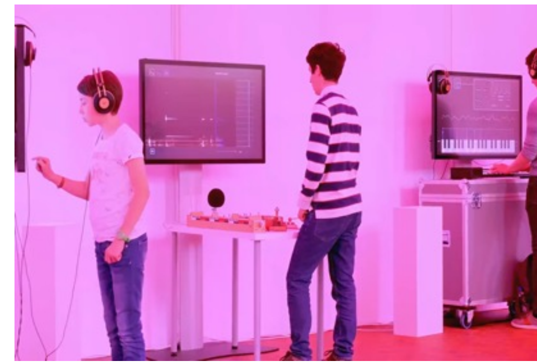
Une visite de l’expo LaLaLab ‘The Mathematics of Music’

(+ quelques idées pour des ateliers maths/musique)

Lycée Blaise Pascal

10 octobre 2022



Moreno ANDREATTA

IRMA/CNRS, Université de Strasbourg
& Ircam/CNRS, Sorbonne Université
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IRMA Institut de Recherche Mathématique Avancée · Université de Strasbourg · cnrs · ircam Centre Pompidou · SORBONNE UNIVERSITÉ


la.la.lab
THE MATHS OF MUSIC
May. 17, 2019 to Dec. 2020
DIE MATHEMATIK
DER MUSIK
Präsentiert von:
HEIDELBERG
LAUREATE FORUM
FOUNDATION


Guerino Mazzola
The Topos of Music
I: Theory
Geometric Logic, Classification, Harmony, Counterpoint, Motives, Rhythm
Second Edition

- **Tour rapide (1 minute) = https://vimeo.com/347620064**
- **Overview en 2 minutes : https://vimeo.com/347619305**

la.la.lab
THE MATHS OF MUSIC

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IMAGINARY
open mathematics

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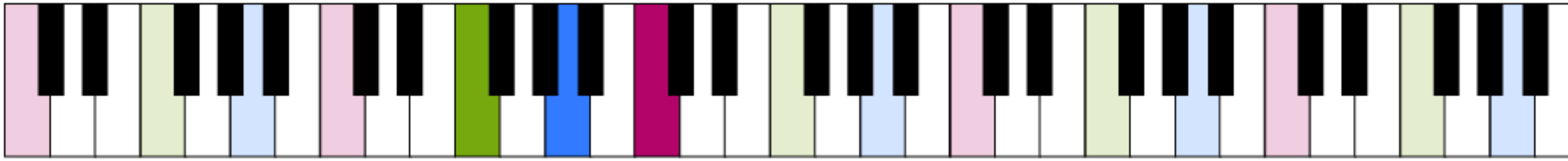
https://imaginary.org/

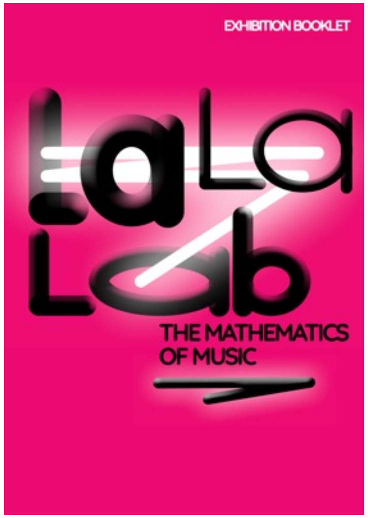


La.La.Lab brings the visitor to an interactive exploration and discovery of music from a mathematical perspective. The exhibition pivots over three axis:

- **Music theory.** Learning what tools build music, and how these tools are used to create art. Basic concepts and historical comments.

- **Current research.** The latest trends of research in the connection of maths and music. Artificial Intelligence, theoretical and new instruments, classification and composition tools.

- **Art and entertainment.** A joyful display of artworks from artists and mathematicians in the field.
Talks/concerts at scheduled events



➔ https://www.imaginary.org/sites/default/files/20190618-lala-booklet-v0.3-web.pdf

Vue d’ensemble des stands/modules disponibles

https://www.imaginary.org/exhibition/la-la-lab-the-mathematics-of-music

Pink Trombone
Program

Show me music
Program

Mind and Music Jukebox
Program

The Spectrum of Sound - Analyzer
Program

The Spectrum of Sound - Synthesizer
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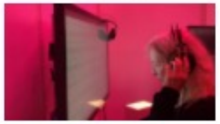


Musical bench
Physical Exhibit / Hands on

The Harmonic Series #3 - 3D display
Program

Gerhard Widmer on Expressive Music Performance, the Boesendorfer CEUS, and a MIDI Theremin
Film

Scale Lab
Program

Whitney Music Box
Program

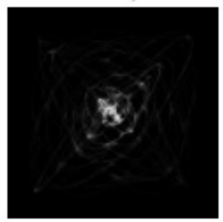
The Harmonic Series #1 - Laser
Physical Exhibit / Hands on

Pentatonic Scales
Program

Tonnetz
Program

Beat Box
Program

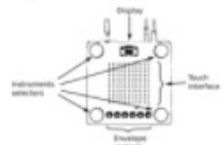
Lissajous figures
Gallery



Note Compass
Program

Con Espressione!
Program

NSynth
Physical Exhibit / Hands on



The Harmonic Series #2 - 3D sculptures
Physical Exhibit / Hands on

JSBach333 - canone permutativo al triangolo from BACH333 - Canon Composition Competition 2018
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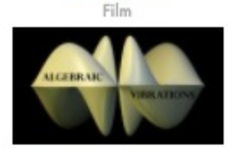
Program

AI Jam
Program

The Sound of Sequences
Program

Algebraic Vibrations
Film



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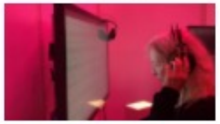

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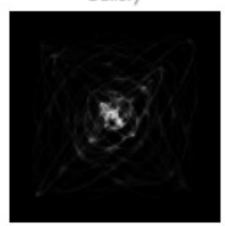

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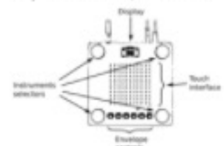

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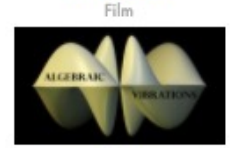

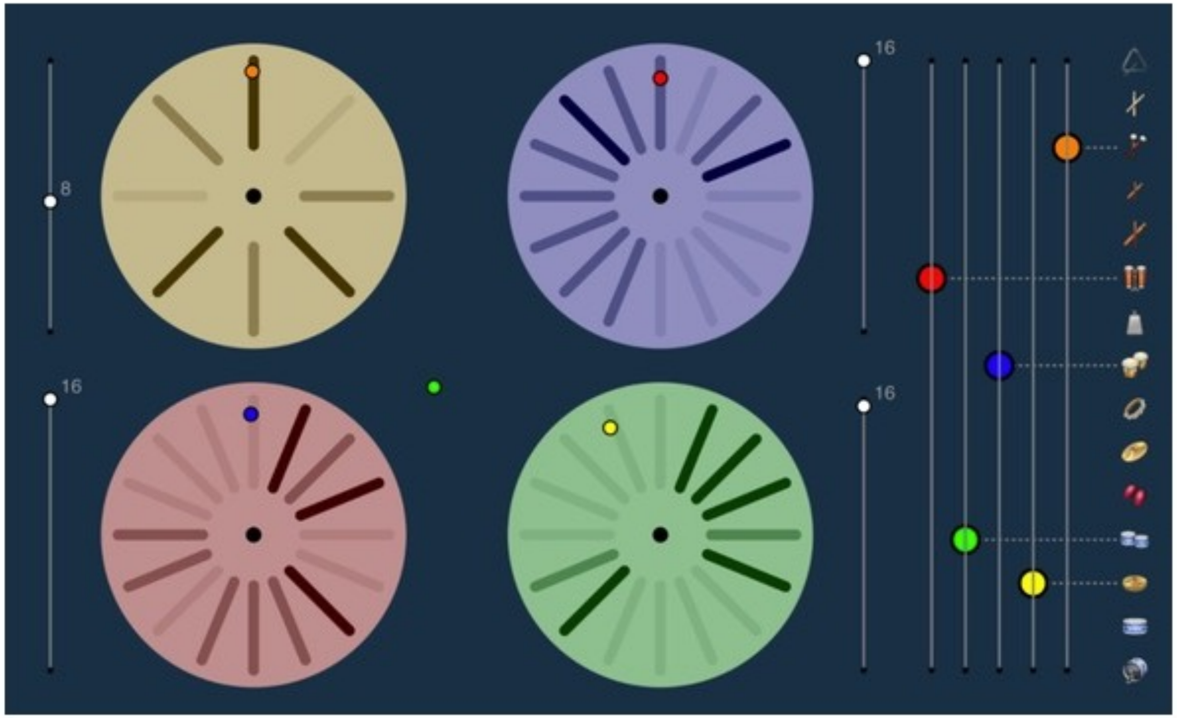

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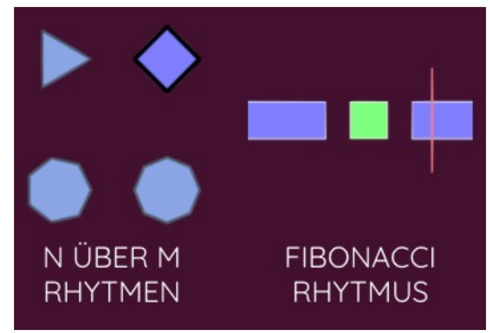
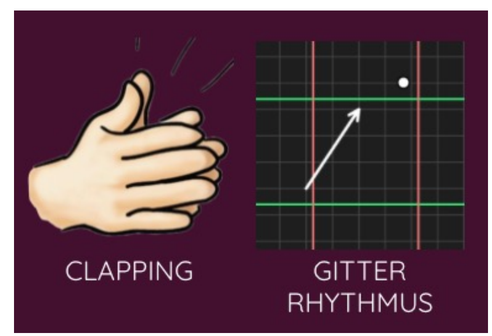


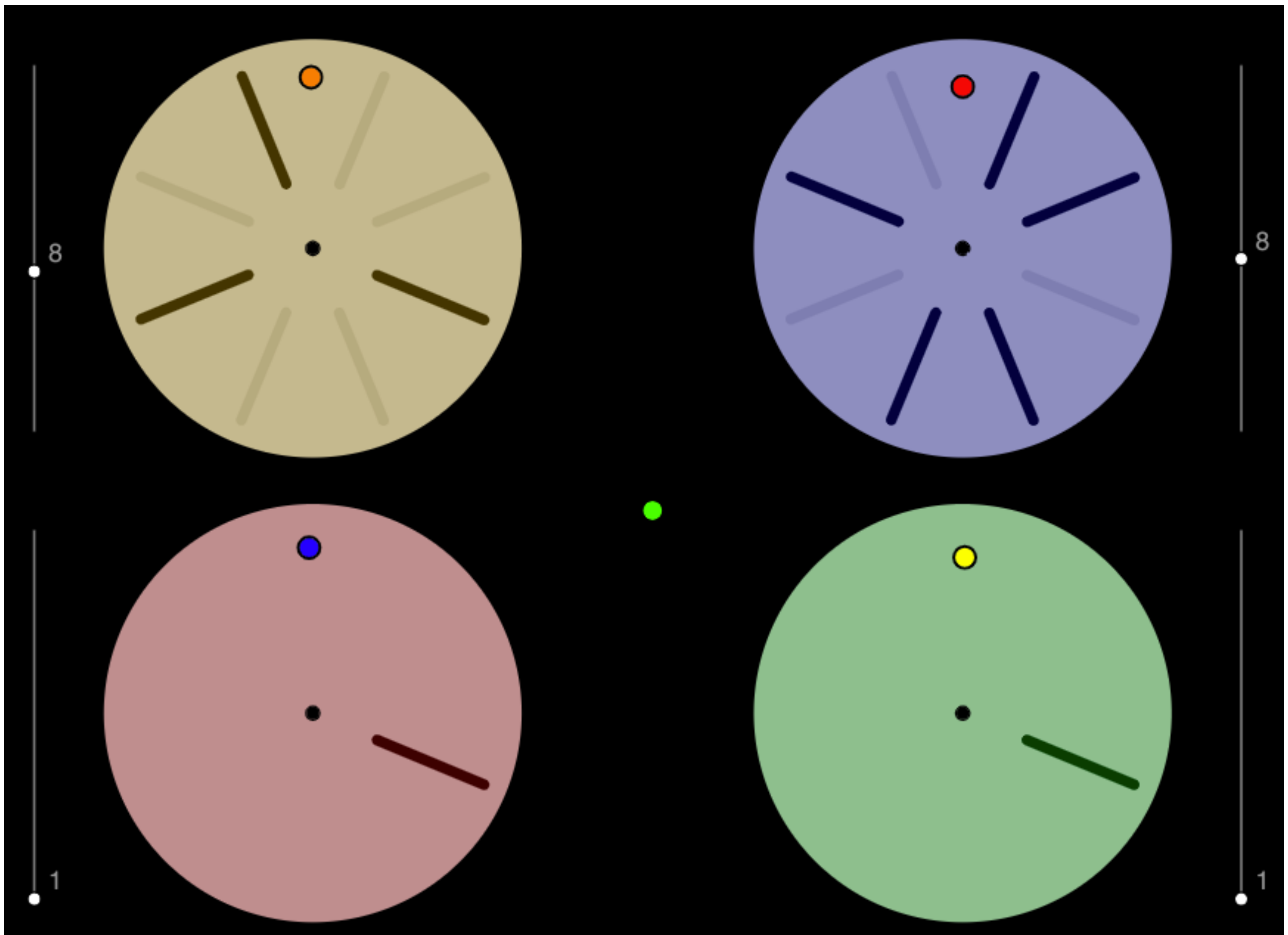


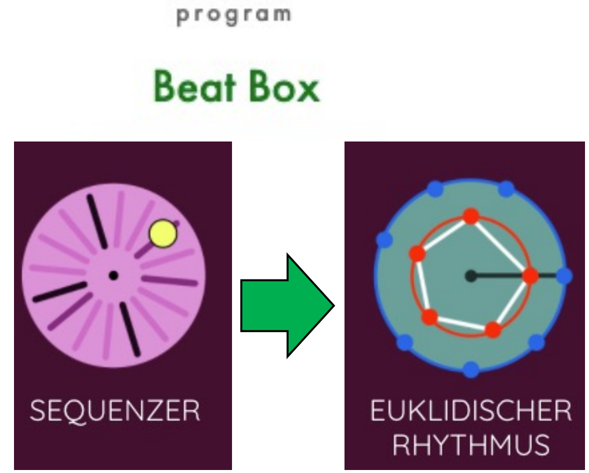
Select an animation, press the “play” button and experiment with the parameters. Try to clap along.

Along with melody and harmony, rhythm is one of the most important components of music. This exhibit explores various ways to create and analyze rhythms by mathematical methods. No instrument is required to play a rhythm, clapping your hands is enough. However, even simple structures may be difficult to perform. In this exhibit, you can explore the relation of fractions and rhythms. See how pixelated lines can be used to create drum patterns. Explore intriguing minimalistic ways to create rhythms and much more.

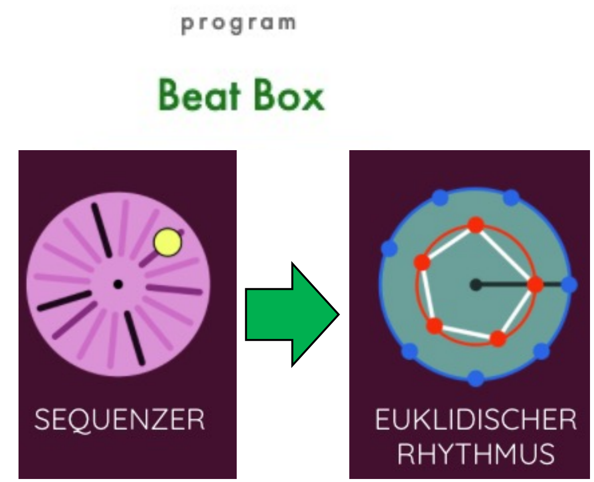
AUTHOR OF THIS EXHIBIT: JÜRGEN RICHTER-GEBERT, TECHNICAL UNIVERSITY OF MUNICH
SOUND ENGINE: PATRICK WILSON AND AARON MONTAG / BASED ON CINDYJS.ORG
TEXT: JÜRGEN RICHTER-GEBERT (TU MUNICH)

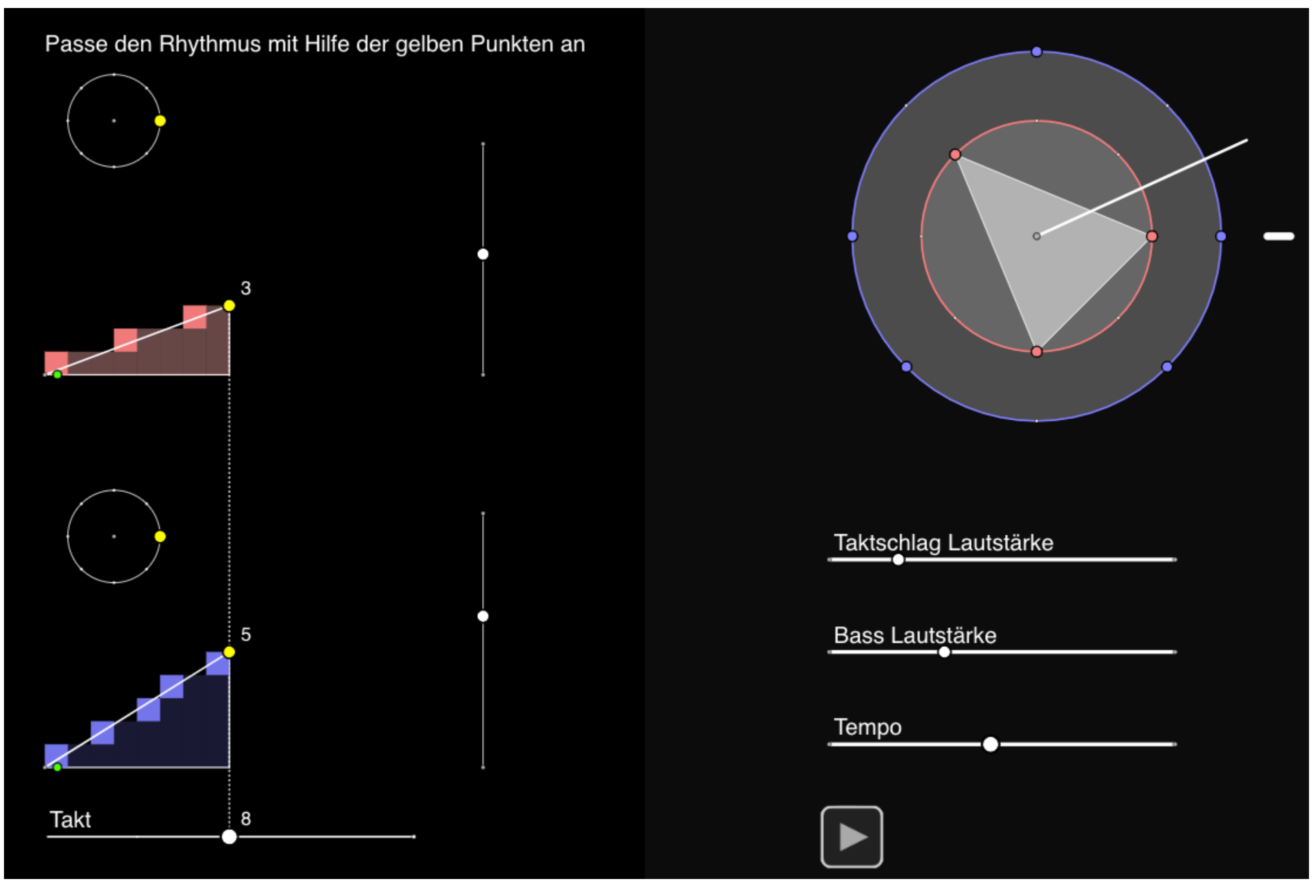
Rythmes complémentaires
le *trecillo* vs le *cinquillo*





Rythmes euclidiens
le *trecillo*, le *cinquillo* et plein d’autres rythmes !
(retrouver les complémentaires)





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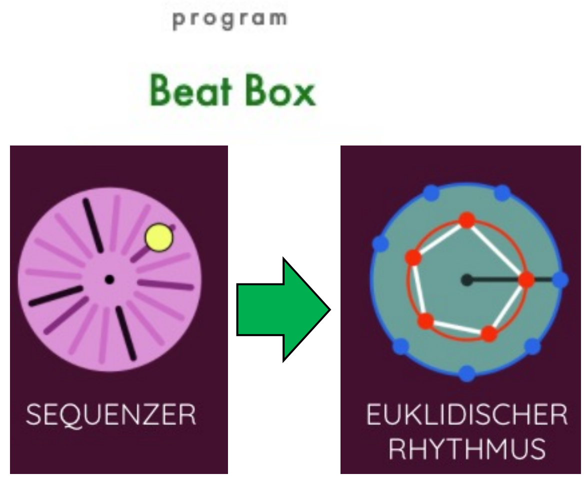
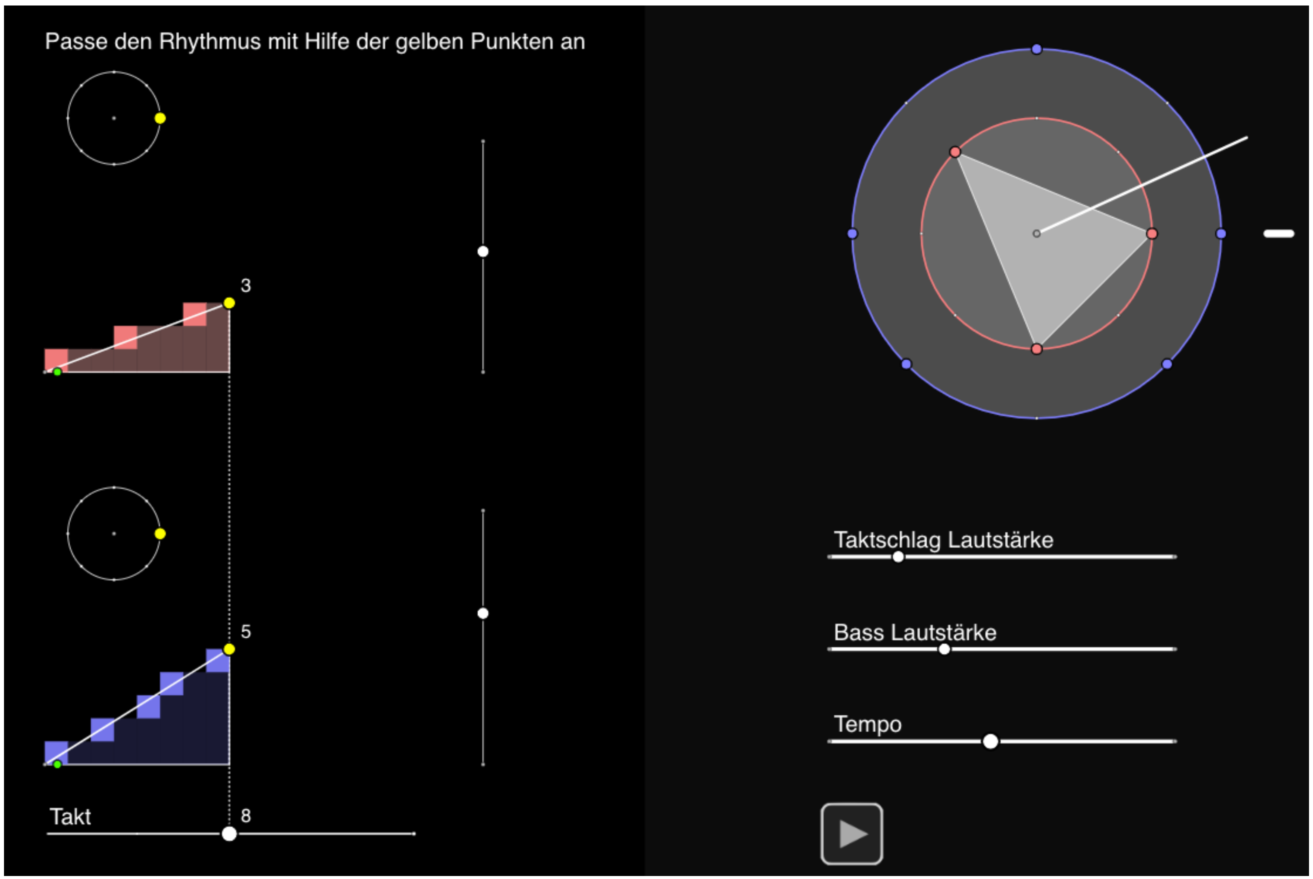
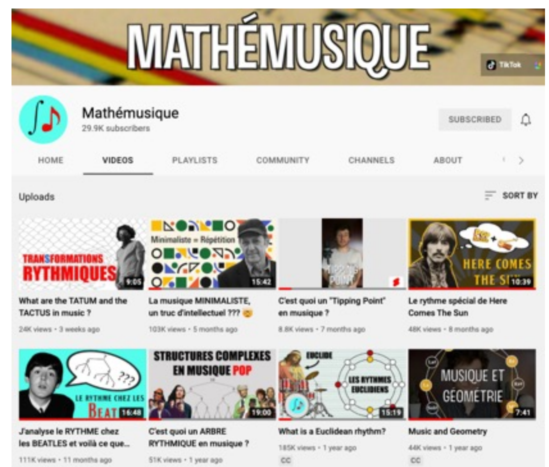
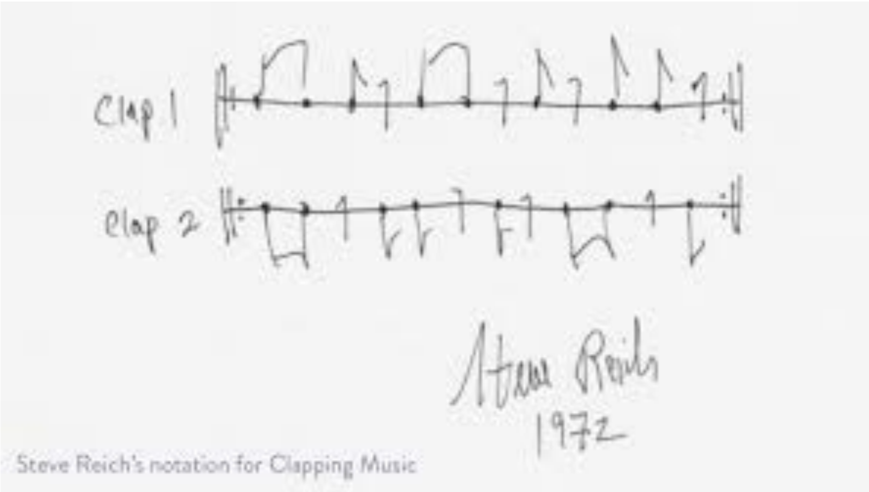




