Master I.C.A.

Traitement interactif de l'image et du son

Méthodes mathématiques pour la création musicale : aspects théoriques, informatiques et cognitifs

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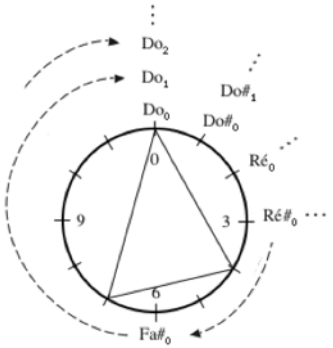
ircam Centre Pompidou

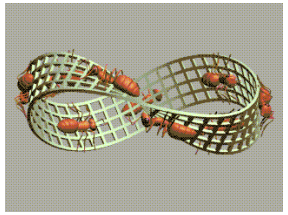
cnrs

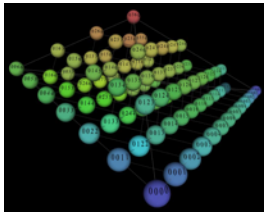
UPMC SORBONNE UNIVERSITÉS

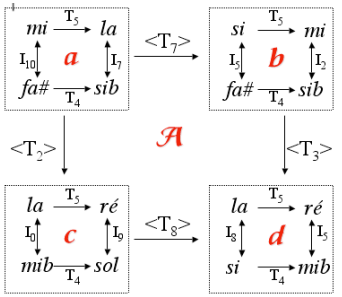
Plan du cours

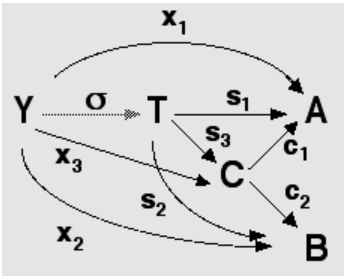
1.) Rappel historique et premiers éléments de musicologie computationnelle
2.) Représentation et énumération des structures musicales : *Set* Theory et théories transformationnelles
3.) Pavages en composition : la construction des canons rythmiques mosaïques
4.) Ramifications philosophiques et cognitives de l'approche algébrique en musique

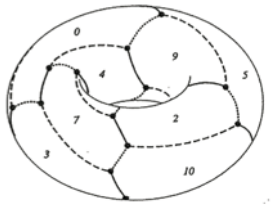


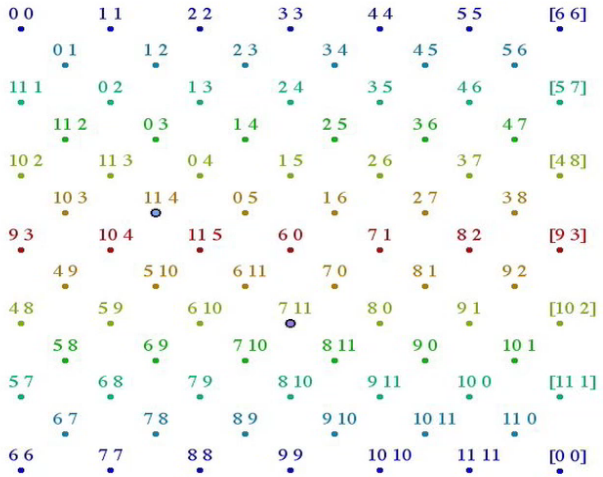


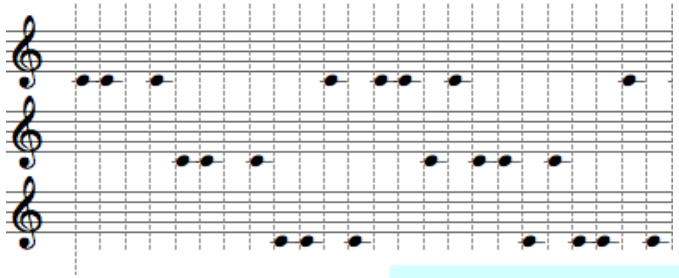


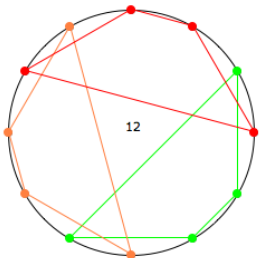


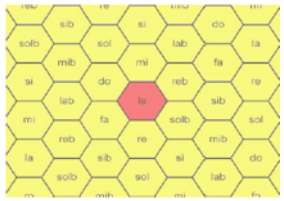










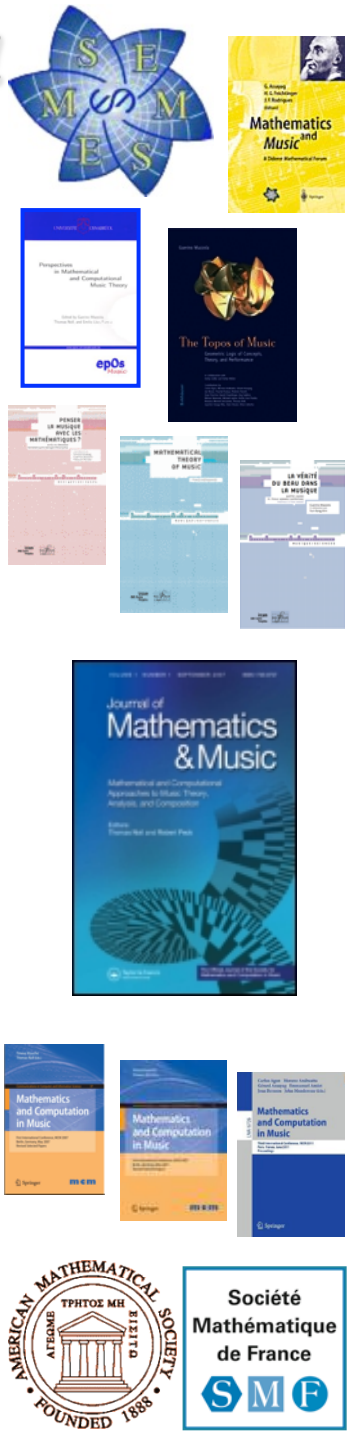


$Df(x)=f(x)-f(x-1).$

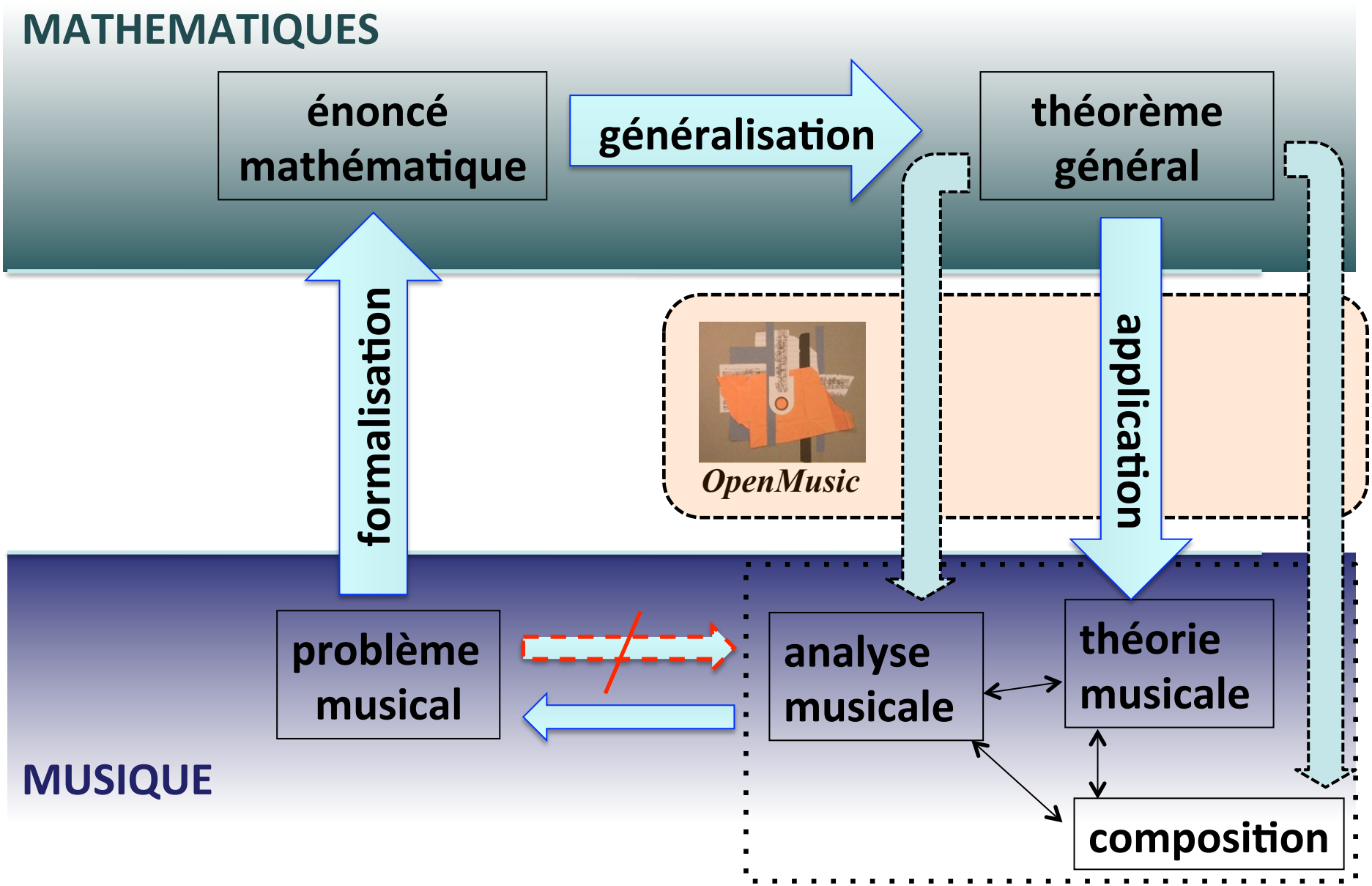
$$
\begin{aligned}
f &= 7\ 11\ 10\ 11\ 7\ 2\ 7\ 11\ 10\ 11\ 7\ 2\ 7\ 11\ldots \\
Df &= 4\ 11\ 1\ 8\ 7\ 5\ 4\ 11\ 1\ 8\ 7\ 5\ 4\ 11\ldots \\
D^2 f &= 7\ 2\ 7\ 11\ 10\ 11\ 7\ 2\ 7\ 11\ 10\ 11\ldots \\
D^3 f &= 7\ 5\ 4\ 11\ 1\ 8\ 7\ 5\ 4\ 11\ 1\ 8\ldots \\
D^k f &= \ldots\ldots
\end{aligned}
$$

Mathematiques/Musique…une histoire récente !

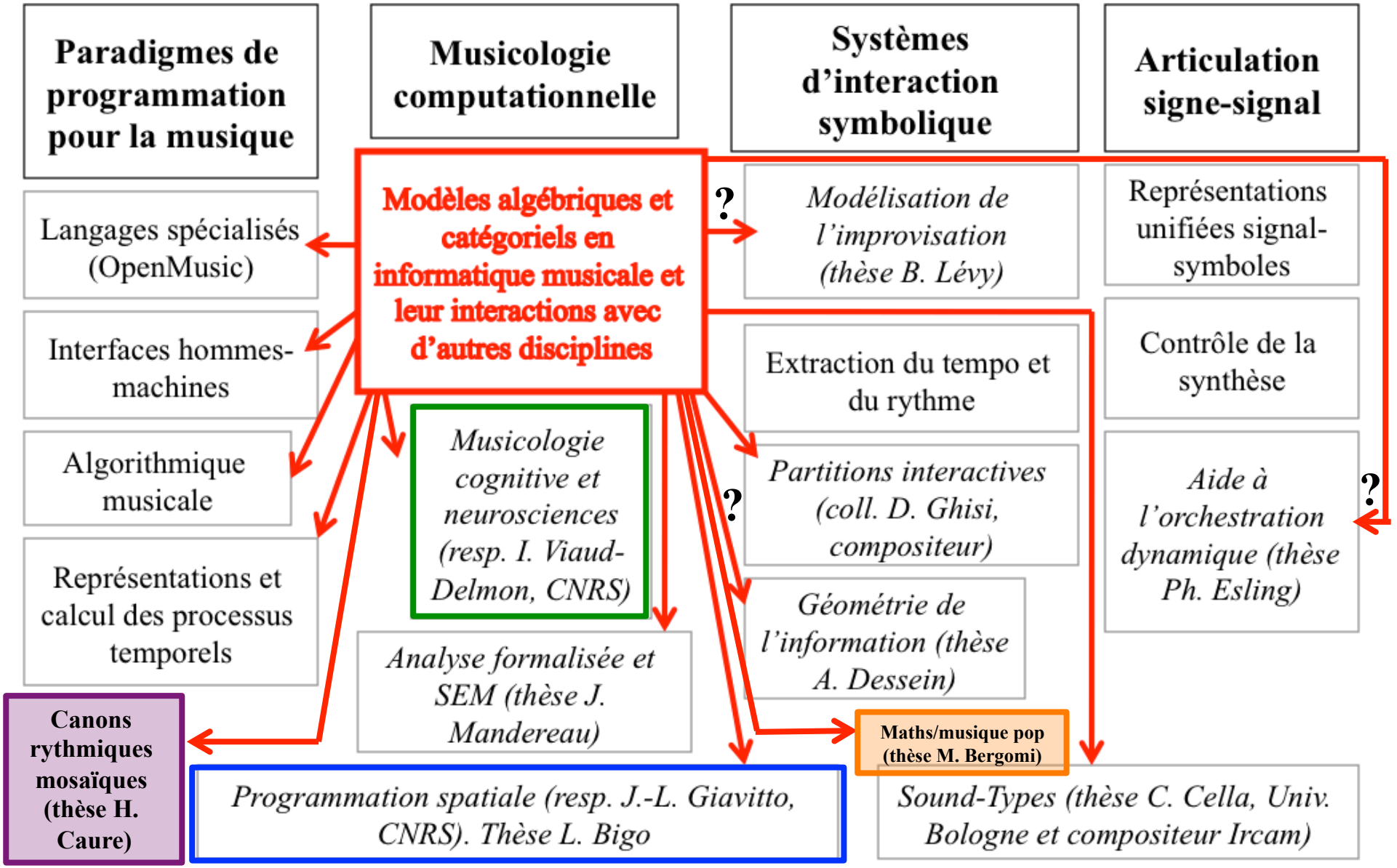
- 1999: 4e Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (G. Assayag, H.G. Feichtinger, J.F. Rodrigues, Springer, 2001)
- 2000-2001: *MaMuPhi Seminar, Penser la musique avec les mathématiques ?* (Assayag, Mazzola, Nicolas éds., Coll. 'Musique/Sciences', Ircam/Delatour, 2006)
- 2000-2003: International Seminar on *MaMuTh* (*Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Luis-Puebla eds, epOs, 2004)
- 2003: *The Topos of Music* (G. Mazzola et al.)
- 2001-…: *MaMuX* Seminar at Ircam
- 2004-…: *mamuphi* Seminar (Ens/Ircam)
- 2006: Collection 'Musique/Sciences' (Ircam/Delatour France)
- 2007: Journal of Mathematics and Music (Taylor & Francis) and SMCM
- 2007: First MCM 2007 (Berlin) and Proceedings by Springer
- 2007-…: AMS Special Session on Mathematical Techniques in Musical Analysis
- 2009: *Computational Music Science* (eds: G. Mazzola, M. Andreatta, Springer)
- 2009: MCM 2009 (Yale University) and Proceedings by Springer
- 2010: Mathematics Subject Classification : 00A65 Mathematics and music
- 2011: MCM 2011 (Ircam, 15-17 June 2011) and Proceedings LNCS Springer
- 2013: MCM 2013 (McGill University, Canada, 12-14 June 2013) - Springer



Double mouvement d'une dynamique mathémusicale



La musicologie computationnelle au sein de RepMus



La place des mathématiques dans la musicologie systématique

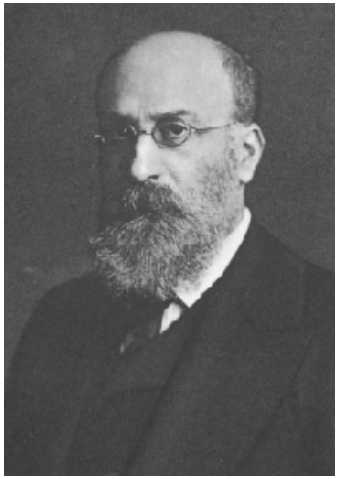
Guido Adler : « Umfang, Methode und Ziel der Musikwissenschaft » (1885)

II. Systematisch.

Aufstellung der in den einzelnen Zweigen der Tonkunst zuhöchst stehenden Gesetze.

A. Erforschung und Begründung derselben in der			B. Aesthetik der Tonkunst.	C. Musikalische Pädagogik und Didaktik	D. Musikologie
1. *Harmonik* (tonal od. tonlich).	2. *Rhythmik* (temporär oder zeitlich).	3. *Melik* (Cohärenz von tonal und temporär).	1. Vergleichung und Werthschätzung der Gesetze und deren Relation mit den appercipirenden Subjecten behufs Feststellung der *Kriterien des musikalisch Schönen.* 2. Complex unmittelbar und mittelbar damit zusammenhängender Fragen.	(Zusammenstellung der Gesetze mit Rücksicht auf den Lehrzweck) 1. Tonlehre, 2. Harmonielehre, 3. Kontrapunkt, 4. Compositionslehre, 5. Instrumentationslehre, 6. Methoden des Unterrichtes im Gesang und Instrumentalspiel.	(Untersuchung und Vergleichung zu ethnographischen Zwecken).

Hilfswissenschaften: Akustik und Mathematik.
Physiologie (Tonempfindungen).
Psychologie (Tonvorstellungen, Tonurtheile und Tongefühle).
Logik (das musikalische Denken).
Grammatik, Metrik und Poetik.
Pädagogik
Ästhetik etc.



G. Adler (1855-1941)

SysMusGraz

R. Parncutt

« La deuxième grande partie de la musicologie est la partie systématique; cette partie se base sur la partie historique. (...) L'accent de l'observation réside dans l'analogie de la méthode musicologique avec la méthode scientifique ».

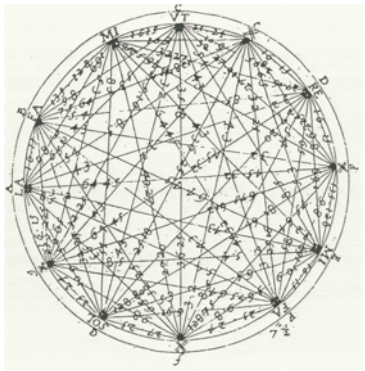
Un court survol historique

MUSIQUE	MATHS
500 av. J. C. Relation hauteur/longueur corde. La musique est source d'inspiration pour la théorie des nombres et la géométrie.	Nombres naturels et rationnels
300 a.J. Invention (théorique) de la gamme chromatique tempérée égale par Atistoxène de Tarente) et **prémonition de la théorie des groupes**. Isomorphismes entre les logarithmes (intervalles musicaux) et les exponentiels (longueur d'une corde)	*Aucune relation.*
1000 Invention de la représentation bidimensionnelle des hauteurs	*Aucune correspondance*
1500 *Aucune reprise des concepts précédents*	Nombres négatifs. Construction des rationnels
1600 *Aucune relation*	Nombres réels et les logarithmes
Marin Mersenne (1588-1648) : combinatoire musicale	Calcul des probabilités
1700 La fugue comme un automate abstrait. Manipulation inconsciente du groupe de Klein	Nombres complexes (Euler, Gauss), les quaternions (Hamilton), continuité (Cauchy), structure de groupe (Galois, Abel)
Leonhard Euler : *Speculum Musicum* (1773)	Théorie des graphes
1900 Libération de la prison de la tonalité (Loquin, Hauer, Schoenberg)	Nombres infinis et transfinis (Cantor). Axiomatique de Peano. Théorie de la mesure (Lebesgue, Borel)
1920 Formalisation radicale des macrostructures à travers le système sériel (Schoenberg)	*Aucun développement de la théorie des nombres.*
Ernst Krenek (1900-1991) : les axiomes dans le système dodécaphonique	*David Hilbert, Les fondements de la géométrie (1899)*

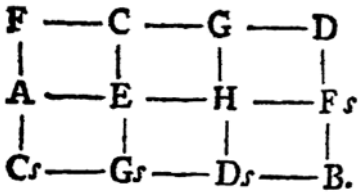
Iannis Xenakis, *Musique. Architecture*, Tournai, Casterman, 1971, 176 p. (New, revised edition: Tournai, Casterman, 1976, 238 p.)

Pythagore et le monocorde, VI^e-V^e siècle av. J. C.

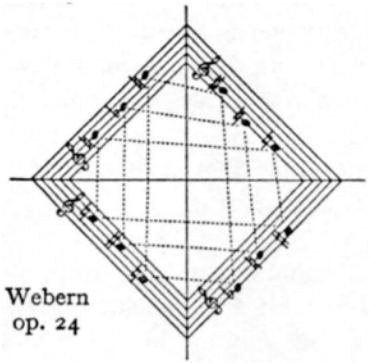
I. Xenakis

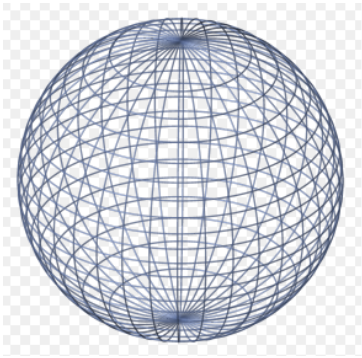


Mersenne, Harmonicorum Libri XII, 1648



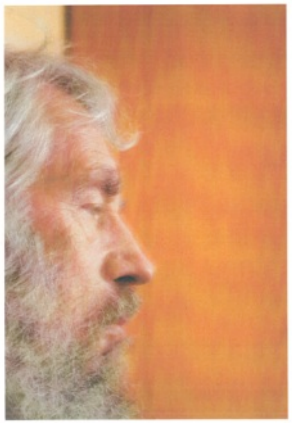
Euler : *Speculum musicum*, 1773

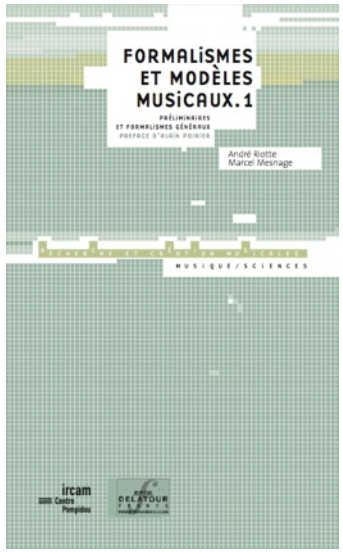


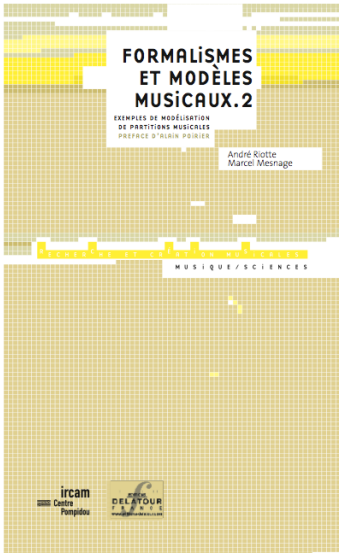


L'analyse formalisée ou les entités formelles en musique

André Riotte e Marcel Mesnage







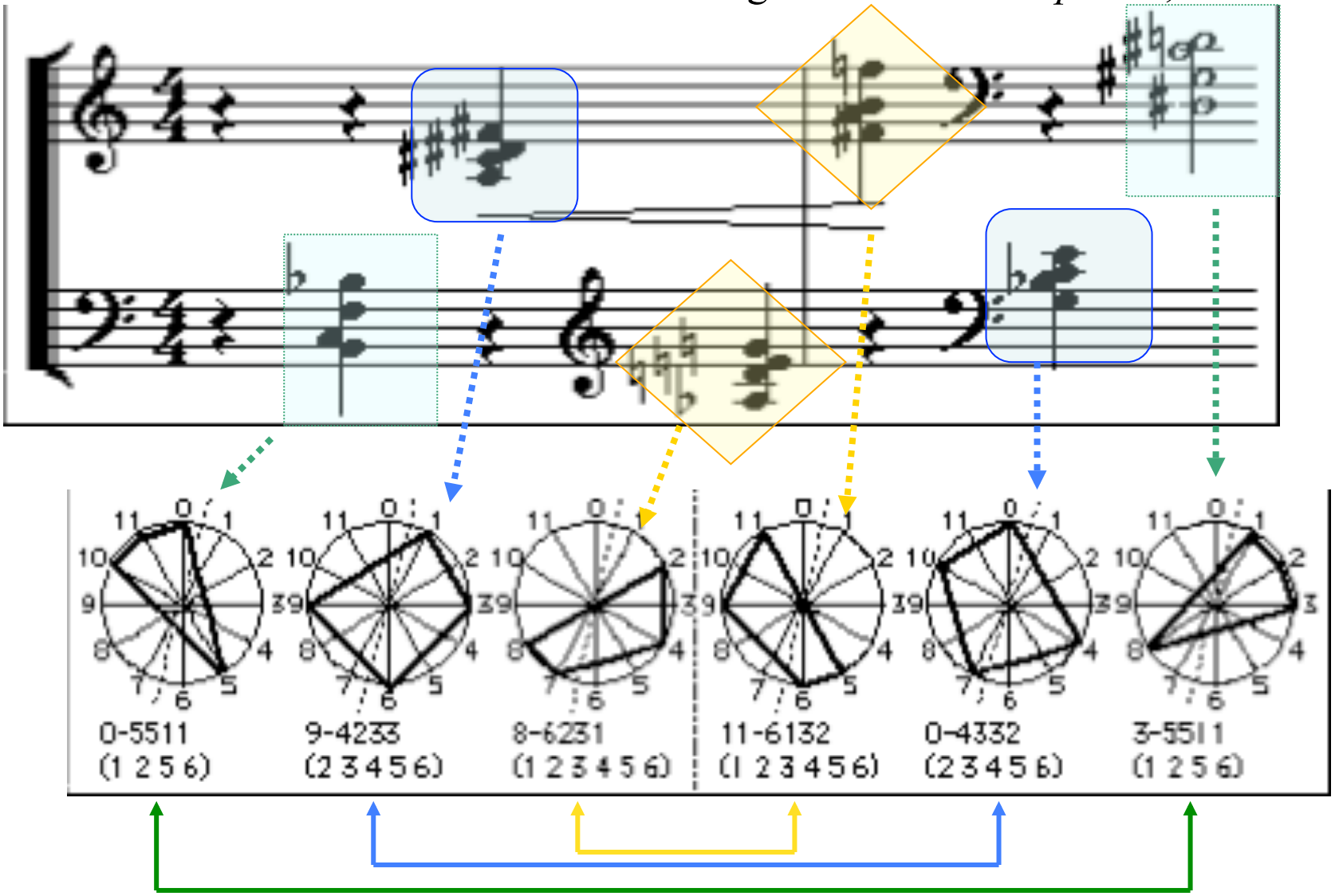
- « Anamorphoses » d'André Riotte
- « La terrasse des audiences du clair de lune » de Claude Debussy : esquisse d'analyse modélisée
- La mise en évidence de régularités locales : le « Mode de valeurs et d'intensités » de Messiaen
- Un exemple d'invention structurelle : le « Mikrokosmos » de Béla Bartok
- Un modèle informatique de la « Pièce pour quatuor à cordes » n°1 de Stravinsky
- Les « Variations pour piano », op. 27, d'Anton Werbern
- L'« Invention à deux voix » n°1 de J.-S. Bach
- Un modèle informatique du « Troisième Regard sur l'Enfant Jésus » d'Olivier Messiaen
- Un modèle de la « Valse sentimentale », Op. 50, n°13, de Franz Schubert
- Un automate musical construit à partir d'une courte pièce de Béla Bartok (Mikrokosmos n°39)

