] & & G_M \arrow[dd, "\mathcal{U}^2"'] \arrow[dr, "\mathcal{U}"] & & C_M \arrow[dd, "\mathcal{U}^2"'] \arrow[dr, "\mathcal{U}"] & & F_M \arrow[dd, "\mathcal{U}^2"'] \arrow[dr, "\mathcal{U}"] & \\
& D_{\text{aug}} \arrow[dl, "\mathcal{U}"] \arrow[rr] & & B_{\text{aug}} \arrow[dl, "\mathcal{U}"] \arrow[rr] & & A\flat_{\text{aug}} \arrow[dl, "\mathcal{U}"] \arrow[rr] & & F_{\text{aug}} \arrow[dl, "\mathcal{U}"] \\
G_m & & C_m & & F_m & & B\flat_m &
\end{tikzcd}
\Longrightarrow
\begin{tikzcd}
\Delta \arrow[rr, "F"] \arrow[dr, "R"', ""{name=R}] & & \mathbf{C} \arrow[dl, "S", ""'{name=S}] \\
& \mathbf{Sets} &
\arrow[from=R, to=S, dashed, bend left, "\phi"]
\end{tikzcd}$$

Popoff A., M. Andreatta, A. Ehresmann, "A Categorical Generalization of Klumpenhouwer Networks", MCM 2015, Queen Mary University, Springer, p. 303-314

MCM 2015 London 22-25 June 2015 Mathematics and Computation in Music

(Unconscious) Algorithmic processes in Pop Music

Muse, "Take a bow" (*Black Holes and Revelations*, 2006)