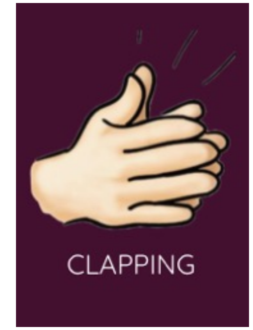


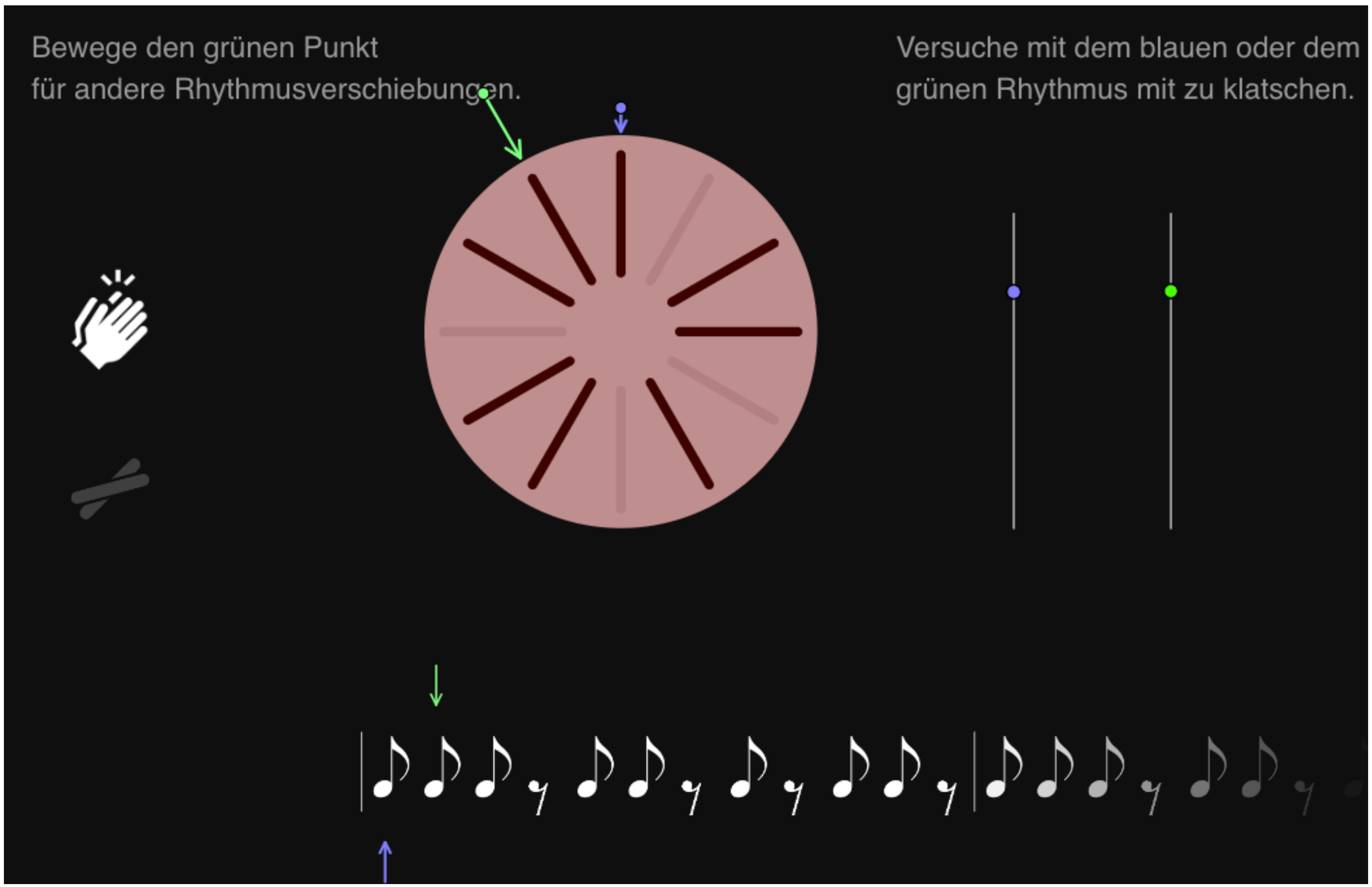
https://www.youtube.com/c/mathemusique



program

Beat Box





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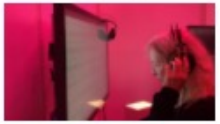

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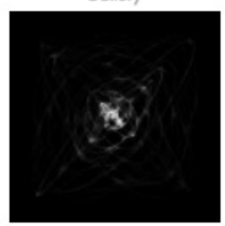

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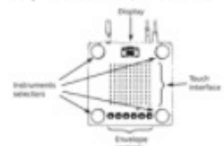

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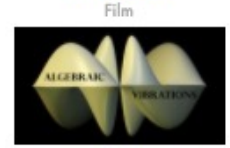

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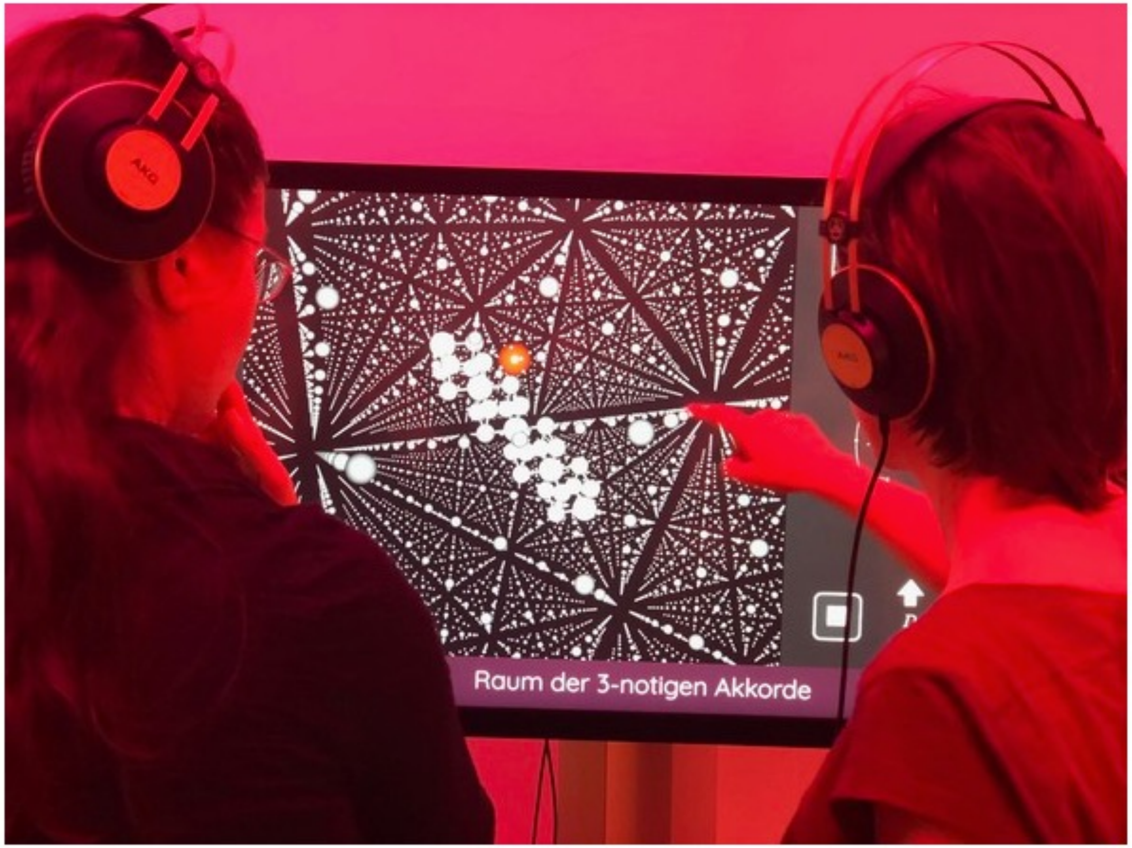

Dive into a multitude of topics that visualize the complex interrelations of melody, harmony and mathematics. Push the “play” button in each visualization and experience the effect of parameters.

Each of the animations looks at a certain musical piece or pattern from a special mathematical viewpoint. Aspects of symmetry, both in time and space help to understand musical ideas. If you look at an animation of a Chopin Prelude, it unveils hidden structures and helps you to understand the music better.

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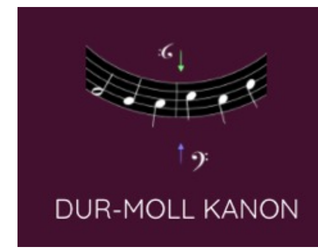

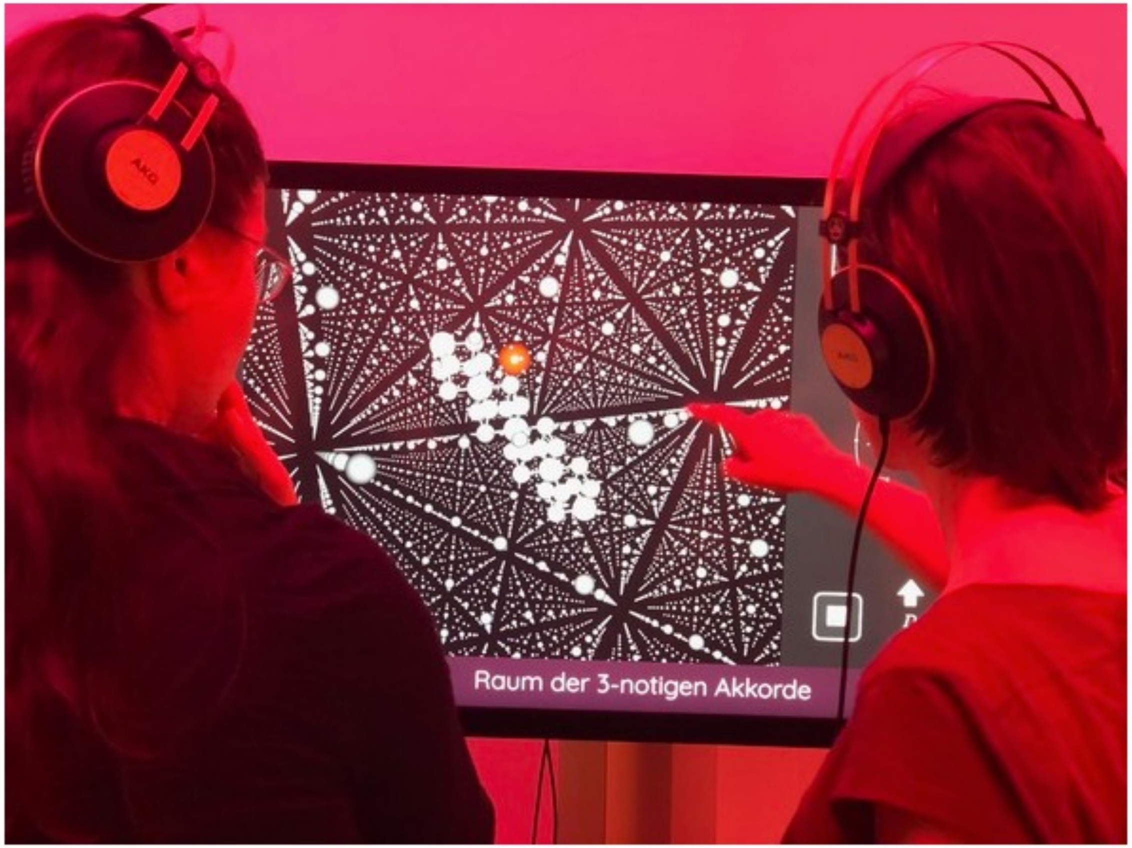

Show me music

ZAHLENTAFEL.
TABLE de CHIFFRES.

Erster Theil. Premiere Partie.

	A	B	C	D	E	F	G	H
2	96	22	141	41	105	122	11	30
3	32	6	128	63	146	46	134	81
4	69	95	158	13	153	55	110	24
5	40	17	113	85	161	2	159	100
6	148	74	163	45	80	97	36	107
7	104	157	27	167	154	68	118	91
8	152	60	171	53	99	133	21	127
9	119	84	114	50	140	86	169	94
10	98	142	42	156	75	129	62	123
11	3	87	165	61	135	47	147	33
12	54	130	10	103	28	37	106	5

Zweiter Theil. Seconde Partie.

	A	B	C	D	E	F	G	H
2	70	121	26	9	112	49	109	14
3	117	39	126	56	174	18	116	83
4	66	139	15	132	73	58	145	79
5	90	176	7	34	67	160	52	170
6	25	143	64	125	76	136	1	93
7	138	71	150	29	101	162	23	151
8	16	155	57	175	43	168	89	172
9	120	88	48	166	51	115	72	111
10	65	77	19	82	137	38	149	8
11	102	4	31	164	144	59	173	78
12	35	20	108	92	12	124	44	131

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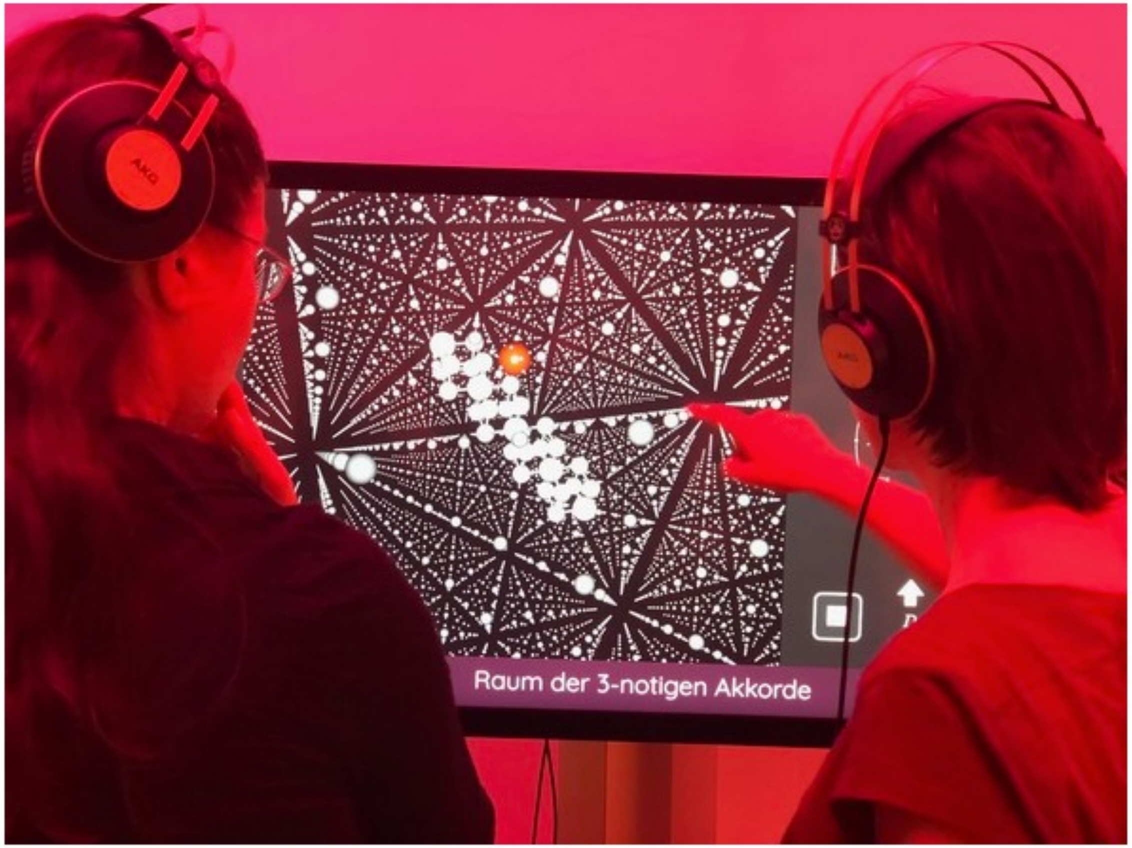

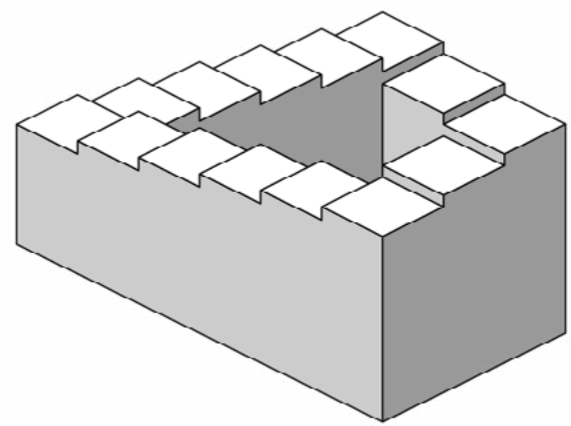

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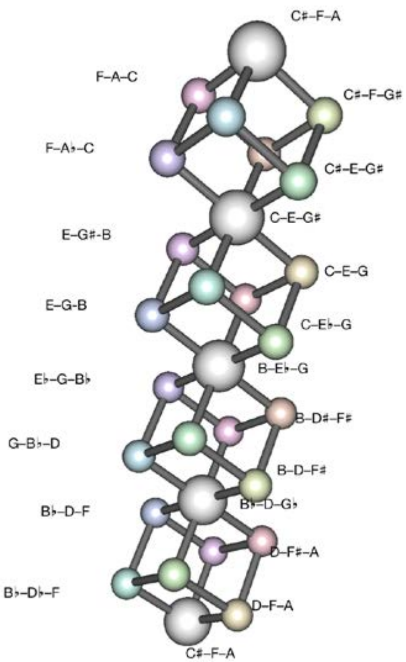

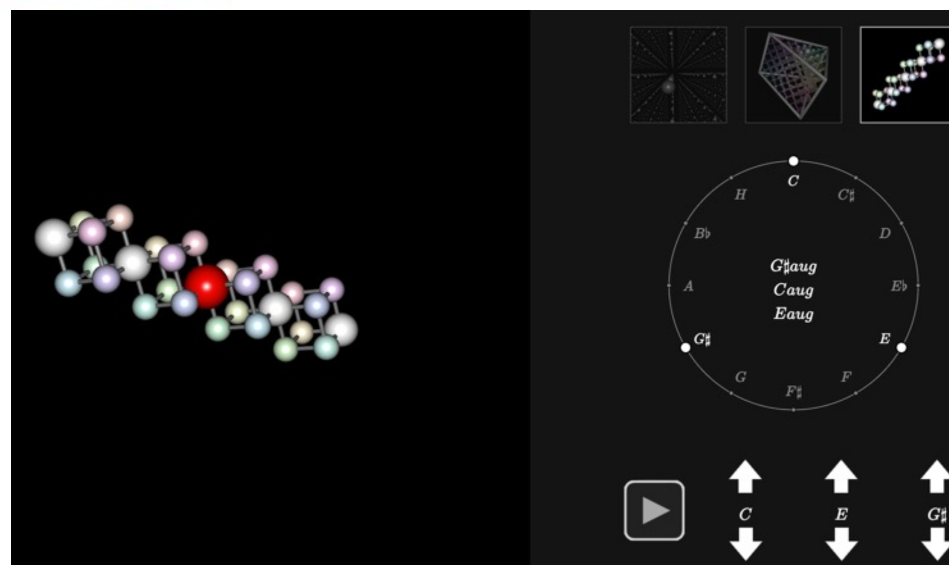

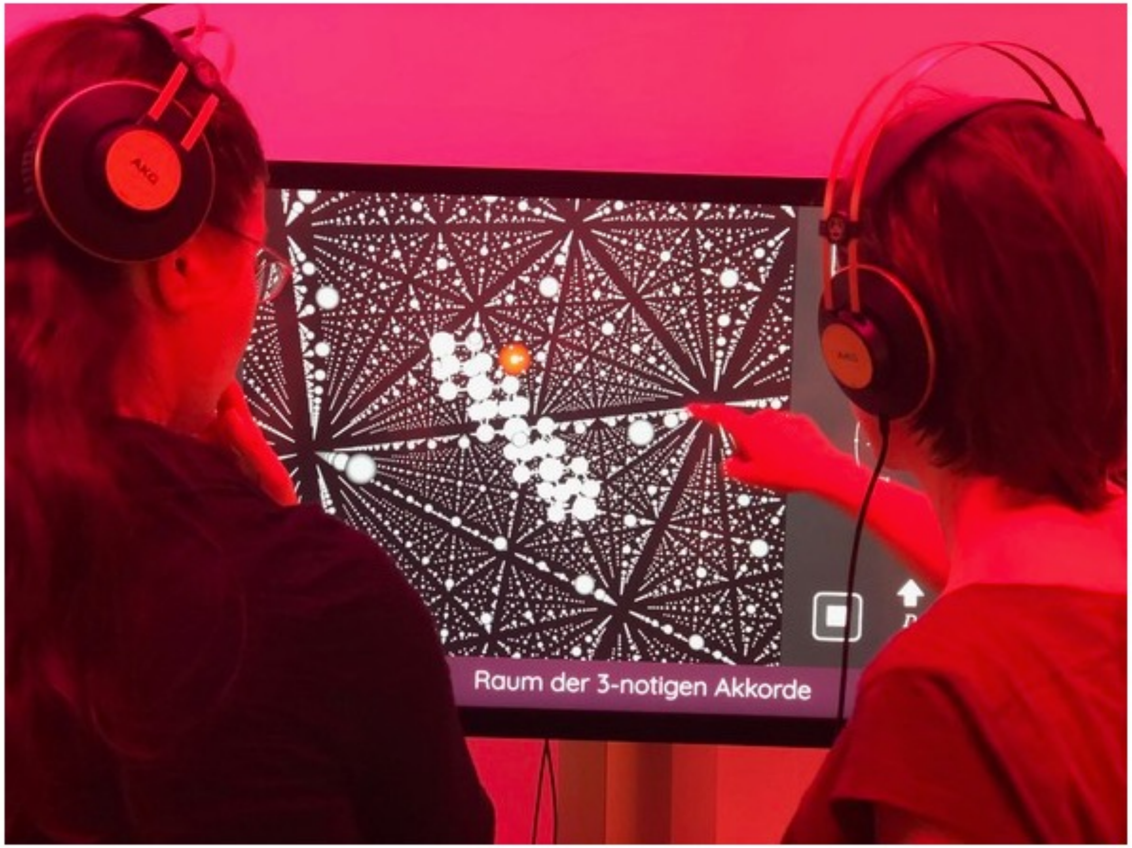

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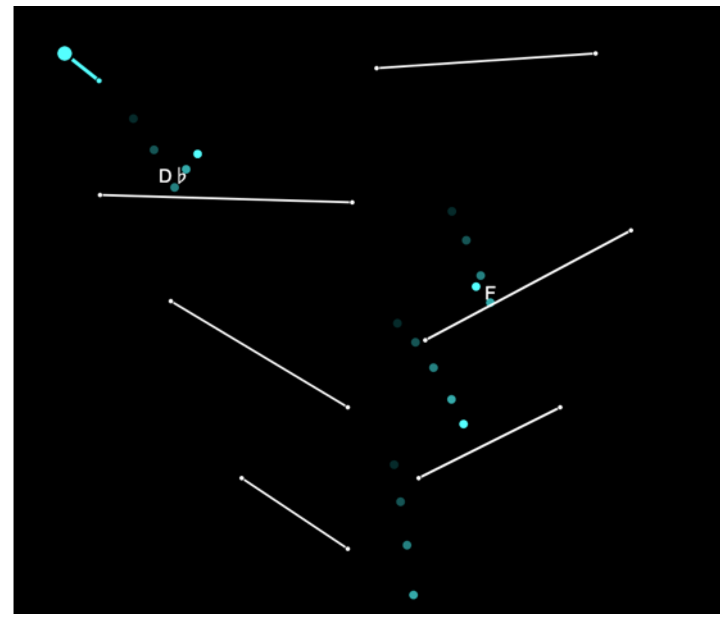

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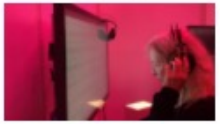

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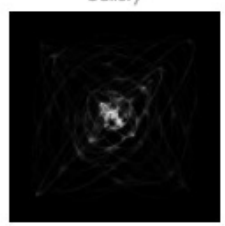

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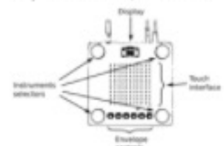

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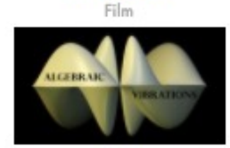

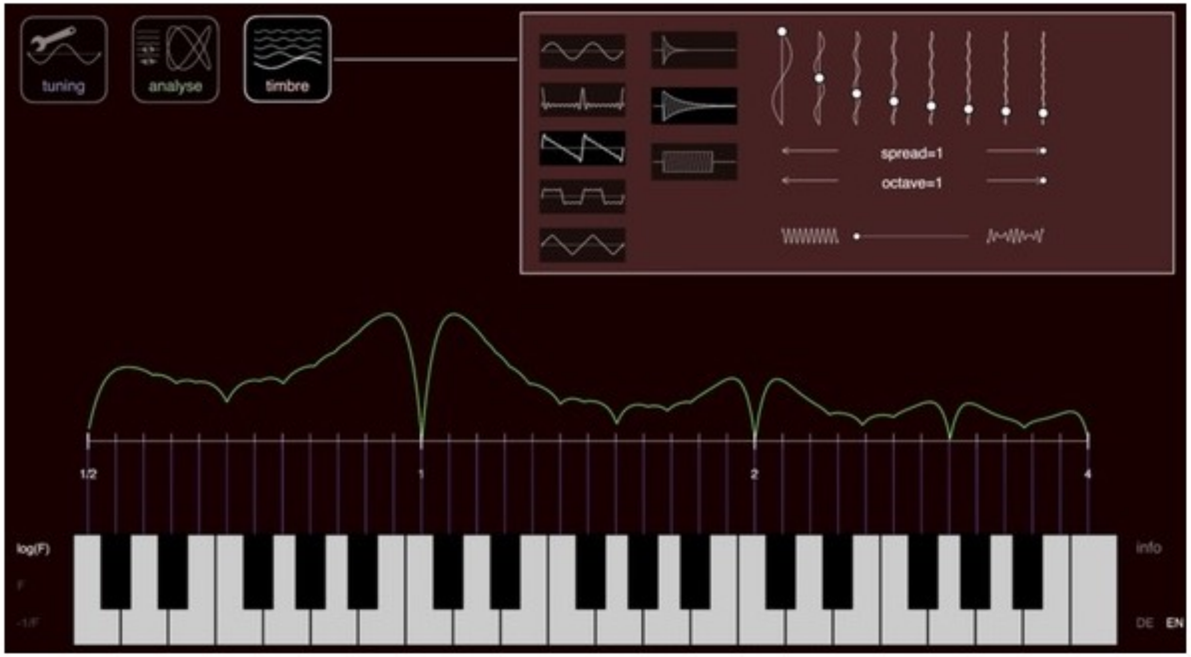

program

Scale Lab

Launch

see all files ↓

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Submitted by

IMAGINARY

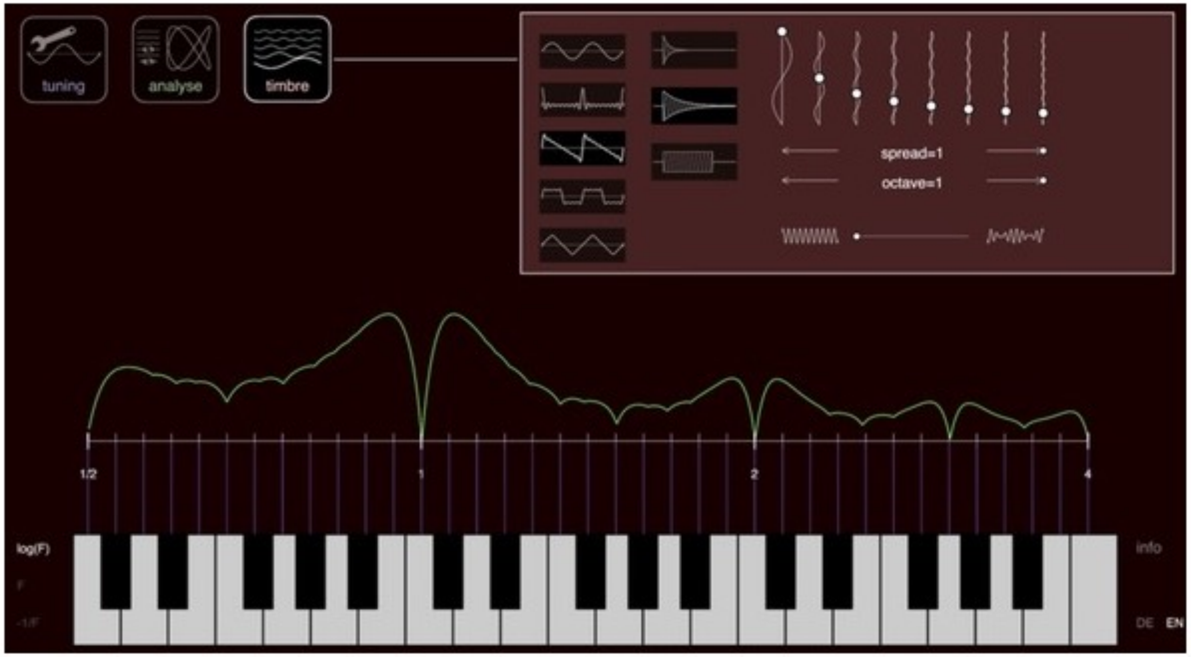
This program is part of
La La Lab - The Mathematics of Music

Credits

Author of this exhibit: Jürgen Richter-Gebert, Technical University of Munich / Sound Engine: Patrick Wilson and Aaron Montag / based on CindyJS.org

Which sounds can be called “notes”, and used to create music? This exhibit is an experimental platform to explore the characteristics of sound and scales.