A. Riotte & M. Mesnage, *Formalismes et modèles musicaux* (en 2 volumes), Collection « Musique/Sciences », Ircam/Delatour France, 2006

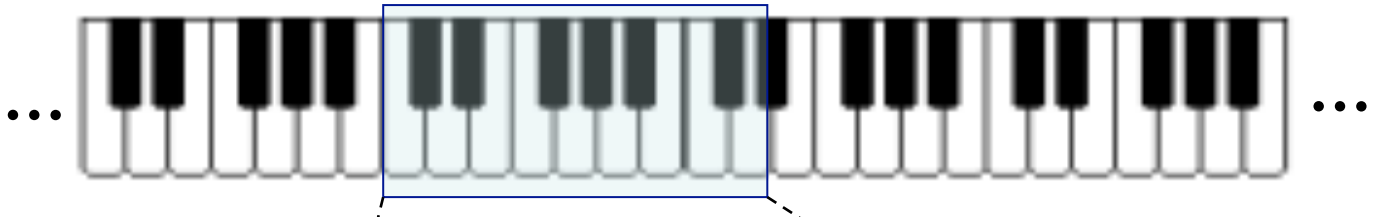
« Entités formelles pour l'analyse musicale »

Marcel Mesnage (1998)

A. Schoenberg : *Klavierstück Op. 33a*, 1929



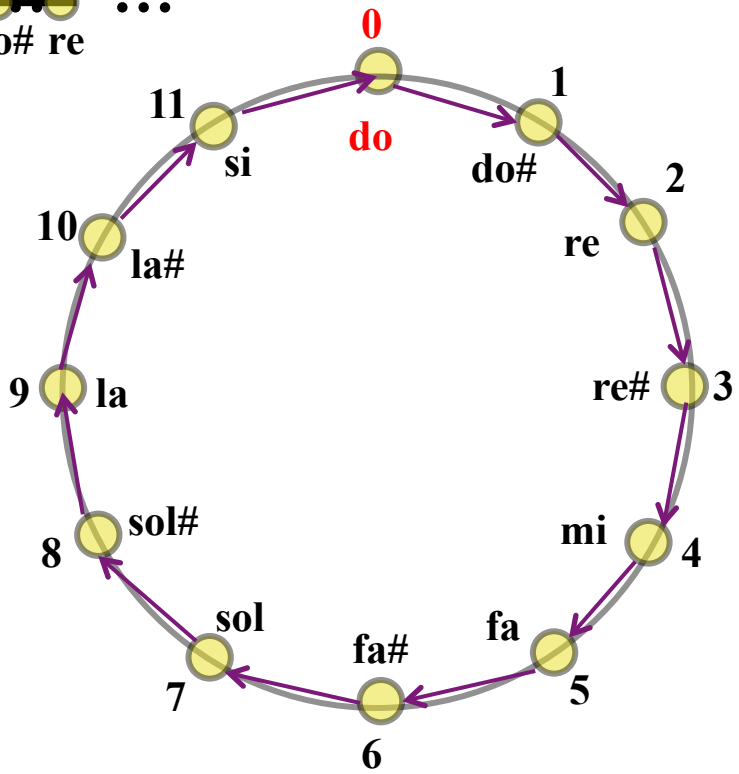
Reduction à l'octave et congruence modulo 12



1

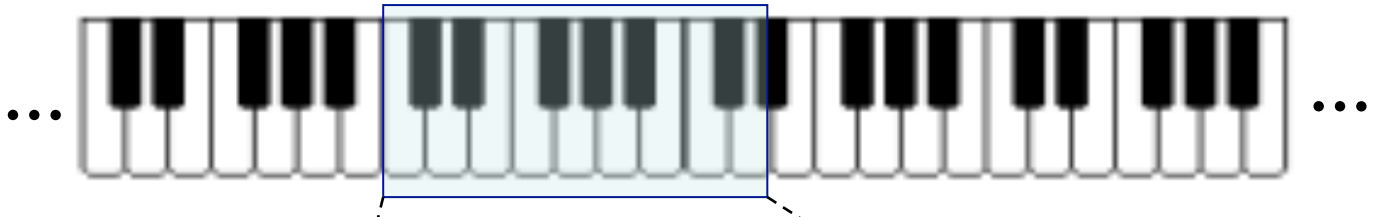
do do# re re# mi fa fa# sol sol# la la# si do do# re

0 1 2 3 4 5 6 7 8 9 10 11 12



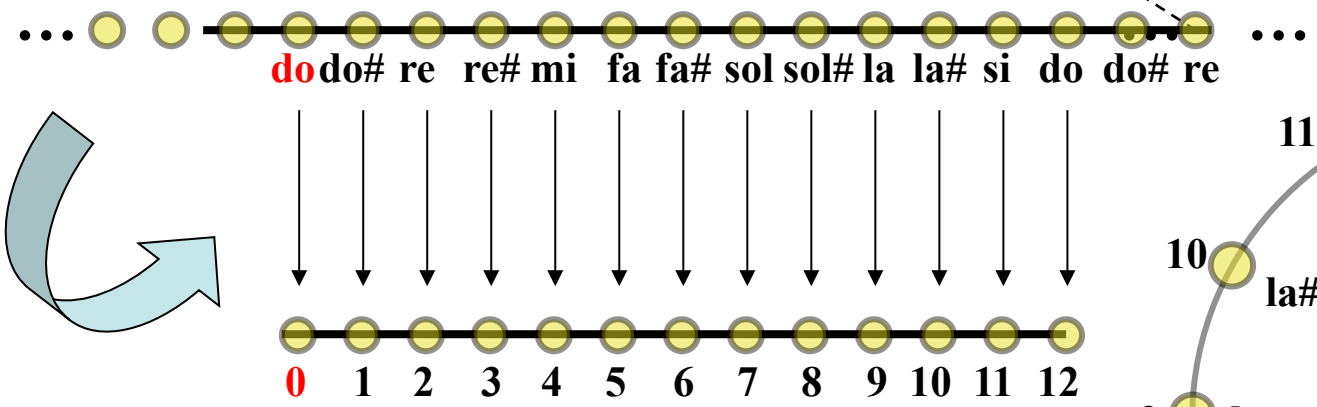
➔ Quels sont les autres entiers qui engendrent le cercle ?

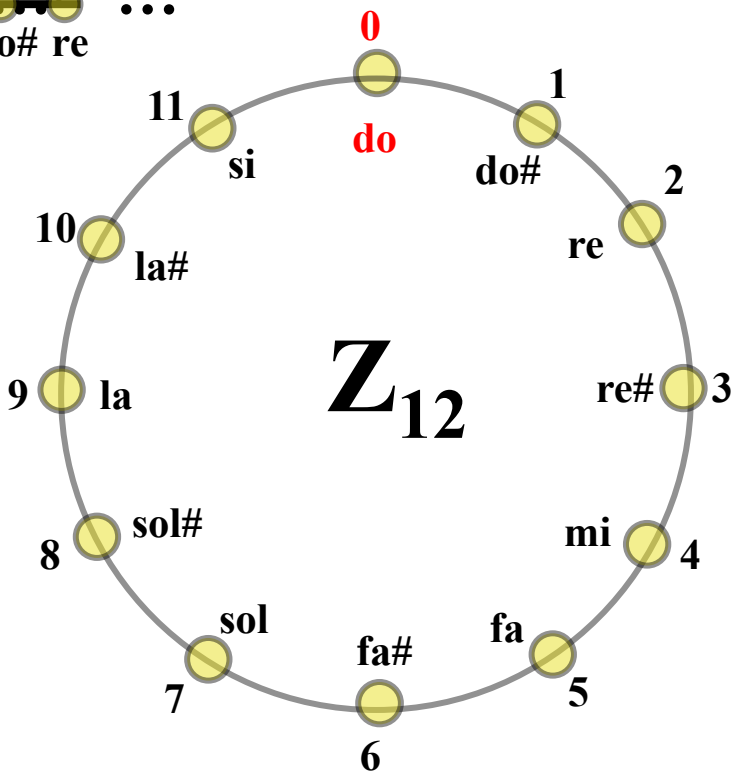
Reduction à l'octave et congruence modulo 12



$$Z_{12} = < T_k \mid (T_k)^{12} = T_0 >$$

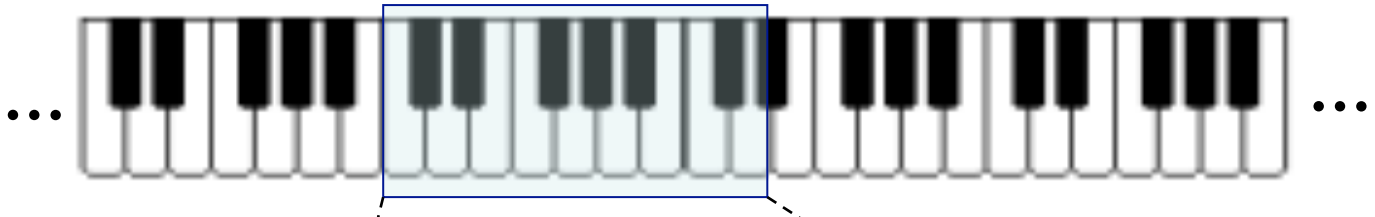
$$T_k : x \rightarrow x+k \bmod 12$$





➔ **Les générateurs du groupe cyclique d'ordre 12 sont les transpositions T_1 , T_5 , T_7 et T_{11}**

Reduction à l'octave et congruence modulo 12



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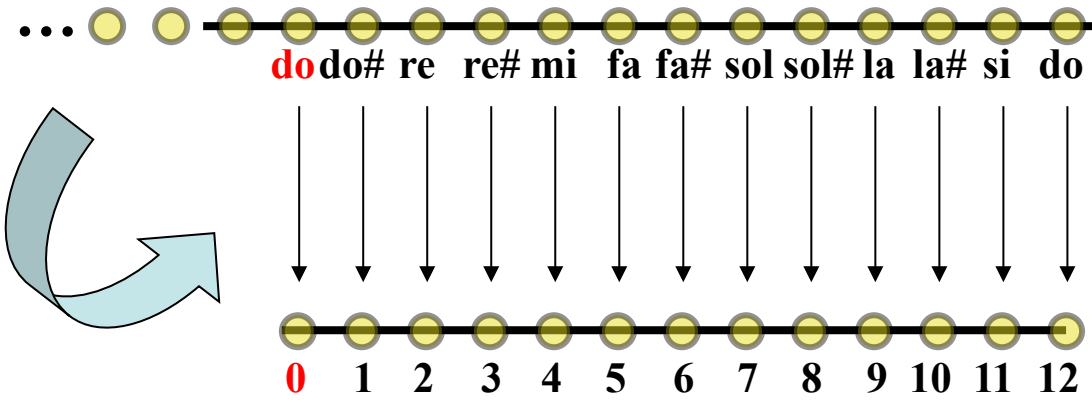

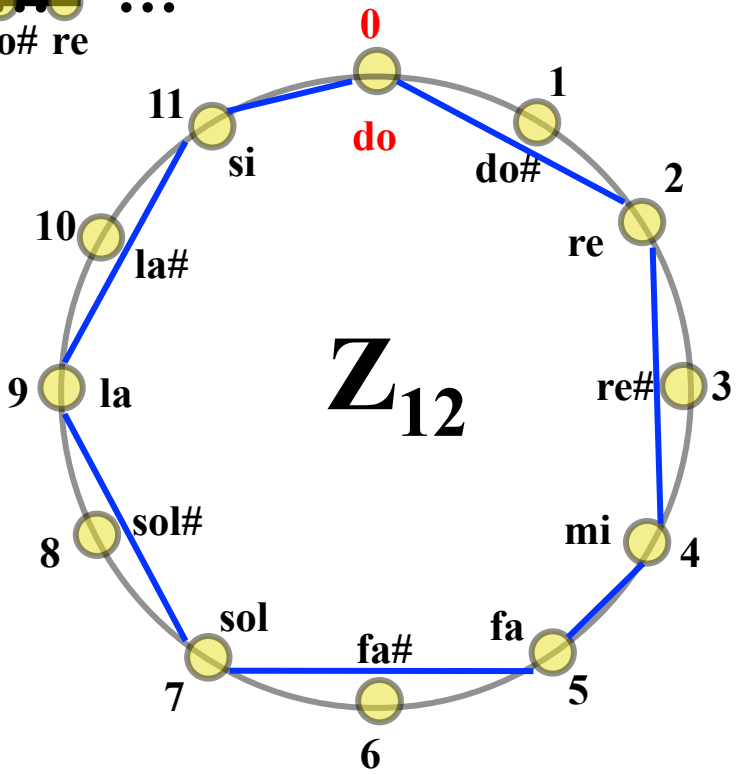

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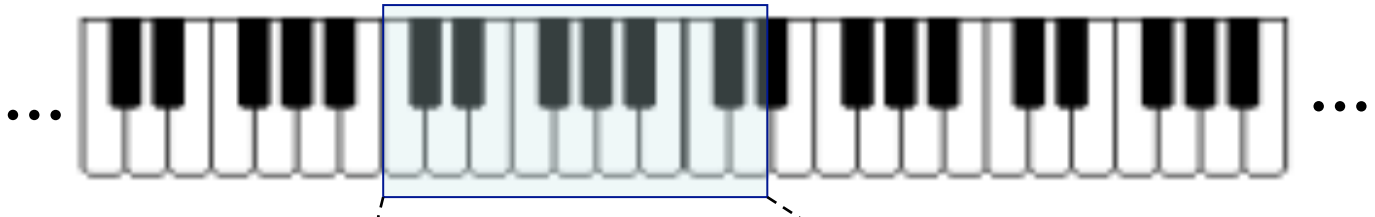
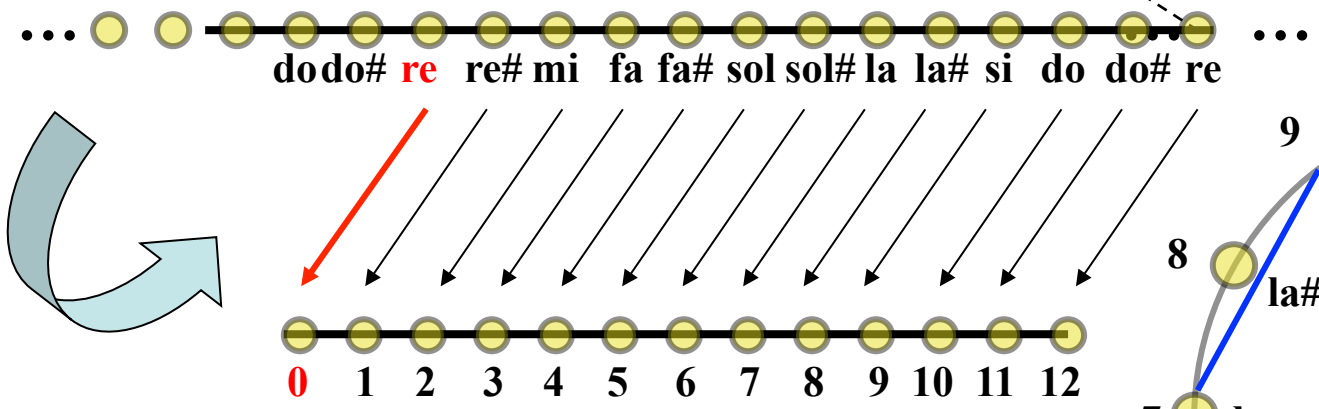
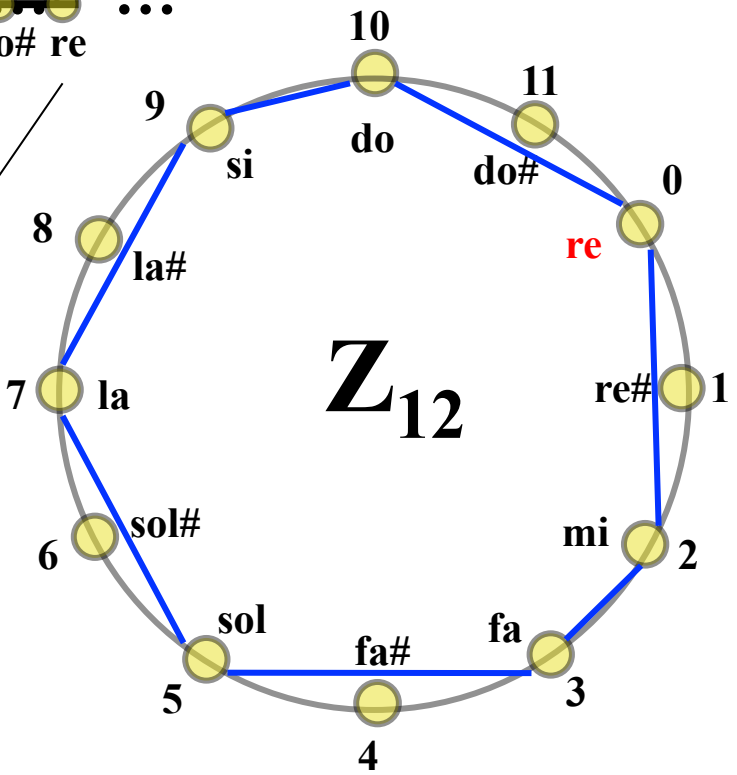


$$Z_{12} = < T_k \mid (T_k)^{12} = T_0 >$$

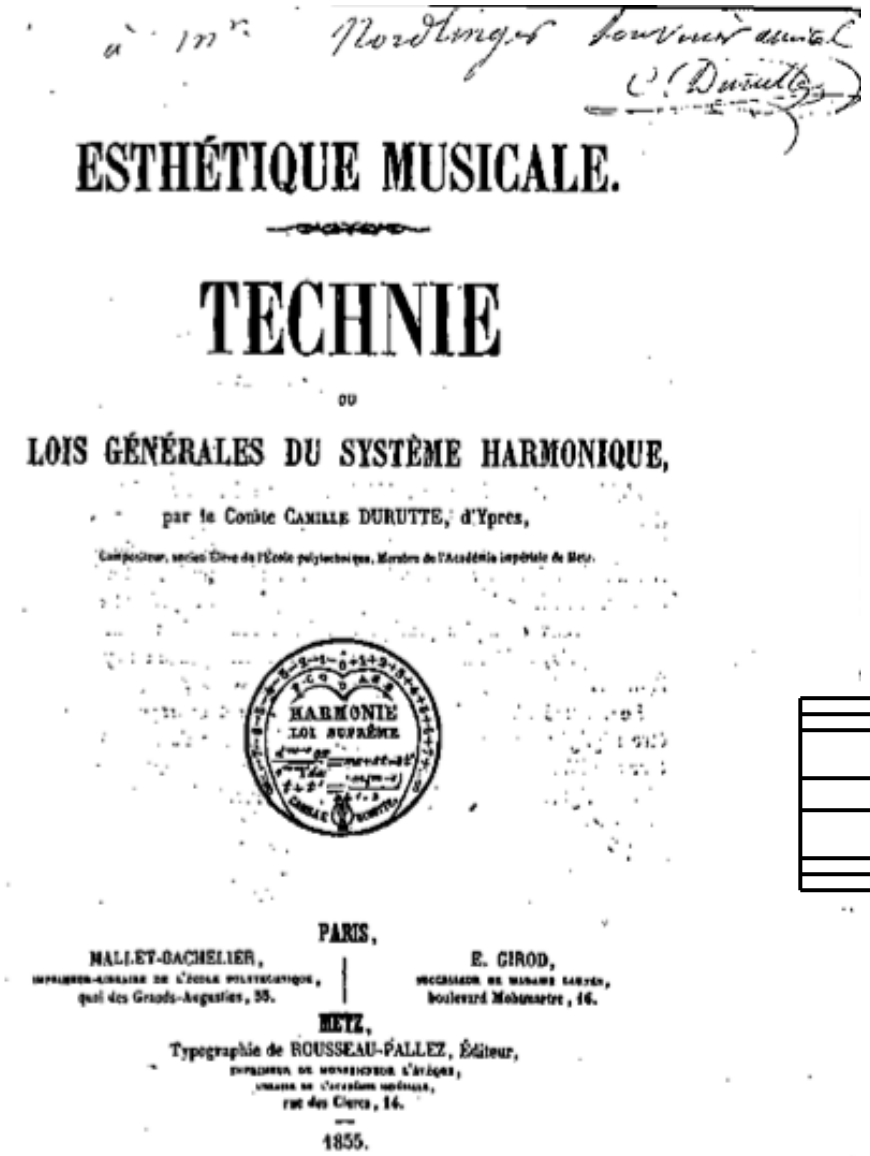
$T_k : x \rightarrow x+k \bmod 12$





➔ **Les générateurs du groupe cyclique d'ordre 12 sont les transpositions T_1 , T_5 , T_7 et T_{11}**

La symétrie du ré…



ESTHÉTIQUE MUSICALE.

TECHNIE

OU

LOIS GÉNÉRALES DU SYSTÈME HARMONIQUE,

par le Comte CAMILLE DURUTTE, d'Ypres,

Compositeur, ancien Élève de l'École polytechnique, Membre de l'Académie impériale de Metz.

HARMONIE
LOI SUPRÊME

PARIS,

MALLET-BACHELIER, E. CIROD,

quai des Grands-Augustins, 55. boulevard Montmartre, 16.

METZ,

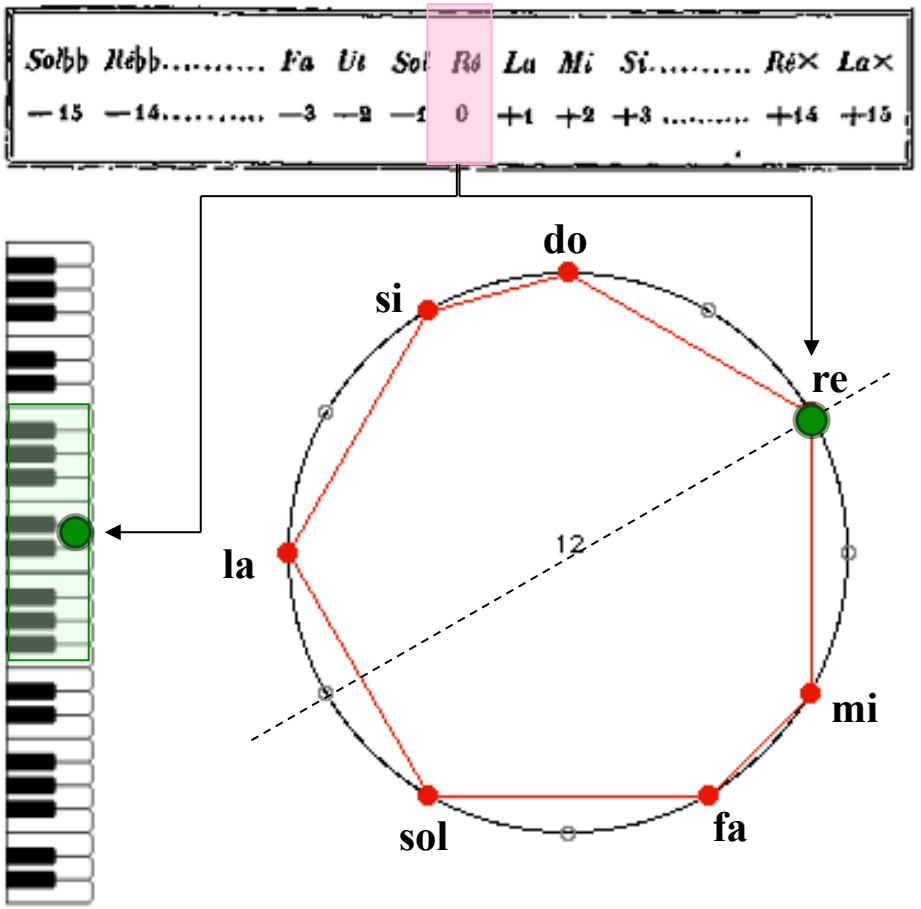
Typographie de ROUSSEAU-PALLEZ, Éditeur,

rue des Clercs, 14.