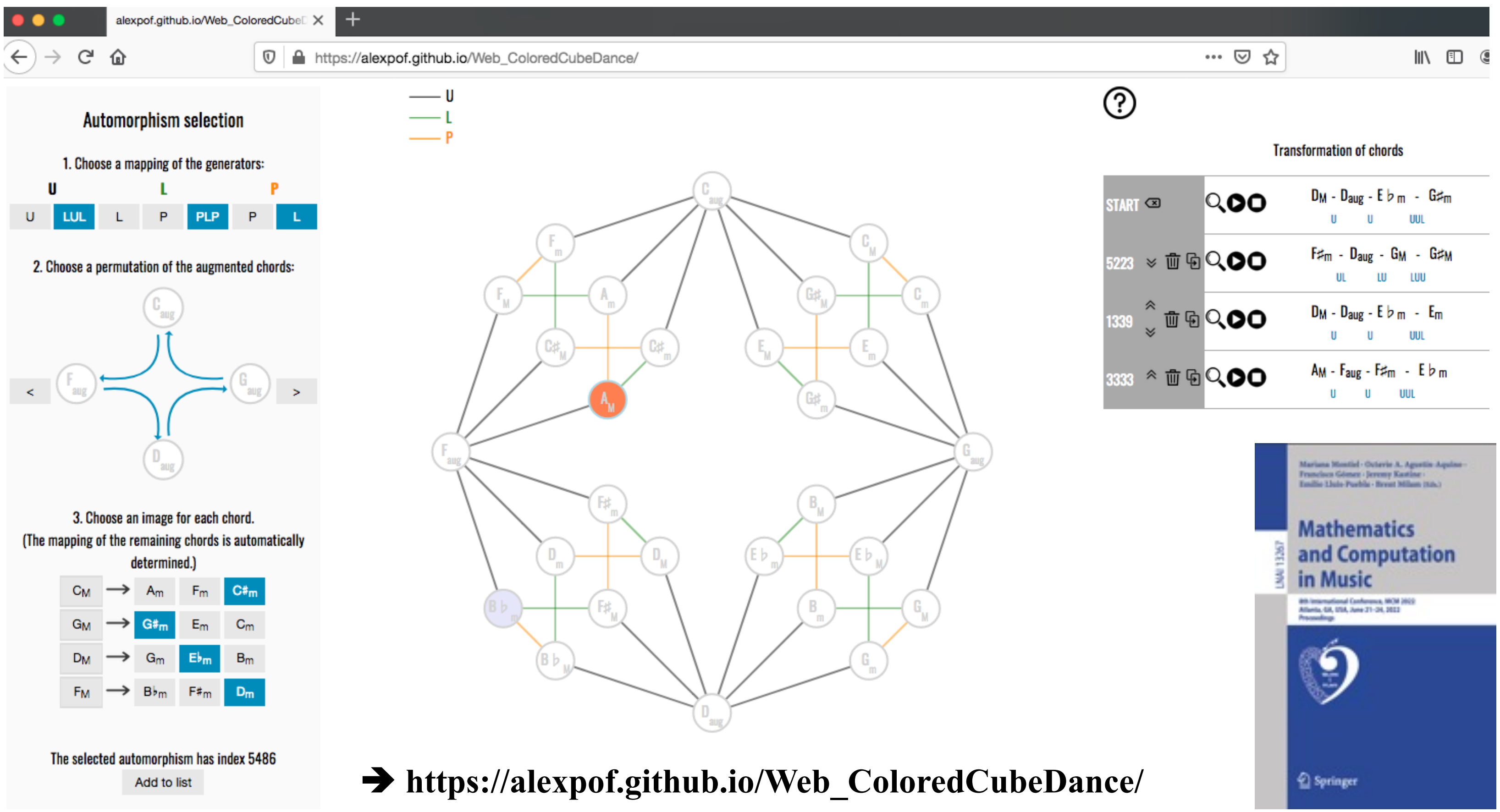
Louis Bigo

Muse - Take A Bow (Tonnetz harmonic analysis) YouTube

30,364 views Jan 20, 2016 Harmonic analysis of the song Take A Bow composed by Matthew Bellamy performed by Muse.

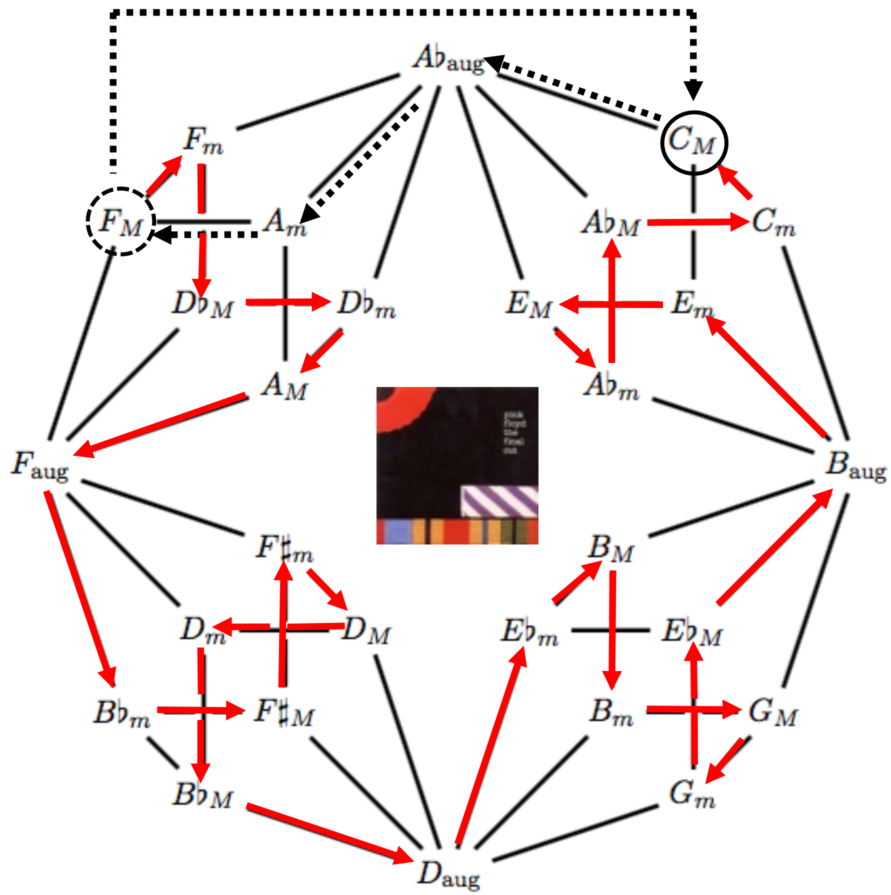
Composing (with) automorphisms

- The group $A \cong (\mathbb{Z}_3{}^4 \rtimes D_8) \rtimes (D_6 \times \mathbb{Z}_2)$ is too complex to be easily manipulated by hand. **(7776 elements)**
- We developed an interactive interface intended for mathemusicians and composers for transforming chord progressions using elements of A.
- HTML/SVG (graphical elements) and Javascript (code) allows one to develop complex interfaces for outreach activities. No installation needed !