We can hear a wide range of sound frequencies, but we don’t use them all to make music. We select some discrete set of them as our notes to form what we call a scale. We try to build scales in such a way that combinations of notes on the scale are harmonious and can be played together. Scale Lab allows you to explore, measure, create and modify both individual sounds and musical scales. This exhibit covers the fundamentals of sound, perception of consonance and dissonance and the creation of scales from single tones.



program

Scale Lab

Launch

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Submitted by

IMAGINARY

This program is part of
La La Lab - The Mathematics of Music

Credits

Author of this exhibit: Jürgen Richter-Gebert, Technical University of Munich / Sound Engine: Patrick Wilson and Aaron Montag / based on CindyJS.org

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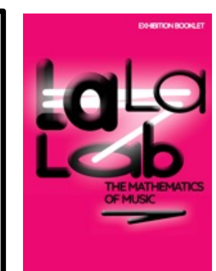
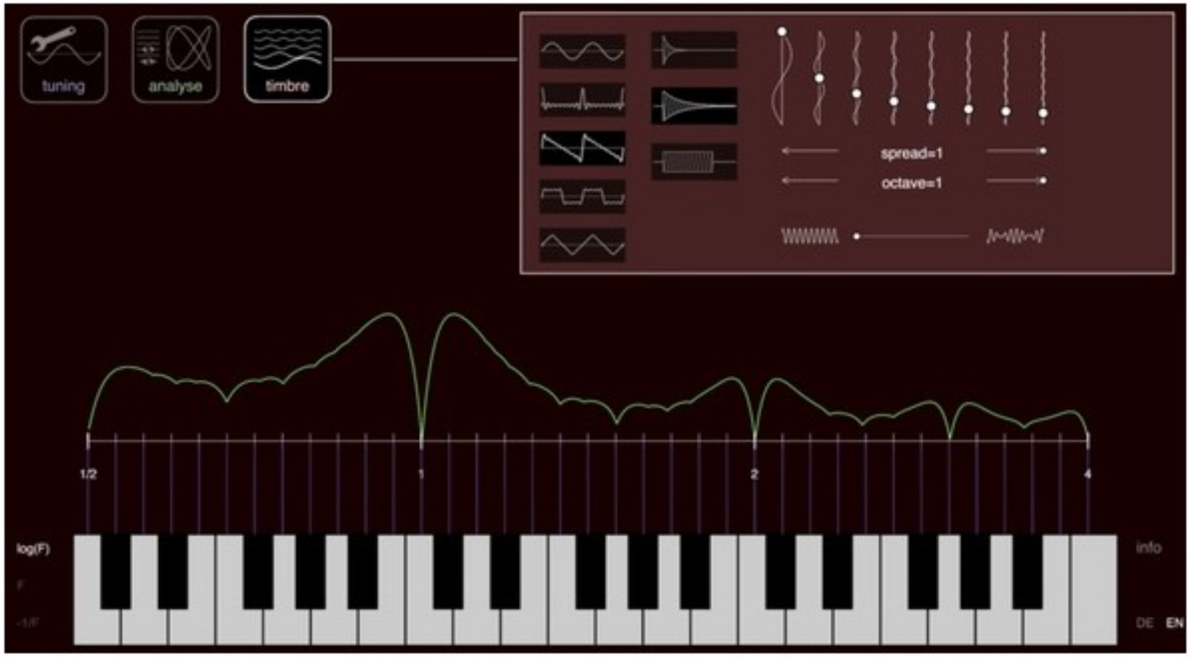
Pythagorean Tuning: Let us start with Pythagorean tuning, which focuses around the idea of having as many perfect fifths as possible. Focus on the seven tones of our C major scale and see what happens. We label the tones by frequency ratios with respect to C and divide by two (drop an octave) every time we become greater than two. The following diagram shows the frequency ratios of notes in the Pythagorean scale and how they are derived.





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Source code: CC BY-NC-SA-4.0

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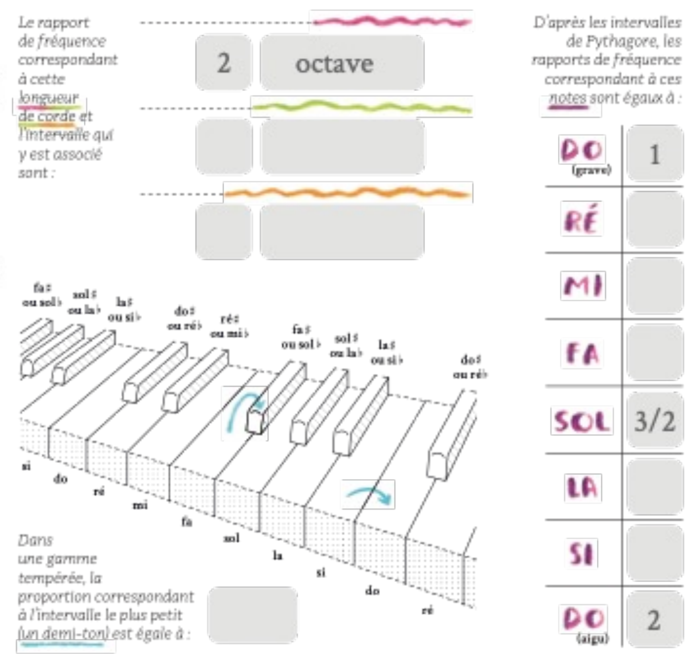
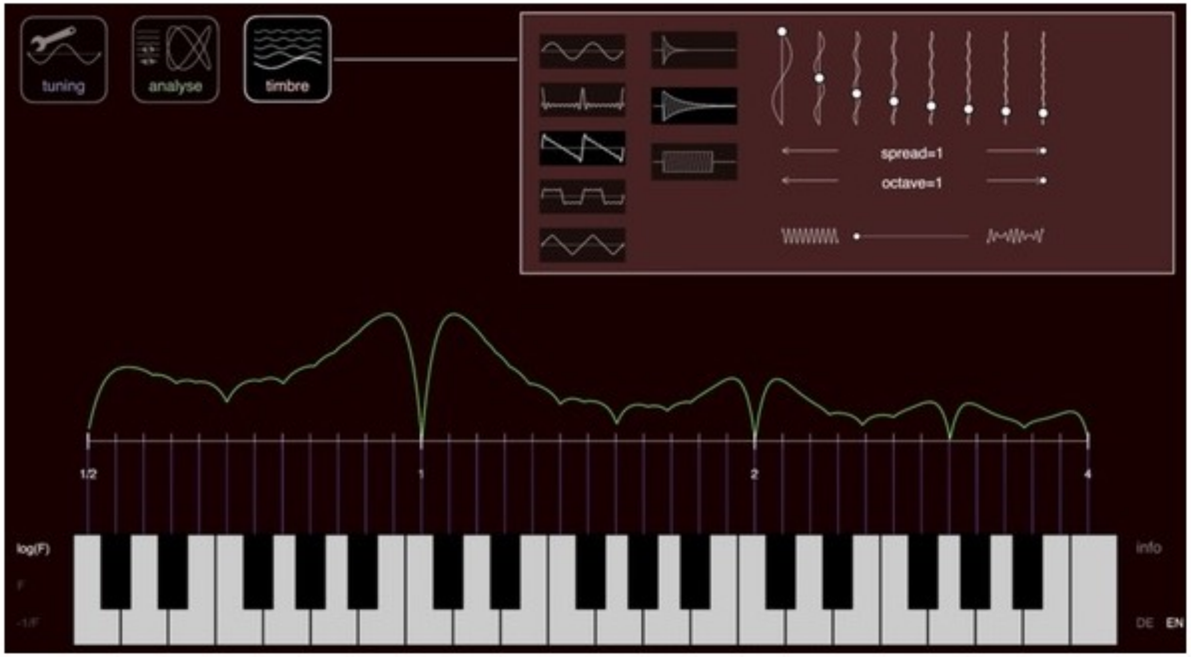
Flyer atelier maths/musique



➔ **http://repmus.ircam.fr/_media/moreno/depliant_atelier_impression.pdf**



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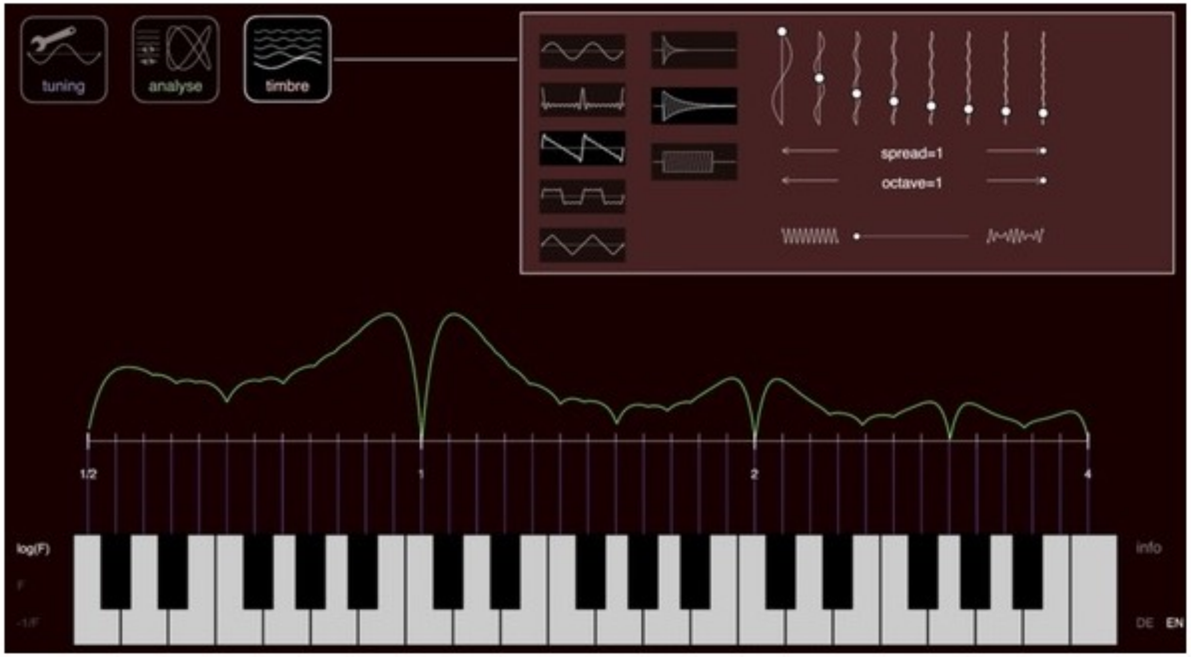
Film pédagogique

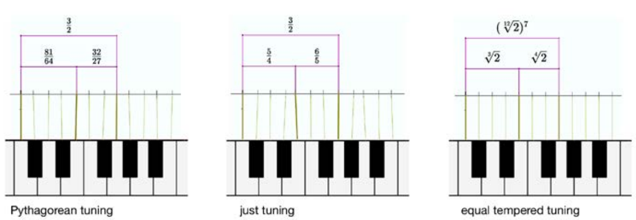
Musique et mathématiques : histoire d’une rencontre

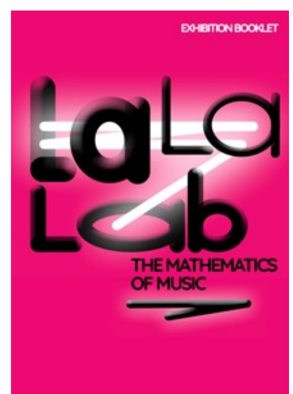
➔ **https://video.math.cnrs.fr/musique-et-mathematiques/**



Which sounds can be called “notes”, and used to create music? This exhibit is an experimental platform to explore the characteristics of sound and scales.

Experiment: Open the tuning box and experiment with different Western tunings. The colors shown in the Tonnetz indicate how close an interval is to a perfect one. It is also very instructive to analyze the different tunings with the ratio tool. It shows the ratios of intervals if they are played on the keyboard. The picture below shows the frequency ratios for a major C chord.





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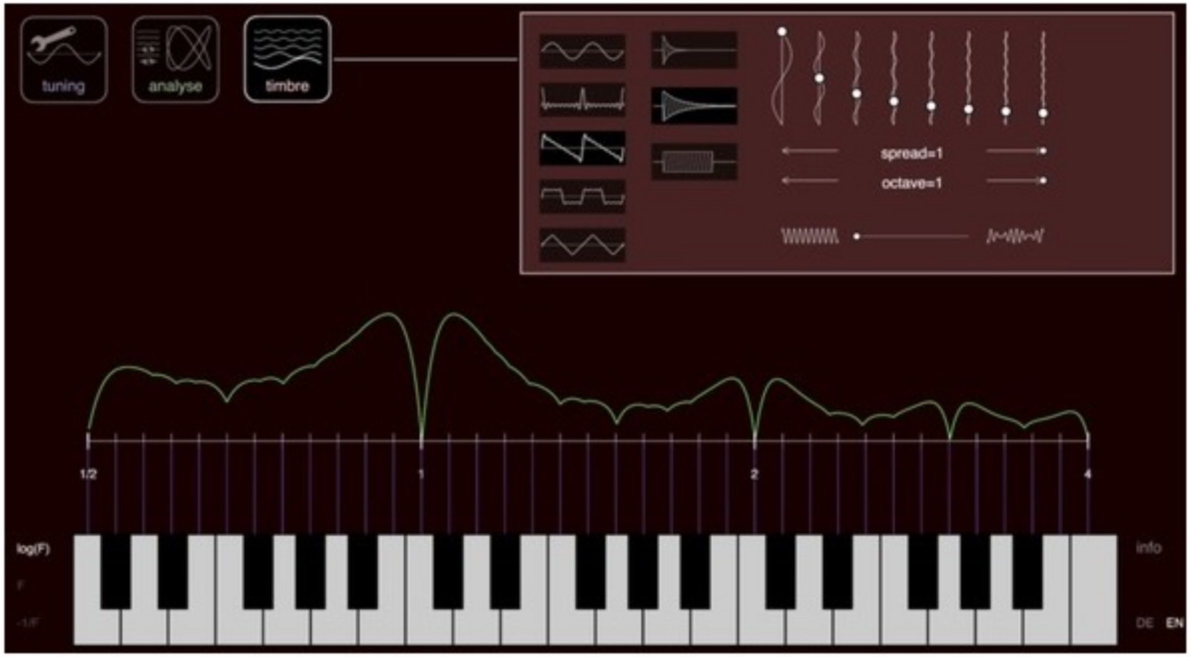

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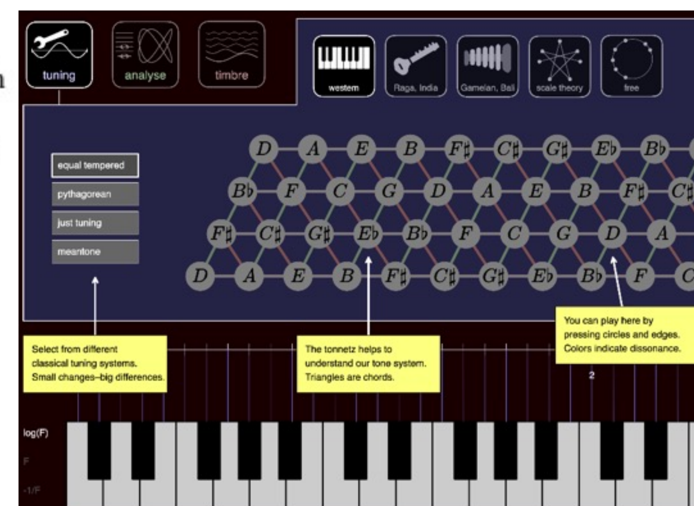


Vue d'ensemble des stands/modules disponibles

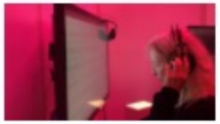
https://www.imaginary.org/exhibition/la-la-lab-the-mathematics-of-music

Pink Trombone
Program


Show me music
Program


Mind and Music Jukebox
Program


The Spectrum of Sound - Analyzer
Program


The Spectrum of Sound - Synthesizer
Program


Musical bench
Physical Exhibit / Hands on


The Harmonic Series #3 - 3D display
Program


Gerhard Widmer on Expressive Music Performance, the Boesendorfer CEUS, and a MIDI Theremin
Film


Scale Lab
Program

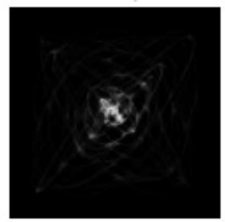

Whitney Music Box
Program


The Harmonic Series #1 - Laser
Physical Exhibit / Hands on


Pentatonic Scales
Program

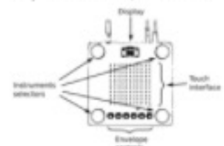

Tonnetz
Program


Beat Box
Program


Lissajous figures
Gallery


Note Compass
Program


Con Espressione!
Program


NSynth
Physical Exhibit / Hands on


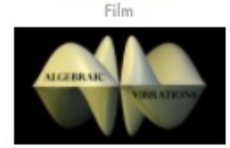
The Harmonic Series #2 - 3D sculptures
Physical Exhibit / Hands on

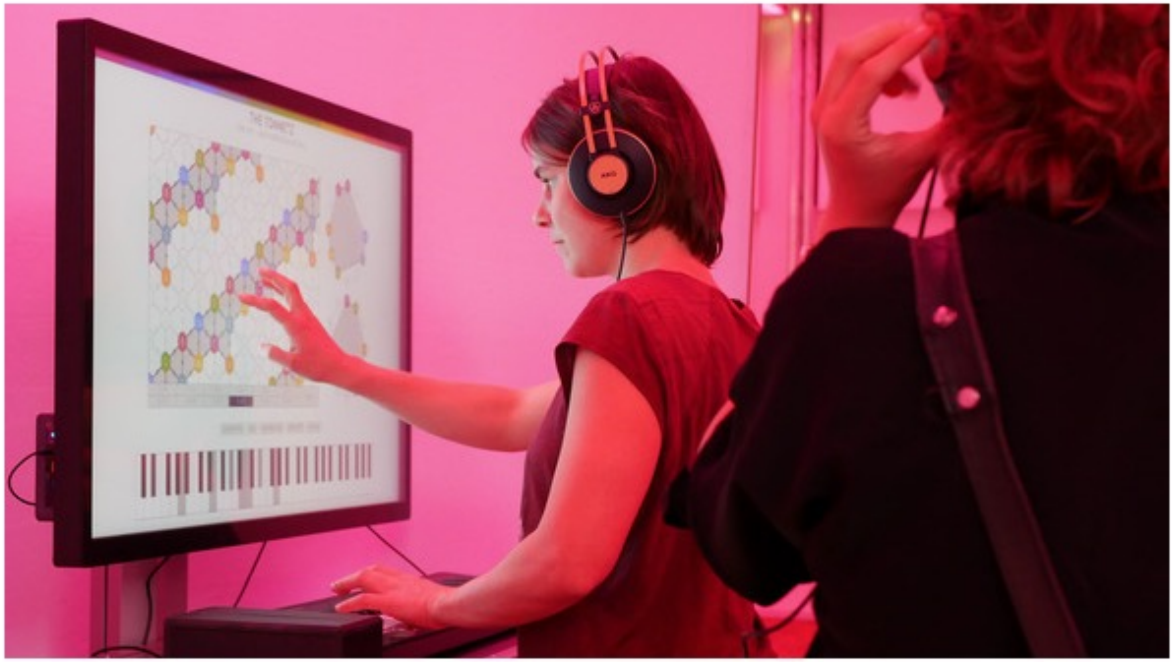

JSBach333 - canone permutativo al triangolo from BACH333 - Canon Composition Competition 2018
Film


The Graph Composer
Program


AI Jam
Program


The Sound of Sequences
Program


Algebraic Vibrations
Film




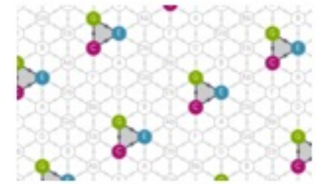
Explore how we can represent the notes in a logical and useful way! The Tonnetz is a pictorial representation of the notes in the plane that reveal affinities and structures between notes and on concrete music pieces.

Written scores and keyboard instruments order notes by pitch. However, notes with close pitch tend to sound bad together, but notes separated by intervals such as the octave (12 semitones), the perfect fifth (7 semitones) or the major third (4 semitones) are more harmonic. The different types of Tonnetz display close together notes by specific intervals. Musicians also use the chromatic circle (ordered by semitones) and the fifths circle (ordered by fifth intervals) as useful tools to compose music.