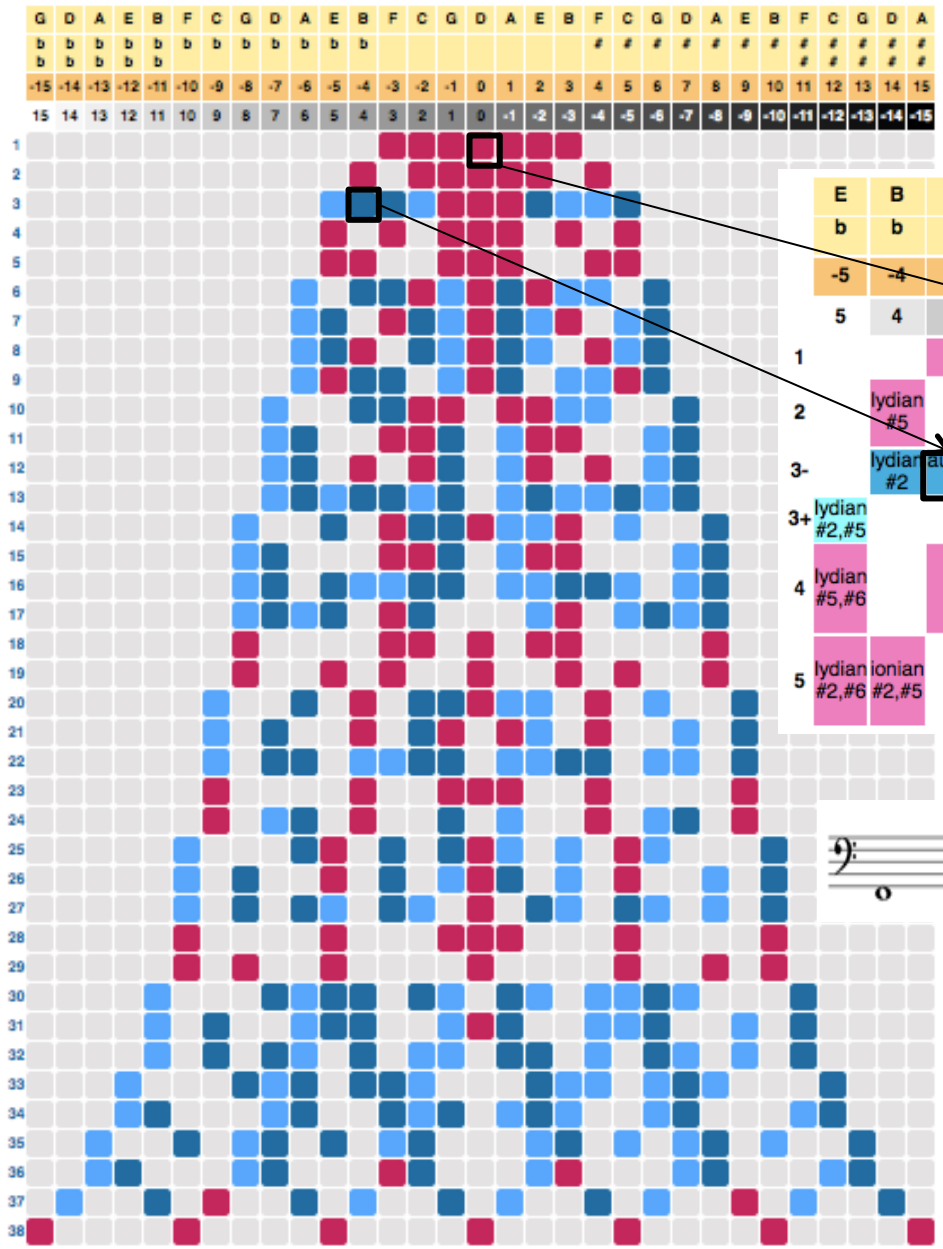
1855.

Camille Durutte:

- *Technie, ou lois générales du système harmonique* (1855)
- *Résumé élémentaire de la Technie harmonique, et complément de cette Technie* (1876)



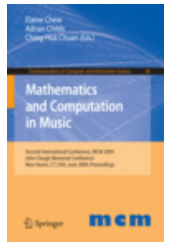
La cloche diatonique (P. Audétat & co.)



	E	B	F	C	G	D	A	E	B	F	C	
	b	b								#	#	
	-5	-4	-3	-2	-1	0	1	2	3	4	5	
	5	4	3	2	1	0	-1	-2	-3	-4	-5	
1			lydian	ionian	mixolydian	dorian	aeolian	phrygian	locrian			diatonic
2		lydian #5		lydian b7	ionian b3	mixolydian b6 or aeolian 3	phrygian 6	locrian 2		locrian b4		minor melodic
3-		lydian #2	augmented ionian		dorian #4	aeolian 7	phrygian 3	locrian 6			locrian b4,bb7	minor harmonic
3+	lydian #2,#5			lydian b3	ionian b6	mixolydian b2	dorian b5		phrygian b4	locrian bb7		major harmonic
4	lydian #5,#6		lydian #5,b7		lydian b6,b7	ionian b2,b3 or phrygian 6,7	locrian 2,3		locrian 2,b4		locrian bb3,b4	unitonic
5	lydian #2,#6	ionian #2,#5			aeolian #4,7	ionian b2,b6 or phrygian 3,7	mixolydian b2,b5			phrygian b4,bb7	locrian bb3,bb7	double harmonic

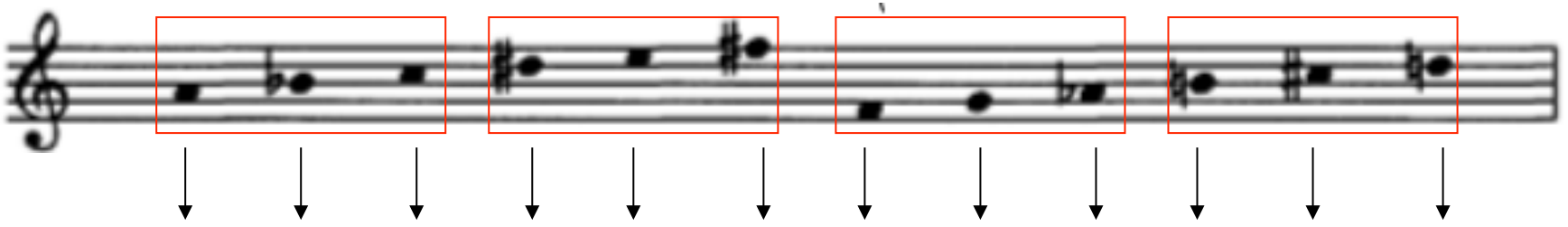
http://www.cloche-diatonique.ch/

Junod, J., Audétat, P., Agon, C., Andreatta, M., « A Generalisation of Diatonicism and the Discrete Fourier Transform as a Mean for Classifying and Characterising Musical Scales », Second International Conference MCM 2009, vol. 38, New Haven, 2009, pp. 166-179

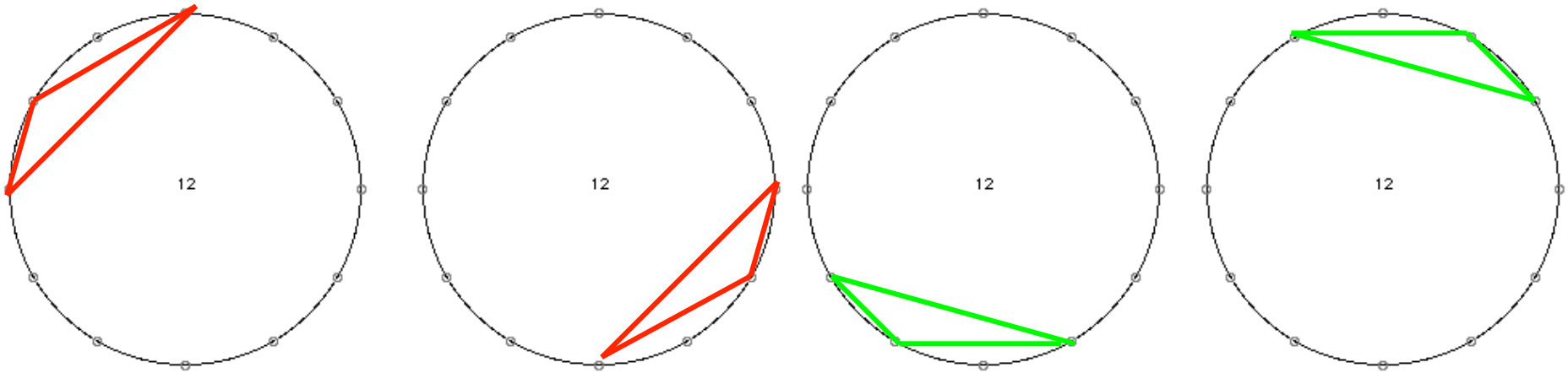


Exercice : retrouver les symétries dans une série (I)

Schoenberg: Serenade Op.24, Mouvement 5



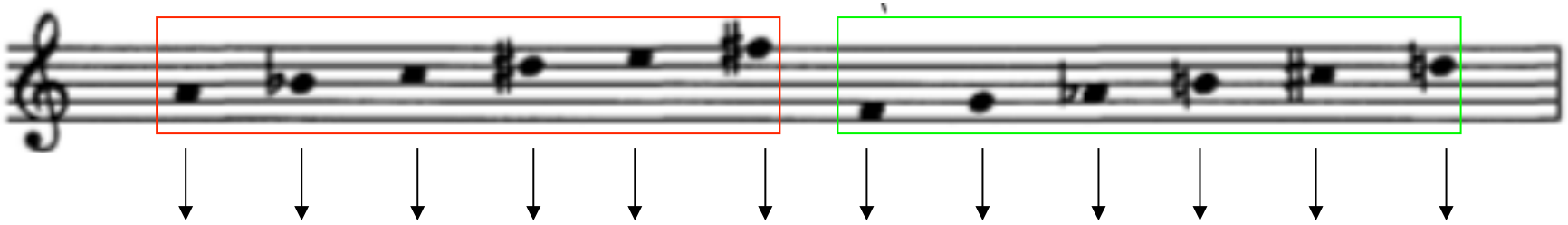
{9 , 10 , 0} {3 , 4 , 6} {5 , 7 , 8} {11 , 1 , 2}



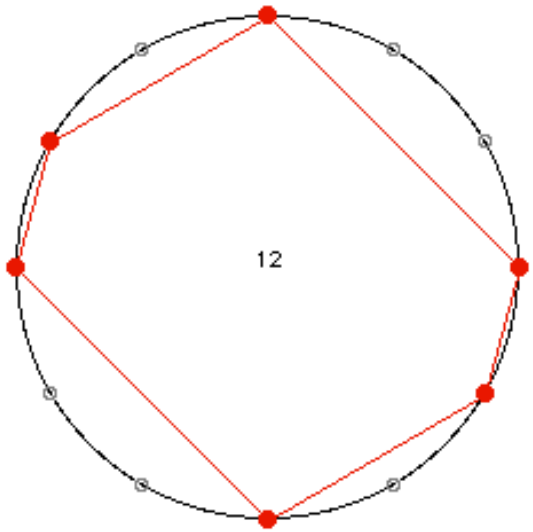
(1 , 2 , 9) (1 , 2 , 9) (2 , 1 , 9) (2 , 1 , 9)

Exercice : retrouver les symétries dans une série (II)

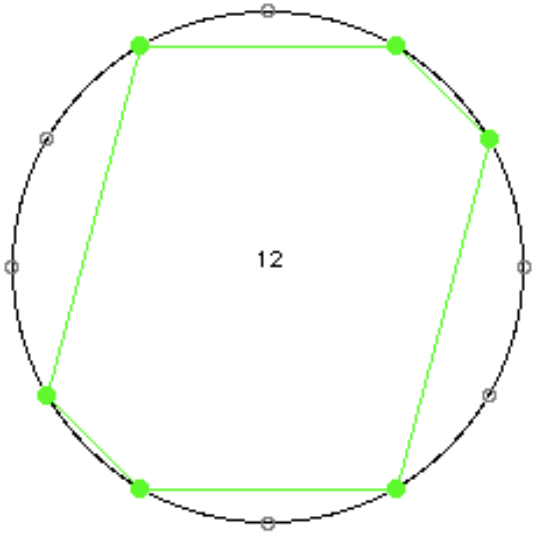
Schoenberg: Serenade Op.24, Mouvement 5



{9 , 10 , 0 , 3 , 4 , 6 } {5 , 7 , 8 , 11 , 1 , 2 }



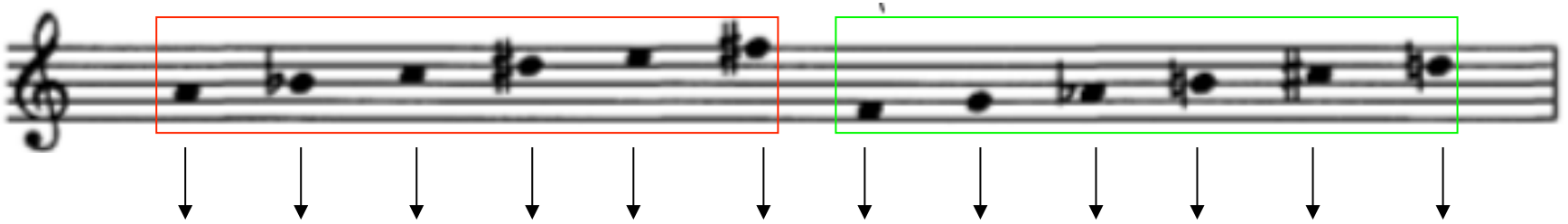
(3, 1, 2, 3, 1, 2)



(2, 1, 3, 2, 1, 3)

“Combinatorialité” et symétrie par transposition

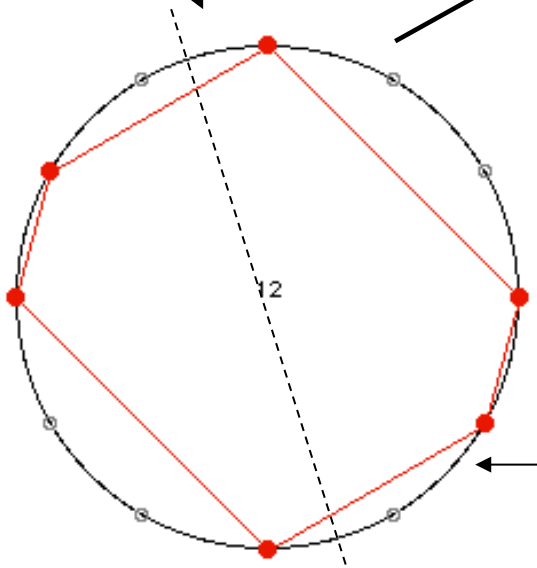
Schoenberg: Serenade Op.24, Mouvement 5



A = {9 , 10 , 0 , 3, 4 , 6} {5 , 7, 8, 11, 1, 2} =A’

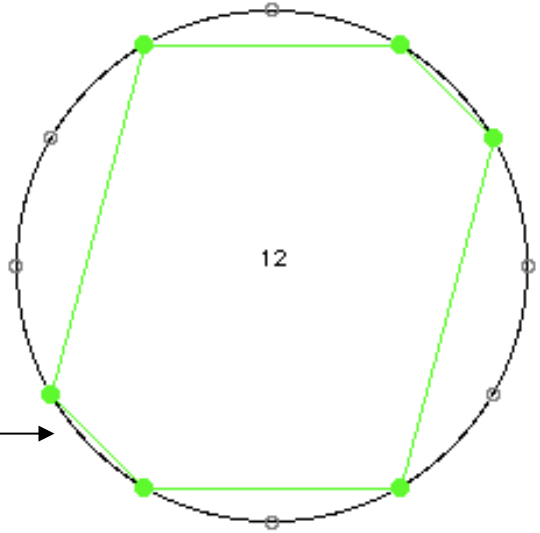
T6{9,10,0,3,4,6}=
={6+9,6+10,6,6+3,6+4,6+6}=
={3,4,6,9,10,0}

T6(A)=A



(3, 1, 2, 3, 1, 2)

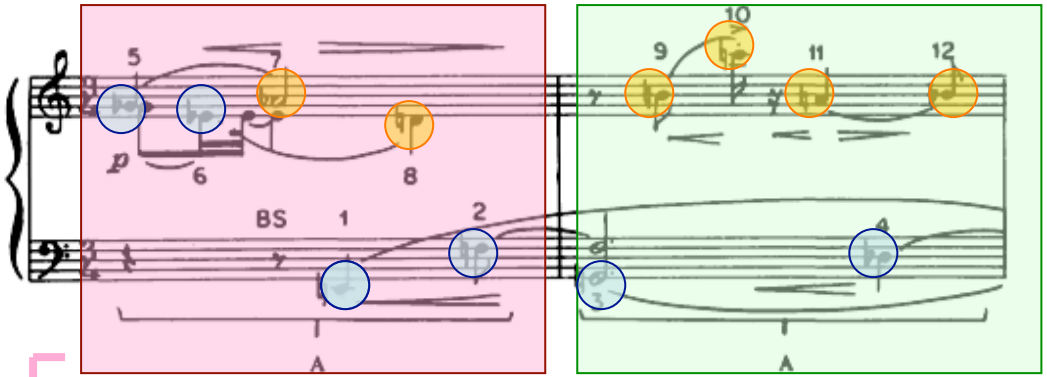
$I_{11} = T_{11}\, I$

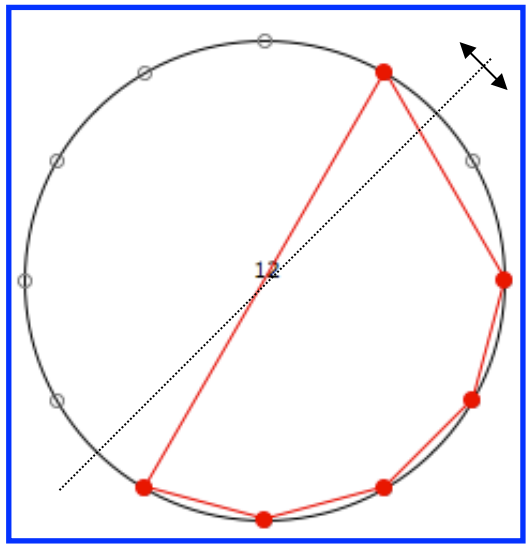


(2, 1, 3, 2, 1, 3)

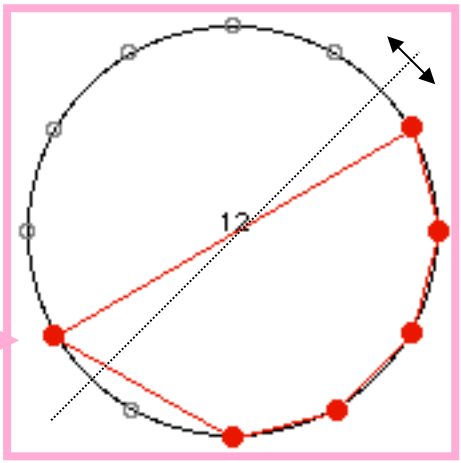
Serialisme and hexachordal combinatoriality

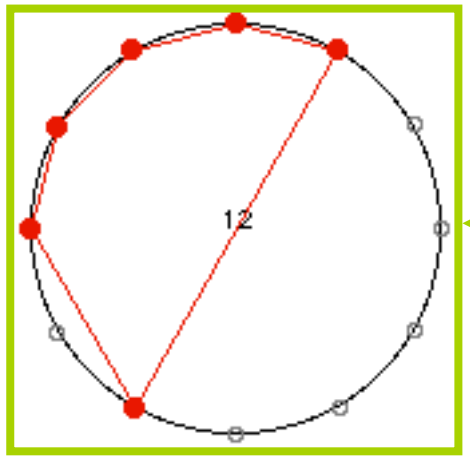
Schoenberg: Suite Op.25, Minuetto

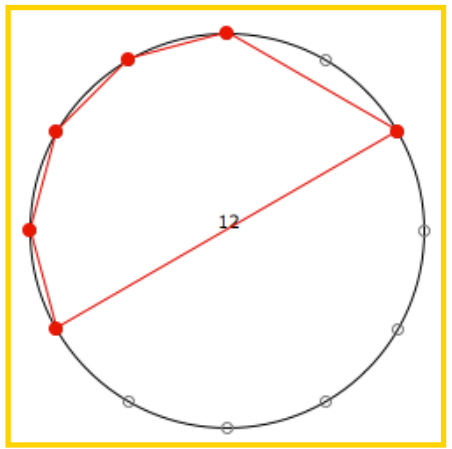




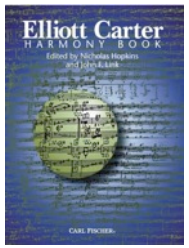
Double combinatoriality





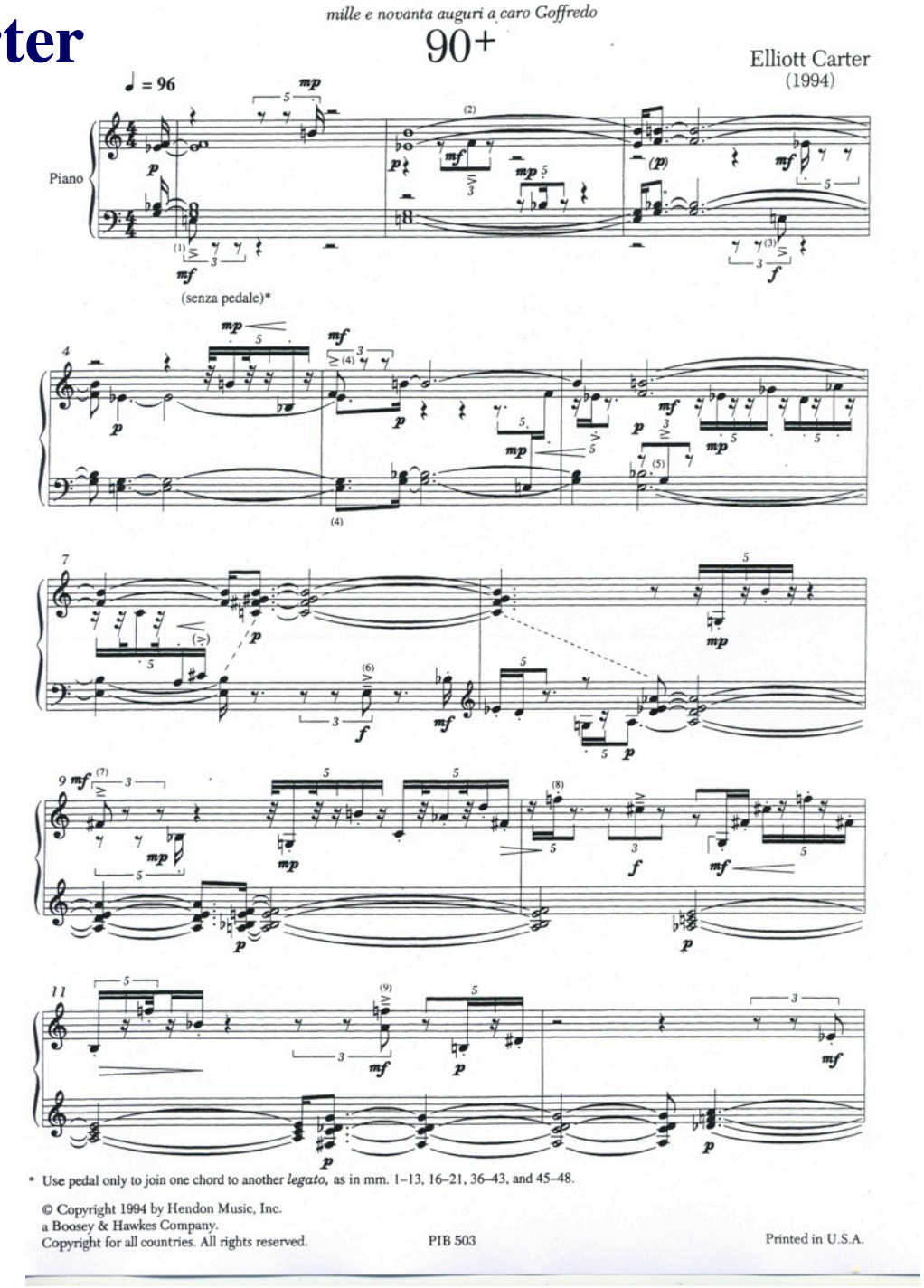


Set Theory chez Elliott Carter



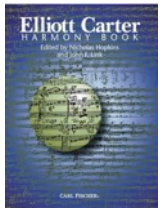
- **Combinatoire d'accords**
  - **Hexacordes**
  - **Tétracordes**
  - **Triades**
  - **Relation Z**
- **Séries tous–intervalles**
  - ***Link–chords***

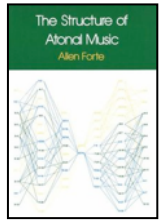
(piano: John Snijders)