➔ **https://alexpof.github.io/Web_ColoredCubeDance/**

Composing with Hamiltonian Cycles in the 'classical' Cube Dance



The Gunner's dream (R. Waters, 1983 / M. Andreatta, 2018)

C C+
Floating down through the clouds
Am F
Memories come rushing up to meet me now.
Fm
In the space between the heavens
C# C#m
and in the corner of some foreign field
A F+ Bbm
I had a dream.
F# F#m D Dm
I had a dream.
Bb
Good-bye Max.
D+
Good-bye Ma.
Ebm B
After the service when you're walking slowly to the car
Bm G
And the silver in her hair shines in the cold November air

Gm
You hear the tolling bell
Eb
And touch the silk in your lapel
G+ Em E G#m
And as the tear drops rise to meet the comfort of the band
G# Cm
You take her frail hand
C
And hold on to the dream.

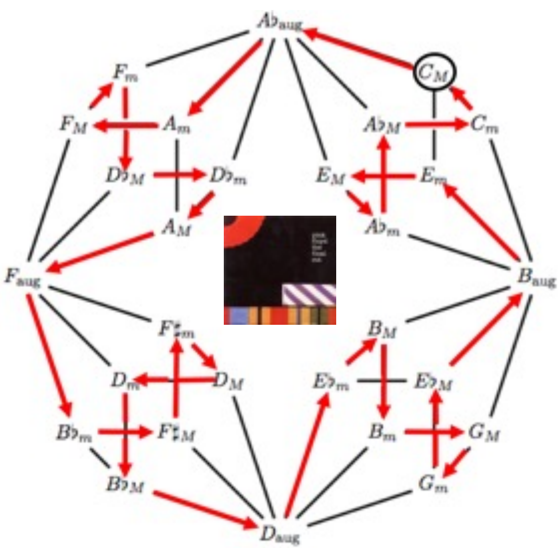
The three Hamiltonian Cycles (C_M = C, C_m = Cm, C_{aug} = C+)

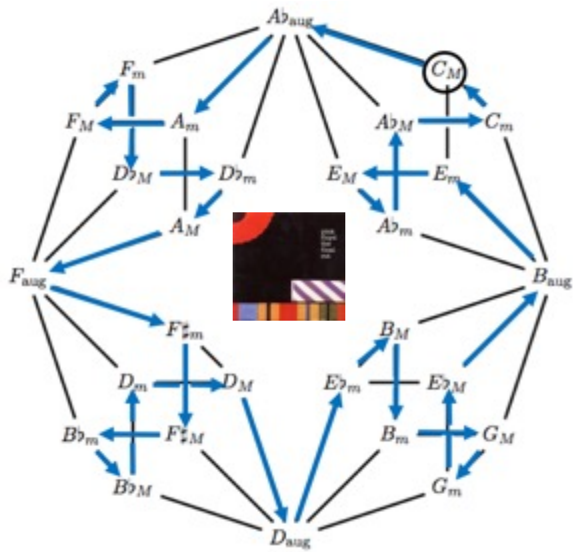
C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->Bbm-->F#-->F#m-->D-->Dm-->Bb-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

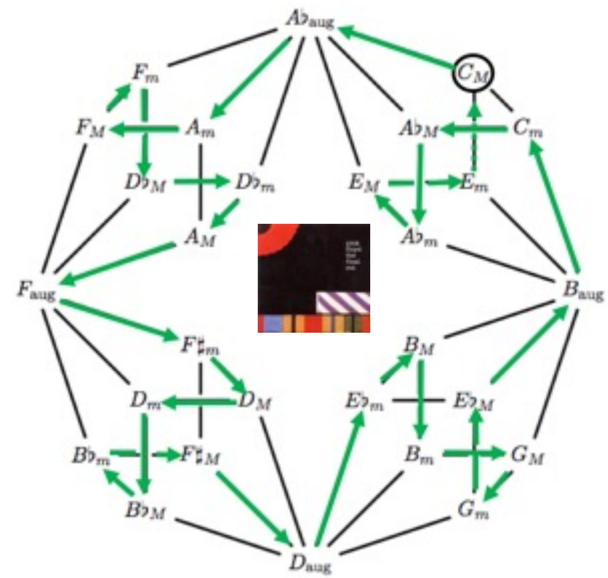
C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->F#-->Bbm-->Bb-->Dm-->D-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->D-->Dm-->Bb-->Bbm-->F#-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Cm-->G#-->G#m-->E-->Em-->C