AUTHORS OF THIS EXHIBIT: MORENO ANDREATTA AND CORENTIN GUICHAOUA (SMIR PROJECT). SUPPORTED BY CNRS/IRCAM/SORBONNE UNIVERSITY, USIAS (UNIVERSITY OF STRASBOURG INSTITUTE FOR ADVANCED STUDY), IRMA/UNIVERSITY OF STRASBOURG. ADAPTED BY PHILIPP LEGNER. TEXT: DANIEL RAMOS (IMAGINARY)

program

Tonnetz



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Licenses

Source code: GPL 3.0

Submitted by

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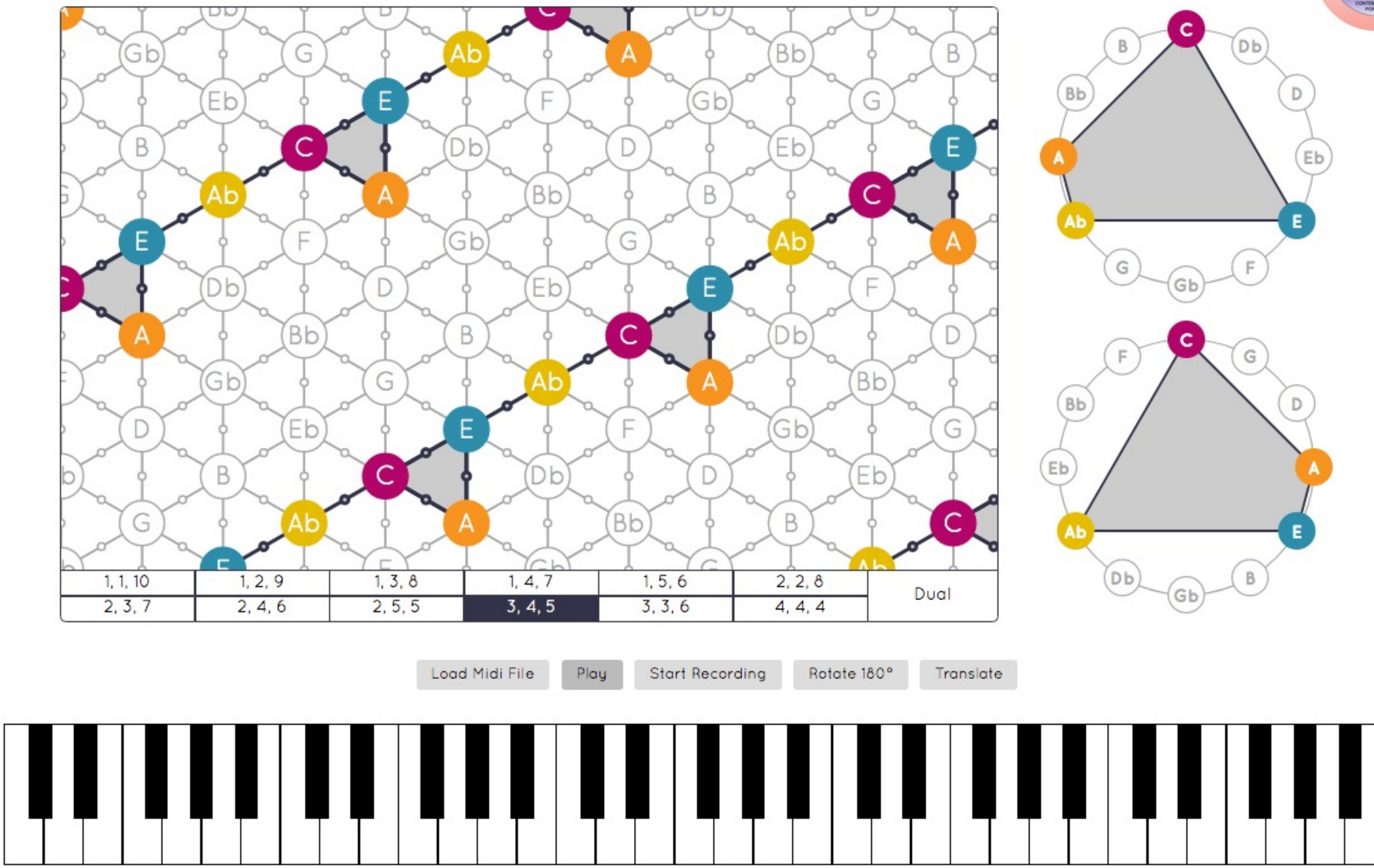
This program is part of
La La Lab - The Mathematics of Music

THE TONNETZ

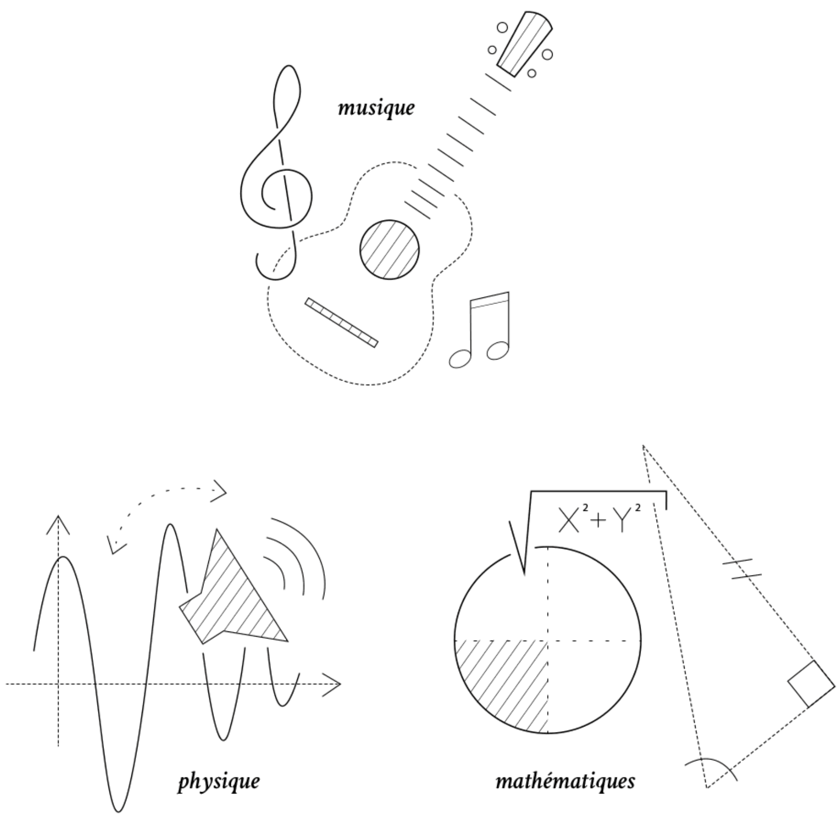
ONE KEY – MANY REPRESENTATIONS



➔ **https://guichaoua.gitlab.io/web-hexachord/**

Quelques idées pour des ateliers maths/musique

La musique entre les maths, la physique et l'info

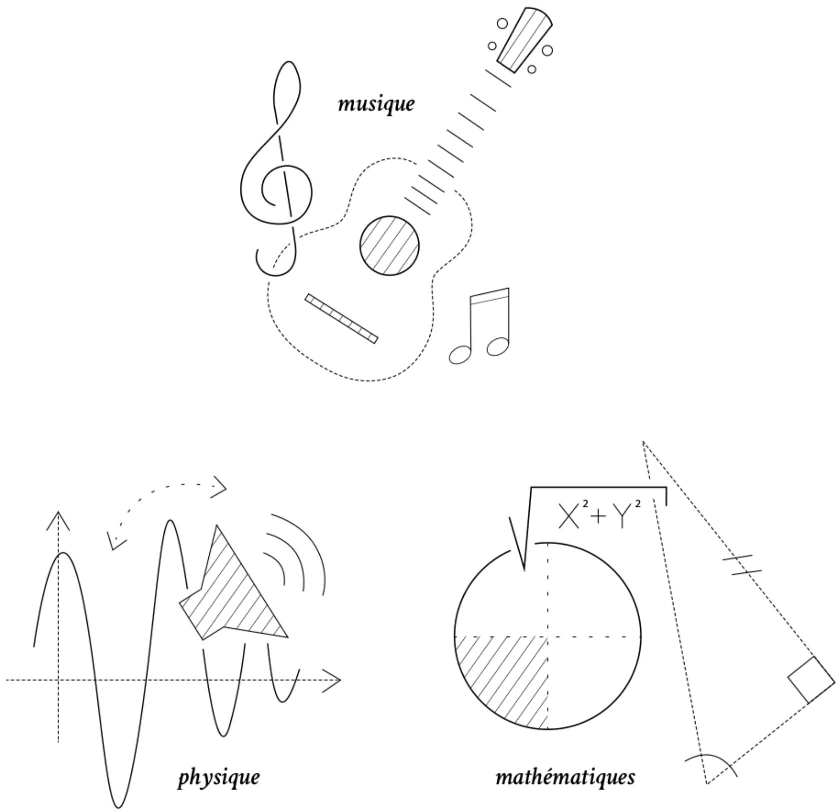


« Mathémusique » dans le programme de l'Education Nationale

(enseignement scientifique, 1ère générale)

L'être humain perçoit le monde à l'aide de **signaux** dont certains sont de nature sonore. De l'Antiquité jusqu'à nos jours, il a combiné les sons de manière harmonieuse pour en faire un art, la musique, qui entretient des liens privilégiés avec **les mathématiques**. L'**informatique** permet aujourd'hui de numériser les sons et la musique. La compréhension des mécanismes auditifs s'inscrit dans une perspective d'éducation à la santé.

La musique entre les maths, la physique et l'info



« Mathémusique » dans le programme de l'Education Nationale

(<u>spécialité musique cycle terminal</u>)

Deux champs de questionnement :

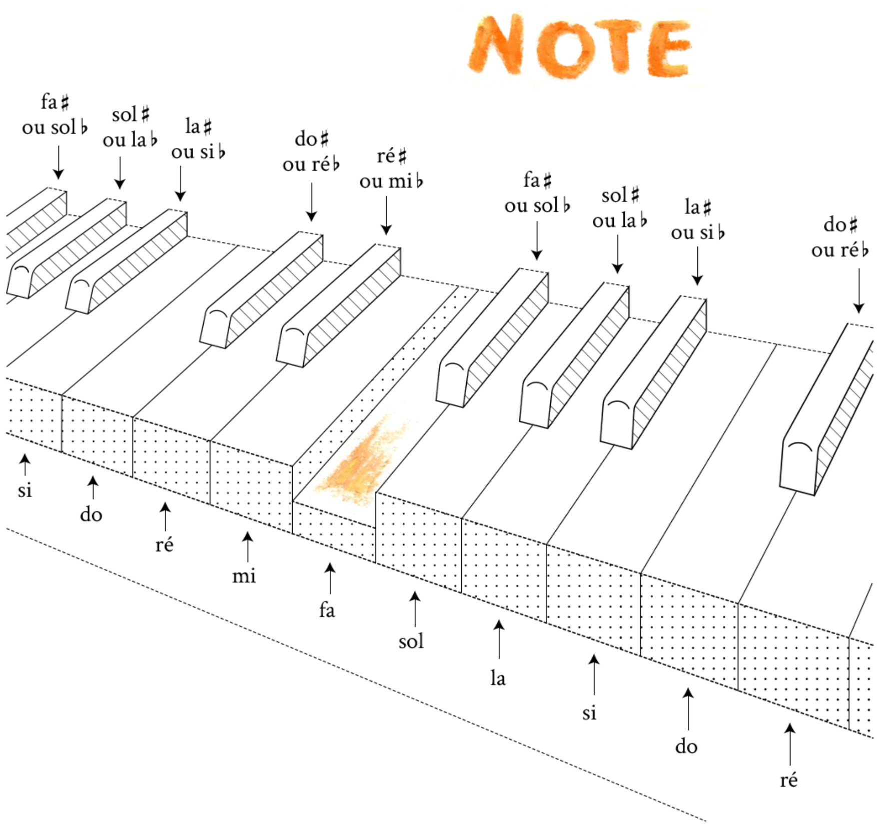
1. Le son, la musique, l'espace et le temps

- La musique art du temps ou de l'espace ?
- La forme musicale (musique savante et populaire)
- Approches algorithmiques en composition
- Les proportions et les modèles des gammes
- Les proportions et les modèles des rythmes

2. La musique, l'homme et la société

- La chanson comme pratique musicale
- Les rapports entre musique et littérature (le cas de la poésie mise en chanson)

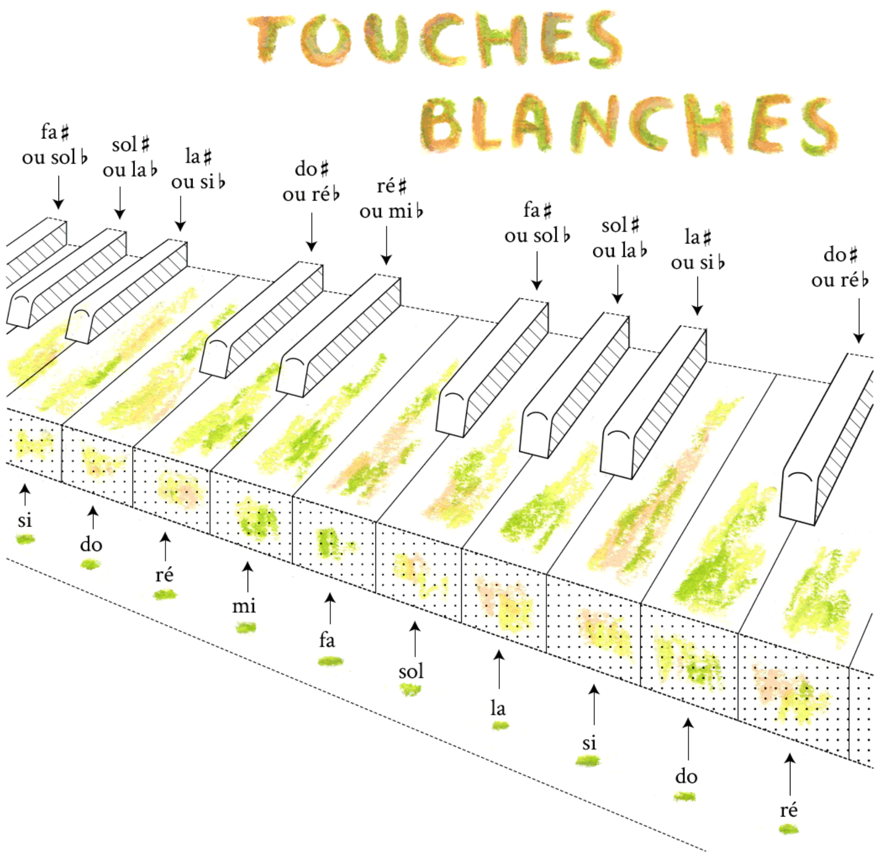
Petit lexique de concepts musicaux



La note

En musique, une note désigne soit un **symbole** permettant de représenter la **hauteur** et la **durée** relative d'un son, soit la hauteur elle-même d'un son.
`(https://www.lire-les-notes.com/)`

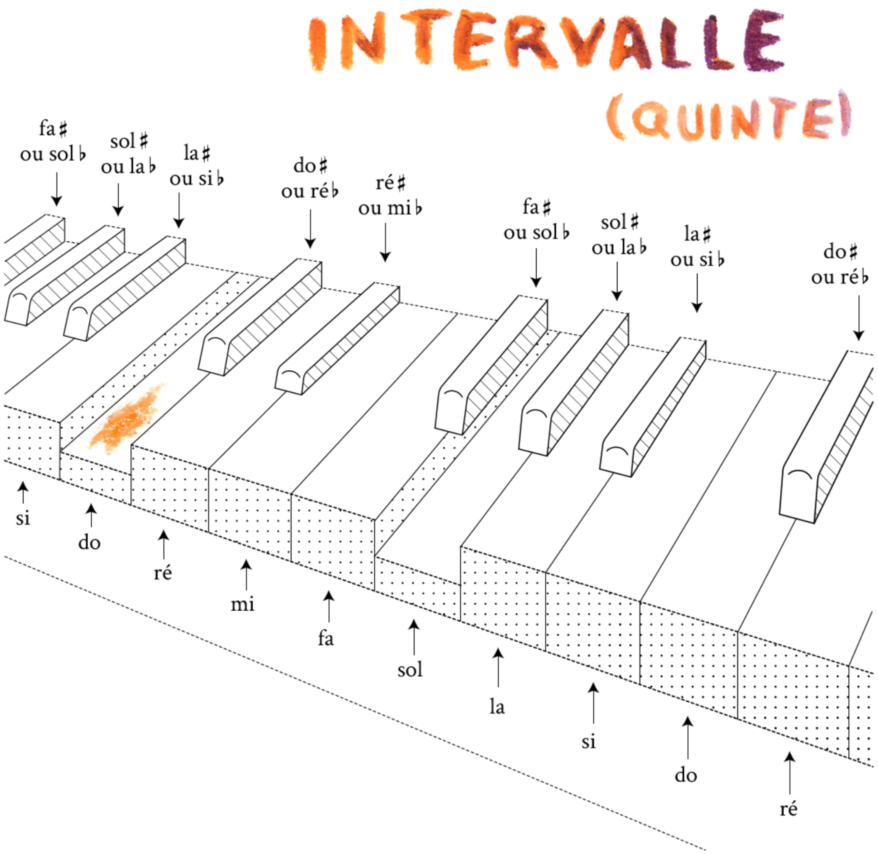
Petit lexique de concepts musicaux



Touches blanches et noires

Le piano est constitué de deux ensembles de notes, à savoir l'ensemble des **touches blanches** et l'ensemble des **touches noires**. Ces deux ensembles sont **disjoints**. Que-ce que cela veut dire ?

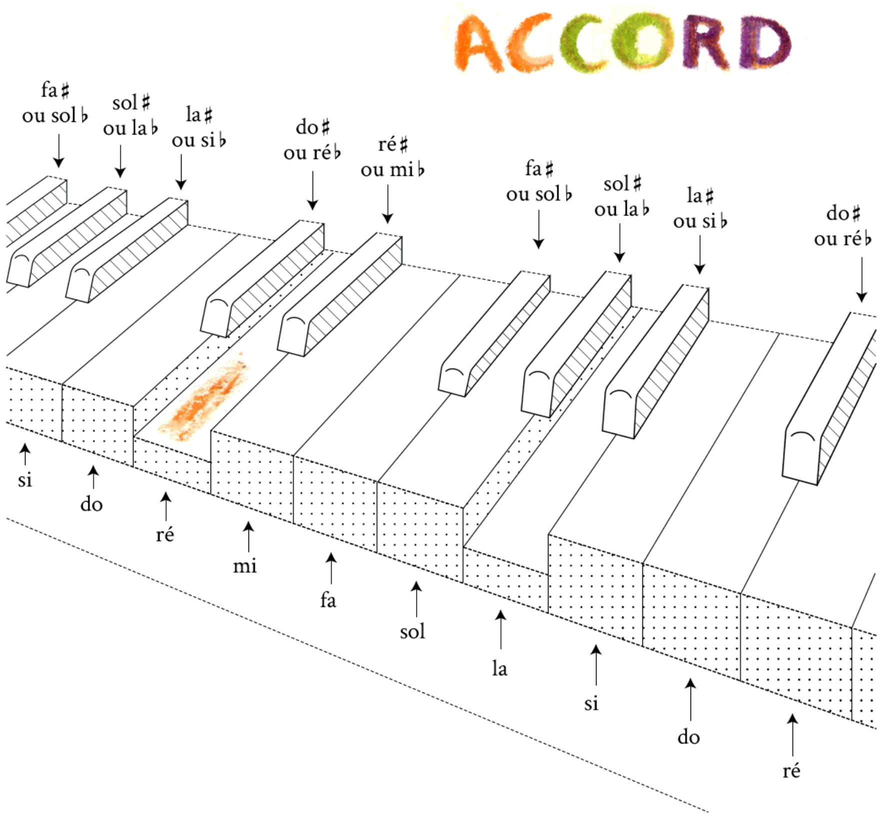
Petit lexique de concepts musicaux



L'intervalle

Un intervalle est une **distance** entre les notes d'une gamme. Par exemple la **quinte** est l'intervalle qui corresponde à une distance de 5 pas sur les touches blanches du piano (ou 7 demi-tons si l'on considère aussi les touches noires).

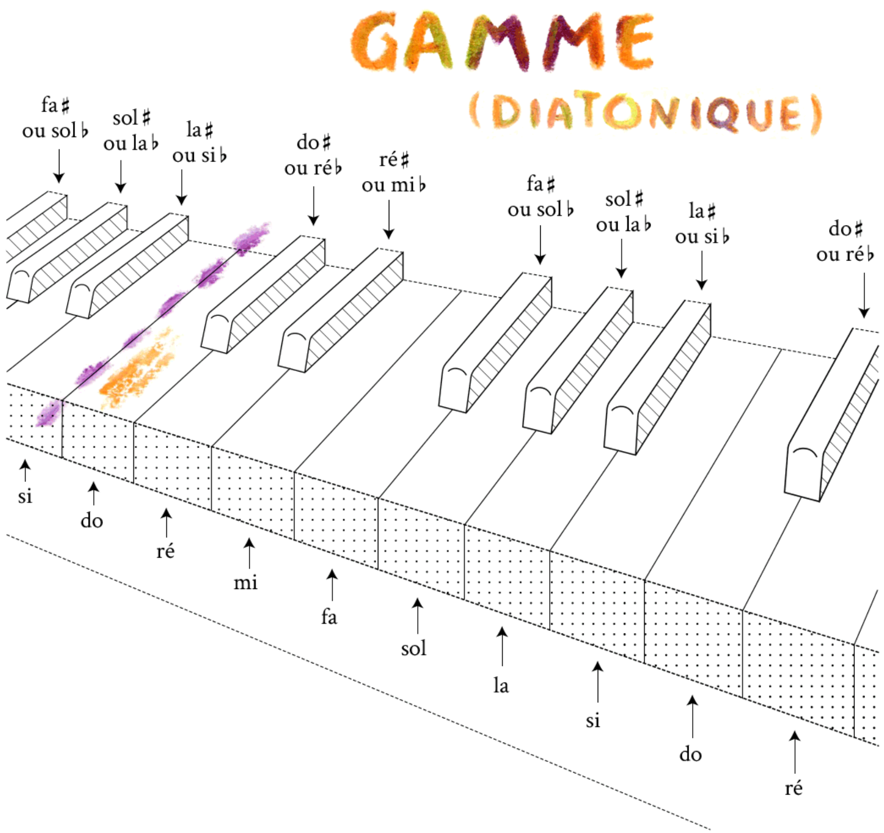
Petit lexique de concepts musicaux



L'accord

Un accord est une **superposition** de plusieurs notes. Par exemple les trois notes en figure constituent l'accord de **ré majeur**.

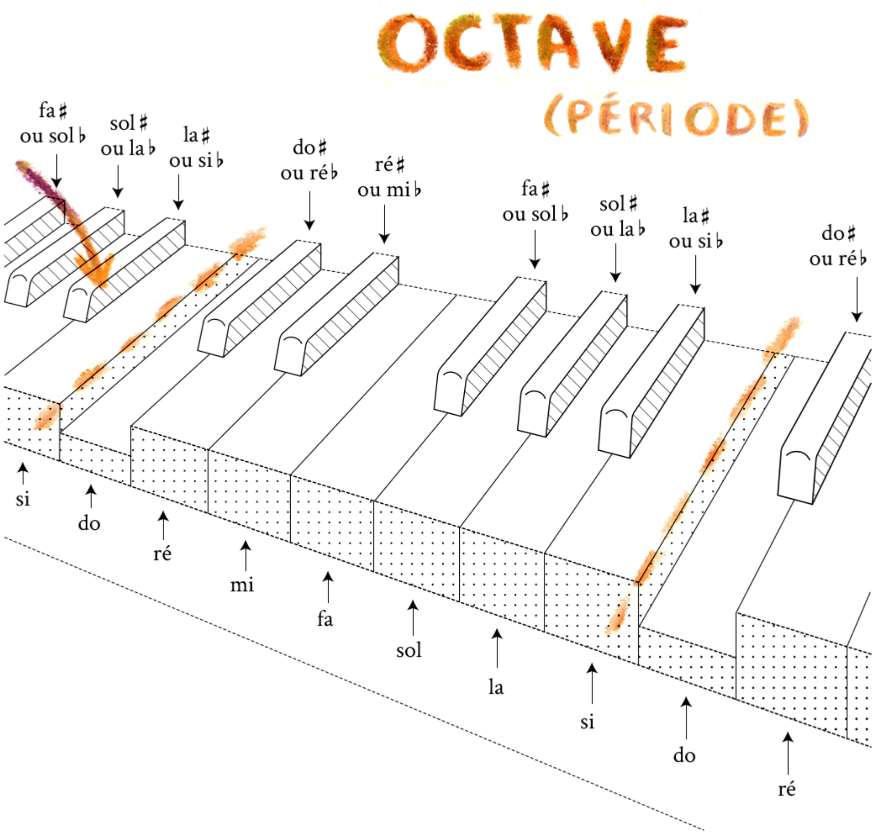
Petit lexique de concepts musicaux



La gamme

Une gamme est une succession ordonnée de notes de musique. Par exemple les touches blanches constituent la **gamme diatonique** et les touches noires forment la **gamme pentatonique.**

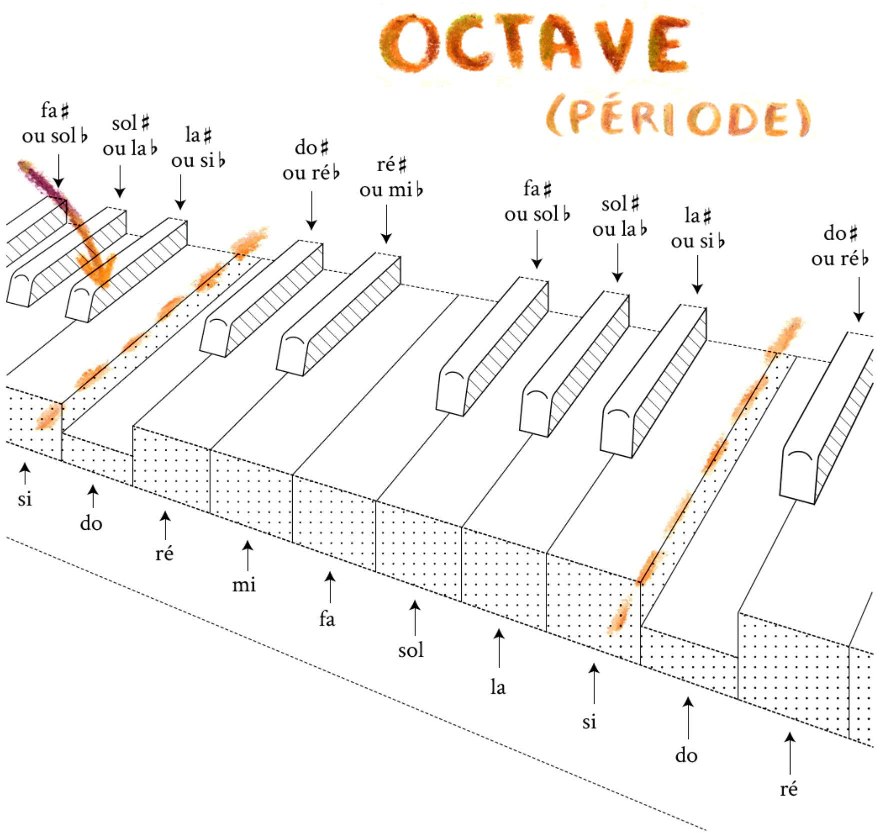
Petit lexique de concepts musicaux



L'octave

L'octave est l'**intervalle** entre deux notes à distance de 12 demi-tons. D'un point de vue physique, l'intervalle d'octave correspond à **doubler la fréquence** d'une note (c'est-à-dire le nombre de vibrations par second de l'onde sonore associée à la note de musique).

Petit lexique de concepts musicaux



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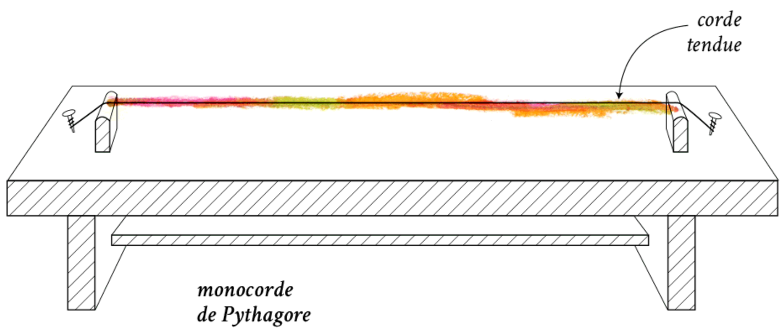
Proportions et gammes musicales

Pythagore et le monocorde

Les intervalles selon le monocorde



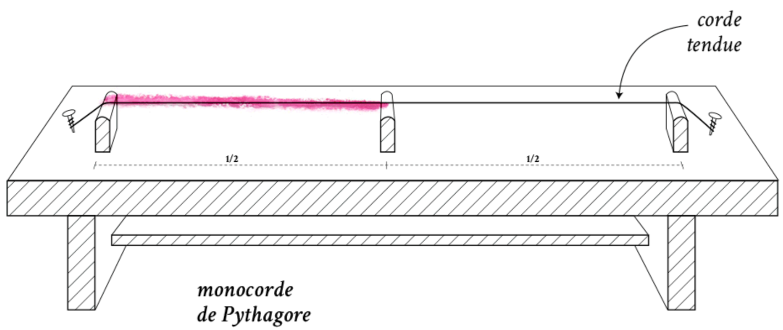
VOICI UNE NOTE

Le monocorde est un instrument contenant une unique corde tendue et une caisse de résonance.

Plus cette corde est courte, plus le son obtenu en la pinçant est aigu, car la **fréquence du son est inversement proportionnelle à la longueur de la corde**.

Ainsi, si le rapport de longueur de la corde est égal à 1 (comme ici), son rapport de fréquence sera égal à 1/1 donc 1 !