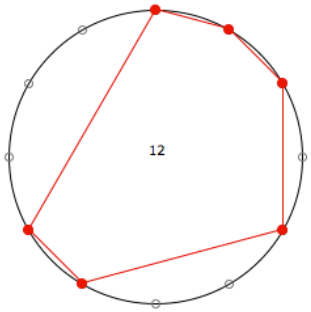
Combinatoire d'hexacordes

$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1



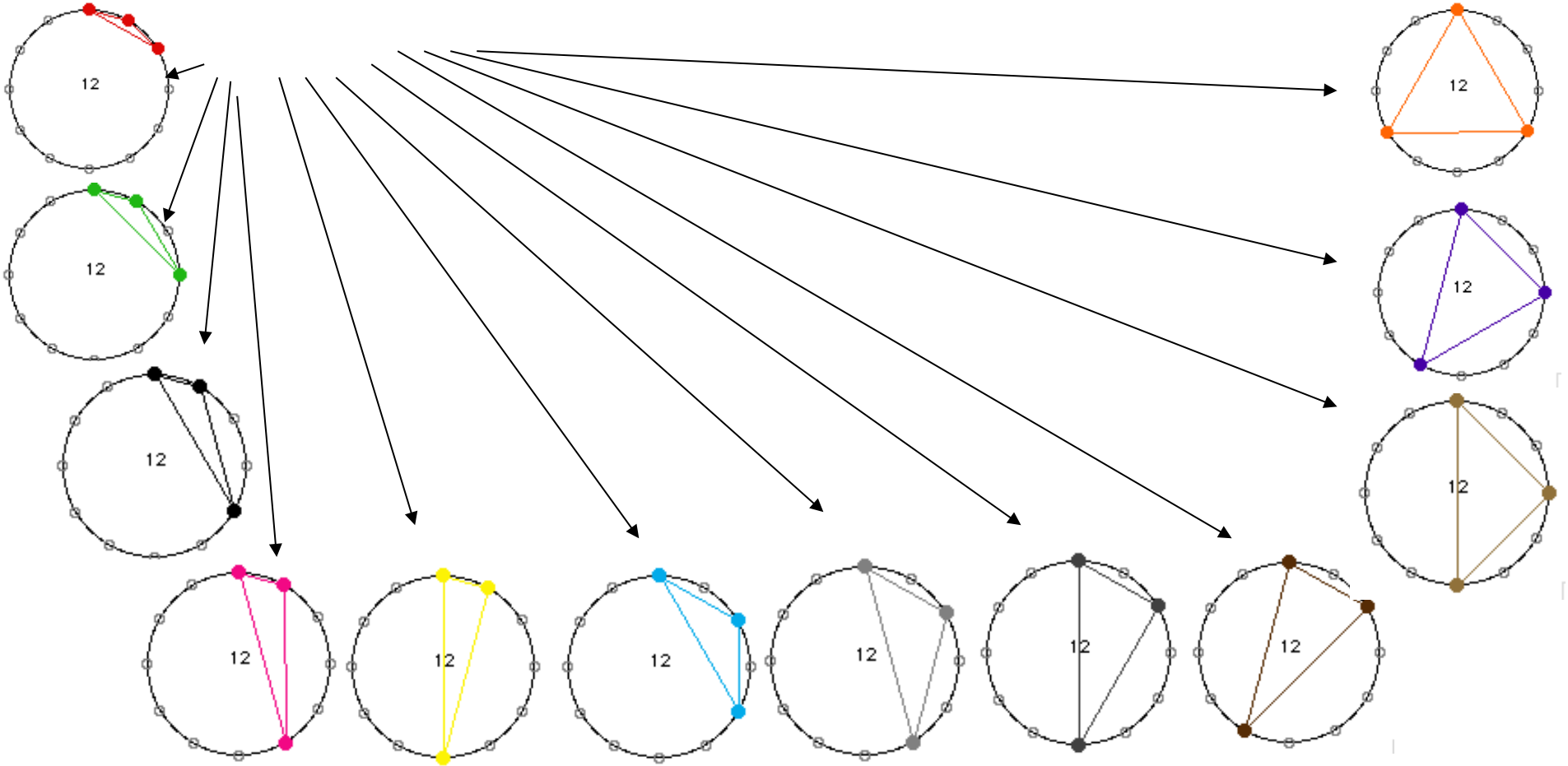


35

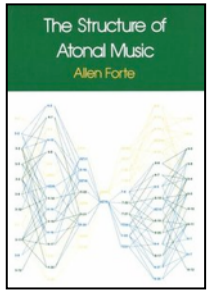


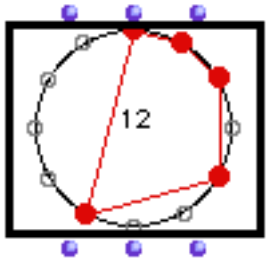
pcset	*vector*	*name*
(0 1 2 4 7 8)	[322332]	6-Z17

All-triad hexachord

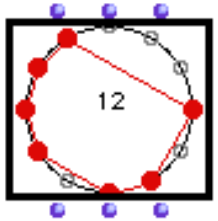


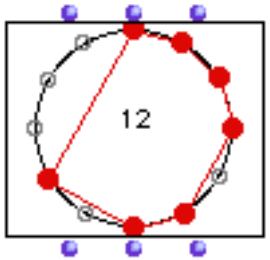
Le catalogue des *pcs* d'Allen Forte (1973) et la relation Z





complémentation





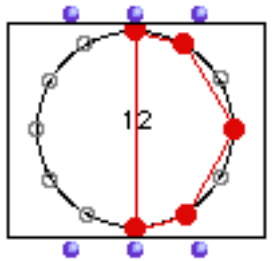
A. Forte (1926-)

5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140
5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211
6-Z4(12)	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442
6-Z37(12)	0,1,2,3,4,8	
6-Z38(12)	0,1,2,3,7,8	
6-Z39	0,2,3,4,5,8	
6-Z40	0,1,2,3,5,8	
6-Z41	0,1,2,3,6,8	
6-Z42(12)	0,1,2,3,6,9	

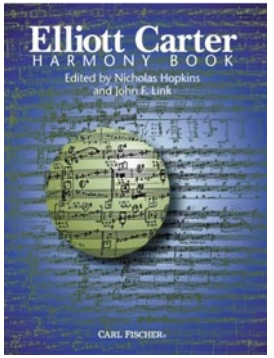
5-Z36 0,1,2,4,7 222121

7-Z36 0,1,2,3,5,6,8 444342

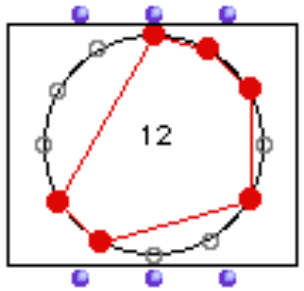
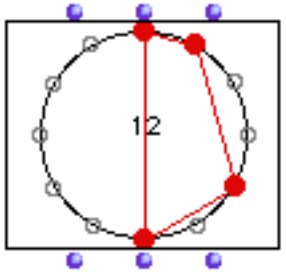
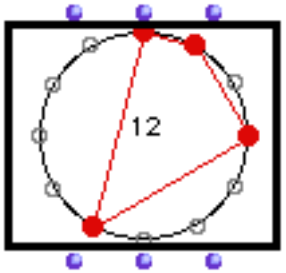


5-Z12

Elliott Carter: 90+ (1994)

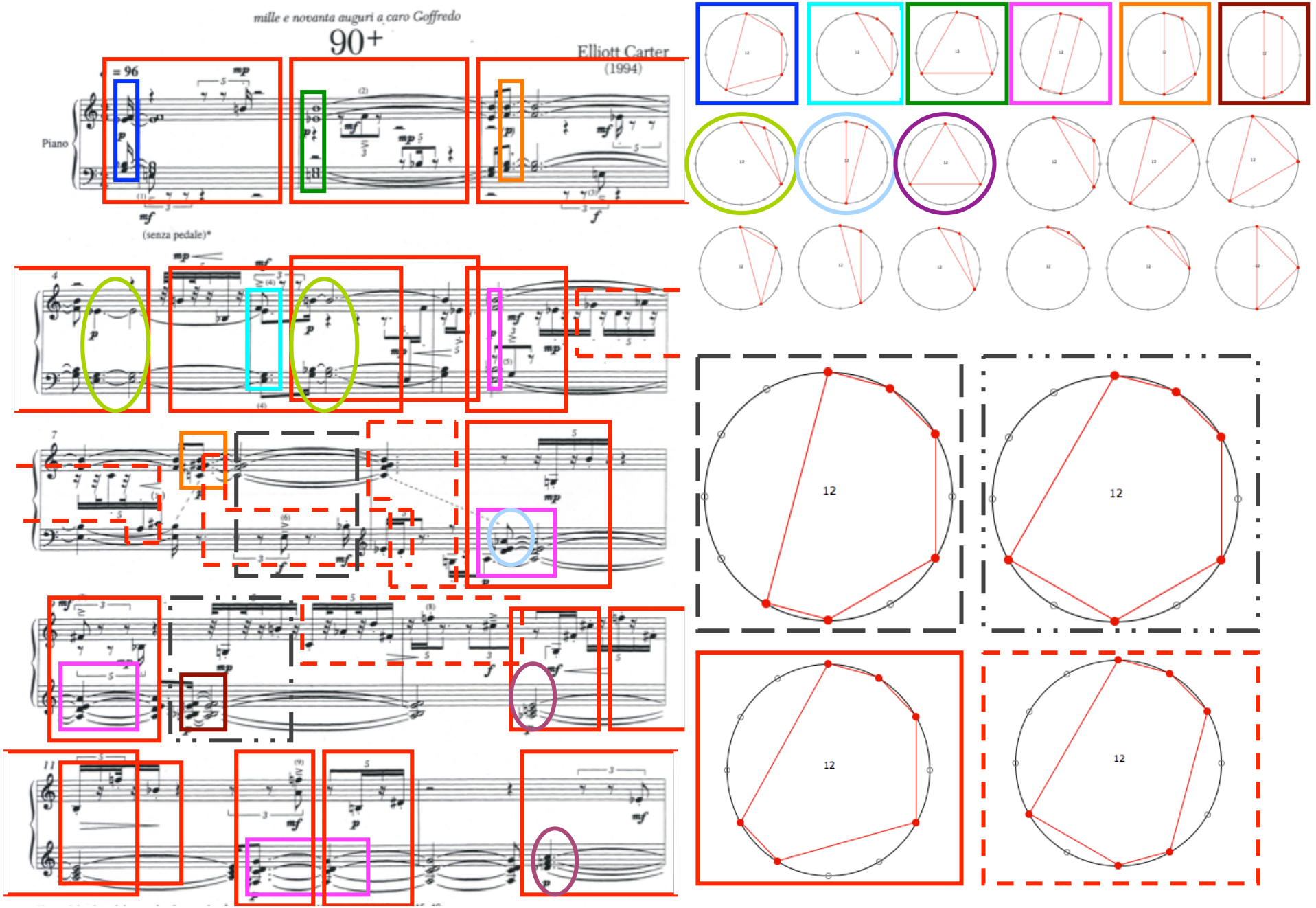


« *From about 1990, I have reduced my vocabulary of chords more and more to the six note chord n° 35 and the four note chords n° 18 and 23, which encompass all the intervals* » (*Harmony Book*, 2002, p. ix)

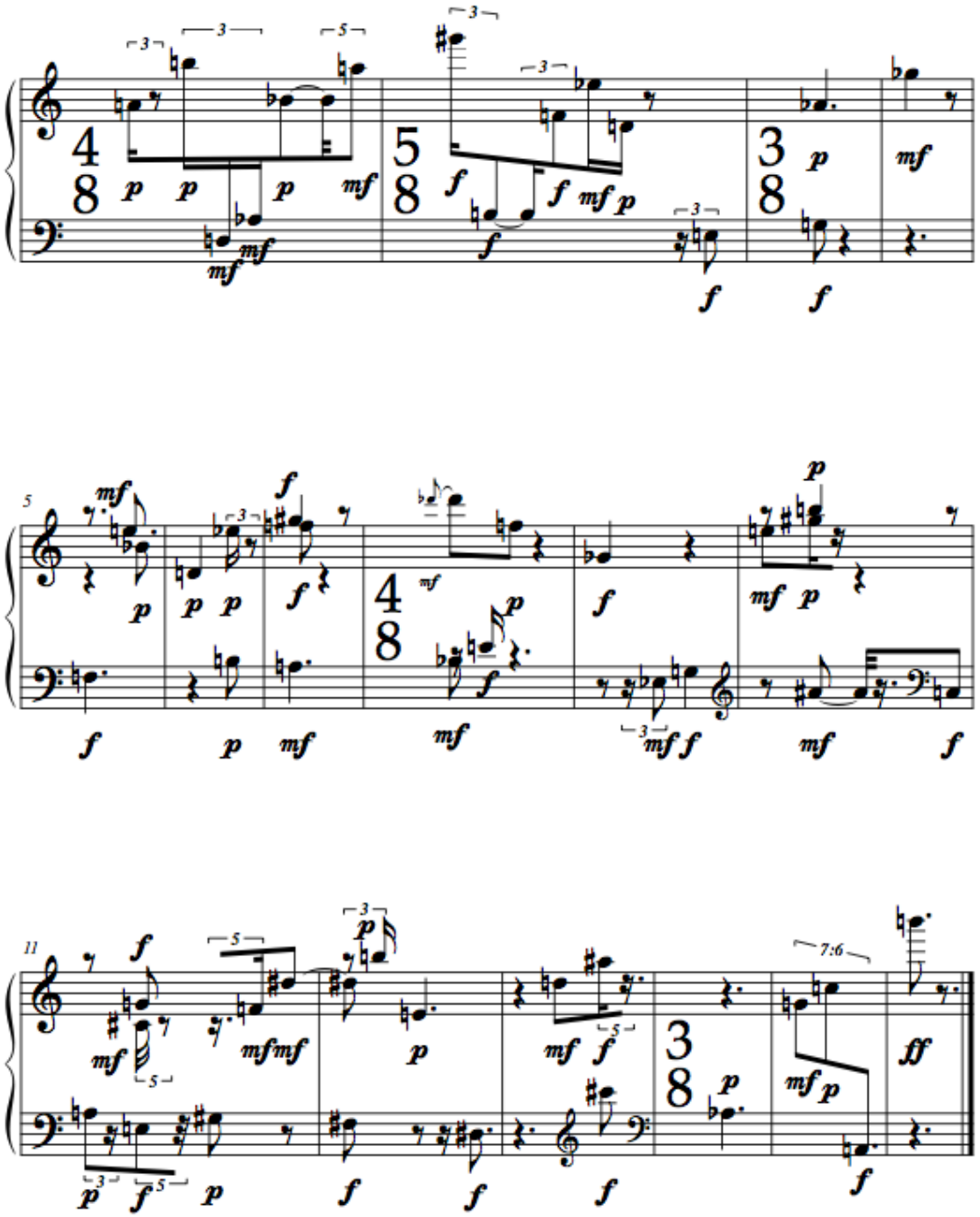
		pcset	*vector*	*name*
35		(0 1 2 4 7 8)	[322332]	6-Z17
18		(0 1 4 6)	[111111]	4-Z15
23		(0 1 3 7)	[111111]	4-Z29

All-triad hexachord

Elliott Carter : 90+ (1994) : combinatoire tetra/tricordale



Set Theory chez K. Stockhausen



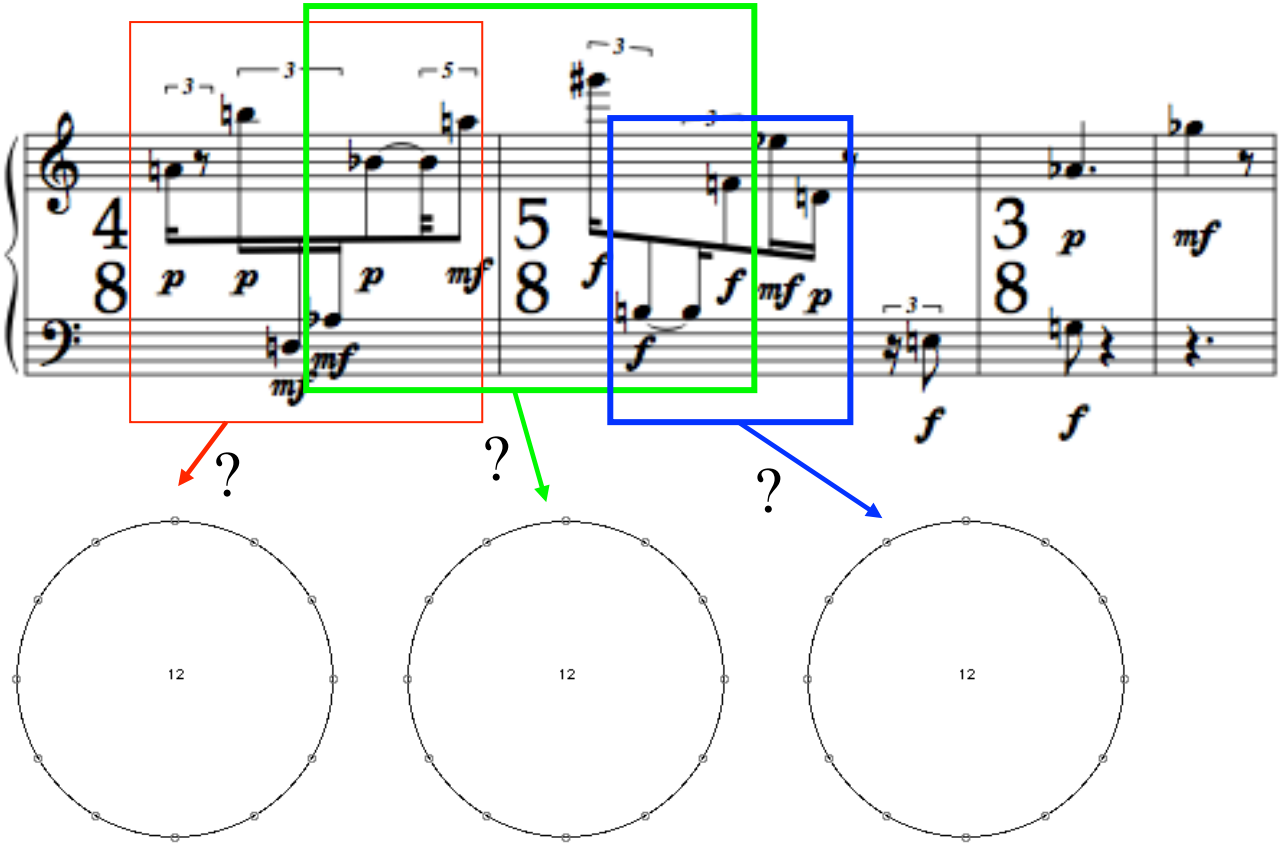
Trois interprétations :

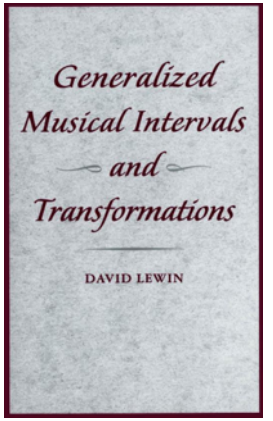
Henck

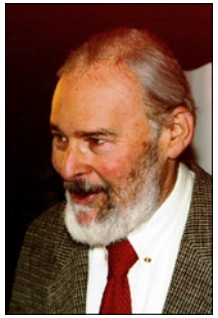
Kontarsky

Tudor

« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »



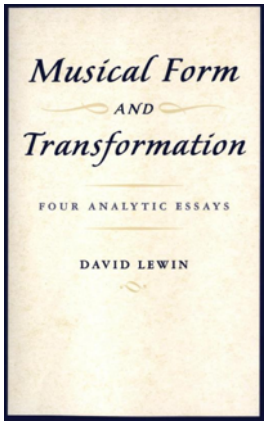




D. Lewin
(1933-2003)

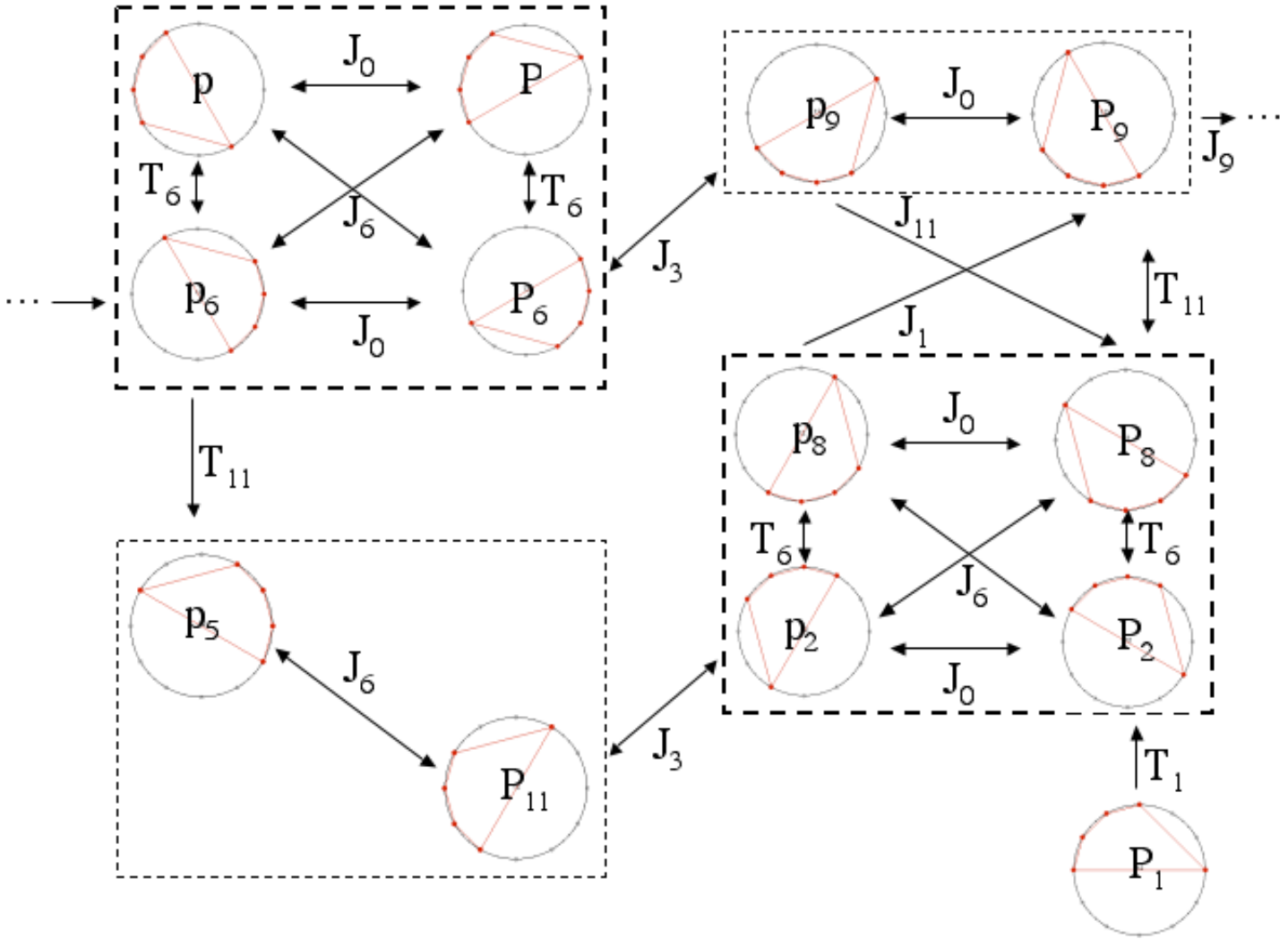
« The most 'theoretical' of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

David Lewin, *Musical Form and Transformation,* YUP 1993



Reseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)



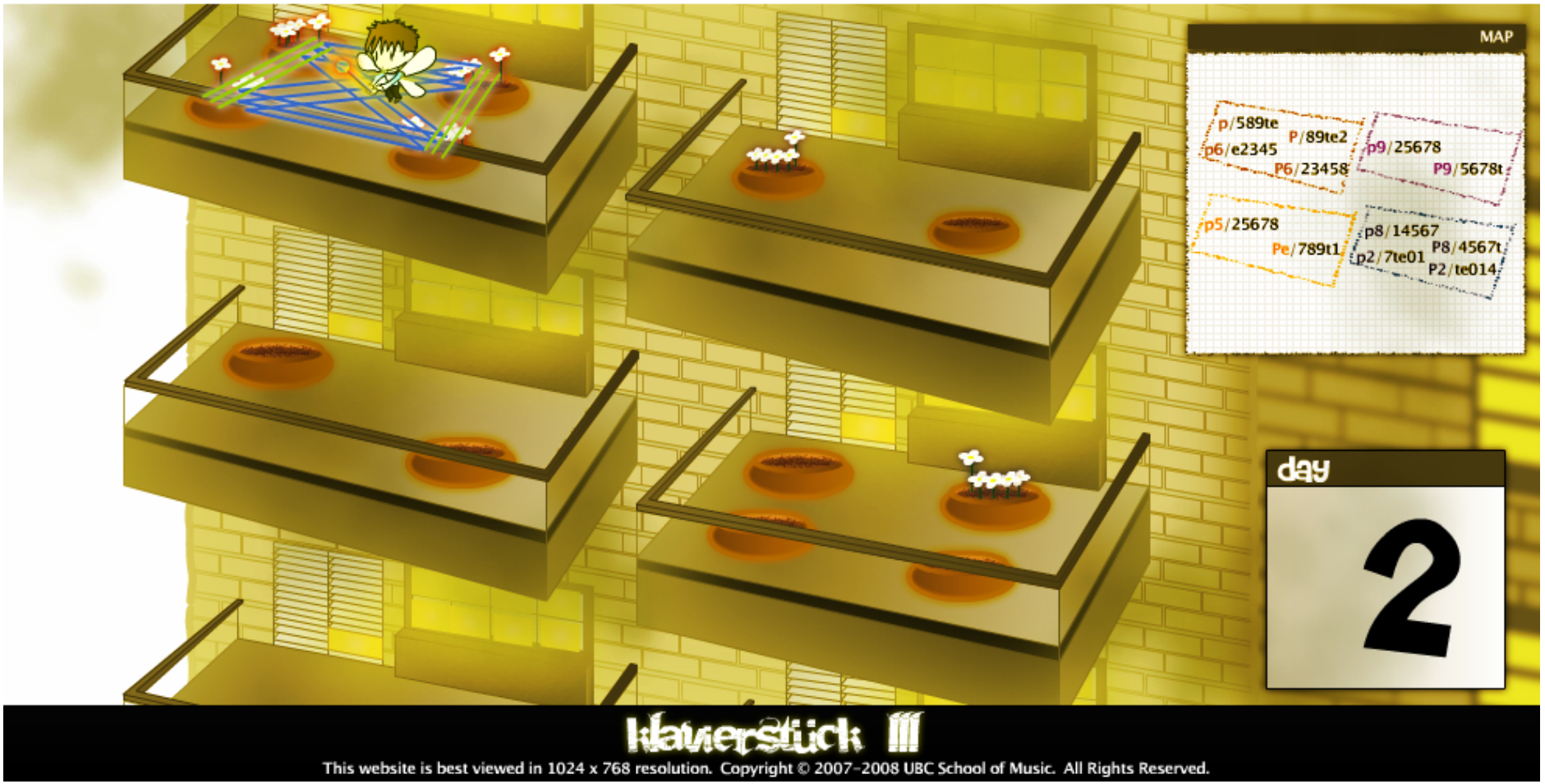
« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their **potentially functional interrelationships**, in a very compactly organized little **spatial configuration**. »

« […] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - **it makes the abstract space of such a world accessible to our sensibilities**. That is to say that the story projects what one would traditionally call *form*. »