Composing with Hamiltonian Cycles in the ‘classical’ Cube Dance







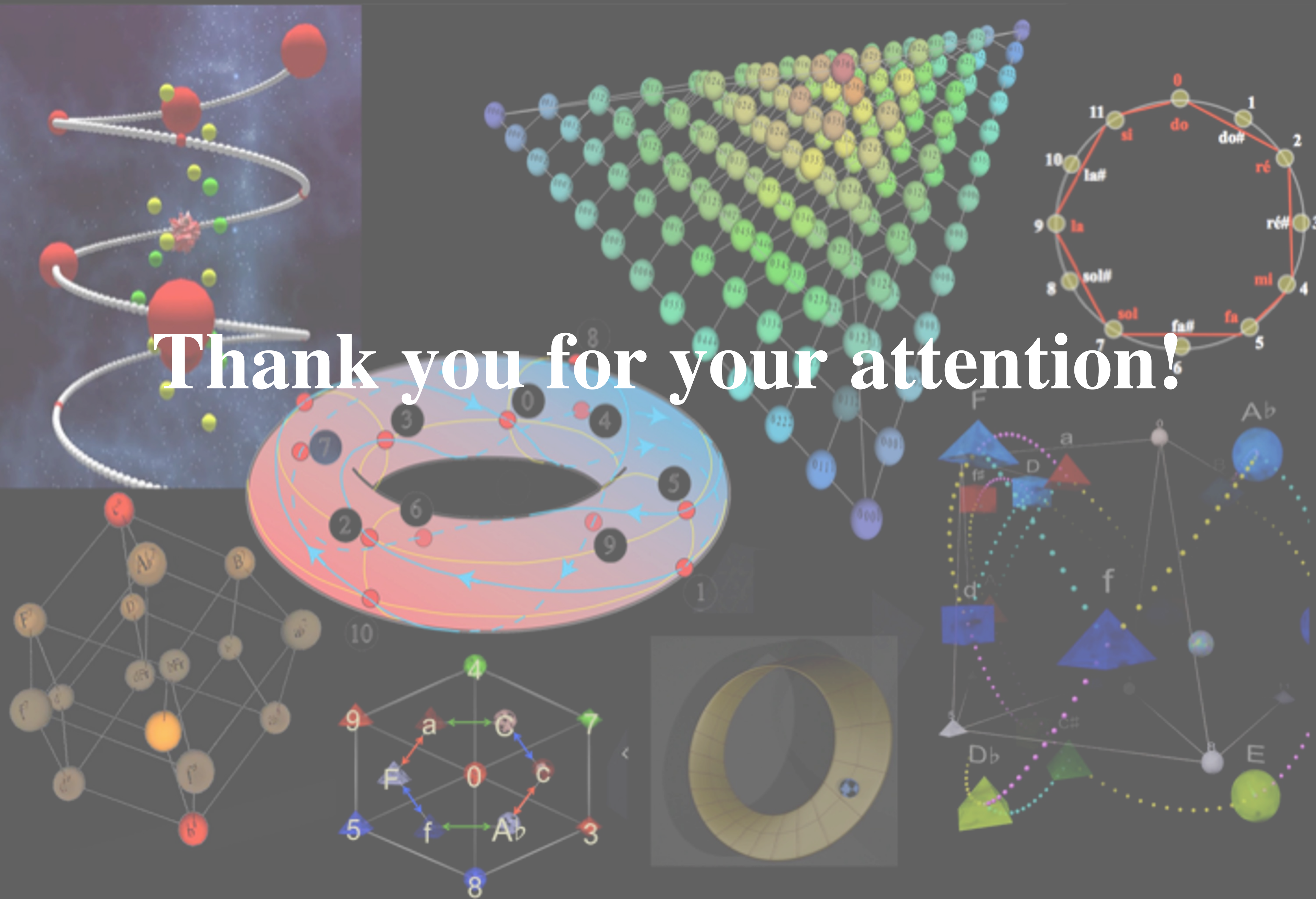
The three Hamiltonian Cycles (C_M = C, C_m = Cm, C_{aug} = C+)

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->Bbm-->F#-->F#m-->D-->Dm-->Bb-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->F#-->Bbm-->Bb-->Dm-->D-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->D-->Dm-->Bb-->Bbm-->F#-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Cm-->G#-->G#m-->E-->Em-->C

➔https://www.youtube.com/watch?v=nz5TYob02B4&ab_channel=MatheMusic4D



Thank you for your attention!