Les intervalles selon le monocorde



Longueur de la corde	Rapport de fréquence	Intervalle correspondant
1/2	2	OCTAVE

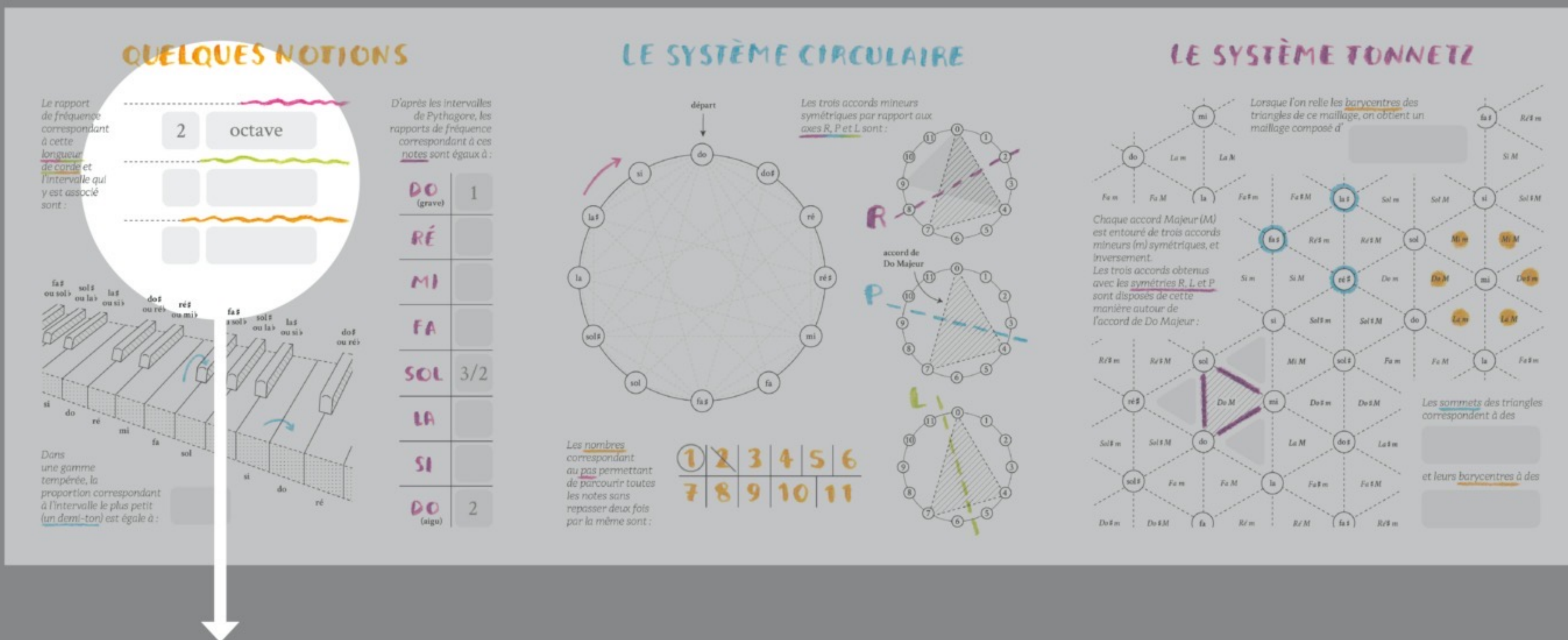
Le monocorde est un instrument contenant une unique corde tendue et une caisse de résonance.

Plus cette corde est courte, plus le son obtenu en la pinçant est aigu, car la **fréquence du son est inversement proportionnelle à la longueur de la corde.**

Ainsi, si le rapport de longueur de la corde est égal à 1 (comme ici), son rapport de fréquence sera égal à 1/1 donc 1 !

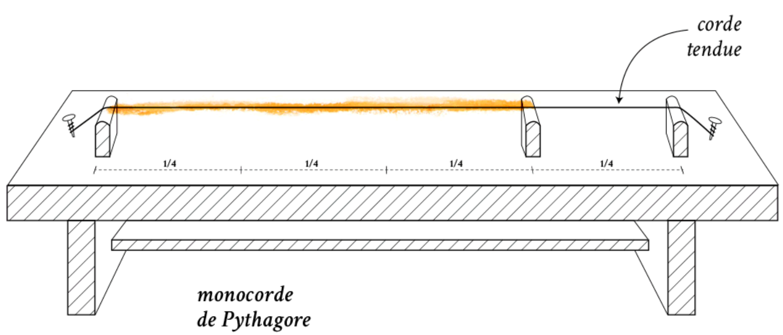
Si le rapport de longueur de la corde est égal à 1/2, son rapport de fréquence sera égal à 1/(1/2) donc 2, ce qu'on appelle l'intervalle d'**octave**

Les intervalles selon le monocorde



Après avoir mesuré la longueur des segments de corde colorés par rapport à la longueur totale de la corde, en déduire le rapport de fréquence qui y correspond

Les intervalles selon le monocorde



Longueur de la corde	Rapport de fréquence	Intervalle correspondant
3/4	4/3	QUARTE

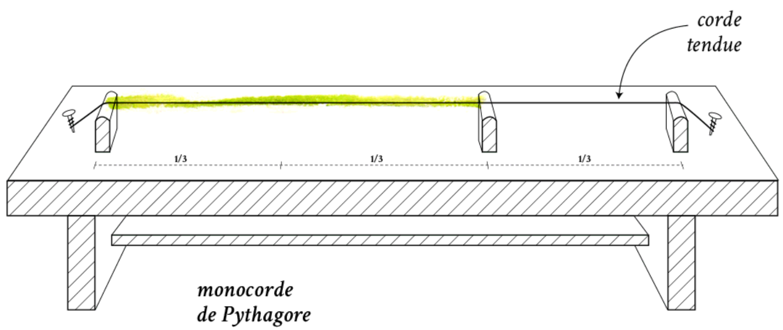
Le monocorde est un instrument contenant une unique corde tendue et une caisse de résonance.

Plus cette corde est courte, plus le son obtenu en la pinçant est aigu, car la **fréquence du son est inversement proportionnelle à la longueur de la corde.**

Ainsi, si le rapport de longueur de la corde est égal à 1 (comme ici), son rapport de fréquence sera égal à 1/1 donc 1 !

Si le rapport de longueur de la corde est égal à 3/4, son rapport de fréquence sera égal à 1/(3/4) donc 4/3, ce qu'on appelle l'intervalle de **quarte.**

Les intervalles selon le monocorde



Longueur de la corde	Rapport de fréquence	Intervalle correspondant
2/3	3/2	QUINTE

Le monocorde est un instrument contenant une unique corde tendue et une caisse de résonance.

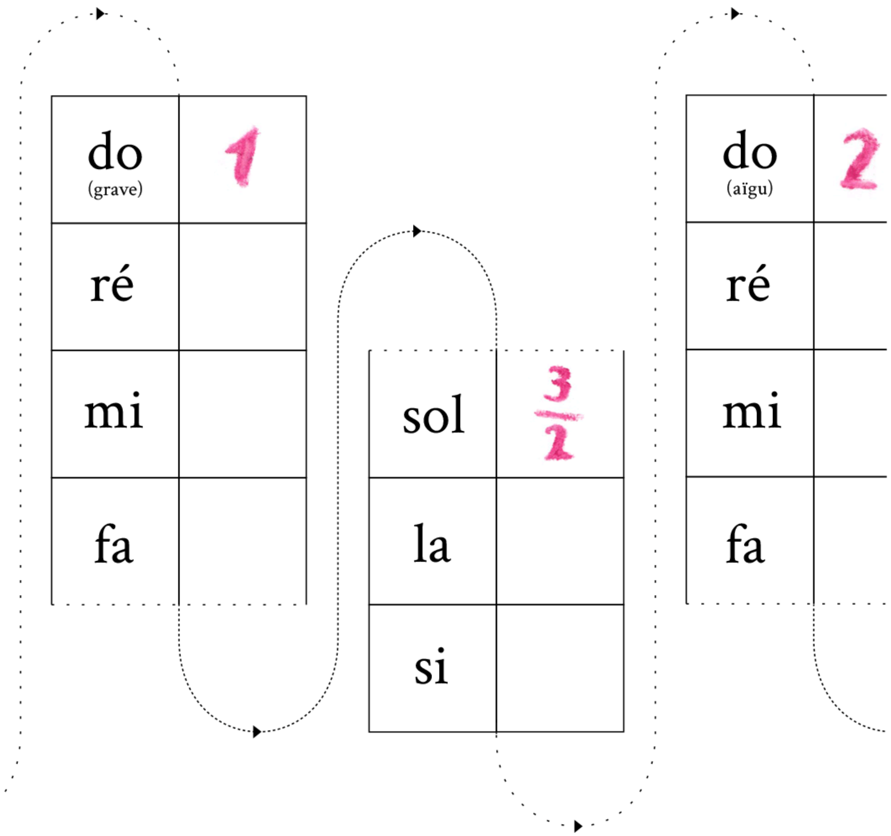
Plus cette corde est courte, plus le son obtenu en la pinçant est aigu, car la **fréquence du son est inversement proportionnelle à la longueur de la corde.**

Ainsi, si le rapport de longueur de la corde est égal à 1 (comme ici), son rapport de fréquence sera égal à 1/1 donc 1 !

Si le rapport de longueur de la corde est égal à 2/3, son rapport de fréquence sera égal à 1/(2/3) donc 3/2, ce qu'on appelle l'intervalle de **quinte.**

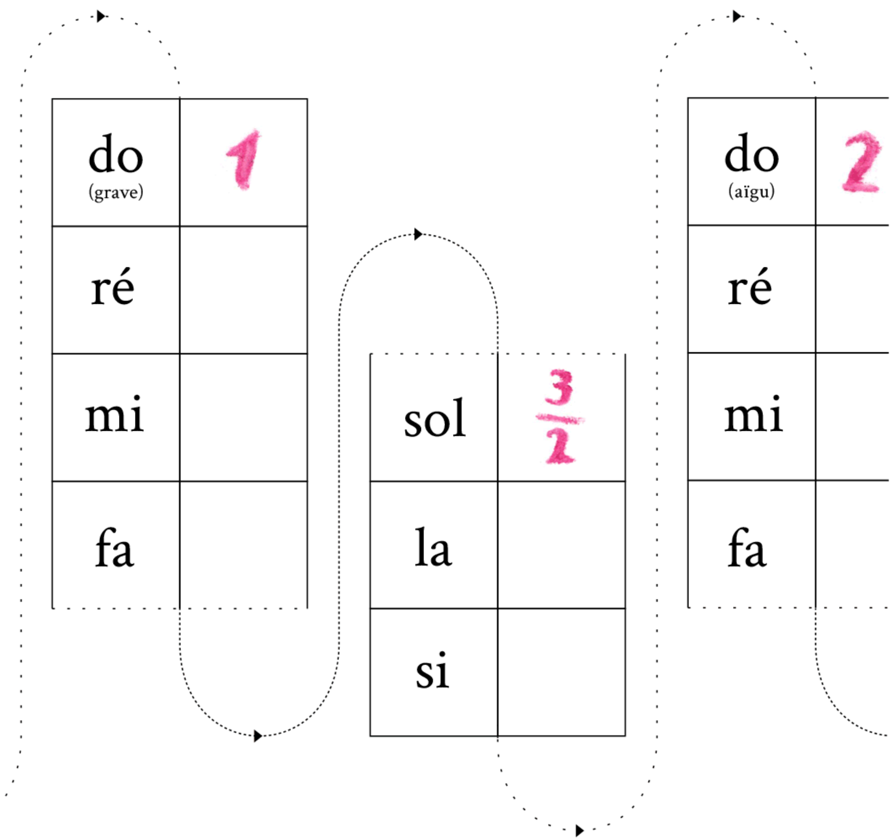
Comment construire une gamme
à partir de la quinte (juste) ?

De quinte en quinte…



Calculer des puissances et des quotients en lien avec le **cycle des quintes**.
Mettre en place un raisonnement mathématique pour **prouver que le cycle des quintes est infini.**

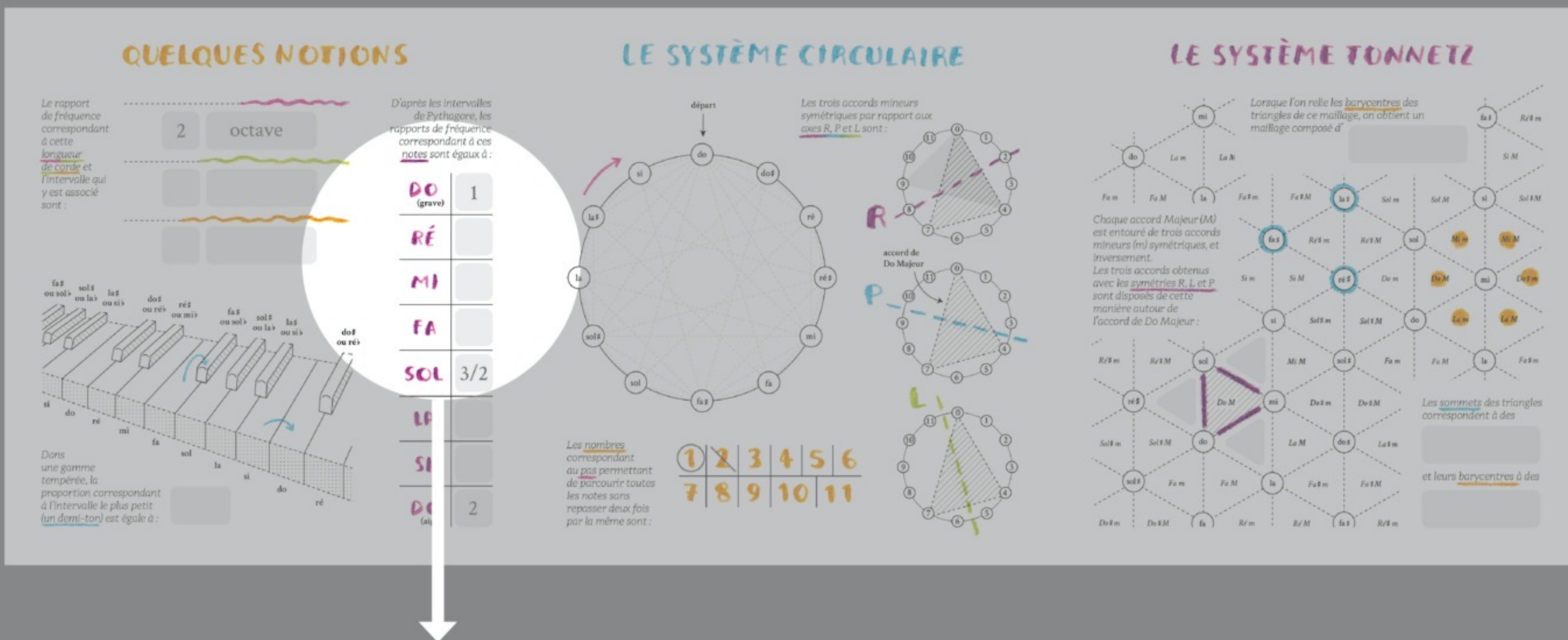
De quinte en quinte…



Calculer des puissances et des quotients en lien avec le **cycle des quintes**.

Mettre en place un raisonnement mathématique pour **prouver que le cycle des quintes est infini.**

De quinte en quinte…



D'après l'intervalle de quinte de Pythagore, calculer les rapports de fréquence des différentes notes de la gamme

Le cycle des quintes et le tempérament pythagoricien

do (grave)	1
ré	$\frac{9}{8}$
mi	
fa	

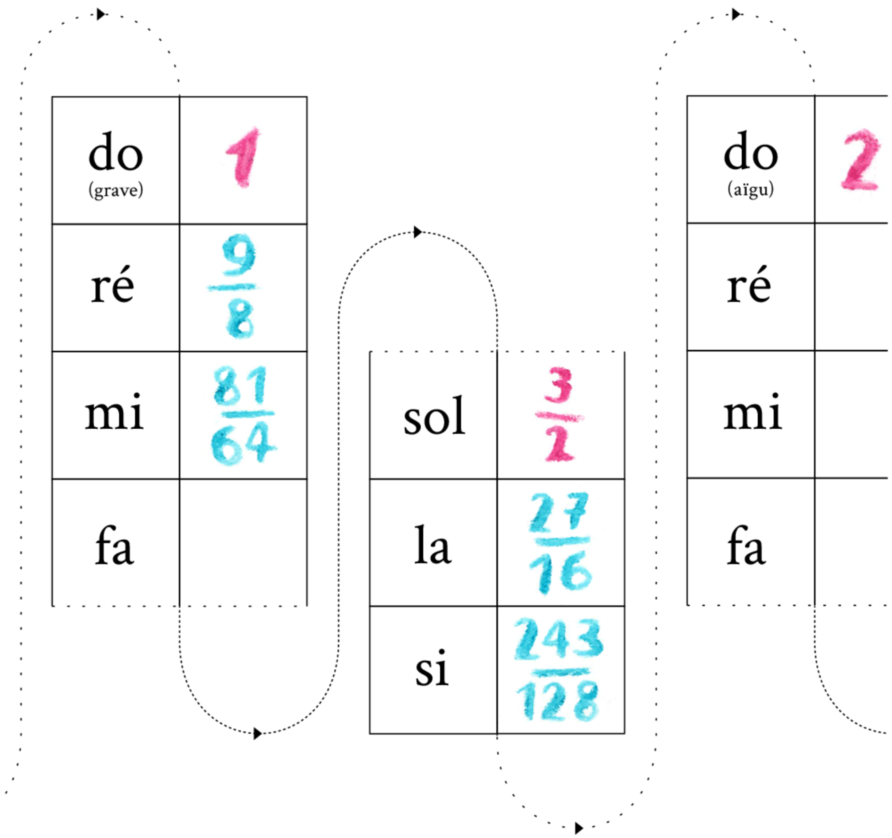
sol	$\frac{3}{2}$
la	
si	

do (aïgu)	2
ré	
mi	
fa	

Calculer des puissances et des quotients en lien avec le **cycle des quintes**.

Mettre en place un raisonnement mathématique pour **prouver que le cycle des quintes est infini.**

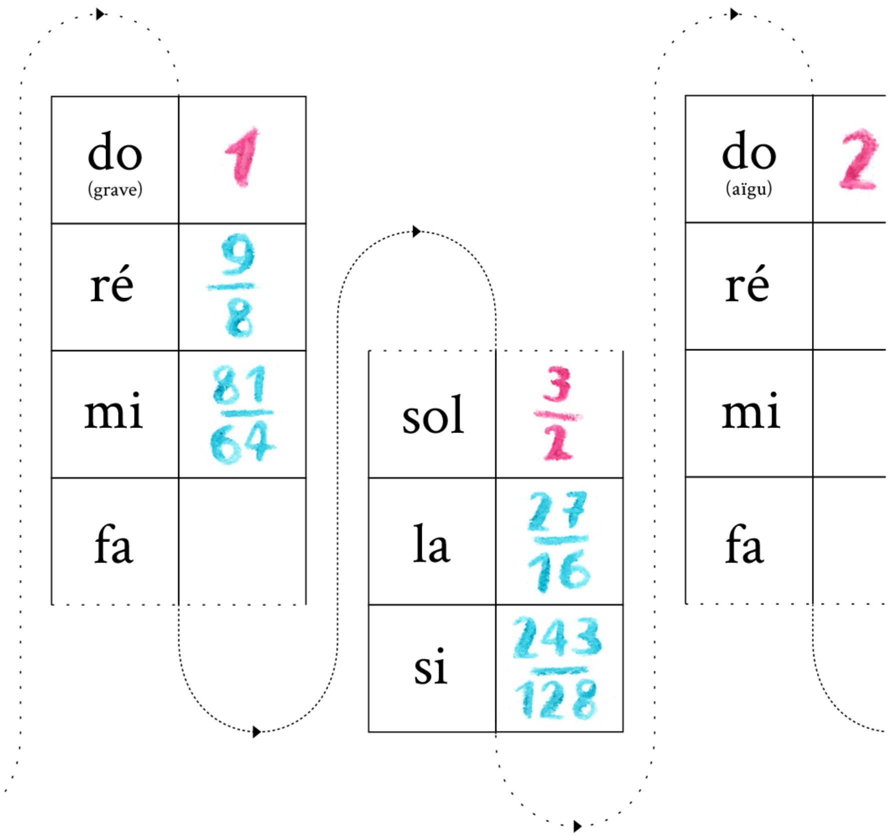
Le cycle des quintes et le tempérament pythagoricien



Calculer des puissances et des quotients en lien avec le **cycle des quintes**.

Mettre en place un raisonnement mathématique pour **prouver que le cycle des quintes est infini.**

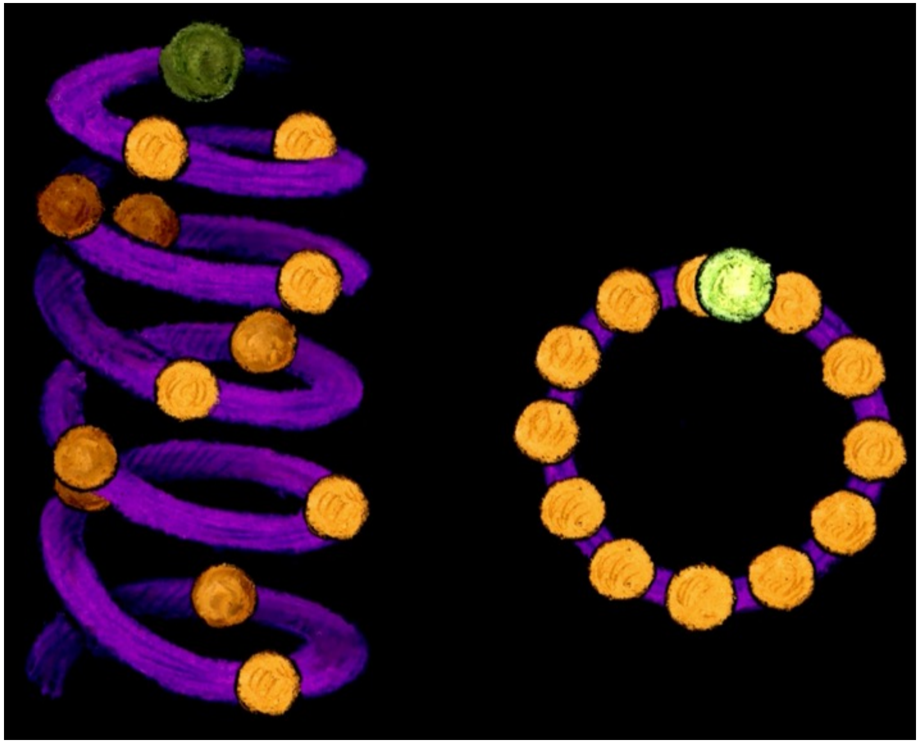
Le cycle des quintes et le tempérament pythagoricien



Calculer des puissances et des quotients en lien avec le **cycle des quintes**.

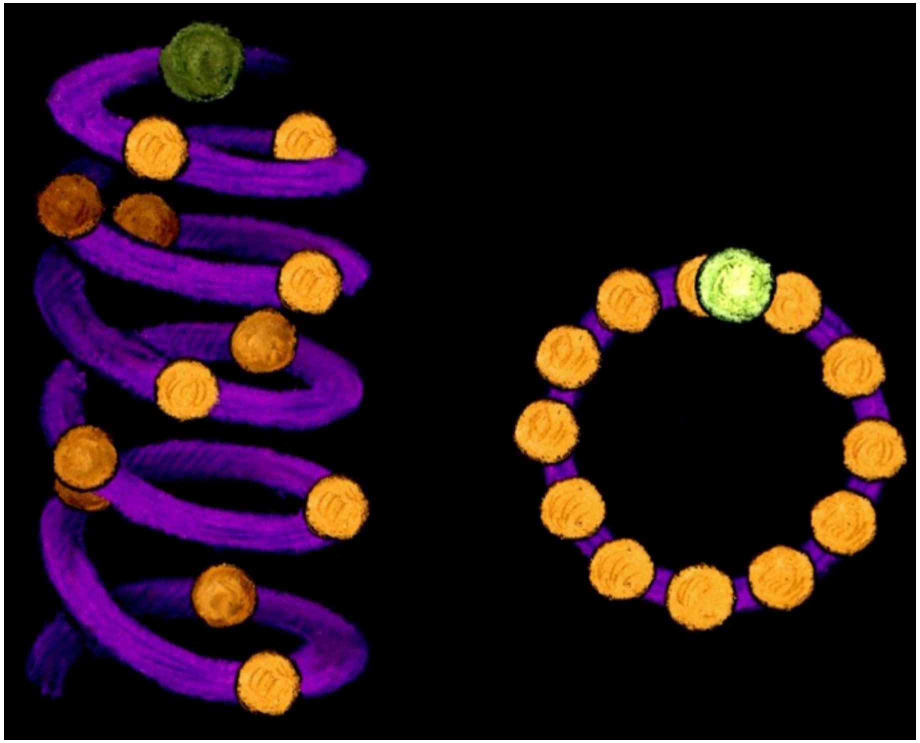
Mettre en place un <u>raisonnement mathématique</u> pour **prouver que le cycle des quintes est infini.**

De quinte en quinte… à l'infini !



Calculer des puissances et des quotients en lien avec le **cycle des quintes**. Mettre en place un raisonnement mathématique pour **prouver que le cycle des quintes est infini.**

De quinte en quinte… à l'infini !