Visualisations multimédia de l'analyse transformationnelle

R. Attas : Metaphors in Motion: Agents and Representation in Transformational Analysis, *MTO*, 15(1), 2009
http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html

Animation 1. Klavierstück III

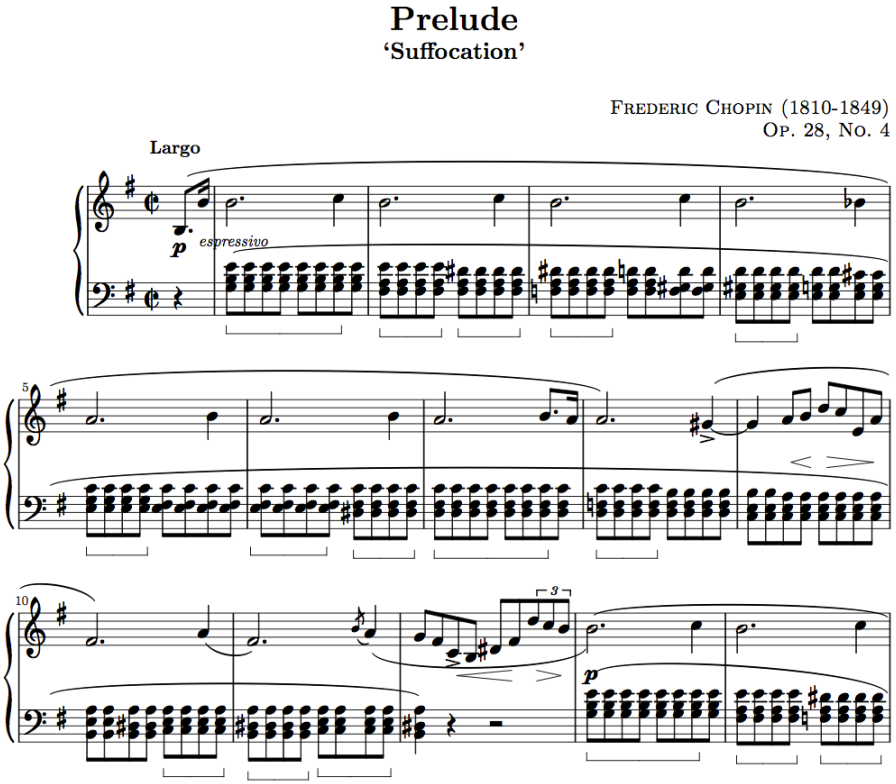


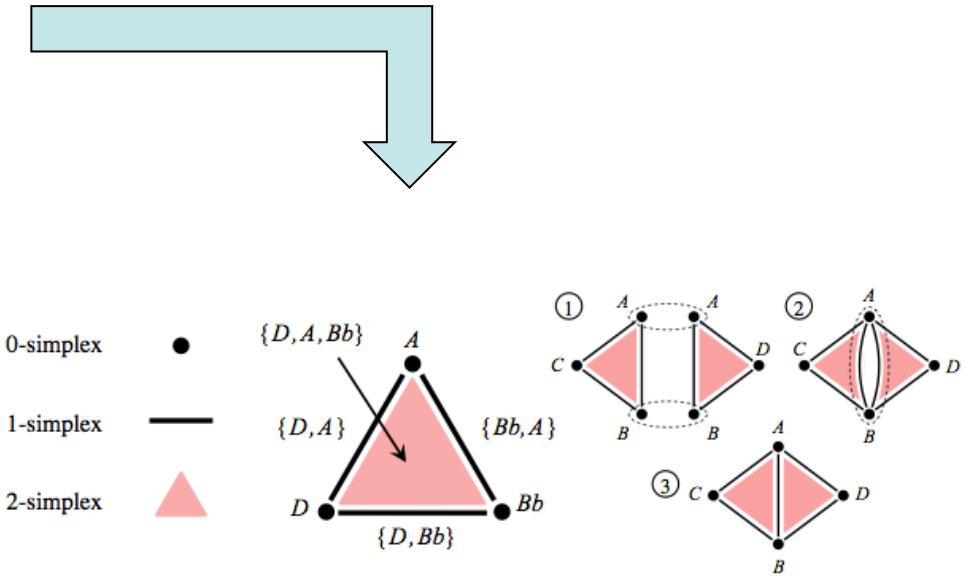
Visualisations multimédia de l'analyse transformationnelle

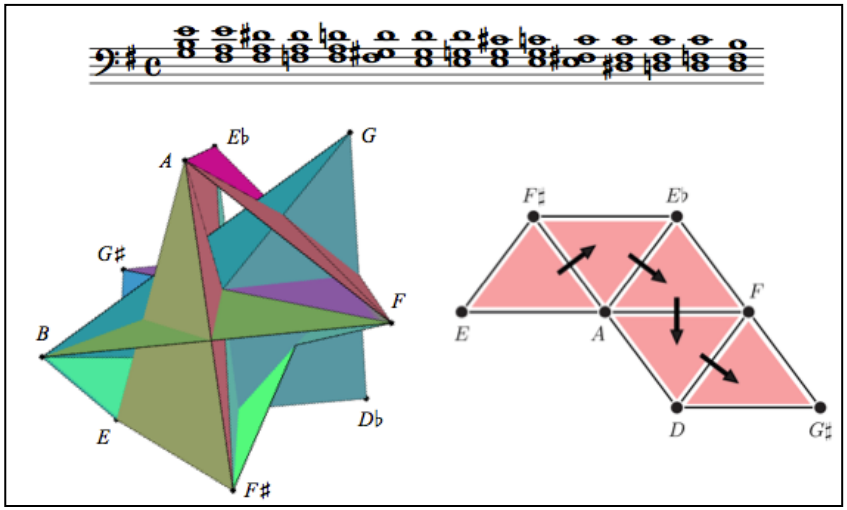
R. Attas : Metaphors in Motion: Agents and Representation in Transformational Analysis, *MTO*, 15(1), 2009
http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html

Animation 2. Grow Your Own Pentachord

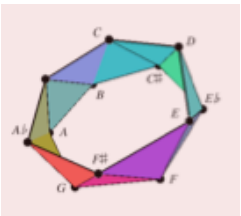
L’espace géométrique d’un *Prélude* de Chopin



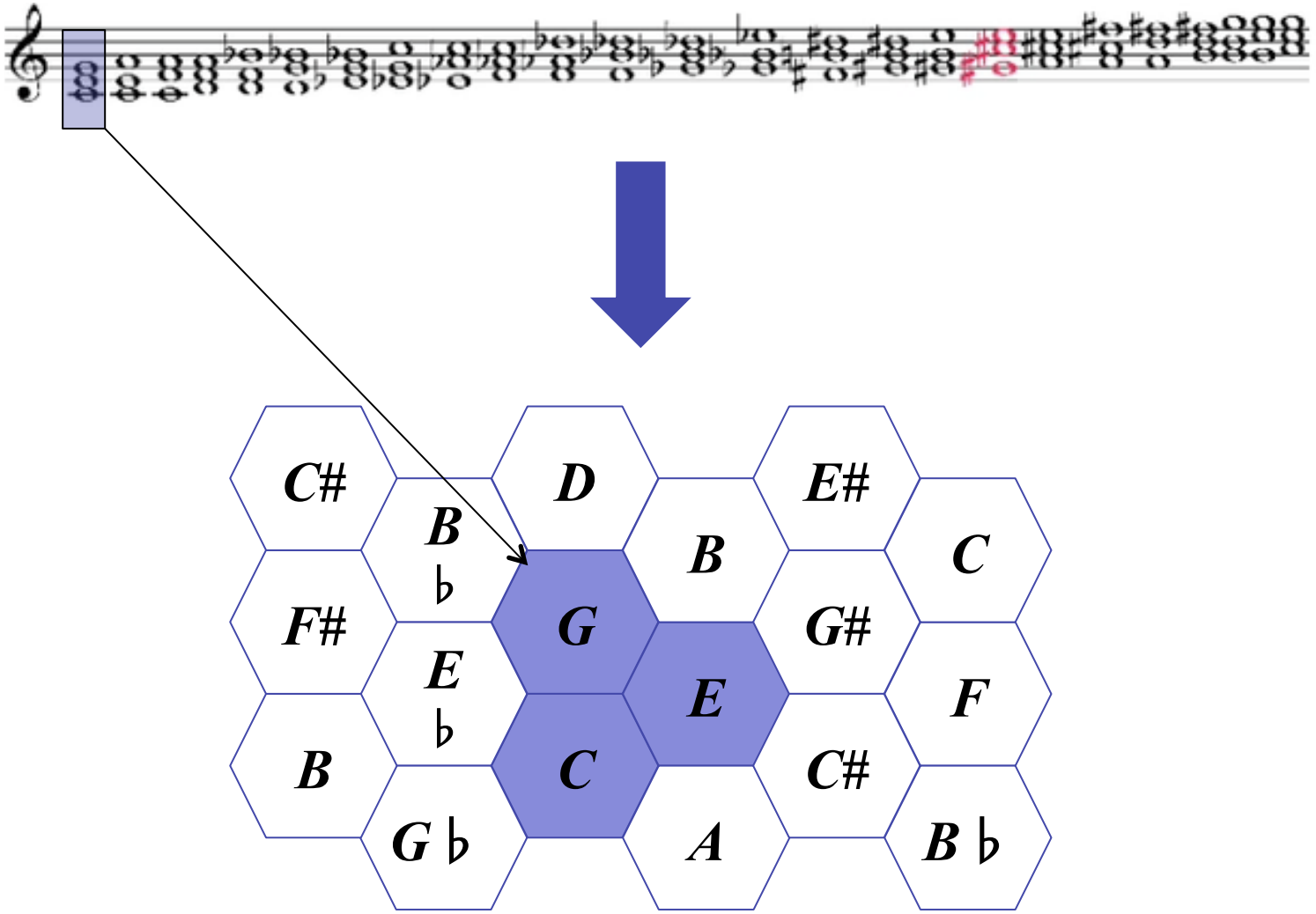




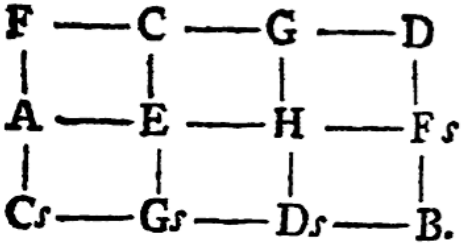
Louis Bigo, *Représentations symboliques musicales et calcul spatial*,
Thèse LAC/Ircam



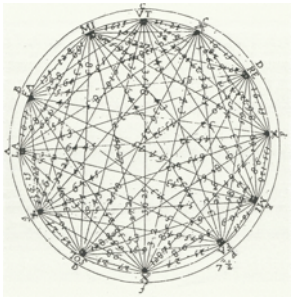
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



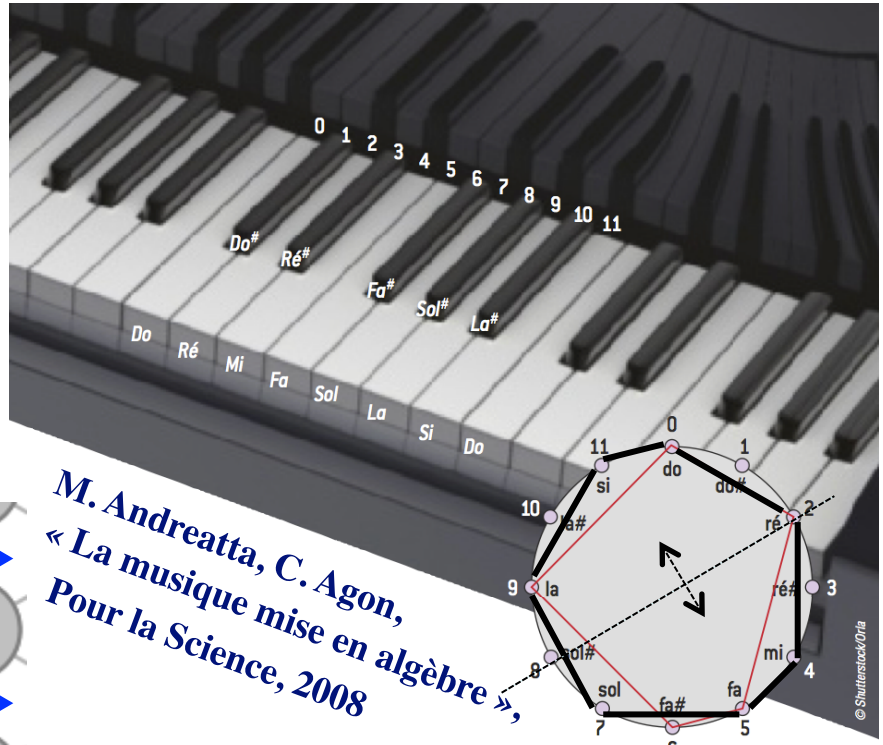
Deux représentations équivalentes : le cercle et le tore



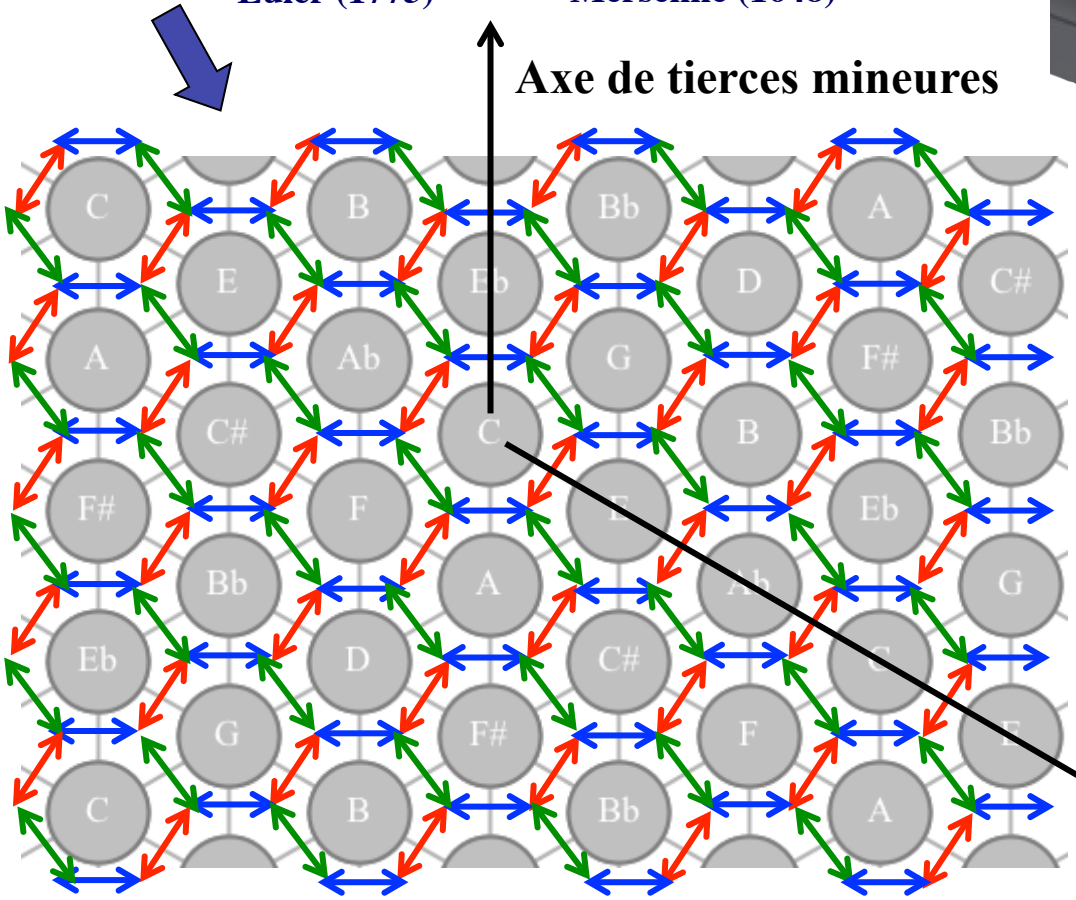
Euler (1773)



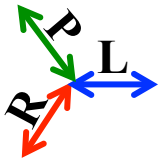
Mersenne (1648)

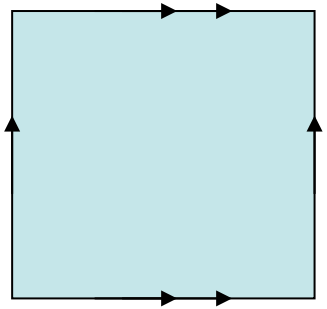


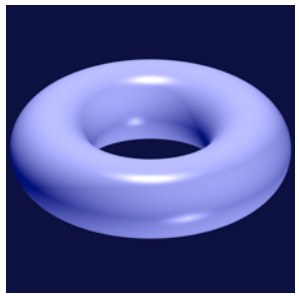
M. Andreatta, C. Agon, « La musique mise en algèbre », Pour la Science, 2008



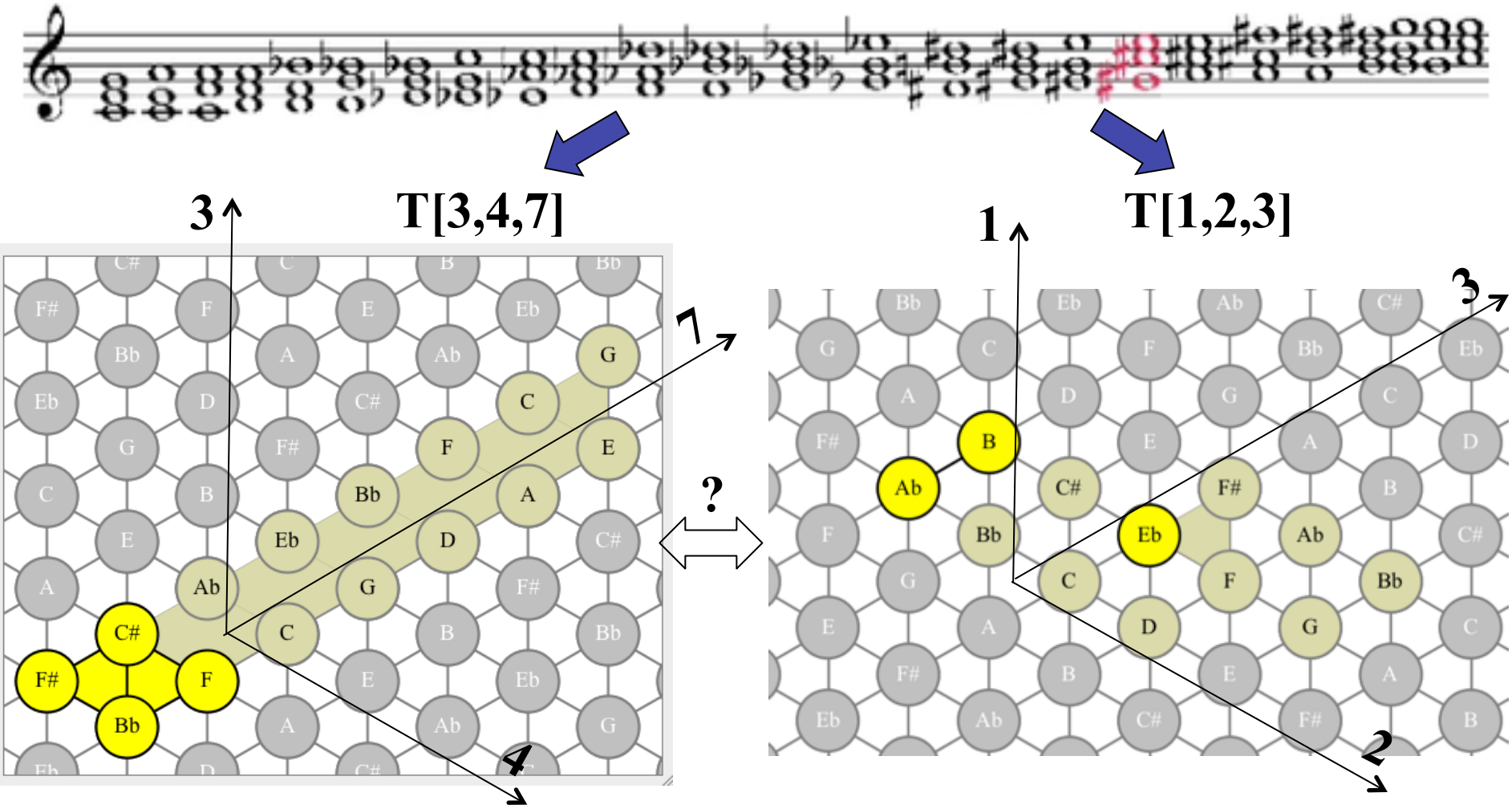
Hexachord (Louis Bigo, 2013)





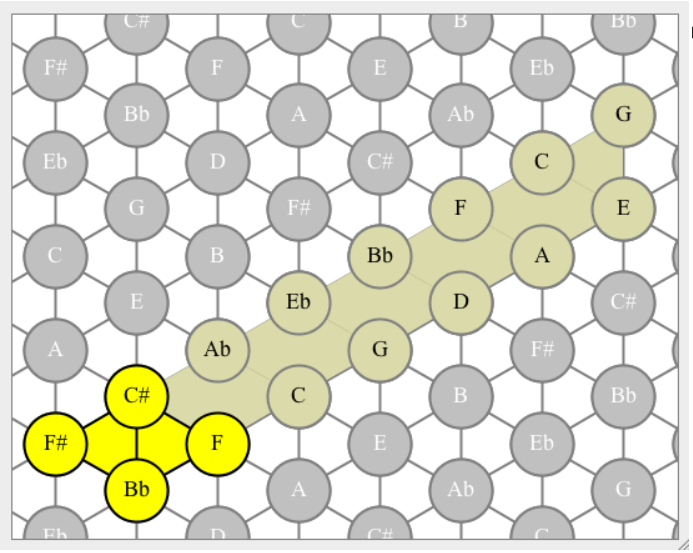


Le caractère spatial de la « logique musicale »



L. Bigo, M. Andreatta, J.-L. Giavitto, O. Michel, et A. Spicher, « Computation and Visualization of Musical Structures in Chord-based Simplicial Complexes », Proceedings Mathematics and Computation in Music Conference 2013 – Montreal, Springer

Le caractère spatial du « style musical »



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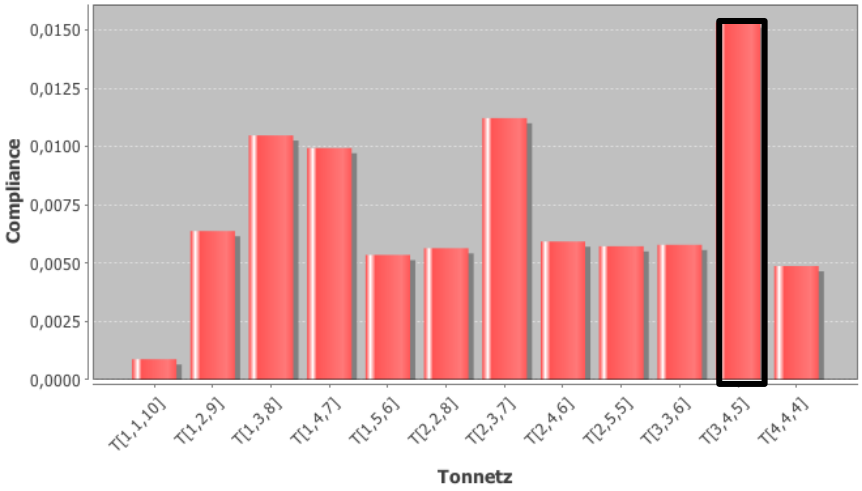
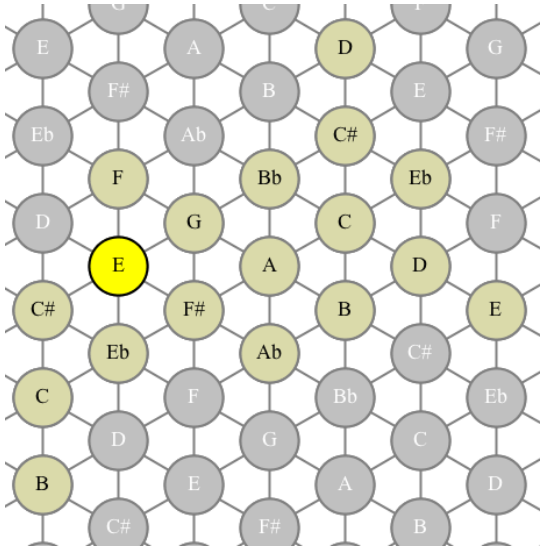


Beethoven, 2e mouvement de la *9e Symphonie*



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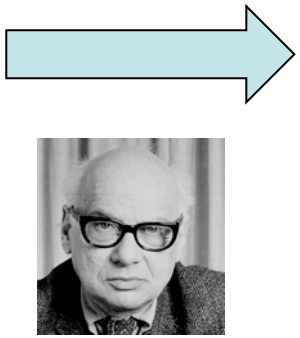
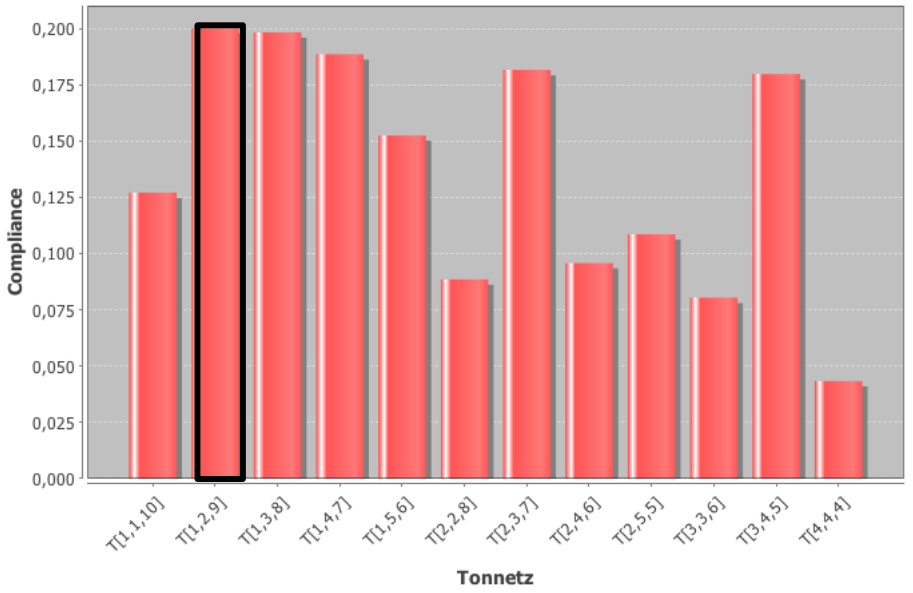
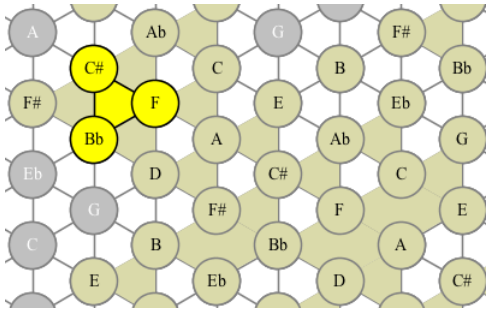


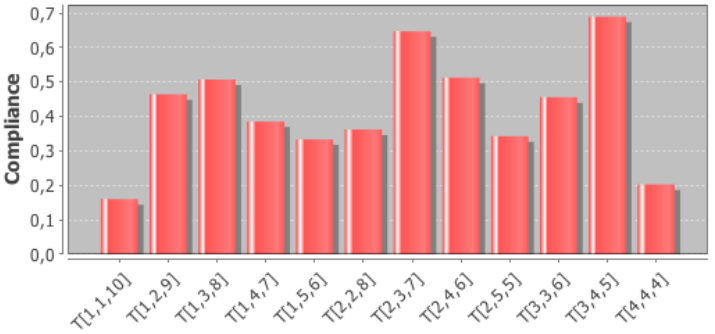


Babbitt, *Semi-Simple Variations*

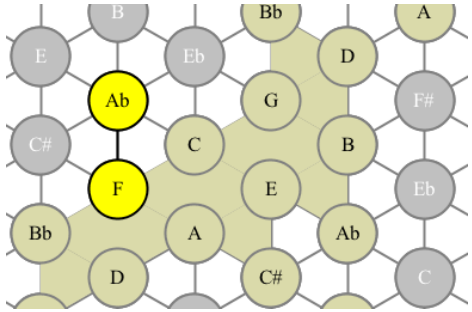
L’espace géométrique comme paramètre de style

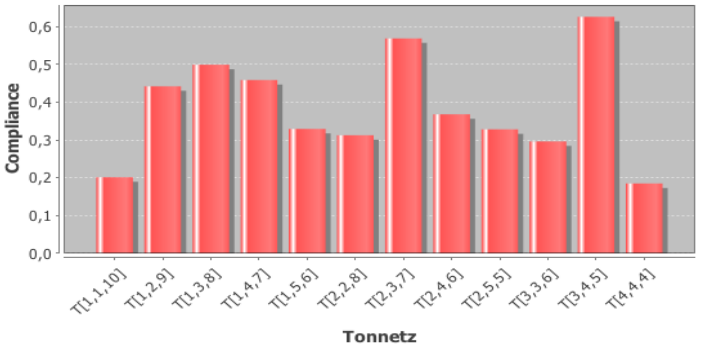
Thelonious Monk, Brilliant Corners



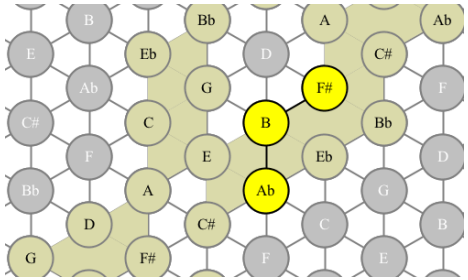


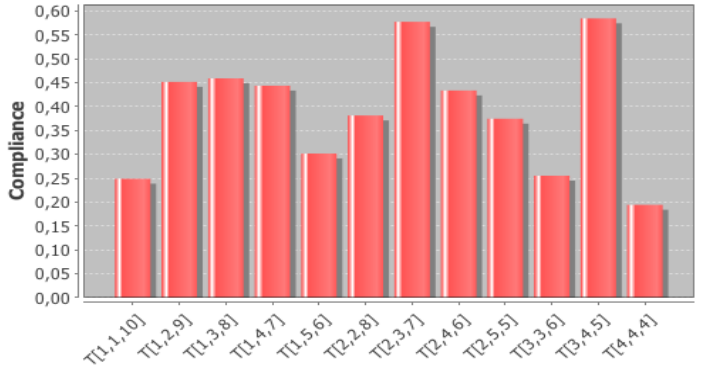
Chick Corea, Eternal Child





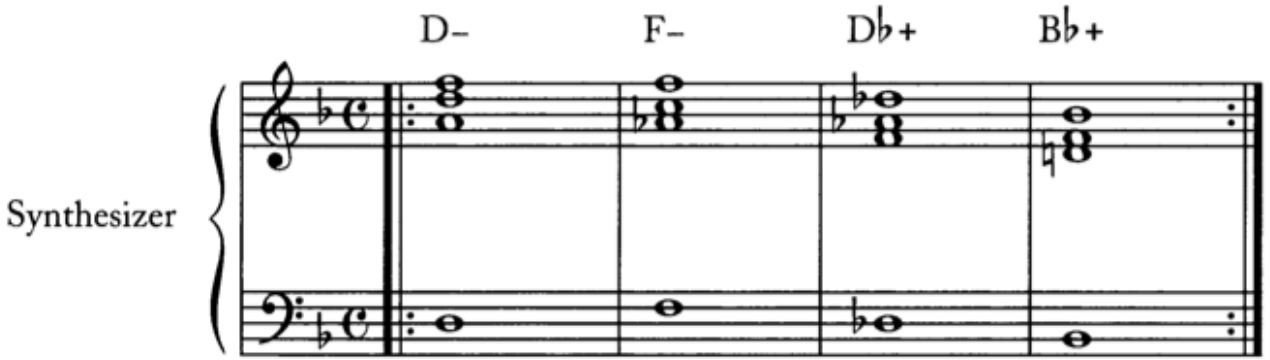
Bill Evans, Turn Out the Stars



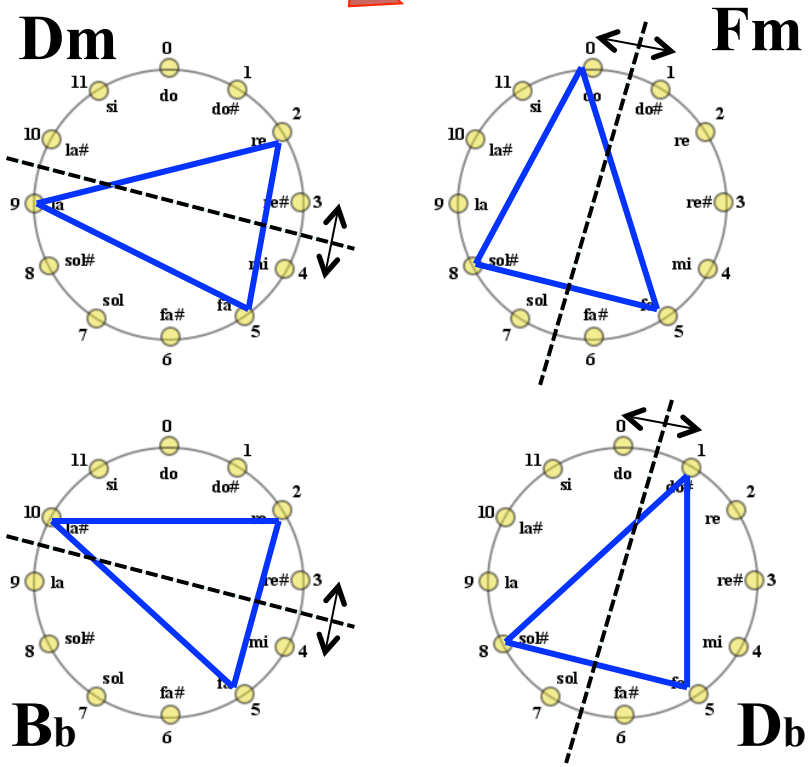


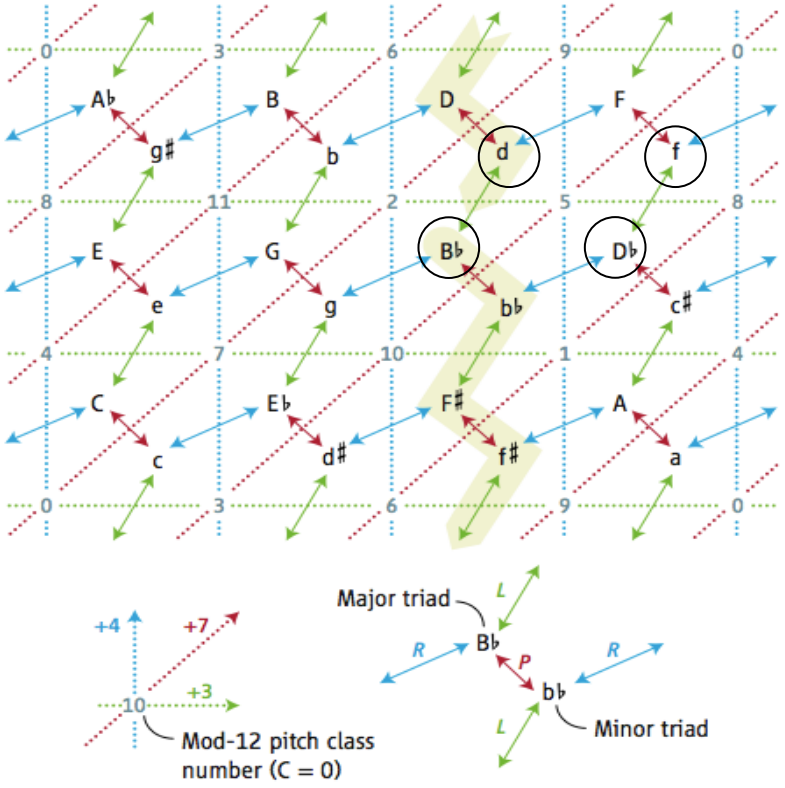
Mise en évidence des symétries dans la musica pop

- Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), p. 177-199, 2004



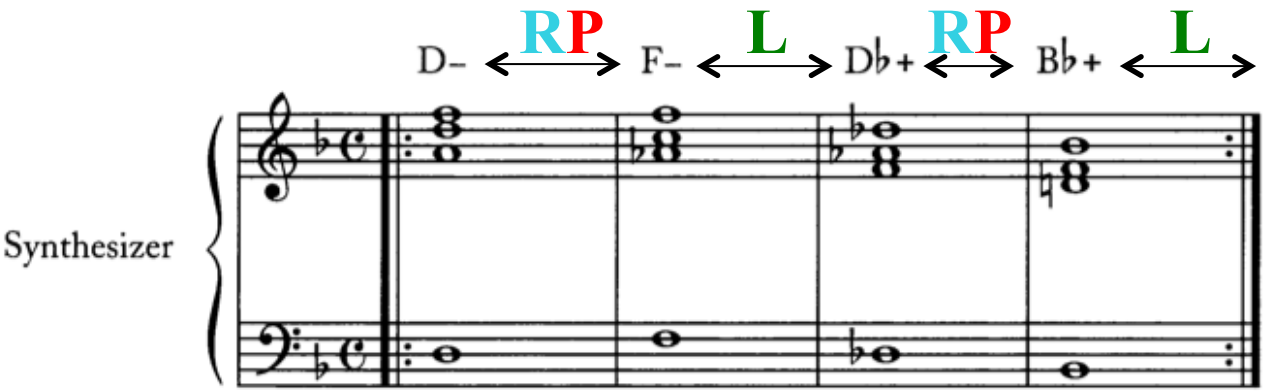
Shake the disease - 1985
(Depeche Mode) – min. 2'17"





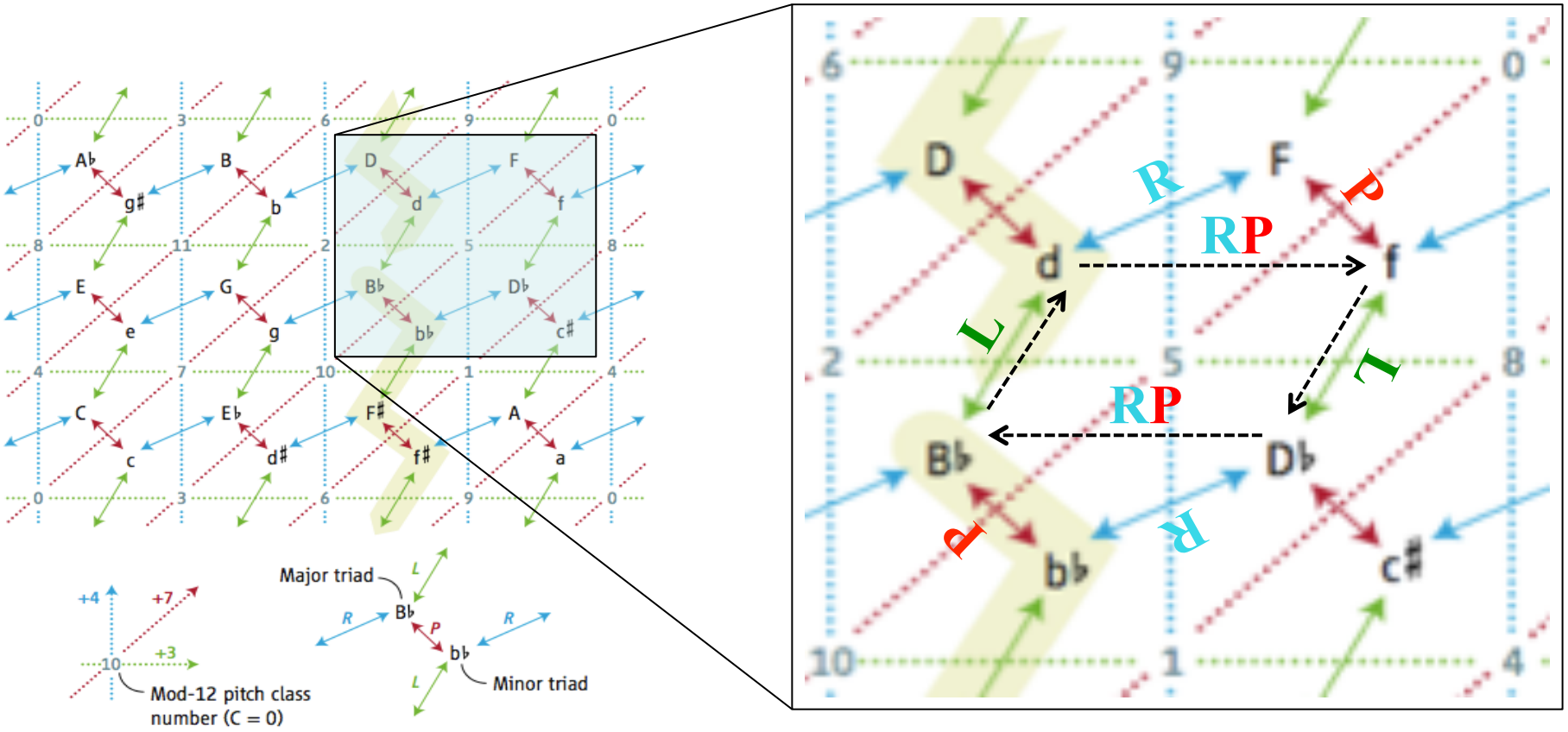
Trajectoire et progression harmonique dans le *Tonnetz*

- Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), p. 177-199, 2004



***Shake the disease* - 1985**
(Depeche Mode)

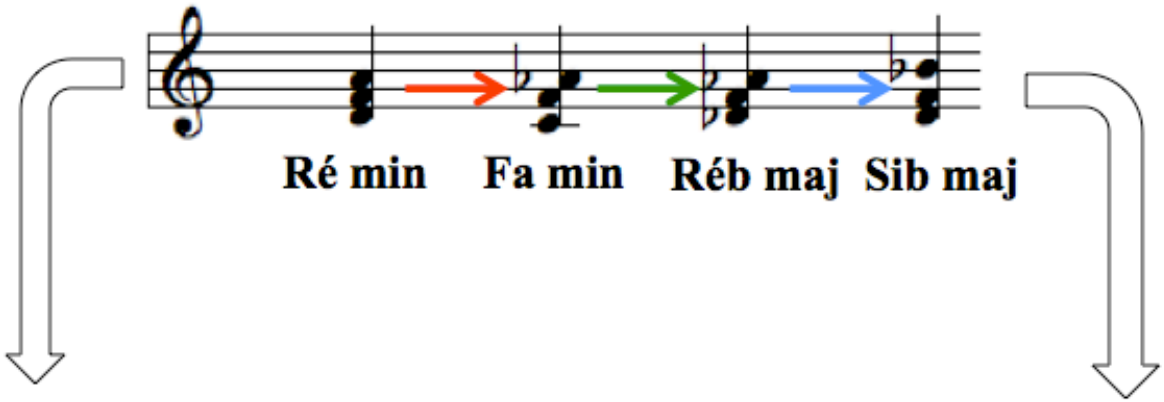
Séquence RPLRPL

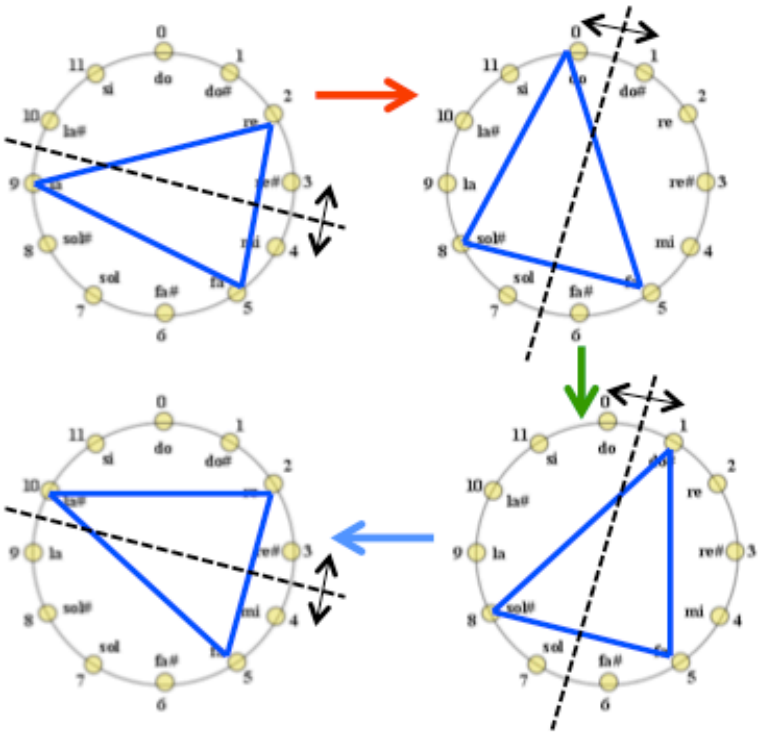


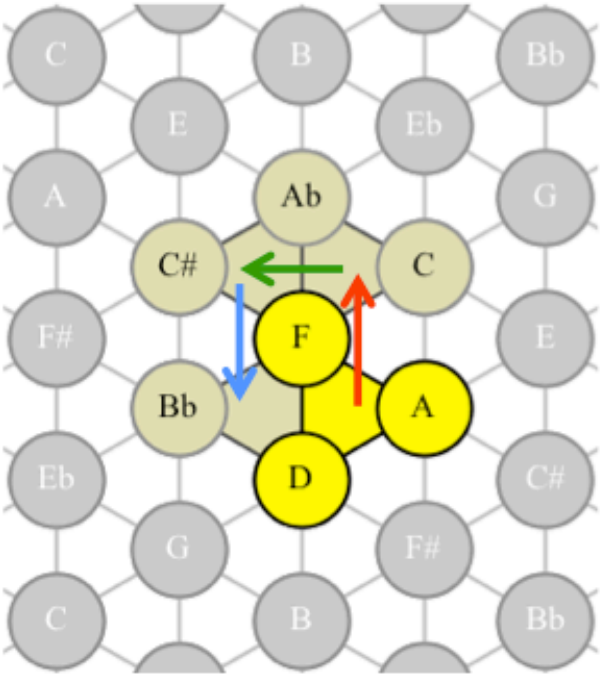
Mise en évidence des symétries dans la musica pop

- **Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), p. 177-199, 2004**

***Shake the disease* - 1985**
(Depeche Mode)



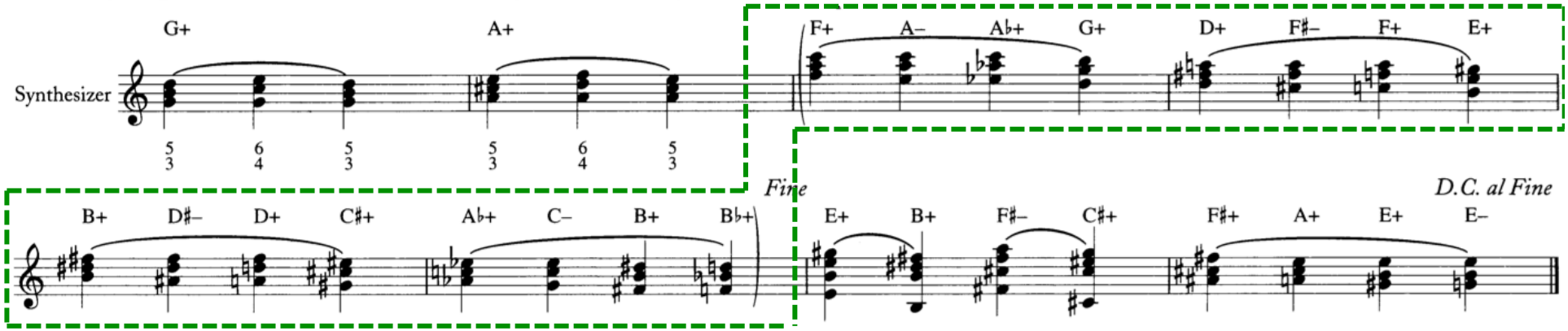




➔ `Hexachord` (by Louis Bigo, 2013)

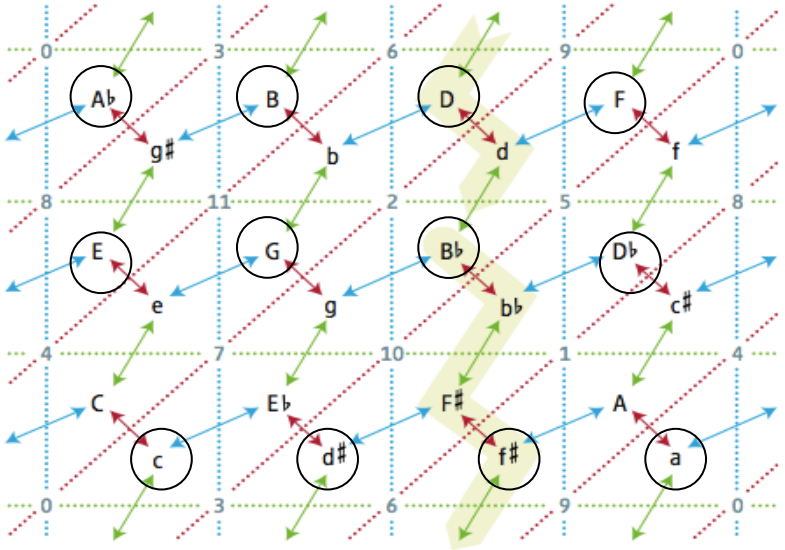
Progressions harmoniques chez Zappa: où sont les symétries ?