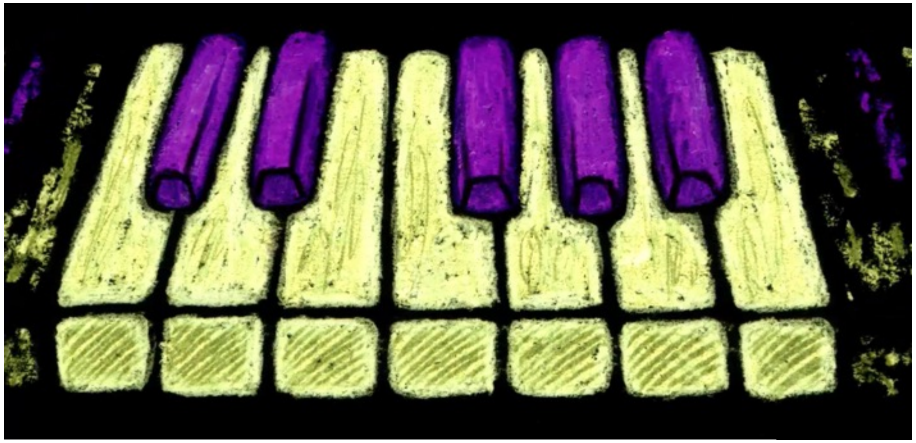


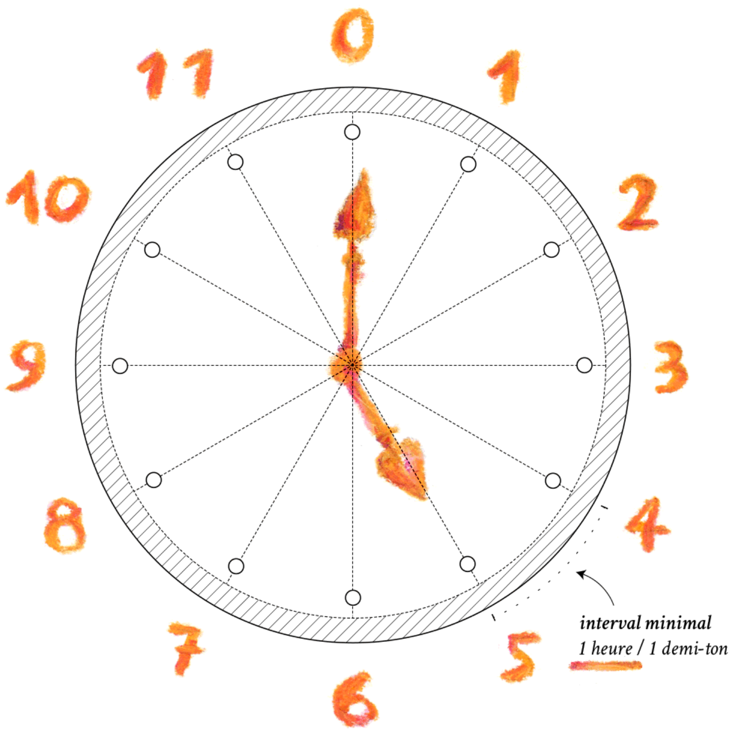
Calculer des puissances et des quotients en lien avec le **cycle des quintes**. Mettre en place un <u>raisonnement mathématique</u> pour **prouver que le cycle des quintes est infini.**

La **preuve par l'absurde** : supposons qu'un multiple de la quinte est égal à un multiple de l'octave et montrons que cela conduit à une contradiction.

Du piano au cercle

Le cadran d’horloge musical et les accords



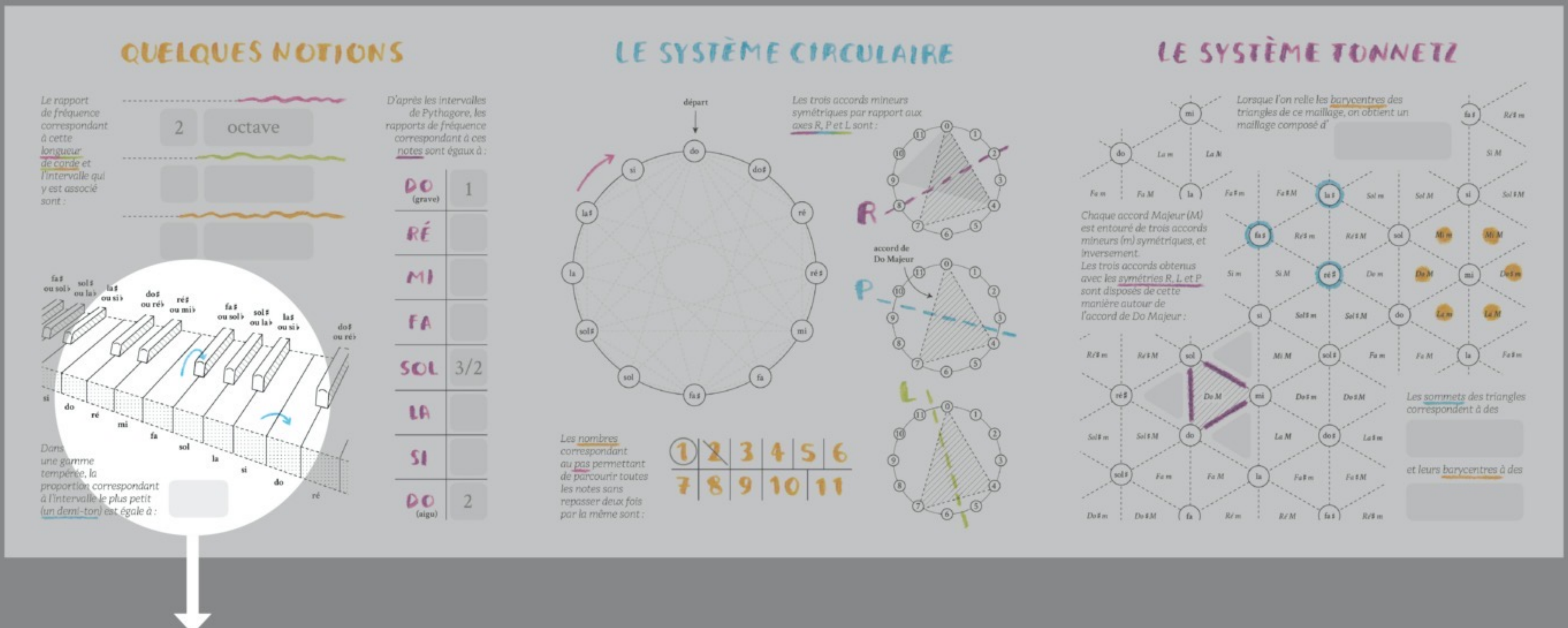


Un accord est un polygone inscrit dans le cercle

L’accord de **ré majeur** correspond par exemple au triangle dessinée dans cette animation.

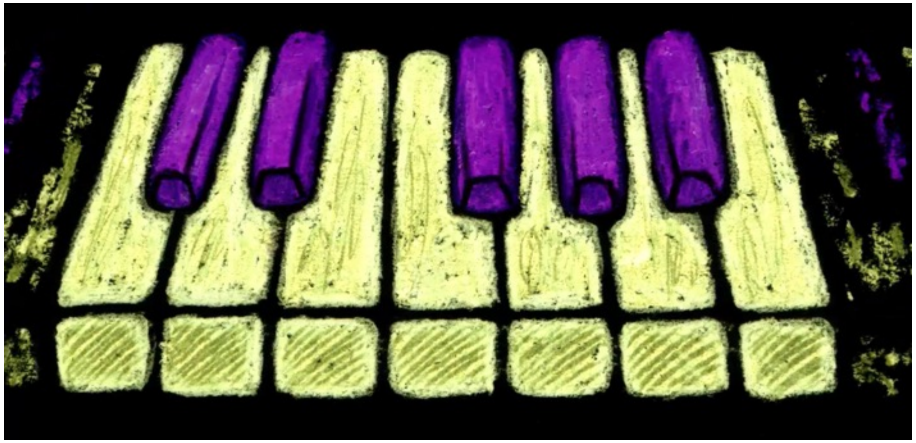
A quoi correspond le demi-ton tempéré ?

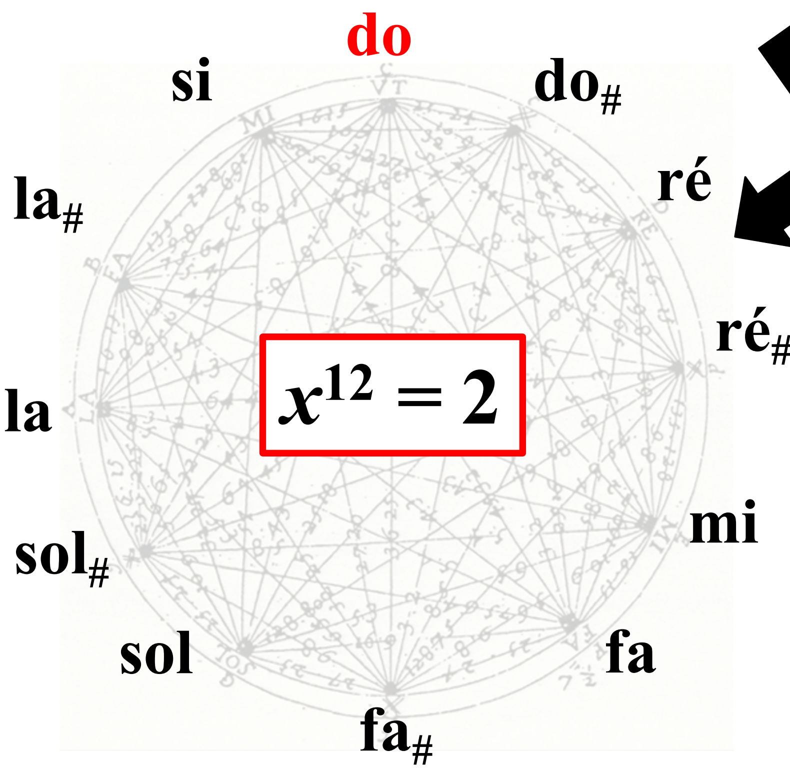
Nombres irrationnels et tempérament égal



Calculer le nombre irrationnel qui correspond au demi-ton tempéré du piano

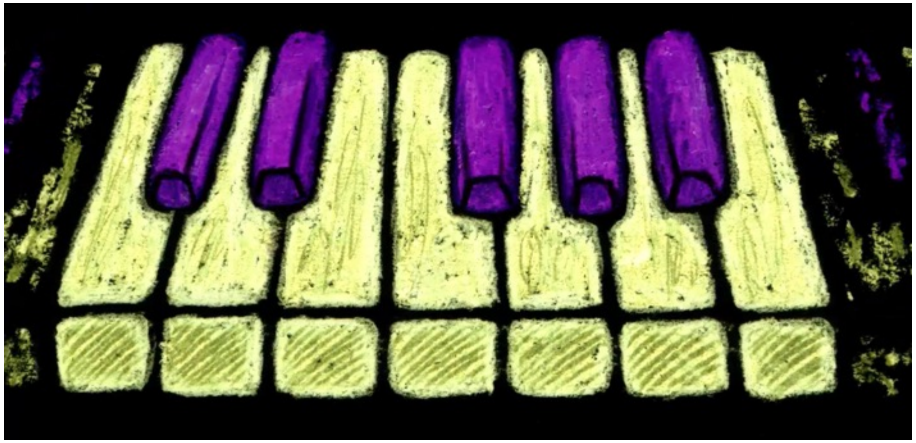
Le tempérament égal : un compromis « combinatoire »

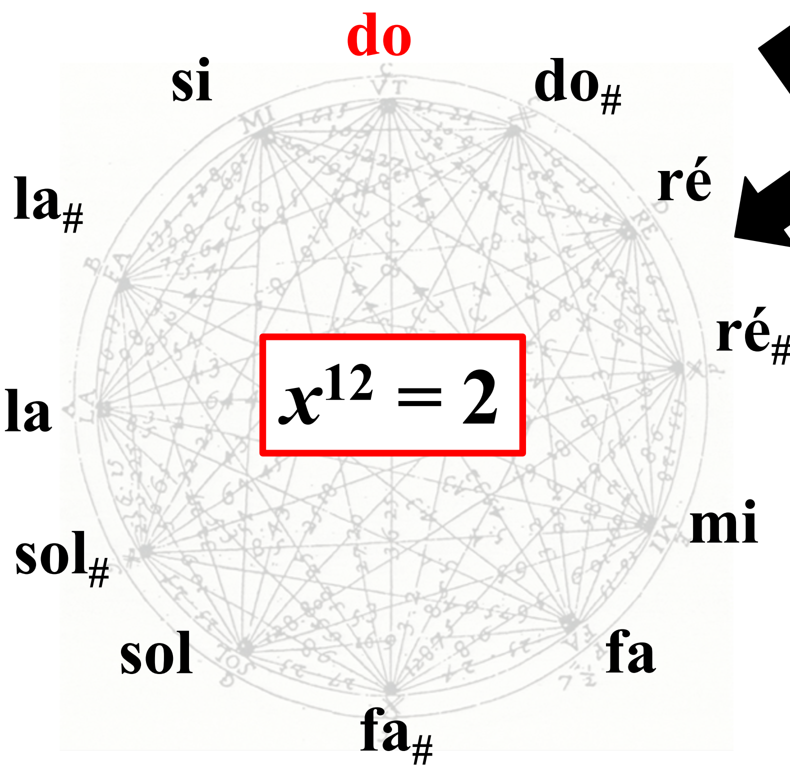




La connaissance des nombres irrationnels a permis, au XVIIe siècle, de construire des **gammes à intervalles égaux**.
L'introduction des gammes « au **tempérament égal** » permet de comprendre en quoi la découverte des nombres irrationnels a des applications en dehors du champ mathématique.

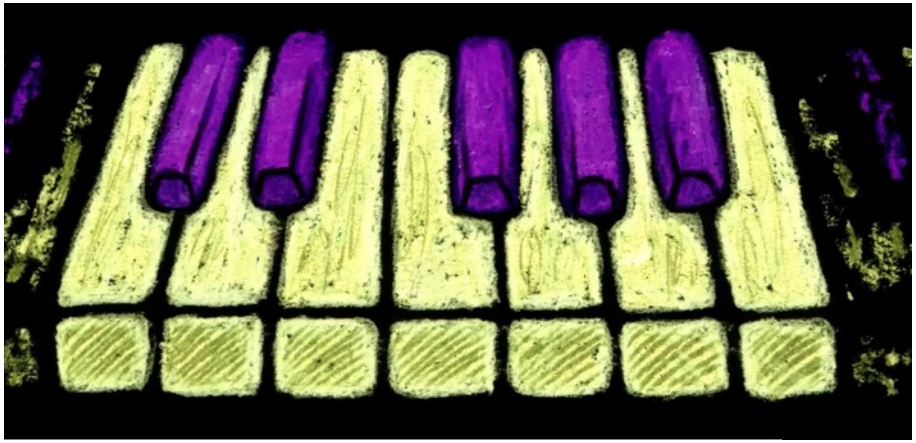
Le tempérament égal : un compromis « combinatoire »

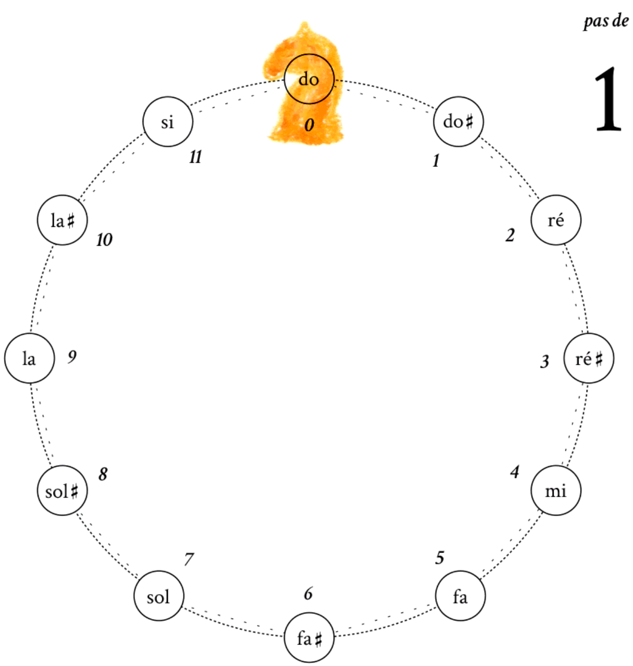




La connaissance des nombres irrationnels a permis, au XVII^e siècle, de construire des **gammes à intervalles égaux**. L’introduction des gammes « au **tempérament égal** » permet de comprendre en quoi la découverte des nombres irrationnels a des applications en dehors du champ mathématique. **Utiliser la racine douzième de 2 pour partager l’octave en douze intervalles égaux.**

Intervalles générateurs du tempérament égal



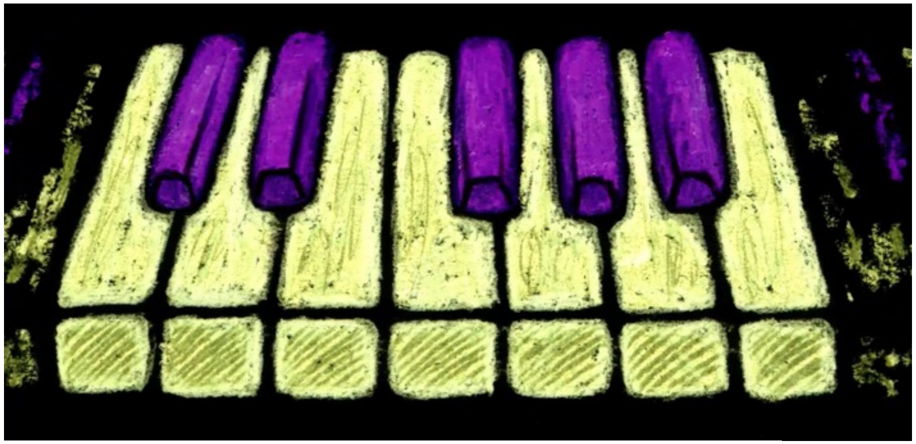


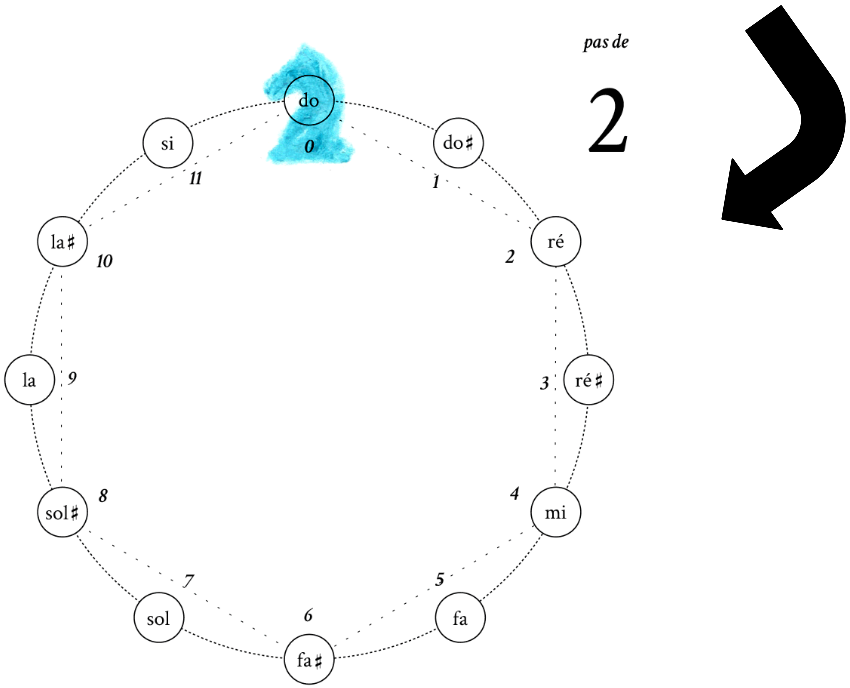
Avec quel pas peut-on engendrer toutes les notes de la gamme tempérée ?

Sans doute ça marche avec le demi-ton donc à pas de 1 car cet intervalle $\boldsymbol{x}$ correspond précisément à la racine douzième de 2 et :

$$\boxed{x^{12} = 2}$$

Intervalles générateurs du tempérament égal

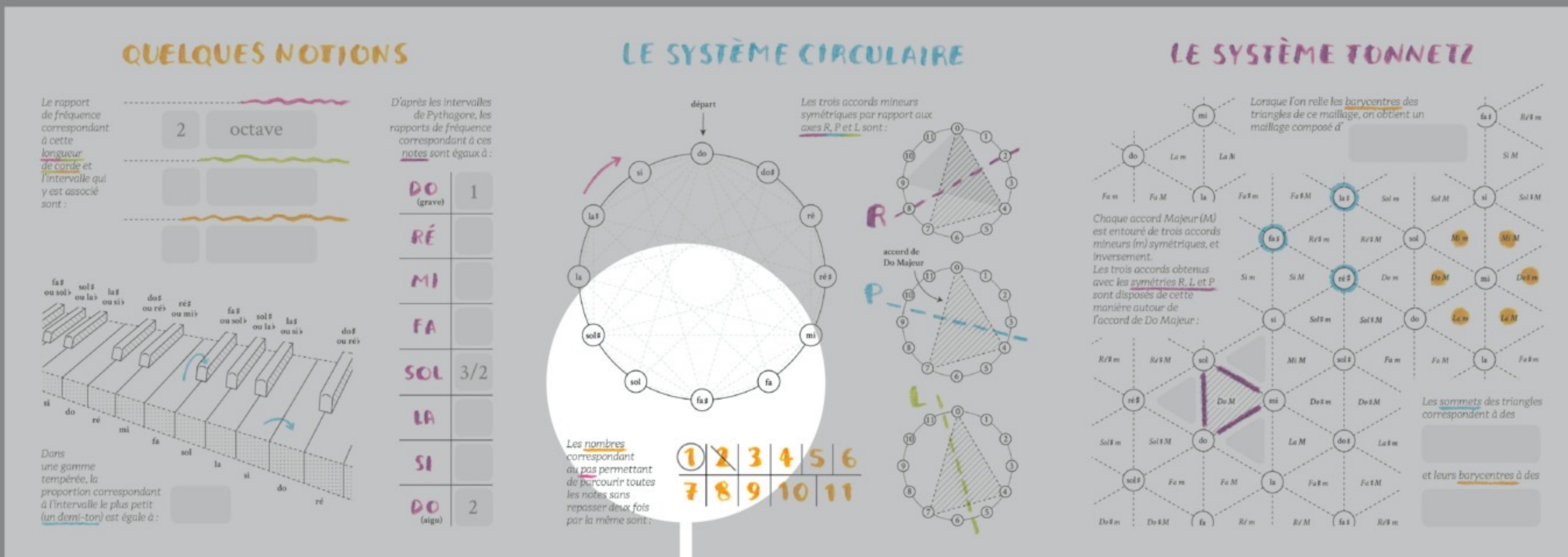


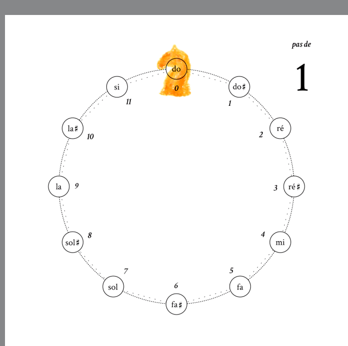


Avec quel pas peut-on engendrer toutes les notes de la gamme tempérée ?

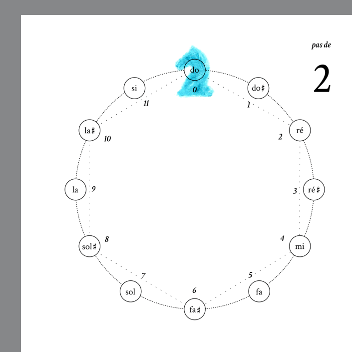
Avec l'intervalle de **ton** ça ne marche pas car au bout de 6 répétitions on retombe sur l'octave et donc on boucle sans pouvoir engendrer les autres notes de la gamme.

Intervalles générateurs du tempérament égal

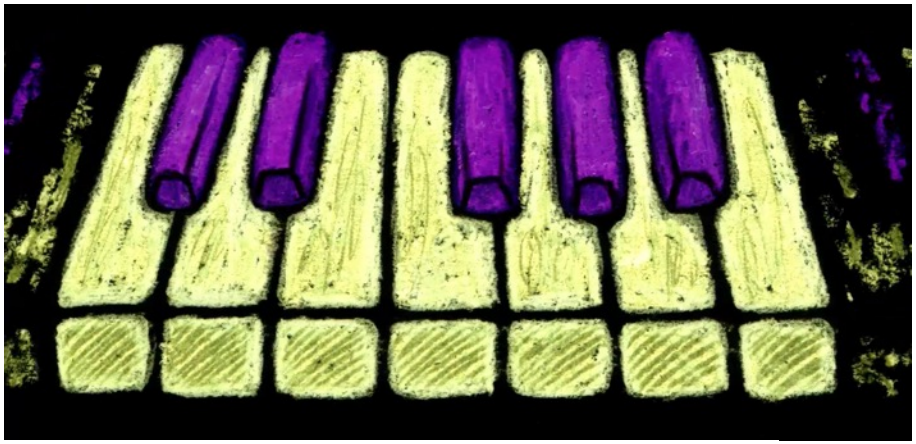


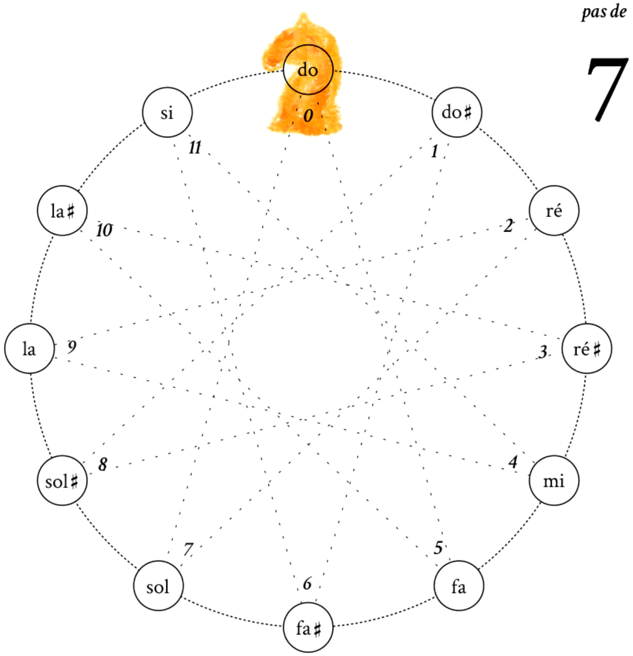


En utilisant le cercle dessiné sur le dépliant, trouvez quels nombres permettent au cavalier de parcourir toutes les notes et revenir au point de départ



Intervalles générateurs du tempérament égal





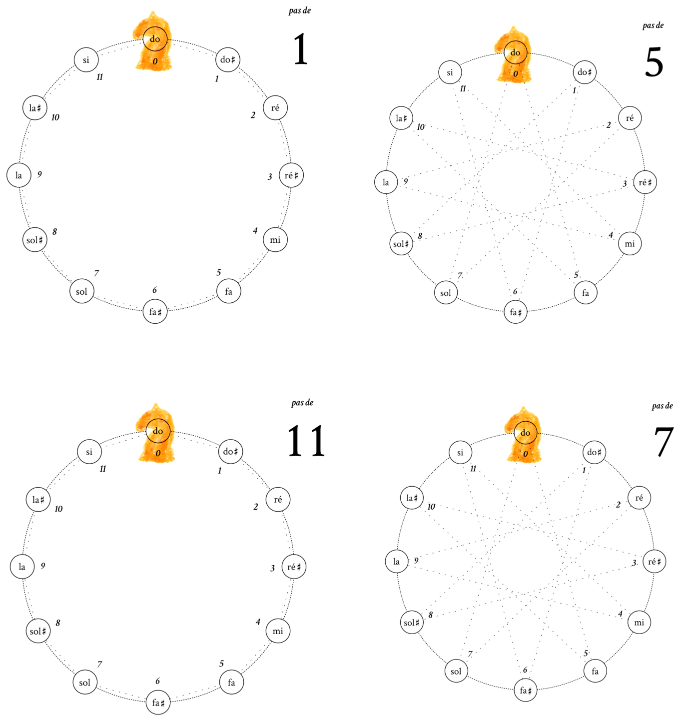
Avec quel pas peut-on engendrer toutes les notes de la gamme tempérée ?