- **Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), p. 177-199, 2004**



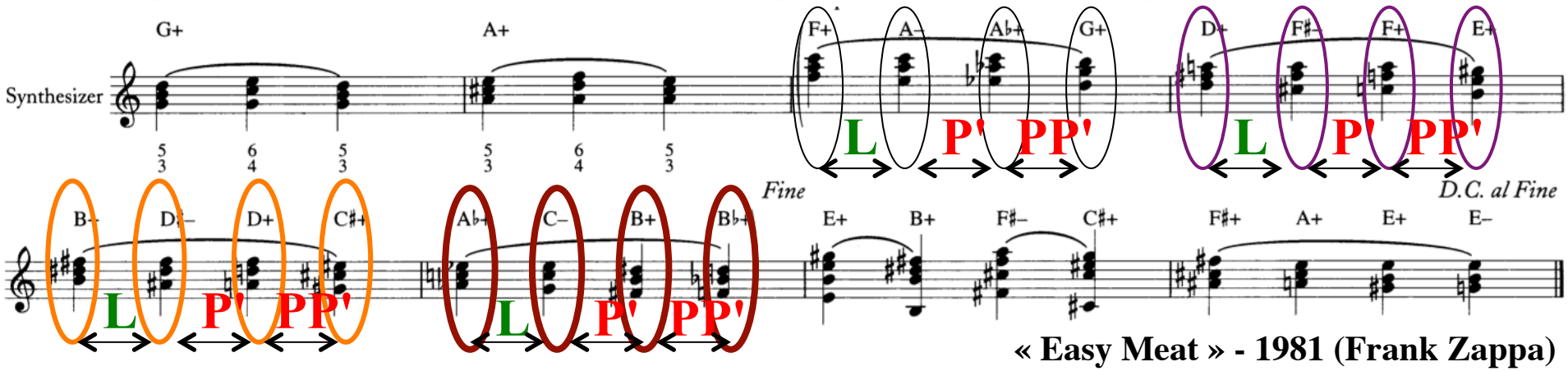
« Easy Meat » - 1981 (Frank Zappa)

(min. 2'29")

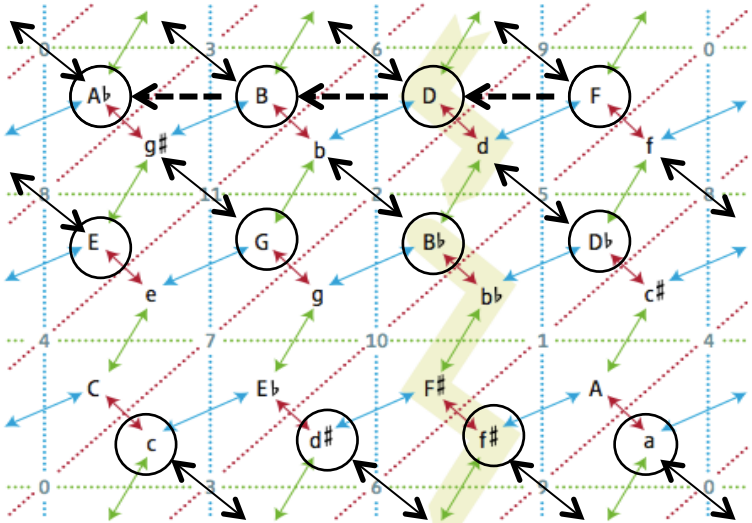


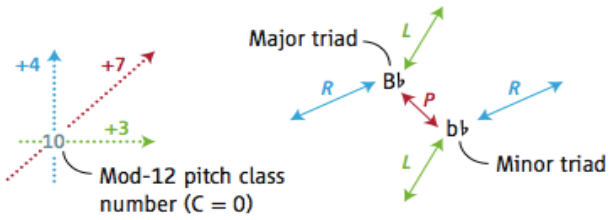
La cellule génératrice et ses translations spatiales

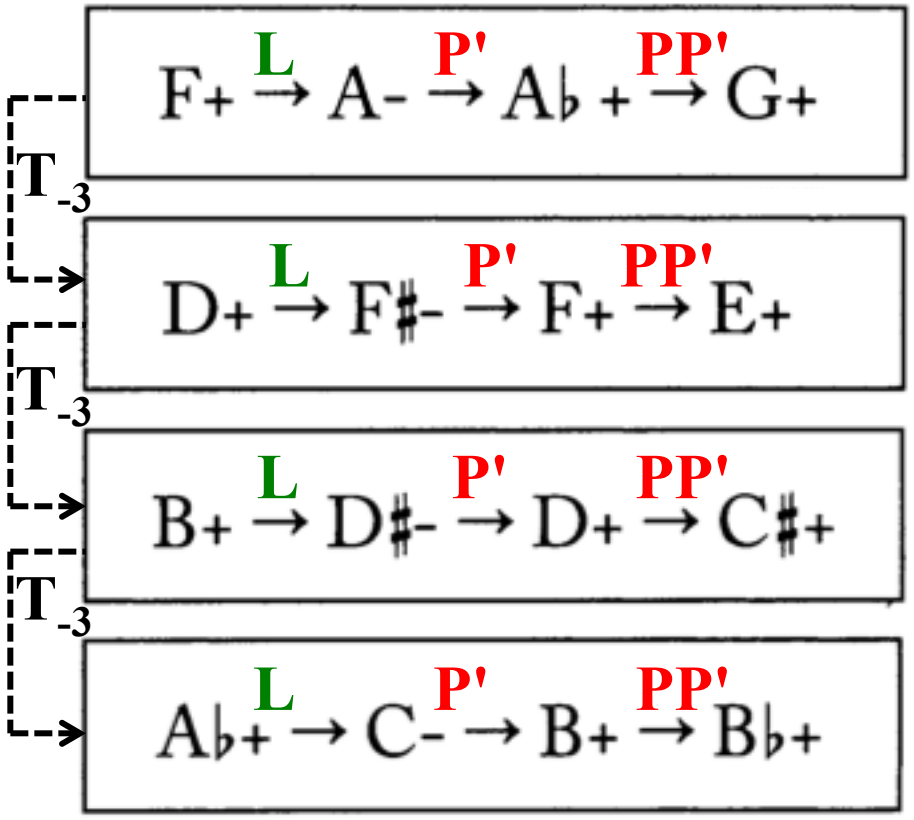
- Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music", Music Theory Spectrum 26(2), p. 177-199, 2004



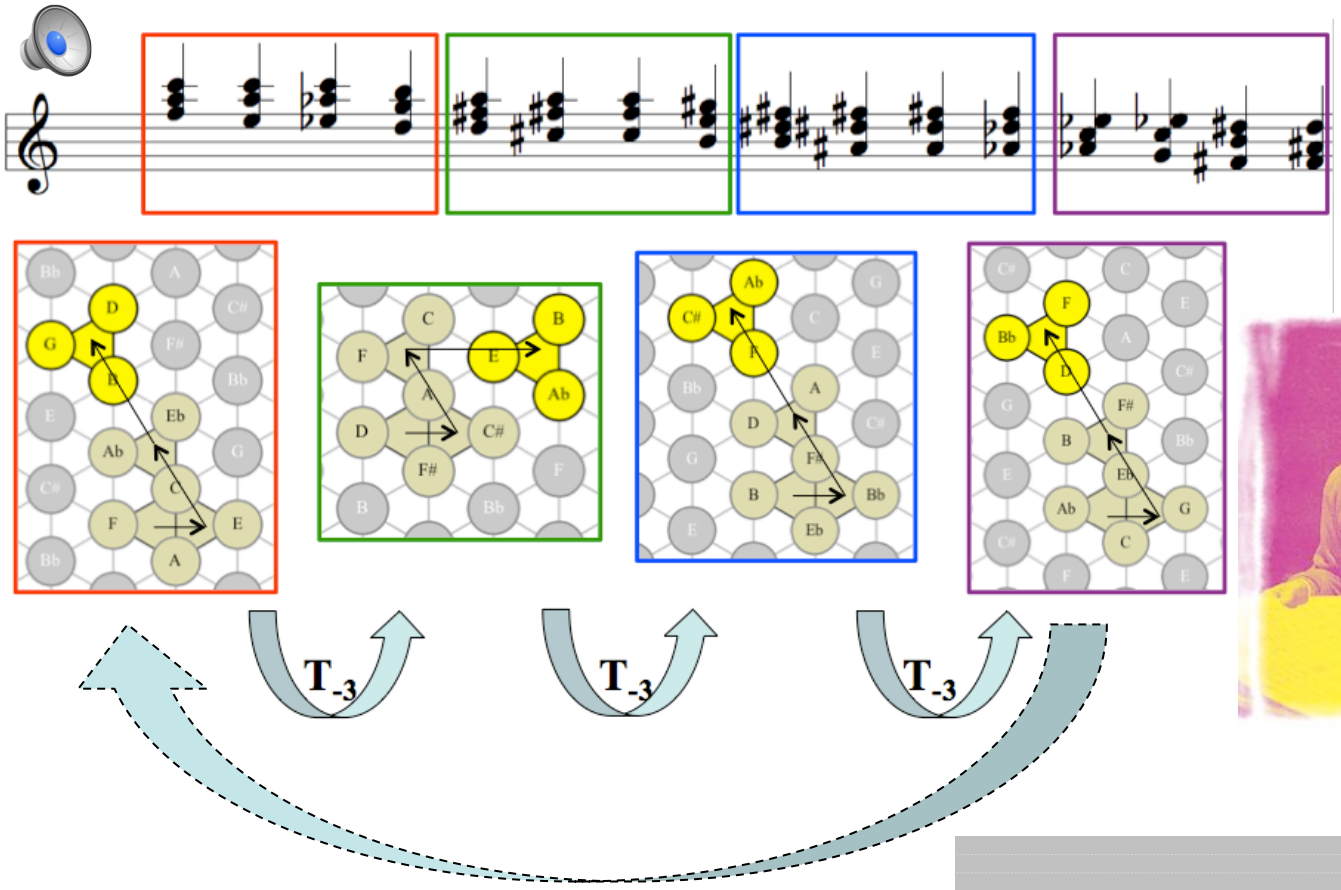
« Easy Meat » - 1981 (Frank Zappa)

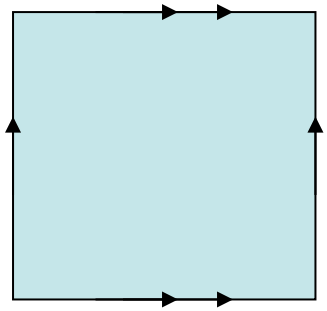


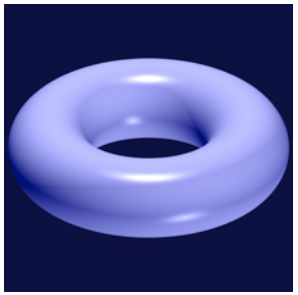


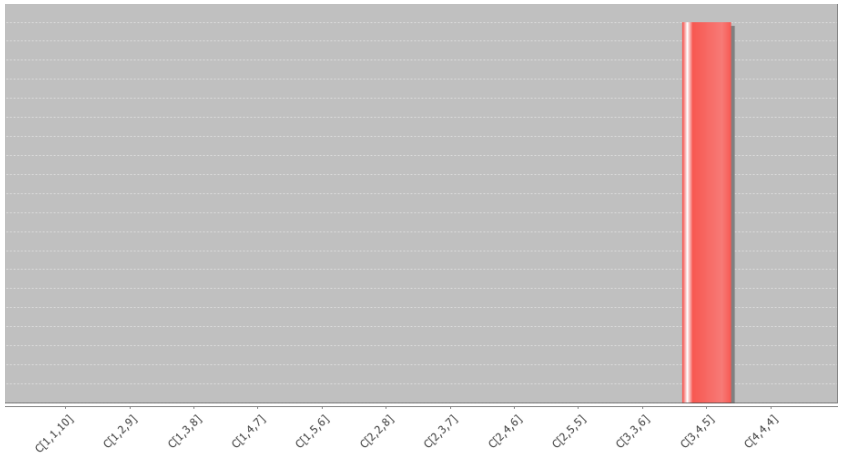


L’espace (de la progression harmonique)

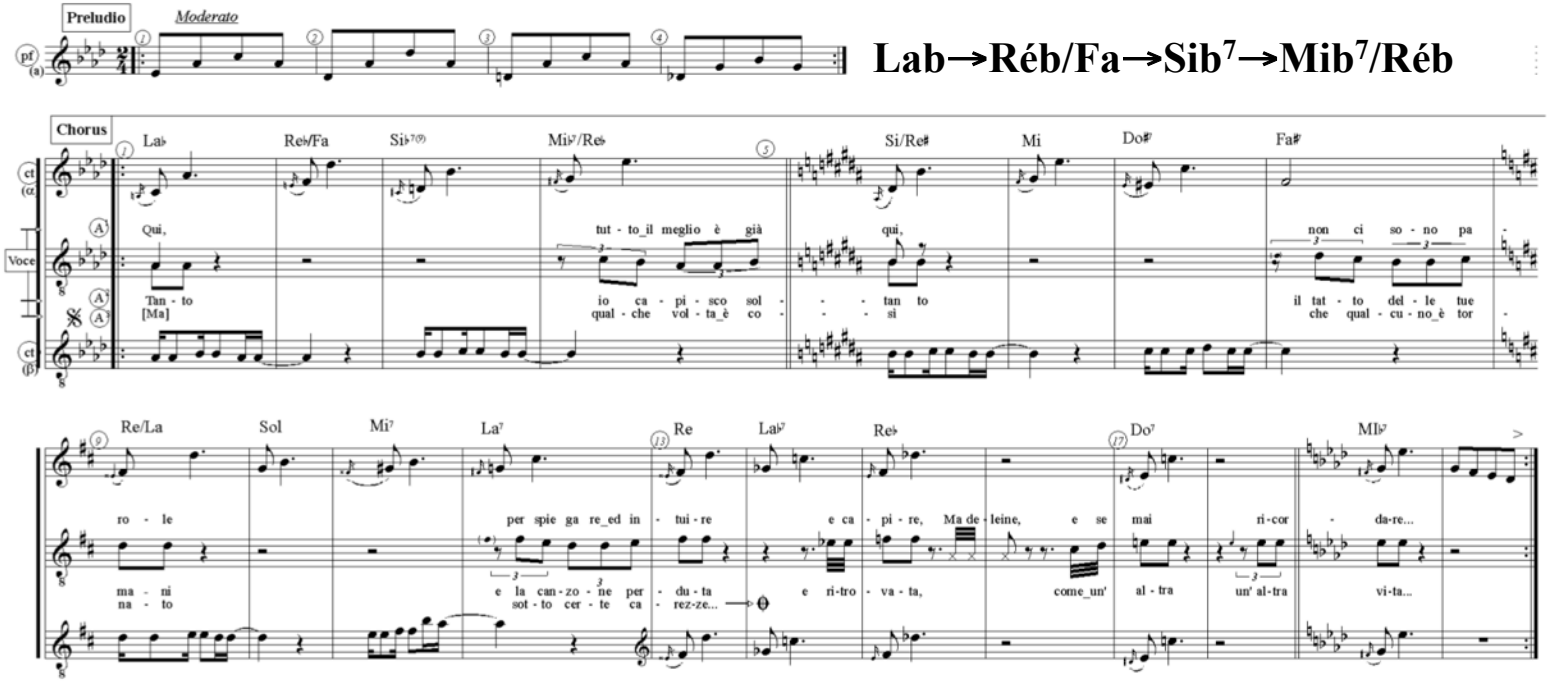






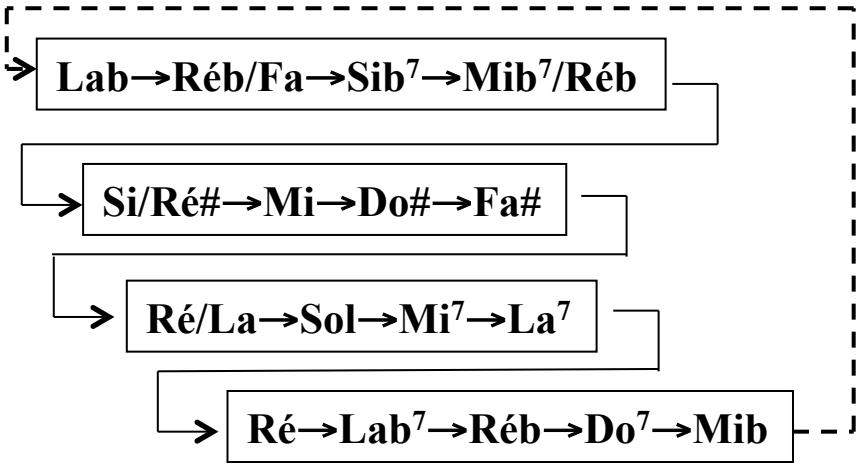


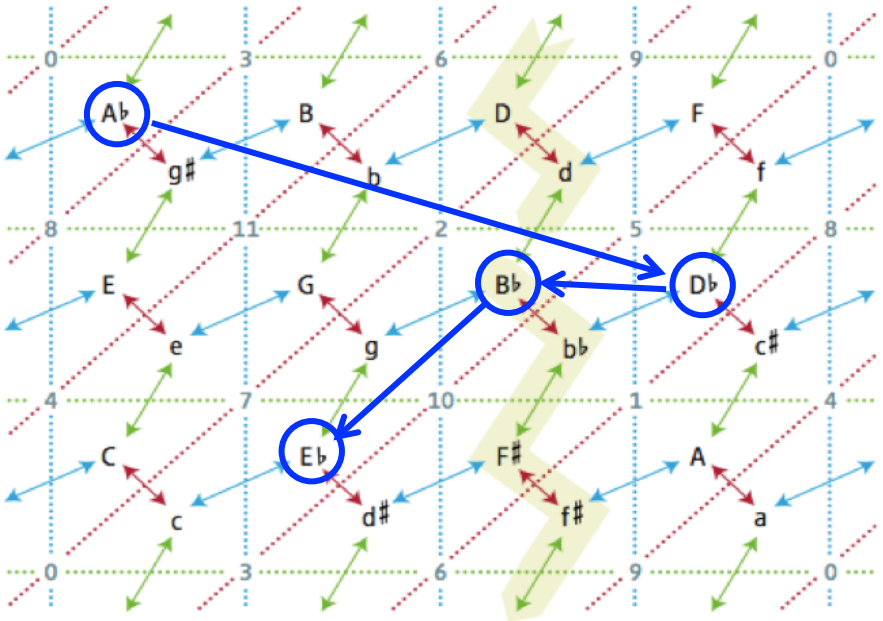
Progressions harmoniques dans « Madeleine » (Paolo Conte)



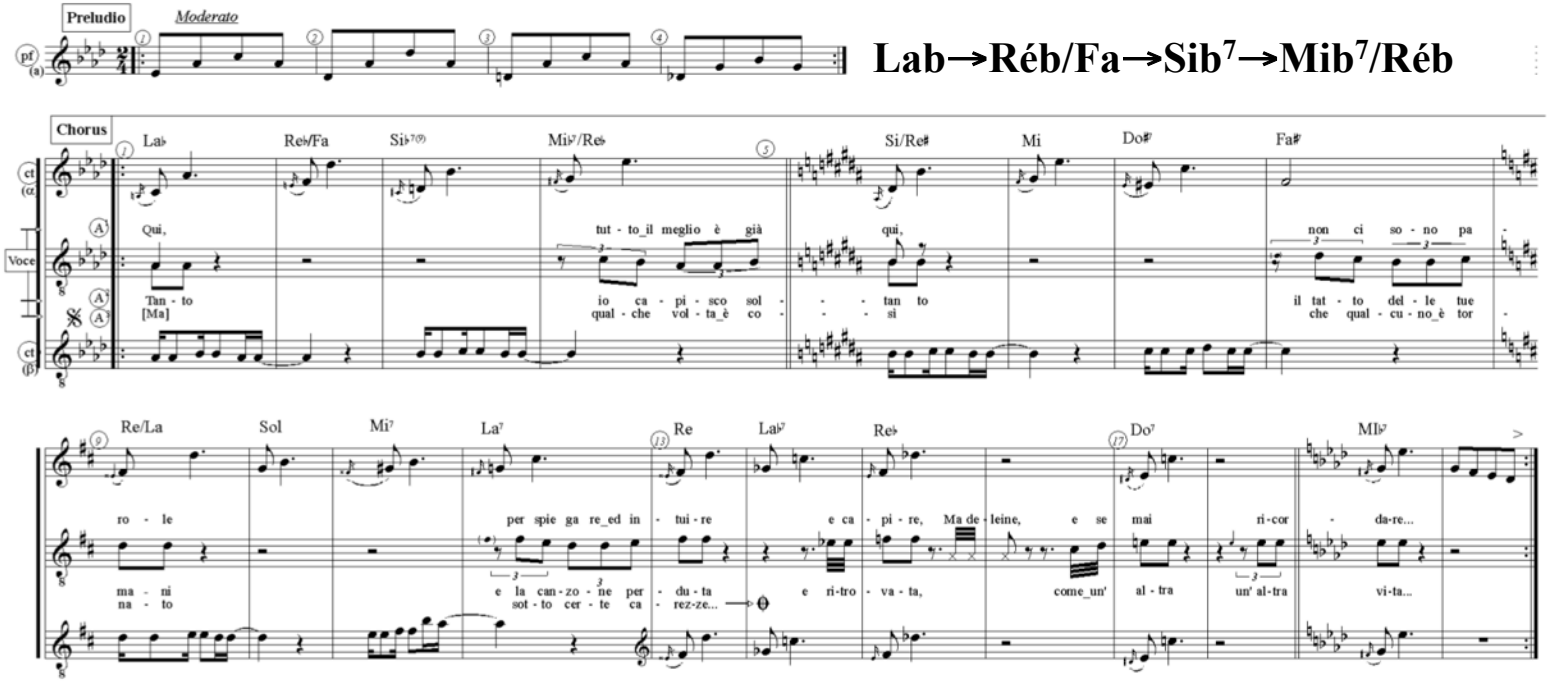
Lab→Réb/Fa→Sib^7→Mib^7/Réb

S. La Via, *Poesia per musica e musica per poesia. Dai trovatori a Paolo Conte*, Carocci, 2006





Progressions harmoniques dans « Madeleine » (Paolo Conte)



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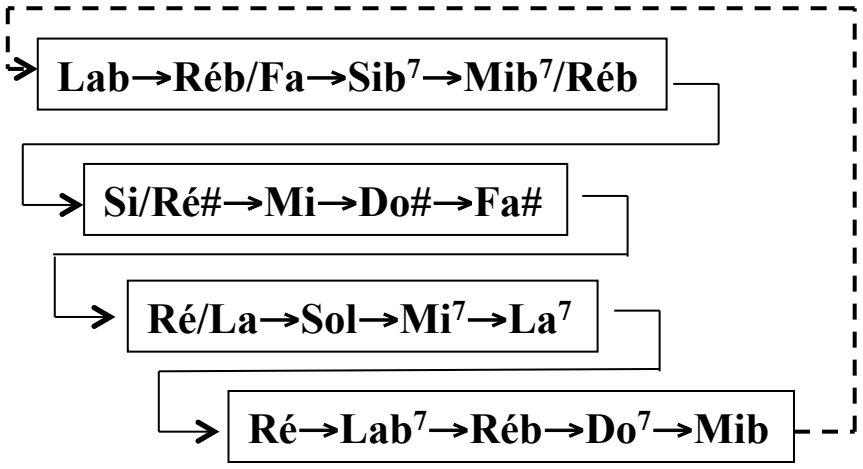
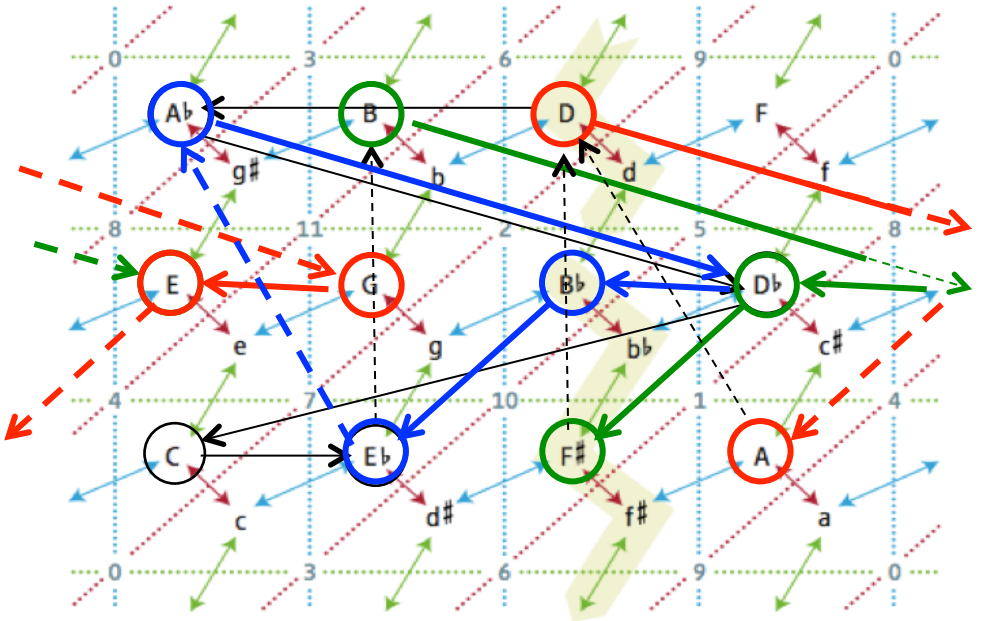
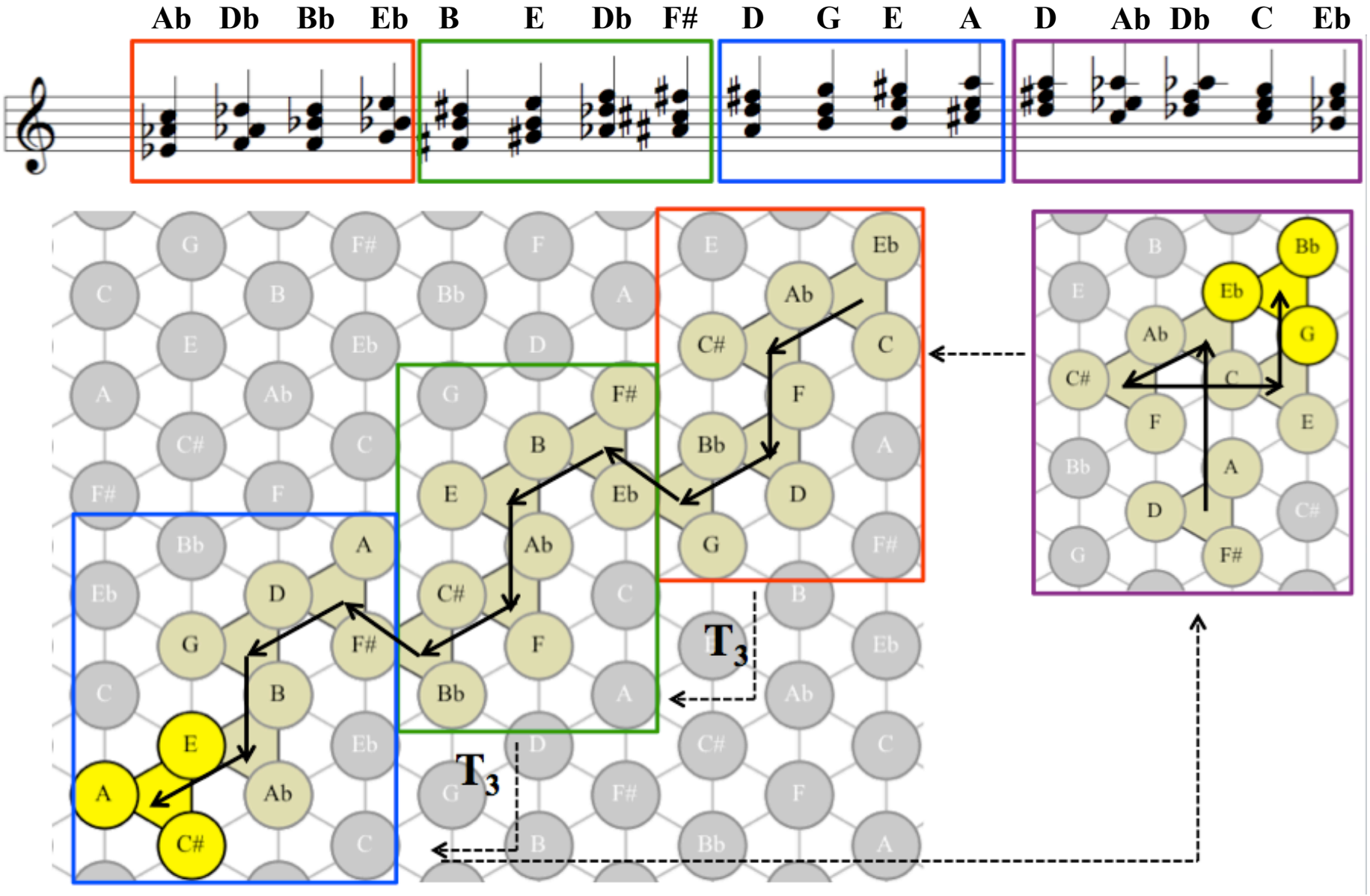
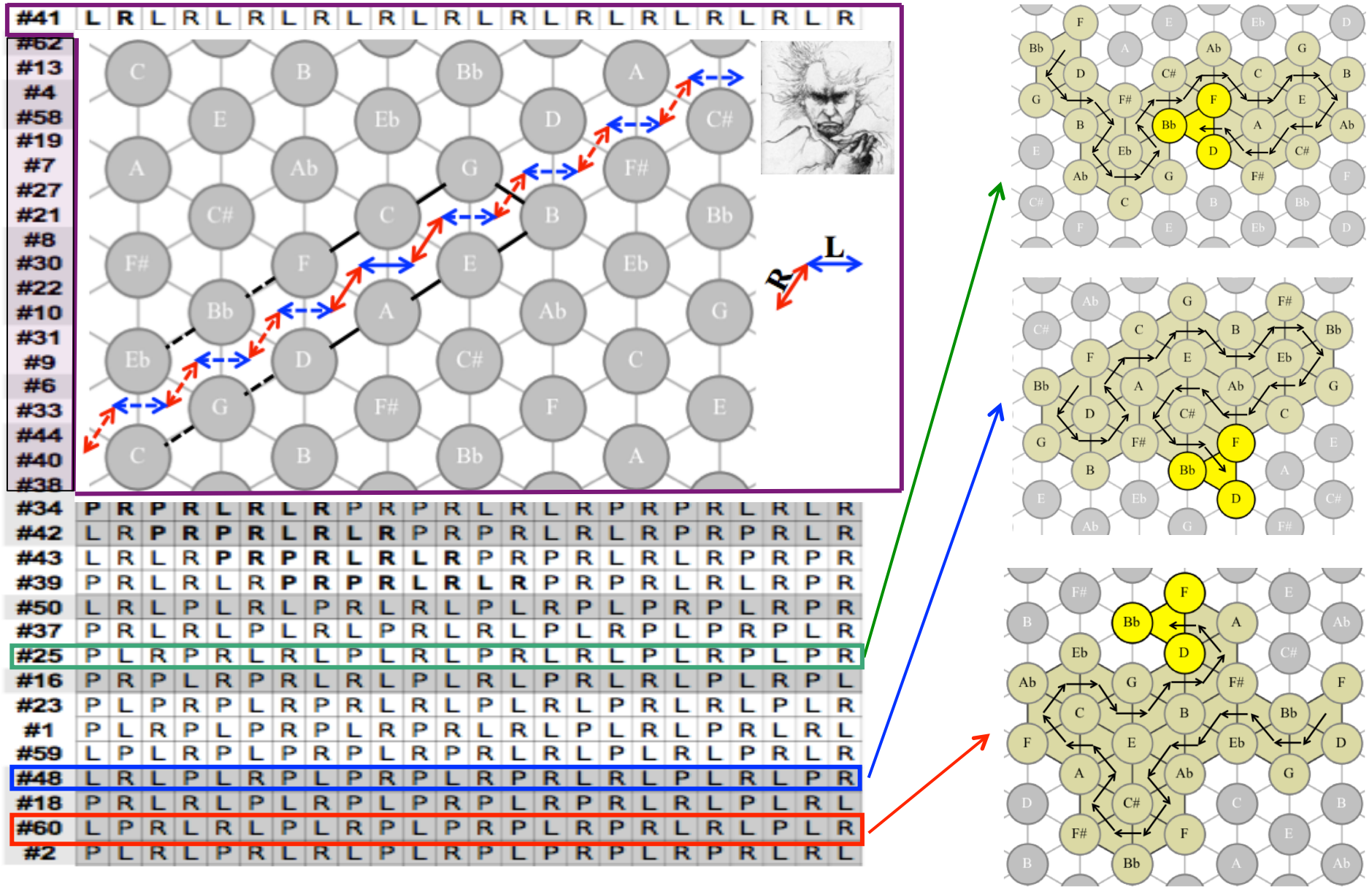




La trajectoire spatiale de « Madeleine » et son caractère ‘hamiltonien’





G. Albini et S. Antonini, « Hamiltonian Cycles in the Topological Dual of the Tonnetz », MCM 2009, Springer

Aprile, chanson hamiltonienne « décadente »

Do←do_m←Sol#←fa_m←Fa←la_m←La←$fa\#_m$←Fa#←sib_m←Do#←$do\#_m$

↘ La ↗ ↗

mi_m→Sol→si_m→Ré→$ré_m$→Sib→sol_m→Mib→mib_m→Si→$sol\#_m$→Mi

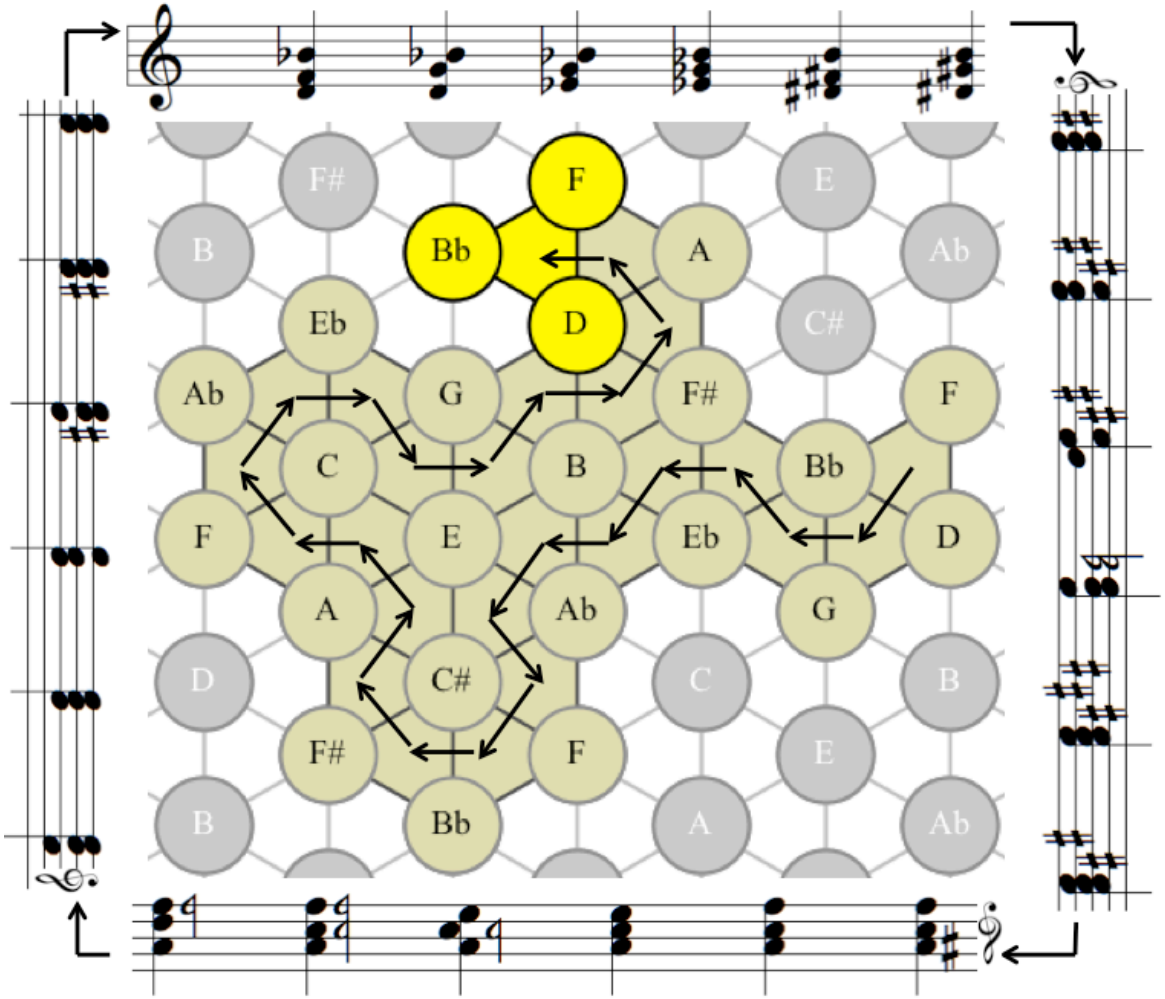
(Gabriele d'Annunzio)

Socchiusa è la finestra, sul giardino.
Un'ora passa lenta, sonnolenta.
Ed ella, ch'era attenta, s'addormenta
a quella voce che già si lamenta,
– che si lamenta in fondo a quel giardino.

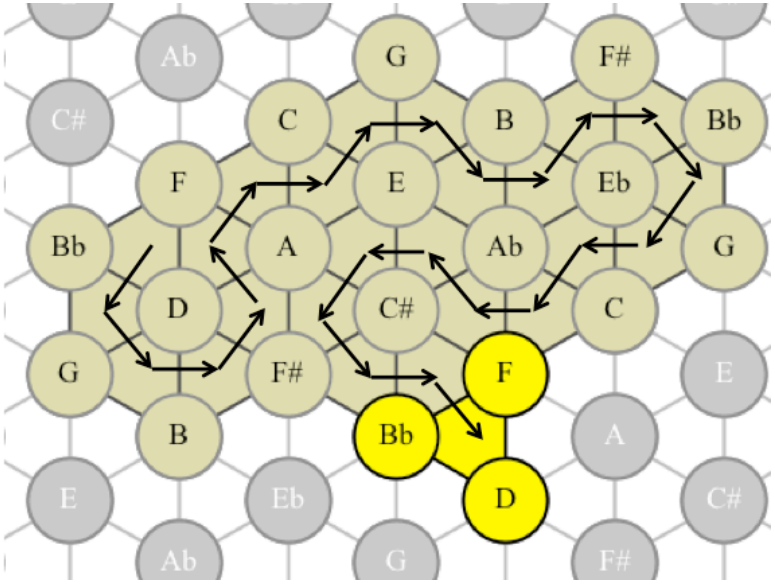
Non è che voce d'acque su la pietra:
e quante volte, quante volte udita!
Quell'amore e quell'ora in quella vita
s'affondan come ne l'onda infinita
stretti insieme il cadavere e la pietra.

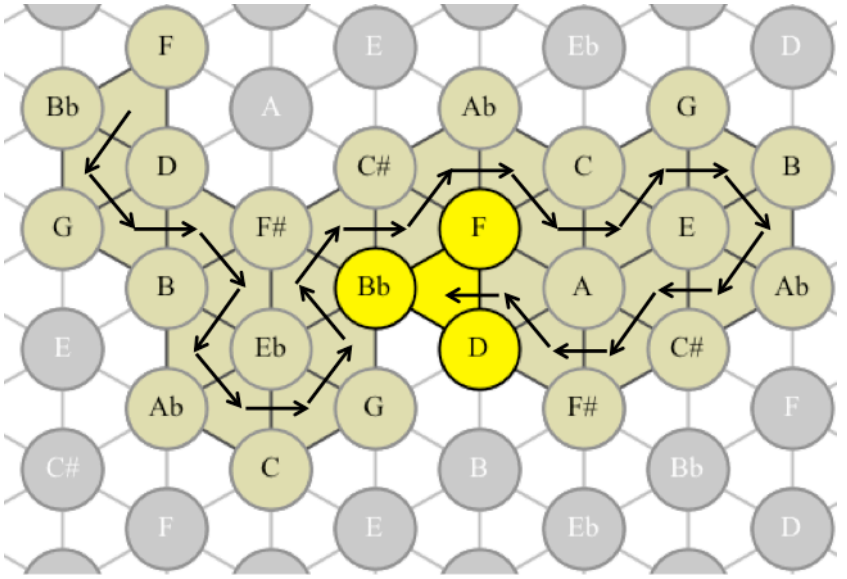
Ella stende l'angoscia sua nel sonno.
L'angoscia è forte, e il sonno è così lieve!
(Par la luce d'april quasi una neve
che sia tiepida.) Ed ella certo deve
soffrire, vagamente, anche nel sonno.

Gabriele D'Annunzio (1863-1938)



Deuxième et troisième cycle hamiltonien





Do→mi_m→Mi→$sol\#_m$→Si→$ré\#_m$→Re#→do_m→Lab→fa_m→Do#→$do\#_m$

↑ ↓

la_m←Fa←$ré_m$←Ré←si_m←Sol←sol_m←Sib←sib_m←Fa#←$fa\#_m$←La

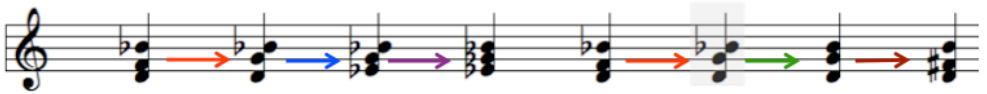
Mi←mi_m←Do←la_m←Fa←fa_m←Reb←sib_m←Fa#←mib_m←Mib←do_m

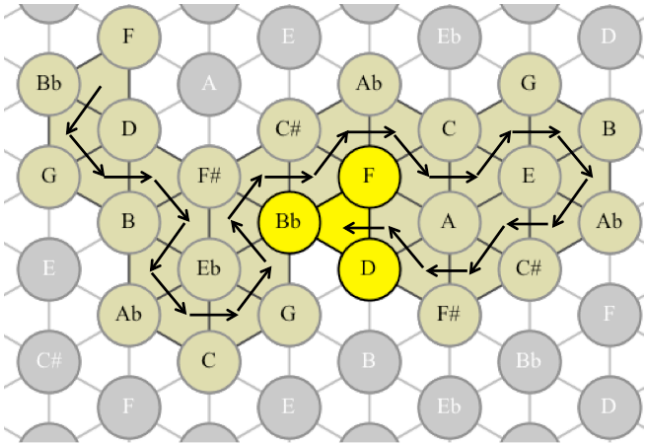
↓ ↑

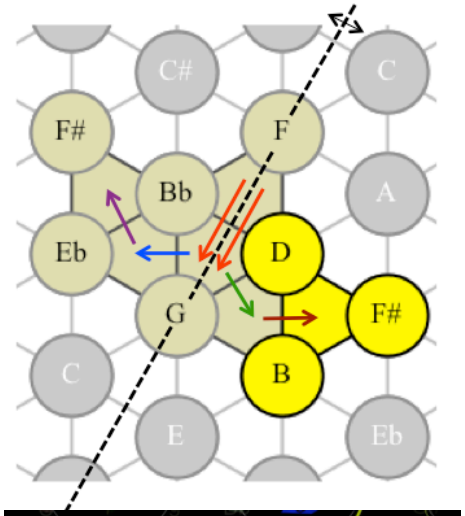
$do\#_m$→La→$fa\#_m$→Ré→$ré_m$→Sib→sol_m→Sol→si_m→Si→$sol\#_m$→Sol#

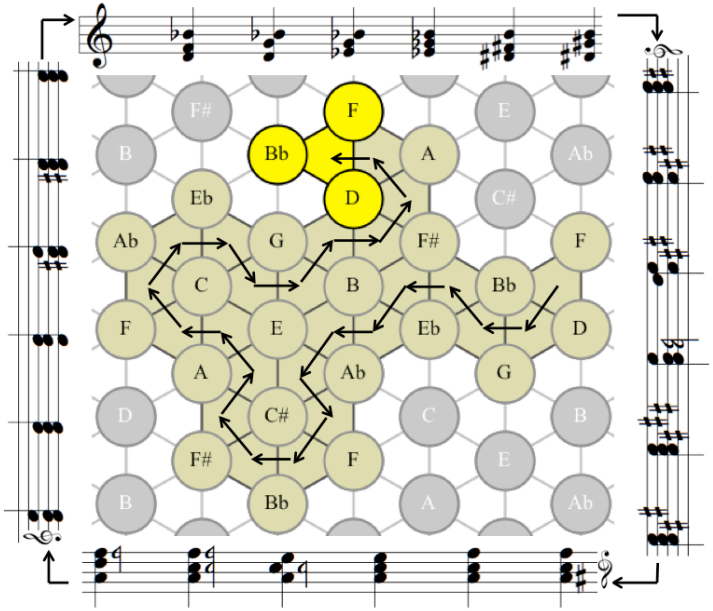
La

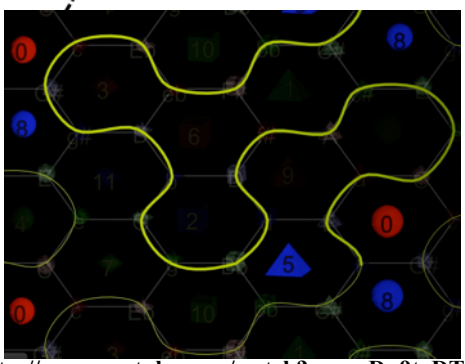
Maths et expériences oumu(po)piennes





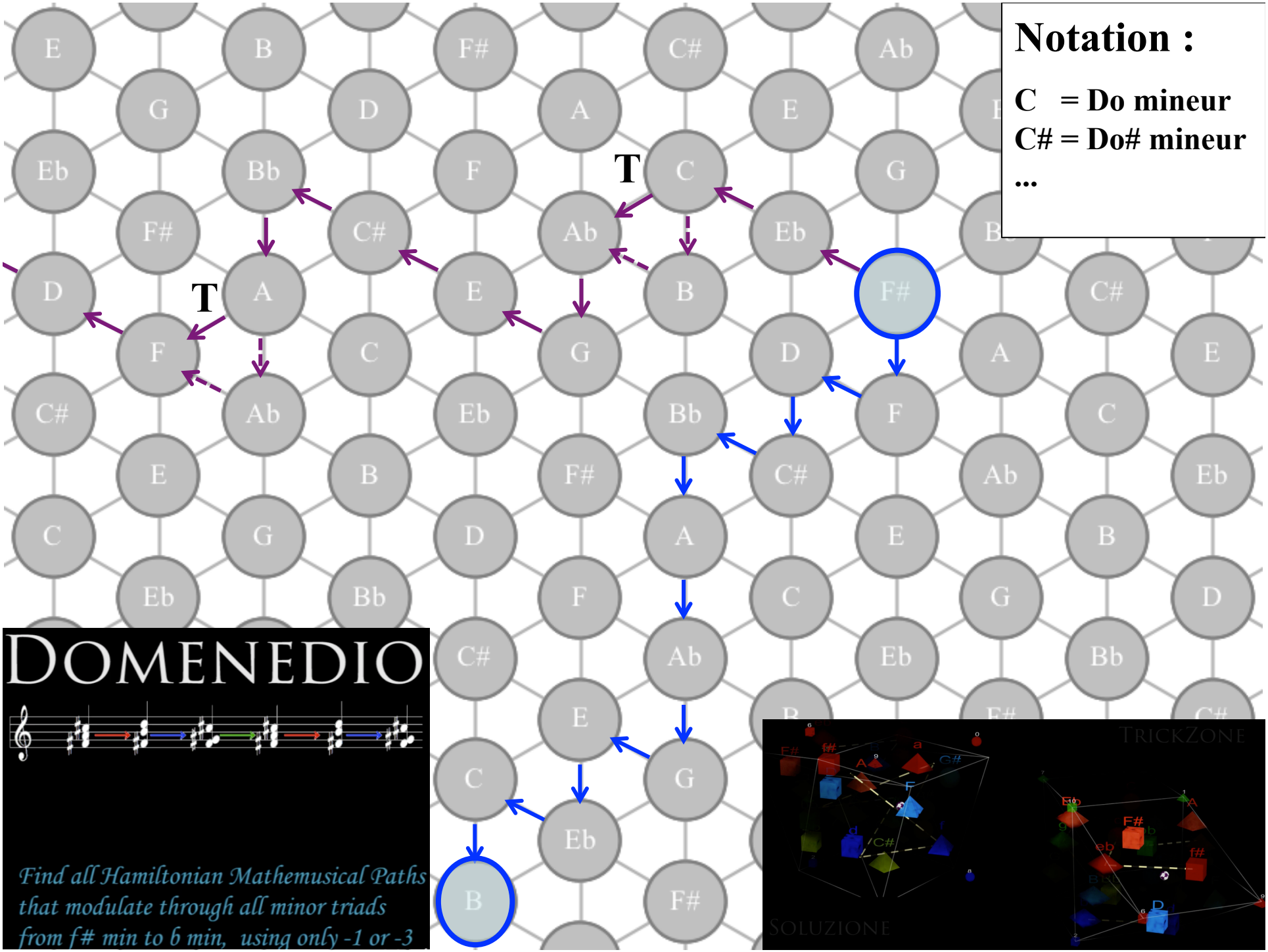




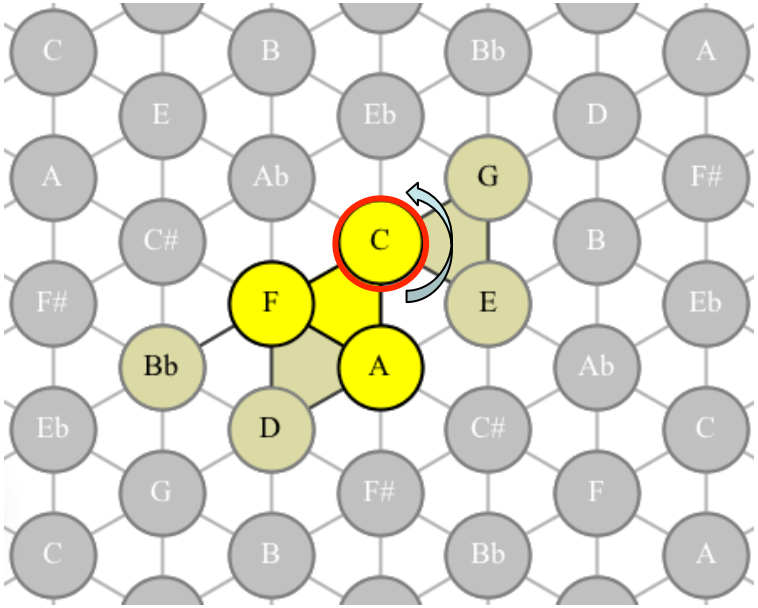


http://www.youtube.com/watch?v=oazDu9t_DTk

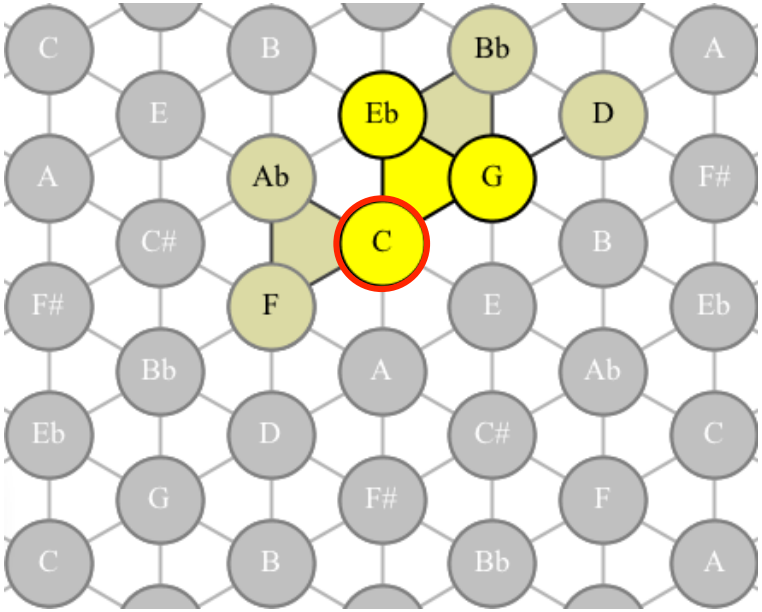
M. Andreatta, « Math'n pop : symétries et cycles hamiltoniens en chanson », *Tangente*, (à paraître)


Notation :
C = Do mineur
C# = Do# mineur
...
T
T
DOMENEDIO
Find all Hamiltonian Mathemusical Paths
that modulate through all minor triads
from f# min to b min, using only -1 or -3
TRICKZONE
SOLUZIONE

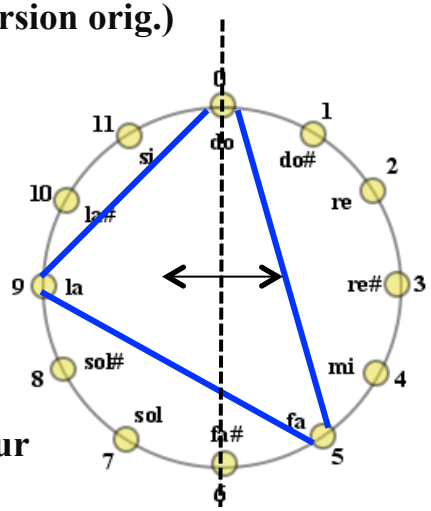
Maths et expériences oumu(po)piennes sur les Beatles



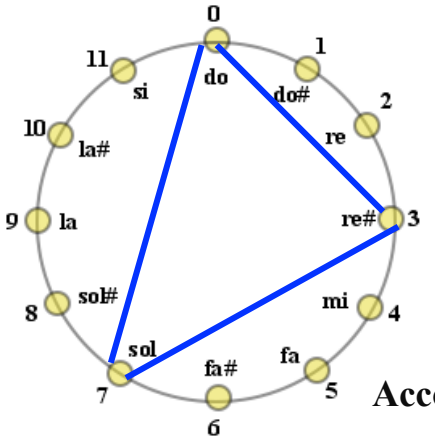
Rotation
(autour du *do*)



Beatles, Hey Jude
(version orig.)



inversion



Beatles, Hey Jude
(version transformée)

Accord majeur

Accord mineur

➔ `http://www.lacl.fr/~lbigo/scw13`