Ca marche avec l'intervalle de **quinte** car la spirale des quinte se renferme dans un cercle pour le tempérament égal.

Et les autres intervalles ?

Intervalles générateurs du tempérament égal

☺



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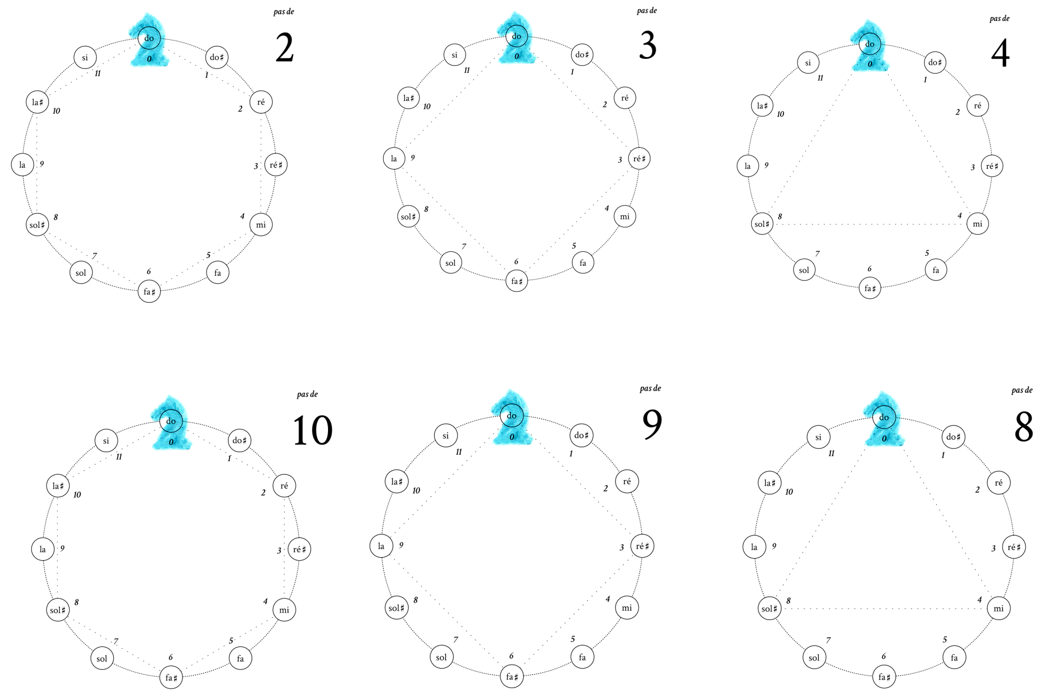
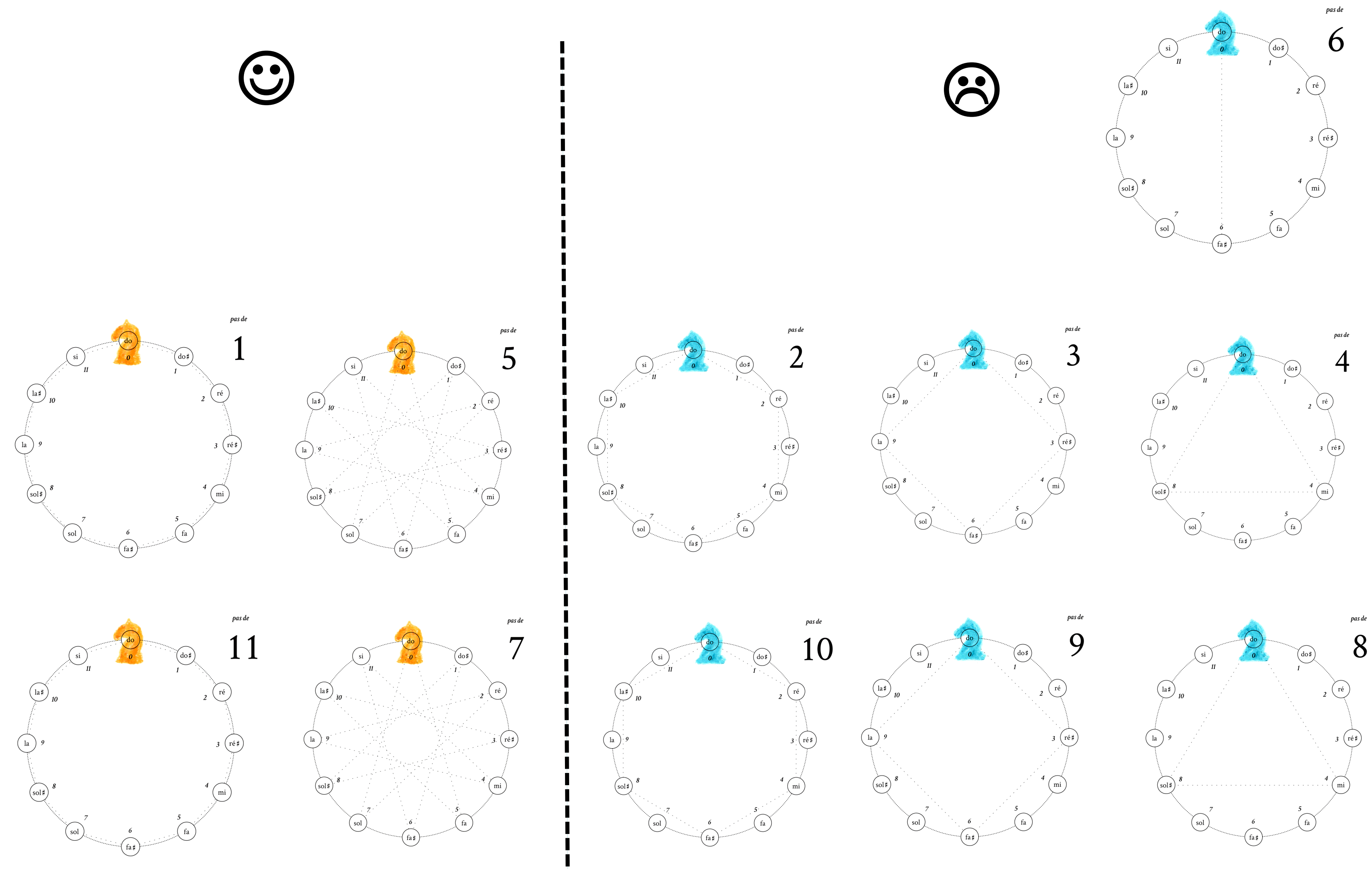
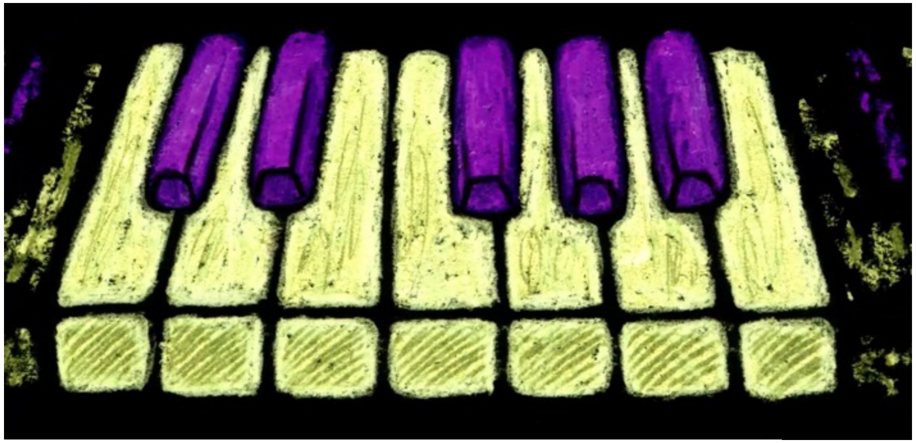


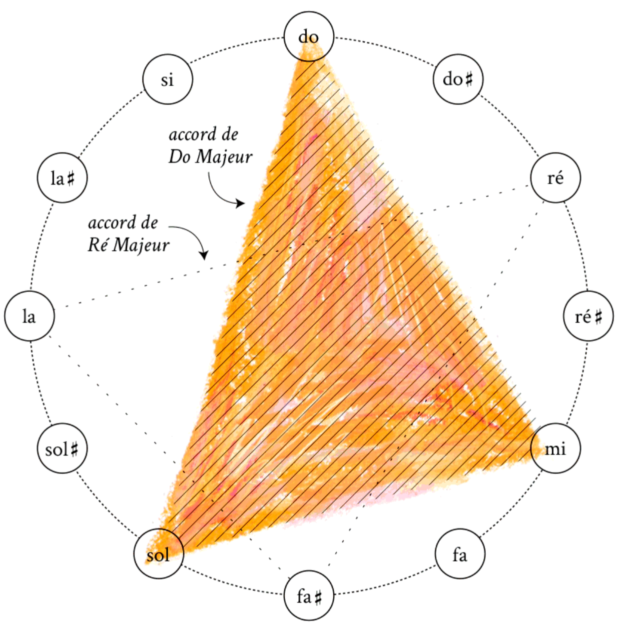
Intervalles générateurs du tempérament égal



Comment transformer les accords dans le cercle ?

Les transpositions musicales sont des rotations

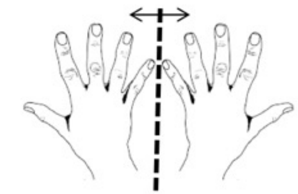


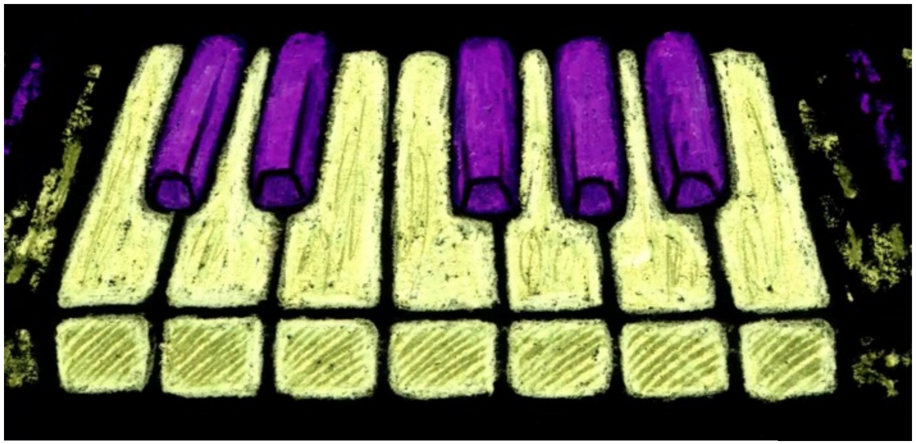


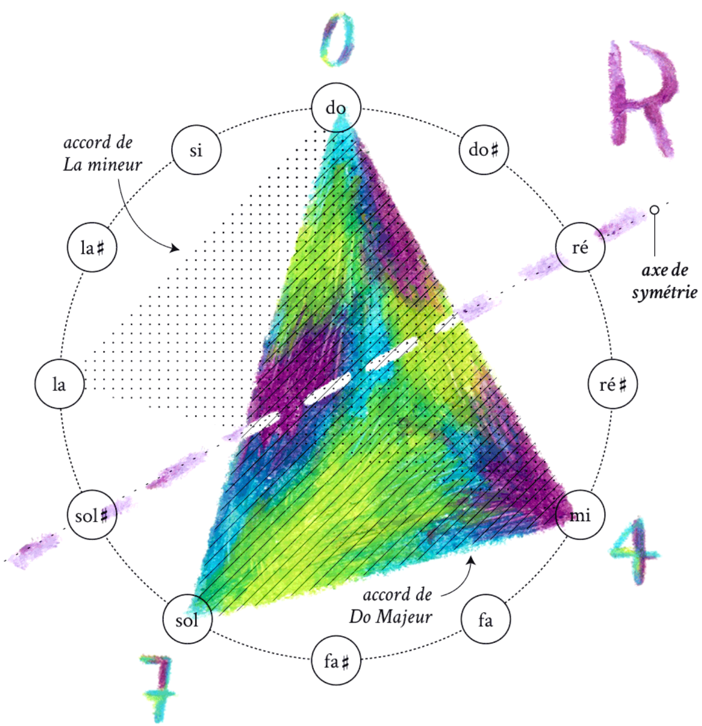
Un accord est un polygone inscrit dans le cercle

L'accord de **ré majeur** correspond par exemple au triangle dessinée dans cette animation et obtenu à partir de l'accord de **do majeur** en le transposant d'un ton.

Un accord majeur et ses symétries





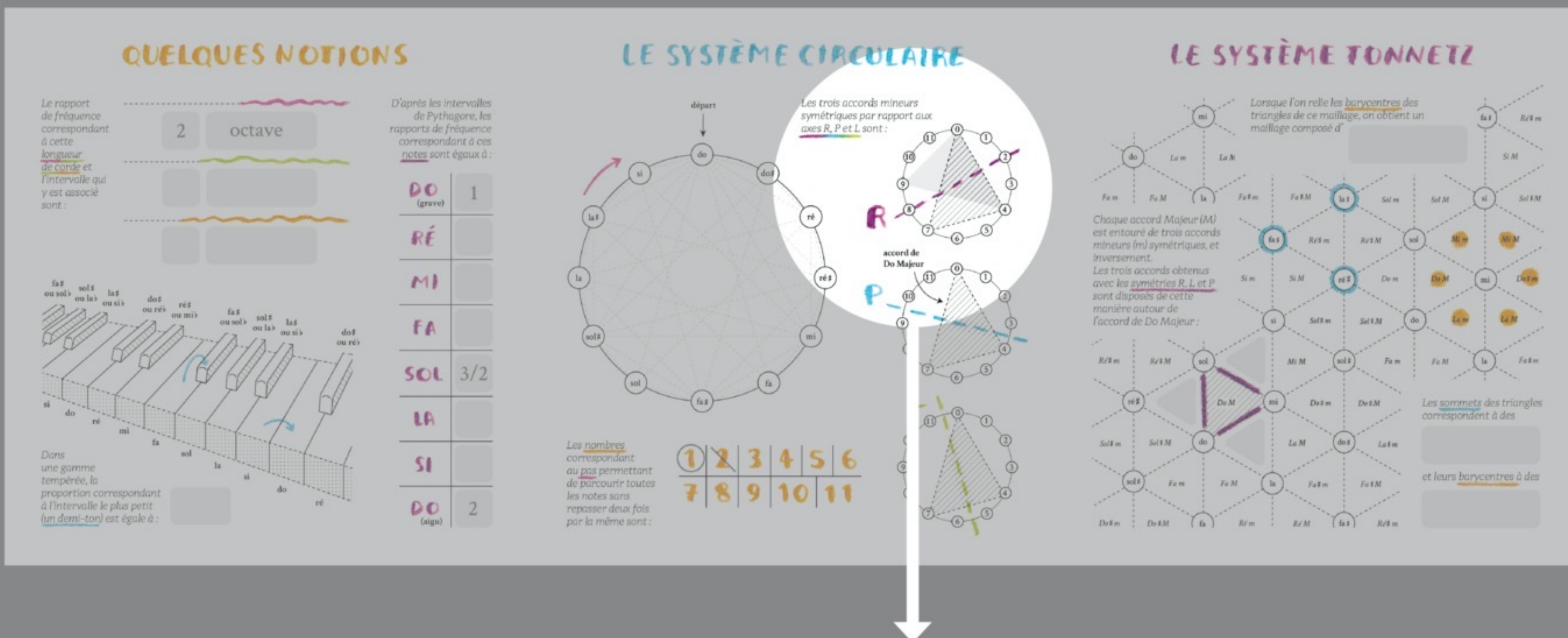


Un accord est un polygone inscrit dans le cercle

L'accord de **ré majeur** correspond par exemple au triangle dessinée dans cette animation et obtenu à partir de l'accord de **do majeur** en le transposant d'un ton.

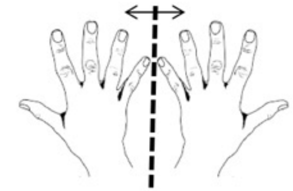
L'**accord mineur** est le symétrique de l'accord majeur ! Celui-ci s'appelle par exemple le **relatif** (d'où la lettre **R**)

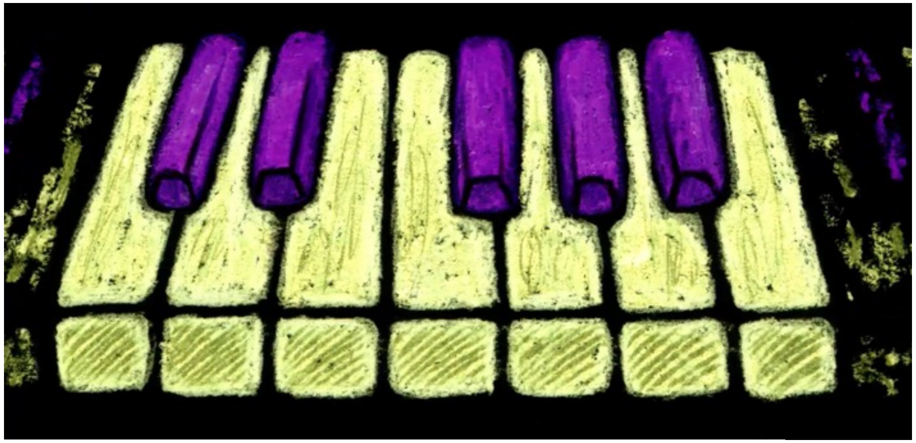
Un accord majeur et ses symétries

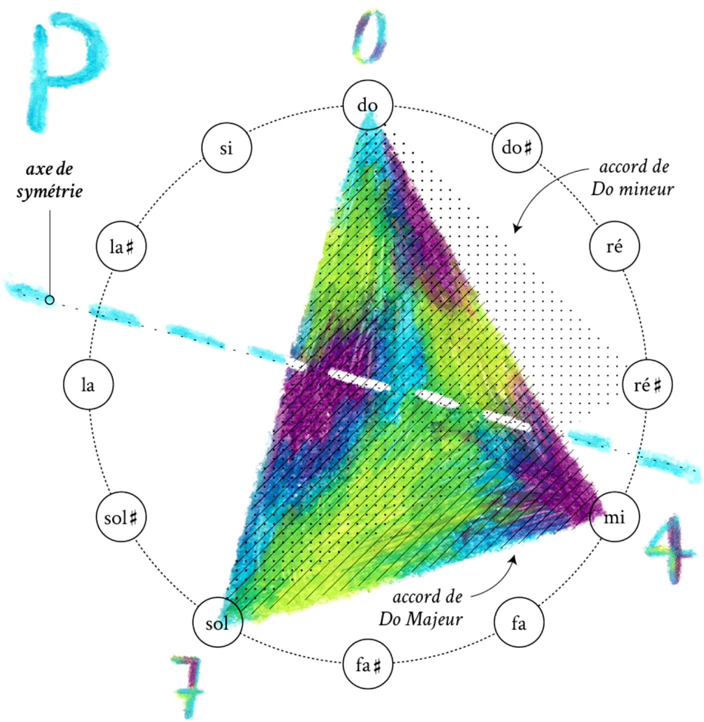


Dessinez les accords mineurs symétriques de l'accord de do majeur par rapport aux axes P et L indiqués en figure ?

Un accord majeur et ses symétries





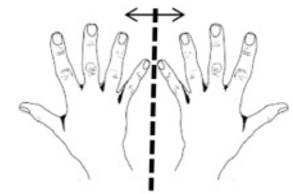


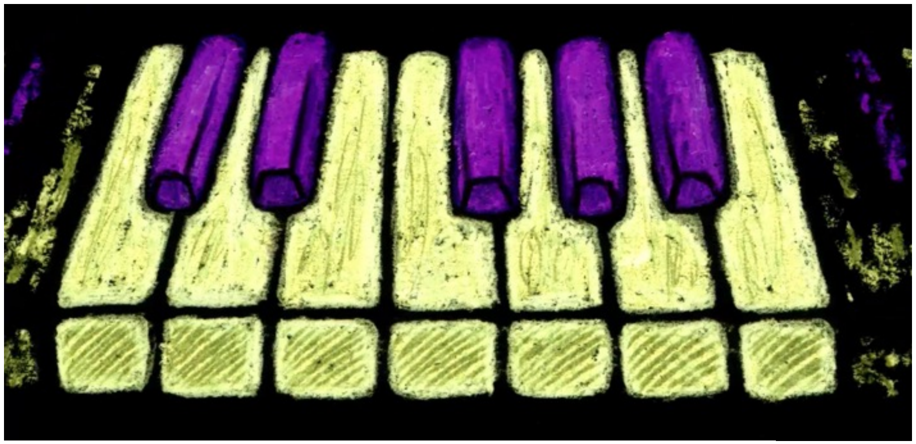
Un accord est un polygone inscrit dans le cercle

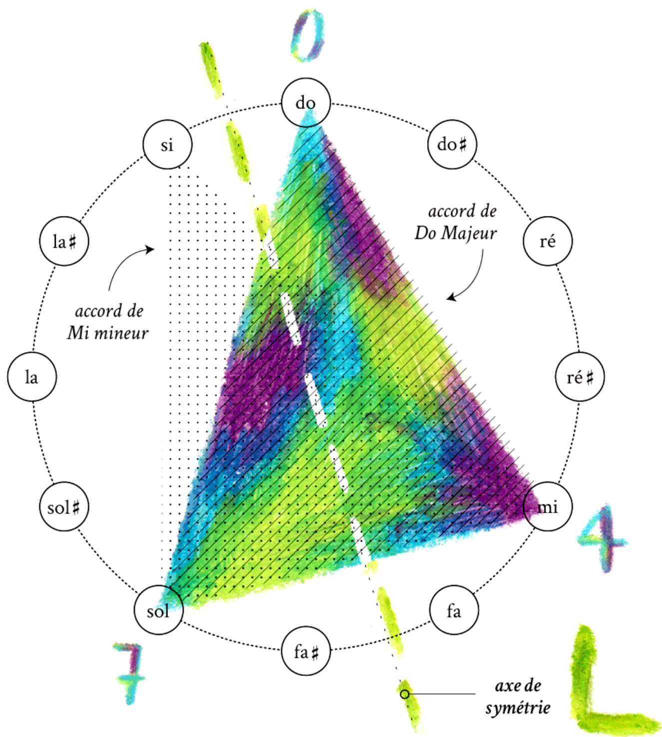
L'accord de **ré majeur** correspond par exemple au triangle dessinée dans cette animation et obtenu à partir de l'accord de **do majeur** en le transposant d'un ton.

L'**accord mineur** est le symétrique de l'accord majeur ! Celui-ci s'appelle en revanche le **parallèle** (d'où la lettre **P**)

Un accord majeur et ses symétries







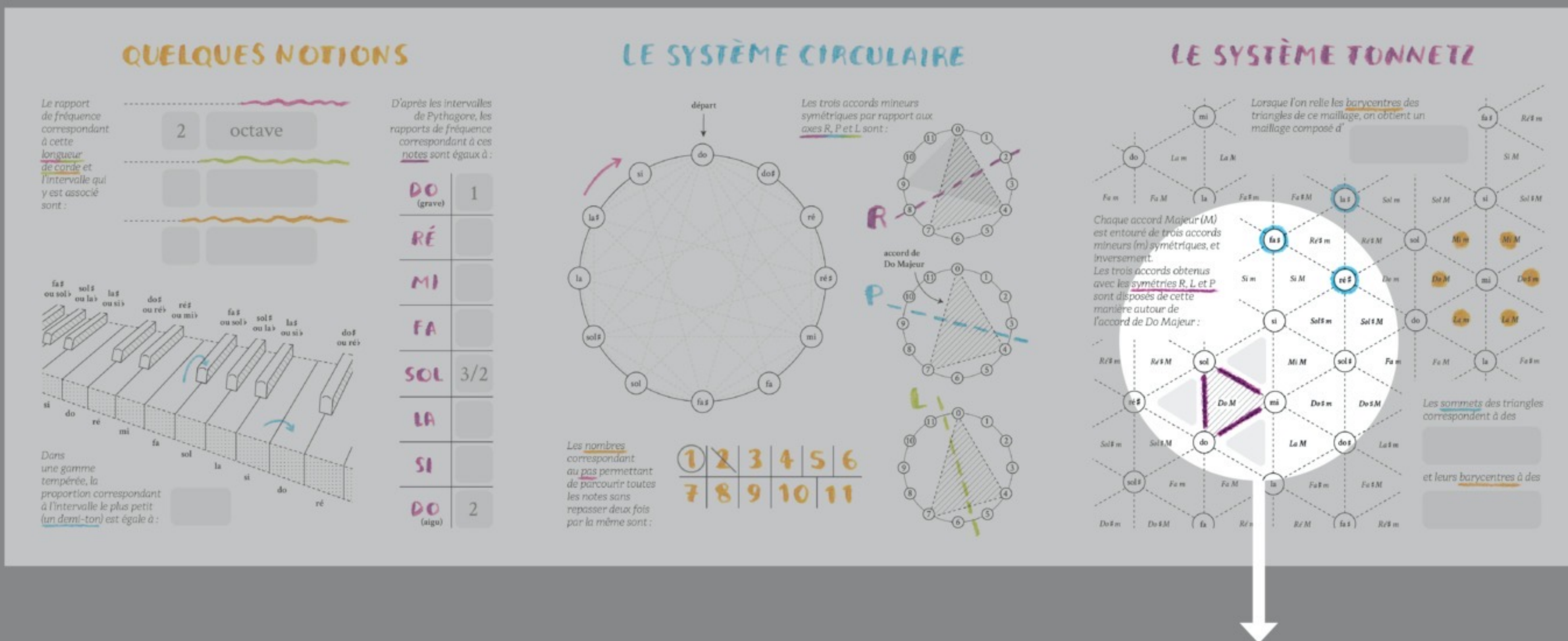
Un accord est un polygone inscrit dans le cercle

L'accord de **ré majeur** correspond par exemple au triangle dessinée dans cette animation et obtenu à partir de l'accord de **do majeur** en le transposant d'un ton.

L'**accord mineur** est le symétrique de l'accord majeur ! Celui-ci a un nom compliqué et on l'indiquera avec **L**.

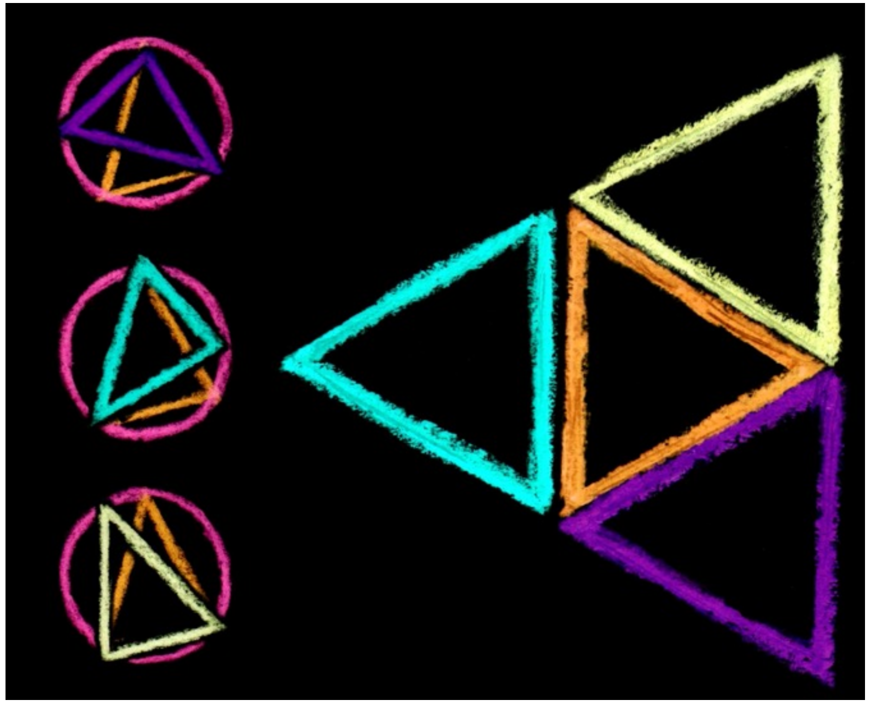
Comment visualiser les symétries dans le plan ?

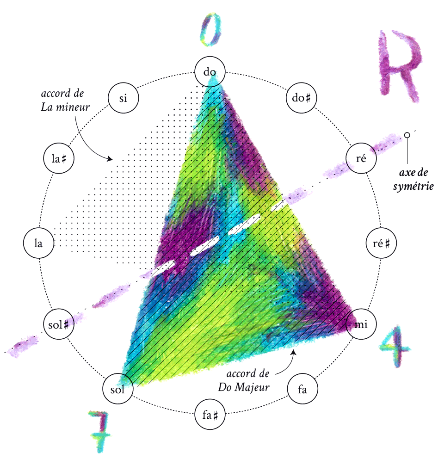
Le système Tonnetz et ses symétries

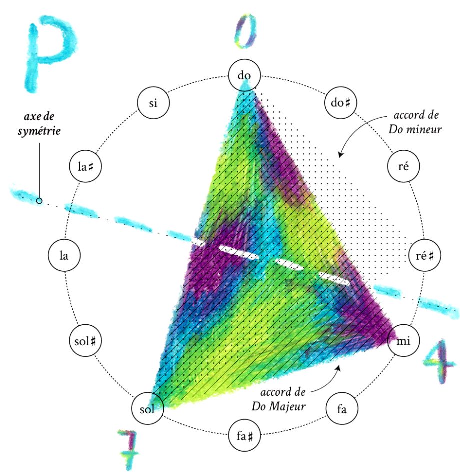


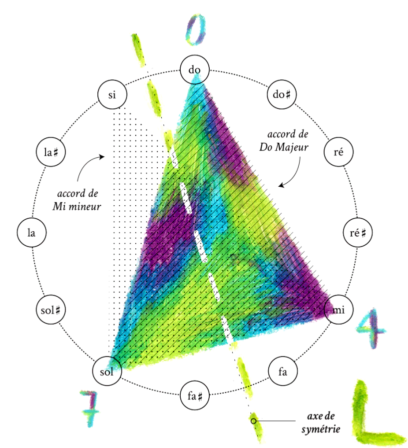
Retrouvez où se placent les accords mineurs symétriques de l’accord majeur dans le Tonnetz

Le système Tonnetz et ses symétries

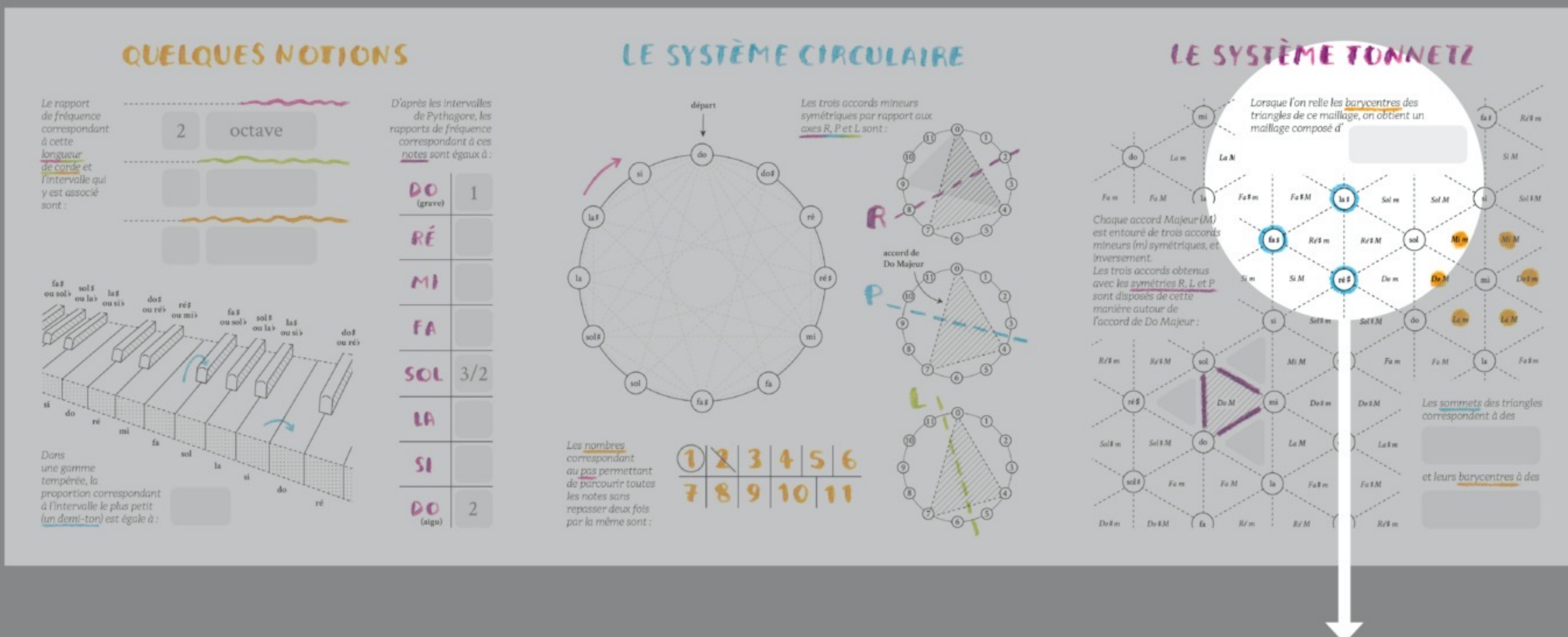






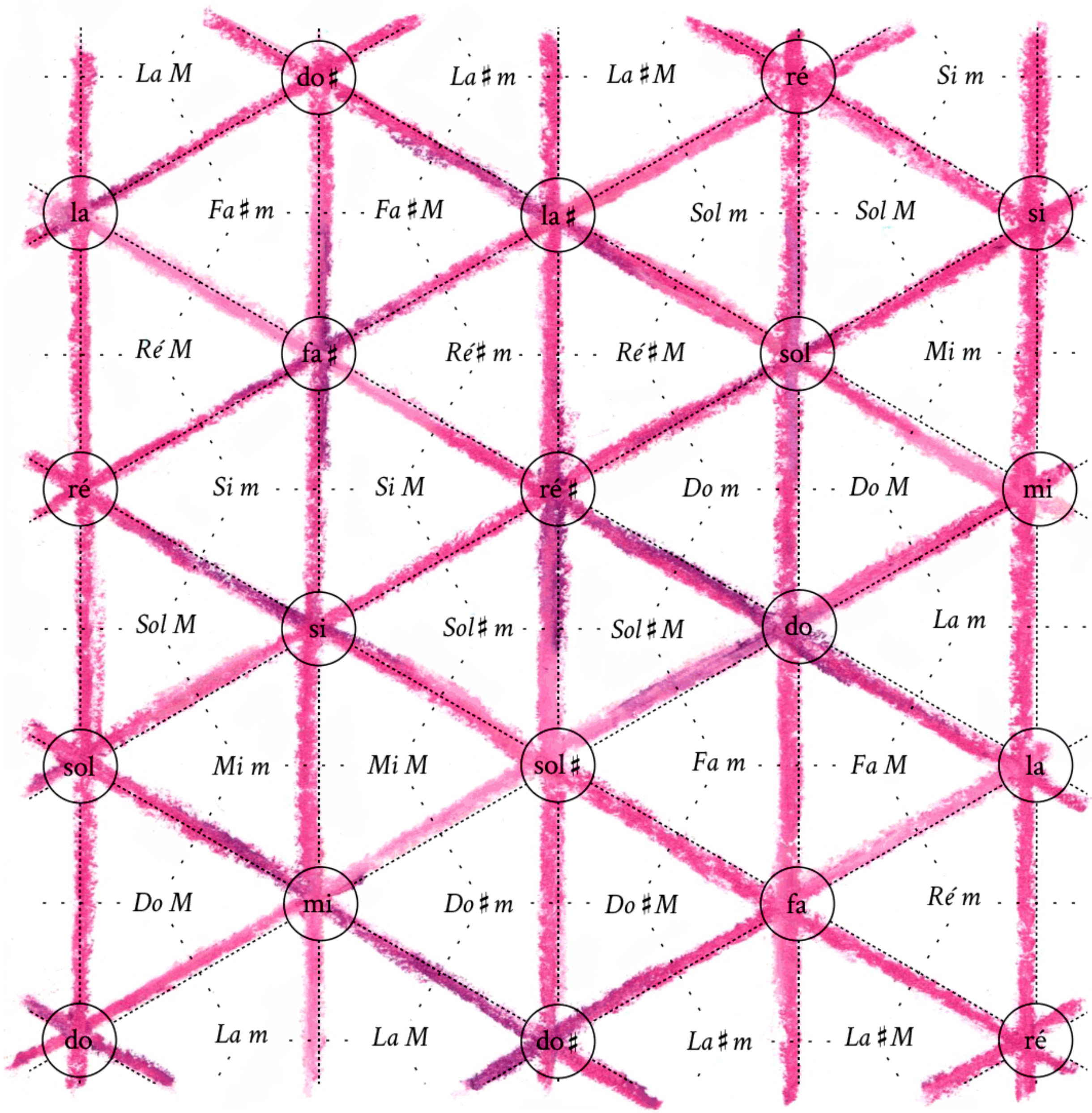


Les deux facettes du système Tonnetz



Complétez le texte

Les deux facettes du système Tonnetz



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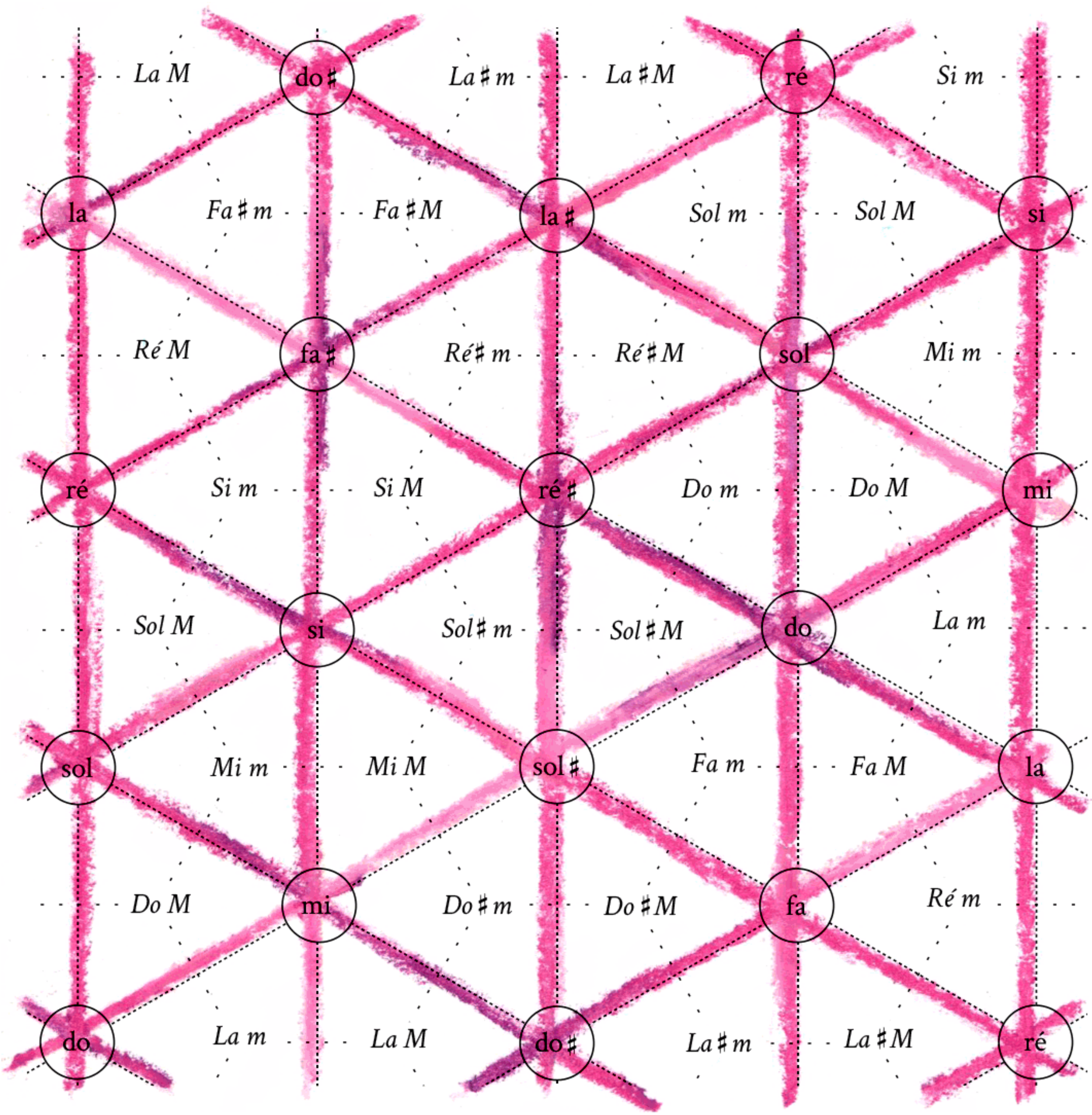

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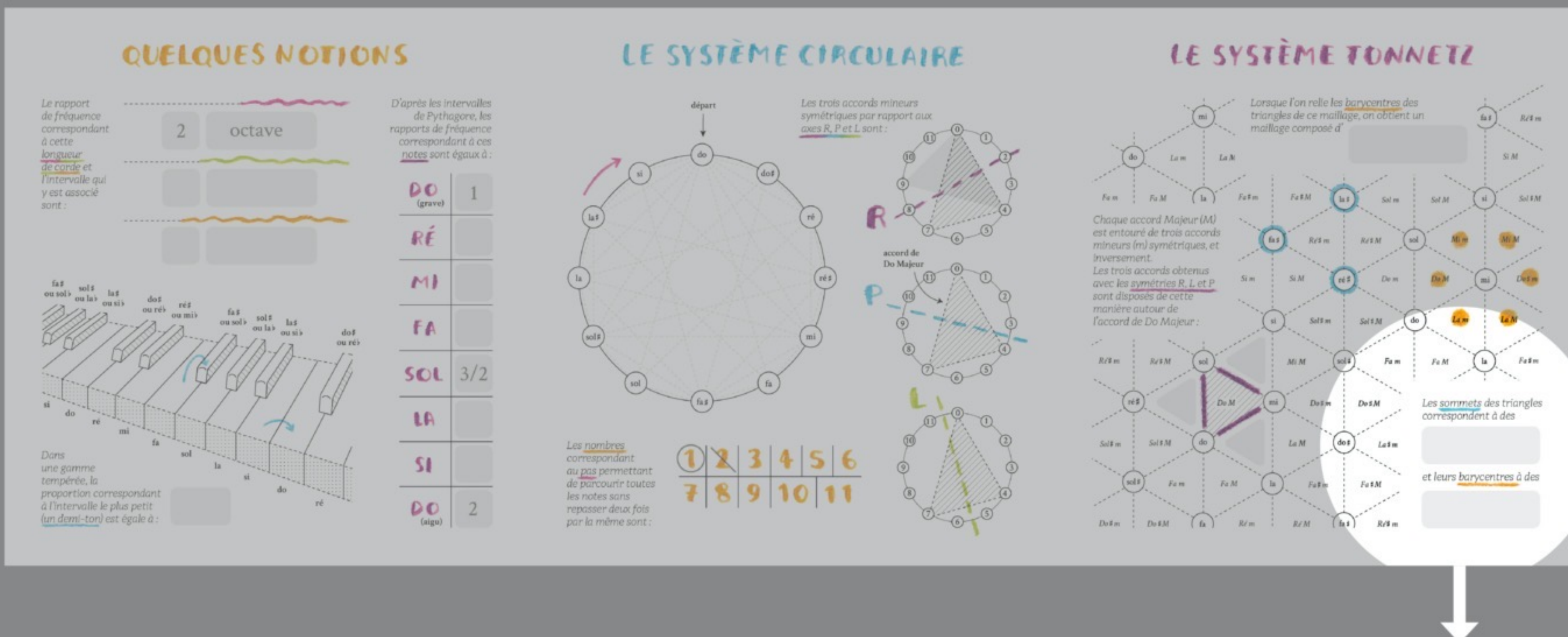
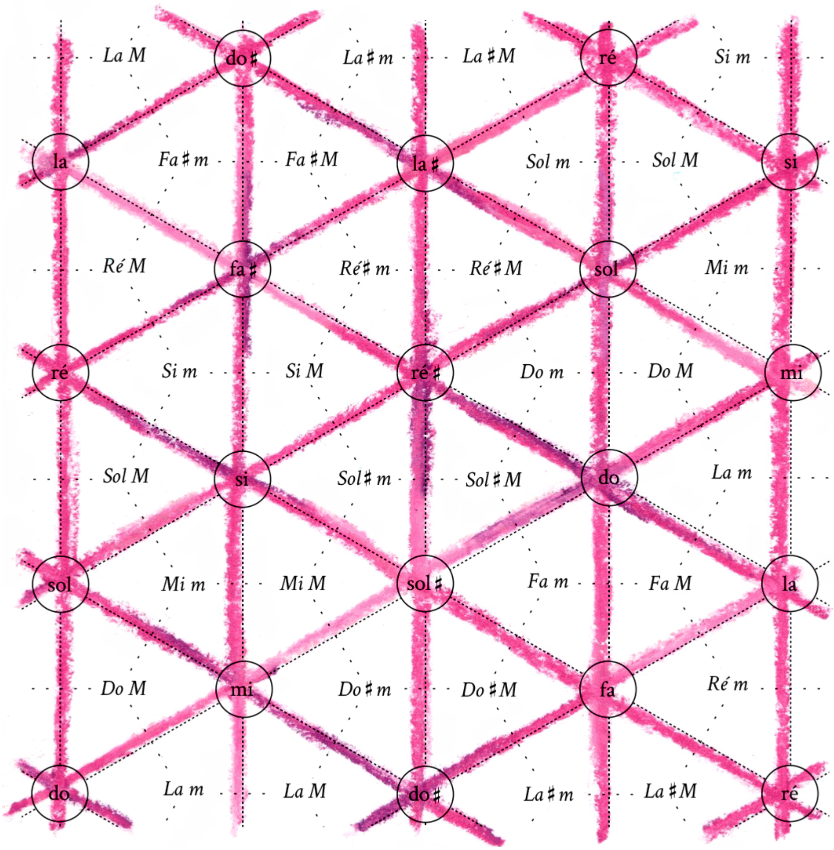
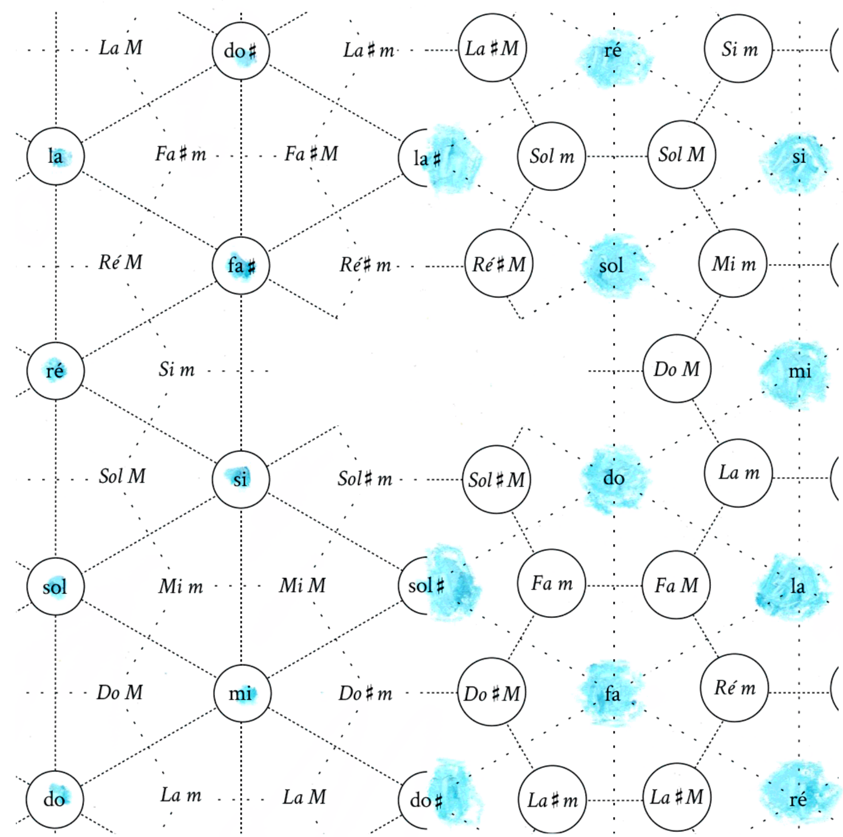


Complétez le texte

Les deux facettes du système Tonnetz





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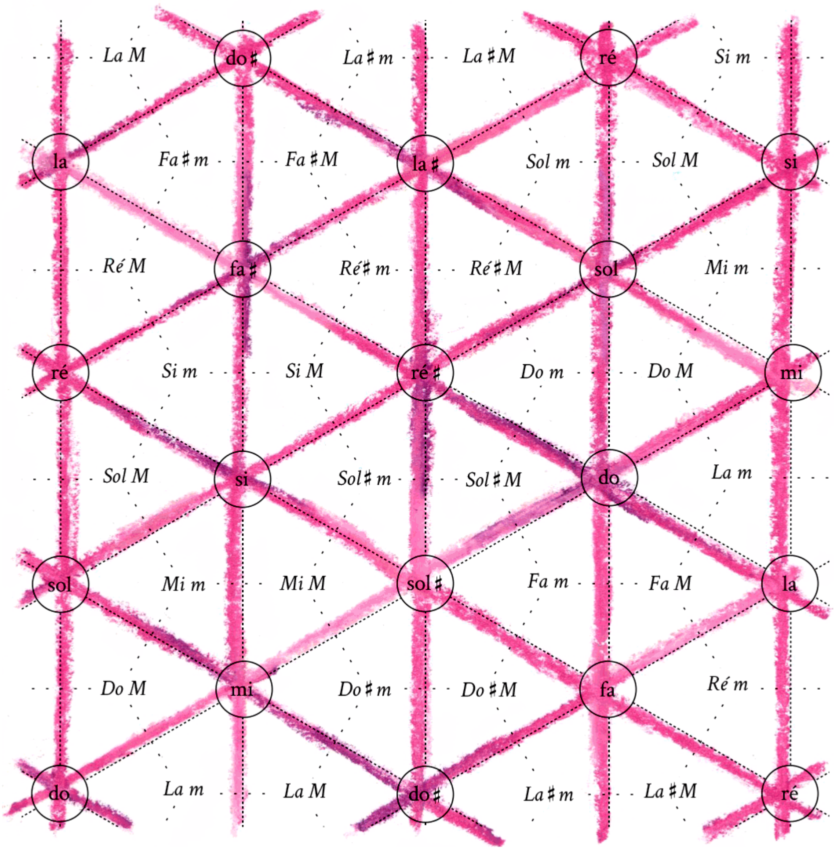

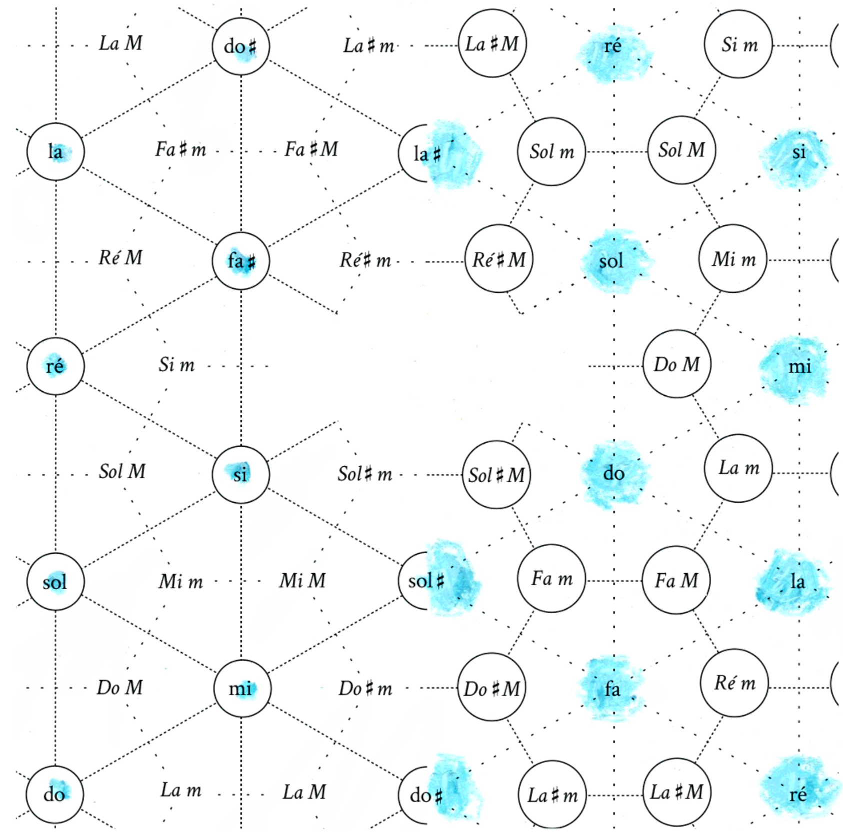

DUALITÉ